

Evolution of multipole resonances in mid-mass open-shell nuclei

ESNT *ab initio* Workshop
Where has the nuclear pairing gone?

Gif-sur-Yvette, May 19th, 2025

Andrea Porro

Technische Universität Darmstadt



TECHNISCHE
UNIVERSITÄT
DARMSTADT

DFG



Introduction

- Physics case and motivation
- Quantities of interest

IMSRG multipole moments

- Sum rule exhaustion
- Strategies for moments evaluation
- Numerical results in SR-IMSRG

Extension to VS-IMSRG

- Computational details
- Multipole moments in Ca isotopes
- A look at Nickel isotopes

Conclusions

Introduction

- Physics case and motivation
- Quantities of interest

IMSRG multipole moments

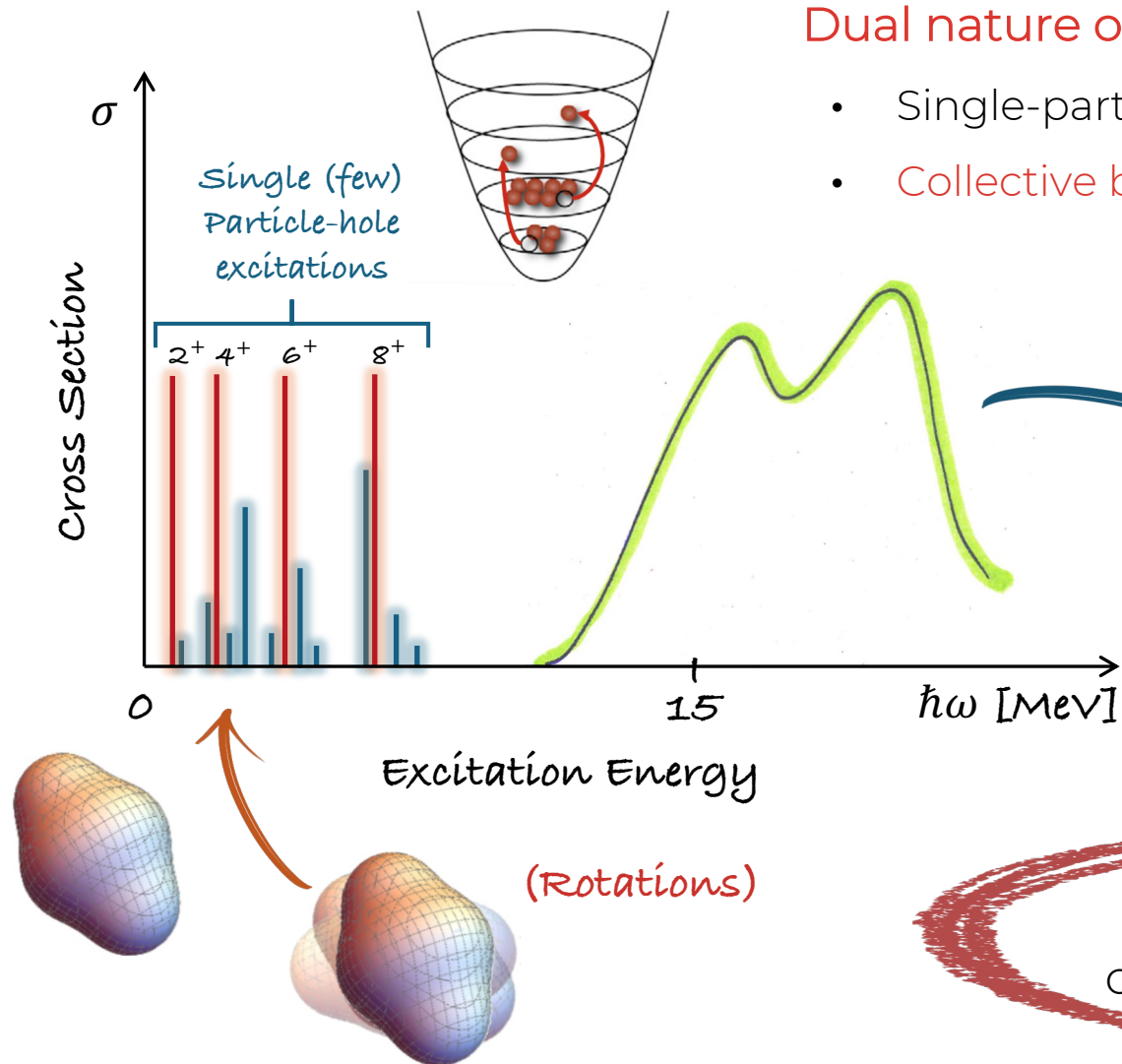
- Sum rule exhaustion
- Strategies for moments evaluation
- Numerical results in SR-IMSRG

Extension to VS-IMSRG

- Computational details
- Multipole moments in Ca isotopes
- A look at Nickel isotopes

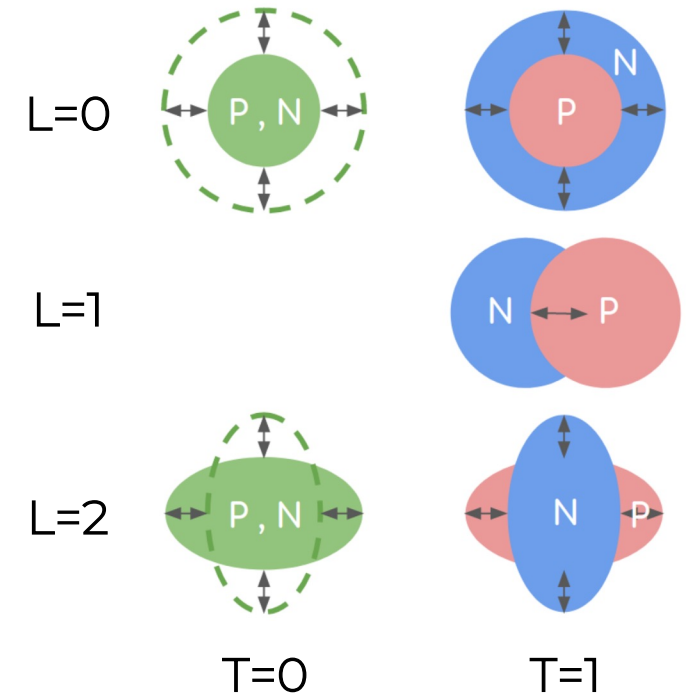
Conclusions

Giant Resonances

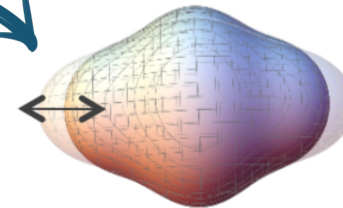


Dual nature of nucleus

- Single-particle features
- Collective behaviour



Giant Resonances



Liquid drop picture
vibrations, oscillations

Giant Resonances (GRs)

clearest manifestation of collective motion

Collective vibrations and pairing

VOLUME 83, NUMBER 11

PHYSICAL REVIEW LETTERS

13 SEPTEMBER 1999

Surface Vibrations and the Pairing Interaction in Nuclei

F. Barranco,¹ R. A. Broglia,^{2,3,4} G. Gori,² E. Vigezzi,³ P. F. Bortignon,^{2,3} and J. Terasaki³

¹*Escuela de Ingenieros Industriales, Universidad de Sevilla, Camino de los Descubrimientos, Sevilla, Spain*

²*Dipartimento de Fisica, Università di Milano, Via Celoria 16, I-20133 Milano, Italy*

³*INFN Sezione di Milano, Via Celoria 16, Milano, I-20133 Italy*

⁴*The Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen, Denmark*
(Received 15 September 1998)

The induced pairing interaction arising from the exchange of low-lying collective surface vibrations among nucleons moving in time reversal states close to a Fermi energy is found to lead to values of the pairing gap which constitute a large fraction of those experimentally observed.

PHYSICAL REVIEW C **80**, 011307(R) (2009)

Role of superfluidity in nuclear incompressibilities

E. Khan

Institut de Physique Nucléaire, Université Paris-Sud, IN2P3-CNRS, F-91406 Orsay Cedex, France

(Received 20 May 2009; published 29 July 2009)

Nuclei are propitious tools to investigate the role of the superfluidity in the compressibility of a Fermionic system. The centroid of the Giant Monopole Resonance (GMR) in Tin isotopes is predicted using a constrained Hartree-Fock Bogoliubov approach, ensuring a full self-consistent treatment. Superfluidity is found to favour the compressibility of nuclei. Pairing correlations explain why doubly magic nuclei such as ²⁰⁸Pb are stiffer compared to open-shell nuclei. Fully self-consistent predictions of the GMR on an isotopic chain should be the way to microscopically extract both the incompressibility and the density dependence of a given energy functional. The macroscopic extraction of K_{sym} , the asymmetry incompressibility, is questioned. Investigations of the GMR in unstable nuclei are called for. Pairing gap dependence of the nuclear matter incompressibility should also be investigated.




Twofold link

- Influence of **low-lying vibrations** on Δ
- Influence of pairing on **GRs** centroids

Correlation seems to reduce pairing importance

PHYSICAL REVIEW LETTERS **131**, 082501 (2023)

Toward a Unified Description of Isoscalar Giant Monopole Resonances in a Self-Consistent Quasiparticle-Vibration Coupling Approach

Z. Z. Li (李征征)^{1,2,3} Y. F. Niu (牛一斐)^{1,2,*} and G. Colò^{3,4,†}

¹*School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China*

²*Frontiers Science Center for Rare Isotope, Lanzhou University, Lanzhou 730000, China*

³*Dipartimento di Fisica, Università degli Studi di Milano, via Celoria 16, 20133 Milano, Italy*

⁴*INFN sezione di Milano, via Celoria 16, 20133 Milano, Italy*

This work: very rich (correlated) **VS-IMSRG** wf

No explicit pairing

Average properties – strength moments

Studied quantity: **multipole strength**

- Transition amplitudes: height of peaks
- Energy difference: position of peaks

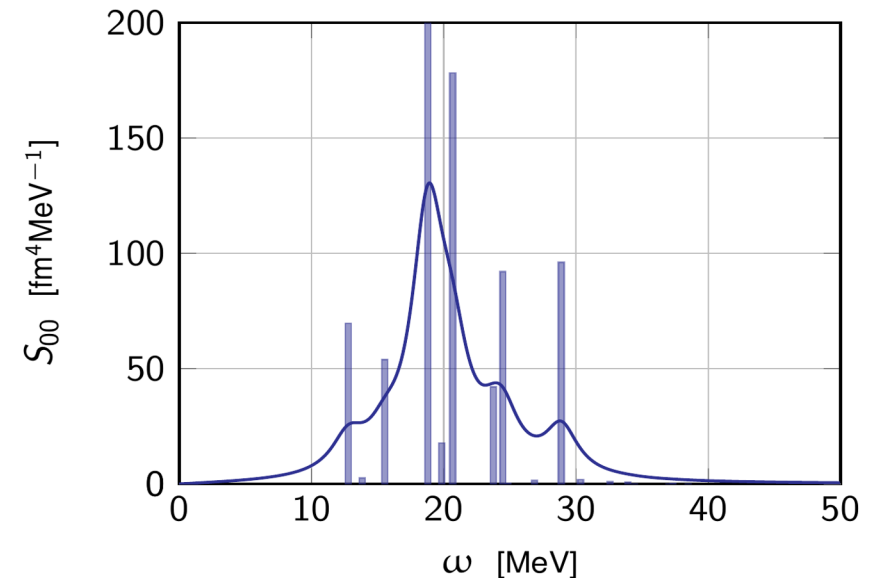
$$S_Q(\omega) \equiv \sum_{\nu} |\langle \Psi_{\nu} | Q | \Psi_0 \rangle|^2 \delta(E_{\nu} - E_0 - \omega)$$

Related **moments**

$$\begin{aligned} m_k(Q) &\equiv \int_0^{\infty} S_Q(\omega) \omega^k d\omega \\ &= \sum_{\nu} (E_{\nu} - E_0)^k |\langle \Psi_{\nu} | Q | \Psi_0 \rangle|^2 \end{aligned}$$

Quantify the **most relevant features** of the strength

$$\bar{E}(Q) \equiv \frac{m_1(Q)}{m_0(Q)} \quad E_k(Q) \equiv \sqrt{\frac{m_k(Q)}{m_{k-2}(Q)}}$$



Introduction

- Physics case and motivation
- Quantities of interest

IMSRG multipole moments

- Sum rule exhaustion
- Strategies for moments evaluation
- Numerical results in SR-IMSRG

Extension to VS-IMSRG

- Computational details
- Multipole moments in Ca isotopes
- A look at Nickel isotopes

Conclusions

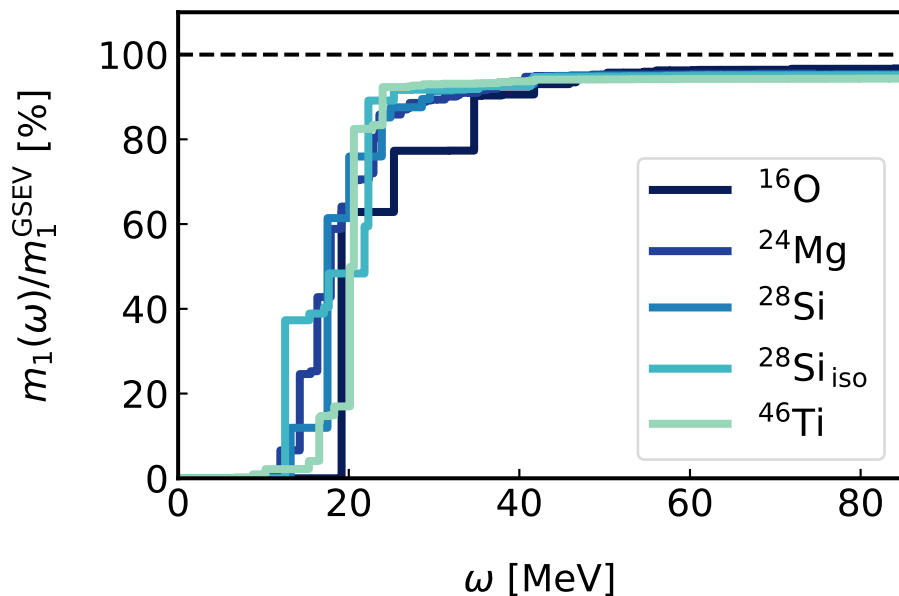
Moment operators

Different evaluation strategies for the moments $S_Q(\omega) \equiv \sum_{\nu} |\langle \Psi_{\nu} | Q | \Psi_0 \rangle|^2 \delta(E_{\nu} - E_0 - \omega)$

$$m_k(Q) \equiv \int_0^{\infty} S_Q(\omega) \omega^k d\omega$$

$$= \sum_{\nu} (E_{\nu} - E_0)^k |\langle \Psi_{\nu} | Q | \Psi_0 \rangle|^2 \longrightarrow \text{Must know excited states} \quad \text{6-7 \% difference in PGCM}$$

$$\equiv \langle \Psi_0 | M_k(Q) | \Psi_0 \rangle \longrightarrow \text{Ground state only}$$



Complexity is shifted to the operator structure

$$M_k(Q) \equiv (-1)^i C_i C_j \quad \forall k \geq 0$$

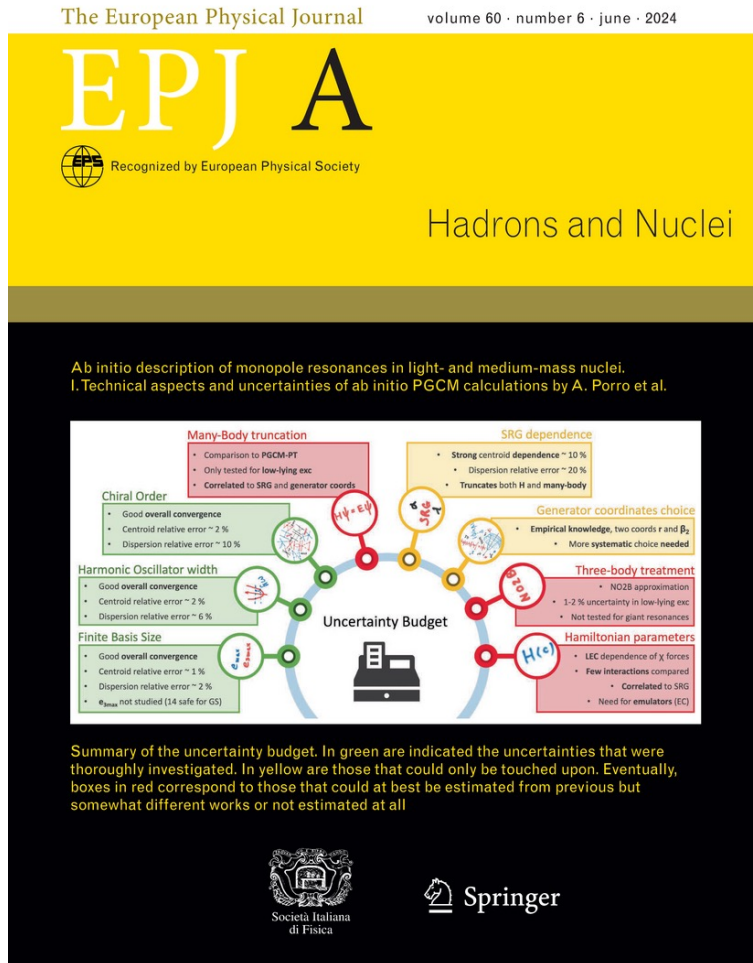
$$M_k(Q) \equiv \frac{1}{2} (-1)^i [C_i, C_j] \quad \forall \text{ odd } k > 0$$

$$C_j \equiv \underbrace{[H, [H, \dots [H, [H, Q]] \dots]]}_{j \text{ times}}$$

Many-body operators

- Exact up to m_1 $H = H^{[1]} + H^{[2]}$

Previous PGCM study



Eur. Phys. J. A (2024) 60:155
<https://doi.org/10.1140/epja/s10050-024-01377-5>

THE EUROPEAN
PHYSICAL JOURNAL A



Regular Article - Theoretical Physics

Ab initio description of monopole resonances in light- and medium-mass nuclei

III. Moments evaluation in ab initio PGCM calculations

A. Porro^{1,2,3,a} , T. Duguet^{3,4}, J.-P. Ebran^{5,6}, M. Frosini⁷, R. Roth^{1,8}, V. Somà³

¹ Department of Physics, Technische Universität Darmstadt, 64289 Darmstadt, Germany

² ExtreMe Matter Institute EMMI, GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany

³ IRFU, CEA, Université Paris-Saclay, 91191 Gif-sur-Yvette, France

⁴ KU Leuven, Department of Physics and Astronomy, Instituut voor Kern- en Stralingsfysica, 3001 Leuven, Belgium

⁵ CEA, DAM, DIF, 91297 Arpajon, France

⁶ Laboratoire Matière en Conditions Extrêmes, Université Paris-Saclay, CEA, 91680 Bruyères-le-Châtel, France

⁷ CEA, DES, IRESNE, DER, SPRC, 13108 Saint-Paul-lès-Durance, France

⁸ Helmholtz Forschungsakademie Hessen für FAIR, GSI Helmholtzzentrum, 64289 Darmstadt, Germany

- I. [EPJA (2024) 60, 133]
- II. [EPJA (2024) 60, 134]
- III. [EPJA (2024) 60, 155]
- IV. [EPJA (2024) 60, 233]

Strategy in the IMSRG framework

Unitary transformation

$$H(s) = U(s) H U^\dagger(s) \\ \equiv H^{\text{d}}(s) + H^{\text{od}} \rightarrow H^{\text{d}}(\infty)$$

↗
Diagonal

↖
Off-diagonal

$$E_{\text{gs}} = \lim_{s \rightarrow \infty} E_0(s) = \langle \Phi | H(s) | \Phi \rangle$$

↖
Slater determinant

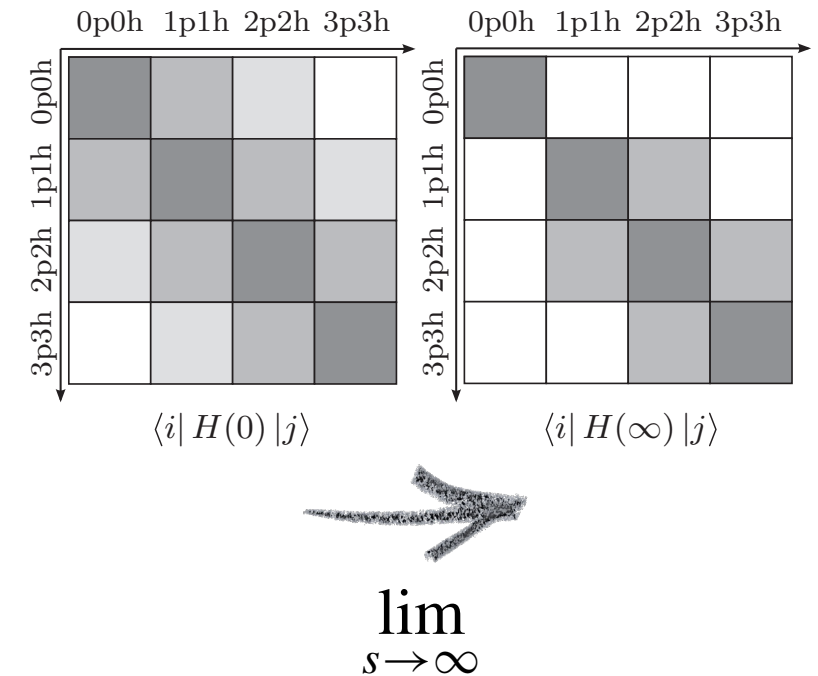
Steps

- Start from the moment operator in the **HO basis**
- Perform an **IMSRG(2)** calculation (VS for open-shell)
- Consistently **evolve** the moment operators using **Magnus**

$$M_1(Q) = \frac{1}{2} [Q^\dagger, [H, Q]]$$

$$M_0(Q) = Q^\dagger Q$$

$$U(s) \equiv e^{\Omega(s)}$$



IMSRG example: Kumar invariants

0th quadrupole moment

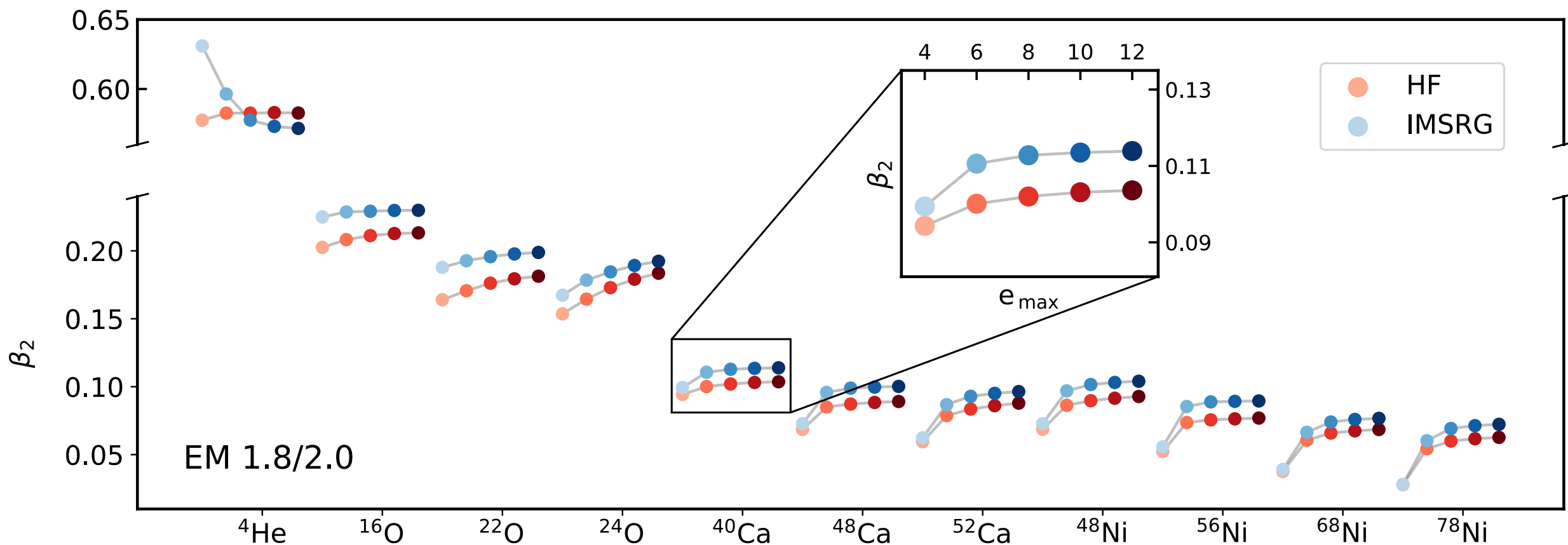
$$m_0(Q_2) = \langle Q_2 \cdot Q_2 \rangle$$

Model-independent deformation «measure»

$$\beta_2 \equiv \frac{4\pi}{3r_0^2} \frac{\langle Q_2 \cdot Q_2 \rangle^{1/2}}{A^{5/3}}$$

Higher invariants also fundamental

[Poves et al., PRC 101 (2020) 054307]



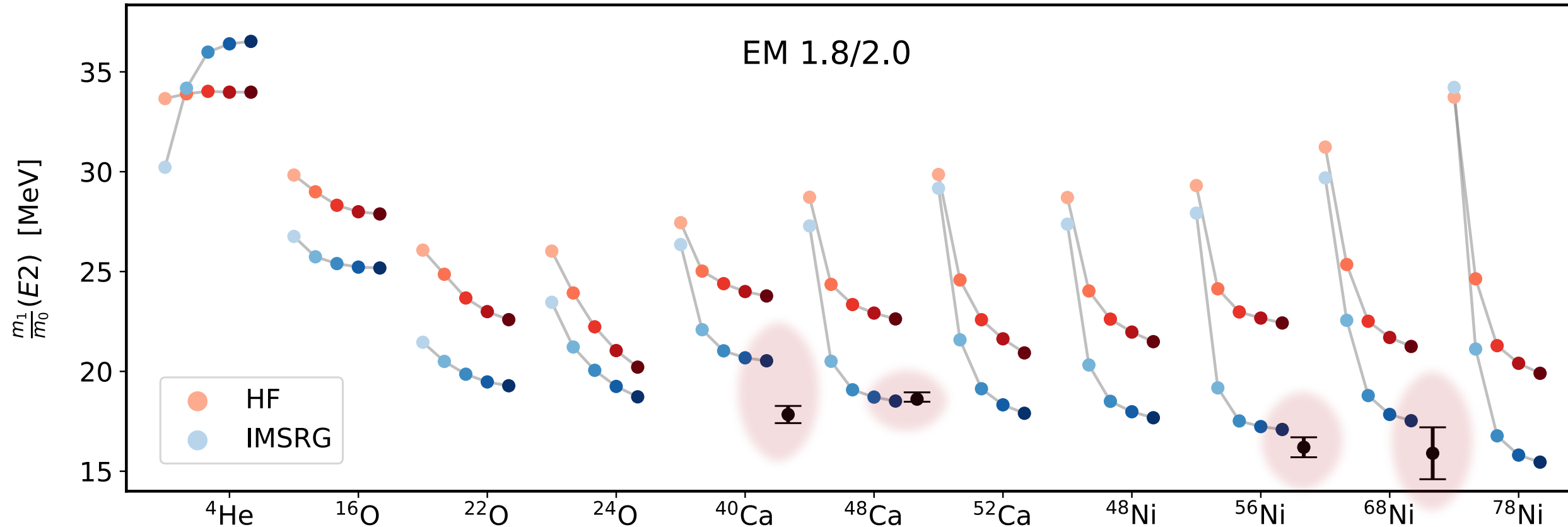
IMSRG example: GQR centroid

Centroid of the quadrupole strength

$$\bar{E}(Q_2) \equiv \frac{m_1(Q_2)}{m_0(Q_2)}$$

IMSRG(2) **GQR study** across the nuclear chart

Disclaimer: finite energy domain in experiments



Introduction

- Physics case and motivation
- Quantities of interest

IMSRG multipole moments

- Sum rule exhaustion
- Strategies for moments evaluation
- Numerical results in SR-IMSRG

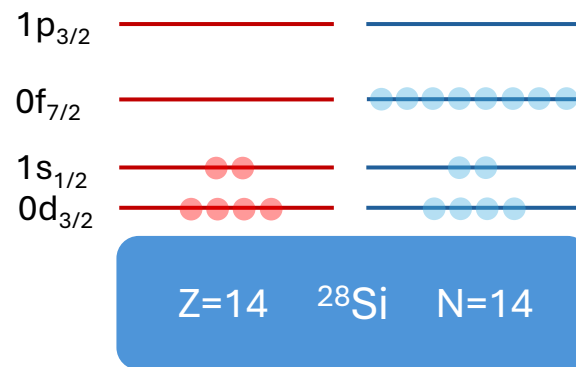
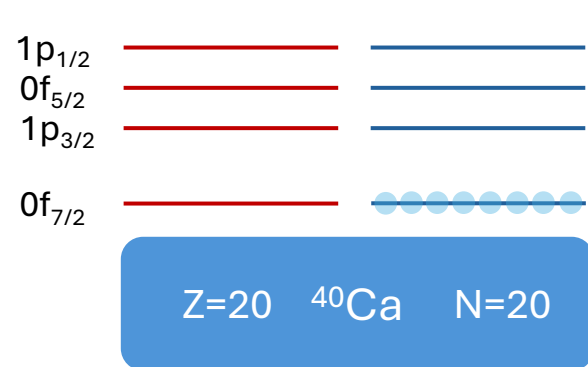
Extension to VS-IMSRG

- Computational details
- Multipole moments in Ca isotopes
- A look at Nickel isotopes

Conclusions

Implementation details

- Moment operators implemented within the **imsrg++** code [github.com/ragnarstroberg/imsrg]
- **J-scheme** expressions of moments 0 and 1 from [Lu and Johnson, PRC 97 (2018) 3, 034330]
- Benchmarked vs **QFAM** code [Beaujeault-Taudière, Frosini et al., PRC 107 (2023), L021302]
- Used interactions EM 1.8/2.0 and $\Delta\text{NNLO}_{\text{go}}$
- HO basis $\hbar\omega = 16$ MeV $e_{3\text{max}}=24$
- NO2B treatment of the 3-body interaction
- VS-IMSRG(2) with two different cores ^{40}Ca and ^{28}Si [Miyagi et al., PRC 102 (2020), 034320]
($\Delta=10$ MeV, $\beta=3$)

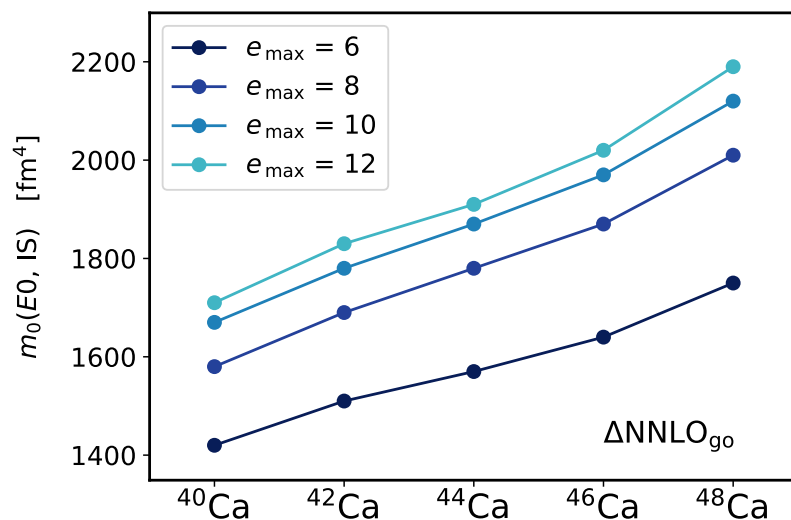
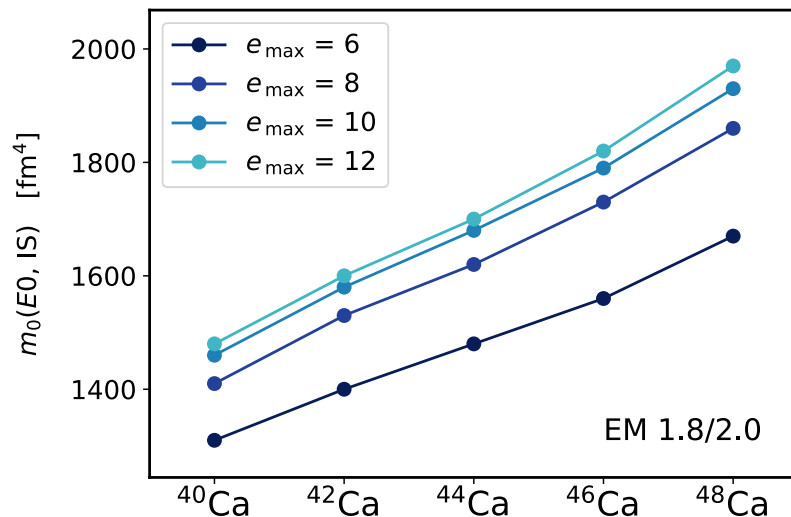


$$H|\Psi_k\rangle = E_k|\Psi_k\rangle \rightarrow H_{\text{eff}}|\Psi_k^{\text{P}}\rangle = E_k|\Psi_k^{\text{P}}\rangle$$

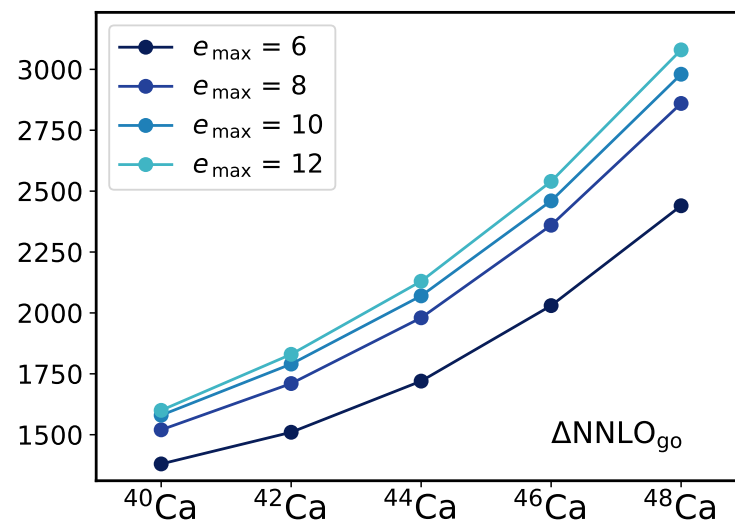
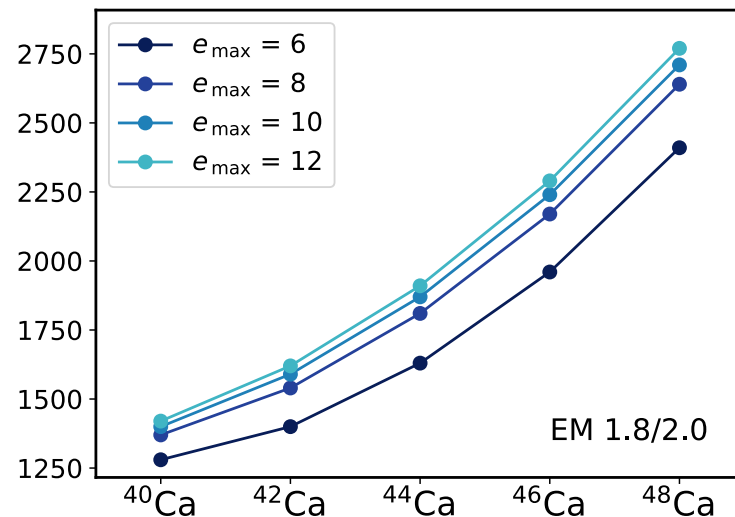
Monopole m_0 (NEWSR)

$$m_0(r^2) = \langle \Psi | r^2 \cdot r^2 | \Psi \rangle$$

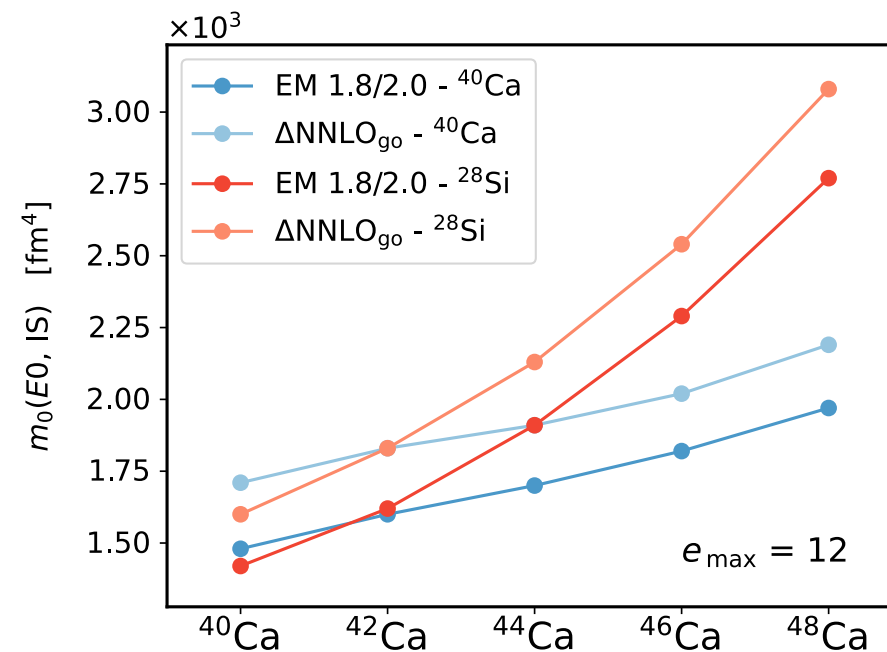
^{40}Ca core



^{28}Si core



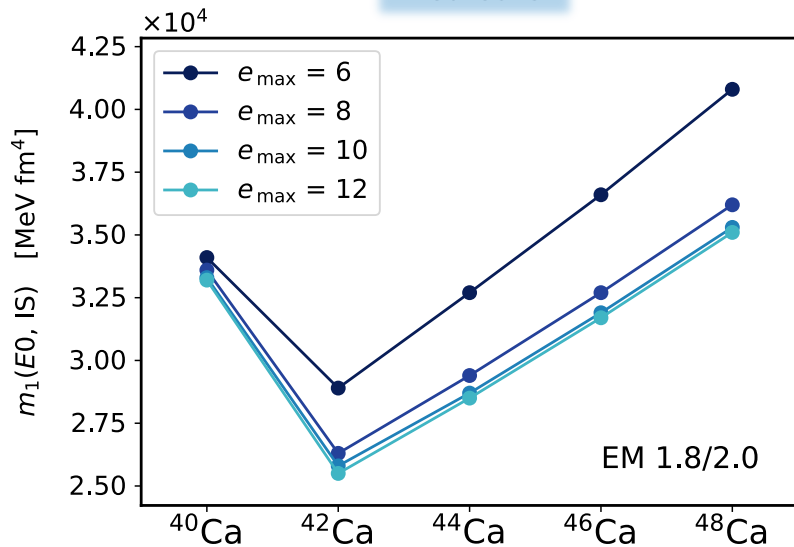
- Qualitatively different trends
- Interactions don't affect trends
- Interaction dependence



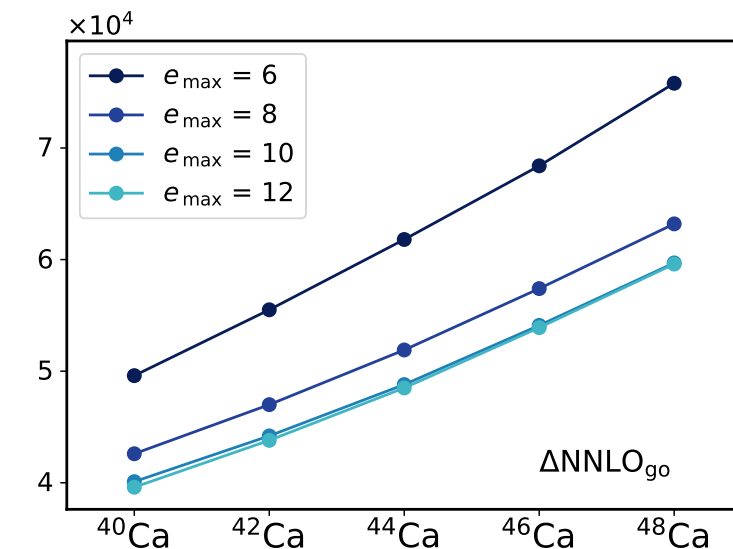
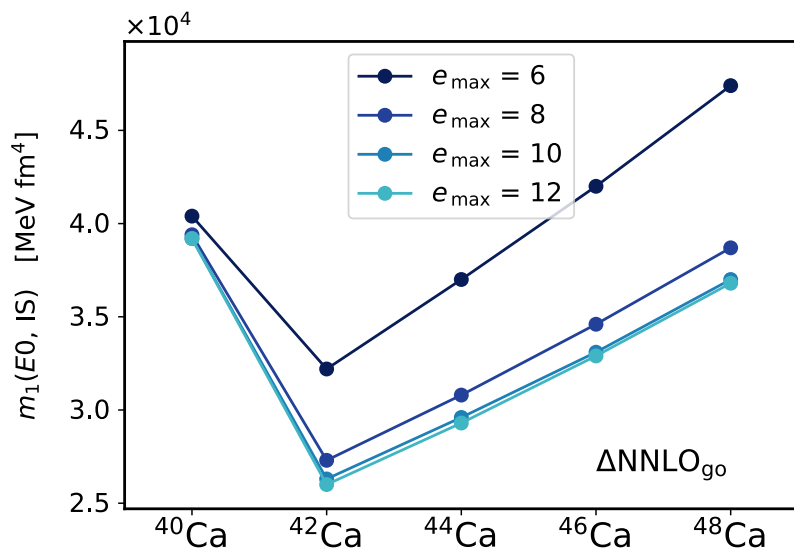
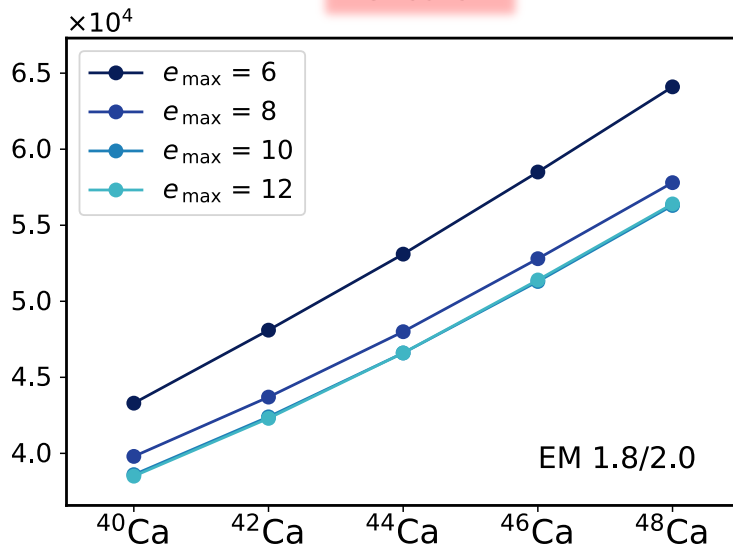
Monopole m_1 (EWSR)

$$m_1(r^2) = \frac{1}{2} \langle \Psi | [r^2, [H, r^2]] | \Psi \rangle$$

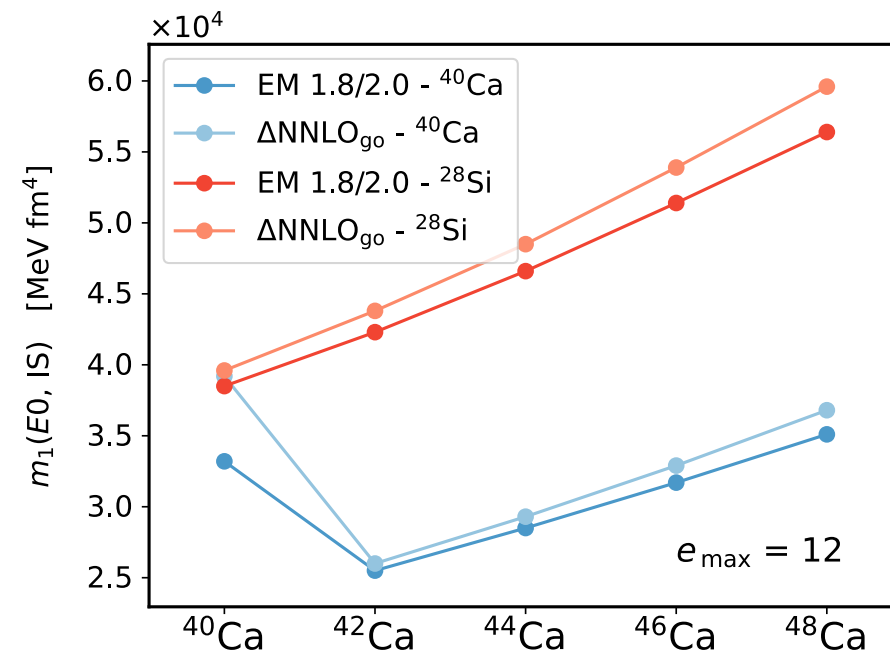
^{40}Ca core



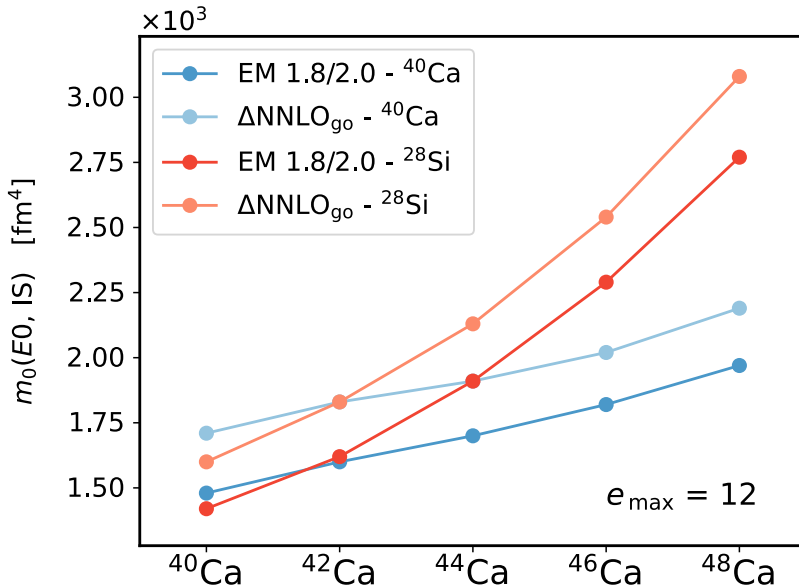
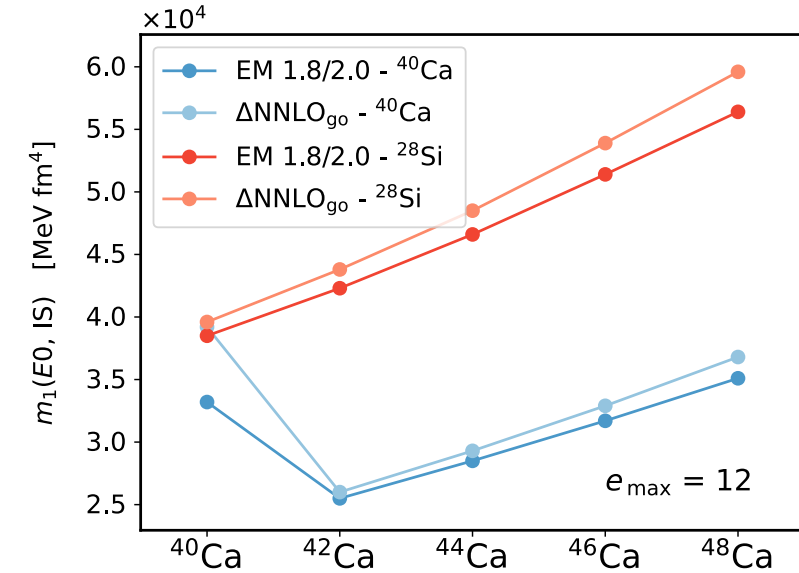
^{28}Si core



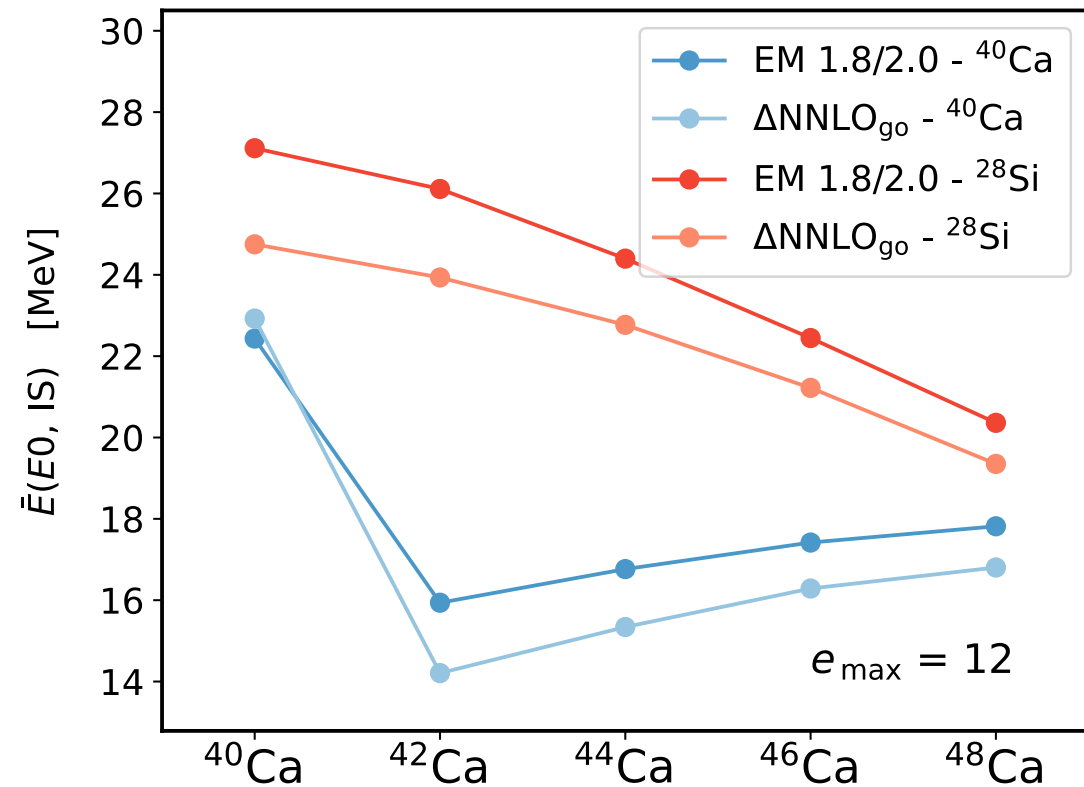
- “Frozen core” effect (^{40}Ca)
- Smaller interaction dependence (commutator)
- Expected linear trend ($\sim A$)



Monopole average energy (m_1/m_0)



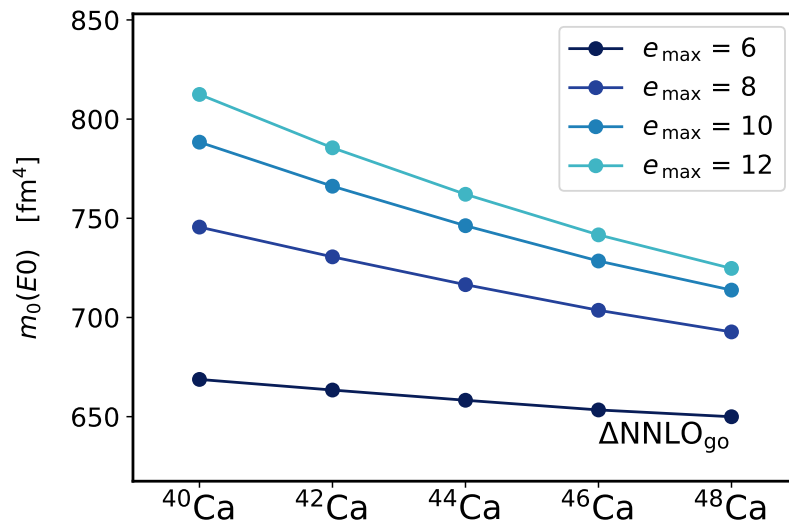
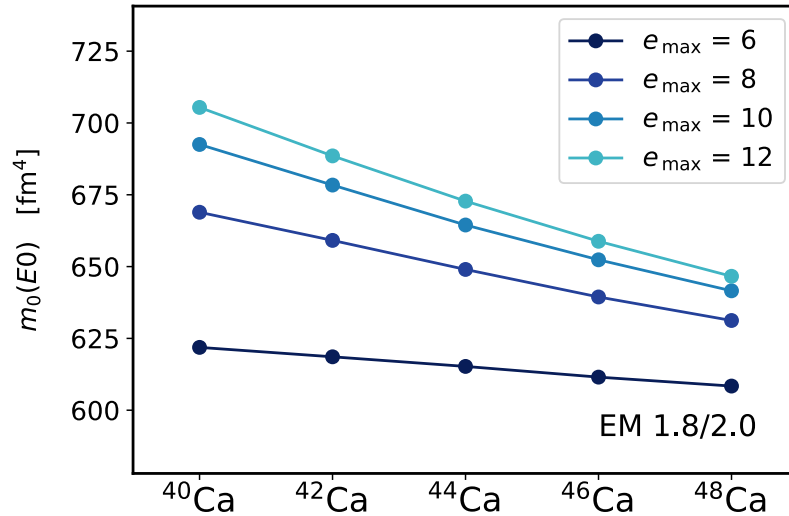
- Large energy difference along the chain
- The GMR (exp) does not display such strong dependence
- Average over all 0⁺ states
- Is the VS large enough ?



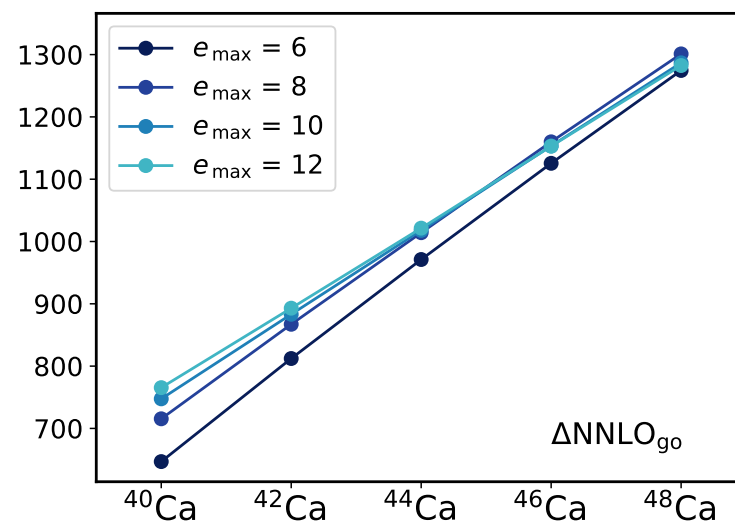
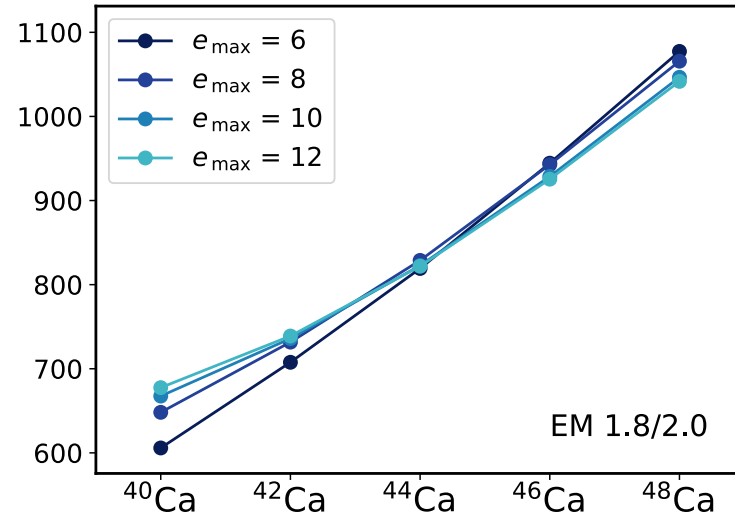
Monopole m_0 (NEWSR) – Protons only

$$m_0(E0) = \sum_{i,j=1}^Z \langle \Psi | r_i^2 \cdot r_j^2 | \Psi \rangle$$

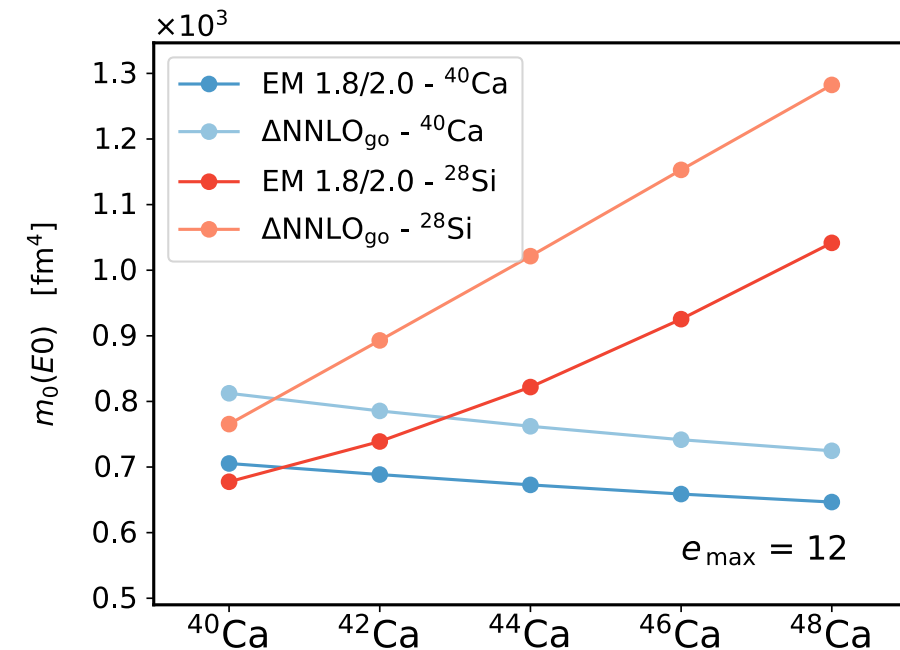
^{40}Ca core



^{28}Si core



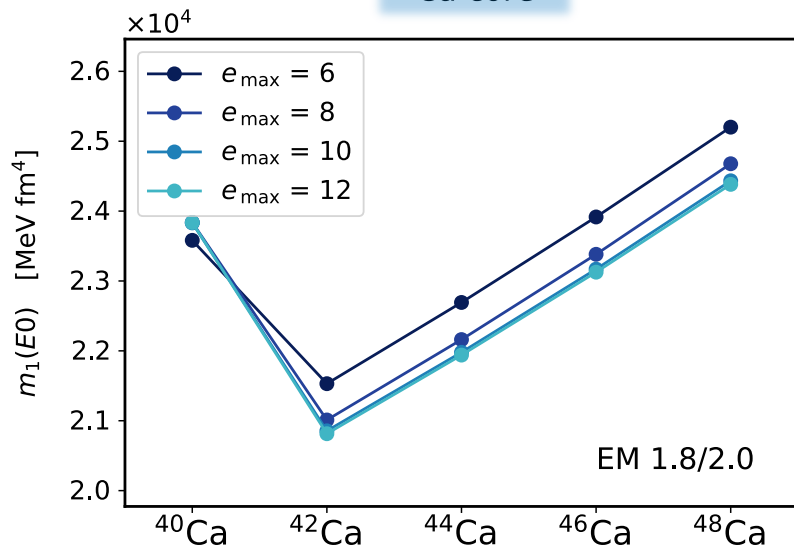
- Completely different trends
- Interactions don't affect trends
- Interaction dependence



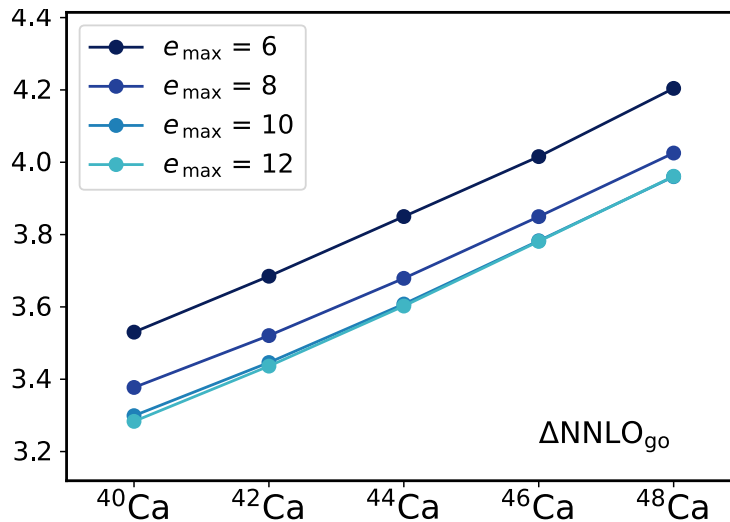
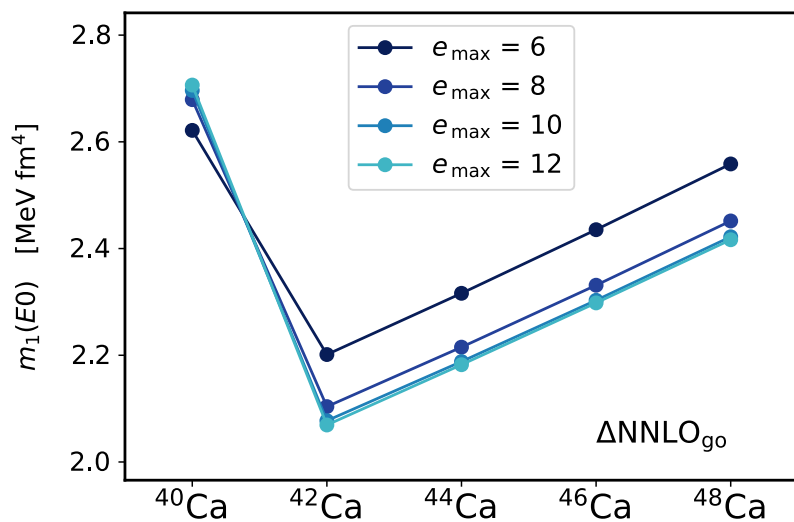
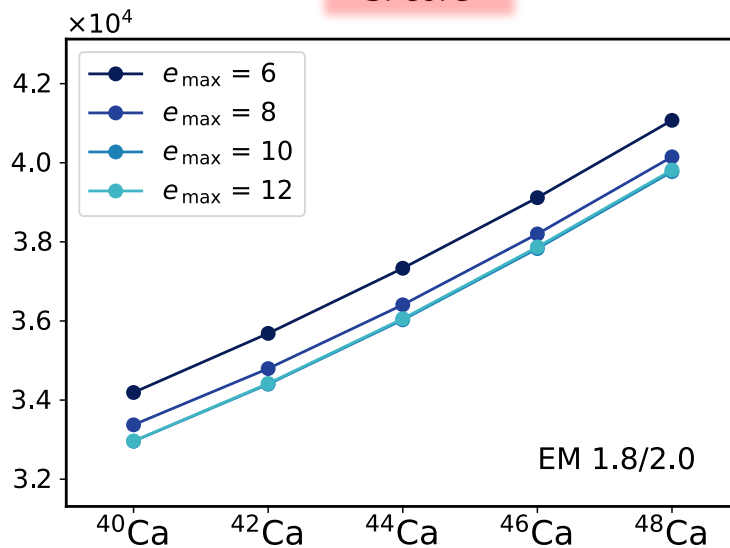
Monopole m_1 (EWSR) – Protons only

$$m_1(E0) = \frac{1}{2} \sum_{i,j=1}^Z \langle \Psi | [r_i^2, [H, r_j^2]] | \Psi \rangle$$

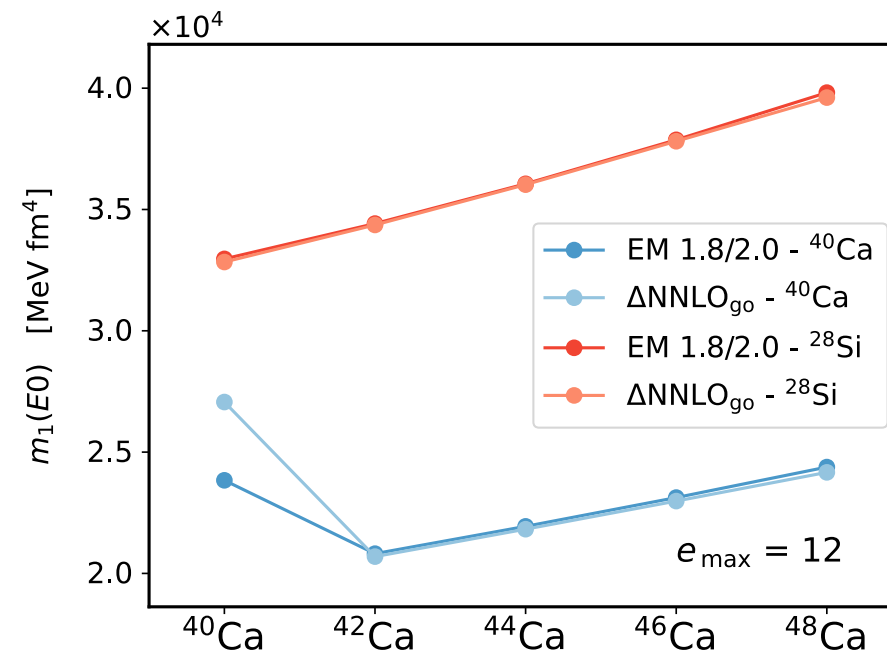
^{40}Ca core



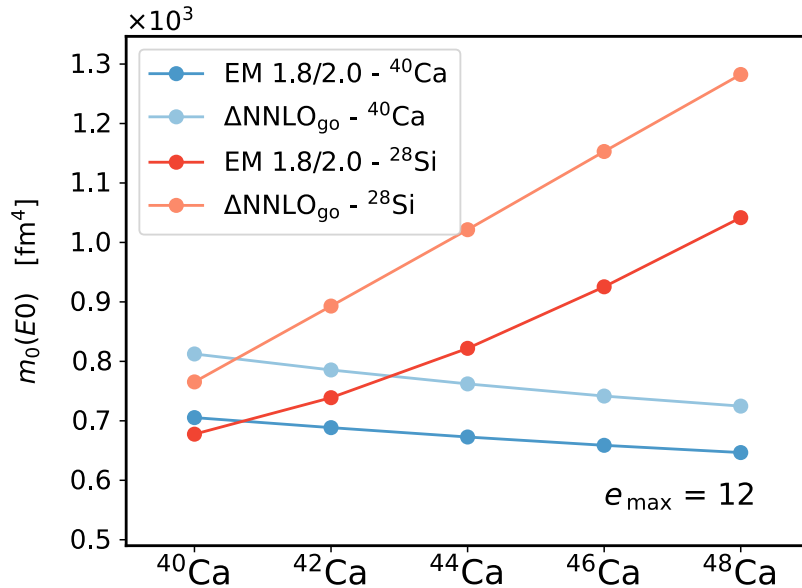
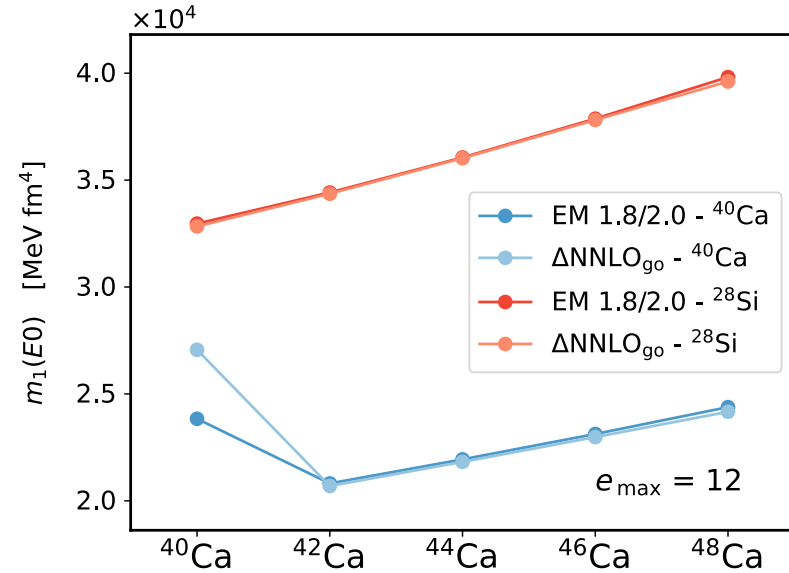
^{28}Si core



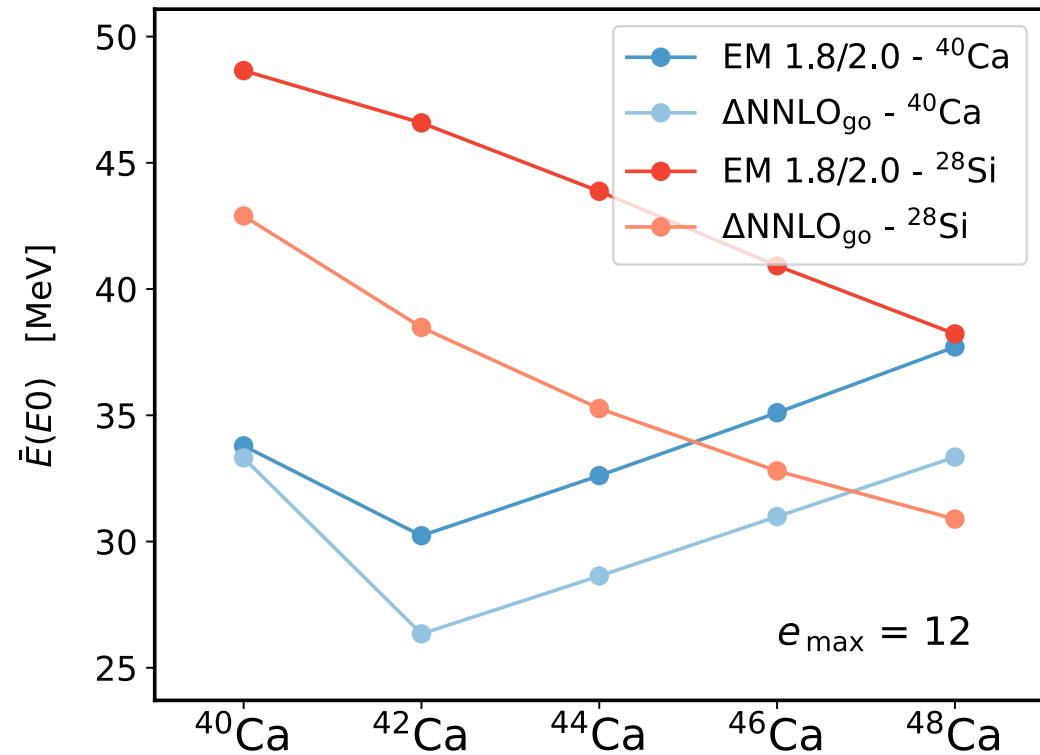
- “Frozen core” effect
- Interactions don’t affect trends
- Interaction independent !



Monopole average energy (m_1/m_0) – Protons only

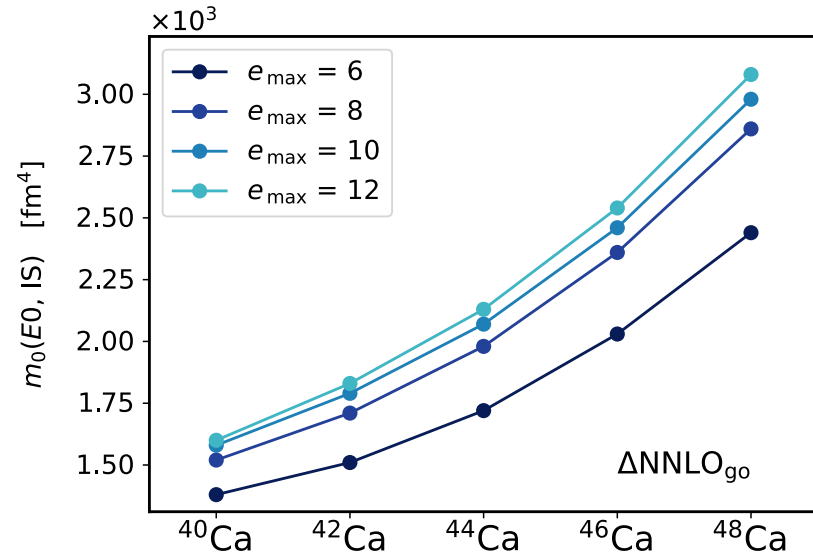


- Difference driven by m_0
- Core needs to be opened
- Is the VS large enough ?
- (Average energy is probably meaningless)

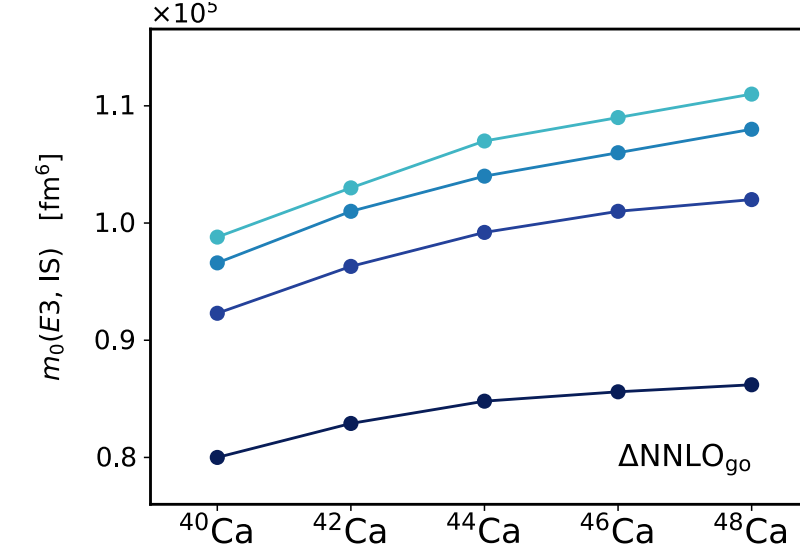
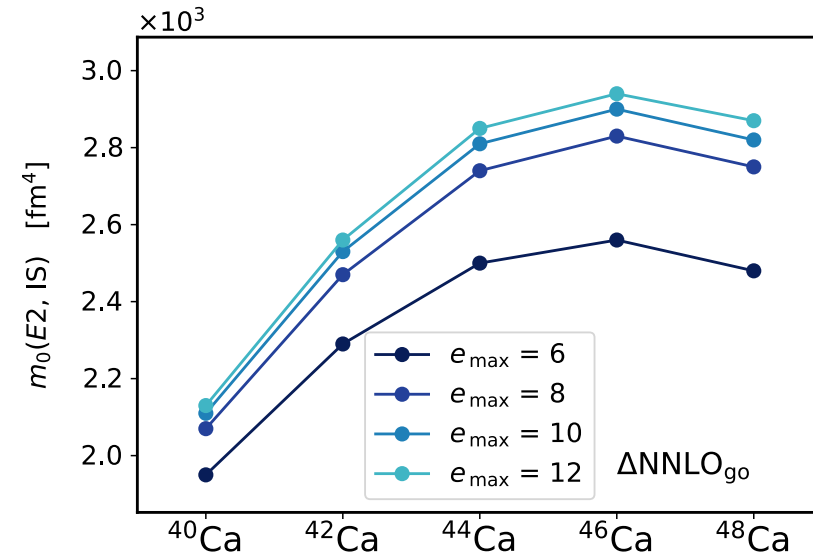
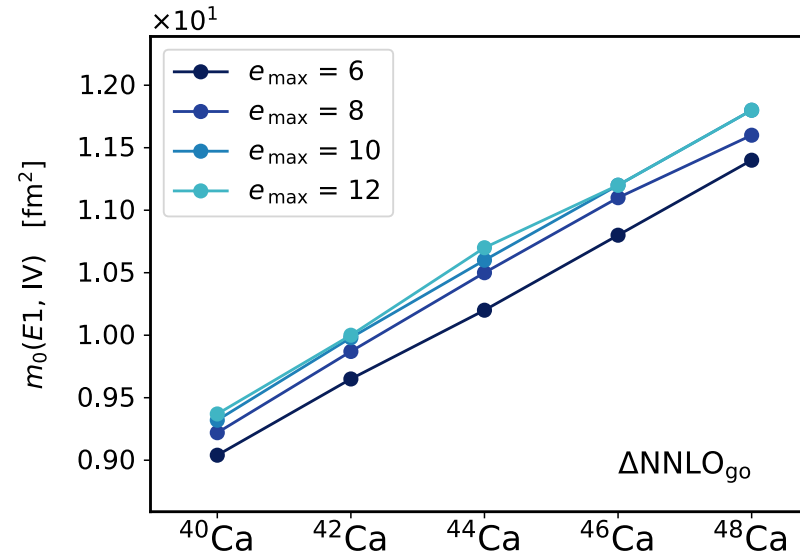


Other multipoles m_0 (NEWSR)

^{28}Si core



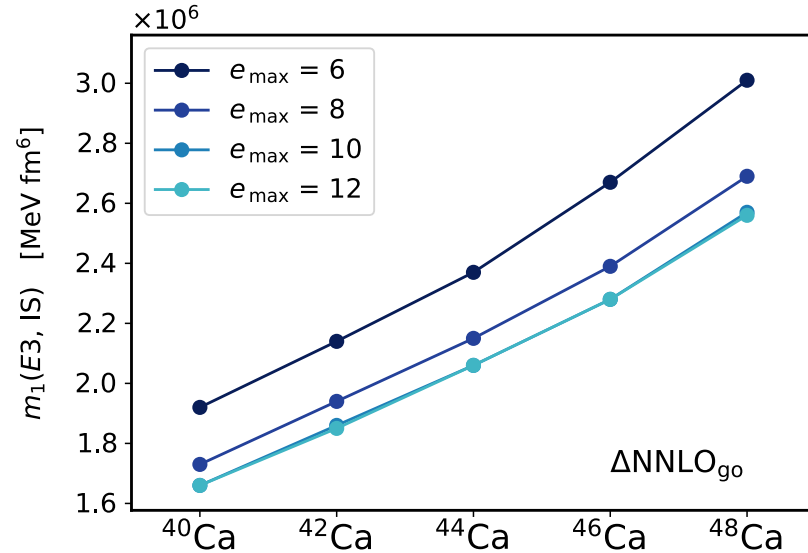
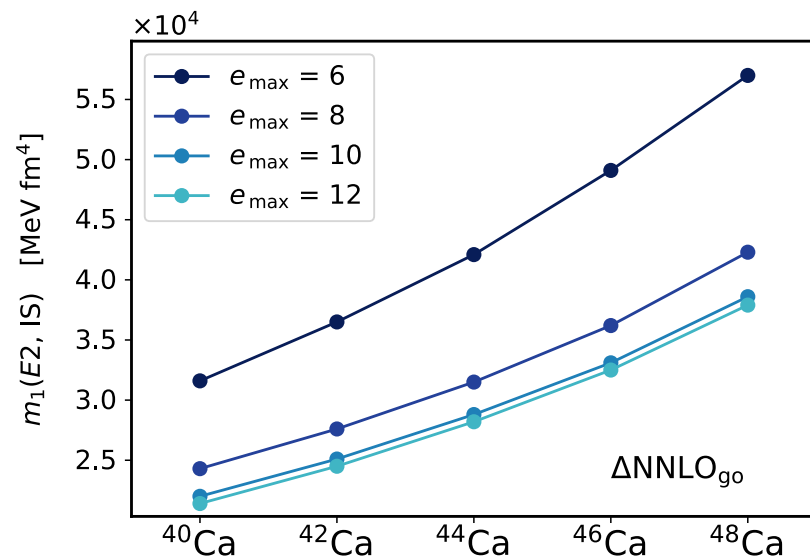
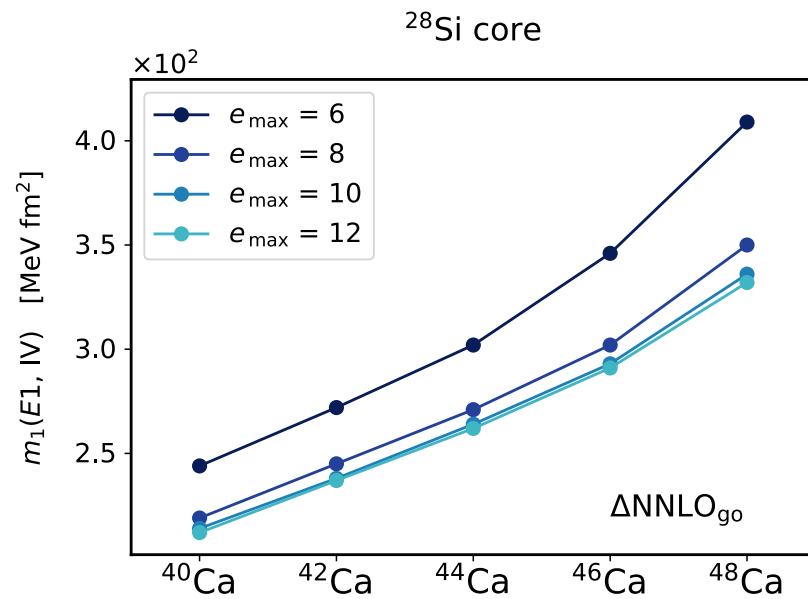
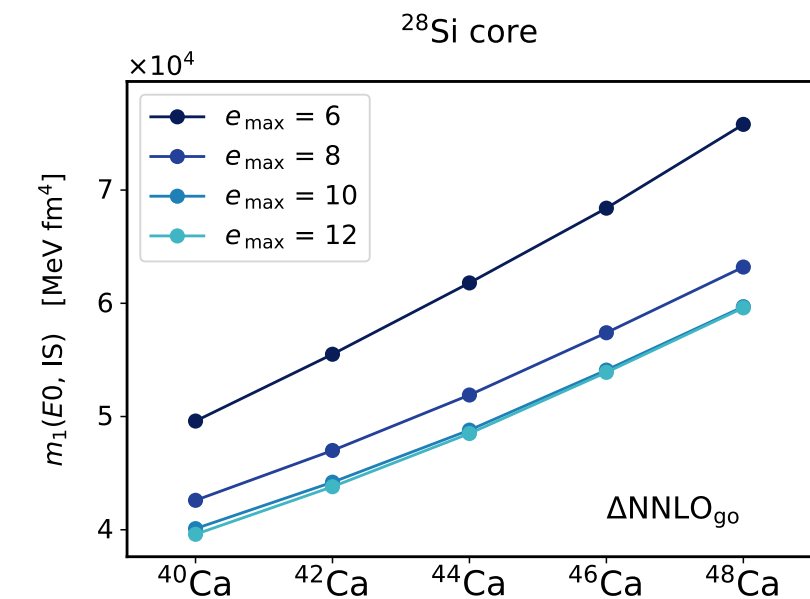
^{28}Si core



$$m_0(Q_\lambda) = \langle \Psi | Q_\lambda \cdot Q_\lambda | \Psi \rangle$$

- More or less similar e_{max} conv (wrt monopole)

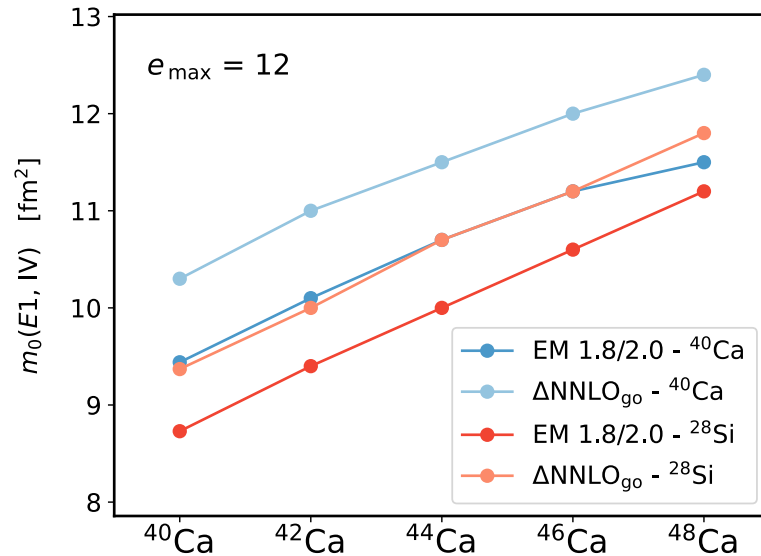
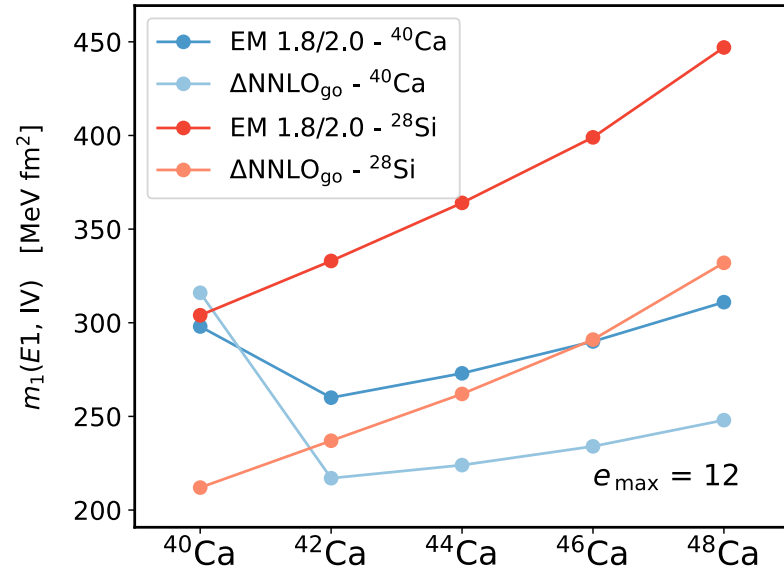
Other multipoles m_1 (EWSR)



$$m_1(Q_\lambda) = \frac{1}{2} \langle \Psi | [Q_\lambda, [H, Q_\lambda]] | \Psi \rangle$$

- Much better e_{max} convergence

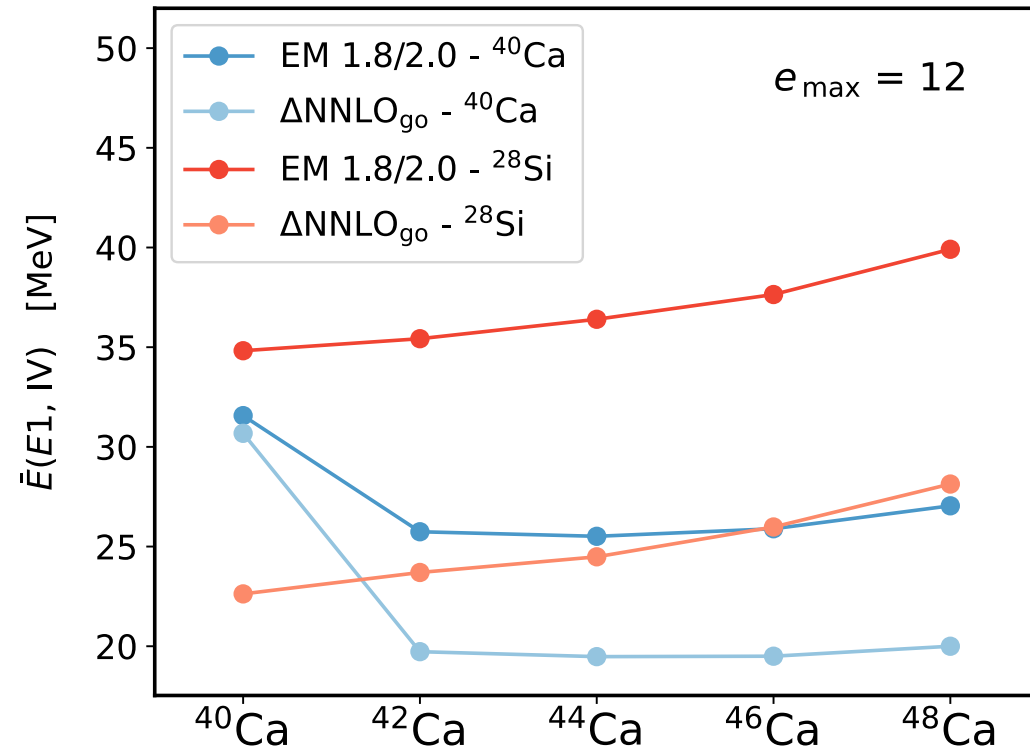
Dipole average energy (m_1/m_0)



- Center of mass motion removed

$$D = \frac{N}{A} \sum_{i=1}^Z r_i Y_{\lambda\mu}(\hat{r}_i) - \frac{Z}{A} \sum_{i=1}^N r_i Y_{\lambda\mu}(\hat{r}_i)$$

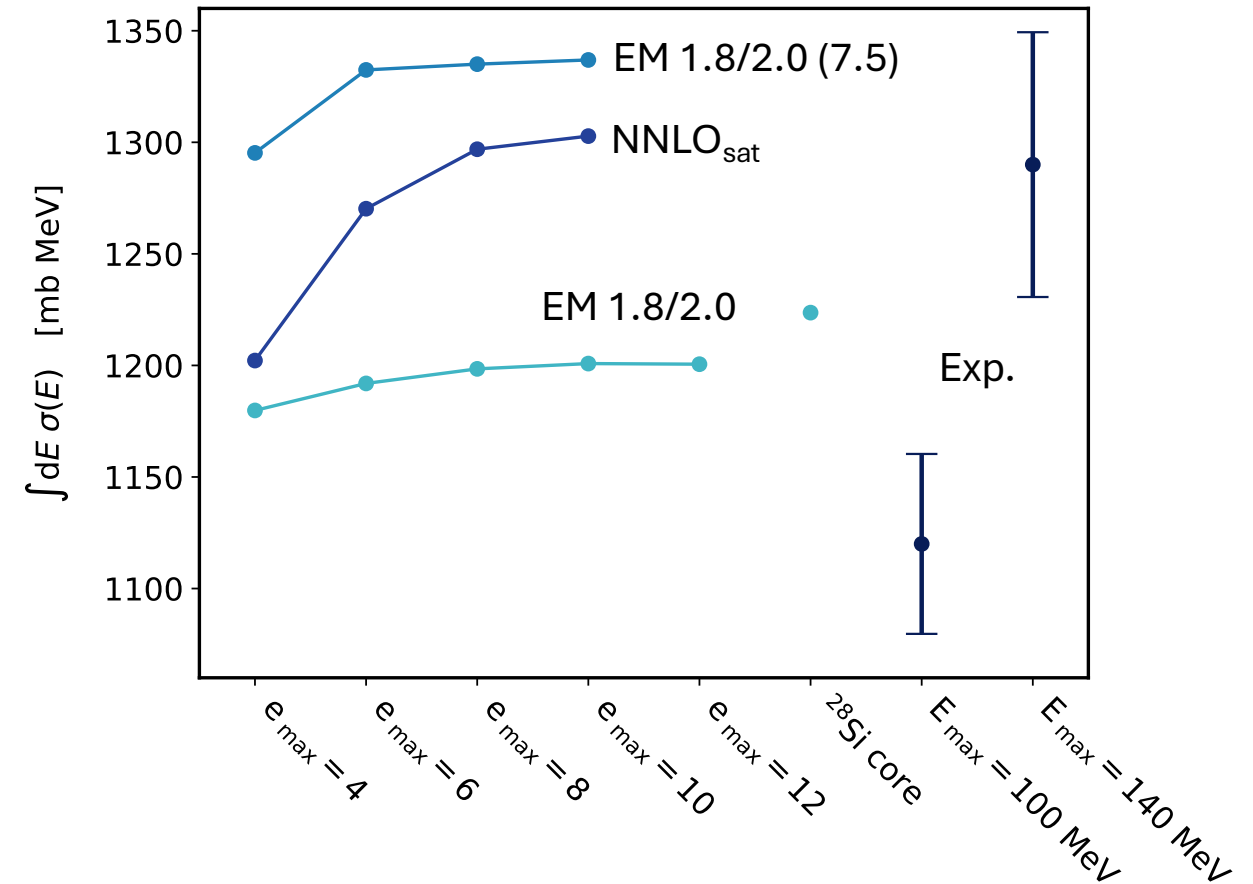
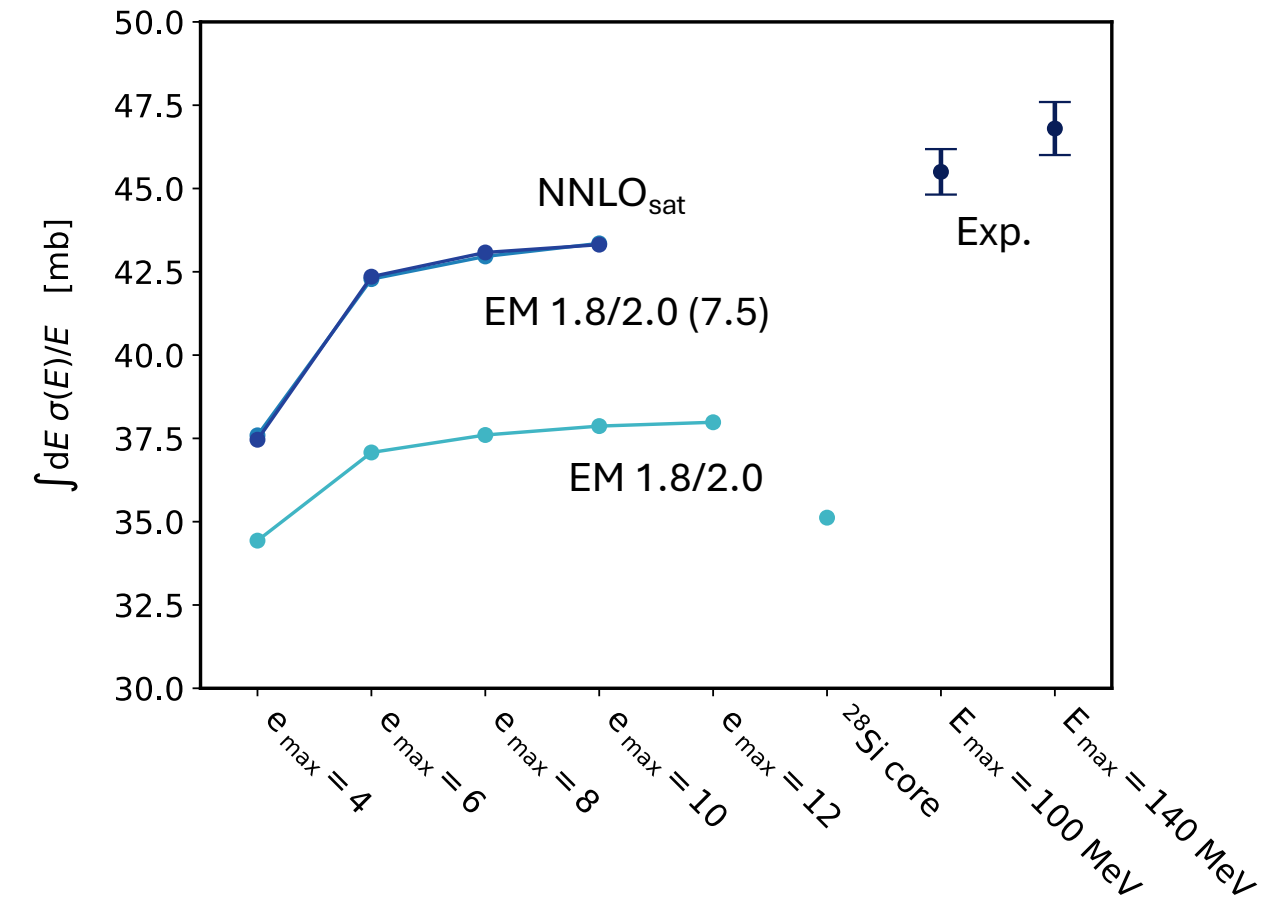
- Integrated quantities can already be compared to exp



Total photonuclear absorption in ^{40}Ca

$$\int_0^{E_{\text{max}}} \frac{\sigma(E)}{E} dE$$

$$\int_0^{E_{\text{max}}} \sigma(E) dE$$



[Ahrens et al., NPA, 1975]

Total photonuclear absorption in ^{40}Ca

TRK sum rule

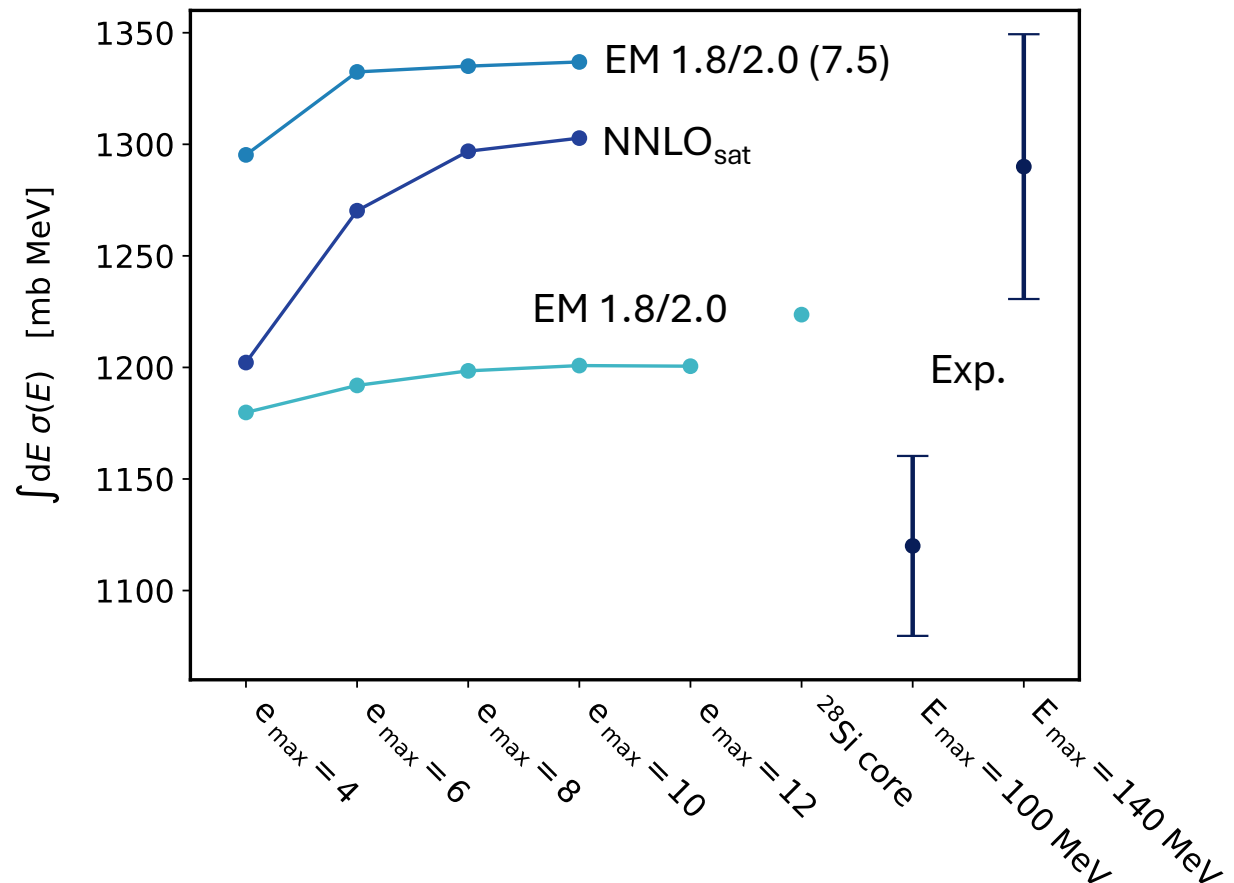
$$\int_0^\infty \sigma(E) dE = \frac{2\pi^2}{\hbar c} \langle \Psi_0 | [D, [H, D]] | \Psi_0 \rangle$$

$$\approx 60 \frac{NZ}{A} (1 + \kappa) \text{ mb} \cdot \text{MeV}$$

Interaction	κ
SV-min	0.05
SV-bas	0.26
SLy6	0.16
SkM*	0.31
SGII	0.33
EM 1.8/2.0	1.01
EM 1.8/2.0 (7.5)	1.24
NNLO _{sat}	1.19
Experiment	1.16

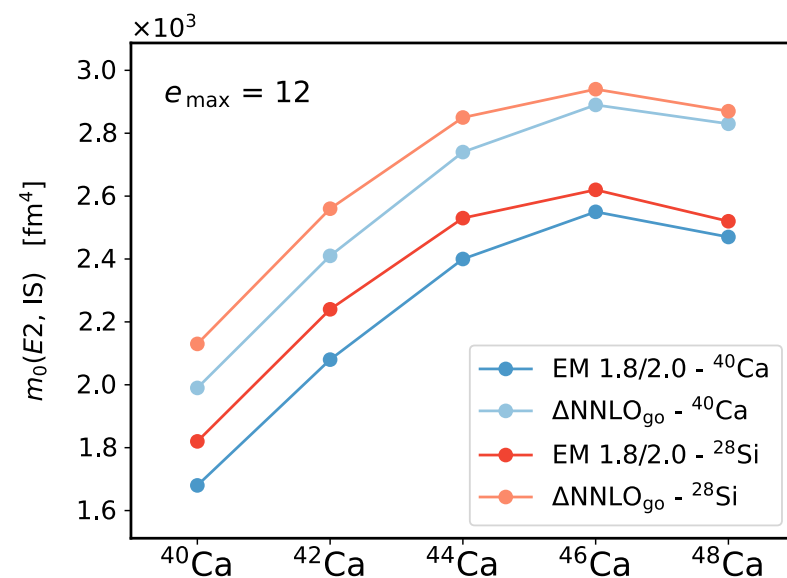
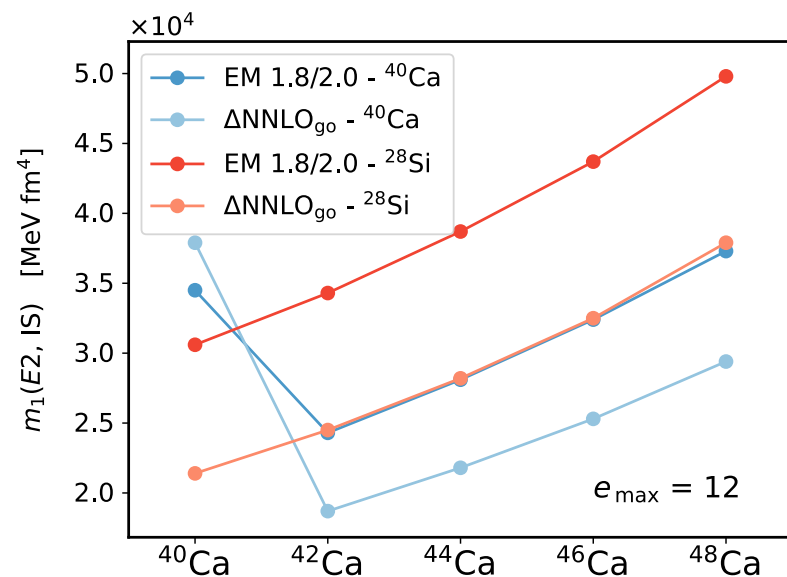
[Courtesy of P.-G. Reinhard]

$$\int_0^{E_{\max}} \sigma(E) dE$$

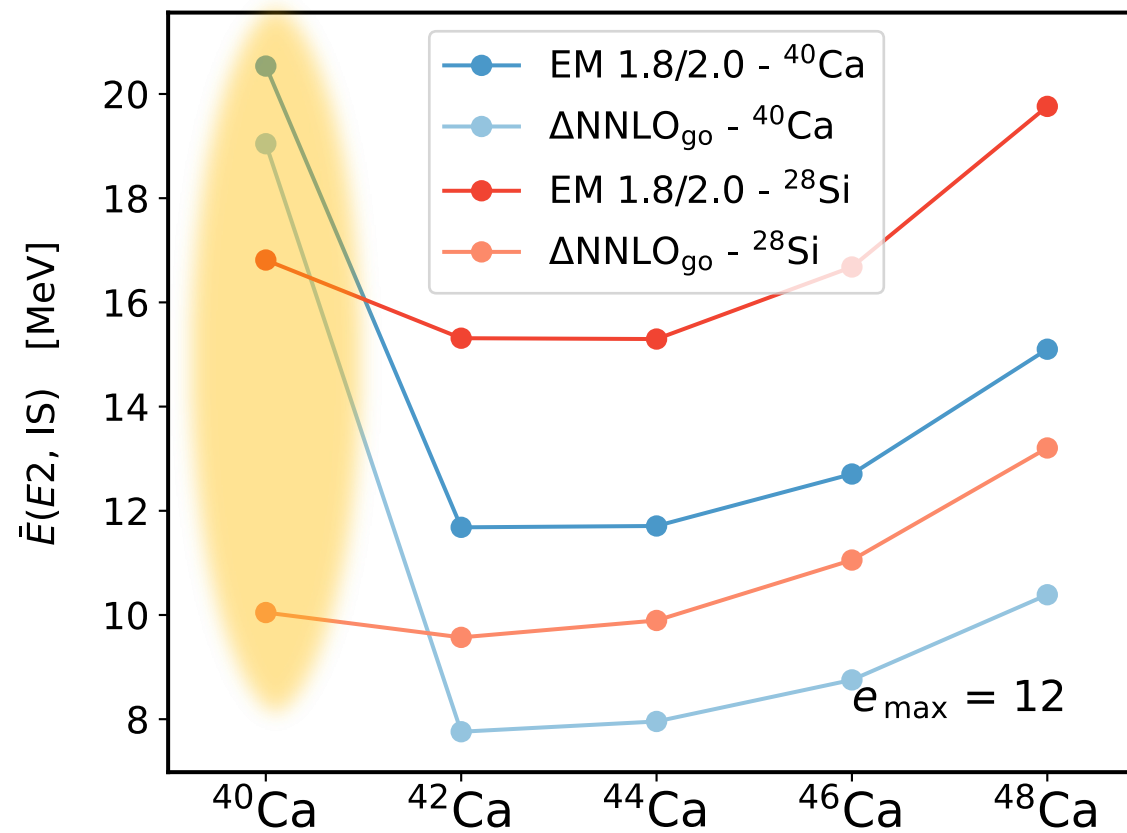


[Ahrens et al., NPA, 1975]

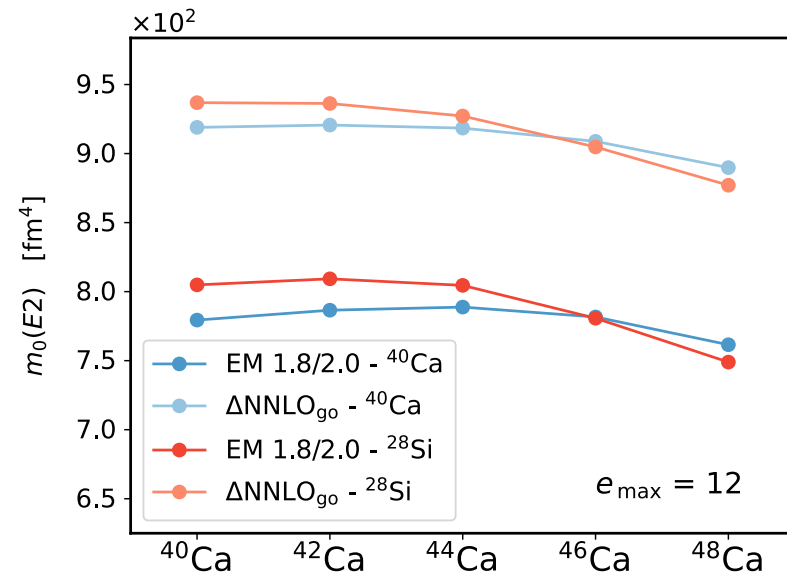
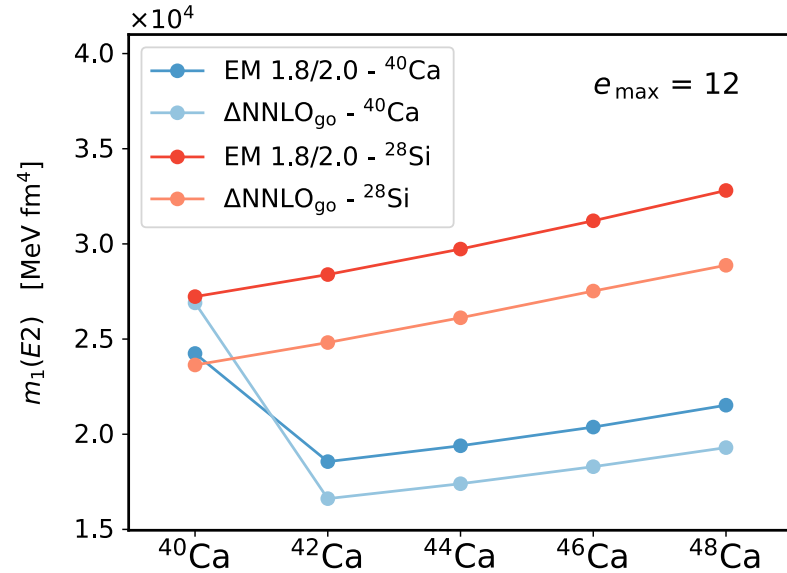
Quadrupole average energy (m_1/m_0)



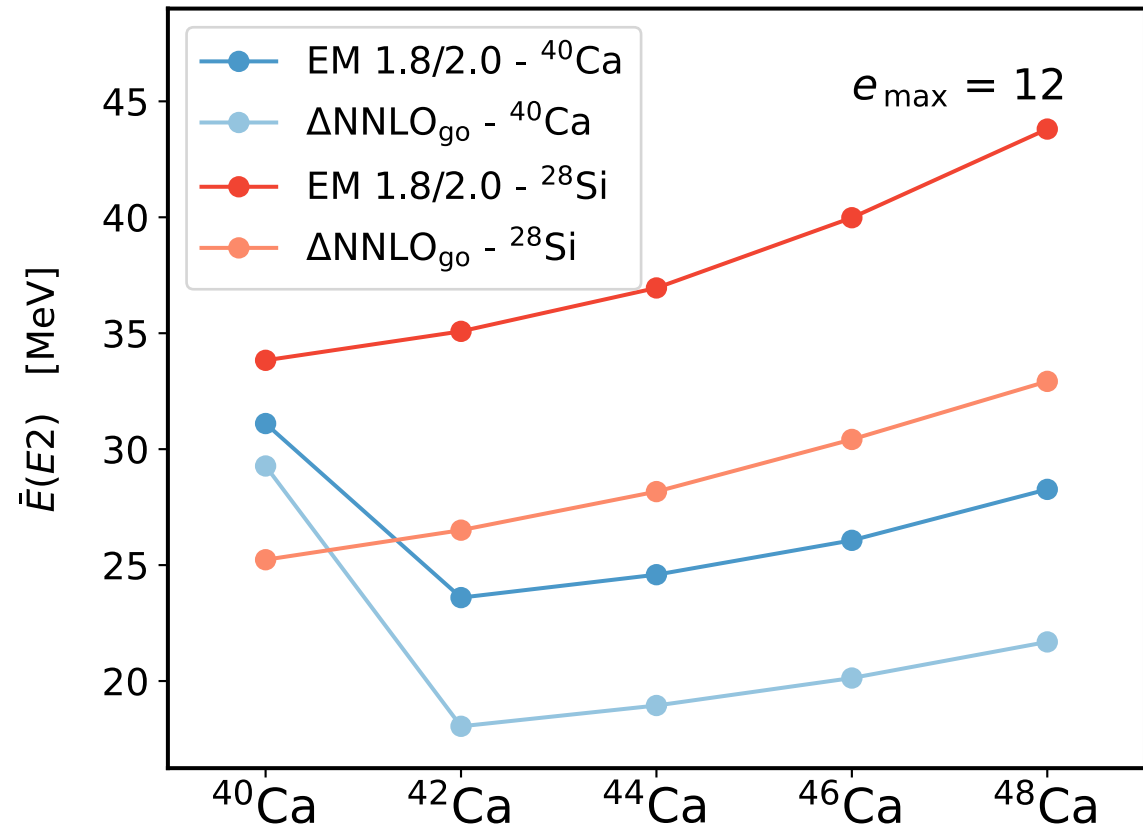
- Low-energy states impact in ^{40}Ca
- Opening the core fundamental for other isotopes aw



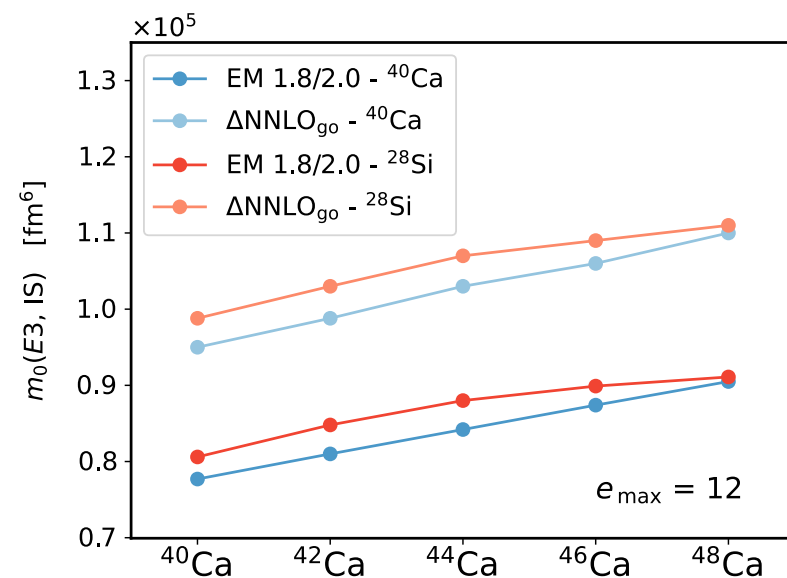
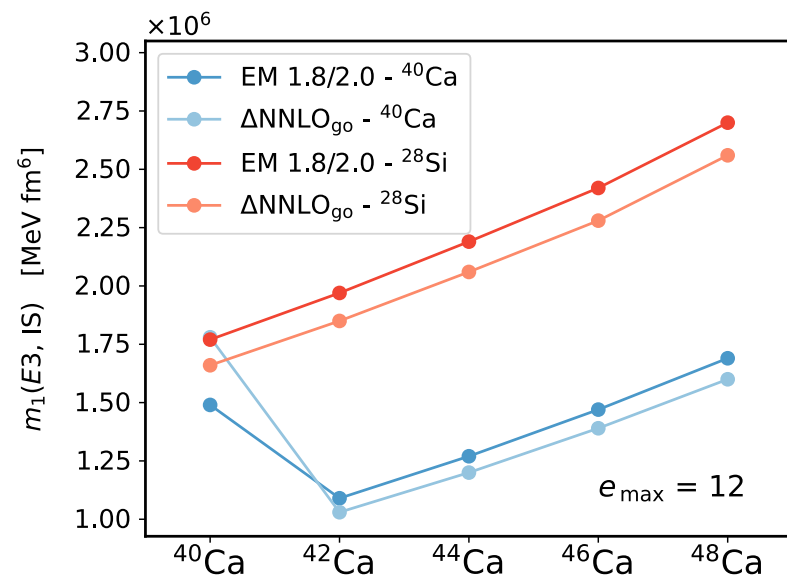
Quadrupole average energy (m_1/m_0) – Protons only



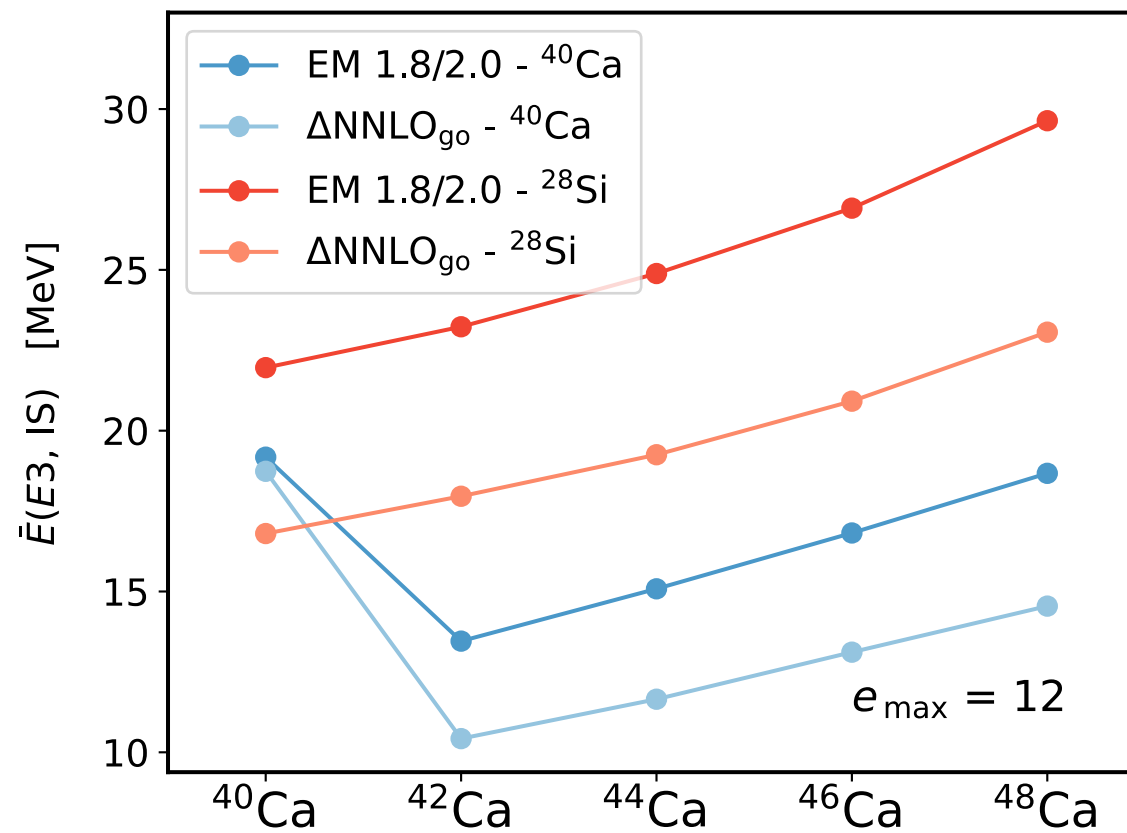
- Less interaction dependence on m_0 and m_1
- Low-lying states are less important



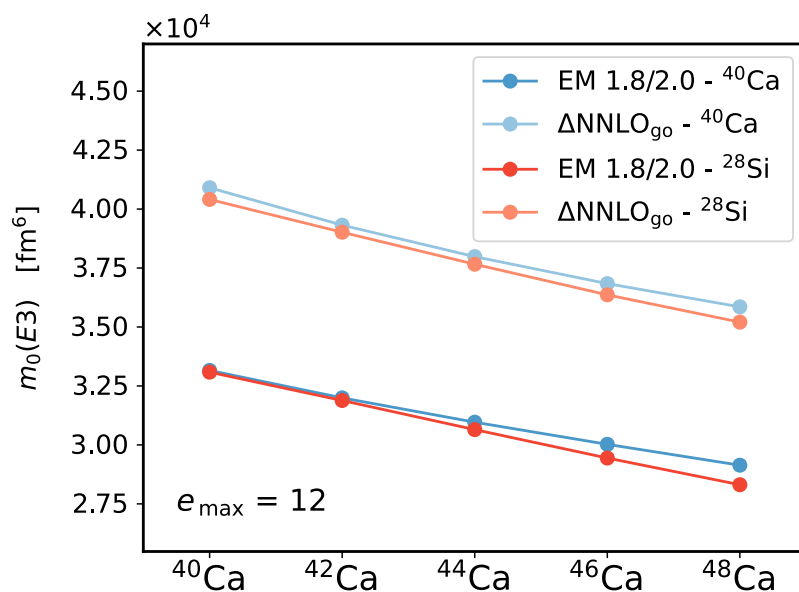
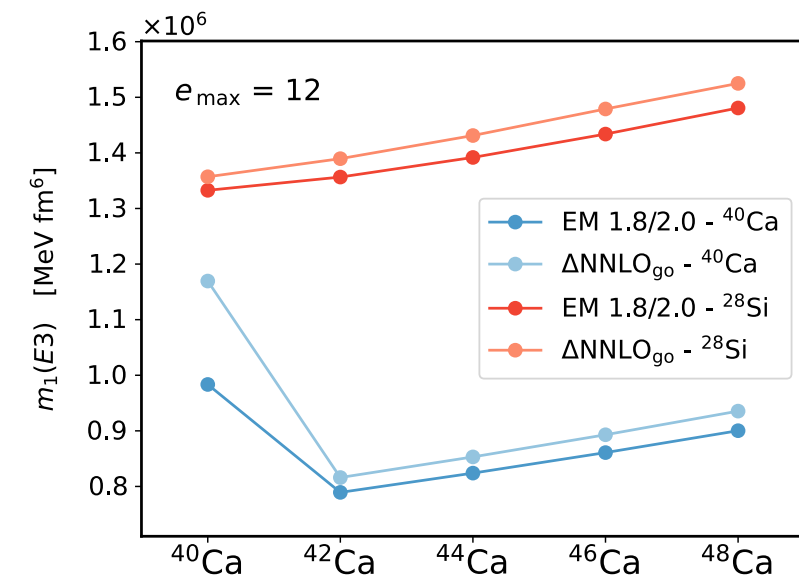
Octupole average energy (m_1/m_0)



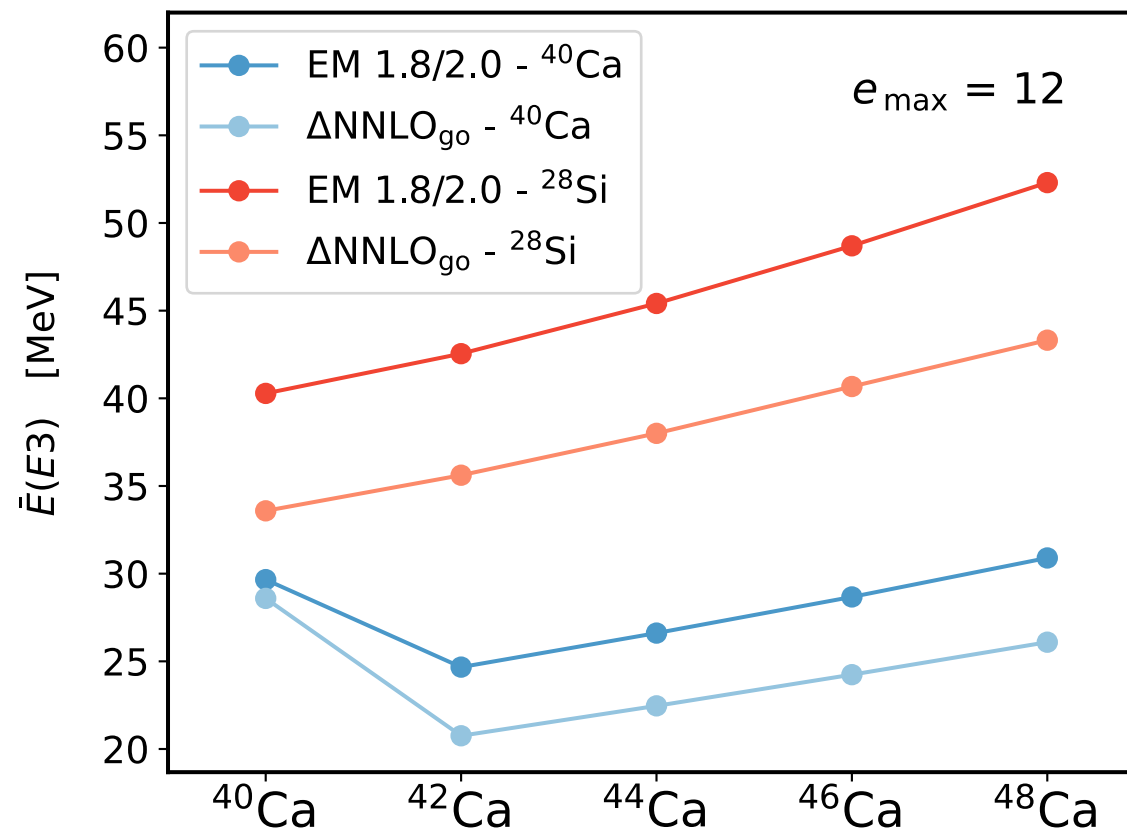
- Little interaction dependence on m_0 and m_1



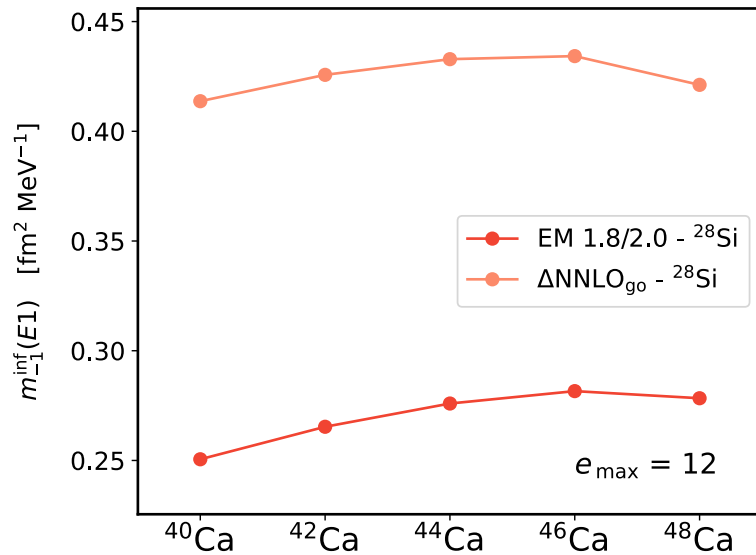
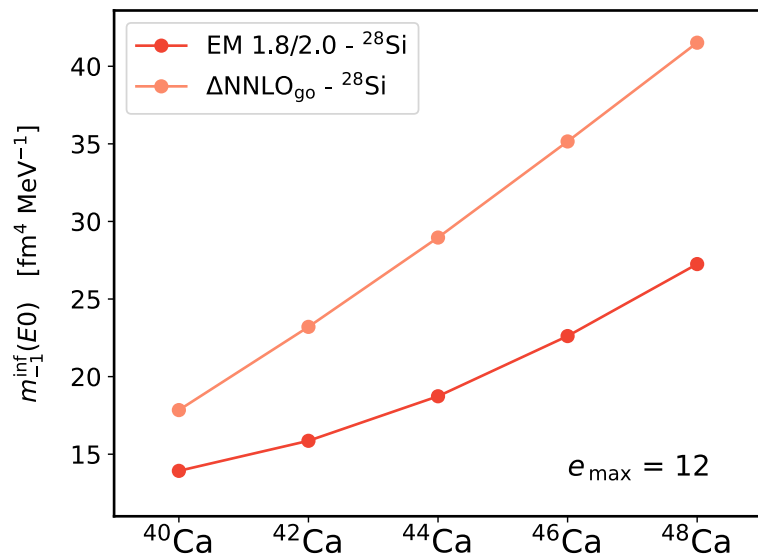
Octupole average energy (m_1/m_0) – Protons only



- Very little interaction dependence on m_0 and m_1



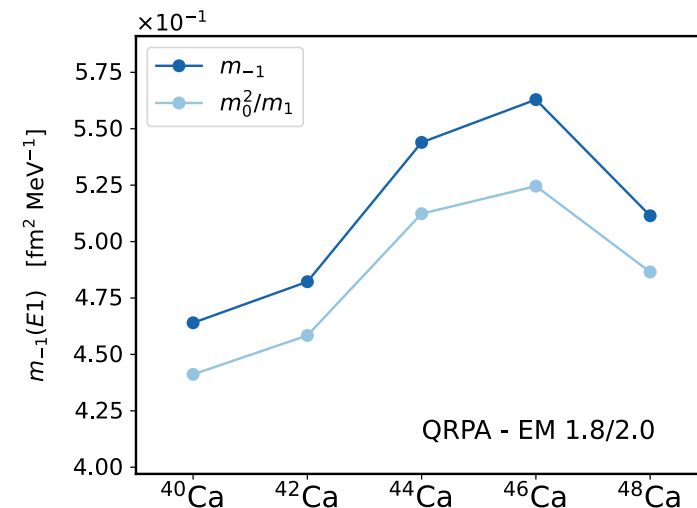
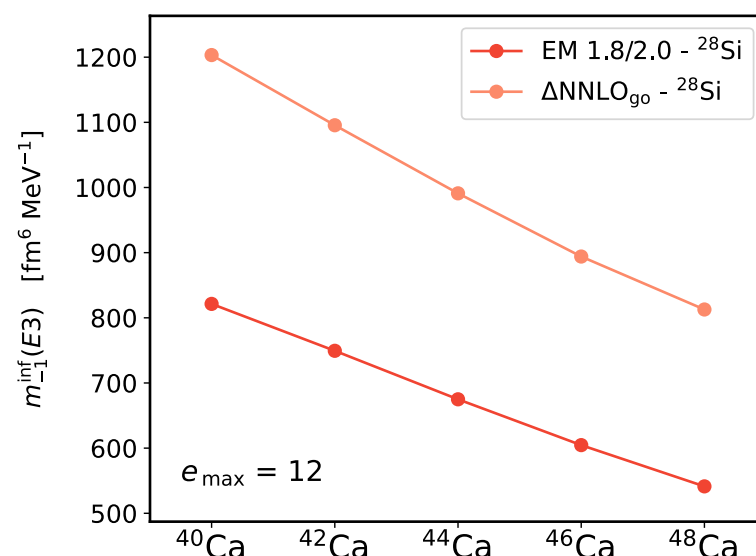
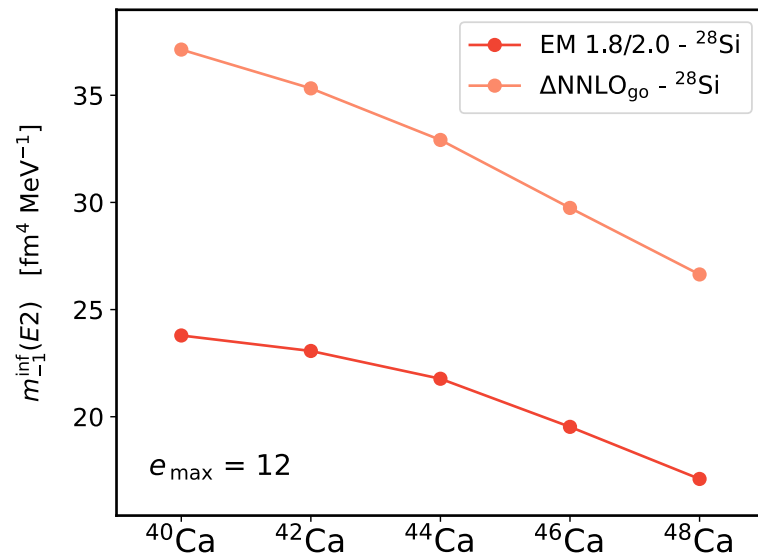
Lower limit on m_{-1} (IEWSR)



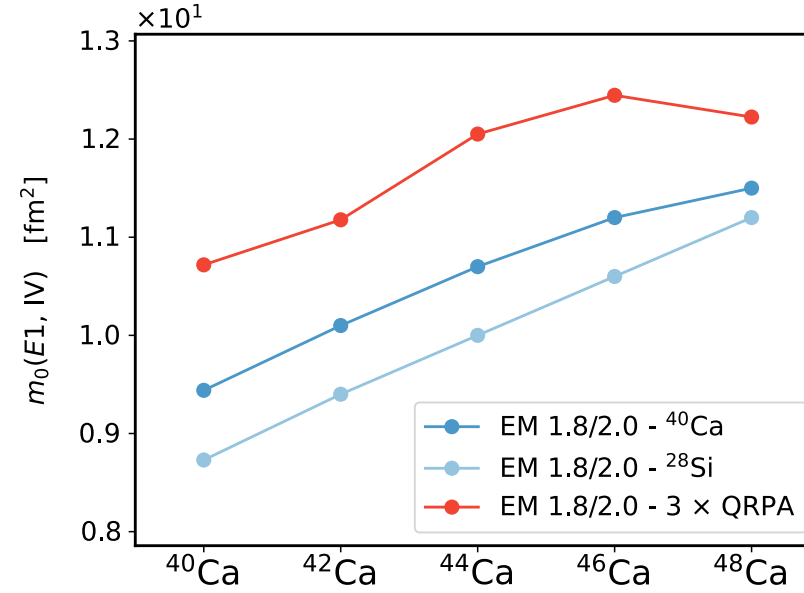
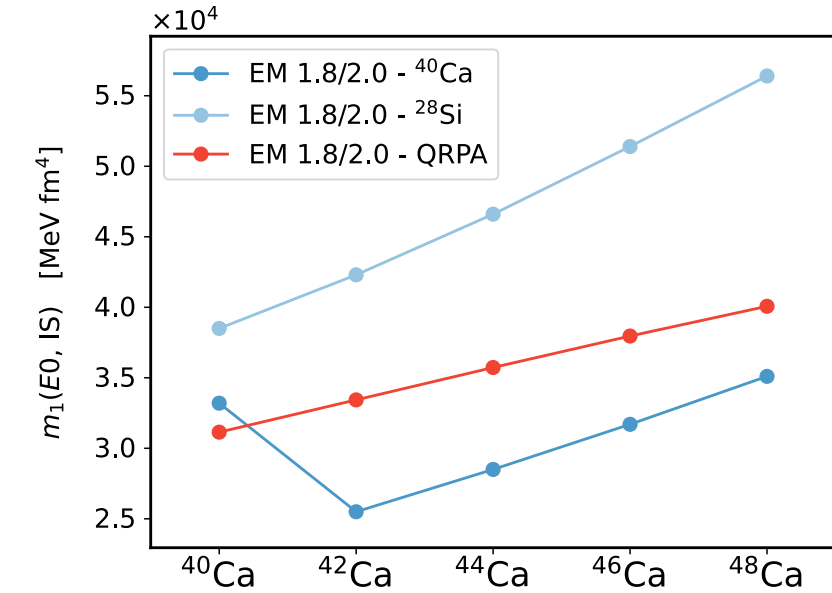
m_0 and m_1 provide a lower limit on m_{-1}
(positive definiteness of the response)

$$m_{-1} \geq \frac{m_0^2}{m_1}$$

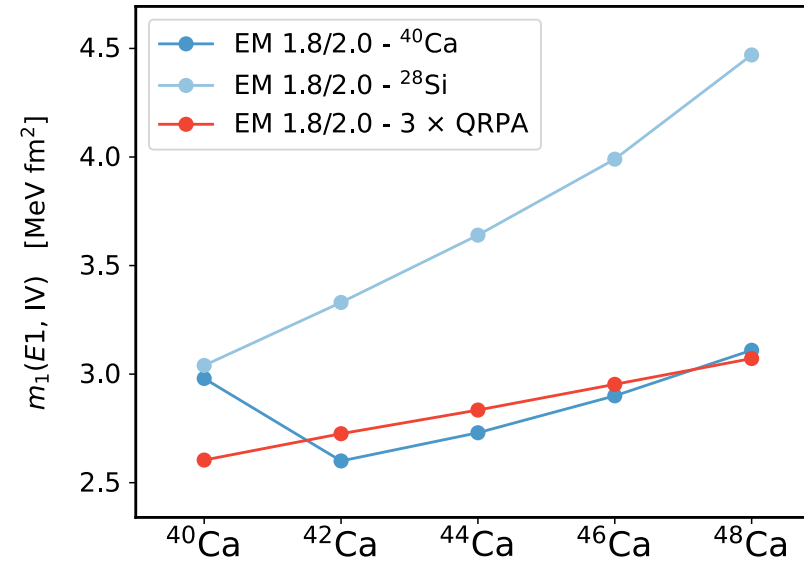
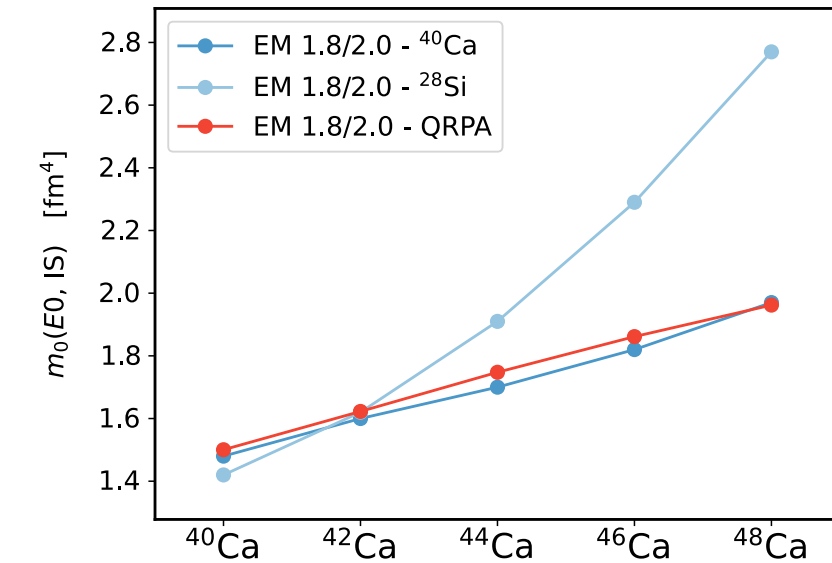
Lower limit on polarisability
(works pretty well in QRPA)



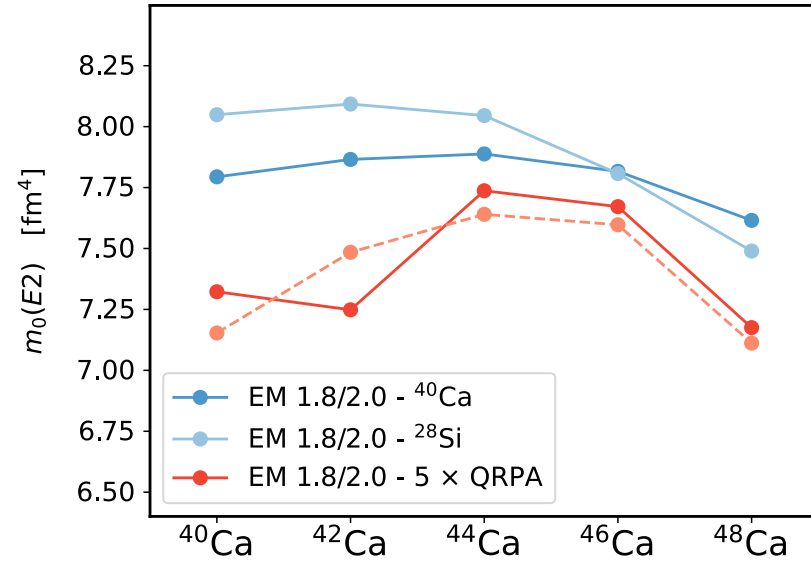
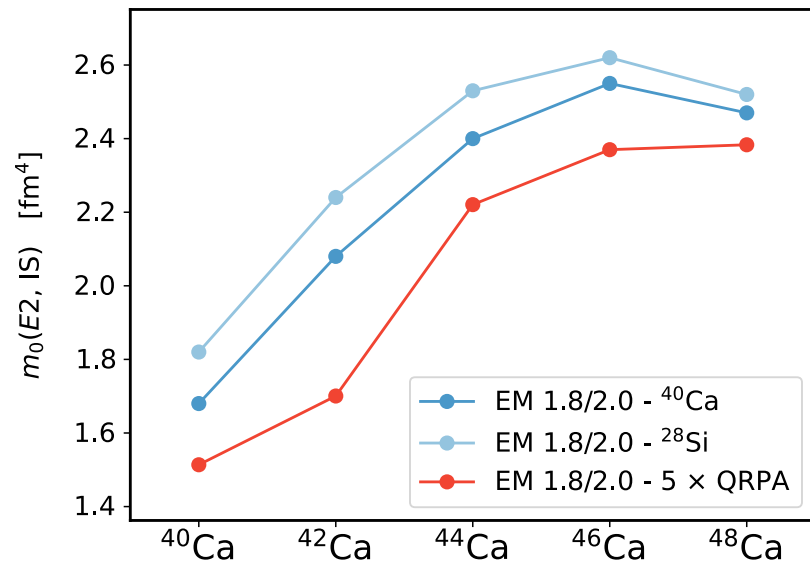
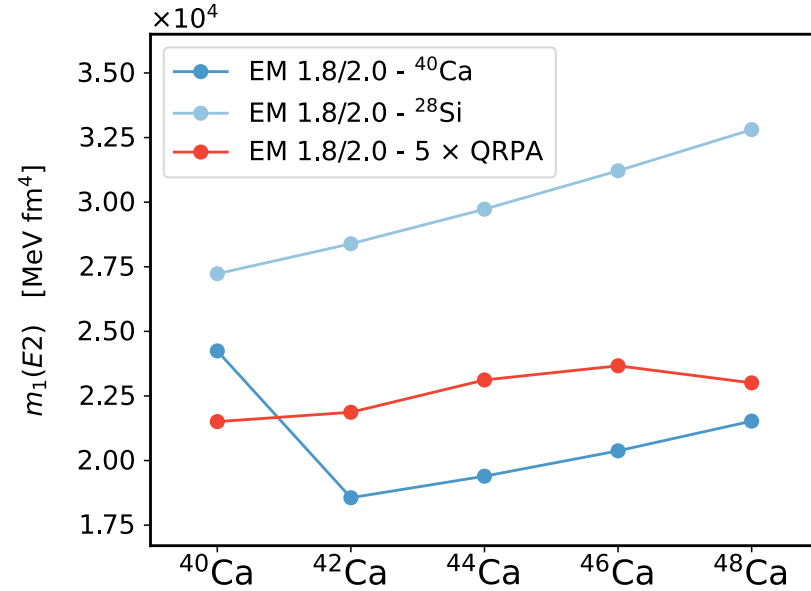
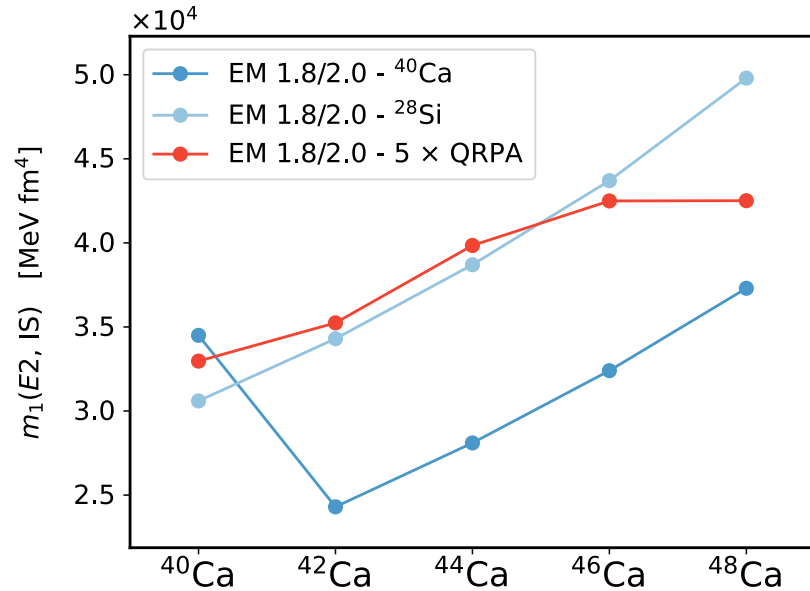
Comparison to QRPA



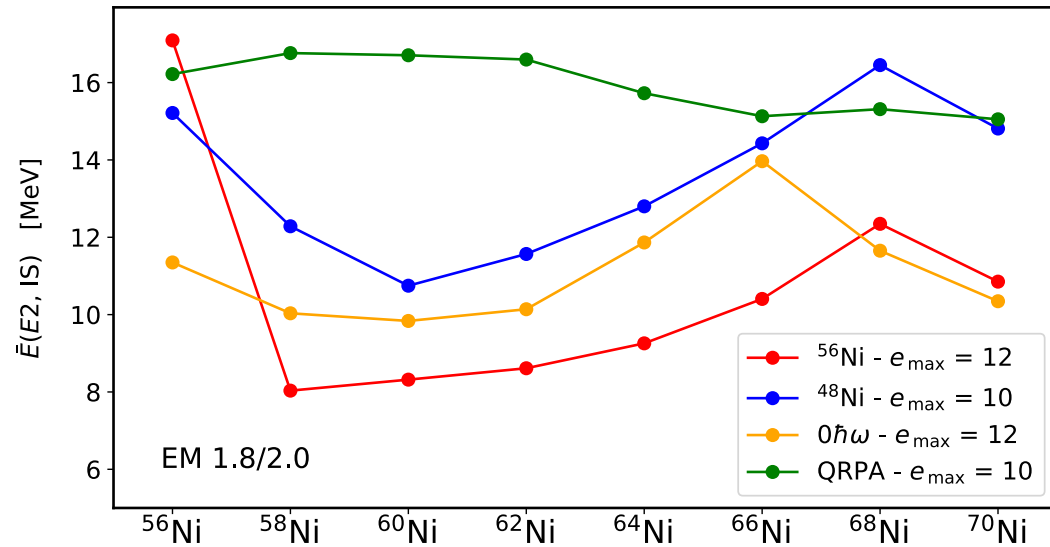
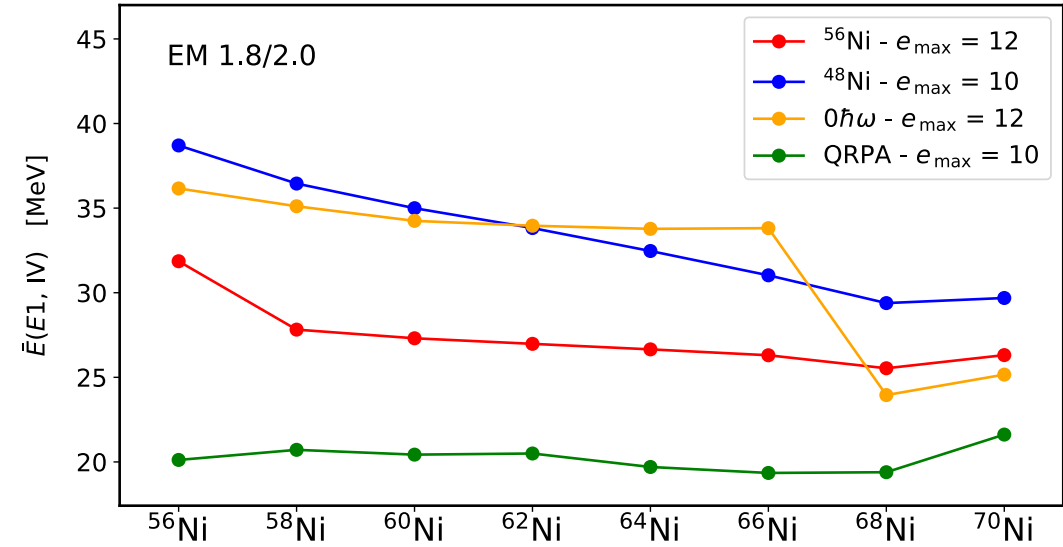
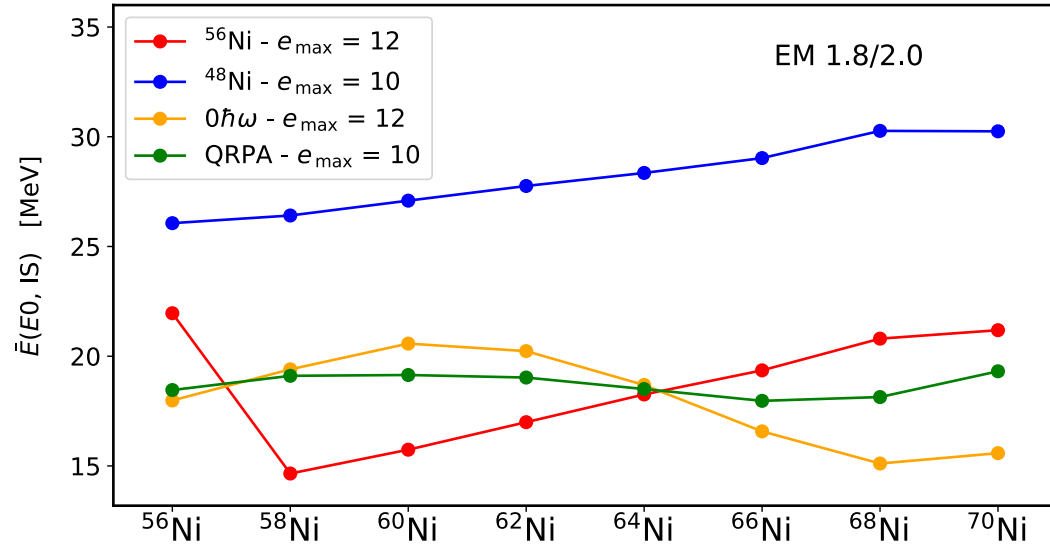
Are the trends supposed to match?



Comparison to QRPA



A look at Nichel isotopes



- Difficult region in terms of VS choice
- More accurate study needed

Introduction

- Physics case and motivation
- Quantities of interest

IMSRG multipole moments

- Sum rule exhaustion
- Strategies for moments evaluation
- Numerical results in SR-IMSRG

Extension to VS-IMSRG

- Computational details
- Multipole moments in Ca isotopes
- A look at Nickel isotopes

Conclusions

Conclusions

General considerations

- Useful to quantify the impact of dynamical correlation (**many-body uncertainty**)
- A consistent VS should be used along isotopic chain (i.e. no core) for integrated quantities

Further investigations

- Better convergence studies wrt chosen VS
- Comparison with more interactions
- More comparisons to std IMSRG (where possible)

Perspectives

- Convergence may be slow in the VS, **m-scheme** necessary?
- EOM-IMSRG ?

Thank you for the attention



Robert Roth
Achim Schwenk
Alexander Tichai



Thomas Duguet
Jean-Paul Ebran
Mikael Frosini
Vittorio Somà



Gianluca Colò



Francesca Bonaiti



Sonia Bacca