



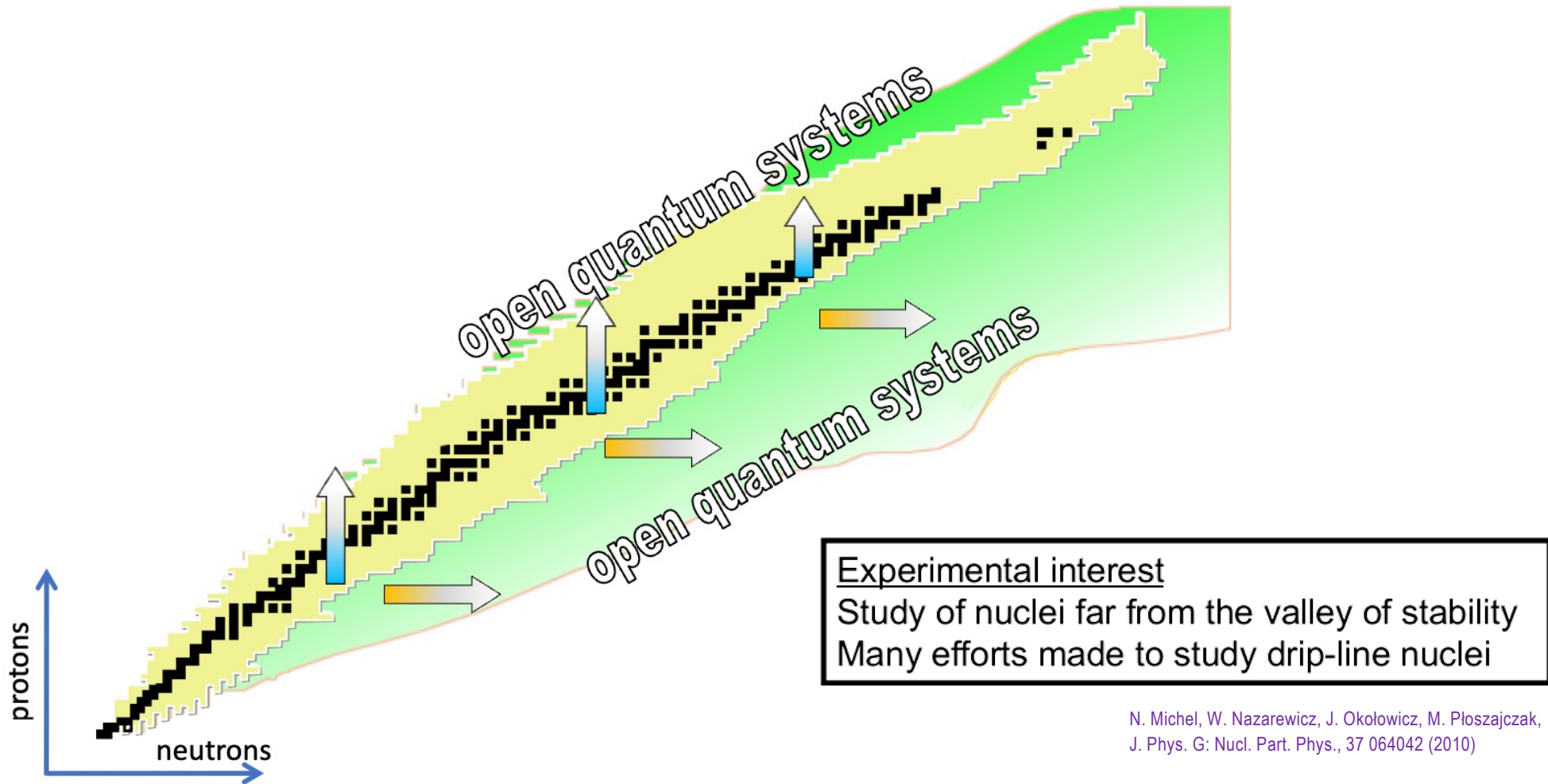
Threshold states and clustering as the emerging phenomenon in open quantum system

Marek Płoszajczak (GANIL)

ESNT Workshop, *Light nuclei between single-particle and clustering features*
Saclay, 3-6 December, 2024

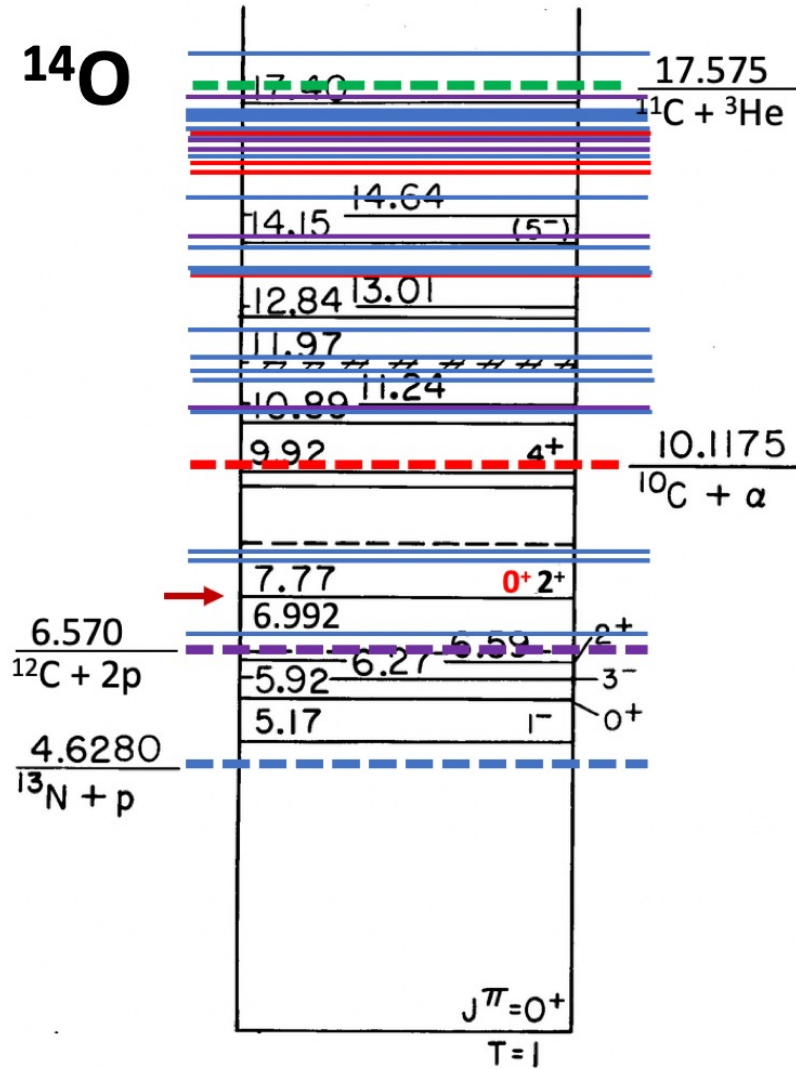
Atomic nucleus: the open quantum system

Scientific context



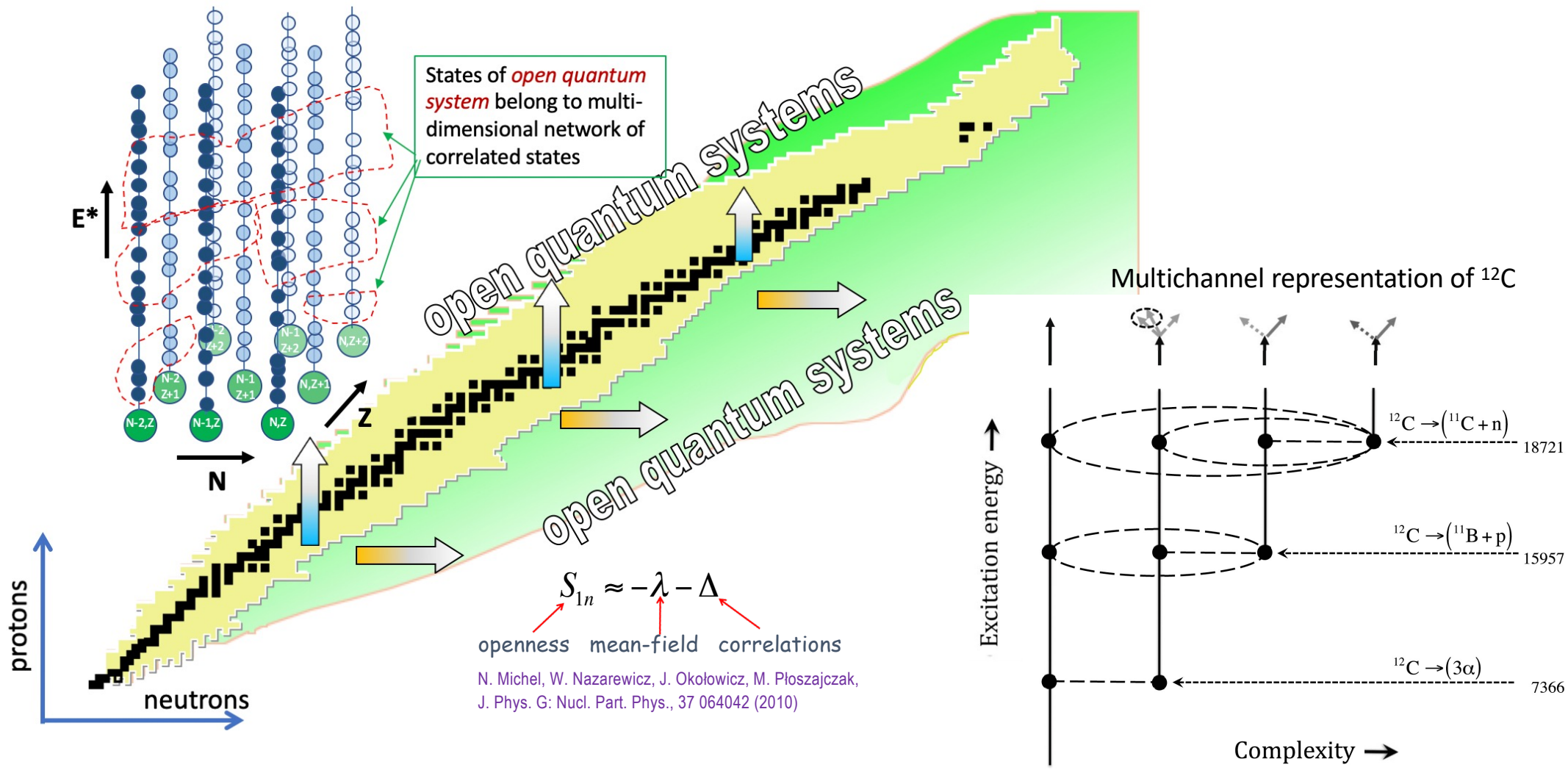
N. Michel, W. Nazarewicz, J. Okołowicz, M. Płoszajczak,
J. Phys. G: Nucl. Part. Phys., 37 064042 (2010)

Scientific context



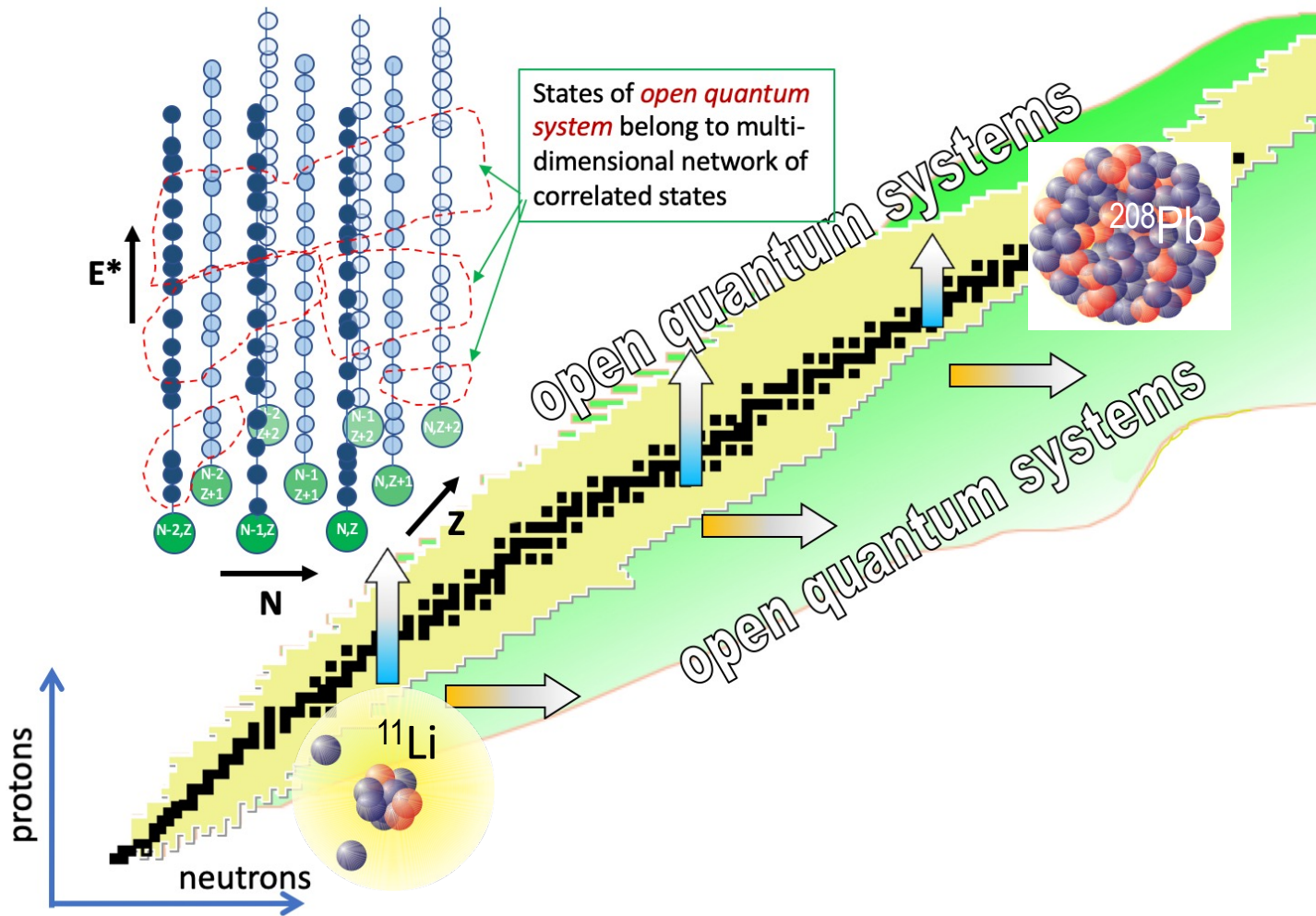
- Nuclear states are *embedded in the scattering continuum*
- Couplings to various particle emission channels are crucial for the properties of near-threshold states
- **Unitarity** is the fundamental property of QM yet 'mainstream' nuclear theory describes nucleus in *unitarity violating schemes*
 \Rightarrow '*Unitarity crisis*' in nuclear theory

Atomic nucleus: the open quantum system

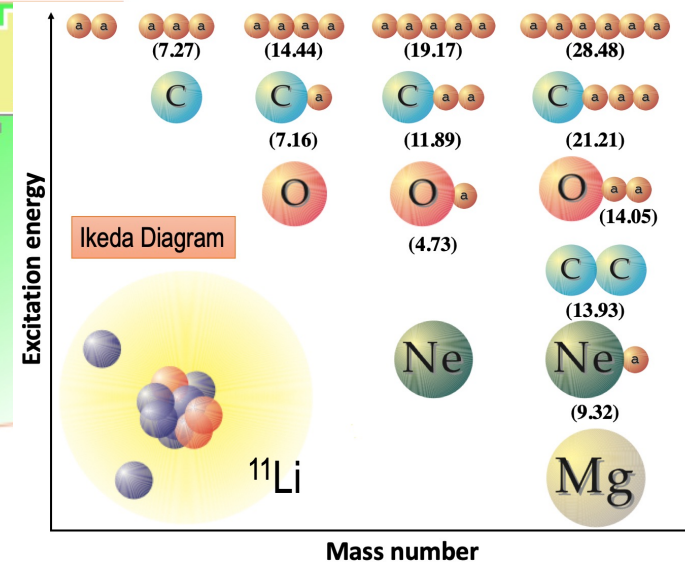


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Atomic nucleus: the open quantum system

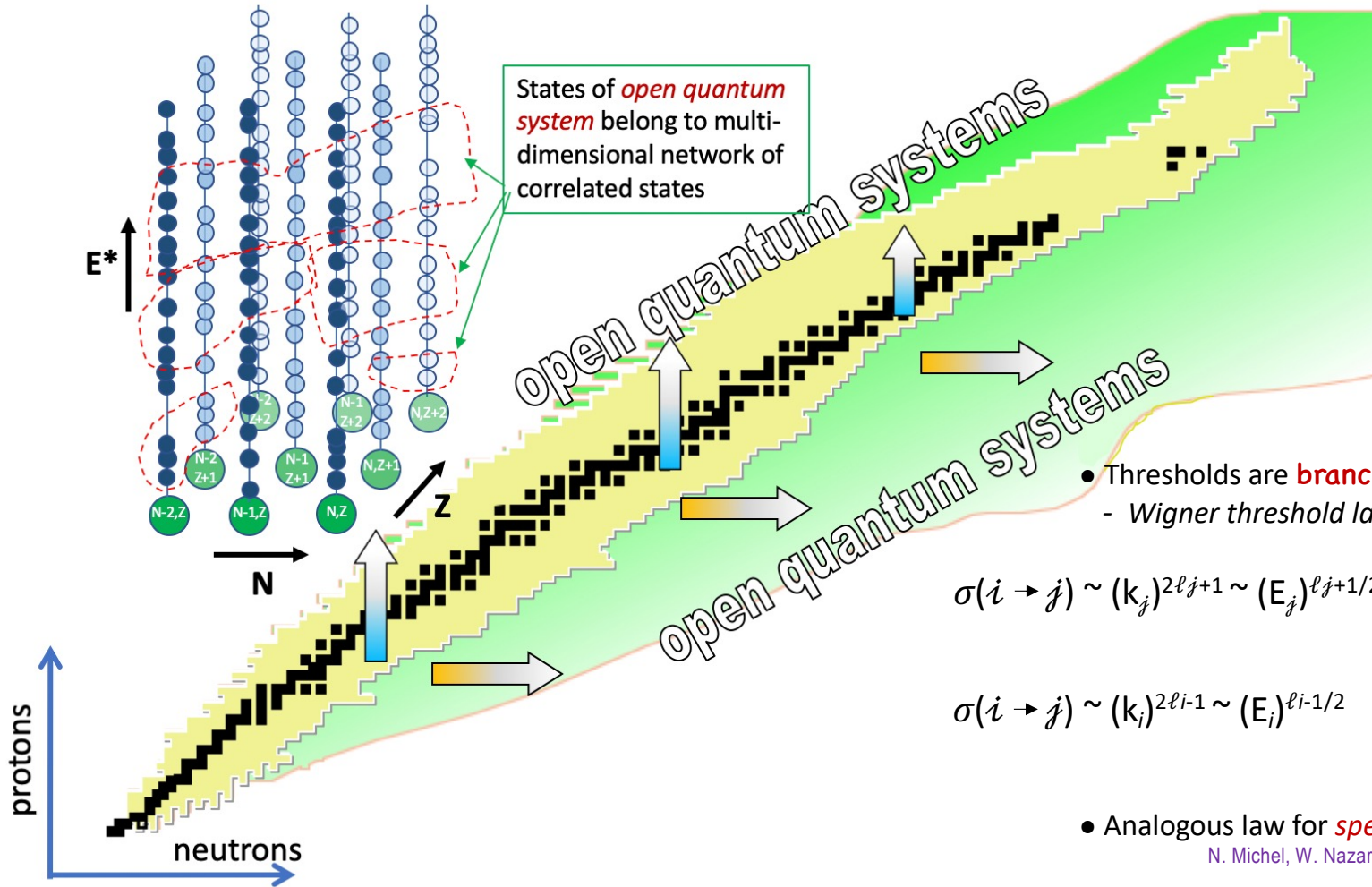


Near-threshold clustering



K. Ikeda, et al., Prog. Theor. Phys. Suppl. E68, 464 (1968)
 W. von Oertzen et al., Eur. Phys. J. A 46, 345 (2010)
 F. Barker, Proc. Phys. Soc. 84, 681 (1964)
 J. Okołowicz et al., Prog. Theor. Phys. Suppl. 196, 230 (2012);
 Fortschr. Phys. 61, 66 (2013)

Atomic nucleus: the open quantum system



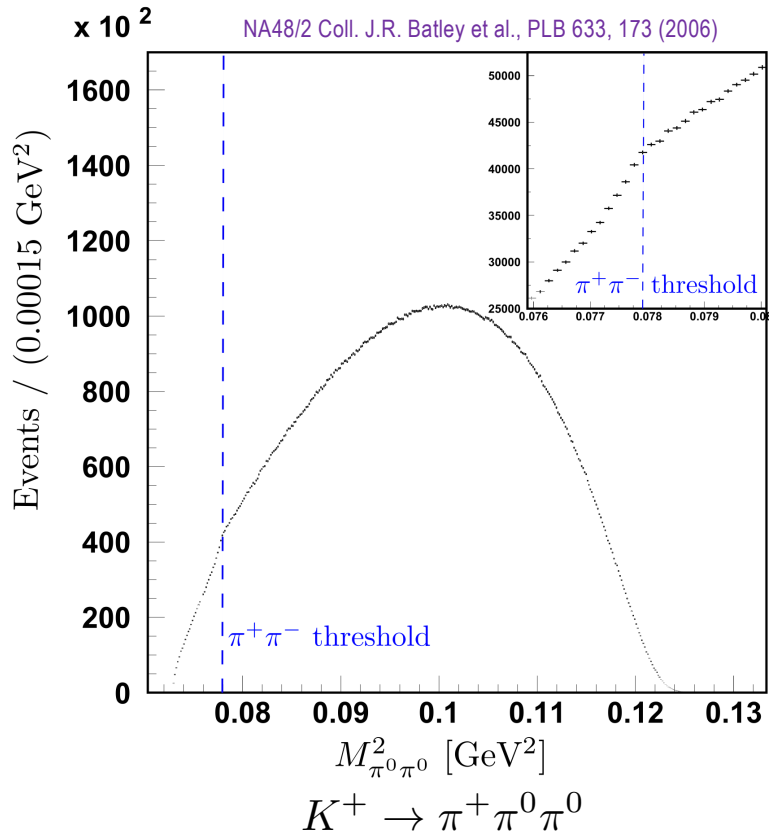
- Thresholds are **branching points** \rightarrow *nonanalytic behavior*
 - Wigner threshold law for elastic and total cross-sections
E.P. Wigner, Phys. Rev. 73, 1002 (1948)

$\sigma(i \rightarrow j) \sim (k_j)^{2\ell_j+1} \sim (E_j)^{\ell_j+1/2}$ for **endoergic reactions**: the production of slow neutral particles

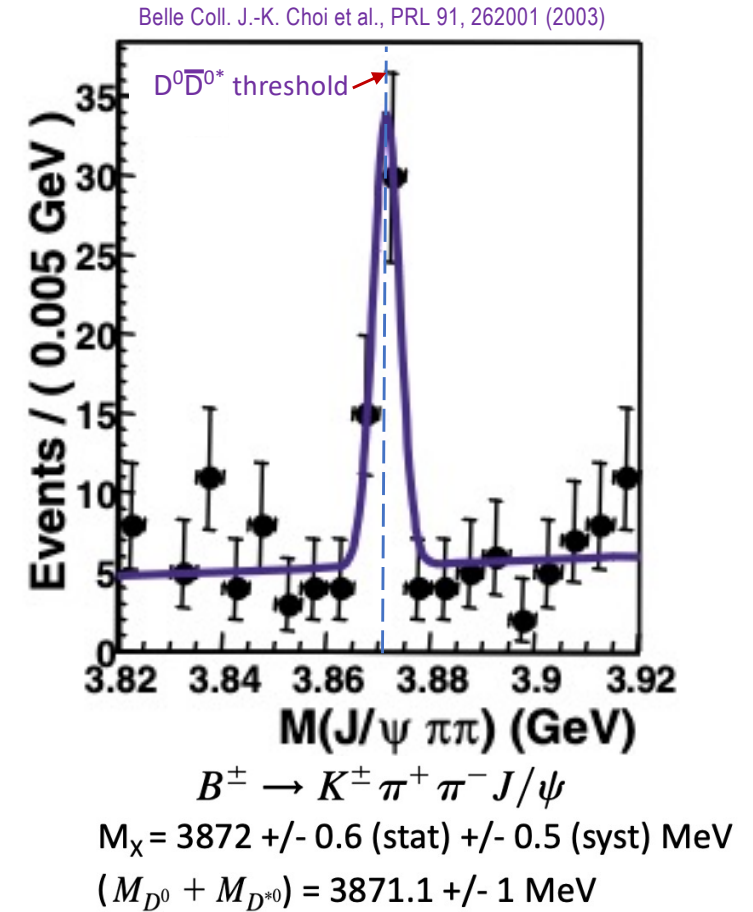
$\sigma(i \rightarrow j) \sim (k_i)^{2\ell_i-1} \sim (E_i)^{\ell_i-1/2}$ for **exoergic reactions**: the absorption of slow neutral particles

- Analogous law for **spectroscopic factors**
N. Michel, W. Nazarewicz., M. Płoszajczak, Phys. Rev. C(R) 75, 031301 (2007)

Threshold effects in hadrons, hadronic molecules, multiquark systems

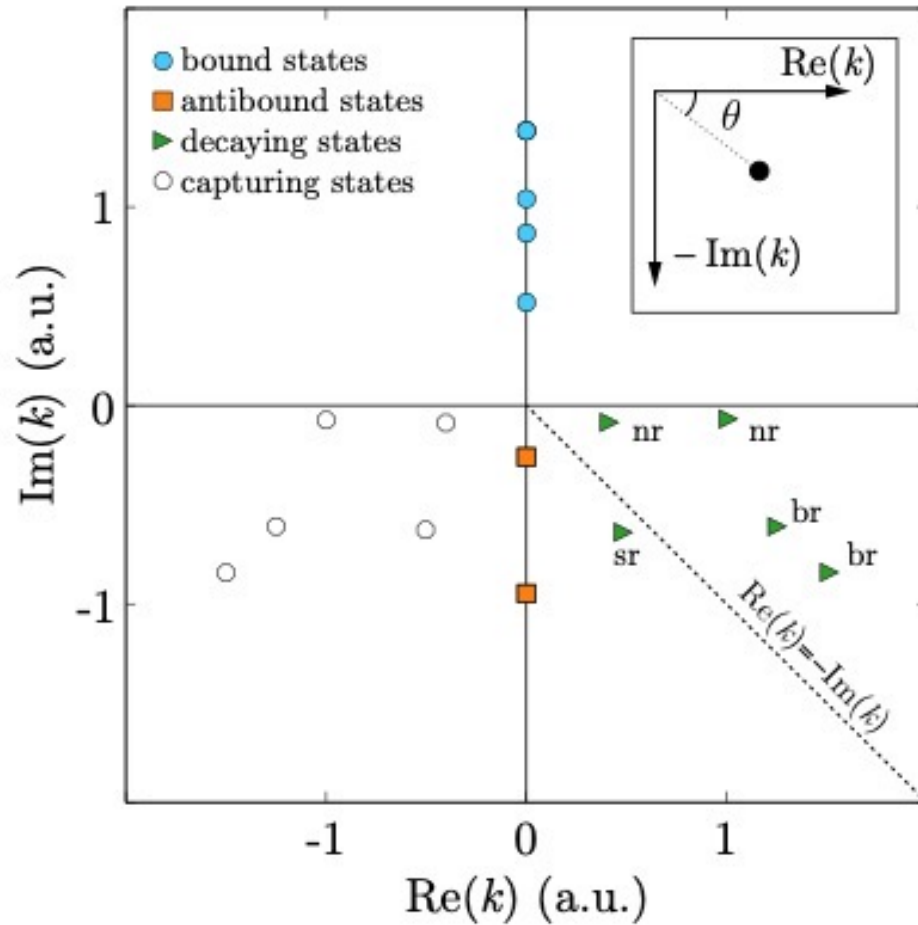


Threshold effects can also result in some resonance-like structures in the pertinent invariant mass spectrum that can be confounded with a genuine resonance states, like molecular states, multiquark states, or hybrid.



Gamow shell model

Resonant states in the complex-k plane



$$E = \frac{\hbar^2 k^2}{2m} \quad f(k) = f(-k^*)$$

(time reversal property)

G. Gamow (1928)

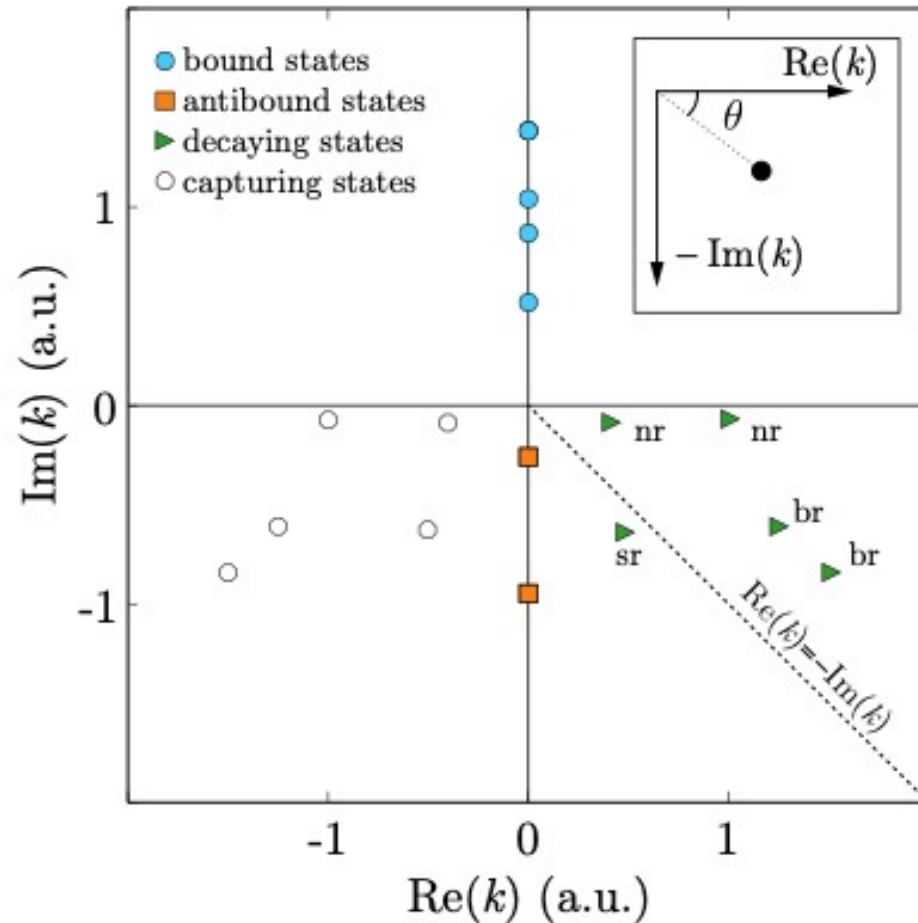
$$i\hbar \frac{\partial}{\partial t} \Phi(r,t) = \hat{H} \Phi(r,t) ; \quad \Phi(r,t) = \tau(t) \Psi(r)$$

$$\hat{H} \Psi = \left(e - i \frac{\Gamma}{2} \right) \Psi \quad \rightarrow \quad \tau(t) = \exp \left(-i \left(e - i \frac{\Gamma}{2} \right) t \right)$$

$$\Psi(0,k) = 0 , \quad \begin{cases} \Psi(\vec{r},k) \rightarrow O_l(kr) \\ \Psi(\vec{r},k) \rightarrow I_l(kr) + O_l(kr) \end{cases}_{r \rightarrow \infty}$$

Only bound states are integrable!

Resonant states in the complex-k plane

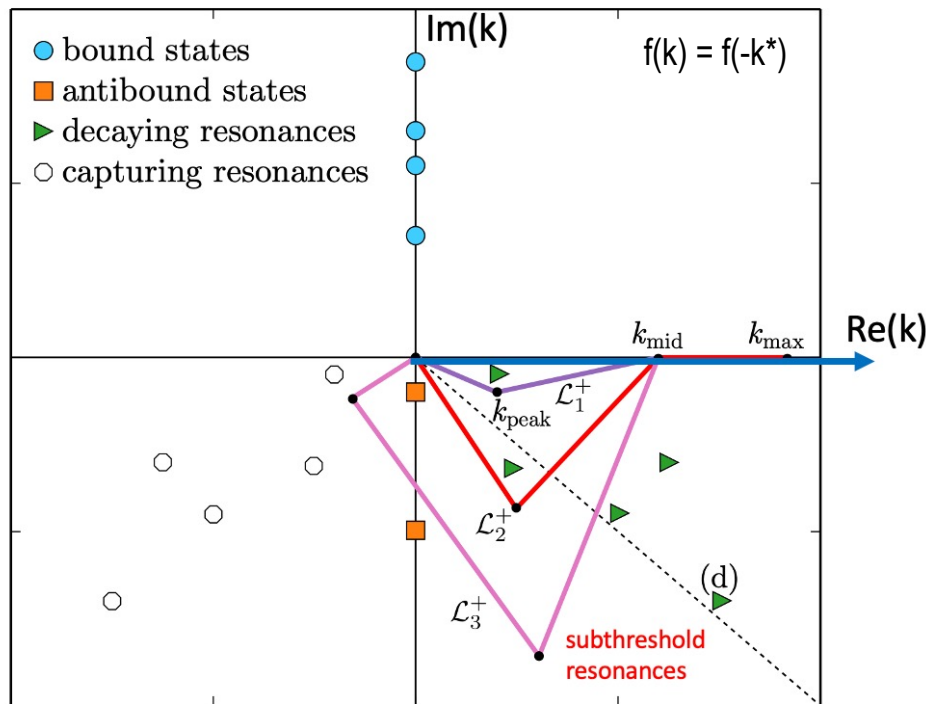


- Resonant states with $\text{Re}(E) > 0$ and small Γ can be associated with *narrow resonances* (nr)
- For *antibound (virtual) states* $\text{Re}(E) < 0$ and $\Gamma = 0$
- For *subthreshold resonant states* $\text{Re}(E) < 0$ and $\Gamma > 0$
- The antibound and subthreshold resonant states lie on the second Riemann energy sheet
- Low-momentum antibound and threshold resonant states result in the low-energy cross-section enhancement. These poles should be viewed as scattering features rather than physical states of the system.

$$E = \frac{\hbar^2 k^2}{2m} \quad f(k) = f(-k^*)$$

(time reversal property)

Gamow shell model: Quasi-stationary extension of standard shell model in the complex k-plane



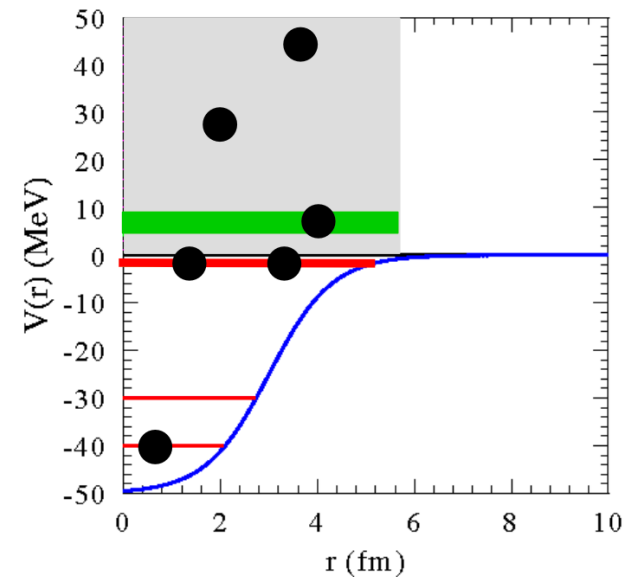
$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L^+} |u_k\rangle\langle\tilde{u}_k| dk = 1 ; \langle u_i | \tilde{u}_j \rangle = \delta_{ij}$$

T. Berggren, Nucl. Phys. A109, 265 (1968)
 K. Maurin, Generalized Eigenfunction Expansion,
 Polish Scientific Publishers, Warsaw (1968)
 T. Lind, Phys. Rev. C47, 1903 (1993)

Efficient discretization of the \mathcal{L}^+ contour with Gauss-Legendre quadrature

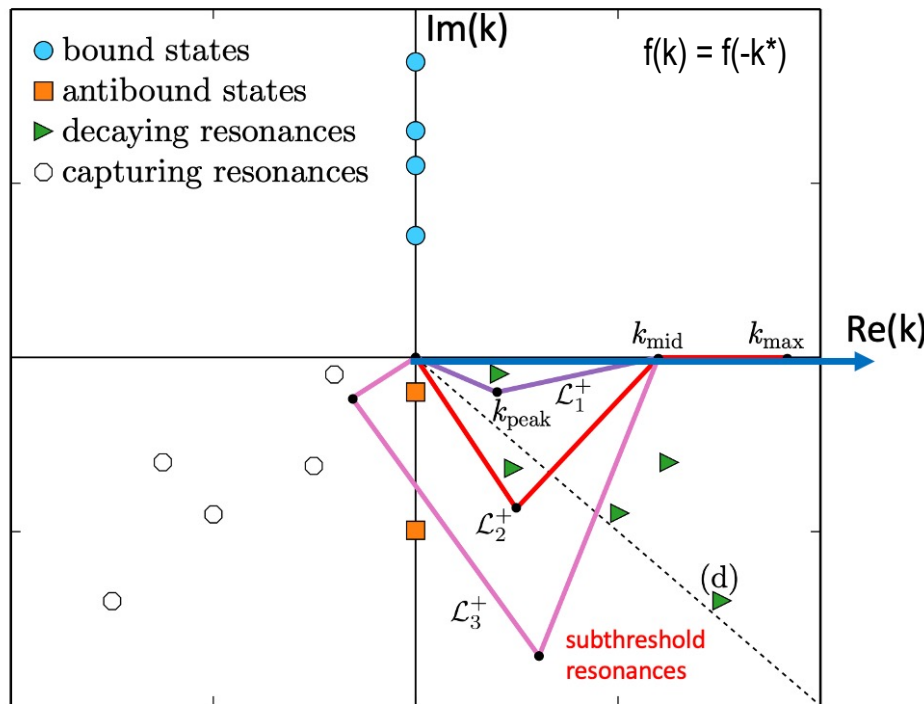
Open quantum system description

N. Michel et al, PRL 89 (2002) 042502
 N. Michel, et al, J. Phys. G37 (2010) 064042



- Localized states
- Halo states of complex structure
- Many-body resonances

Gamow shell model: Quasi-stationary extension of standard shell model in the complex k-plane



Gamow shell model (GSM)

$$|SD_i\rangle = |u_{i_1} \dots u_{i_A}\rangle \rightarrow \sum_k |SD_k\rangle \langle SD_k| \cong 1$$

N. Michel et al, PRL 89, 042502 (2002)
 N. Michel, et al, J. Phys. G37, 064042 (2010)

- Calculation in the relative coordinates of core cluster SM coordinates Y. Suzuki, K. Ikeda, PRC 38 (1988) 410
- Center-of-mass handled by recoil term:

$$H \rightarrow H + \frac{1}{M_{\text{core}}} \sum_{(i < j) \in \text{val}} \mathbf{p}_i \cdot \mathbf{p}_j$$

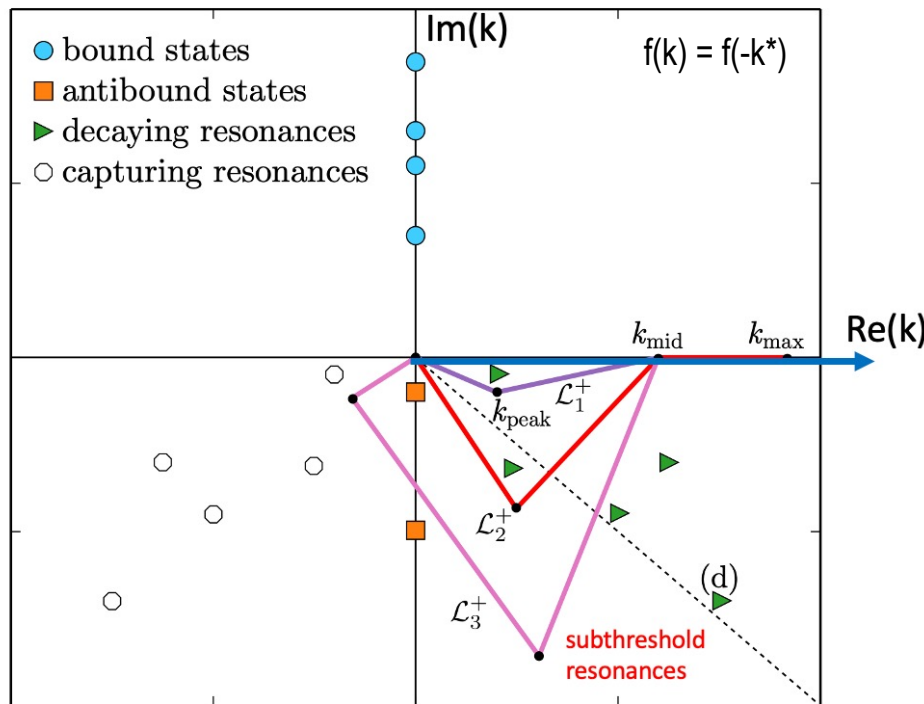
in the Hamiltonian

- **Unitary formulation** of the nuclear Shell Model

$$\sum_n |u_n\rangle \langle \tilde{u}_n| + \int_{L_+} |u_k\rangle \langle \tilde{u}_k| dk = 1 ; \langle u_i | \tilde{u}_j \rangle = \delta_{ij}$$

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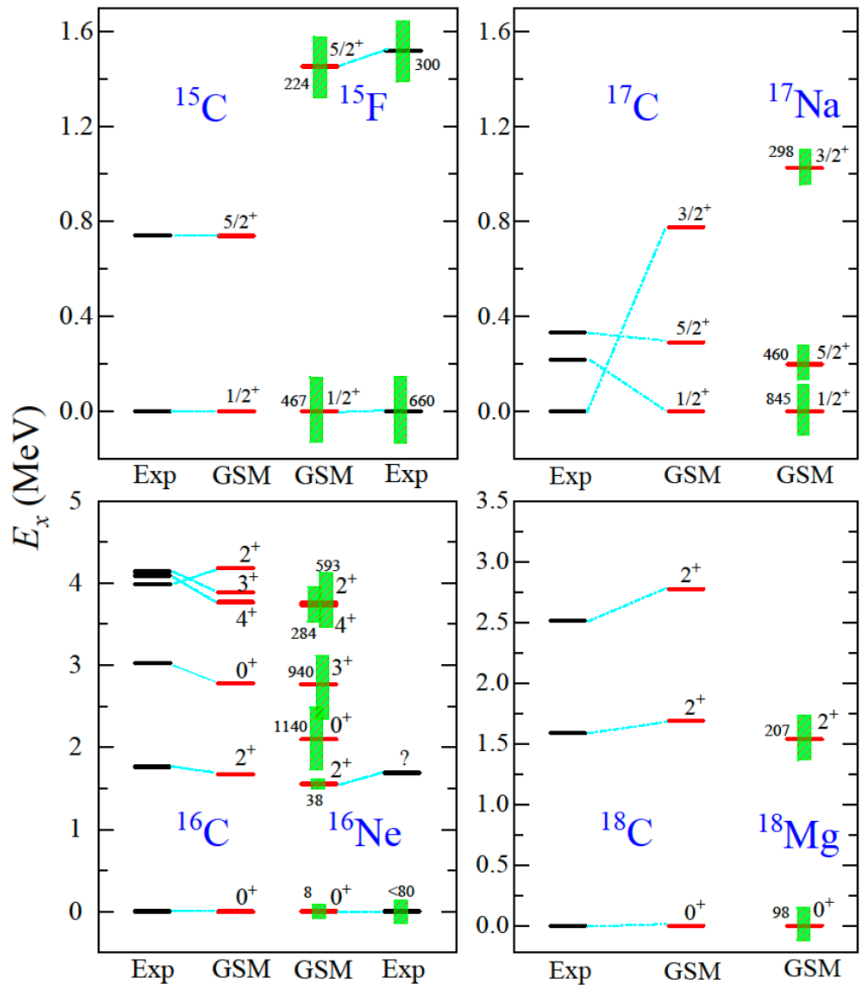
- **Unitary formulation** of the nuclear Shell Model

Resonant states of the NN system

- np bound state (deuteron): $k = +i0.2315 \text{ fm}^{-1}$ **T=0**
- np virtual state (deuteron): $k = -i0.044 \text{ fm}^{-1}$ **T=1**
- nn virtual state: $k = -i0.0559(33) \text{ fm}^{-1}$ **T=1**
 V.A. Babenko, N.M. Petrov, Phys. At. Nucl. 76, 684 (2013)
- pp threshold resonant state: $k = (0.0647 - i0.0870) \text{ fm}^{-1}$ **T=1**
 L.P. Kok, Phys. Rev. Lett. 45, 427 (1980)

Gamow shell model: Quasi-stationary extension of standard shell model in the complex k-plane

Example: Carbon isotopes and isotones



Gamow shell model (GSM)

$$|SD_i\rangle = |u_{i_1} \dots u_{i_A}\rangle \rightarrow \sum_k |SD_k\rangle \langle SD_k| \equiv 1$$

N. Michel et al, PRL 89, 042502 (2002)
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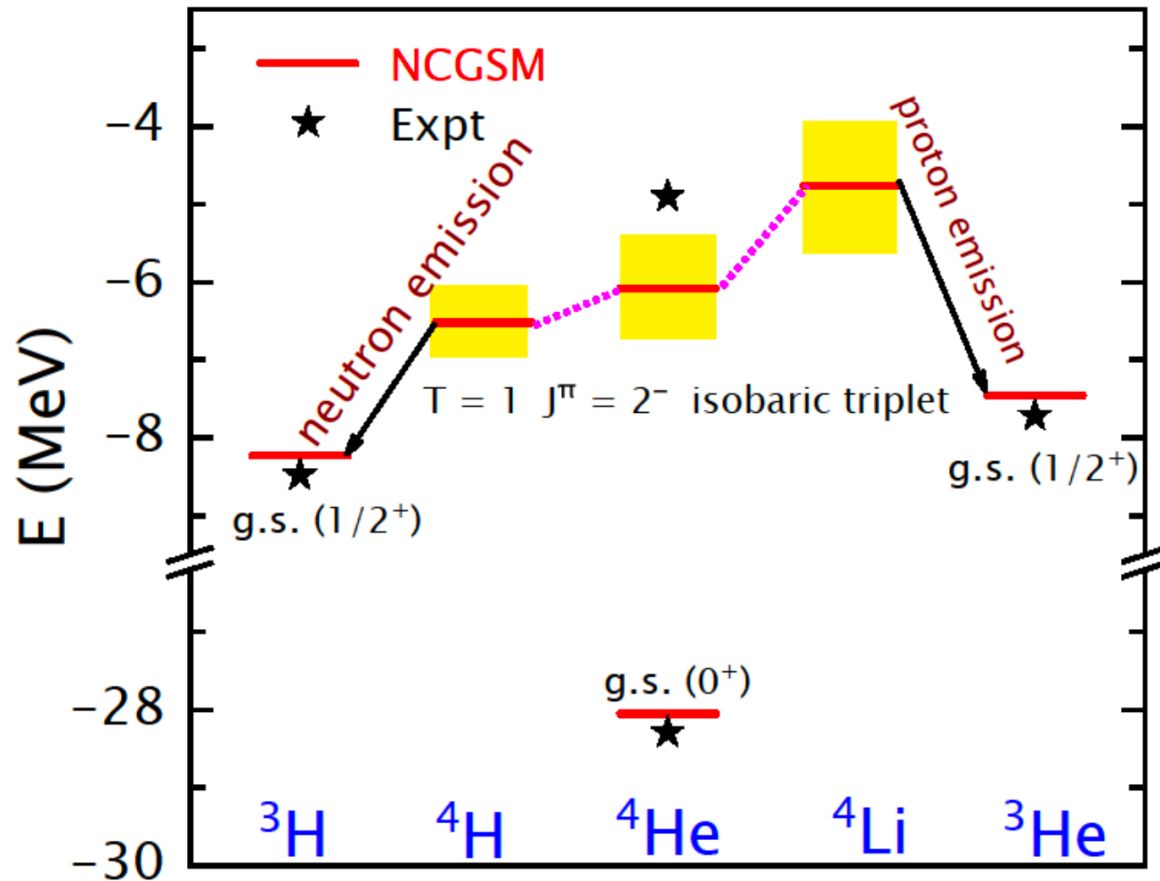
- in the Hamiltonian
- **Unitary formulation** of the nuclear Shell Model

- GSM with a core of ^{14}O
- EFT interaction in psd Berggren basis L. Huth et al., PRC 98, 044301 (2018)
- Carbon isotopes/isotones well bound/unbound

N. Michel et al., PRC 103, 044319 (2021)

Gamow shell model: Quasi-stationary extension of standard shell model in the complex k-plane

Example: Unbound hydrogen isotopes



Gamow shell model (GSM)

$$|SD_i\rangle = |u_{i_1} \dots u_{i_A}\rangle \rightarrow \sum_k |SD_k\rangle \langle SD_k| \cong 1$$

N. Michel et al, PRL 89, 042502 (2002)
N. Michel, et al, J. Phys. G37, 064042 (2010)

- Calculation in the relative coordinates of core cluster SM coordinates Y. Suzuki, K. Ikeda, PRC 38 (1988) 410
- Center-of-mass handled by recoil term:

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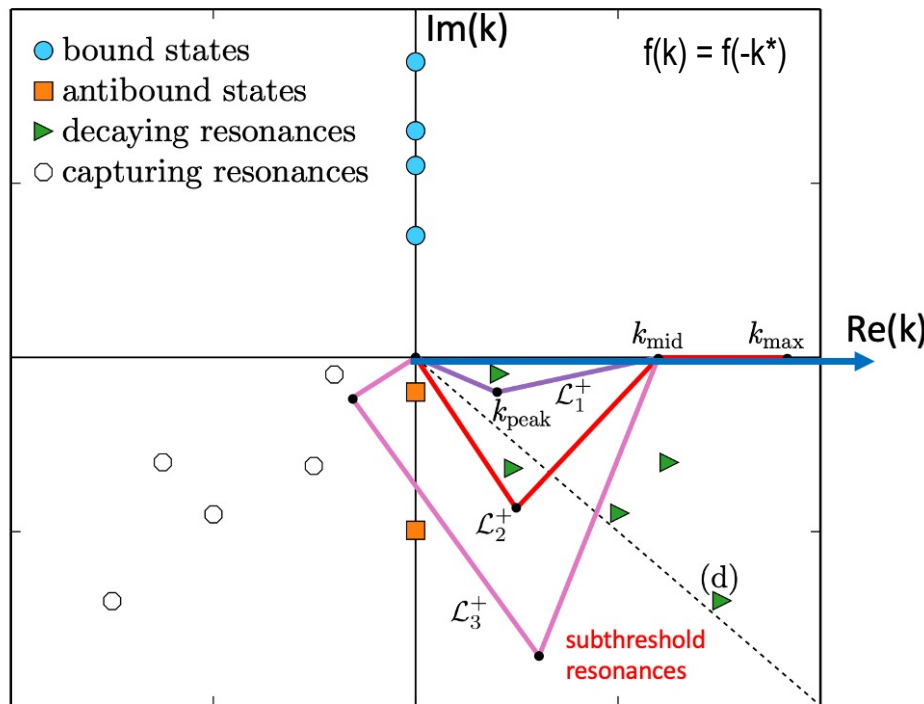
- in the Hamiltonian
- *Unitary formulation* of the nuclear Shell Model

- $T=1, J^\pi=2^-$ many-body states; Isospin multiplet in $A=4$ nuclei
- Broad resonances in $T=1$ multiplet
- $T=1$ in ${}^4\text{H}$ and ${}^4\text{Li}$
- $T \sim 0.71$ in ${}^4\text{He}$

Isospin symmetry strongly broken

N. Michel et al., PRC 104, 024319 (2021)

Gamow shell model: Quasi-stationary extension of standard shell model in the complex k-plane



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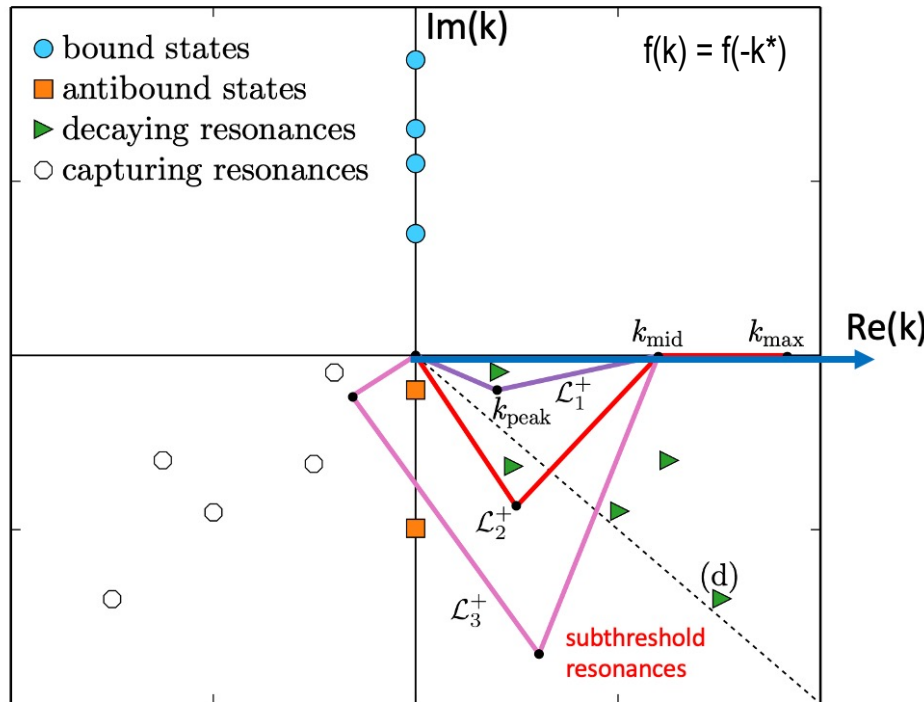
- **Unitary formulation** of the nuclear Shell Model

In the Slater determinant representation of GSM
the reaction channels are not identified!

$$\sum_n |u_n\rangle \langle \tilde{u}_n| + \int_{L_+} |u_k\rangle \langle \tilde{u}_k| dk = 1 ; \langle u_i | \tilde{u}_j \rangle = \delta_{ij}$$

T. Berggren, Nucl. Phys. A109, 265 (1968)
 K. Maurin, Generalized Eigenfunction Expansion,
 Polish Scientific Publishers, Warsaw (1968)
 T. Lind, Phys. Rev. C47, 1903 (1993)

Gamow shell model – Coupled-channel representation



$$|\Psi_M^J\rangle = \sum_c \int_0^{+\infty} |(c, r)_M^J\rangle \frac{u_c^{JM}(r)}{r} r^2 dr$$

$$\downarrow$$

$$H |\Psi_M^J\rangle = E |\Psi_M^J\rangle \rightarrow \sum_c \int_0^\infty r^2 (H_{cc'}(r, r') - EN_{cc'}(r, r')) \frac{u_c(r)}{r} = 0$$

$$H_{cc'}(r, r') = \langle (c, r) | \hat{H} | (c', r') \rangle$$

$$N_{cc'}(r, r') = \langle (c, r) | (c', r') \rangle$$

- Entrance and exit reaction channels defined
→ Unification of nuclear structure and reactions
- Reaction channels with different (binary) mass partitions
- Core is arbitrary

Y. Jaganathen et al, PRC 88, 044318 (2014)
 K. Fossez et al., PRC 91, 034609 (2015)
 A. Mercenne et al., PRC 99, 044606 (2019)

N. Michel, M. P.,
 «Gamow Shell Model: The Unified Theory of
 Nuclear Structure and Reactions »
 Lecture Notes in Physics, Vol. 983, (Springer Verlag, 2021)

$$\sum_n |u_n\rangle \langle \tilde{u}_n| + \int_{L_+} |u_k\rangle \langle \tilde{u}_k| dk = 1 ; \langle u_i | \tilde{u}_j \rangle = \delta_{ij}$$

T. Berggren, Nucl. Phys. A109, 265 (1968)
 K. Maurin, Generalized Eigenfunction Expansion,
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Effective interaction in open quantum system

NN interaction in different regimes of binding

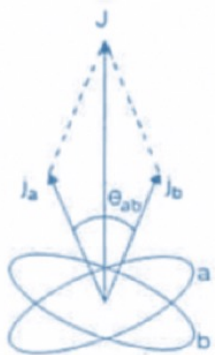
- ★ $B(j_a, j_b) = -10$ MeV
- ★ $B(j_a, j_b) = -1$ MeV $\ell = p, d, f, g, h$
- ★ $B(j_a, j_b) = +1$ MeV Minnesota interaction

$$\Re(V_{12}) = E_n / \langle E_n \rangle; \quad E_n = E - e_a - e_b$$

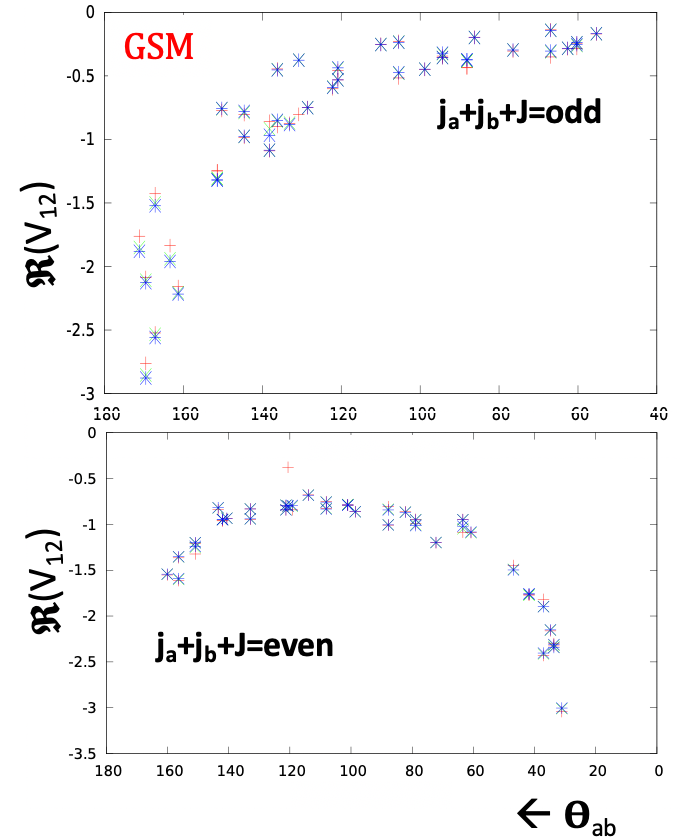
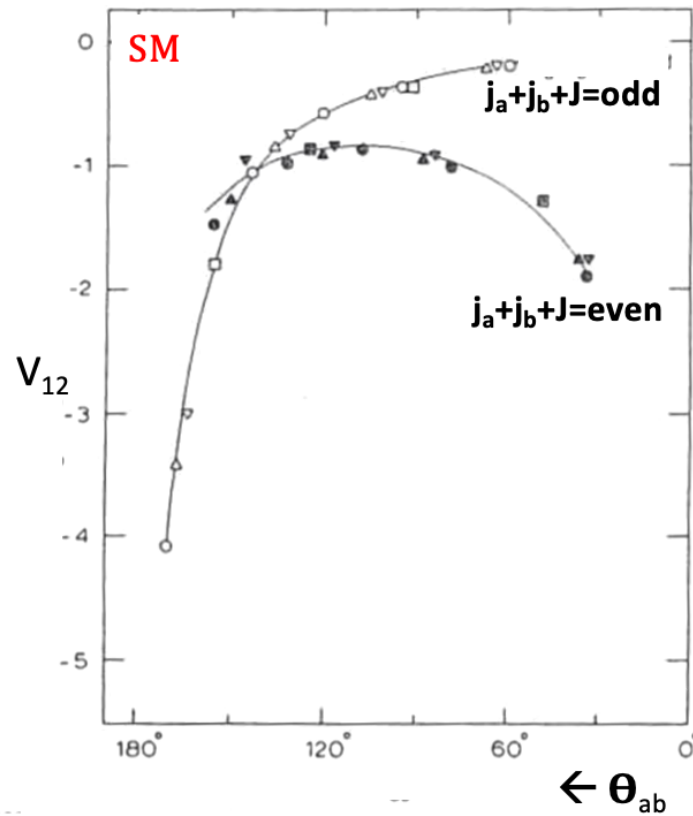
$$\langle E_n \rangle = \left| \frac{\sum_J (2J+1) (E - e_a - e_b)}{\sum_J (2J+1)} \right|$$

$$\Im(V_{12}) = \Gamma_n / \langle \Gamma_n \rangle; \quad \Gamma_n = \Gamma - \gamma_a - \gamma_b$$

$$\langle \Gamma_n \rangle = \left| \frac{\sum_J (2J+1) (\Gamma - \gamma_a - \gamma_b)}{\sum_J (2J+1)} \right|$$



$$\cos(\theta) = \frac{J(J+1) - j_a(j_a+1) - j_b(j_b+1)}{2\sqrt{j_a(j_a+1)j_b(j_b+1)}}$$



- Similar qualitative dependence of the TBMEs on angle θ_{ab} in SM and GSM
- *TBMEs are complex* in weakly bound/unbound nuclei

NN interaction in different regimes of binding

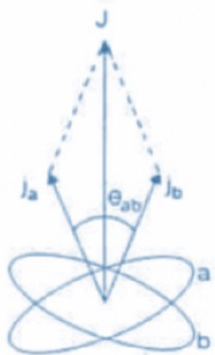
- ★ $B(j_a, j_b) = -10$ MeV
- ★ $B(j_a, j_b) = -1$ MeV $\ell = p, d, f, g, h$
- ★ $B(j_a, j_b) = +1$ MeV Minnesota interaction

$$\mathcal{R}(V_{12}) = E_n / \langle E_n \rangle; \quad E_n = E - e_a - e_b$$

$$\langle E_n \rangle = |\Sigma_j (2J+1) (E - e_a - e_b) / \Sigma_j (2J+1)|$$

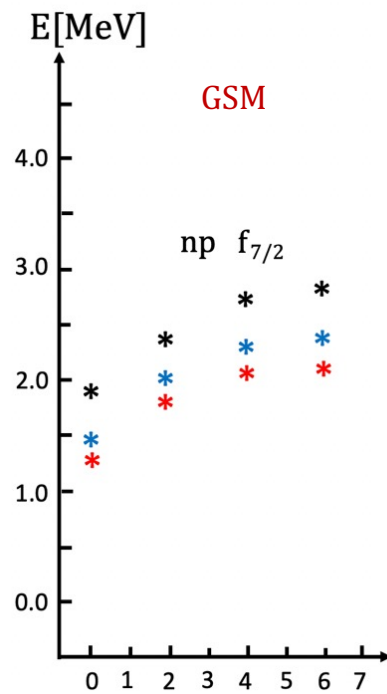
$$\mathcal{S}(V_{12}) = \Gamma_n / \langle \Gamma_n \rangle; \quad \Gamma_n = \Gamma - \gamma_a - \gamma_b$$

$$\langle \Gamma_n \rangle = |\Sigma_l (2J+1) (\Gamma - \gamma_a - \gamma_b) / \Sigma_l (2J+1)|$$



$$\cos(\theta) = \frac{J(J+1) - j_a(j_a+1) - j_b(j_b+1)}{2\sqrt{j_a(j_a+1)j_b(j_b+1)}}$$

- * $(B(j_a), B(j_b)) = (-10, -10)$ MeV
- * $(B(j_a), B(j_b)) = (-1, -10)$ MeV
- * $(B(j_a), B(j_b)) = (+1, -10)$ MeV



Strong reduction of np interaction
in weakly bound/unbound nuclei:
~50% reduction in p-shell

Dependence of V_{nn}/V_{pp} on $S_n - S_p$ asymmetry

ℓ_j	J^π	S_p [MeV]	S_n [MeV]	V_{nn}/V_{pp}
$P_{1/2}$	2^+	10	-1	0.39
		1	-1	0.58
$d_{5/2}$	2^+	10	-1	0.83
		1	-1	0.835
	4^+	10	-1	0.75
		1	-1	0.84

- Strong asymmetry of V_{nn} and V_{pp} for large $|S_n - S_p|$ and low ℓ_j
- If $S_n \ll S_p$, then $V_{pp} > V_{nn}$, i.e. protons in the neutron-rich environment interact stronger than neutrons
 → Proton SF is reduced with respect to neutron SF (and vice versa) if $S_p \ll S_n$ ($S_p \gg S_n$)

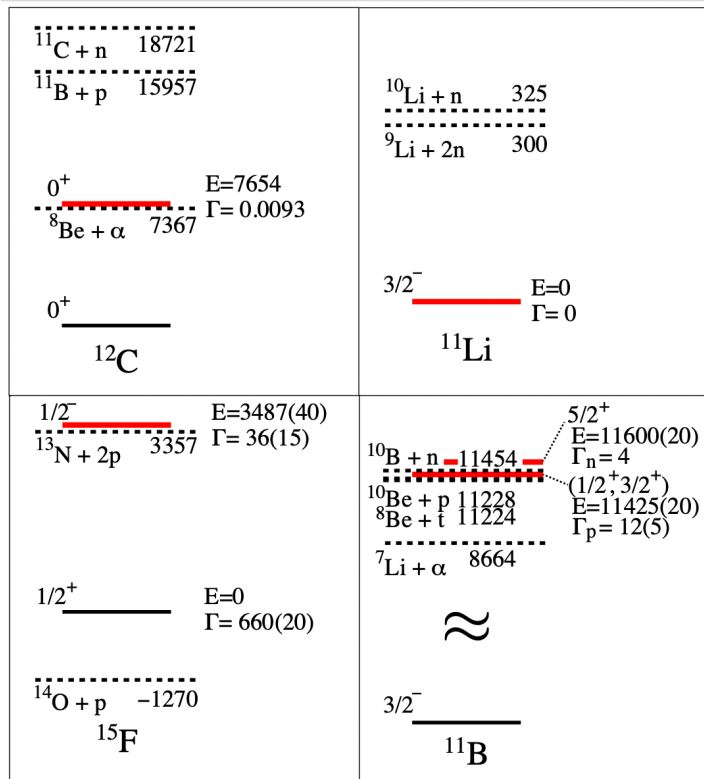
Origin of clustering in near-threshold states

Near-threshold states and origin of clustering

α -clustering “... α -cluster states can be found in the proximity of α -particle decay threshold...”

K. Ikeda, N. Takigawa, H. Horiuchi (1968)

But this is only the tip of the iceberg!



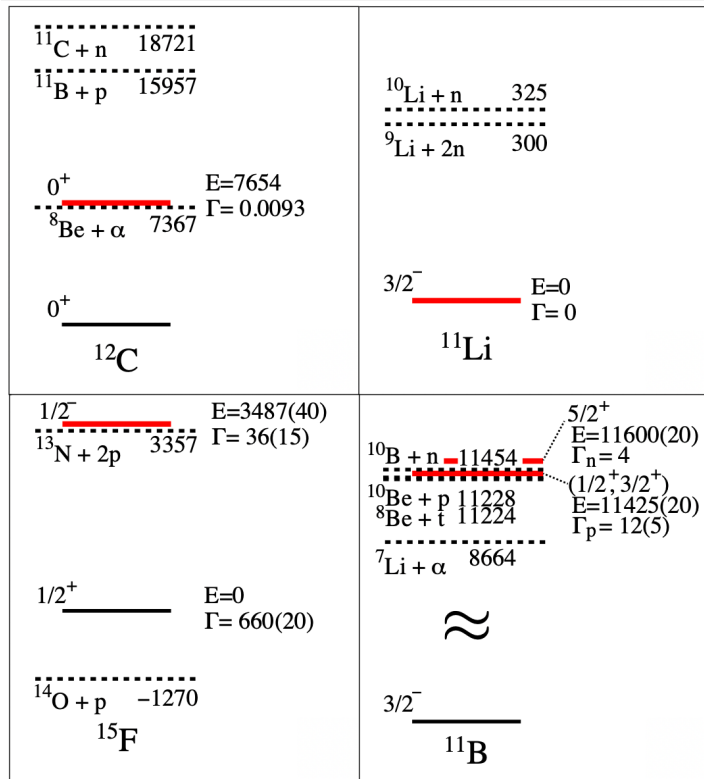
- Other cases: ^6He , ^6Li , ^7Be , ^7Li , ^{11}O , ^{11}C , ^{17}O , ^{20}Ne , ^{26}O , ^{24}Mg ,...
- *Various clusterings*: ^2H , ^3He , ^3H , $2p$, $2n$
- *Astrophysical relevance* of near-threshold resonances for α - and proton-capture reactions of nucleosynthesis

Near-threshold states and origin of clustering

α -clustering “... α -cluster states can be found in the proximity of α -particle decay threshold...”

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But this is only the tip of the iceberg!



- ‘Fortuitous’ appearance of correlated states close to open channels?
 → They cannot result from any particular feature of the NN interaction or any dynamical symmetry of the nuclear many-body problem

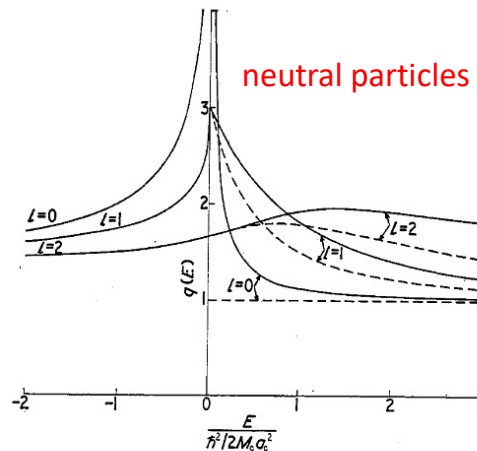


Figure 1. Enhancement factors for neutron channels with orbital angular momenta $l = 0, 1$ and 2 and reduced widths $\gamma_{\alpha c}^2 = \hbar^2/M_c a_c^2$ as functions of channel energy E (in units of $\hbar^2/2M_c a_c^2 \simeq 1$ meV). Full curves give values of $q(E)$, broken curves values of $q_l(E)$.

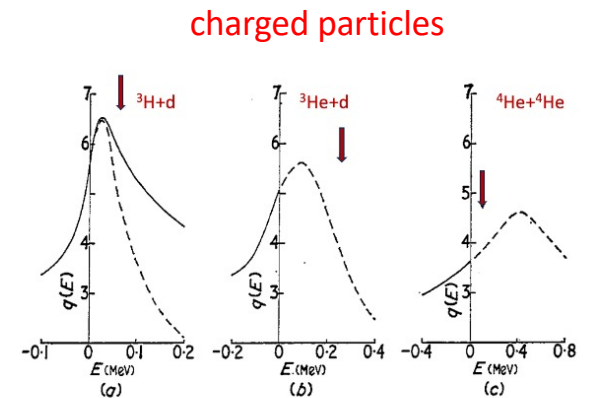


Figure 2. Enhancement factors for channels (a) $^3\text{H} + \text{d}$, (b) $^3\text{He} + \text{d}$, (c) $^4\text{He} + ^4\text{He}$, all with $l = 0$ and with values of a_c and $\gamma_{\alpha c}^2$ given in the text. Full curves give values of $q(E)$, broken curves values of $q_l(E)$. Arrows indicate energies of observed levels of ^8He , ^8Li and ^8Be .

F. Barker, Proc. Phys. Soc. 84, 681 (1964)

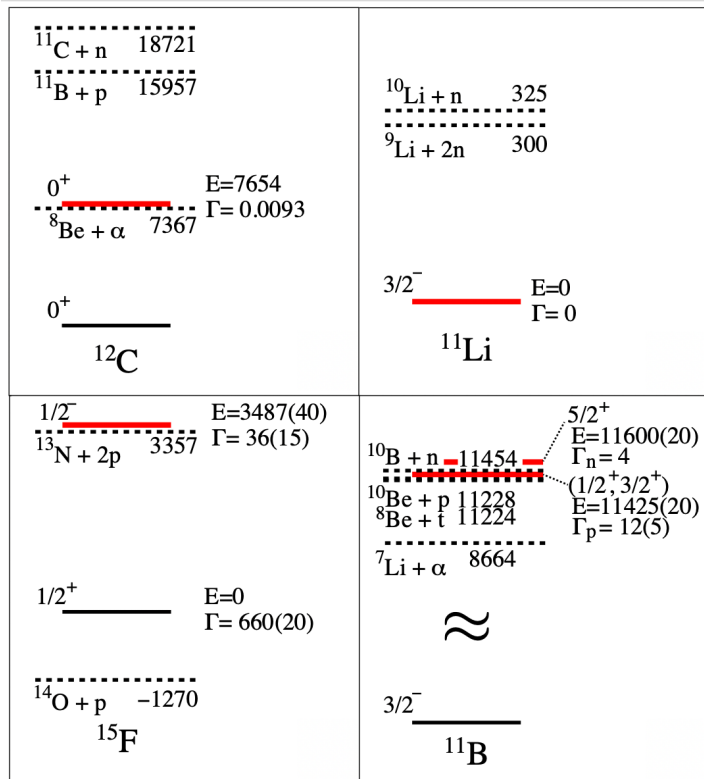
- Other cases: ^6He , ^6Li , ^7Be , ^7Li , ^{11}O , ^{11}C , ^{17}O , ^{20}Ne , ^{26}O , ^{24}Mg ,...
- Various clusterings: ^2H , ^3He , ^3H , $2p$, $2n$
- Astrophysical relevance of near-threshold resonances for α - and proton-capture reactions of nucleosynthesis

- The appearance of near-threshold resonances can be explained in terms of the increased level density: $g_l(E) = \frac{1}{\pi} \frac{d\delta_l(E)}{dE}$
- The enhancement of the level density is largest for low-barrier potentials

Near-threshold states and origin of clustering

α -clustering “... α -cluster states can be found in the proximity of α -particle decay threshold...” K. Ikeda, N. Takigawa, H. Horiuchi (1968)

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 - They cannot result from any particular feature of the NN interaction or any dynamical symmetry of the nuclear many-body problem

Continuum shell model perspective

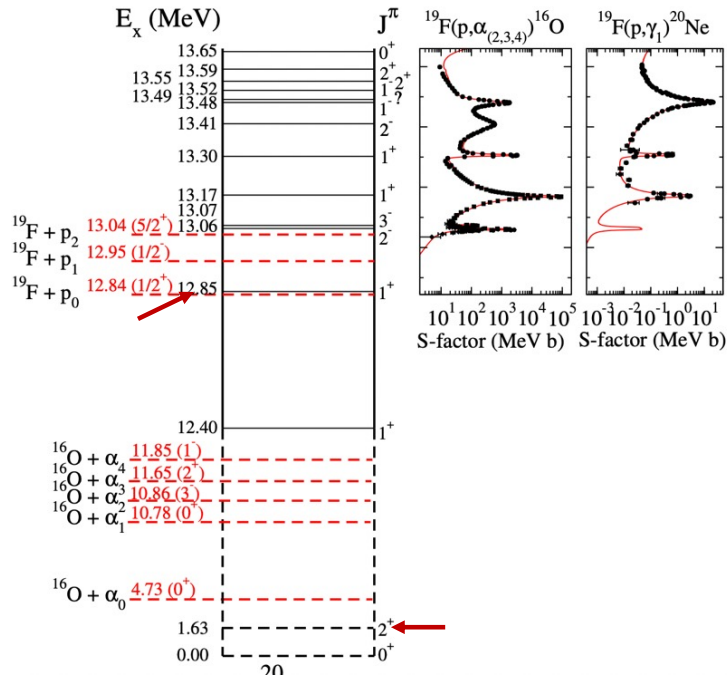
J. Okołowicz, M. Płoszajczak, W. Nazarewicz, Prog. Theor. Phys. Suppl. 196, 230 (2012);
Fortschr. Phys. 61, 66 (2013)

- The appearance of correlated (cluster) states close to open channels is the generic *open quantum system phenomenon* related to the collective rearrangement of SM wave functions due to the coupling via the continuum
- Specific aspects:
 - Energetic order of particle emission thresholds depends on (nuclear) Hamiltonian
 - Absence of stable cluster entirely composed of like nucleons

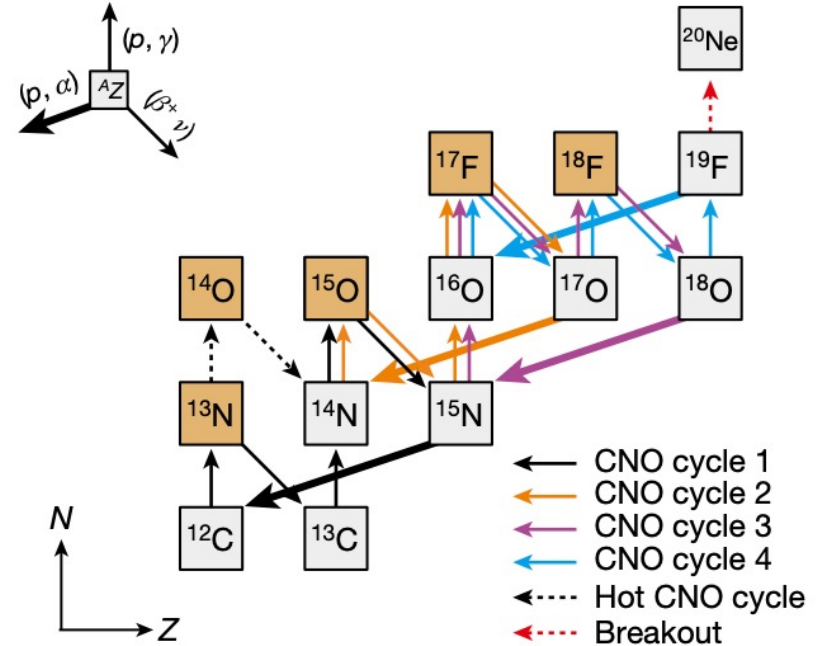
- Other cases: ^6He , ^6Li , ^7Be , ^7Li , ^{11}O , ^{11}C , ^{17}O , ^{20}Ne , ^{26}O , ^{24}Mg ,...
- *Various clusterings*: ^2H , ^3He , ^3H , $2p$, $2n$
- *Astrophysical relevance* of near-threshold resonances for α - and proton-capture reactions of nucleosynthesis

Astrophysical relevance for α - and proton-capture reactions of nucleosynthesis

R.J. DeBoer et al, Nature 610, 656 (2022)



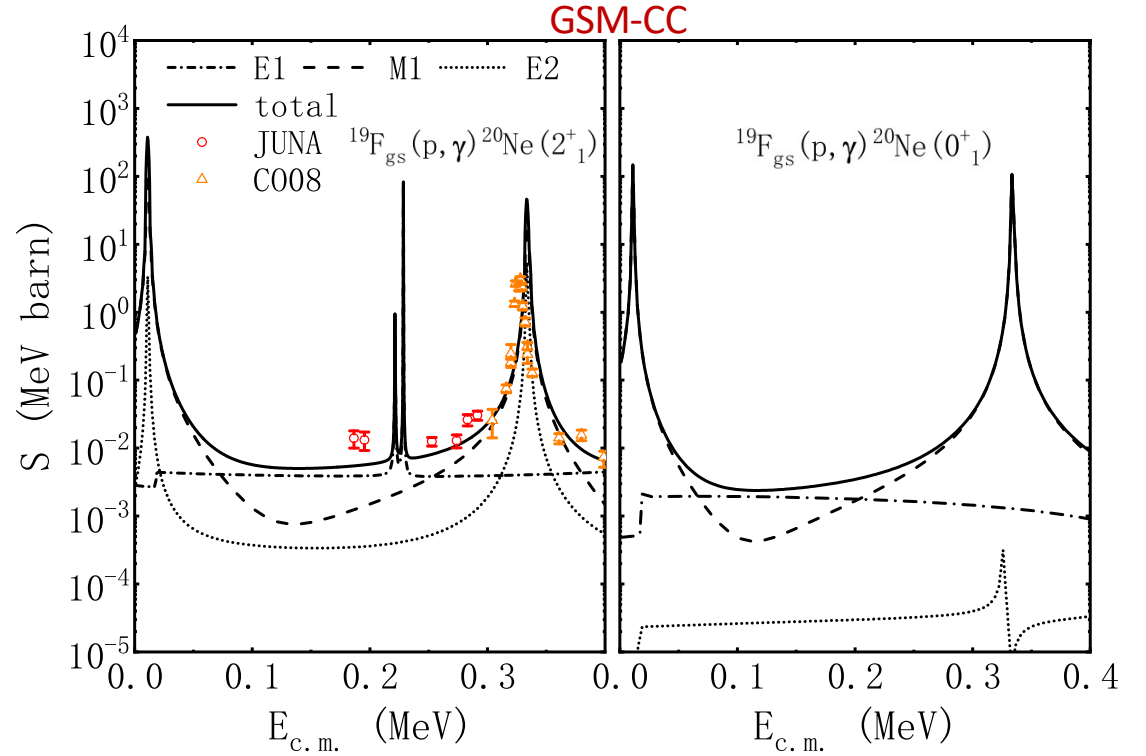
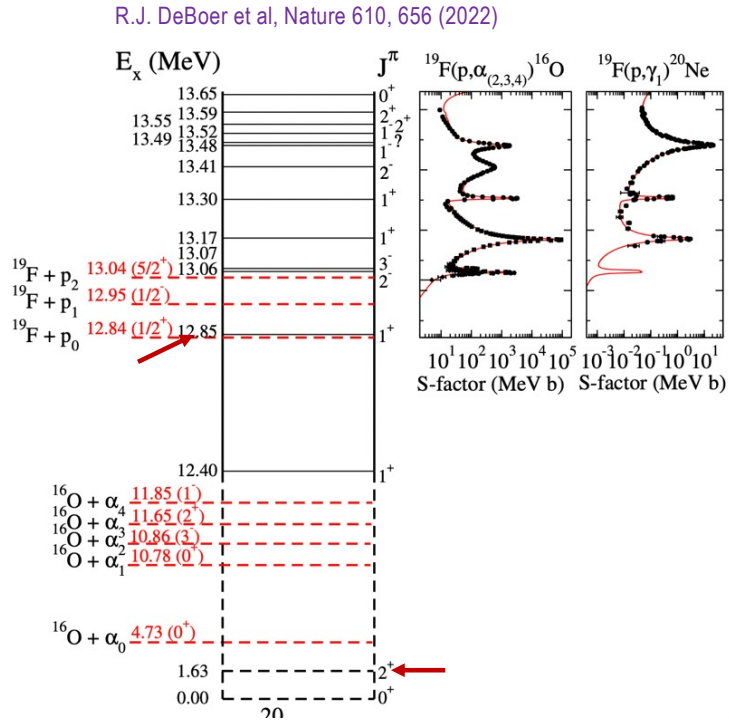
What is the effect of 1^+ resonance at ~ 10 keV above the proton emission threshold on the S-factor?



Liyong Zhang et al., Nature 610, 656 (2022)

Does $^{19}\text{F}(p,\gamma)^{20}\text{Ne}$ breakout reaction from the CNO cycle overcome $^{19}\text{F}(p,\alpha)^{16}\text{O}$ back-process reaction cross section becoming a source of the Ca abundance in the first generation stars?

Astrophysical relevance for α - and proton-capture reactions of nucleosynthesis



Exp: Liyong Zhang et al., Nature 610, 656 (2022)

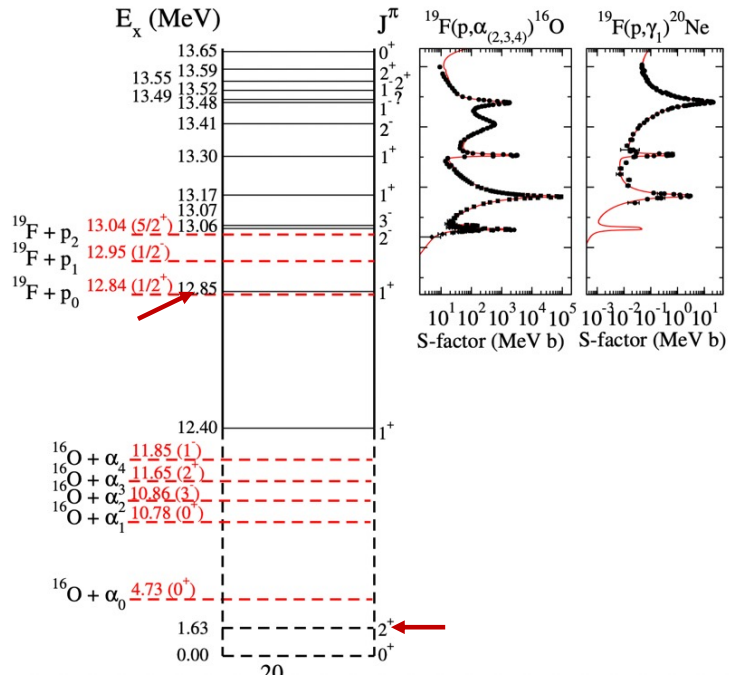
What is the effect of 1^+ resonance at ~ 10 keV above the proton emission threshold on the S-factor?

- S(0) astrophysical factor increases by more than 2 orders of magnitude!
- The decay to the 2^+ first excited state in ^{20}Ne dominates

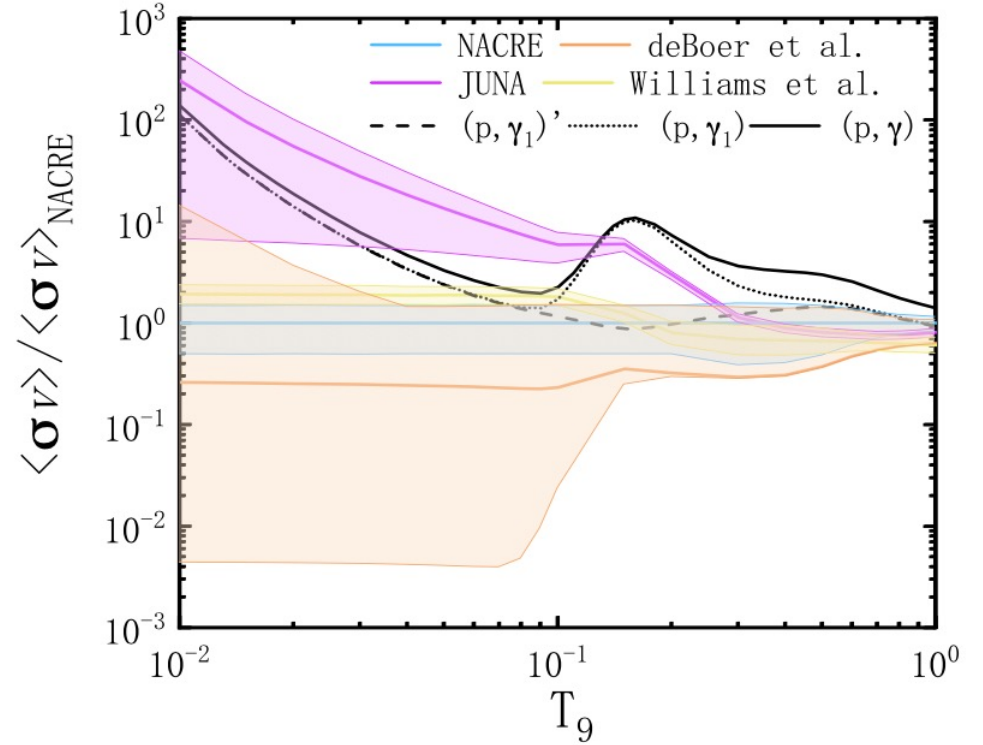
X.B. Wang, G.X. Dong, N. Michel, M. Ploszajczak, arXiv:2411.17243

Astrophysical relevance for α - and proton-capture reactions of nucleosynthesis

R.J. DeBoer et al, Nature 610, 656 (2022)



Exp: Liyong Zhang et al., Nature 610, 656 (2022)



What is the effect of 1^+ resonance at ~ 10 keV above the proton emission threshold on the S-factor?

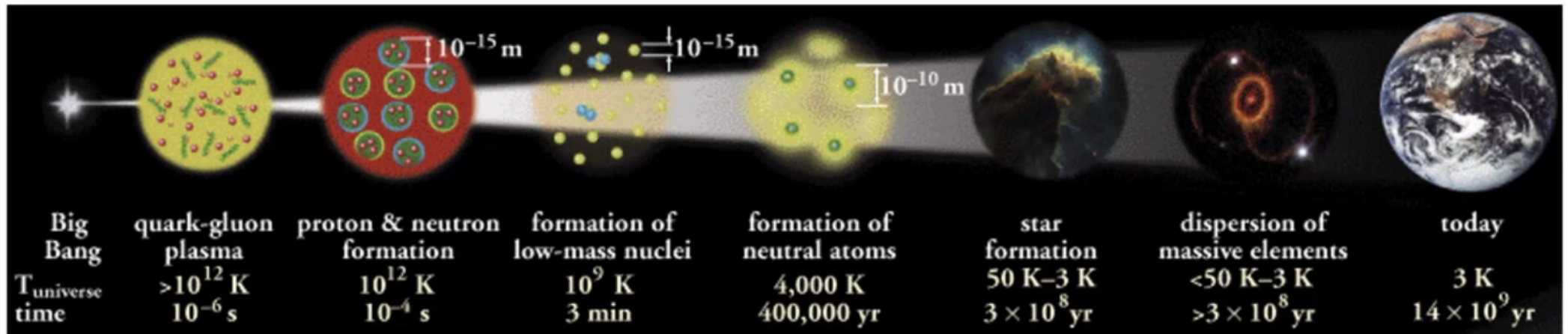
- GSM-CC reaction rates are significantly larger than in NACRE and comparable with JUNA data
- $^{19}\text{F}(p, \alpha)^{16}\text{O}$ back-process reaction should be remeasured to verify the hypothesis of breaking from hot-CNO cycle

X.B. Wang, G.X. Dong, N. Michel, M. Ploszajczak, arXiv:2411.17243

Mimicry mechanism of clustering

Ubiquitous process of clustering

Clustering is one of the most *mysterious* processes in Physics. It happens at all scales in time, distances and energies: from the micro scales of hadrons and nuclei to the macro scales of living organisms and clusters of galaxies, from the high excitation energies to cold systems



Generic mechanisms of the clusterization, independent of individual features of the studied system:

- *statistical mechanism* rooted in the Central Limit Theorem

- Random fragmentation or random aggregation?

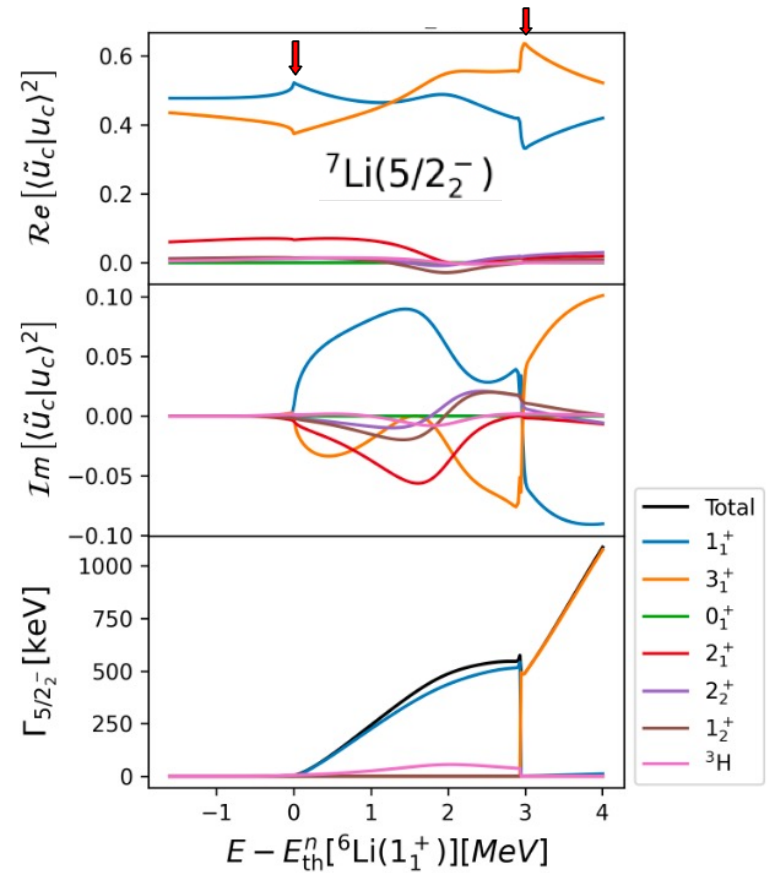
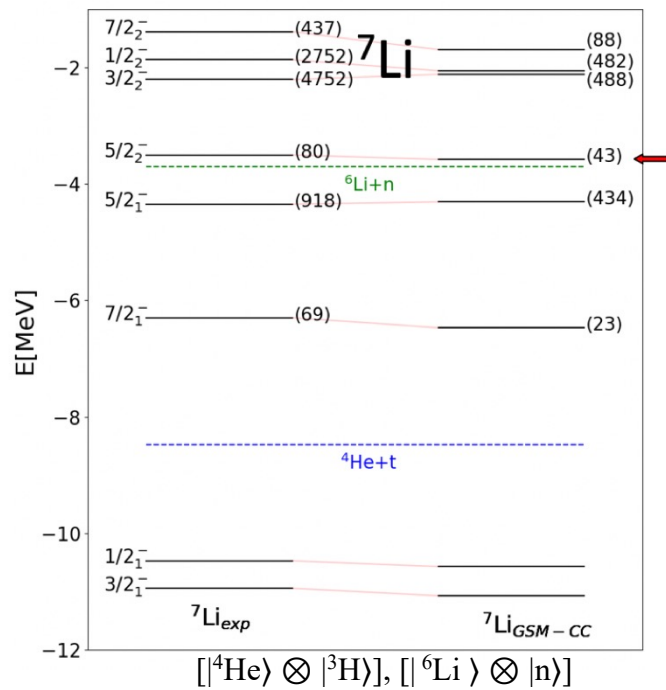
- In central HI collisions energies, nuclear fragments are produced in the *aggregation process*

R. Botet, M. Ploszajczak and INDRA Coll., Phys. Rev. Lett. 86, 3514 (2001)

- *mimicry mechanism* due to the interaction with the environment

- ...

Mimicry mechanism of clusterization



- Hamiltonian: 1-body potential, 2-body FHT interaction

H. Furutani et al, Prog. Theor. Phys. 62, 981 (1979)

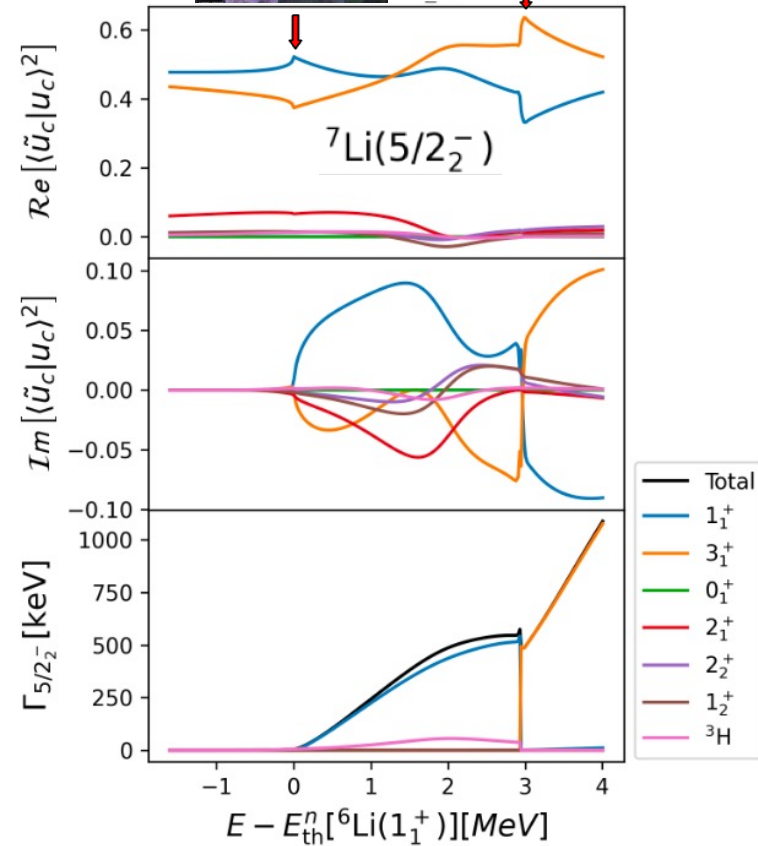
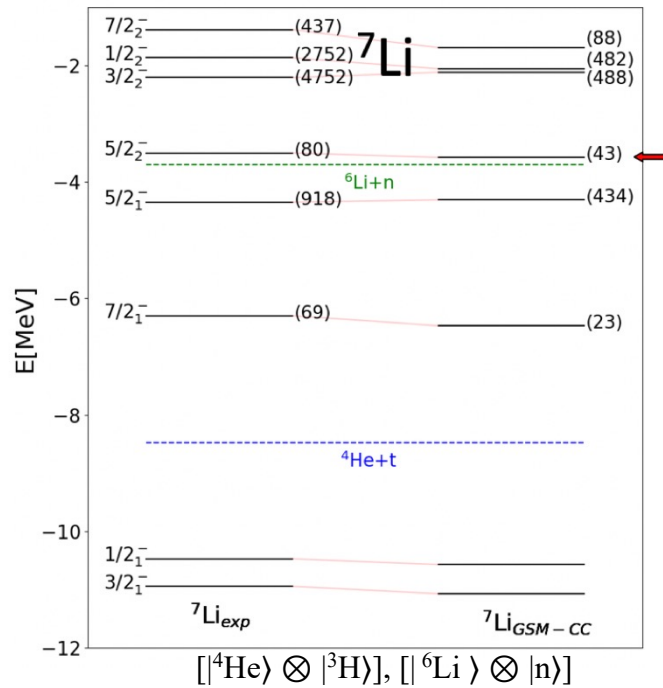
^3H wave functions calculated using $\text{N}^3\text{LO}_{(2\text{-body})}$ interaction

- Channels: $^6\text{Li}(K^\pi)$: $K^\pi=1_1^+, 1_2^+, 3_1^+, 0_1^+, 2_1^+, 2_2^+$

n : $\ell_j = s_{1/2}, p_{1/2}, p_{3/2}, d_{3/2}, d_{5/2}, f_{5/2}, f_{7/2}$

$^3\text{H}(L)$: $L \equiv {}^2\text{J}_{\text{int}+1}[\text{L}_{\text{CM}}]_{\text{JP}} = {}^2\text{S}_{1/2}, {}^2\text{P}_{1/2}, {}^2\text{P}_{3/2}, {}^2\text{D}_{3/2}, {}^2\text{D}_{5/2}, {}^2\text{F}_{5/2}, {}^2\text{F}_{7/2}$

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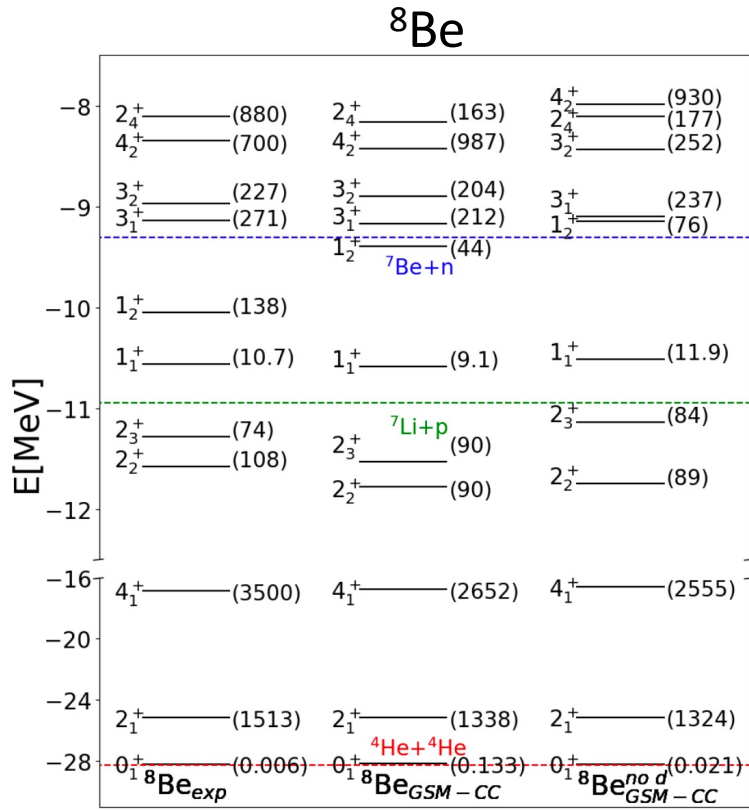
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- The resonance (*chameleon*) changes its structure (*skin color*) as a result of the alignment (*mimicry*) with the nearby new reaction channel (*changing environment*)

J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

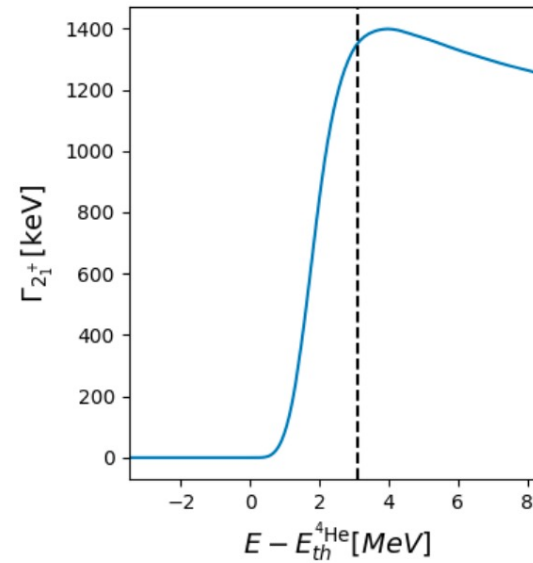
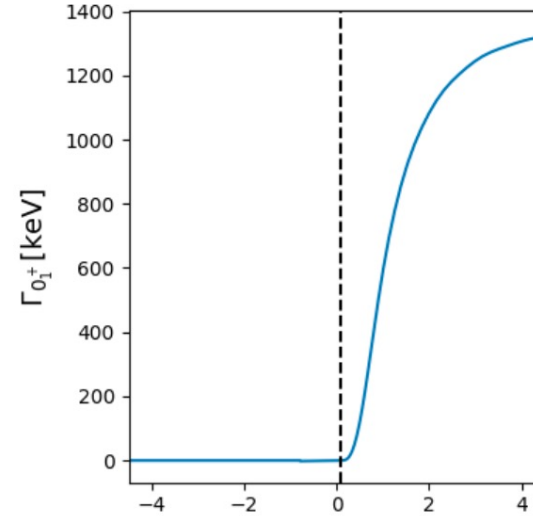
Near-threshold clustering in ^8Be



Mass partitions:

$[[^4\text{He}] \otimes ^4\text{He}]$, $[[^7\text{Li}] \otimes |p\rangle]$, $[[^7\text{Be}] \otimes |n\rangle]$, $[[^6\text{Li}] \otimes |d\rangle]$

J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

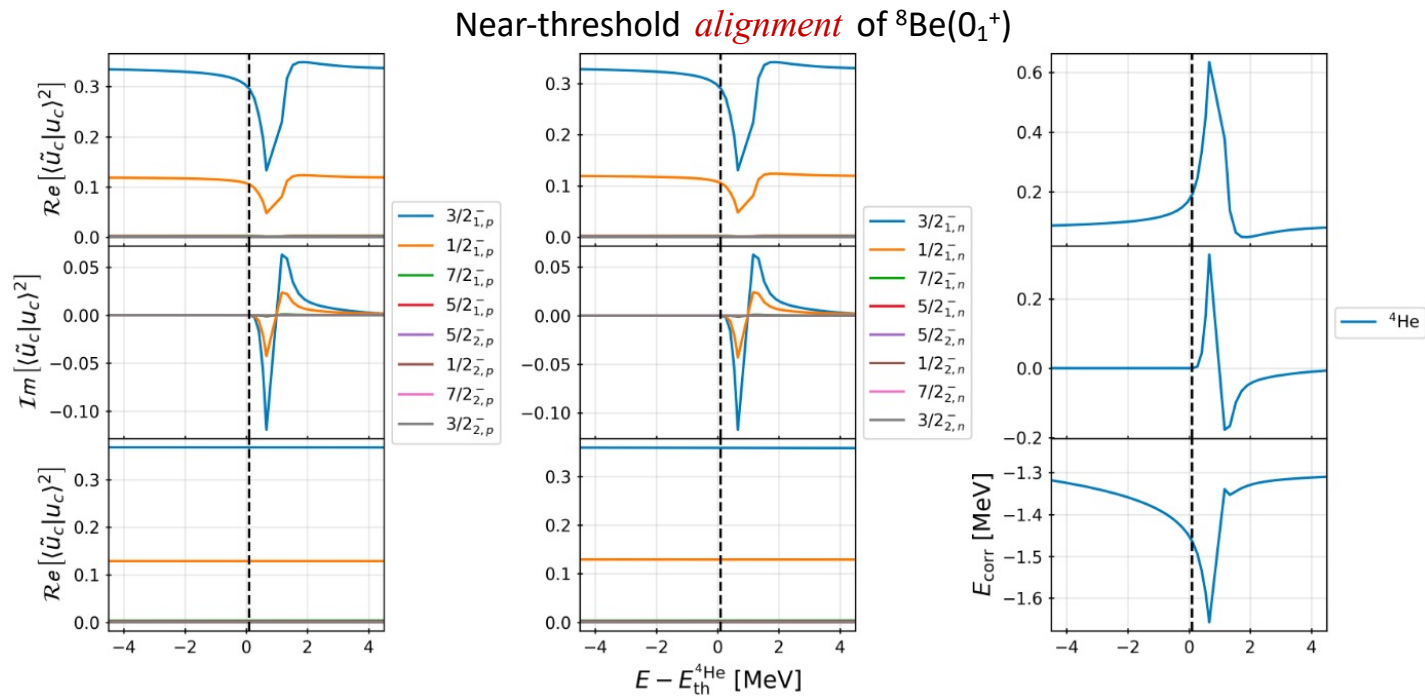
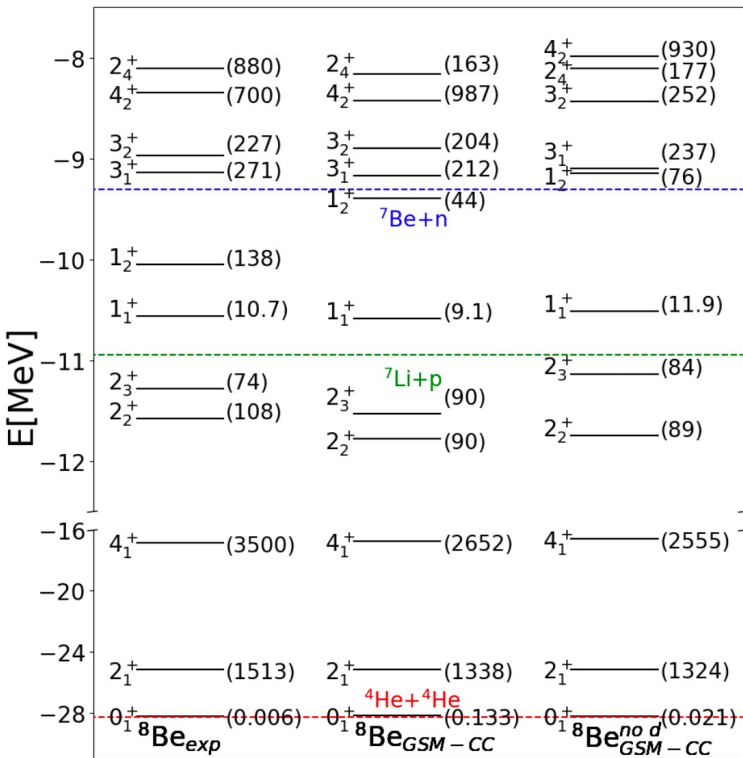


Near-threshold clustering in ^8Be

Continuum coupling correlation energy $\rightarrow E_{J\pi,M}^{(\text{corr})} = \langle \tilde{\Psi}_M^J | H | \Psi_M^J \rangle - \langle \tilde{\Phi}_M^{J;(\alpha)} | H | \Phi_M^{J;(\alpha)} \rangle \equiv \mathcal{E}_{J\pi,M} - \mathcal{E}_{J\pi,M}^{(\alpha)}$

$$|\Phi_M^{J;(\alpha)}\rangle = \sum_{c; c \neq \alpha} \int_0^{+\infty} |(c,r)_M^J\rangle \frac{\bar{u}_c^{JM}(r)}{r} r^2 dr$$

^8Be



Mass partitions:

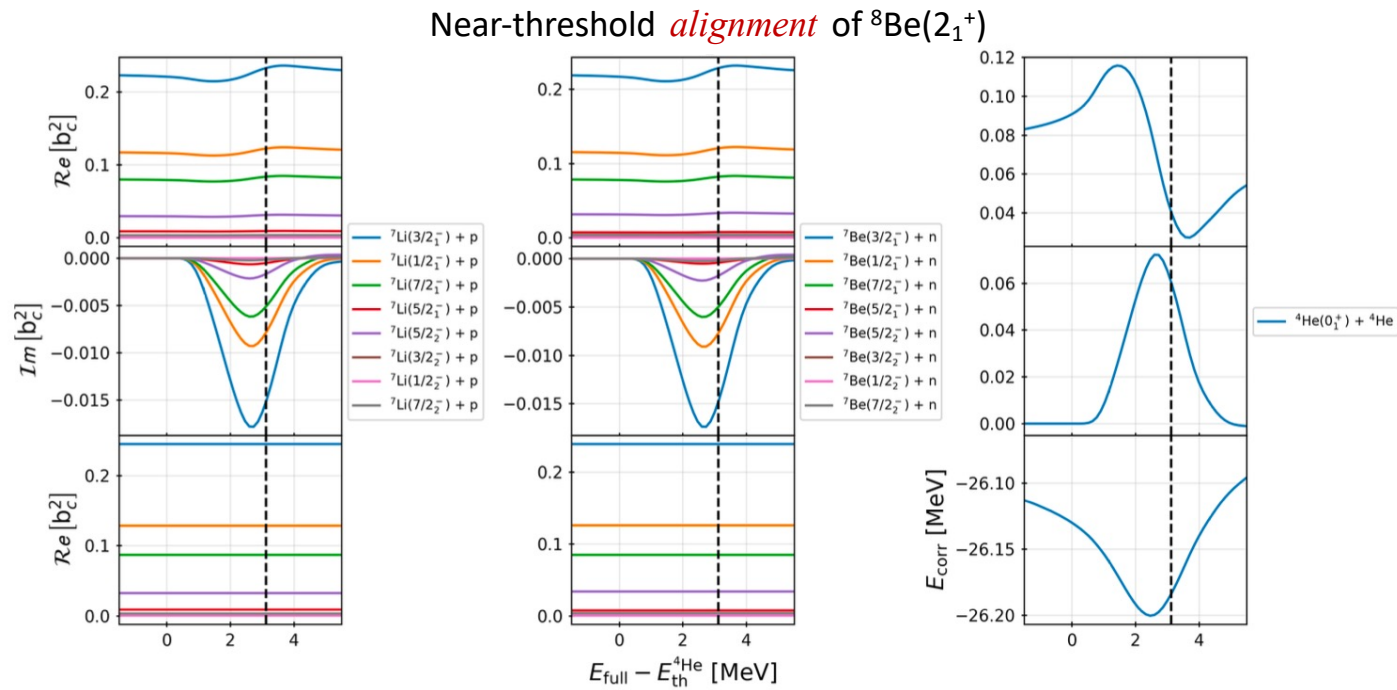
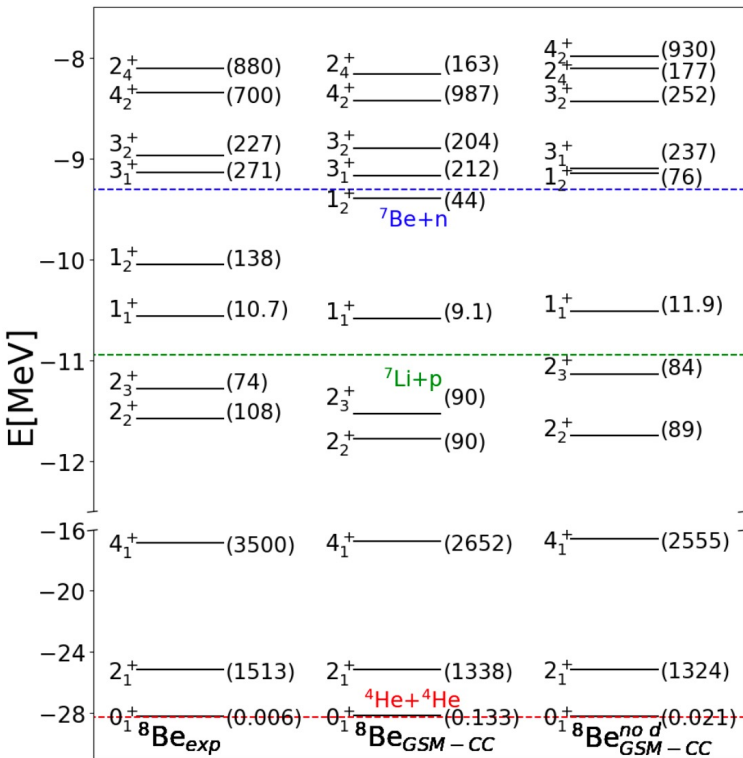
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Near-threshold clustering is the *emergent phenomenon*

J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

Conclusions

- Quantum systems in the vicinity of a particle emission threshold belong to the category of *open quantum systems* having unique properties which distinguish them from well-bound *closed quantum systems*
- Proximity of the threshold (branching point) induces the collective mixing of eigenstates resulting in a single *aligned eigenstate* of the open quantum system Hamiltonian (\rightarrow *chameleon resonance*)
- Clustering is the *emergent phenomenon* associated with the branch point singularity at the particle emission threshold
- Near-threshold phenomena are *terra incognita* of the nuclear physics:
 - *Collectivization* of wave functions due to the coupling to decay channel(s)
 - Formation of clusters/correlations: ${}^2\text{H}$, ${}^3\text{H}$, ${}^3\text{He}$, ${}^3\text{n}$, ${}^4\text{n}$, ... which carry an imprint of nearby decay channel(s)
 - Modification of NN interaction/spectroscopic factors
 - Effects of *coalescing resonances* in nuclear spectroscopy and reactions
 -

Essential role of *unitarity*!

In collaboration with:

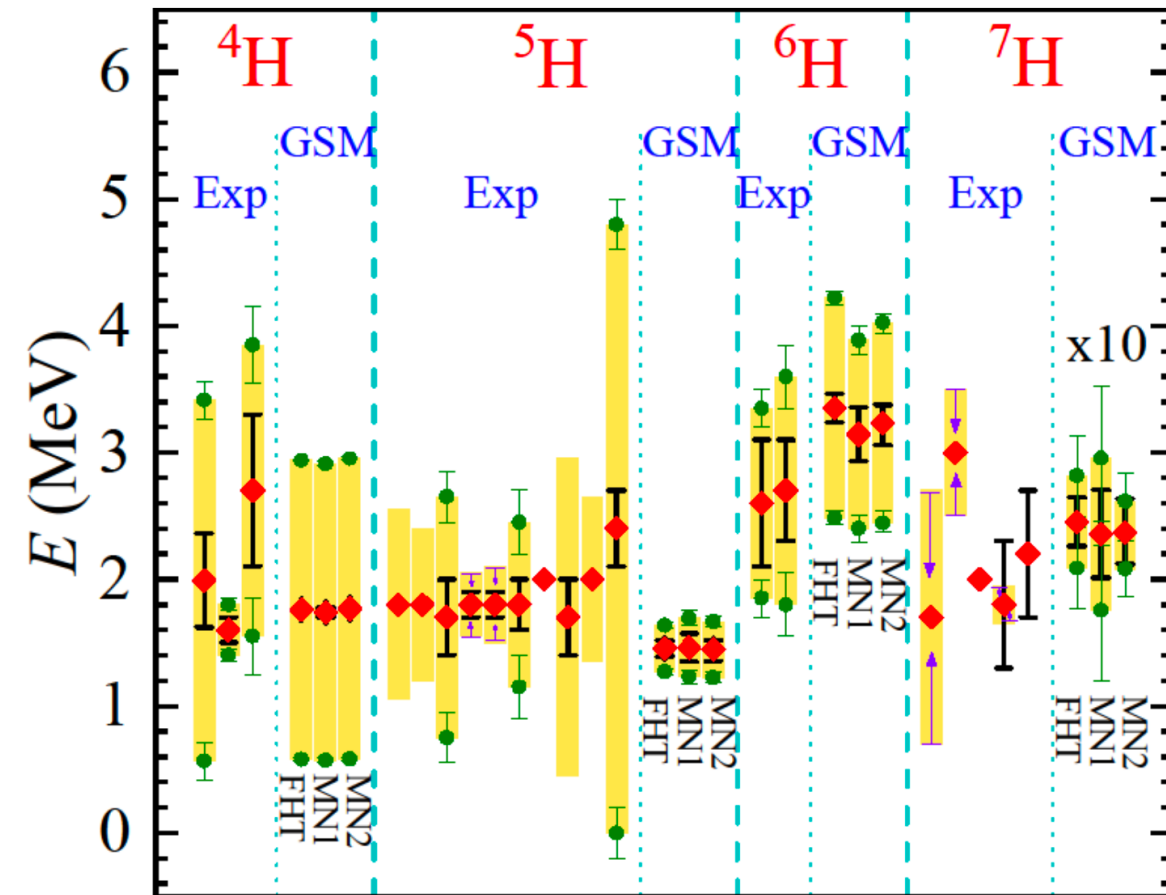
Nicolas Michel
Witek Nazarewicz
Jacek Okołówicz
Jose Pablo Linares
Xiaobao Wang
Guoxiang Dong

IMP/CAS Lanzhou/Beijin, China
MSU/FRIB East Lansing, USA
GANIL, INP Kraków, Poland
LSU Baton Rouge, USA
Huzhou University, China
Huzhou University, China

Thank You

Gamow shell model: Quasi-stationary extension of standard shell model in the complex k-plane

Example: Unbound hydrogen isotopes



Gamow shell model (GSM)

$$|SD_i\rangle = |u_{i_1} \dots u_{i_A}\rangle \rightarrow \sum_k |SD_k\rangle \langle SD_k| \cong 1$$

N. Michel et al, PRL 89, 042502 (2002)
 N. Michel, et al, J. Phys. G37, 064042 (2010)

- Calculation in the relative coordinates of core cluster SM coordinates Y. Suzuki, K. Ikeda, PRC 38 (1988) 410
- Center-of-mass handled by recoil term:

$$H \rightarrow H + \frac{1}{M_{\text{core}}} \sum_{(i < j) \in \text{val}} \mathbf{p}_i \cdot \mathbf{p}_j$$

- in the Hamiltonian
- *Unitary formulation* of the nuclear Shell Model

- GSM with a core of ^3H
- FHT and Minnesota interactions in spdf/spd space with Berggren basis
- Two-body interaction from a fit to the He chain
- Large widths for $^4, ^6\text{H}$, smaller widths for ^5H (~500 keV) and ^7H (10-250 keV) – to be checked in future experiments

H.H. Li, J.G. Li, N. Michel, W. Zuo, PRC 104, L061306 (2021)