Institut de Ciències del Cosmos
UNIVERSITAT DE BARCELONA

EXCELENCIA MARÍA DE MAEZTU

Impact of correlations on the phase diagram of nuclear matter

M Drissi

TECHNISCHE UNIVERSITÄT DARMSTADT

C Barbieri

UNIVERSITÀ DEGLI STUDI DI MILANO

Drissi, Rios & Barbieri, Ann. Phys. **469** 169729 (2024)
Drissi, Rios & Barbieri, Ann. Phys. **469** 169730 (2024)
Drissi & Rios, EPJA **58** 90 (2022)

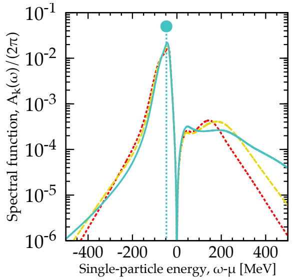
Dr Arnau Rios Huguet
Institute of Cosmos Sciences
Universitat de Barcelona

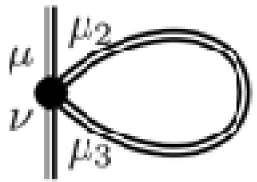
ESNT Pairing Workshop
21 May 2025

UNIVERSITY OF SURREY
UNIVERSITAT DE BARCELONA
Departament de Física Quàntica i Astrofísica
@HadNucAtUB

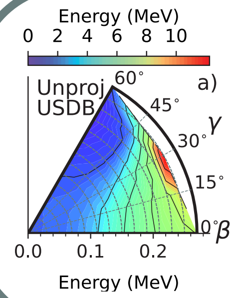
Nuclear Physics in Barcelona

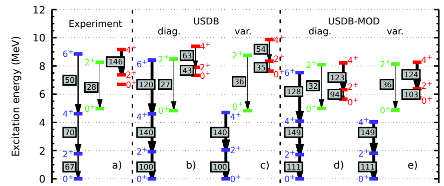
Dense matter





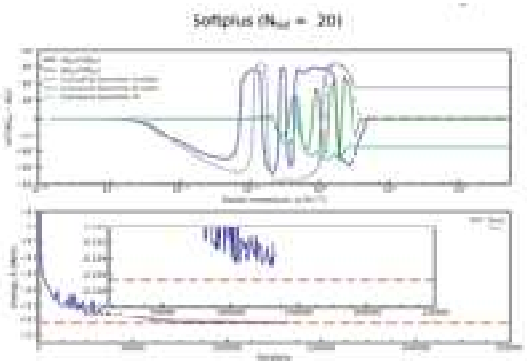
Nuclear structure





Frycz et al PRC 110, 054326 (2024)

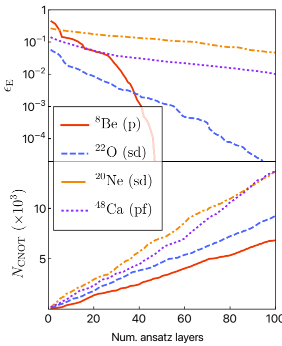
Machine learning

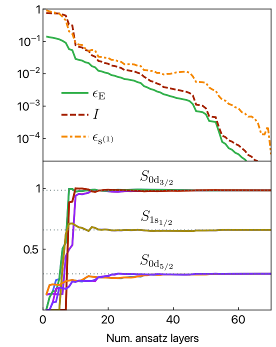


JWT Keeble

J Rozalén

Quantum computing





AM Romero

J Menéndez

Barcelona Supercomputing Center

Fellowships

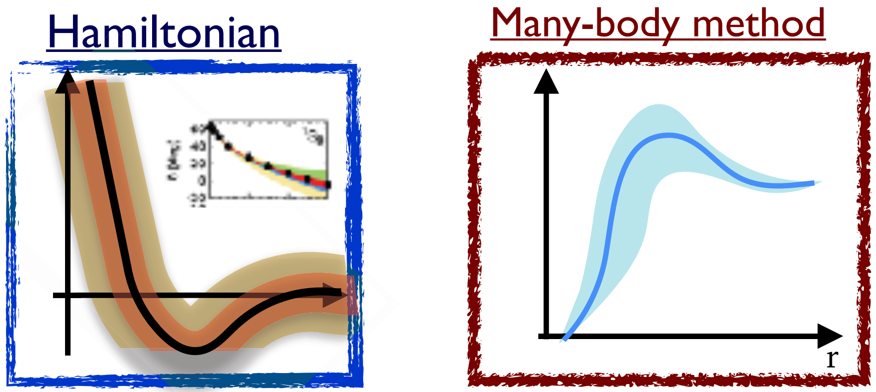
Fundación "la Caixa"

Investigación Programa Ramón y Cajal

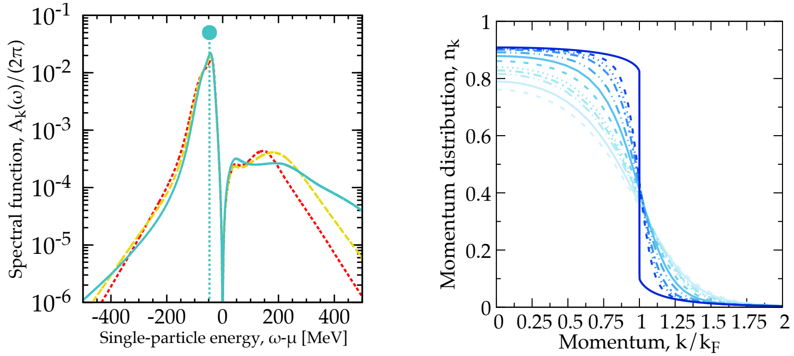
bp

Outline

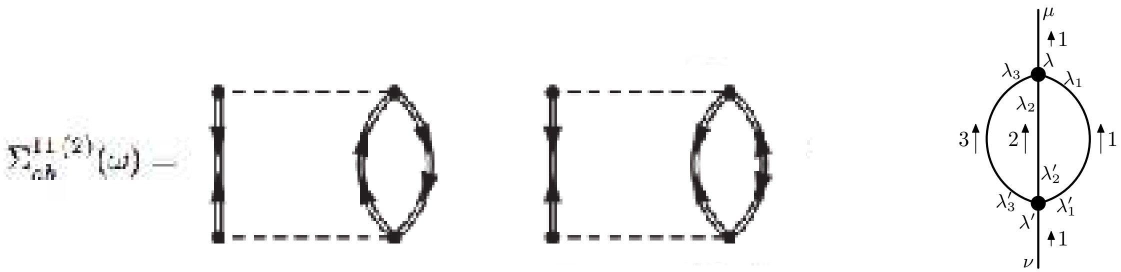
- **Motivation**



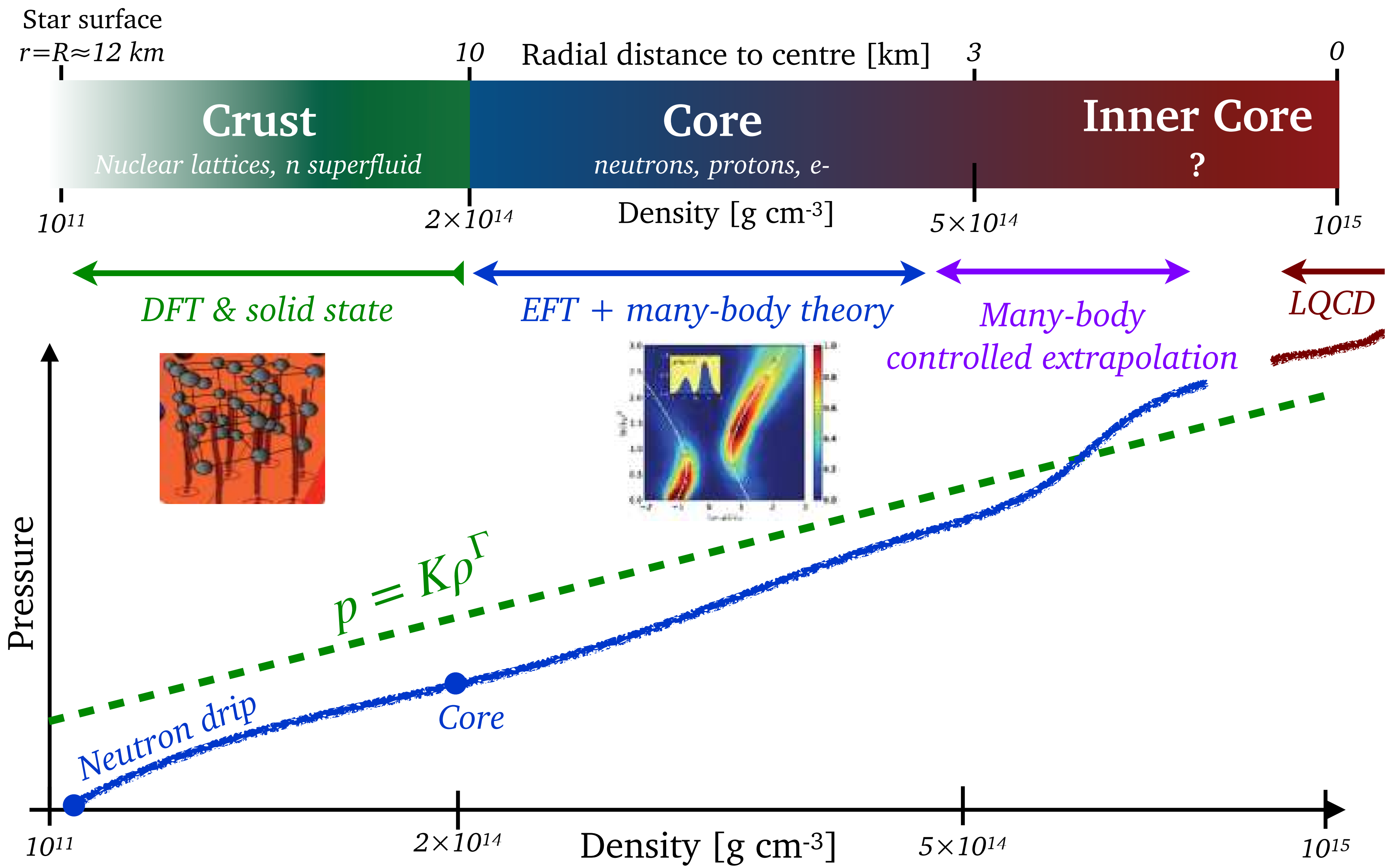
- "Normal" self-consistent Green's functions



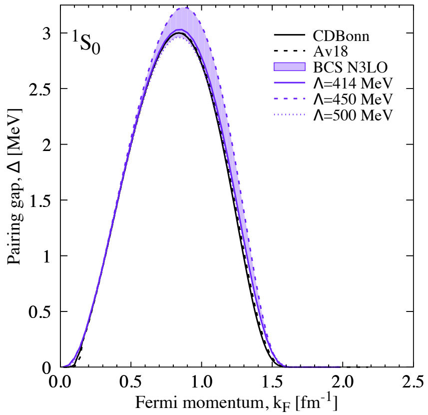
- Nambu-Covariant Green's functions

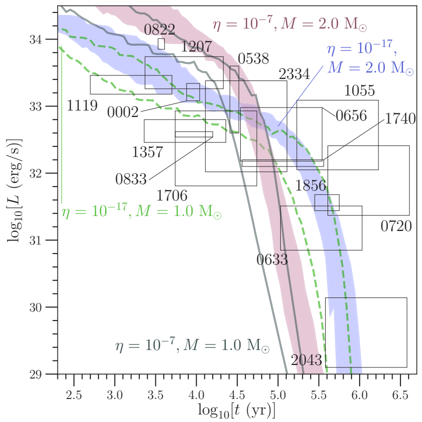


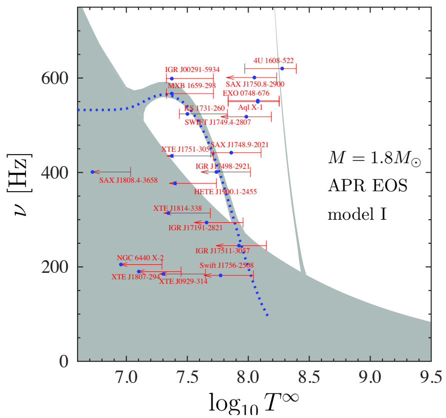
Neutron-star modelling



Why are pairing gaps necessary?







- **Glitches** in period
  - Pinning of neutron superfluid, crustal physics
  - Proton superconductor?

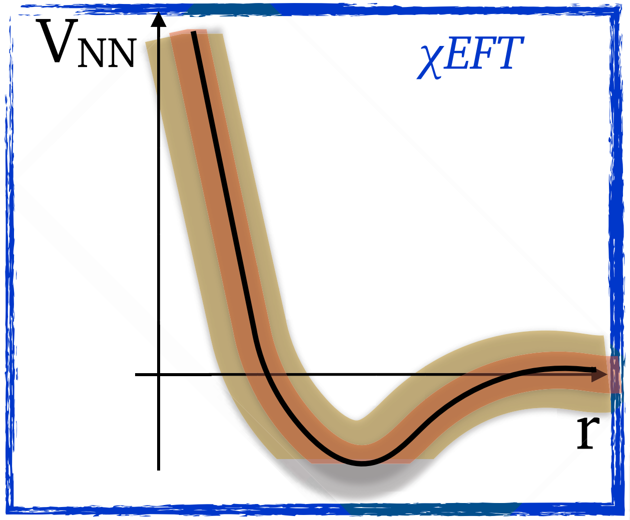
- Neutron star **cooling**
  - Neutrino rates through pair-breaking

- **Astereoseismology**
  - Superfluid modes affected
  - Potential observable signal in GW in BNS

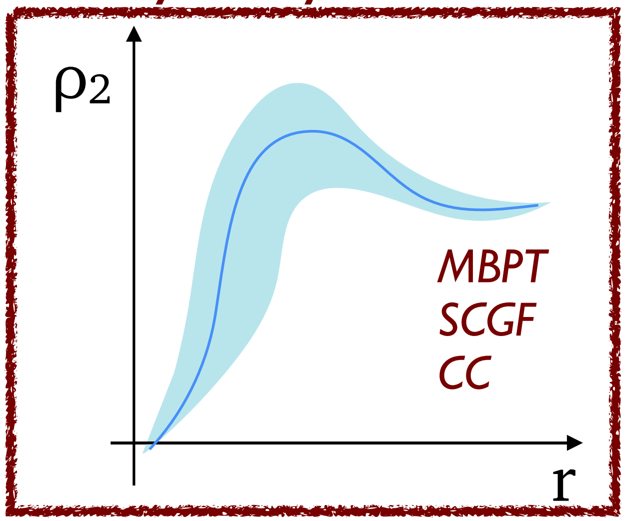
Kantor et al. Phys. Rev. Lett. **125** 151101 (2020),
Rau & Wasserman, MNRAS **481**, 4427-4444 (2018)

Nuclear uncertainty quantification

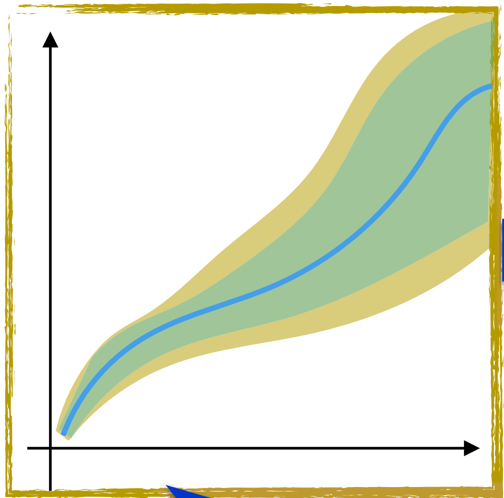
Hamiltonian



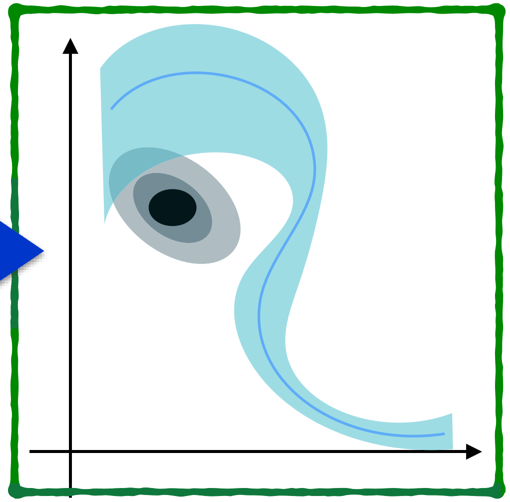
Many-body method



Astronuclear property



Neutron star observations

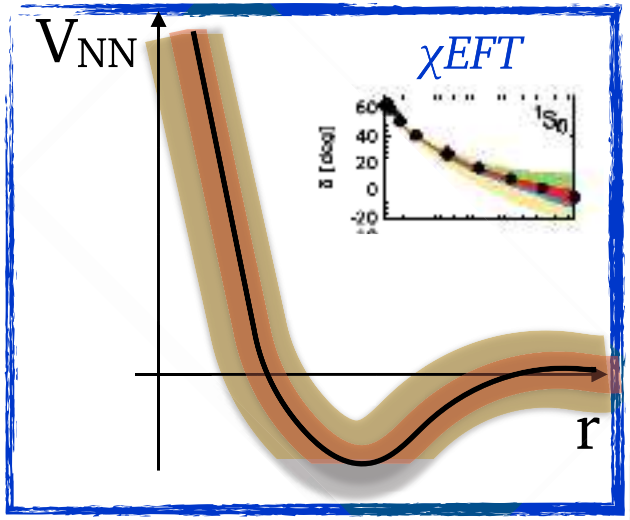


Forwards modelling

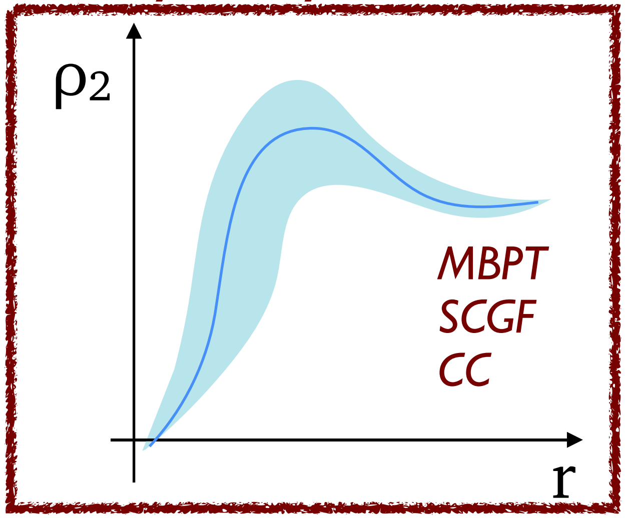
- Statistical propagation
- Bayesian analysis
- Emulators

Nuclear uncertainty quantification

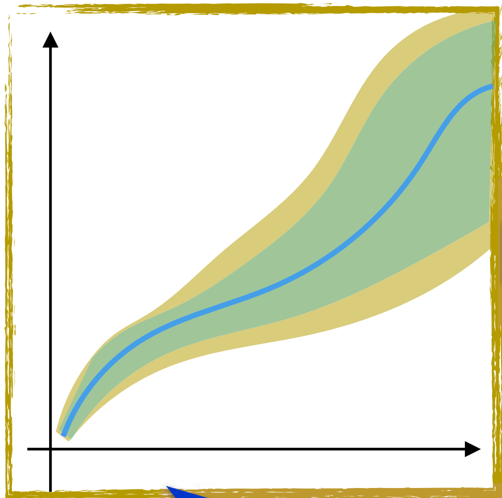
Hamiltonian



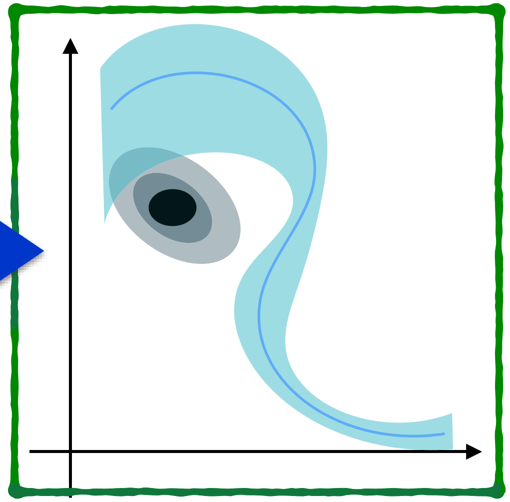
Many-body method



Astronuclear property



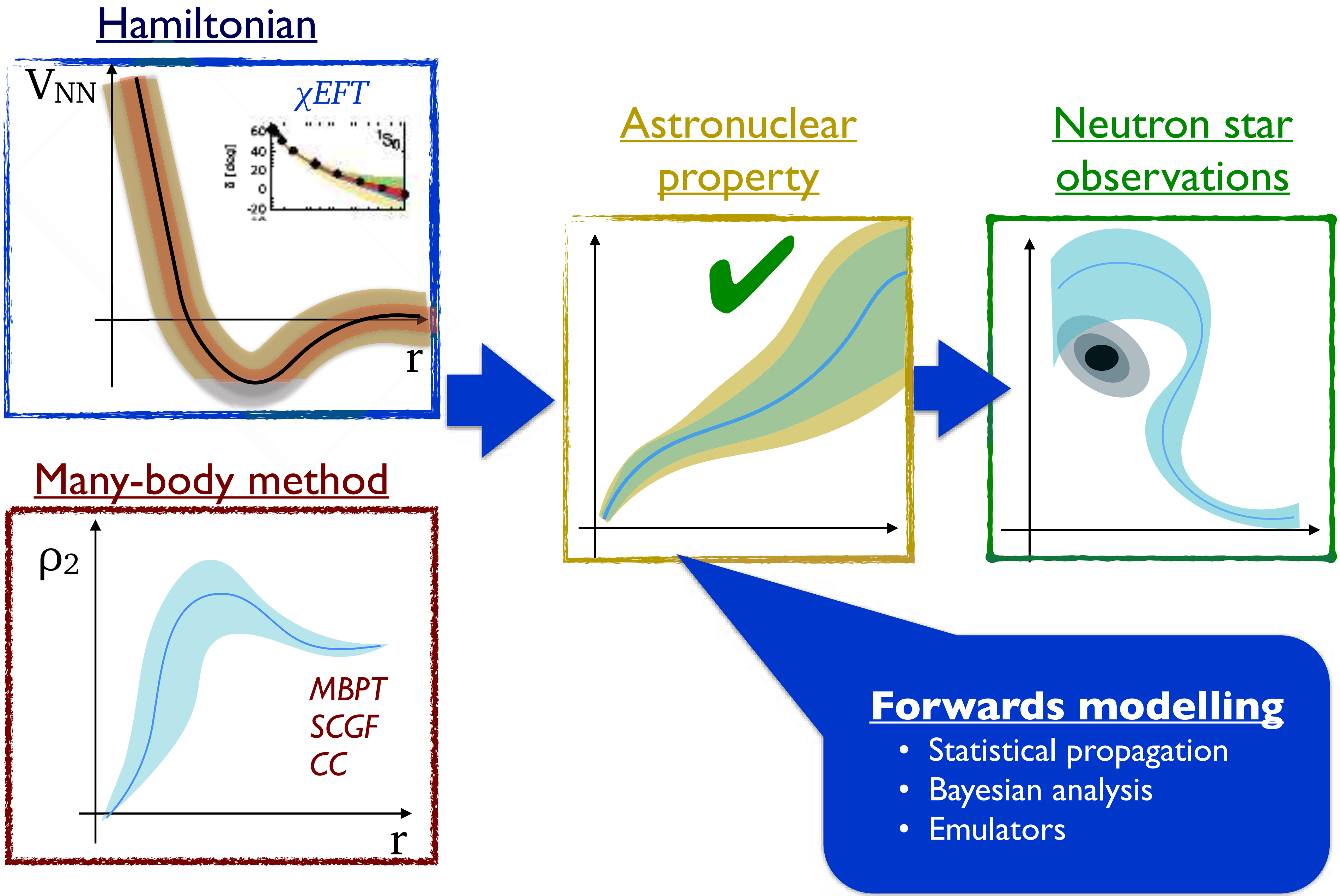
Neutron star observations



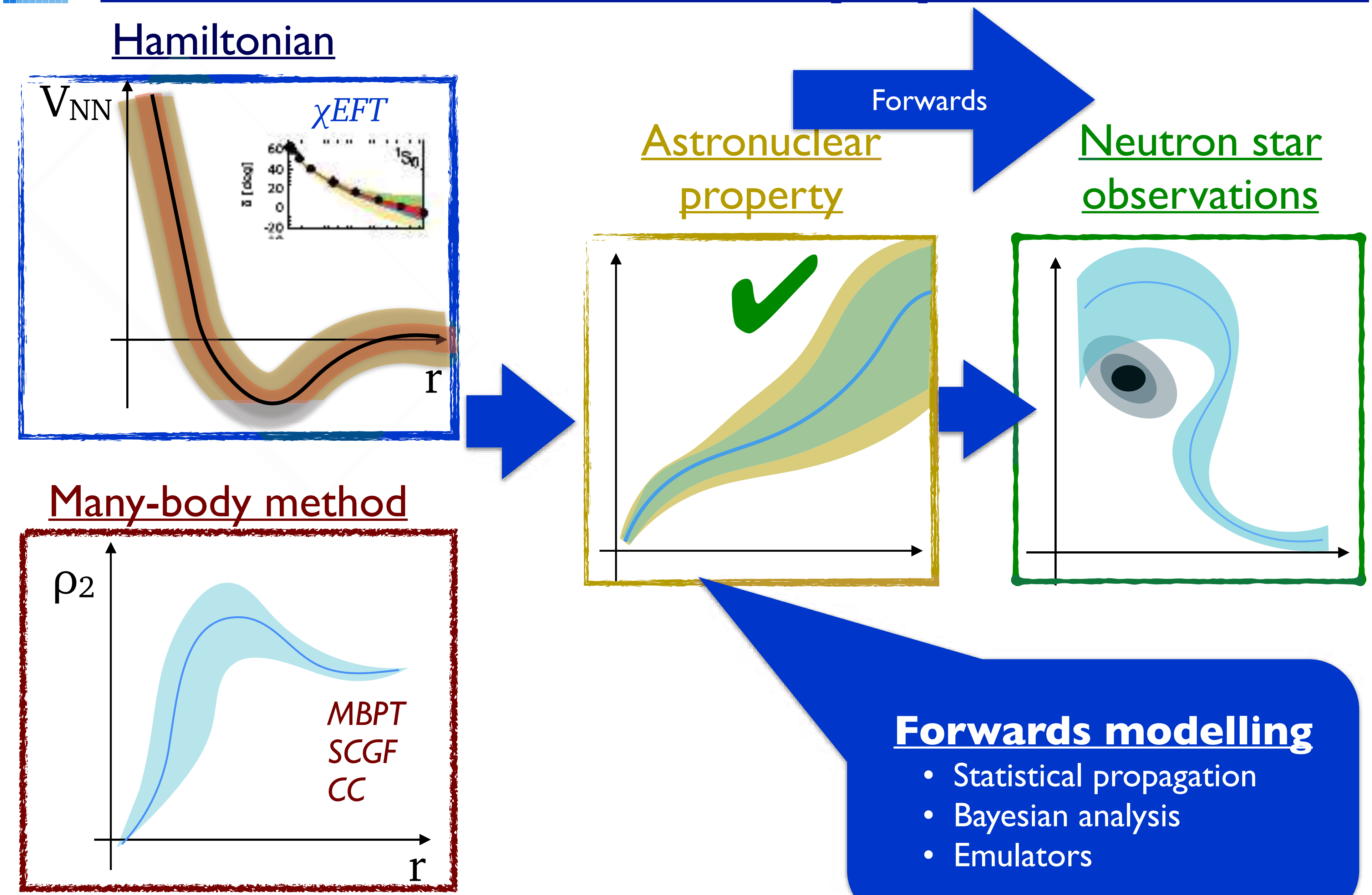
Forwards modelling

- Statistical propagation
- Bayesian analysis
- Emulators

Nuclear uncertainty quantification



Nuclear uncertainty quantification



Forwards modelling

- Statistical propagation
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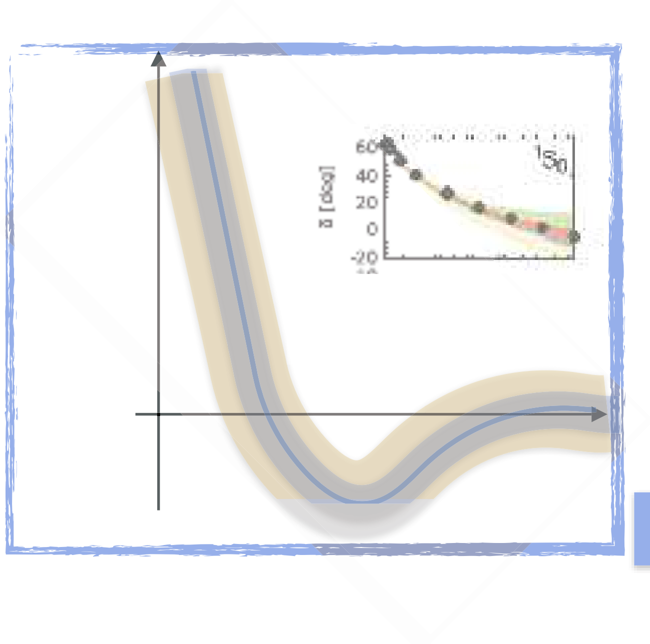
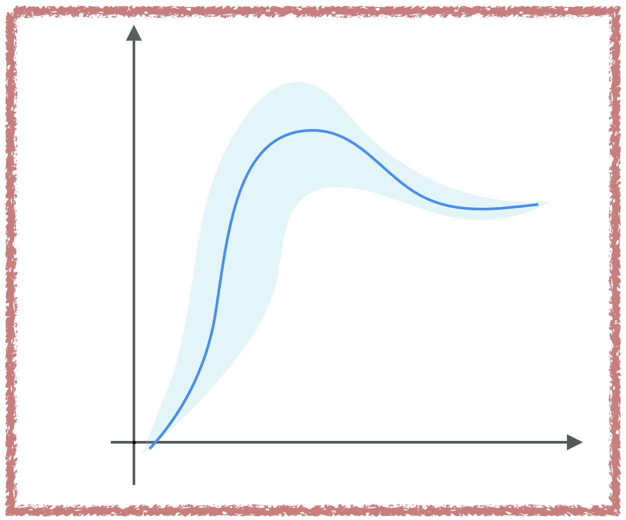
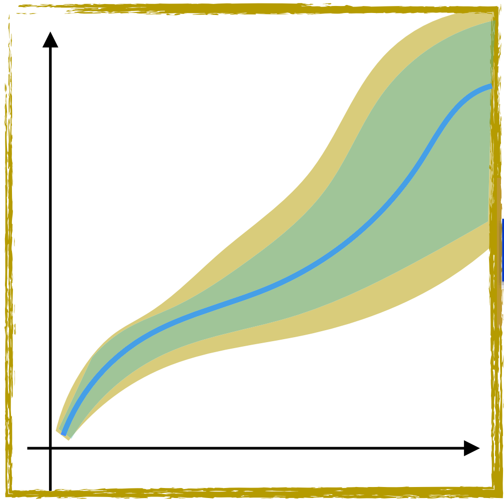
Nuclear uncertainty quantification

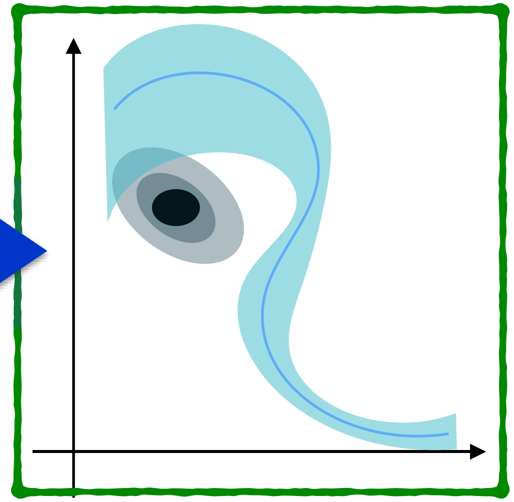




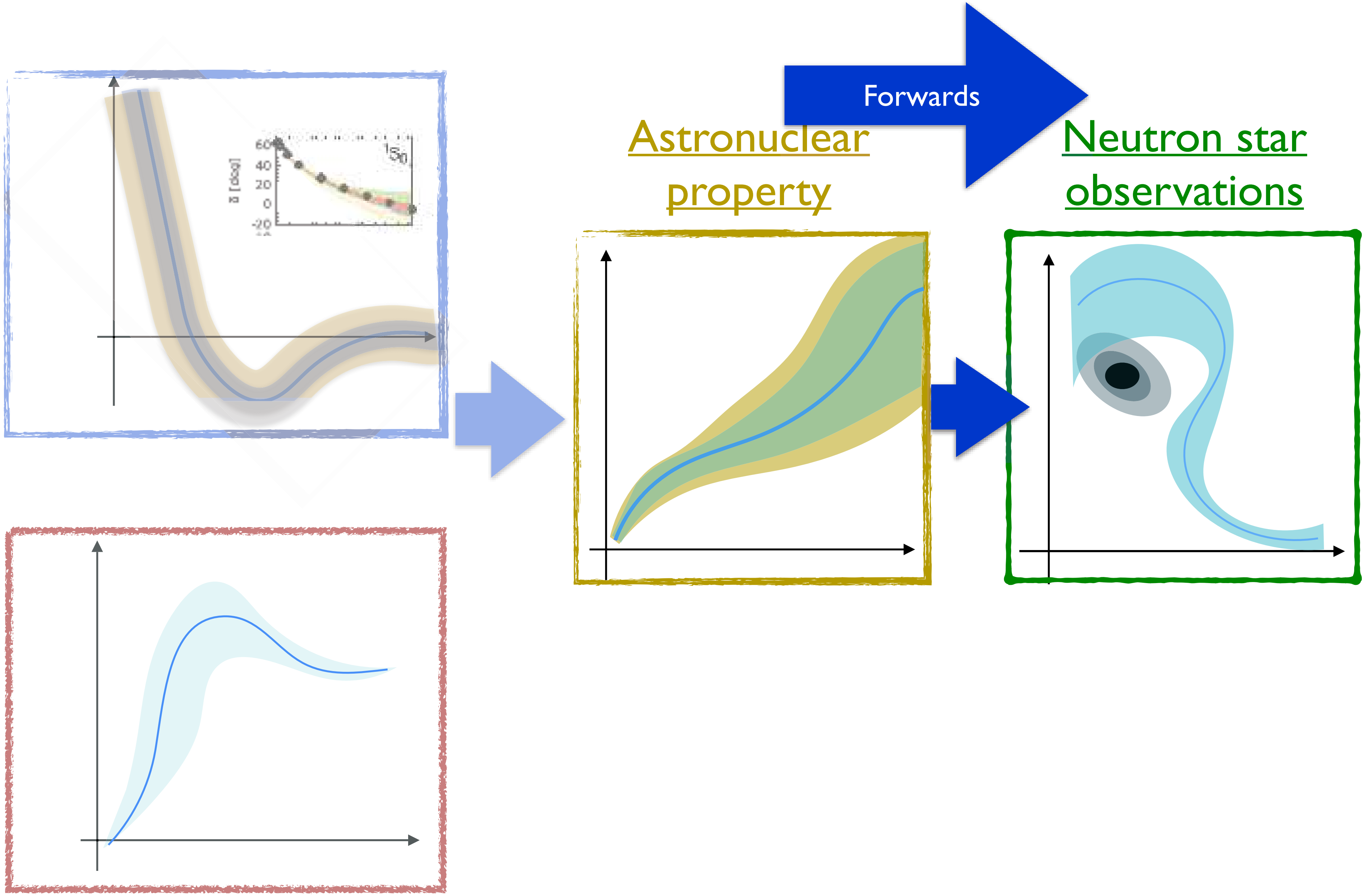
Astronuclear property



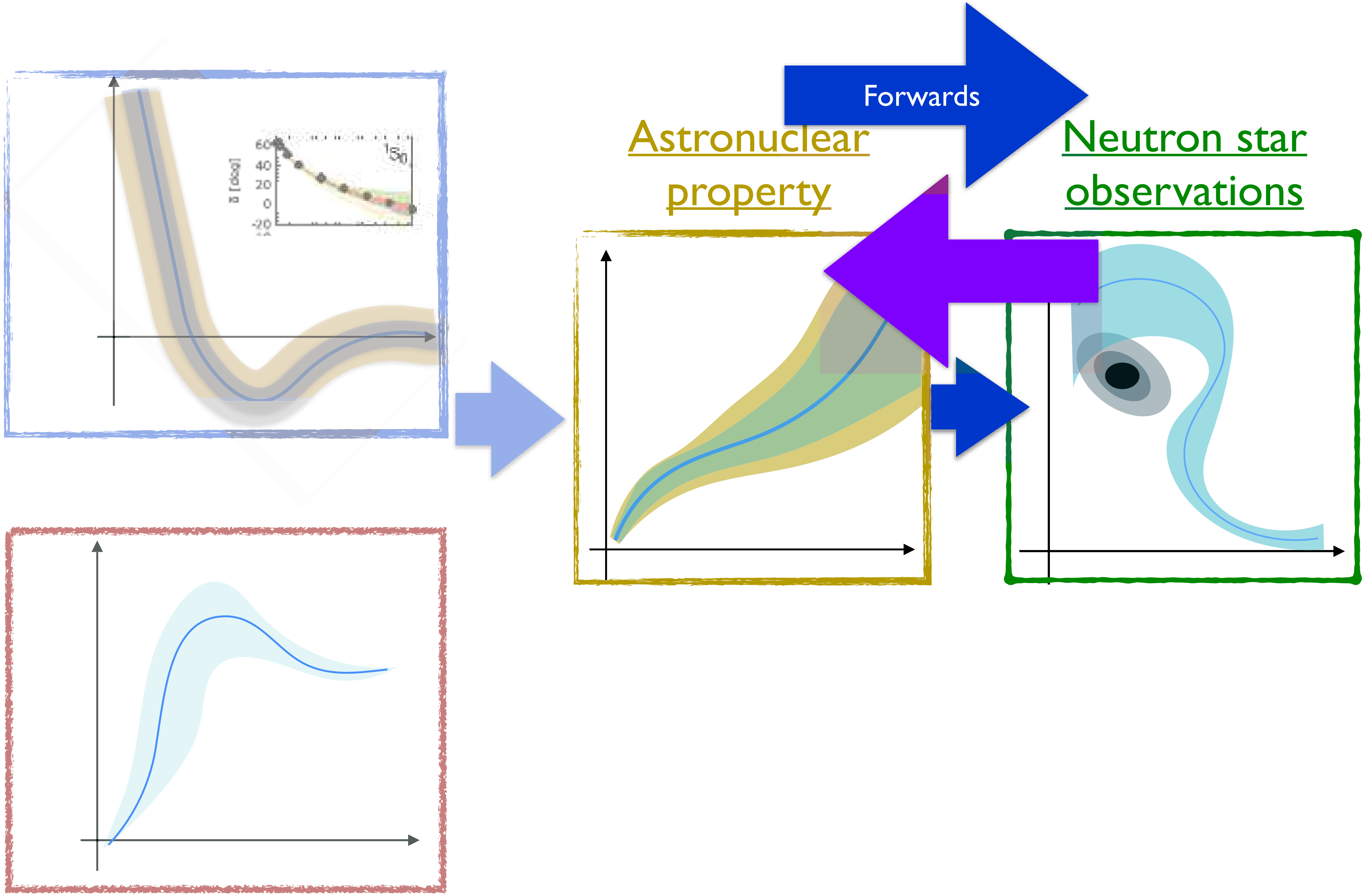
Neutron star observations



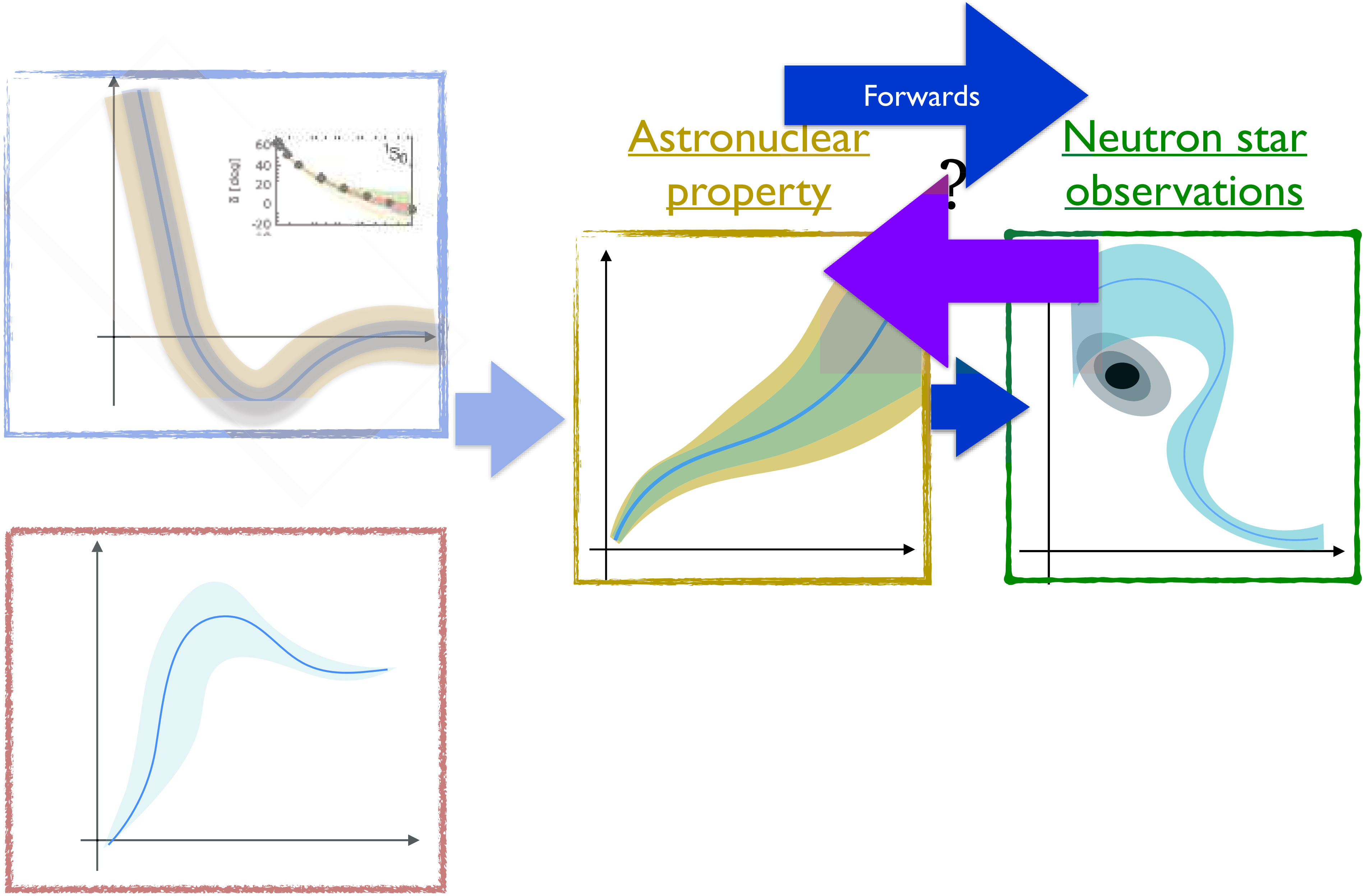
Nuclear uncertainty quantification



Nuclear uncertainty quantification



Nuclear uncertainty quantification



From M-R to EoS

Backwards

Tolman-Oppenheimer-Volkov equations

$$\frac{dP(r)}{dr} = -\frac{G}{c^2}\frac{\left[m_<(r) + 4\pi P(r) r^3\right]\left[\epsilon(r) + P(r)\right]}{1 - \frac{2Gm_<(r)}{rc^2}}$$

$$\frac{dm_<(r)}{dr} = 4\pi r^2 \rho(r)$$

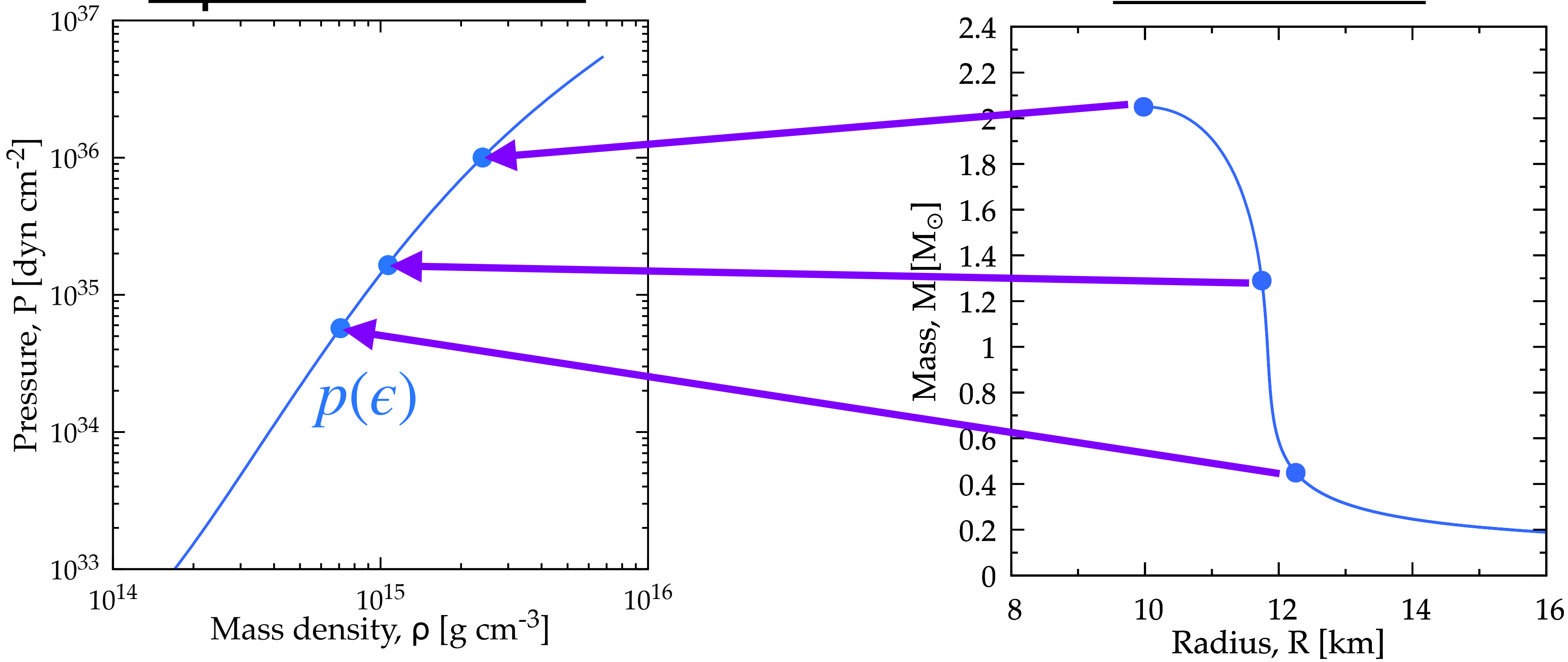
$+$

EoS

$$P \equiv P(\rho)$$

$$\epsilon = \rho c^2$$

Equation of State | **Mass-Radius**



From M-R to EoS

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$$P \equiv P(\rho)$$

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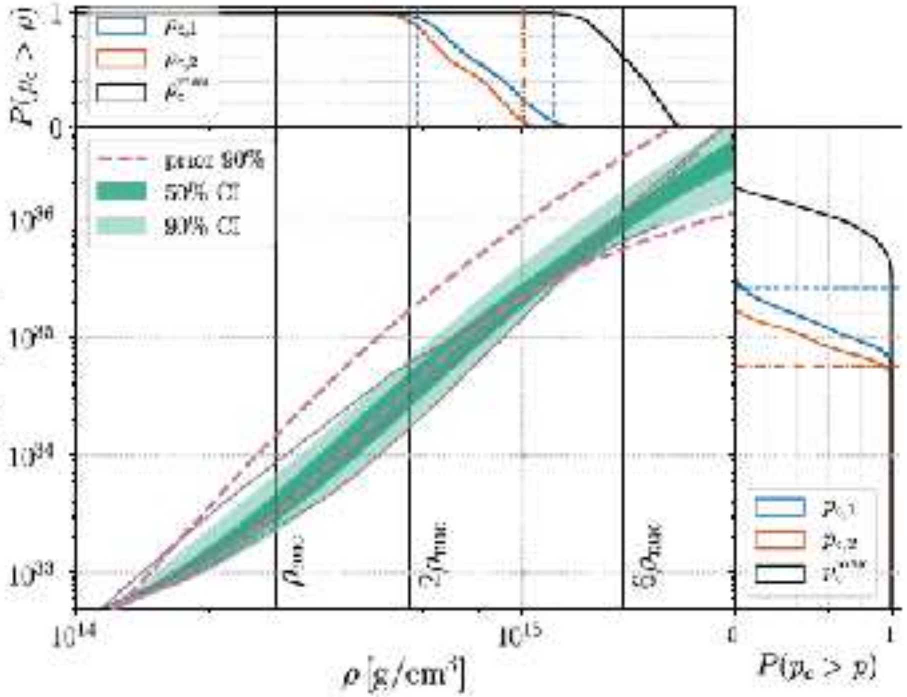
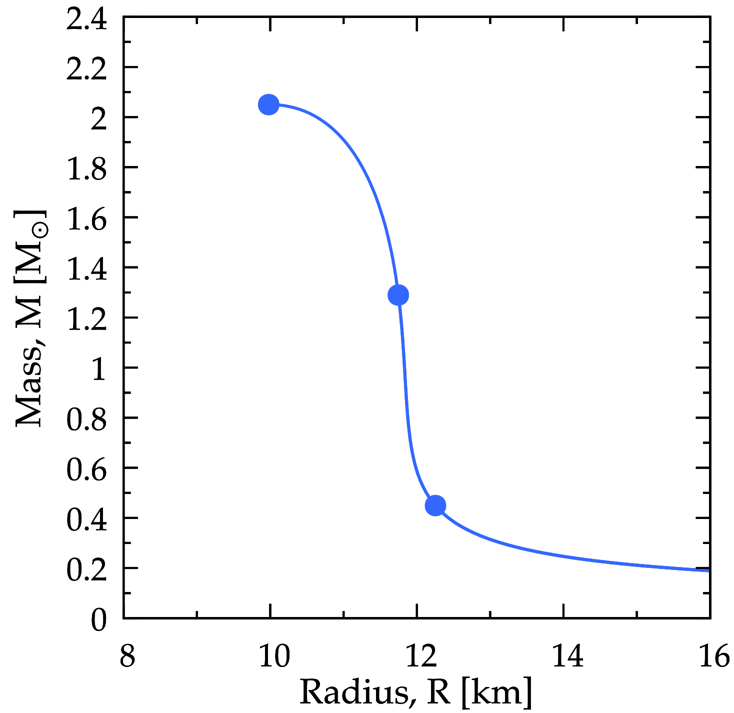
Equation of State

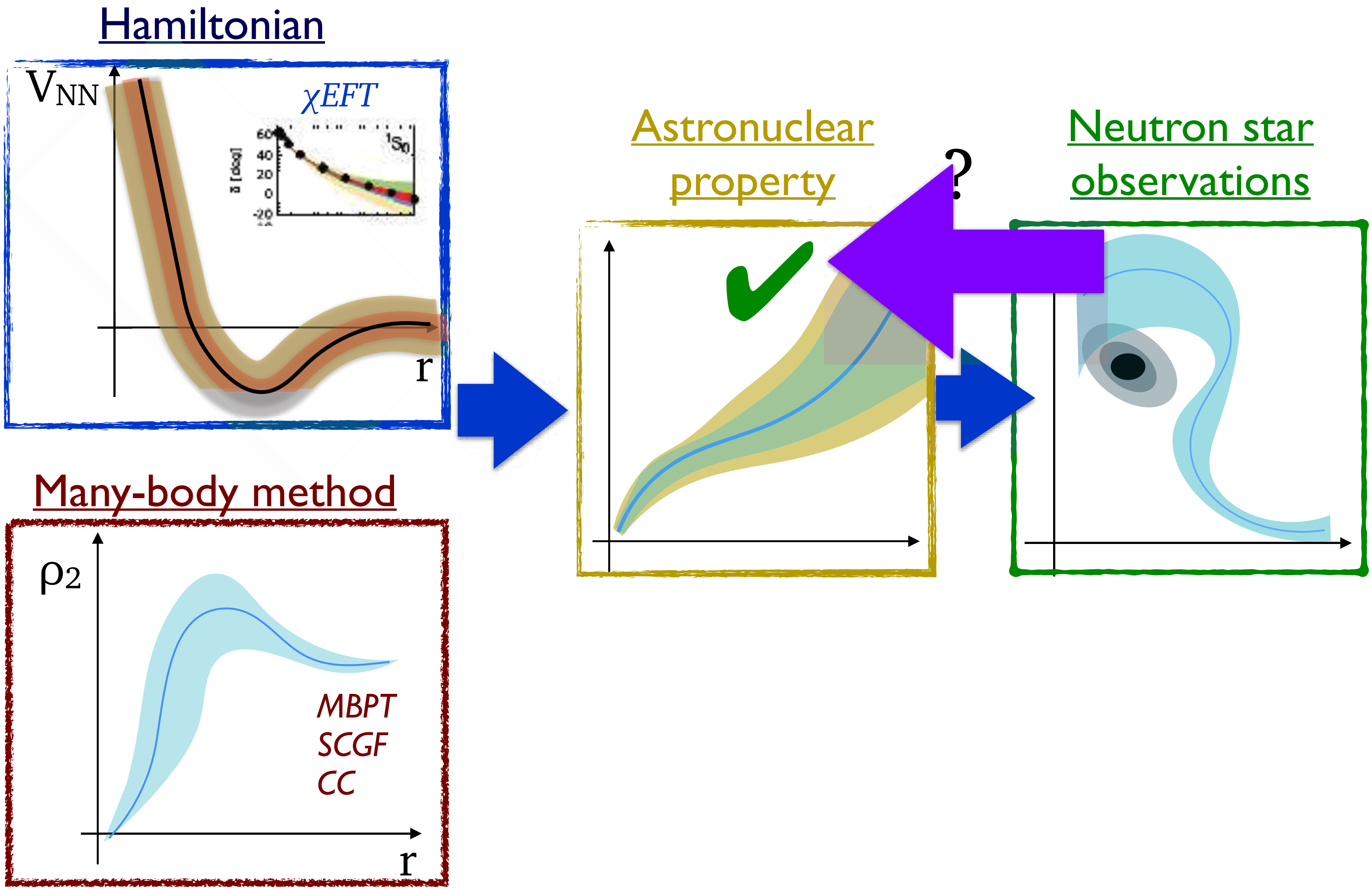
LIGO/VIRGO PRL **121** 161101 (2018)



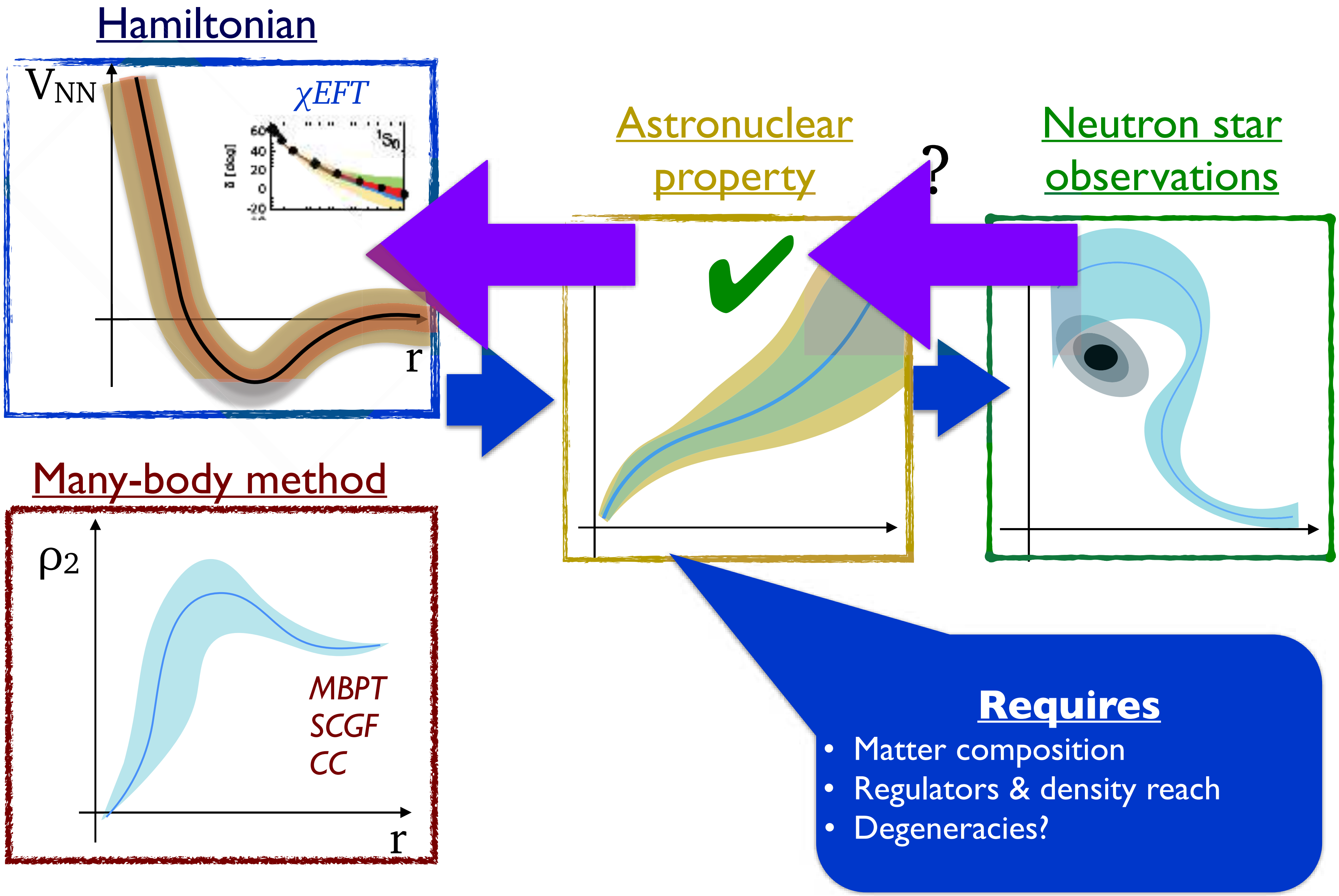
Mass-Radius



Nuclear uncertainty quantification



Nuclear uncertainty quantification

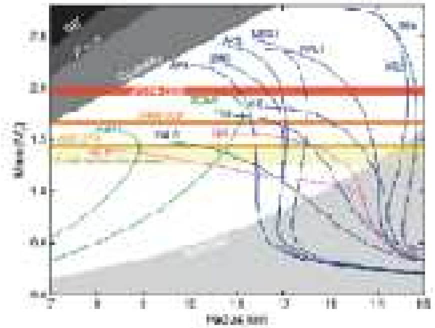


Neutron star modelling

Input #1
EoS

→

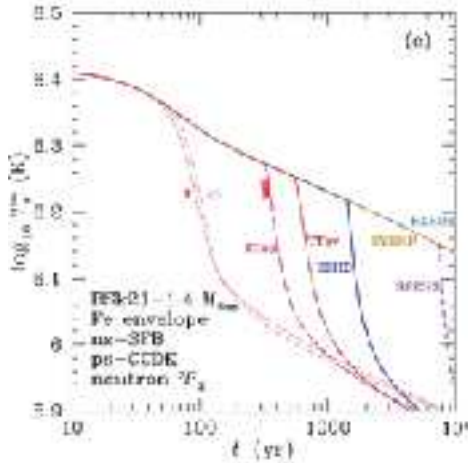
Observable #1
Mass-Radius relation



Input #2
Pairing gap

→

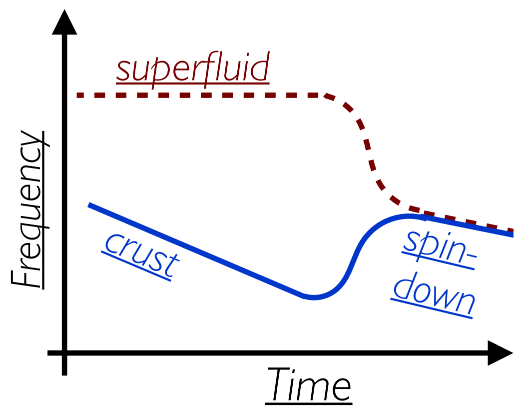
Observable #2
Cooling curve



Input #3
Crust-core

→

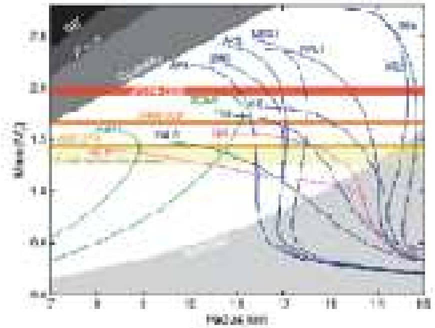
Observable #3
Glitching



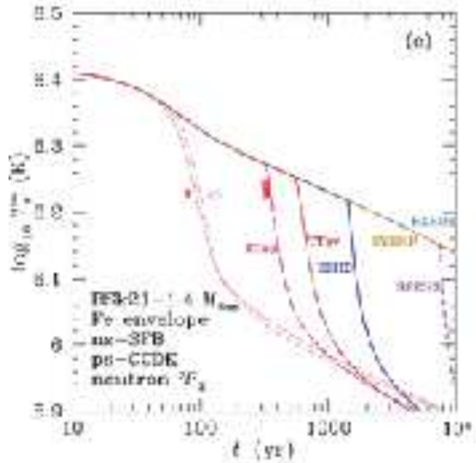
Neutron star modelling

Input
Consistent many-body theory

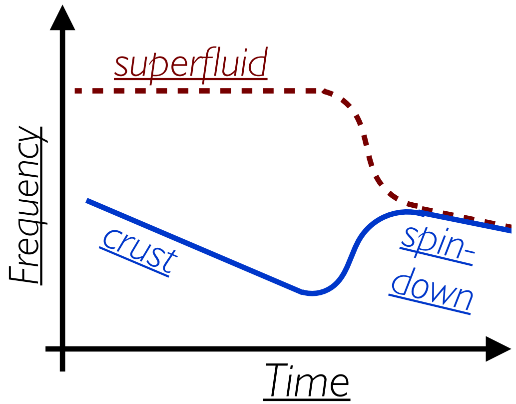
Observable #1
Mass-Radius relation



Observable #2
Cooling curve



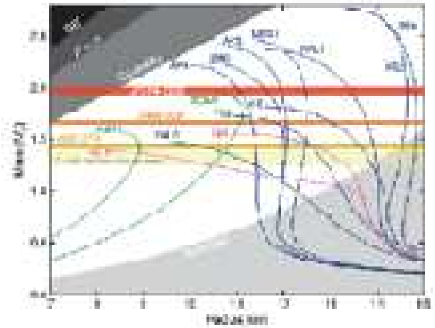
Observable #3
Glitching



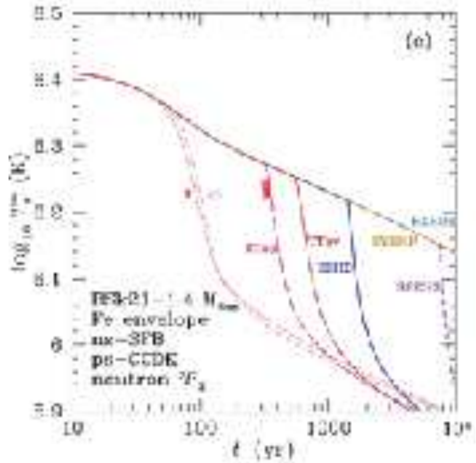
Neutron star modelling

Input
Consistent many-body theory

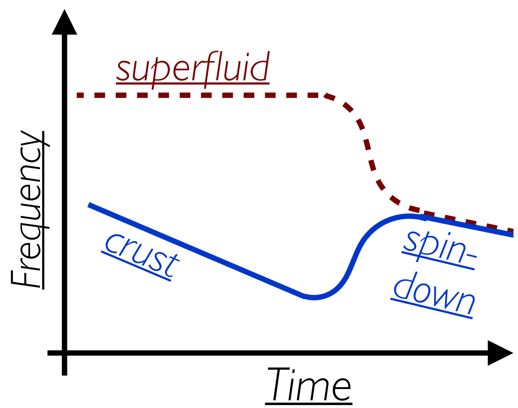
Observable #1
Mass-Radius relation
"Normal"



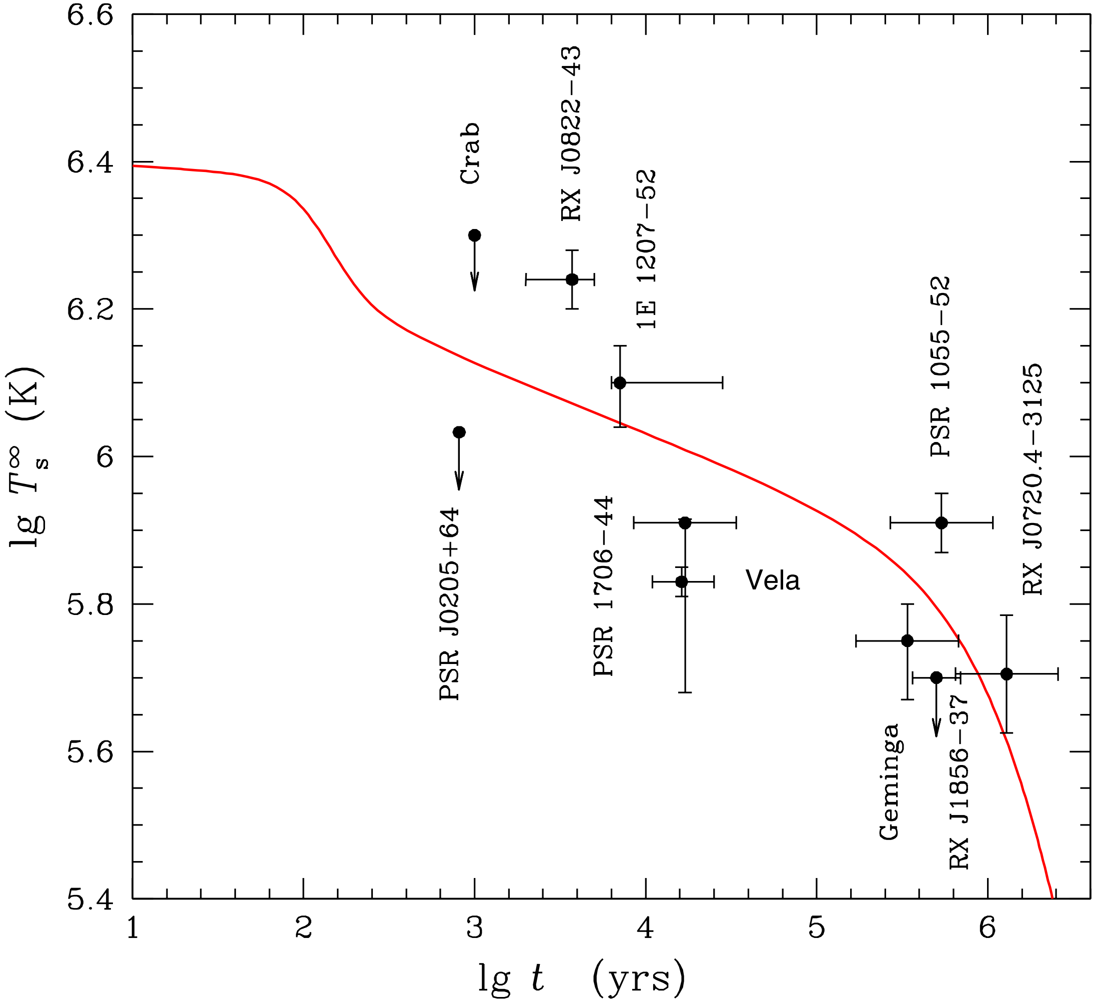
Observable #2
Cooling curve
"Superfluid"



Observable #3
Glitching
"Superfluid"



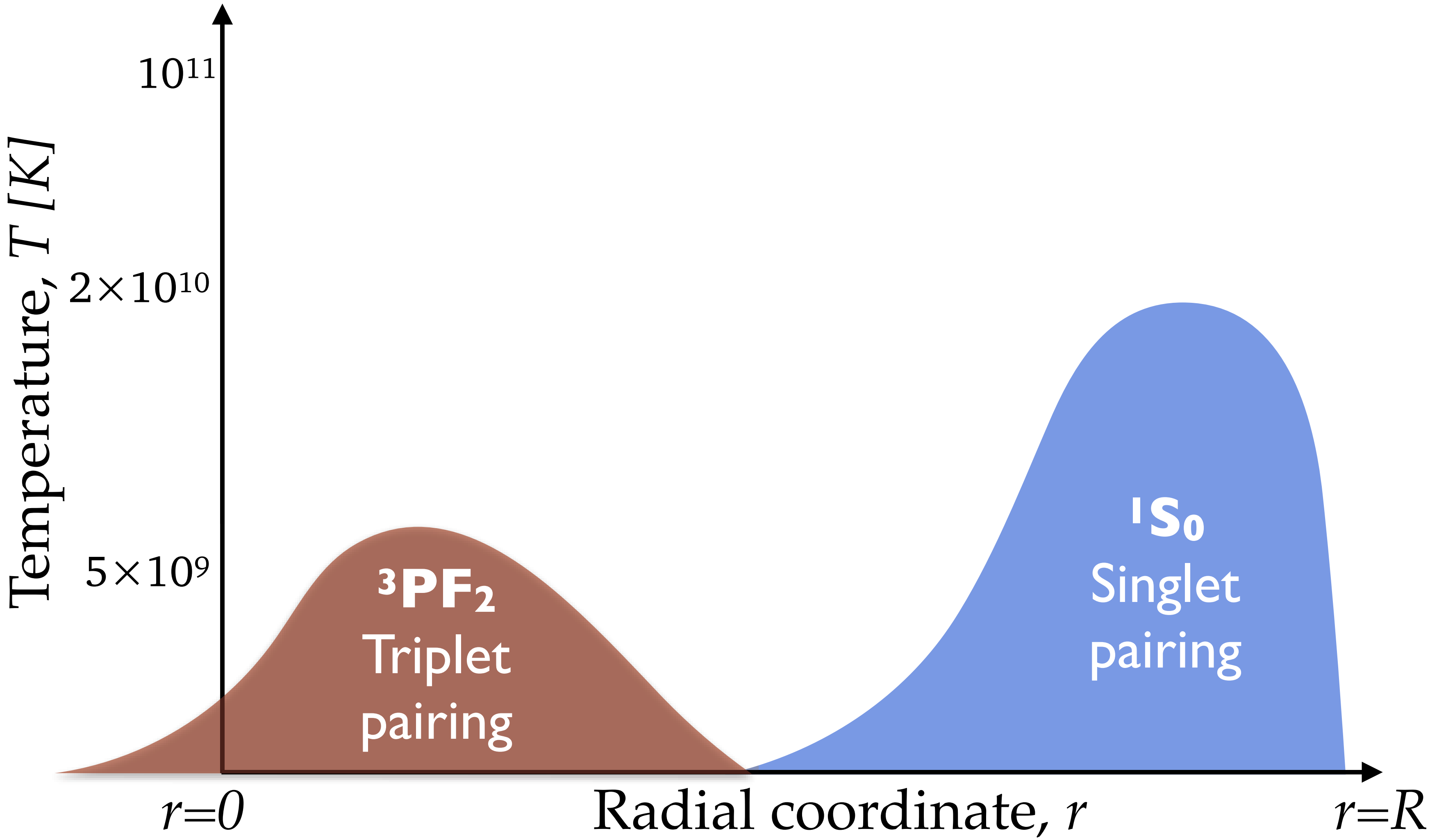
Cooling curve of neutron stars



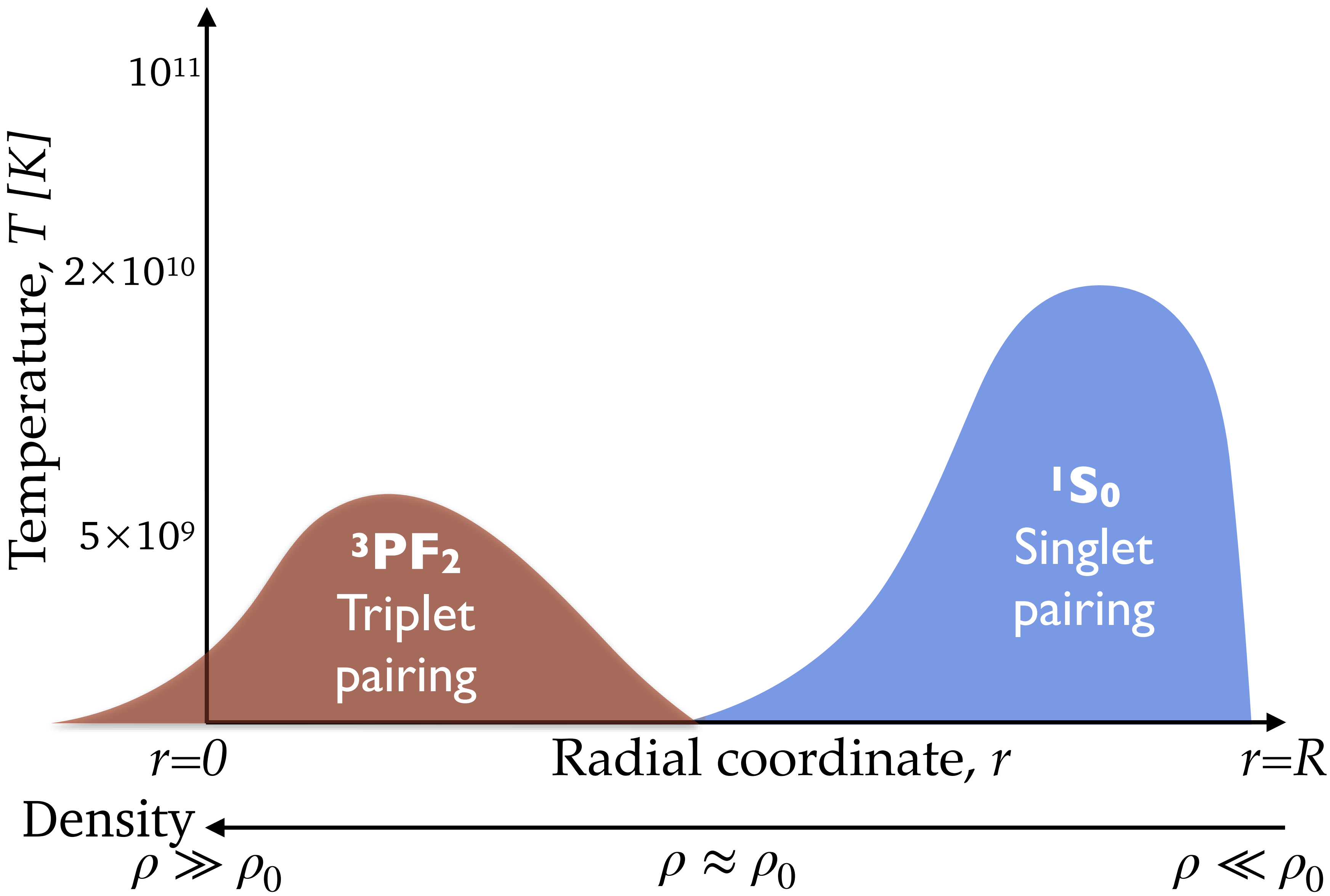
- Observational data available for a handful of NS
- Sensitive to **interior** physics (mostly **pairing)**

Yakovlev & Pethick, ARAA **42** 169 (2004)

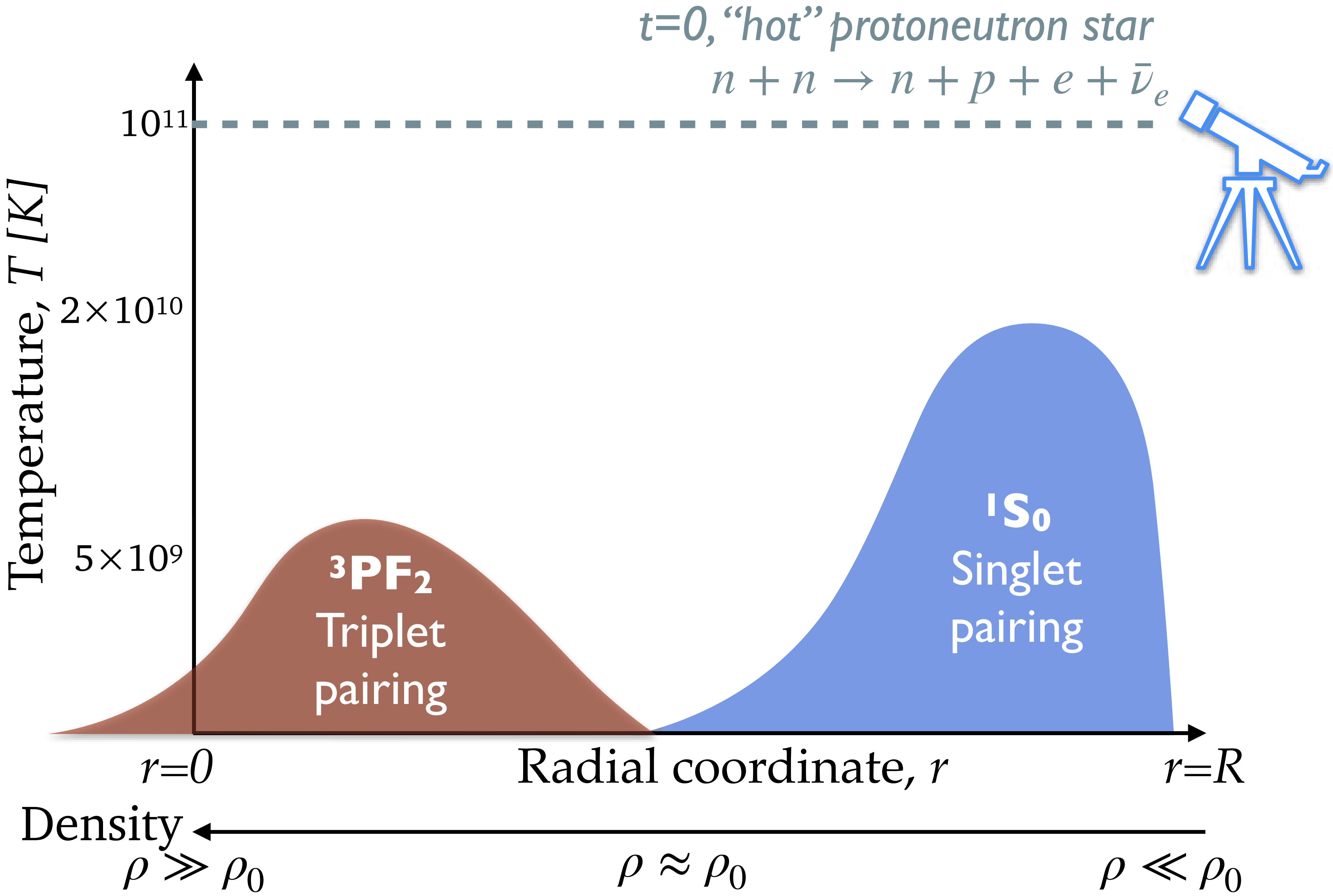
Pairing gaps & cooling



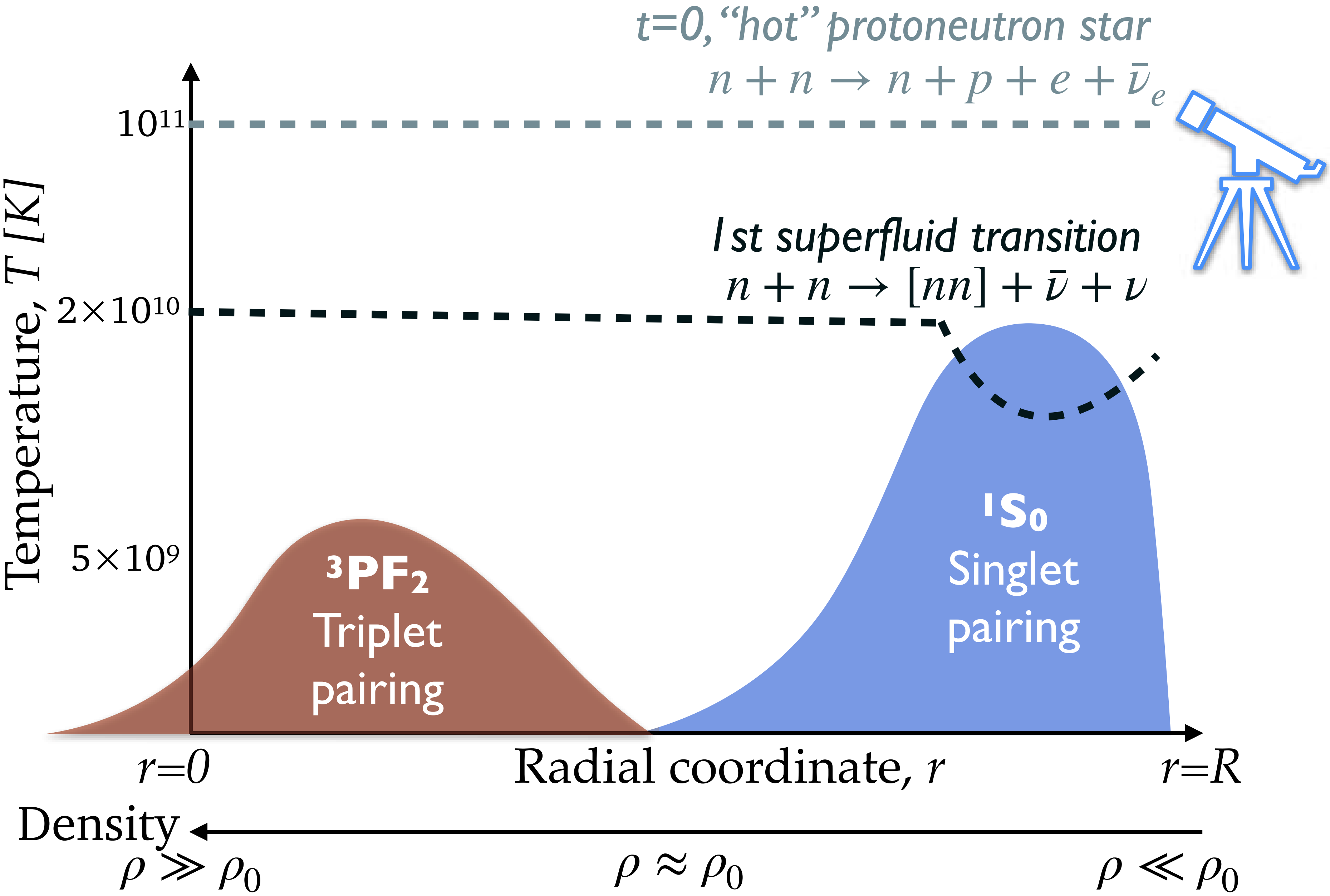
Pairing gaps & cooling



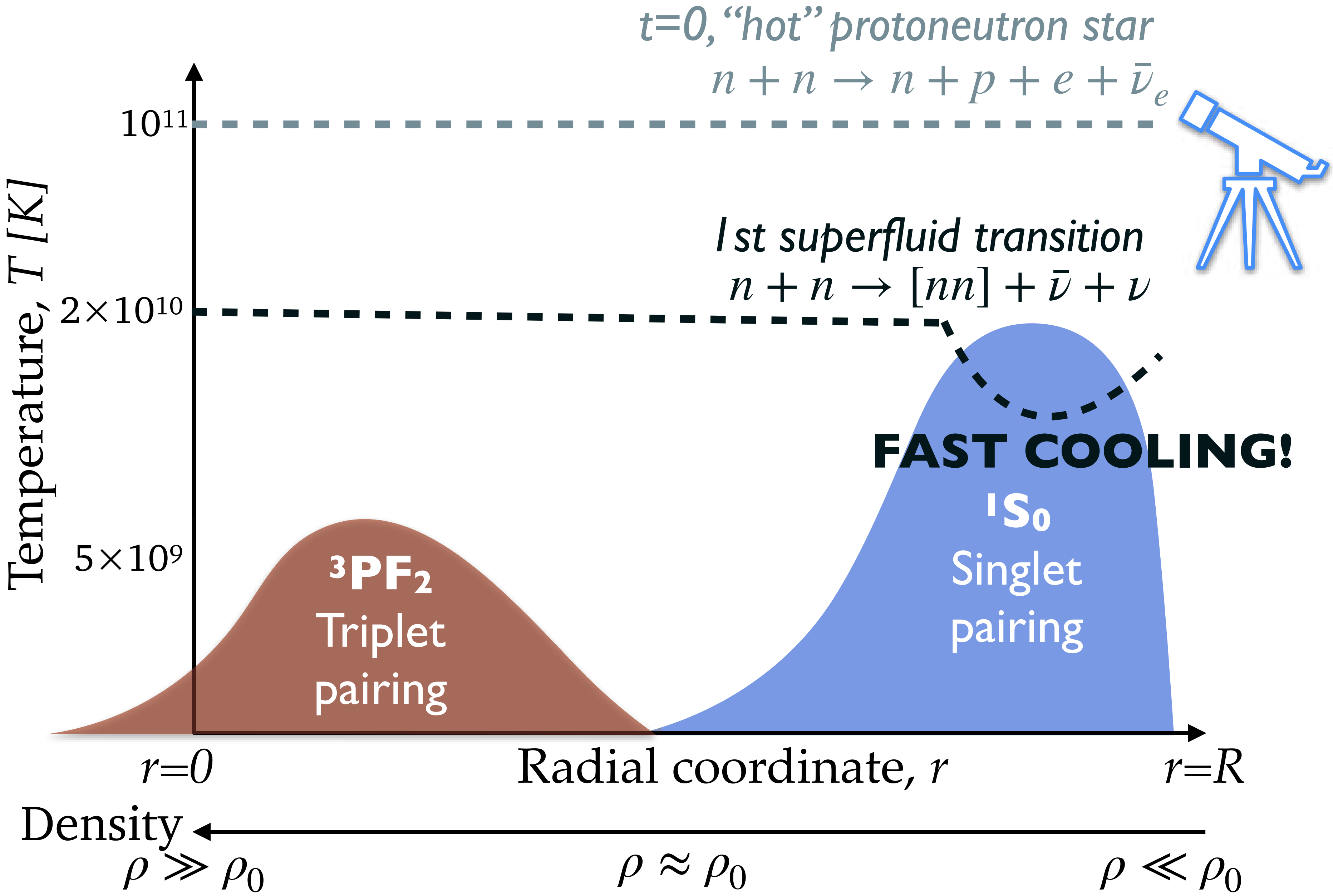
Pairing gaps & cooling



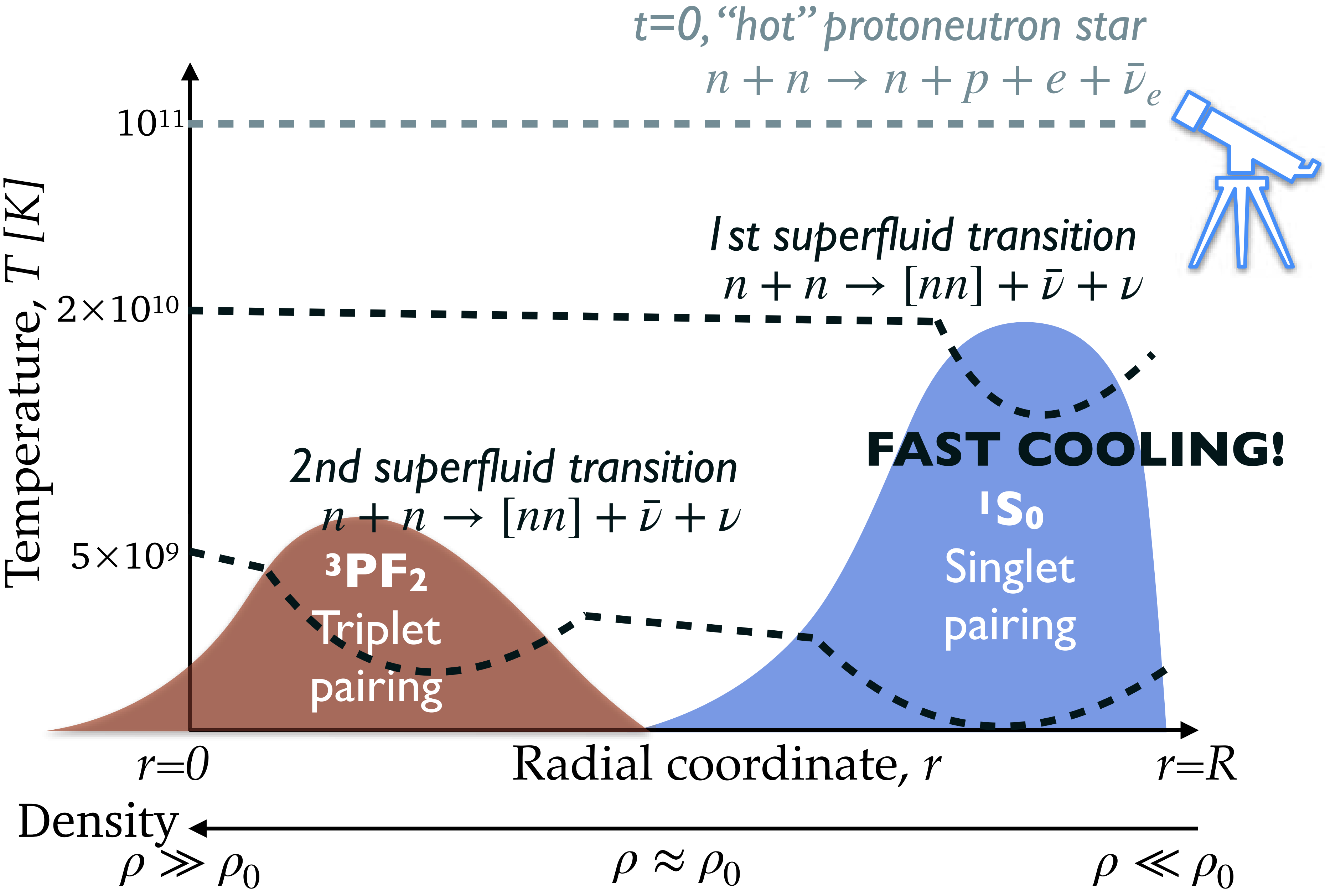
Pairing gaps & cooling



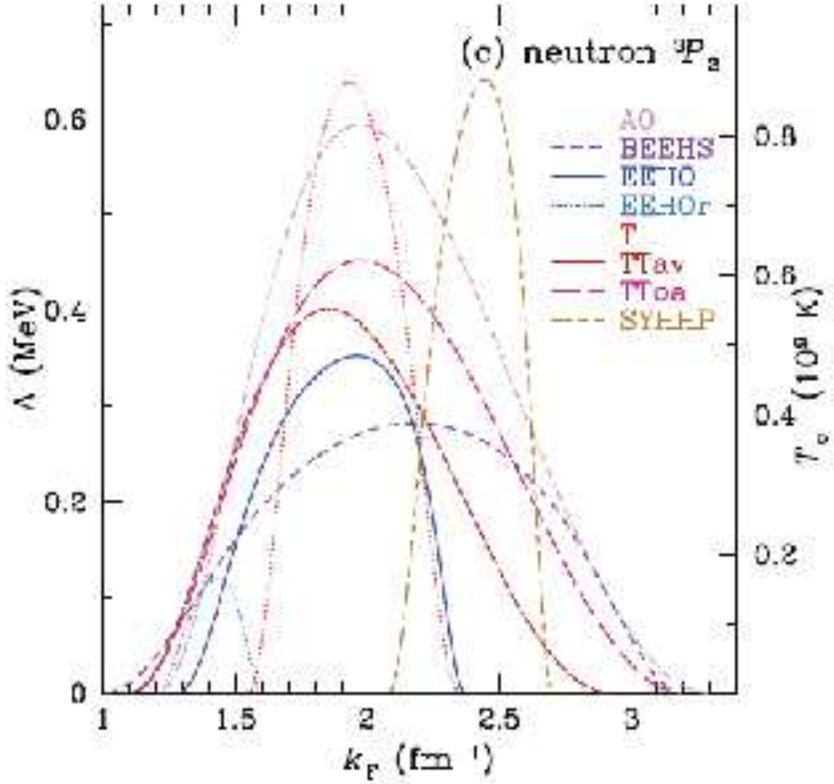
Pairing gaps & cooling

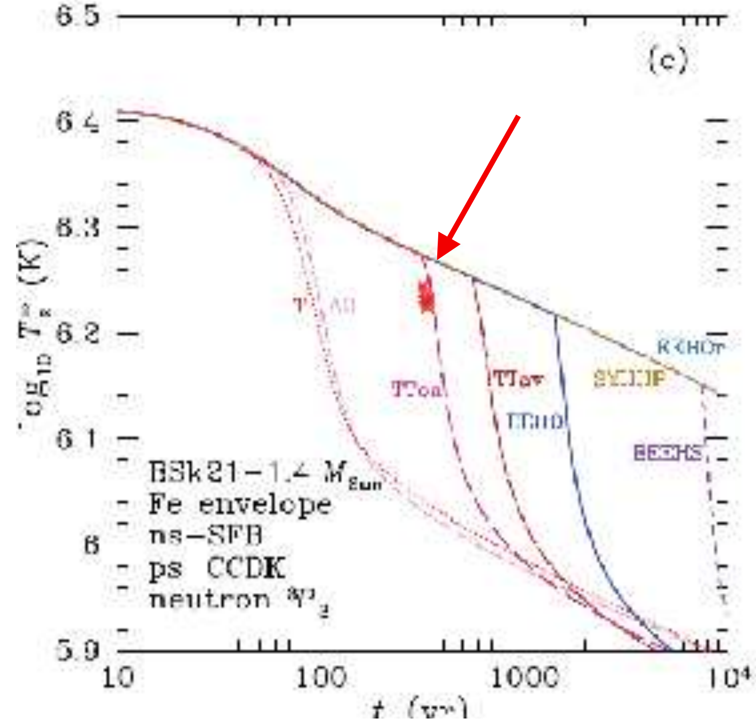


Pairing gaps & cooling



Cooling: Cas A





Ho, *et al.*, PRC **91** 015806 (2015)
Page, *et al.*, *PRL* **106** 081101 (2011)

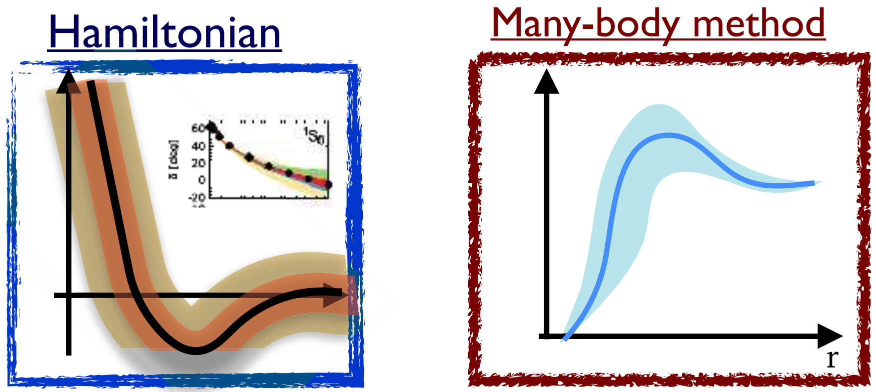
Name	Process	Emissivity (erg cm^{-3} s^{-1})
Modified Urca (neutron branch)	$n+n\to n+p+e^-+\bar{\nu}_e$ $n+p+e^-\to n+n+\nu_e$	$\sim 2\times10^{21}\,RT_9^8$
Modified Urca (proton branch)	$p+n\to p+p+e^-+\nu_e$ $p+p+e^-\to p+n+\nu_e$	$\sim 10^{21}\,RT_9^8$
Bremsstrahlungs	$n+n\to n+n+\nu+\bar{\nu}$ $n+p\to n+p+\nu+\bar{\nu}$ $p+p\to p+p+\nu+\bar{\nu}$	$\sim 10^{19}\,RT_9^8$
Cooper pair	$n+n\to[nn]+\nu+\bar{\nu}$ $p+p\to[pp]+\nu+\bar{\nu}$	$\sim 5\times10^{21}\,RT_9^7$ $\sim 5\times10^{19}\,RT_9^7$
Direct Urca (nucleons)	$n\to p+e^-+\bar{\nu}_e$ $p+e^-\to n+\nu_e$	$\sim 10^{27}\,RT_9^6$

Ingredients

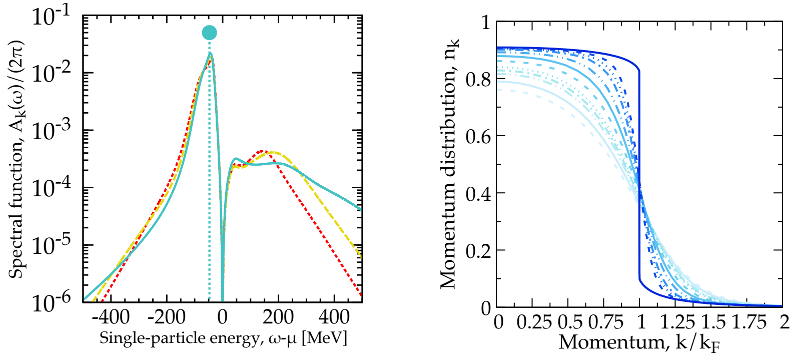
(a) Mass of pulsar
(b) EoS (determines radius)
(c) Internal composition
(d) **Pairing gaps** ($^1S_0$ & ^{3}**PF**$_2$ channels)
(e) Atmosphere composition

Outline

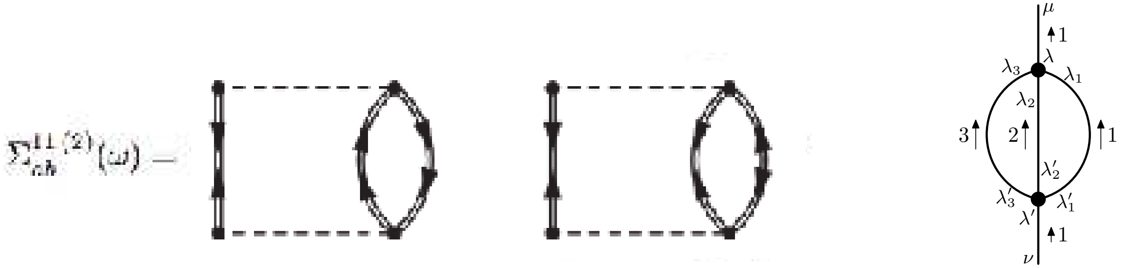
- Motivation



- **“Normal” self-consistent Green’s functions**



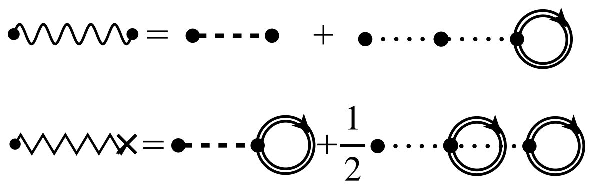
- Nambu-Covariant Green’s functions



Self-Consistent Green's Functions

(ρ, T)

2N & 3N forces



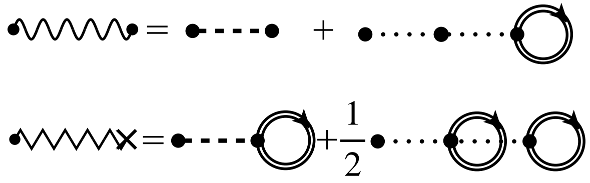
Carbone, Rios & Polls PRC **88** 044302 (2013);
PRC **90**, 054322 (2014);
Carbone PhD Thesis

Forward

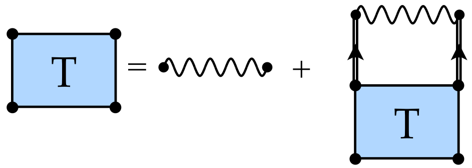
Self-Consistent Green's Functions

(ρ, T)

2N & 3N forces



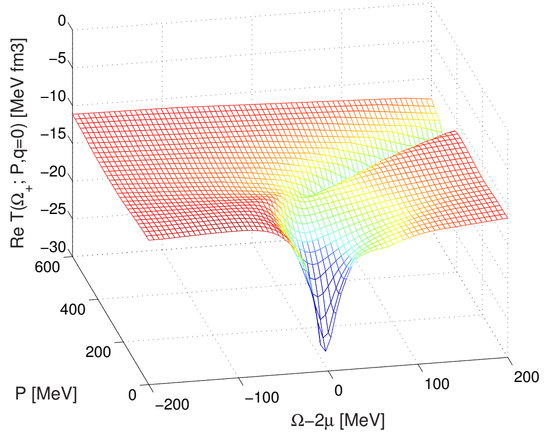
In-medium interaction



Carbone, Rios & Polls PRC **88** 044302 (2013);
PRC **90**, 054322 (2014);
Carbone PhD Thesis

Forward

T-matrix at T=5 MeV



Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
Alm *et al.*, PRC **53** 2181 (1996)
Dewulf *et al.*, PRL **90** 152501 (2003)
Frick & Muther, PRC **68** 034310 (2003)
Rios, PhD Thesis, U. Barcelona (2007)
Soma & Bozek, PRC **78** 054003 (2008)
Rios & Soma PRL **108** 012501 (2012)

Self-Consistent Green's Functions

Carbone, Rios & Polls PRC **88** 044302 (2013);
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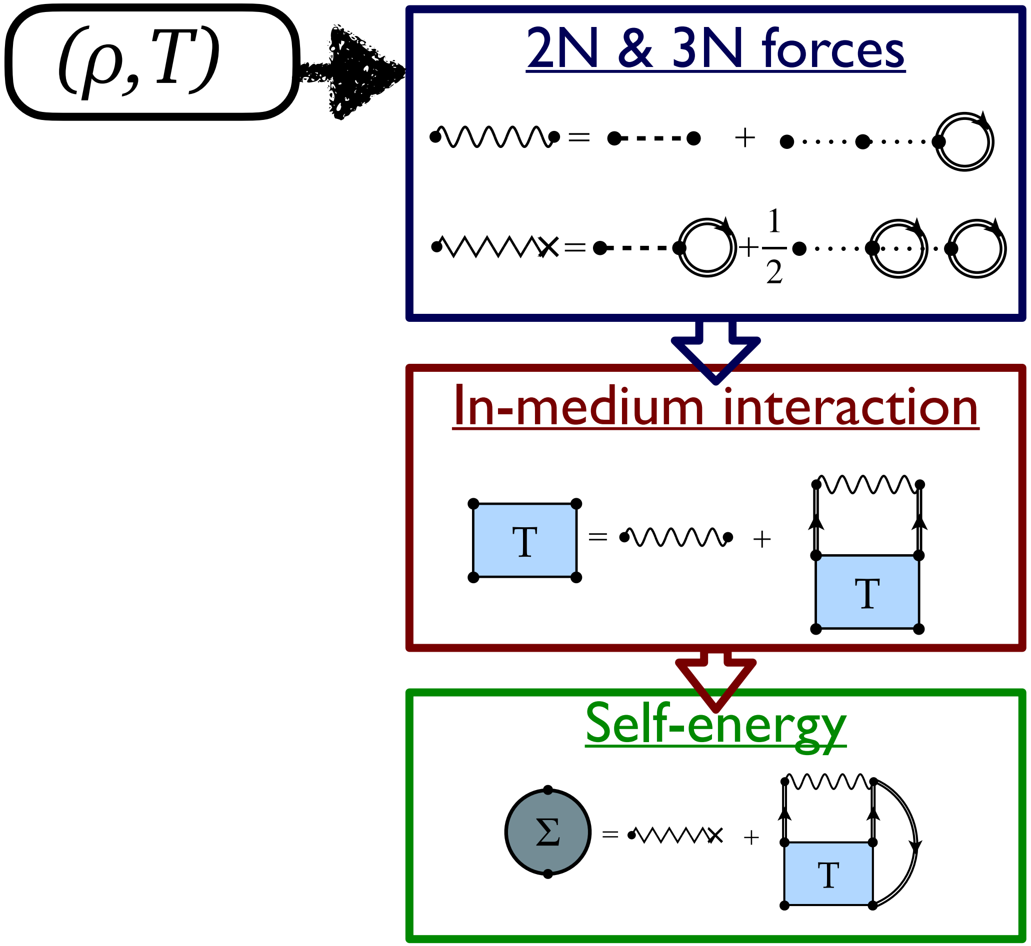
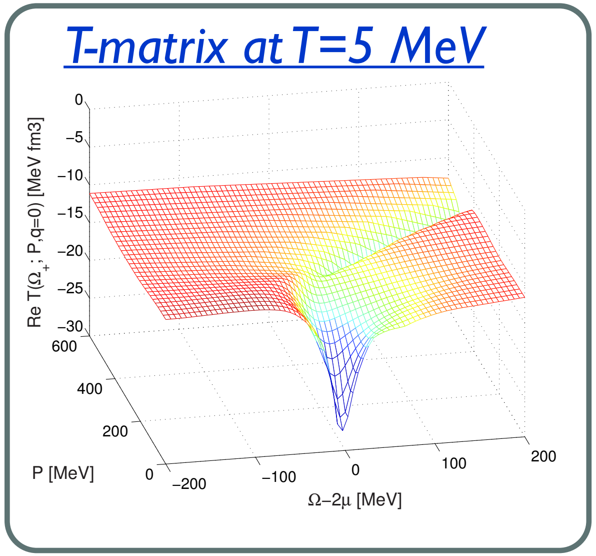
Forward





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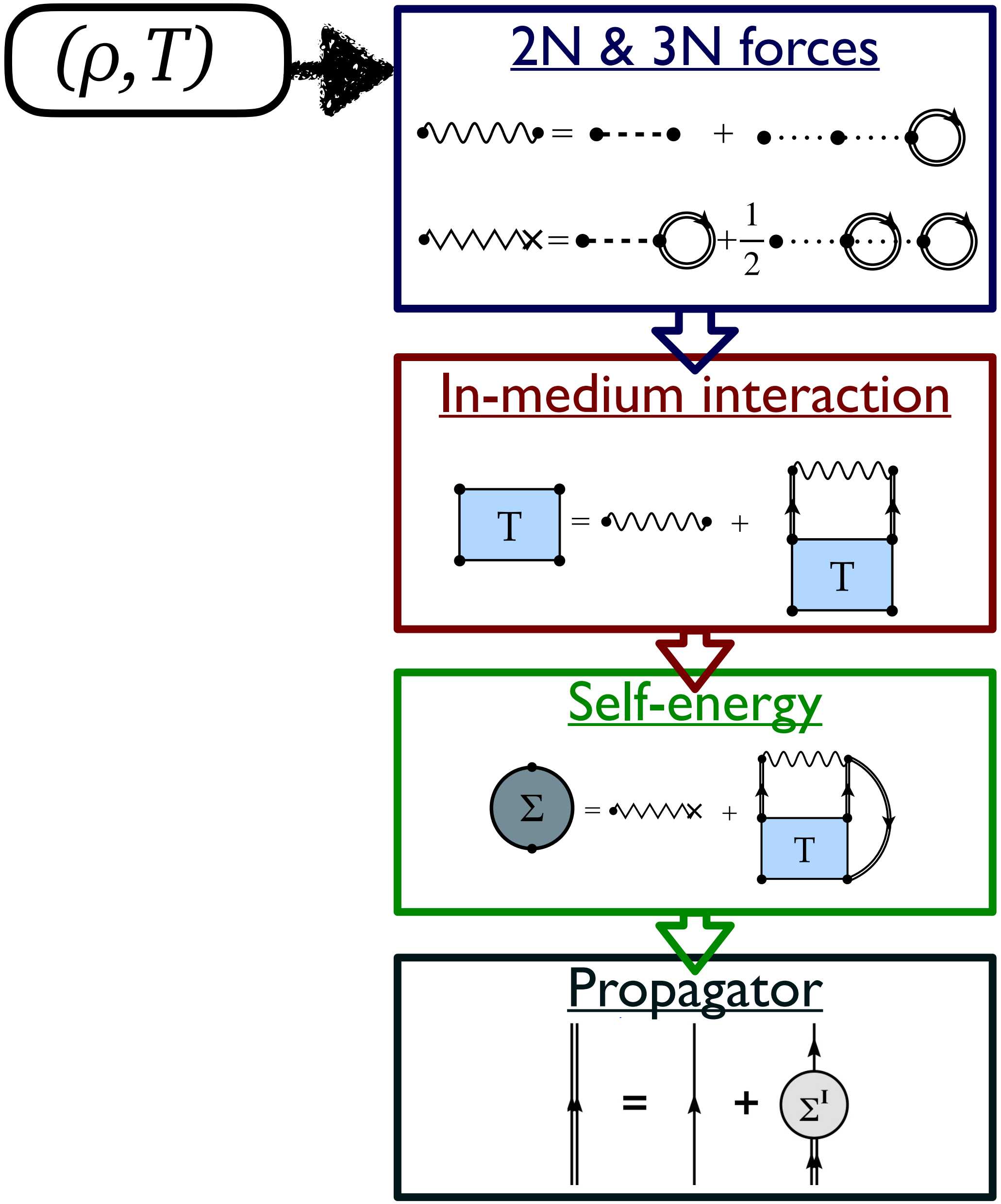

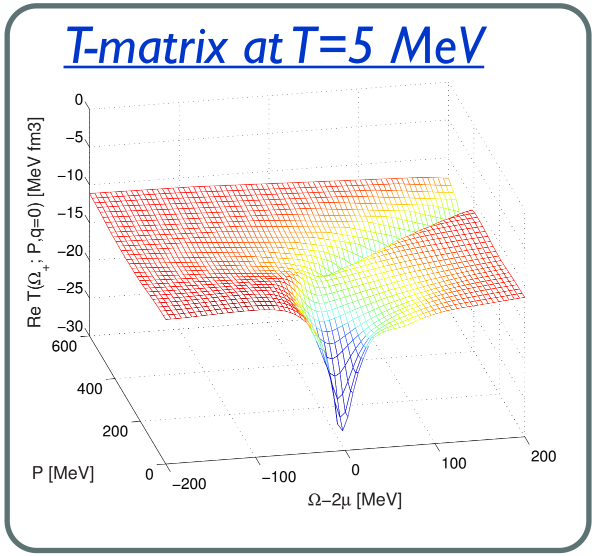

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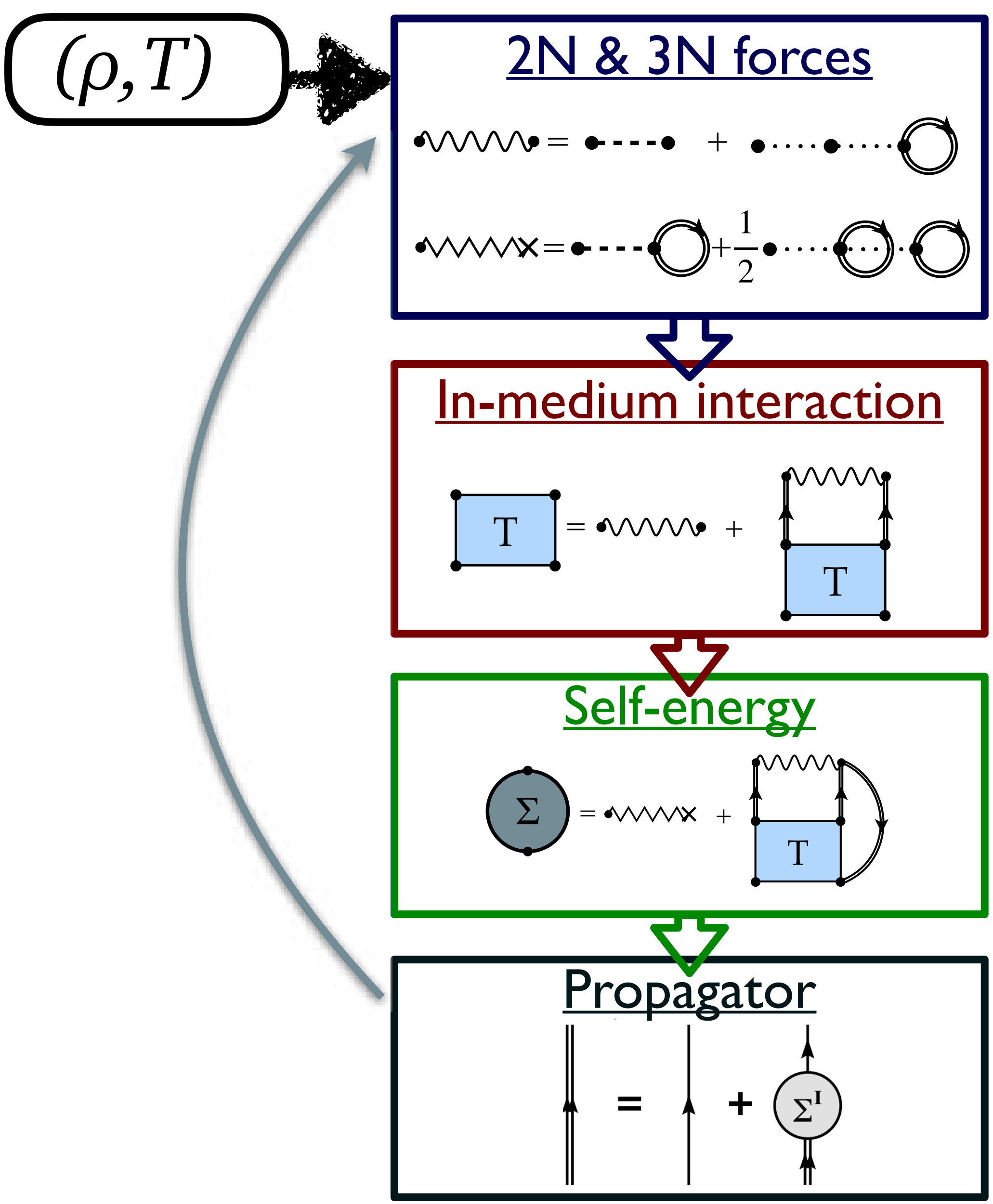

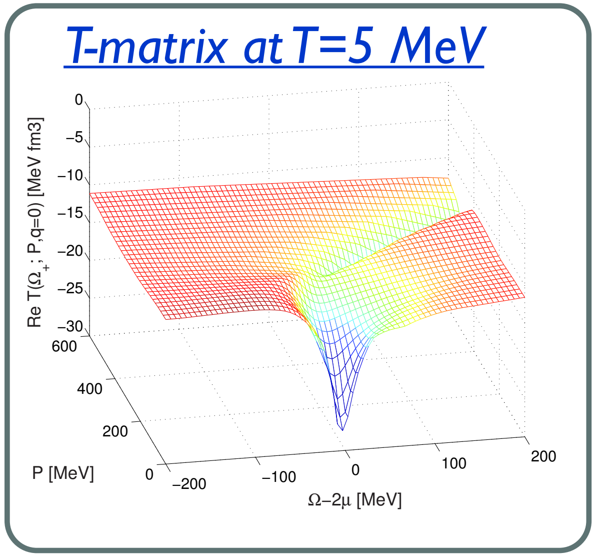

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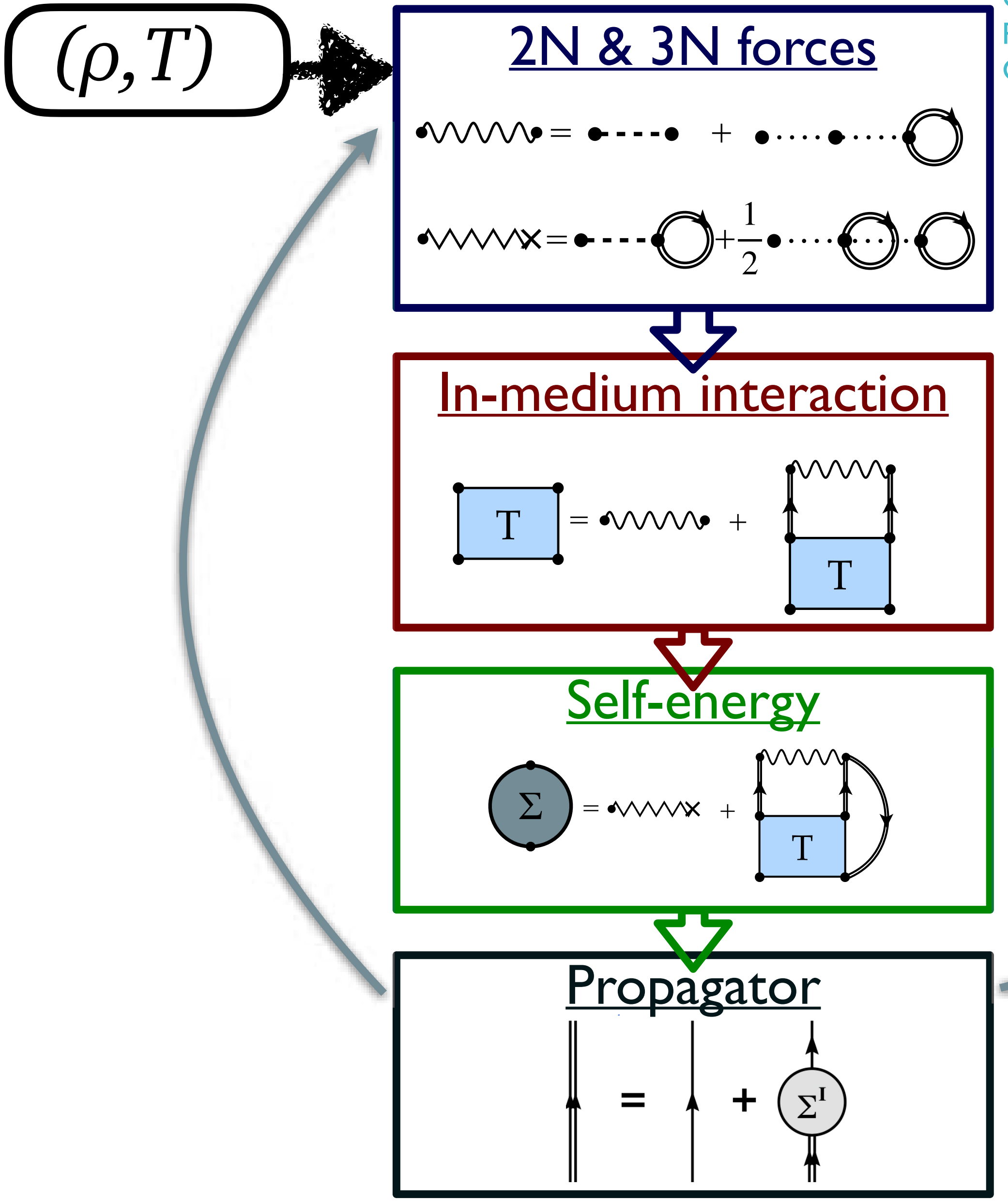

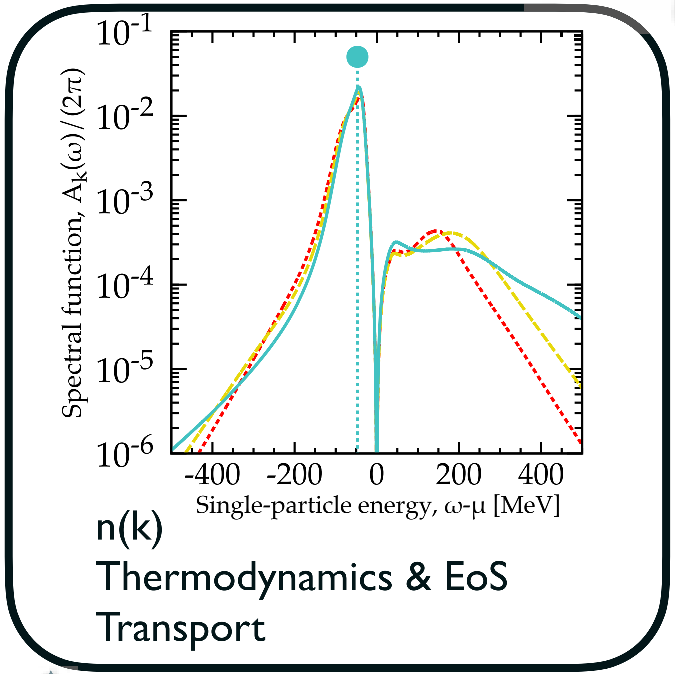

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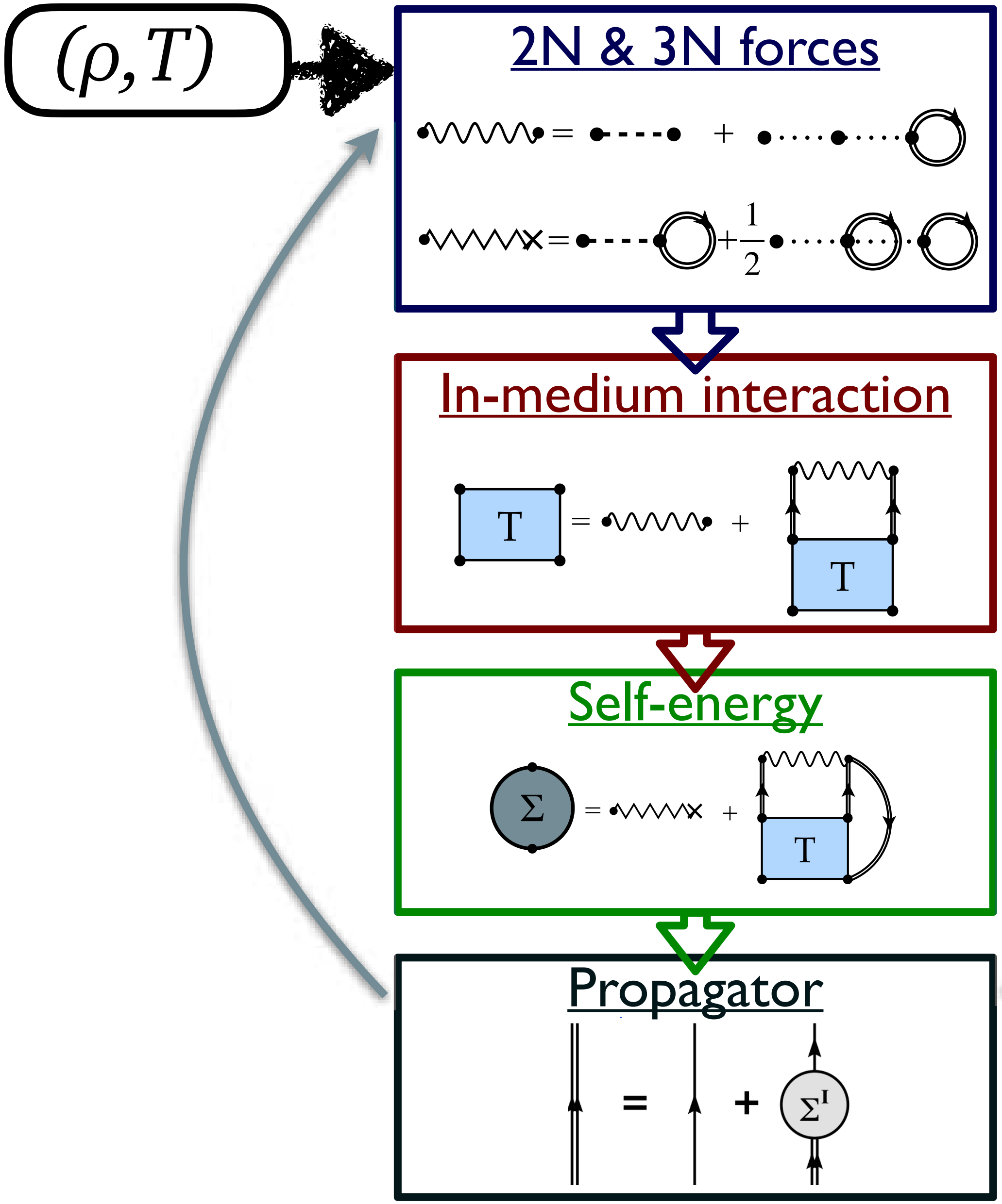
Self-Consistent Green's Functions



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Carbone PhD Thesis

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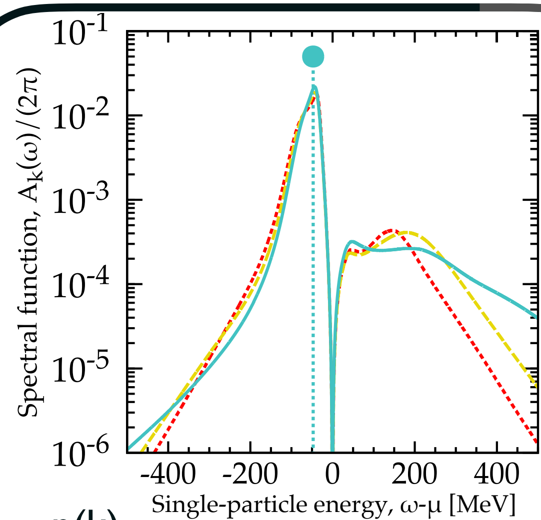
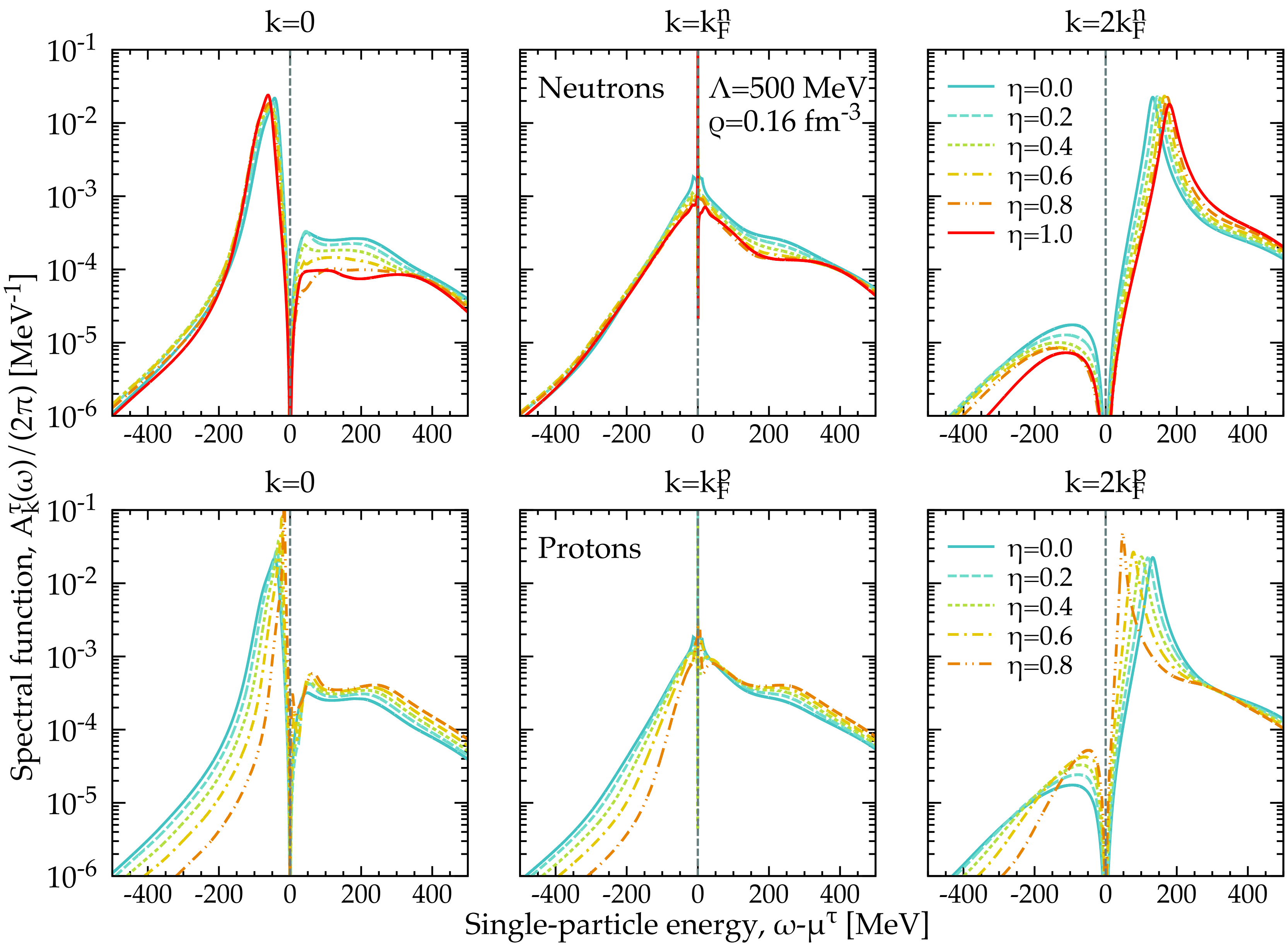


n(k)
Thermodynamics & EoS
Transport

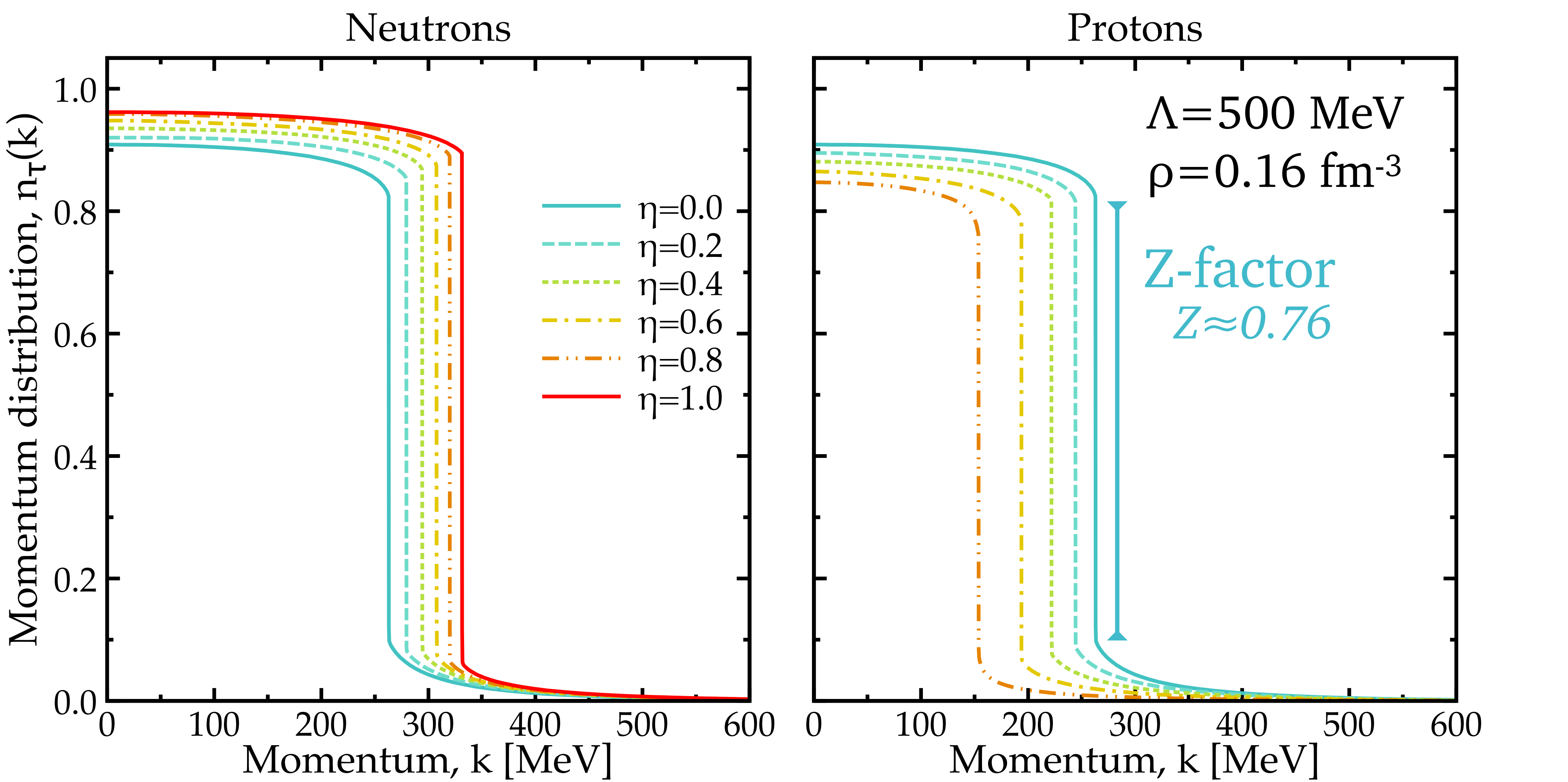
- Off-shell ✔
- Matsubara formalism ✔
- Φ-derivable ✔

Ramos, Polls & Dickhoff, NPA **503** 1 (1989)
Alm *et al.*, PRC **53** 2181 (1996)
Dewulf *et al.*, PRL **90** 152501 (2003)
Frick & Muther, PRC **68** 034310 (2003)
Rios, PhD Thesis, U. Barcelona (2007)
Soma & Bozek, PRC **78** 054003 (2008)
Rios & Soma PRL **108** 012501 (2012)

Isospin-asymmetric spectral functions



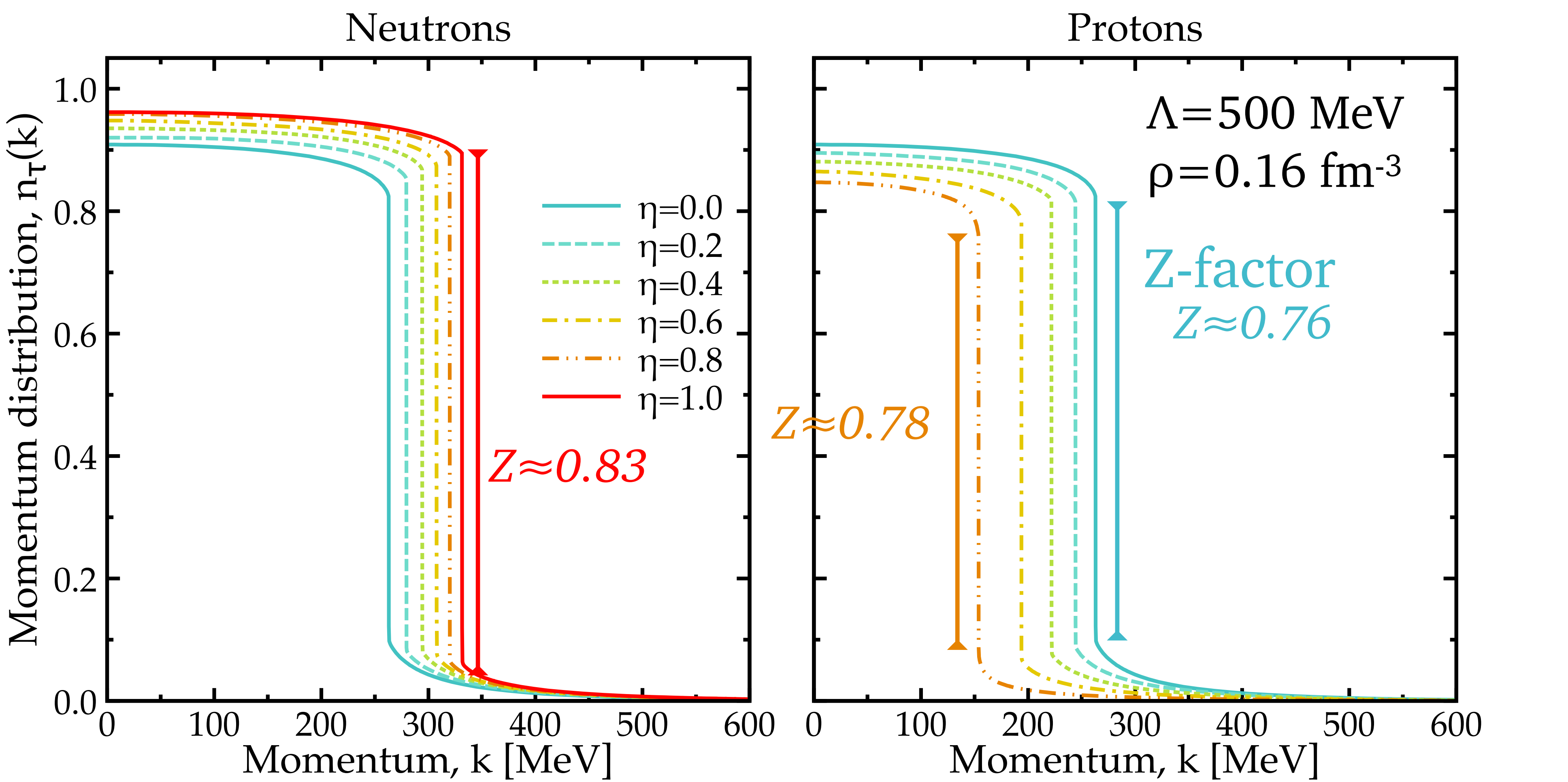
Momentum distribution



Forward

[Rios, Frontiers Physics fphy.2020.00387 (2020)
[arXiv:2006.10610]]

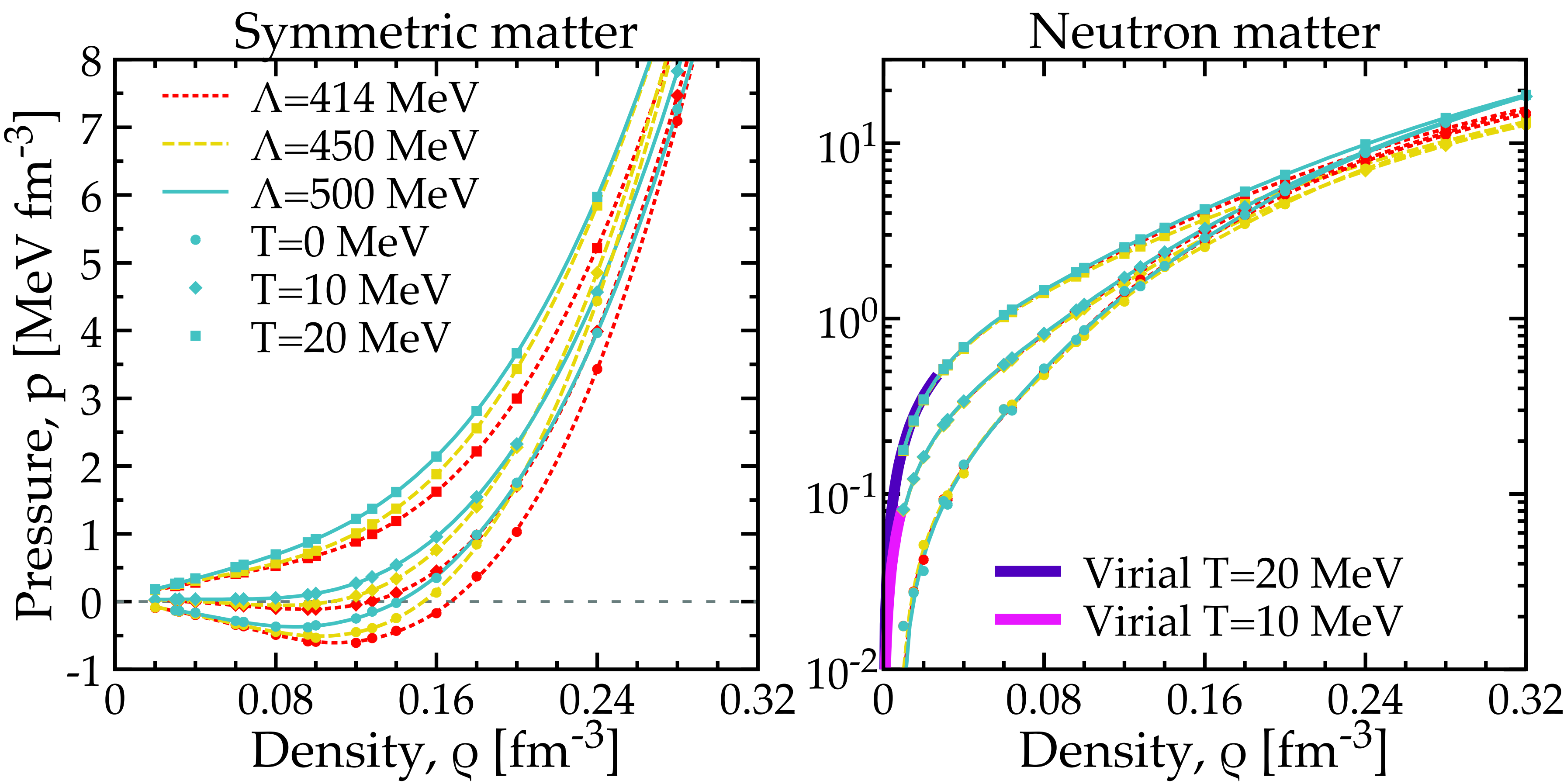
Momentum distribution



Forward

[Rios, Frontiers Physics fphy.2020.00387 (2020)]
[[arXiv:2006.10610]]

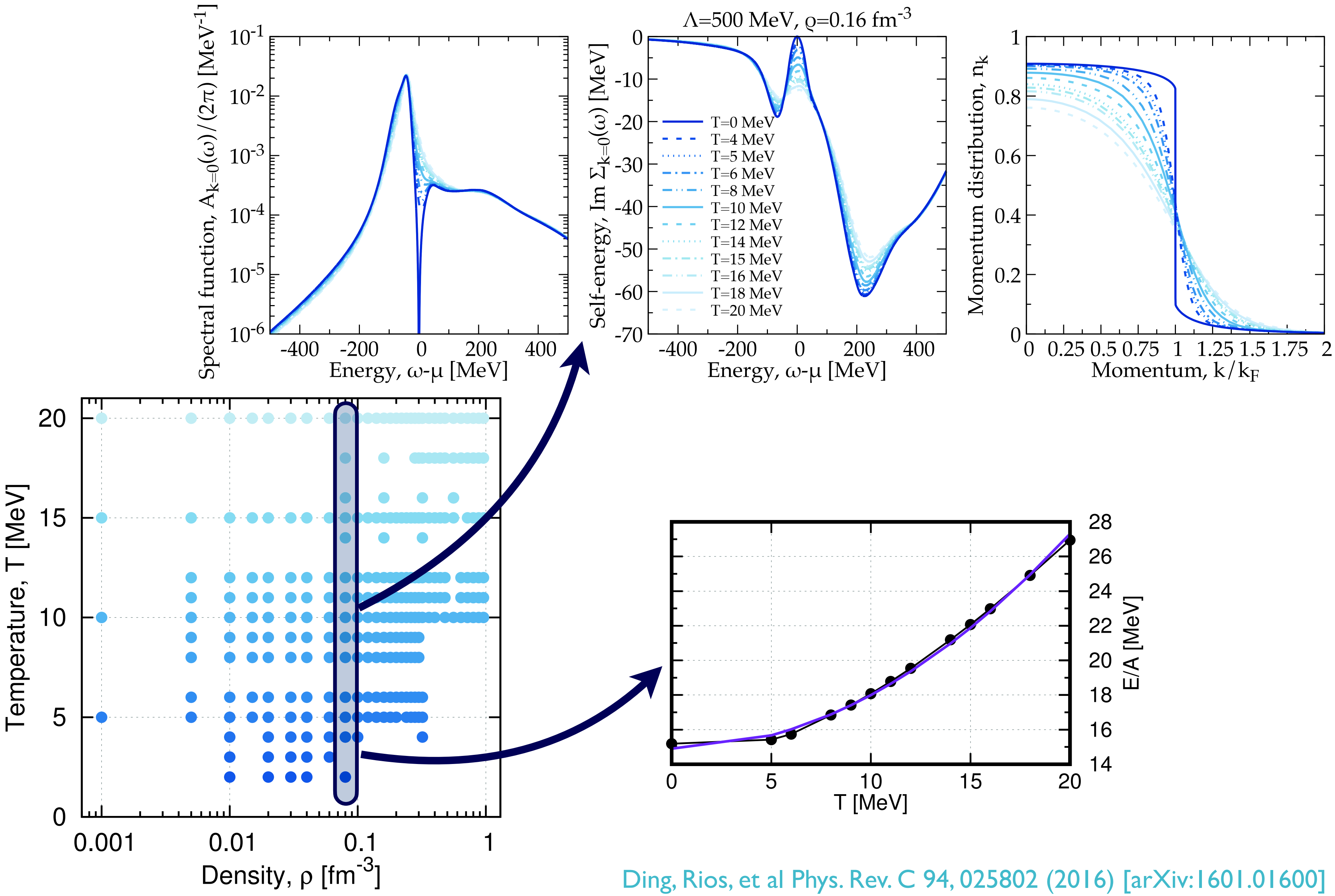
EoS at finite temperature



- Relevant & **necessary** for binary NS simulations
- **Parameter-free** first principles calculation
- Reproduces **virial** at low density

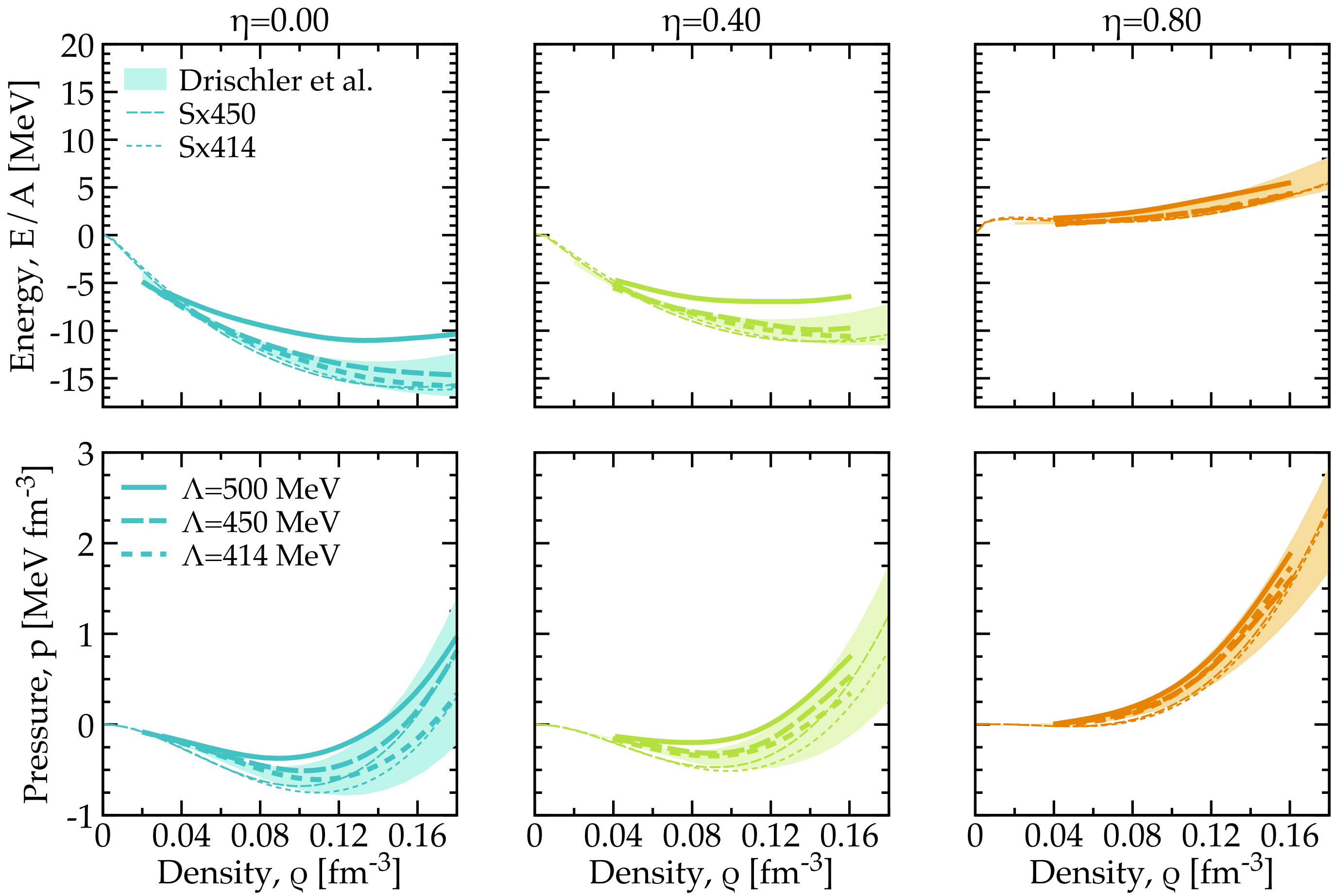
Rios, Frontiers Physics fphy.2020.00387 (2020)
[arXiv:2006.10610]

Zero temperature extrapolation



Ding, Rios, et al Phys. Rev. C 94, 025802 (2016) [arXiv:1601.01600]

Self-Consistent Green's Functions



[Rios, Frontiers Physics fphy.2020.00387 (2020)
[arXiv:2006.10610]]

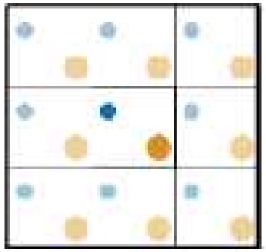
SCGF can treat:

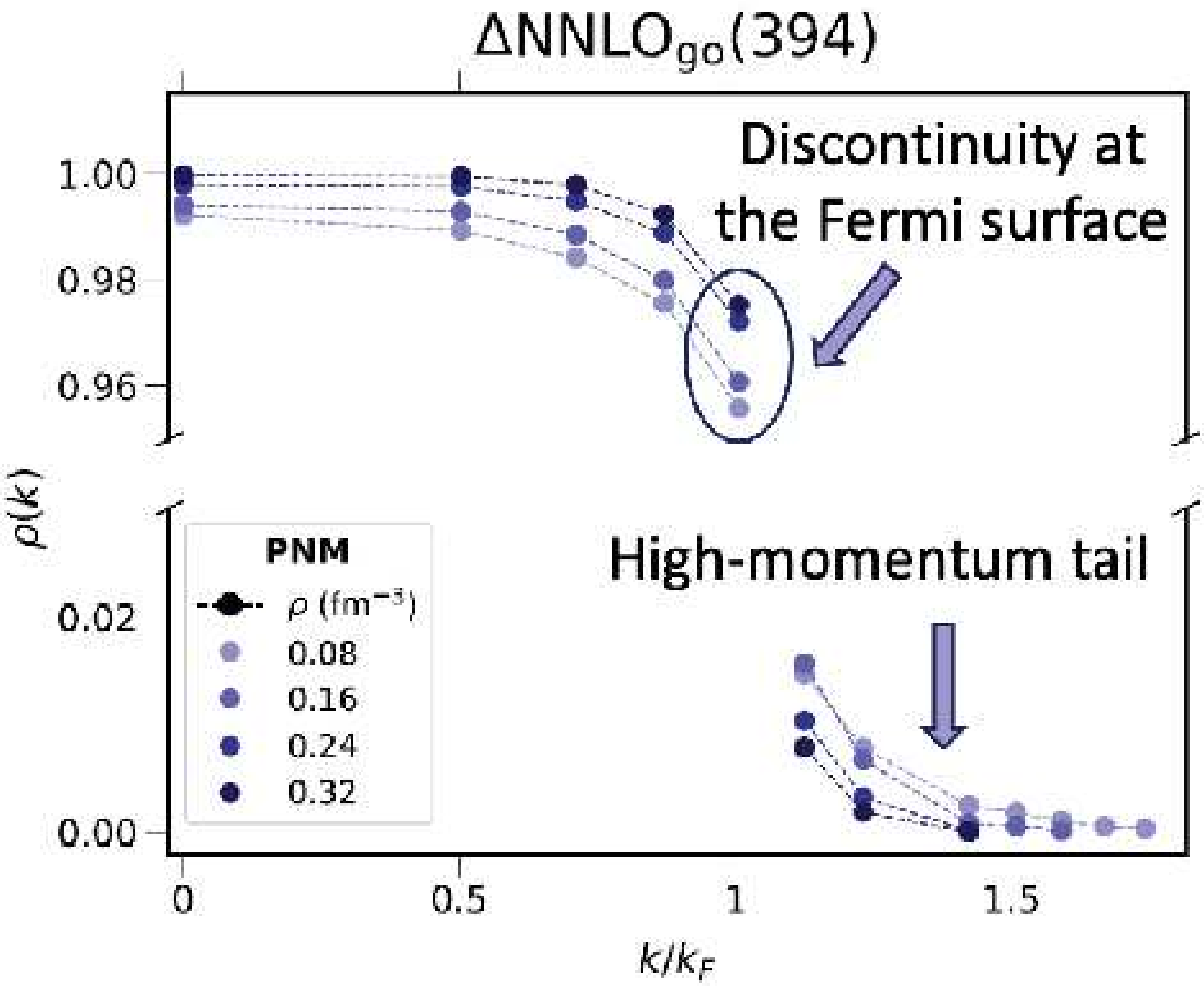
- Explicitly asymmetric matter ✔
- Finite temperature ✔
- Systematic expansion ✔
- 3 nucleon forces ✔

but:

- Numerically intensive
- Pairing?

ADC(3) for matter

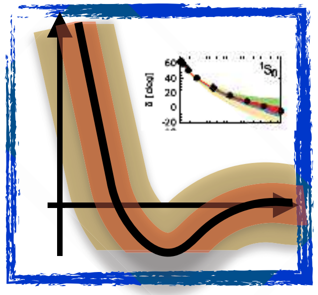




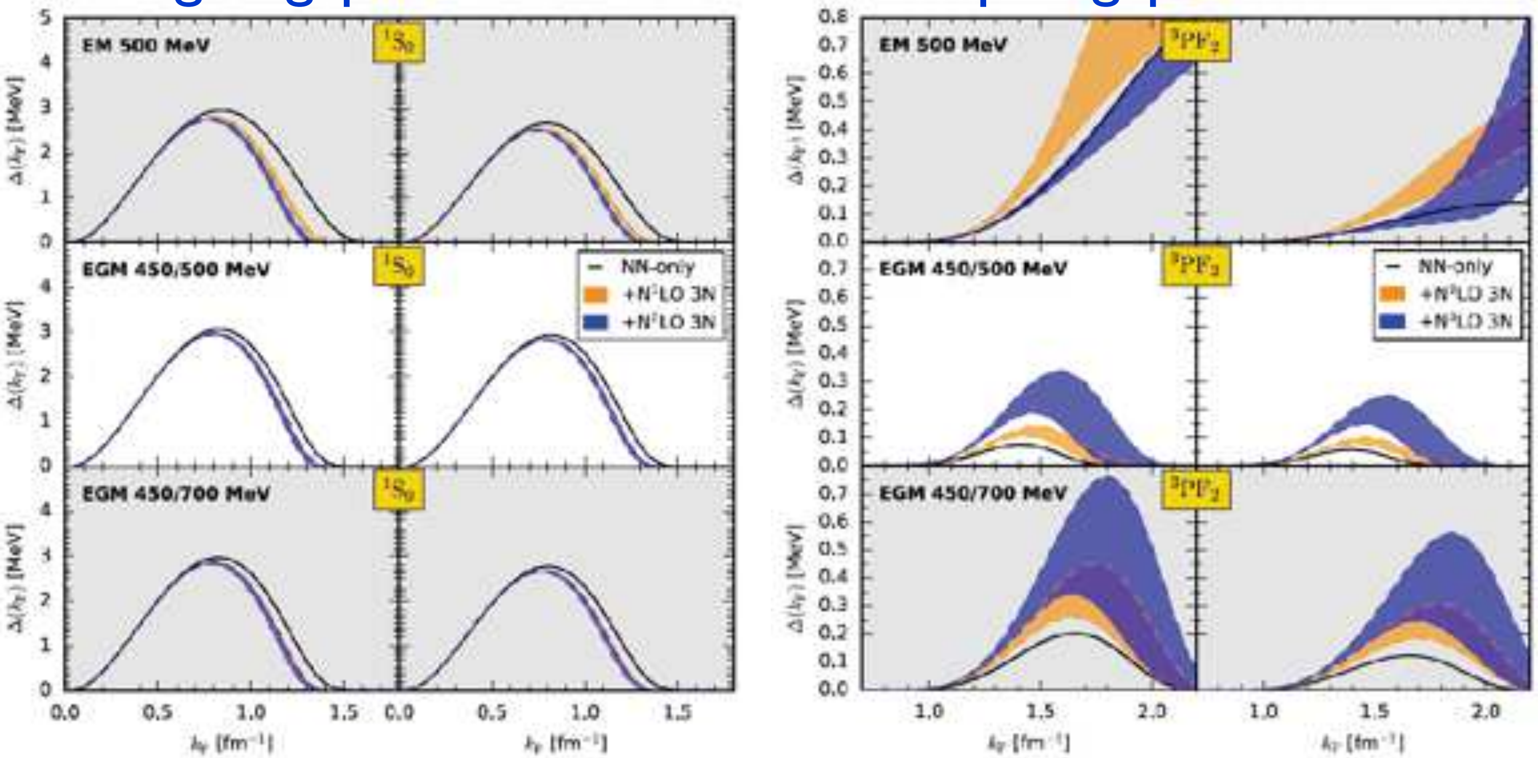
F. Marino et al.

BCS+HF gaps in neutron matter

Hamiltonian



Singlet gaps with 3NF | Triplet gaps with 3NF



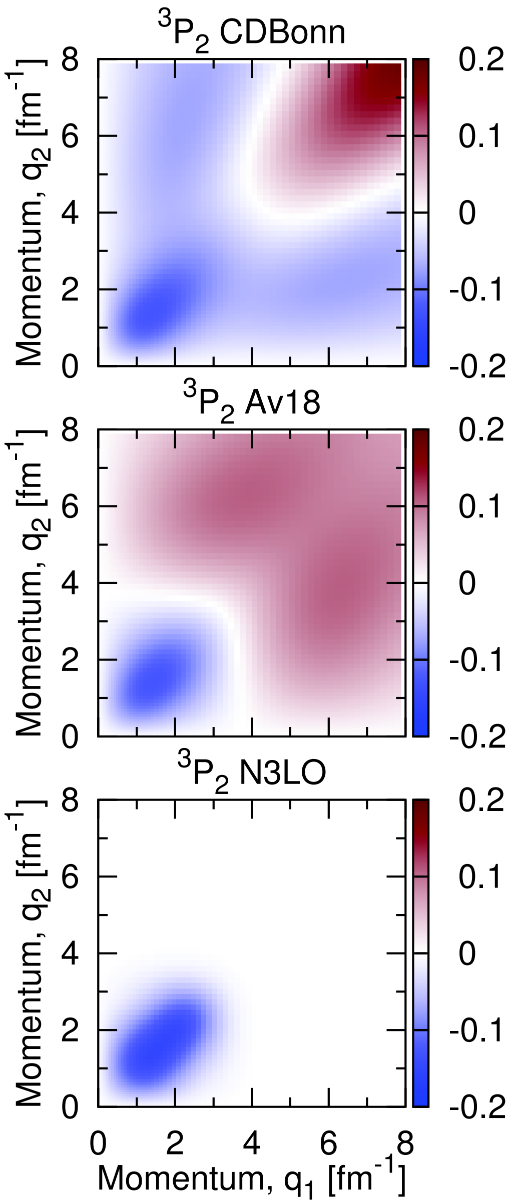
Drischler, Kruger, Hebeler, Schwenk, *PRC* 95 024302 (2017) [arXiv:1610.05213]

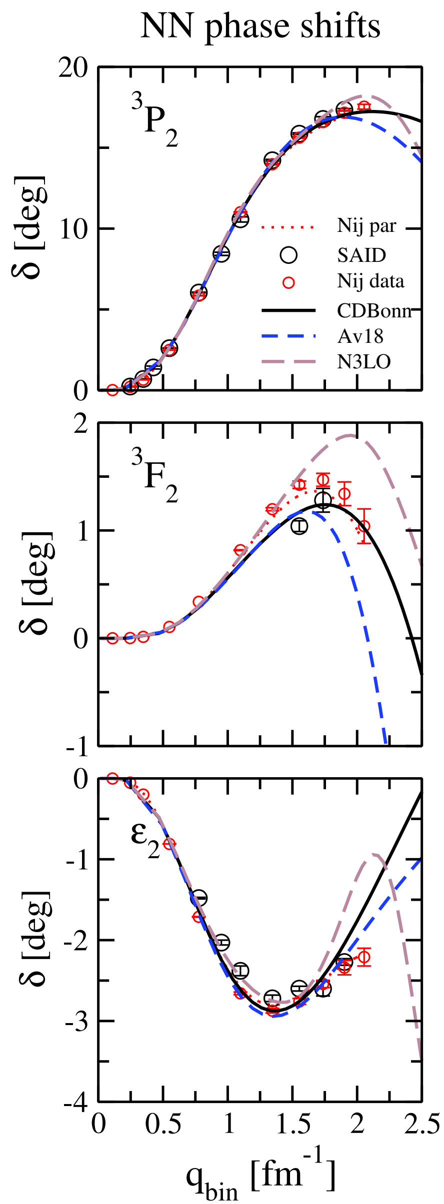
BCS

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad + \quad \begin{aligned} \chi_k &= \varepsilon_k - \mu \\ \varepsilon_k &= \frac{k^2}{2m} + U(k) - \mu \end{aligned}$$

- **Error estimates** from nuclear force (chiral expansion) ✓
- **Many-body uncertainty?** ✗

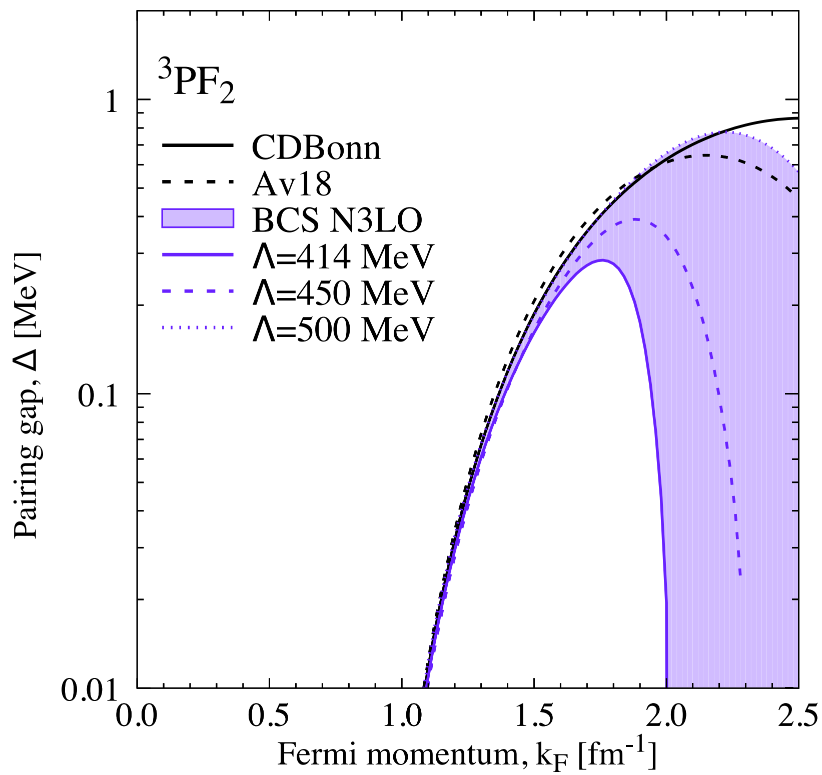
Triplet channel: limits of EFT





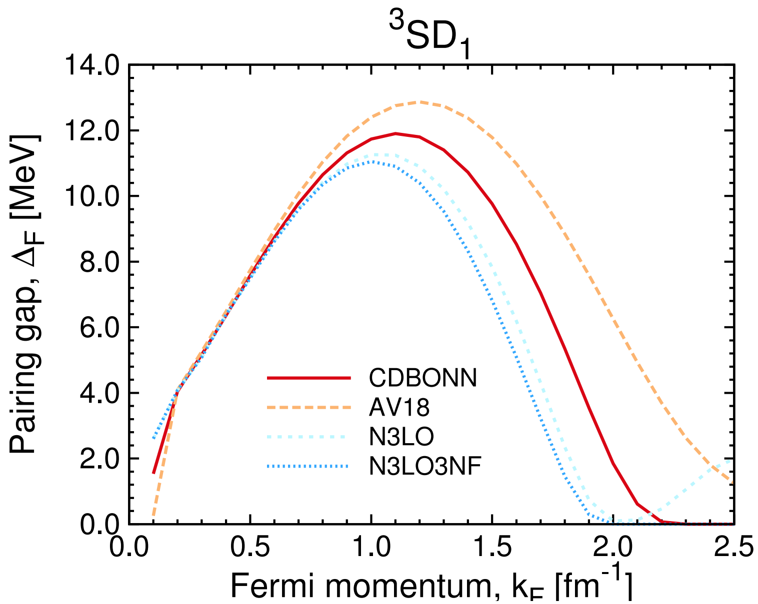
BCS equation

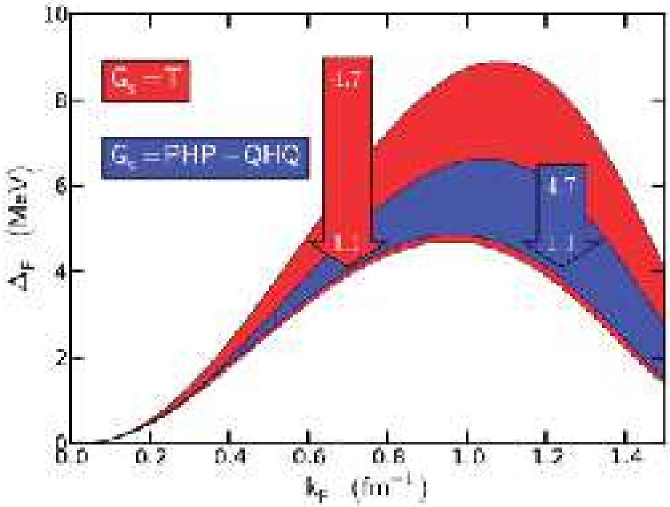
$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$



Triplet pairing: symmetric matter

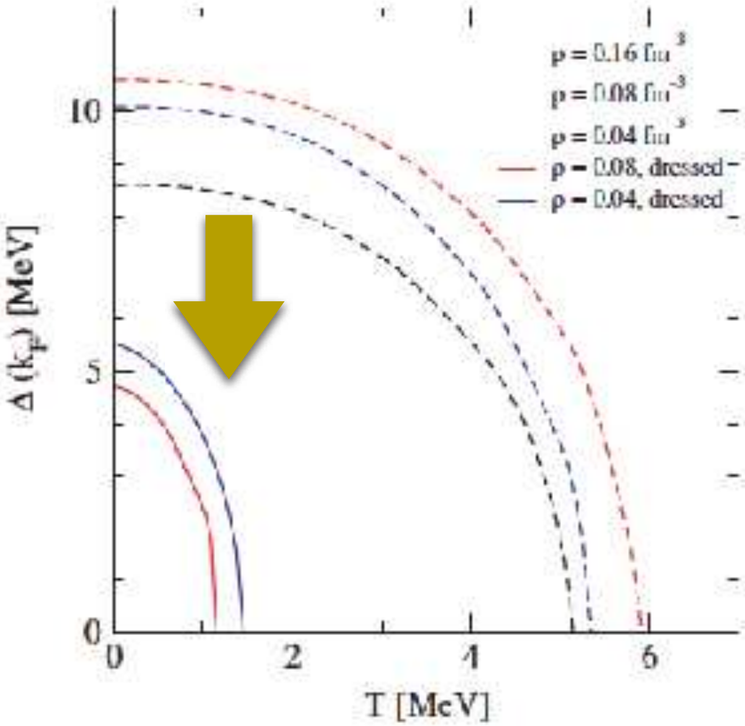
$^3SD_1$ nuclear matter BCS gaps





Maurizio, Holt & Finelli, PRC **90**, 044003 (2014)

SRC-depleted $^3SD_1$ gaps

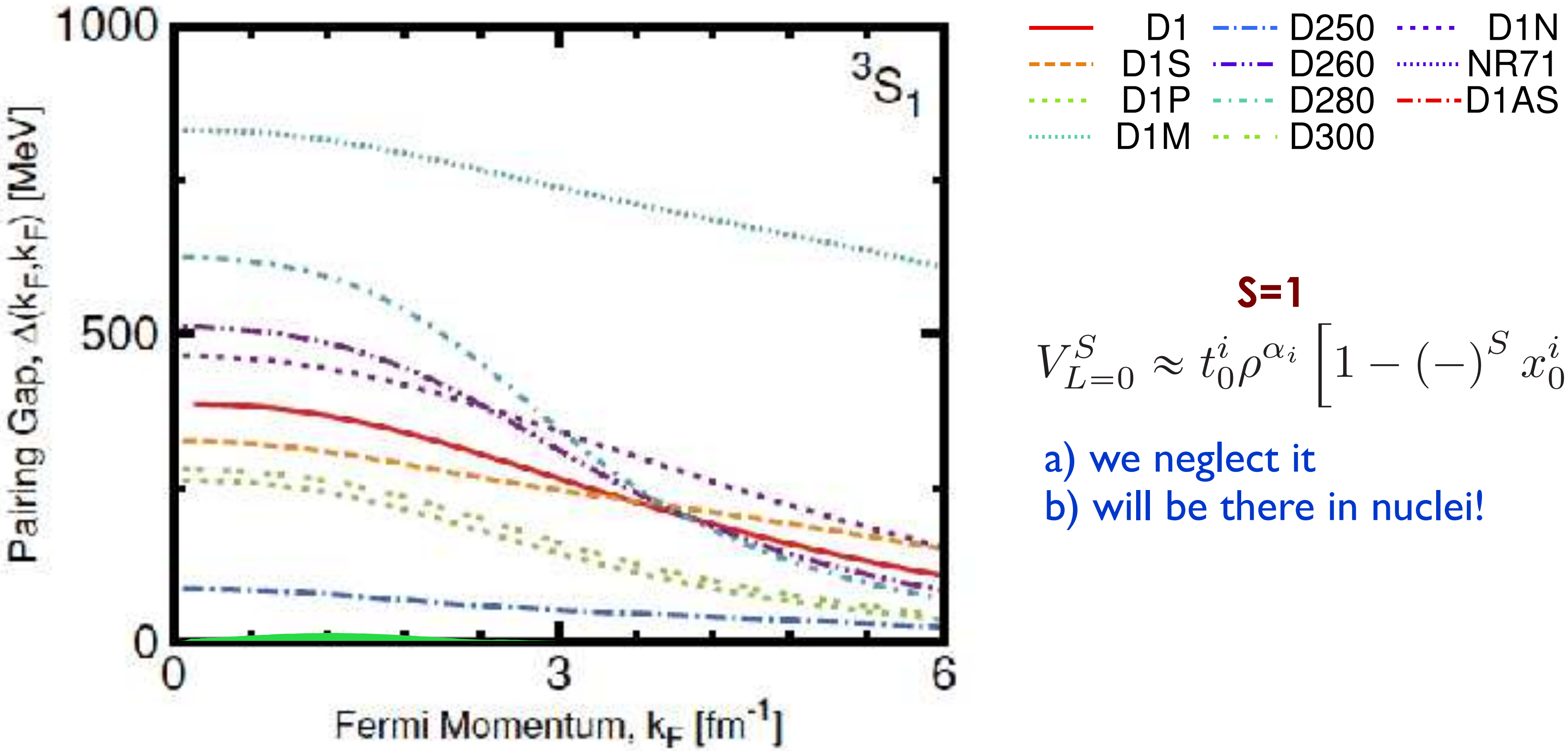


Muether & Dickhoff, PRC **72** 054313 (2005)

- **Massive** gaps ^{3}**SD$_1$** channel but…
- No evidence of strong *np* nuclear pairing
- Short-range correlations **deplete** gap
- 3BF effect? Short-range effects? Deformation?

Gezerli's talk

Triplet pairing: Gogny?



S=1

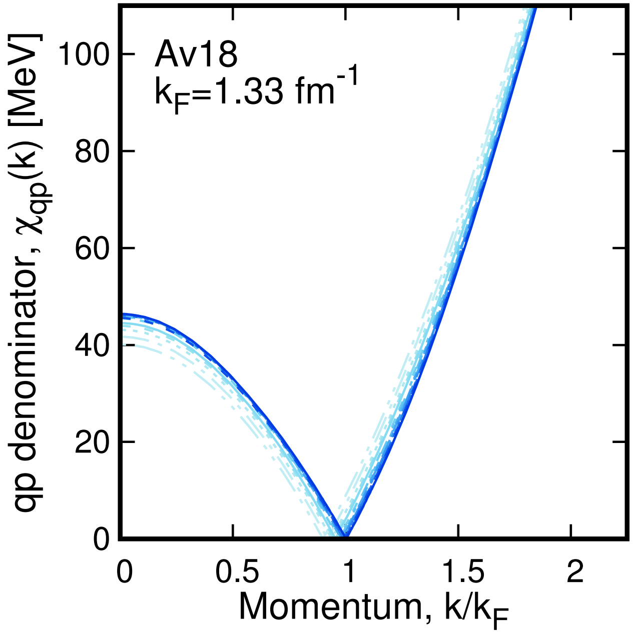
$$V^S_{L=0} \approx t_0^i \rho^{\alpha_i} \left[1 - (-)^S x_0^i \right]$$

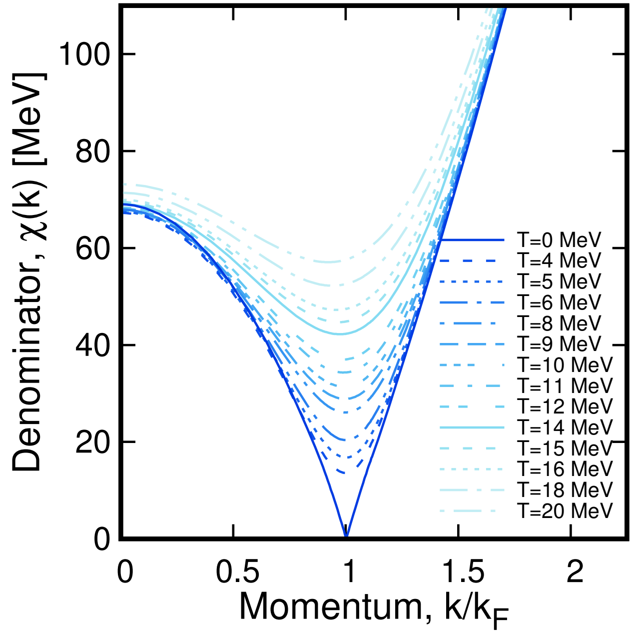
a) we neglect it
b) will be there in nuclei!

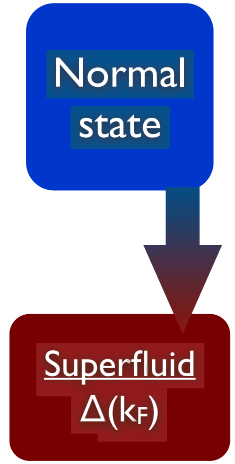
BCS equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'} \quad \mathbf{+} \quad \chi_k = \varepsilon_k - \mu$$

Beyond-BCS: SRCs







BCS gap equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

$$+ \quad \frac{1}{2\bar{\chi}_k} = \frac{1}{2|\varepsilon_k - \mu|}$$

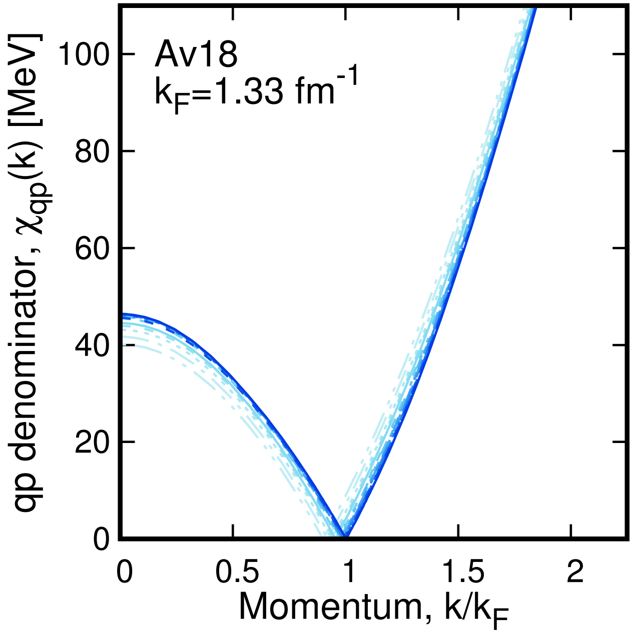
BCS+SRC gap equation

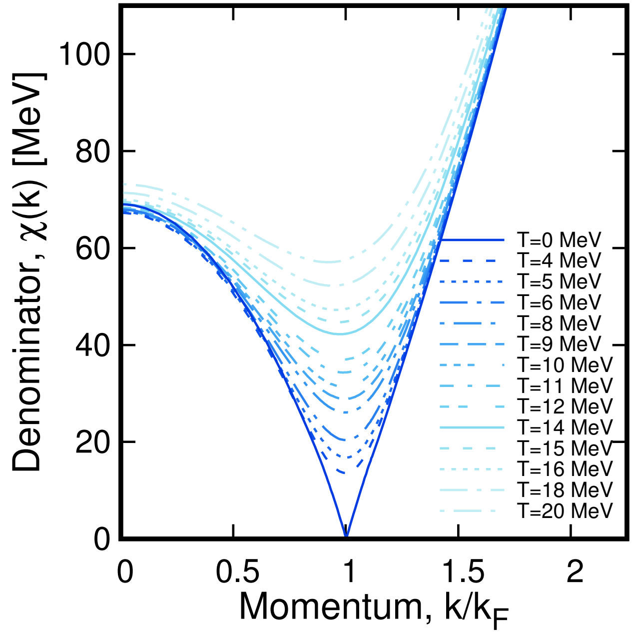
$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\bar{\chi}_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

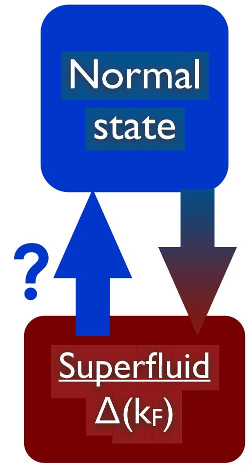
$$+ \quad \frac{1}{2\bar{\chi}_k} = \int_\omega \int_{\omega'} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A(k,\omega) A_s(k,\omega')$$

Bozek, PRC **62** 054316 (2000)
Muther & Dickhoff, PRC **72** 054313 (2005)
Rios, Polls & Dickhoff, J Low T Phys **189** 234 (2017)

Beyond-BCS: SRCs







BCS gap equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\chi_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

$$+ \quad \frac{1}{2\bar{\chi}_k} = \frac{1}{2|\varepsilon_k - \mu|}$$

BCS+SRC gap equation

$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\langle k | V_{nn}^{LL'} | k' \rangle}{2\sqrt{\bar{\chi}_{k'}^2 + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

$$+ \quad \frac{1}{2\bar{\chi}_k} = \int_\omega \int_{\omega'} \frac{1 - f(\omega) - f(\omega')}{\omega + \omega'} A(k,\omega) A_s(k,\omega')$$

Bozek, PRC **62** 054316 (2000)
Muther & Dickhoff, PRC **72** 054313 (2005)
Rios, Polls & Dickhoff, J Low T Phys **189** 234 (2017)

Beyond-BCS: LRCs

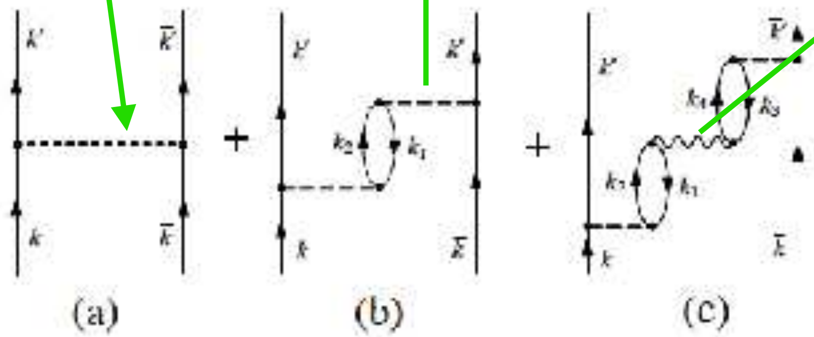
$$\Delta_k^L = -\sum_{L'} \int_{k'} \frac{\boxed{\langle k | V_{nn}^{LL'} | k' \rangle}}{2\sqrt{\boxed{\chi_{k'}^2} + |\Delta_{k'}|^2}} \Delta_{k'}^{L'}$$

?

ph recoupled G-matrix

Effective Landau parameters

$\mathcal{V}_{\text{pair}}=$



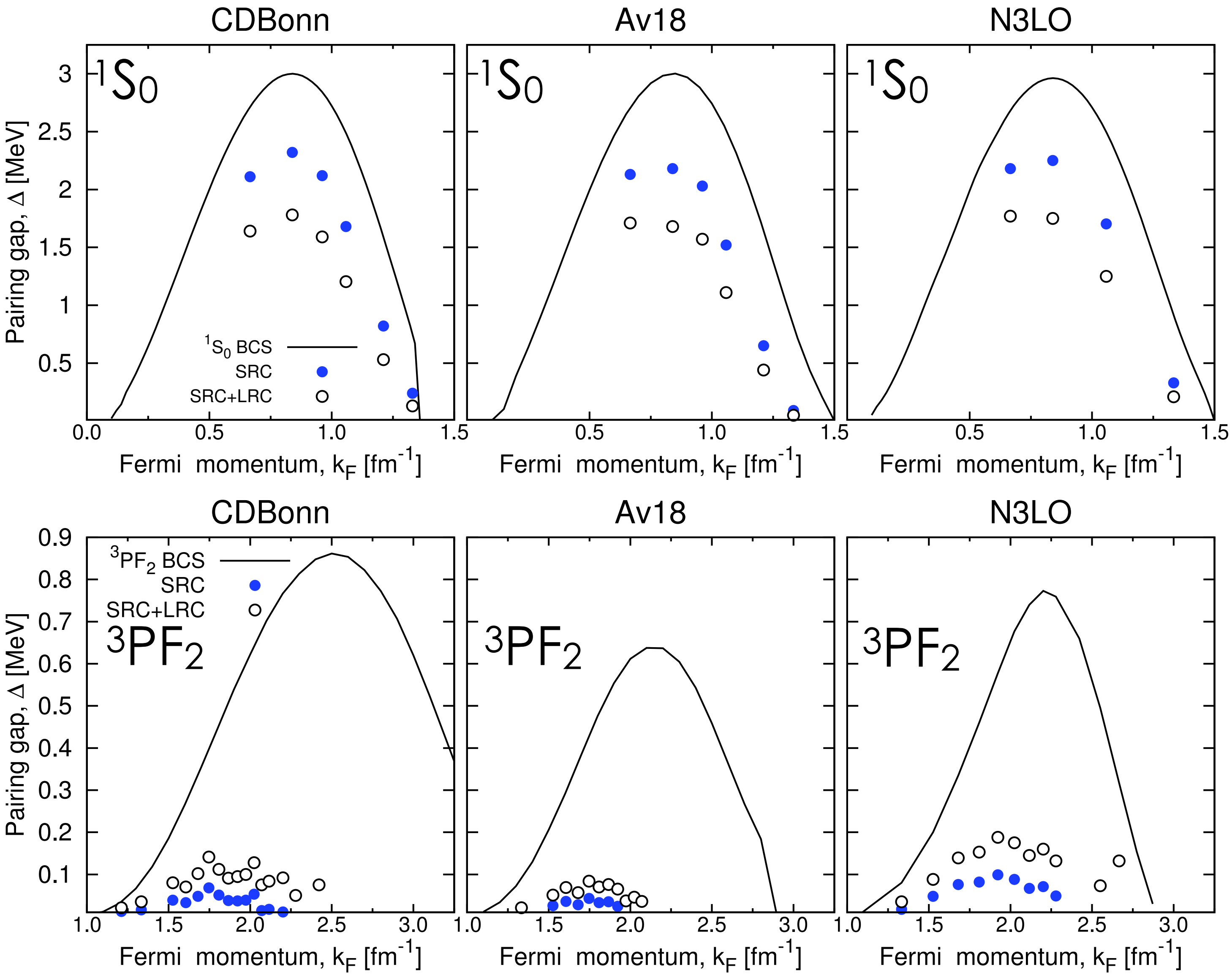
$$\langle 1\bar{1}|\mathcal{V}|1\bar{1}\rangle = \frac{1}{4}\sum_{2,2'}\sum_{S,T}(-)^S(2S+1)\langle 12|G_{ST}^{\text{ph}}|1'2'\rangle_A \langle 2'\bar{1}|G_{ST}^{\text{ph}}|2\bar{1}'\rangle_A \Lambda(22')$$

$$\Lambda_{ST}(q) = \frac{\Lambda_{ST}^0(q)}{1-\Lambda_{ST}^0(q)\times F_{ST}}$$

- Bare NN potential only is **not** the only possible interaction
- Diagram (a): nuclear interaction
- Diagram (b): in-medium interaction, density and spin fluctuations
- Diagram (c): included by Landau parameters

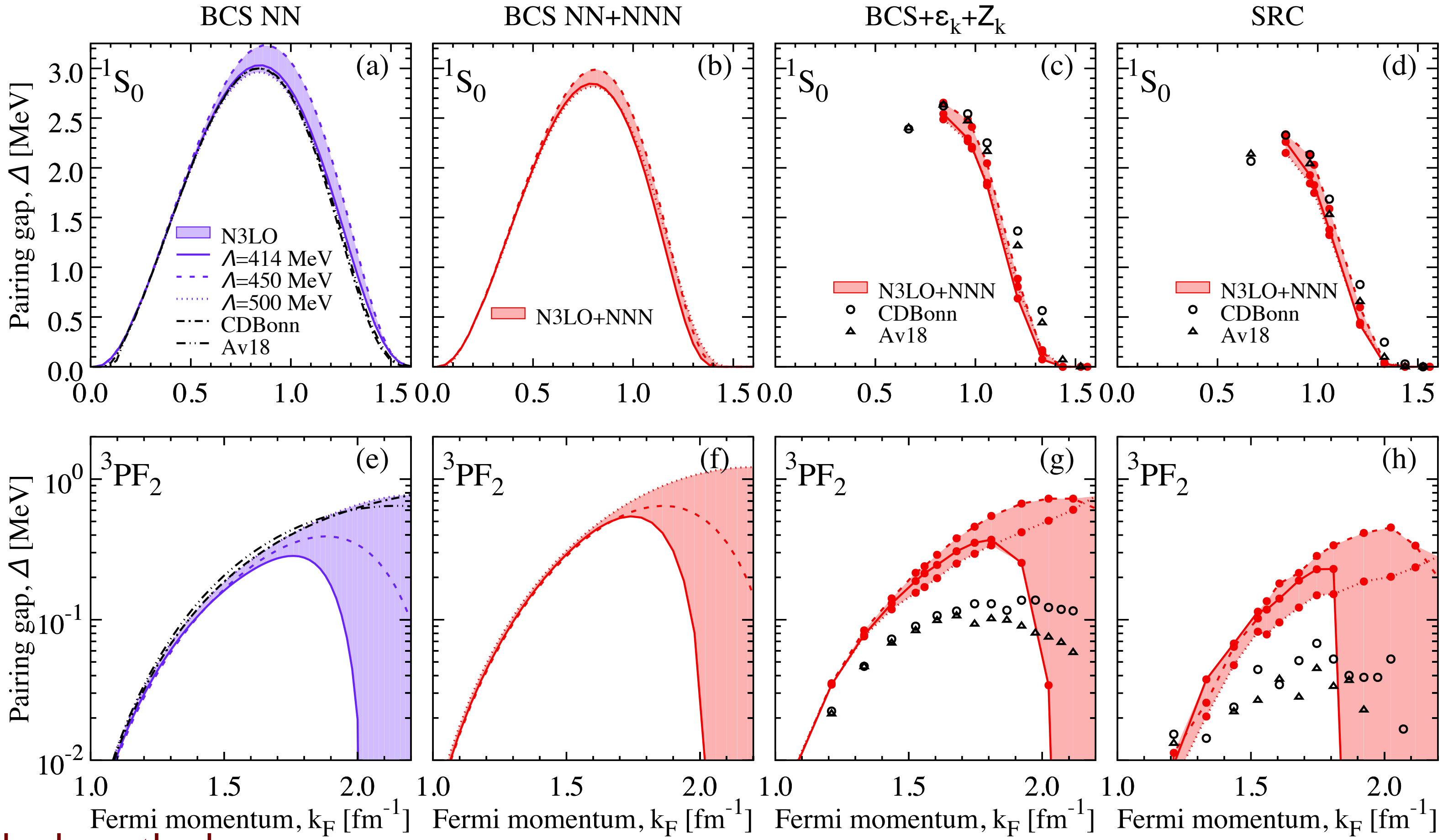
Cao, Lombardo & Schuck, *PRC* **74** 064301 (2006)

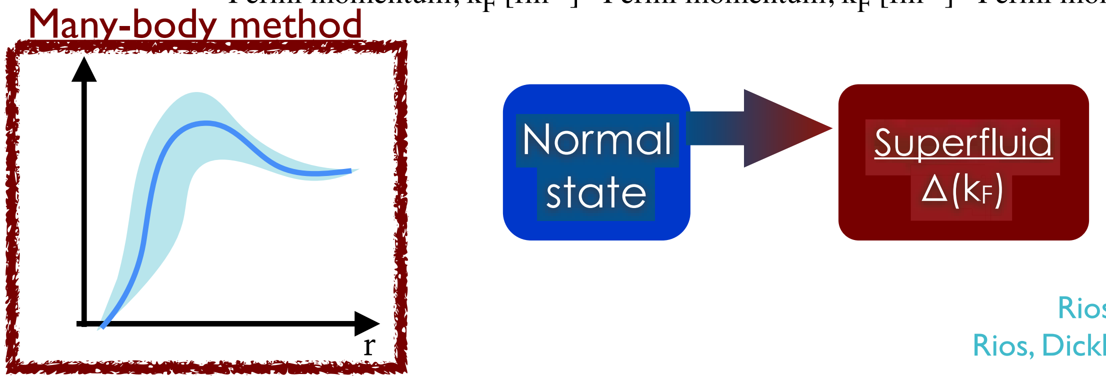
Results



Ding, Rios, Dussan, Dickhoff, Witte, Carbone & Polls, PRC **94** 025802 (2016)

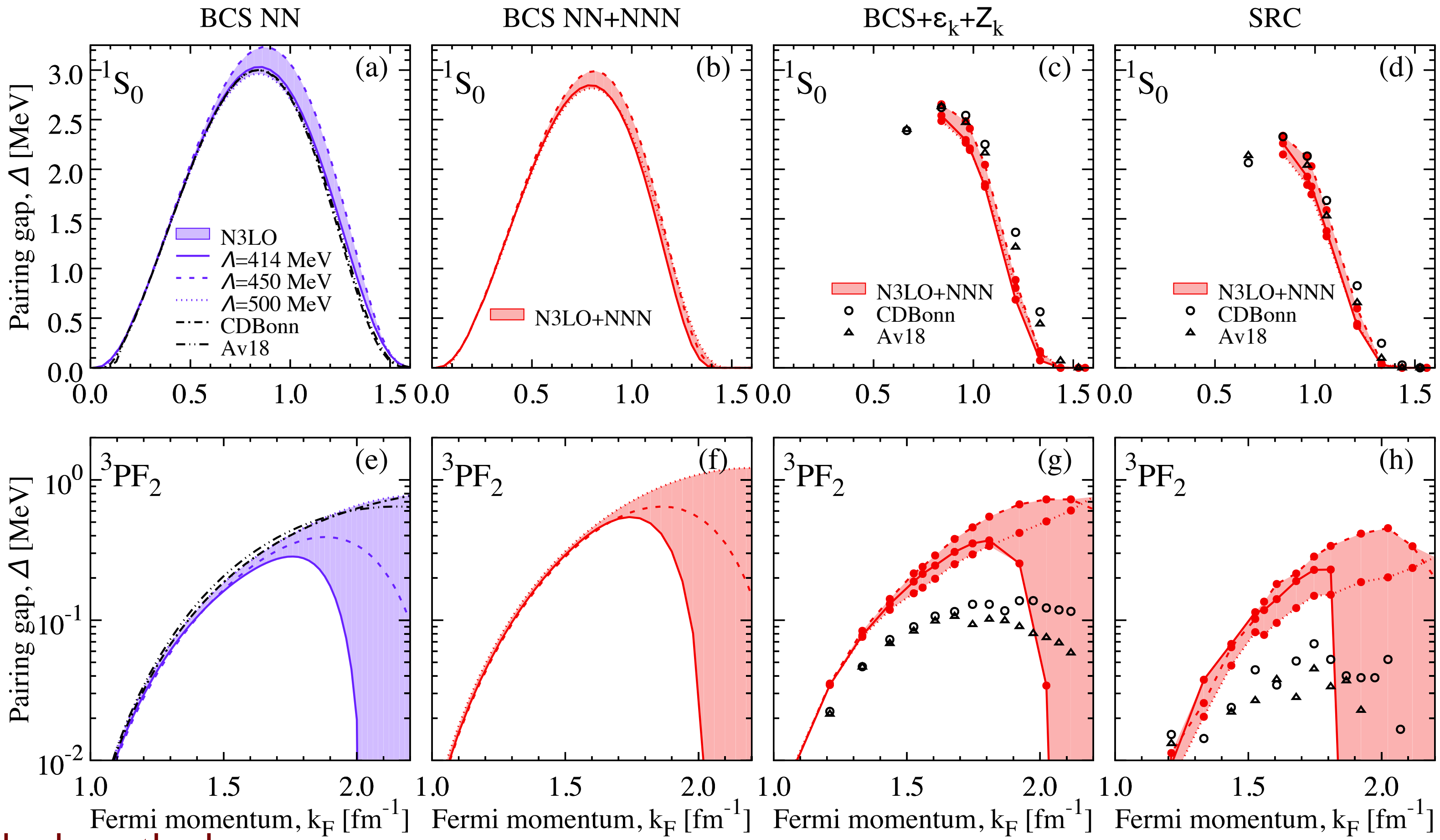
Beyond-BCS in neutron matter: SRC

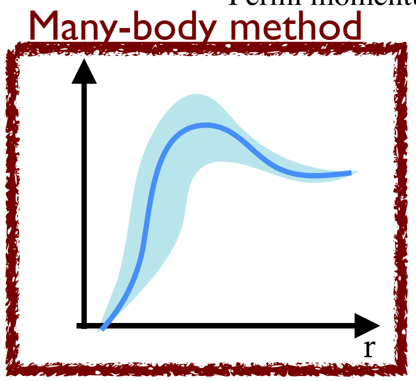


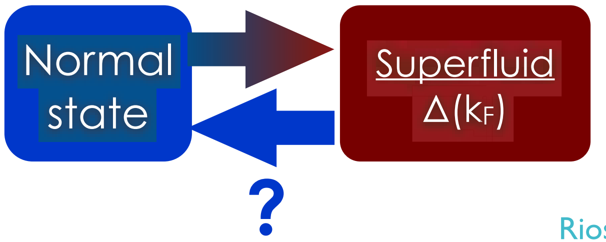


Rios, Dickhoff, Polls, JLTP **189**, 234 (2017) [arXiv:1707.04140]
Rios, Dickhoff, Polls, et al PRC **94,** 025802 (2016 [arxiv:1601.01600]

Beyond-BCS in neutron matter: SRC

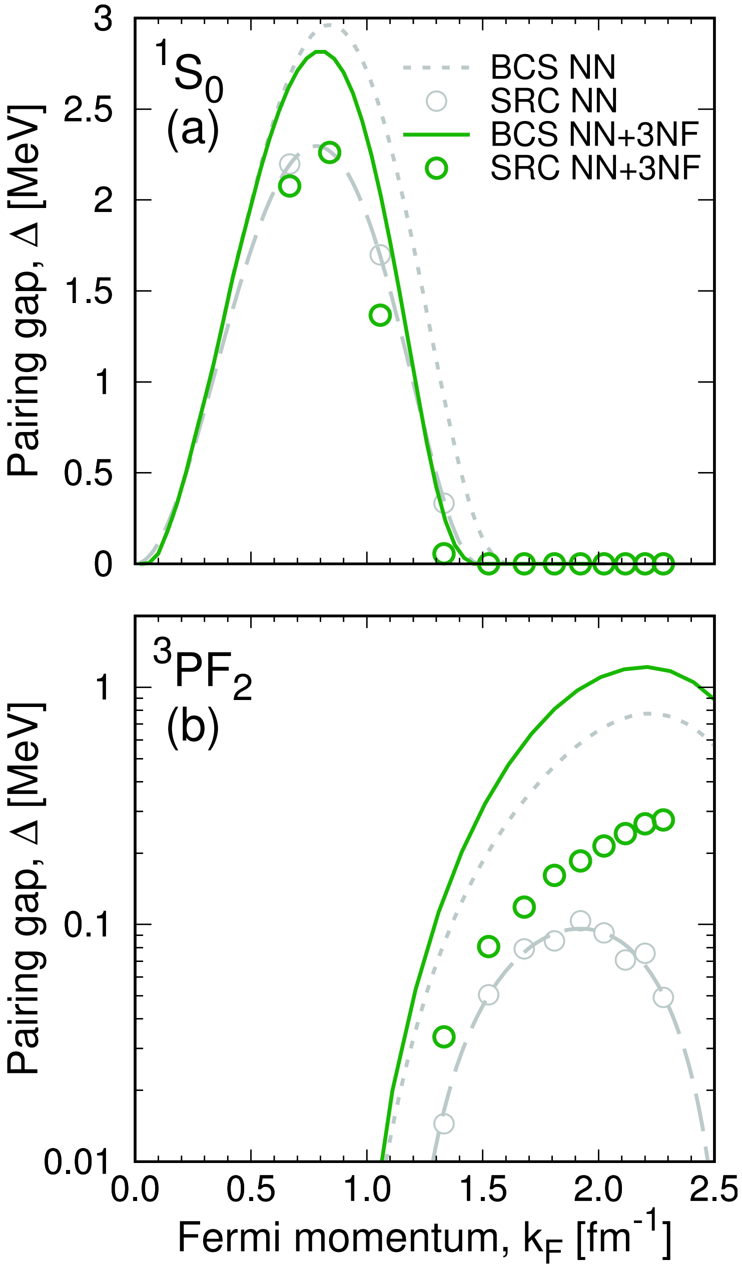




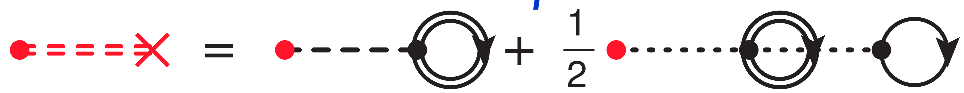


Rios, Dickhoff, Polls, JLTP **189**, 234 (2017) [arXiv:1707.04140]
Rios, Dickhoff, Polls, et al PRC **94,** 025802 (2016 [arxiv:1601.01600]

Role of 3NFs



Effective one-body force ⇒*spectrum*

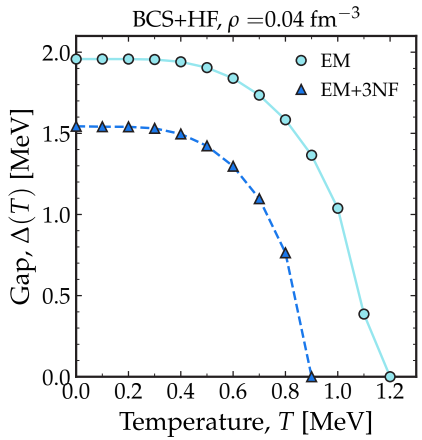


Effective two-body force ⇒*NN forces*

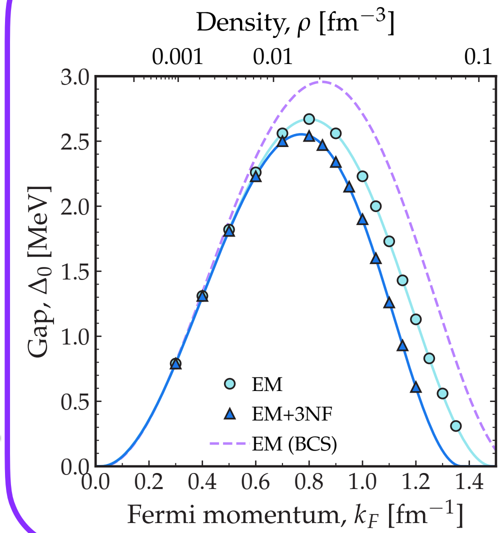
- Singlet gap: 3NF **reduce** closure
- Triplet gap: 3NF **increase** gap
- **Model dependence** to be explored
- Use **SRG** for systematics (Michael!)

Ding, Rios, Dussan, Dickhoff, Witte, Carbone & Polls, PRC **94** 025802 (2016)

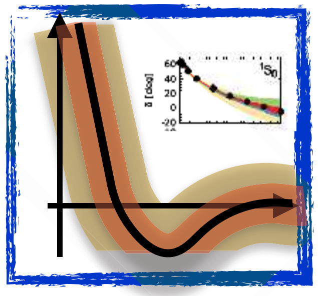
Finite Temperature BCS



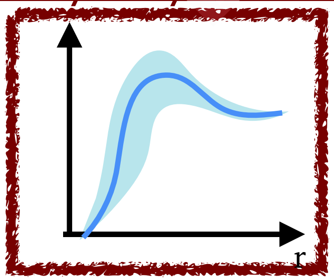
BCS+HF



Hamiltonian

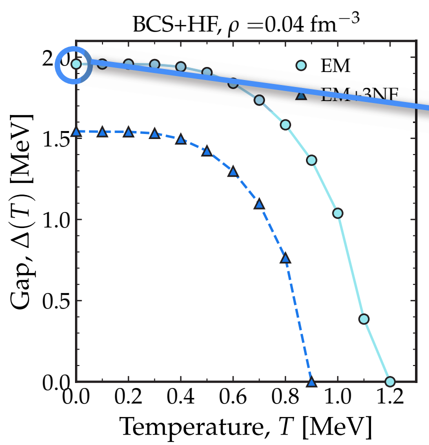


Many-body method

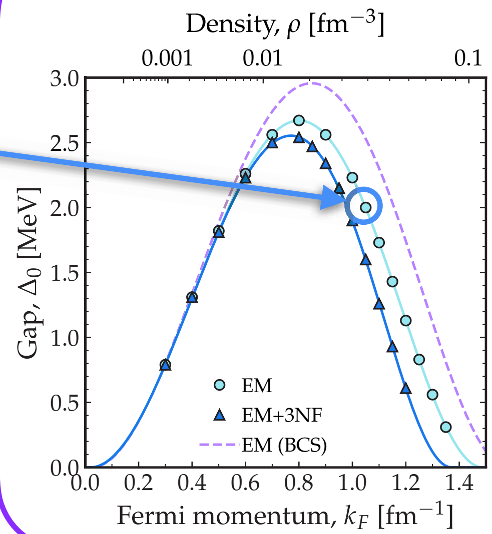


Drissi & Rios, Eur. Phys. J. A 58, 90 (2022) [arXiv:2202.07501]

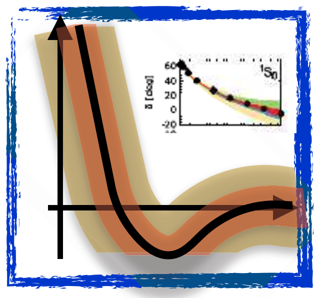
Finite Temperature BCS



BCS+HF



Hamiltonian



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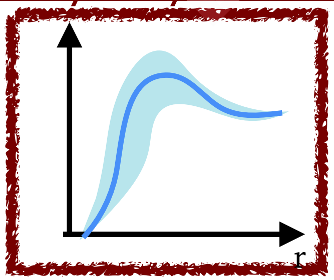

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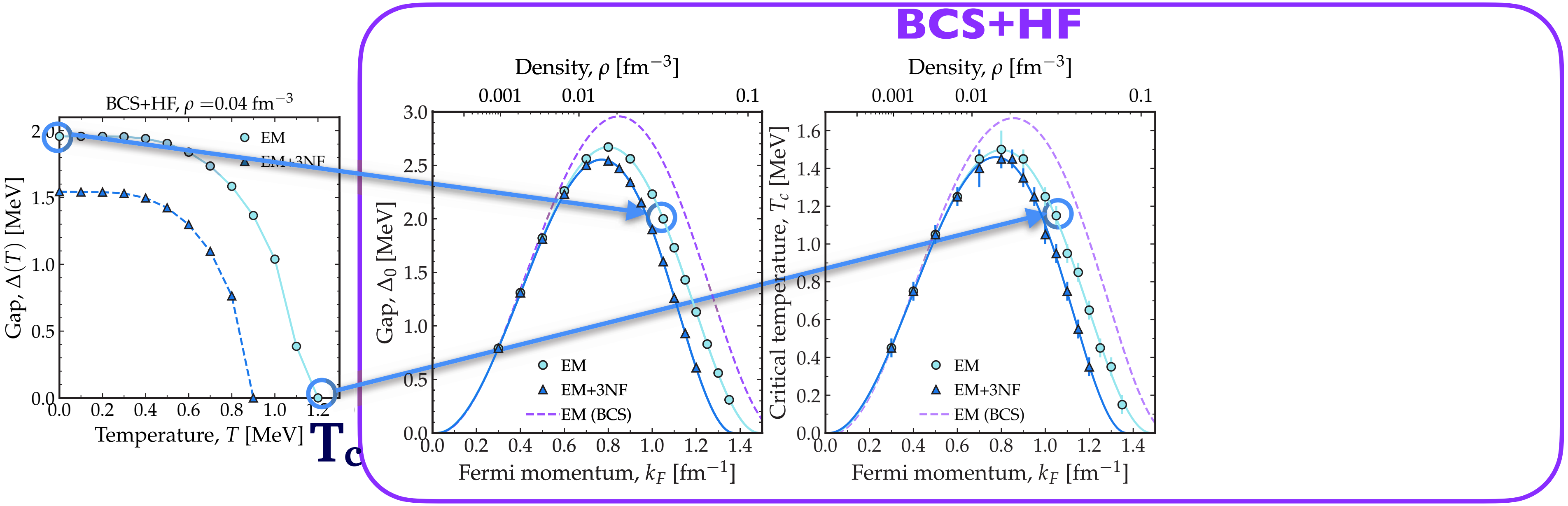

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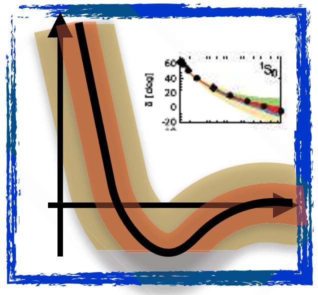

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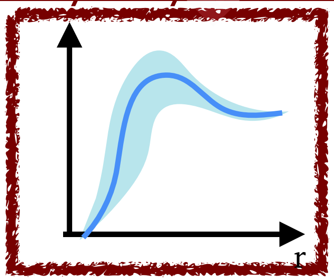

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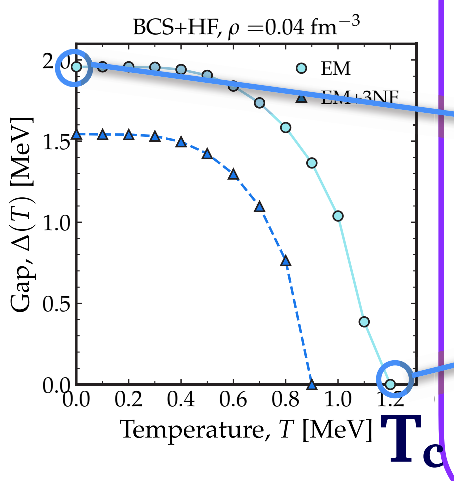
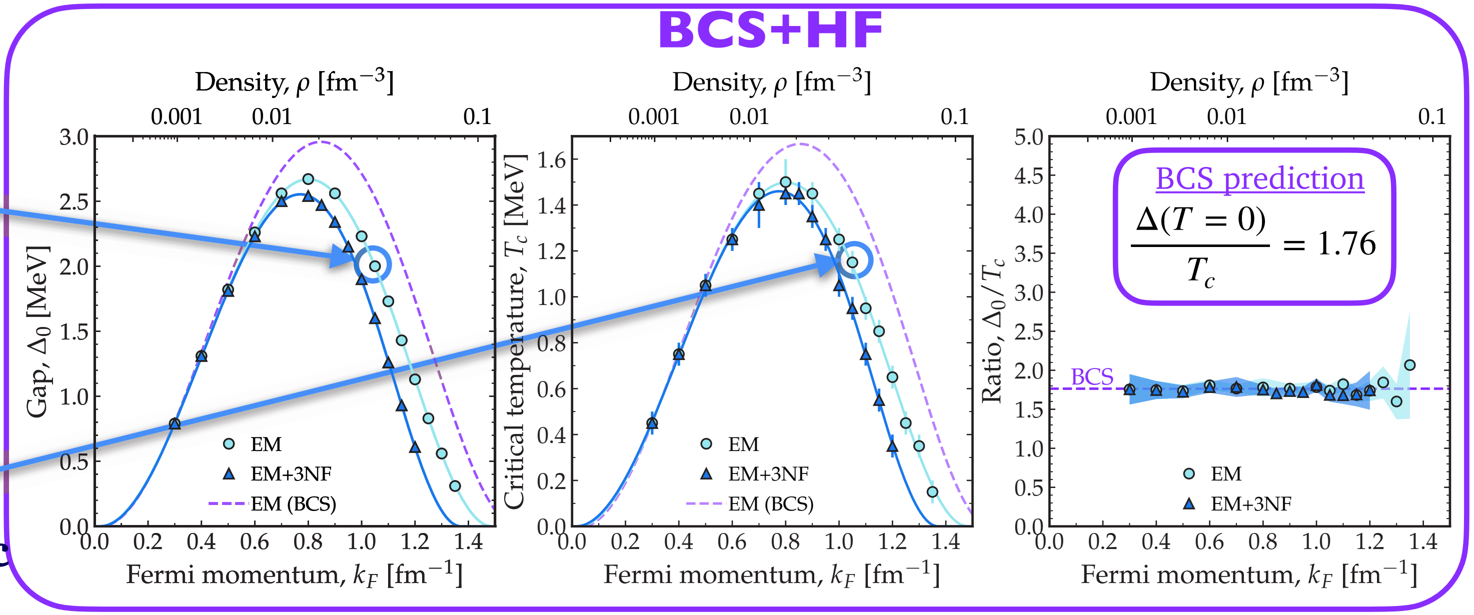
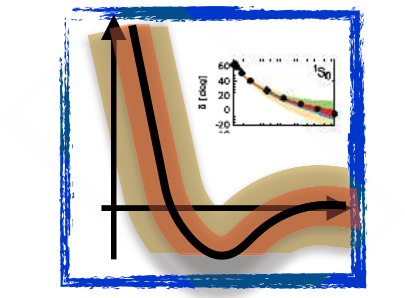
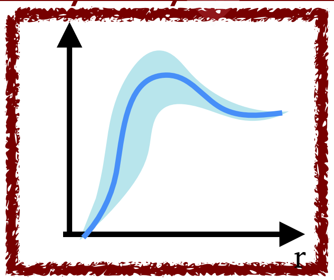




Hamiltonian



Many-body method



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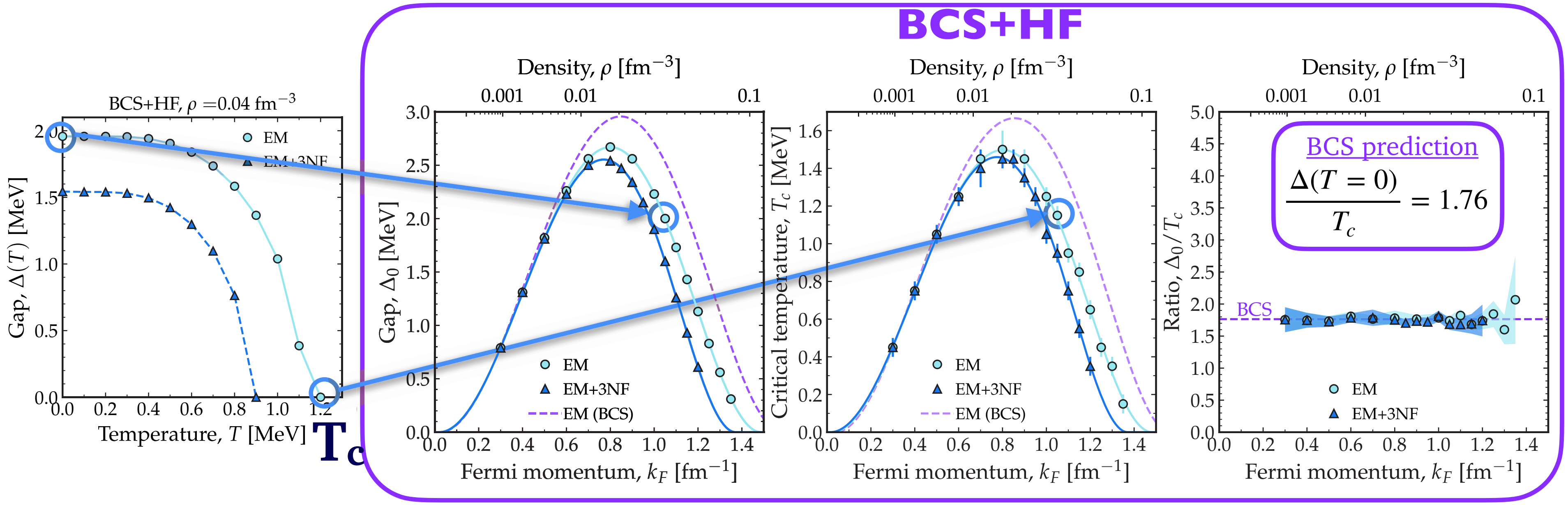
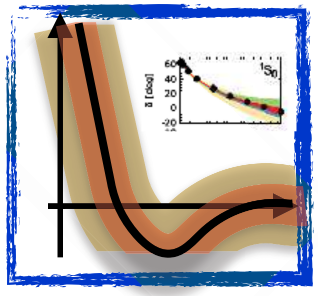
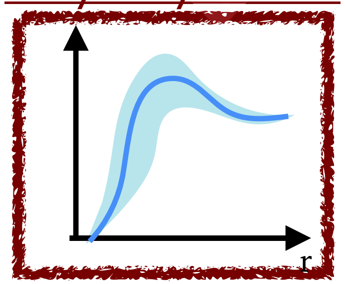
Finite Temperature BCS



Hamiltonian



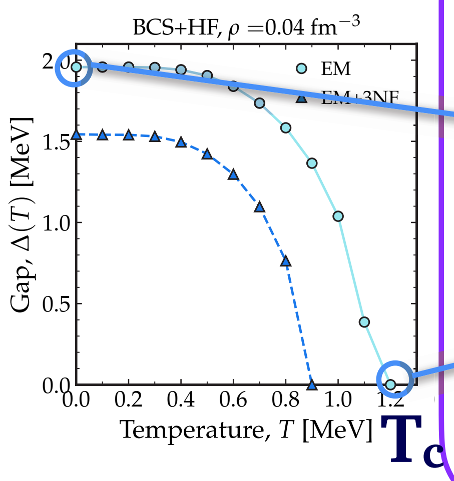
Many-body method



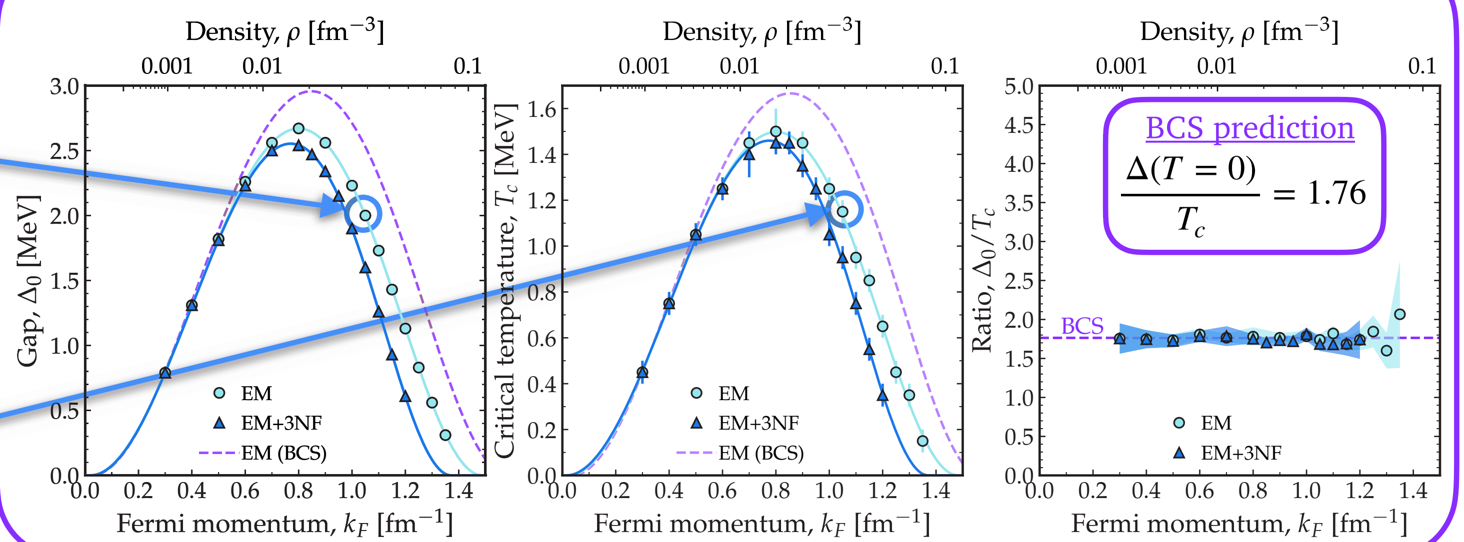
- **BCS** prediction based on **constant V**
- Full chiral **NN** interactions
- Full **HF spectrum**
- 3NFs do not change the ratio!

Drissi & Rios, Eur. Phys. J.A 58, 90 (2022) [arXiv:2202.07501]

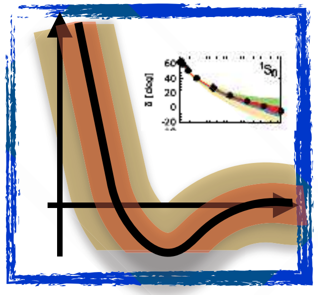
Finite Temperature beyond-BCS



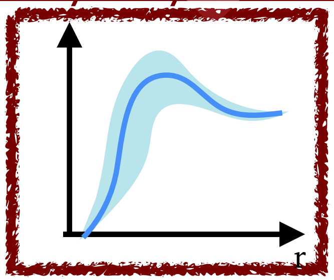
BCS+HF



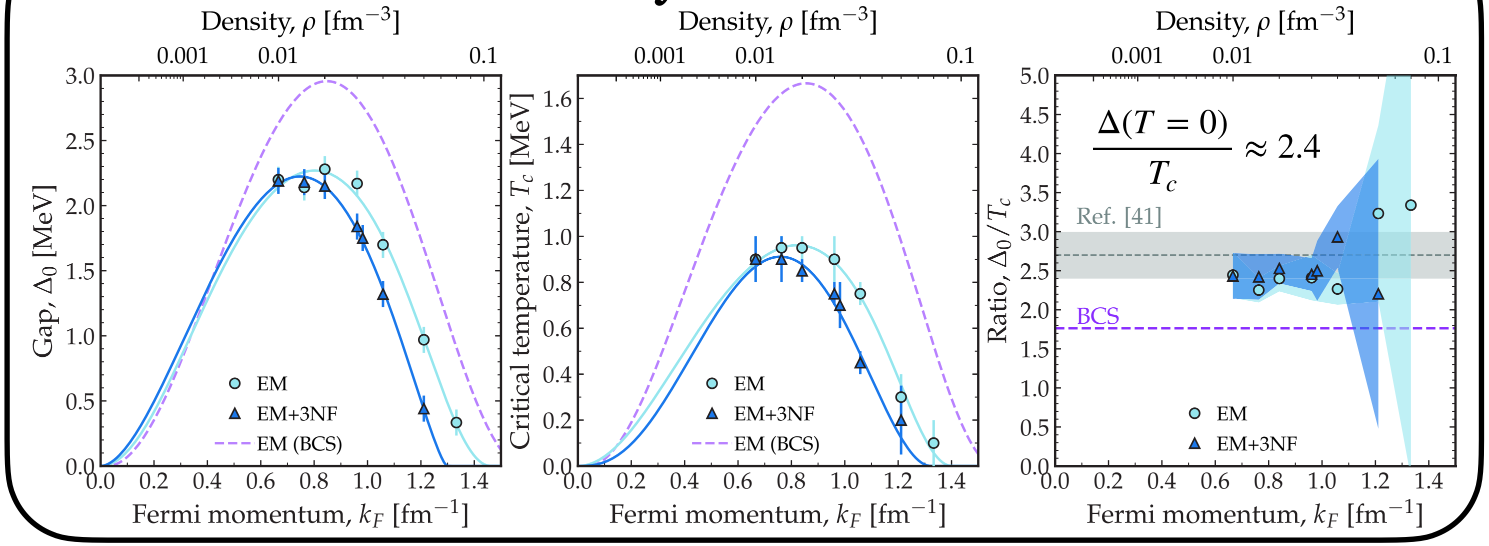
Hamiltonian



Many-body method

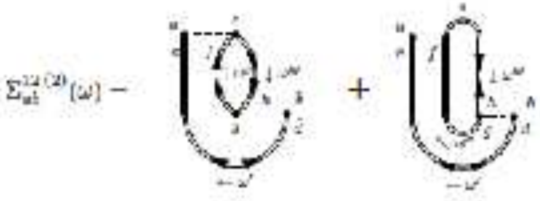


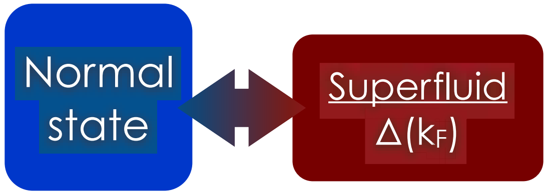
"Beyond"-BCS



Drissi & Rios, Eur. Phys. J. A 58, 90 (2022) [arXiv:2202.07501]

Ho do we go consistently beyond BCS?





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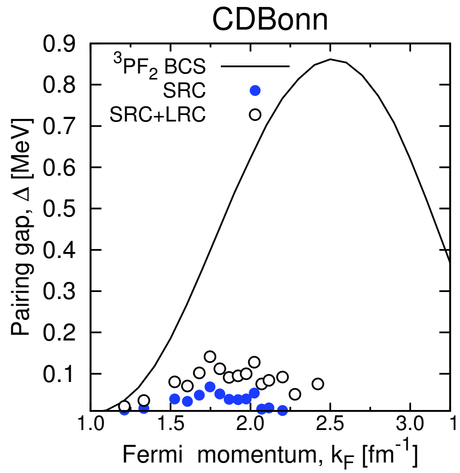

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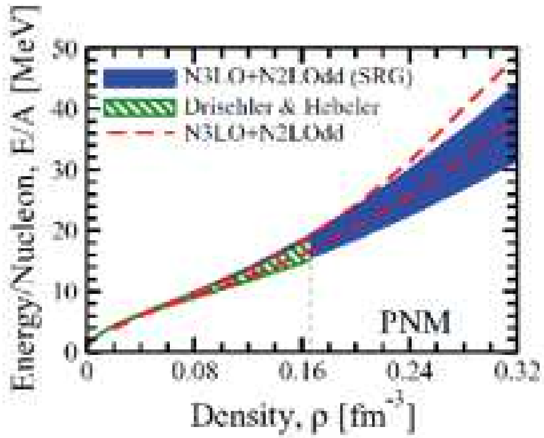
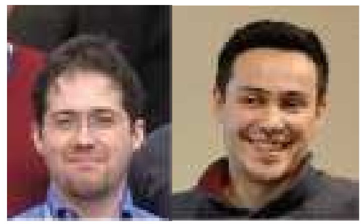
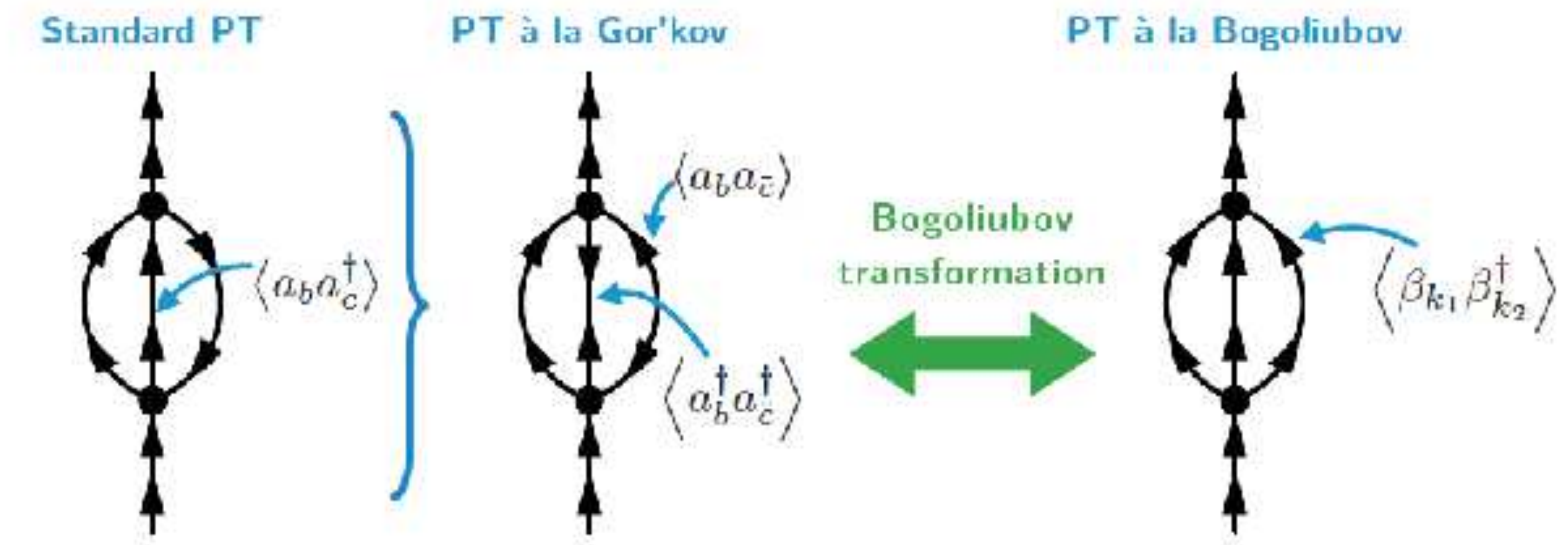


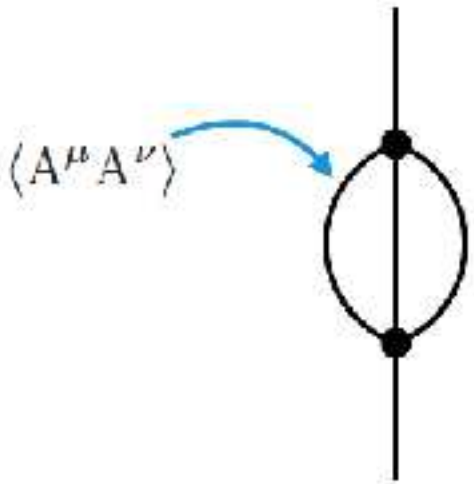
- **Existing frameworks** difficult to generalise
- **Nambu-covariant SCGF technique**
  - Symmetry breaking ✔
  - Finite temperature ✔
  - Systematic expansion w diagrams ✔
  - 3 nucleon forces ✔



Barbieri, Drissi

What was the issue before?





Reformulation

- Based on Nambu fields
- Propagators transform contravariantly
- Vertices transform covariantly
- Un-oriented diagrammatic

Drissi, Rios & Barbieri, Paper 1, Ann. Phys. 469, 169729 (2024) + arxiv:2107.09759

Nambu-Covariant Perturbation Theory

Double dimension H space

$$\mathscr{H}_1^e \equiv \mathscr{H}_1 \times \mathscr{H}_1^\dagger$$

Basis

$$\mathcal{B}^e \equiv \mathcal{B} \cup \bar{\mathcal{B}}$$

$$|b\rangle \qquad \langle \bar{b}|$$

Elements

$$\begin{pmatrix} |\Psi_1\rangle \\ \langle \Psi_1'| \end{pmatrix}$$

$$\mu = (b, l) \qquad \bar{\cdot} : 1 \mapsto \bar{1} = 2$$

$$\bar{\mu} \equiv (b, \bar{l}) \qquad 2 \mapsto \bar{2} = 1$$

$$|b, 1\rangle \equiv \begin{pmatrix} |b\rangle \\ 0 \end{pmatrix} \quad |b, 2\rangle \equiv \begin{pmatrix} 0 \\ \langle \bar{b}| \end{pmatrix}$$

Product & metric tensor

$$g\left(\begin{pmatrix} |\Psi_1\rangle \\ \langle \Psi_1'| \end{pmatrix}, \begin{pmatrix} |\Psi_2\rangle \\ \langle \Psi_2'| \end{pmatrix} \right) \equiv \langle \Psi_2' \mid \Psi_1 \rangle + \langle \Psi_1' \mid \Psi_2 \rangle$$

$$g_{\mu\nu} \equiv g\left(|\mu\rangle, |\nu\rangle\right) = \delta_{\mu\bar{\nu}}$$

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Nambu fields

Nambu fields

$$\mathrm{A}^{(b,1)} \equiv a_b \ , \qquad \mathrm{A}^{(b,2)} \equiv \bar{a}_b \ , \qquad \bar{\mathrm{A}}_{(b,1)} \equiv \bar{a}_b \ , \qquad \bar{\mathrm{A}}_{(b,2)} \equiv a_b \ .$$

$$\mathrm{A}^{\mu} \equiv A^{(b,g)} = \begin{pmatrix} a_b \\ a^{\dagger}_{\bar{b}} \end{pmatrix}$$

$$\bar{\mathrm{A}}_{\mu} \equiv \bar{A}_{(b,g)} = \begin{pmatrix} a^{\dagger}_{\bar{b}} & a_b \end{pmatrix}$$

$$\boxed{\bar{a}_b = a^{\dagger}_{\bar{b}} \neq a^{\dagger}_{b}}$$

Commutator relations

(On extended indices!)

$$\{ \mathrm{A}^{\mu}, \mathrm{A}^{\nu} \} = g^{\mu\nu} \ ,$$
$$\{ \mathrm{A}^{\mu}, \bar{\mathrm{A}}_{\nu} \} = g^{\mu}{}_{\nu} \ ,$$
$$\{ \bar{\mathrm{A}}_{\mu}, \mathrm{A}^{\nu} \} = g_{\mu}{}^{\nu} \ ,$$
$$\{ \bar{\mathrm{A}}_{\mu}, \bar{\mathrm{A}}_{\nu} \} = g_{\mu\nu}$$

Co- or contravariant

$$\bar{\mathrm{A}}_{\mu} = \sum_{\nu} g_{\mu\nu} \ \mathrm{A}^{\nu} \ ,$$
$$\bar{\mathrm{A}}_{\mu} = \sum_{\nu} g_{\mu}{}^{\nu} \ \bar{\mathrm{A}}_{\nu} \ ,$$
$$\mathrm{A}^{\mu} = \sum_{\nu} g^{\mu\nu} \ \bar{\mathrm{A}}_{\nu} \ ,$$
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Perturbative construction

Hamiltonian partitioning

$$\Omega = \Omega_0 + \Omega_1$$

$$\Omega_0 = \frac{1}{2} \sum_{\mu\nu} U_{\mu\nu} \mathrm{A}^{\mu} \mathrm{A}^{\nu}$$

$$\Omega_1 = \sum_{k=1}^{n} \frac{1}{(2k)!} \sum_{\mu_1 \dots \mu_{2k}} v^{(k)}_{\mu_1 \dots \mu_{2k}} \mathrm{A}^{\mu_1} \dots \mathrm{A}^{\mu_{2k}}$$

Covariant **k-body** vertices

Fully antisymmetric vertex

- **Definition**

$$v^{(k)}_{[\mu_1\, \mu_2\, \dots\, \mu_{2k-1}\, \mu_{2k}]} \equiv \frac{1}{(2k)!} \sum_{\sigma \in S_{2k}} \epsilon(\sigma)\; v^{(k)}_{\mu_{\sigma(1)}\, \mu_{\sigma(2)}\, \dots\, \mu_{\sigma(2k-1)}\, \mu_{\sigma(2k)}}$$

- **Antisymmetrisation** defines a new *(0,2k)*-tensor
- ***Not*** the case in a *mixed* representation

Green's functions

- Contravariant **k-body** Green's function

$$(-1)^k \, \mathcal{G}^{\mu_1 \dots \mu_{2k}}(\tau_1, \dots, \tau_{2k}) \equiv \left\langle \mathrm{T} \left[\mathrm{A}^{\mu_1}(\tau_1) \;\dots\; \mathrm{A}^{\mu_{2k}}(\tau_{2k}) \right] \right\rangle$$

with $\langle \,.\, \rangle = \mathrm{Tr} \left(.\, \rho \right)$ and $\rho \equiv \dfrac{e^{-\beta\Omega}}{\mathrm{Tr}\left(e^{-\beta\Omega}\right)}$

- Unperturbed case: $\Omega \longleftrightarrow \Omega_0$

Propagators

$-\mathcal{G}^{\mu\nu}(\omega_p) =$

$-\left(\mathcal{G}^{(0)}\right)^{\mu\nu}(\omega_p) =$

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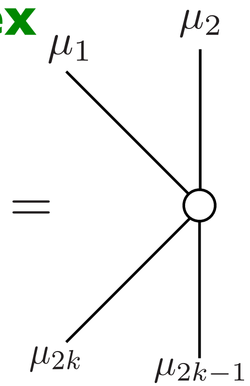
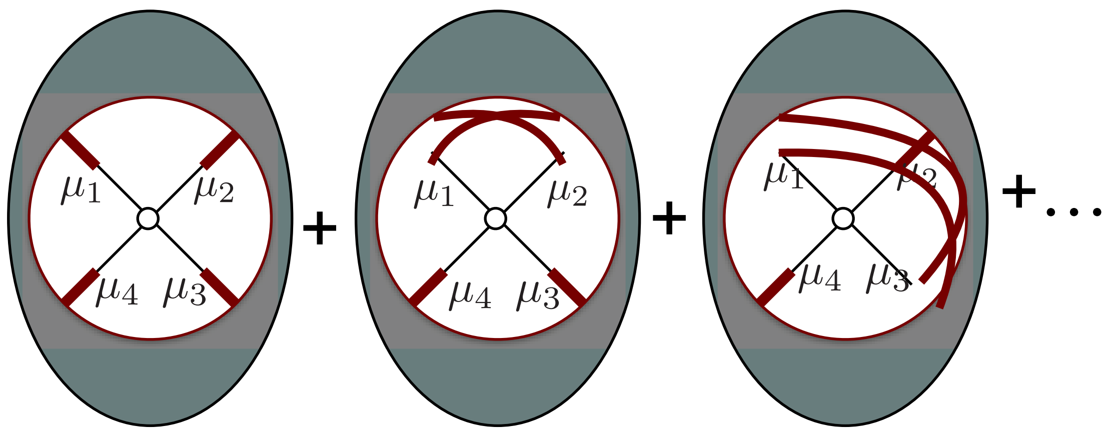
Why antisymmetric vertices?

Un-symmetrised vertex

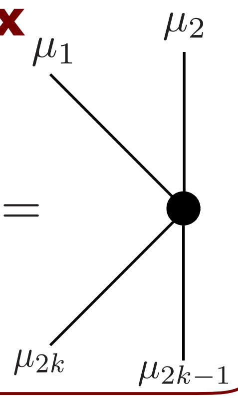
$$v^{(k)}_{\mu_1 \mu_2 \dots \mu_{2k-1} \mu_{2k}} =$$





Antisymmetrized vertex

$$v^{(k)}_{[\mu_1 \mu_2 \dots \mu_{2k-1} \mu_{2k}]} =$$



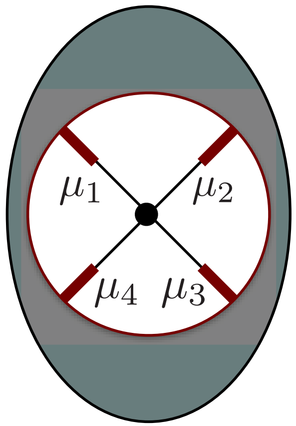


Diagram factorisation

- Derivations rely on
  - Wick theorem $\Rightarrow$ sum over pairing
  - Sum over single-particle and Nambu indices

➡ **Extends Hugenholtz antisymmetrisation**

- Antisym is a **one-off pre-computing** cost

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Perturbative expansion

Order n graphical rules

- Draw all topologically distinct **connected unlabelled** diagrams
  - with ***2k* external** legs
  - with ***n* vertices** (for order n contributions)

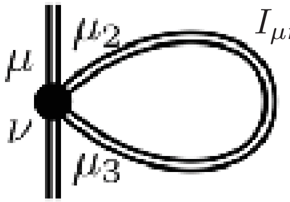
Feynman rules

1. Label vertices from *1* to n
   - S is the number of vertex labels permutations leaving the diagram invariant
2. For each line multiply by $-\left(\mathscr{G}^{(0)}\right)^{\mu\nu}(\omega_e)$
3. For each k-body vertex multiply by $v^{(k)}_{[\mu_1\ \mu_2\ \ldots\ \mu_{2k-1}\ \mu_{2k}]}$
4. Sum over each internal μ index and each independent ω_e frequency
5. Multiply by $\dfrac{(-1)^{n+L}}{S \times 2^T\ \prod_{l=2}^{l\text{max}} (l!)^m}$

Gaudin rules

- These simplify **Matsubara sums**
- Require **spanning trees**

Tadpoles are exceptional



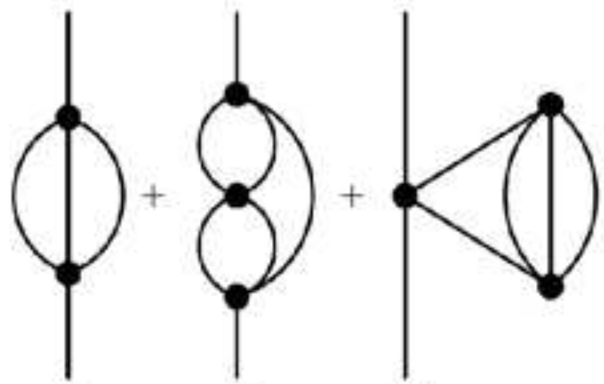
$$I_{\mu\nu} = \sum_{\mu_2\ldots\mu_{2k-1}} \frac{(-1)^k}{2^{k-1}(k-1)!} v^{(k=2)}_{[\mu\ \dot{\mu}_2\ \dot{\mu}_3\ \nu]} \times \frac{1}{\beta}\sum_{\omega_e} -\mathcal{G}^{\mu_2\mu_3}(\omega_e) e^{-i\omega_e \eta_p}$$

- **Partially antisymmetrized** vertices needed:

$$v^{(k)}_{[\mu_1\ldots\dot{\mu}_x\ldots\dot{\mu}_y\ldots\mu_{2k}]} \equiv \frac{2^p p!}{(2k)!} \sum_{\sigma\in S_{2k}/S_2^p\times S_p} \epsilon(\sigma)\ v^{(k)}_{\mu_{\sigma(1)}\ldots\dot{\mu}_x\ldots\dot{\mu}_y\ldots\mu_{\sigma(2k)}}$$

- p internal lines are **fixed**
- ***k*-body generalisation** works

HFB partitioning 3rd order

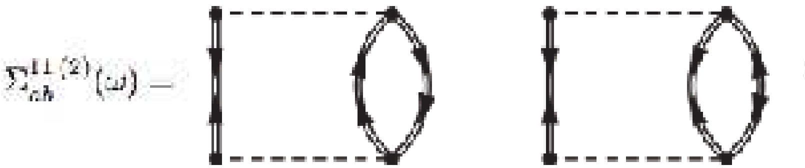


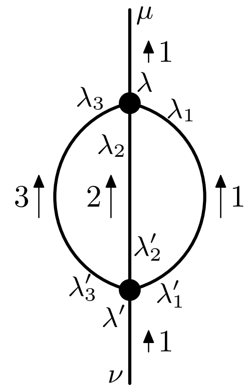
$\mathscr{G}_2$ + $\mathscr{G}_3$ + $\mathscr{G}'_3$

Advantages vs Gorkov

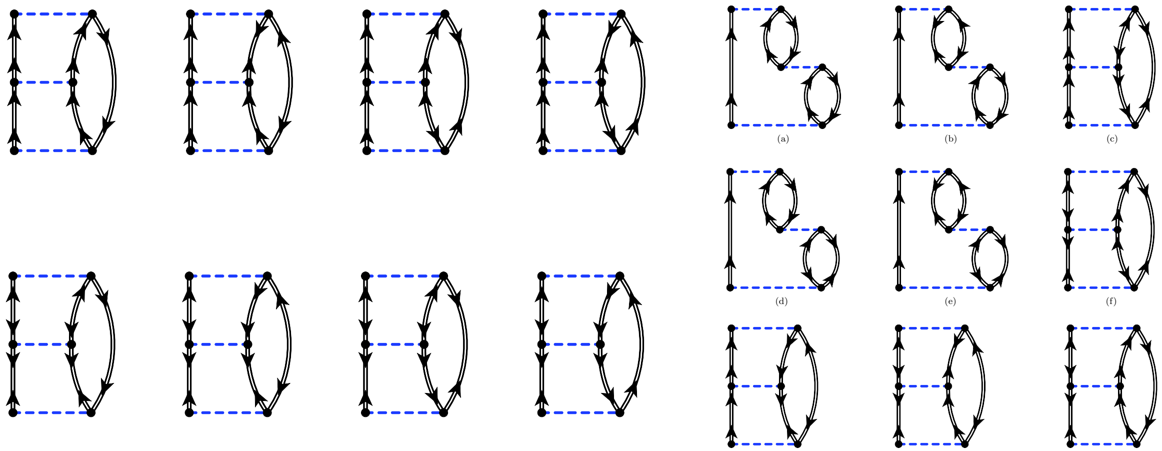
Gorkov GF	NCGF

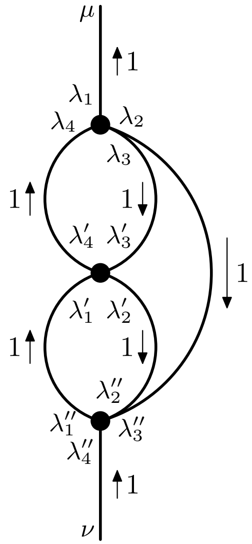
Order 2





Order 3





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Self-consistent Green's function resummation

Dyson equation

- Partitioning considered

$$\Omega = \underbrace{\frac{1}{2!}\sum_{\mu\nu} U_{\mu\nu}\ \mathrm{A}^{\mu}\mathrm{A}^{\nu}}_{\Omega_0} + \underbrace{\frac{1}{4!}\sum_{\alpha\beta\gamma\delta} v^{(2)}_{\alpha\beta\gamma\delta}\ \mathrm{A}^{\alpha}\mathrm{A}^{\beta}\mathrm{A}^{\gamma}\mathrm{A}^{\delta}}_{\Omega_1}$$

- Dyson equation

$$\mathscr{G}^{\mu\nu}(\omega_n) = \mathscr{G}^{(0)\mu\nu}(\omega_n) + \sum_{\lambda_1\lambda_2} \mathscr{G}^{(0)\mu\lambda_1}(\omega_n)\ \Sigma_{\lambda_1\lambda_2}(\omega_n)\ \mathscr{G}^{\lambda_2\nu}(\omega_n)$$

Diagrammatic expansion of $\Sigma_{\mu\nu}(\omega_n)$

- with unperturbed propagators

$$\Sigma_{\mu\nu}(\omega_n) = \frac{\mathscr{I}_{\mu\nu}(\omega_n) - \mathscr{I}_{\nu\mu}(-\omega_n)}{2}$$

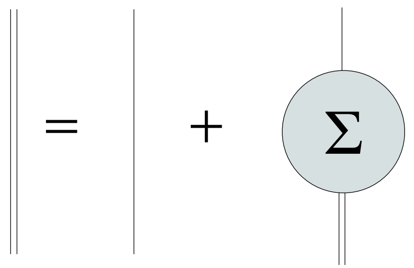
$$\mathscr{I}_{\mu\nu}(\omega_n) = \sum \text{1PI diagrams with } \mathscr{G}^{(0)}$$

- with self-consistent propagators

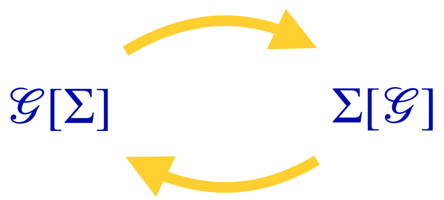
$$\Sigma_{\mu\nu}(\omega_n) = \frac{\mathscr{J}_{\mu\nu}(\omega_n) - \mathscr{J}_{\nu\mu}(-\omega_n)}{2}$$

$$\mathscr{J}_{\mu\nu}(\omega_n) = \sum \text{2PI diagrams with } \mathscr{G}\ \left(= \mathscr{J}_{\mu\nu}(\omega_n)\right)$$

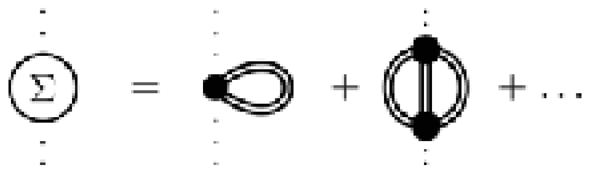
Diagrammatic representation



SCGF cycle



Self-energy expression



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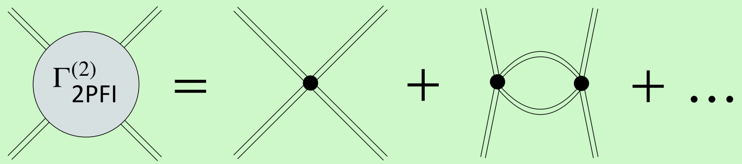
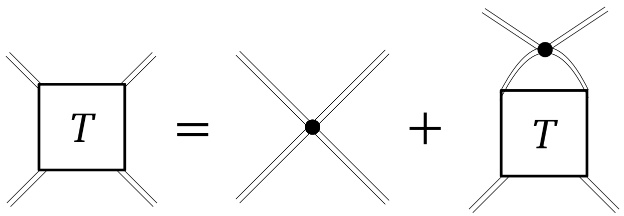
T-matrix: ladders

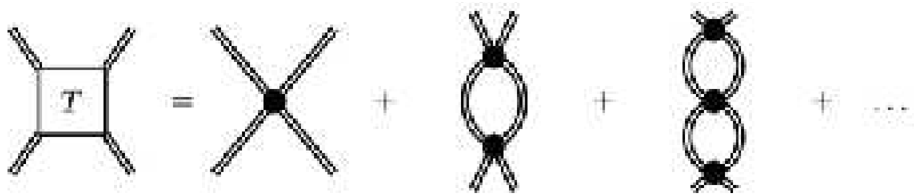
Approximations on $\Gamma^{(2)}_{\mathbf{2PFI}}$

- Sum of all possible rungs



T-matrix $\equiv \Gamma^{(2)}$ in ladder approximation





Ladder approximation

- Analytic/Retarded/Advanced/Sp function $\Rightarrow$ as usual
- T-matrix equation

$$T_{MN}(Z) \;=\; V^{(2)}_{MN} \;+\; \frac{1}{2}\sum_{LL'} V^{(2)}_{ML}\,\Pi^{LL'}(Z)\,T_{L'N}(Z)$$

where

$$V^{(2)}_{MN} \equiv v^{(2)}_{[\mu_1\mu_2\nu_1\nu_2]},\, M \equiv (\mu_1,\mu_2)\ \&\ N \equiv (\nu_1,\nu_2)$$

Solving the ladder

- Spectral representation

$$T_{MN}(Z) \equiv V^{(2)}_{MN} + \int_{-\infty}^{+\infty} \frac{\mathrm{d}\Omega}{2\pi}\,\frac{\mathcal{T}_{MN}(\Omega)}{Z-\Omega}$$

- Solution

$$\mathcal{T}(\Omega) = iV^{(2)}\left\{\left(gg - \frac{1}{2}\Pi^{R}(\Omega)V^{(2)}\right)^{-1} - \left(gg - \frac{1}{2}\Pi^{A}(\Omega)V^{(2)}\right)^{-1}\right\}$$

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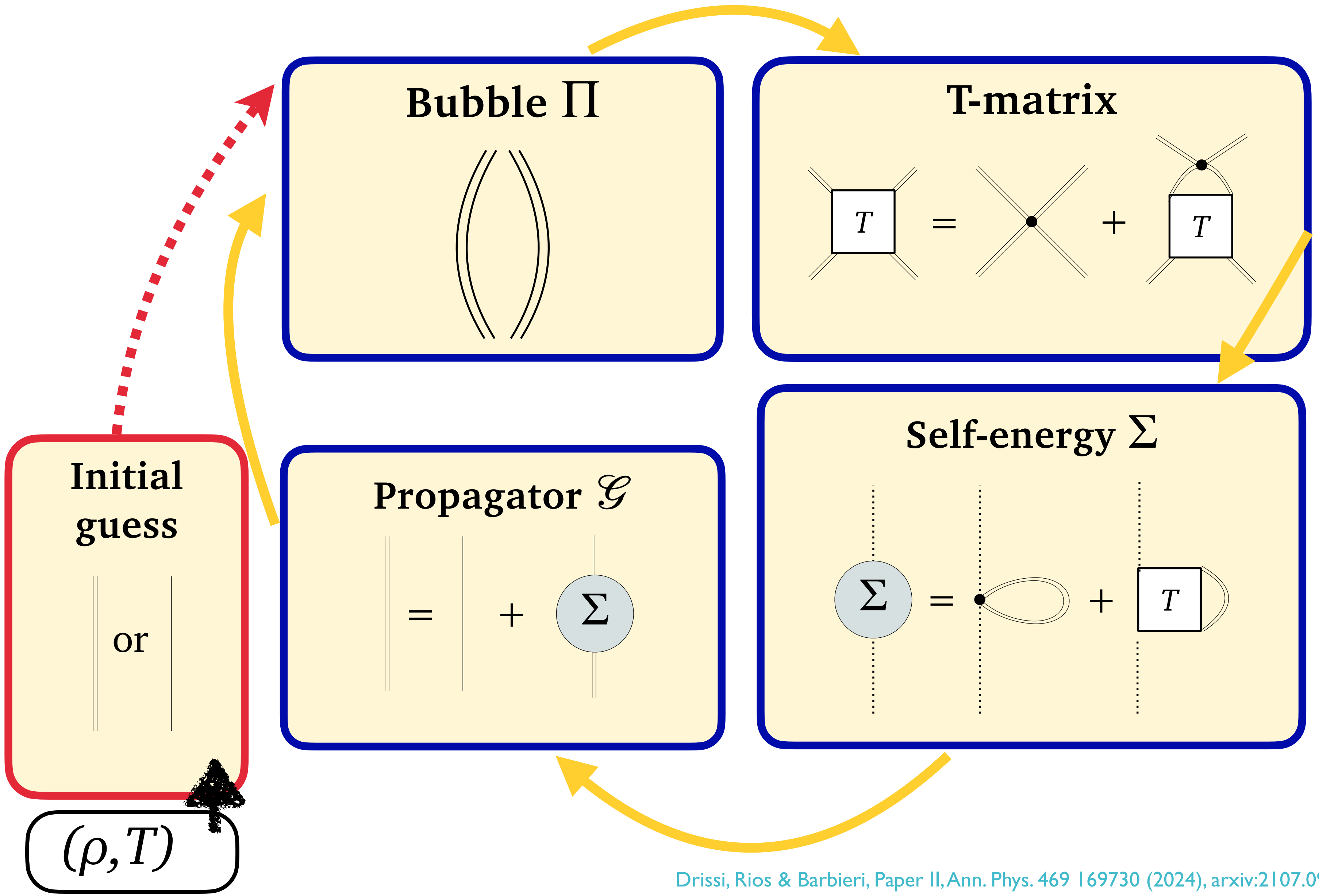
Nambu-Covariant Ladders



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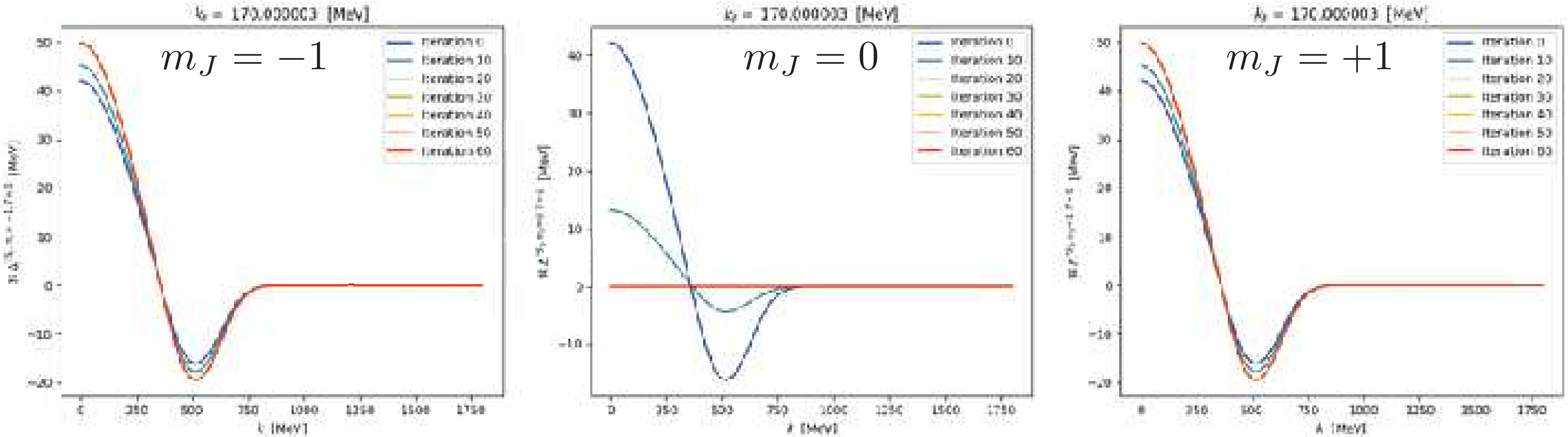
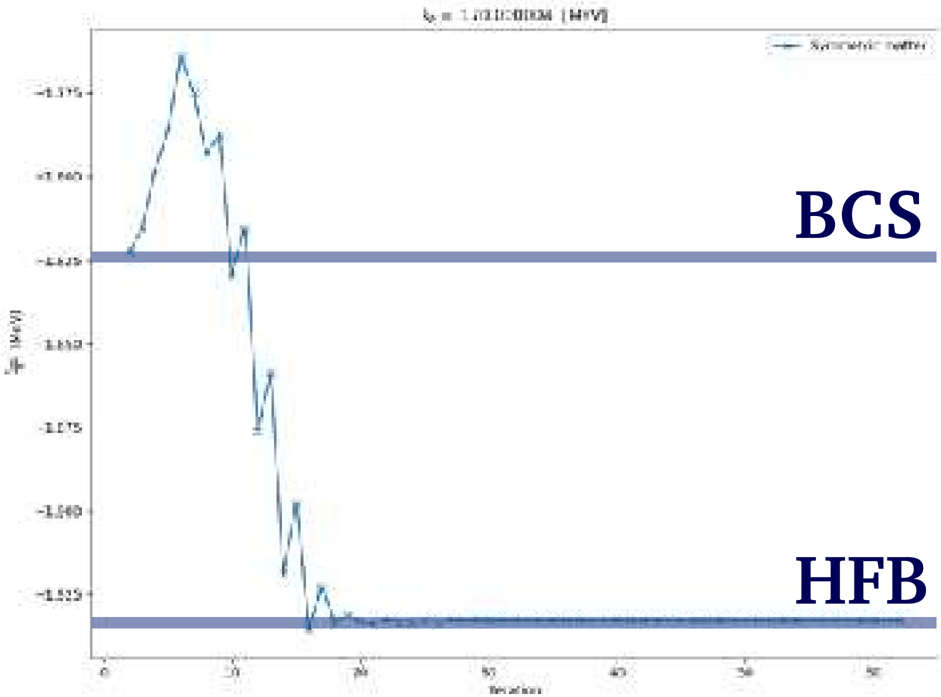
First results

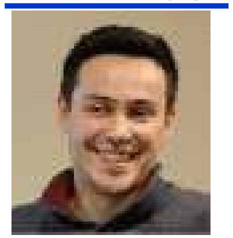




Preliminary!

- From BCS to HFB
- $^3SD_1$ channel
- N3LO EM
- T=0.2 MeV

[M Drissi]

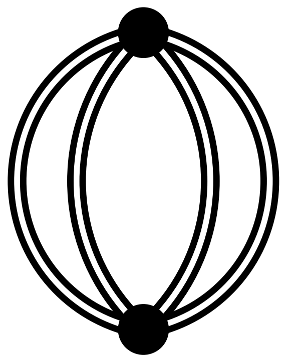


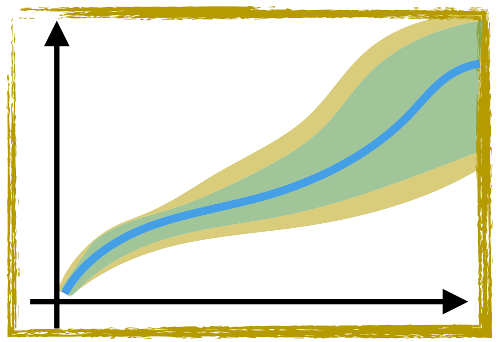
Conclusions

1) Thermal & microscopic properties

2) Nuclear uncertainty quantification

3) New superfluid extensions





Next:

Numerical implementation

Uncertainties in predictions?

Thank you!

Drissi, Rios & Barbieri, Ann. Phys. **469** 169729 (2024)
Drissi, Rios & Barbieri, Ann. Phys. **469** 169730 (2024)
Drissi & Rios, EPJA **58** 90 (2022)

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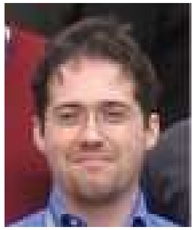

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