Resonance properties in light nuclei from structure methods: NCSM with complex scaling

Osama Yaghi
Supervisors: Marcella Grasso
Guillaume Hupin

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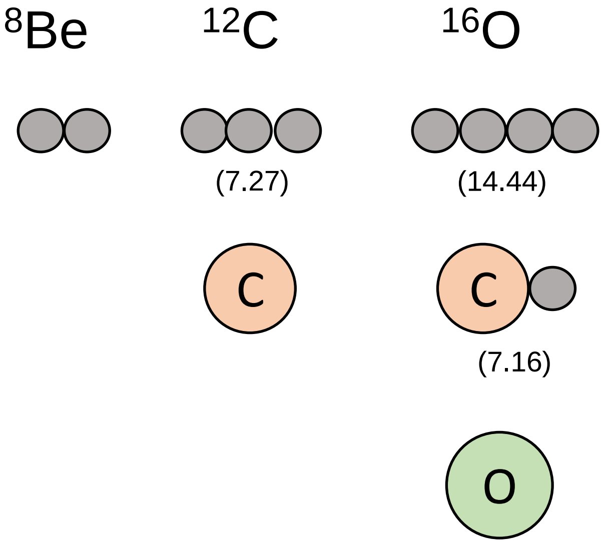
IJCLab, CNRS/IN2P3 & Université Paris-Saclay, Orsay, France

Connection with clustering

- Ikeda picture: clustering is connected to excited states near the threshold (Ikeda).
- Requires a theoretical tool that deals with structure and continuum.



Ideal tool:

- Nucleonic d.o.f .
- Ab-initio.

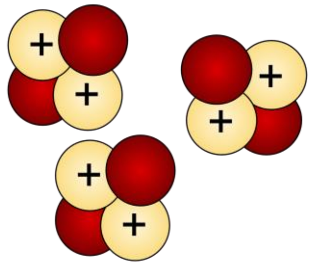
We want to develop a unique tool applicable to both nuclear structure and reactions, to enhance our understanding of the strong force at low energy.

Structure

Extraction of resonance properties for:

- Fine tuning interactions.
  ^{4}He
- Applications to astrophysics.
  ^{6}Li ^{11}Be ^{15}N
- Overlap with clustering wavefunction.

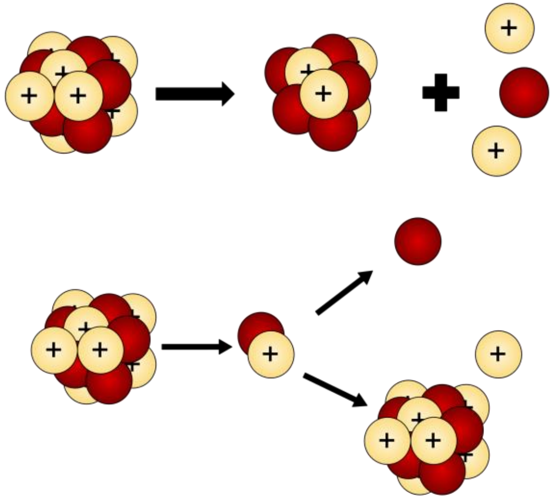
^{4}He ^{4}Li ^{4}H 4n



[Wiescher, M., Clarkson, O., deBoer, R.J. *et al. Eur. Phys. J. A* **57**, 24 (2021).]

Reactions

Enabling us to study nuclear breakup reactions including Final State Interaction (FSI) and calculation of complex charged (and multi-neutron) nuclear decay.



Outline

1. Method.

2. Proof of principle.
   1. A=2,A=3,A=4
3. Applications.
   1. ^{4}He ^{4}Li ^{4}H 4n

No Core Shell Model (NCSM)

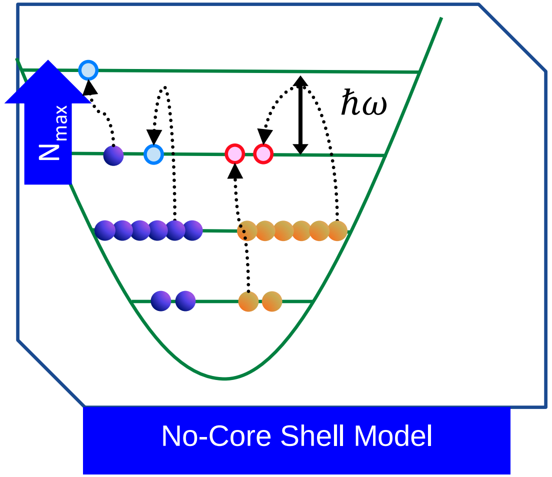
- Configuration Interaction (CI):
  - Eigen-value problem → Matrix diagonalization: $\hat{H}\phi_n = \varepsilon_n \phi_n$

$$\Psi^{(A)}_{NCSM} = |A\lambda J^{\pi} T\rangle = \sum_{\alpha} c_{\alpha} \, |A\alpha j_z{}^{\pi} t_z\rangle$$

Mixing coefficients (unknown)

A-body harmonic oscillator states

- No Core Shell Model (NCSM):
  - HO wavefunctions.
  - Center of mass is factorized.
  - Easy to transform from single particle basis to jacobi basis.
  - Best for well bound states!



Basis parameters: $\hbar\omega$, N_{max}

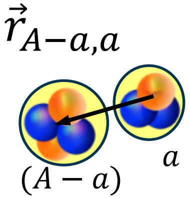
- Nucleonic d.o.f.
- Variational.
- Tracking uncertainties.

NCSMC (with continuum)

- RGM:
  - ➢ Introduce a reaction channel.
  - ➢ Treat relative motion with a continuous function.

$$\Psi^{(A)}_{RGM} = \sum_{\nu} \int d\vec{r}\, g_\nu(\vec{r})\, \hat{A}_\nu \left| \Phi^{(A-a,a)}_{\nu\vec{r}} \right\rangle$$

Relative wave function (unknown) — Antisymmetrizer — Channel basis



$$\psi^{(A-a)}_{\alpha_1} \psi^{(a)}_{\alpha_2}\, \delta(\vec{r} - \vec{r}_{A-a,a})$$

Cluster expansion technique

- NCSM with continuum (NCSMC):
  - ➢ For computing reactions and exotic nuclei.
  - ➢ Extend NCSM with RGM.

$$\left| \Psi^{J^\pi T}_{A} \right\rangle = \sum_\lambda c^{J^\pi T}_\lambda \left| A\lambda J^\pi T \right\rangle + \sum_\nu \int \mathrm{d}r\, r^2 \frac{\gamma^{J^\pi T}_\nu(r)}{r} \hat{\mathcal{A}}_\nu \left| \Phi^{J^\pi T}_{\nu r} \right\rangle$$

NCSM — RGM

Limitations:

- ➢ Resonances properties are not accessible directly.
- ➢ Reaction channels need to be introduced manually.

Complex Scaling (CS)

- Resonances are associated with complex poles of the S-matrix.
- The scattering solution associated with a resonance diverges.
- CS transform these solutions to become square integrable → accessible with bound-state methods, e.g. NCSM.
- definition of the CS operator $\hat{S}$:

$$\hat{S} f(r) = f\left(r e^{i\theta}\right)$$

$$\hat{S} = e^{i\theta \frac{\partial}{\partial r}}$$

Complex scaling transforms resonances into bound-state-like structure.

$$\phi_{sc}(r \to \infty) = A(k) e^{-ikr} + B(k) e^{+ikr}$$
$$\simeq e^{-ikr} + F(k) e^{+ikr}$$
$$S(k) = \frac{B(k)}{A(k)}$$

$$k_{res} = k_r - i k_i$$

↓

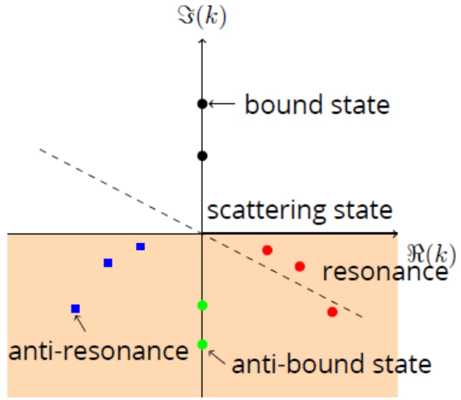
$$\phi_n^{res}(r \to \infty) = B(k_n) e^{+i|k_n| e^{-i\theta_c} r}$$
$$= B(k_n) e^{i a_n r} e^{+b_n r} \to \infty$$

$$\hat{H} \phi_n = (\varepsilon_n - (i/2)\Gamma_n) \phi_n$$

$$(\hat{S}\hat{H}\hat{S}^{-1})(\hat{S}\phi_n^{res}) = (\varepsilon_n - (i/2)\Gamma_n)(\hat{S}\phi_n^{res})$$

divergent

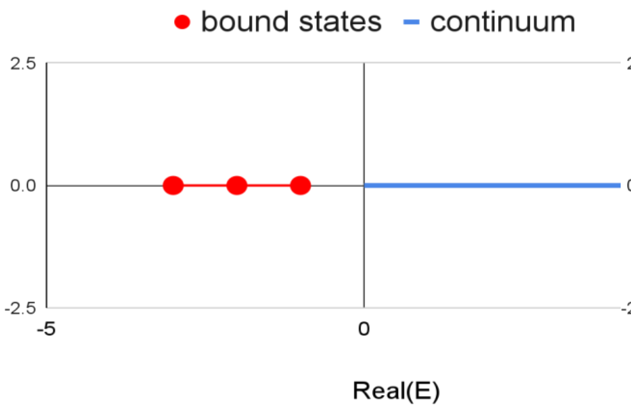
convergent

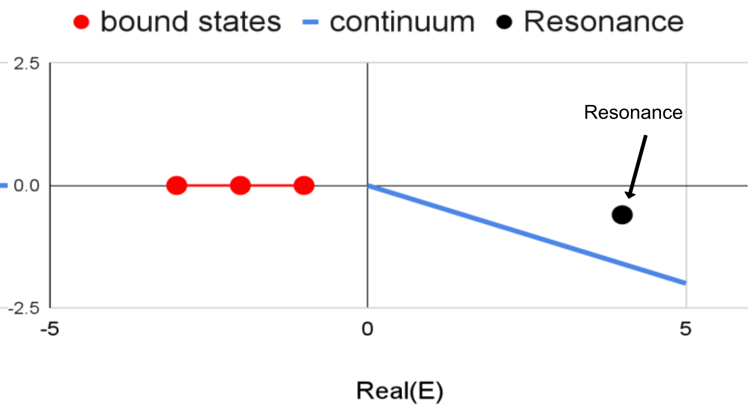


*[Aguilar, Balslev and Combes. Communications in Mathematical Physics, 22(4), 280–294.]

$$\hat{S}^{-1} = \hat{S}^{*}$$ Not unitary!

$$H_\theta = {H_\theta}^T$$ Not hermitian!



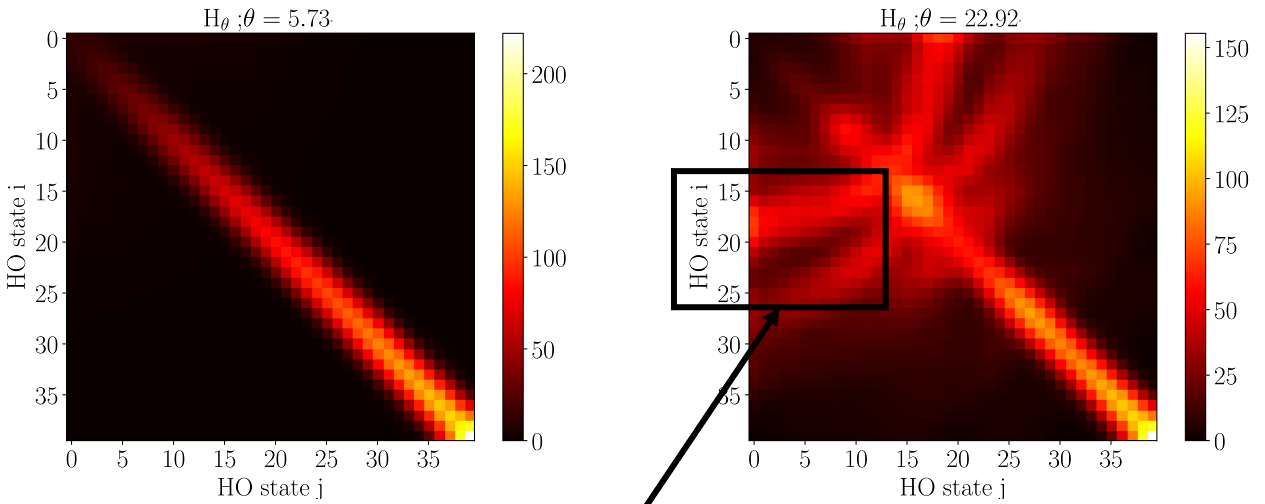


$$E_{res} = \varepsilon_n - i\frac{\Gamma_n}{2}$$

$$\theta_c = \frac{1}{2}\operatorname{atan}\left(\frac{\Gamma_n}{2\varepsilon_n}\right)$$

- Known problem with realistic NN interactions.
- Complex-Scaling leads to larger off-diagonal coupling .
- Slow convergence (Large N_{max} needed).

A=2 Hamiltonian matrix elements with complex values *



Large off-diagonal coupling

Off-diagonal coupling is problematic for many-body calculations!

* The absolute value of the elements are shown

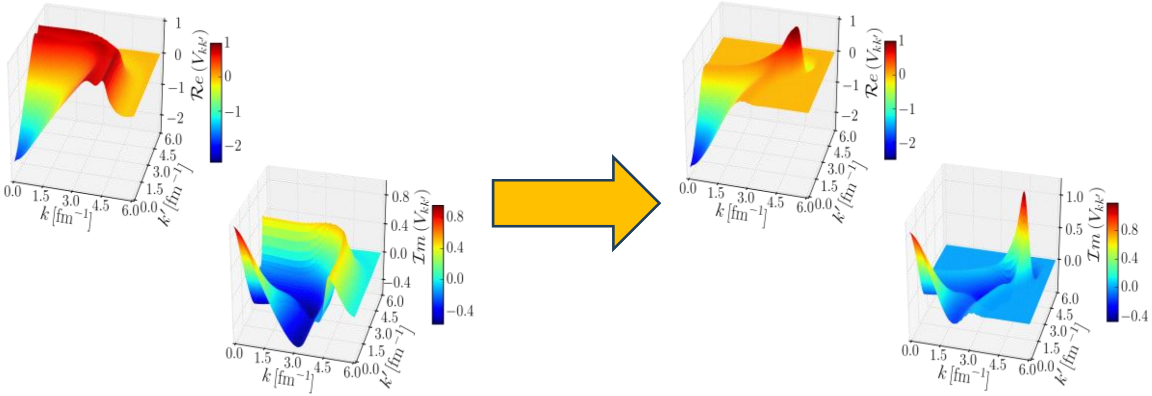
- We use SRG to soften the interaction.
- SRG is applied as a flow equation
- Unitarily transform the interaction (soften it).
- Disadvantages:
  - Induced interactions in the 3-body, 4-body … A-body space.

SRG works as well with CS Hamiltonians.

$$H_\lambda(\theta) = U_\lambda H(\theta) U_\lambda^T$$

Orthonormal transformation

$$\frac{dH_\lambda(\theta)}{d\lambda} = -\frac{4}{\lambda^5}[\eta(\lambda), H_\lambda(\theta)]$$



- Bound state and continuum follow the expected pattern.
- Previous application were limited to :

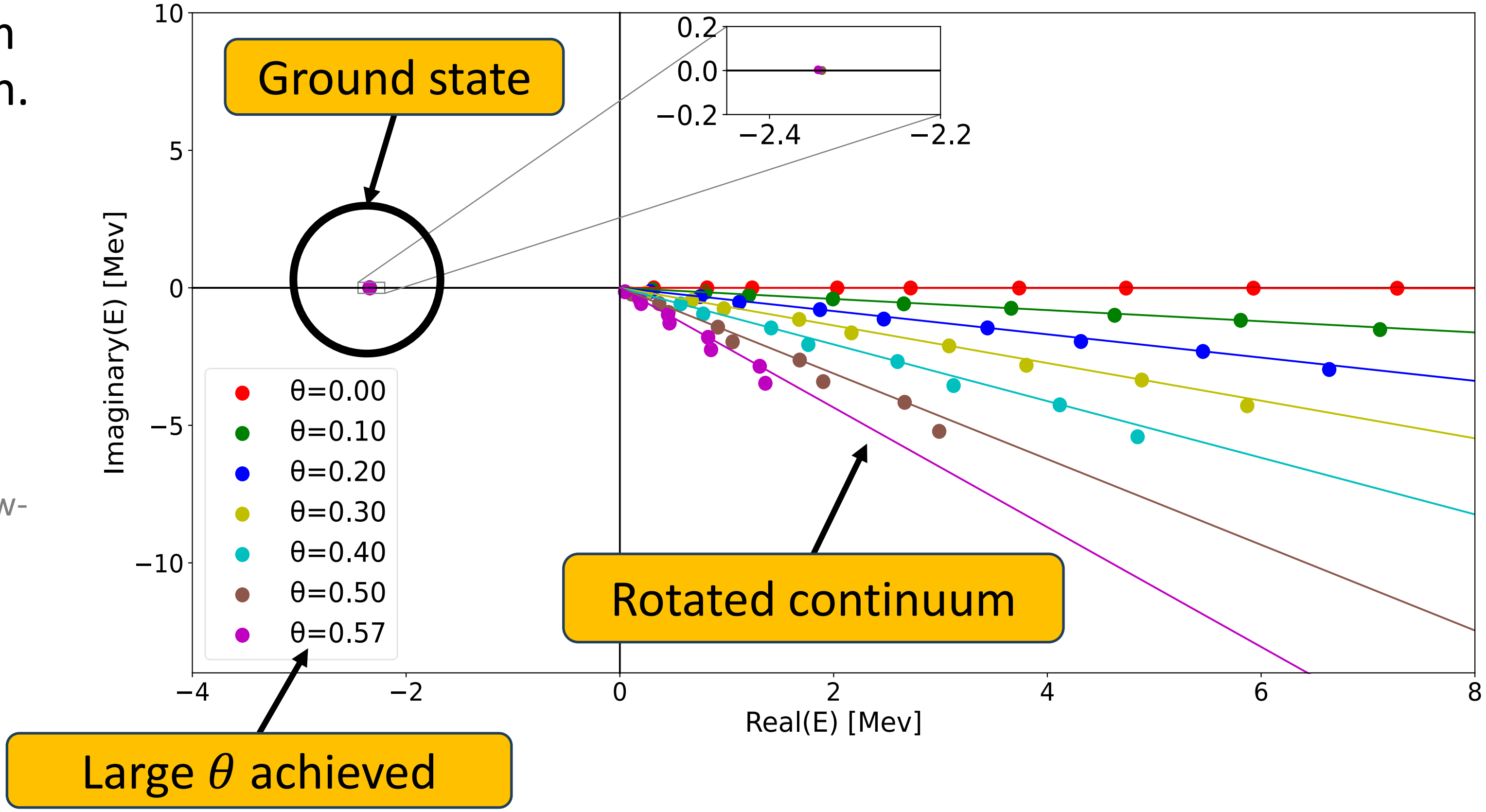
$$\theta < [0.16 - 0.3]\,rad.$$

[Papadimitriou, G. and Vary, J. P. Phys. Rev.C.91.2]

[Lazauskas, R., & Carbonell, J. (2013). Few-Body Systems, 54(7), 967-972.]

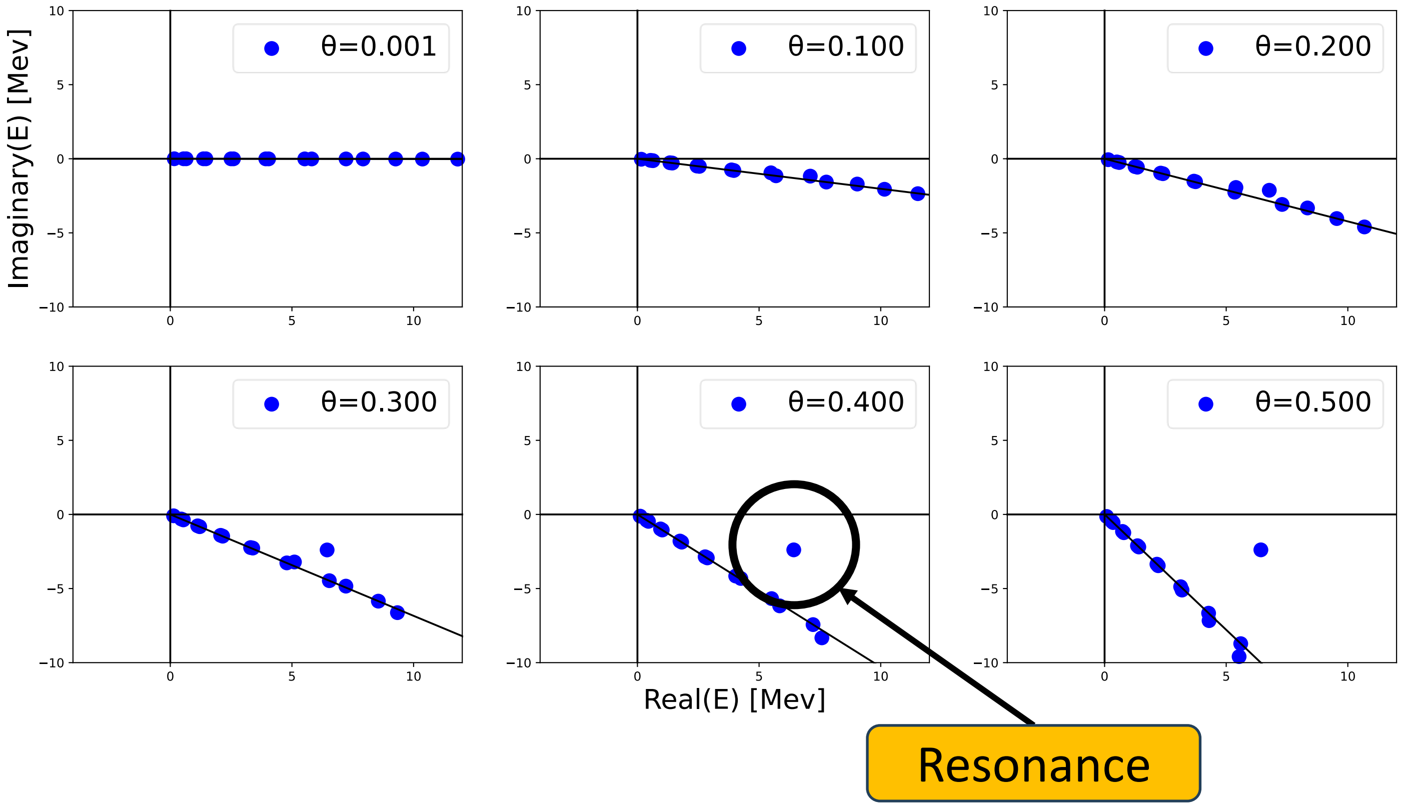
Large values of θ are possible!

A=2 CS-Hamiltonian eigenvalues



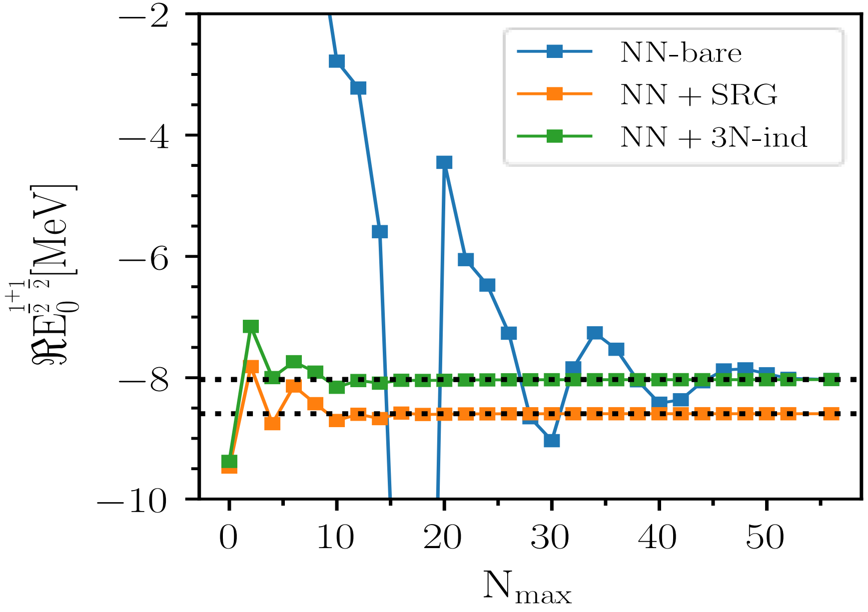
A=2 Toy model for resonance

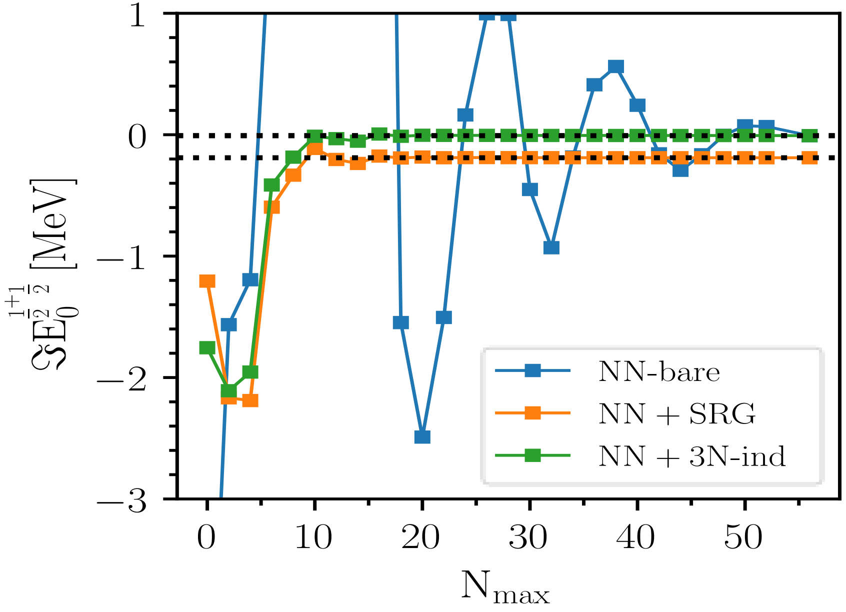
- An interaction with known resonance was used to test the approach.
- The resonance was extracted as θ exceeded the critical angle (θ_c) .



The approach works well with resonances!

3H Spectrum (T=1/2, J=1/2 , θ=0.3 rad)





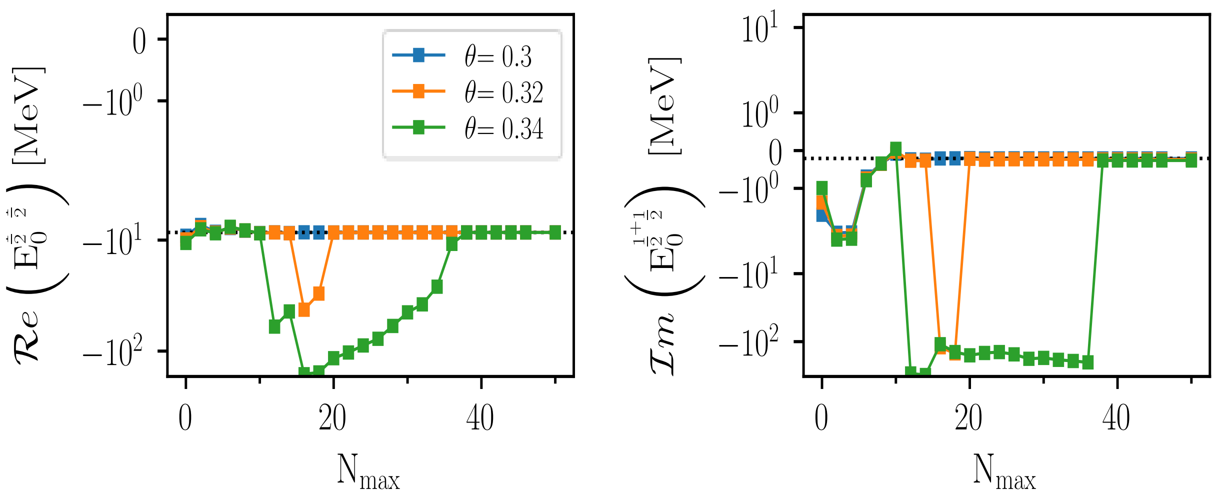
Conclusion:

- SRG leads to faster convergence.
- It works also with CS Hamiltonian.

Convergence issues

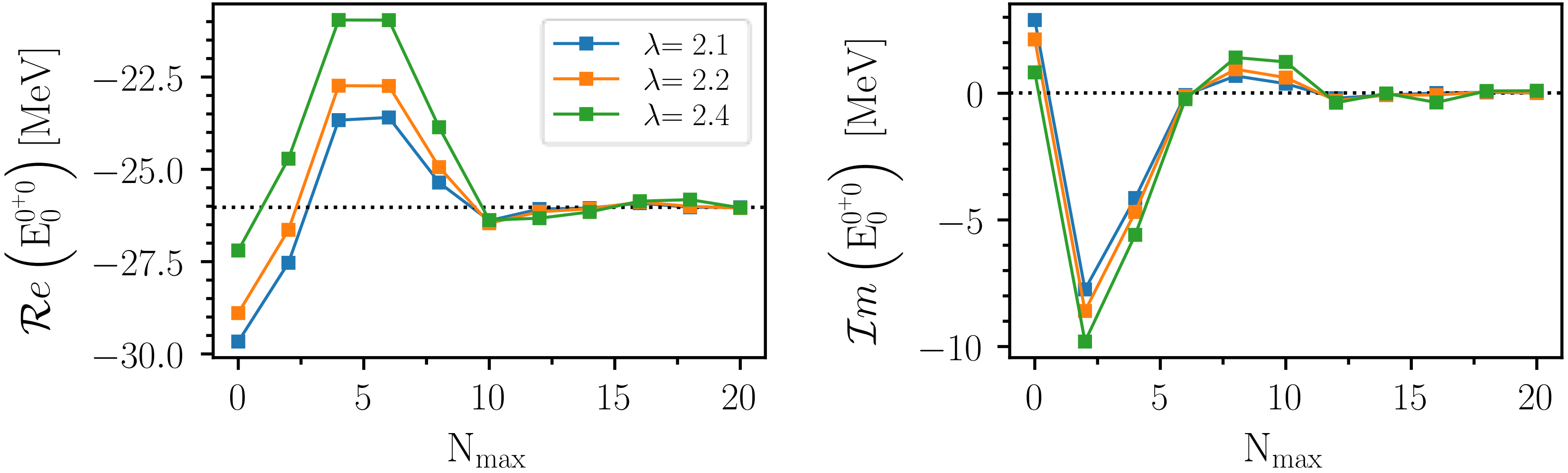
- A similar issue was reported in connection with large bound states (~100 Mev).
- Could be related to spurious resonances in NN interaction at high energy.

3H Spectrum (T=1/2, J=1/2 , θ=0.3 rad)



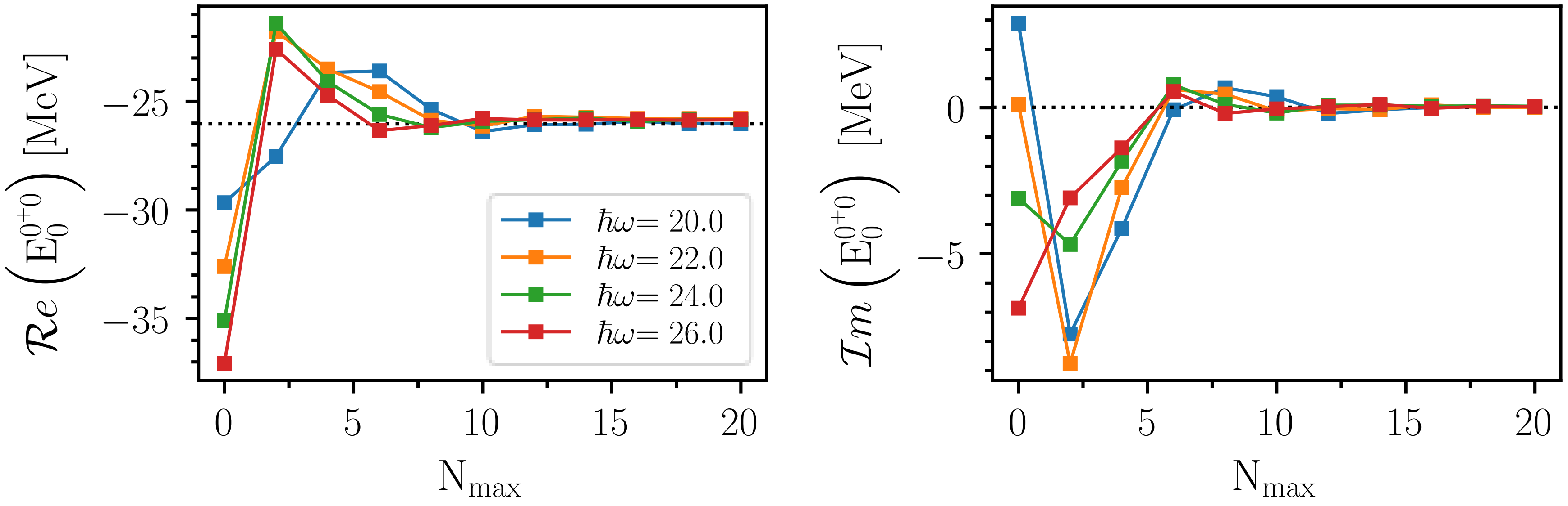
SRG limits us to below $\theta = 0.3$ rad.

^{4}He : 0^+ ground state, NN+3N-induced



Induced 4b-interaction is not significant here.

4He: 0^+ ground state

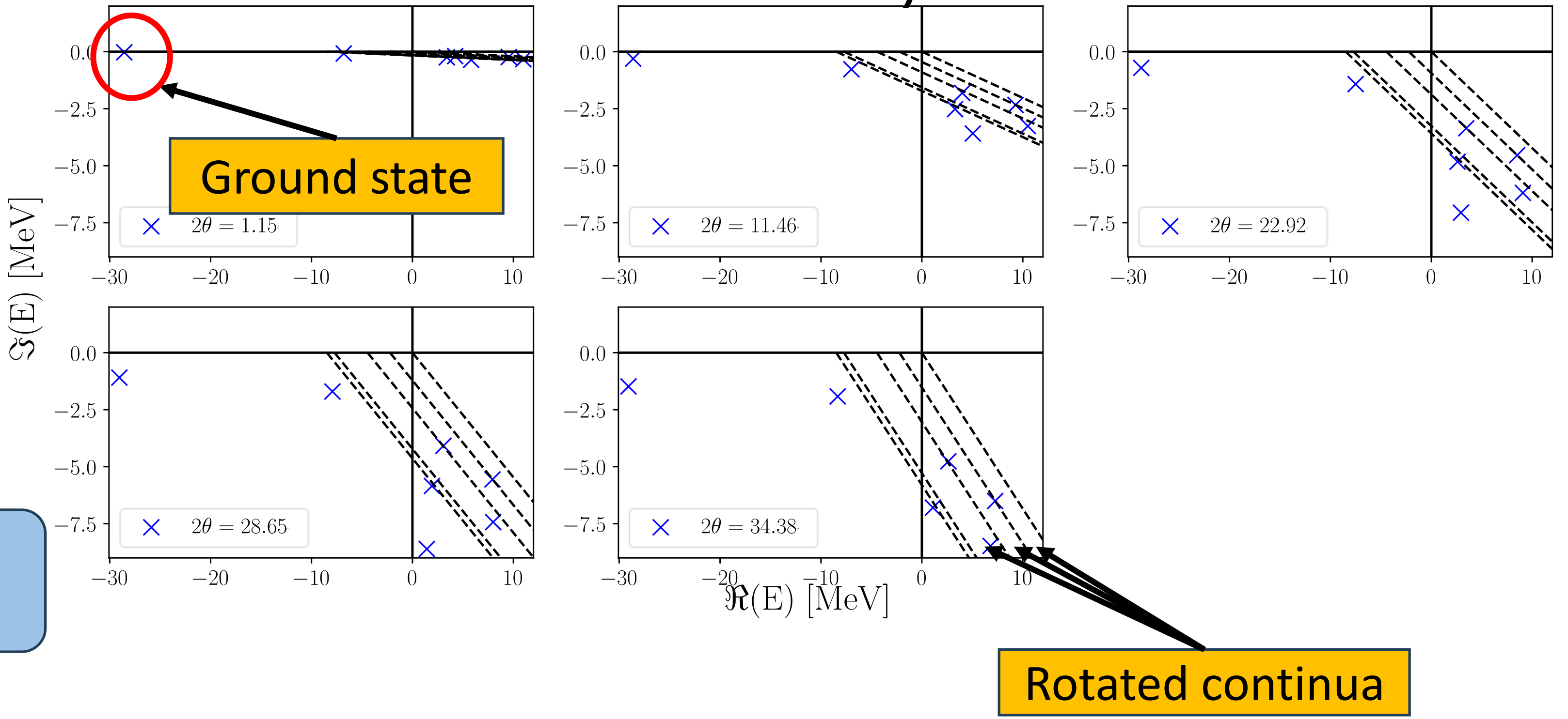


Well converged for different $\hbar\omega$

Spectrum of the alpha particle

- Multiple thresholds for different breakup/decay channel.
- Effects of SRG induced interactions are visible.

4He Spectrum ($J = 0, T = 0,\ \lambda = 2.0\ fm^{-1}$) using NN+SRG (no 3N-induced)

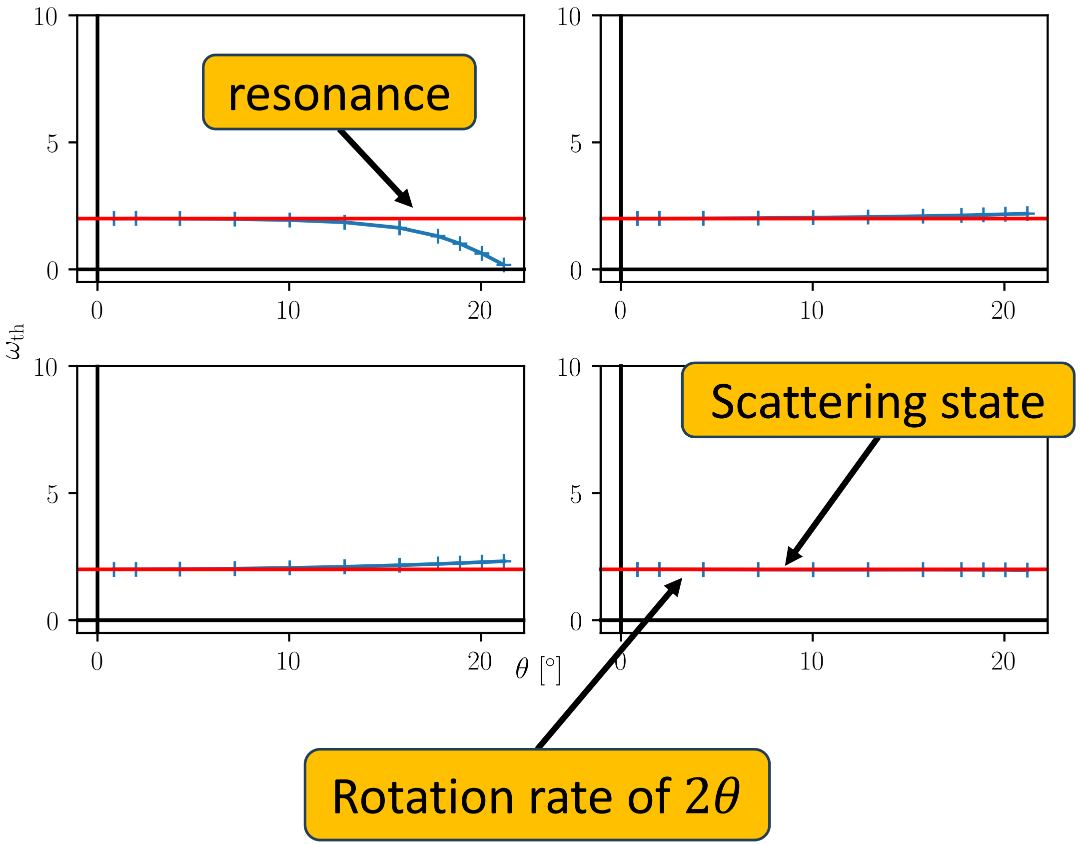


Identifying resonances is harder visually!

- We vary θ and track the change in eigen values.
- Scattering states rotate with the continuum at 2θ .
- Resonances rotate at 2θ below their critical angle.
- They become fixed for $\theta > \theta_c$.

It is necessary to go above θ_c !

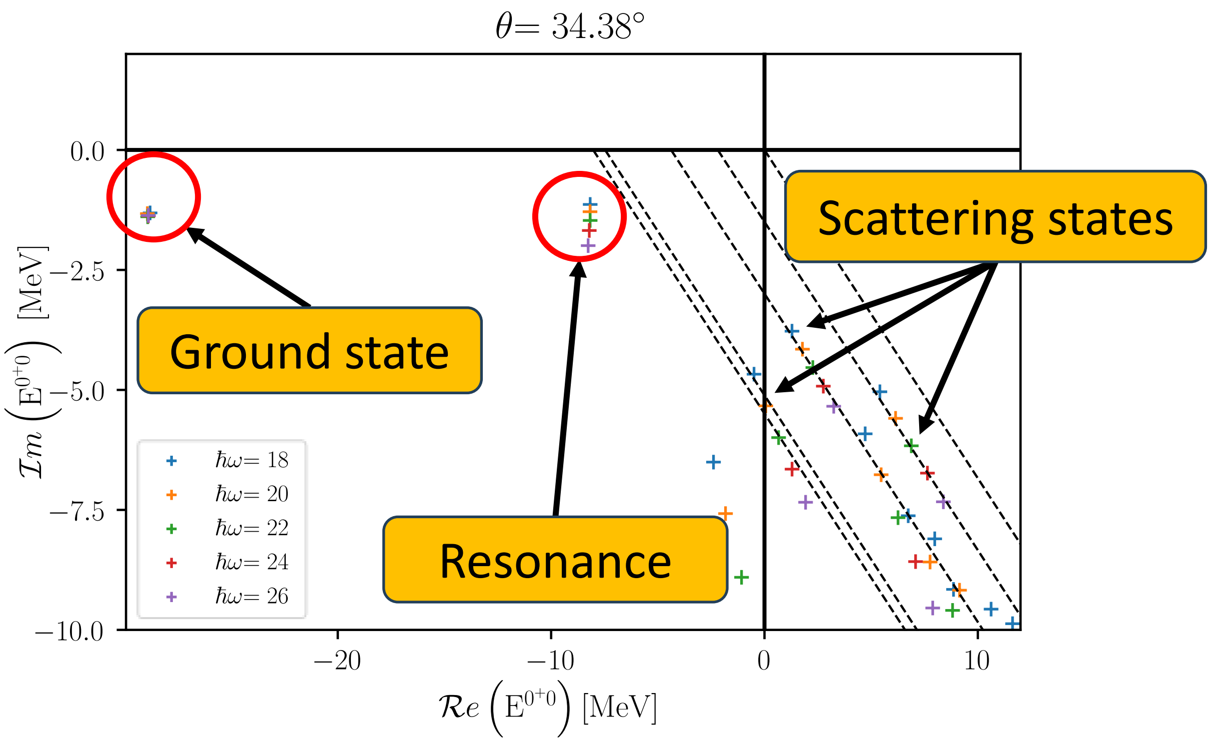
Rotation rate relative to the threshold



- We calculate the eigen values for different values of $\hbar\omega$.
- Bound states are relatively invariant to $\hbar\omega$.
- Scattering states vary significantly with $\hbar\omega$ and they move along the continuum line (2θ).
- Resonances show progressively less variation as we increase θ.

We can identify resonances below θ_c !

4He 0^+ Spectrum ($J = 0, T = 0,\ \lambda = 2.1\ fm^{-1}$) using NN+SRG (no 3b-induced)

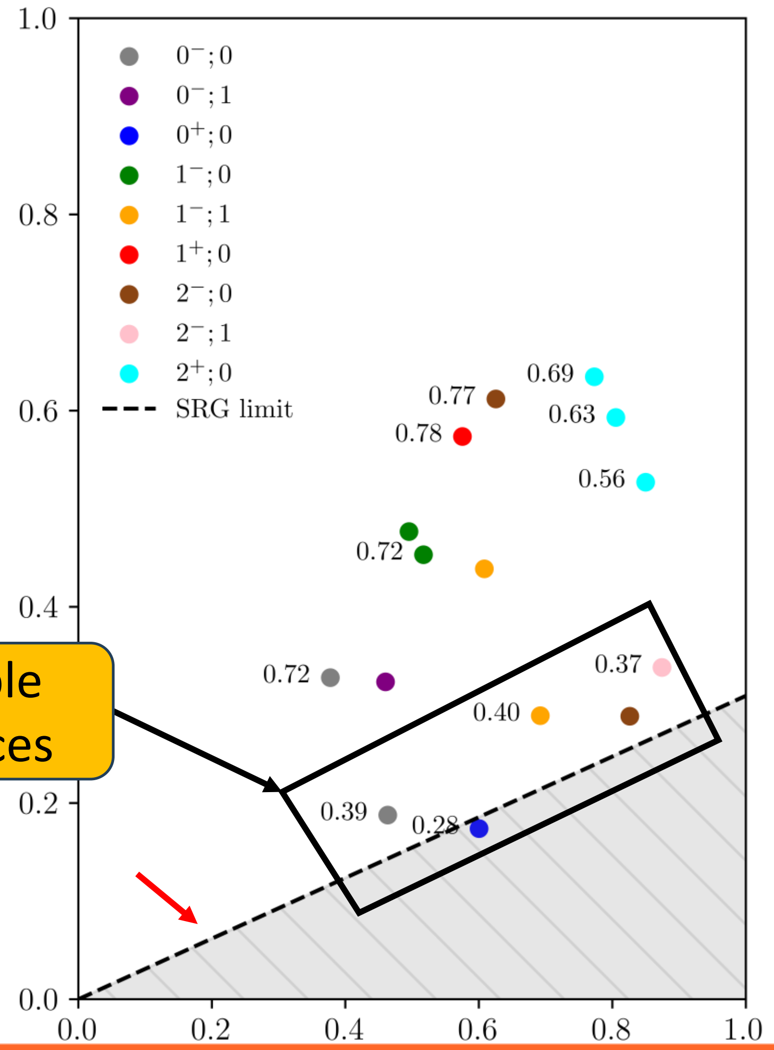


Experimental values

- Experimental data gives us an estimation of the critical angle of ^{4}He resonances.
- SRG constraints us to the $\theta < 0.3\ rad$ region.
- Only one resonance is in this region.
- Four resonances are close to it.

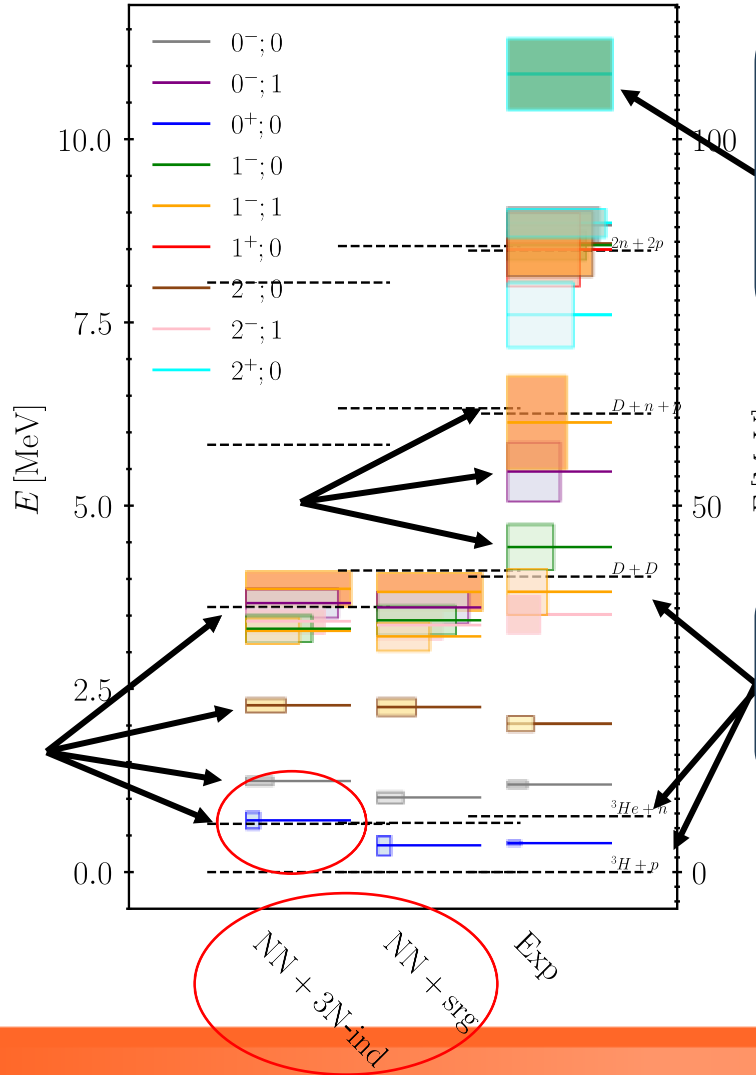
We expect to access 5 resonances.

Accessible resonances



^{4}He energy levels

- We use NN+SRG with and without induced 3N interactions.
- Four resonances are reproduced well.
- Three resonances are shifted several MeV below.
- Width of the first 0^+ resonance predicted to be higher.



Bar width indicates the width of the resonance

Dashed lines indicated thresholds

Visualizing errors

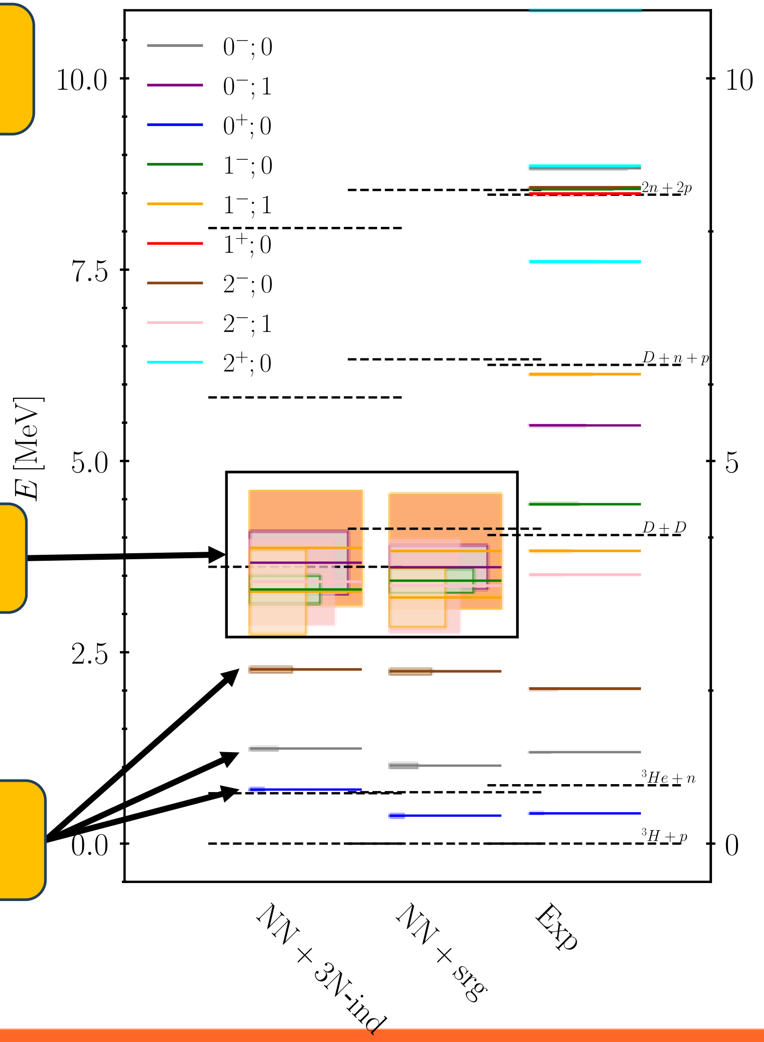
- We use NN+SRG with and without induced 3N interactions.
- Narrower resonances are well converged.
- Very wide resonances are not converged.

Uncertainty quantification

Truncation Error ($\mathbf{N_{max}}$)

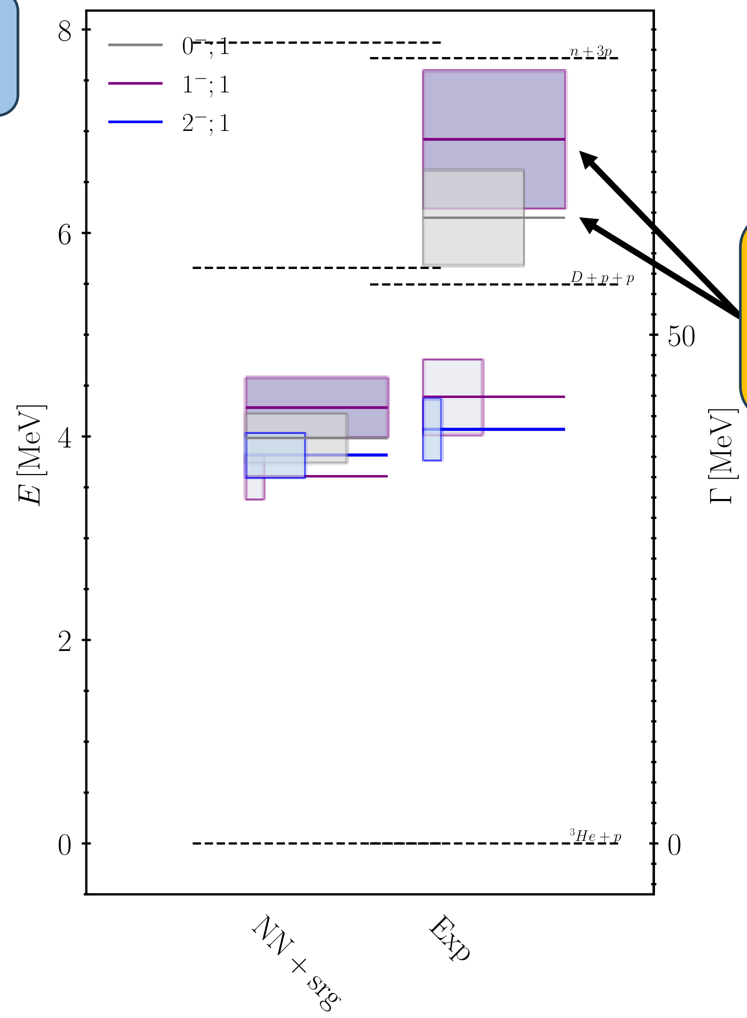
Unconverged resonances

Well converged resonances



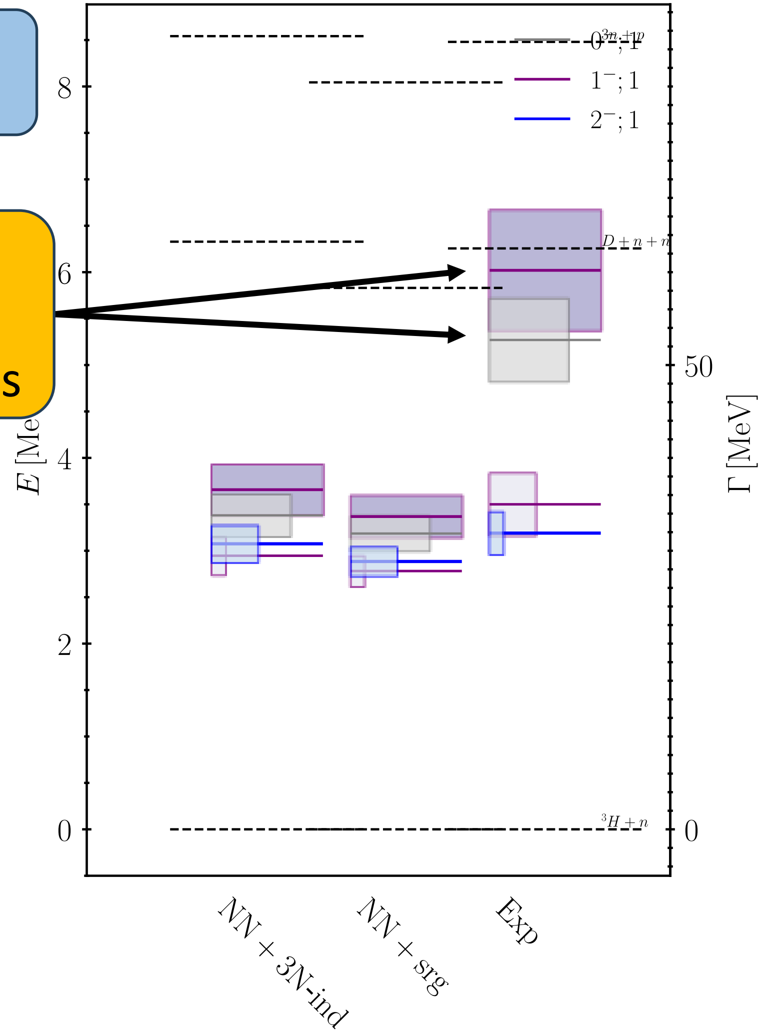
Other A=4 nuclei

^{4}Li



Significant deviation for two resonances

^{4}H



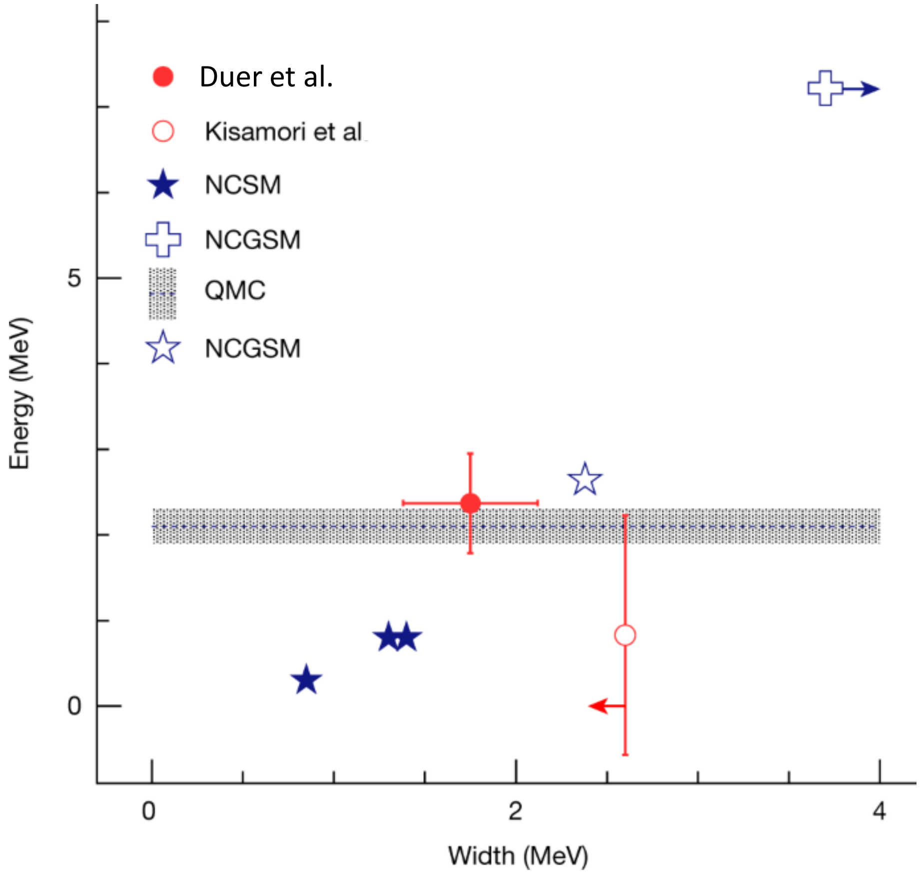
4 neutron resonance ?

- Several theoretical and experimental studies claim the existence of a resonance in the 4-neuteron system (Tetraneuteron).
- A study by M.duer et al claim the detection of this resonance at :

$$E_r = 2.37 \pm 0.38 \pm 0.44 \; MeV$$
$$\Gamma_r = 1.75 \pm 0.22 \pm 0.3 \; MeV$$

[Duer, M., Aumann, T., Gernhäuser, R. et al. Observation of a correlated free four-neutron system. Nature 606, 678–682 (2022). https://doi.org/10.1038/s41586-022-04827-6]

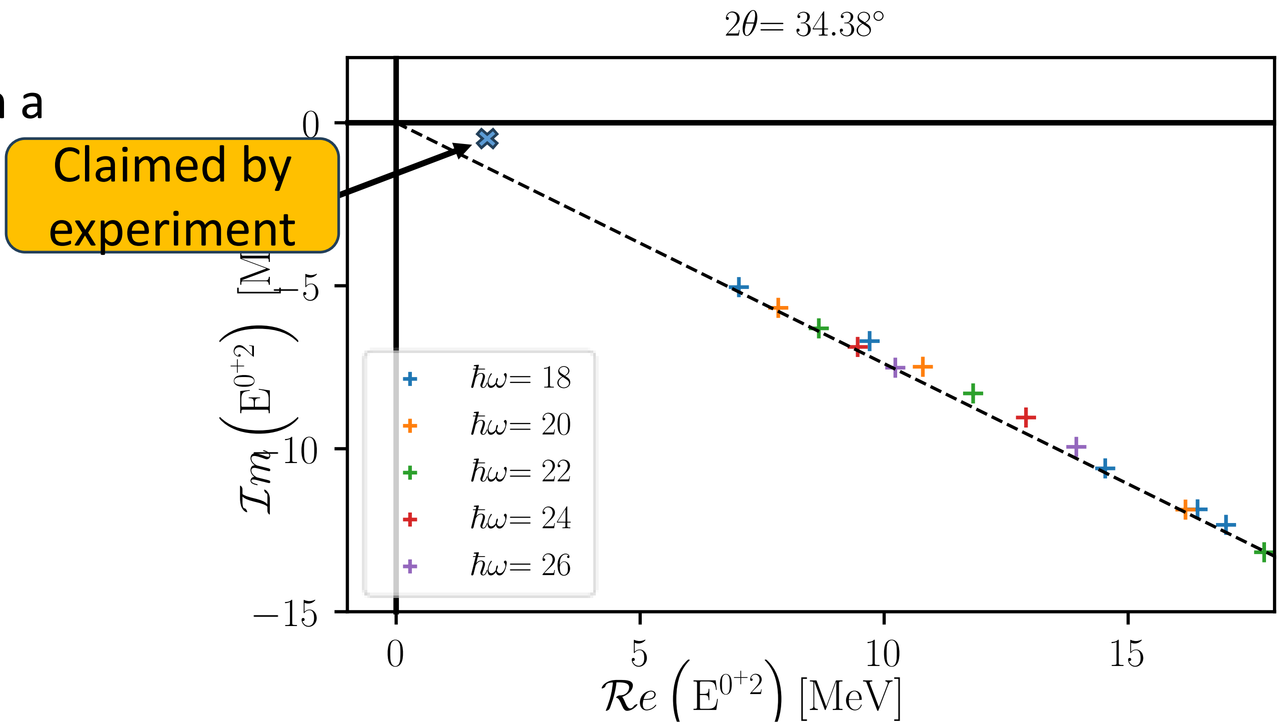
- Contradictory prediction between different models.



- CS shows no indication of such a resonance in $1^+, 1^-, 0^+$ or 0^-.
- Lower bound of :
$$\Gamma_r = 1.9\, E_r$$
- Or :
$$\Gamma_r = 4.5\, MeV$$

Resonance not found

4neutron Spectrum ($\lambda = 2.1\ fm^{-1}$) using NN+SRG +3N-induced

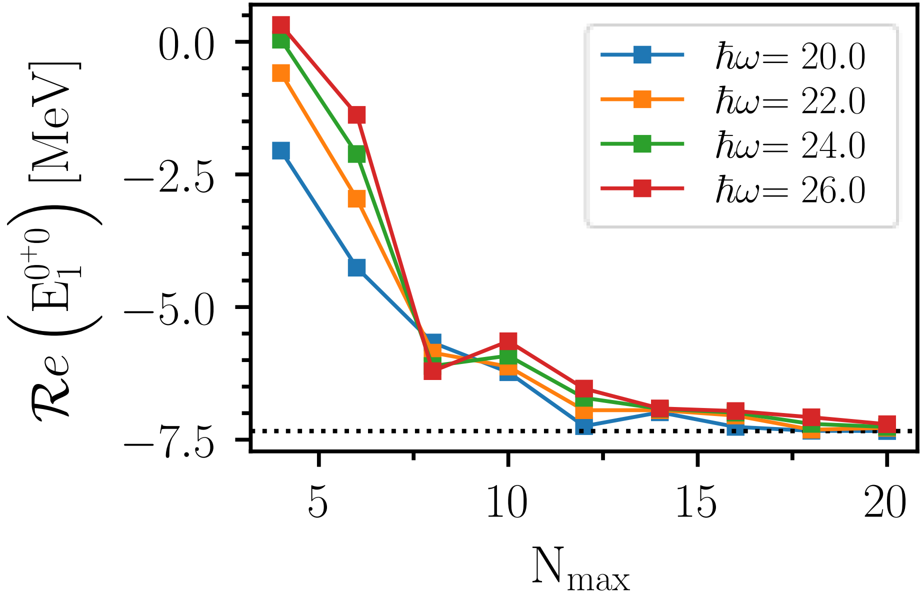


Conclusions

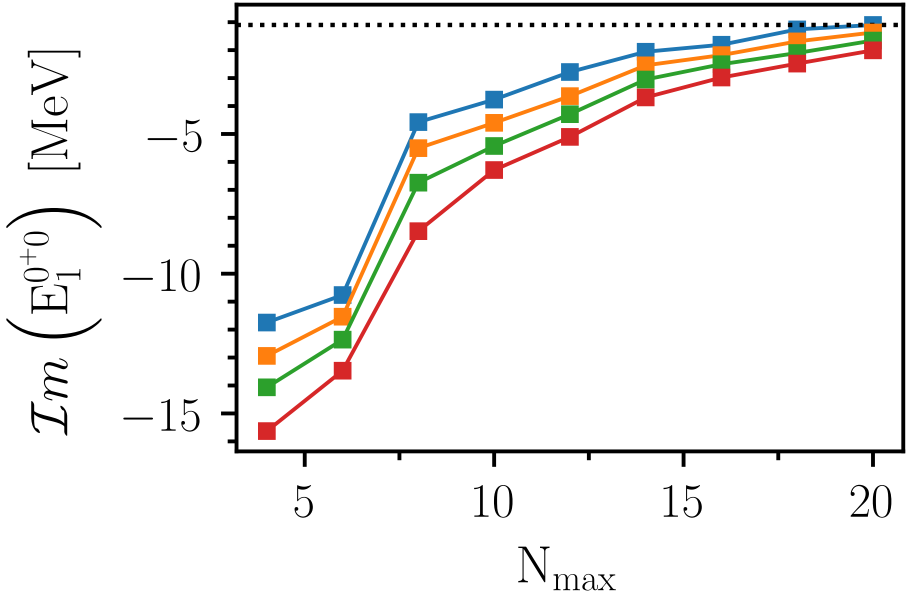
- This approach has been demonstrated on the resonances of A=4 nuclei.
- Current limitation on θ is from SRG convergence issues ($\theta < 0.3\, rad$).
- Next steps:

1. Including true 3N force.
2. Study of SRG on complex scaled Hamiltonian and search for better generator choice.
3. Applications in heavier systems to study systems where clustering is expected (^{12}C, ^{8}Be).

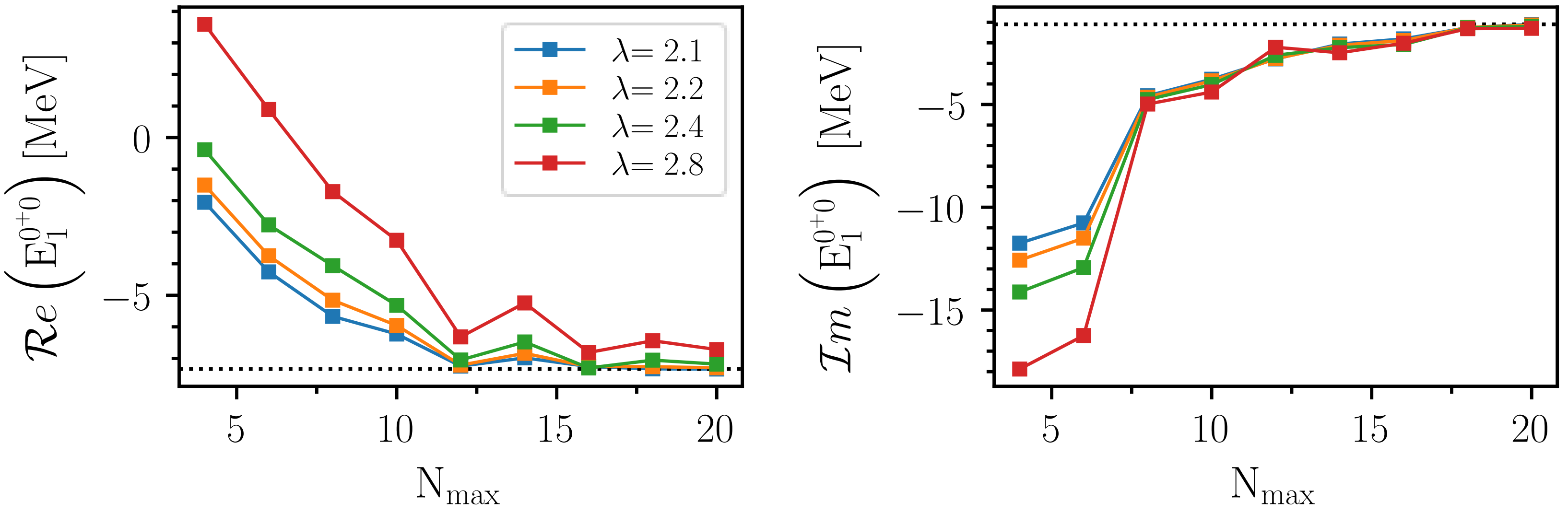
Thanks for listening



0.0
−2.5
−5.0
−7.5
5
10
15
20
N_{max}
$\mathcal{R}e\left(E_1^{0^+0}\right)$ [MeV]
$\hbar\omega$= 20.0
$\hbar\omega$= 22.0
$\hbar\omega$= 24.0
$\hbar\omega$= 26.0



−5
−10
−15
5
10
15
20
N_{max}
$\mathcal{I}m\left(E_1^{0^+0}\right)$ [MeV]



$\mathcal{R}e\left(E_1^{0^+0}\right)$ [MeV]
0
−5
5
10
15
20
N_{max}
λ= 2.1
λ= 2.2
λ= 2.4
λ= 2.8
$\mathcal{I}m\left(E_1^{0^+0}\right)$ [MeV]
−5
−10
−15
5
10
15
20
N_{max}