

# *Ab initio* calculations of electric dipole, Schiff, and anapole moments in atomic nuclei

Espace de Structure et de réactions Nucléaires Théorique

Theoretical and experimental developments for symmetry-violating nuclear properties

CEA Saclay, June 23-27, 2025

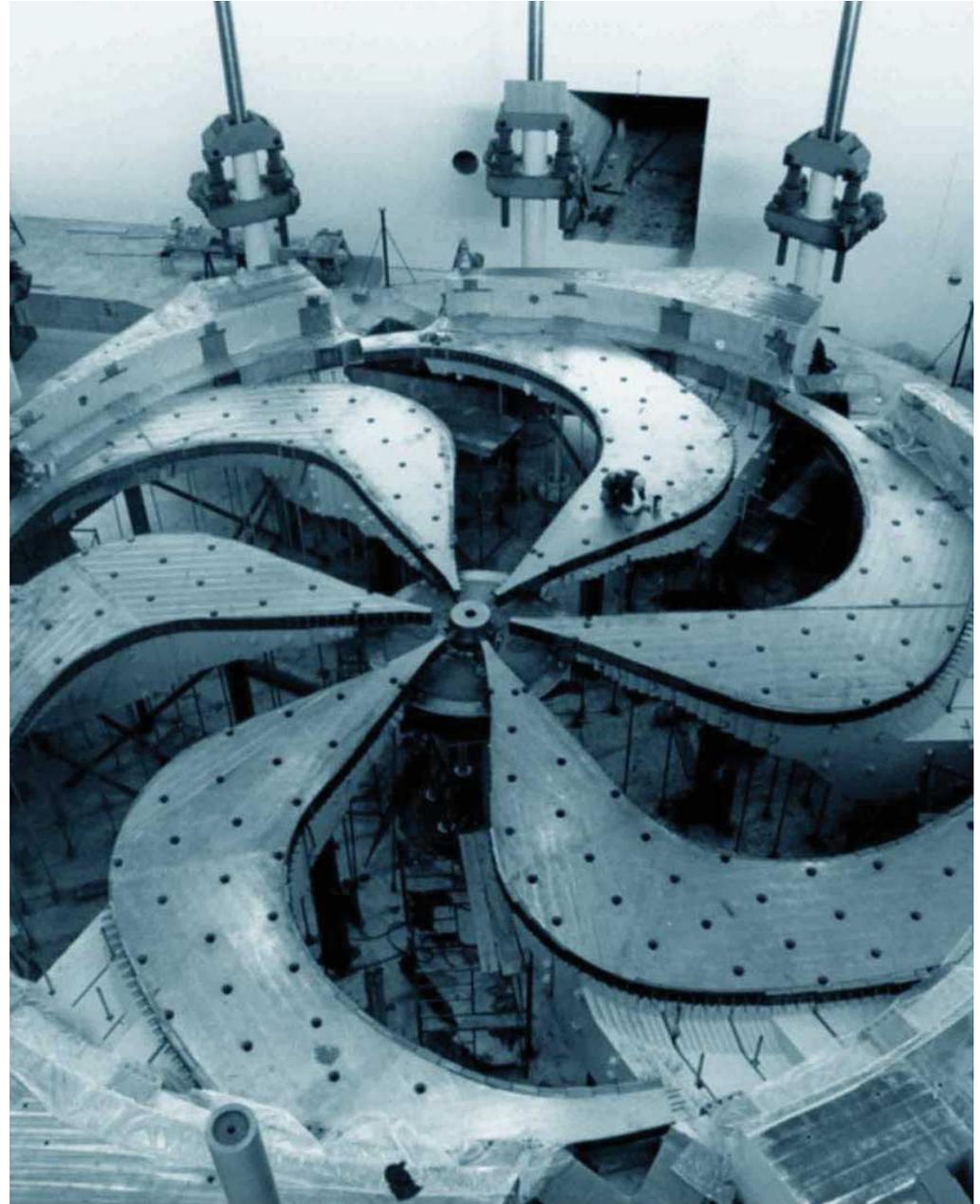
Petr Navratil

TRIUMF

Collaborator:

Stephan Foster (TRIUMF, McMaster U)

2025-06-24

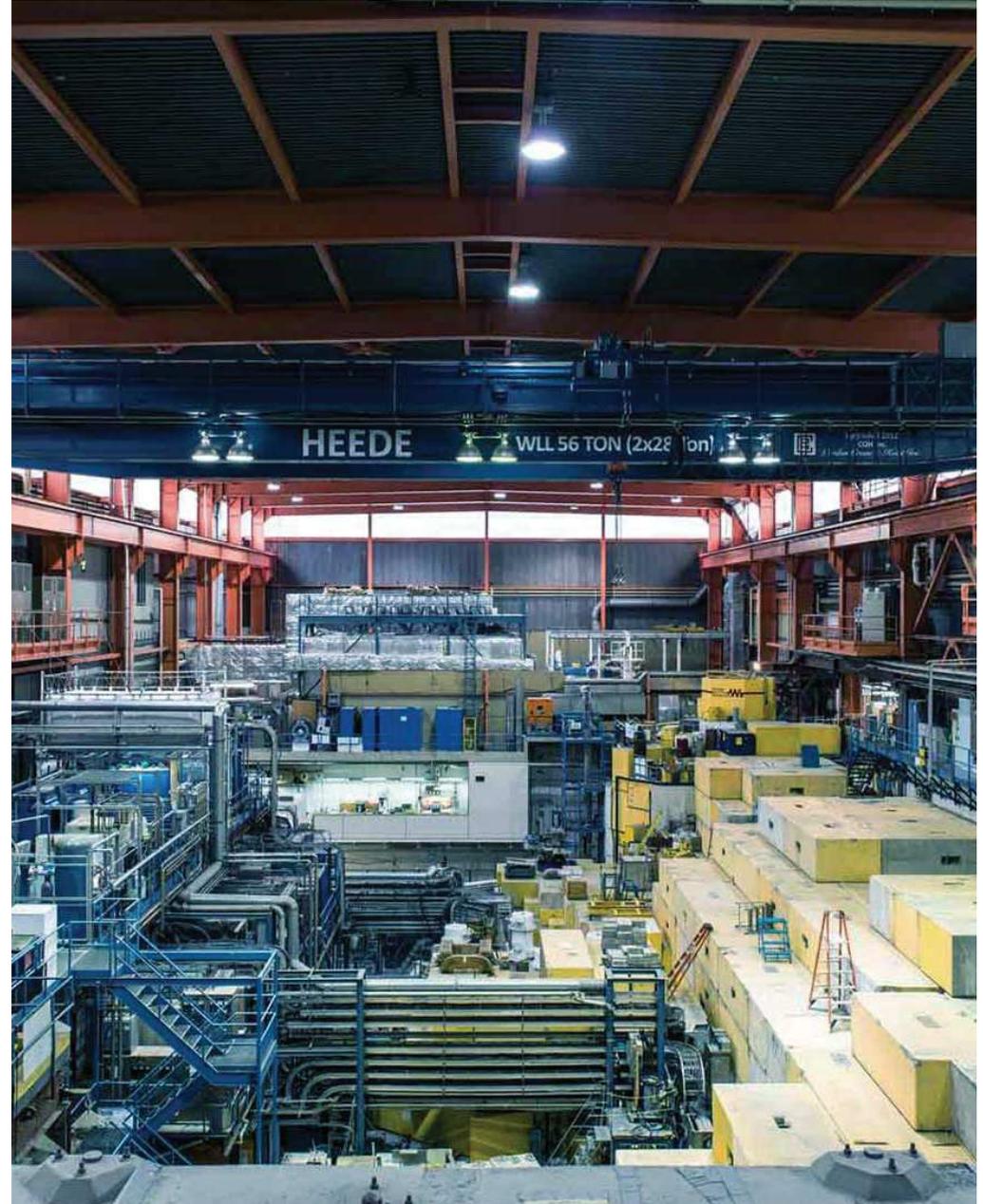


## Outline

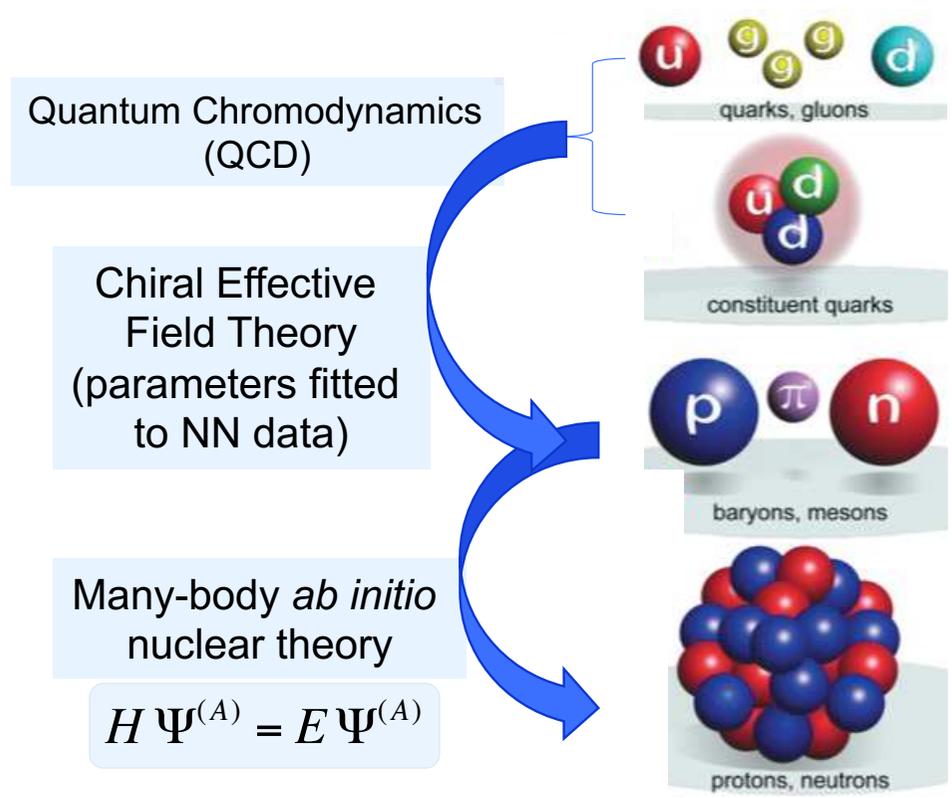
- Introduction – *Ab initio* nuclear theory – **no-core shell model (NCSM)**
- *Ab initio* calculations of parity-violating moments 
  - Parity violating and parity & time-reversal violating NN interactions
  - Calculations of anapole, electric dipole, and nuclear Schiff moments
    - Schiff moment of  $^{19}\text{F}$
- Conclusions

*Ab initio* nuclear theory -  
no-core shell model (NCSM)

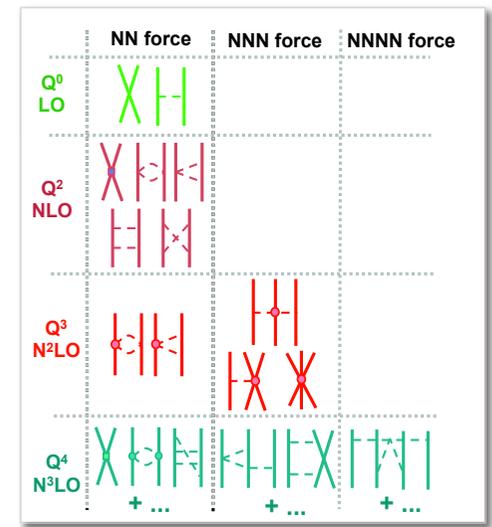
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# First principles or *ab initio* nuclear theory



$$H \Psi^{(A)} = E \Psi^{(A)}$$



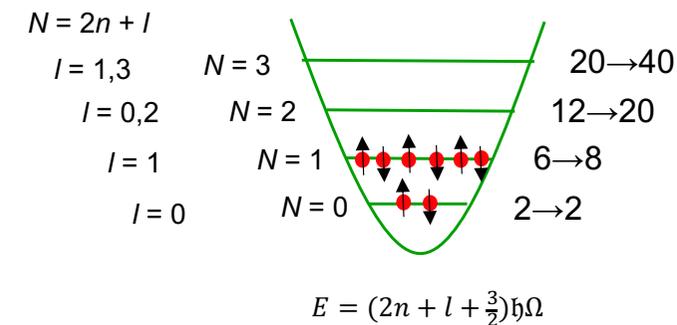
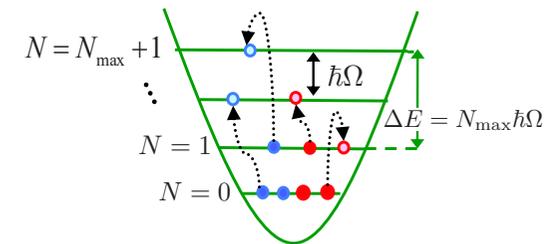


## Conceptually simplest *ab initio* method: No-Core Shell Model (NCSM)

- Basis expansion method (CI)
  - Harmonic oscillator (HO) basis truncated in a particular way ( $N_{\max}$ )
    - HO frequency variational parameter
  - Why HO basis?
    - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 –  $^4\text{He}$ ,  $^{16}\text{O}$ ,  $^{40}\text{Ca}$ )
    - Equivalent description in relative (Jacobi)-coordinate and Slater determinant basis – nuclei self-bound,  $[\text{H}, \text{P}_{\text{CM}}]=0$ 
      - Exact factorization of CM and intrinsic eigenfunctions at each  $N_{\max}$

$$\Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^{HO}(\vec{\eta}_1, \vec{\eta}_2, \dots, \vec{\eta}_{A-1})$$

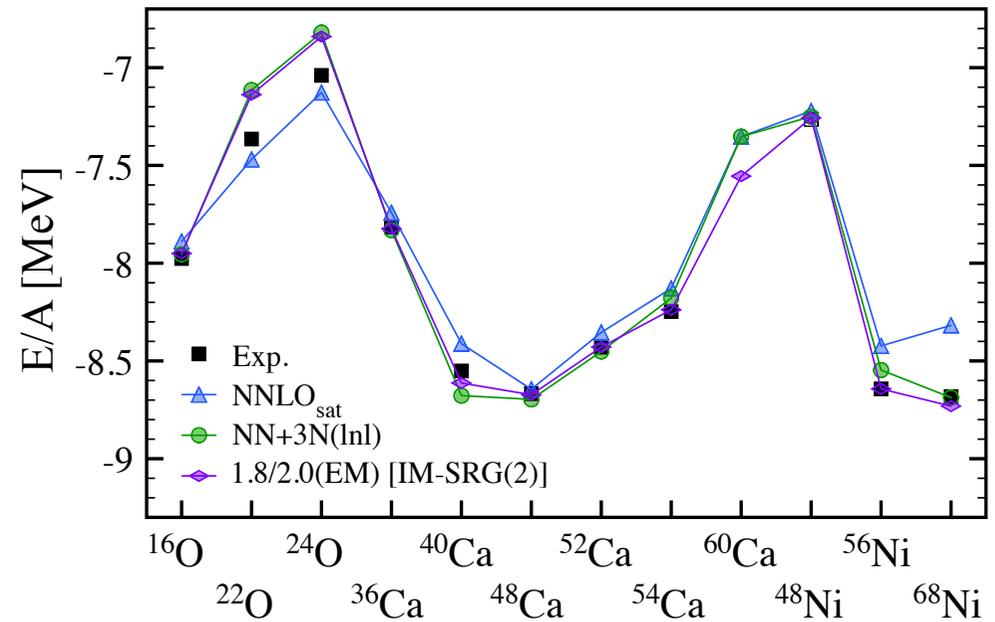
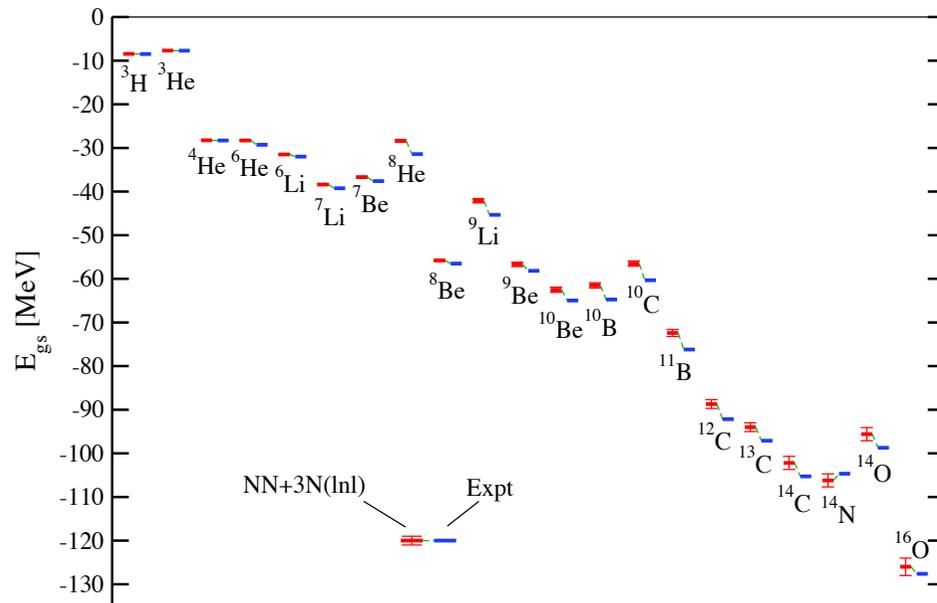
$$\Psi_{\text{SD}}^A = \sum_{N=0}^{N_{\max}} \sum_j c_{Nj}^{\text{SD}} \Phi_{\text{SD}Nj}^{HO}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{\text{CM}})$$



## Binding energies of atomic nuclei from nuclear forces from chiral Effective Field Theory

- Quite reasonable description of binding energies across the nuclear charts becomes feasible
  - **The Hamiltonian fully determined in  $A=2$  and  $A=3,4$  systems**
    - Nucleon–nucleon scattering, deuteron properties,  $^3\text{H}$  and  $^4\text{He}$  binding energy,  $^3\text{H}$  half life
  - Light nuclei – NCSM
  - Medium mass nuclei – Self-Consistent Green’s Function method

NN N<sup>3</sup>LO (Entem-Machleidt 2003)  
3N N<sup>2</sup>LO w local/non-local regulator

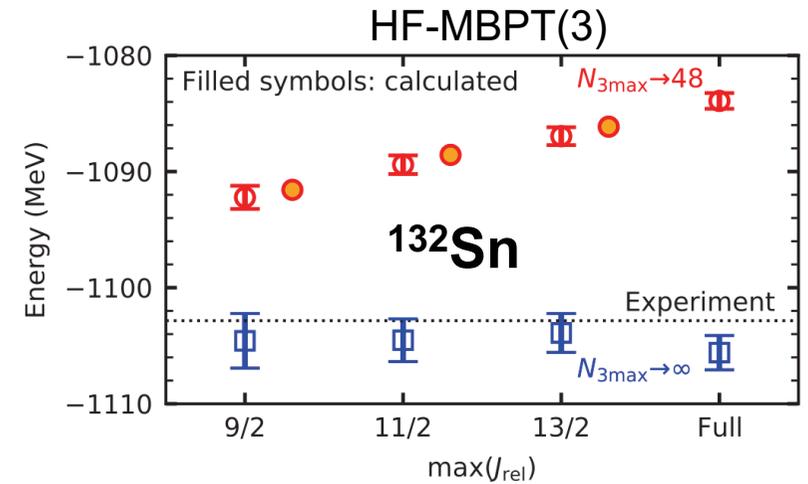
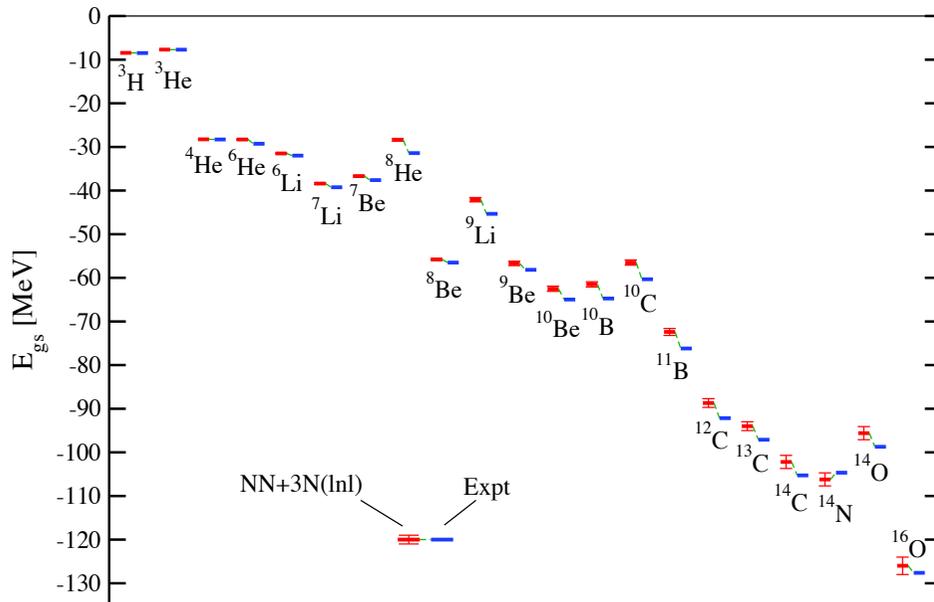


1.8/2.0 (EM) results: J. Simonis, S. R. Stroberg, K. Hebeler, J. D. Holt, and A. Schwenk, Phys. Rev. C 96, 014303 (2017).

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  - Light nuclei – NCSM
  - Heavy nuclei – HF-MBPT(3)

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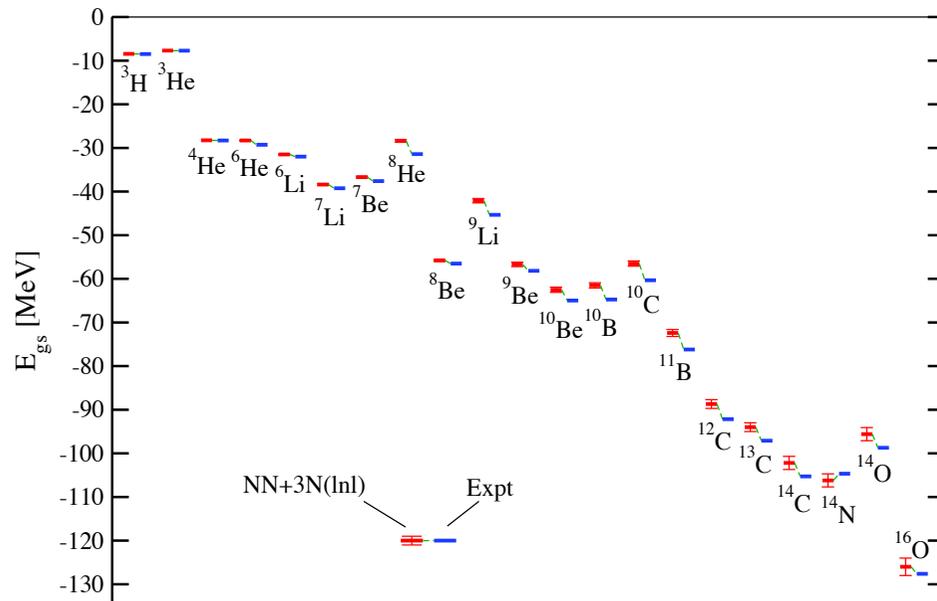


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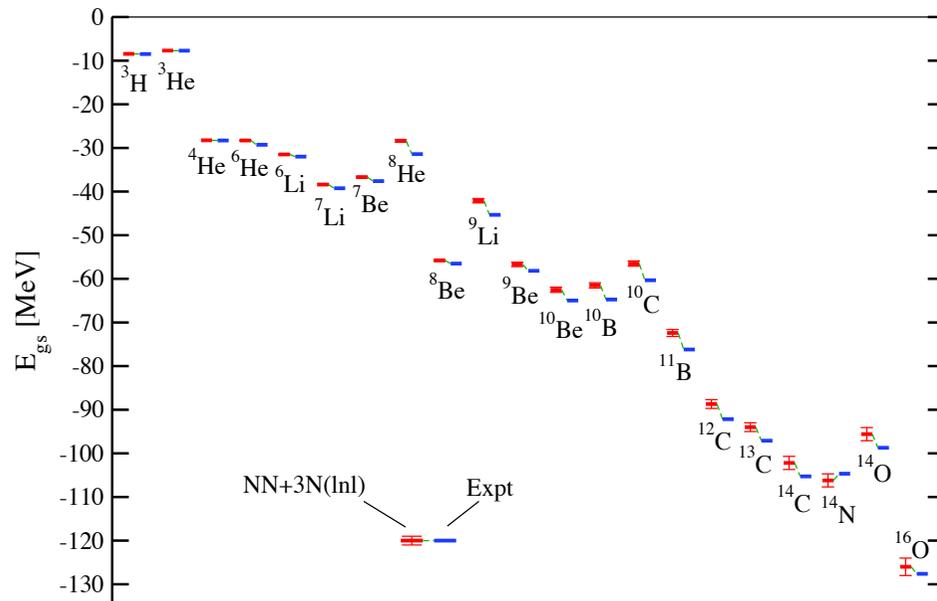


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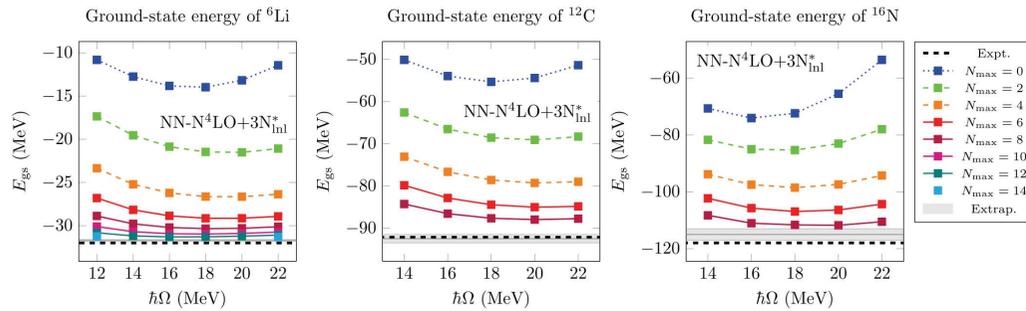


- A new version denoted as NN  $N^4\text{LO} + 3N_{\text{InlE7}}$ 
  - NN  $N^4\text{LO} 500$  (Entem-Machleidt-Nosyk 2017)
  - 3N  $N^2\text{LO}$  w local/non-local regulator
  - 3N subleading spin-orbit contact term (Girlanda 2011)
    - new LEC ( $E_7$ ) fitted to improve excitation levels in  $^6\text{Li}$
    - reproduces well  $P$ -wave resonances in  $^5\text{He}$
  - Successfully applied to  $^7\text{Be}(p,\gamma)^8\text{B}$  (PLB 845,138156) and muon capture on  $^6\text{Li}$ ,  $^{12}\text{C}$ , and  $^{16}\text{O}$  (PRC 109, 065501)
  - Applied here for  $^{19}\text{F}$  EDM, Schiff moment calculations

## Binding energies of atomic nuclei from nuclear forces from chiral Effective Field Theory

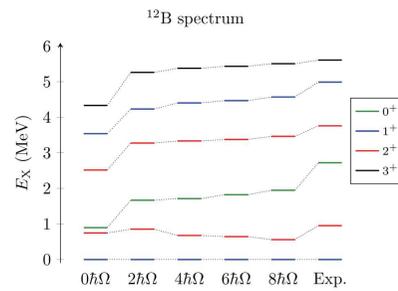
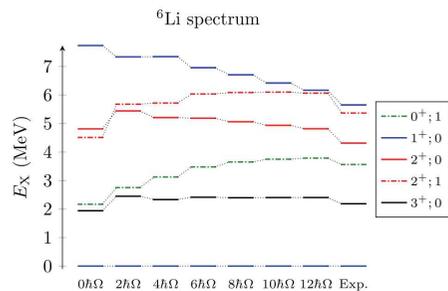
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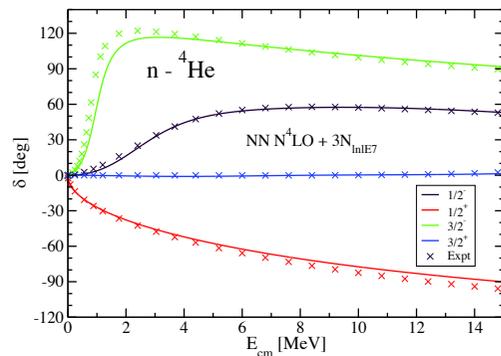


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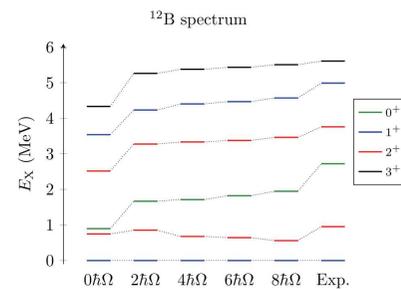
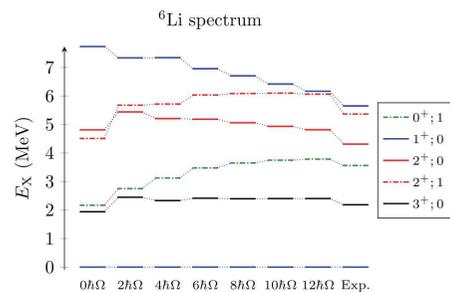
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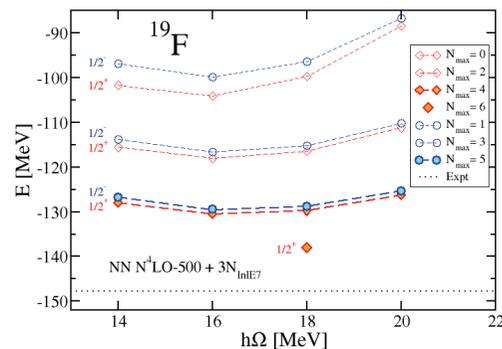


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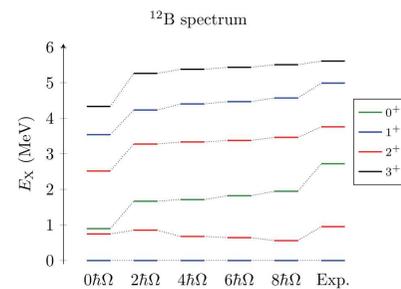
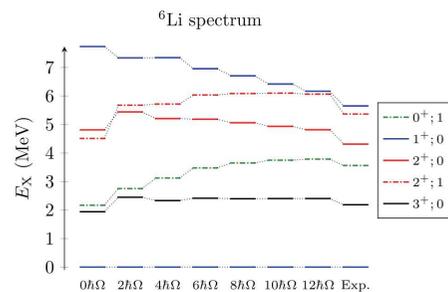
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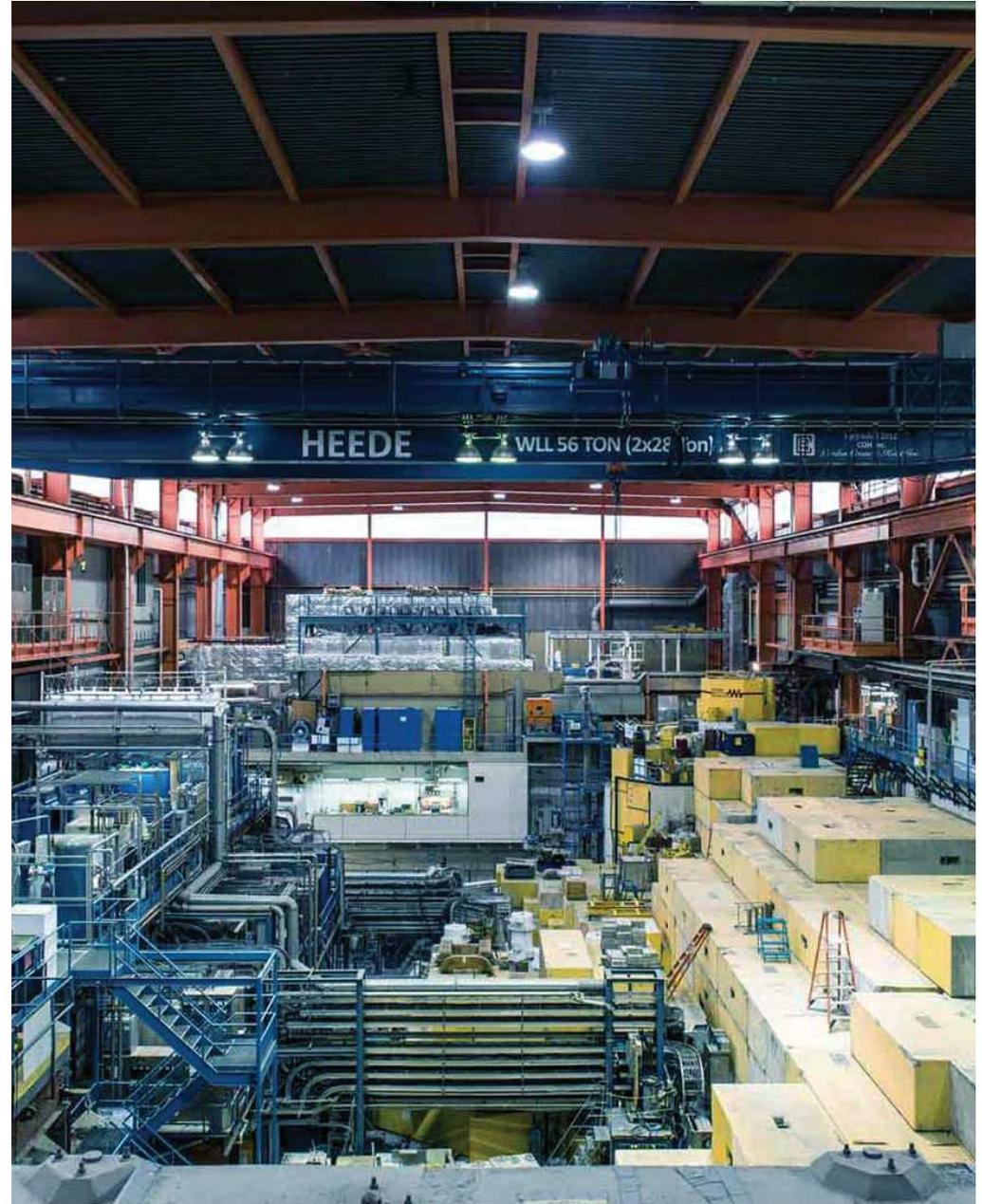
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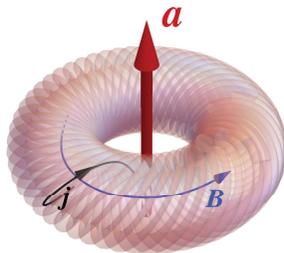
*Ab initio* calculations  
of parity-violating moments  
-  
anapole moment  
electric dipole moment (EDM)  
Schiff moment

2025-06-24



## Why investigate parity violation in atomic and molecular systems and the nuclear anapole moment?

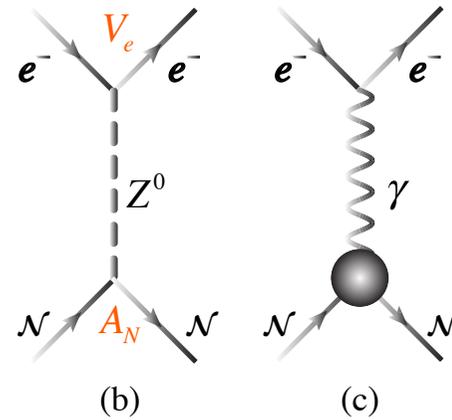
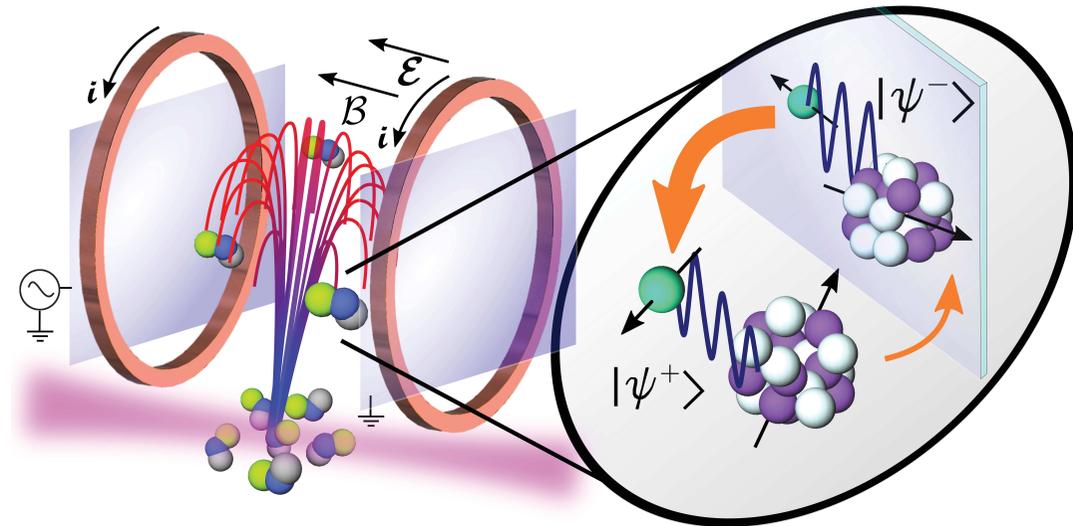
- Parity violation in atomic and molecular systems sensitive to a variety of “new physics”
  - Probes electron-quark electroweak interaction
  - Best limits on the  $Z'$  boson parity violating interaction with electrons and nucleons



# Nuclear spin dependent parity violating effects in light polyatomic molecules

- Experiments proposed for  $^9\text{BeNC}$ ,  $^{25}\text{MgNC}$
- To extract the underlying physics, atomic, molecular, and **nuclear** structure effects must be understood
  - Ab initio* calculations

- Spin dependent parity violation
  - Z-boson exchange between nucleon axial-vector and electron-vector currents (b)
  - Electromagnetic interaction of atomic electrons with the nuclear anapole moment (c)

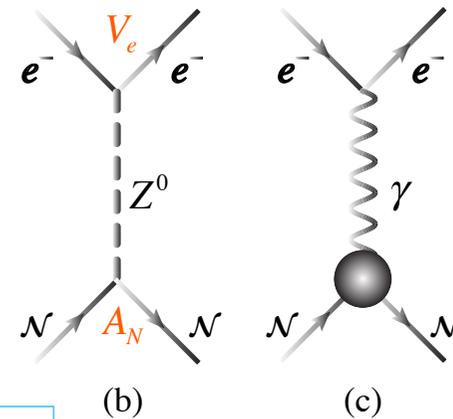
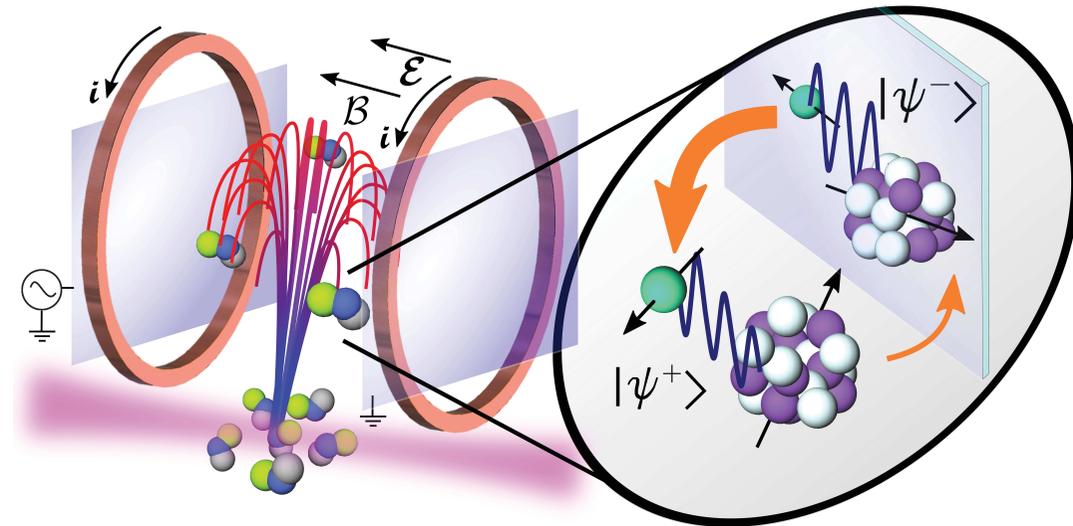


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Anapole moment measurements also planned in  ${}^{137}\text{BaF}$  molecule

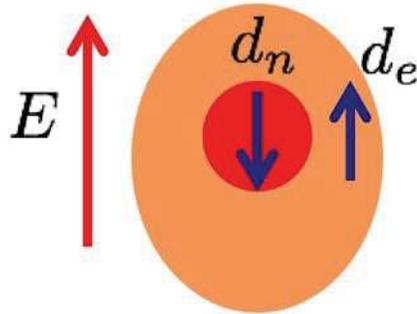
## Why investigate the Electric Dipole Moment (EDM) and nuclear Schiff Moment?

- Unsolved problem in physics: matter-antimatter asymmetry of the universe
- Standard model predicts some CP violation, not enough to explain this asymmetry
- The EDM and nuclear Schiff moment is a promising probe for CP violation beyond the standard model, as well as CP violating QCD  $\bar{\theta}$  parameter
  
- Nuclear EDMs can be measured in storage rings (CERN feasibility study: arXiv:1912.07881)
- Nuclear Schiff moments can be measured using (radioactive) molecules

Nuclear Schiff moment measurements planned in  $^{227}\text{ThF}^+$ , RaF, and FrAg molecules

To understand the nuclear EDM and Schiff moment, nuclear structure effects must be understood

## What is the nuclear Schiff moment?



Schiff Moment

$$\vec{S} = \frac{\langle er^2 \vec{r} \rangle}{10} - \frac{\langle r^2 \rangle \langle e\vec{r} \rangle}{6}$$

Leonard Schiff's Theorem (1963):

- Any permanent dipole moment of the nucleus is perfectly shielded by its electron cloud
- True for point-like nuclei, non-relativistic electrons

However, the "Schiff moment" is not shielded by this effect

- Zero for point-like, spherical nuclei
- Arises from deformations in the nucleus or its constituent nucleons
- Very large in nuclei with both a quadrupole and octupole deformation

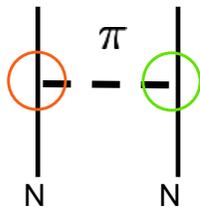
**Look for heavy nuclei with large quadrupole and octupole deformations!**

Slide by Matthew R. Dietrich (ANL)

## Parity violating (PV) and parity & time-reversal (PTV) violating nucleon-nucleon (NN) interaction

- Anapole moment arises due to PV NN interaction (weak force - imaginary), EDM and Schiff moment due to PTV NN interaction (real)
- Parity non-conserving PV or PTV  $V_{NN}^{PNC}$  interaction
  - Conserves total angular momentum /
  - Mixes opposite parities
  - Has isoscalar, isovector and isotensor components

Meson-exchange picture – one vertex PC strong force, one vertex PV (weak) force



$$\mathcal{H}_{MNN}^{p.v.} = (2)^{-1/2} f_{\pi} \bar{N} (\vec{\tau} \times \vec{\phi}^{\pi})^3 N$$

$$+ \bar{N} \left[ h_{\rho}^0 \vec{\tau} \cdot \vec{\phi}_{\mu}^{\rho} + h_{\rho}^1 \phi_{\mu}^{\rho 3} + h_{\rho}^2 \frac{(3\tau^3 \phi_{\mu}^{\rho 3} - \vec{\tau} \cdot \vec{\phi}_{\mu}^{\rho})}{2(6)^{1/2}} \right] \gamma^{\mu} \gamma_5 N$$

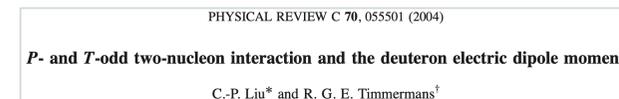
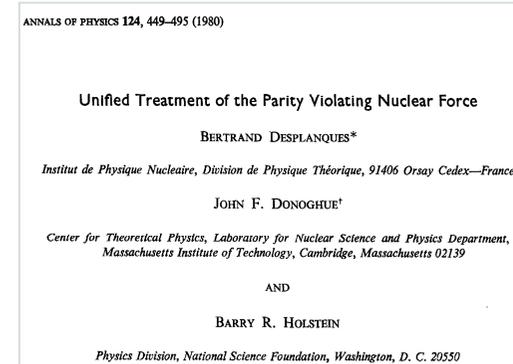
$$+ \bar{N} [h_{\omega}^0 \phi_{\mu}^{\omega} + h_{\omega}^1 \tau^3 \phi_{\mu}^{\omega}] \gamma^{\mu} \gamma_5 N$$

$$- h_{\rho}^1 \bar{N} (\vec{\tau} \times \vec{\phi}_{\mu}^{\rho})^3 \frac{\sigma^{\mu\nu} k_{\nu}}{2M} \gamma_5 N.$$

$$\mathcal{H}_{MNN}^{p.c.} = ig_{\pi NN} \bar{N} \gamma_5 \vec{\tau} \cdot \vec{\phi}^{\pi} N + g_{\rho} \bar{N} \left( \gamma_{\mu} + \frac{i\chi_V}{2M} \sigma_{\mu\nu} k^{\nu} \right) \vec{\tau} \cdot \vec{\phi}^{\mu} N$$

$$+ g_{\omega} \bar{N} \left( \gamma_{\mu} + \frac{i\chi_S}{2M} \sigma^{\mu\nu} k_{\nu} \right) \phi_{\mu}^{\omega} N$$

Include  $\pi$ ,  $\rho$ ,  $\omega$  meson exchanges



## Parity violating (PV) and parity & time-reversal (PTV) violating nucleon-nucleon (NN) interaction

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- Anapole moment arises due to PV NN interaction (weak force - imaginary), EDM and Schiff moment due to PTV NN interaction (real)
- Parity non-conserving PV or PTV  $V_{NN}^{\text{PNC}}$  interaction
  - Conserves total angular momentum  $l$
  - Mixes opposite parities
  - Has isoscalar, isovector and isotensor components

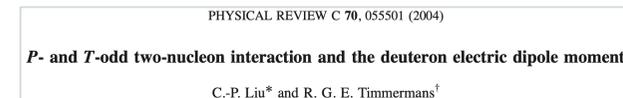
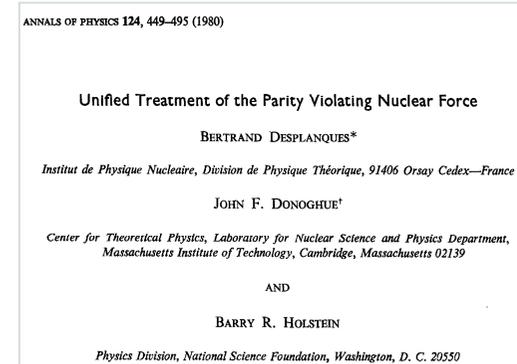
Meson-exchange picture – one vertex PC strong force, one vertex PV (weak) force

$$H_{PV} \propto \left[ \frac{\vec{p}}{M}, y_x(r) \right] \dots + \dots \left\{ \frac{\vec{p}}{M}, y_x(r) \right\}$$

$$H_{PTV} \propto i \left[ \frac{\vec{p}}{M}, y_x(r) \right]$$

$$y_x(r) = e^{-m_x r} / (4\pi r)$$

Include  $\pi$ ,  $\rho$ ,  $\omega$  meson exchanges



$$\begin{aligned}
 H_{PTV}(\mathbf{r}) = & \frac{1}{2m_n} \boldsymbol{\sigma}_- \cdot \nabla \left( -\bar{G}_\omega^0 y_\omega(r) \right) \\
 & + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \boldsymbol{\sigma}_- \cdot \nabla \left( \bar{G}_\pi^0 y_\pi(r) - \bar{G}_\rho^0 y_\rho(r) \right) \\
 & + \frac{\tau_+^z}{2} \boldsymbol{\sigma}_- \cdot \nabla \left( \bar{G}_\pi^1 y_\pi(r) - \bar{G}_\rho^1 y_\rho(r) - \bar{G}_\omega^1 y_\omega(r) \right) \\
 & + \frac{\tau_-^z}{2} \boldsymbol{\sigma}_+ \cdot \nabla \left( \bar{G}_\pi^1 y_\pi(r) + \bar{G}_\rho^1 y_\rho(r) - \bar{G}_\omega^1 y_\omega(r) \right) \\
 & + (3\tau_1^z \tau_2^z - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \boldsymbol{\sigma}_- \cdot \nabla \left( \bar{G}_\pi^2 y_\pi(r) - \bar{G}_\rho^2 y_\rho(r) \right)
 \end{aligned}$$

- Based on one meson exchange model

$$\sigma_\pm = \sigma_1 \pm \sigma_2$$

- $y_x(r) = e^{-m_x r} / (4\pi r)$

$$\tau_\pm^z = \tau_1^z \pm \tau_2^z$$

## Parity violating (PV) and parity & time-reversal (PTV) violating nucleon-nucleon (NN) interaction

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PHYSICAL REVIEW C 70, 055501 (2004)

*P*- and *T*-odd two-nucleon interaction and the deuteron electric dipole moment

C.-P. Liu\* and R. G. E. Timmermans†

$$\begin{aligned}
 H_{PTV}(\mathbf{r}) = & \frac{1}{2m_n} \boldsymbol{\sigma}_- \cdot \nabla \left( -\bar{G}_\omega^0 y_\omega(r) \right) \\
 & + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \boldsymbol{\sigma}_- \cdot \nabla \left( \bar{G}_\pi^0 y_\pi(r) - \bar{G}_\rho^0 y_\rho(r) \right) \\
 & + \frac{\tau_+^z}{2} \boldsymbol{\sigma}_- \cdot \nabla \left( \bar{G}_\pi^1 y_\pi(r) - \bar{G}_\rho^1 y_\rho(r) - \bar{G}_\omega^1 y_\omega(r) \right) \\
 & + \frac{\tau_-^z}{2} \boldsymbol{\sigma}_+ \cdot \nabla \left( \bar{G}_\pi^1 y_\pi(r) + \bar{G}_\rho^1 y_\rho(r) - \bar{G}_\omega^1 y_\omega(r) \right) \\
 & + (3\tau_1^z \tau_2^z - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \boldsymbol{\sigma}_- \cdot \nabla \left( \bar{G}_\pi^2 y_\pi(r) - \bar{G}_\rho^2 y_\rho(r) \right)
 \end{aligned}$$

- Based on one meson exchange model

$$\sigma_\pm = \sigma_1 \pm \sigma_2$$

- $y_x(r) = e^{-m_x r} / (4\pi r)$

$$\tau_\pm^z = \tau_1^z \pm \tau_2^z$$

- Coupling constants

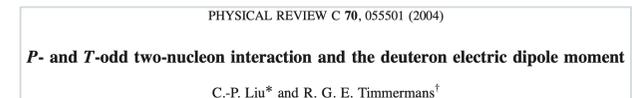
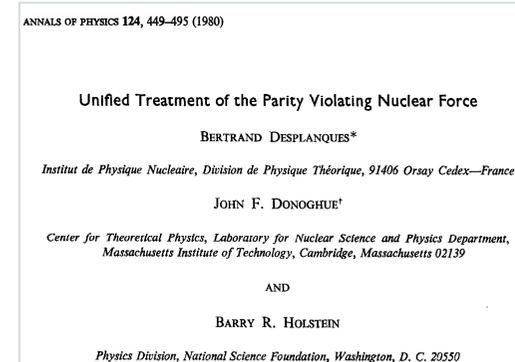
$$\bar{G}_\pi^t = g_{\pi NN} \bar{g}_{\pi NN}^t \quad (g_{\pi NN} \sim 13.3)$$

## Parity violating (PV) and parity & time-reversal (PTV) violating nucleon-nucleon (NN) interaction

23

- Anapole moment arises due to PV NN interaction (weak force - imaginary), EDM and Schiff moment due to PTV NN interaction (real)
- Parity non-conserving PV or PTV  $V_{\text{NN}}^{\text{PNC}}$  interaction
  - Conserves total angular momentum  $I$
  - Mixes opposite parities
  - Has isoscalar, isovector and isotensor components
  - Admixes unnatural parity states in the ground state

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \times \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$



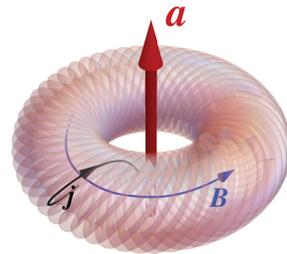
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- Anapole moment operator dominated by spin contribution

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \times \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

$$\mathbf{a} = -\pi \int d^3r r^2 \mathbf{j}(\mathbf{r})$$



$$\hat{\mathbf{a}}_s = \frac{\pi e}{m} \sum_{i=1}^A \mu_i (\mathbf{r}_i \times \boldsymbol{\sigma}_i)$$

$$\mu_i = \mu_p(1/2 + t_{z,i}) + \mu_n(1/2 - t_{z,i})$$

$$a_s = \langle \psi_{\text{gs}} I I_z = I | \hat{a}_{s,0}^{(1)} | \psi_{\text{gs}} I I_z = I \rangle$$

- Anapole moment calculation:

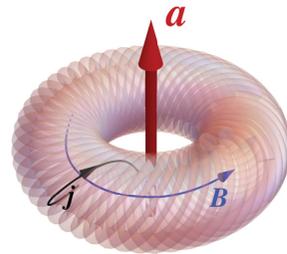
$$\kappa_A = \frac{\sqrt{2}e}{G_F} a_s \quad \kappa_A = -i4\pi \frac{e^2}{G_F} \frac{\hbar}{mc} \frac{(II10|II)}{\sqrt{2I+1}} \sum_j \langle \psi_{\text{gs}} I^\pi | \sqrt{4\pi/3} \sum_{i=1}^A \mu_i r_i [Y_1(\hat{r}_i) \sigma_i]^{(1)} | \psi_j I^{-\pi} \rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

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$$\mathbf{a} = -\pi \int d^3r r^2 \mathbf{j}(\mathbf{r})$$



$$\hat{\mathbf{a}}_s = \frac{\pi e}{m} \sum_{i=1}^A \mu_i (\mathbf{r}_i \times \boldsymbol{\sigma}_i)$$

$$\mu_i = \mu_p(1/2 + t_{z,i}) + \mu_n(1/2 - t_{z,i})$$

$$a_s = \langle \psi_{\text{gs}} I I_z = I | \hat{a}_{s,0}^{(1)} | \psi_{\text{gs}} I I_z = I \rangle$$

Low lying states of opposite parity can lead to enhancement!

- Anapole moment calculation:

$$\kappa_A = \frac{\sqrt{2}e}{G_F} a_s \quad \kappa_A = -i4\pi \frac{e^2}{G_F} \frac{\hbar}{mc} \frac{(II10|II)}{\sqrt{2I+1}} \sum_j \langle \psi_{\text{gs}} I^\pi | \sqrt{4\pi/3} \sum_{i=1}^A \mu_i r_i [Y_1(\hat{r}_i) \sigma_i]^{(1)} | \psi_j I^{-\pi} \rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

## Parity violating (PV) and parity & time-reversal (PTV) violating nucleon-nucleon (NN) interaction

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- EDM and Schiff moment operators

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \times \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

$$\hat{D}_z = \frac{e}{2} \sum_{i=1}^A (1 + \tau_i^z) z_i$$

$$\mathbf{S} = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 \mathbf{r}_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} \mathbf{r}_i \right)$$

- EDM and Schiff moment calculation
  - Nuclear EDM is dominated by and the Schiff moment determined by the polarization contribution:

$$D^{(pol)} = \langle \psi_{\text{gs}} I^\pi | \hat{D}_z | \psi_{\text{gs}} I \rangle + c. c.$$

$$\mathbf{S} = \langle \psi_{\text{gs}} I^\pi | \mathbf{S} | \psi_{\text{gs}} I \rangle + c. c.$$

**NCSM applications to parity-violating moments:**  
How to calculate the sum of intermediate unnatural parity states?

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

## NCSM applications to parity-violating moments:

How to calculate the sum of intermediate unnatural parity states?

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

Sum over all possible  
intermediate states

**NCSM applications to parity-violating moments:**  
How to calculate the sum of intermediate unnatural parity states?

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

- Solving Schroedinger equation with inhomogeneous term

$$(E_{\text{gs}} - H) |\psi_{\text{gs}} I\rangle = V_{\text{NN}}^{\text{PNC}} |\psi_{\text{gs}} I^\pi\rangle$$

- To invert this equation, we apply the Lanczos algorithm

## NCSM applications to parity-violating moments:

### How to calculate the sum of intermediate unnatural parity states?

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

- Solving Schroedinger equation with inhomogeneous term

$$(E_{\text{gs}} - H) |\psi_{\text{gs}} I\rangle = V_{\text{NN}}^{\text{PNC}} |\psi_{\text{gs}} I^\pi\rangle$$

- To invert this equation, we apply the Lanczos algorithm
  - Bring matrix to tri-diagonal form ( $\mathbf{v}_1, \mathbf{v}_2 \dots$  orthonormal,  $H$  Hermitian)

$H\mathbf{v}_1 = \alpha_1\mathbf{v}_1 + \beta_1\mathbf{v}_2$
$H\mathbf{v}_2 = \beta_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \beta_2\mathbf{v}_3$
$H\mathbf{v}_3 = \beta_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 + \beta_3\mathbf{v}_4$
$H\mathbf{v}_4 = \beta_3\mathbf{v}_3 + \alpha_4\mathbf{v}_4 + \beta_4\mathbf{v}_5$

- $n^{\text{th}}$  iteration computes  $2n^{\text{th}}$  moment
- Eigenvalues converge to extreme (largest in magnitude) values
- $\sim 150$ - $200$  iterations needed for 10 eigenvalues (even for  $10^9$  states)

$$|\psi_{\text{gs}} I\rangle = |\psi_{\text{gs}} I^\pi\rangle + \sum_j |\psi_j I^{-\pi}\rangle \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

- Solving Schroedinger equation with inhomogeneous term

$$(E_{\text{gs}} - H) |\psi_{\text{gs}} I\rangle = V_{\text{NN}}^{\text{PNC}} |\psi_{\text{gs}} I^\pi\rangle$$

- To invert this equation, we apply the Lanczos algorithm

$$|\mathbf{v}_1\rangle = V_{\text{NN}}^{\text{PNC}} |\psi_{\text{gs}} I^\pi\rangle$$

**Ab initio calculations of electric dipole moments of light nuclei**

Paul Froese\*  
 TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada  
 and Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T 1Z1, Canada

Petr Navrátil\*  
 TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

## $^3\text{He}$ EDM Benchmark Calculation

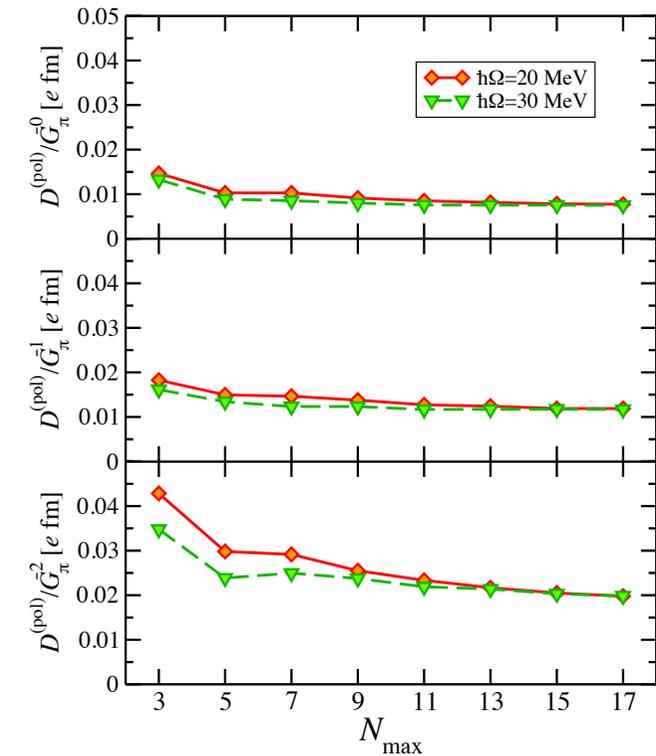
Discrepancy between calculations?

	PLB 665:165-172 (2008) (NN EFT)	PRC 87:015501 (2013)	PRC 91:054005 (2015)	Our calculation (NN EFT)
$\bar{G}_\pi^0$	0.015	(x 1/2)	(x 1/2)	0.0073 (x 1/2)
$\bar{G}_\pi^1$	0.023	(x 1/2)	(x 1/2)	0.011 (x 1/2)
$\bar{G}_\pi^2$	0.037	(x 1/5)	(x 1/2)	0.019 (x 1/2)
$\bar{G}_\rho^0$	-0.0012	(x 1/2)	(x 1/2)	-0.00062 (x 1/2)
$\bar{G}_\rho^1$	0.0013	(x 1/2)	(x 1/2)	0.00063 (x 1/2)
$\bar{G}_\rho^2$	-0.0028	(x 1/5)	(x 1/2)	-0.0014 (x 1/2)
$\bar{G}_\omega^0$	0.0009	(x 1/2)	(x 1/2)	0.00042 (x 1/2)
$\bar{G}_\omega^1$	-0.0017	(x 1/2)	(x 1/2)	-0.00086 (x 1/2)

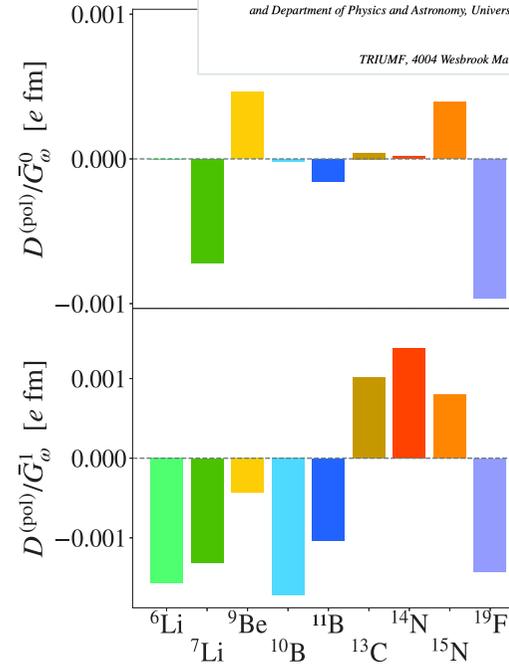
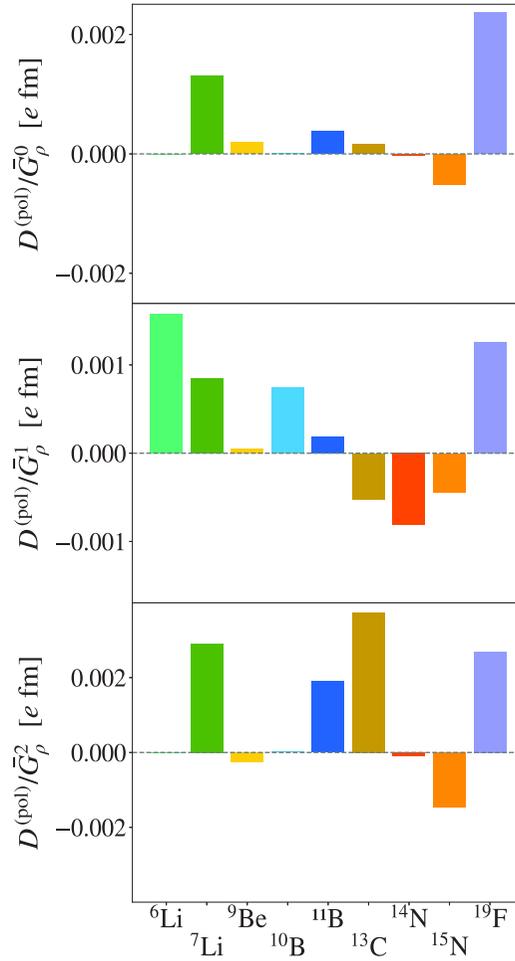
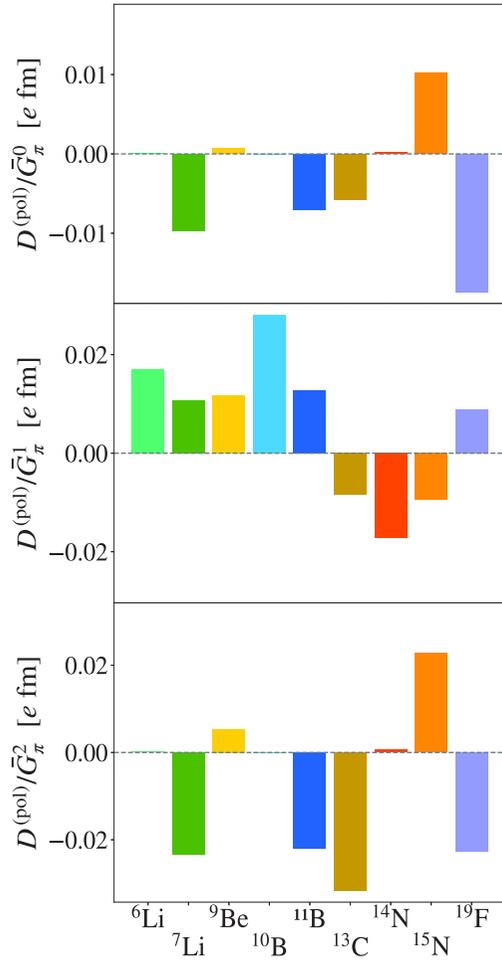
Our results confirm those of Yamanaka and Hiyama, PRC 91:054005 (2015)

## $N_{\text{max}}$ convergence for $^3\text{He}$

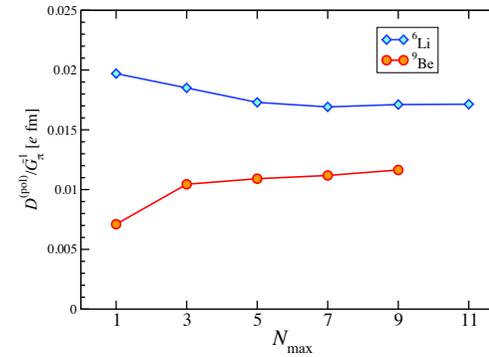
$\text{N}^3\text{LO NN}$



# NCSM applications to parity-violating moments: EDMs of light stable nuclei



Examples of  $N_{\text{max}}$  convergence



## Ab initio calculations of electric dipole moments of light nuclei

Paul Froese\*

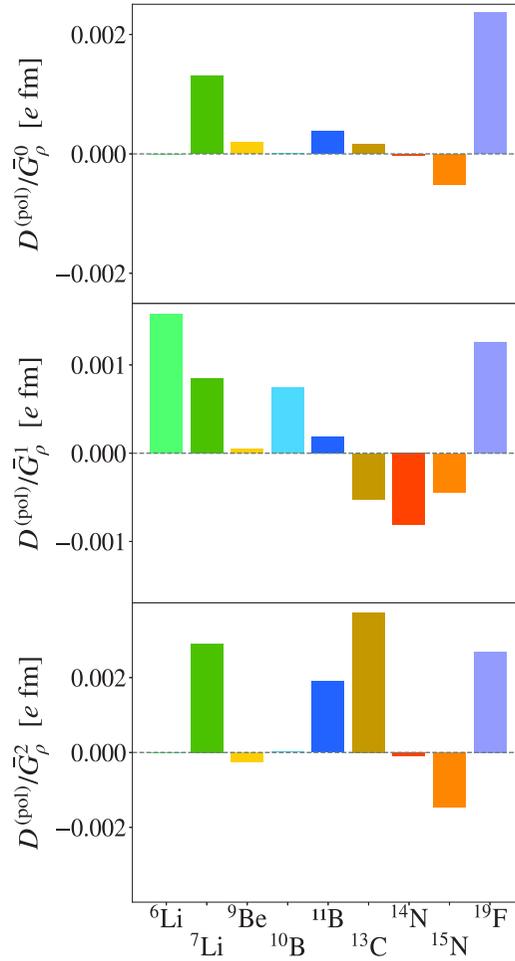
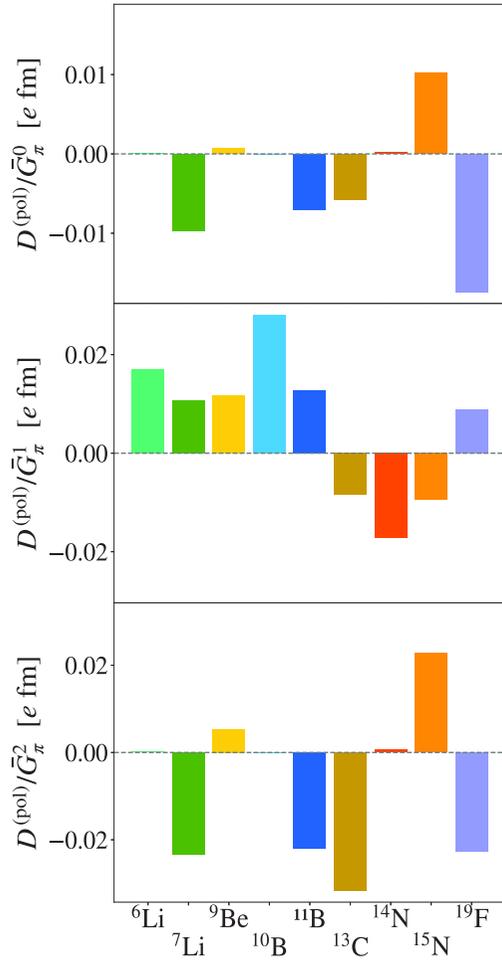
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada  
and Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T 1Z1, Canada

Petr Navrátil\*

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# NCSM applications to parity-violating moments:

## EDMs of light stable nuclei

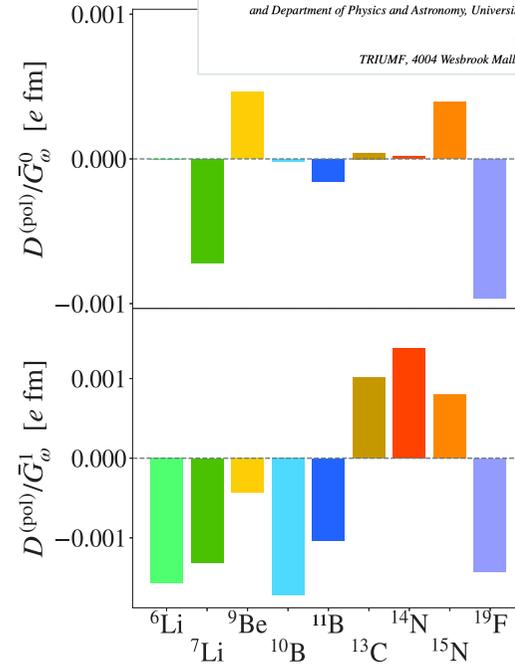


PHYSICAL REVIEW C **104**, 025502 (2021)

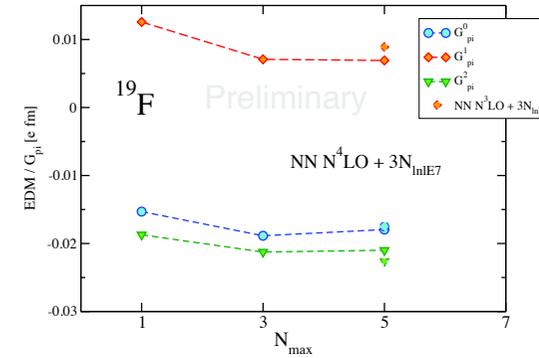
***Ab initio* calculations of electric dipole moments of light nuclei**

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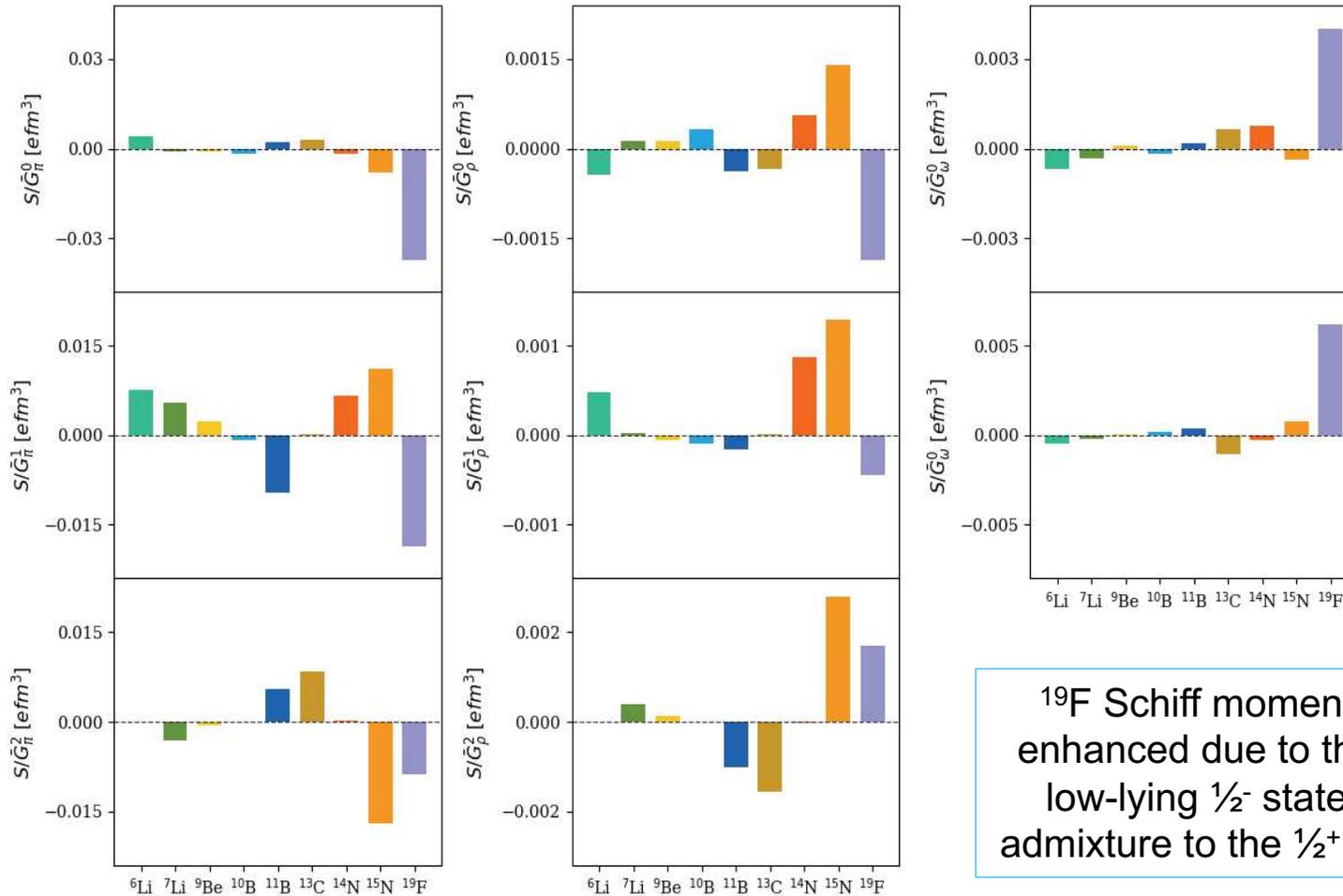


### Examples of $N_{\text{max}}$ convergence



# NCSM applications to parity-violating moments: Schiff moments of light stable nuclei

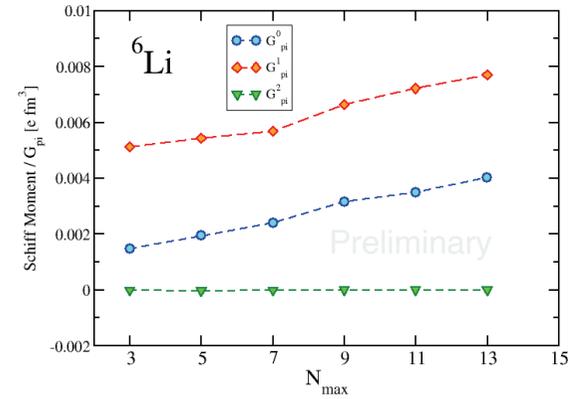
Results preliminary



<sup>19</sup>F Schiff moment enhanced due to the low-lying 1/2<sup>-</sup> state admixture to the 1/2<sup>+</sup> gs

Work in progress with Stephan Foster, McMaster University undergraduate student

Examples of N<sub>max</sub> convergence

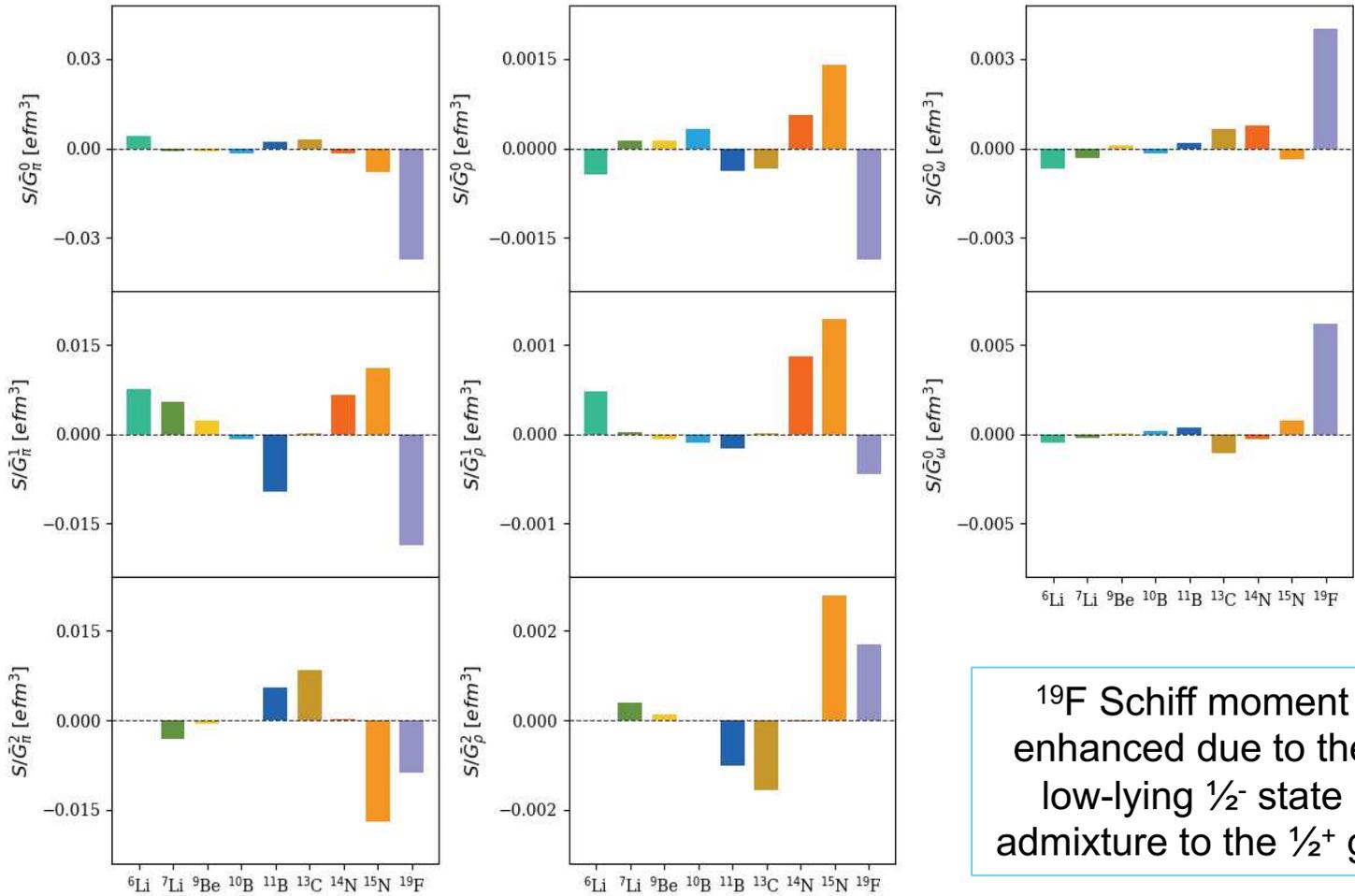


Convergence more challenging due to a destructive contribution of the two terms and the long-range r<sup>3</sup> dependence

$$S = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 \mathbf{r}_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} \mathbf{r}_i \right)$$

# NCSM applications to parity-violating moments: Schiff moments of light stable nuclei

Results preliminary



Work in progress  
with Stephan Foster,  
McMaster University  
undergraduate student

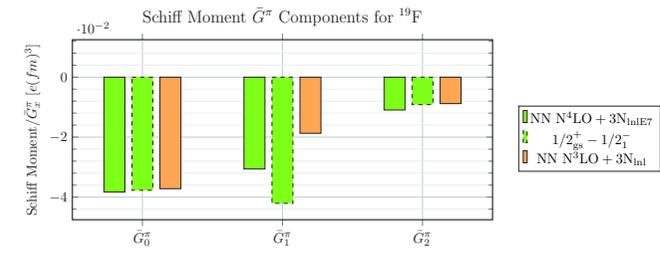


Figure 1: Comparison of  $^{19}\text{F}$   $\tilde{G}_n^\pi$  components for different interactions and included states.

$^{19}\text{F}$  Schiff moment  
enhanced due to the  
low-lying  $1/2^-$  state  
admixture to the  $1/2^+$  gs

$$S = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 \mathbf{r}_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} \mathbf{r}_i \right)$$

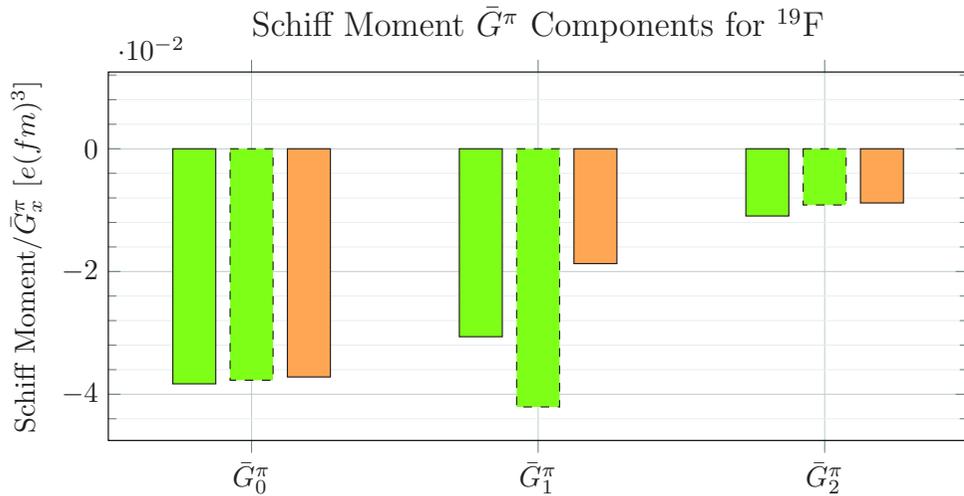
# NCSM applications to parity-violating moments: Schiff moments of light stable nuclei

Results preliminary

Calculated  $1/2^-$  state energies shifted to match the  $1/2^-$  excitation energy

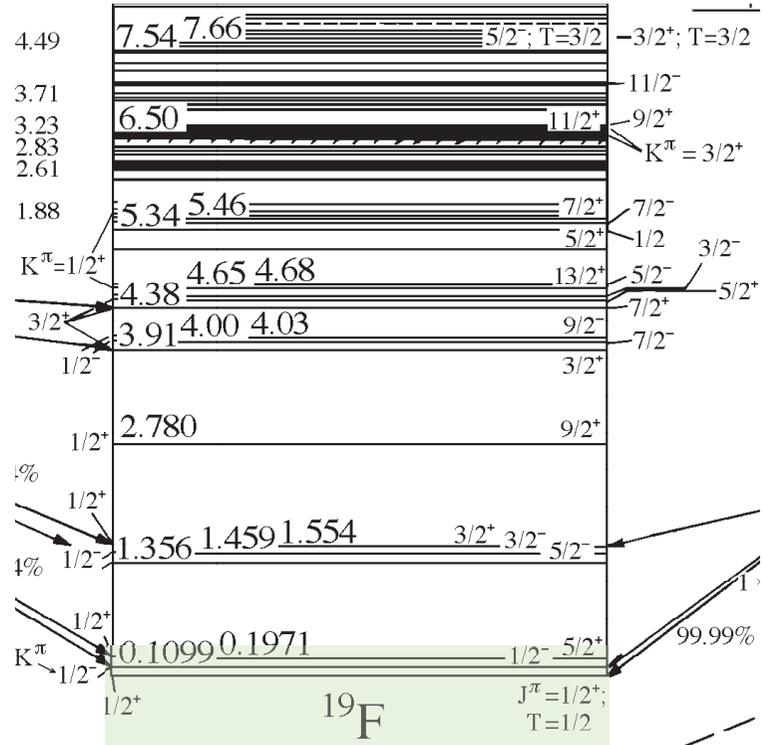
Relevant for planned nuclear Schiff moment measurements in  $^{227}\text{ThF}^+$  at TRIUMF

Work in progress with Stephan Foster, McMaster University undergraduate student



■ NN  $N^4\text{LO} + 3N_{\text{lnIE7}}$   
▨  $1/2_{\text{gs}}^+ - 1/2_1^-$   
■ NN  $N^3\text{LO} + 3N_{\text{lnl}}$

Figure 1: Comparison of  $^{19}\text{F}$   $\bar{G}^\pi$  components for different interactions and included states.



$$S = \langle \psi_{\text{gs}} | I^\pi | S | \psi_{\text{gs}} | I \rangle + c. c.$$

$$|\psi_{\text{gs}} | I \rangle = |\psi_{\text{gs}} | I^\pi \rangle + \sum_j |\psi_j | I^{-\pi} \rangle \times \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j | I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} | I^\pi \rangle$$

$^{19}\text{F}$  Schiff moment enhanced due to the low-lying  $1/2^-$  state admixture to the  $1/2^+$  gs

$$s = \frac{e}{10} \sum_{i=1}^z \left( r_i^2 r_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} r_i \right)$$

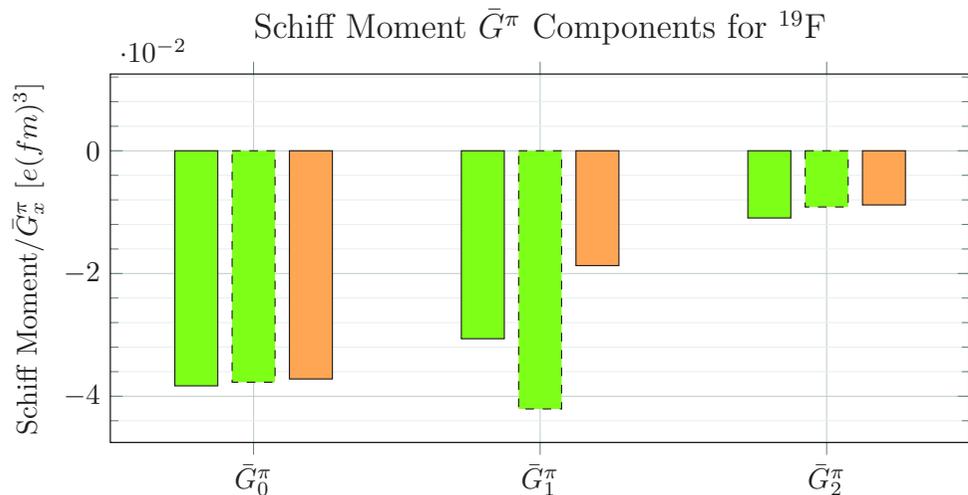
# NCSM applications to parity-violating moments: Schiff moments of light stable nuclei

Results preliminary

Calculated  $1/2^-$  state energies shifted to match the  $1/2^-$  excitation energy

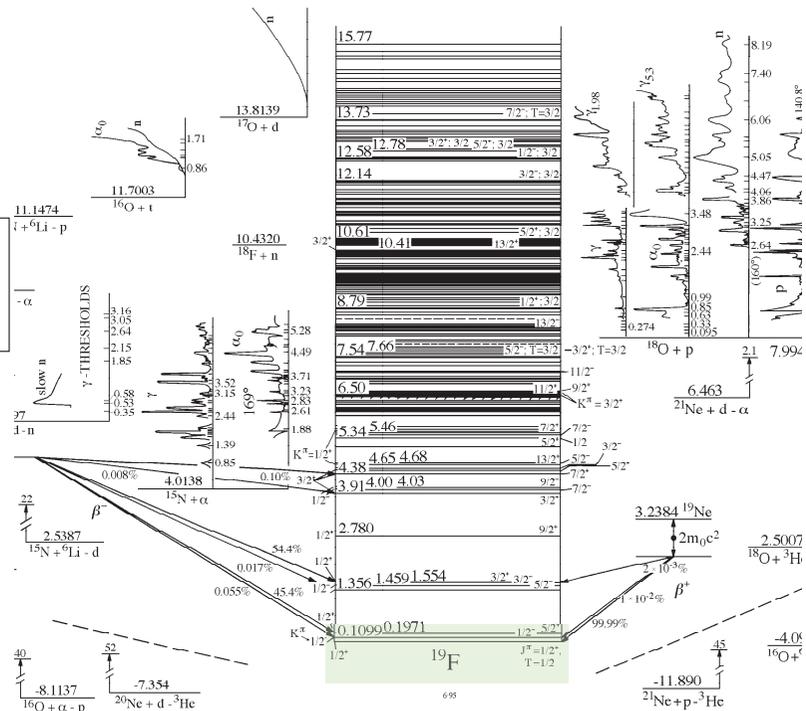
Relevant for planned nuclear Schiff moment measurements in  $^{227}\text{ThF}^+$  at TRIUMF

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■ NN  $N^4\text{LO} + 3N_{\text{lnIE7}}$   
▨  $1/2^+_{\text{gs}} - 1/2^-_1$   
■ NN  $N^3\text{LO} + 3N_{\text{lnl}}$

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$^{19}\text{F}$  Schiff moment enhanced due to the low-lying  $1/2^-$  state admixture to the  $1/2^+$  gs

$$S = \langle \psi_{\text{gs}} | I^\pi | S | \psi_{\text{gs}} | I \rangle + c.c.$$

$$|\psi_{\text{gs}} | I \rangle = |\psi_{\text{gs}} | I^\pi \rangle + \sum_j |\psi_j | I^{-\pi} \rangle \times \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j | I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} | I^\pi \rangle$$

$$s = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 r_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} r_i \right)$$

# NCSM applications to parity-violating moments: Schiff moments of light stable nuclei

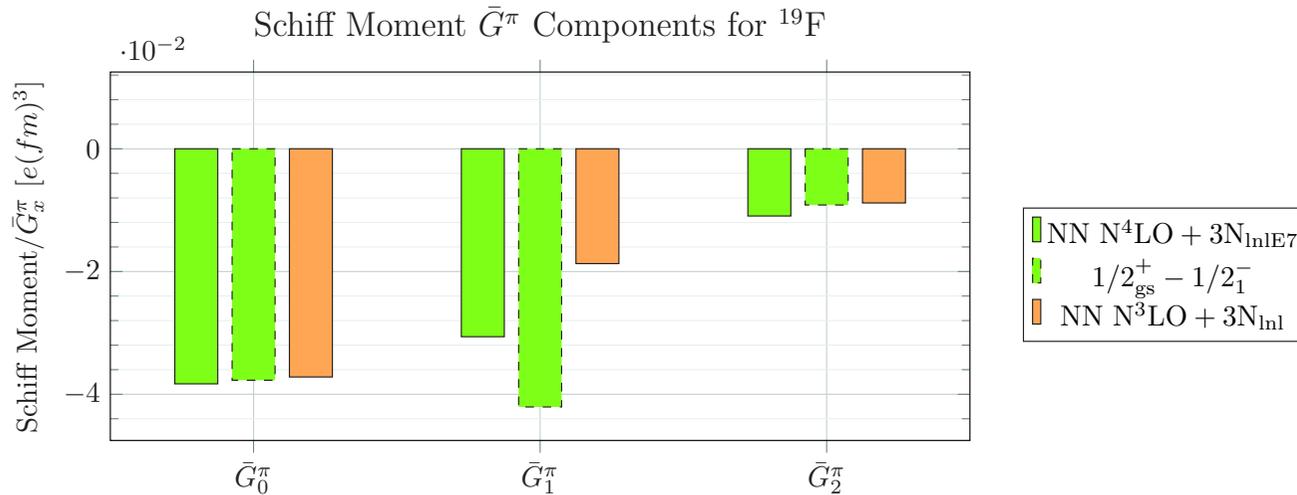
Results preliminary

Calculated  $1/2^-$  state energies shifted to match the  $1/2^-_{-1}$  excitation energy

Relevant for planned nuclear Schiff moment measurements in  $^{227}\text{ThF}^+$  at TRIUMF

Work in progress with Stephan Foster, McMaster University undergraduate student

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$^{19}\text{F}$  Schiff moment comparable to  $^{129}\text{Xe}$  Schiff moment calculated within the nuclear shell model

PHYSICAL REVIEW C **102**, 065502 (2020)

Large-scale shell-model calculations of nuclear Schiff moments of  $^{129}\text{Xe}$  and  $^{199}\text{Hg}$

Kota Yanase\* and Noritaka Shimizu\*

TABLE II. The NSM coefficients of  $^{129}\text{Xe}$  in units of  $10^{-2}e\text{fm}^3$ . Our final results are given in bold.

	$a_0$	$a_1$	$a_2$
IPM ( $m_\pi \rightarrow \infty$ )	-9.9	-9.9	-19.8
IPM	-4.6	-4.6	-9.2
LSSM (SN100PN, $m_\pi \rightarrow \infty$ )	-8.7	-8.2	-15.8
LSSM (SNV, $m_\pi \rightarrow \infty$ )	-8.6	-8.3	-16.2
LSSM (SN100PN)	-3.7	-4.1	-8.0
<b>LSSM (SNV)</b>	<b>-3.8</b>	<b>-4.1</b>	<b>-8.1</b>
IPM ( $m_\pi \rightarrow \infty$ ) [35,36]	-11	-11	-22
IPM [38]	-6	-6	-12
RPA [38]	-0.8	-0.6	-0.9
PTSM [41]	0.05	-0.04	0.19
PTSM [42]	0.3	-0.1	0.4

Figure 1: Comparison of  $^{19}\text{F}$   $\bar{G}^\pi$  components for different interactions and included states.

$$S = \langle \psi_{\text{gs}} I^\pi | S | \psi_{\text{gs}} I \rangle + c. c.$$

$$|\psi_{\text{gs}} I \rangle = |\psi_{\text{gs}} I^\pi \rangle + \sum_j |\psi_j I^{-\pi} \rangle \times \frac{1}{E_{\text{gs}} - E_j} \langle \psi_j I^{-\pi} | V_{\text{NN}}^{\text{PNC}} | \psi_{\text{gs}} I^\pi \rangle$$

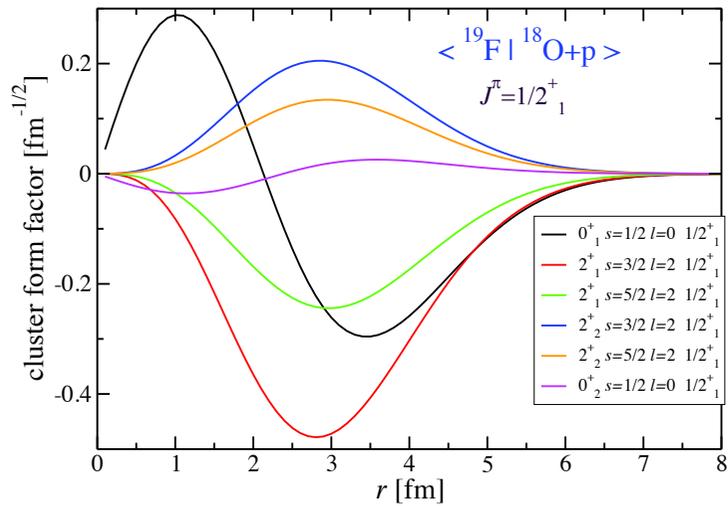
$^{19}\text{F}$  Schiff moment enhanced due to the low-lying  $1/2^-$  state admixture to the  $1/2^+$  gs

$$S = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 r_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} r_i \right)$$

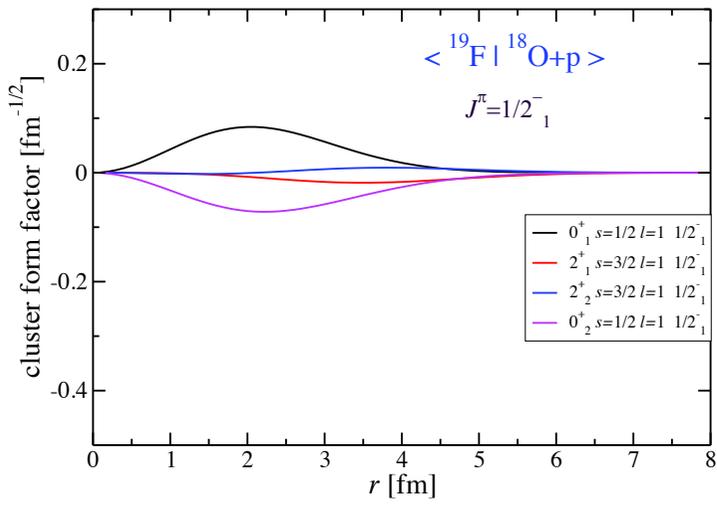
# NCSM applications to parity-violating moments: Schiff moments of light stable nuclei

Work in progress  
with Stephan Foster,  
McMaster University  
undergraduate student

$^{19}\text{F}$  Schiff moment dominated by the contribution of the lowest  $\frac{1}{2}^-$  state  
However, its contribution to the EDM of  $^{19}\text{F}$  is negligible.  
This is due to very different structure of the  $\frac{1}{2}^+$  g.s. and the  $\frac{1}{2}^-$  state



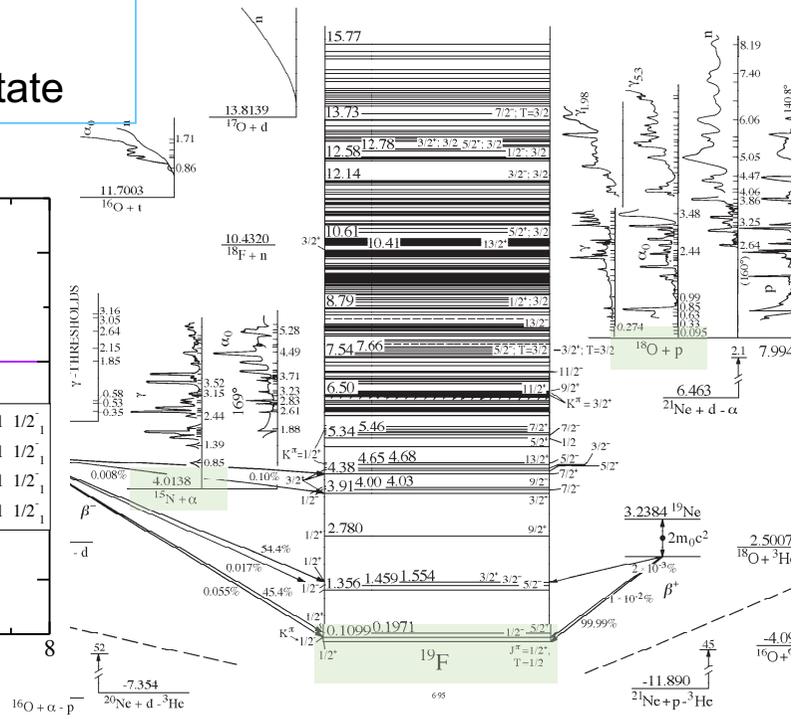
$^{18}\text{O}+p$  (shell-model-like)



$^{15}\text{N}+^4\text{He}$  – alpha-clustering

E1 matrix element small  
S matrix element large due to the  $r^3$  term

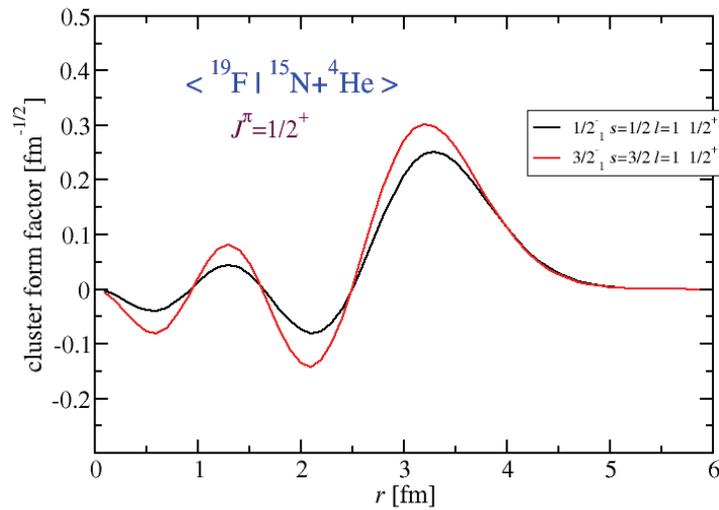
$$\hat{D}_z = \frac{e}{2} \sum_{i=1}^A (1 + \tau_i^z) z_i \quad \mathbf{s} = \frac{e}{10} \sum_{i=1}^Z \left( r_i^2 \mathbf{r}_i - \frac{5}{3} \langle r^2 \rangle_{\text{ch}} \mathbf{r}_i \right)$$



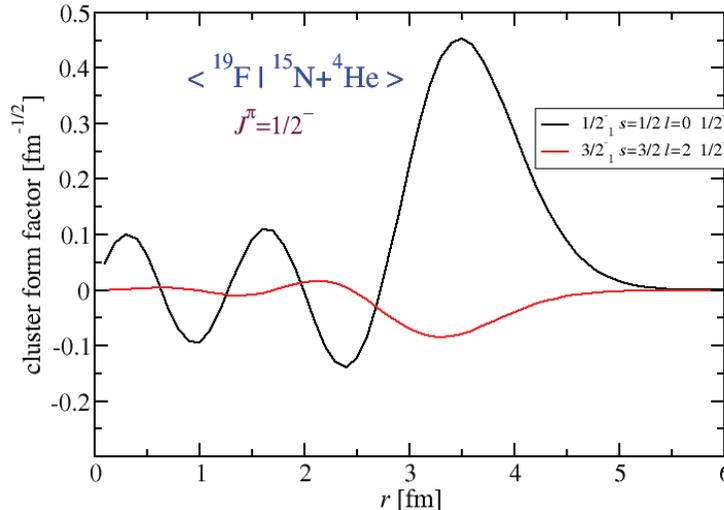
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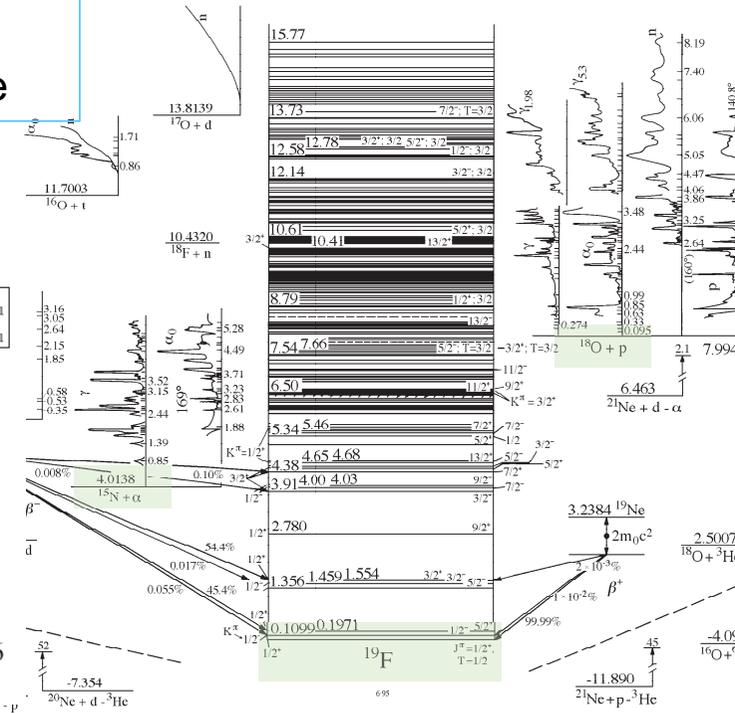


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## Nuclear spin-dependent parity-violating effects from NCSM

- Contributions from nucleon axial-vector and the anapole moment

$$\kappa_{ax} \simeq -2C_{2p}\langle s_{p,z} \rangle - 2C_{2n}\langle s_{n,z} \rangle \simeq -0.1\langle s_{p,z} \rangle + 0.1\langle s_{n,z} \rangle$$

$$\langle s_{\nu,z} \rangle \equiv \langle \psi_{\text{gs}} I^\pi I_z = I | \hat{s}_{\nu,z} | \psi_{\text{gs}} I^\pi I_z = I \rangle$$

$$C_{2p} = -C_{2n} = g_A(1 - 4\sin^2 \theta_W)/2 \simeq 0.05$$

## Conclusions

- *Ab initio* nuclear theory
  - Makes connections between the low-energy QCD and many-nucleon systems
- No-core shell model is an *ab initio* configuration interaction method
  - Applicable to nuclear structure, reactions including those relevant for astrophysics, electroweak processes, tests of fundamental symmetries
  - In combination with the Lanczos strength method provides robust results for electroweak observables and nuclear structure dependent corrections