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# Predominance of $J=0$ ground states: A fingerprint of pairing?

Calvin W. Johnson

“This material is based upon work supported by the U.S. Department of Energy, Office of Science, Office of Nuclear Physics, under Award Number DE-FG02-96ER40985.”



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Part 1: Random shell-model two-body interactions and (?) pairing

Part 2: Proton-neutron pairing and Shannon entropy

Finale: A statistical 'explanation' for  $J=0$  g.s. dominance



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## Part 1: Random shell-model 2b interactions and (?) pairing

“Orderly spectra from random interactions,” **CWJ**, Bertsch, and Dean, PRL **80** (1998) 2749

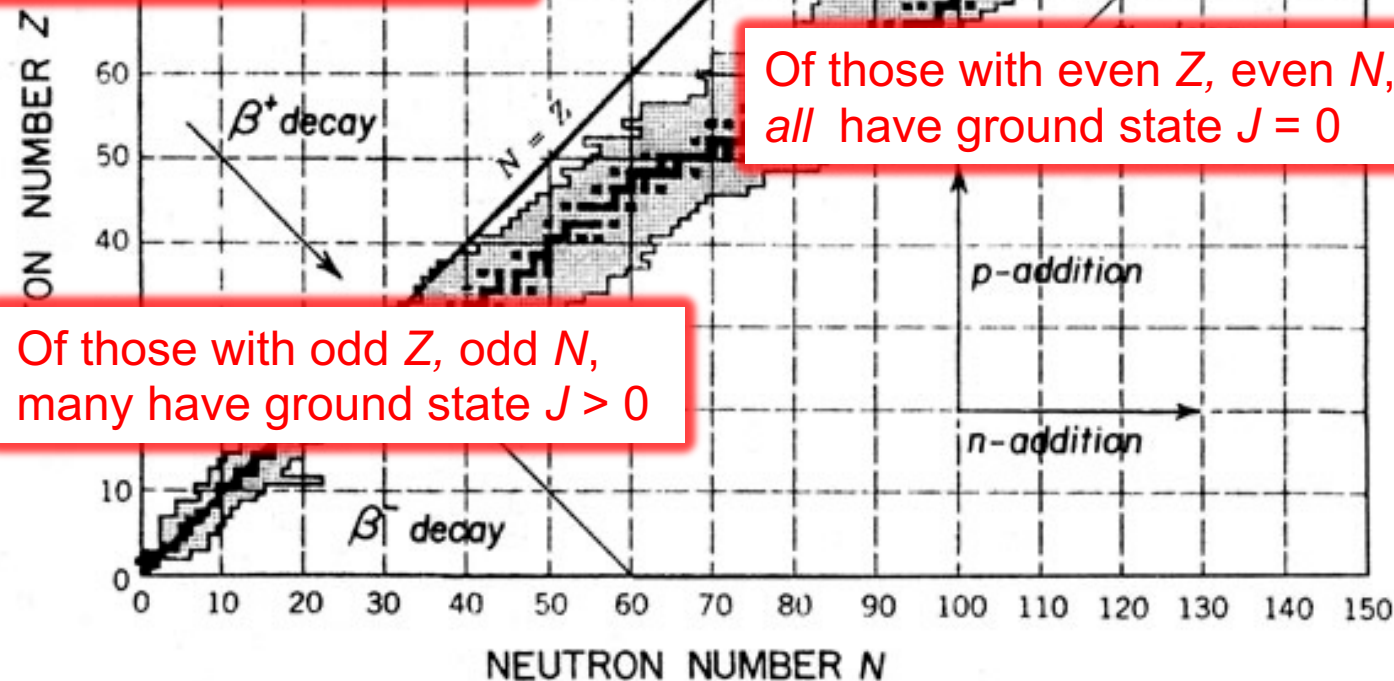
“Generalized seniority from random interactions,” **CWJ**, Bertsch, Dean, and Talmi, PRC **61**, 014311 (2000)

For example....



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There are some 3000 known  
isotopes....another 6000  
thought to exist



Of those with even  $Z$ , even  $N$ ,  
all have ground state  $J = 0$

Of those with odd  $Z$ , odd  $N$ ,  
many have ground state  $J > 0$

Why?



According to textbooks:

$J = 0$  in ground states are due to specifics of nuclear forces,  
in particular, nucleon-nucleon *pairing*



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Is this true?

Let's test



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First, let's review  
shell-model  
calculations



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Configuration-interaction converts the  
Schrödinger eqn into a matrix eigenvalue  
equation

$$\sum_B H_{AB} V_B = E_A V_A$$

One chooses a basis of approx  $10^4 - 10^{10}$  states  
(Slater determinants)

**Key point:** Once a basis is chosen, the two-body interaction is reduced to  
integrals between single-particle states and is **stored as a list of real numbers**  
(the *two-body matrix elements*)

A *shell model program* then computes the *many-body matrix elements*  
from the *two-body matrix elements* and solves for eigenvalues/vectors.



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The two-body matrix elements *in principle but not in practice* depend on the single-particle wfns:

$$\langle ab; JT | \hat{H} | cd; JT \rangle = \int d^3r \int d^3r' \varphi_a^*(r) \varphi_b^*(r')$$

$$V(r, r') (\varphi_c(r) \varphi_d(r') - \varphi_c(r') \varphi_d(r))$$

But only the final  
number is read in!

Interaction File

# of TBME				Single Particle Energies		
				J	T	V
63				1.6465800	-3.9477999	-3.1635399
a	b	c	d	J	T	V
1	1	1	1	0	1	-2.1845000
1	1	1	1	1	0	-1.4151000
1	1	1	1	2	1	-0.0665000
1	1	1	1	3	0	-2.8842001
2	1	1	1	1	0	0.5647000
2	1	1	1	2	1	-0.6149000
2	1	1	1	3	0	2.0337000
2	1	2	1	1	0	-6.5057998
2	1	2	1	1	1	1.0334001
2	1	2	1	2	0	-3.8253000
2	1	2	1	2	1	-0.28
2	1	2	1	3	0	

Single Particle States

iso
3
0. 2. 1.5 2 ! orbits
0. 2. 2.5 4
1. 0. 0.5 6

(1s<sub>1/2</sub>)

(0d<sub>5/2</sub>)

(0d<sub>3/2</sub>)



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2	1	2	1	2	1	-0.2845000
2	1	2	1	3	0	0.0000000

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iso				
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0.	2.	1.5	2	! orbits
0.	2.	2.5	4	
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Those numbers  
look  
random  
(but are not)

Interaction File

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a	b	c	d	J	T	V
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Single Particle States

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(1s<sub>1/2</sub>)  
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$J = 0$  in ground states are due to specifics of nuclear forces,  
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Is this true?

Let's test with a null hypothesis: *random interactions*



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Is this true?

Let's test with a null hypothesis: *random interactions*

A numerical  
experiment:



Draw interaction from  
“two-body random ensemble”



General many-body  
structure code



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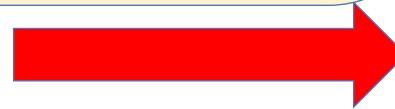


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This is an interesting thought  
experiment...

How far can we 'simplify'  
(nuclear) physics?

General many-body  
structure code





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Bellini, *Madonna  
and Child*



Renoir, *Country Road*

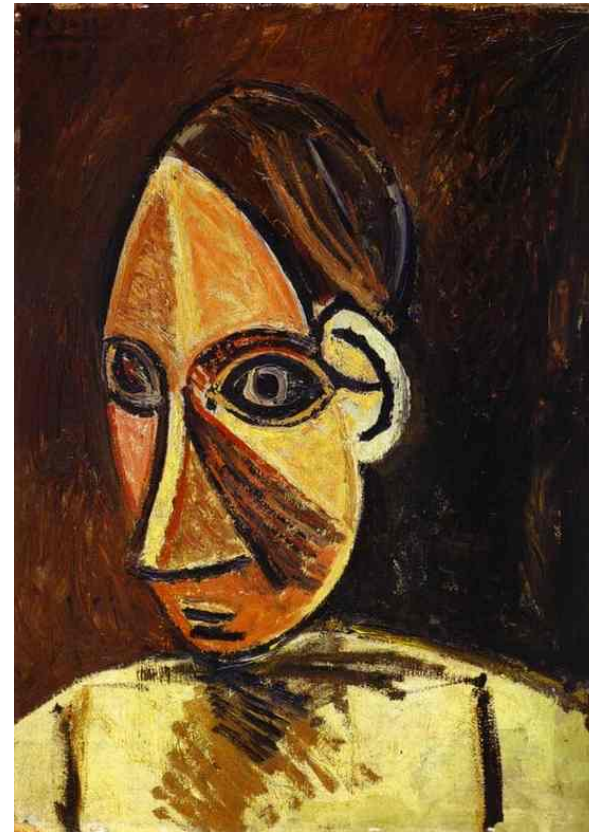
ESNT Workshop: Where has the pairing gone? May 16, 2025



We're not satisfied to *merely* represent reality...  
in art (and science) we explore how far we can  
stray and yet still “represent” some aspects



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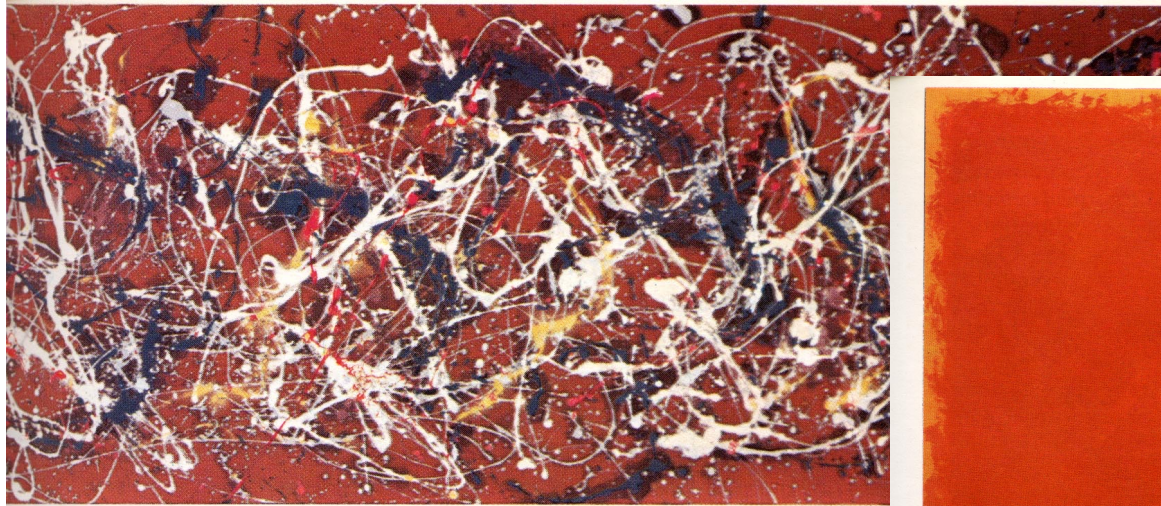


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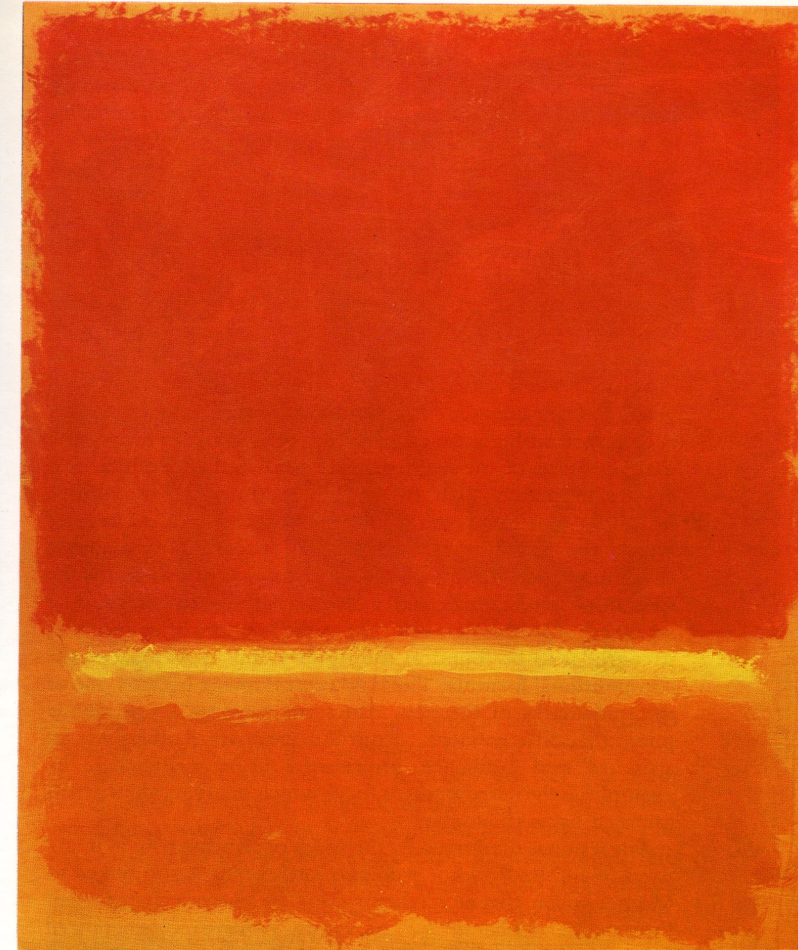


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14 JACKSON POLLOCK *Number 2* 1949

Very simple systems may  
not seem realistic, but they  
probe the fundamentals in a  
way we can come to appreciate  
as beautiful

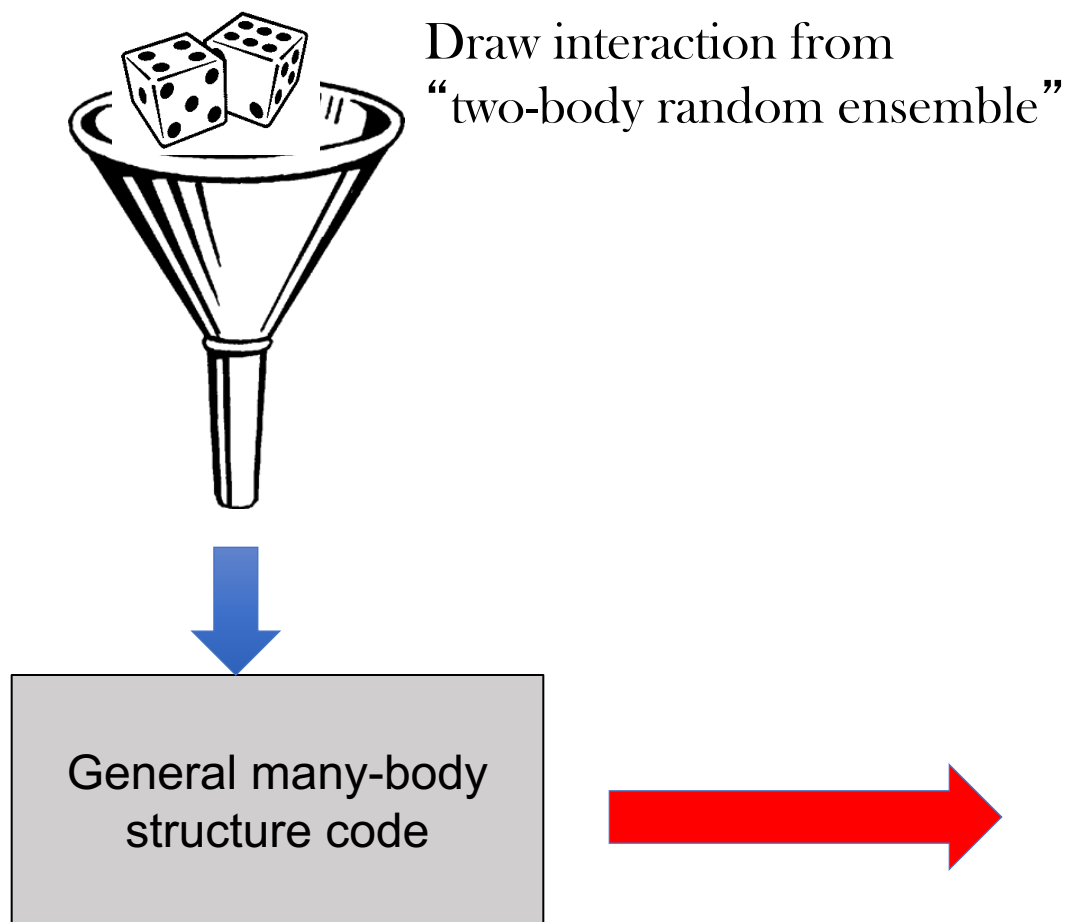


20 MARK ROTHKO *Orange Yellow Orange* 1969



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Back to our numerical experiment:



C. W. J, G. F. Bertsch, and D.J.Dean ,  
Phys. Rev. Lett. **80** (1998) 2749.





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We could randomly replace the carefully calculated  
two-body matrix elements with random numbers

$$\langle ab; JT | \hat{H} | cd; JT \rangle = \int d^3r \int d^3r' \varphi_a^*(r) \varphi_b^*(r') V(r, r') (\varphi_c(r) \varphi_d(r') - \varphi_c(r') \varphi_d(r))$$



randomly drawn from  
a Gaussian distribution\*  
(symmetric about zero)

\*not very sensitive to details  
of distribution

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0	2	1.5	2	!	orbits
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(1s<sub>1/2</sub>)  
(0d<sub>5/2</sub>)  
(0d<sub>3/2</sub>)





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**What do  
we expect?**

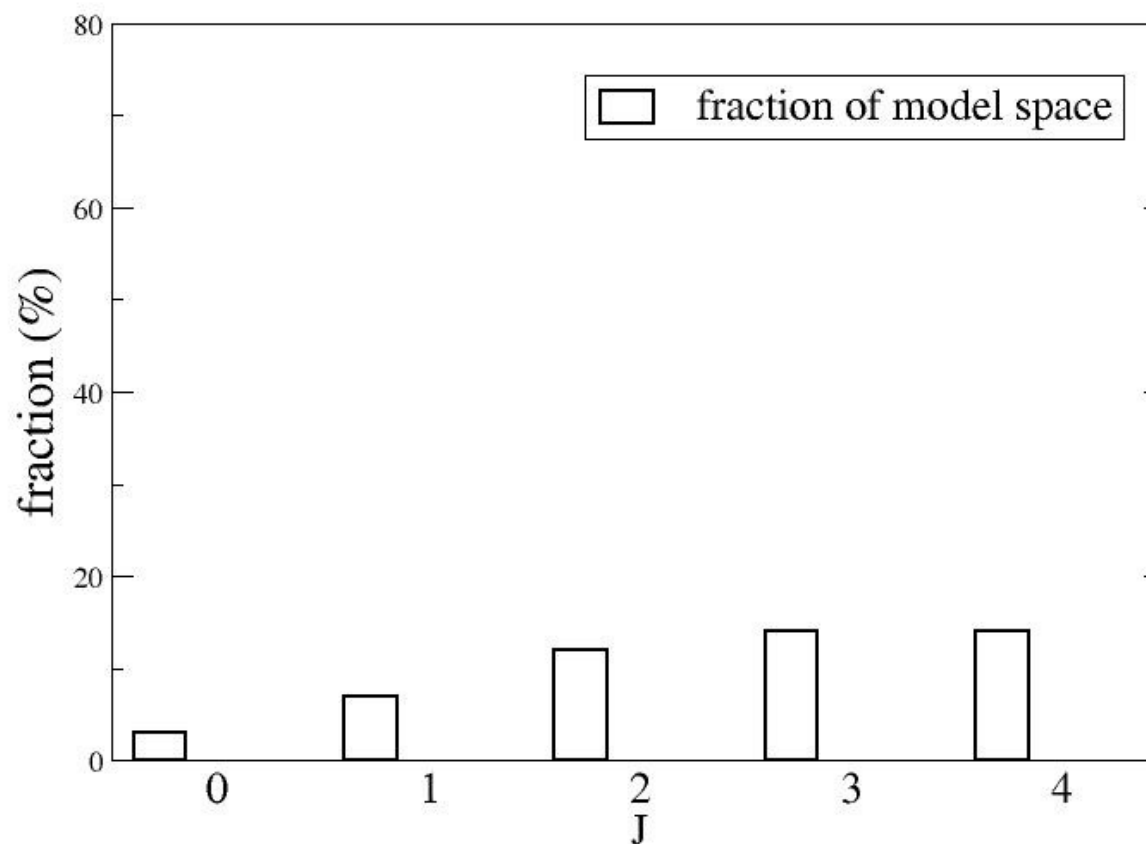


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What do  
we expect?

$^{48}\text{Ca}$  in  $pf$  shell (8 neutrons)





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TABLE I. Percentage of ground states for selected random ensembles that have  $J=0$  for our target nuclides, as compared to the percentage of all states in the model spaces that have these quantum numbers. (Statistical error is approximately 1 – 3%.) Entries with dashes were not computed.

Nucleus	$J=0$ (total space)	$J=2$ (total space)
$^{22}\text{O}$	11.1%	14.8%
$^{24}\text{O}$	9.8%	13.4%
$^{44}\text{Ca}$	11.1%	14.8%
$^{46}\text{Ca}$	5.0%	9.6%
$^{48}\text{Ca}$	3.5%	8.1%
$^{50}\text{Ca}$	2.9%	7.6%
$^{50}\text{Ca}$	2.7%	7.1%
$^{24}\text{Mg}$	4%	16%
$^{26}\text{Mg}$	4%	15%
$^{28}\text{Mg}$	4%	16%

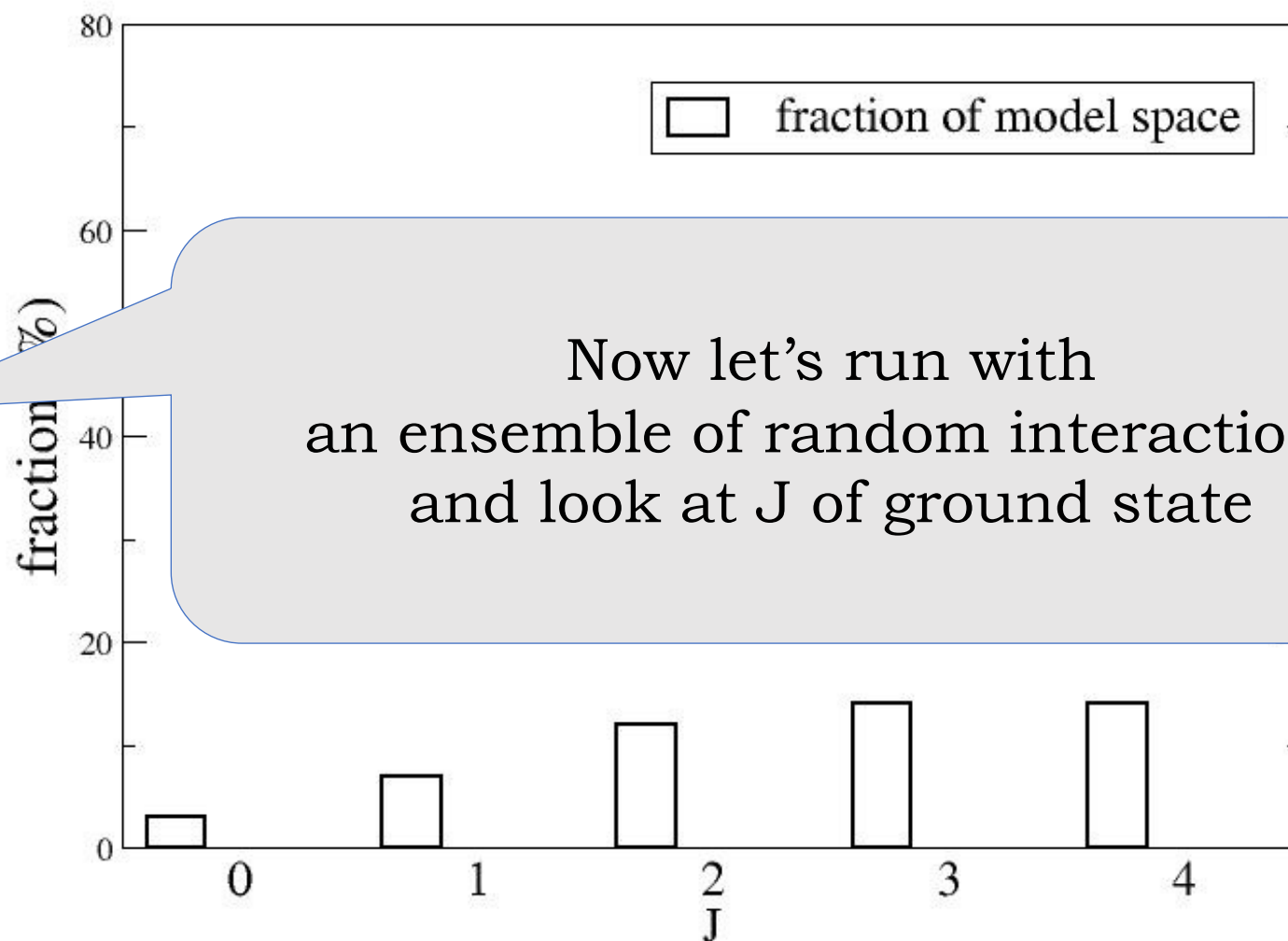


The fraction of states  
with  $J = 0$  is quite  
small...you have a  
higher chance of  
randomly getting  $J = 2$



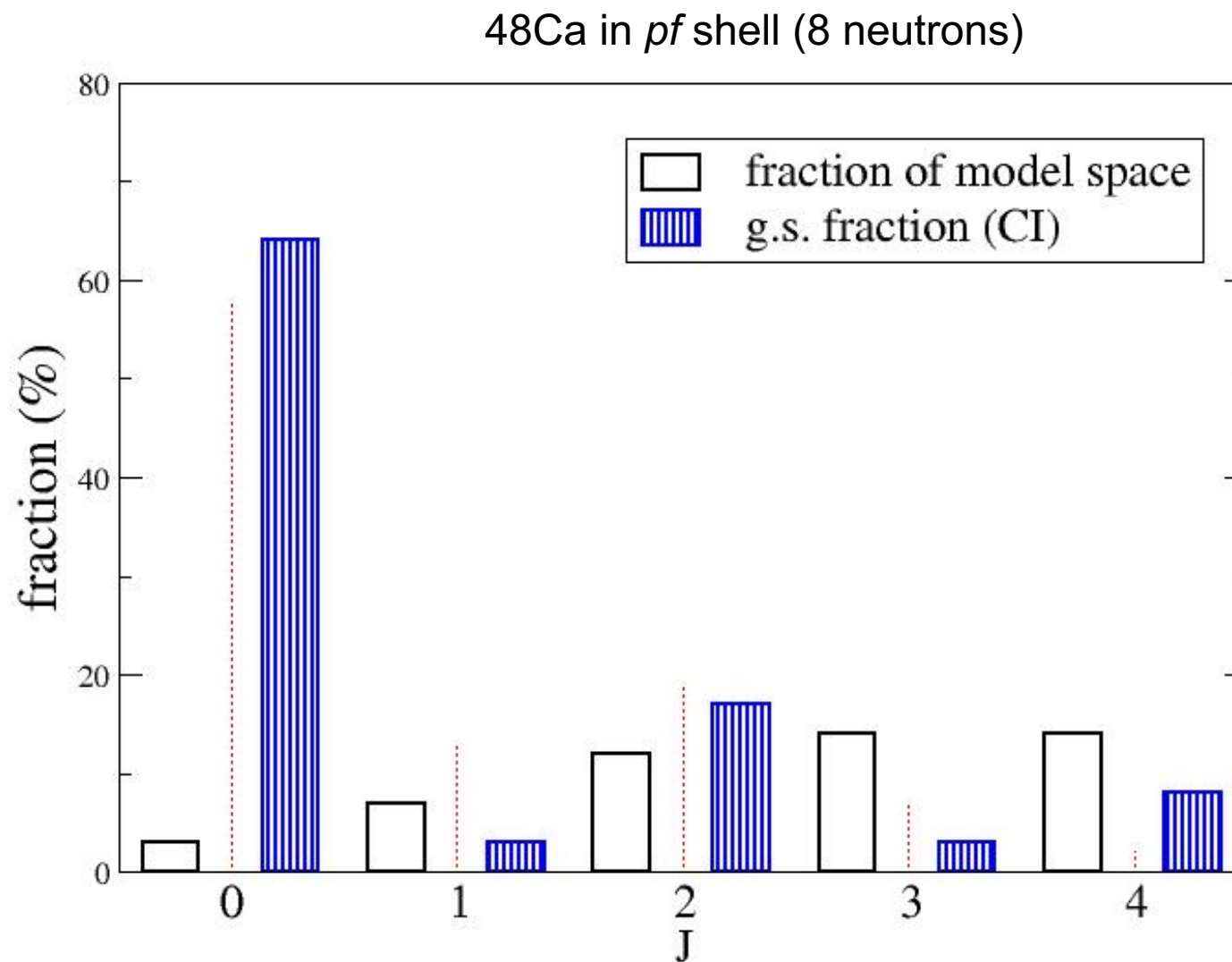
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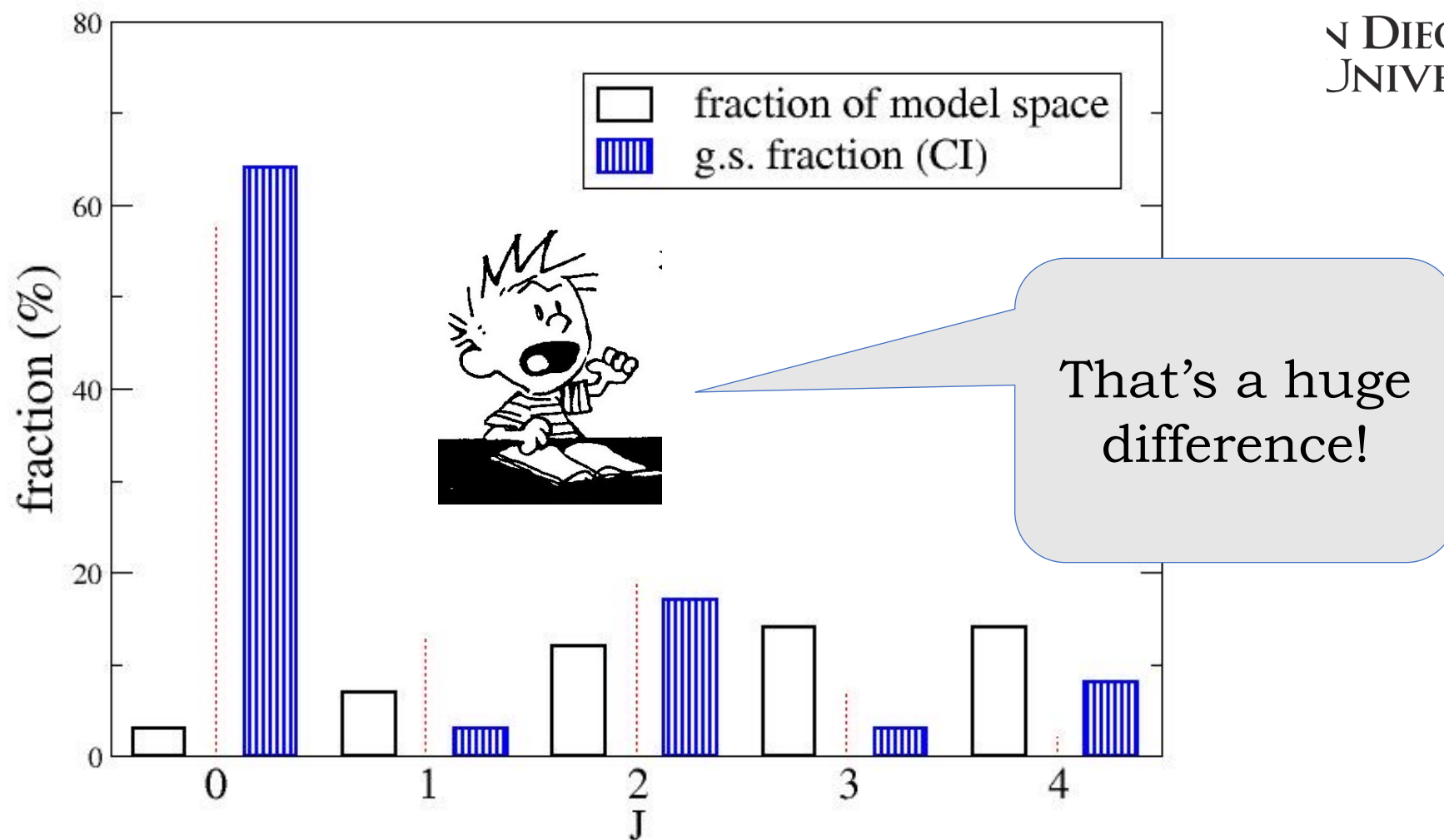


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This is ***amazing!***

Do we understand  
this?

“...the simple question of symmetry and chaos  
asks for a simple answer which  
is still missing.”

- A. Volya, PRL **100**, 162501 (2008).



Maybe random  
interactions lead to  
pairing...?



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“Generalized seniority from random interactions,”  
**CWJ**, Bertsch, Dean, and Talmi, PRC **61**, 014311 (2000)



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ESNT Workshop: Where has the pairing gone? May 16, 2025

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### Evidence for pairing-like behavior:

- $J=0$  ground states
- Seniority-like spectra
- Odd-even staggering
- Large pair-transfer collectivity



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Nucleus	RQE	RQE-NP	TBRE	RQE-SPE	$J=0$ (total space)	$J=2$ (total space)
$^{20}\text{O}$	68%	50%	50%	49%	11.1%	14.8%
$^{22}\text{O}$	72%	68%	71%	77%	9.8%	13.4%
$^{24}\text{O}$	66%	51%	55%	78%	11.1%	14.8%
$^{44}\text{Ca}$	70%	46%	41%	70%	5.0%	9.6%
$^{46}\text{Ca}$	76%	59%	56%	74%	3.5%	8.1%
$^{48}\text{Ca}$	72%	53%	58%	71%	2.9%	7.6%
$^{50}\text{Ca}$	65%	45%	51%	61%	2.7%	7.1%
$^{24}\text{Mg}$	66%	–	44%	54%	4%	16%
$^{26}\text{Mg}$	62%	52%	48%	56%	4%	15%
$^{28}\text{Mg}$	59%	46%	44%	54%	4%	16%

‘RQE’ is a different  
 weighting of choosing  
 the ensemble

NP = remove all  
 $J=0$  two-body matrix  
 elements (‘no pairing’)

SPE = add realistic  
 USDB single particle  
 energies



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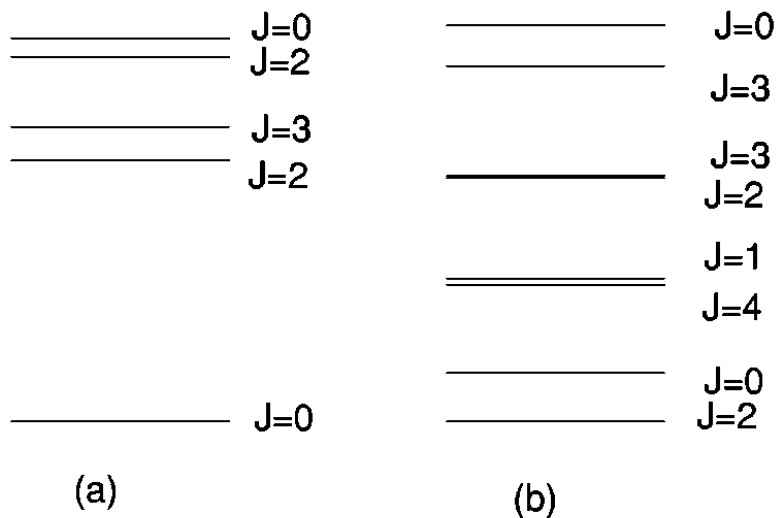


FIG. 1. Typical spectra for  $^{22}\text{O}$  with an RQE Hamiltonian: (a) an example with the  $J=0$  ground state; (b) an example with the  $J \neq 0$  ground state. Note the absence of a ground-state gap in the  $J \neq 0$  case. (Energy scale is arbitrary.)

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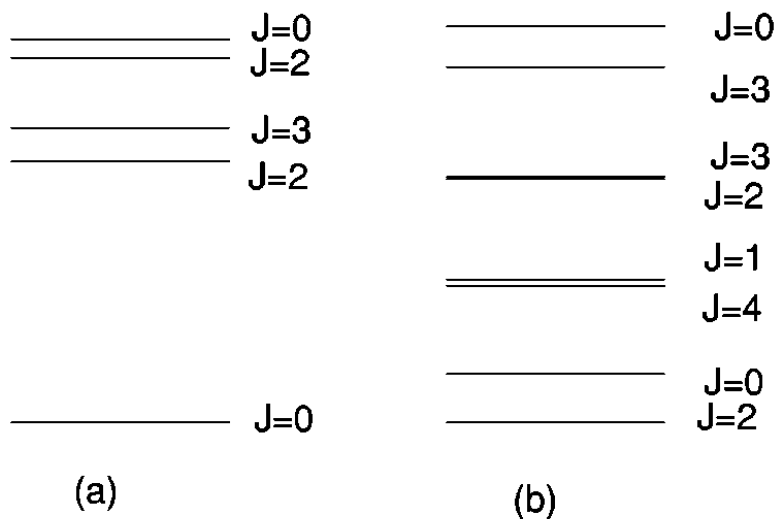


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TABLE II. Average gap between  $J=0$  ground state and first excited states  $\langle s \rangle$  scaled by the local level spacing (computed from the first and second excited states). The same quantity computed for  $J > 0$  ground states is between 0.9 and 1.2 for all cases considered.

Nucleus	RQE	RQE-NP	TBRE	RQE-SPE
$^{20}\text{O}$	2.7	2.5	2.3	2.3
$^{22}\text{O}$	3.2	2.8	2.8	3.4
$^{24}\text{O}$	2.9	2.5	2.3	3.7
$^{44}\text{Ca}$	3.1	2.6	2.4	3.1
$^{46}\text{Ca}$	3.8	3.2	3.0	3.6
$^{48}\text{Ca}$	3.4	3.0	3.5	3.5
$^{50}\text{Ca}$	3.5	3.0	3.0	3.4

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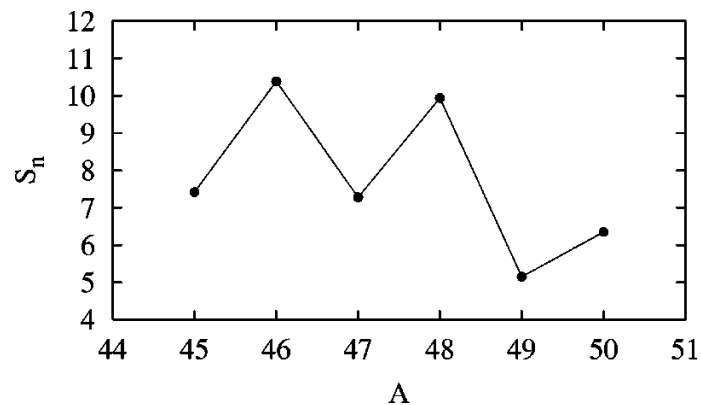


FIG. 3. Experimental neutron separation energies of Ca isotopes in the range  $A=45-50$ .

Odd-even staggering

cf. T. Duguet's talk  
on 5/13

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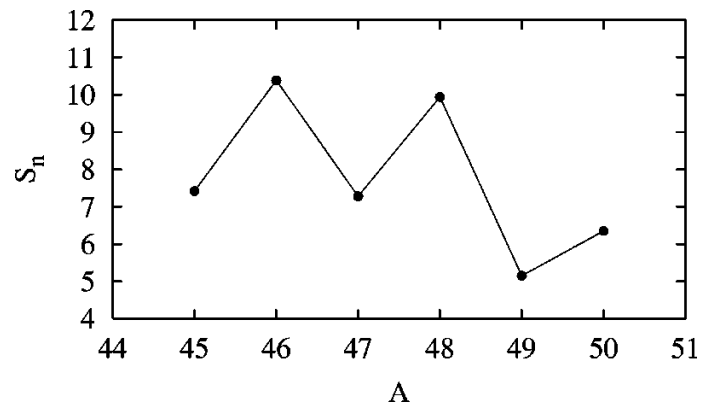


FIG. 3. Experimental neutron separation energies of Ca isotopes in the range  $A=45-50$ .

cf. T. Duguet's talk  
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Odd-even staggering



This topic is  
worth a little  
detour....



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J-projected HF  
captures much  
of OES for  
light-medium  
nuclides

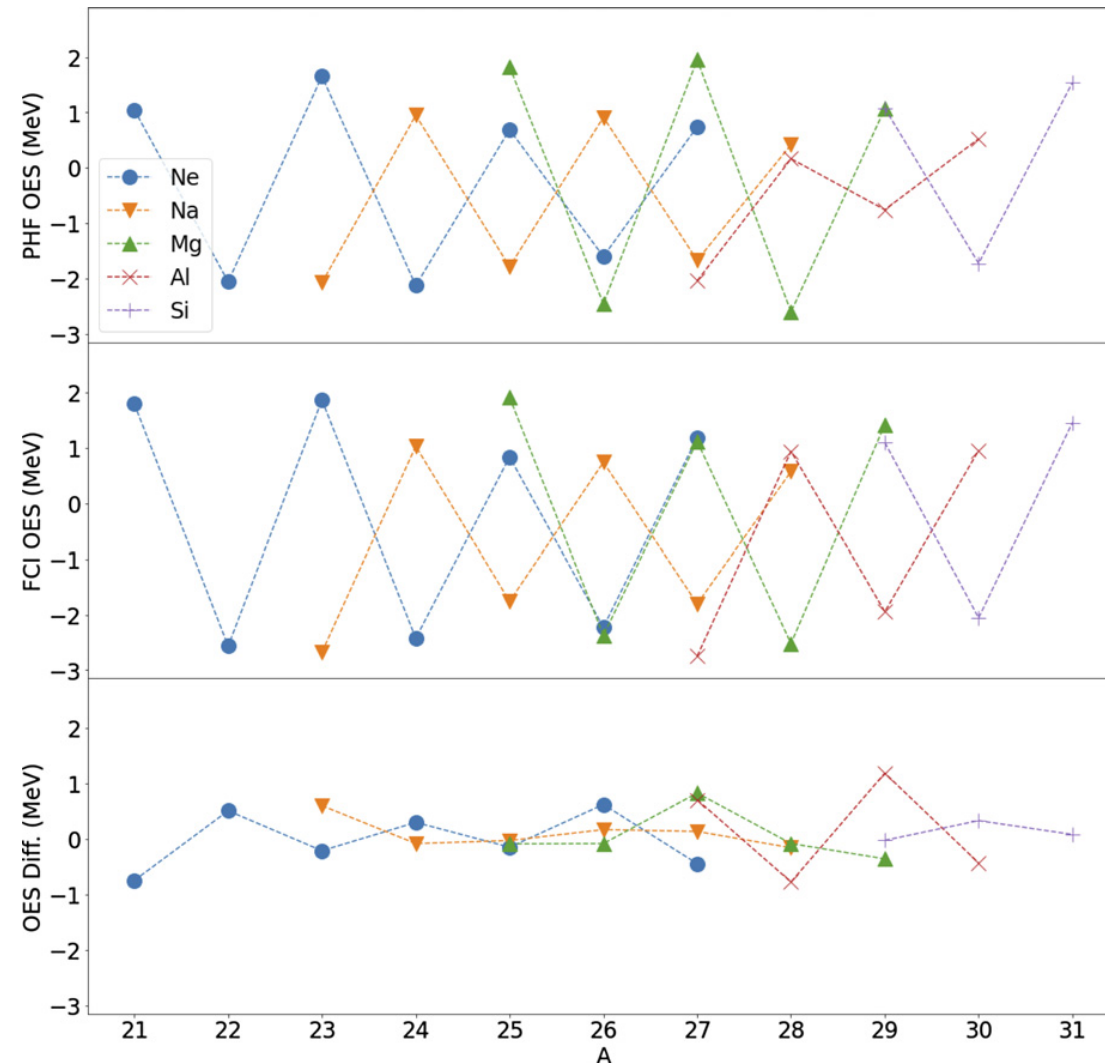
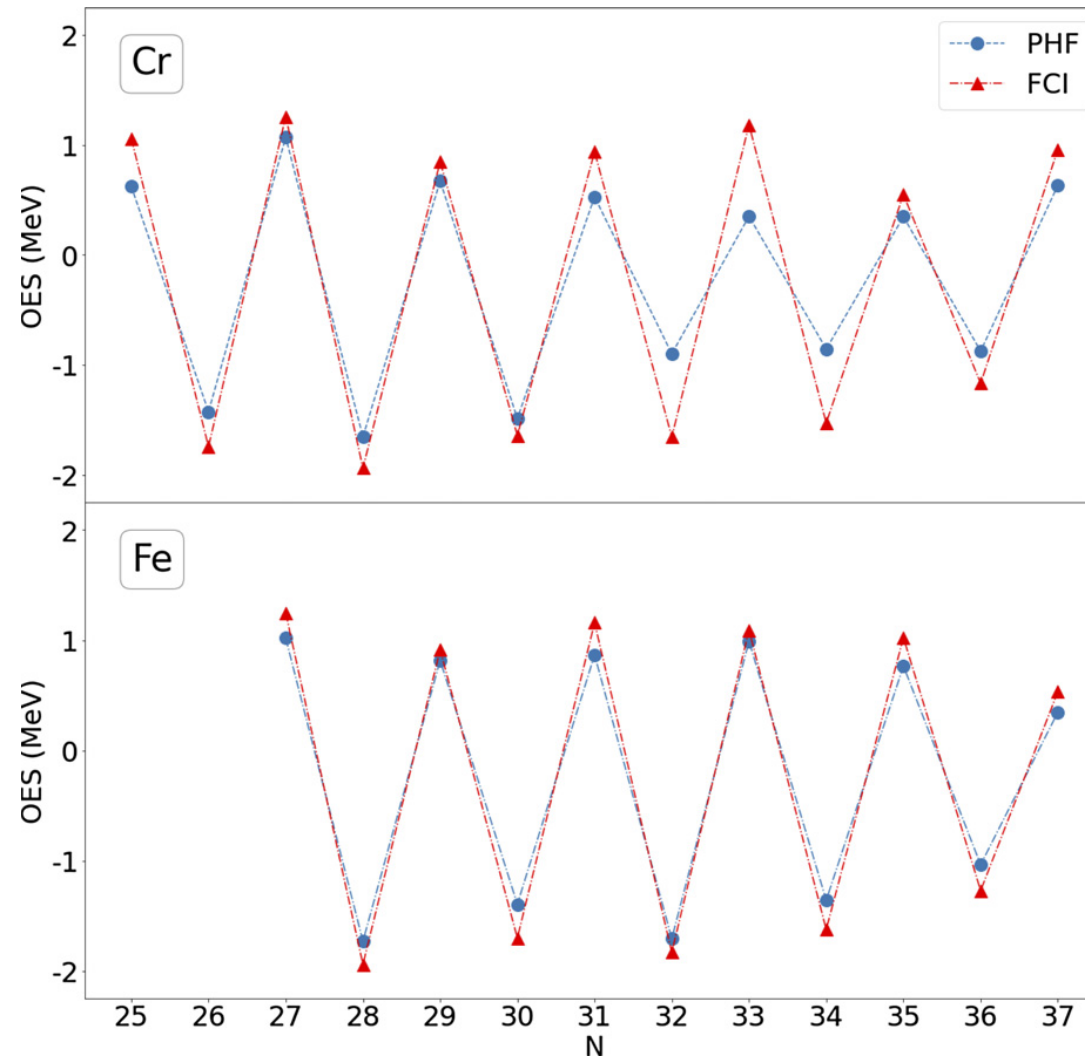


Figure 8. OES in *sd*-shell nuclei.



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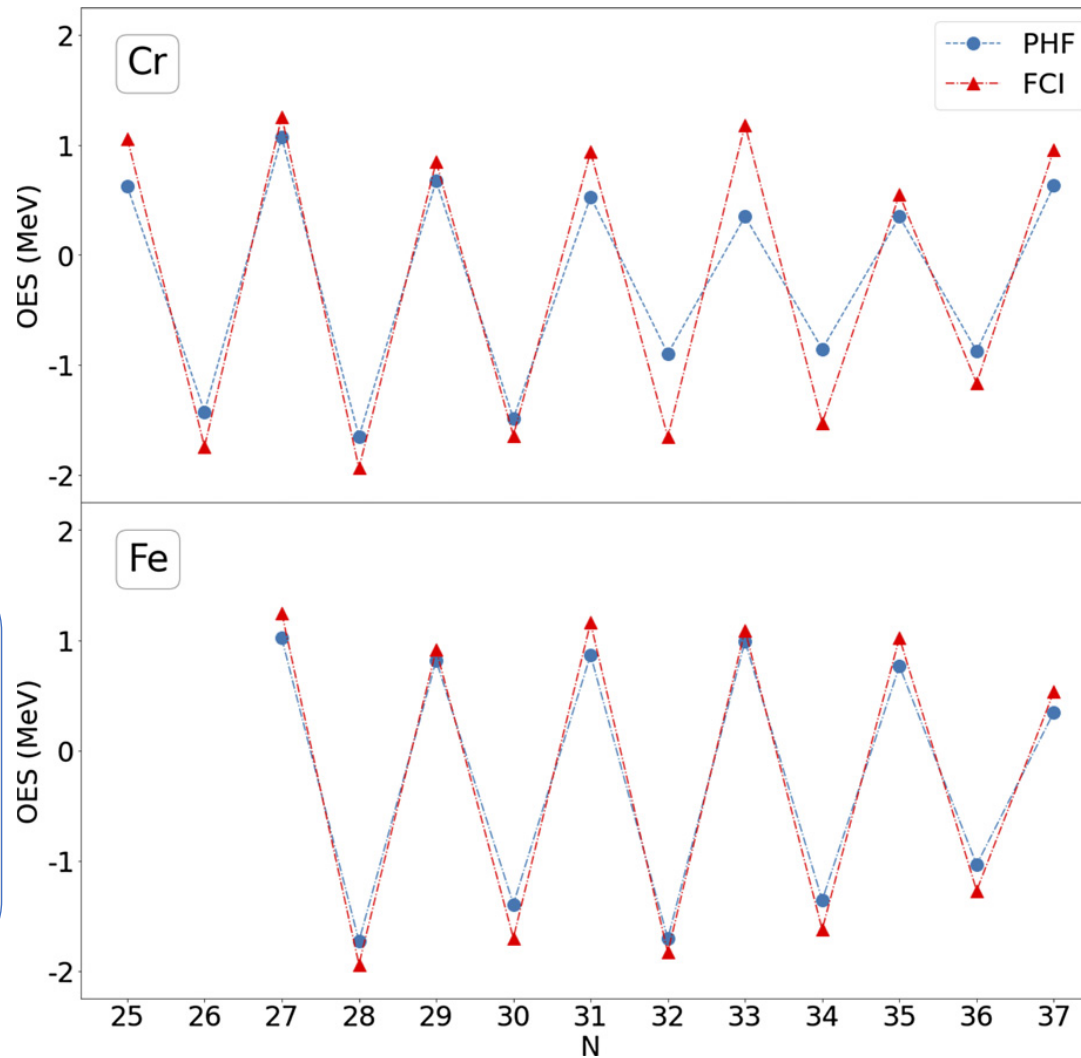
**Figure 9.** OES along the chromium (upper panel) and iron (lower panel) isotopic chains.



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If we artificially  
enhance pairing,  
the PHF results  
degrade

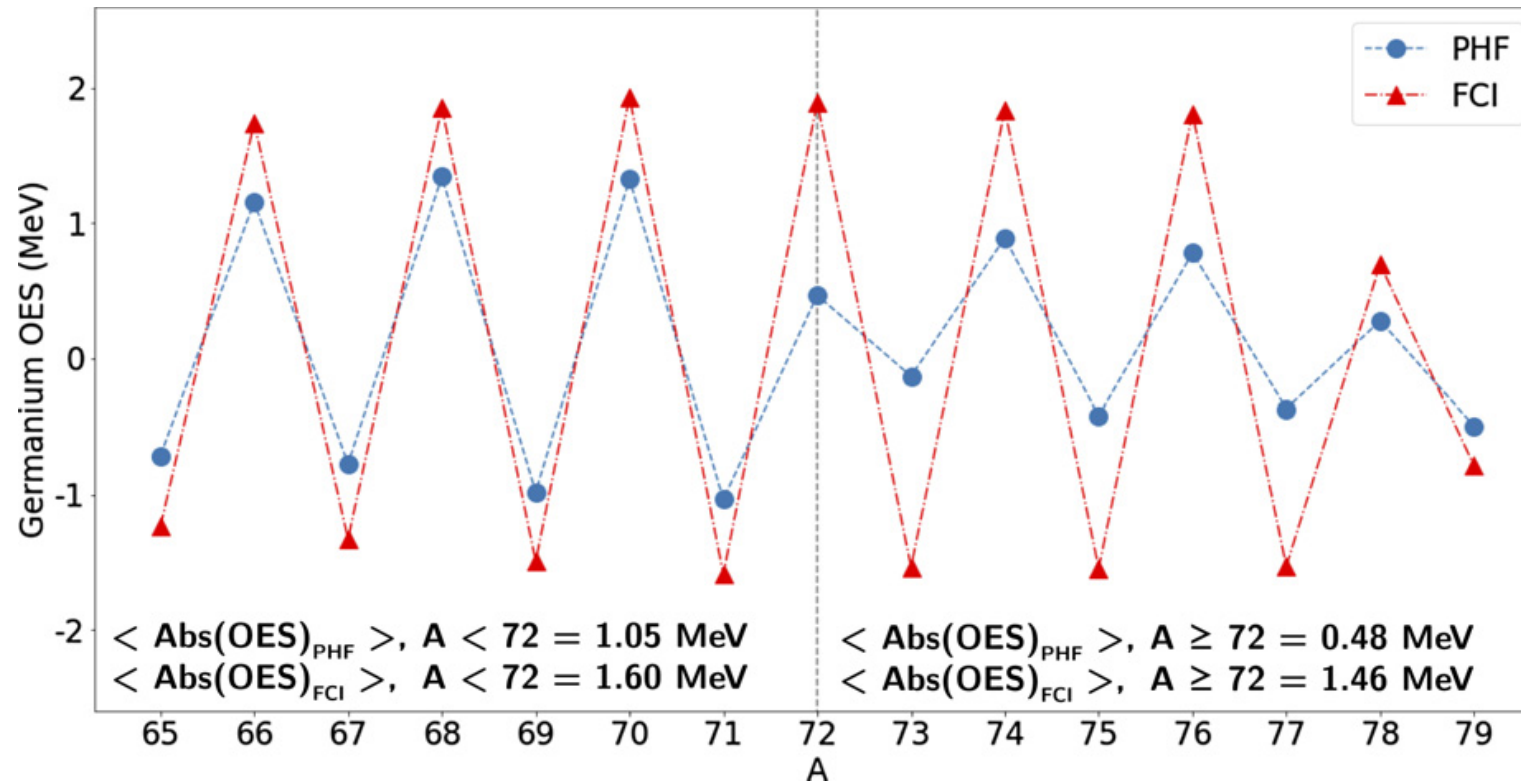


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**Figure 10.** OES along the germanium isotopic chain.

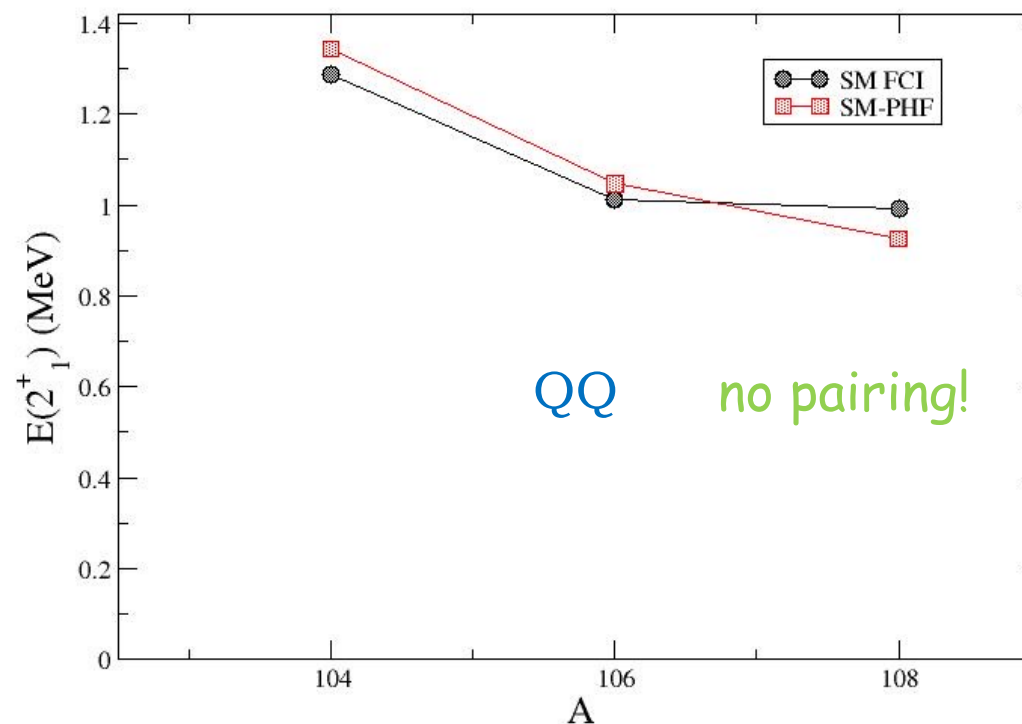
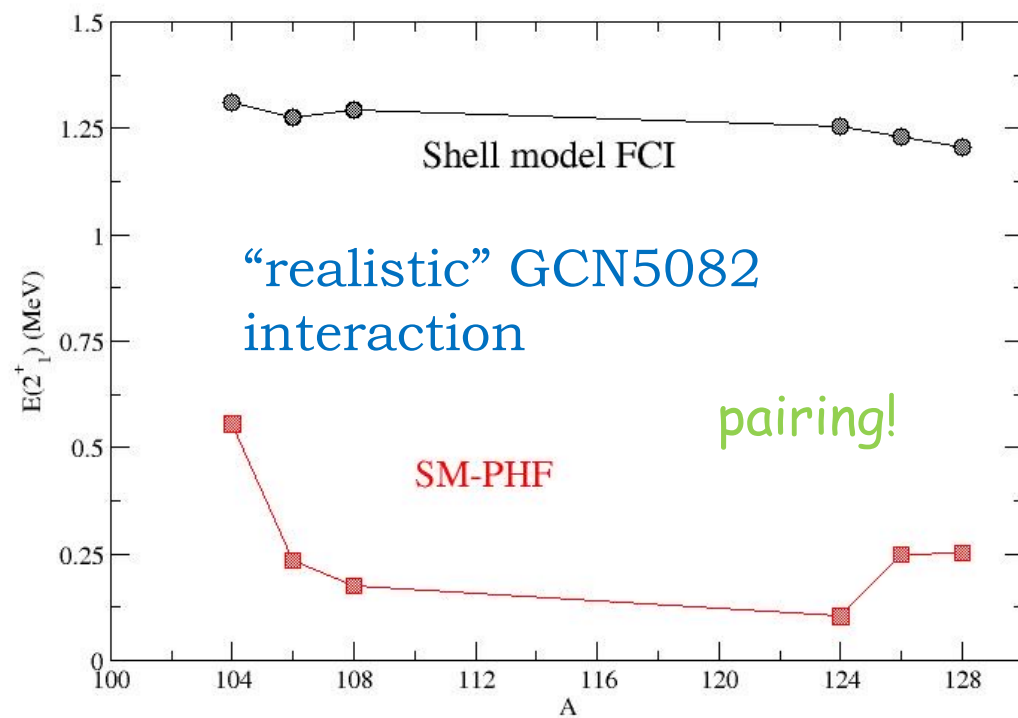


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# Evidence that pairing becomes more important in heavier nuclides with realistic interactions

Sn isotopes in 50-82 space

*New results this week!*



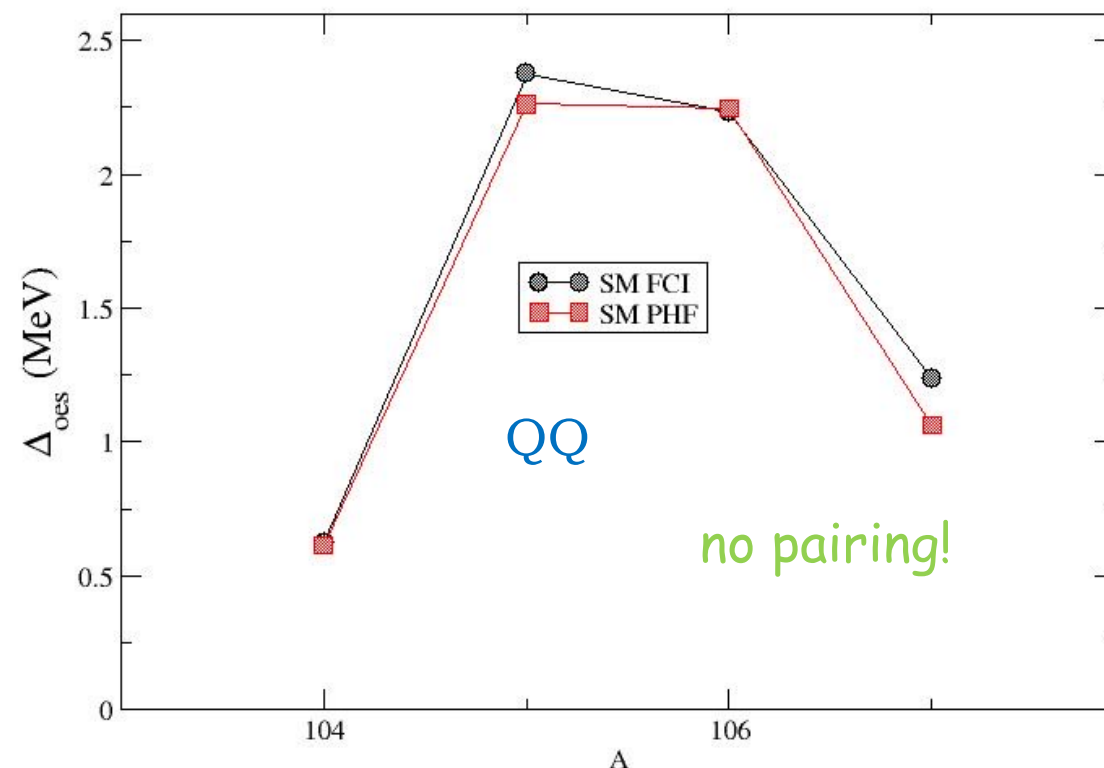
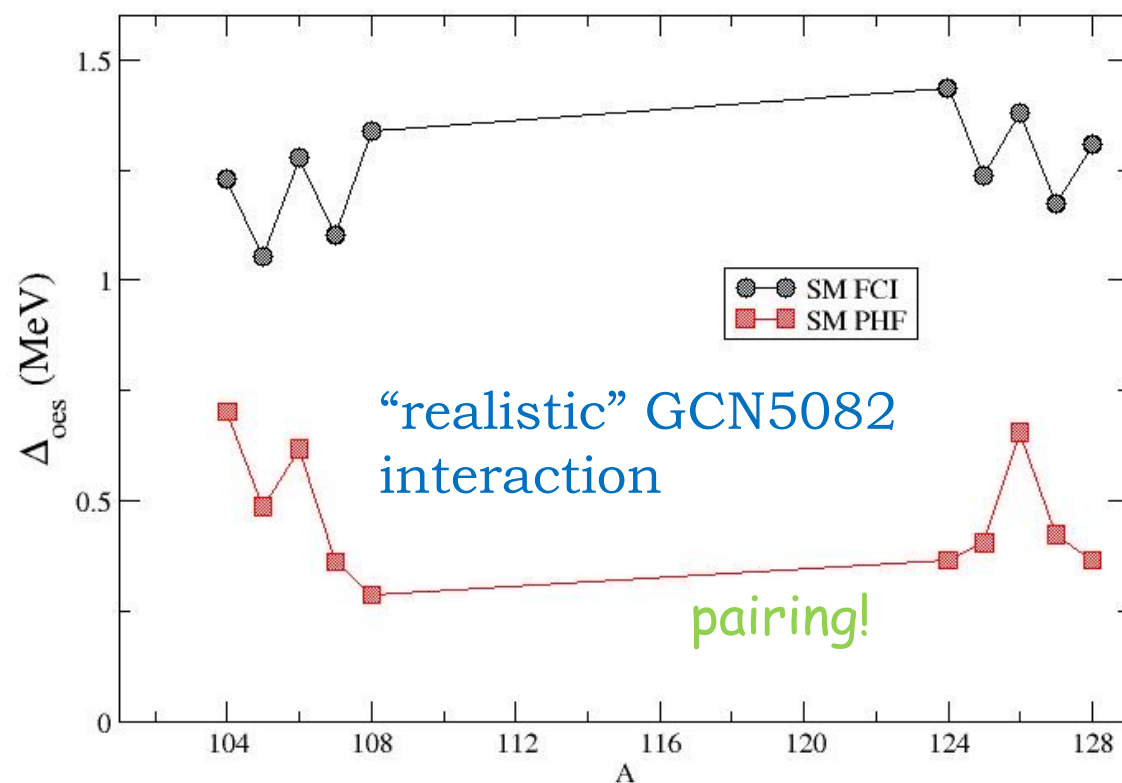


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Evidence that pairing becomes more important  
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*New results this week!*

Sn isotopes in 50-82 space





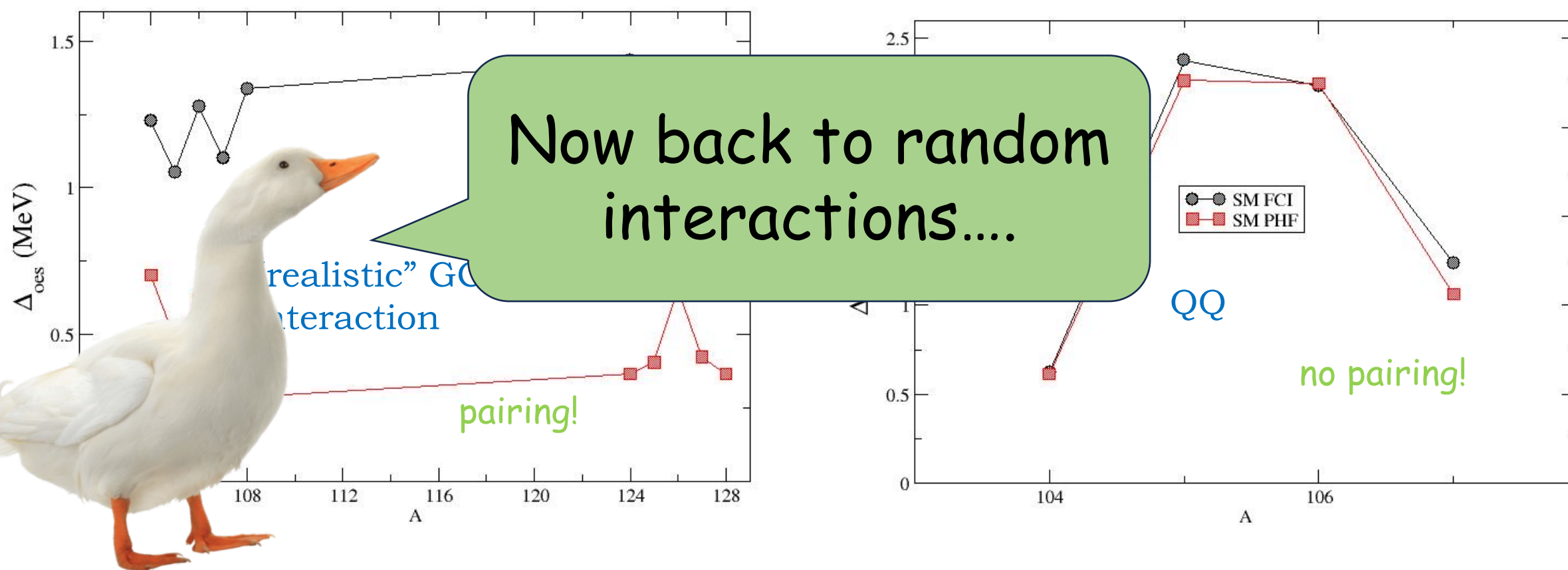


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Evidence that pairing becomes more important  
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*New results this week!*

Sn isotopes in 50-82 space



ESNT Workshop: Where has the pairing gone? May 16, 2025

“Generalized seniority from random interactions,”  
 CWJ, Bertsch, Dean, and Talmi, PRC **61**, 014311 (2000)



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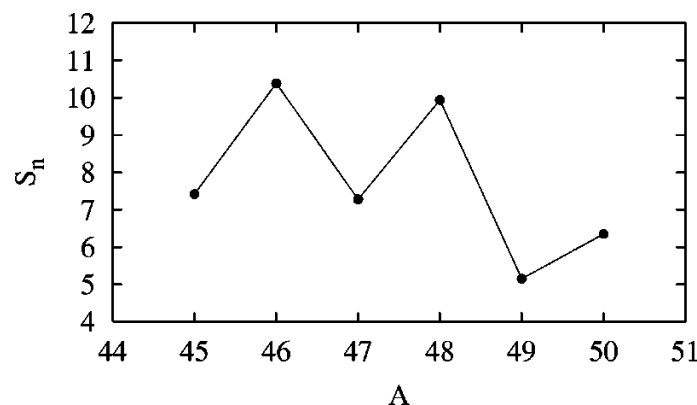


FIG. 3. Experimental neutron separation energies of Ca isotopes in the range  $A=45-50$ .

cf. T. Duguet’s talk  
 on 5/13

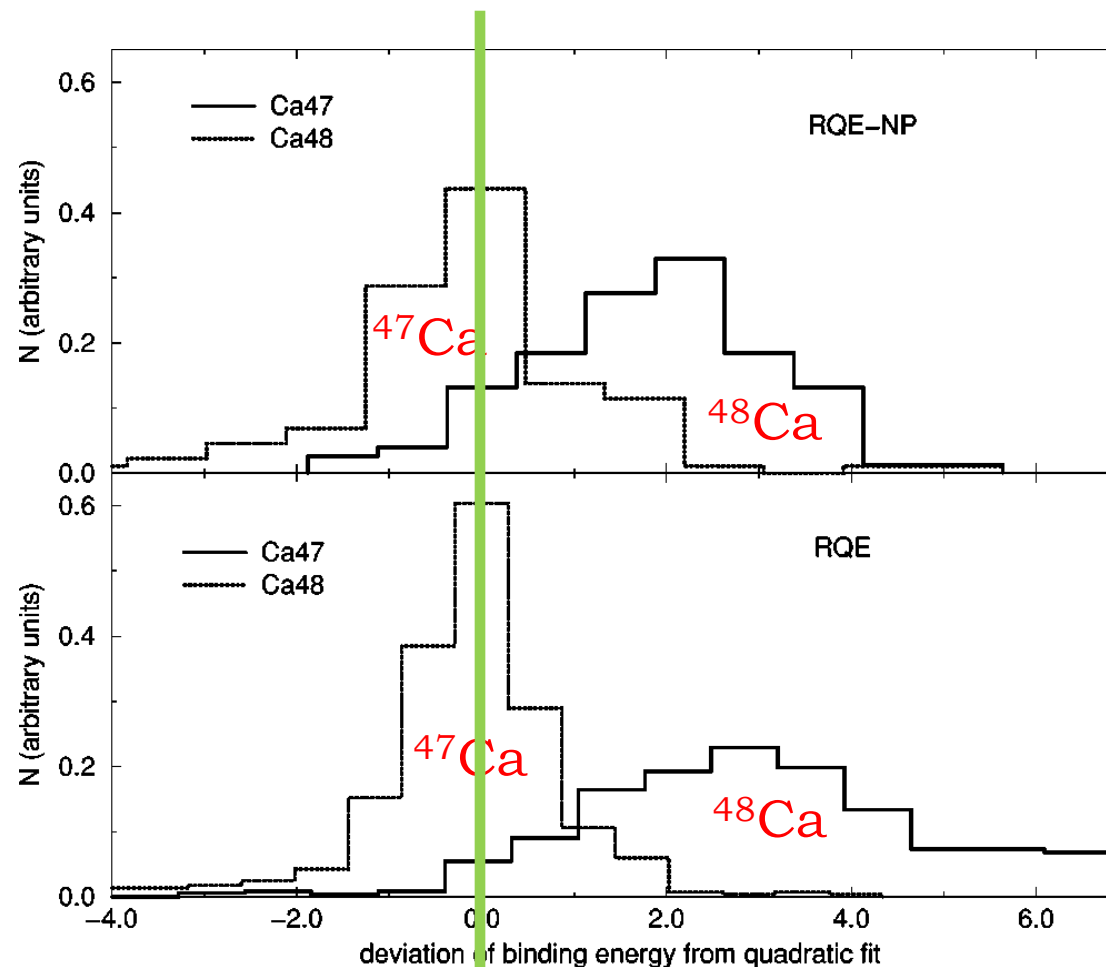


FIG. 4. Even-odd staggering effect in the RQE and RQE-NP for 4–10 neutrons in the  $pf$  shell.

“Generalized seniority from random interactions,”  
CWJ, Bertsch, Dean, and Talmi, PRC **61**, 014311 (2000)



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### Evidence for pairing-like behavior:

- $J=0$  ground states
- Seniority-like spectra
- Odd-even staggering
- **Large pair-transfer collectivity**



“Generalized seniority from random interactions,”  
CWJ, Bertsch, Dean, and Talmi, PRC **61**, 014311 (2000)



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Let  $S$  remove a pair of like nucleons:

$$\hat{S}|A\rangle = |A - 2\rangle$$

cf. S. Frauendorf's  
talk on 5/13

optimized for this isotopic chain:

$$\hat{S} = \sum_a S_a [c_a \otimes c_a]_{J=0}$$

The  $S_a$  are chosen to maximize  $\langle A - 2 | S | A \rangle$

“Generalized seniority from random interactions,”  
CWJ, Bertsch, Dean, and Talmi, PRC **61**, 014311 (2000)



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Fractional pair transfer  
collectivity:

$$f_p = \frac{\langle A - 2 | S | A \rangle^2}{\langle A | S^\dagger S | A \rangle}.$$

“Generalized seniority from random interactions,”  
CWJ, Bertsch, Dean, and Talmi, PRC **61**, 014311 (2000)



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$$f_p = \frac{\langle A - 2 | S | A \rangle^2}{\langle A | S^\dagger S | A \rangle}.$$

TABLE III. Average value of fractional pair-transfer collectivity  $f_{\text{pair}}$  between nuclides  $A$  and  $A - 2$ . Realistic denotes Wildenthal interaction for  $sd$  shell nuclides and KB3 interaction for  $pf$  shell nuclides. GOE denotes pair-transfer amplitudes between random wave functions; that is,  $A$  and  $A - 2$  were computed using different members of the RQE ensemble.

Nucleus initial → final	Realistic	GOE	RQE	RQE-NP	TBRE	RQE-SPE
$^{24}\text{O} \rightarrow ^{22}\text{O}$	0.99	0.25	0.77	0.75	0.78	0.86
$^{22}\text{O} \rightarrow ^{20}\text{O}$	0.86	0.22	0.65	0.59	0.62	0.77
$^{50}\text{Ca} \rightarrow ^{48}\text{Ca}$	0.98	0.032	0.57	0.42	0.47	0.58
$^{48}\text{Ca} \rightarrow ^{46}\text{Ca}$	0.86	0.036	0.51	0.34	0.38	0.53
$^{46}\text{Ca} \rightarrow ^{44}\text{Ca}$	0.94	0.070	0.48	0.28	0.30	0.48
$^{28}\text{Mg} \rightarrow ^{26}\text{Mg}$	0.57		0.26	0.15		0.27
$^{26}\text{Mg} \rightarrow ^{24}\text{Mg}$	0.72		0.39	0.27		0.47

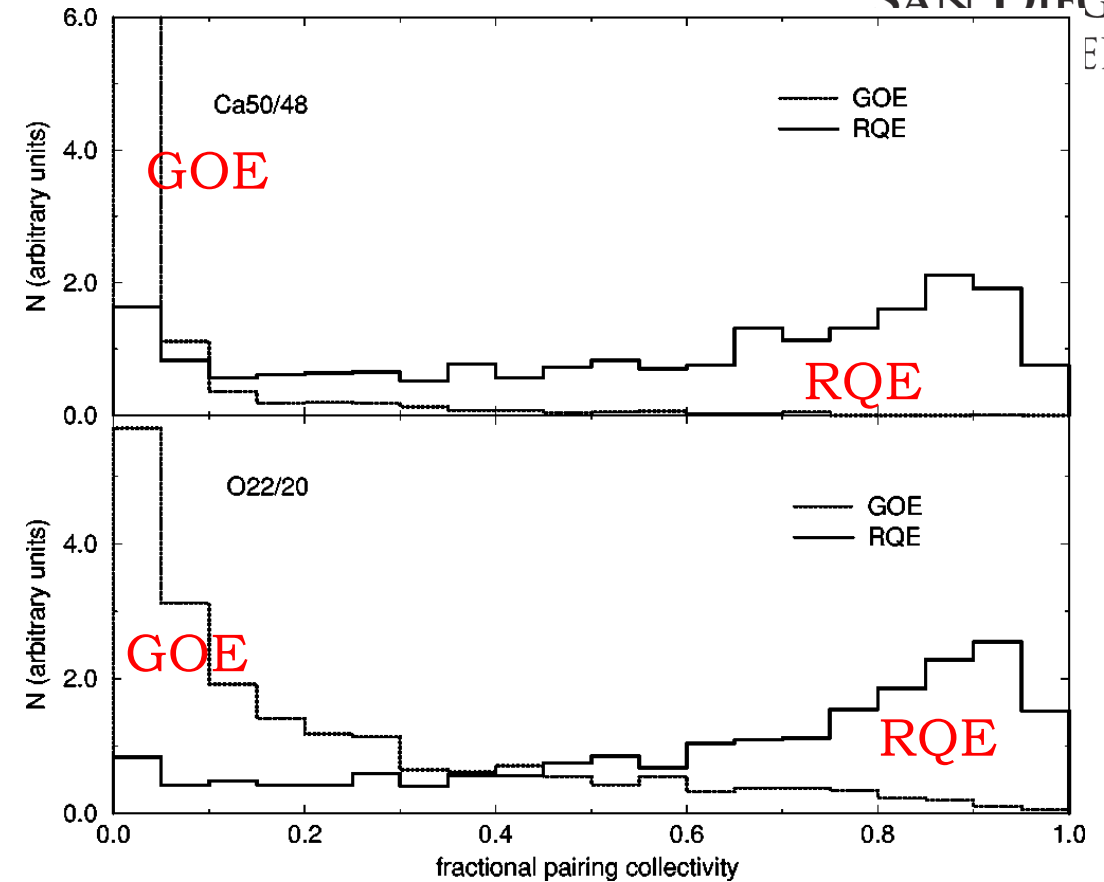


FIG. 5. Distribution of fractional pair-transfer collectivity  $f_p$  for selected isotopes and ensembles.

“GOE” = initial, final states  
derived from different interactions

“Generalized seniority from random interactions,”  
 CWJ, Bertsch, Dean, and Talmi, PRC 61, 014311 (2000)



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$$f_p = \frac{\langle A - 2|S|A \rangle^2}{\langle A|S^\dagger S|A \rangle}.$$

Looks like an open-and-shut  
 case of generalized seniority!

TABLE III. Average value of fractional  $f_{\text{pair}}$  between nuclides  $A$  and  $A-2$ . Realistic denotes Wildenthal interaction for  $sd$  shell nuclides and KB3 interaction for  $pf$  shell nuclides. GOE denotes pair-transfer amplitudes between random wave functions; that is,  $A$  and  $A-2$  were computed using different members of the RQE ensemble.

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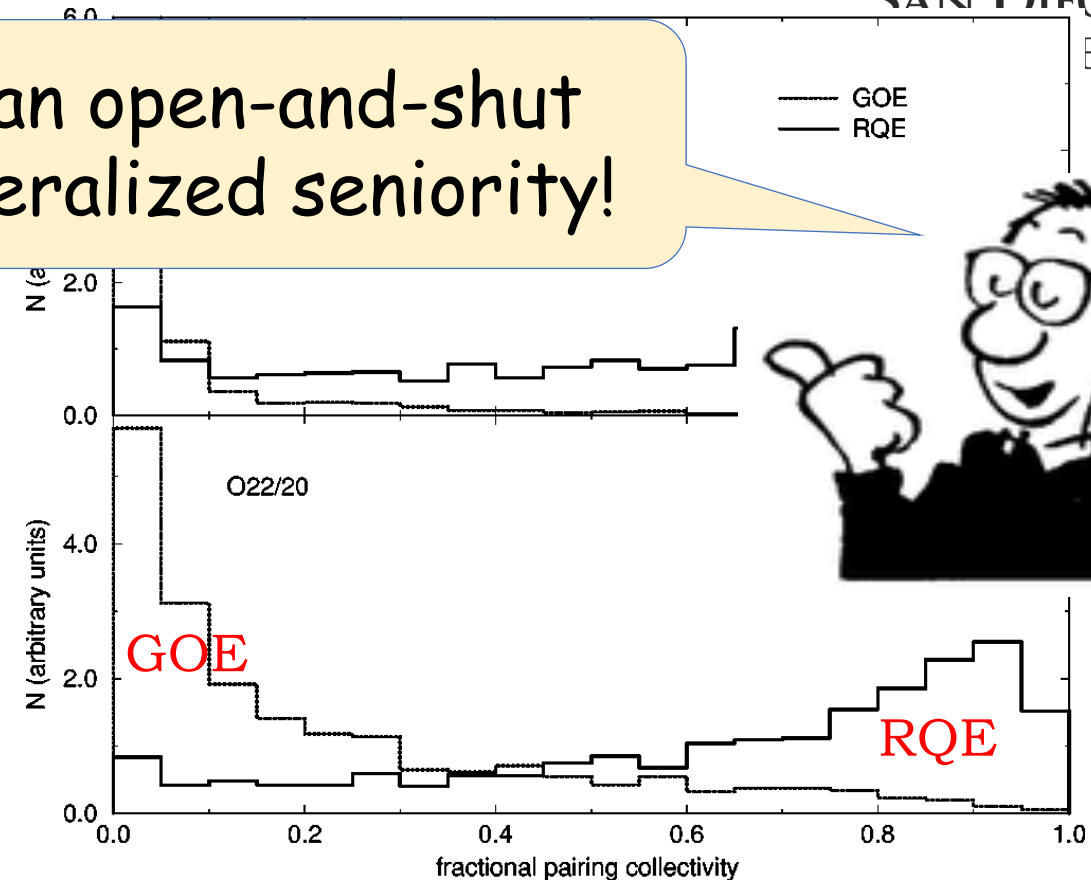


FIG. 5. Distribution of fractional pair-transfer collectivity  $f_p$  for selected isotopes and ensembles.



“Generalized seniority from random interactions,”  
CWJ, Bertsch, Dean, and Talmi, PRC 61, 014311 (2000)

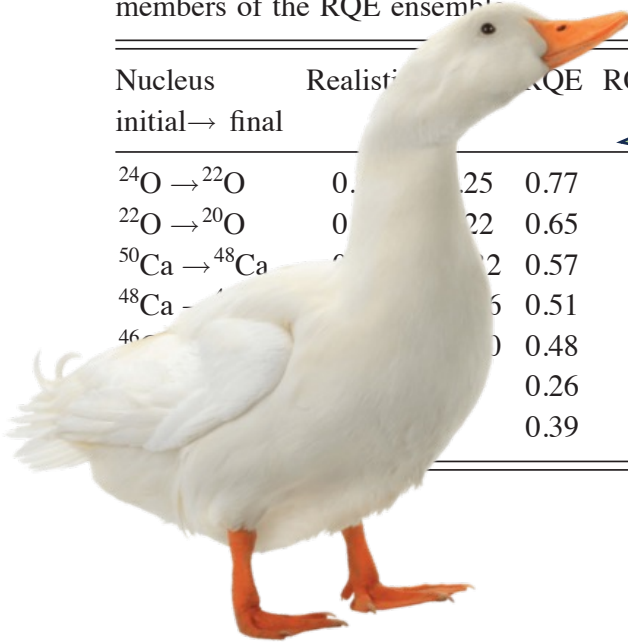


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$$f_p = \frac{\langle A-2 | S | A \rangle^2}{\langle A | S^\dagger S | A \rangle}$$

Looks like an open-and-shut  
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TABLE III. Average value of fractional  $f_{\text{pair}}$  between nuclides  $A$  and  $A-2$ . Realistic denotes Wildenthal interaction for  $sd$  shell nuclides and KB3 interaction for  $sd$  shell nuclides. GOE denotes pair-transfer amplitudes between wave functions; that is,  $A$  and  $A-2$  were computed as members of the RQE ensemble.



Nucleus initial → final	Realistic	KB3	RQE	RQE-NP	Wildenthal	KB3
$^{24}\text{O} \rightarrow ^{22}\text{O}$	0.25	0.77	0.75	0.75	0.75	0.75
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$^{42}\text{Ca} \rightarrow ^{40}\text{Ca}$	0.39	0.27	0.27	0.47	0.47	0.47

Not so fast!

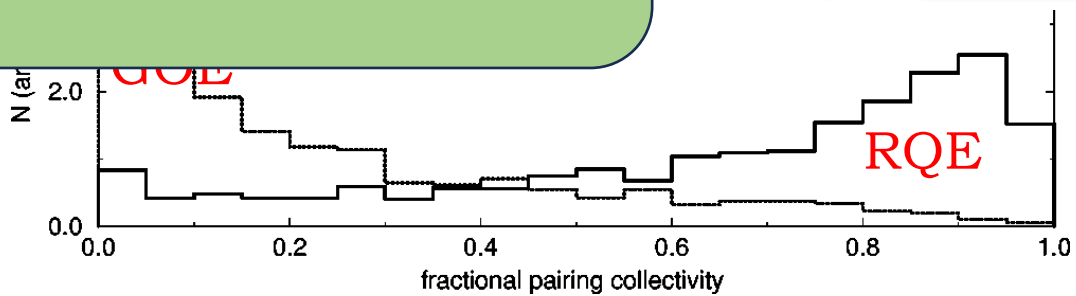


FIG. 5. Distribution of fractional pair-transfer collectivity  $f_p$  for selected isotopes and ensembles.



Zhao, Arima, and Yoshinga, PRC **66**, 064322 (2002)



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Evidence **against** pairing-like behavior:

No substantial seniority = 0 component  
in single- $j$  shell calculation

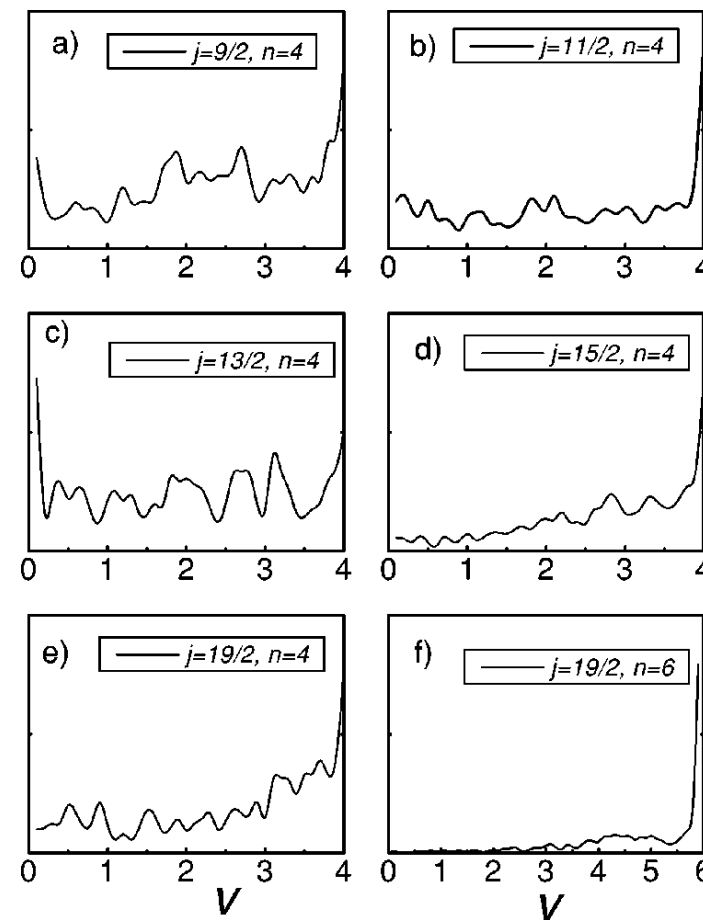


FIG. 8. The seniority distribution in the angular momentum  $I = 0$  ground states. No bias of low seniority is observed in these four and six fermions in a single- $j$  shell, which indicates that the contribution to the total 0 g.s. beyond a low seniority chain may be more important.

Zhao, *et al*, PRC **70**, 054322 (2004)



Evidence **against** pairing-like behavior:

No substantial seniority = 0 component  
in multi-shell calculations

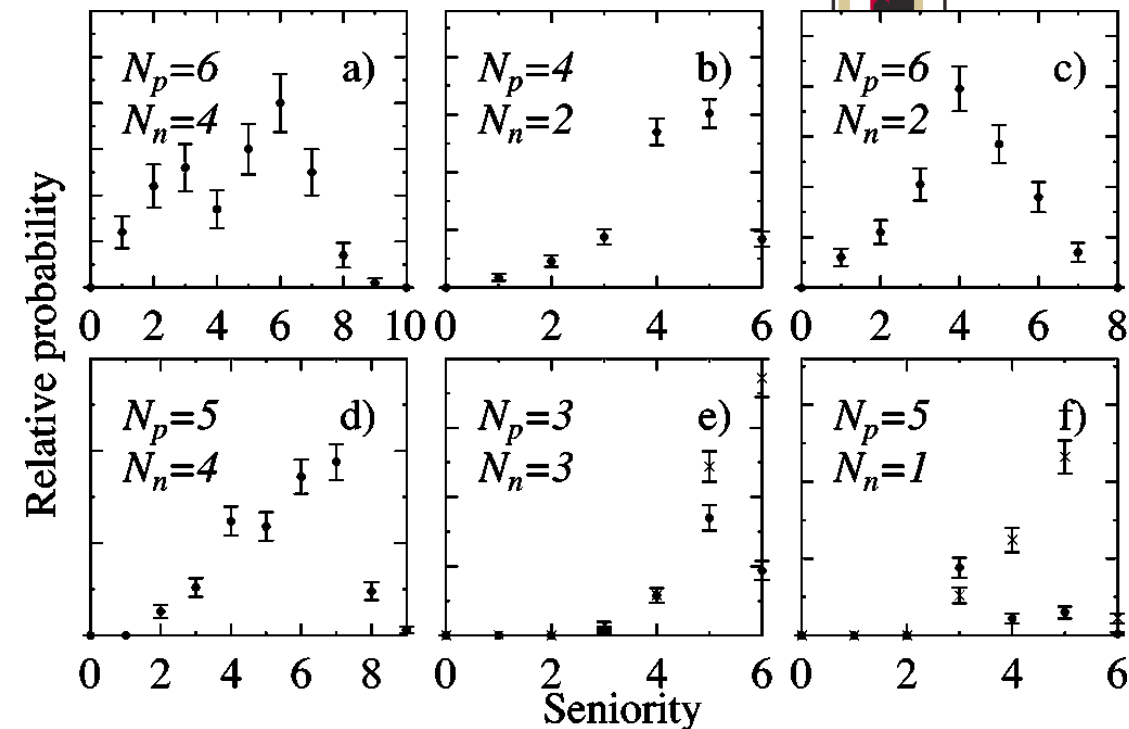


FIG. 2. Distribution of seniority in the ground states with spin zero for even-even nuclei [refer to panels (a), (b), and (c)], spin  $I=j_1, j_2, j_3$  for the odd- $A$  case [refer to panel (d)], or spin  $I=1, 0$  for odd-odd nuclei [refer to panels (e) and (f)]. The error bar is defined by the square root of the count (statistics) for each seniority bin (step width is 1). The dominance of seniority zero components of ground states is not observed.



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But: splitting of single-particle levels  
in multi-shell calculations enhances  
generalized-seniority =0 components

(Lei *et al*, PRC **83**, 024302 (2011))

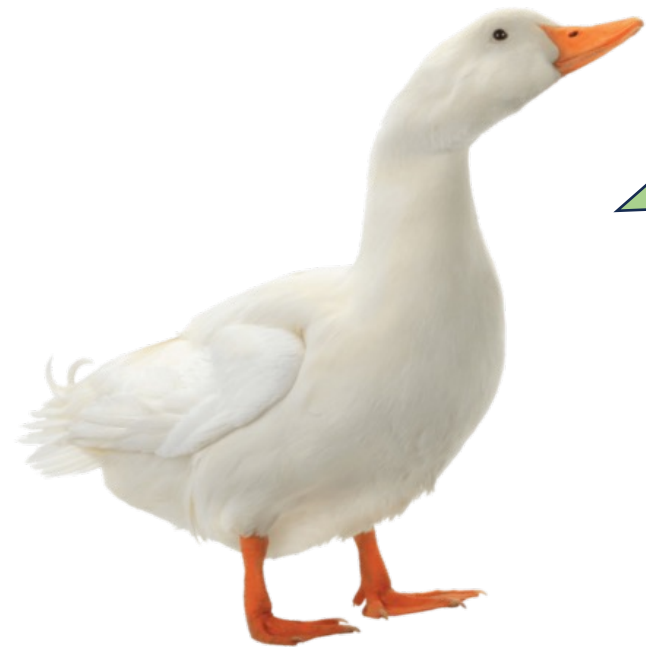




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The evidence is mixed:  
*observables* behave as if we have  
generalized seniority...

...but analyzing wave functions  
provide mixed evidence



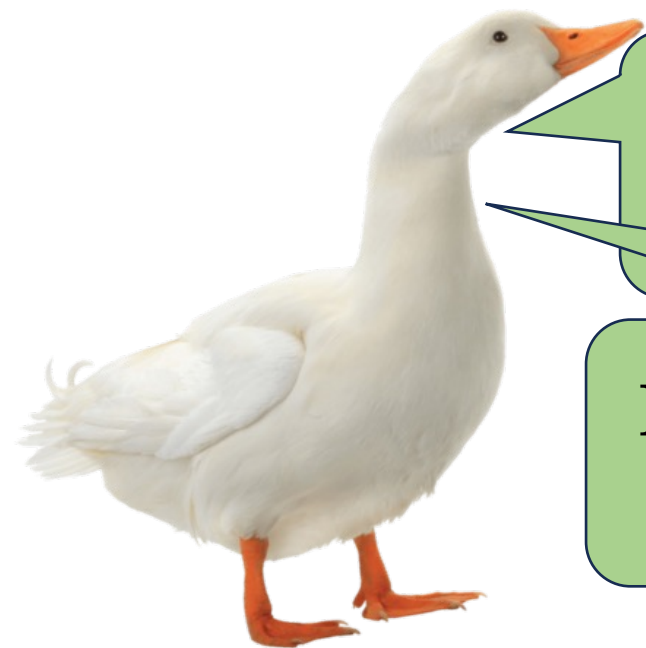


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In other words, pairing *produces* many phenomena:  $J=0$  g.s., OES, pair transfer collectivity, reduction of moment of inertia...

...but many of these can be produced *without* explicit pairing.

Model-independent evidence for pairing may be hard to come by.





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I have another 'explanation'  
for  $J=0$  dominance, to be  
discussed at the end







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## Part 2: Proton-neutron pairing and entanglement

“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen,  
PRC **111**, 024310 (2025)

“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen,  
PRC **111**, 024310 (2025)



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Like-nucleon ( $T=1$ ) pairing is a ubiquitous phenomenon  
(with realistic interactions)

what about proton-neutron ( $T=0$ ) pairing?

There have been many investigations...

“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen,  
PRC **111**, 024310 (2025)



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In this study:

Nucleon-pair condensates: a particle-number-conserving  
alternative to HFB

“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen, PRC **111**, 024310 (2025)



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In this study:

Nucleon-pair condensates: a particle-number-conserving  
alternative to HFB

$$\hat{A}^\dagger = \sum_{i,j} A_{ij} \hat{c}_i^\dagger \hat{c}_j^\dagger$$

general pair creation operator  
(not necessarily good J)

“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen, PRC **111**, 024310 (2025)



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In this study:

Nucleon-pair condensates: a particle-number-conserving  
alternative to HFB

skew-symmetric matrix  
(to be optimized)

$$\hat{A}^\dagger = \sum_{i,j} A_{ij} \hat{c}_i^\dagger \hat{c}_j^\dagger$$

general pair creation operator  
(not necessarily good J)

can be 2 protons, 2 neutrons or proton-neutron

“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen, PRC **111**, 024310 (2025)



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Nucleon-pair condensates: a particle-number-conserving alternative to HFB

$$\hat{A}^\dagger = \sum_{i,j} \hat{c}_i^\dagger \hat{c}_j^\dagger$$

general pair creation operator  
(not necessarily good J)

$$|\psi\rangle = (\hat{A}^\dagger)^{n/2} |0\rangle \quad \longrightarrow \quad |\psi\rangle = \overset{\text{p-p pair}}{(\hat{P}^\dagger)^{Z/2}} \overset{\text{n-n pair}}{(\hat{N}^\dagger)^{N/2}} |0\rangle$$

or

$$|\psi\rangle = \overset{\text{p-n pair}}{(\hat{P}\hat{N}^\dagger)^{A/2}} |0\rangle$$



“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen, PRC **111**, 024310 (2025)



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Nucleon-pair condensates: a particle-number-conserving alternative to HFB

$$\hat{A}^\dagger = \sum_{i,j} A_{ij} \hat{c}_i^\dagger \hat{c}_j^\dagger$$

general pair creation operator  
(not necessarily good J)

$$|\psi\rangle = (\hat{A}^\dagger)^{n/2} |0\rangle$$

Now (before J-projection) minimize

$$\frac{\langle\psi|\hat{H}|\psi\rangle}{\langle\psi|\psi\rangle}$$

by varying the  $A_{ij}$

“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen, PRC 111, 024310 (2025)



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Nucleon-pair condensates: a particle-number-conserving alternative to HFB

$\hat{A}^\dagger$

This is probably equivalent, or at least very similar, to number-projected HFB

$|\psi\rangle = (\hat{A}^\dagger)^N |\Phi\rangle$

Now (before J-projection) minimize

$$\frac{\langle\psi|\hat{H}|\psi\rangle}{\langle\psi|\psi\rangle}$$

by varying the  $A_{ij}$



“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen, PRC **111**, 024310 (2025)



## Nucleon-pair condensates: a particle-number-conserving alternative to HFB

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The final step: project out good J and solve a generalized eigenvalue problem

$$|\psi\rangle = (\hat{A}^\dagger)^{n/2} |0\rangle$$

$$N_{J,MK} = \langle \psi | \hat{P}_{MK}^J | \psi \rangle,$$

J-projection operator

$$H_{J,MK} = \langle \psi | \hat{P}_{MK}^J \hat{H} | \psi \rangle,$$

“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen, PRC **111**, 024310 (2025)



## Nucleon-pair condensates: a particle-number-conserving alternative to HFB

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J-projection operator

$$H_{J,MK} = \langle \psi | \hat{P}_{MK}^J \hat{H} | \psi \rangle,$$

Solve

$$\sum_K H_{J,MK} g_{J,K} =$$

$$E \sum_K N_{J,MK} g_{J,K} =$$

“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen, PRC 111, 024310 (2025)



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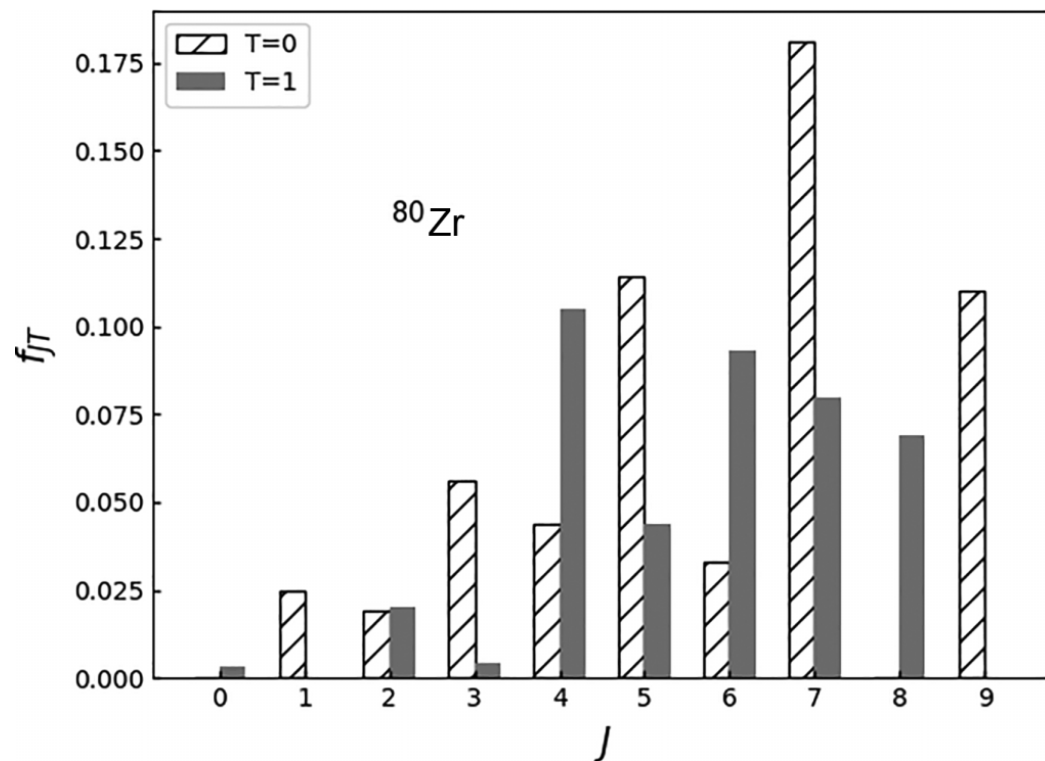
Nucleon-pair condensates: a particle-number-conserving alternative to HFB

In this project, we were particularly interested in analyzing the properties of the structure coefficient  $A_{ij}$  :

- Decompose  $A$  into pairs of good J and T
- Compute ‘entropy’ as an alternative way to characterize beyond-mean-field correlations



- Decompose  $A$  into pairs of good  $J$  and  $T$



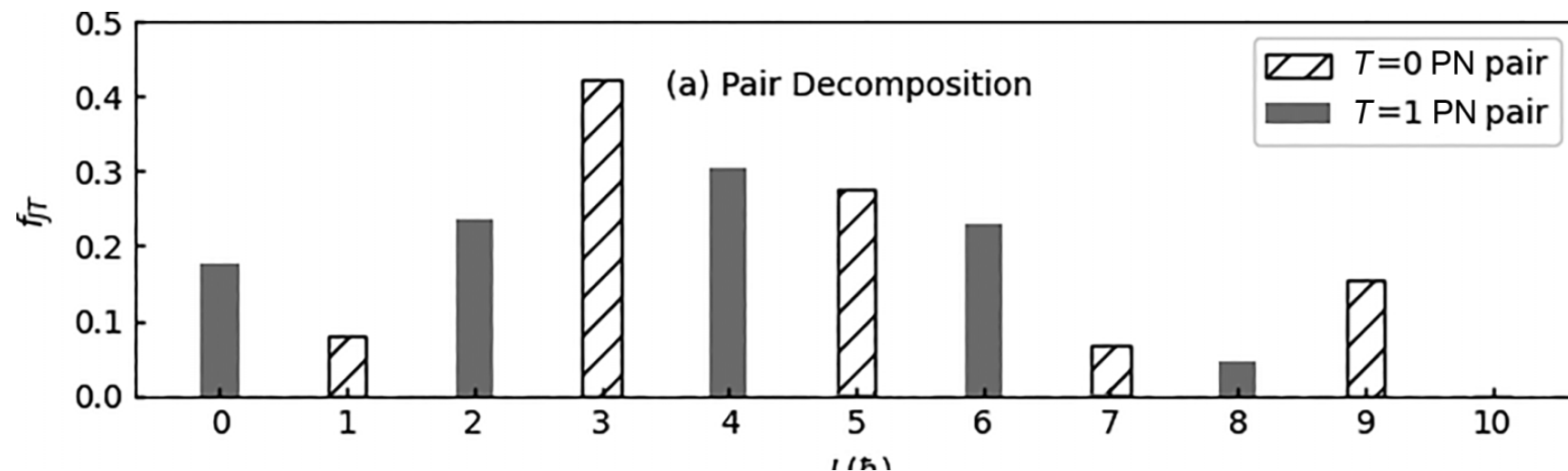
Decomposition of  $^{80}\text{Zr}$   
optimized p-n pair

Note very small  $T=1, J=0$   
(and not large  $T=0, J=1$ )

FIG. 2. Decomposition of the optimized proton-neutron pair for  $^{80}\text{Zr}$  in the  $1p0f_{5/2}0g_{9/2}$  major shell.



- Decompose  $A$  into pairs of good  $J$  and  $T$



Decomposition of  $^{92}\text{Pd}$   
optimized (but constrained  $T=0,1$ ) p-n pairs



“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen, PRC **111**, 024310 (2025)



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- Compute ‘entropy’ (of the structure coefficient  $A_{ij}$ ) as an alternative way to characterize beyond-mean-field correlations



- Compute ‘entropy’ (of the structure coefficient  $A_{ij}$ )  
as an alternative way to characterize  
beyond-mean-field correlations

Put structure coefficient  $A_{ij}$  into ‘canonical form’  
(skew-symmetric anti-diagonal)  $\rightarrow$  equivalent to diagonalizing  
Hermitian matrices

$$\hat{A}^\dagger = \frac{1}{2} \sum_{\alpha\beta} A_{\alpha\beta} \hat{c}_\alpha^\dagger \hat{c}_\beta^\dagger = \sum_{k>0} v_k a_k^\dagger a_{\bar{k}}^\dagger,$$

“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen, PRC **111**, 024310 (2025)



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Canonical form:

$$\hat{A}^\dagger = \frac{1}{2} \sum_{\alpha\beta} A_{\alpha\beta} \hat{c}_\alpha^\dagger \hat{c}_\beta^\dagger = \sum_{k>0} v_k a_k^\dagger a_{\bar{k}}^\dagger,$$

where  $p(k_1 k_2 \dots k_n)$  is the probability of finding the configuration  $a_{k_1}^\dagger a_{\bar{k}_1}^\dagger a_{k_2}^\dagger a_{\bar{k}_2}^\dagger \dots a_{k_n}^\dagger a_{\bar{k}_n}^\dagger |0\rangle$  in  $|PC, n\rangle$ ,

$$p(k_1 k_2 \dots k_n) = \frac{v_{k_1}^2 v_{k_2}^2 \dots v_{k_n}^2}{\langle PC, n | PC, n \rangle}, \quad (17)$$

then define the Shannon entropy as:

$$S = \sum_{\{k_1, k_2, \dots, k_n\}} -p(k_1 k_2 \dots k_n) \log_d p(k_1 k_2 \dots k_n),$$

“Shannon entropy of optimized proton-neutron pair condensates,” Liang, Lu, Lei, CWJ, Fu, Shen, PRC **111**, 024310 (2025)



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One can show  
that  $S = 0$   
= single Slater det  
and seniority = 0  
 $\rightarrow S = 1$

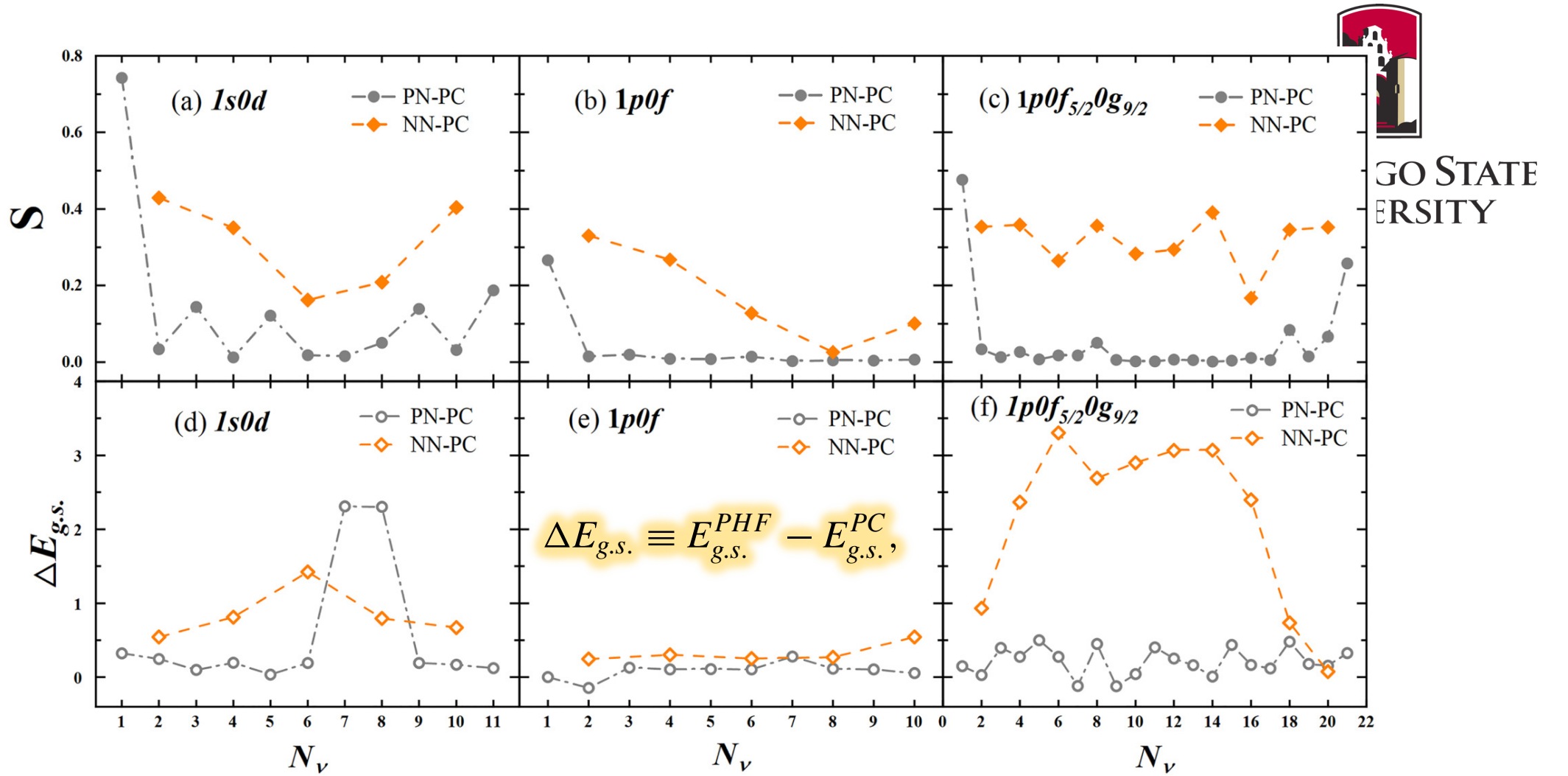


FIG. 7. (a) Entropy of optimized PN-PCs for  $1s0d$ -shell nuclei from  $^{18}\text{F}$  to  $^{38}\text{K}$ , in comparison with semi-magic oxygen isotopes. “PN-PC” denotes proton-neutron pair condensates, while “NN-PC” denotes neutron-neutron pair condensates; all those condensates are optimized by variation with shell model interactions. (b) Entropy for  $1p0f$ -shell  $N = Z$  nuclei from  $^{42}\text{Sc}$  to  $^{60}\text{Zn}$ , semimagic nuclei  $^{42}\text{Ca}$  to  $^{50}\text{Ca}$ . (c) Entropy for  $1p0f_{5/2}0g_{9/2}$ -shell  $N = Z$  nuclei from  $^{58}\text{Cu}$  to  $^{98}\text{In}$ , semimagic nuclei  $^{58}\text{Ni}$  to  $^{76}\text{Ni}$ . (d)–(f) The difference between the ground state energy from PCs and that from the PHF method.

Most p-n condensates had very low entropy, and the corresponding condensate energy was nearly the same as PHF



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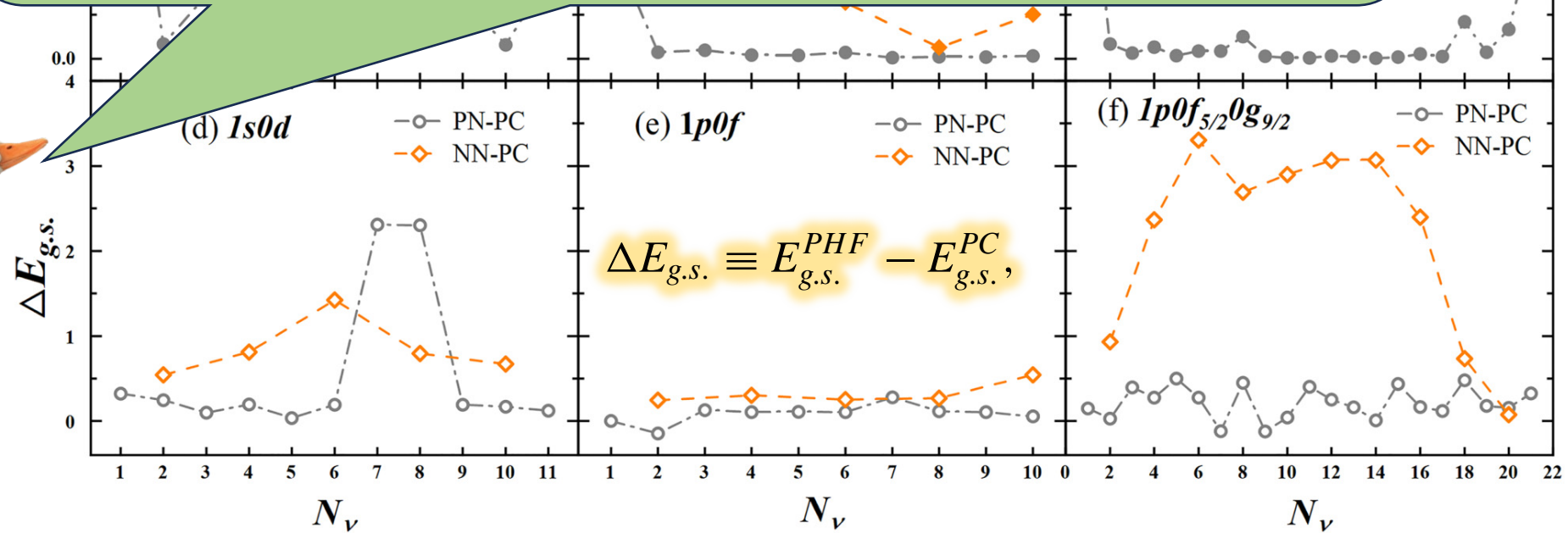


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Finally, artificially enhance/suppress  
T=0/1 components in a realistic interaction (USDB)



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Finally, artificially enhance/suppress  
T=0/1 components in a realistic interaction (USDB)

$$\hat{H}' = \hat{H}_0^{USDB} + (1 - x)\hat{H}_{T=1}^{USDB} + (1 + x)\hat{H}_{T=0}^{USDB},$$



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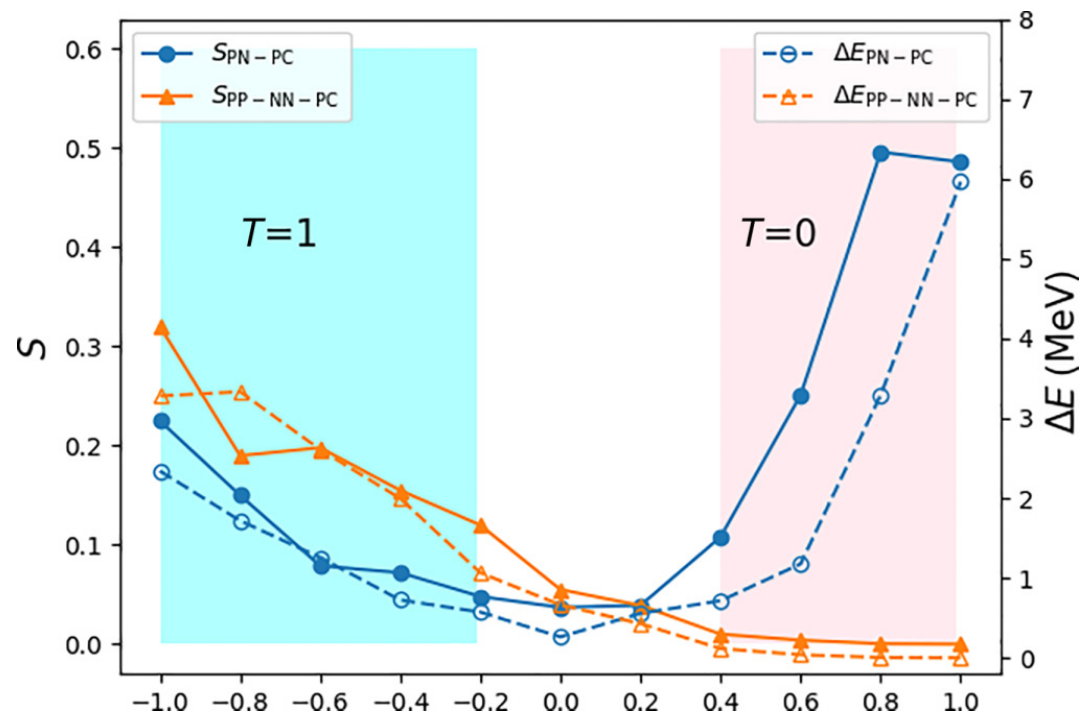


FIG. 8. Entropy of optimized pair condensates for  $^{24}\text{Mg}$  in the  $1s0d$  major shell, under artificial Hamiltonian  $\hat{H} = \hat{H}_0^{USDB} + (1+x)\hat{H}_{T=0}^{USDB} + (1-x)\hat{H}_{T=1}^{USDB}$ , with  $x \in [-1, 1]$ . “PN-PC” stands for proton-neutron pair condensate, while “PP-NN-PC” stands for proton-proton neutron-neutron pair condensate. When  $x = 0$  the physical situation has low correlation entropy for both PN-PC and PP-NN-PC; when  $x$  goes away from 0, the pair condensates onset  $T = 0$  and  $T = 1$  “phase” transitions. The corresponding energetic descent of the pair condensates due to correlation further than a Hartree-Fock solution is denoted as  $\Delta E_{\text{PN-PC}}$  and  $\Delta E_{\text{PP-NN-PC}}$ , respectively.

$$\Delta E_{\text{PN-PC}} = E_{\min}^{\text{HF}} - E_{\min}^{\text{PN-PC}},$$

$$\Delta E_{\text{PP-NN-PC}} = E_{\min}^{\text{HF}} - E_{\min}^{\text{PP-NN-PC}},$$

Finally, artificially  
enhance/suppress  
 $T=0/1$  components  
in a realistic interaction (USDB)

$$\hat{H}' = \hat{H}_0^{USDB} + (1-x)\hat{H}_{T=1}^{USDB} + (1+x)\hat{H}_{T=0}^{USDB},$$



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$$\hat{H}' = \hat{H}_0^{USDB} + (1 - x)\hat{H}_{T=1}^{USDB} + (1 + x)\hat{H}_{T=0}^{USDB},$$

Finally, let's decompose the optimized p-n pairs  
in the  $X = -1, 0, +1$  limits

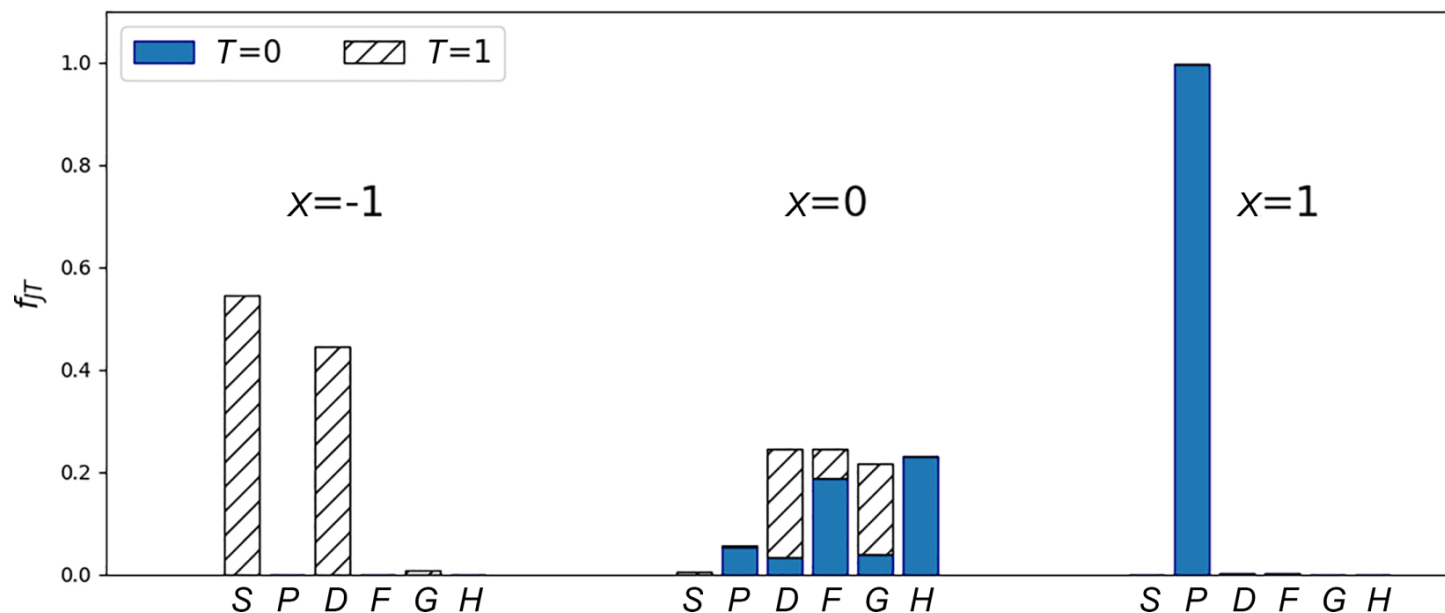


FIG. 9. Fractions of different pairs with  $J = 0, \dots, 5$  and  $T = 0, 1$  in the optimized pair of the proton-neutron pair condensate for  $^{24}\text{Mg}$ , given the artificial Hamiltonian in Eq. (23). When  $x = 0$  the physical solution has all kinds of pairs of  $T = 0$  and  $T = 1$  mixed up; when  $x = -1$ ,  $T = 1$  S and D pairs dominate; when  $x = 1$  the  $T = 0$  P pairs dominate. S, P, D, F, G, H stand for  $J = 0, 1, 2, 3, 4, 5$  pairs, respectively.



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## CONCLUSIONS



We introduced a generalized, J-projected optimized pair condensate ( $\sim$  projected HFB, with variation *after* number projection but *before* J-projection)

Proton-neutron pair condensates did not naturally optimize to  $T=1, J=0$  or  $T=0, J=1$  pairs,

and those condensates were not much different from J-projected Hartree-Fock.



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## CONCLUSIONS



We could **force** domination by  $T=0$  or  $T=1$  pairs, but for a 'realistic' Hamiltonian it was an admixture (and, again, close to HF).

Overall, in this study, the evidence for strong proton-neutron pairing beyond the mean-field was not found.



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# FINALE

An alternate explanation (?) for  $J=0$  dominance

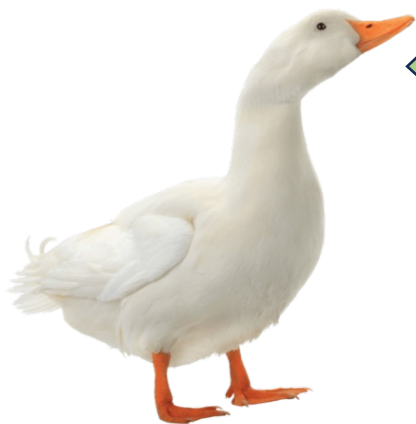


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# FINALE

An alternate explanation (?) for  $J=0$  dominance

Can we analyze the behavior of  
random two-body interactions  
with random (GOE) matrices?







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Mapping random matrices onto many-body simulations is not trivial:

- Different  $J$  spaces have different dimensions
- Level densities is Gaussian, not semicircle (GOE)

To account for this, look at  
 $\text{rms } \textit{matrix element}$



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J-projected centroids:

$$d_J = \frac{1}{N_J} \text{tr}(P_J H)$$

J-projected widths:

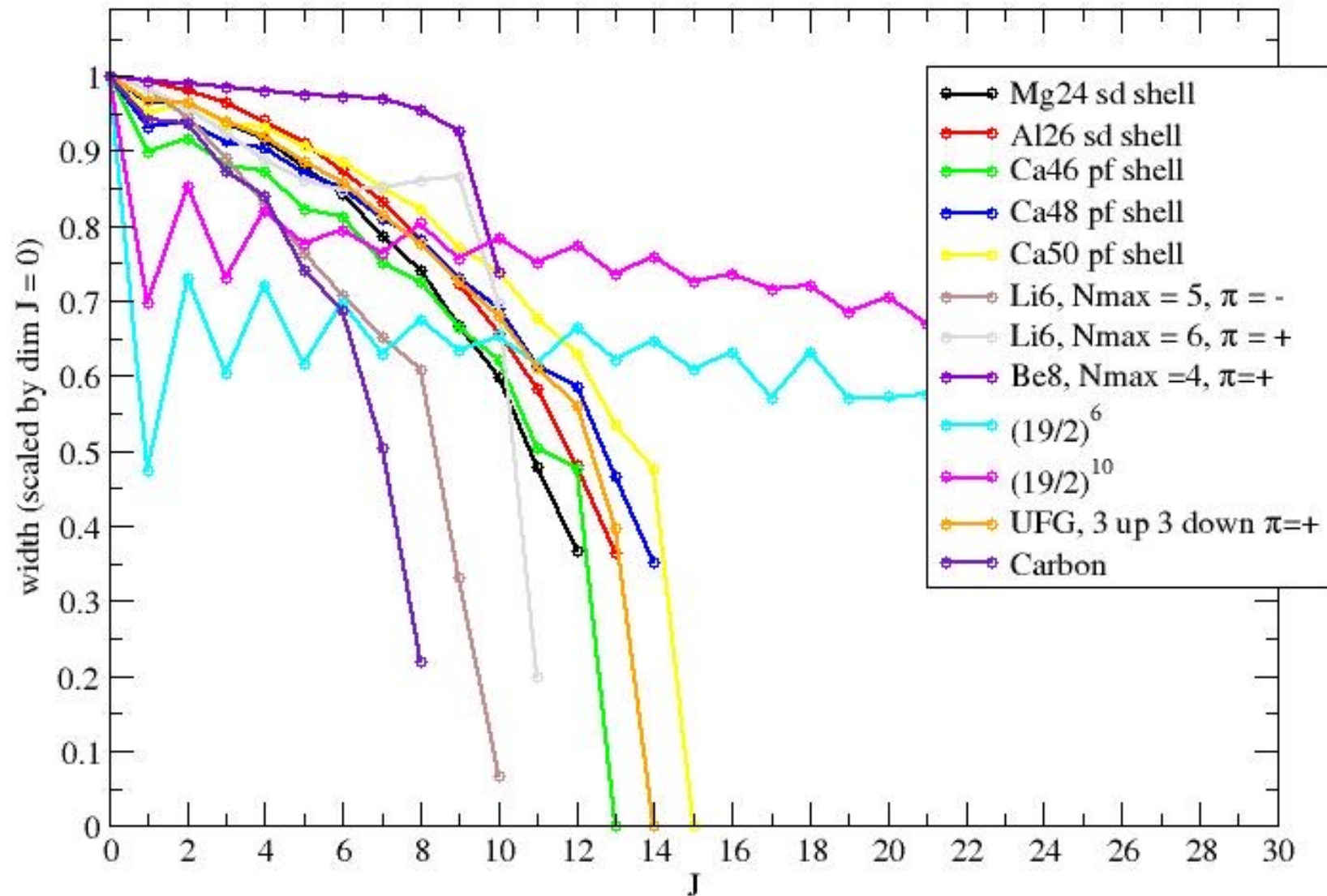
$$\sigma_J^2 = \frac{1}{N_J} \text{tr}(P_J H^2) - d_J^2$$

scaled widths:

$$\sigma_J / \sigma_0$$



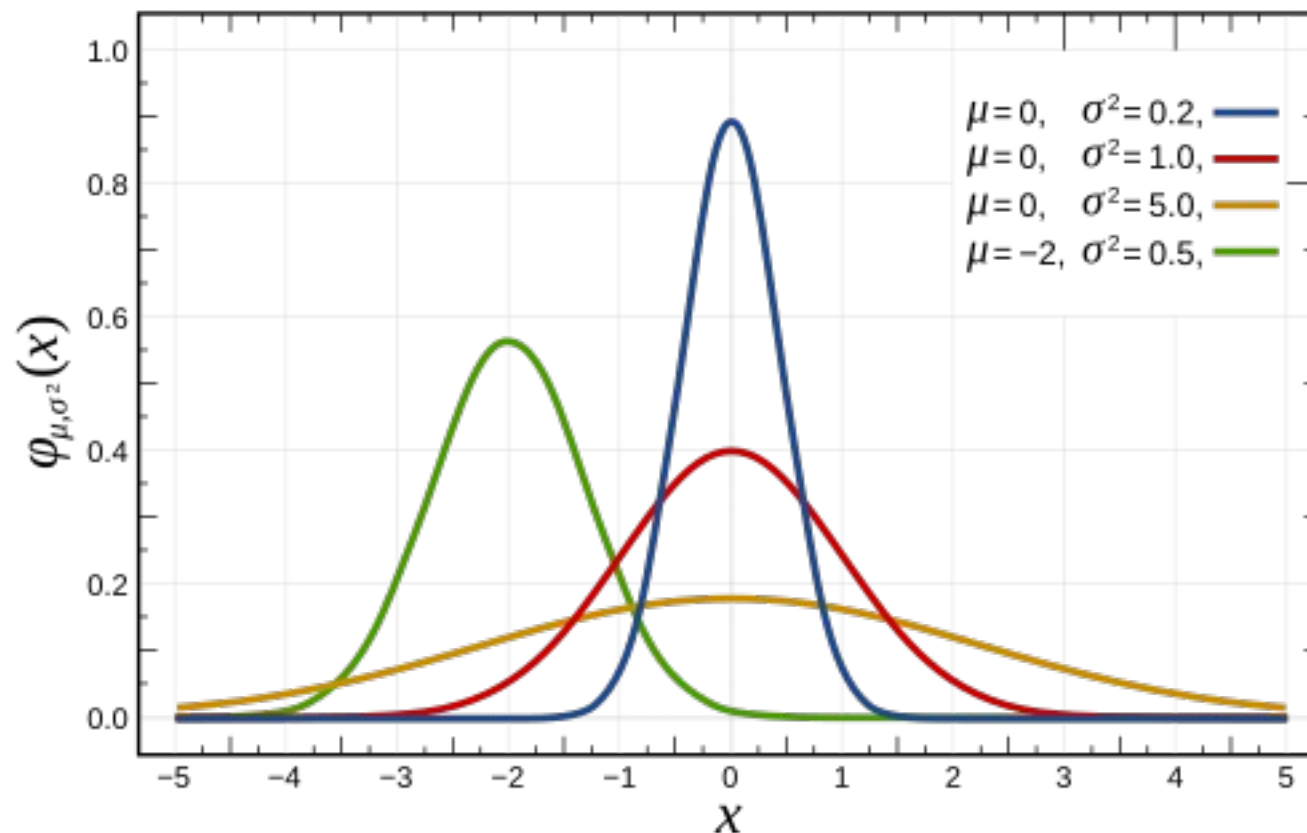
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This partially explains  
J=0 g.s. dominance:  
J=0 space, despite  
smaller dimensions,  
have a greater width

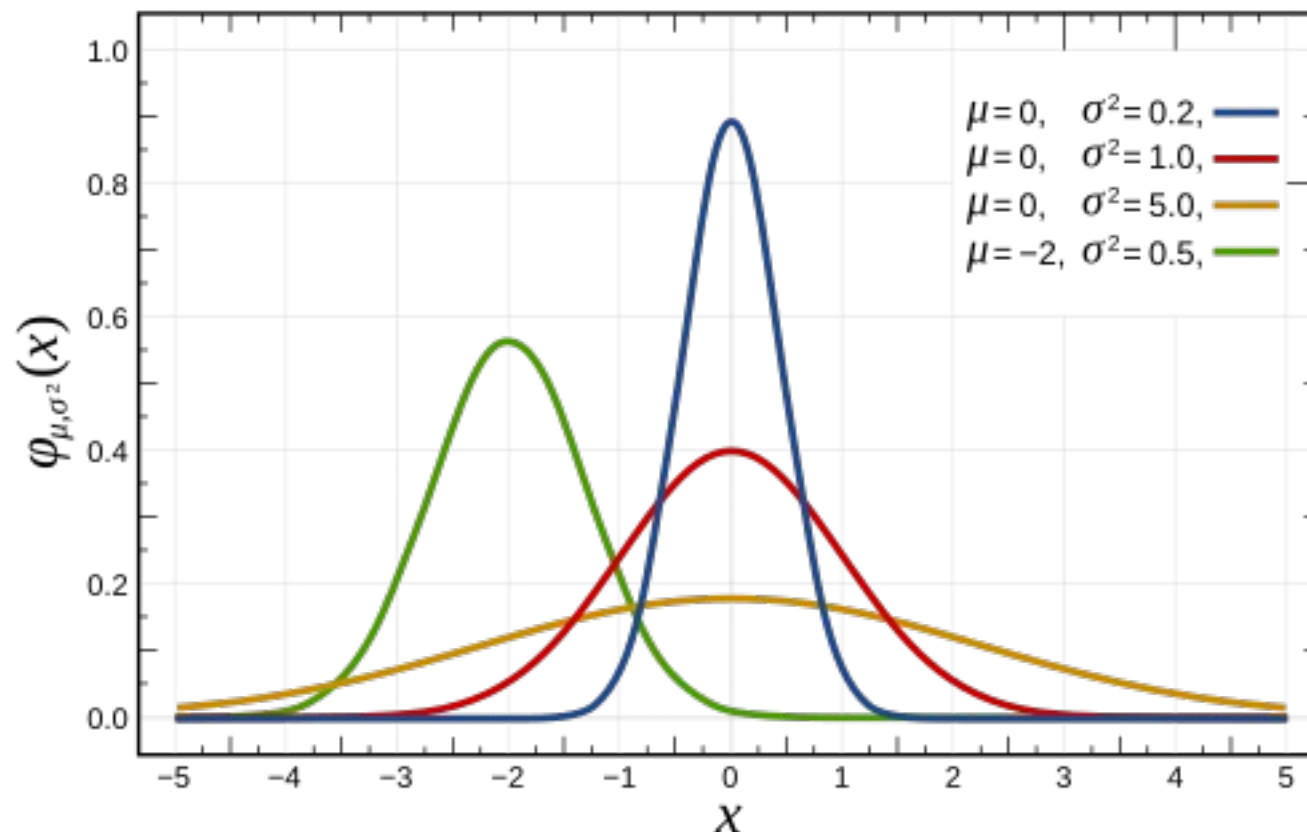




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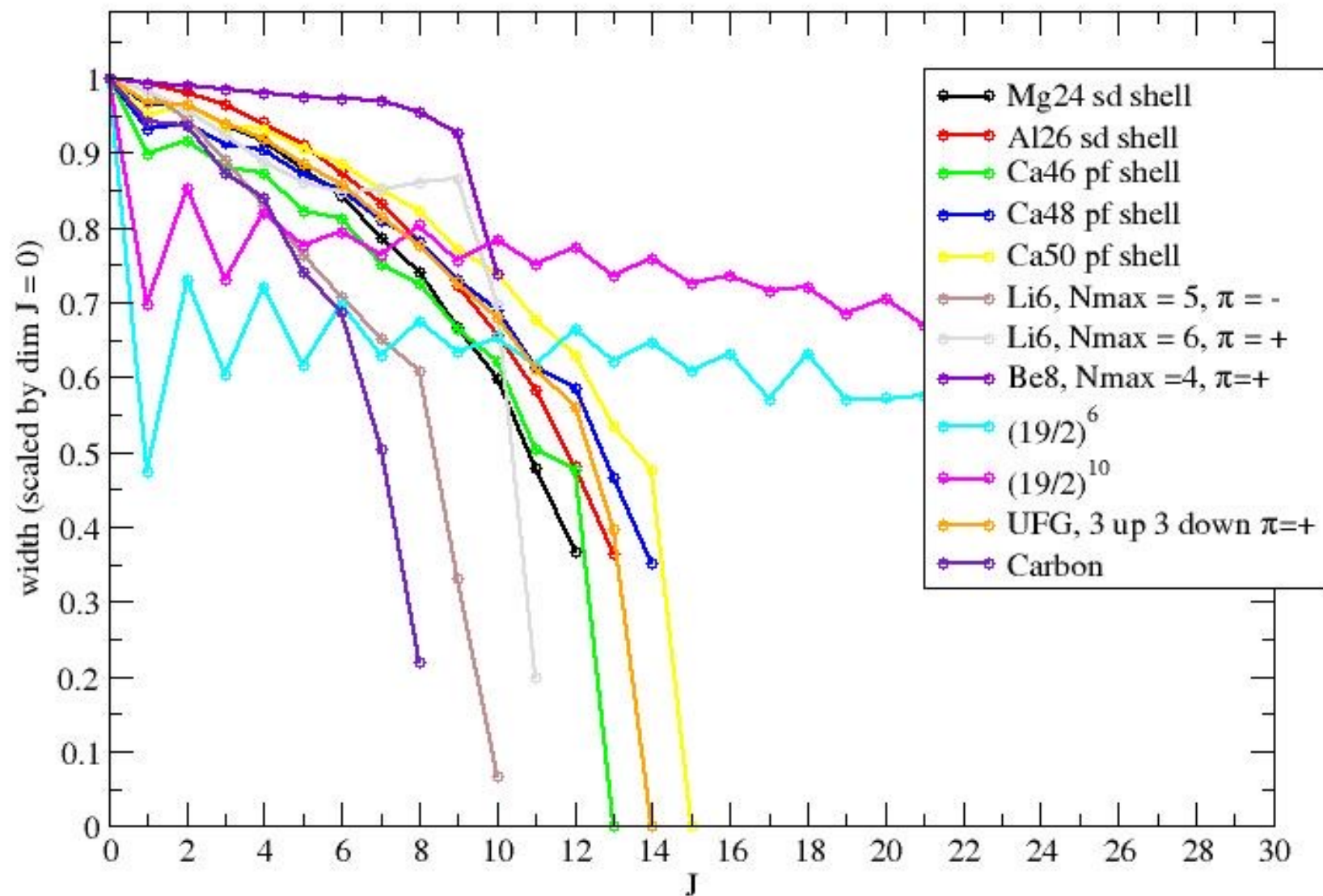
This partially explains  
J=0 g.s. dominance:  
J=0 space, despite  
smaller dimensions,  
have a greater width

...but why?



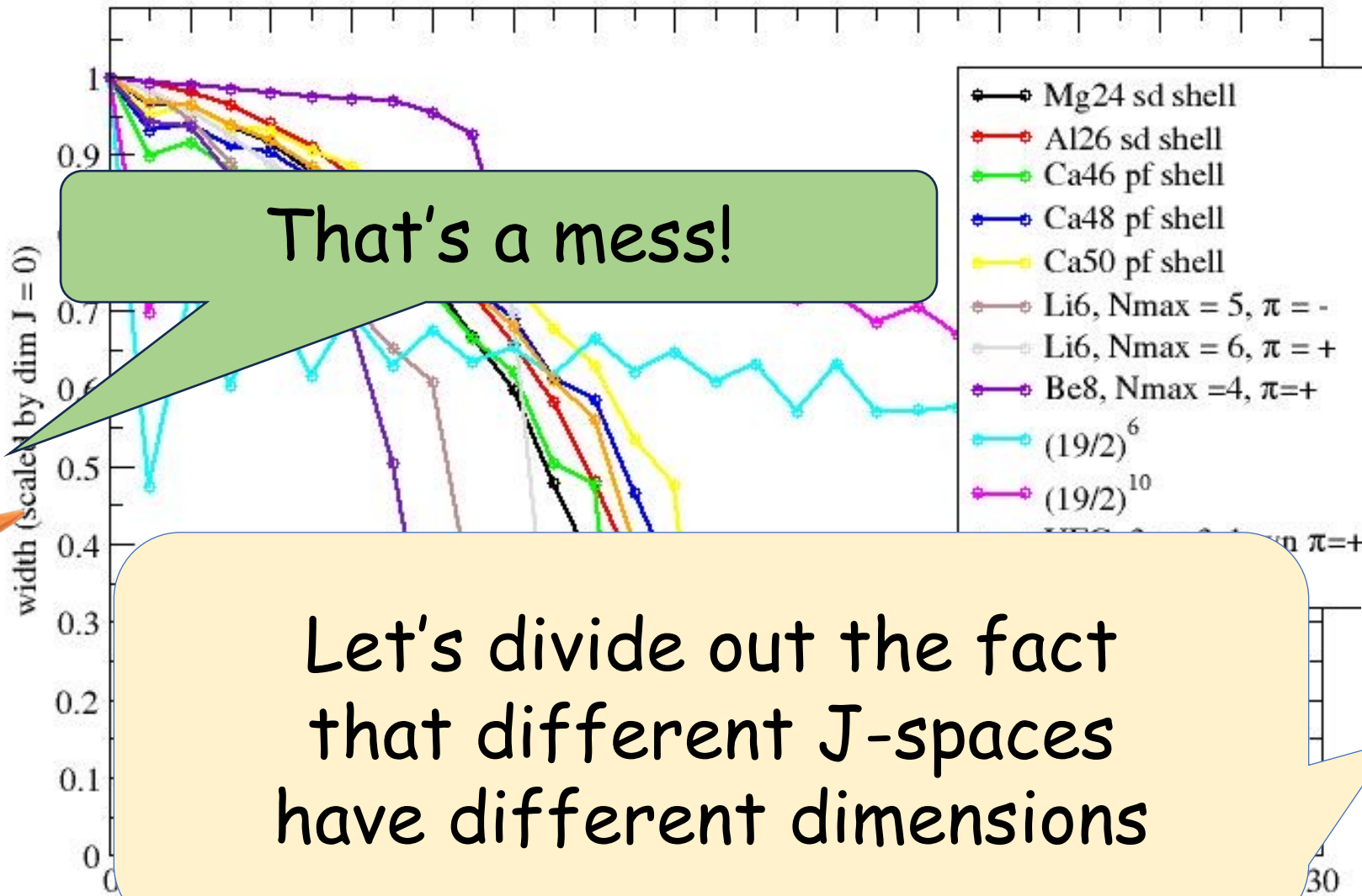


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rms matrix elements (of Hamiltonian)

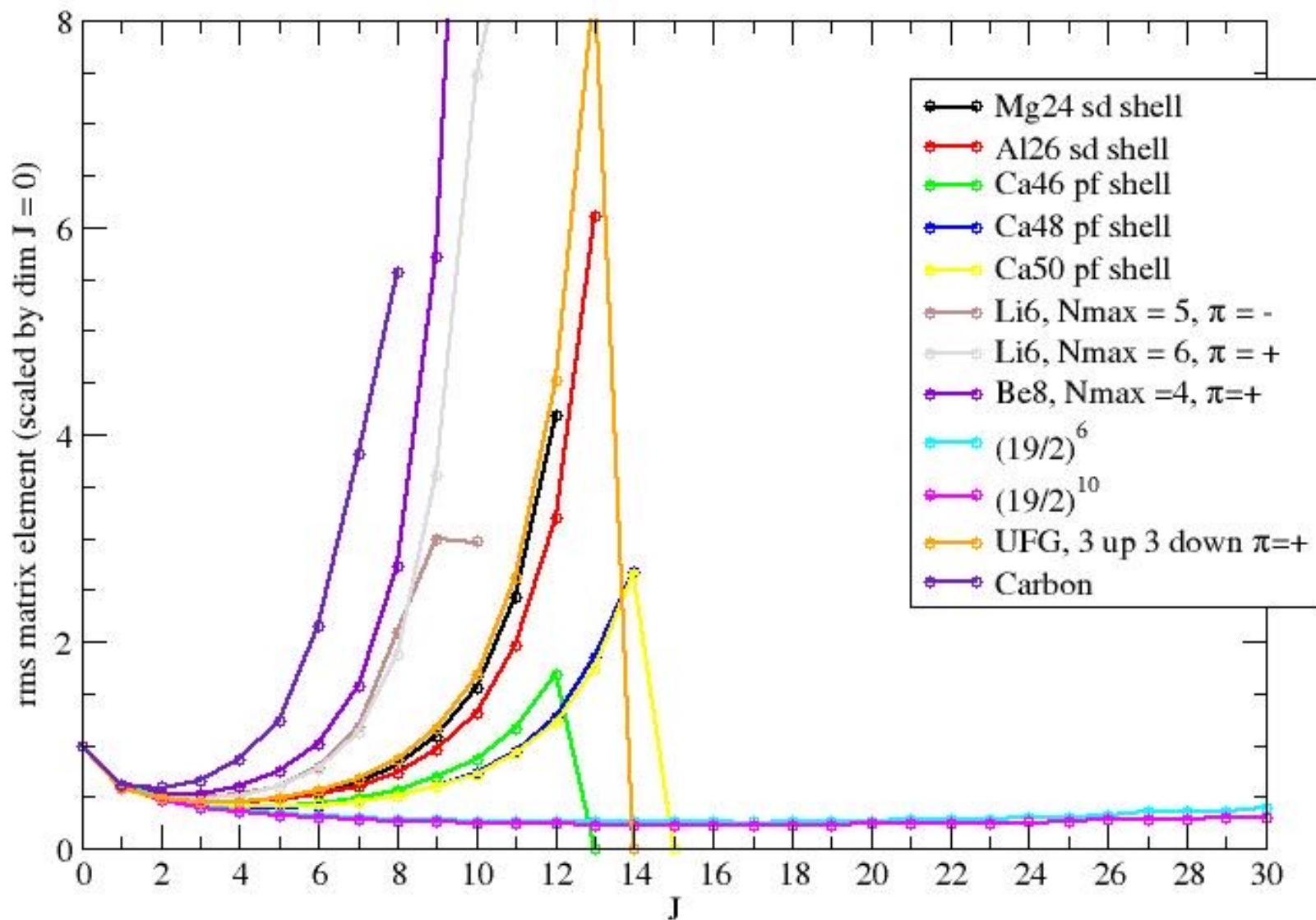
$$\gamma_J^2 = \frac{\sigma_j^2}{N_J} \approx \frac{1}{N_J^2} \text{tr}(P_J H^2)$$

Scaled rms matrix element:

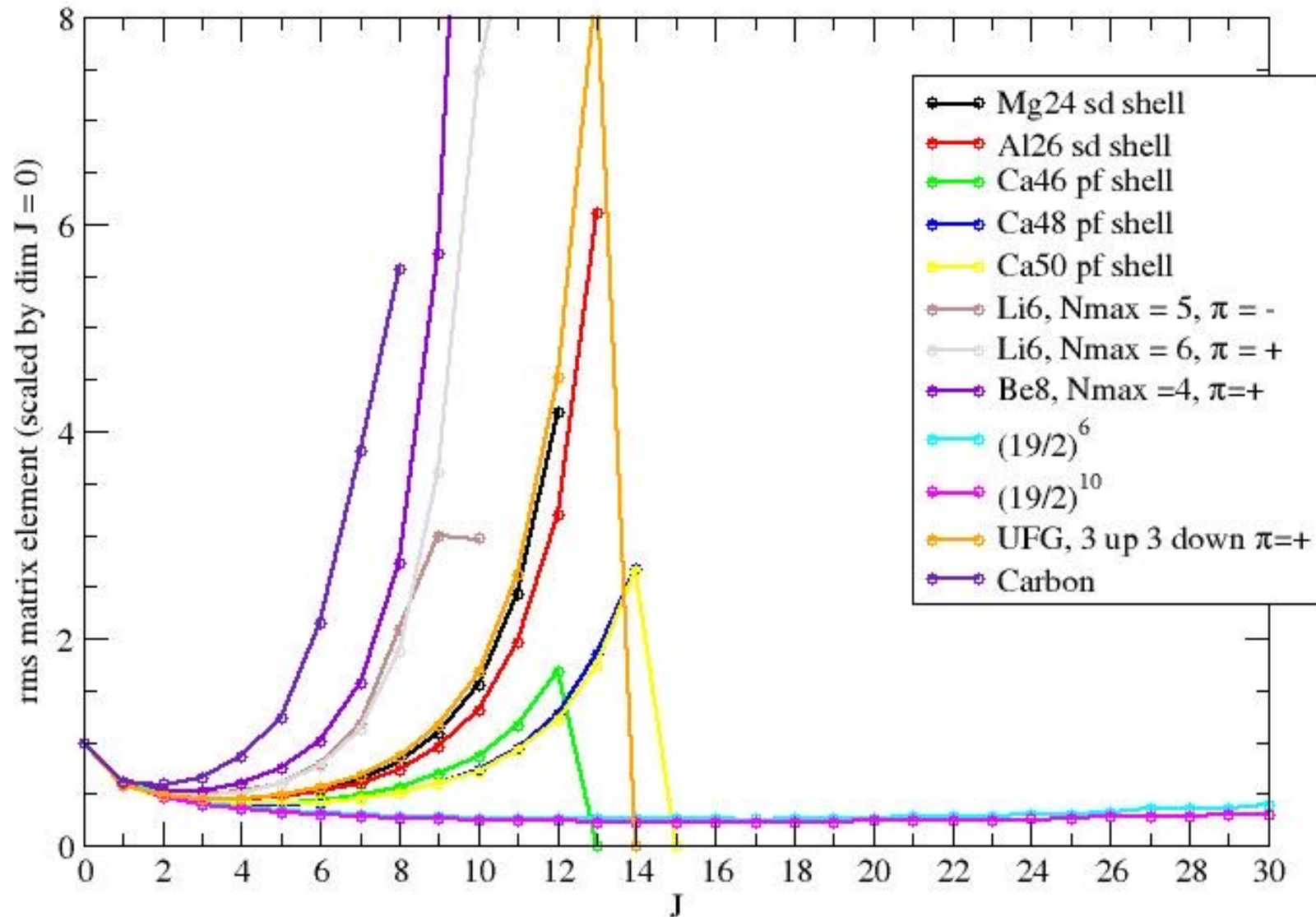
$$\gamma_J / \gamma_0$$



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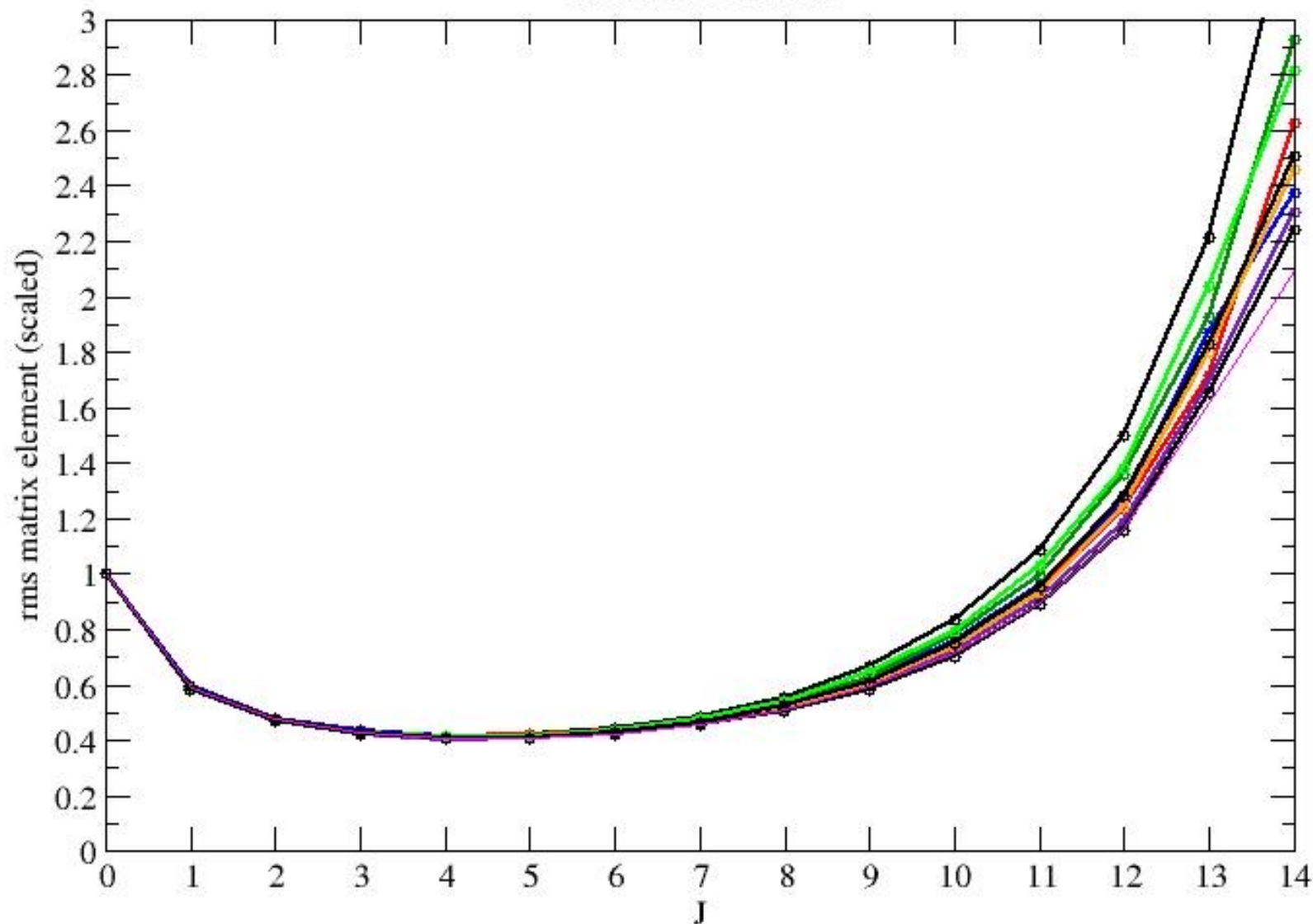
These  
systems  
have  
different  
max J



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# Ca 50, pf shell

random interactions

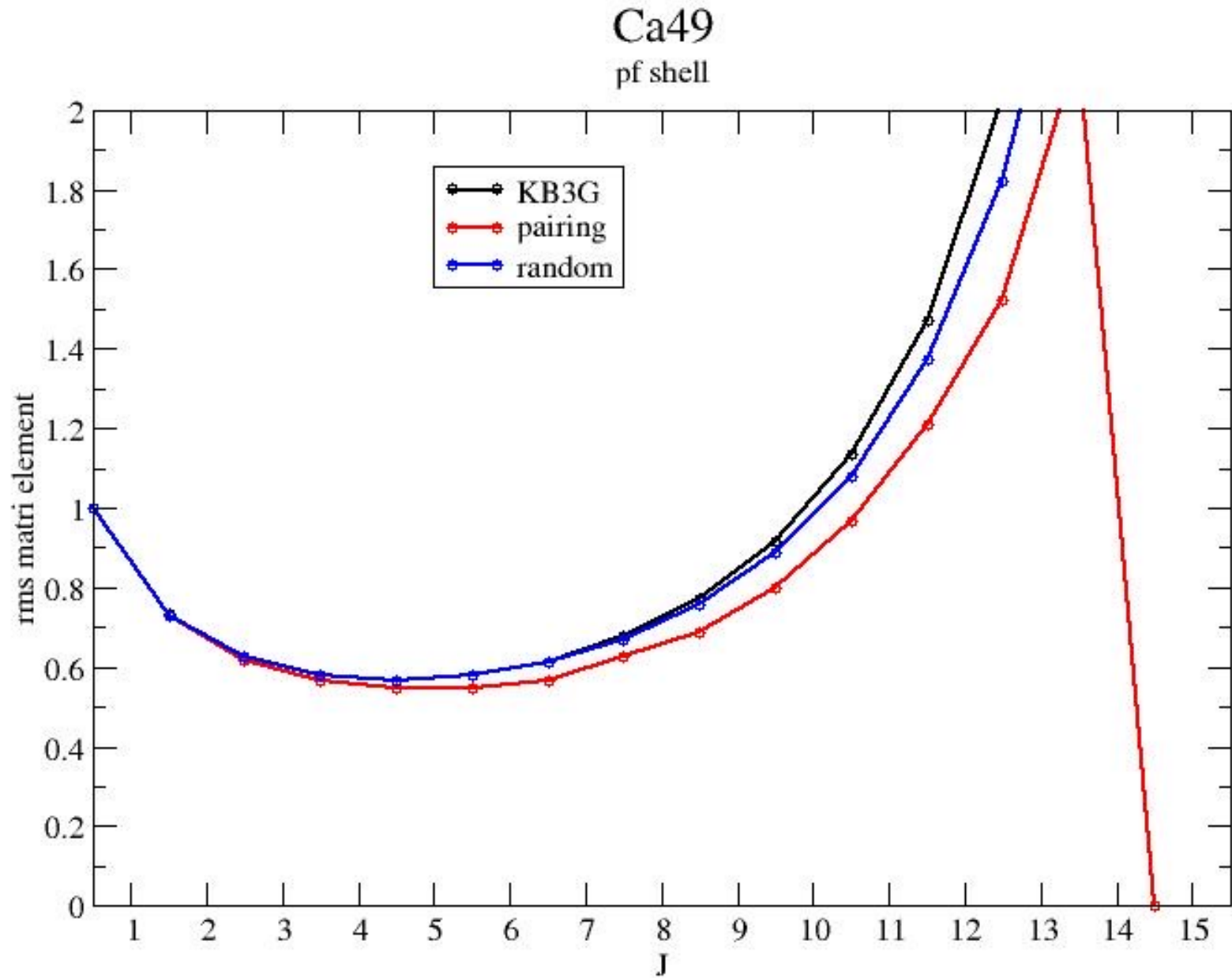


Fixed  
system,  
fixed  
max J



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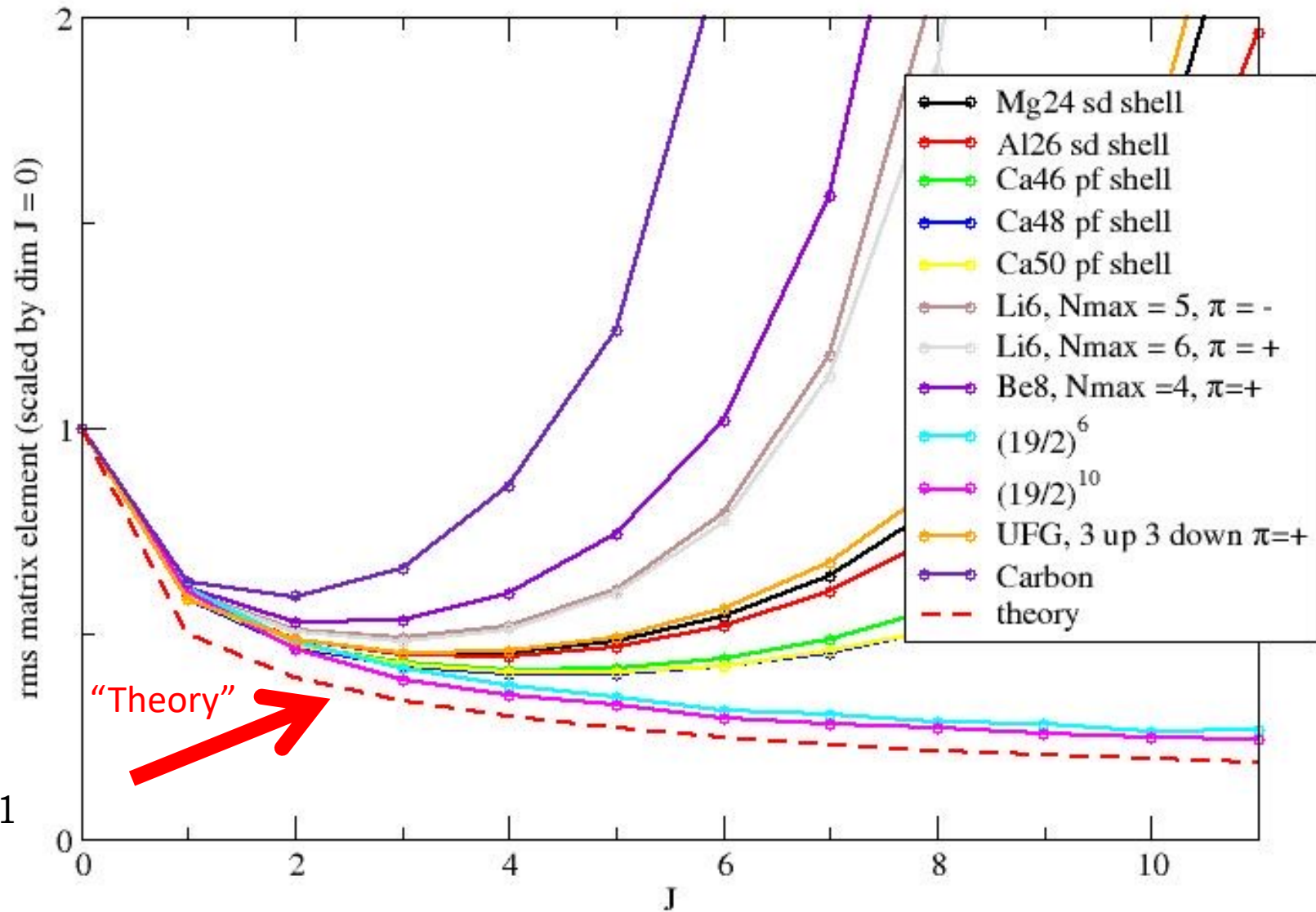
Fixed  
system,  
fixed  
max J



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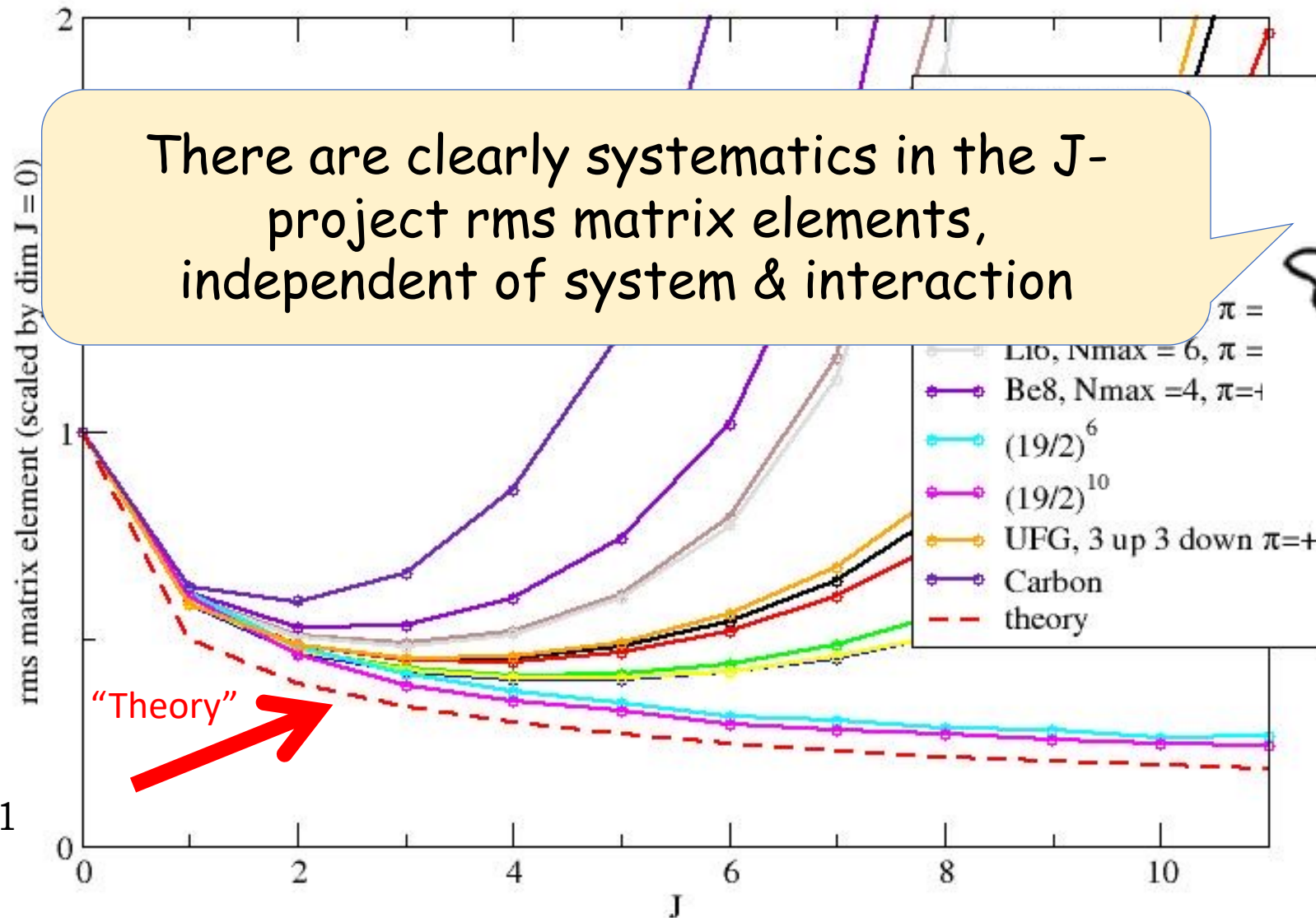


cf. CWJ,  
arxiv:1103.4161





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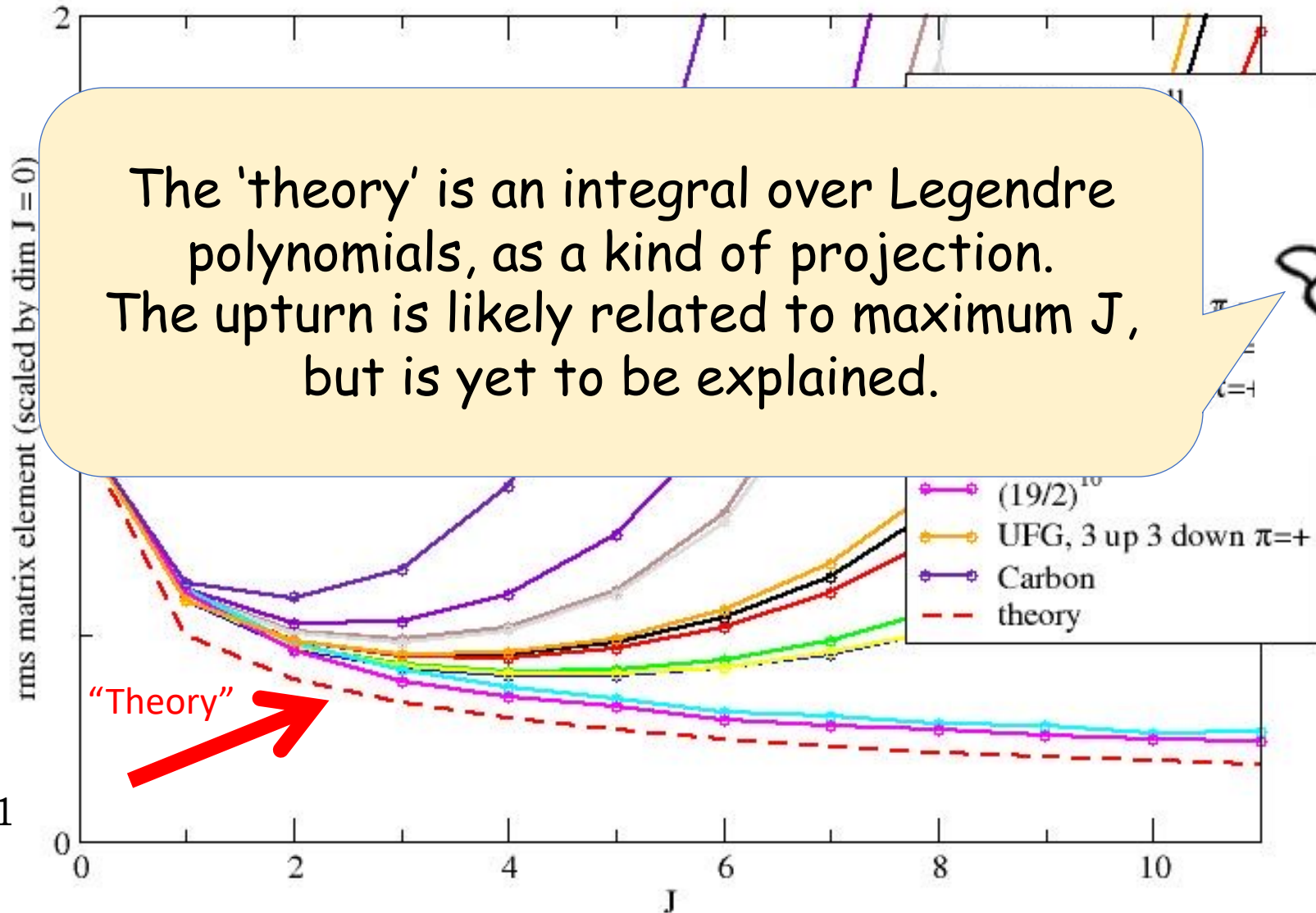
cf. CWJ,  
arxiv:1103.4161



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The 'theory' is an integral over Legendre polynomials, as a kind of projection.  
The upturn is likely related to maximum  $J$ , but is yet to be explained.



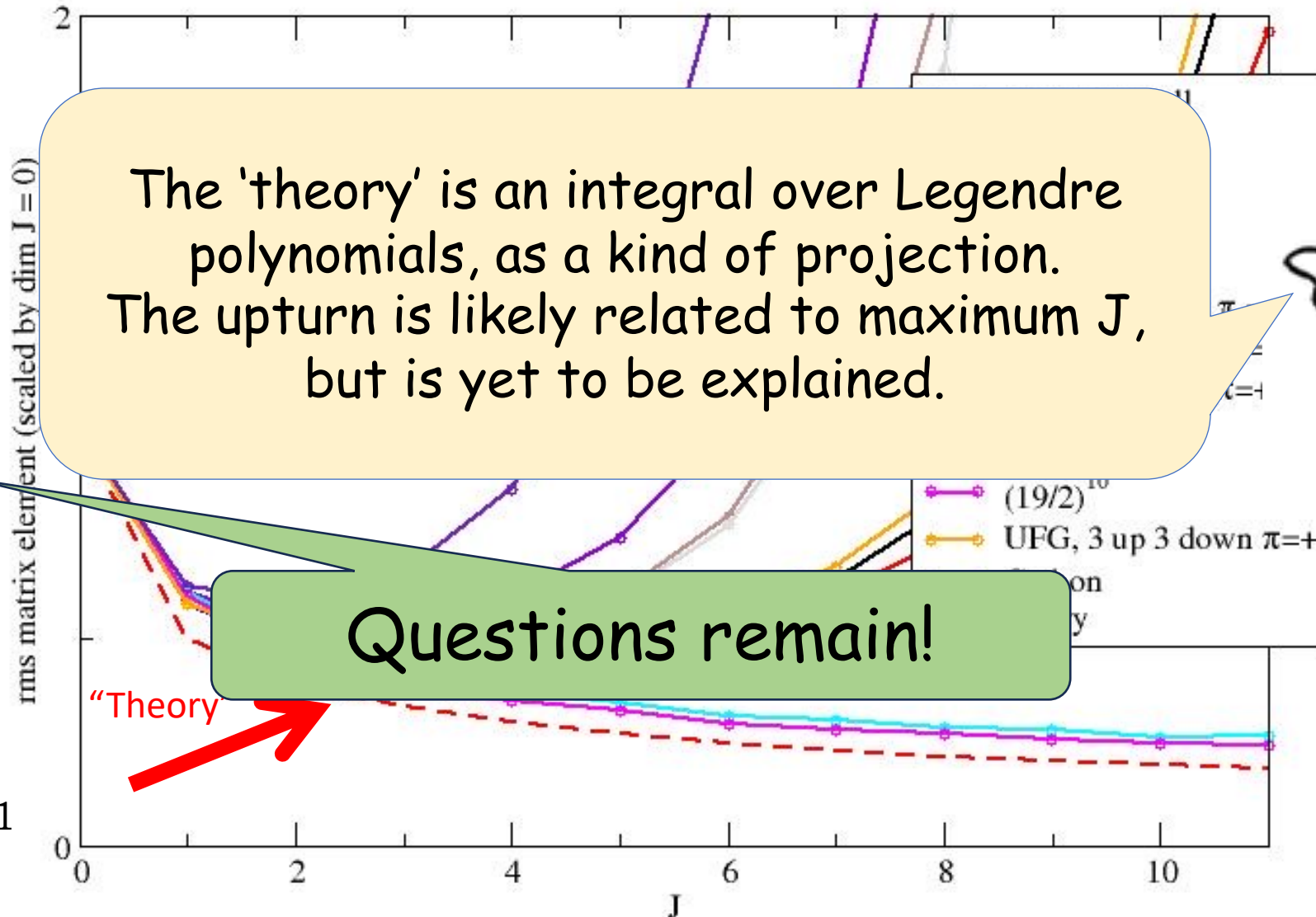
"Theory"

cf. CWJ,  
arxiv:1103.4161





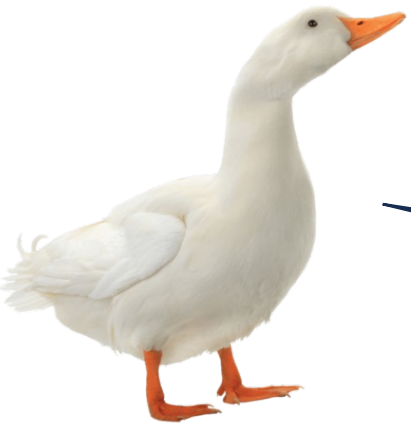
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The 'theory' is an integral over Legendre polynomials, as a kind of projection.  
The upturn is likely related to maximum  $J$ , but is yet to be explained.

Questions remain!

cf. CWJ,  
arxiv:1103.4161





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What questions  
do *you* have?

