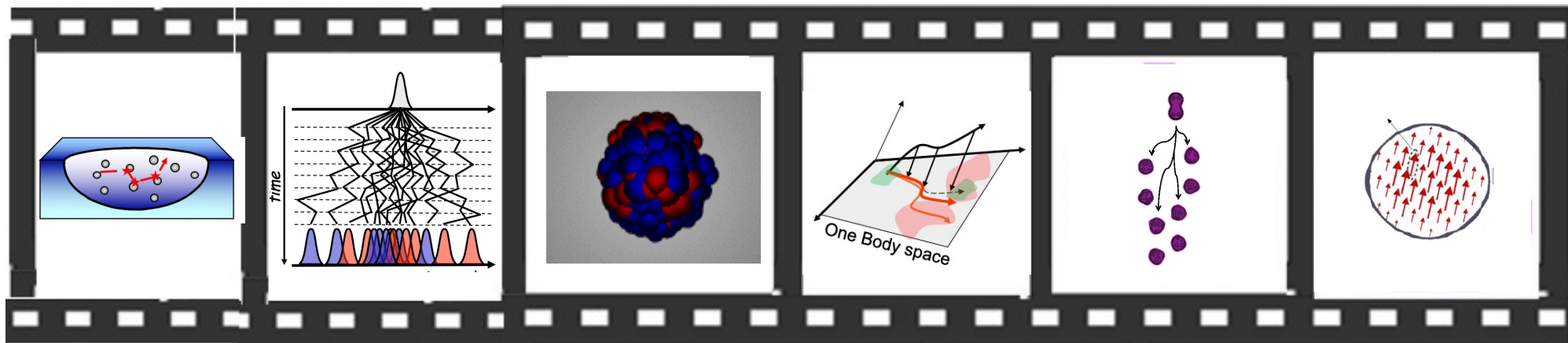


# *Time-dependent* microscopic description of collective nuclear excitations

Denis Lacroix (IJCLab, Orsay, France)

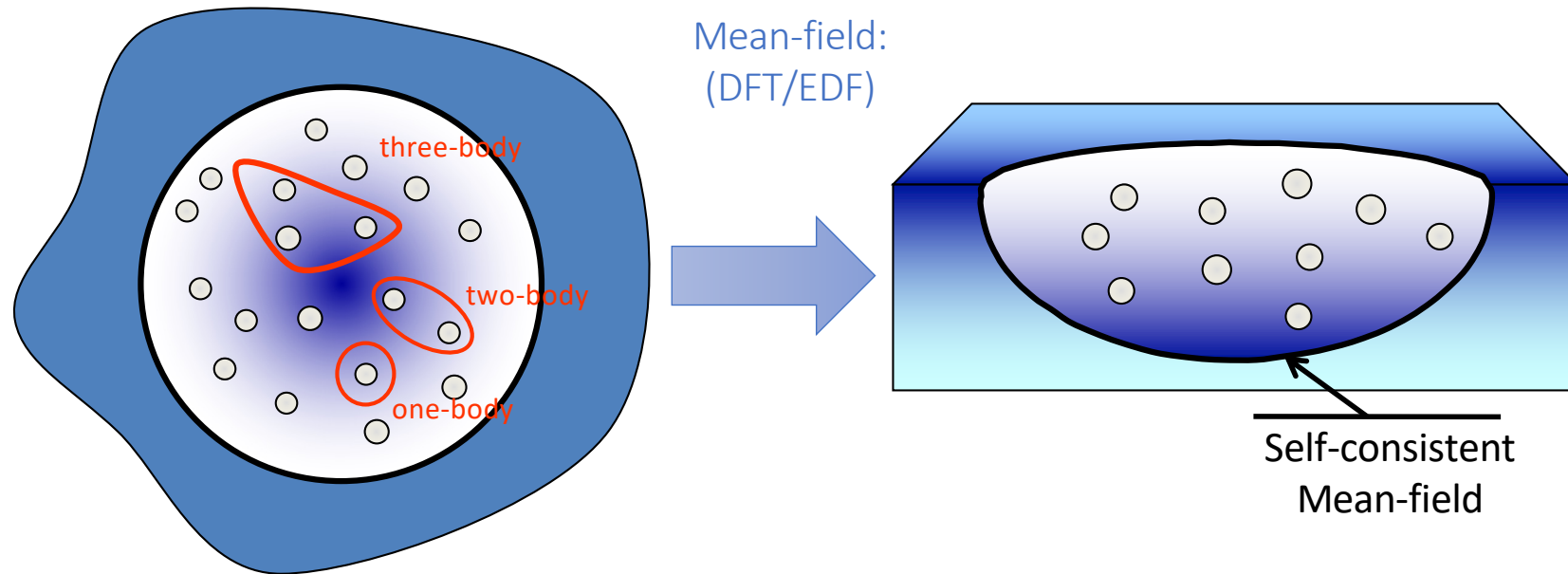


# The Nuclear Energy Density Functional: Goal

Starting point:

$$H = \sum_i T(i) + \sum_{i<j} V^{(2)}(i,j) + \sum_{i<j<k} V^{(3)}(i,j,k)$$

Map the nuclear many-body problem into an “independent” (quasi-)particle problem



“Simple” Trial state:

$$\Psi(r_1, \dots, r_{12}, \dots, r_{123}, \dots)$$

In the EDF, particles behaves as Independent particles interacting through an effective average potential

Strategy

- ➡ Identify relevant degrees of freedom (one-body DOF)
- ➡ Use appropriate trial states in the variational principle (Slater Det. wave-function)

# Hartree-Fock (HF) and time-dependent Hartree-Fock (TDHF) theory

From variational principle

$$S = \int_{t_0}^{t_1} ds \langle \Psi(t) | i\hbar \partial_t - H | \Psi(t) \rangle \Rightarrow S = \int_{t_0}^{t_1} dt \sum_{\alpha} \int_{\mathbf{r}} d^3\mathbf{r} \left\{ i\hbar \phi_{\alpha}^*(i) \partial_t \phi_{\alpha}(i) - \mathcal{H}(\phi_{\alpha}, \phi_{\alpha}^*) \right\}$$

For two-body hamiltonian

$$\mathcal{H} = \sum_{ij\alpha} t_{ij} \phi_{\alpha}^*(i) \phi_{\alpha}(j) + \frac{1}{2} \sum_{ijkl\alpha\beta} \tilde{v}_{ij,kl} \phi_{\alpha}^*(i) \phi_{\beta}^*(j) \phi_{\alpha}(k) \phi_{\beta}(l)$$

Mean-field equation of motion (in r-space)

$$\Rightarrow i\hbar \partial_t \phi_{\alpha}(\mathbf{r}) = -\frac{\hbar^2}{2m} \Delta \phi_{\alpha}(\mathbf{r}) + U_{\text{H}}(\mathbf{r}) \phi_{\alpha}(\mathbf{r}) + \int d\mathbf{r}' U_{\text{ex}}(\mathbf{r}, \mathbf{r}') \phi_{\alpha}(\mathbf{r}')$$

$$\text{Direct term } U_{\text{H}}(\mathbf{r}) = \int d\mathbf{r}' v(\mathbf{r} - \mathbf{r}') \rho(\mathbf{r}', \mathbf{r}')$$

$$\text{Exchange term } U_{\text{ex}}(\mathbf{r}, \mathbf{r}') = -v(\mathbf{r} - \mathbf{r}') \rho(\mathbf{r}, \mathbf{r}')$$

From Ehrenfest.

$$i\hbar \frac{d\langle A_{\alpha} \rangle}{dt} = \langle [A_{\alpha}, H] \rangle \Rightarrow i\hbar \frac{d}{dt} \langle a_i^{\dagger} a_j \rangle = \langle [a_i^{\dagger} a_j, H] \rangle \Rightarrow i\hbar \partial_t \rho = [h_{\text{MF}}[\rho], \rho]$$

## Many-Body states

Slater determinant

$$|\Phi_0\rangle = \prod_{i=1}^N a_i^\dagger |0\rangle$$

Quasi-particle vacuum

$$|\Phi_0\rangle = \mathcal{C}_0 \prod_v \alpha_v |0\rangle$$

$$\alpha_v^\dagger = \sum_i (U_{iv}^0 a_i^\dagger + V_{iv}^0 a_i)$$

- Wick theorem still apply
- Particle number is broken – U(1) sym.

## Observables

One-body density

$$\rho_{ij} = \langle \Phi_0 | a_j^\dagger a_i | \Phi_0 \rangle$$

One-body density and anomalous density

$$\rho_{ij} = \langle \Phi_0 | a_j^\dagger a_i | \Phi_0 \rangle$$

$$\kappa_{ij} = \langle \Phi_0 | a_j a_i | \Phi_0 \rangle \quad \kappa_{ij}^* = \langle \Phi_0 | a_i^\dagger a_j^\dagger | \Phi_0 \rangle$$

Correlations  $\propto \kappa \kappa^*$

## Density evolution

TDHF

$$i\hbar \frac{d\rho}{dt} = [h(\rho), \rho]$$

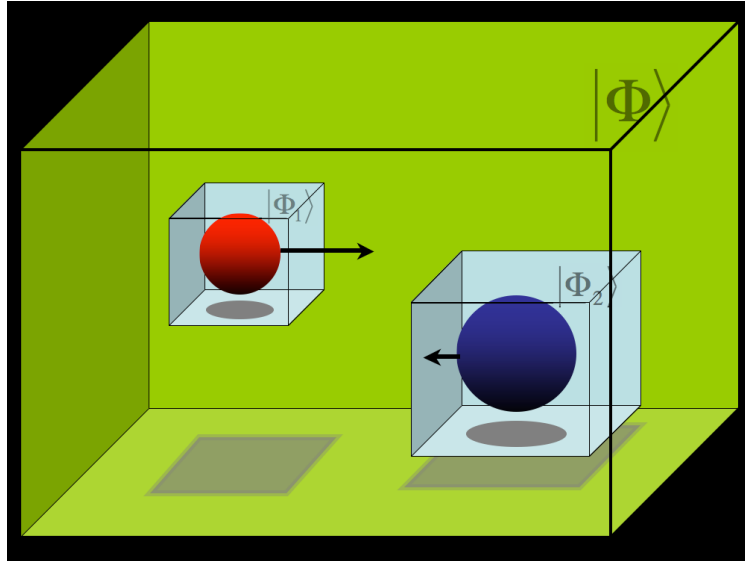
TDHFB

$$i\hbar \frac{d}{dt} \mathcal{R} = [\mathcal{H}(\mathcal{R}), \mathcal{R}]$$

$$\mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & 1 - \rho \end{pmatrix}$$

# Pairing: from independent particles to independent quasi-particles picture

Nuclear reaction with normal/superfluid nuclei on a mesh



TDHF is a standard tool  $|\Phi_i\rangle$  : Slater

$$i\hbar \frac{d\rho}{dt} = [h(\rho), \rho] \quad \longrightarrow \quad \text{Single-particle evolution}$$

Simenel, Lacroix, Avez, arXiv:0806.2714v2

Introduction of pairing: TDHFB

$$i\hbar \frac{d}{dt} \mathcal{R} = [\mathcal{H}(\mathcal{R}), \mathcal{R}] \quad \mathcal{R} = \begin{pmatrix} \rho & \kappa \\ -\kappa^* & 1 - \rho \end{pmatrix}$$

$\longrightarrow$  Quasi-particle evolution

(Active Groups: France, US, Japan, England...)

$$\text{TDHFB} = 1000 * (\text{TDHF})$$

$\longrightarrow$  Full TDHFB (Skyrme-spherical symmetry)

Avez, Simenel, Chomaz, PRC 78 (2008).

Full TDHFB (Skyrme-symmetry unrestricted )  
(Gogny-axial symmetry)

Stetcu, Bulgac, Magierski, and Roche, PRC 84 (2011)

Hashimoto, PRC 88 (2013).

$\longrightarrow$  Symmetry unrestricted TDBCS limit of TDHFB (also called Canonical basis TDHFB)

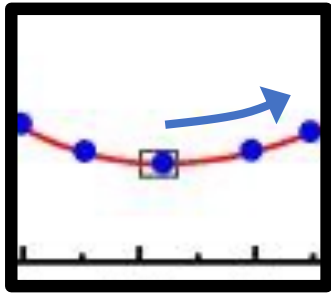
Neglect  $\Delta_{ij} \longrightarrow |\Phi(t)\rangle = \prod_{k>0} \left( u_k(t) + v_k(t) a_k^\dagger(t) a_{\bar{k}}^\dagger(t) \right) |-\rangle.$

$$\text{TDBCS} = 2-3 * (\text{TDHF})$$

Ebata, Nakatsukasa et al, PRC82 (2010)  
Scamps, Lacroix, PRC88 (2013).

$\longrightarrow$  Very good predictive power

New comers in the last 10 years: China (Relativistic and non-relativistic version), Croatia



First step towards non equilibrium

Constrained calculations

Nuclei at various shapes

$$\delta\langle\Psi|H - \lambda Q - E|\Psi\rangle = 0$$

Thermodynamics of nuclei

$$\delta\langle\Psi|H - TS - \mu N|\Psi\rangle = 0$$

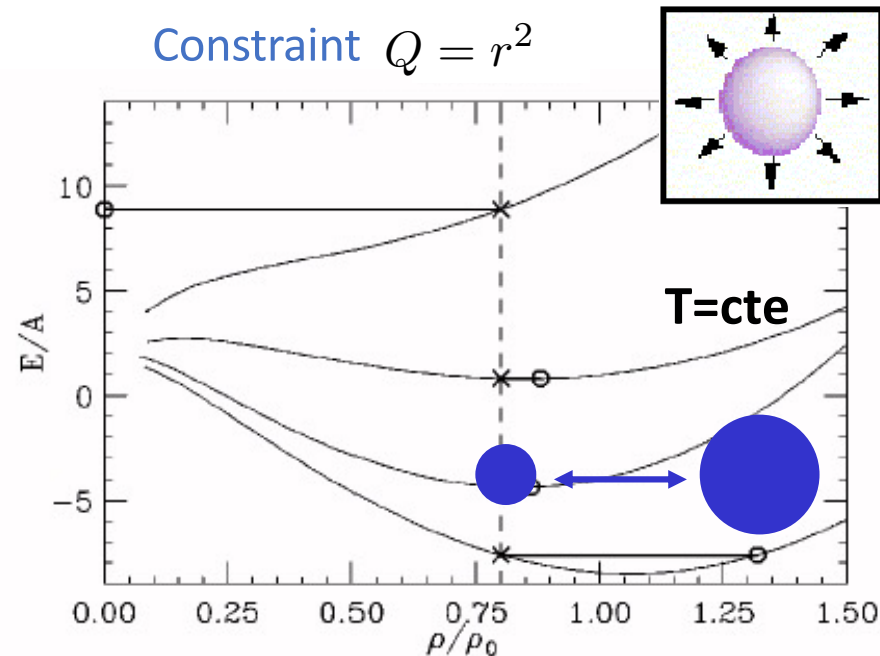
with  $S = -\text{Tr}(D \ln D)$

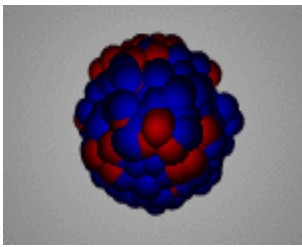
Here  $\rho = \sum |i\rangle n_i \langle i|$   $S[n_i] = - \sum_i [n_i \log(n_i) + (1 - n_i) \log(1 - n_i)]$

$\Rightarrow n_i = 1/(1 + \exp\{(\varepsilon_i - \mu)/T\})$

Monopole vibration

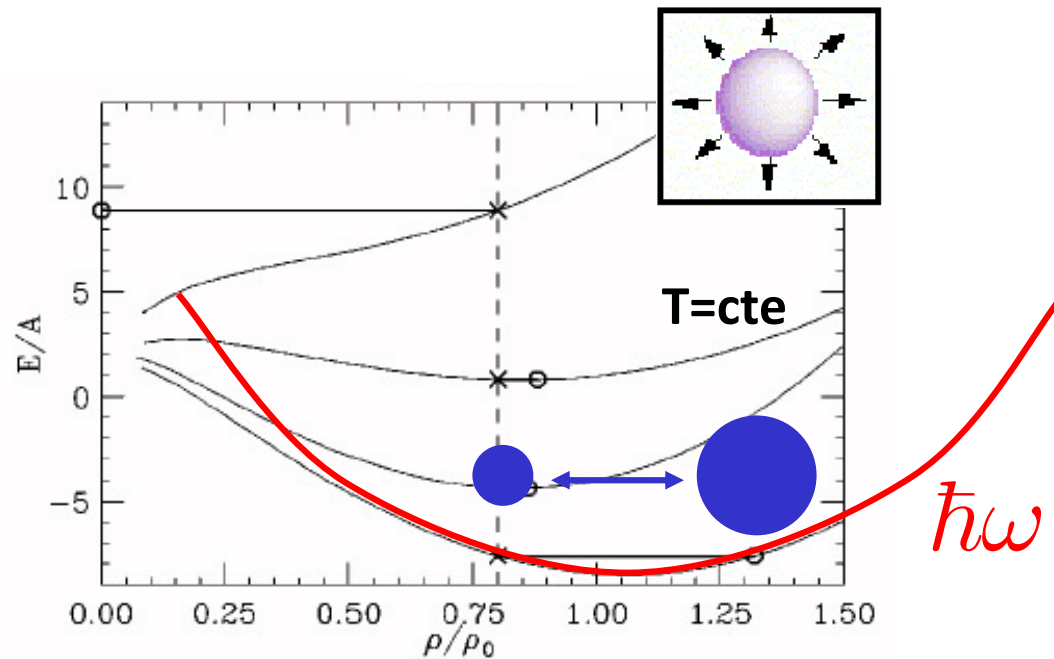
Constraint  $Q = r^2$



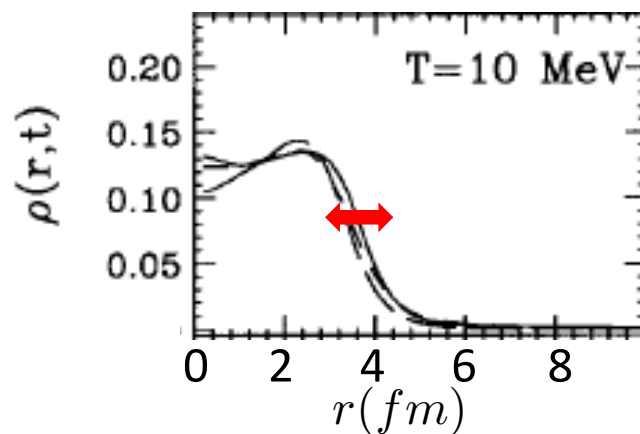


## Collective motion

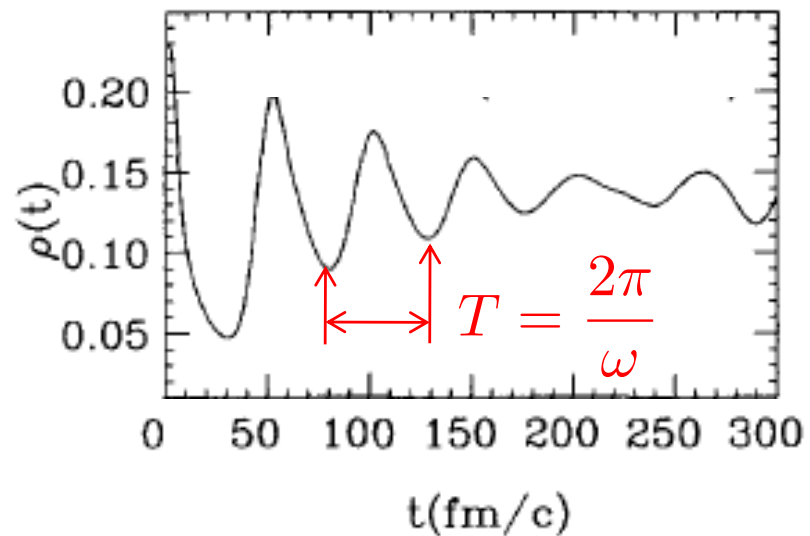
### Constrained mean-field versus dynamics



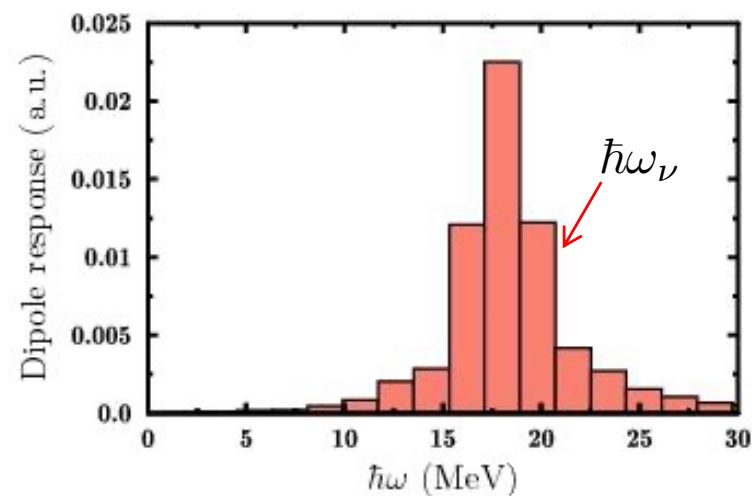
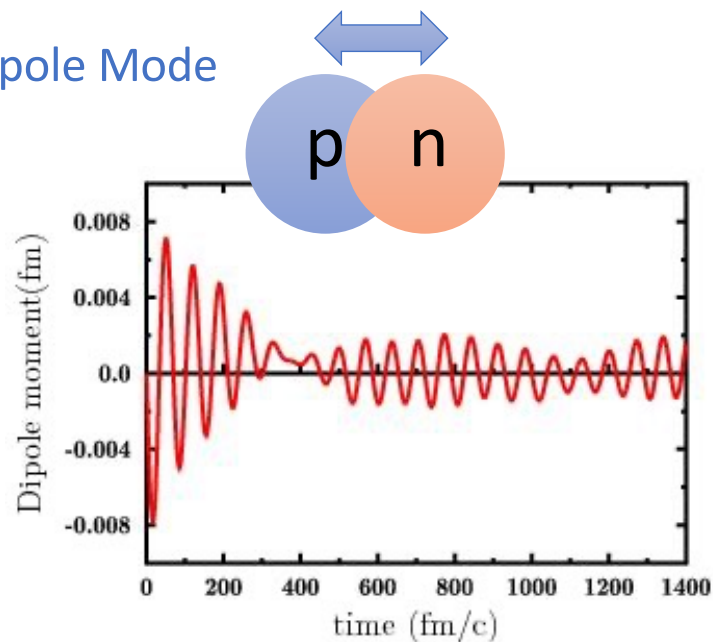
### Time evolution



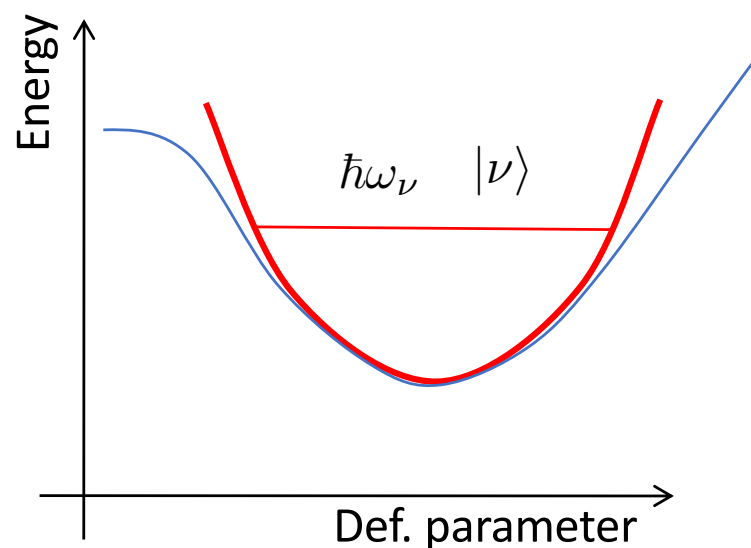
Lacroix, Chomaz, Nucl. Phys. A636 (1998)



## Dipole Mode

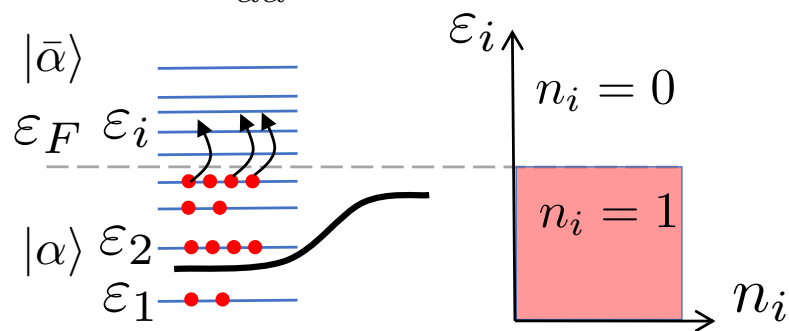


## Nature of the collective states



## Particle-hole decomposition

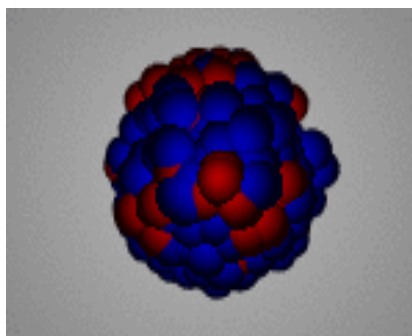
$$|\nu\rangle \propto \sum_{\alpha\bar{\alpha}} X_{\bar{\alpha}\alpha}^\nu a_{\bar{\alpha}}^\dagger a_\alpha |\Psi_0\rangle$$



➡ Normally identical to (Q)RPA [but evaporation]

## An illustration of a recent application: Pairing effect on nuclear collective motion

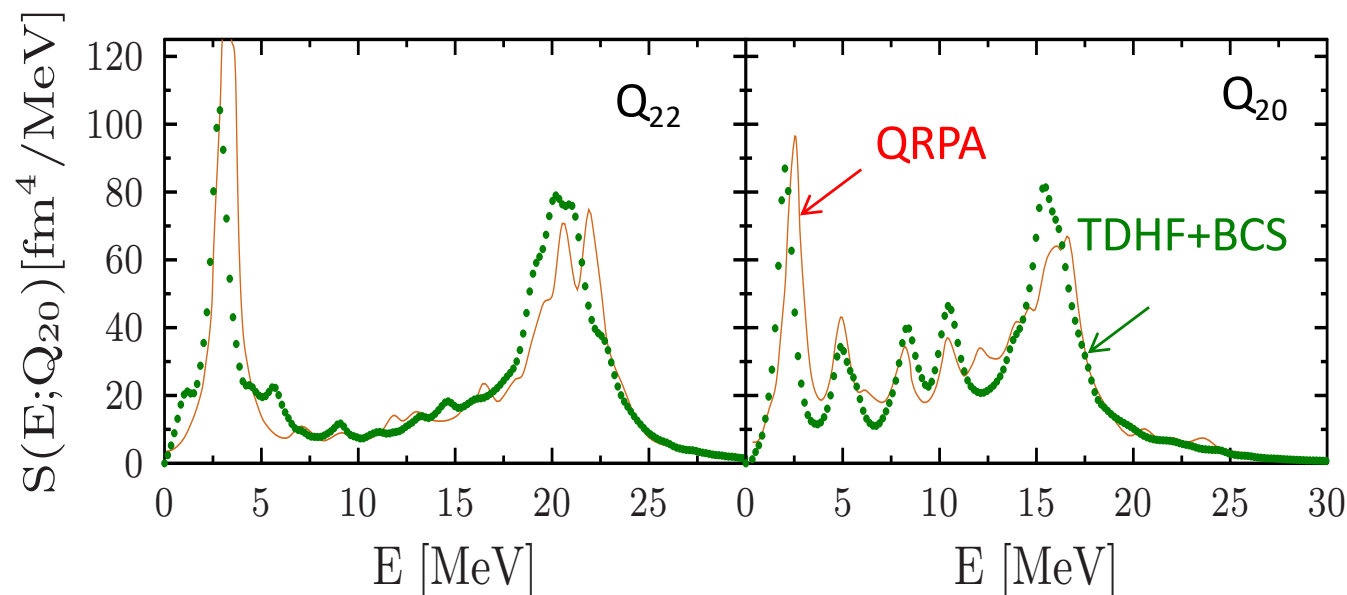
Illustration with the GQR



Comparison TDHF+BCS / QRPA

- ➔ Inclusion of pairing provides realistic ground state deformation
- ➔ It allows the description of mid-shell nuclei
- ➔ It includes partially correlation effects

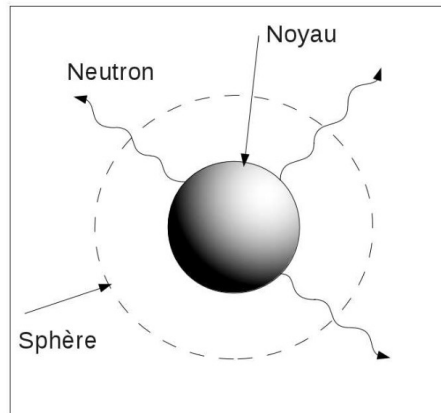
Strength distribution in deformed  $^{34}\text{Mg}$



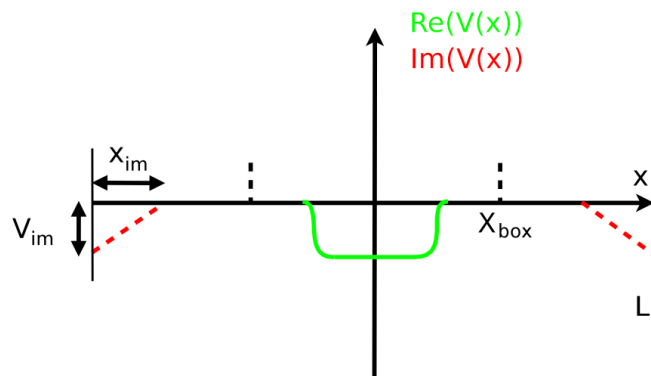
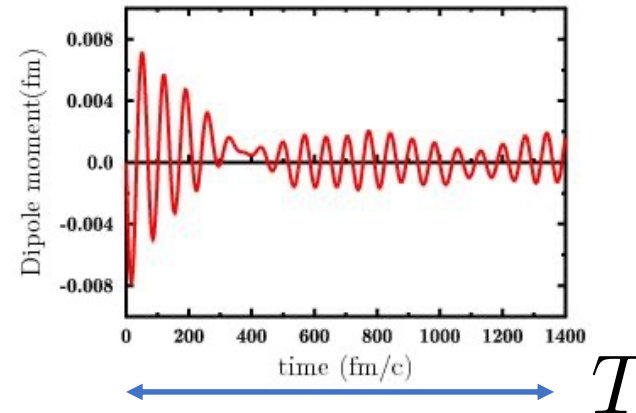
QRPA: C. Losa, et al PRC 81, (2010).

- ➔ Almost no difference between TDHF+BCS and TDHFB (QRPA)
- ➔ Main effect of pairing is to set the deformation

## Particle evaporation



## Finite time evolution



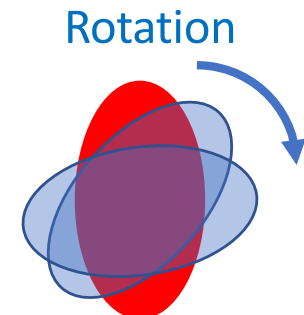
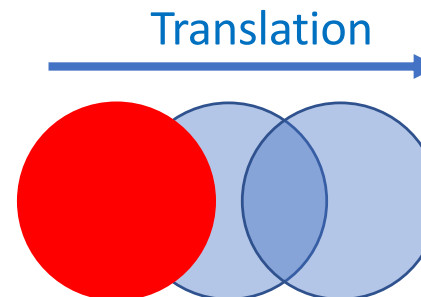
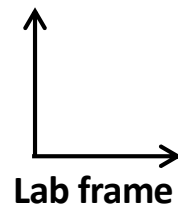
$$S_Q(\omega) = -\frac{1}{\pi\lambda\hbar} \text{Im} \int_0^T e^{-\frac{\Gamma_C}{2\hbar}t} [Q(t) - Q(0)] \sin(\omega t) dt$$

$$= \frac{1}{\pi} \sum_v |\langle \Psi_v | \hat{Q} | \Psi_0 \rangle|^2 \left\{ \frac{-\frac{\Gamma_C}{2} \left[ e^{-\frac{\Gamma_C T}{2\hbar}} \cos\left(\frac{(E-E_v)T}{\hbar}\right) - 1 \right] + (E-E_v) e^{-\frac{\Gamma_C T}{2\hbar}} \sin\left(\frac{(E-E_v)T}{\hbar}\right)}{(\frac{\Gamma_C}{2})^2 + (E-E_v)^2} \right.$$

$$\left. - \frac{-\frac{\Gamma_C}{2} \left[ e^{-\frac{\Gamma_C T}{2\hbar}} \cos\left(\frac{(E+E_v)T}{2\hbar}\right) - 1 \right] + (E+E_v) e^{-\frac{\Gamma_C T}{2\hbar}} \sin\left(\frac{(E+E_v)T}{\hbar}\right)}{(\frac{\Gamma_C}{2})^2 + (E+E_v)^2} \right\}.$$

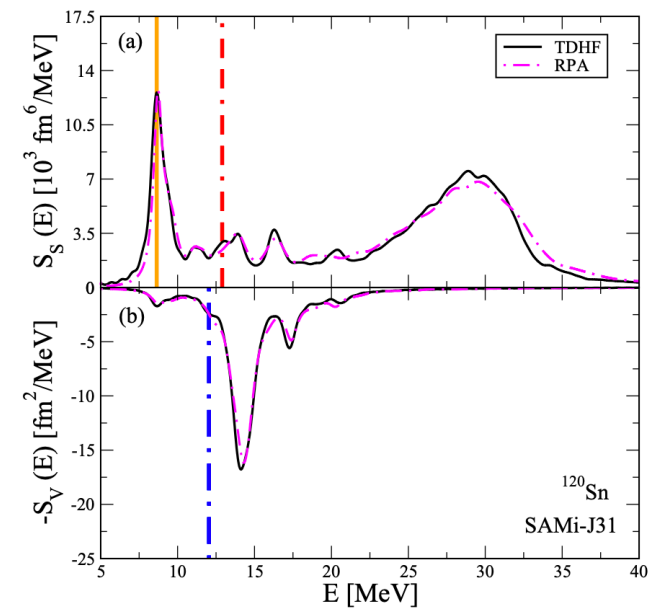
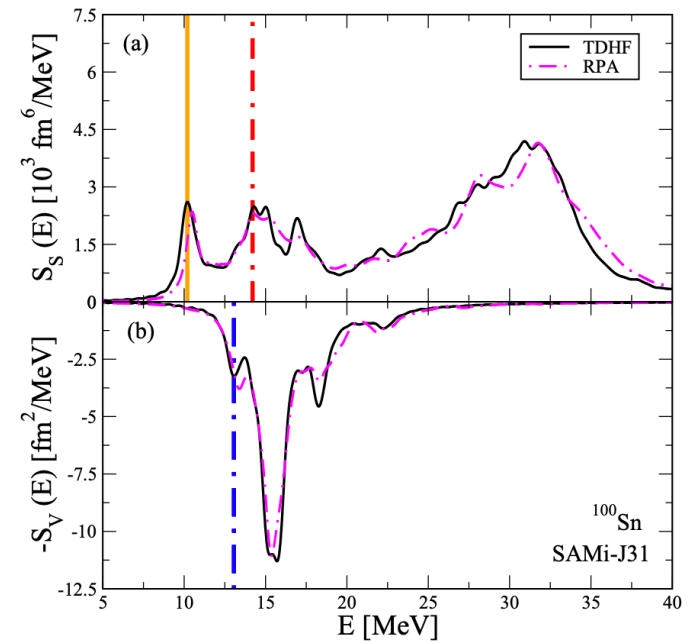
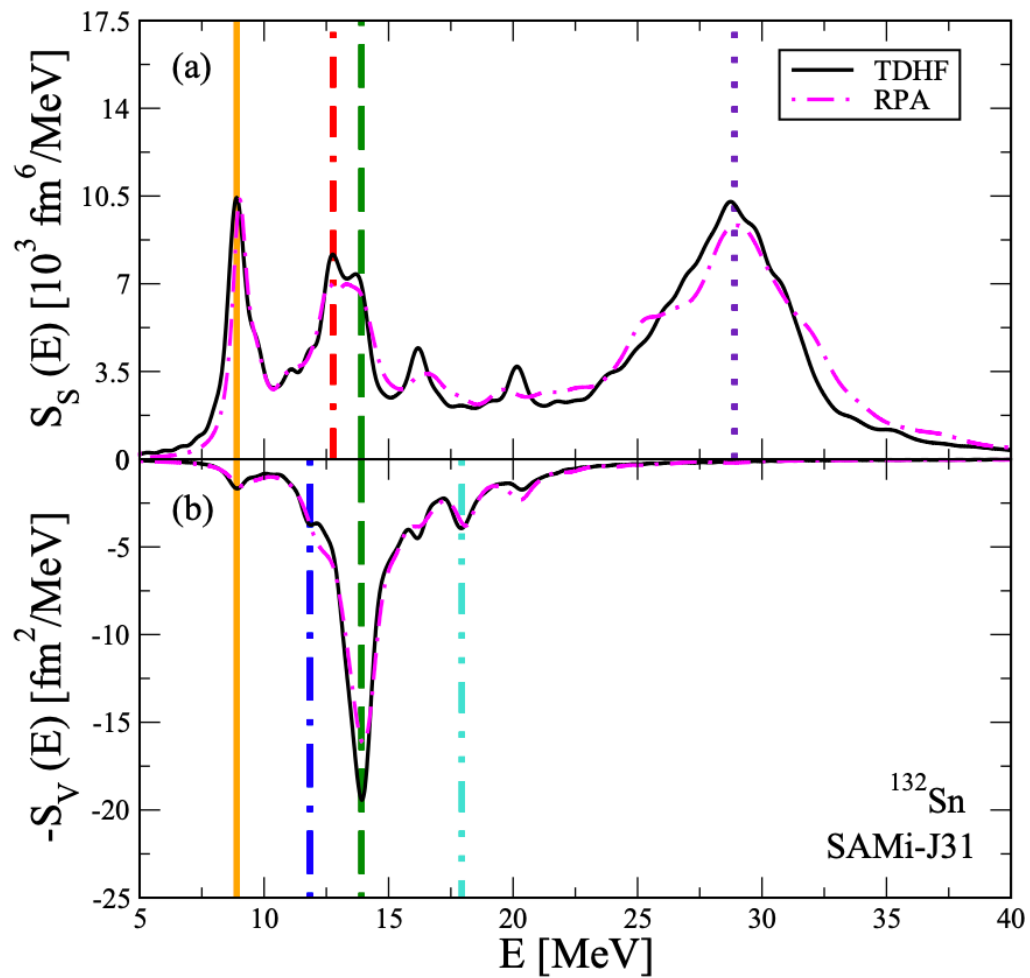
Scamps, Lacroix, PRC88 (2013)

## coupling between modes



# Interplay between low-lying isoscalar and isovector dipole modes: A comparative analysis between semiclassical and quantum approaches

S. Burrello,<sup>1</sup> M. Colonna,<sup>1</sup> G. Colò,<sup>2,3</sup> D. Lacroix,<sup>4</sup> X. Roca-Maza,<sup>2,3</sup> G. Scamps,<sup>5,6</sup> and H. Zheng<sup>1,7</sup>

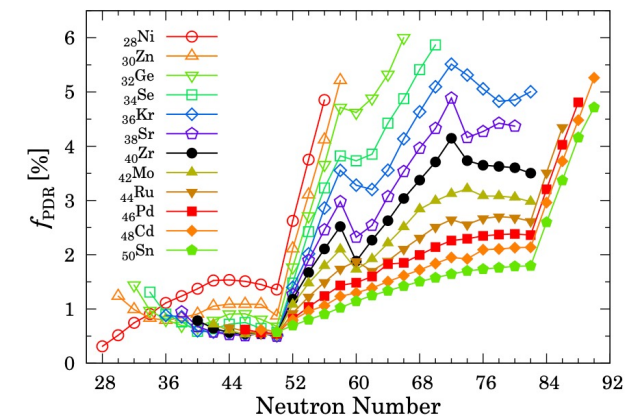
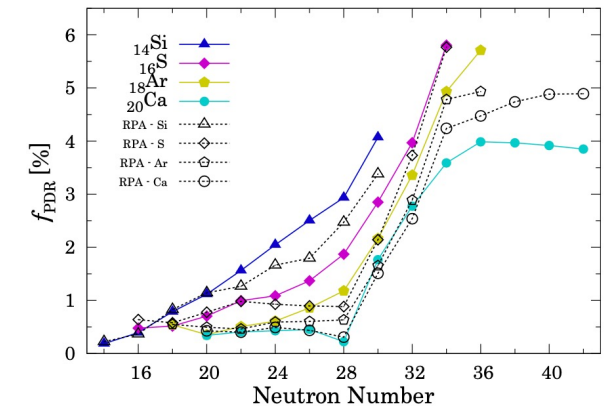
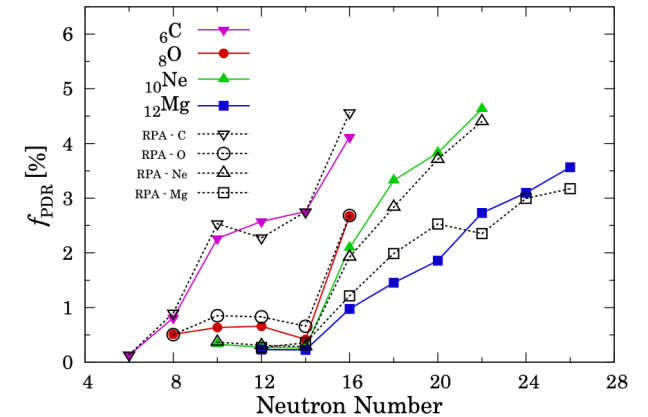
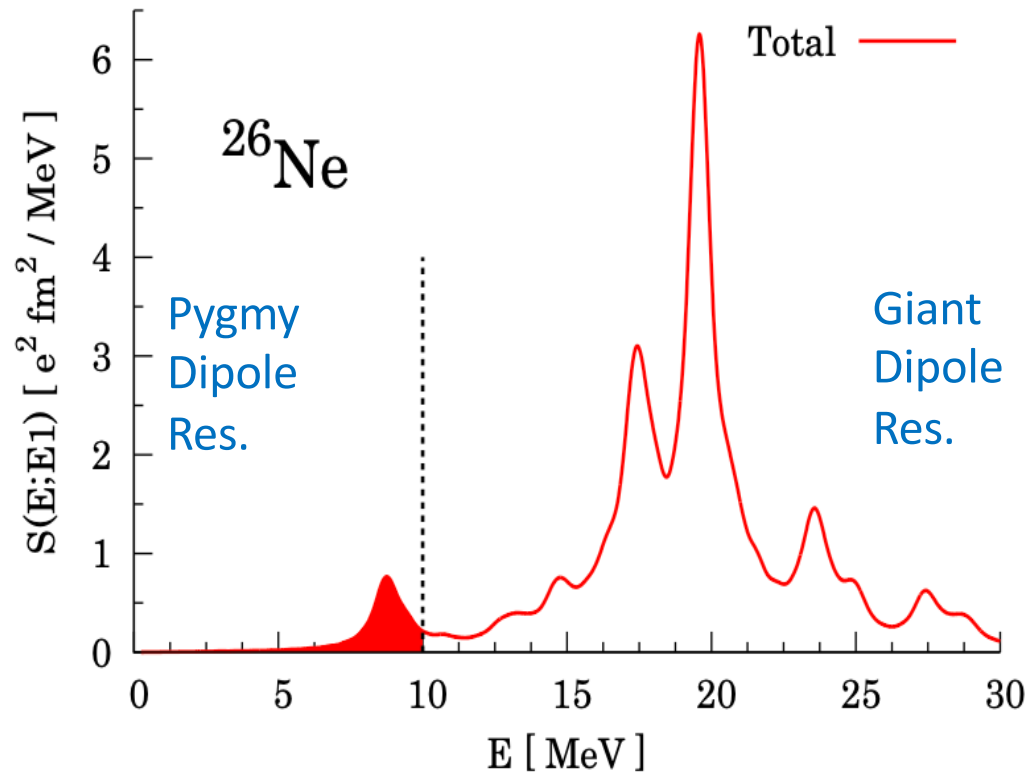


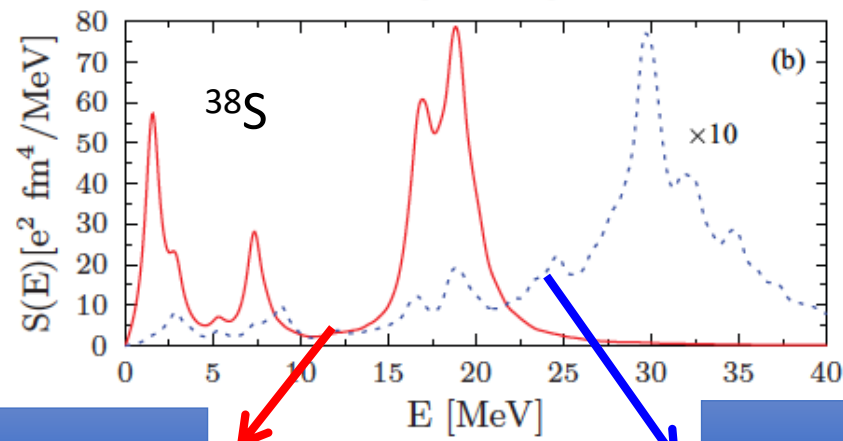
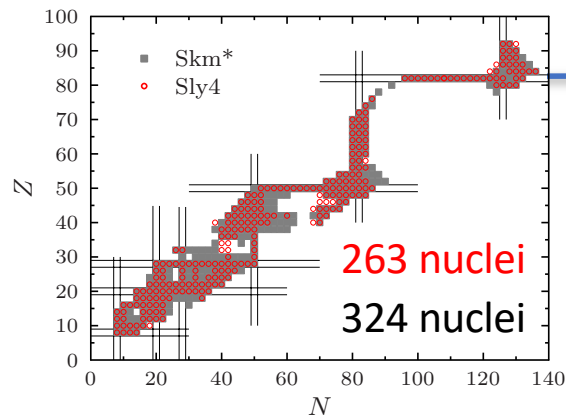
# Time-dependent methods with (BCS) pairing are now rather mature and allows for systematic investigations

PHYSICAL REVIEW C **90**, 024303 (2014)

## Systematic investigation of low-lying dipole modes using the canonical-basis time-dependent Hartree-Fock-Bogoliubov theory

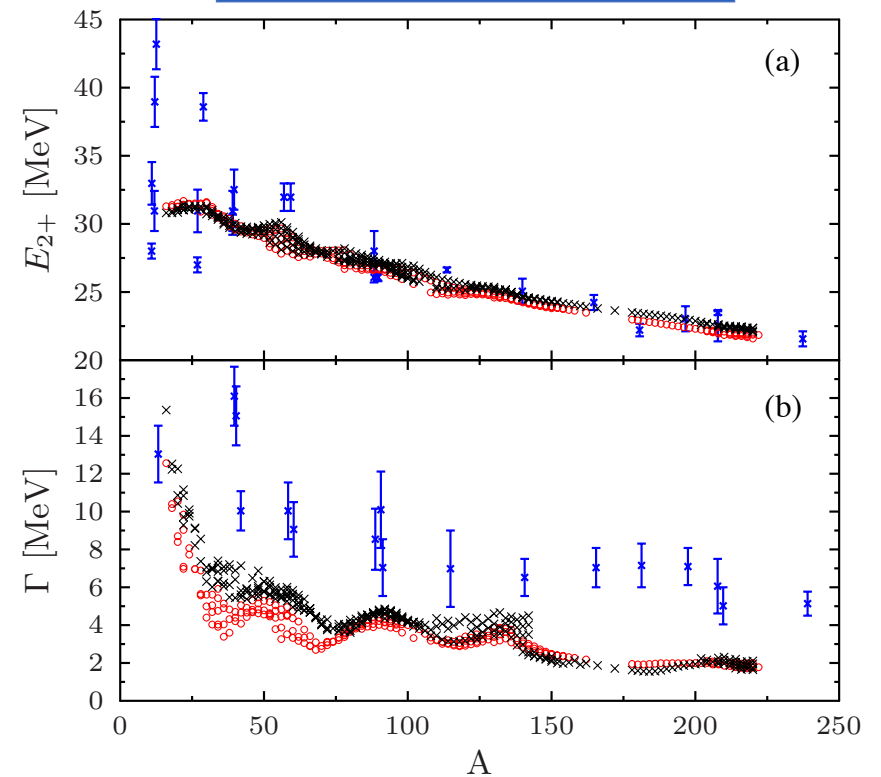
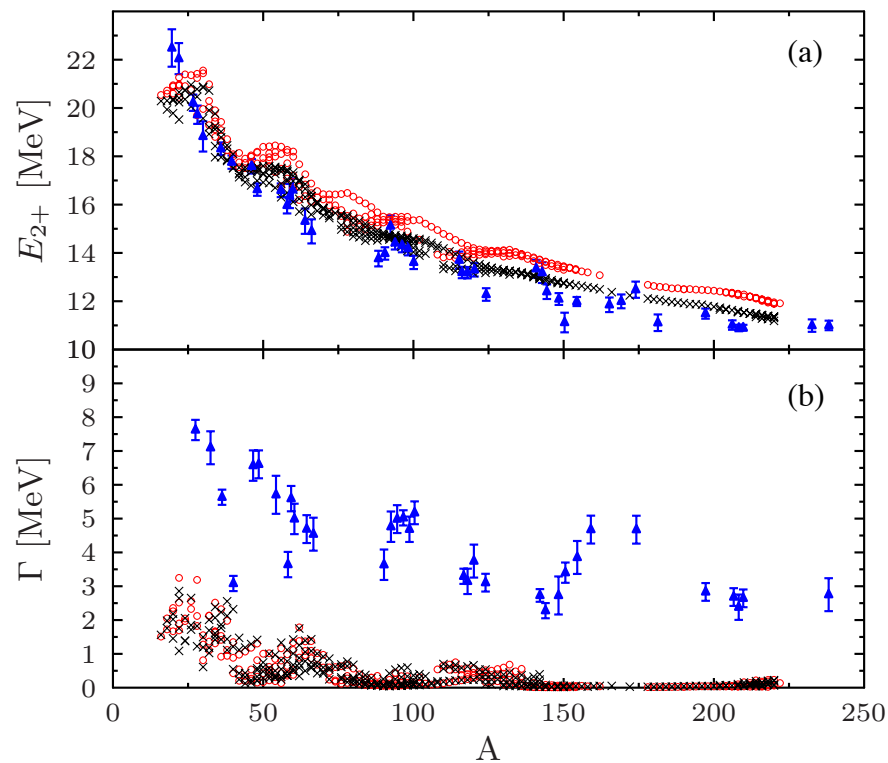
Shuichiro Ebata,<sup>1,2,3</sup> Takashi Nakatsukasa,<sup>3,4</sup> and Tsunenori Inakura<sup>3,5</sup>

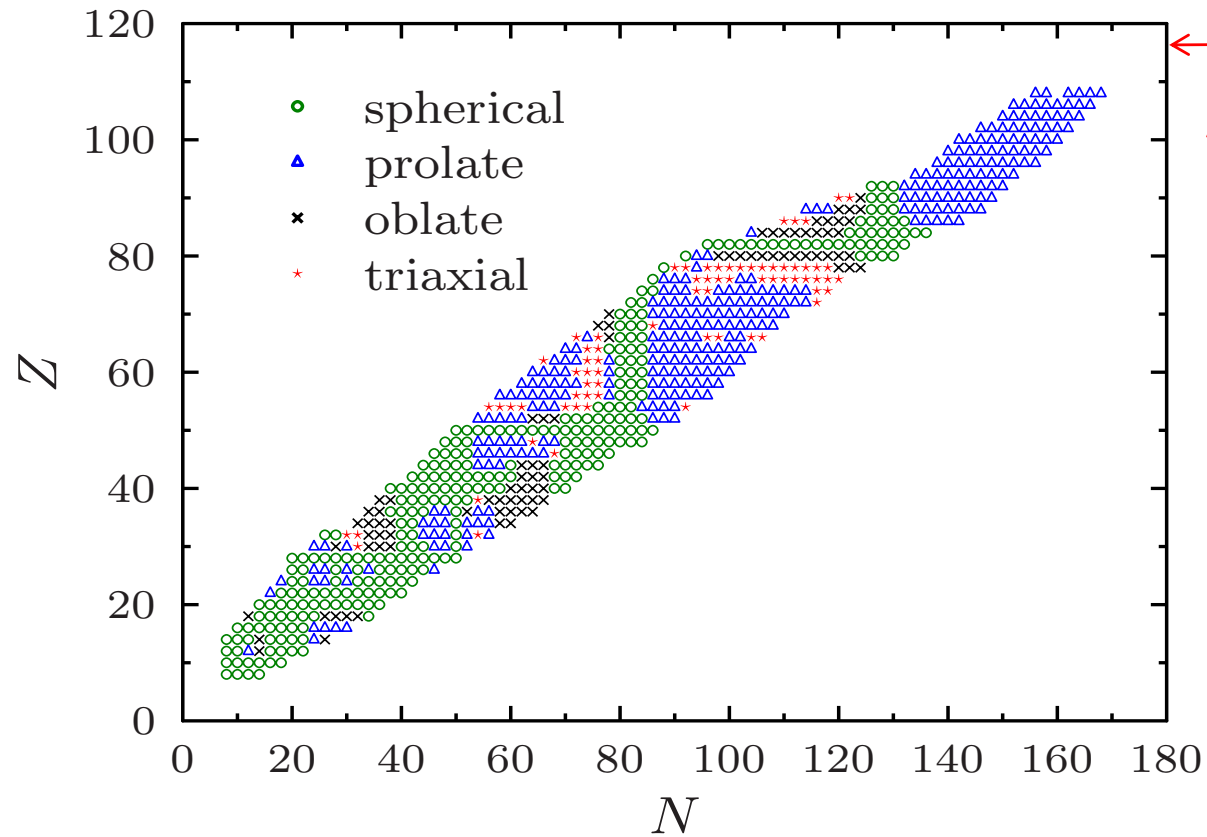




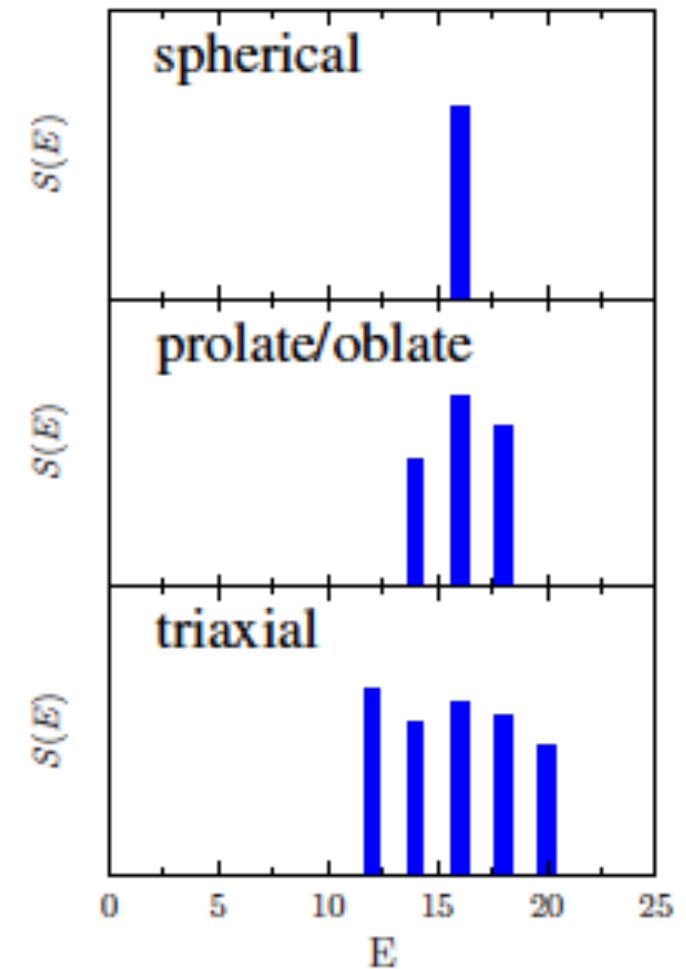
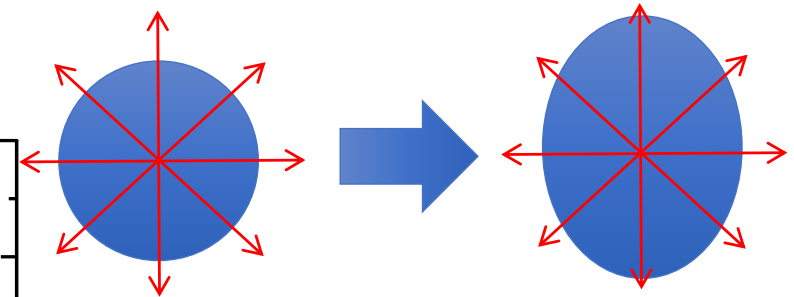
Isoscalar GQR

Isovector GQR





Scamps, Lacroix, PRC89 (2014)

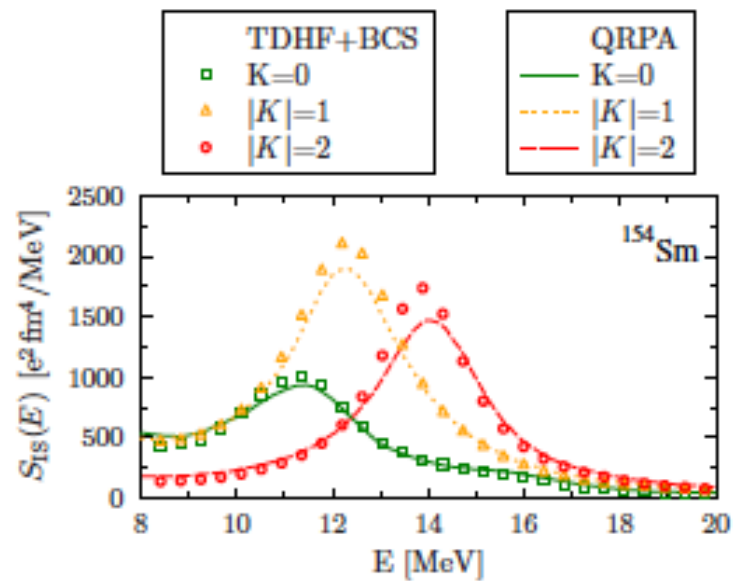


Excitation operators

$$Q_{2K}$$

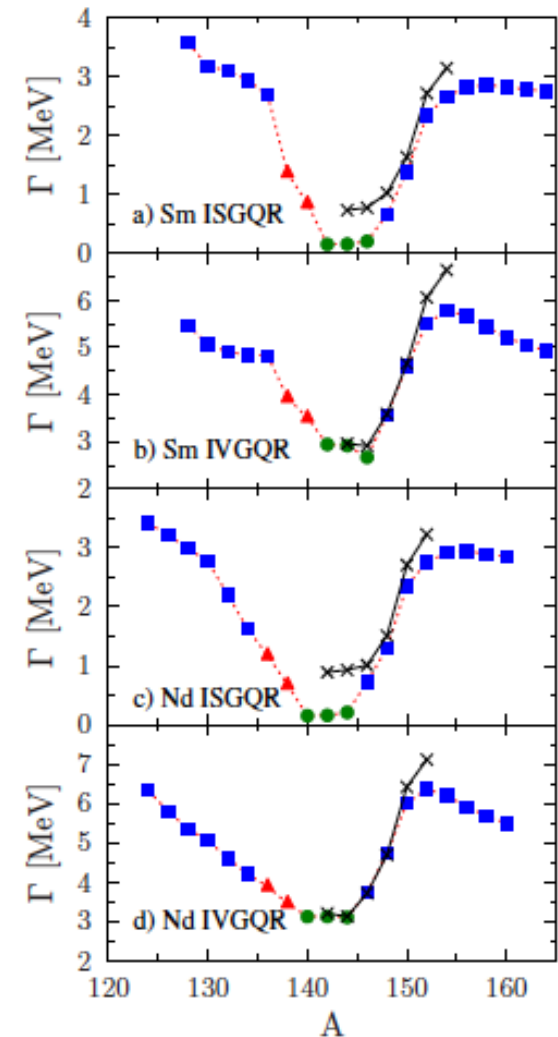
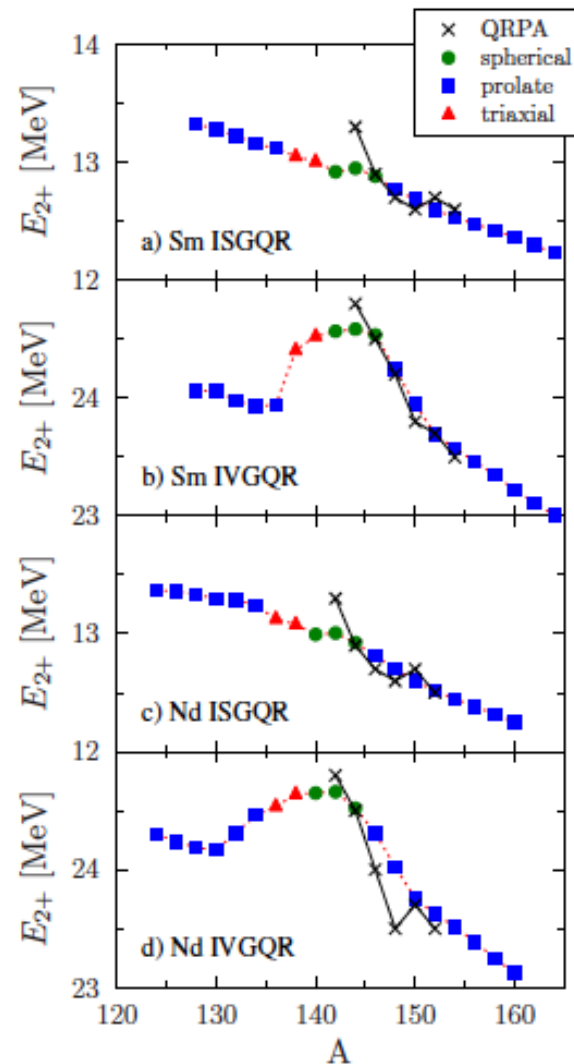
$$K = -2, -1, 0, 1, 2$$

## Collective energy



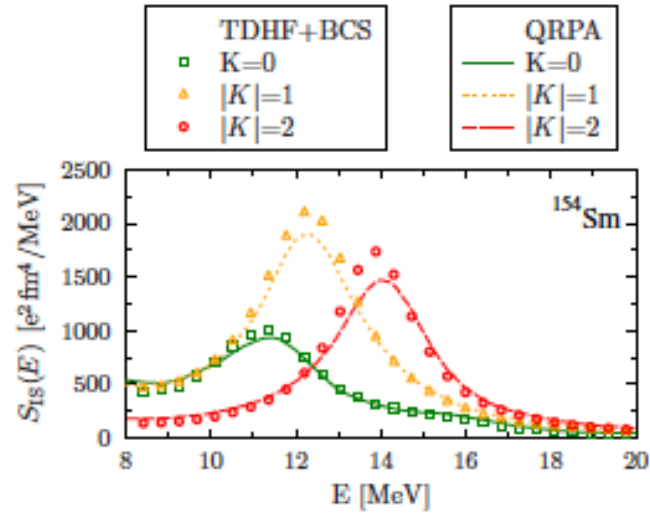
QRPA: Yoshida, Nakatsukasa,  
PRC88 (2013)

## Damping width



# Systematic in deformed nuclei: fragmentation and damping

Damping is more complex:

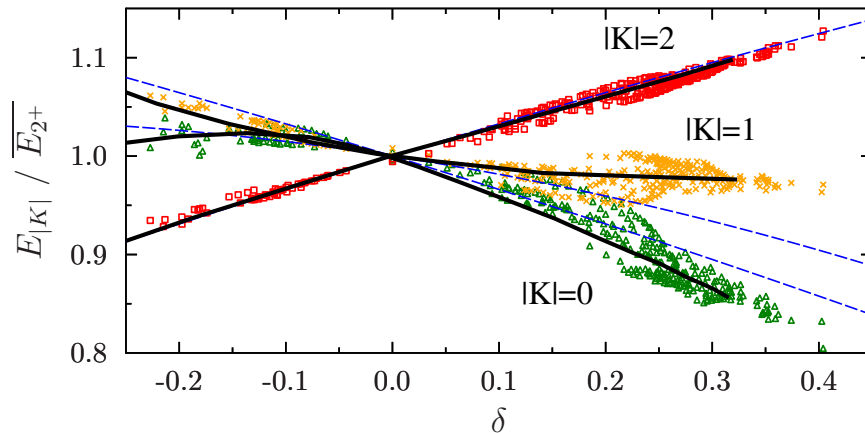


Energy splitting:

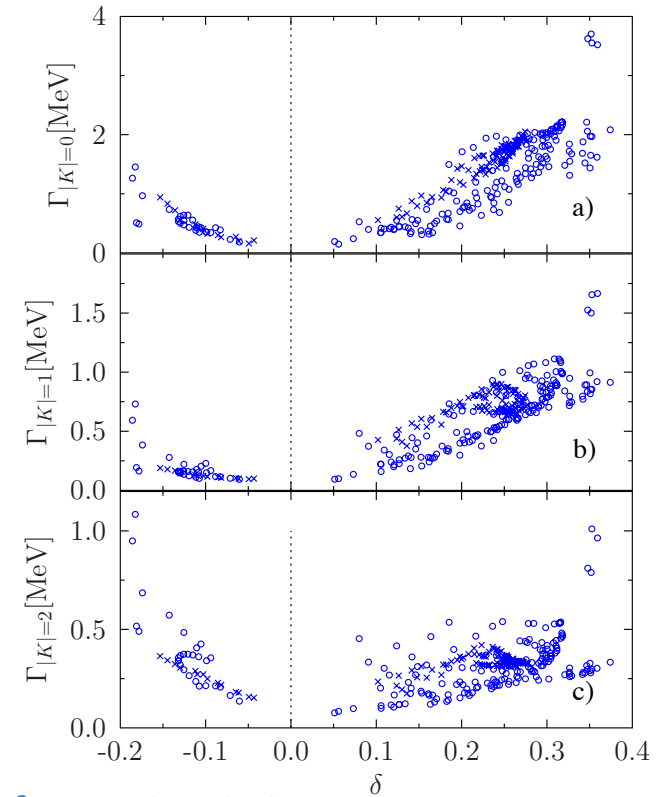


oblate

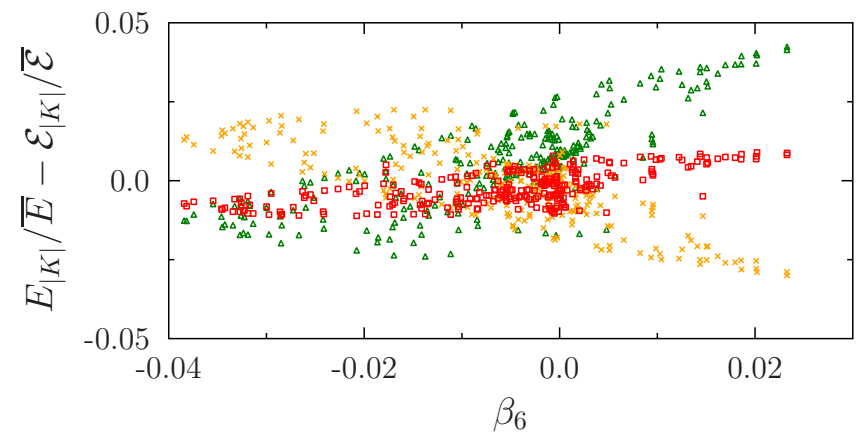
prolate

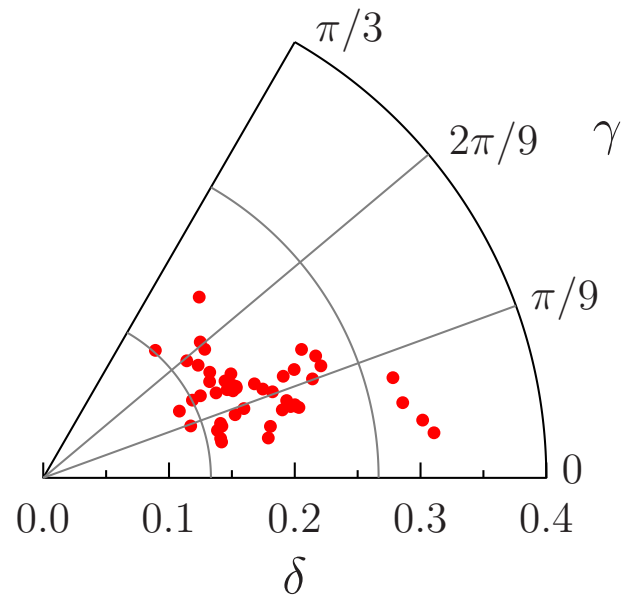
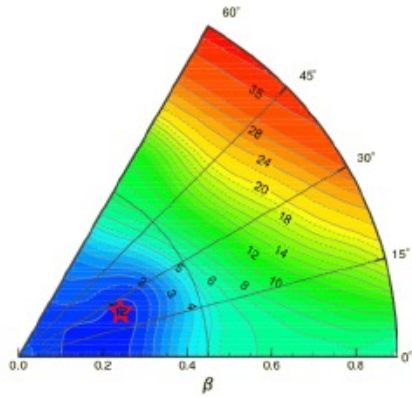


Scamps, Lacroix, PRC89 (2014)



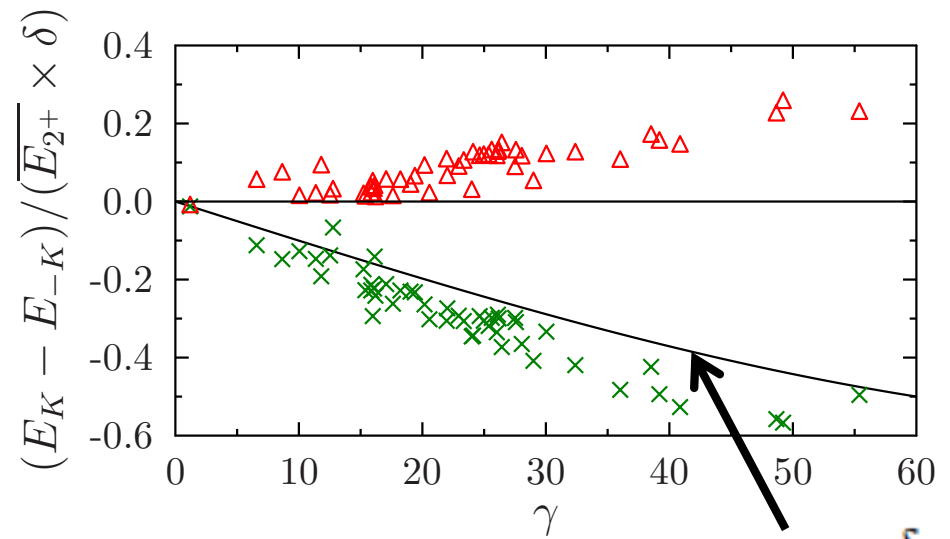
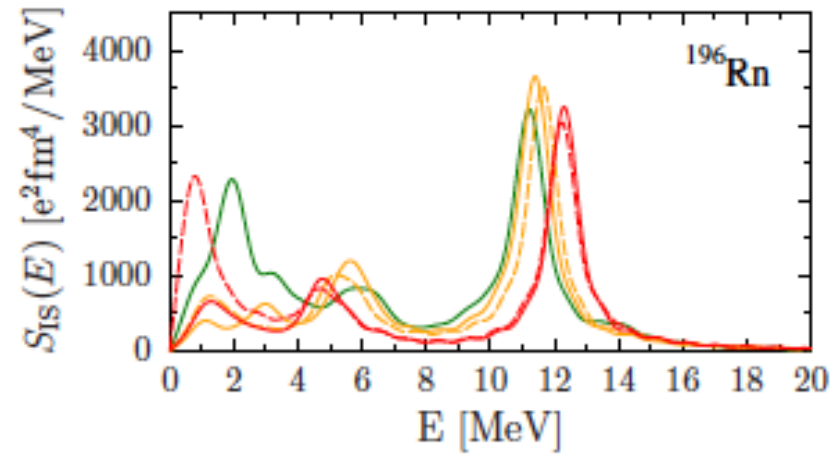
High order deformation is important



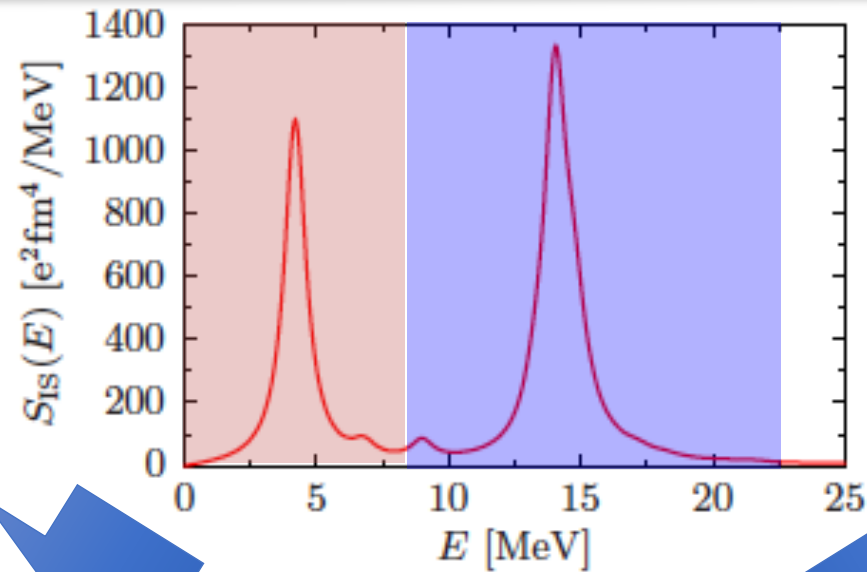


54 triaxial nuclei

Scamps, Lacroix, PRC89 (2014)

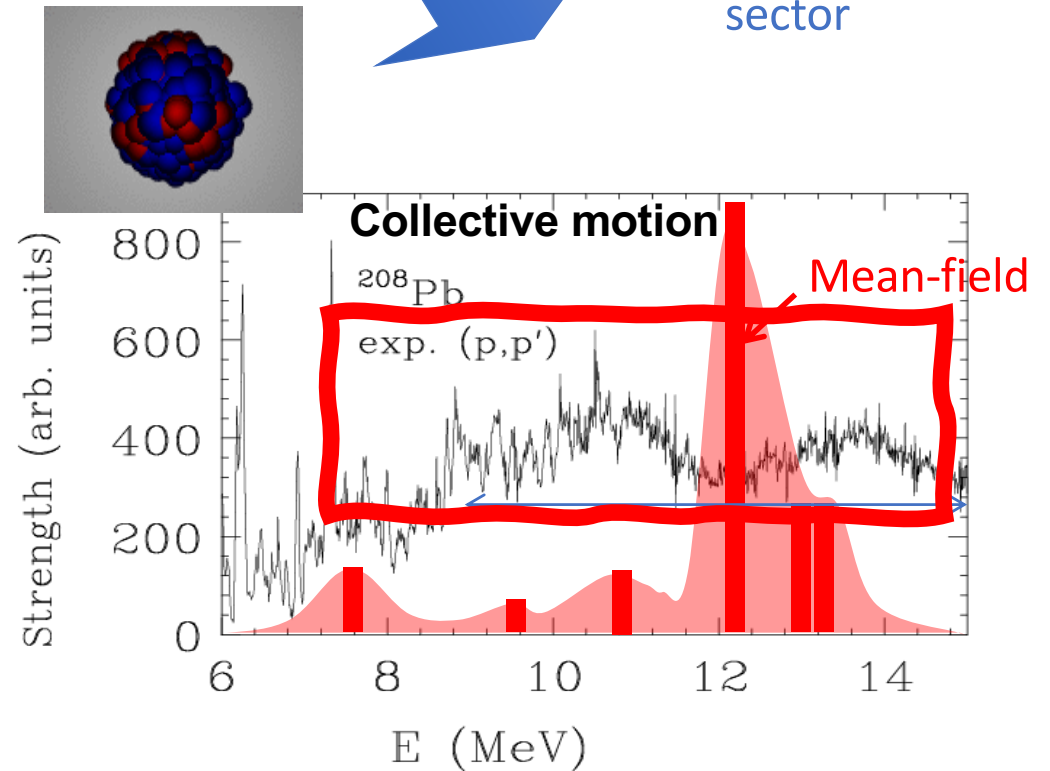
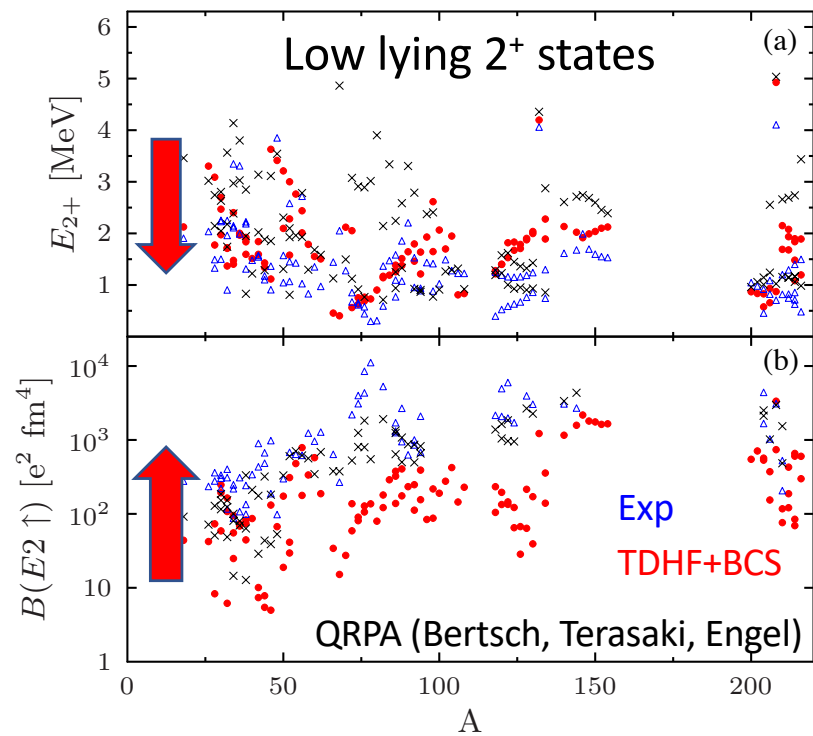


$$\Delta E_1 \simeq -E_{\text{sph}} \frac{\delta}{\sqrt{3}} \sin \gamma$$



Low-lying  
sector

Collective  
sector



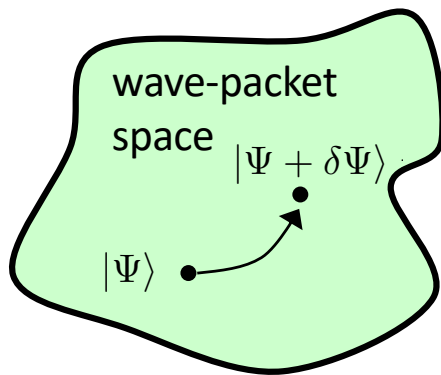
## From Ehrenfest

$$i\hbar \frac{d\langle A_\alpha \rangle}{dt} = \langle [A_\alpha, H] \rangle \quad \longrightarrow \quad i\hbar \partial_t \rho = [h_{\text{MF}}[\rho], \rho]$$

Obtained from  $\langle AB \rangle \simeq \langle A \rangle \langle B \rangle$

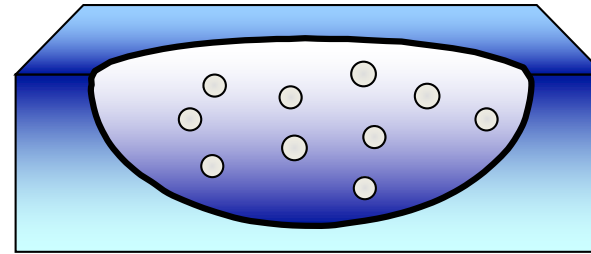
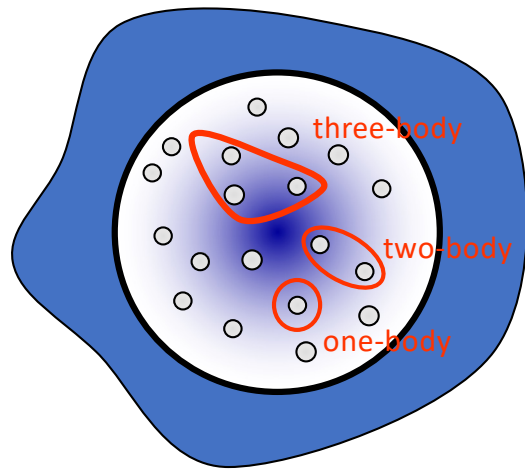
→ Can severely underestimate the quantum fluctuations both in static and dynamical Evolution.

## From General coherent state argument



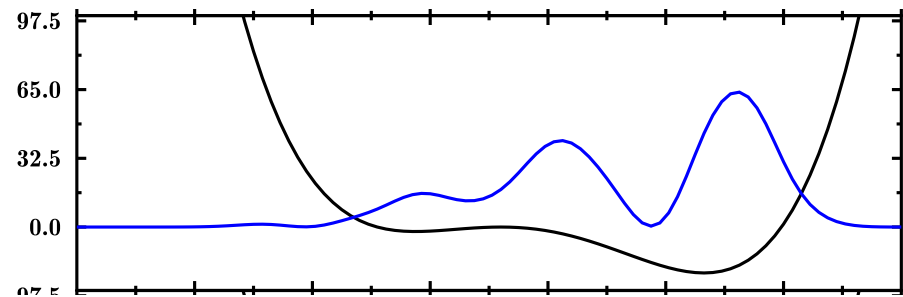
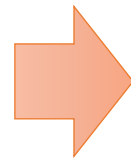
$$|\Psi + \delta\Psi\rangle = e^{\sum_{\beta\bar{\beta}} \delta Z_{\beta\bar{\beta}} a_{\bar{\beta}}^\dagger a_{\beta}} |\Psi\rangle = e^{\hat{Z}} |\Psi\rangle$$

Quasi-particle vacua are coherent fermionic states for the many-body problem. As such they behave likely as the most classical objects for interacting particles



quantum

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \left\{ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right\} \Psi(x, t)$$



Wave

Density Matrix

Observable space

$$|\Psi(t)\rangle \longrightarrow D(t) = |\Psi(t)\rangle \langle \Psi(t)| \longrightarrow \langle O(t) \rangle = \text{Tr}(OD(t))$$

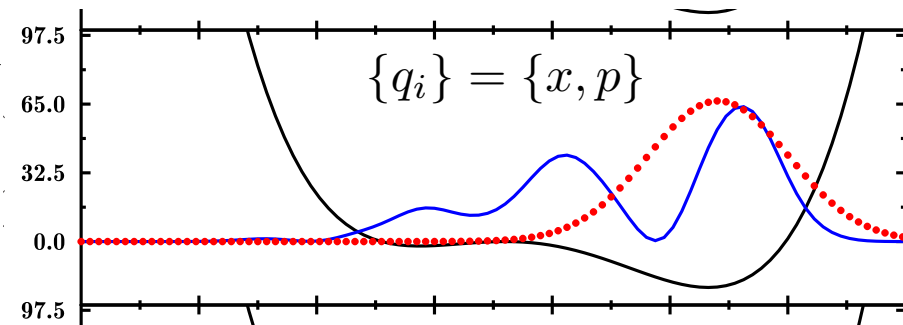
Complexity reduction

$$|\Psi + \delta\Psi\rangle = e^{\sum_{\alpha} \delta q_{\alpha} A_{\alpha}} |\Psi\rangle$$

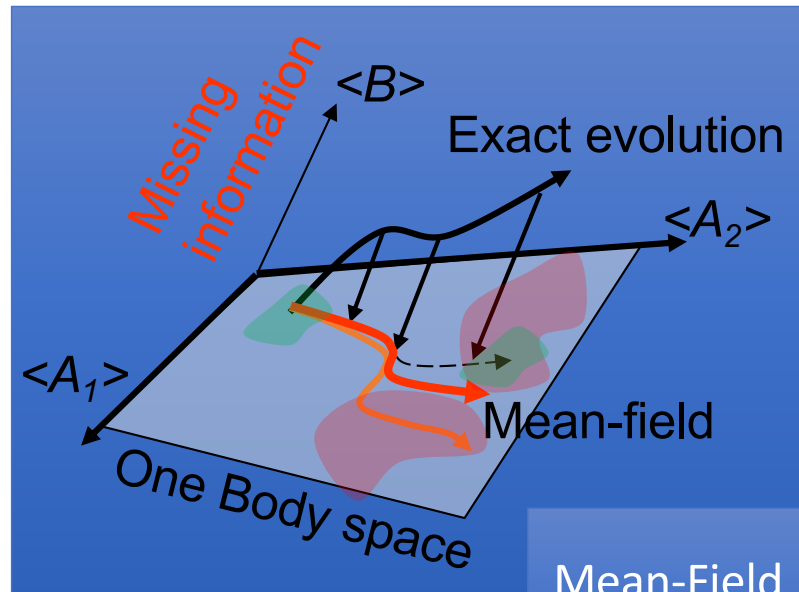
A diagram showing a path from  $|\Psi(t_0)\rangle$  to  $|\Psi(t_1)\rangle$  in the state space. The path is a thick black line, with several thinner grey lines branching off from it, representing different possible evolutions.

$$\Psi_i \longrightarrow \{q_i\}$$

Optimal dyn.  
for the  $\langle A_{\alpha} \rangle$



# Alternative stochastic methods to treat correlations Beyond Hartree-Fock / TDHF



Mean-Field  
State: Slater det, QP vacuum  
information: one-body DOFs



Correct for the improper  
Evolution of initial quantum  
Fluctuations with  
Phase-space approaches



TDGCM



Phase-space  
Methods

Correlation that built up in time



Ex: BBGKY inspired  
( $\rho_1, \rho_2, \dots$ )

Stochastic  
unraveling

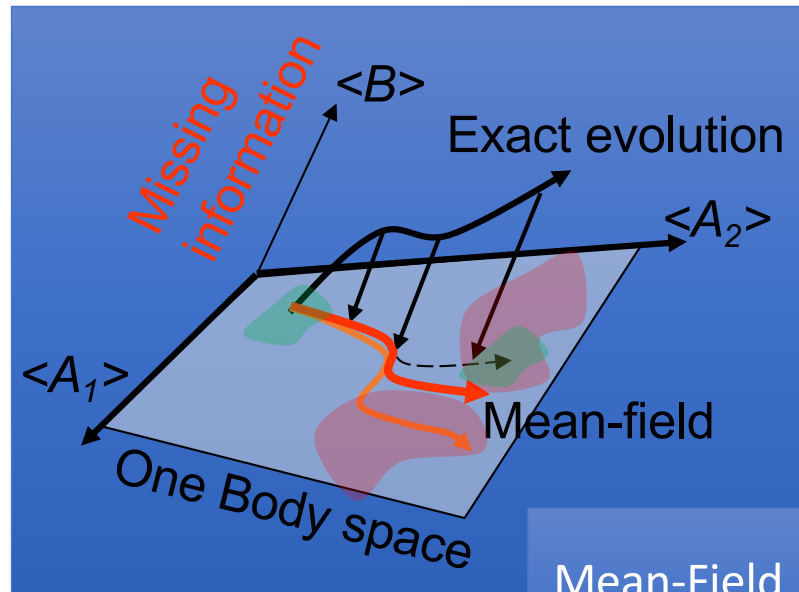


Many-proposals (ETDHF, 2RDM, ...)

Replace the initial complex problem by an  
ensemble of simpler problem  
(mean-field like)

Quantum Monte-Carlo,  
Quantum state diffusion...

# Alternative stochastic methods to treat correlations Beyond Hartree-Fock / TDHF



Mean-Field  
State: Slater det, QP vacuum  
information: one-body DOFs

Correct for the improper  
Evolution of initial quantum  
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Phase-space approaches

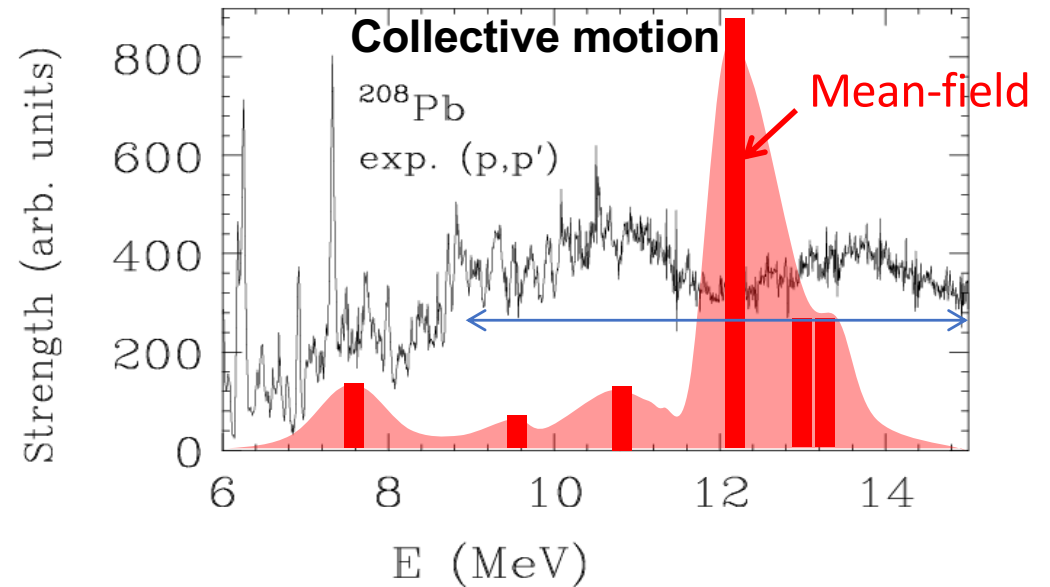
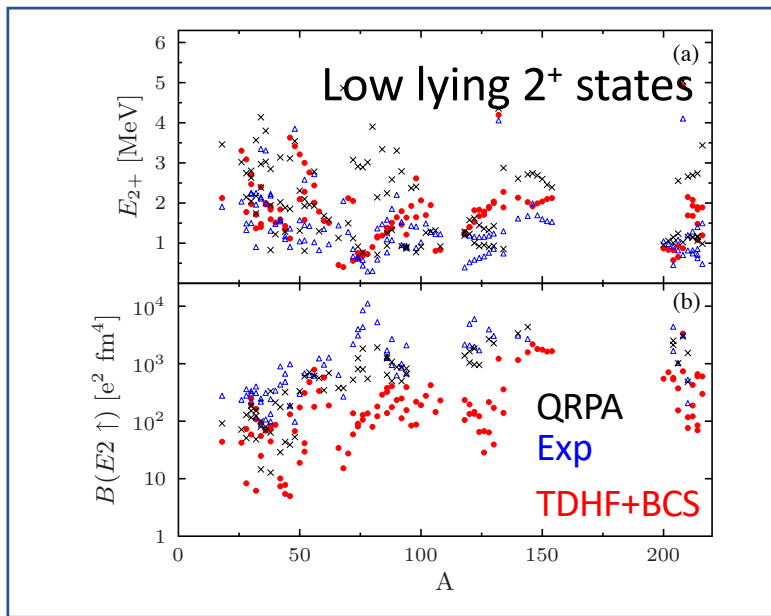
Correlation that built up in time

Ex: BBGKY inspired  
( $\rho_1, \rho_2, \dots$ )

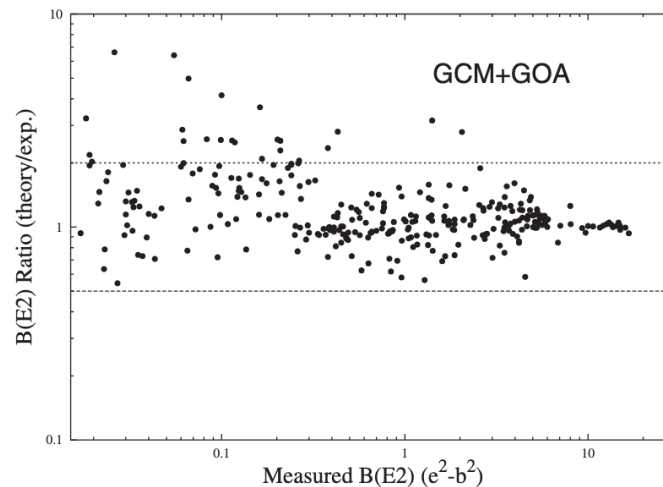
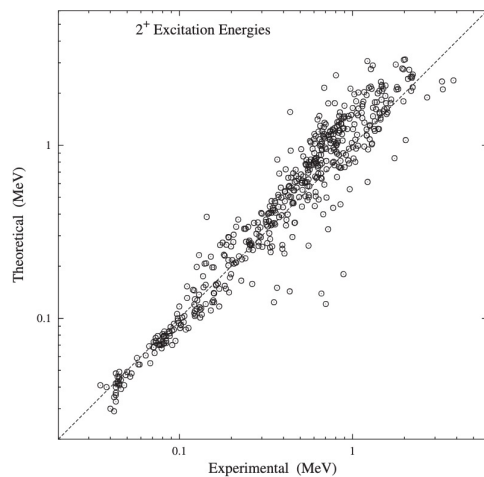
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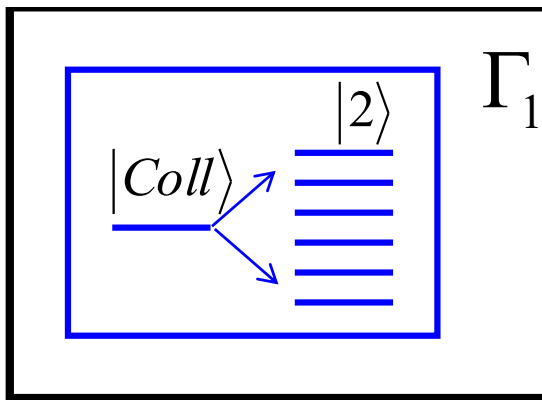
BUT for collective excitations  
what are the important  
effects beyond mean-field ?



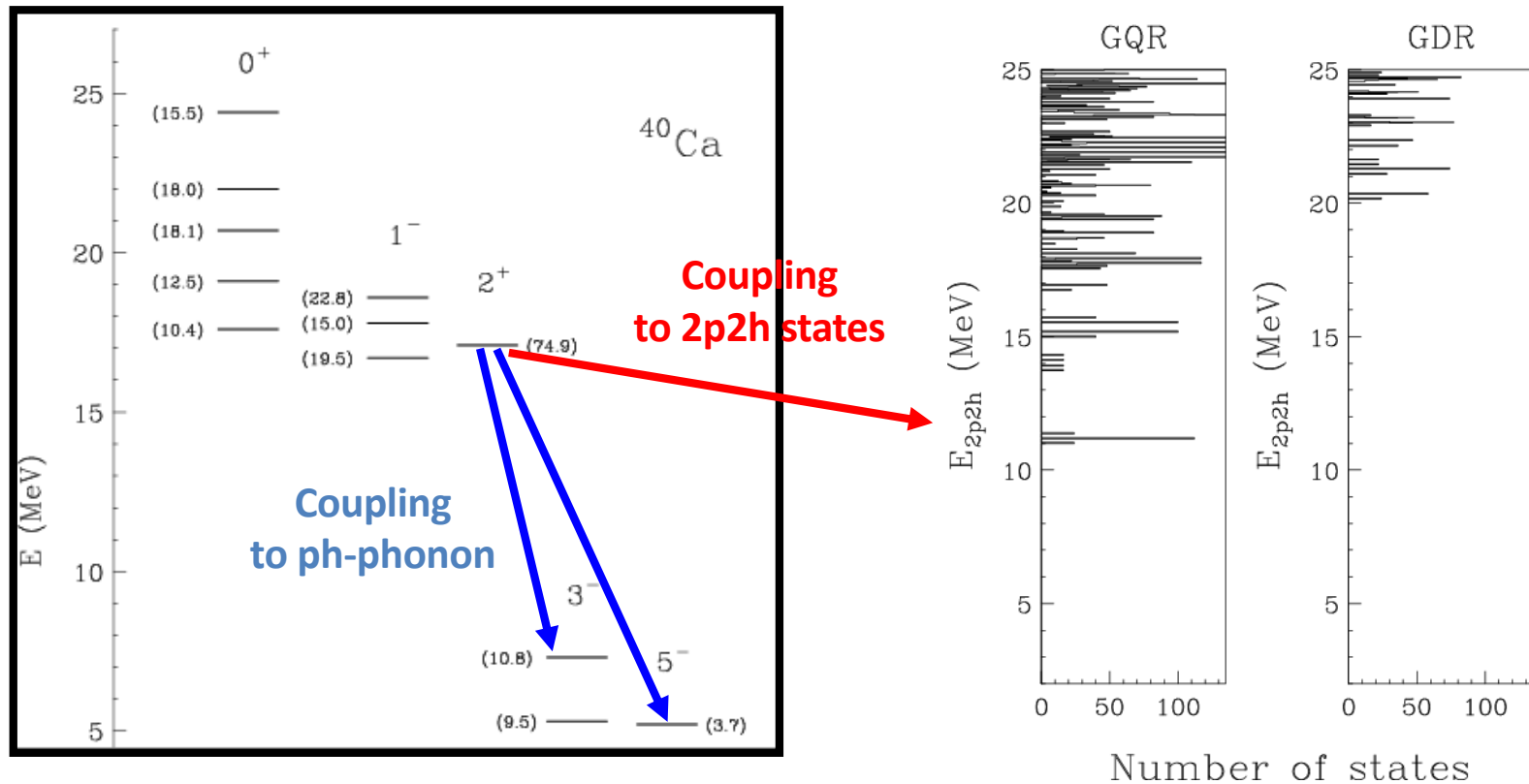
Empirical Shell model is usually better. Probably we are missing important Ground state correlation (GCM like).



Origin of decay width and fine structure ?

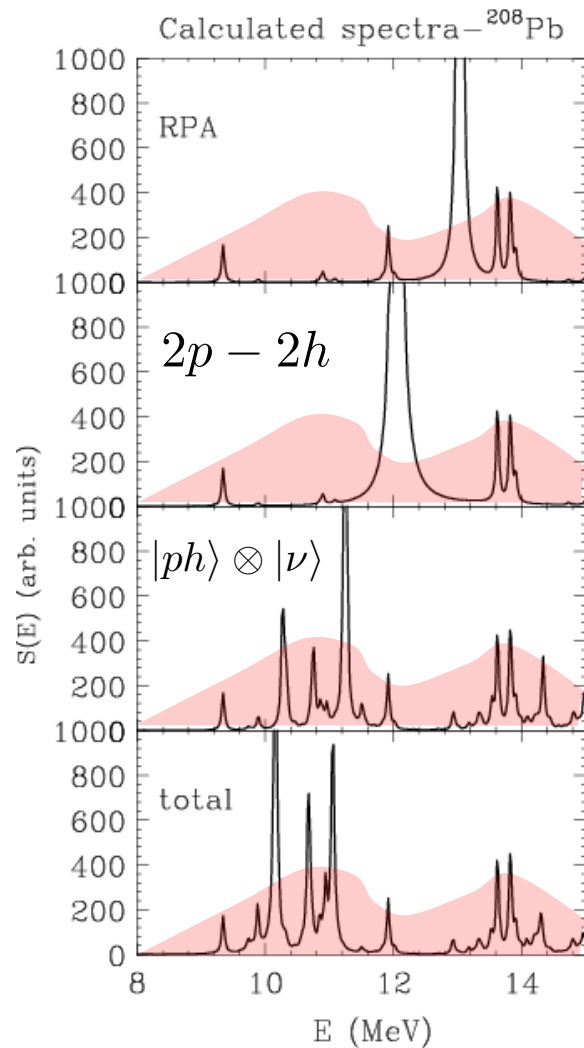


### Standard RPA states

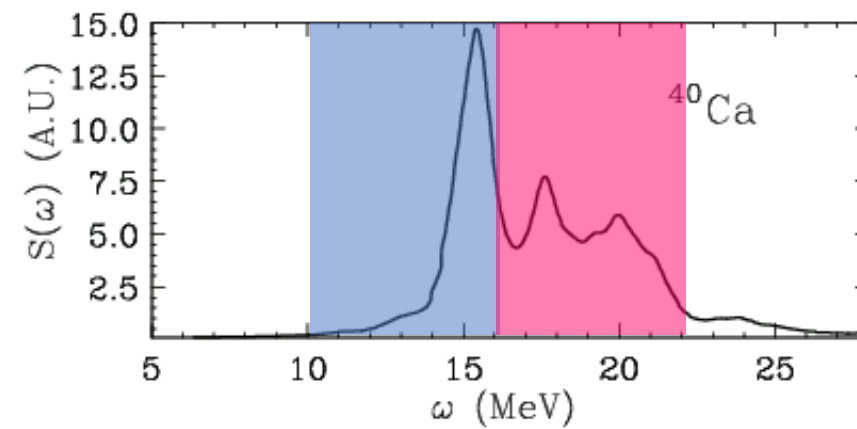


Bertsch, Bortignon, Broglia, Rev. Mod. Phys. 55, 287 (1983)  
 Lacroix, Ayik, Chomaz, Prog. Part and Nucl. Phys. (2004)

## GQR in $^{208}\text{Pb}$



## GQR in $^{40}\text{Ca}$



## EWSR

$^{40}\text{Ca} / 2^+$	RPA	coherent only	incoherent only	total	Experiment
% EWSR	%	%	%	%	%
[0 - 40]	92.4	86.6	84.7	87.6	
[10 - 16]	0.0	26.6	39.1	31.0	$33 \pm 7$ ( $e, e'x$ ) [80, 89] $60 \pm 15$ ( $\alpha, \alpha'\alpha_0$ ) [81]
[16 - 22]	72.5	51.6	34.7	33.6	$28.6 \pm 7$ ( $p, p'$ ) [91] $\sim 40$ ( $\alpha, \alpha'\alpha_0$ ) [81] 44 ( $p, p'$ ) [92]

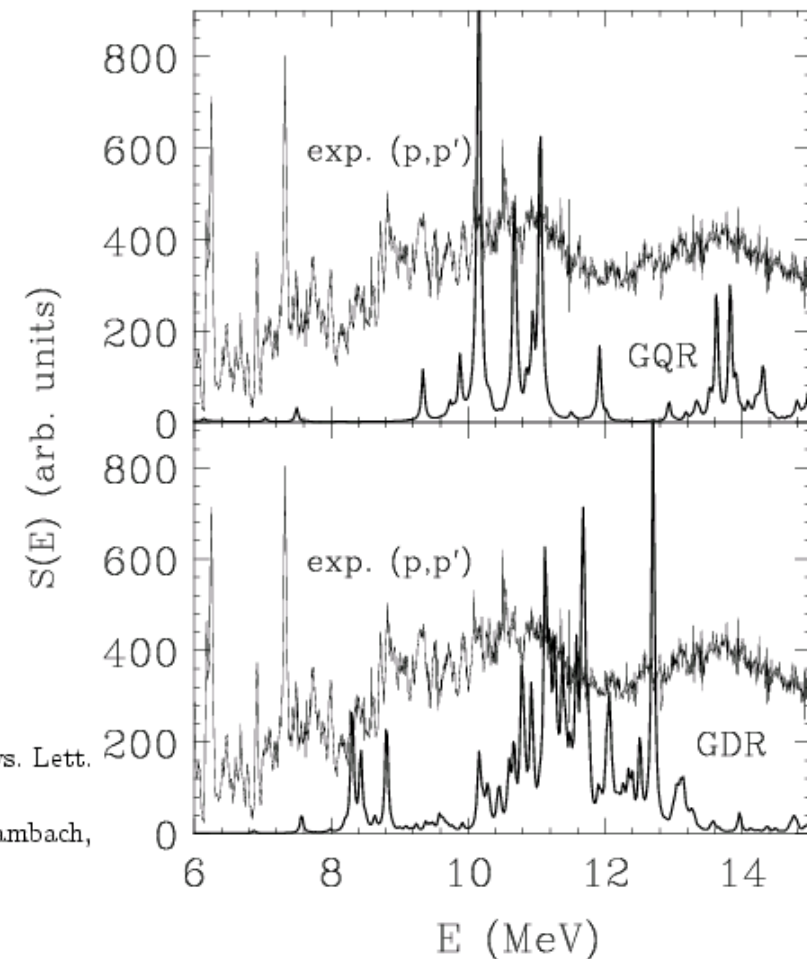
## Comparison of the structures positions :

(e,e') [6]	(p,p') [94]	(p,p') [7]	incoherent and coherent	coherent only	incoherent only only
8.9	8.9	8.9			
9.4	9.3	<b>9.4</b>	<b>9.3</b>	9.3	9.3
9.6		<b>9.6</b>	<b>9.9</b>	9.9	
10.1		<b>10.1</b>	<b>10.2</b>	10.3	
10.7	10.6	<b>10.7</b>	<b>10.7</b>	10.8	
11.5		<b>11.0</b>	<b>11.0</b>	11.3	
					11.9

[6] G. Kühner, D. Meuer, S. Müller, A. Richter, E. Spamer, O. Titze, and W. Knüpfer, Phys. Lett. **104B**, 189 (1981).

[7] S. Kamerdzhiev, J. Lisantti, P. von Neumann-Cosel, A. Richter, G. Tertychny and J. Wambach, Phys. Rev. **C55** (1997) 2101.

[94] F.E. Bertrand, *et al*, Phys. Rev. **C34** (1986) 45.



From a BBGKY picture

One-body  $i\hbar \frac{\partial}{\partial t} \rho_1 - [h_1, \rho_1] = \text{Tr}_2 [v_{12}, C_{12}]$

Correlations  $i\hbar \frac{\partial}{\partial t} C_{12} - [h_1 + h_2, C_{12}] = F_{12}$

Integrating correlation effects

$$C_{12}(t) = -\frac{i}{\hbar} \int_{t_0}^t U_{12}(t, s) F_{12}(s) U_{12}^\dagger(t, s) ds + \delta C_{12}(t)$$

where

$$\delta C_{12}(t) = U_{12}(t, t_0) C_{12}(t_0) U_{12}^\dagger(t, t_0)$$

➡ Propagated initial correlations

$$U_{12}(t, s) F_{12}(s) U_{12}^\dagger(t, s) = (1 - \rho_1)(1 - \rho_2) v_{12}(t, s) \widetilde{\rho_1} \widetilde{\rho_2} \cdot \\ - \widetilde{\rho_1} \widetilde{\rho_2} v_{12}(t, s) (1 - \rho_1)(1 - \rho_2)$$

➡ Two-body effect projected on the one-body space

Molecular Chaos assumption

Treat initial fluctuations as random objects

$$i\hbar \frac{\partial}{\partial t} \rho^{(n)} - [h(\rho^{(n)}), \rho^{(n)}] = K_I(\rho^{(n)}) + \delta K^{(n)}(t)$$

Lacroix, Ayik, EPJA (Review) 50 (2014)

where

$$\delta K^{(n)}(t) = \text{Tr}_2[v_{12}, \delta C_{12}^{(n)}(t)]$$



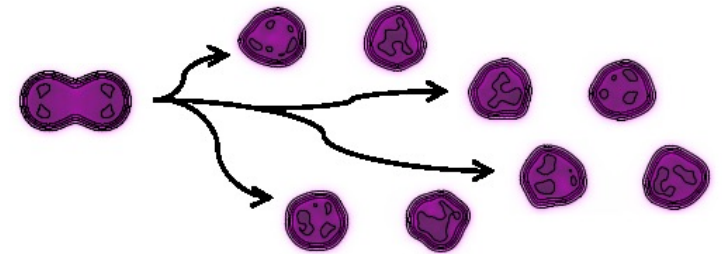
Linked to the  
phase-space  
technique

$$K_I(\rho_1) = -\frac{i}{\hbar} \int_{t_0}^t ds \text{Tr}_2[v_{12}, U_{12}(t, s) F_{12}(s) U_{12}^\dagger(t, s)].$$

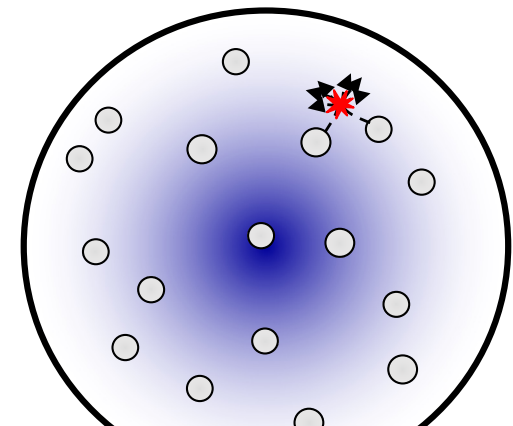


Extended TDHF  
with a two-body collision term

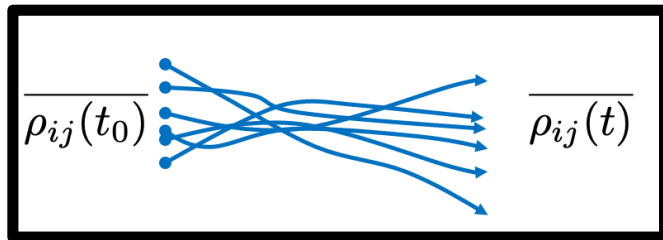
My PhD (long ago)



Tanimura, Lacroix, Ayik, PRL 2017



$$i\hbar \frac{\partial}{\partial t} \rho^{(n)} - [h(\rho^{(n)}), \rho^{(n)}] = K_I(\rho^{(n)}) + \delta K^{(n)}(t)$$

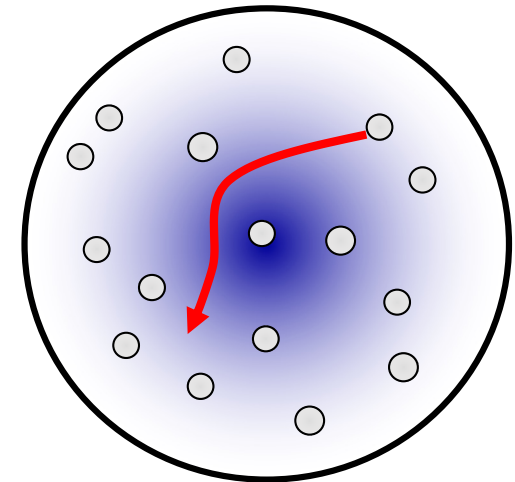


then

Averaging over trajectories

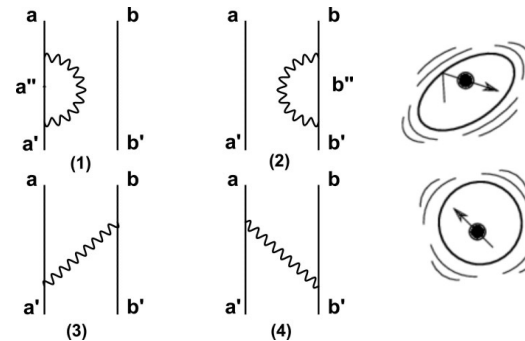
Incoherent nucleon-nucleon collision term.

$$K_I(\rho_1) = -\frac{i}{\hbar} \int_{t_0}^t ds \text{Tr}_2 [v_{12}, U_{12}(t, s) F_{12}(s) U_{12}^\dagger(t, s)].$$



Coherent contribution

$$K_C(\rho) = \overline{[\delta h^{(n)}(t), \delta \rho^{(n)}(t)]}$$



## Going to linear response

How to get this in a dynamical mean-field theory?

$$(\hbar\omega - \hbar\omega_\lambda)z_\lambda^+ - \sum_\mu \{K_{\lambda\mu}(\omega)z_\lambda^+ - \widetilde{K}_{\lambda\mu}(\omega)z_\lambda^-\} = +A_\lambda$$

### Incoherent damping

$$K_{\lambda\mu}^I(\omega) = -\frac{1}{4} \sum_{ijkl} \frac{\langle k\ell | [O_\lambda, v] | ij \rangle \langle ij | [O_\mu^\dagger, v] | k\ell \rangle}{\hbar\omega + i\eta - \Delta\varepsilon_{ijkl}} \mathcal{N}_{ijkl}$$

$$\Delta\varepsilon_{ijkl} = \varepsilon_i + \varepsilon_j - \varepsilon_k - \varepsilon_\ell$$

$$\mathcal{N}_{ijkl} = (1 - n_i)(1 - n_j)n_k n_\ell - n_i n_j(1 - n_k)(1 - n_\ell)$$

Ph. Chomaz, D. Lacroix, S. Ayik, and M. Colonna  
PRC 62, 024307 (2000)

### Coupling to 2p-2h states

$$\Gamma_{2p2h} \propto \sum \frac{|\langle 2p2h | V | Coll \rangle|^2}{\hbar\omega - E_{2p2h}}$$

### Coherent damping

$$K_{\lambda\mu}^C(\omega) = -\sum_{\nu ij} \frac{\langle i | [Q_\lambda, h_\nu^\dagger] | j \rangle \langle j | [Q_\mu^\dagger, h_\nu] | i \rangle}{\hbar\omega + i\eta - \hbar\omega_\nu - \varepsilon_j + \varepsilon_i} \mathcal{M}_{\nu,ij} \\ + \sum_{\nu ij} \frac{\langle i | [Q_\lambda, h_\nu] | j \rangle \langle j | [Q_\mu^\dagger, h_\nu^\dagger] | i \rangle}{\hbar\omega + i\eta + \hbar\omega_\lambda - \varepsilon_j + \varepsilon_i} \mathcal{M}_{\nu,ji}$$

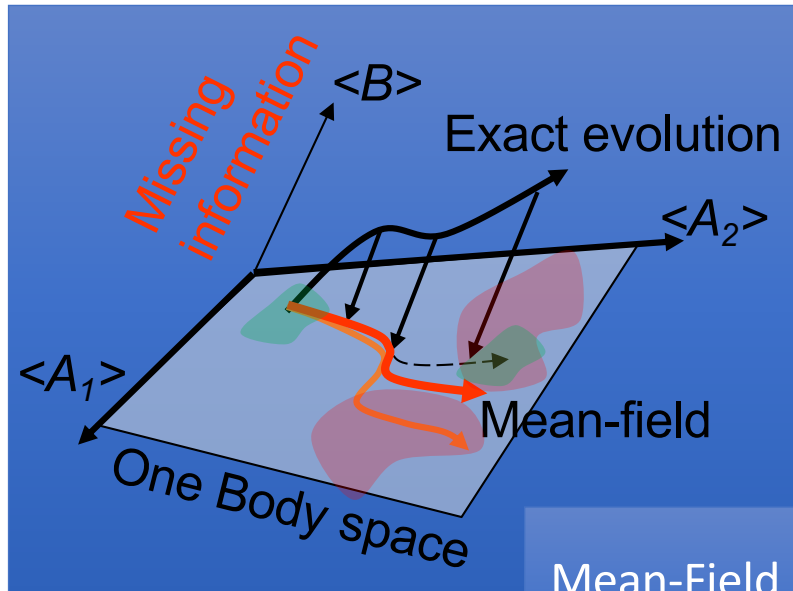
$$\mathcal{M}_{\nu,ij} = (N_\nu + 1)(1 - n_j)n_i - N_\nu n_j(1 - n_i)$$

S. Ayik and Y. Abe, PRC 64, 024609 (2001).

### Coupling to ph-phonon states

$$\Gamma_{\lambda \otimes ph} \propto \sum \frac{|\langle \lambda \otimes ph | V | Coll \rangle|^2}{\hbar\omega - E_{\lambda \otimes ph}}$$

[CQFD]



BUT for collective excitations  
what are the important  
effects beyond mean-field ?

## Intermediate status

Mean-Field  
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Correlation that built up in time

Ex: BBGKY inspired  
( $\rho_1, \rho_2, \dots$ )

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Replace the initial complex problem by an  
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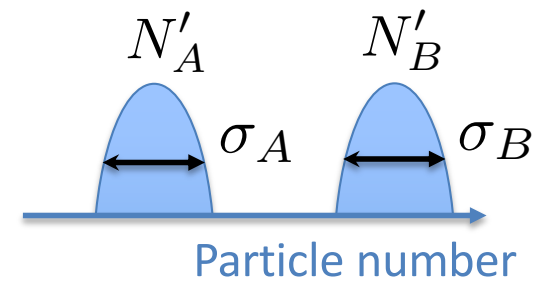
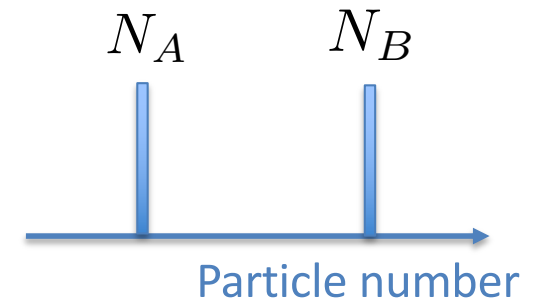
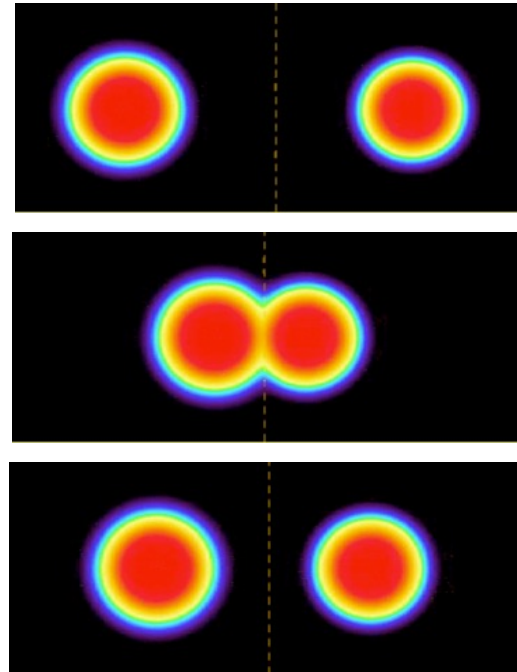
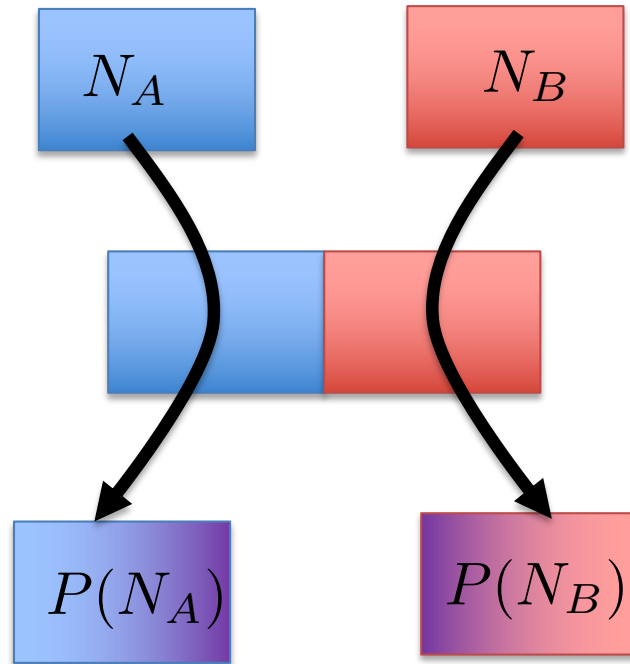
Interferences  
[2019-Now]

Phase-space (full glory)  
[2014-Now]

Pairing is ok  
2-body collisions is less clear  
largely uncontrolled.

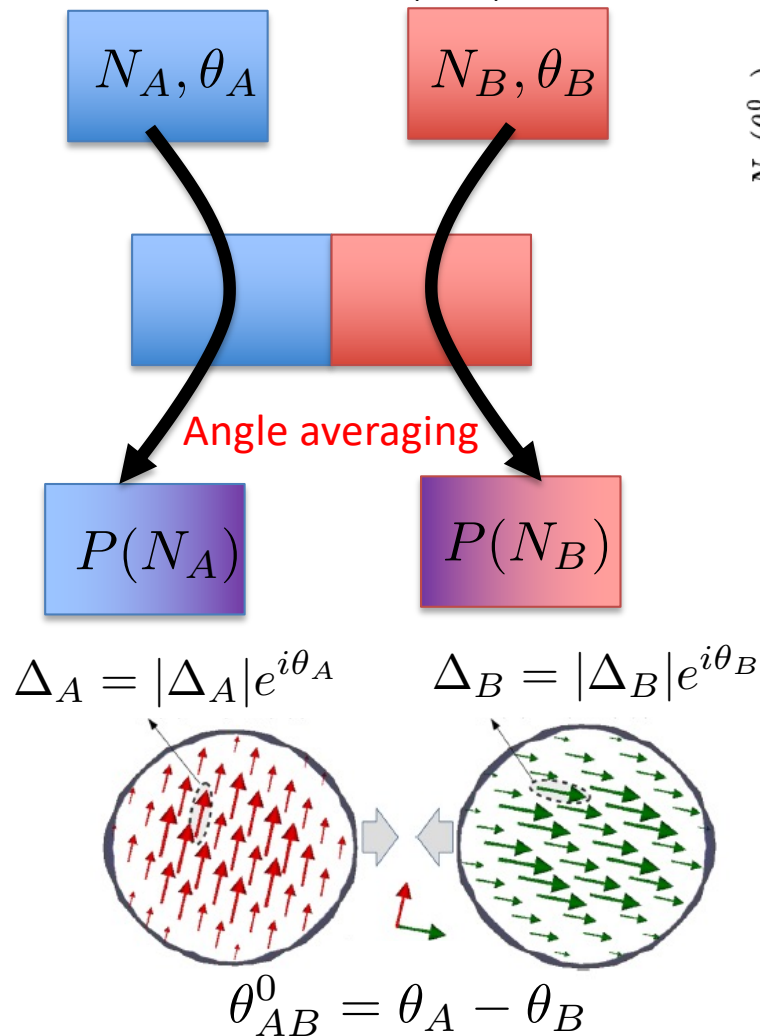
A minimal reaction model

Dietrich, PLB 32 (1970).

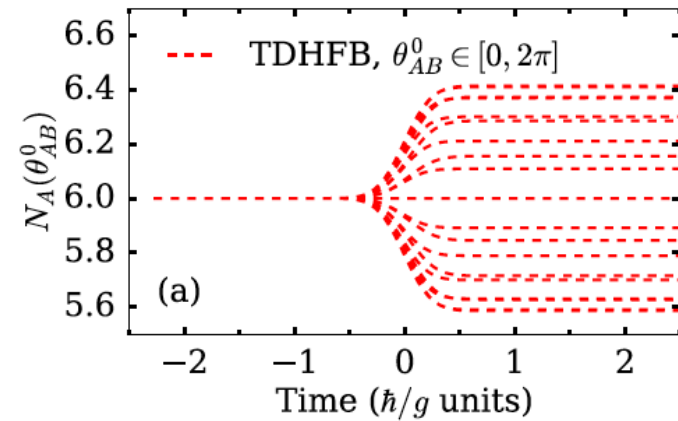


# A minimal reaction model

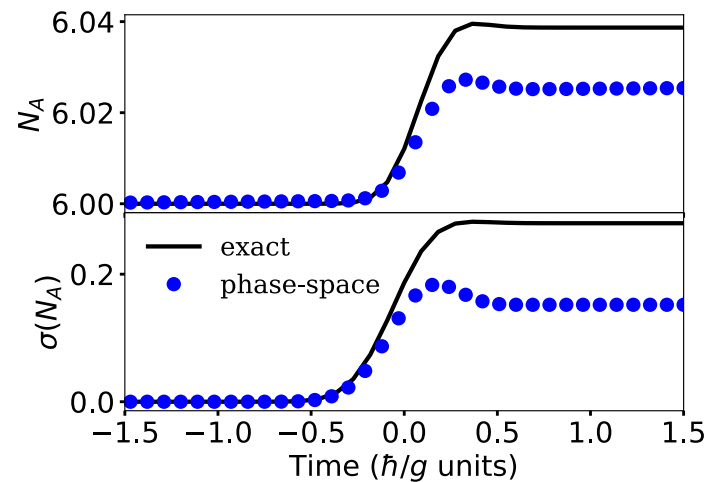
K. Dietrich, PLB 32 (1970).



Regnier, Lacroix, Phys. Rev. C 97 (2018)



Semi-classical  
Phase-space  
distribution



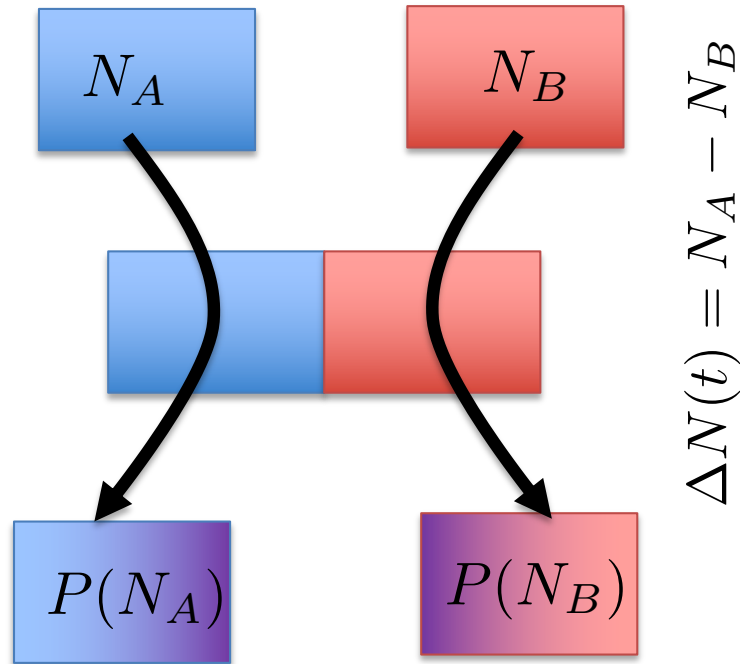
The naïve phase-space picture does not work so well

# Emergent dynamical pairing phenomena close to the Coulomb barrier

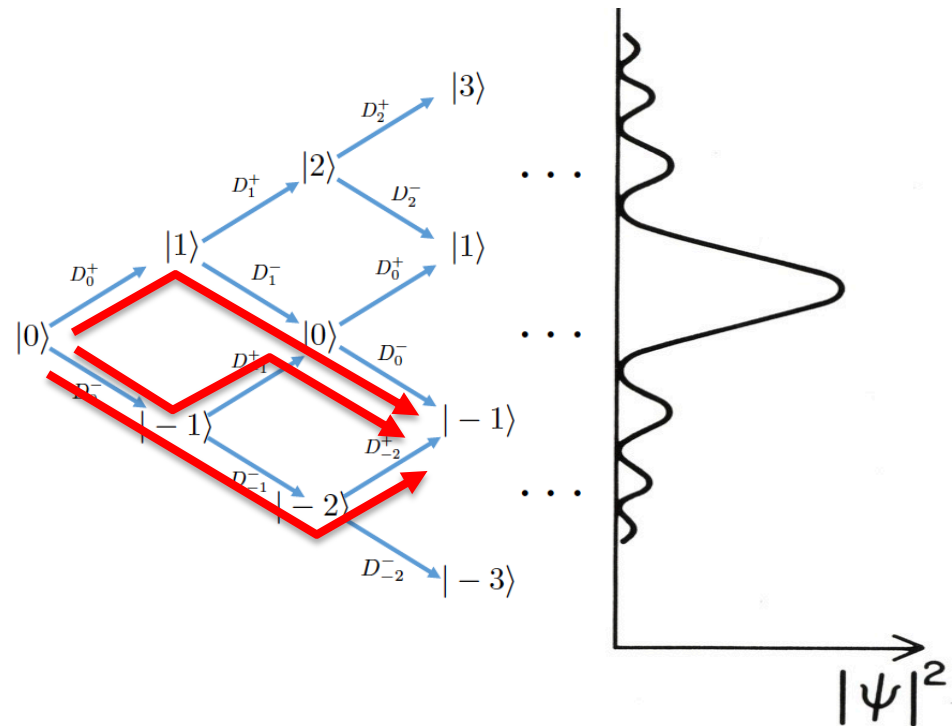
## Effects beyond the mean-field

### A minimal reaction model

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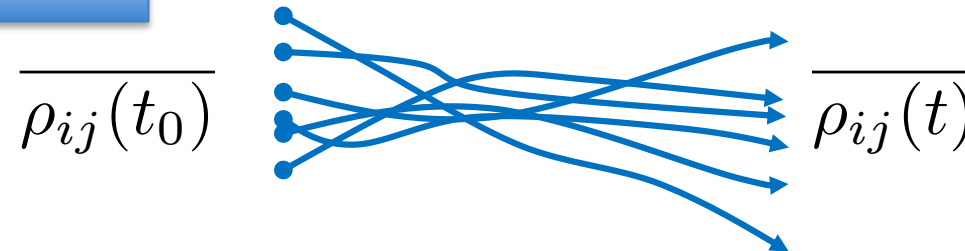


→ The naïve mean-field picture does not work

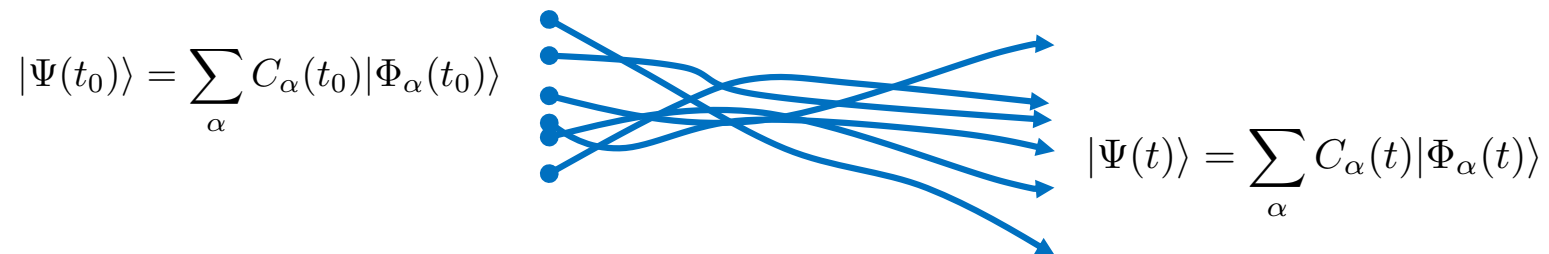


Mean-field trajectories interfere with each other and should be re-quantized.

Phase-space methods



Dynamique beyond the phase-space: the TDGCM



$$i\hbar\partial_t|\Psi\rangle = i\hbar\sum_{\alpha}\dot{C}_{\alpha}|\Phi_{\alpha}(t)\rangle + i\hbar\sum_{\alpha}C_{\alpha}(t)|\dot{\Phi}_{\alpha}(t)\rangle$$

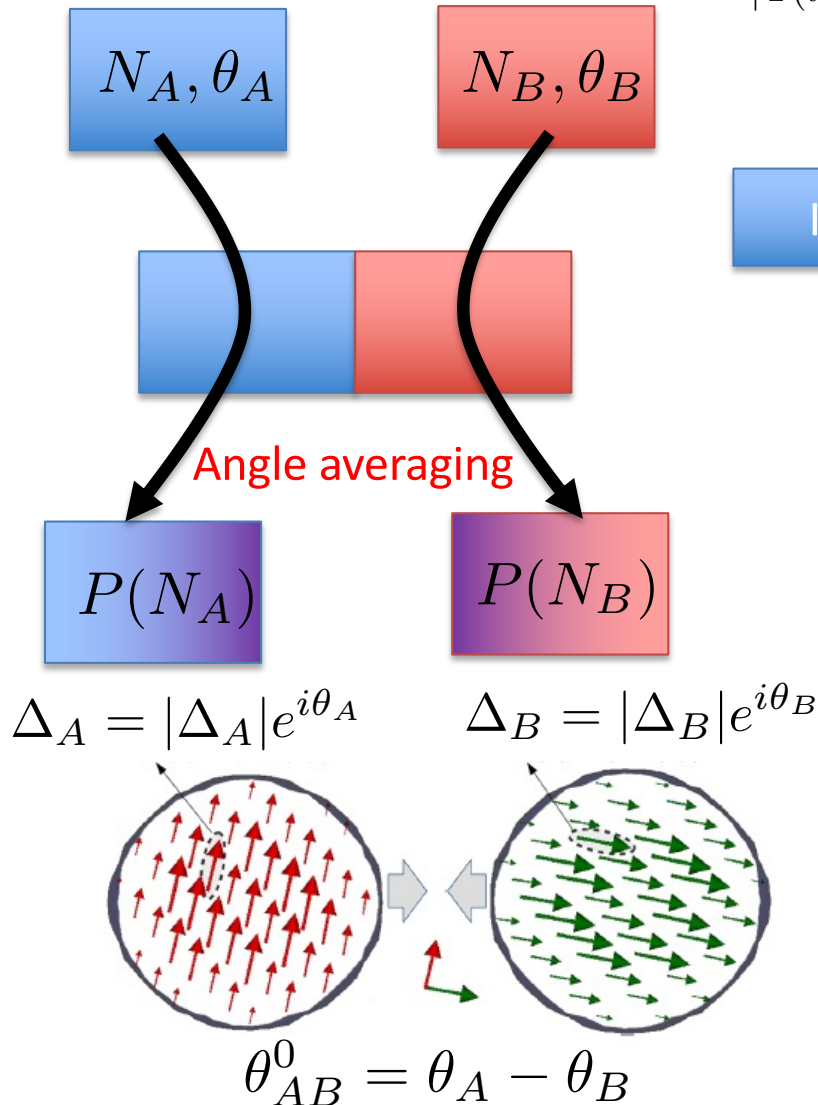
Assumed to be independent TDHF or TDHFB trajectories

+ solution of the Hill-Wheeler equations

## Back to the transfer between two superfluid systems

## A minimal reaction model

K. Dietrich, Phys. Lett. B 32 (1970).



$$|\Psi(t_0)\rangle = \sum_{\alpha} C_{\alpha}(t_0) |\Phi_{\alpha}(t_0)\rangle$$

$$|\Psi(t)\rangle = \sum_{\alpha} C_{\alpha}(t) |\Phi_{\alpha}(t)\rangle$$

## Initial states

$$|\Phi(t_0)\rangle = \prod (U_L + V_L a_R^{\dagger} a_R) \otimes \prod (U_R + V_R b_L^{\dagger} b_L) |0\rangle$$

$$\uparrow e^{2i\theta_A}$$

$$\uparrow e^{2i\theta_B}$$

$$|\Psi(t_0)\rangle = P_{N_A} P_{N_B} |\Phi(t_0)\rangle$$

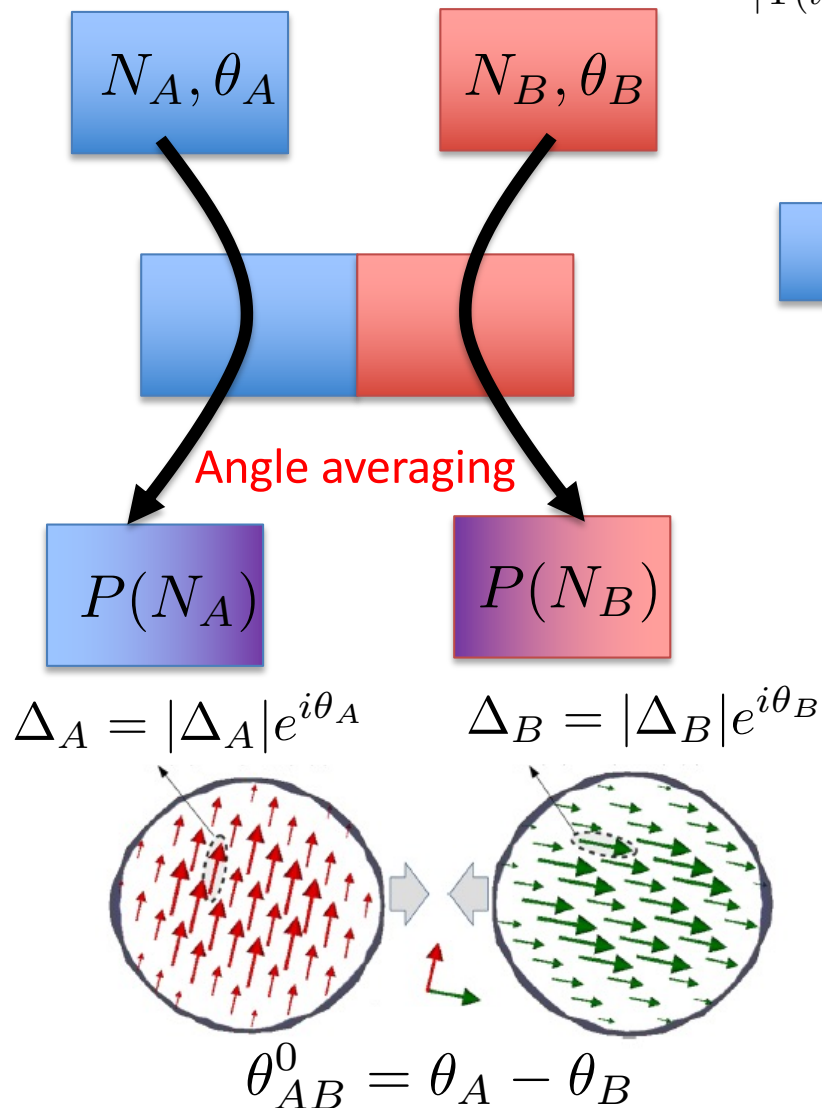
$$|\Psi(t_0)\rangle = \iint d\theta_A d\theta_B C_{\theta_A \theta_B}(t_0) |\Phi(\theta_A, \theta_B)\rangle$$

Coupled  
equationIndependent  
TDHFB  
evolution

## Back to the transfer between two superfluid systems

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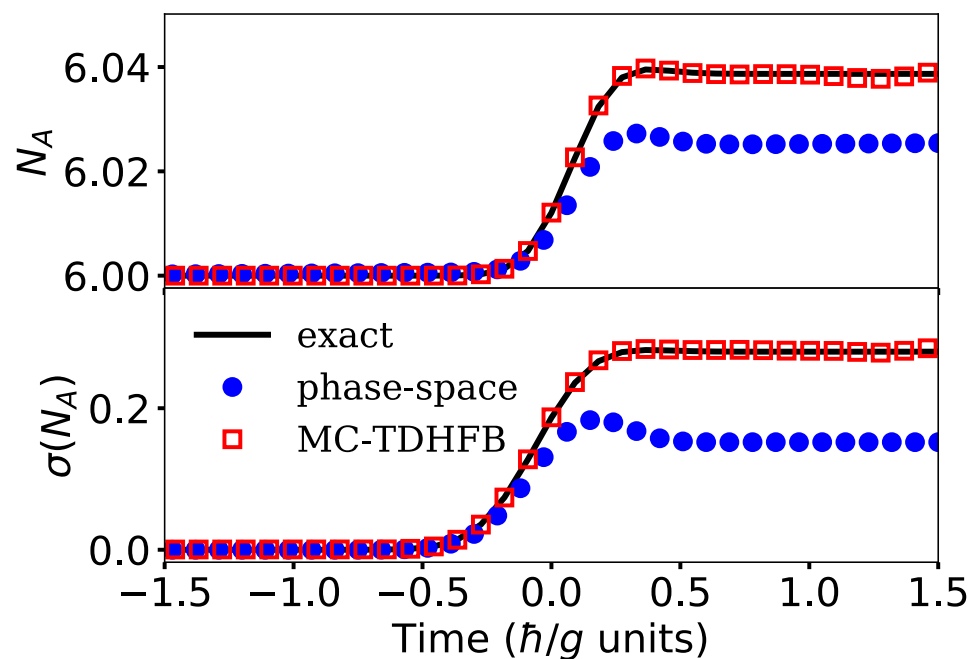


$$|\Psi(t_0)\rangle = \sum_{\alpha} C_{\alpha}(t_0) |\Phi_{\alpha}(t_0)\rangle$$

$$|\Psi(t)\rangle = \sum_{\alpha} C_{\alpha}(t) |\Phi_{\alpha}(t)\rangle$$

The diagram shows the evolution of the wavefunction components over time. It features a set of blue dots on the left, representing the initial components  $|\Phi_{\alpha}(t_0)\rangle$ , and a set of blue dots on the right, representing the final components  $|\Phi_{\alpha}(t)\rangle$ . Blue lines connect the dots, showing the mixing and evolution of the components over time.

## Results



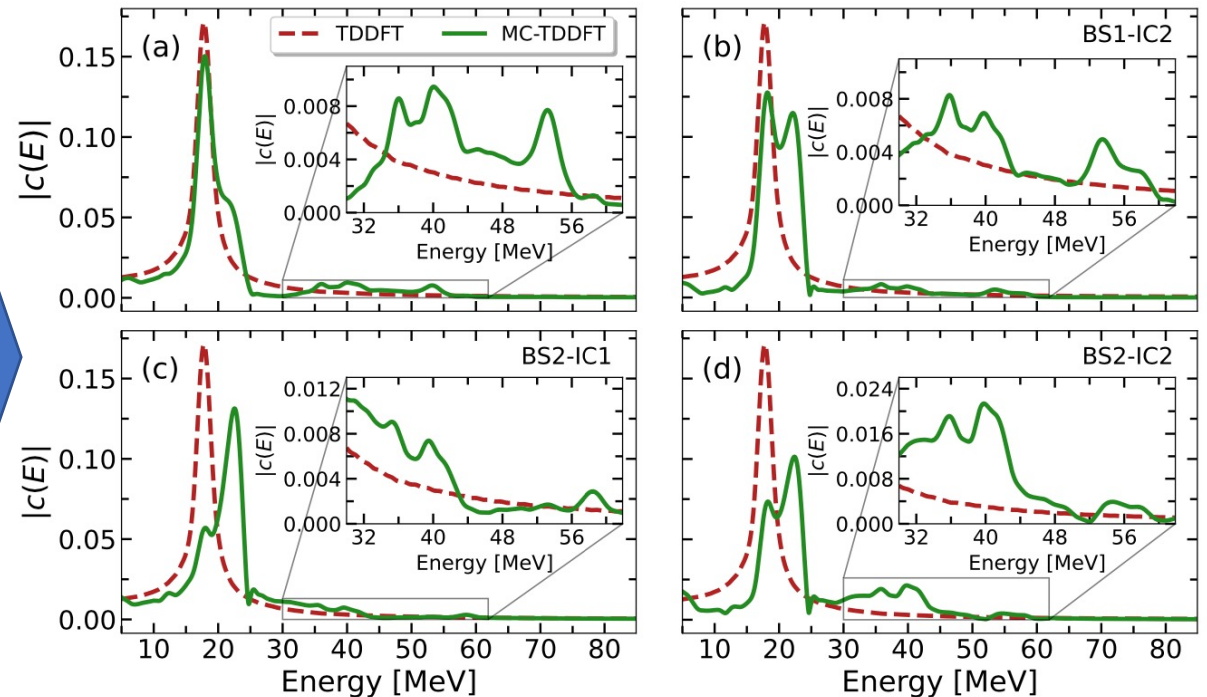
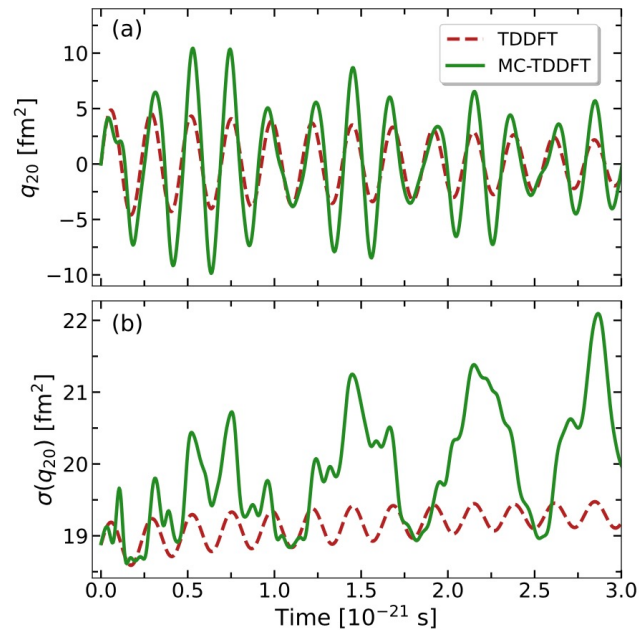
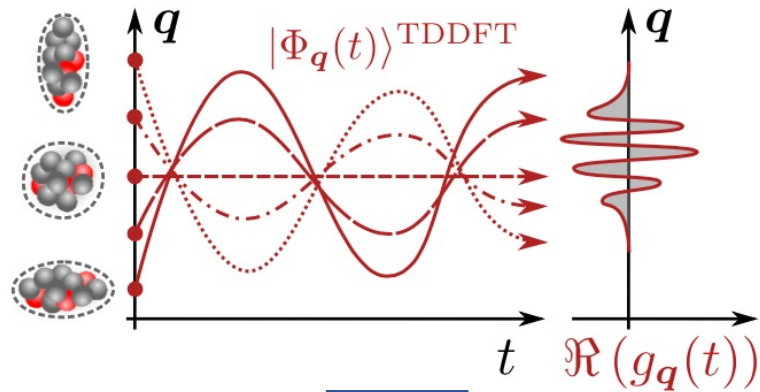
# Quantum fluctuations induce collective multiphonons in finite Fermi liquids

Petar Marević, David Regnier, and Denis Lacroix

Phys. Rev. C **108**, 014620 (2023) - Published 31 July 2023

(also Marevic et al, EPJA (2024))

Realistic application to atomic nuclei



$$S = \int_{t_0}^{t_1} dt \langle \Psi(t) | \hat{H} - i\hbar \partial_t | \Psi(t) \rangle$$

with

$$|\Psi(t)\rangle = \sum_{\mathbf{q}} f_{\mathbf{q}}(t) |\Phi_{\mathbf{q}}(t)\rangle$$

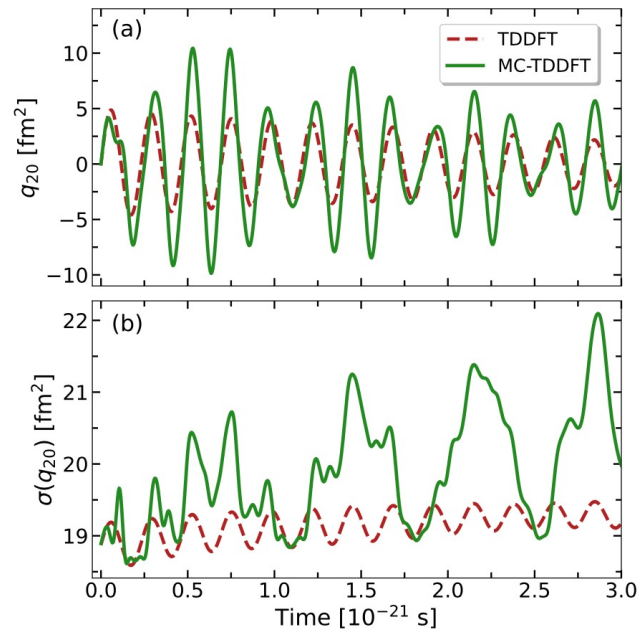
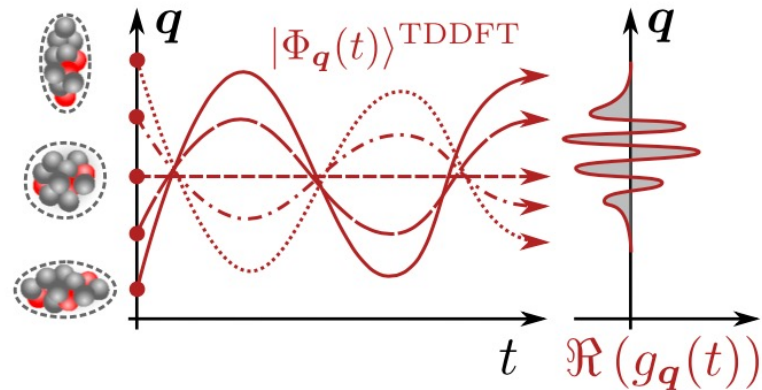
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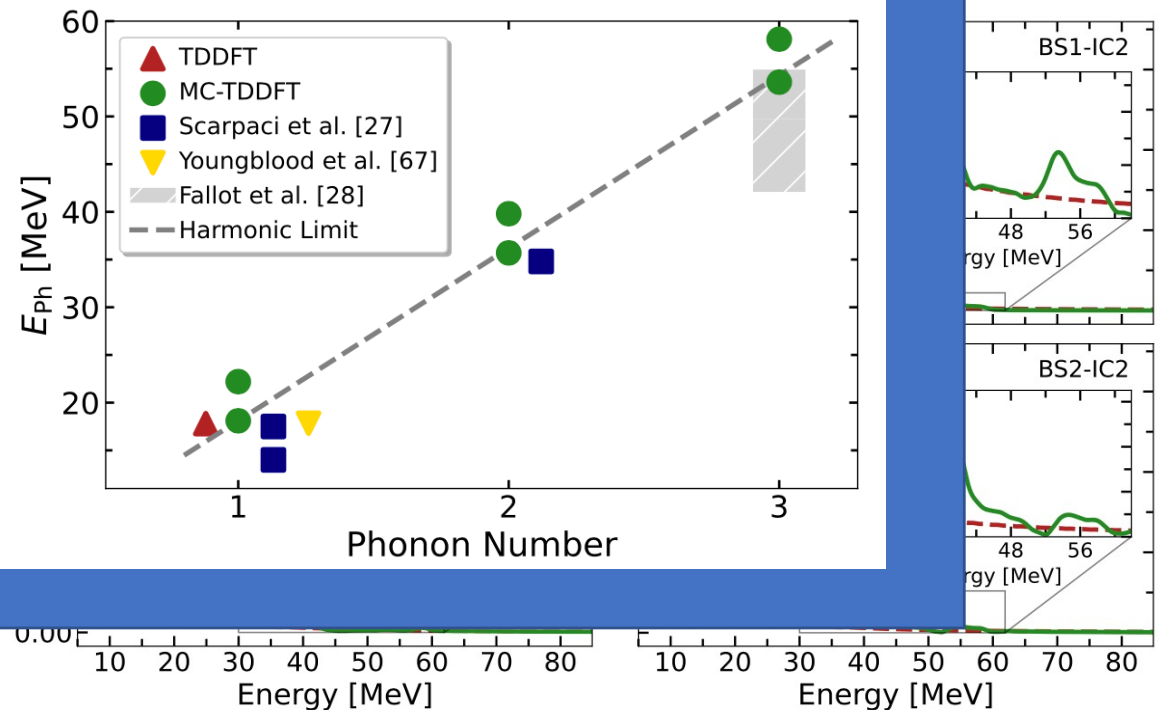
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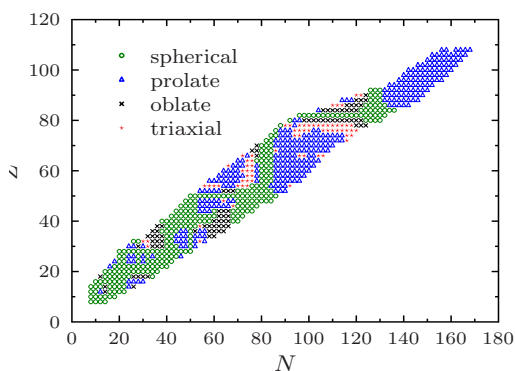
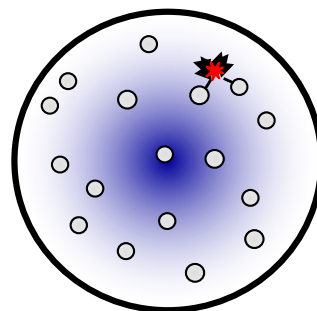
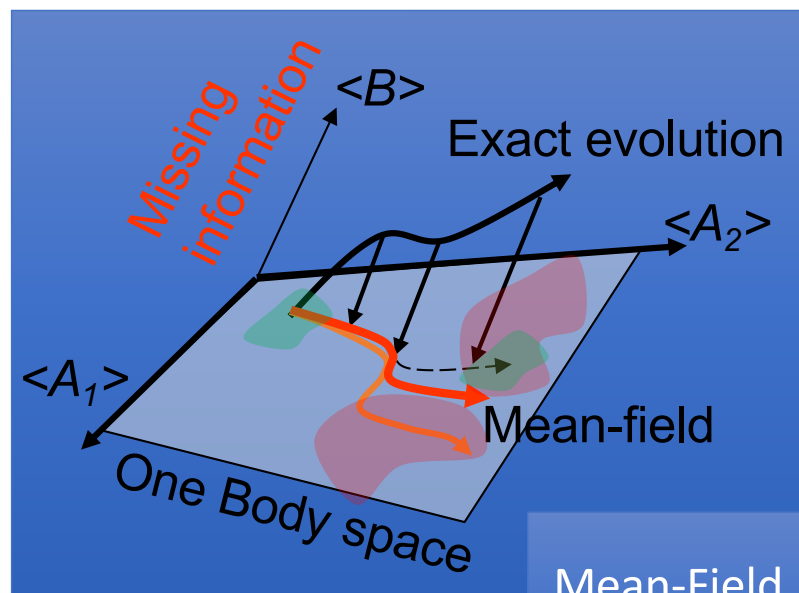


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Thank You !