Core excitation in one-neutron halo nuclei using halo effective field theory

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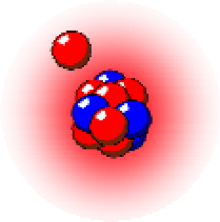
Halo nuclei

- Light, neutron-rich nuclei with large matter radius
- Low $\mathbf{S}_\mathrm{n}$ or $\mathbf{S}_\mathrm{2n}$: one or two loosely-bound neutrons
- Clusterised structure: neutrons can tunnel far from the core
  → halo-nucleus ≡ compact core + valence neutron(s)



- **Our case study** : $^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be} + \mathrm{n}$
- Short-lived → studied via reactions (e.g. **breakup**)
  → need of an **effective few-body** model for reaction calculations
  → **Halo-EFT**

Halo-EFT description of ^{11}Be

- Halo-structure → separation of scales
  → small parameter $\eta = \frac{R_{core}}{R_{halo}} \simeq 0.4 < 1$
  → expansion of the core-neutron Hamiltonian along η,
  i.e. reproducing the low-energy (viz. long distance) behaviour of the system
  [Bertulani, Hammer, van Kolck, NPA 712, 37 (2002)]
  Review: **[Hammer, Ji, Phillips, JPG 44, 103002 (2017)]**
- ^{11}Be $=^{10}$Be(0^+)+n [**core has no internal structure**]
  → **single-particle description**: $H(\mathbf{r}) = \mathrm{T}_{\mathbf{r}} + \mathrm{V}_{\mathrm{cn}}(\mathbf{r})$
- Effective Gaussian potentials in each partial wave ℓj @NLO ($\ell \leqslant 1$):

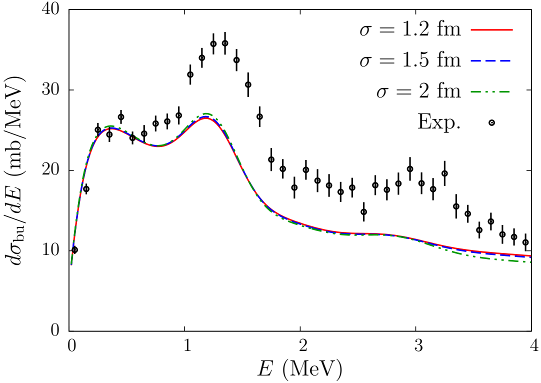
$$\mathrm{V}_{\mathrm{cn}}(\mathrm{r}) = \mathrm{V}^{(0)}_{\ell \mathrm{j}} \mathrm{e}^{-\frac{\mathrm{r}^2}{2\sigma^2}} + \mathrm{V}^{(2)}_{\ell \mathrm{j}} \mathrm{r}^2 \mathrm{e}^{-\frac{\mathrm{r}^2}{2\sigma^2}}$$

$\mathrm{V}^{(0)}_{\ell \mathrm{j}}$ and $\mathrm{V}^{(2)}_{\ell \mathrm{j}}$ fitted to reproduce:
→ $\mathbf{S}_{\mathrm{n}}$ & asymptotic normalization coefficient (**ANC**) for bound states
→ effective range parameters for continuum states

$\sigma :=$ cut-off → evaluates sensitivity to short-range physics

What is the problem ?

- Assumption: ^{10}Be remains in its 0^+ ground state still valid ?
  → Nuclear breakup: $^{11}\mathrm{Be}+\mathrm{C} \rightarrow {}^{10}\mathrm{Be}+\mathrm{n}+\mathrm{C}$



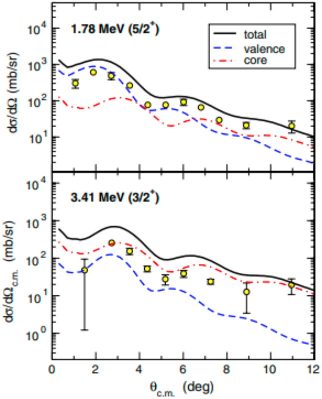
Exp: [Fukuda *et al.* PRC 70, 054606 (2004)]

Th.: **[L.-P. Kubushishi and P. Capel, arXiv:2406.10168 (2024)]**

⇒ Missing peaks at $\frac{5}{2}^+$ and $\frac{3}{2}^+$ resonances → is **s.p.** enough ?

Nuclear breakup & core excitation

- Origin of these missing strengths ? → a missing degree of freedom [^{10}Be(2^+)]
  ⇒ ^{10}Be core can be excited to its first 2^+state **[Moro** & **Lay, PRL 109, 232502 (2012)]**



- To better understand **structure effects on reaction calculations**

we have developed a **Halo-EFT** few-body model including **core excitation**

Core excitation within Halo-EFT

- **Extension of Halo-EFT to include core excitation:**

$$H(\mathbf{r},\xi) = \mathrm{T}_{\mathbf{r}} + \mathrm{V}_{\mathrm{cn}}(\mathbf{r},\xi) + \mathrm{h}_{\mathrm{core}}(\xi)$$

$\mathrm{h}_{\mathrm{core}}(\xi)$:= intrinsic Hamiltonian of the core with eigenstates $\chi_{\mathrm{I}}^{\mathrm{c}}(\xi)$

- **Halo-EFT particle-rotor model** [Bohr and Mottelson (1975)]:

$$\mathrm{V}_{\mathrm{cn}}(\mathbf{r},\xi) = \mathrm{V}_{\mathrm{cn}}(\mathrm{r}) + \beta\sigma \mathrm{Y}_2^0(\hat{\mathrm{r}})\frac{\mathrm{d}}{\mathrm{d}\sigma}\mathrm{V}_{\mathrm{cn}}(\mathrm{r})$$

- Set of radial **coupled-channel** Schrödinger equations:

$$\left[\mathrm{T}_{\mathrm{r}}^{\ell} + \mathrm{V}_{\alpha\alpha}(\mathrm{r}) + \epsilon_\alpha - \mathrm{E}\right]\psi_\alpha(\mathrm{r}) = -\sum_{\alpha'\neq\alpha} \mathrm{V}_{\alpha\alpha'}(\mathrm{r})\psi_{\alpha'}(\mathrm{r})$$

with $\mathrm{V}_{\alpha\alpha'}(\mathrm{r}) = \mathcal{Y}_\alpha(\hat{\mathrm{r}})\chi_\alpha(\xi)\mathrm{V}_{\mathrm{cn}}(\mathbf{r},\xi)\mathcal{Y}_{\alpha'}(\hat{\mathrm{r}})\chi_{\alpha'}(\xi)$, α=$\{\ell,\mathrm{s},\mathrm{j},\mathrm{I}\}$

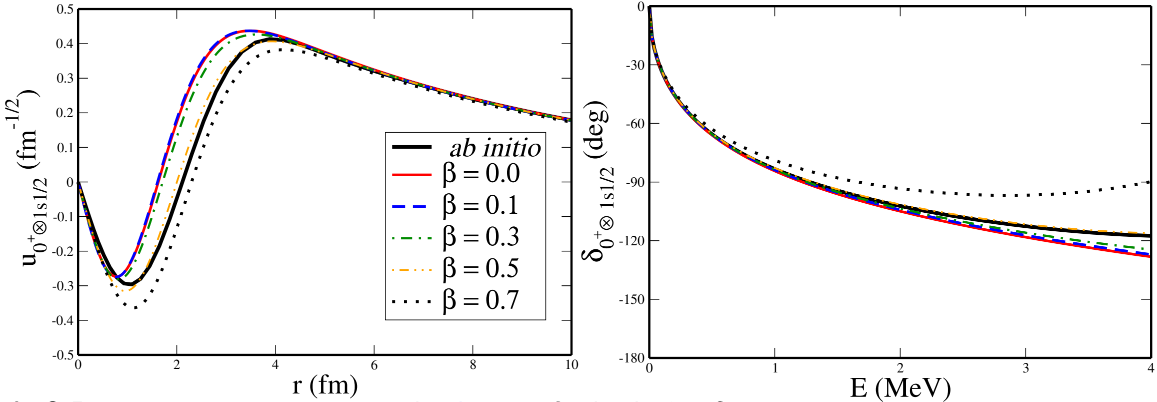
→ solved within the **R-Matrix method** on a Lagrange mesh
[D. Baye, Physics Reports 565 (2015) 1]

→ study impact of core excitation on: ψ_α, δ_α

Ground state: $\frac{1}{2}^+$

Compare to *ab initio* predictions [Calci et al., PRL 117, 242501 (2016)]

- $\Psi_{1/2^+} = \psi_{1s1/2}(\mathbf{r}) \otimes \chi_{0^+}^{^{10}\mathrm{Be}} + \psi_{0d5/2}(\mathbf{r}) \otimes \chi_{2^+}^{^{10}\mathrm{Be}} + \psi_{0d3/2}(\mathbf{r}) \otimes \chi_{2^+}^{^{10}\mathrm{Be}}$
- NLO potentials **fitted to** reproduce **S**$_\mathrm{n}$ and *ab initio* **ANC** for $\neq \beta$

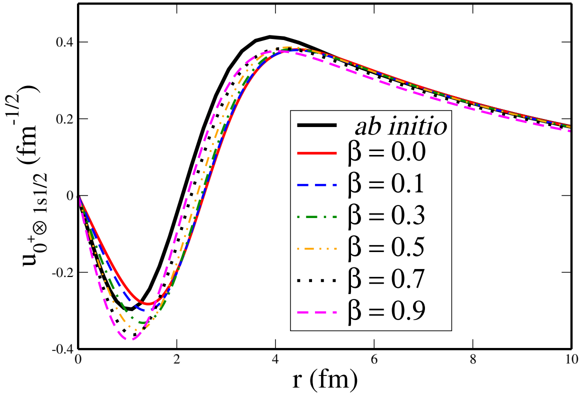


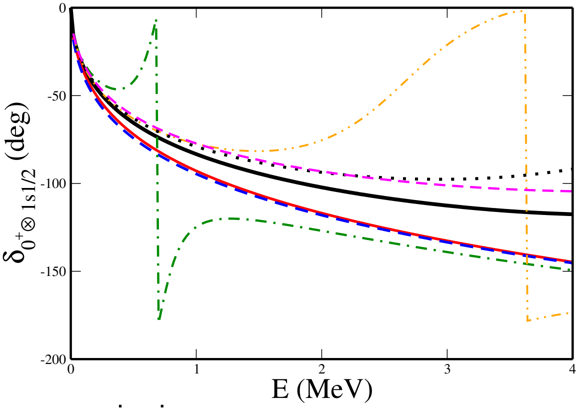
β=**0.5** in excellent agreement with *ab initio* for both ψ_α, δ_α

⇒ Including core **dof** improves both ψ_α, δ_α with 1 added parameter: β

Ground state: $\frac{1}{2}^+$ - Type 2 solution

- $\Psi_{1/2^+} = \psi_{1s1/2}(\mathbf{r}) \otimes \chi_{0^+}^{^{10}\mathrm{Be}} + \psi_{0d5/2}(\mathbf{r}) \otimes \chi_{2^+}^{^{10}\mathrm{Be}} + \psi_{0d3/2}(\mathbf{r}) \otimes \chi_{2^+}^{^{10}\mathrm{Be}}$
- Another type of solutions can be found:
  → when potential hosts a $0\mathrm{d}$ **bound state** (expected in shell model)



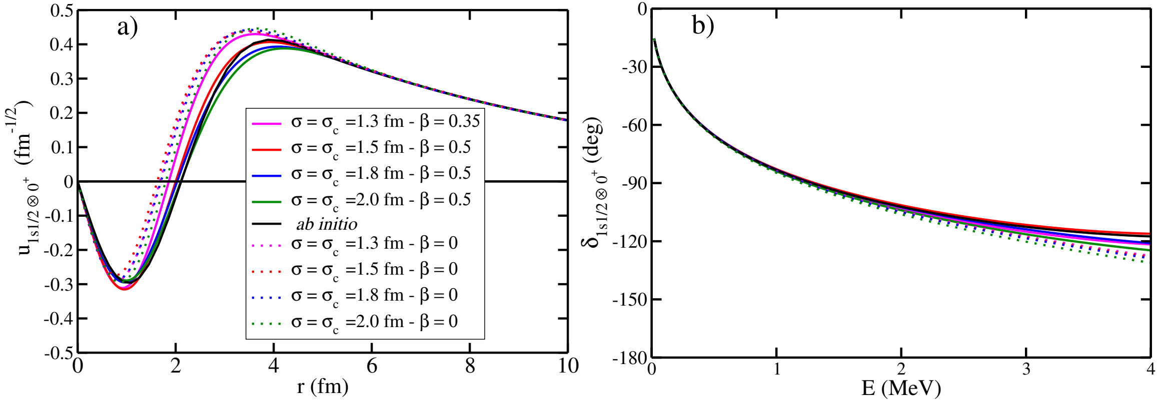


- Results less good than calculations without core excitation
  → this solution is rejected

Ground state and σ-dependency

Q: In the spirit of the Halo-EFT, are our calculations σ-independent ?

Idea: compare coupled-channel [**Type 1 solution**] to s.p. NLO results

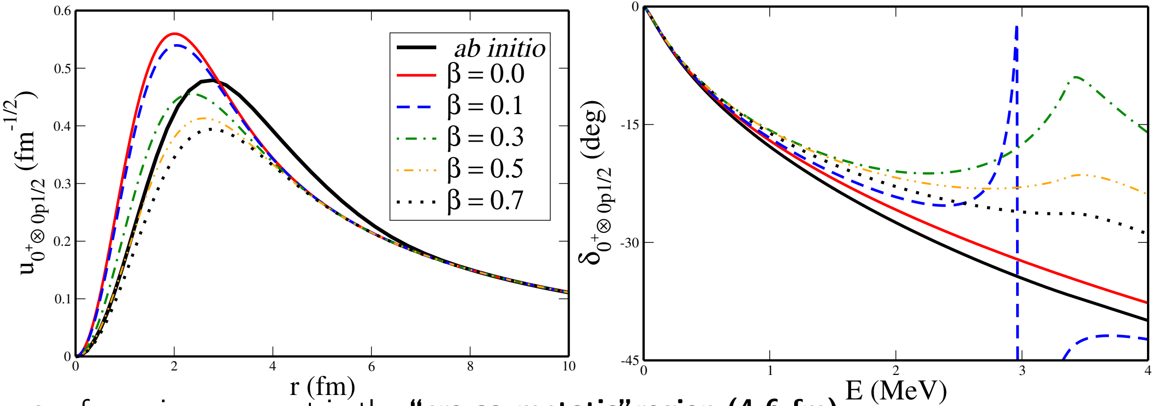


Coupled-channel calculations reduce σ-dependency for both ψ_α and δ_α

$\rightarrow$ **new degree of freedom decreases σ-dependency**

Bound excited state: $\frac{1}{2}^-$

- $\Psi_{1/2^-} = \psi_{0p1/2}(\mathbf{r}) \otimes \chi_{0^+}^{^{10}\mathrm{Be}} + \psi_{0p3/2}(\mathbf{r}) \otimes \chi_{2^+}^{^{10}\mathrm{Be}} + \psi_{0f5/2}(\mathbf{r}) \otimes \chi_{2^+}^{^{10}\mathrm{Be}}$
- NLO potentials **fitted to** reproduce $\mathbf{S}_\mathrm{n}$ and *ab initio* **ANC** for $\neq \beta$

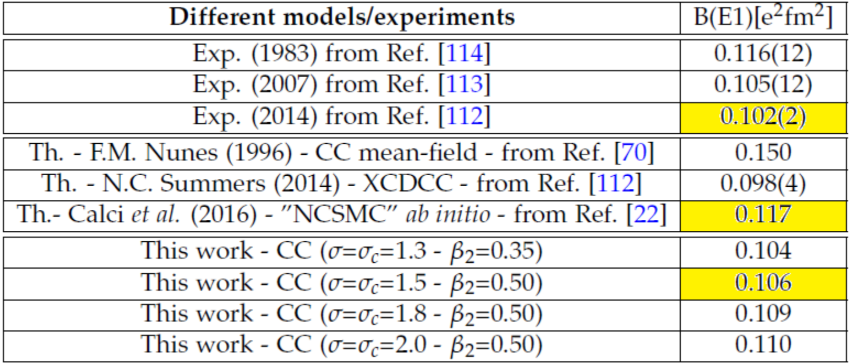


- wfs: no improvement in the **"pre-asymptotic" region (4-6 fm)**
- phase shifts: less good than without core excitation
- **No "type 2" solution** because $\mathrm{E}_{0p3/2}$ not at the right energy

Electric dipole transition probability: B(E1)

Observable to test our predictions: **E1** transition from bound state to bound state: $\frac{1}{2}^+ \to \frac{1}{2}^-$

$$\mathcal{B}(\mathrm{E}\lambda; \mathrm{i} \to \mathrm{f}) = \frac{2\mathrm{J_f}+1}{2\mathrm{J_i}+1} \left|\langle \mathrm{J_f} || \mathcal{M}(\mathrm{E}\lambda) || \mathrm{J_i} \rangle\right|^2 ,$$



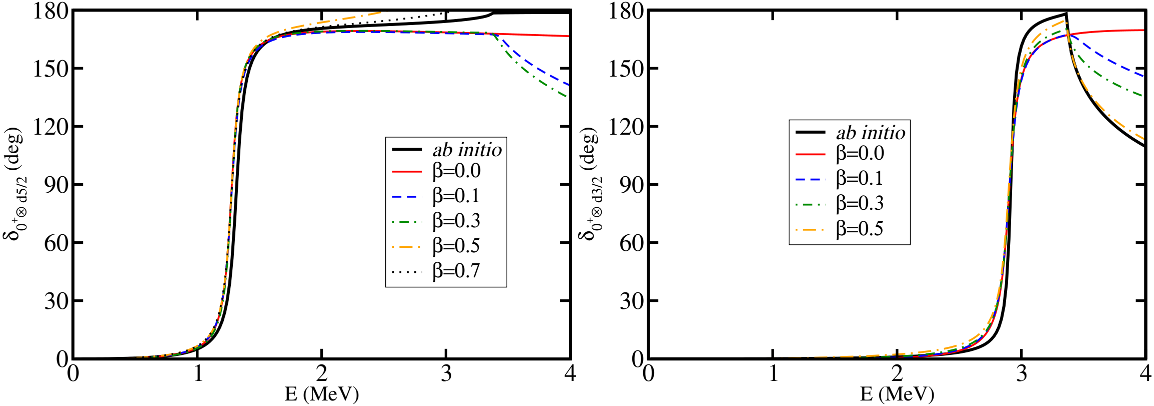
Different models/experiments	B(E1)[e^2fm^2]
Exp. (1983) from Ref. [114]	0.116(12)
Exp. (2007) from Ref. [113]	0.105(12)
Exp. (2014) from Ref. [112]	0.102(2)
Th. - F.M. Nunes (1996) - CC mean-field - from Ref. [70]	0.150
Th. - N.C. Summers (2014) - XCDCC - from Ref. [112]	0.098(4)
Th.- Calci *et al.* (2016) - "NCSMC" *ab initio* - from Ref. [22]	0.117
This work - CC ($\sigma=\sigma_c$=1.3 - β_2=0.35)	0.104
This work - CC ($\sigma=\sigma_c$=1.5 - β_2=0.50)	0.106
This work - CC ($\sigma=\sigma_c$=1.8 - β_2=0.50)	0.109
This work - CC ($\sigma=\sigma_c$=2.0 - β_2=0.50)	0.110

- Good agreement with experimental data & discrepancy with *ab initio* value
- Ab initio **overestimates** the strength of the transition

Resonances @NLO: $\frac{5}{2}^+$, $\frac{3}{2}^-$, $\frac{3}{2}^+$

Compare to *ab initio* predictions [Calci et al., PRL 117, 242501 (2016)]

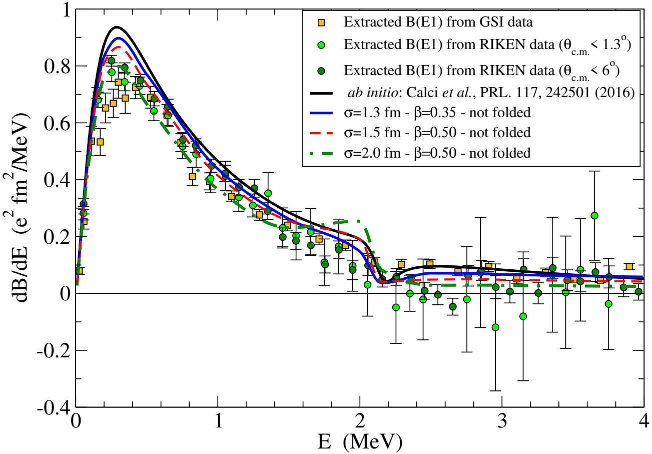
- NLO potentials **fitted to** reproduce exp. $\mathbf{E}_{\mathrm{res}}$ and Γ_{res} for $\neq \beta$



- Excellent agreement with *ab initio* results → probing **nature of resonances** $[\Gamma_{0^+}, \Gamma_{2^+}]$
- Direct access to scattering wfs, phase shifts → $\frac{\mathrm{dB(E1)}}{\mathrm{dE}}$, cross sections,...

B(E1) distributions

- **E1** transition from $\frac{1}{2}^+$ bound state to the continuum with **final-state interactions**



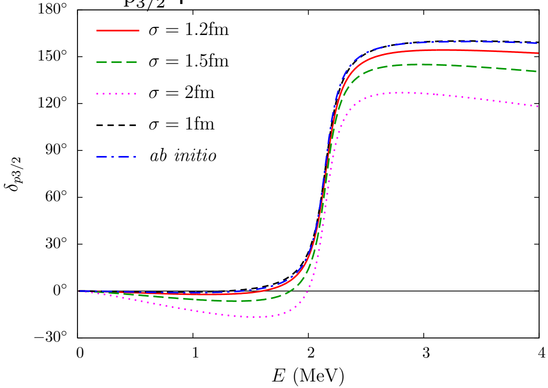
- **Good agreement** with exp. data reproduced but **overshoot at low** E (like *ab initio*)
- σ-dependency in $\mathrm{B(E1)}$ distributions: $\neq \sigma \rightarrow \neq$ scattering properties

s.p. NLO: σ-dependency in p-waves

Sensitivity already seen in NLO calculations:

[Capel, Phillips, Hammer, PRC 98, 034610 (2018)]

$\delta_{\mathrm{p}_{3/2}}$ phaseshifts



- @NLO: σ-dependency in δ

s.p. NLO: σ-dependency in p-waves

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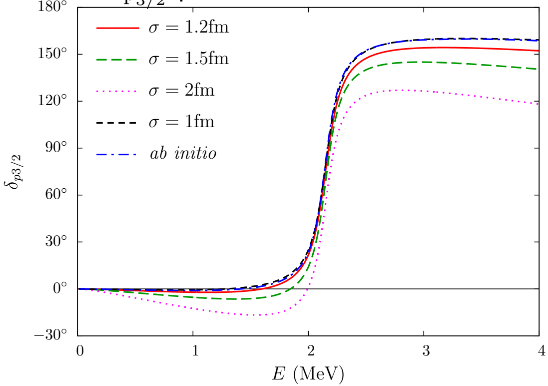
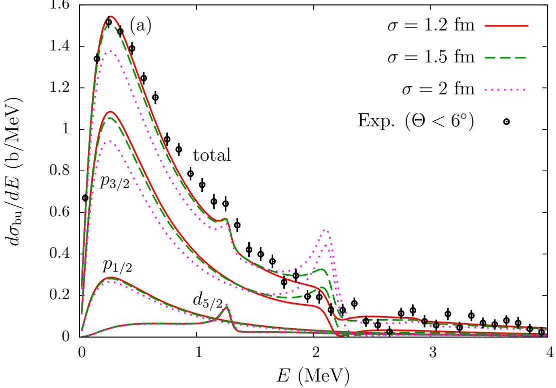


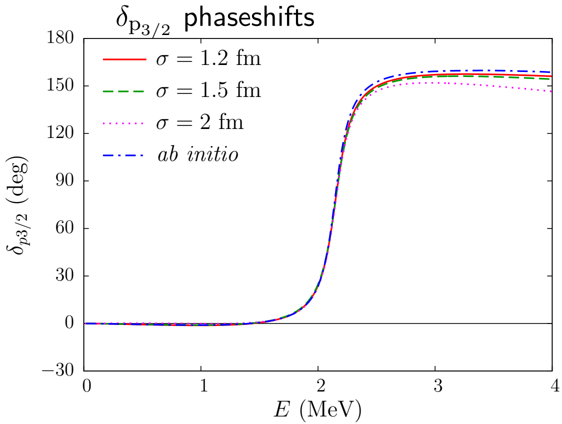
$^{11}\mathrm{Be} + \mathrm{Pb} \rightarrow {}^{10}\mathrm{Be} + \mathrm{n} + \mathrm{Pb}$

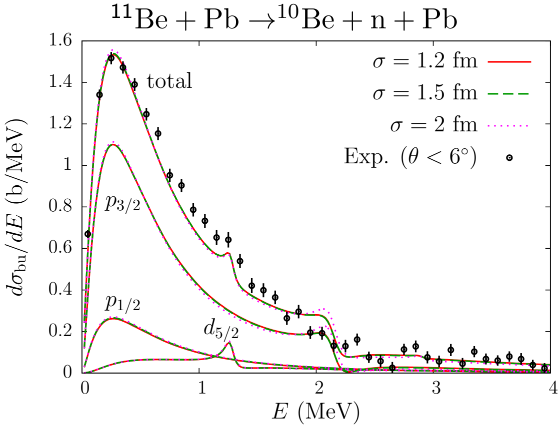


- @NLO: σ-dependency in δ leads to σ-dependency on cross sections
- @N^{2}LO: strong reduction of the σ-dependency in $\delta_{\mathrm{p}_{1/2}}$ and $\delta_{\mathrm{p}_{3/2}}$?

s.p. N^{2}LO: σ-dependency in p-waves

Description of ^{11}Be @N^{2}LO:



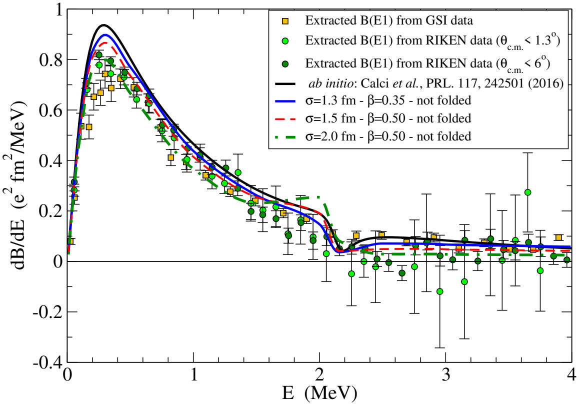


- Suppression σ-dependency in p-waves phaseshifts
  → same cross sections for all σ
  [L.-P. Kubushishi and P. Capel, arXiv:2406.10168 (2024)]

Coulomb breakup & Equivalent Photon Method (EPM)

- Coulomb breakup: $^{11}\text{Be}+\text{Pb} \rightarrow {}^{10}\text{Be}+\text{n}+\text{Pb}$ @69AMeV → **E1**-dominated
- **EPM:** cross section $\propto \mathrm{B(E1)}$ through number of equivalent photons $\mathrm{N_{E1}(E)}$:

$$\frac{\mathrm{d}\sigma}{\mathrm{dE}} = \frac{16\pi^3}{9\hbar c}\mathrm{N_{E1}(E)}\frac{\mathrm{dB(E1)}}{\mathrm{dE}}$$



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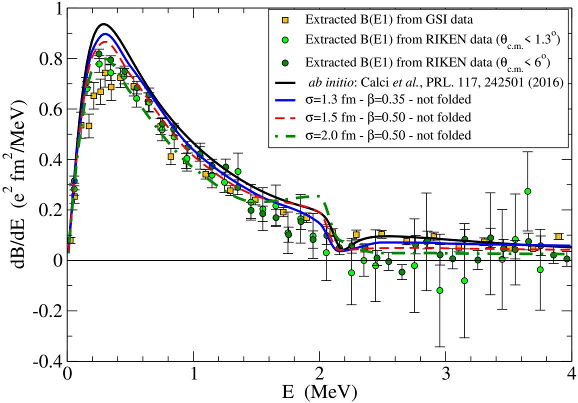

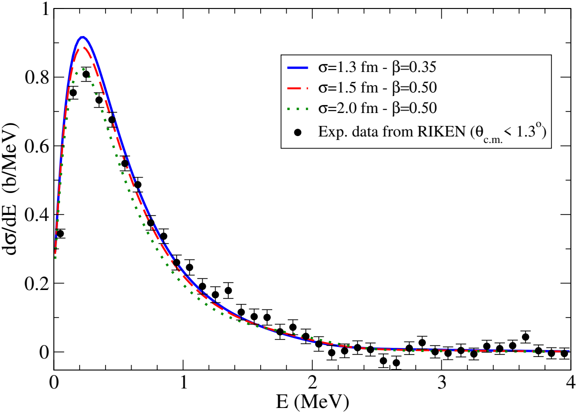

→ B(E1) distribution overshoots reflected on cross-sections (**which are folded**)

Conclusion

I want to study reactions involving **one-neutron halo nuclei** :

- need of a **realistic few-body** model for reaction calculations
  → Halo-EFT

My model of one-neutron halo nuclei [^{11}Be] provides:

- explicit inclusion of **core excitation within Halo-EFT**
- $\frac{1}{2}^{+}$ **state:** core excitation improves its few-body description
  → both wave function and phase shift
- $\frac{1}{2}^{-}$ **state:** core excitation does not improve its few-body description
- realistic description of low-lying resonances of ^{11}Be

[L.-P. Kubushishi and P. Capel, (2024), (in preparation)]
[L.-P. Kubushishi, (2024), (in preparation)]

Outlook:

- same formalism to study structure and breakup of ^{17}C (**Juan's talk**), ^{19}C, ^{31}Ne, ^{37}Mg,...
- development of an halo-EFT for light deformed nuclei (ongoing)
- include my model in reaction codes (**nuclear** breakup, knock-out,...)

Thanks to my collaborators!

- Pierre Capel (JGU Mainz)
- Daniel R. Phillips (Ohio University)