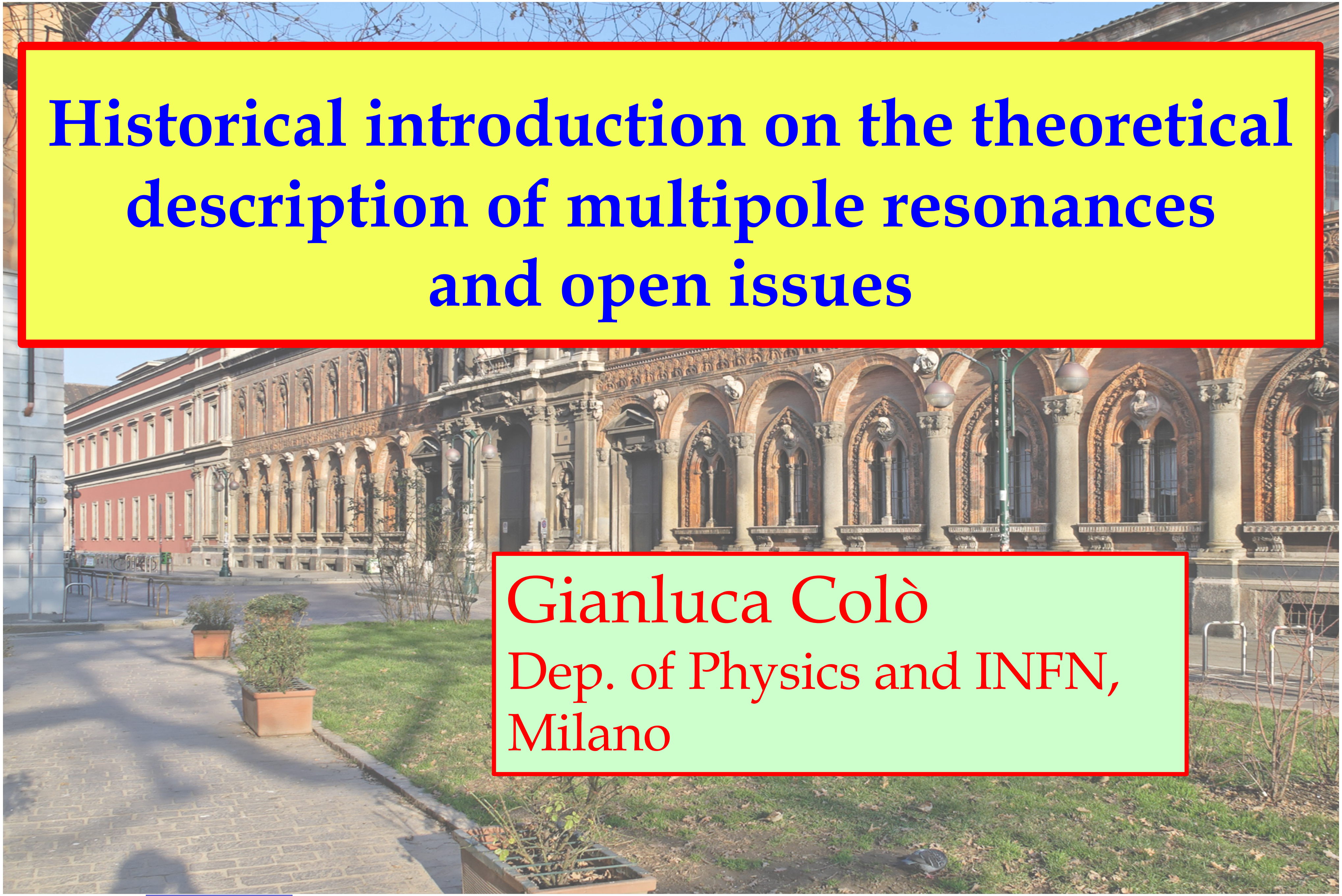


Historical introduction on the theoretical description of multipole resonances and open issues

Gianluca Colò
Dep. of Physics and INFN, Milano

CEA ENST, 19/11/2024

What can I discuss about theory of collective motion without boring you?

Theoretical Methods for Giant Resonances

Gianluca Colò

I. Tanihata et al. (eds.), *Handbook of Nuclear Physics*,
https://doi.org/10.1007/978-981-15-8818-1_72-1

Why should we study nuclear collective states?

- **Test new models** – either new energy density functionals (EDFs), or chiral Hamiltonians (for their emergent properties)
- Find the **nature of elusive/new modes** ("pygmy" modes, toroidal modes)
- There is still much to understand about **GR decay**
- **"Applications"**: the Equation of State (EoS), astrophysics, matrix elements for β- or ββ-decay

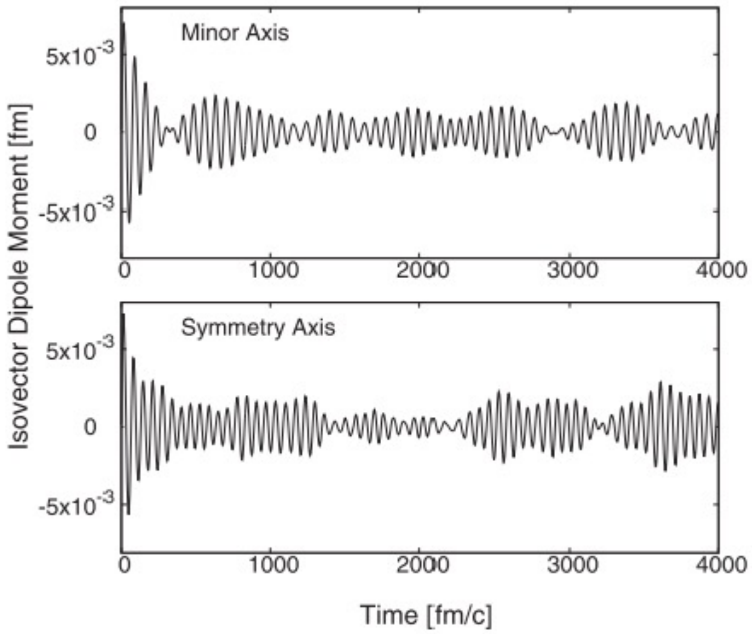
Outline of this talk

- Linear response theory vs time-dependent approaches
- RPA vs SM
- Giant resonances and their decay (continuum coupling)
- Beyond the linear response: SRPA and (Q)PVC
- Spin- and spin-isospin modes (and β-decay)
- Drip line nuclei, “pygmies” vs threshold effects
- ”Service” for particle physics, astrophysics
- Novelties, conclusions (mainly apologies for omitted topics)

Time-dependent Hartree-Fock or Kohn-Sham

$$\boxed{h\phi_i = \varepsilon_i \phi_i}$$

From: P. Stevenson (U. Surrey)



In the time-dependent case, one can solve the **evolution equation for the density** directly:

$$h(t) = h + f(t) \qquad [h(t), \rho(t)] = i\hbar\, \dot{\rho}(t)$$

$$\rho(t=0) \neq \rho_{\mathrm{g.s.}}$$

$$\rho(t=\Delta t) = U(t=0, t=\Delta t)\rho(t=0) \qquad U = e^{-i\frac{\Delta t}{\hbar}\cdot h}$$

This approach allows also studying large-amplitude motion (e.g. reactions).

If the equation for the density is **linearized** (small amplitude limit or linear response): **Random Phase Approximation or RPA**.

How to derive the RPA equations (I)

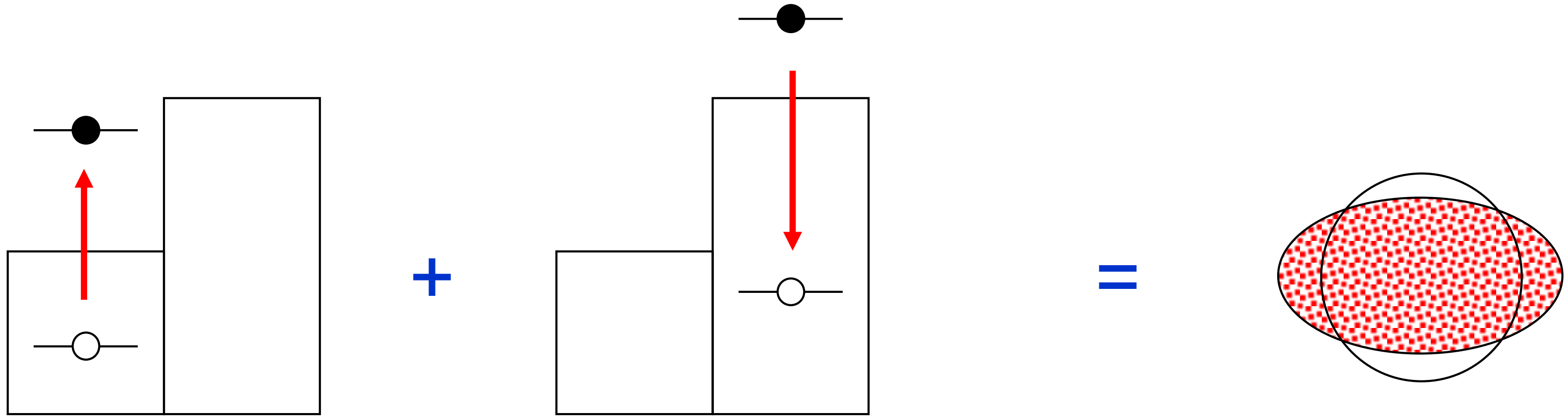
$$h(t) = h + f(t) \qquad [h(t), \rho(t)] = i\hbar\, \dot{\rho}(t)$$

The so-called **Random Phase Approximation**(*) should be better called "linearization of the equation of motion". We truncate the equation of motion at first order.

(*) The name comes from plasma physics, cf. e^{ikr}…

$$\rho = \rho^{(0)} + \delta\rho \qquad h = h^{(0)} + \delta h$$

In the **Random Phase Approximation,** the linearized equation of motion is written on a p-h basis and becomes a matrix equation.



$$|n\rangle = \sum_{\rm ph} X_{\rm ph}|ph^{-1}\rangle - Y_{\rm ph}|hp^{-1}\rangle$$

Pairing → QRPA

Matrix RPA and Finite Amplitude Method (FAM)

$\rho = \rho^{(0)} + \delta\rho \qquad h = h^{(0)} + \delta h$

Small amplitude Harmonic approx.

$\delta\rho = \delta\rho(\vec{r})\mathrm{e}^{-i\omega t} + \mathrm{h.c.}$

$$\hbar\omega\delta\rho(\omega) = \left[h^{(0)}, \delta\rho(\omega)\right] + \left[\delta h(\omega), \rho^{(0)}\right] + \left[f, \rho^{(0)}\right]$$

Standard definition of the "forward" and "backward" amplitudes:

$$X_{\mathrm{ph}} = \langle \mathrm{ph}^{-1} | \delta\rho | 0 \rangle \qquad Y_{\mathrm{ph}} = \langle \mathrm{hp}^{-1} | \delta\rho | 0 \rangle$$

$$(\varepsilon_p - \varepsilon_h - \omega)\, X_{\mathrm{ph}} + \delta h^{\mathrm{ph}}(\omega) = -f^{\mathrm{ph}}(\omega)$$
$$(\varepsilon_p - \varepsilon_h + \omega)\, Y_{\mathrm{ph}} + \delta h^{\mathrm{hp}}(\omega) = -f^{\mathrm{hp}}(\omega)$$

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \hbar\omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

FAM: the calculation of the two-body matrix elements is avoided

Matrix formulation

$$\delta h = \sum \frac{\delta h}{\delta \rho} \delta\rho$$

G.C. et al., Computer Physics Commun. 184, 142 (2013).

RPA and TDHF

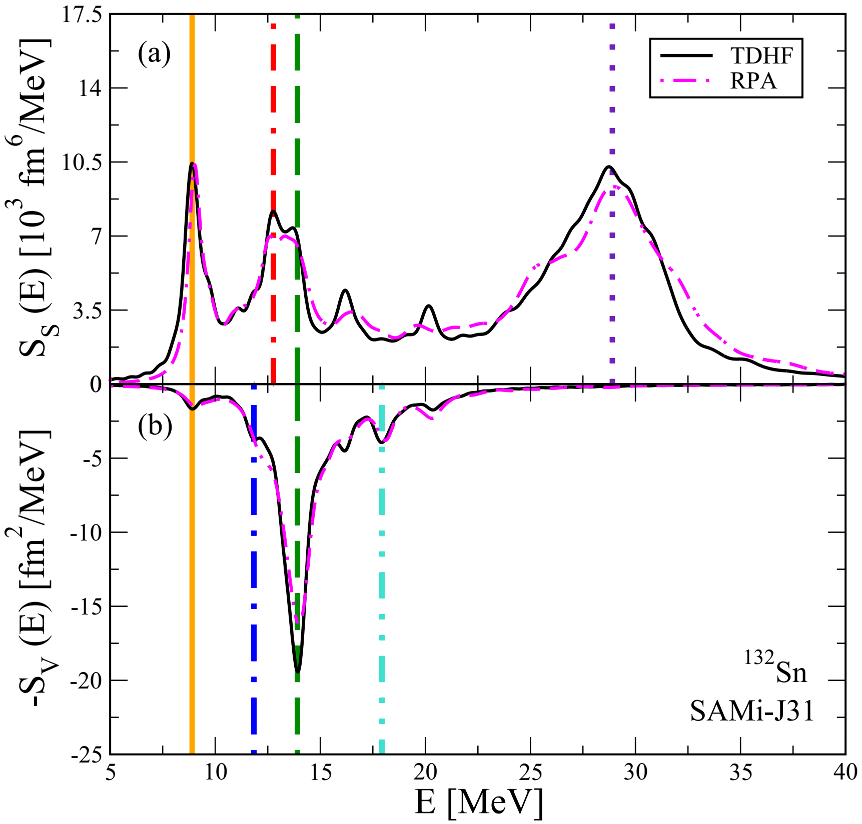


FIG. 9. The strength function of the IS (a) and IV (b) response for ^{132}Sn with SAMi-J31 as obtained in TDHF or in RPA calculation, with $L_{\mathrm{box}} = 20$ or 30 fm, respectively. The vertical lines indicate the energy of the modes selected for the transition density analysis (see Sec. III E).

This comparison between TDHF and RPA (using Skyrme EDFs) is taken from:

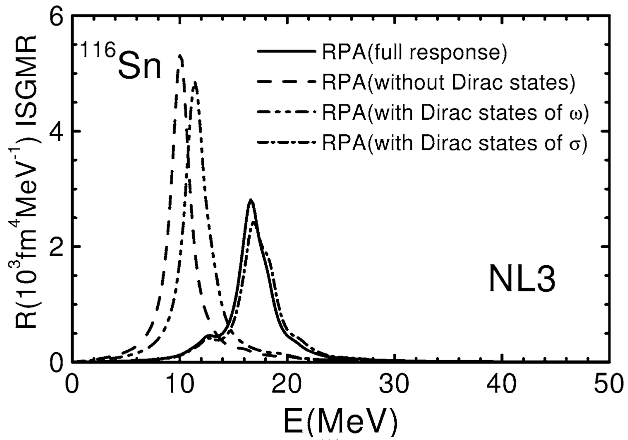
S. Burrello *et al.*, Phys. Rev. C99, 054314 (2019).

Relativistic RPA and time-dependent Hartree

At the turn of the century, there was a puzzle: relativistic RPA seemed to be very different from time-dependent RMF (i.e. Hartree) calculations.

It turned out that this was an effect of insufficient model space. Not only p-h but **excitations from the Dirac sea of anti-particles are needed** to achieve the (mathematical) completeness of the basis.

A fully consistent relativistic random phase approximation (RRPA) is investigated for giant resonances in finite nuclei. It can be concluded in the framework of the nucleon-meson field theory that a consistent treatment of RRPA built on the relativistic mean field(RMF) ground state has to include not only the positive energy particle-hole pairs, but also the pairs formed from the Dirac states and occupied Fermi states. A schematic model is introduced to understand the role of the Dirac state contributions from the pairs of Fermi to Dirac sea states. It is found that the main contribution of those states are through the exchange of the scalar meson, while the vector mesons play a negligible role. The static polarizability in isoscalar monopole modes is studied in a constrained RMF, which provides the energy inverse weighted sum rule. It is found that the sum rule is fulfilled only in the case where the Dirac state contributions are included.

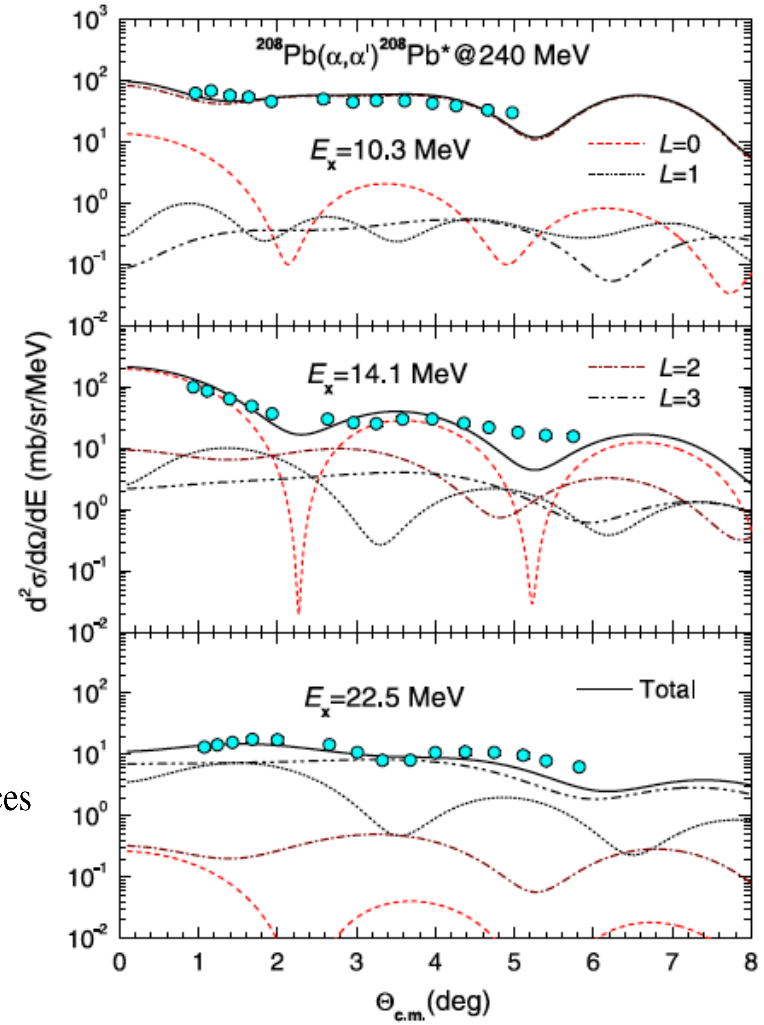


Z.Y. Ma *et al.*, NPA 687 (2001) 64c; P. Ring *et al.*, NPA 694 (2001) 249

Wave function and strength function

Using the RPA wave function, it is straightforward to calculate the **strength function associated with various operators** (backup slide about this).

- Using the RPA (but also other) wave functions **one can extract inelastic cross sections** using DWBA or CC
- The RPA+DWBA calculations can **reasonably reproduce the RCNP data** at E_α=240 MeV
- The strengths show **quantitative differences** from those "extracted" by the exp. analysis



Microscopic study of the isoscalar giant resonances in ^{208}Pb induced by inelastic α scattering

Do Cong Cuong [a], Dao T. Khoa [a,*], Gianluca Colò [b]

Nuclear Physics A 836 (2010) 11–42

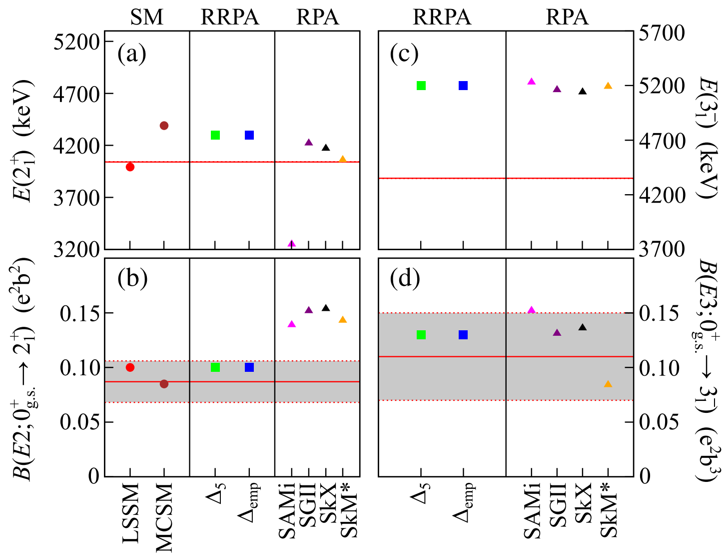
RPA and the nuclear Shell Model (SM)

PHYSICAL REVIEW LETTERS **121,** 252501 (2018)

Enhanced Quadrupole and Octupole Strength in Doubly Magic ^{132}Sn

D. Rosiak,[1] M. Seidlitz,[1,*] P. Reiter,[1] H. Naïdja,[2,3,4] Y. Tsunoda,[5] T. Togashi,[5] F. Nowacki,[2,3] T. Otsuka,[6,5,7,8,9] G. Colò,[10,11] K. Arnswald,[1] T. Berry,[12] A. Blazhev,[1] M. J. G. Borge,[13,†] J. Cederkäll,[14] D. M. Cox,[15,16] H. De Witte,[8] L. P. Gaffney,[13] C. Henrich,[17] R. Hirsch,[1] M. Huyse,[8] A. Illana,[8] K. Johnston,[13] L. Kaya,[1] Th. Kröll,[17] M. L. Lozano Benito,[13] J. Ojala,[15,16] J. Pakarinen,[15,16] M. Queiser,[1] G. Rainovski,[18] J. A. Rodriguez,[13] B. Siebeck,[1] E. Siesling,[13] J. Snäll,[14] P. Van Duppen,[8] A. Vogt,[1] M. von Schmid,[17] N. Warr,[1] F. Wenander,[13] and K. O. Zell[1]

(MINIBALL and HIE-ISOLDE Collaborations)

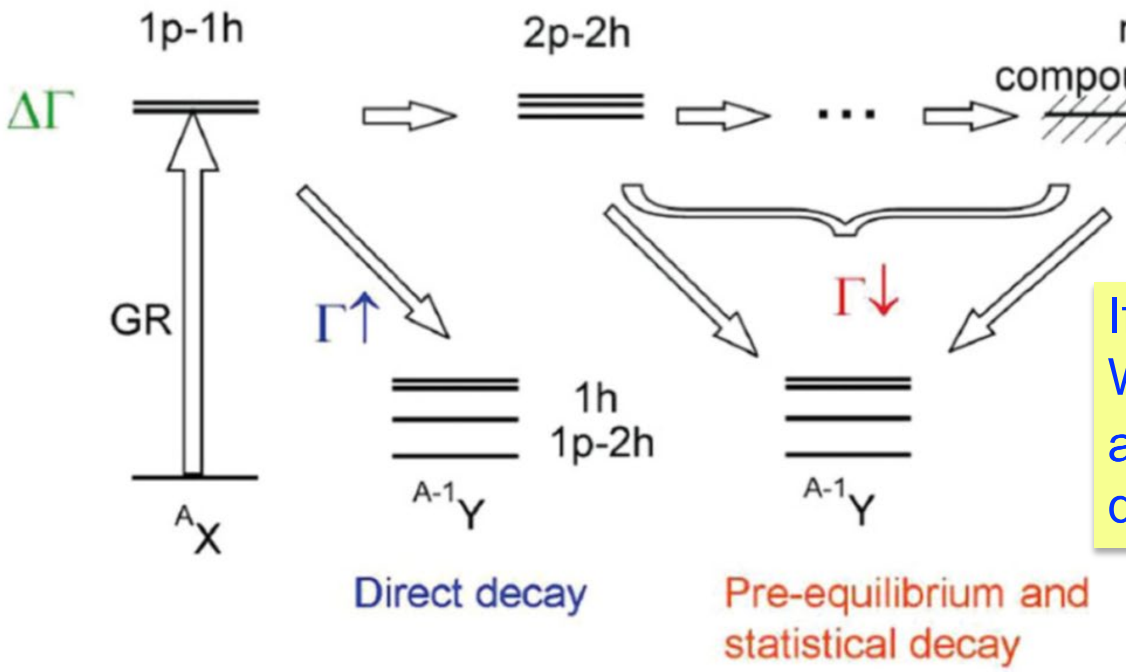


theory [11,12]. Because of the computational limits of the valence space, the SM approaches do not provide information on the 3_1^- state. The RPA and RRPA calculations

In addition, the shell model cannot provide response at high energy (cross sections for high-E neutrinos, just to make an example, are doable within RPA and QRPA but not SM).

Giant resonances are **NOT** sharp states but have a large width (several MeV)

$$\Gamma = \underbrace{\Gamma_{\text{Landau}} + \Gamma^{\downarrow}}_{\text{Internal mixing}} + \underbrace{\Gamma^{\uparrow} + \Gamma_{\gamma}}_{\text{Decay}}$$



If this picture clear in all details? We do not have systematic data and full understanding (cf. also β-delayed neutron emission).

This is a **challenge for theory** and may not simply call for parameter tuning but is related to fundamental questions (many-body theory, open quantum systems, the concept of thermalization...).

Ideally, we should perform calculations in large model spaces (not only 1p-1h but 2p-2h, 3p-3h...) and include continuum coupling.

All decay channels are correlated.

Continuum can be included at the level of RPA: continuum-RPA exists. This is possible due to Green’s function formalism (there is an exact expression for the single-particle propagator in the continuum, provided one can define an effective potential).

To describe the “**spreading width**”: SRPA, PVC are the main approaches.

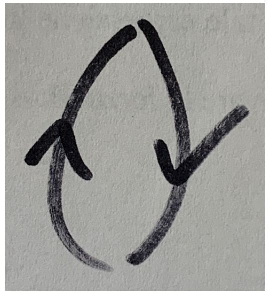
Advantages of SRPA: large model spaces, no approximations;
Advantages of PVC: more diagrams included, extended to the case with pairing.

How to derive the RPA equations (II)

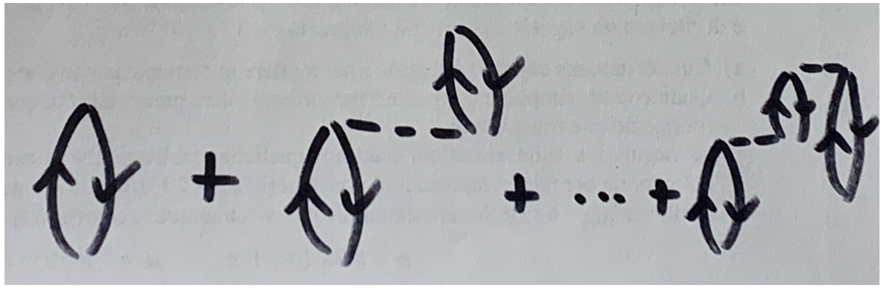
$G(r,t,r',t')$ The **Green's function**, represents the probability amplitude that a particle is found in r, at time t, after having been introduced in r' at time t'.

$G^0(r,t,r',t')$ HF or KS approximation.

$G(r,r',\omega)$ $G(k,\omega)$



$$\Pi^0(1,2) = -iG^0(1,2)G^0(2,1)$$



$$\Pi = \Pi^0 + \Pi^0 V \Pi^0 + \Pi^0 V \Pi^0 V \Pi^0 + \dots$$

Continuum can be included

$$\boxed{\Pi = \Pi^0 + \Pi^0 V \Pi}$$

Second RPA calculations

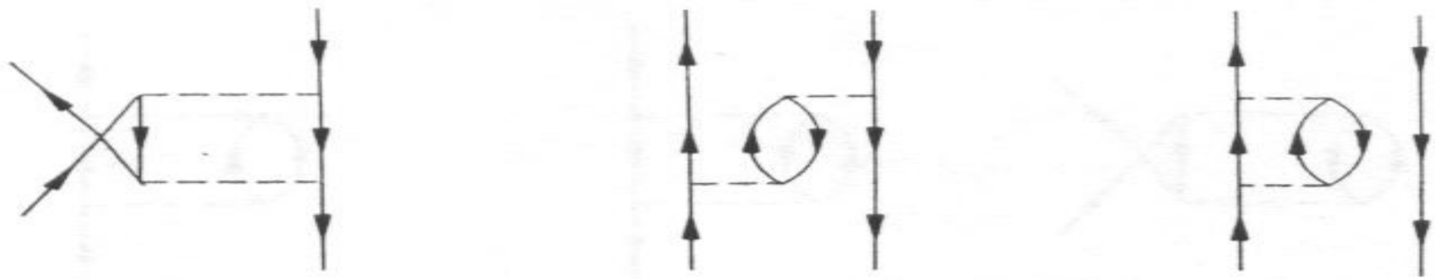
- The wave function of the vibrational states is enriched by adding 2 particle-2 hole components on top of the 1 particle-1 hole already present in RPA.

$$X_{\rm ph}|ph^{-1}\rangle - Y_{\rm ph}|hp^{-1}\rangle + X^{(2)}_{\rm php'h'}|ph^{-1}p'h'^{-1}\rangle - Y^{(2)}_{\rm php'h'}|hp^{-1}hp'^{-1}\rangle$$

- If one projects on the 1p-1h space, assuming the “complicated” states are not interacting, one gets a very manageable equation

$$\begin{pmatrix} A+\Sigma(E) & B \\ -B & -A-\Sigma^*(-E) \end{pmatrix} \qquad \Sigma_{\rm php'h'}(E) = \sum_\alpha \frac{\langle ph|V|\alpha\rangle\langle\alpha|V|p'h'\rangle}{E-E_\alpha+i\eta}$$

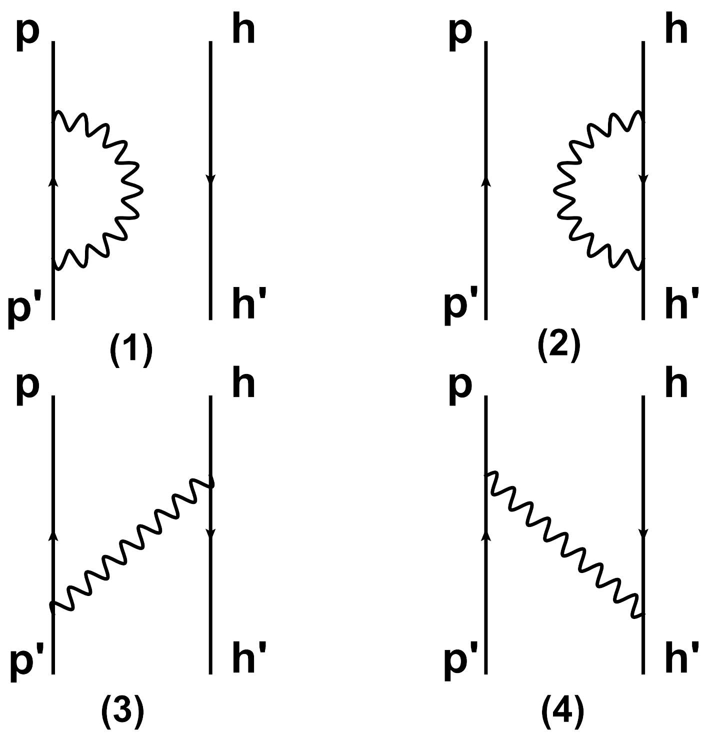
- Full calculations by D. Gambacurta *et al.* go beyond this approximation.



Particle-vibration coupling (PVC) calculations

$$\begin{pmatrix} A+\Sigma(E) & B \\ -B & -A-\Sigma^*(-E) \end{pmatrix} \qquad \Sigma_{\mathrm{php'h'}}(E) = \sum_\alpha \frac{\langle ph|V|\alpha\rangle\langle\alpha|V|p'h'\rangle}{E-E_\alpha+i\eta}$$

The state α is 1p-1h plus one phonon.



The scheme is very effective to produce GR widths. It also produces a downward shift of the GRs.

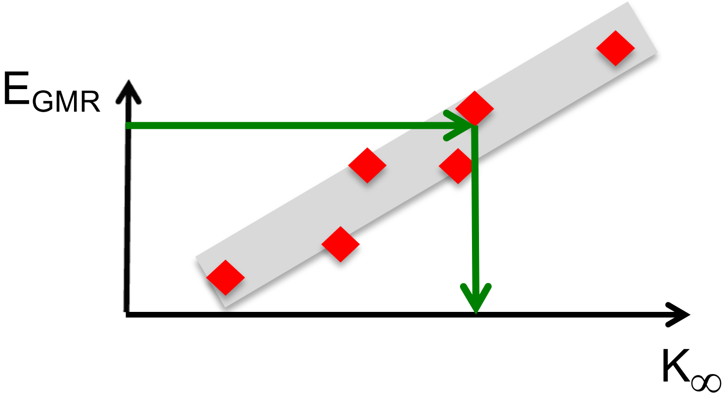
$$\Sigma(E) \approx \int dE' \, \frac{V^2}{E-E'+i\epsilon}$$

$$\frac{1}{E-E'+i\epsilon} \to \frac{1}{E-E'} - i\pi\delta(E-E')$$

WE HAVE A SCHEME INCLUDING PAIRING for all GRs

Why is tin so fluffy?

In even-even $^{112\text{-}124}$Sn, the ISGMR centroid energy is overestimated by about 1 MeV by the same models, which reproduce the ISGMR energy well in ^{208}Pb.



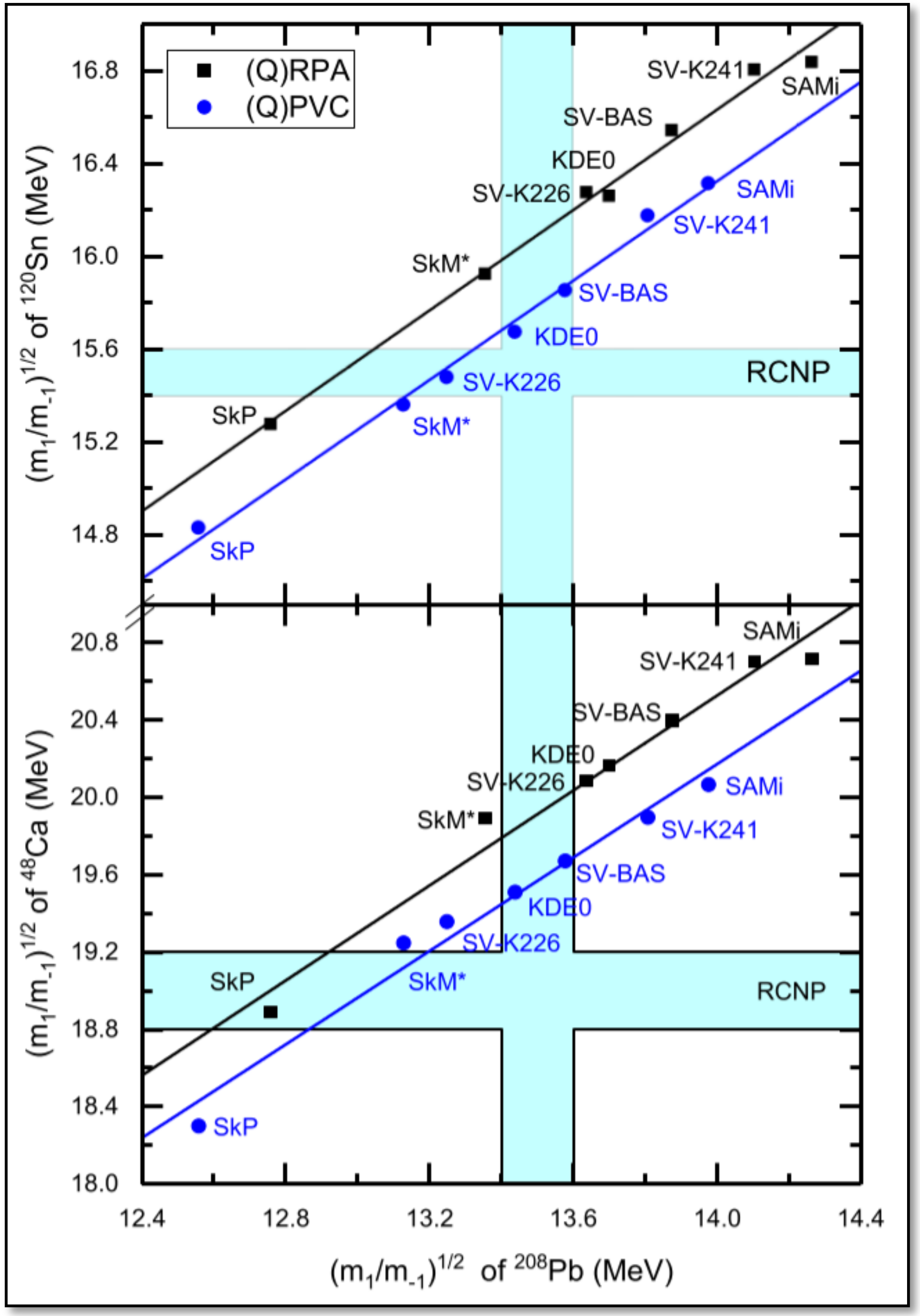
Only models that treat **uniform matter and the response of finite nuclei on equal footing** allow extracting K_∞

J.P. Blaizot, Phys. Rep. 64, 171 (1980)

There are different sources of model dependence in this procedure.

.

QRPA → QPVC



In our work, we have been able, for the first time, to analyse **in a systematic manner** the consistency between ISGMR energies in different nuclei.

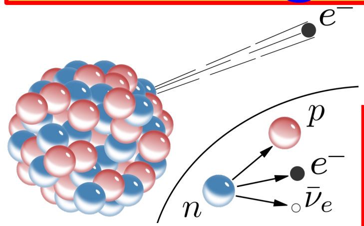
We have used many Skyrme EDFs.

With the inclusion of QPVC effects, a big improvement is achieved.

Within QPVC, the ISGMR energy in ^{208}Pb is consistent with ^{120}Sn.

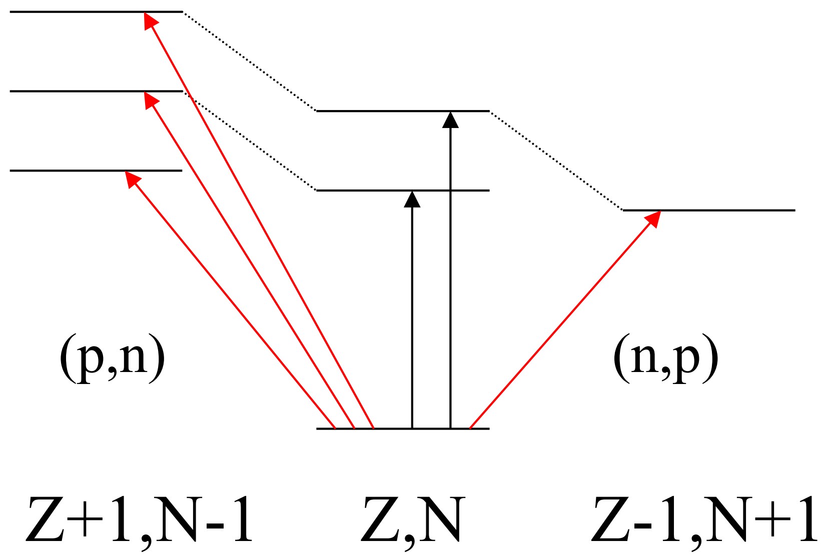
Z.Z. Li, Y.F. Niu, GC, Phys. Rev. Lett. 131, 082501 (2023)

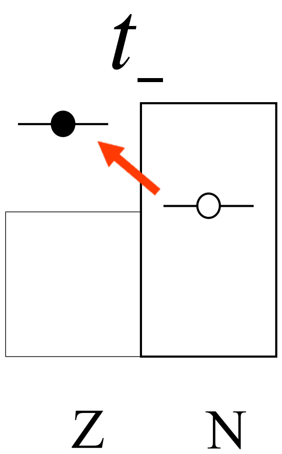
Charge-exchange transitions and β-decay



They are induced by reactions, like (p,n) or (^{3}He,t).

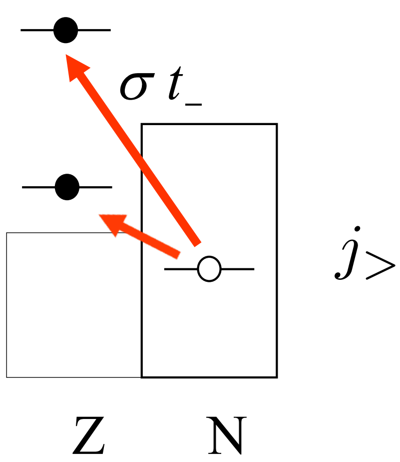
Some transitions may be inside the allowed β-decay window.





$$j_< = \ell - \frac{1}{2}$$

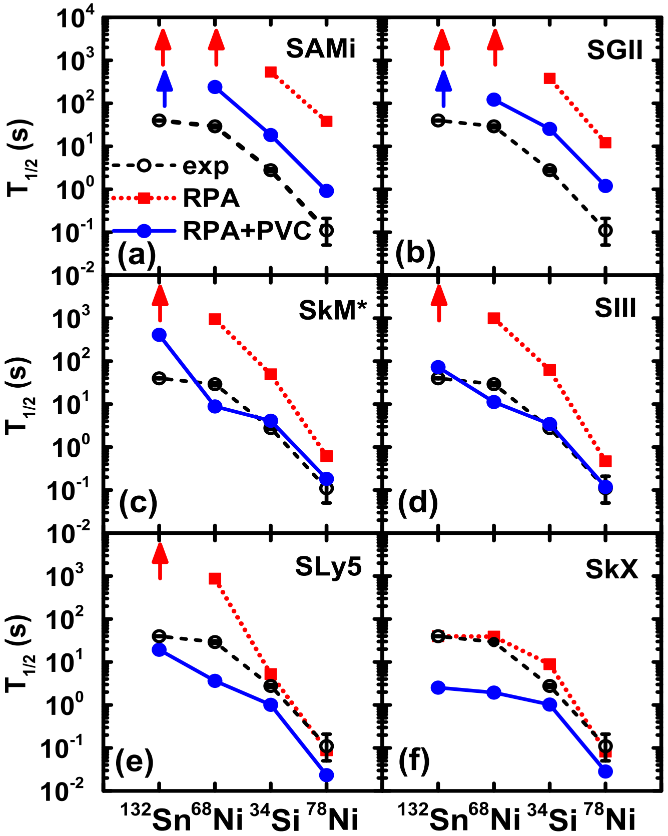
$$j_> = \ell + \frac{1}{2}$$



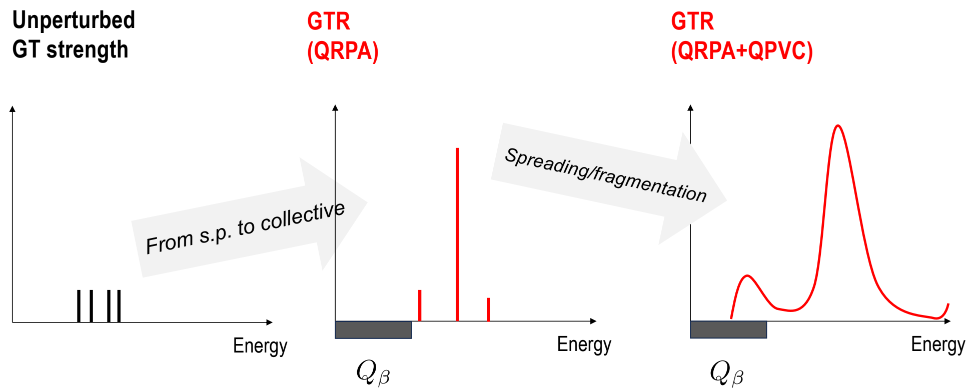
Isobaric Analog State:
n changed into p

Gamow-Teller Resonance:
also spin-flip

More applications of QPVC: β-decay



Y. F. Niu *et al.*, Phys. Rev. Lett. 114, 142501



While QRPA collects the simple two-quasiparticle excitation in a main peak, it does not account for spread and fragmentation of the strength. QPVC remedies to this shortcoming.

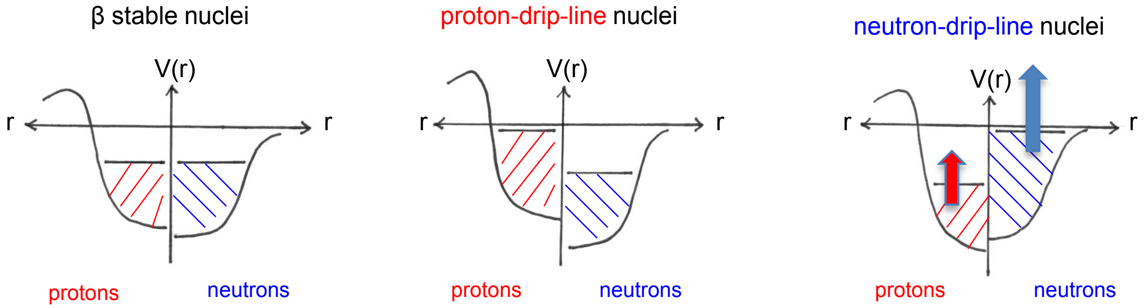
In the case of β-decay, this is particularly important because of the phase-space factor.

The width and decay channels of the GTR

Decay channel	Theory			Experiment
	Only $W^\uparrow$	$W^\uparrow + W^\downarrow$		
		(a)	(b)	[4]
$p_{1/2}$	0.223	0.033	0.018	0.013±0.002
$p_{3/2}$	0.418	0.035	0.019	0.023±0.003
$i_{13/2}$	0.014	0.003	0.001	0.002±0.002
$f_{5/2}$	0.319	0.013	0.007	included in $p_{3/2}$
$f_{7/2}$	0.016	0.010	0.003	0.003±0.002
$h_{9/2}$	0.010	0.001	$< 10^{-3}$	
$\sum_c B_c$	1.0	0.095	0.048	0.041±0.009

One of the few experiments that measured the decay of a resonance to different hole states of the A-1 nucleus (in this case ^{208}Bi decaying by proton emission)

Neutron-rich and neutron-deficient nuclei



If the neutron number increases, neutrons occupies higher levels – protons become more bound due to the dominance of the p-n interaction.

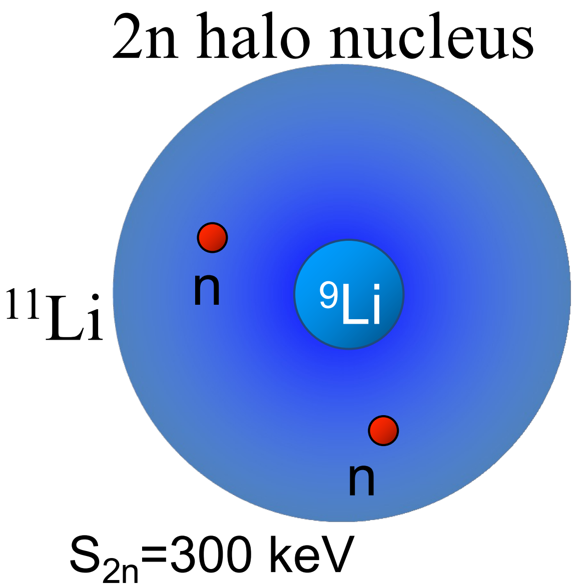
If neutrons occupy levels that are close to the continuum and protons are in deeply bound levels, **the neutron and proton excitations may decouple from each other.**

Very weakly bound neutrons can have wave functions with long tail that overlap with continuum states.

The corresponding transition matrix elements, i.e. the low-lying strength, is strongly enhanced.

Threshold effect

Threshold effect in the dipole strength of ^{11}Li



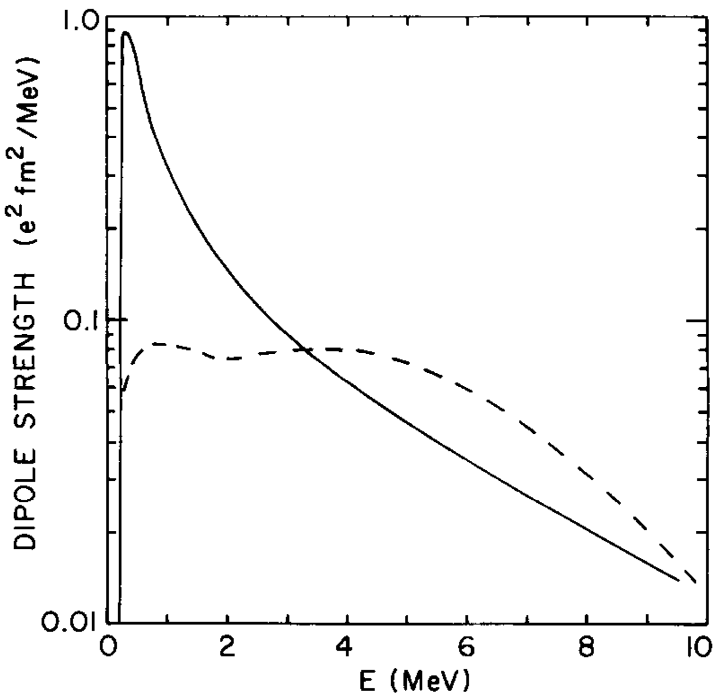


FIG. 10. Dipole strength of the valence neutrons in ^{11}Li as a function of the excitation energy. Both curves are independent particle responses, obtained as described in the text for a radial cutoff at 12.5 fm (dashed curve) and at 40 fm (fully drawn curve).

“Pygmy” dipole resonance (?)

Progress in Particle and Nuclear Physics 70 (2013) 210–245

Contents lists available at SciVerse ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

Experimental studies of the Pygmy Dipole Resonance

D. Savran [a,b,*], T. Aumann [c,d], A. Zilges [e]

Progress in Particle and Nuclear Physics 129 (2023) 104006

Contents lists available at ScienceDirect

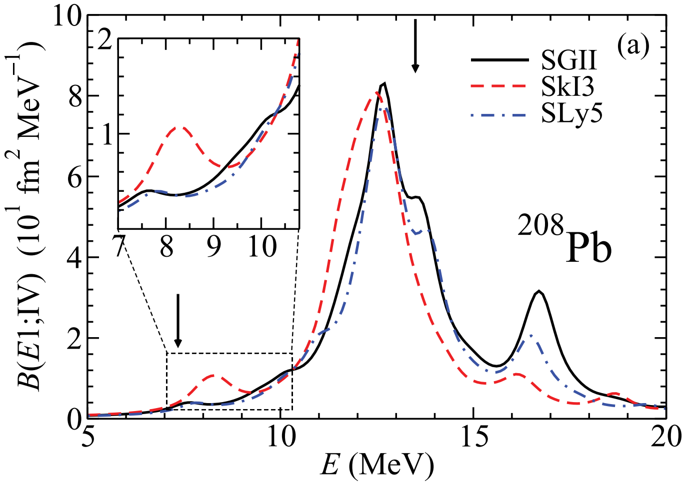
Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

Theoretical studies of Pygmy Resonances

E.G. Lanza [a,b,*], L. Pellegri [c,d], A. Vitturi [e,f], M.V. Andrés [g]



There have been strong discussions if it really exists.

“Resonance” or SPS?

Impact for (n,γ)?

The nuclear EoS and the symmetry energy

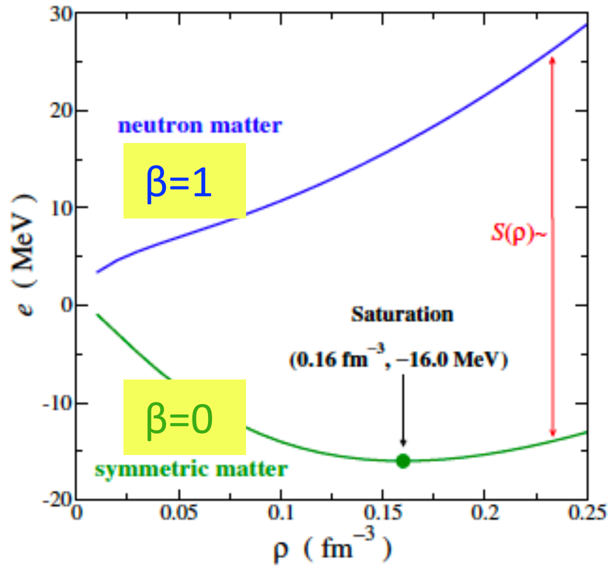
Nuclear matter EOS

Symmetric matter EOS

Symmetry energy S

$$\frac{E}{A}(\rho,\beta) = \frac{E}{A}(\rho,\beta = 0) + S(\rho)\beta^2$$

$$\beta \equiv \frac{\rho_n - \rho_p}{\rho}$$



$$S \equiv \frac{E}{A}(\text{neutron matter}) - \frac{E}{A}(\text{symmetric matter})$$

In turn, the symmetry energy can be expanded around saturation density.

$$S(\rho_0) \equiv J$$
$$S'(\rho_0) \equiv L/3\rho_0$$
$$S''(\rho_0) \equiv K_{\text{sym}}/9\rho_0^2$$

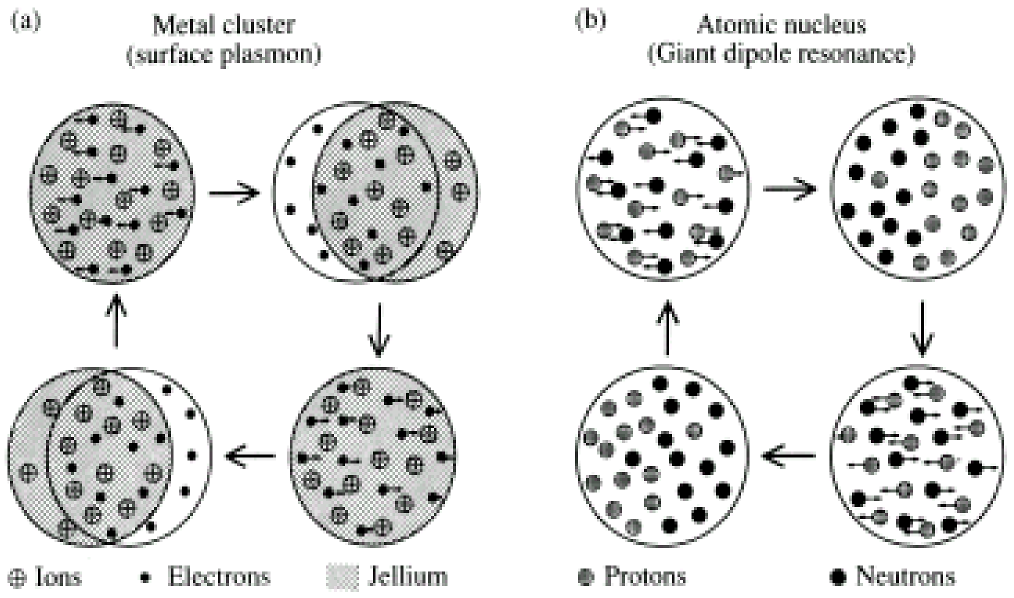
M. Oertel *et al.*, RMP **89**, 015007 (2017); B.A. Li et al., Prog. Part. Nucl. Phys. **99**, 29 (2018)

Symmetry energy from IV vibrations

Neutrons and protons oscillate in opposition of phase.

$$E_{\mathrm{IVGR}} \approx \sqrt{\frac{\partial^2 E}{\partial \beta^2}} \approx \sqrt{S(\rho)}$$

$$\beta \equiv \frac{\rho_n - \rho_p}{\rho}$$



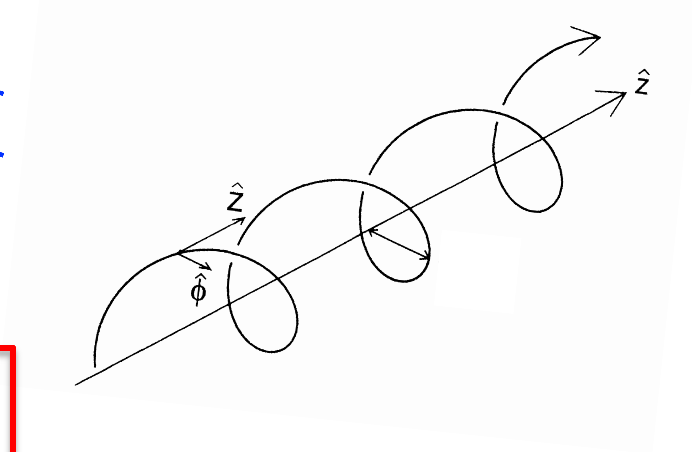
Promising observables to extract the properties of the symmetry energy.

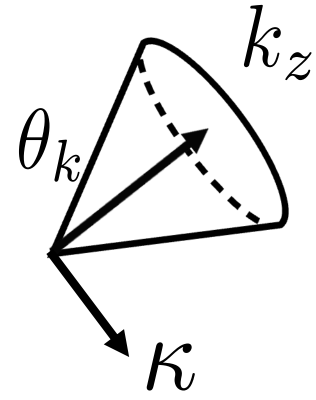
Problems:
the nucleus is not a homogeneous system, it has a shell structure, and there is isoscalar/isovector mixing.

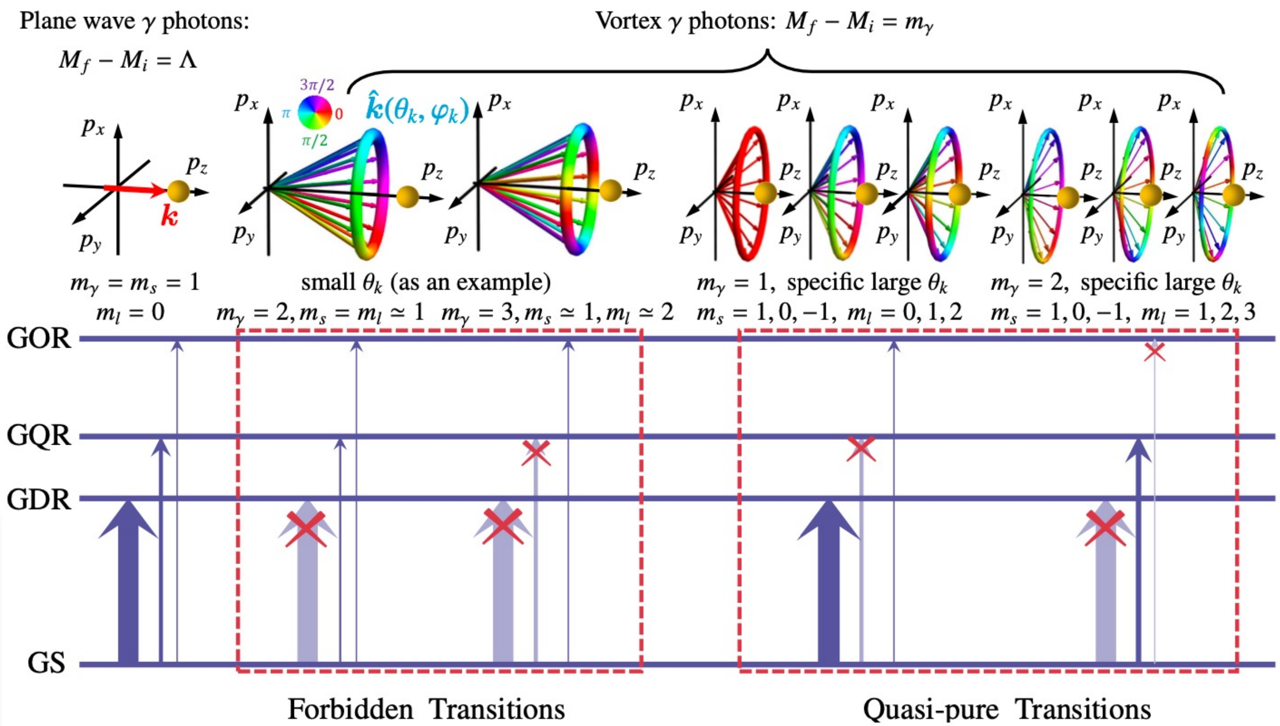
Vortex photons

Use of photons with orbital angular momentum can open new avenues for the physics of collective states.

$$\boxed{M_f - M_i = m_\gamma = m_\ell + m_s}$$







Z.W. Lu *et al.*, Phys. Rev. Lett. 131, 202502 (2023)

Conclusions

Nuclear collective motion has many **different facets**. The hydrodynamic, simple picture may not be always valid (there are spin- and spin-isospin oscillations, surface vs volume oscillations, collective vs non-collective – whatever this means...)

The **decay** is an intriguing feature: direct or statistical?

Many questions have not been addressed: does toroidal motion exist? Do “pair” modes exist and can we describe then? Finite-temperature and Brink-Axel hypothesis: can we say it is all understood? Is there a unique theory for small- and large-amplitude motion? ...

Thank you for listening!

Backup slides

Strength function and operators

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \, \delta(E - E_n)$$

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \frac{\Gamma_n}{(E - E_n)^2 + \frac{\Gamma_n^2}{4}}$$

$$F_{\mathrm{IS}} = \sum_i r_i^L Y_{LM}(\hat{r}_i),$$

$$F_{\mathrm{IV}} = \sum_i r_i^L Y_{LM}(\hat{r}_i)\tau_z(i).$$

$$F_{\mathrm{IS}} = \sum_i r_i^L \left[Y_{LM}(\hat{r}_i) \otimes \sigma(i)\right]_J,$$

$$F_{\mathrm{IV}} = \sum_i r_i^L \left[Y_{LM}(\hat{r}_i) \otimes \sigma(i)\right]_J \tau_z(i).$$

$$F_{\mathrm{ISGMR}} = \sum_i r_i^2,$$

$$F_{\mathrm{IVGMR}} = \sum_i r_i^2 \tau_z(i).$$

$$F_{\mathrm{IVGDR}} = \frac{eN}{A}\sum_{i=1}^{Z} r_i Y_{1M}(\hat{r}_i) - \frac{eZ}{A}\sum_{i=1}^{N} r_i Y_{1M}(\hat{r}_i).$$

RPA and collectivity: schematic model (I)

Schematic 2 x 2 case

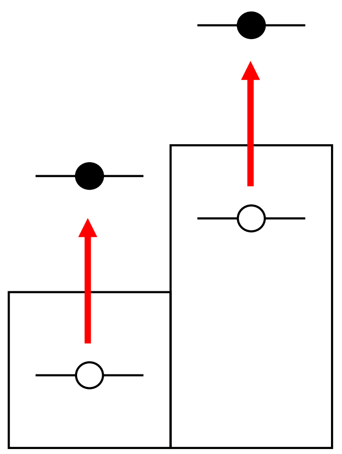
$$\left(\begin{array}{cc} \varepsilon + v & v \\ v & \varepsilon + v \end{array} \right)$$

$$\hbar\omega_1 = \varepsilon, \qquad X^{(1)} = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ -1 \end{array} \right),$$

$$\hbar\omega_2 = \varepsilon + v \qquad X^{(2)} = \frac{1}{\sqrt{2}} \left(\begin{array}{c} 1 \\ 1 \end{array} \right).$$

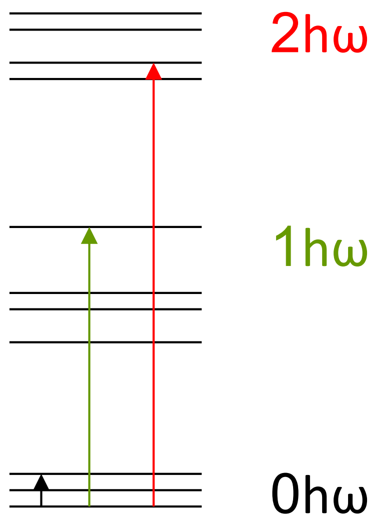
Magnetic spin-flip states (M1)

^{208}Pb : $h_{11/2} \rightarrow h_{9/2}$ (proton)
$i_{13/2} \rightarrow i_{11/2}$ (neutron)



RPA and collectivity: schematic model (II)

Schematic *N* x *N* case

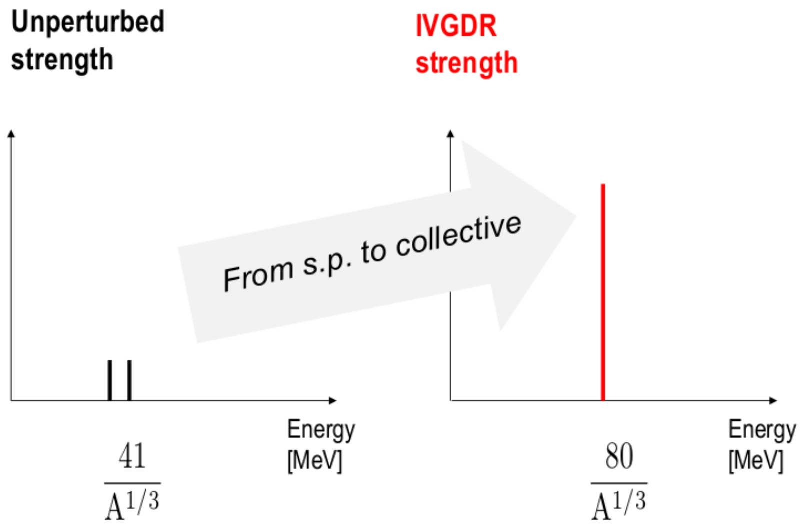


There is one "coherent state":

$$\frac{1}{\sqrt{N}}\begin{pmatrix} 1 \\ 1 \\ \cdots \\ 1 \end{pmatrix}$$

Its transition amplitude is enhanced:

$$\langle n|F|0\rangle = \sum_{ph} X_{ph}\langle p|F|h\rangle \approx N\frac{1}{\sqrt{N}}M = \sqrt{N}\ M$$



G. C., *Theoretical Methods for Giant Resonances*, in: Handbook of Nuclear Physics, edited by I. Tanihata, H. Toki and T. Kajino (Springer, 2022).

How to derive the RPA equations (III)

Equation of motion method

We define a “phonon” as a boson operator made with pairs of fermions

$$\Gamma_n^\dagger = \sum_{ph} X_{ph} a_p^\dagger a_h - Y_{ph} a_h^\dagger a_p$$

The equation of motion to solve is

$$\left[H, \Gamma^\dagger\right] = \hbar\omega\Gamma^\dagger$$

Additional requirements are the quasi-boson approximation and the definition of the ground-state as “phonon vacuum”

$$\left[\Gamma, \Gamma^\dagger\right] = 1$$

$$\Gamma_n |RPA\rangle = 0$$