

## *Ab initio* calculations of electric dipole, Schiff, and anapole moments in atomic nuclei

Espace de Structure et de réactions Nucléaires Théorique

Theoretical and experimental developments for symmetry-violating nuclear properties

CEA Saclay, June 23-27, 2025

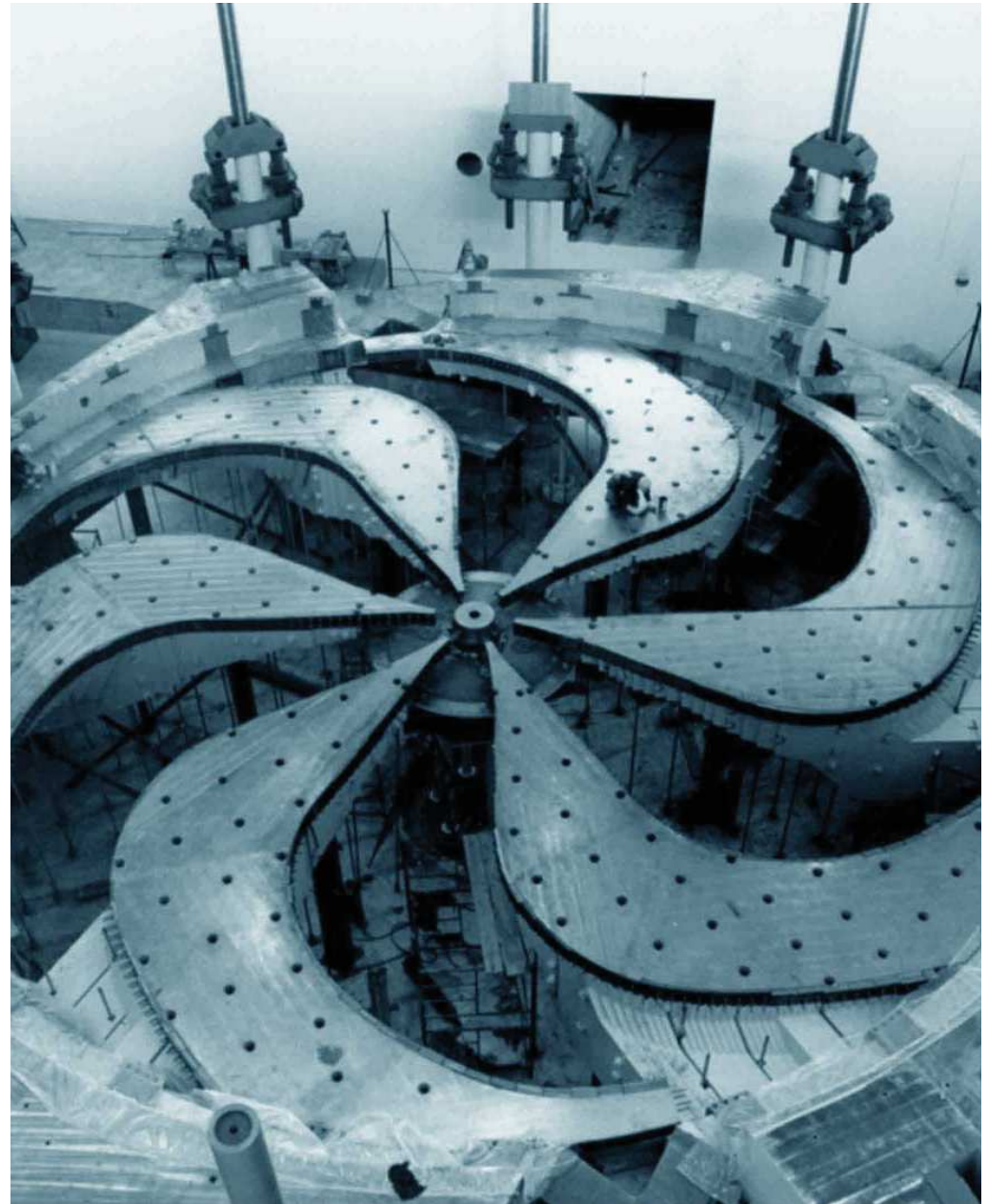
Petr Navratil

TRIUMF


Collaborator:

Stephan Foster (TRIUMF, McMaster U)

2025-06-24



## Outline

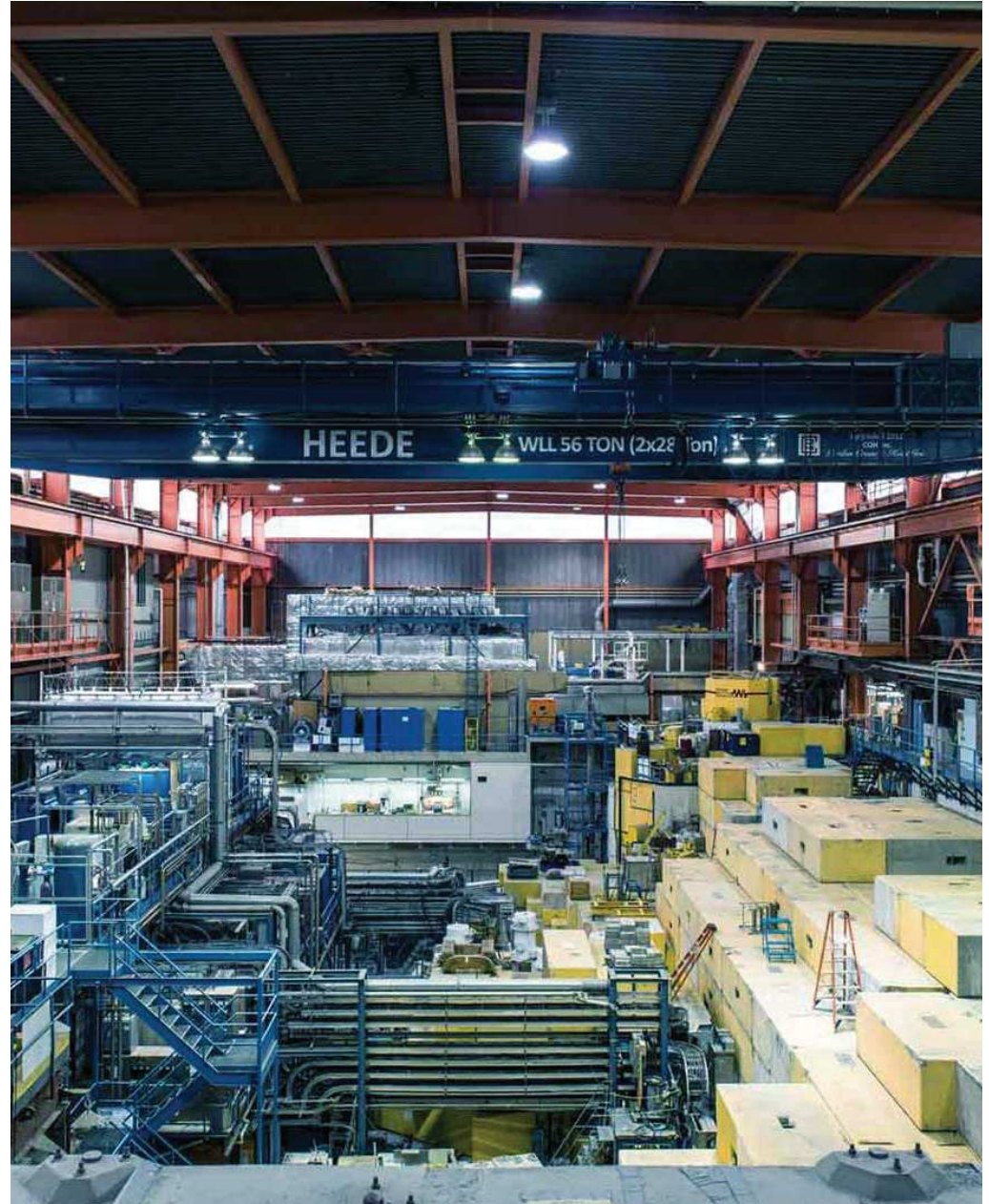
- Introduction – *Ab initio* nuclear theory – no-core shell model (NCSM)
- *Ab initio* calculations of parity-violating moments 

Lanczos strength method

  - Parity violating and parity & time-reversal violating NN interactions
  - Calculations of anapole, electric dipole, and nuclear Schiff moments
    - Schiff moment of  $^{19}\text{F}$
- Conclusions

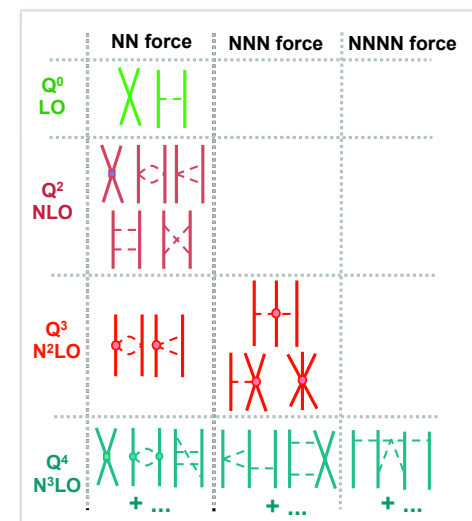
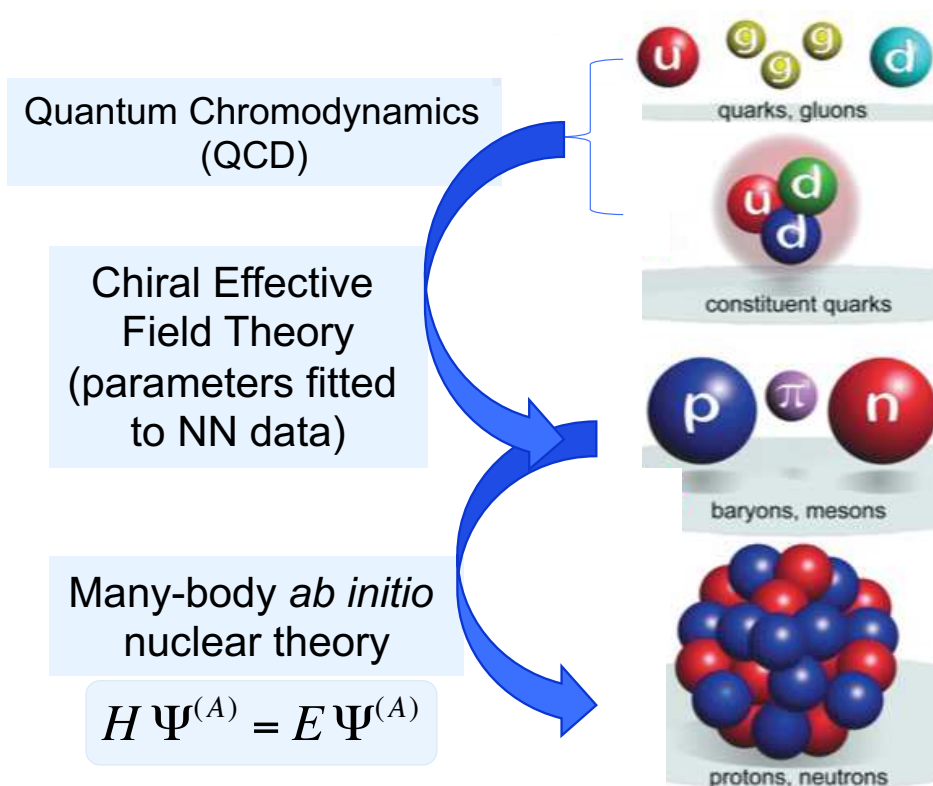
*Ab initio* nuclear theory -  
no-core shell model (NCSM)

2025-06-24



# First principles or *ab initio* nuclear theory

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Review

*Ab initio* no core shell modelBruce R. Barrett<sup>a</sup>, Petr Navrátil<sup>b</sup>, James P. Vary<sup>c,\*</sup>

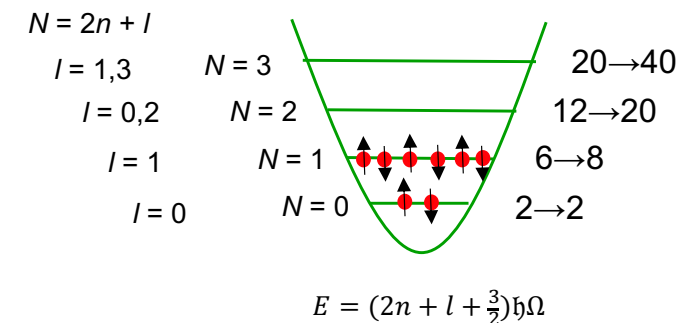
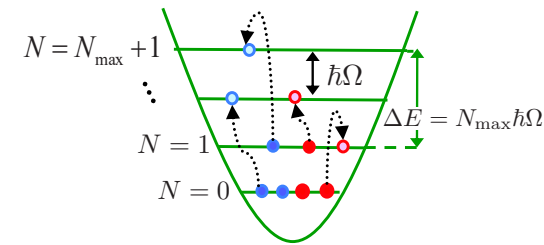
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## Conceptually simplest *ab initio* method: No-Core Shell Model (NCSM)

- Basis expansion method (CI)
  - Harmonic oscillator (HO) basis truncated in a particular way ( $N_{\max}$ )
    - HO frequency variational parameter
  - Why HO basis?
    - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 –  $^4\text{He}$ ,  $^{16}\text{O}$ ,  $^{40}\text{Ca}$ )
    - Equivalent description in relative (Jacobi)-coordinate and Slater determinant basis – **nuclei self-bound**,  $[\text{H}, \text{P}_{\text{CM}}]=0$ 
      - Exact factorization of CM and intrinsic eigenfunctions at each  $N_{\max}$

$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^{HO}(\vec{\eta}_1, \vec{\eta}_2, \dots, \vec{\eta}_{A-1})$$

$$\Psi_{\text{SD}}^A = \sum_{N=0}^{N_{\max}} \sum_j c_{Nj}^{\text{SD}} \Phi_{\text{SD}Nj}^{HO}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{\text{CM}})$$

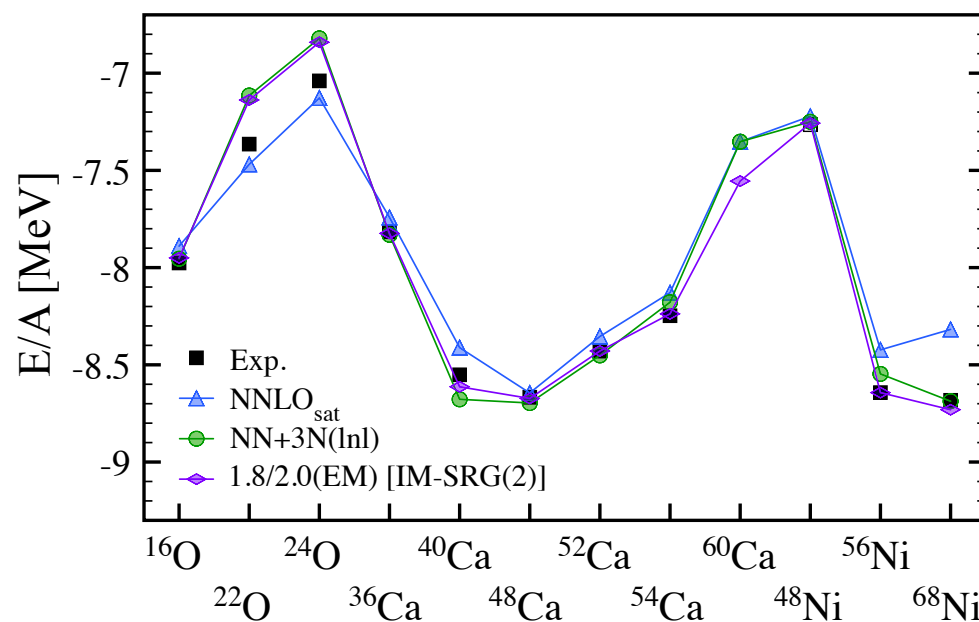
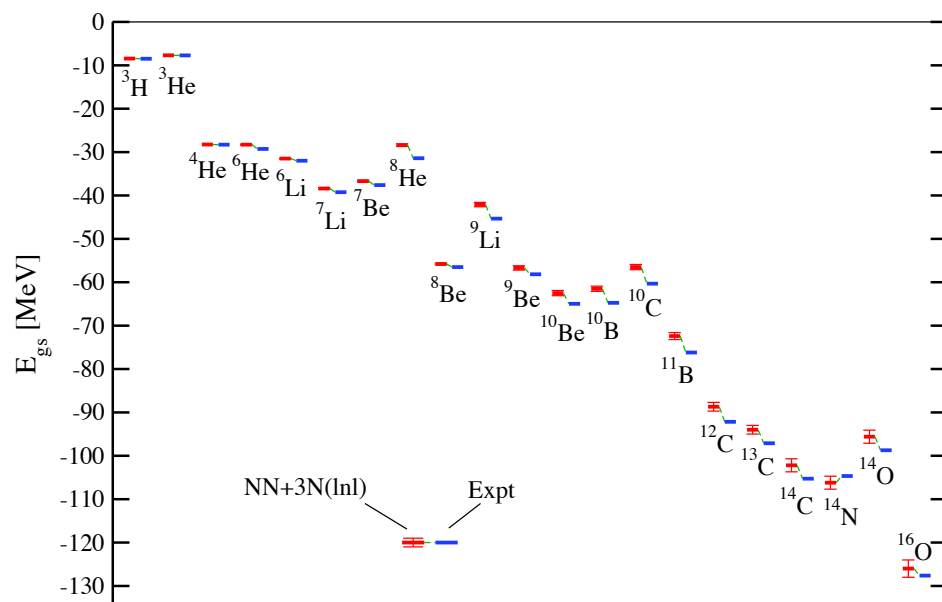


## Binding energies of atomic nuclei from nuclear forces from chiral Effective Field Theory

6

- Quite reasonable description of binding energies across the nuclear charts becomes feasible
  - The Hamiltonian fully determined in  $A=2$  and  $A=3,4$  systems**
    - Nucleon–nucleon scattering, deuteron properties,  $^3\text{H}$  and  $^4\text{He}$  binding energy,  $^3\text{H}$  half life
  - Light nuclei – NCSM
  - Medium mass nuclei – Self-Consistent Green's Function method

NN N<sup>3</sup>LO (Entem-Machleidt 2003)  
3N N<sup>2</sup>LO w local/non-local regulator



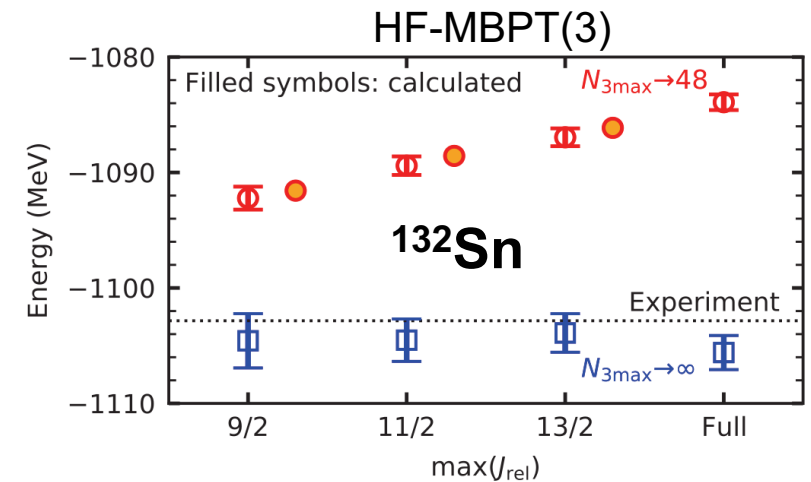
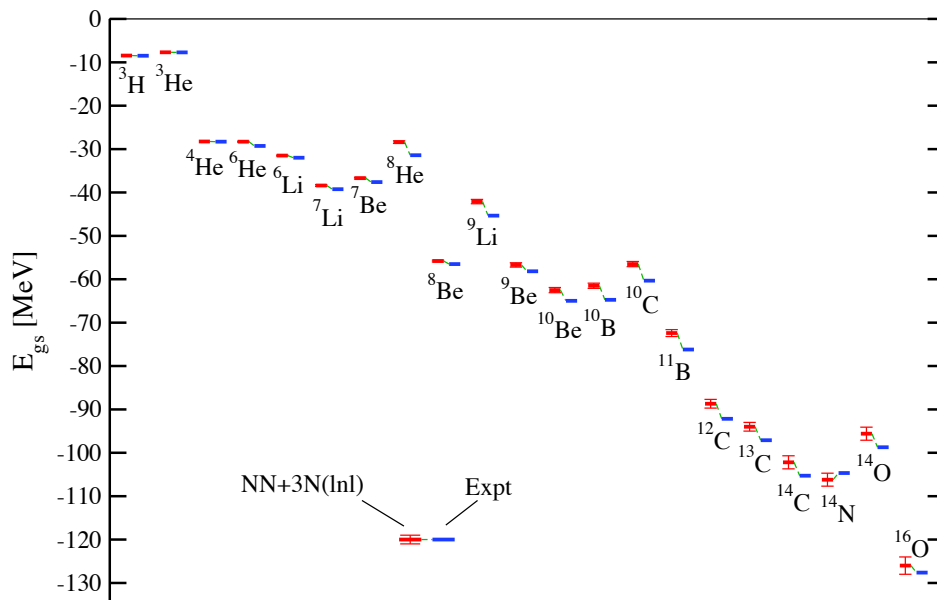
1.8/2.0 (EM) results: J. Simonis, S. R. Stroberg, K. Hebeler, J. D. Holt, and A. Schwenk, Phys. Rev. C 96, 014303 (2017).

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  - Light nuclei – NCSM
  - Heavy nuclei – HF-MBPT(3)

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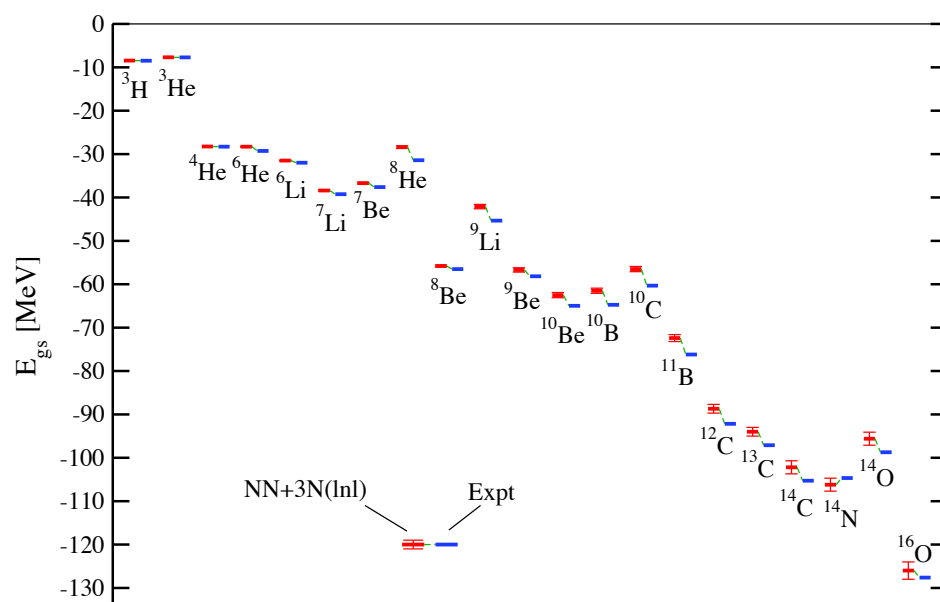


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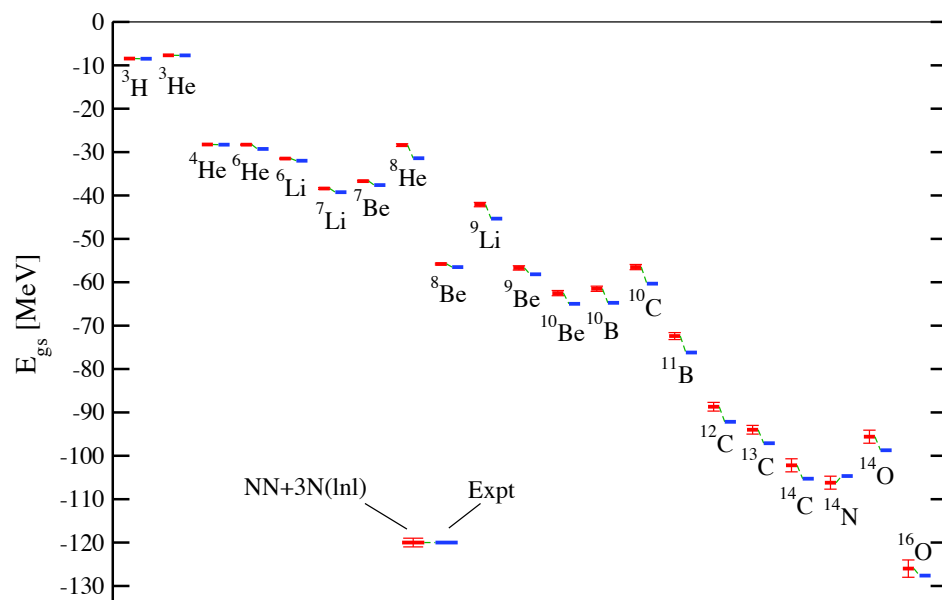


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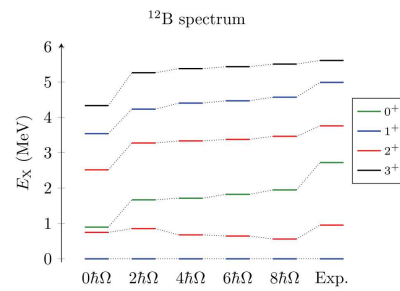
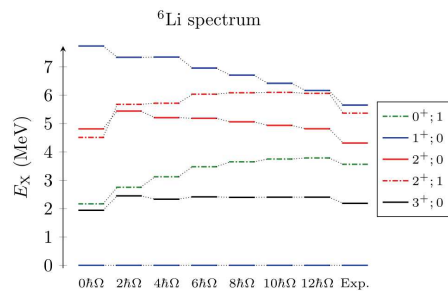
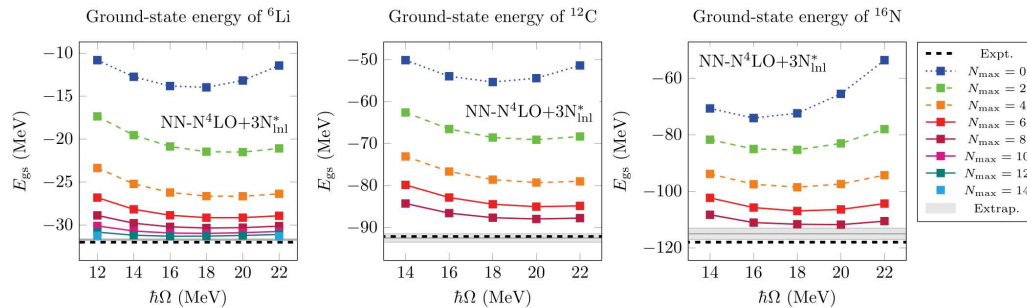
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  - NN  $N^4\text{LO}$  500 (Entem-Machleidt-Nosyk 2017)
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  - 3N subleading spin-orbit contact term (Girlanda 2011)
    - new LEC ( $E_7$ ) fitted to improve excitation levels in  $^6\text{Li}$
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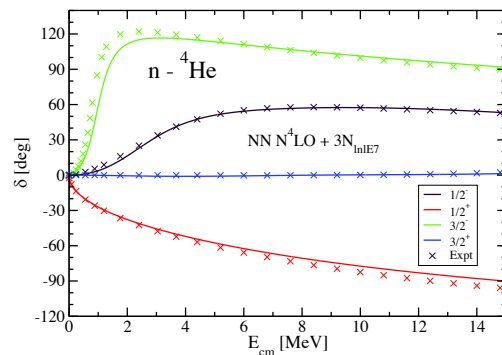
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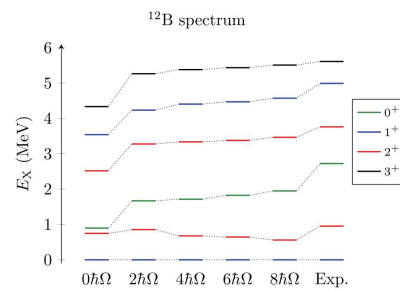
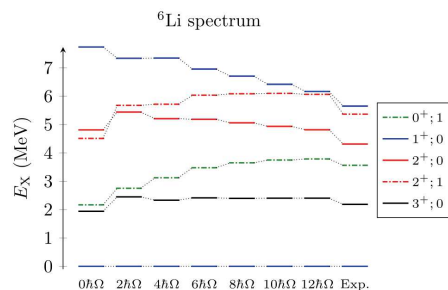
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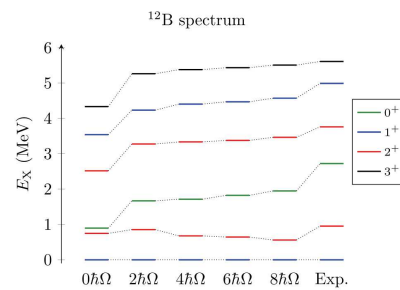
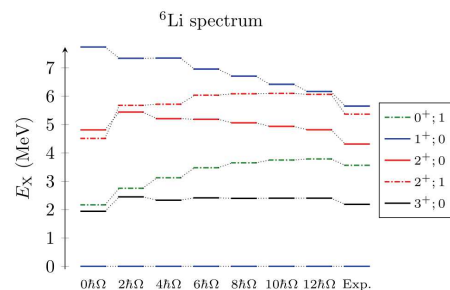
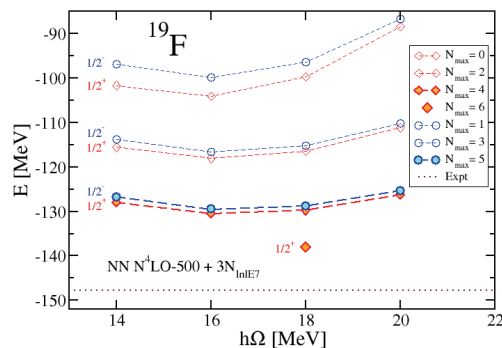


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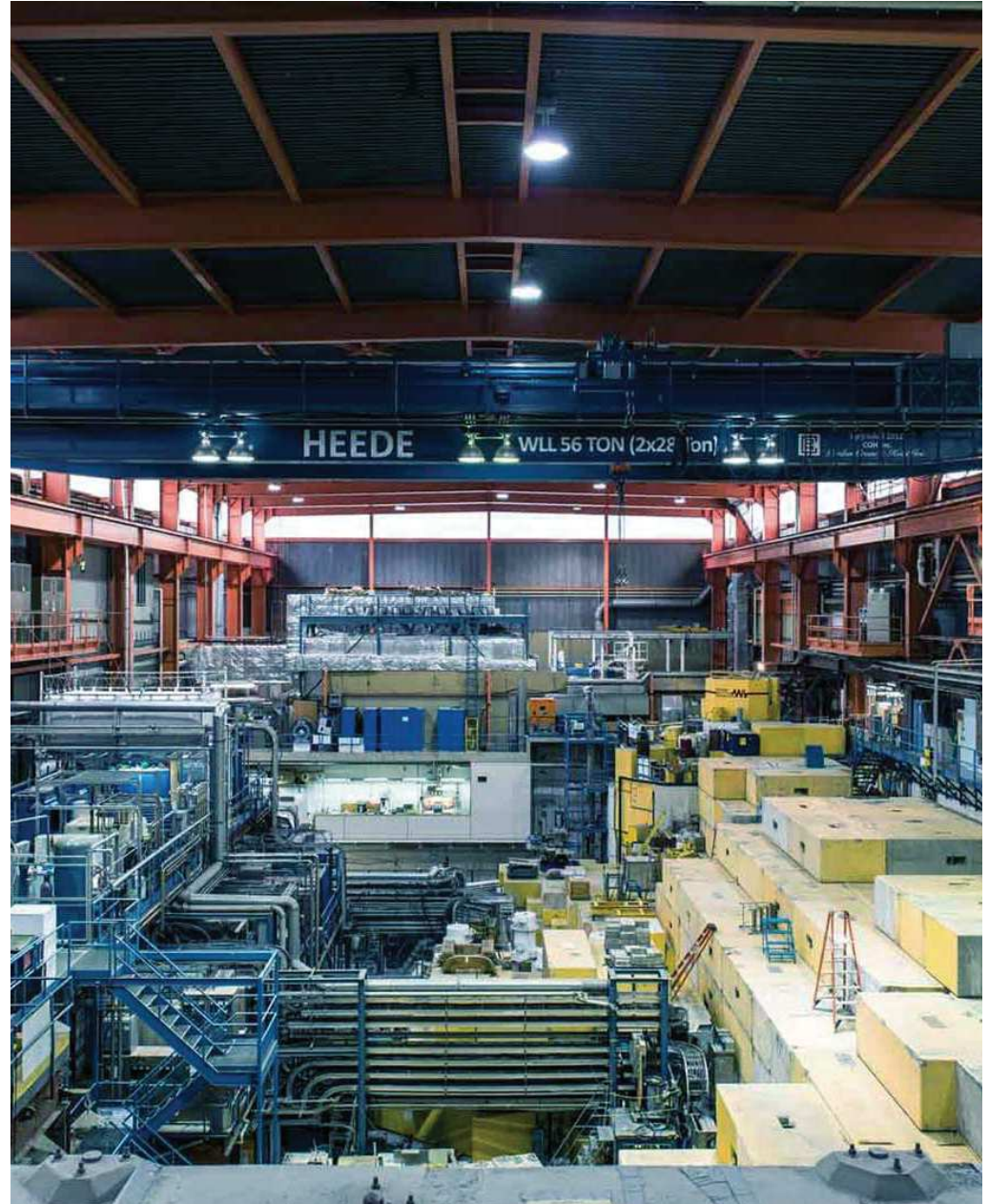
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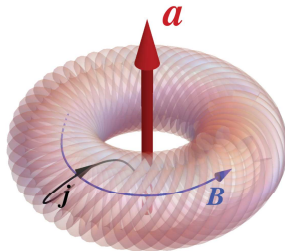
*Ab initio* calculations  
of parity-violating moments  
-  
anapole moment  
electric dipole moment (EDM)  
Schiff moment

2025-06-24



## Why investigate parity violation in atomic and molecular systems and the nuclear anapole moment?

- Parity violation in atomic and molecular systems sensitive to a variety of “new physics”
  - Probes electron-quark electroweak interaction
  - Best limits on the  $Z'$  boson parity violating interaction with electrons and nucleons

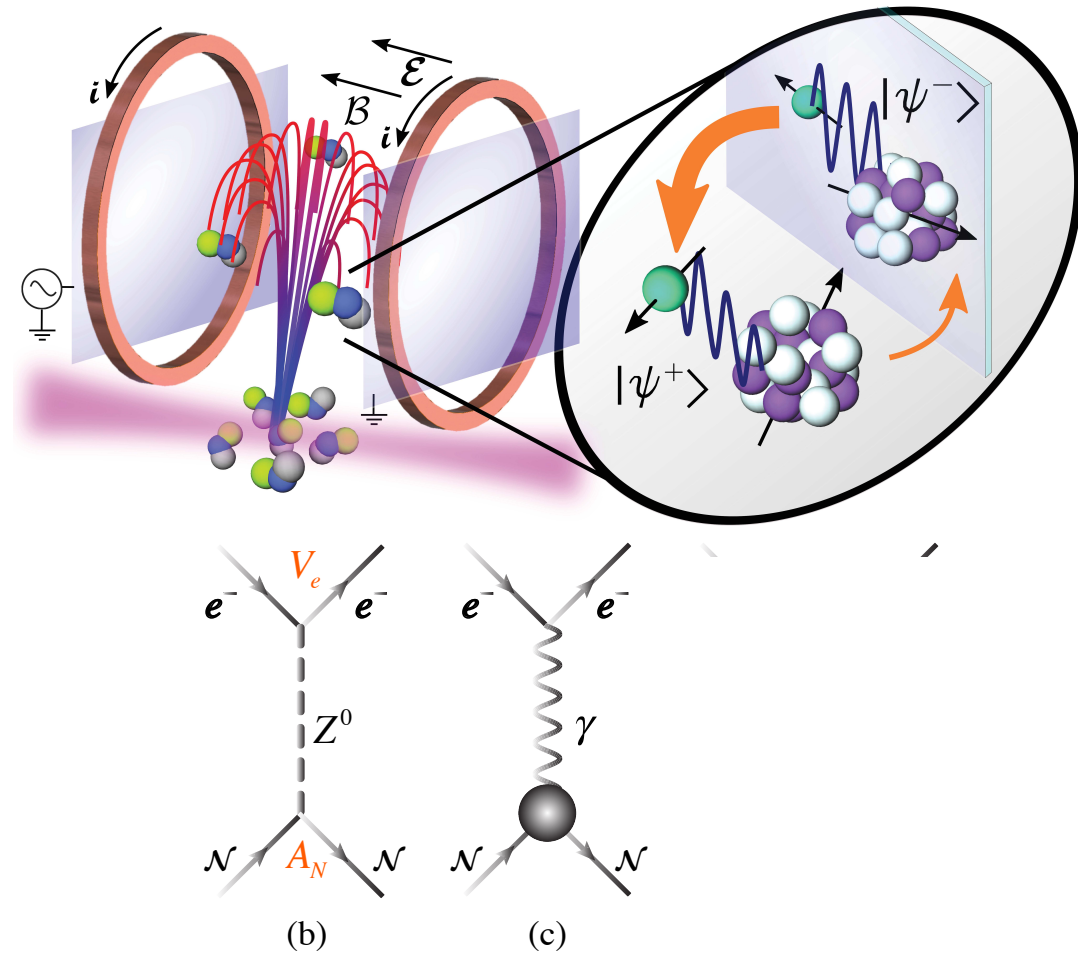




# Nuclear spin dependent parity violating effects in light polyatomic molecules

15

- Experiments proposed for  $^9\text{BeNC}$ ,  $^{25}\text{MgNC}$
- To extract the underlying physics, atomic, molecular, and **nuclear** structure effects must be understood
  - Ab initio* calculations
- Spin dependent parity violation
  - Z-boson exchange between nucleon axial-vector and electron-vector currents (b)
  - Electromagnetic interaction of atomic electrons with the nuclear anapole moment (c)

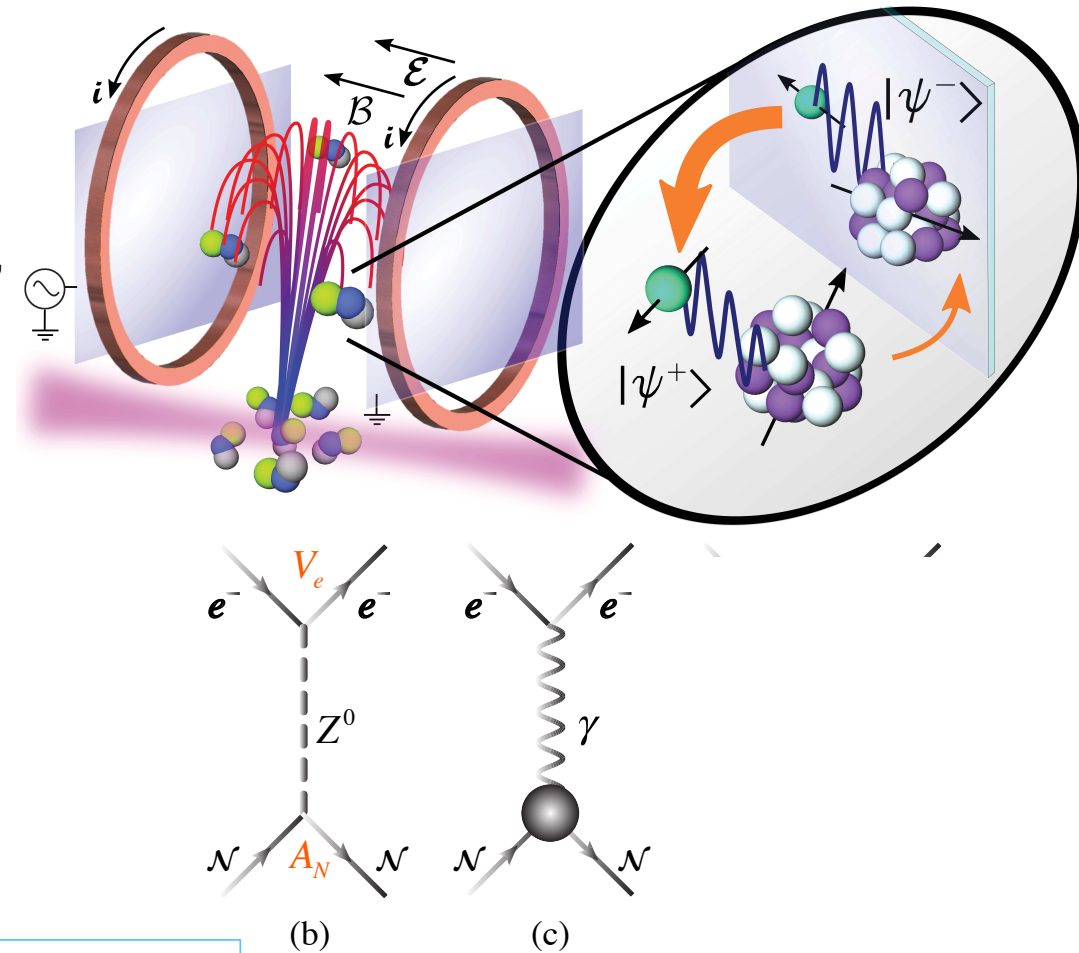


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Anapole moment measurements also planned in  $^{137}\text{BaF}$  molecule

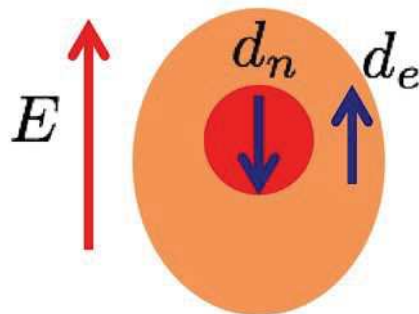
## Why investigate the Electric Dipole Moment (EDM) and nuclear Schiff Moment?

- Unsolved problem in physics: matter-antimatter asymmetry of the universe
- Standard model predicts some CP violation, not enough to explain this asymmetry
- The EDM and nuclear Schiff moment is a promising probe for CP violation beyond the standard model, as well as CP violating QCD  $\bar{\theta}$  parameter
- Nuclear EDMs can be measured in storage rings (CERN feasibility study: arXiv:1912.07881)
- Nuclear Schiff moments can be measured using (radioactive) molecules

Nuclear Schiff moment measurements planned in  $^{227}\text{ThF}^+$ , RaF, and FrAg molecules

To understand the nuclear EDM and Schiff moment, nuclear structure effects must be understood

## What is the nuclear Schiff moment?



Schiff Moment

$$\vec{S} = \frac{\langle e r^2 \vec{r} \rangle}{10} - \frac{\langle r^2 \rangle \langle e \vec{r} \rangle}{6}$$

Leonard Schiff's Theorem (1963):

- Any permanent dipole moment of the nucleus is perfectly shielded by its electron cloud
- True for point-like nuclei, non-relativistic electrons

However, the "Schiff moment" is not shielded by this effect

- Zero for point-like, spherical nuclei
- Arises from deformations in the nucleus or its constituent nucleons
- Very large in nuclei with both a quadrupole and octupole deformation

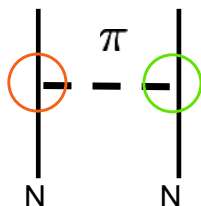
**Look for heavy nuclei with large quadrupole and octupole deformations!**

Slide by Matthew R. Dietrich (ANL)

## Parity violating (PV) and parity & time-reversal (PTV) violating nucleon-nucleon (NN) interaction

- Anapole moment arises due to PV NN interaction (weak force - imaginary), EDM and Schiff moment due to PTV NN interaction (real)
- Parity non-conserving PV or PTV  $V_{NN}^{\text{PNC}}$  interaction
  - Conserves total angular momentum /
  - Mixes opposite parities
  - Has isoscalar, isovector and isotensor components

Meson-exchange picture – one vertex PC strong force, one vertex PV (weak) force



$$\begin{aligned} \mathcal{H}_{MNN}^{\text{p.v.}} = & (2)^{-1/2} f_\pi \bar{N} (\vec{\tau} \times \vec{\phi}^\pi)^3 N \\ & + \bar{N} \left[ h_\rho^0 \vec{\tau} \cdot \vec{\phi}_\mu^\rho + h_\rho^1 \phi_\mu^{\rho 3} + h_\rho^2 \frac{(3\tau^3 \phi_\mu^{\rho 3} - \vec{\tau} \cdot \vec{\phi}_\mu^\rho)}{2(6)^{1/2}} \right] \gamma^\mu \gamma_5 N \\ & + \bar{N} [h_\omega^0 \phi_\mu^\omega + h_\omega^1 \tau^3 \phi_\mu^\omega] \gamma^\mu \gamma_5 N \\ & - h_\rho^1 \bar{N} (\vec{\tau} \times \vec{\phi}_\mu^\rho)^3 \frac{\sigma^{\mu\nu} k_\nu}{2M} \gamma_5 N. \end{aligned}$$

$$\begin{aligned} \mathcal{H}_{MNN}^{\text{p.c.}} = & i g_{\pi NN} \bar{N} \gamma_5 \vec{\tau} \cdot \vec{\phi}^\pi N + g_\rho \bar{N} \left( \gamma_\mu + \frac{i\chi_V}{2M} \sigma_{\mu\nu} k^\nu \right) \vec{\tau} \cdot \vec{\phi}^{\mu\rho} N \\ & + g_\omega \bar{N} \left( \gamma_\mu + \frac{i\chi_S}{2M} \sigma^{\mu\nu} k_\nu \right) \phi_\mu^\omega N \end{aligned}$$

Include  $\pi$ ,  $\rho$ ,  $\omega$  meson exchanges

ANNALS OF PHYSICS 124, 449–495 (1980)

### Unified Treatment of the Parity Violating Nuclear Force

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JOHN F. DONOGHUE†

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AND

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*Physics Division, National Science Foundation, Washington, D. C. 20550*

PHYSICAL REVIEW C 70, 055501 (2004)

### P- and T-odd two-nucleon interaction and the deuteron electric dipole moment

C.-P. Liu\* and R. G. E. Timmermans†

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in Physics

REVIEW  
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doi: 10.3389/fphy.2020.00218



### Parity- and Time-Reversal-Violating Nuclear Forces

Jordy de Vries<sup>1,2</sup>, Evgeny Epelbaum<sup>1</sup>, Luca Girlanda<sup>1,3</sup>, Alex Gnech<sup>4</sup>, Emanuele Mereghetti<sup>1</sup> and Michele Viviani<sup>1\*</sup>

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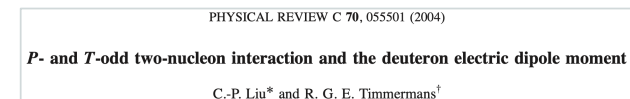
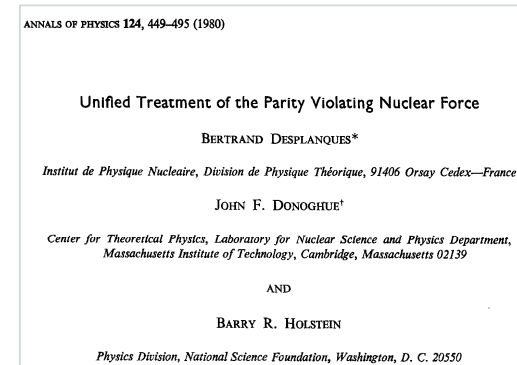
Meson-exchange picture – one vertex PC strong force, one vertex PV (weak) force

$$H_{PV} \propto \left[ \frac{\vec{p}}{M}, y_x(r) \right] \dots + \dots \left\{ \frac{\vec{p}}{M}, y_x(r) \right\}$$

$$H_{PTV} \propto i \left[ \frac{\vec{p}}{M}, y_x(r) \right]$$

$$y_x(r) = e^{-m_x r} / (4\pi r)$$

Include  $\pi$ ,  $\rho$ ,  $\omega$  meson exchanges





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PHYSICAL REVIEW C **70**, 055501 (2004)

***P*- and *T*-odd two-nucleon interaction and the deuteron electric dipole moment**

C.-P. Liu\* and R. G. E. Timmermans†

$$\begin{aligned}
 H_{PTV}(\mathbf{r}) = & \frac{1}{2m_n} \boldsymbol{\sigma}_- \cdot \nabla \left( -\bar{G}_\omega^0 y_\omega(r) \right) \\
 & + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \boldsymbol{\sigma}_- \cdot \nabla \left( \bar{G}_\pi^0 y_\pi(r) - \bar{G}_\rho^0 y_\rho(r) \right) \\
 & + \frac{\tau_+^z}{2} \boldsymbol{\sigma}_- \cdot \nabla \left( \bar{G}_\pi^1 y_\pi(r) - \bar{G}_\rho^1 y_\rho(r) - \bar{G}_\omega^1 y_\omega(r) \right) \\
 & + \frac{\tau_-^z}{2} \boldsymbol{\sigma}_+ \cdot \nabla \left( \bar{G}_\pi^1 y_\pi(r) + \bar{G}_\rho^1 y_\rho(r) - \bar{G}_\omega^1 y_\omega(r) \right) \\
 & + (3\tau_1^z \tau_2^z - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \boldsymbol{\sigma}_- \cdot \nabla \left( \bar{G}_\pi^2 y_\pi(r) - \bar{G}_\rho^2 y_\rho(r) \right)
 \end{aligned}$$

- Based on one meson exchange model

$$\sigma_\pm = \sigma_1 \pm \sigma_2$$

- $y_x(r) = e^{-m_x r} / (4\pi r)$

$$\tau_\pm^z = \tau_1^z \pm \tau_2^z$$

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$$\begin{aligned}
 H_{PTV}(\mathbf{r}) = & \frac{1}{2m_n} \boldsymbol{\sigma}_- \cdot \nabla \left( -\bar{G}_\omega^0 y_\omega(r) \right) \\
 & + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \boldsymbol{\sigma}_- \cdot \nabla \left( \bar{G}_\pi^0 y_\pi(r) - \bar{G}_\rho^0 y_\rho(r) \right) \\
 & + \frac{\tau_+^z}{2} \boldsymbol{\sigma}_- \cdot \nabla \left( \bar{G}_\pi^1 y_\pi(r) - \bar{G}_\rho^1 y_\rho(r) - \bar{G}_\omega^1 y_\omega(r) \right) \\
 & + \frac{\tau_-^z}{2} \boldsymbol{\sigma}_+ \cdot \nabla \left( \bar{G}_\pi^1 y_\pi(r) + \bar{G}_\rho^1 y_\rho(r) - \bar{G}_\omega^1 y_\omega(r) \right) \\
 & + (3\tau_1^z \tau_2^z - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \boldsymbol{\sigma}_- \cdot \nabla \left( \bar{G}_\pi^2 y_\pi(r) - \bar{G}_\rho^2 y_\rho(r) \right)
 \end{aligned}$$

- Based on one meson exchange model

$$\sigma_\pm = \sigma_1 \pm \sigma_2$$

- $y_x(r) = e^{-m_x r} / (4\pi r)$

$$\tau_\pm^z = \tau_1^z \pm \tau_2^z$$

- Coupling constants

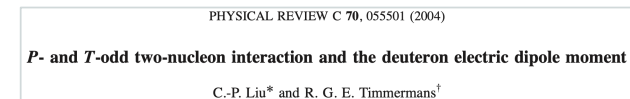
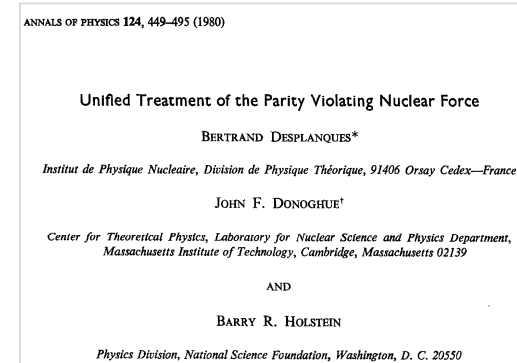
$$\bar{G}_\pi^t = g_{\pi NN} \bar{g}_{\pi NN}^t \quad (g_{\pi NN} \sim 13.3)$$

## Parity violating (PV) and parity & time-reversal (PTV) violating nucleon-nucleon (NN) interaction

23

- Anapole moment arises due to PV NN interaction (weak force - imaginary), EDM and Schiff moment due to PTV NN interaction (real)
- Parity non-conserving PV or PTV  $V_{\text{NN}}^{\text{PNC}}$  interaction
  - Conserves total angular momentum  $I$
  - Mixes opposite parities
  - Has isoscalar, isovector and isotensor components
  - Admixes unnatural parity states in the ground state

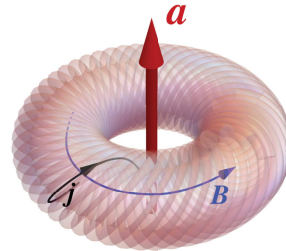
$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \times \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$



## Parity violating (PV) and parity & time-reversal (PTV) violating nucleon-nucleon (NN) interaction

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  - Admixes unnatural parity states in the ground state
- Anapole moment operator dominated by spin contribution

$$\mathbf{a} = -\pi \int d^3r r^2 \mathbf{j}(\mathbf{r})$$



$$\hat{\mathbf{a}}_s = \frac{\pi e}{m} \sum_{i=1}^A \mu_i (\mathbf{r}_i \times \boldsymbol{\sigma}_i)$$

$$\mu_i = \mu_p(1/2 + t_{z,i}) + \mu_n(1/2 - t_{z,i})$$

$$a_s = \langle \psi_{\text{gs}} I I_z = I | \hat{a}_{s,0}^{(1)} | \psi_{\text{gs}} I I_z = I \rangle$$

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle$$

$$\times \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

- Anapole moment calculation:

$$\kappa_A = \frac{\sqrt{2}e}{G_F} a_s \quad \kappa_A = -i4\pi \frac{e^2}{G_F} \frac{\hbar}{mc} \frac{(II10|II)}{\sqrt{2I+1}} \sum_j \langle \psi_{\text{gs}} I^\pi | \sqrt{4\pi/3} \sum_{i=1}^A \mu_i r_i [Y_1(\hat{r}_i) \sigma_i]^{(1)} | \psi_j I^{-\pi} \rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

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$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \times \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

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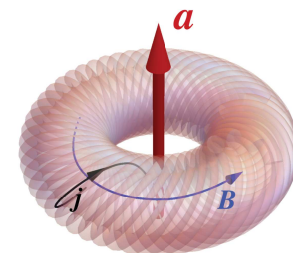
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$$\mu_i = \mu_p(1/2 + t_{z,i}) + \mu_n(1/2 - t_{z,i})$$

$$a_s = \langle \psi_{\text{gs}} I I_z = I | \hat{a}_{s,0}^{(1)} | \psi_{\text{gs}} I I_z = I \rangle$$



Low lying states of opposite parity can lead to enhancement!

## Parity violating (PV) and parity & time-reversal (PTV) violating nucleon-nucleon (NN) interaction

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  - Admixes unnatural parity states in the ground state
- EDM and Schiff moment operators

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \times \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

$$\hat{D}_z = \frac{e}{2} \sum_{i=1}^A (1 + \tau_i^z) z_i$$

$$\mathbf{S} = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 \mathbf{r}_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} \mathbf{r}_i \right)$$

- EDM and Schiff moment calculation
  - Nuclear EDM is dominated by and the Schiff moment determined by the polarization contribution:

$$D^{(pol)} = \langle \psi_{\text{gs}} I^\pi | \hat{D}_z | \psi_{\text{gs}} I \rangle + c. c.$$

$$\mathbf{S} = \langle \psi_{\text{gs}} I^\pi | \mathbf{S} | \psi_{\text{gs}} I \rangle + c. c.$$



**NCSM applications to parity-violating moments:**

How to calculate the sum of intermediate unnatural parity states?

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

## NCSM applications to parity-violating moments:

How to calculate the sum of intermediate unnatural parity states?

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

Sum over all possible  
intermediate states

## NCSM applications to parity-violating moments:

How to calculate the sum of intermediate unnatural parity states?

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$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

- Solving Schroedinger equation with inhomogeneous term

$$(E_{\text{gs}} - H) |\psi_{\text{gs}} I\rangle = V_{\text{NN}}^{\text{PNC}} |\psi_{\text{gs}} I^\pi\rangle$$

- To invert this equation, we apply the Lanczos algorithm

## NCSM applications to parity-violating moments:

### How to calculate the sum of intermediate unnatural parity states?

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

- Solving Schroedinger equation with inhomogeneous term

$$(E_{\text{gs}} - H) |\psi_{\text{gs}} I\rangle = V_{\text{NN}}^{\text{PNC}} |\psi_{\text{gs}} I^\pi\rangle$$

- To invert this equation, we apply the Lanczos algorithm
  - Bring matrix to tri-diagonal form ( $\mathbf{v}_1, \mathbf{v}_2 \dots$  orthonormal,  $H$  Hermitian)

$$\begin{aligned} H\mathbf{v}_1 &= \alpha_1 \mathbf{v}_1 + \beta_1 \mathbf{v}_2 \\ H\mathbf{v}_2 &= \beta_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 + \beta_2 \mathbf{v}_3 \\ H\mathbf{v}_3 &= \beta_2 \mathbf{v}_2 + \alpha_3 \mathbf{v}_3 + \beta_3 \mathbf{v}_4 \\ H\mathbf{v}_4 &= \beta_3 \mathbf{v}_3 + \alpha_4 \mathbf{v}_4 + \beta_4 \mathbf{v}_5 \end{aligned}$$

- $n^{\text{th}}$  iteration computes  $2n^{\text{th}}$  moment
- Eigenvalues converge to extreme (largest in magnitude) values
- $\sim 150$ - $200$  iterations needed for 10 eigenvalues (even for  $10^9$  states)

Journal of Research of the National Bureau of Standards Vol. 45, No. 4, October 1950 Research Paper 2133  
 An Iteration Method for the Solution of the Eigenvalue Problem of Linear Differential and Integral Operators<sup>1</sup>  
 By Cornelius Lanczos

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

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$$(E_{\text{gs}} - H) |\psi_{\text{gs}} I\rangle = V_{\text{NN}}^{\text{PNC}} |\psi_{\text{gs}} I^\pi\rangle$$

- To invert this equation, we apply the Lanczos algorithm

$$|\mathbf{v}_1\rangle = V_{\text{NN}}^{\text{PNC}} |\psi_{\text{gs}} I^\pi\rangle$$

## $^3\text{He}$ EDM Benchmark Calculation

Discrepancy between calculations?

	PLB 665:165-172 (2008) (NN EFT)	PRC 87:015501 (2013)	PRC 91:054005 (2015)	Our calculation (NN EFT)
$\bar{G}_\pi^0$	0.015	(x 1/2)	(x 1/2)	0.0073 (x 1/2)
$\bar{G}_\pi^1$	0.023	(x 1/2)	(x 1/2)	0.011 (x 1/2)
$\bar{G}_\pi^2$	0.037	(x 1/5)	(x 1/2)	0.019 (x 1/2)
$\bar{G}_\rho^0$	-0.0012	(x 1/2)	(x 1/2)	-0.00062 (x 1/2)
$\bar{G}_\rho^1$	0.0013	(x 1/2)	(x 1/2)	0.00063 (x 1/2)
$\bar{G}_\rho^2$	-0.0028	(x 1/5)	(x 1/2)	-0.0014 (x 1/2)
$\bar{G}_\omega^0$	0.0009	(x 1/2)	(x 1/2)	0.00042 (x 1/2)
$\bar{G}_\omega^1$	-0.0017	(x 1/2)	(x 1/2)	-0.00086 (x 1/2)

Our results confirm those of Yamanaka and Hiyama, PRC 91:054005 (2015)

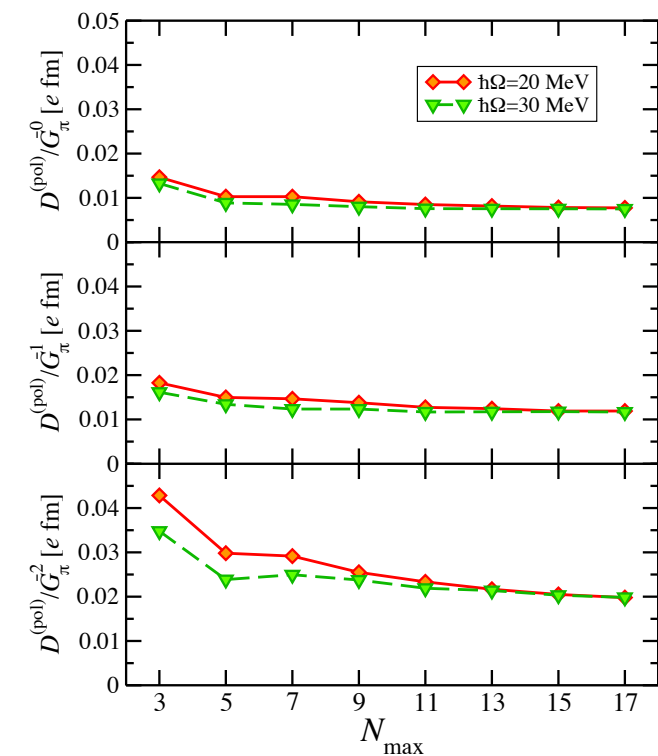
PHYSICAL REVIEW C **104**, 025502 (2021)

***Ab initio* calculations of electric dipole moments of light nuclei**

Paul Froese\*  
 TRIUMF, 4004 Westbrook Mall, Vancouver, British Columbia V6T 2A3, Canada  
 and Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T 1Z1, Canada

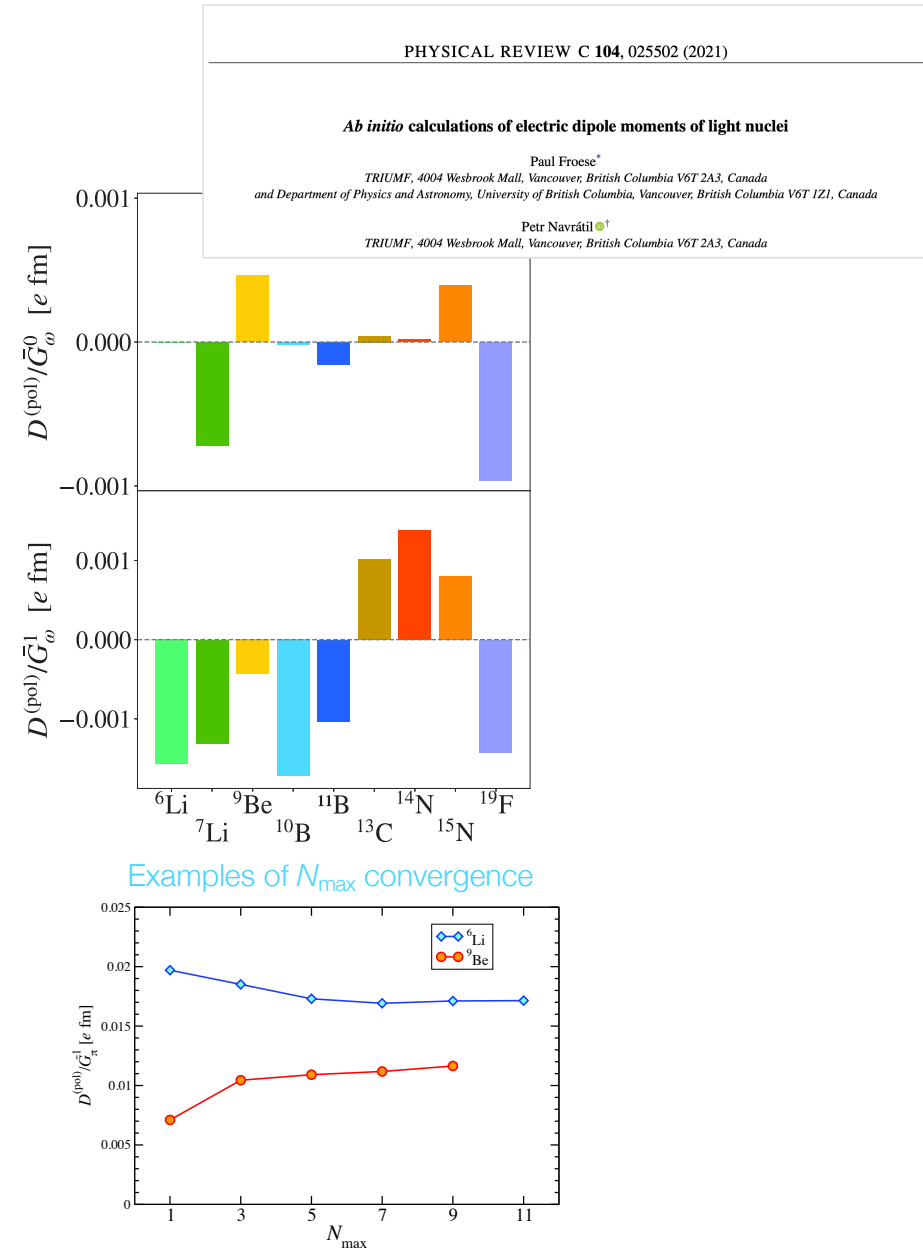
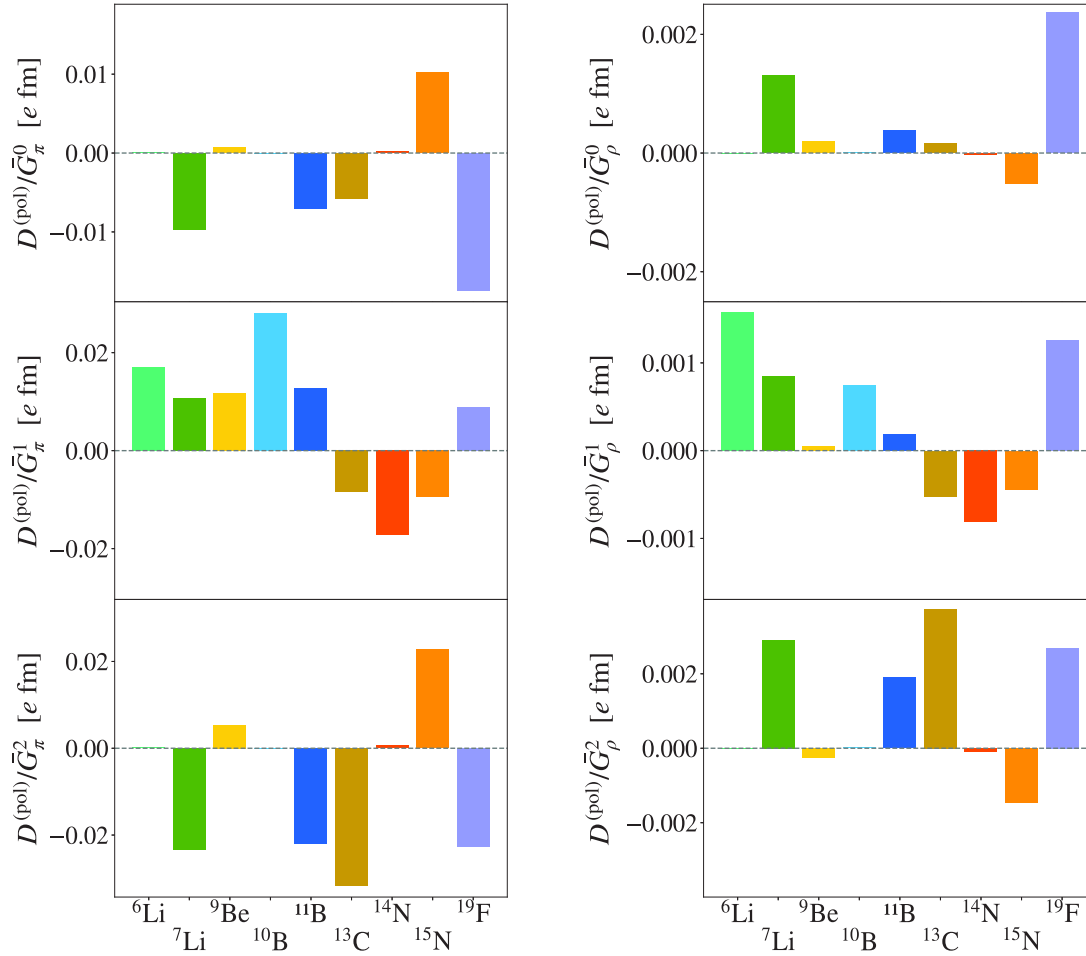
Petr Navrátil\*  
 TRIUMF, 4004 Westbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

## $N_{\text{max}}$ convergence for $^3\text{He}$ $N^3\text{LO NN}$

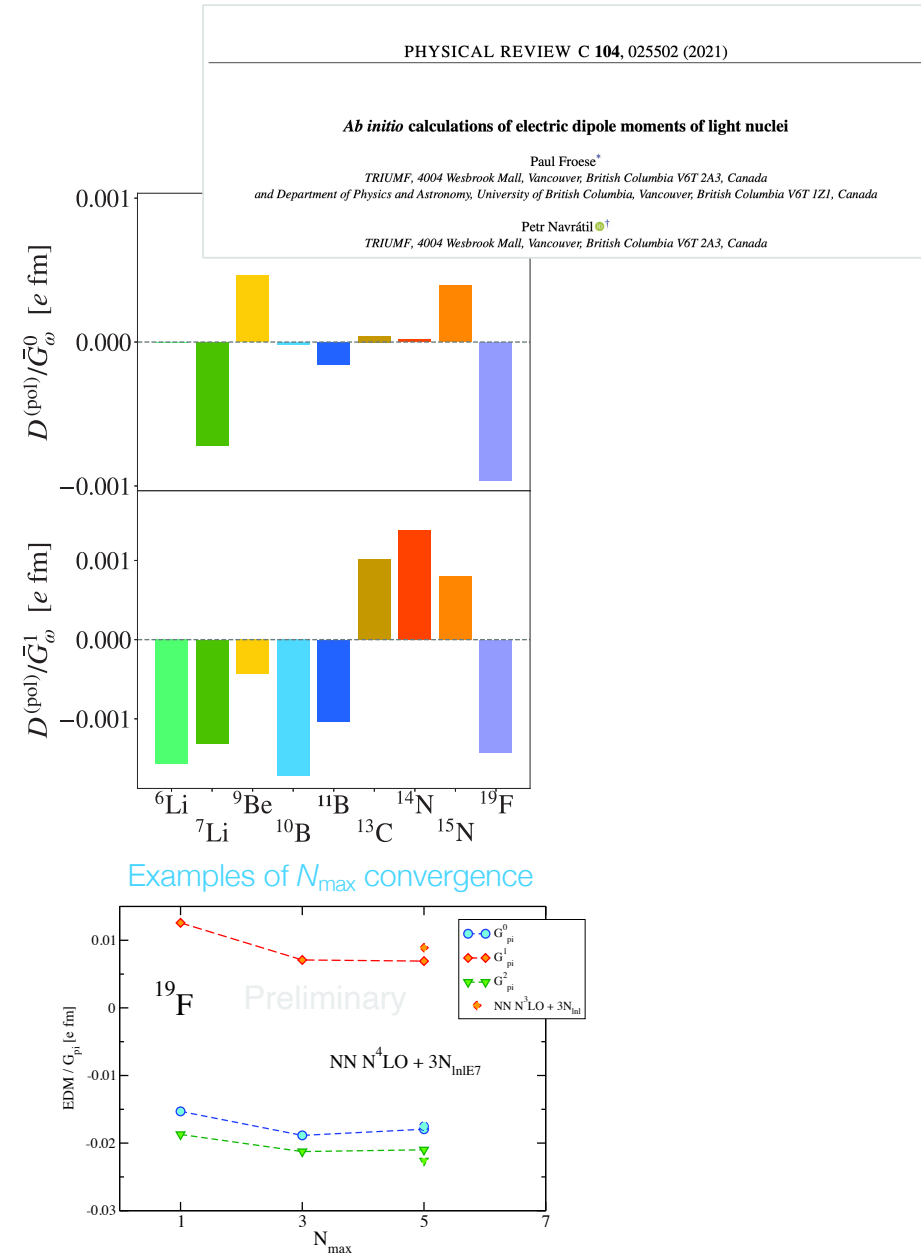
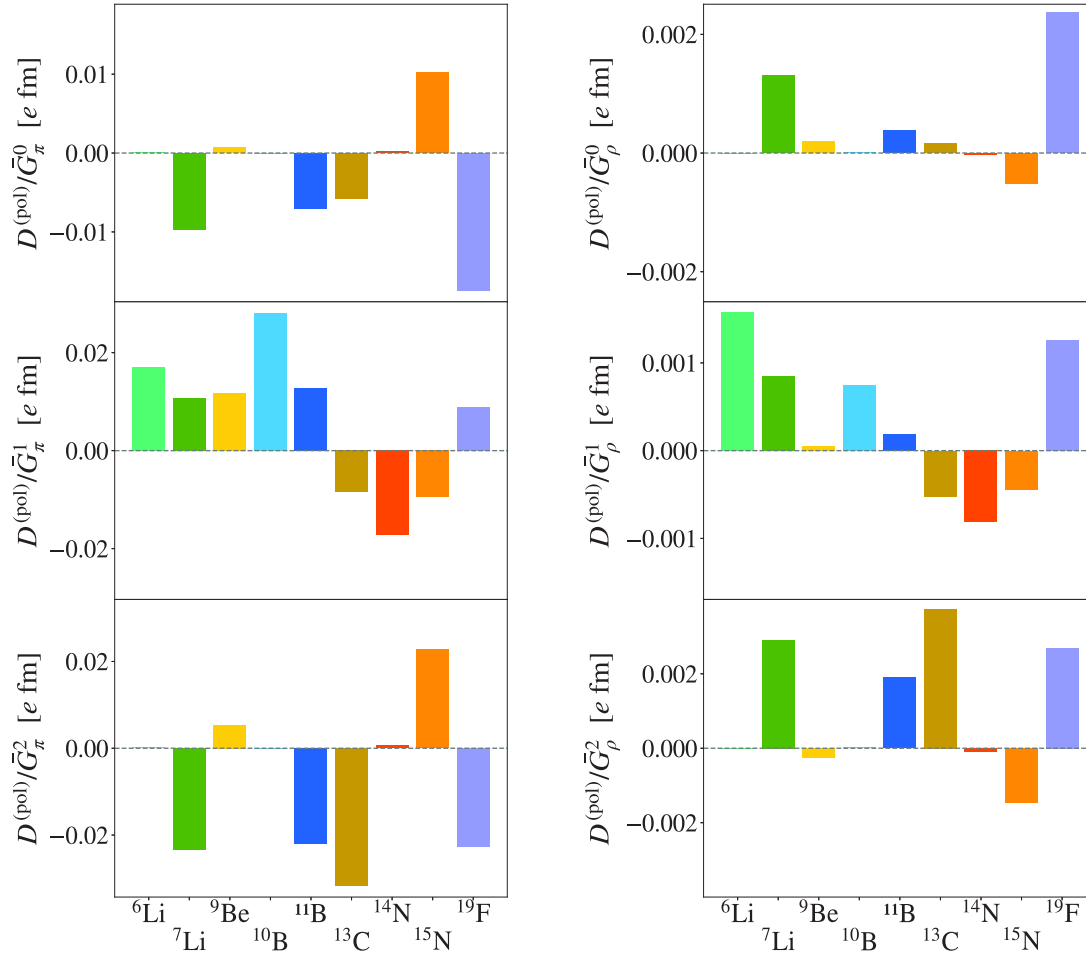




# NCSM applications to parity-violating moments: EDMs of light stable nuclei

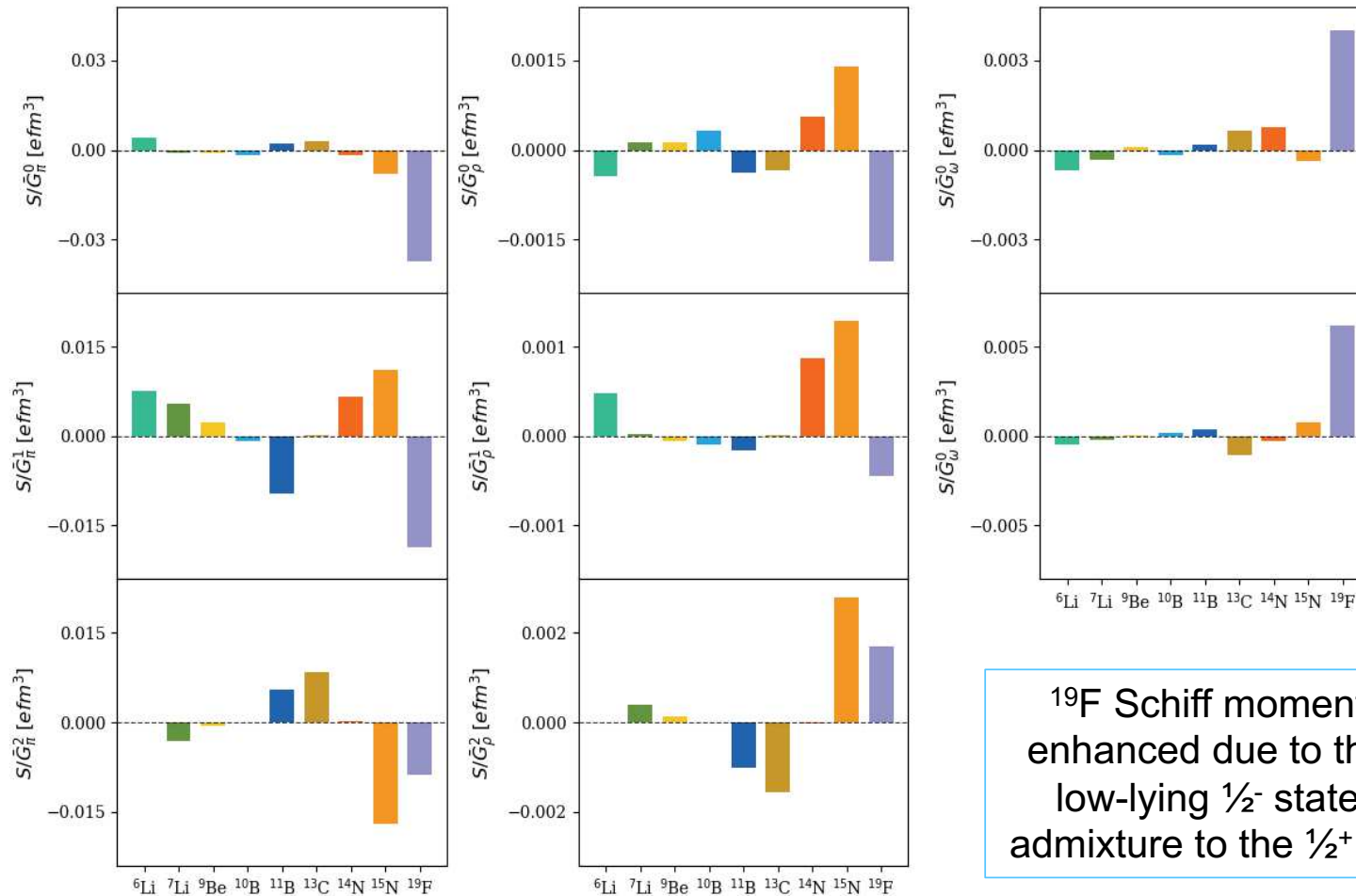


# NCSM applications to parity-violating moments: EDMs of light stable nuclei



# NCSM applications to parity-violating moments: Schiff moments of light stable nuclei

Results preliminary

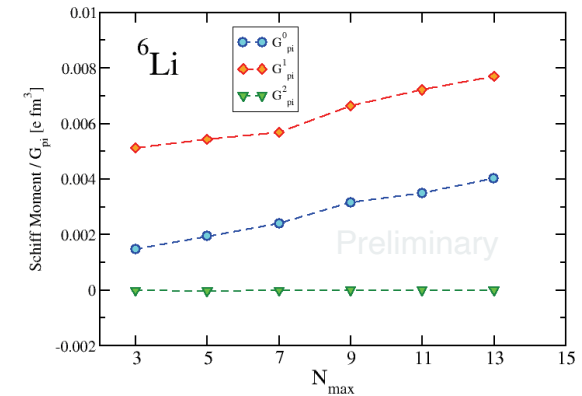


$^{19}\text{F}$  Schiff moment enhanced due to the low-lying  $\frac{1}{2}^-$  state admixture to the  $\frac{1}{2}^+$  gs

Work in progress with Stephan Foster, McMaster University undergraduate student

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Examples of  $N_{\text{max}}$  convergence

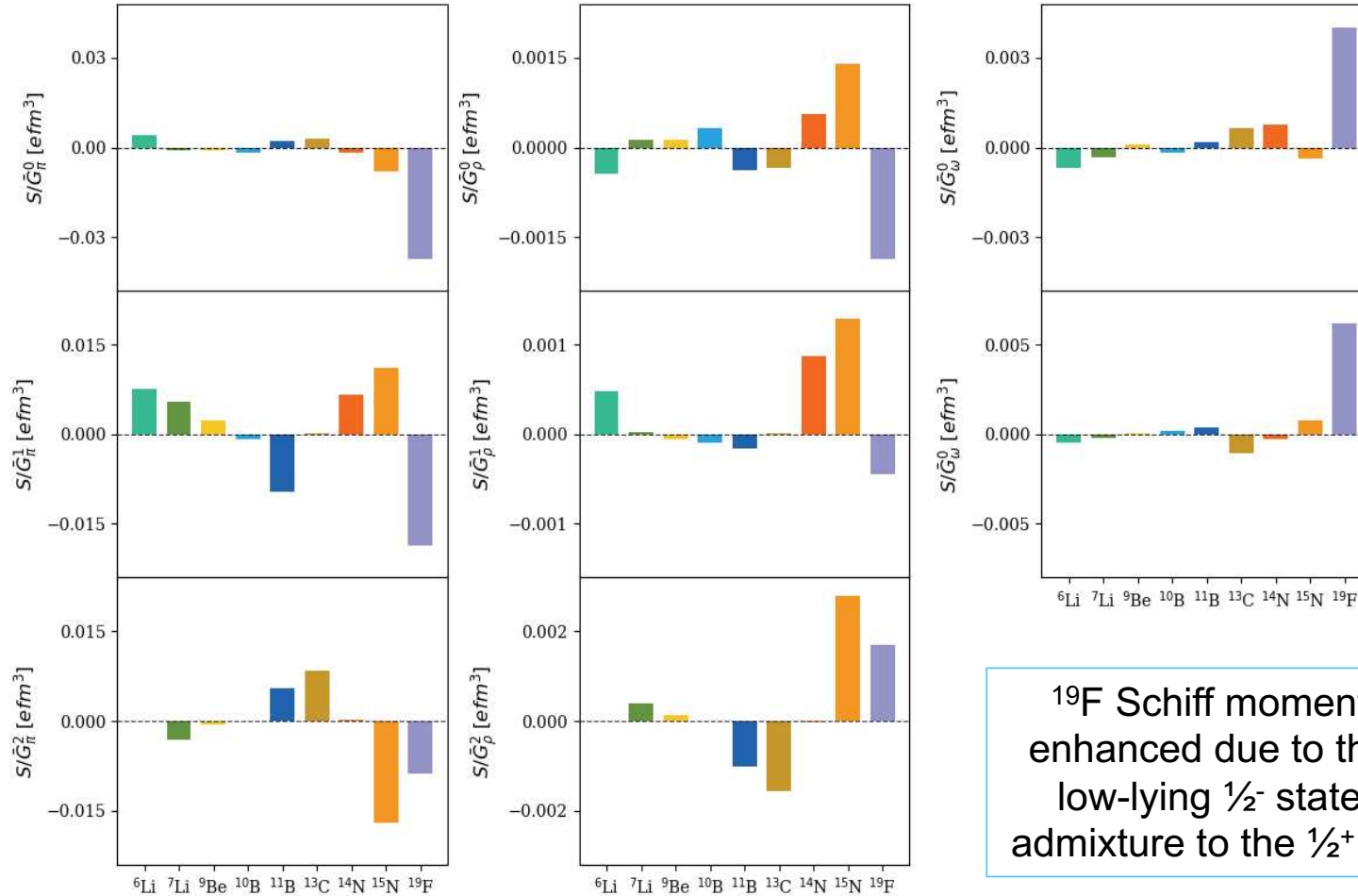


Convergence more challenging due to a destructive contribution of the two terms and the long-range  $r^3$  dependence

$$S = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 \mathbf{r}_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} \mathbf{r}_i \right)$$

# NCSM applications to parity-violating moments: Schiff moments of light stable nuclei

Results preliminary



${}^{19}\text{F}$  Schiff moment  
enhanced due to the  
low-lying  $\frac{1}{2}^-$  state  
admixture to the  $\frac{1}{2}^+$  gs

Work in progress  
with Stephan Foster,  
McMaster University  
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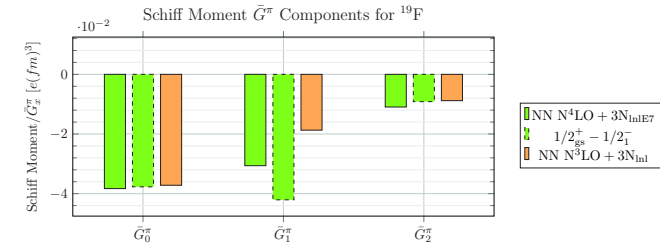


Figure 1: Comparison of  ${}^{19}\text{F}$   $\tilde{G}^\pi$  components for different interactions and included states.

$$S = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 \mathbf{r}_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} \mathbf{r}_i \right)$$

# NCSM applications to parity-violating moments: Schiff moments of light stable nuclei

Results preliminary

Calculated  $\frac{1}{2}^-$  state energies shifted to match the  $\frac{1}{2}^-_{-1}$  excitation energy

Relevant for planned nuclear Schiff moment measurements in  $^{227}\text{ThF}^+$  at TRIUMF

Work in progress with Stephan Foster, McMaster University undergraduate student

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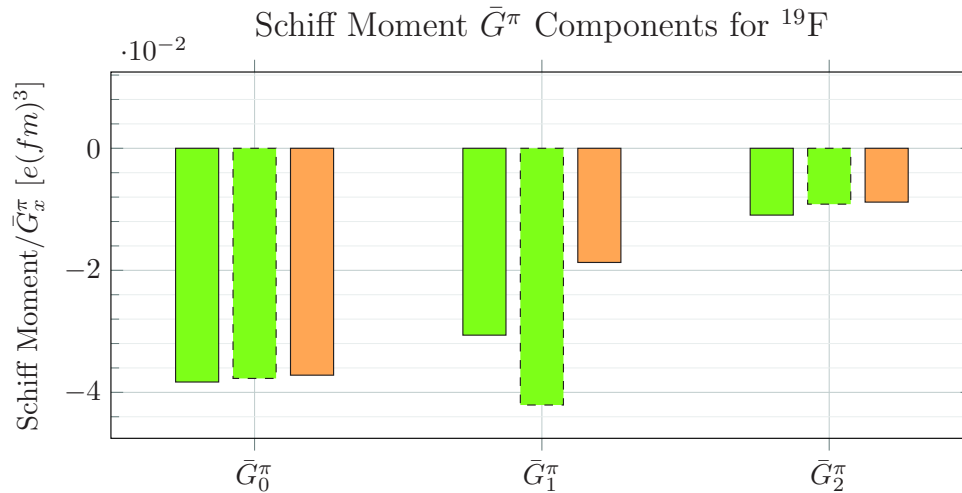
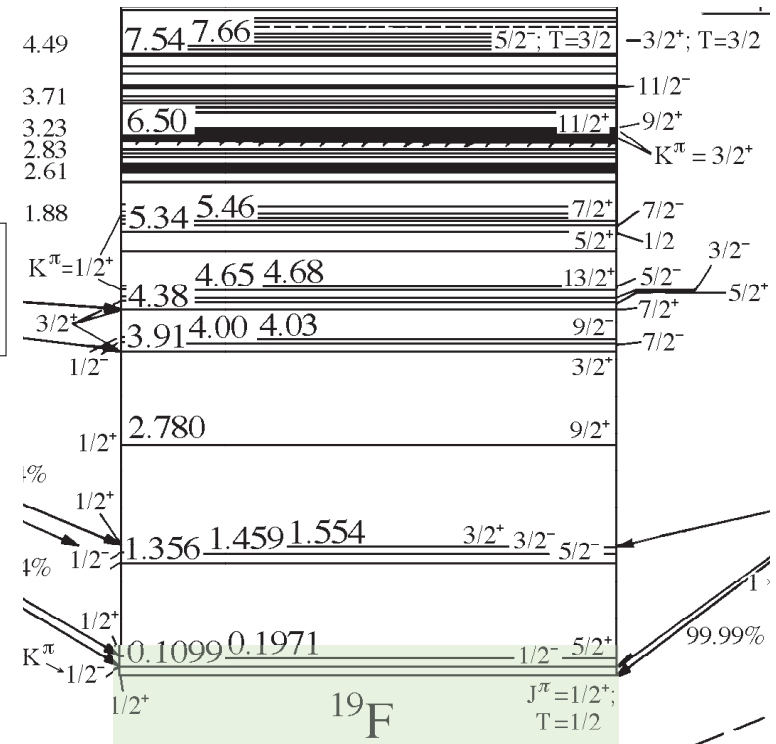


Figure 1: Comparison of  $^{19}\text{F}$   $\bar{G}^\pi$  components for different interactions and included states.

$$S = \langle \psi_{\text{gs}} | I^\pi | S | \psi_{\text{gs}} | I \rangle + c. c.$$

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$$s = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 \mathbf{r}_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} \mathbf{r}_i \right)$$

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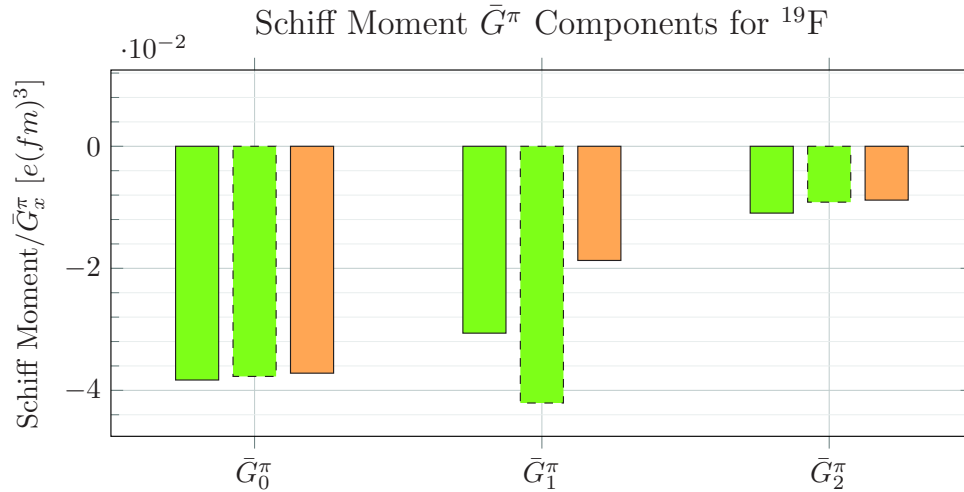
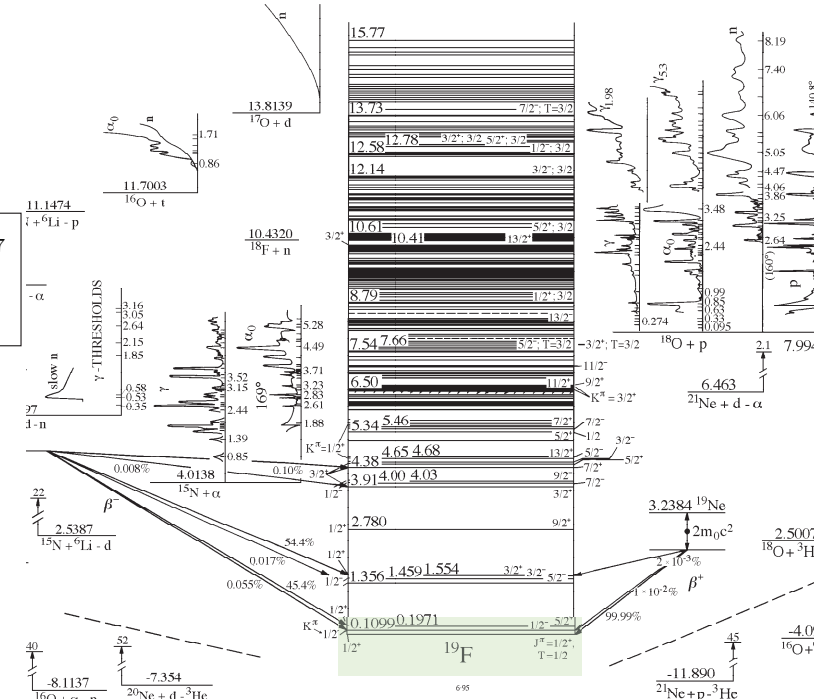


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$^{19}\text{F}$  Schiff moment enhanced due to the low-lying  $\frac{1}{2}^-$  state admixture to the  $\frac{1}{2}^+$  gs



## NCSM applications to parity-violating moments:

### Schiff moments of light stable nuclei

Results preliminary

Calculated  $\frac{1}{2}^-$  state energies shifted to match the  $\frac{1}{2}^-_{-1}$  excitation energy

Relevant for planned nuclear Schiff moment measurements in  $^{227}\text{ThF}^+$  at TRIUMF

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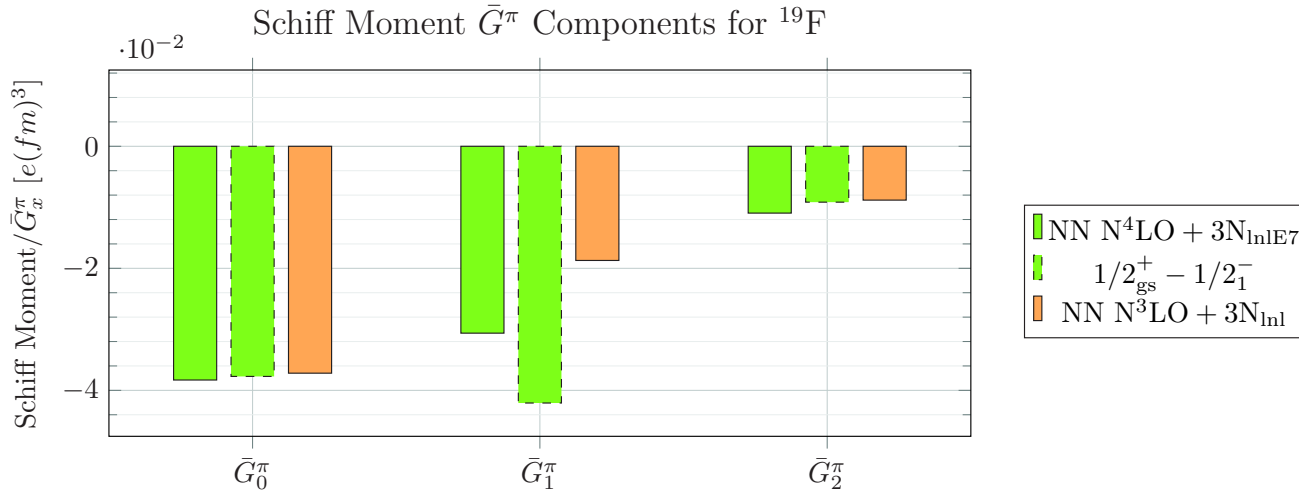


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$$\times \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j | I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} | I^\pi \rangle$$

$^{19}\text{F}$  Schiff moment enhanced due to the low-lying  $\frac{1}{2}^-$  state admixture to the  $\frac{1}{2}^+$  gs

$^{19}\text{F}$  Schiff moment comparable to  $^{129}\text{Xe}$  Schiff moment calculated within the nuclear shell model

PHYSICAL REVIEW C **102**, 065502 (2020)

Large-scale shell-model calculations of nuclear Schiff moments of  $^{129}\text{Xe}$  and  $^{199}\text{Hg}$

Kota Yanase\* and Noritaka Shimizu\*

TABLE II. The NSM coefficients of  $^{129}\text{Xe}$  in units of  $10^{-2} e \text{fm}^3$ . Our final results are given in bold.

	$a_0$	$a_1$	$a_2$
IPM ( $m_\pi \rightarrow \infty$ )	-9.9	-9.9	-19.8
IPM	-4.6	-4.6	-9.2
LSSM (SN100PN, $m_\pi \rightarrow \infty$ )	-8.7	-8.2	-15.8
LSSM (SNV, $m_\pi \rightarrow \infty$ )	-8.6	-8.3	-16.2
LSSM (SN100PN)	-3.7	-4.1	-8.0
<b>LSSM (SNV)</b>	<b>-3.8</b>	<b>-4.1</b>	<b>-8.1</b>
IPM ( $m_\pi \rightarrow \infty$ ) [35,36]	-11	-11	-22
IPM [38]	-6	-6	-12
RPA [38]	-0.8	-0.6	-0.9
PTSM [41]	0.05	-0.04	0.19
PTSM [42]	0.3	-0.1	0.4

$$S = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 \mathbf{r}_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} \mathbf{r}_i \right)$$

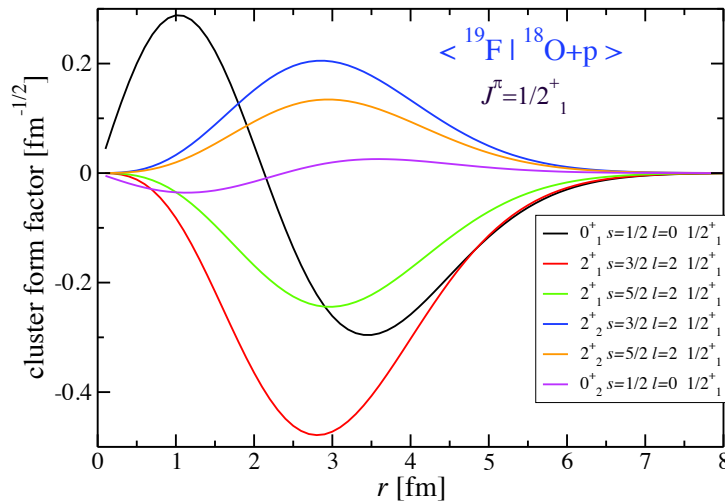
# NCSM applications to parity-violating moments: Schiff moments of light stable nuclei

Work in progress  
with Stephan Foster,  
McMaster University  
undergraduate student

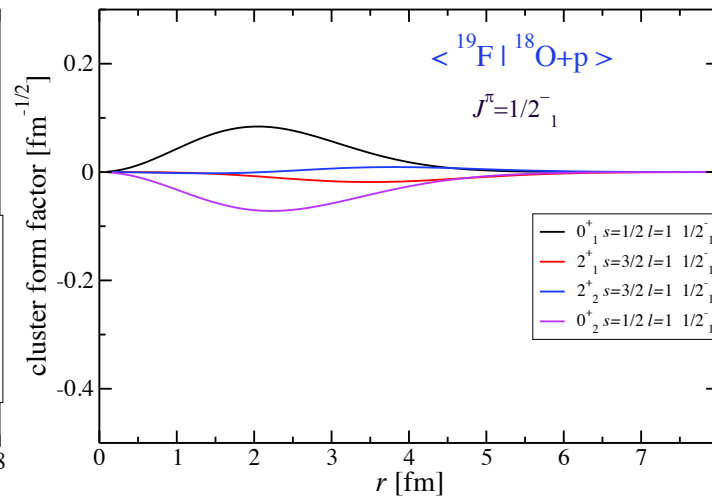
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$^{19}\text{F}$  Schiff moment dominated by the contribution of the lowest  $\frac{1}{2}^-$  state  
However, its contribution to the EDM of  $^{19}\text{F}$  is negligible.

This is due to very different structure of the  $\frac{1}{2}^+$  g.s. and the  $\frac{1}{2}^-$  state



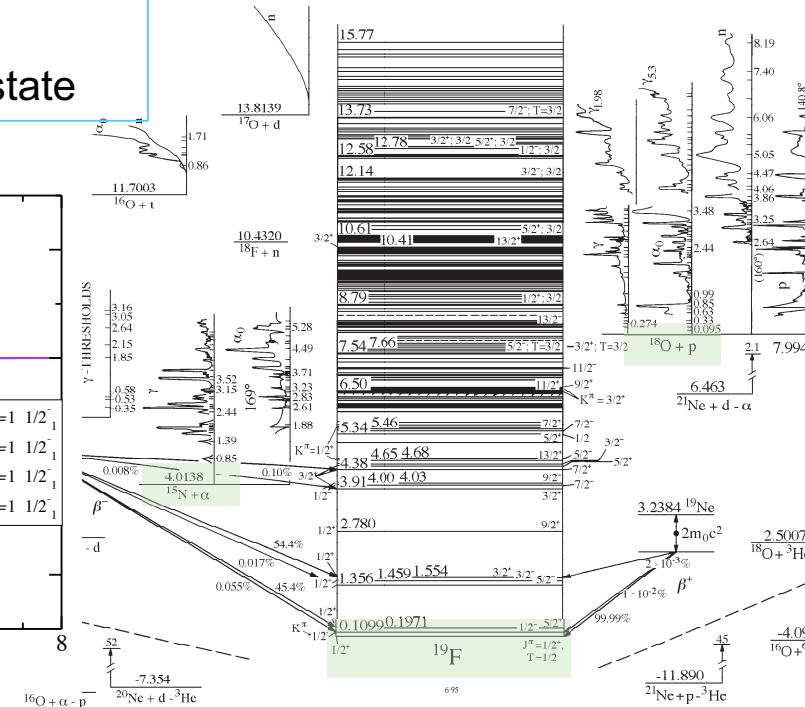
$^{18}\text{O}+p$  (shell-model-like)



$^{15}\text{N}+^4\text{He}$  – alpha-clustering

E1 matrix element small  
S matrix element large due to the  $r^3$  term

$$\hat{D}_z = \frac{e}{2} \sum_{i=1}^A (1 + \tau_i^z) z_i \quad S = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 r_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} r_i \right)$$





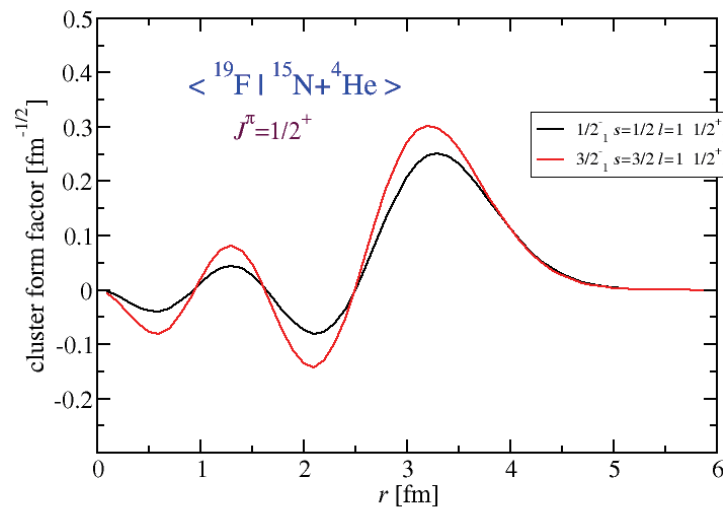
# NCSM applications to parity-violating moments: Schiff moments of light stable nuclei

Work in progress  
with Stephan Foster,  
McMaster University  
undergraduate student

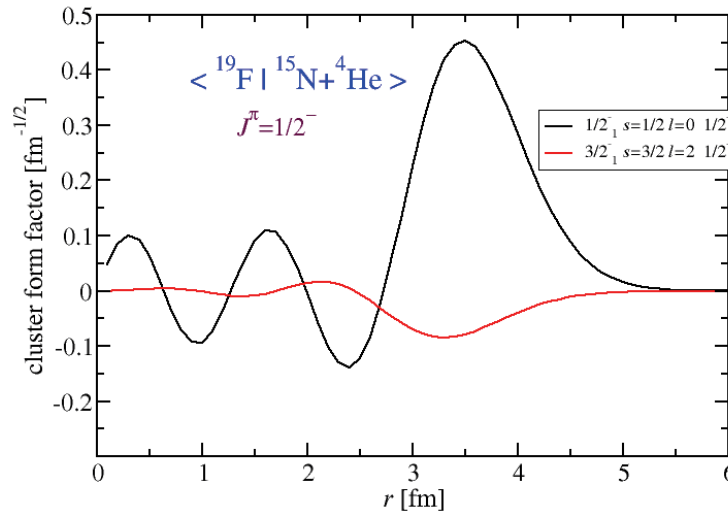
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$^{19}\text{F}$  Schiff moment dominated by the contribution of the lowest  $\frac{1}{2}^-$  state  
However, its contribution to the EDM of  $^{19}\text{F}$  is negligible.

This is due to very different structure of the  $\frac{1}{2}^+$  g.s. and the  $\frac{1}{2}^-$  state



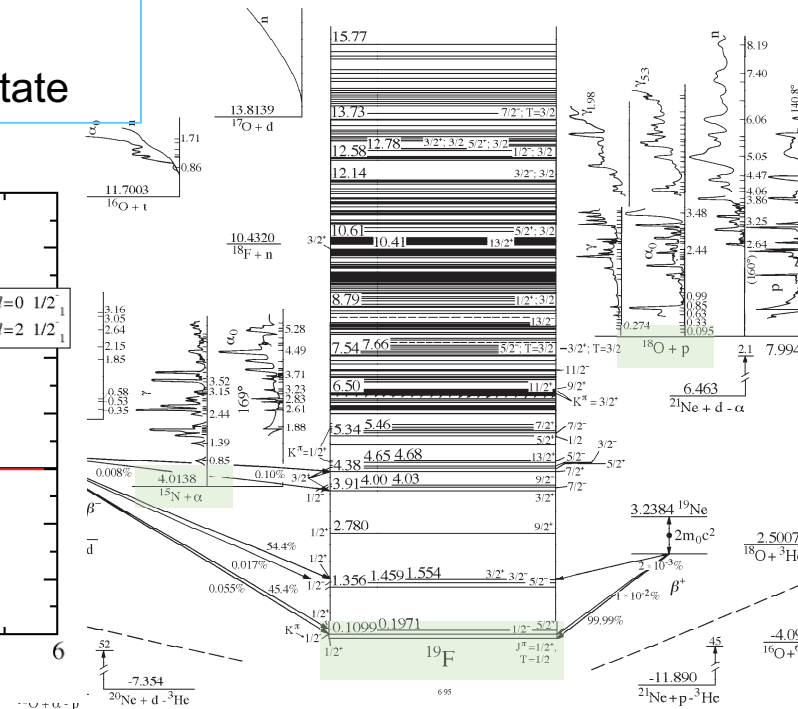
$^{18}\text{O} + \text{p}$  (shell-model-like)



$^{15}\text{N} + ^4\text{He}$  – alpha-clustering

E1 matrix element small  
S matrix element large due to the  $r^3$  term

$$\hat{D}_z = \frac{e}{2} \sum_{i=1}^A (1 + \tau_i^z) z_i \quad S = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 r_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} r_i \right)$$



## Nuclear spin-dependent parity-violating effects from NCSM

- Contributions from nucleon axial-vector and the anapole moment

$$\kappa_{ax} \simeq -2C_{2p}\langle s_{p,z} \rangle - 2C_{2n}\langle s_{n,z} \rangle \simeq -0.1\langle s_{p,z} \rangle + 0.1\langle s_{n,z} \rangle$$

$$\langle s_{\nu,z} \rangle \equiv \langle \psi_{\text{gs}} \, I^\pi I_z = I | \hat{s}_{\nu,z} | \psi_{\text{gs}} \, I^\pi I_z = I \rangle$$

$$C_{2p} = -C_{2n} = g_A(1 - 4\sin^2 \theta_W)/2 \simeq 0.05$$

## Conclusions

- *Ab initio* nuclear theory
  - Makes connections between the low-energy QCD and many-nucleon systems
- No-core shell model is an *ab initio* configuration interaction method
  - Applicable to nuclear structure, reactions including those relevant for astrophysics, electroweak processes, tests of fundamental symmetries
  - In combination with the Lanczos strength method provides robust results for electroweak observables and nuclear structure dependent corrections