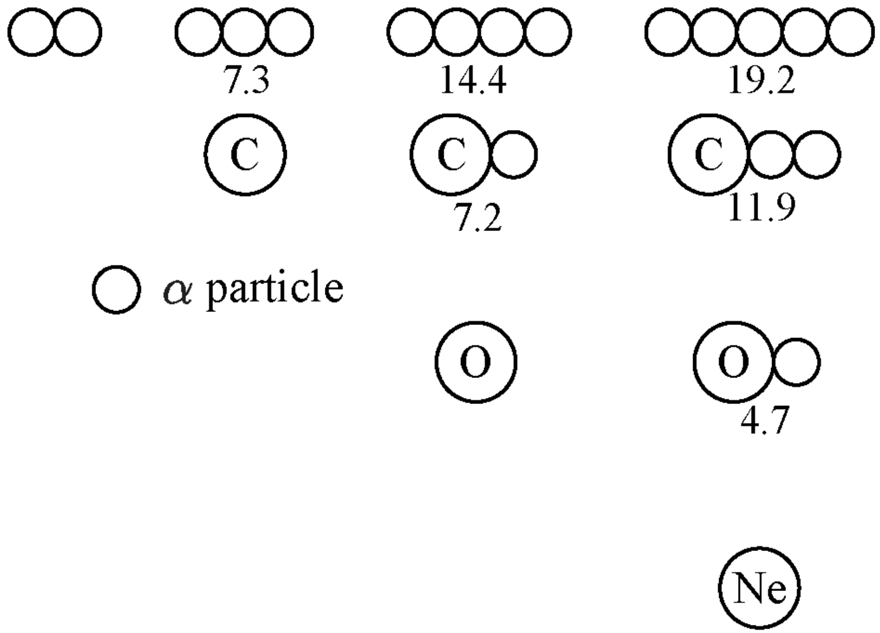
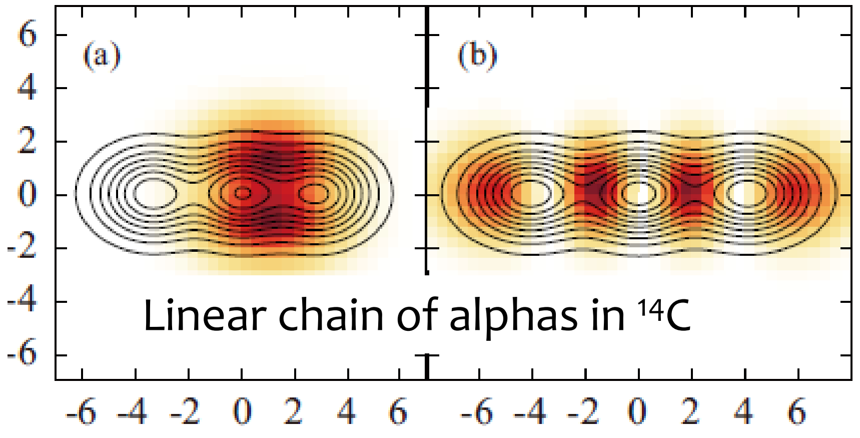
# Alpha clusters in the ground states of nuclei

M. Kimura (RIKEN)

Ikeda diagram asserts the clusters above the threshold energy but says nothing about the ground states




We have discussed clusters in the excited states




T. Baba and M.K., PRC96, 064318 (2017).

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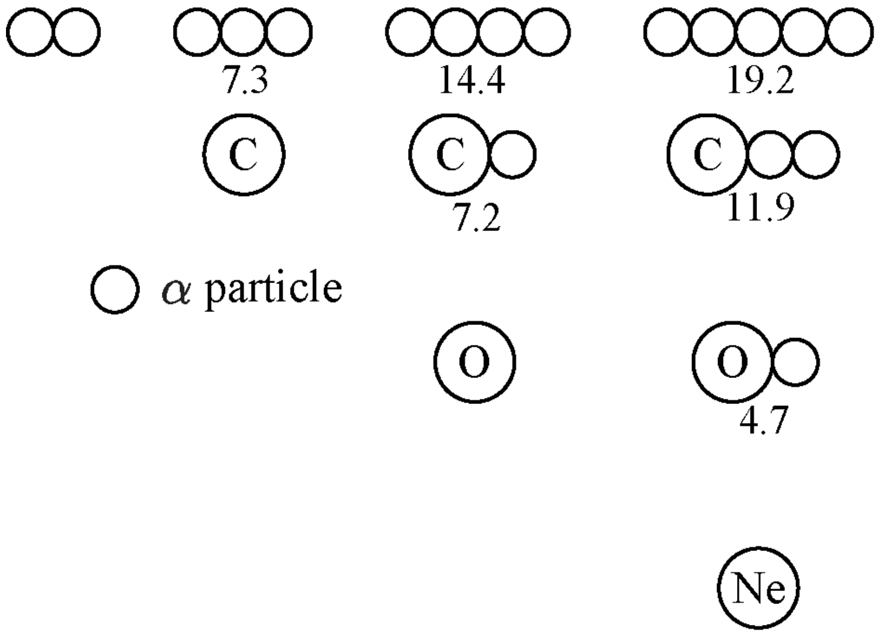
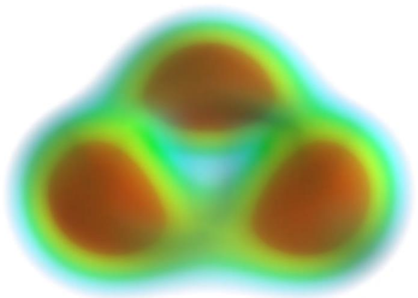
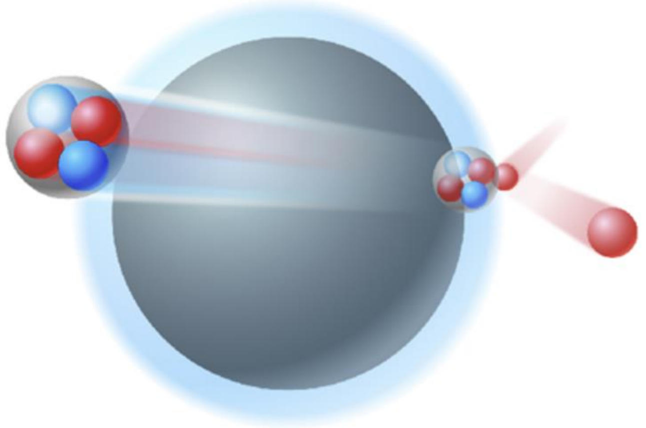



Recently, I've been convinced that
we can also observe clusters in the ground state.

Reason #1: Electron scattering data of $^{12}C$

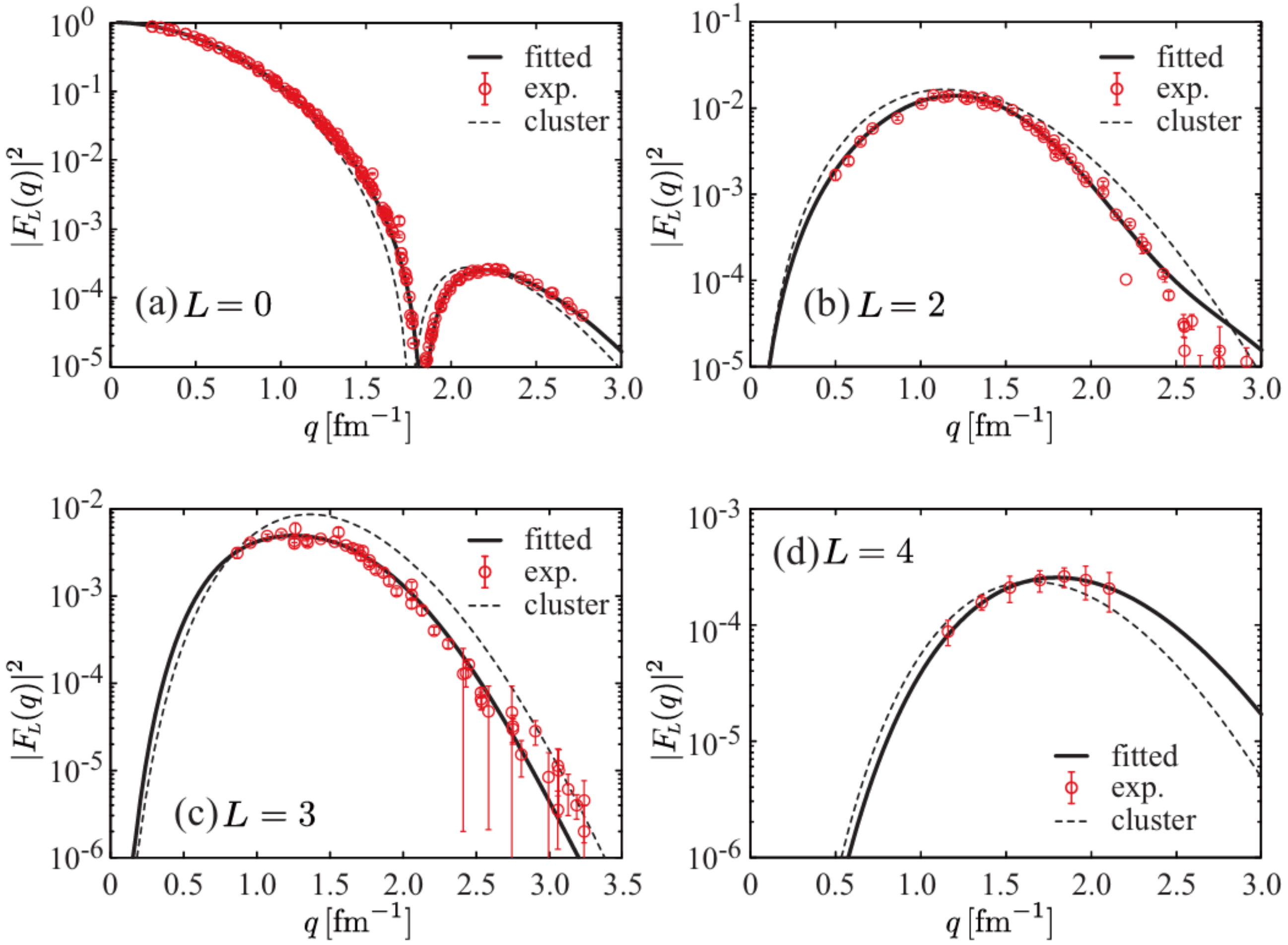
Reason #2: Alpha knockout reaction data

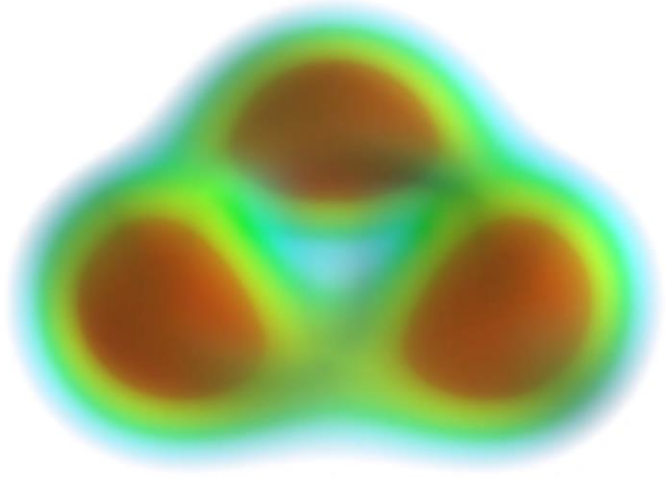




# Reason #1: Shape of $^{12}$C from electron scattering data

M. Kimura and Y. Taniguchi, EPJA60, 77 (2024).



# Spinning triangle of 3 Alphas

## Spinning triangle of Alphas

Three alpha particles locate
in a triangle configuration ($D_{3h}$ symmetry)

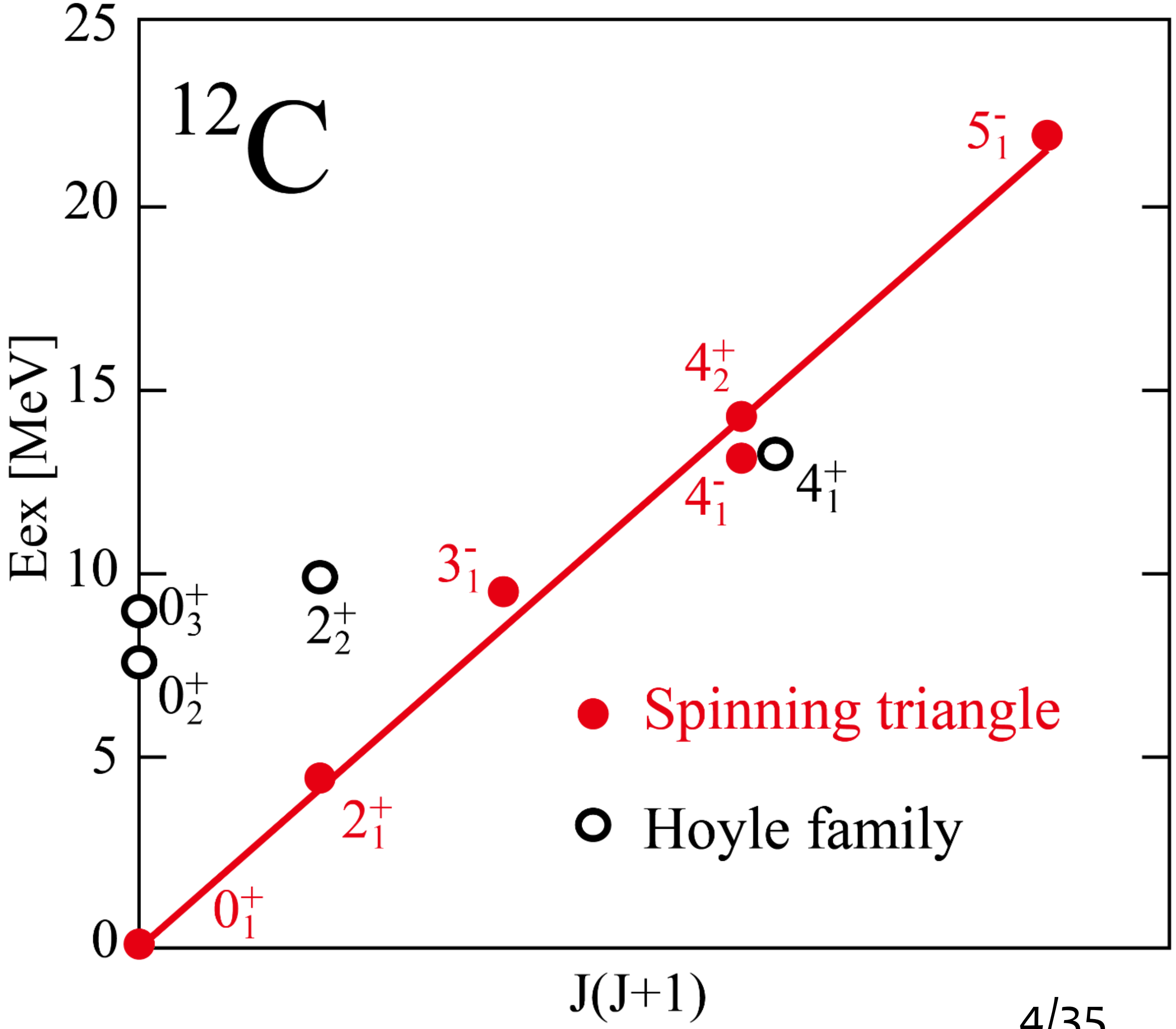
R. Bijker and F. Iachello, PRC61, 067305 (2000).

◎ Due to the symmetry, $0^+$, $2^+$, $3^-$, $4^{\pm}$, $5^-$ states constitute "ground band"

◎ $4^+$ and $4^-$ states should degenerate due to "symmetry"

I wish to discuss shape of $^{12}C$ only from exp. data with minimal assumption

D.J. Marín-Lámbarri, et al. PRL113, 012502(2014)



# Rebuilding nuclear shape from Exp. data

M. Kimura and Y. Taniguchi, EPJA60, 77 (2024).

# Rebuilding nuclear shape from observables

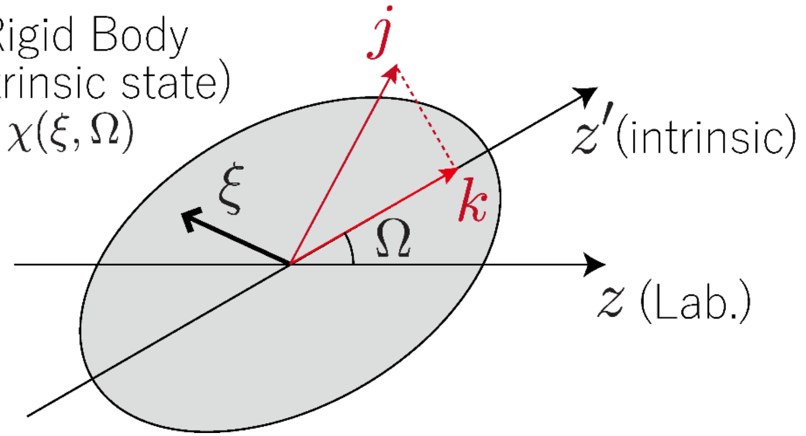
I assume that **the $0^+$, $2^+$, $3^-$ and $4^+$ states constitute "the ground band" sharing the same intrinsic state**
(This is the only assumption I made)

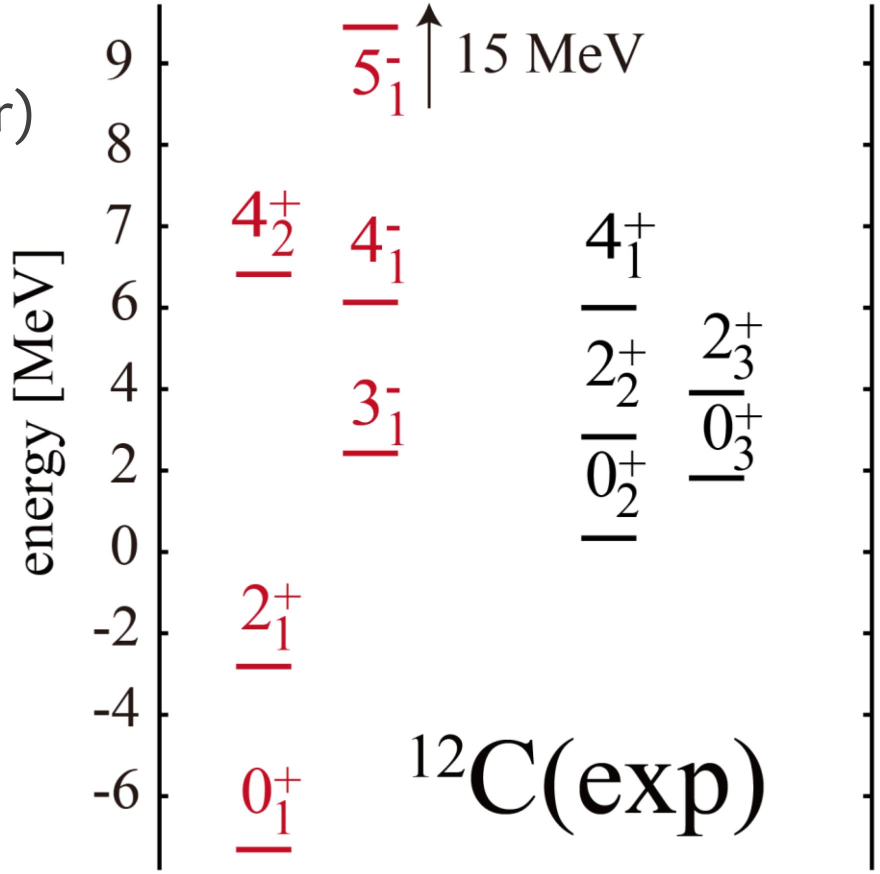
Non-axial and parity-asymmetric rigid rotor (a textbook matter)

$$\Psi^{L\Pi}_{L_zK} = \sqrt{\frac{2L+1}{16\pi^2(1+\delta_{K0})}}\,\chi(\xi,\Omega)\left\{ D^{L}_{L_zK}(\Omega) + \Pi(-)^{L+K} D^{L}_{L_z-K}(\Omega) \right\}$$

**laboratory (observable)**

**intrinsic (unobservable)**



# Rebuilding nuclear shape from observables

Consider the diagonal/transition densities (observed by electron scatt.)

$$\rho^{0^+\to L^\Pi}(\boldsymbol{r}) := \langle L^\Pi, 0|\rho(\boldsymbol{r})|0_1^+, 0\rangle$$

$$\frac{d\sigma_L^{\rm obs}}{d\theta} = |F_L(q)|^2 \frac{d\sigma_L^{\rm Mott}}{d\theta}, \qquad F_L(q) := \frac{\sqrt{4\pi(2L+1)}}{Z}\int_0^\infty r^2 dr\, j_L(qr)\, \rho^{0\to L^\Pi}(\boldsymbol{r}) / Y_{L0}(\hat{r}),$$

observable

transition density

1. Insert the rigid-rotor wave function into this definition, one gets

$$\rho^{0\to L^\Pi}(\boldsymbol{r}) = \frac{1}{8\pi^2}\left(\frac{2L+1}{2(1+\delta_{K0})}\right)^{1/2}\int d\Omega \left\{ D^{L^*}_{L_z K}(\Omega) + \Pi(-)^K D^{L^*}_{L_z -K}(\Omega)\right\} \langle \chi(\xi,\Omega)|\rho(\boldsymbol{r})|\chi(\xi,\Omega)\rangle.$$

transition density (observable) rigid-body density (unobservable)

2. Using the multipole decomp. of rigid-body density, the integral can be calculated analytically.

$$\langle \chi(\xi,\Omega=0)|\rho(\boldsymbol{r})|\chi(\xi,\Omega=0)\rangle = \frac{1}{2}\sum_{lm} \rho^{\rm rigid}_{lm}(r)\left\{Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r})\right\}.$$

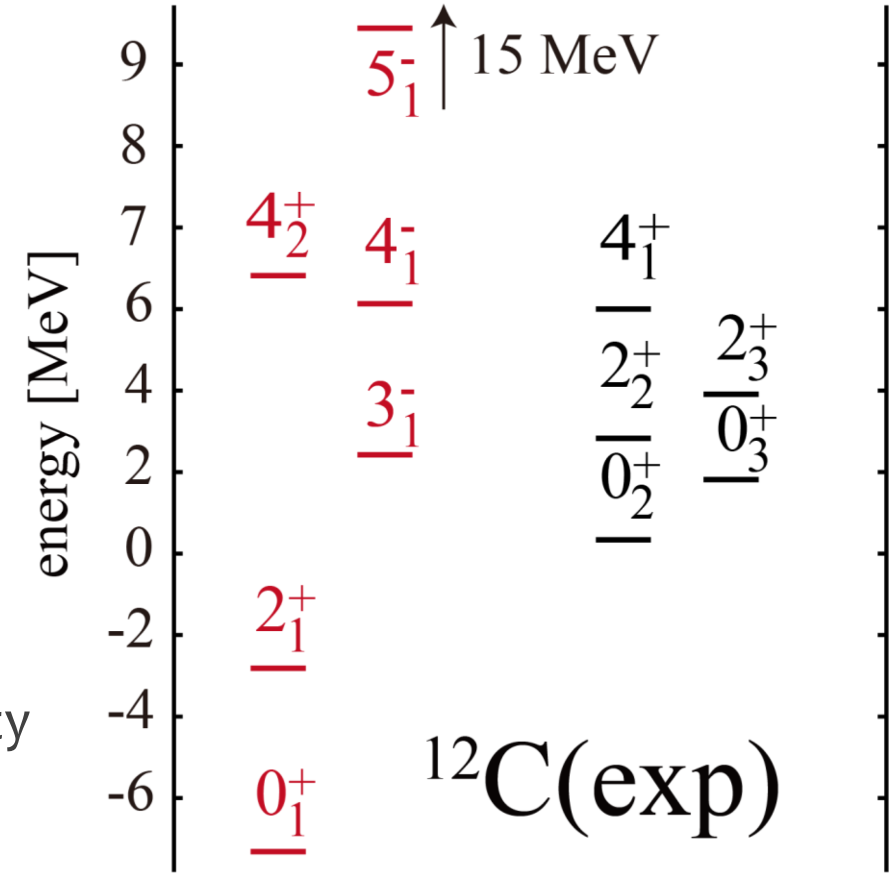
$$\rho^{0\to L^\Pi}(\boldsymbol{r}) = \frac{1}{\sqrt{2L+1}}\left(\frac{1+\Pi(-)^L}{1+\delta_{K0}}\right)^{1/2} \rho^{\rm rigid}_{LK}(r) Y_{L0}(\hat{r}),$$

transition density (observable) rigid-body density (unobservable)

# Rebuilding nuclear shape from observables

Summary of our assumption and numerical procedure

Assumption: The $0^+$, $2^+$, $3^-$ and $4^+$ states constitute "the ground band" sharing the same intrinsic state




① Get the multipole decomposition of the rigid-body density from the observed transition densities

$$\rho^{0\to L^\Pi}(\boldsymbol{r}) = \frac{1}{\sqrt{2L+1}}\left(\frac{1+\Pi(-)^L}{1+\delta_{K0}}\right)^{1/2}\rho^{\rm rigid}_{LK}(r)Y_{L0}(\hat{r}),$$

**transition density (observable)** **rigid-body density (unobservable)**

② Sum up all the multipole to reconstruct the rigid-body density

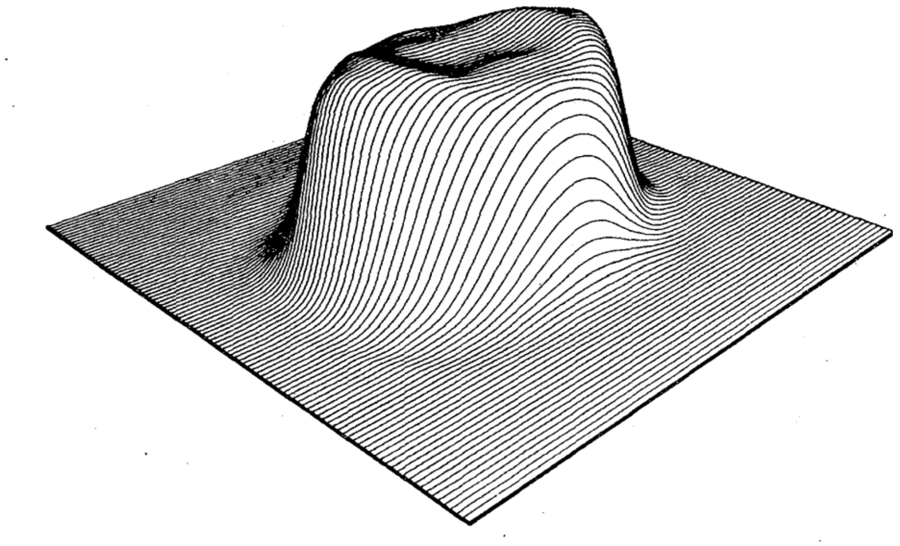
$$\rho^{\rm rigid}(\boldsymbol{r}) = \frac{1}{2}\sum_{lm}\rho^{\rm rigid}_{lm}(r)\left\{\,Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r})\,\right\}$$

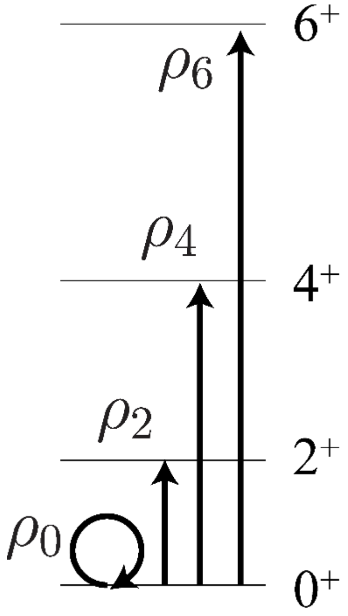
# Rebuilding nuclear shape from observables

Actually, this is not a new idea.
In 1970's, it has been already applied to axial and parity-symmetric shapes.

$^{152}$Sm; intrinsic density

L.S. Carcman et al., PRC78, 1388 (1978).



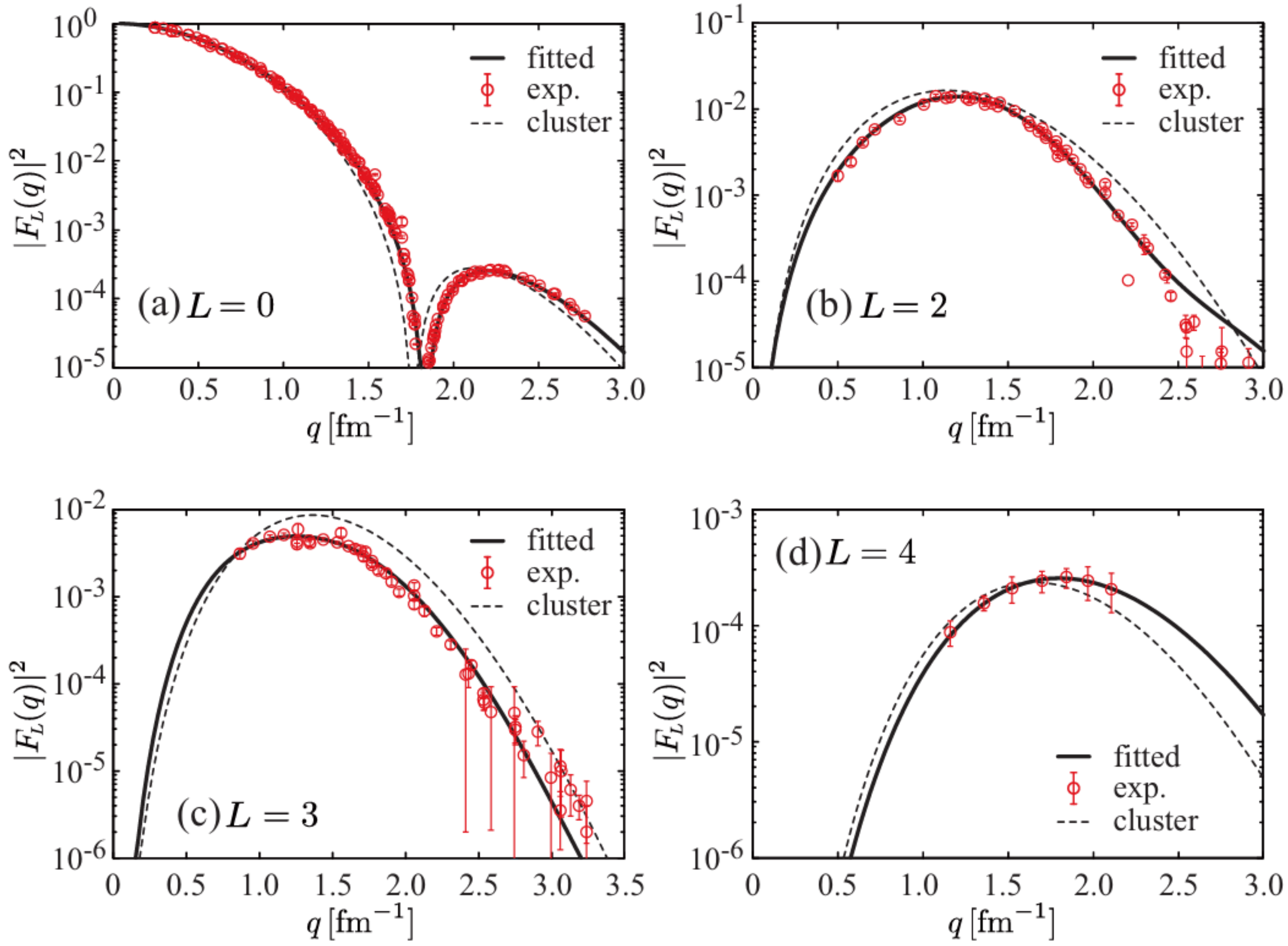


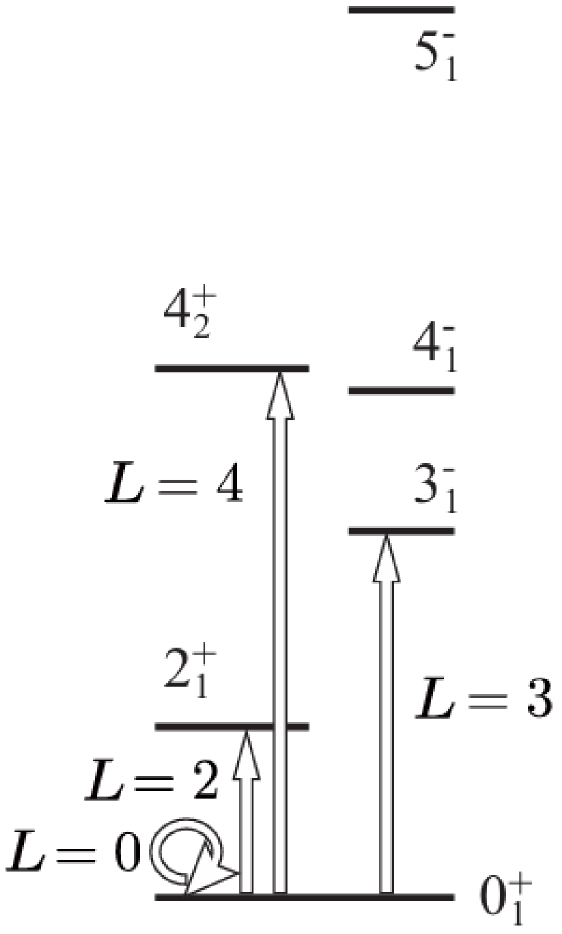

Here, we have extended this to the Non-axial and parity-asymmetric cases

# Rebuilding shape of $^{12}$C

Diagonal/Transition formfactors were measured.

$$\frac{d\sigma_L^{\mathrm{obs}}}{d\theta} = |F_L(q)|^2 \frac{d\sigma_L^{\mathrm{Mott}}}{d\theta},$$

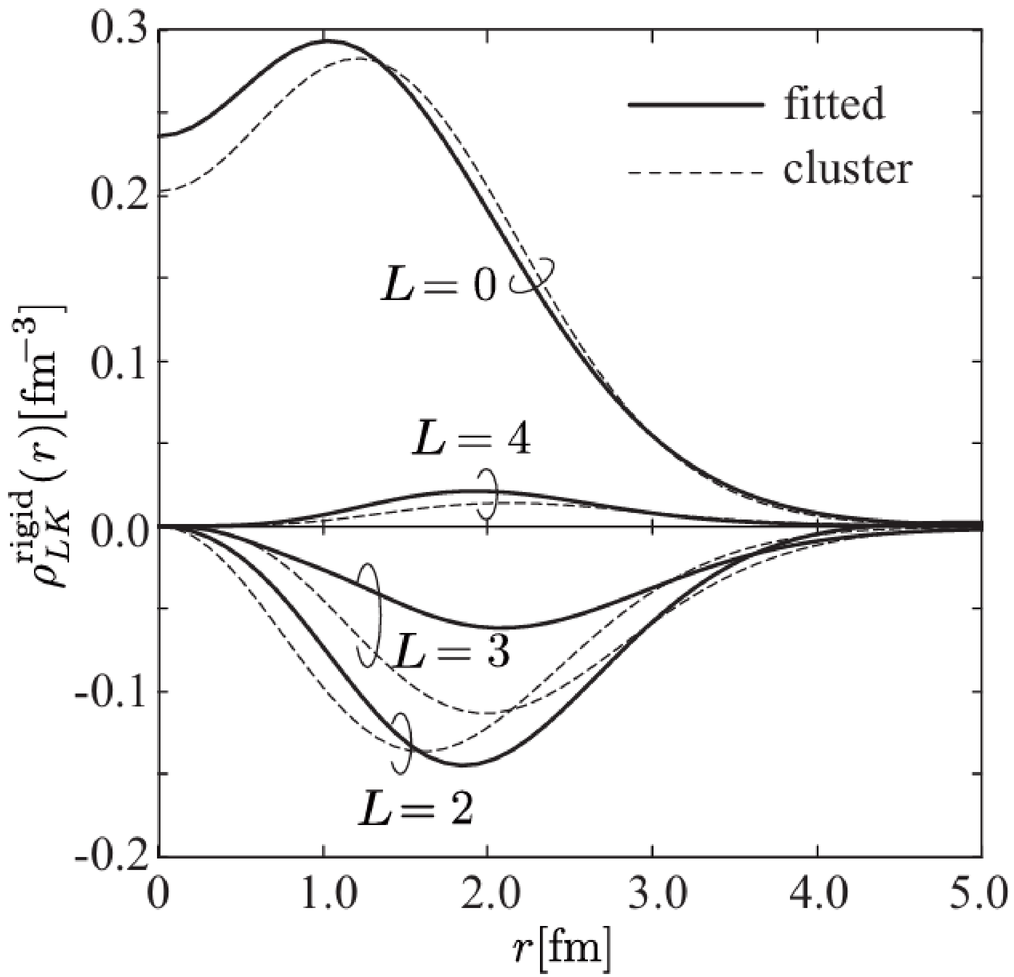


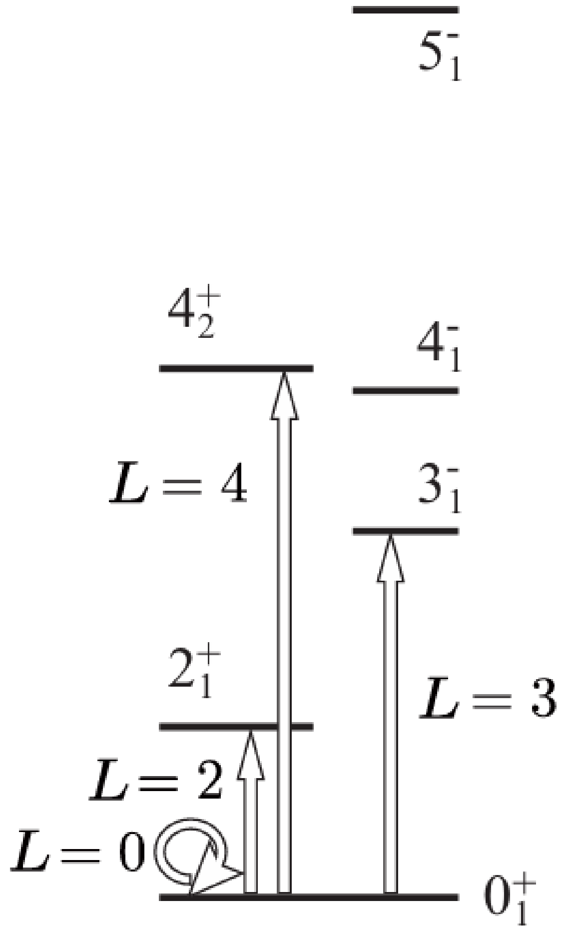



# Rebuilding shape of $^{12}$C

And inverse Fourier transform to the densities.

$$F_L(q) := \frac{\sqrt{4\pi(2L+1)}}{Z}\int_0^\infty r^2 dr\ j_L(qr)\,\rho^{0\to L^\Pi}(\boldsymbol{r})/Y_{L0}(\hat{r}),$$



# Rebuilding shape of $^{12}$C

We rebuild the rigid density of $^{12}$C by superposing the "observed" transition densities

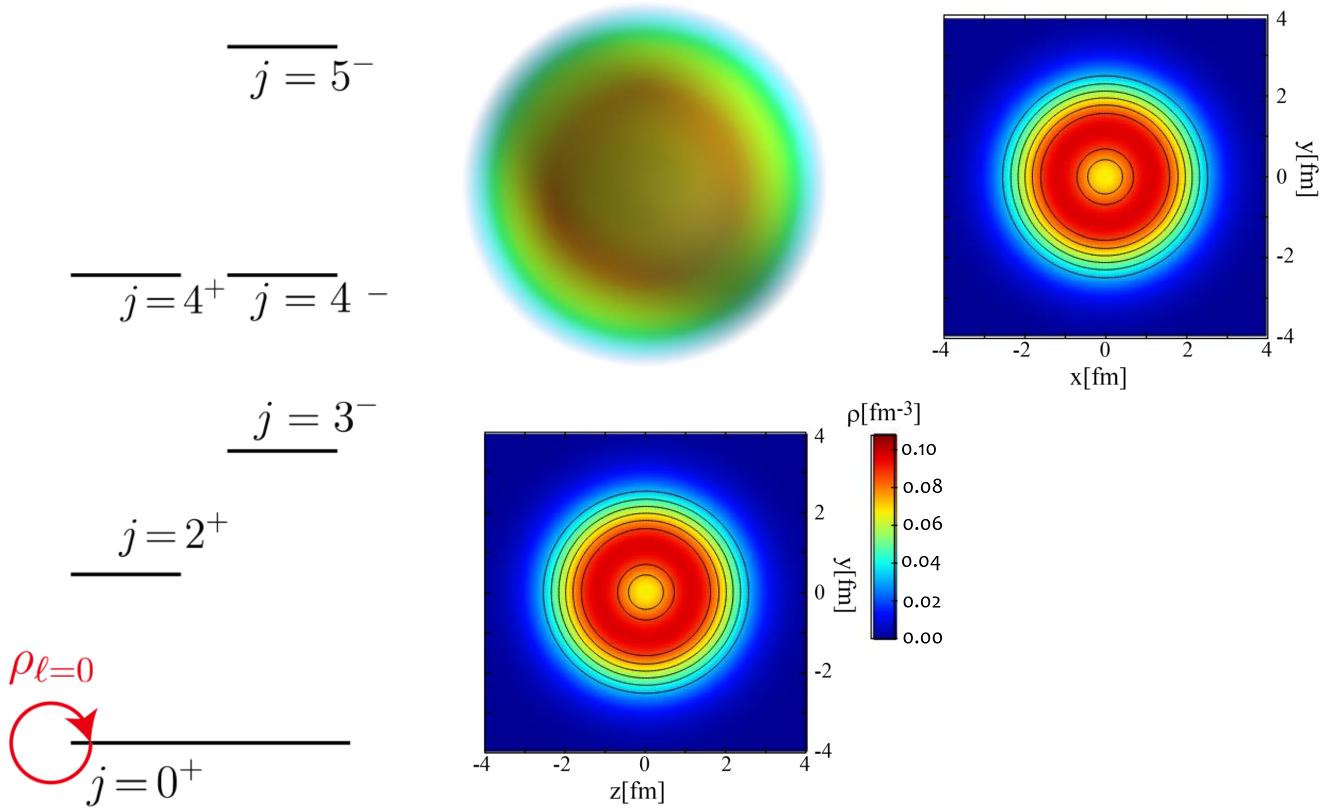
$$\rho^{\text{rigid}}(\boldsymbol{r}) = \frac{1}{2}\sum_{lm} \rho^{\text{rigid}}_{lm}(r)\left\{ Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r}) \right\}$$

**rigid-body density (unobservable)**

**multipole decomp. rigid-body density (unobservable)**

$\ell = 0$

- With only the $\ell = 0$ density, it is spherical



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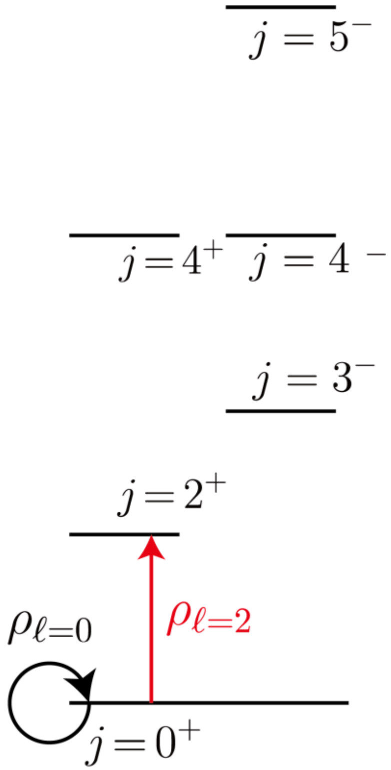
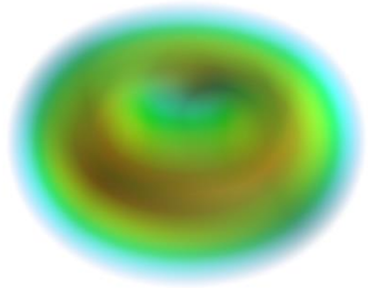
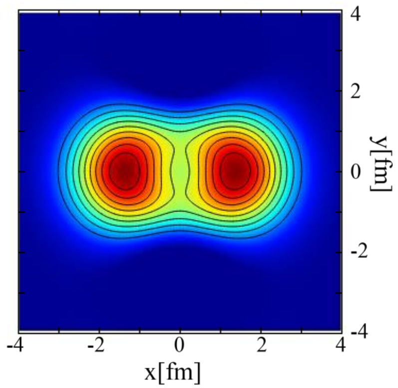
$$\underbrace{\rho^{\mathrm{rigid}}(\boldsymbol{r})}_{\text{rigid-body density (unobservable)}} = \frac{1}{2}\sum_{lm} \underbrace{\rho^{\mathrm{rigid}}_{lm}(r)}_{\text{multipole decomp. rigid-body density (unobservable)}} \{\, Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r}) \,\}$$

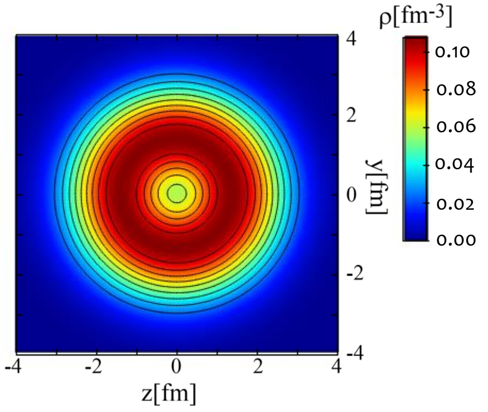
$$\ell = 0 + 2$$

- With only the $\ell = 0$ density, it is spherical
- The $\ell = 2$ density makes it toroidal shape





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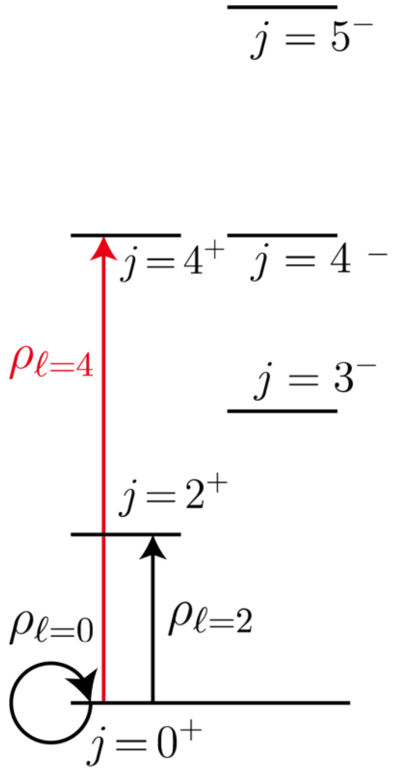
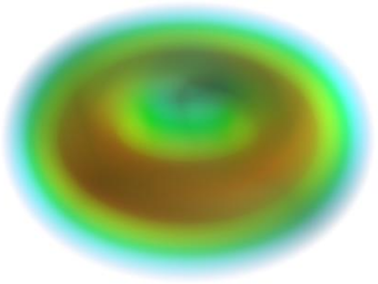
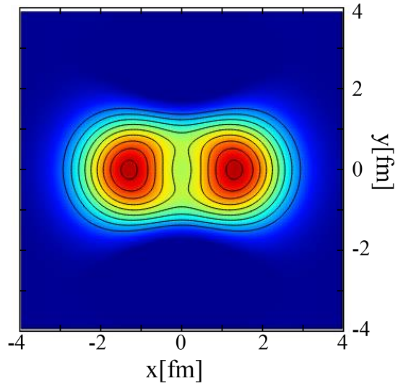
$$\underbrace{\rho^{\text{rigid}}(\boldsymbol{r})}_{\text{rigid-body density (unobservable)}} = \frac{1}{2}\sum_{lm} \underbrace{\rho^{\text{rigid}}_{lm}(r)}_{\text{multipole decomp. rigid-body density (unobservable)}} \{\, Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r}) \,\}$$

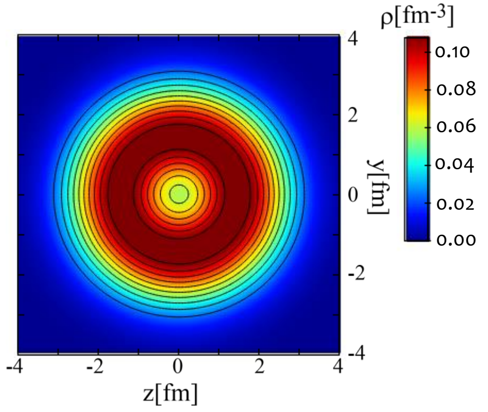
$$\ell = 0^+ + 2^+ + 4^+$$

- With only the $\ell = 0$ density, it is spherical
- The $\ell = 2$ density makes it toroidal shape
- The $\ell = 4$ density emphasizes toroidal shape





# Rebuilding shape of $^{12}$C

We rebuild the rigid density of $^{12}$C by superposing the "observed" transition densities

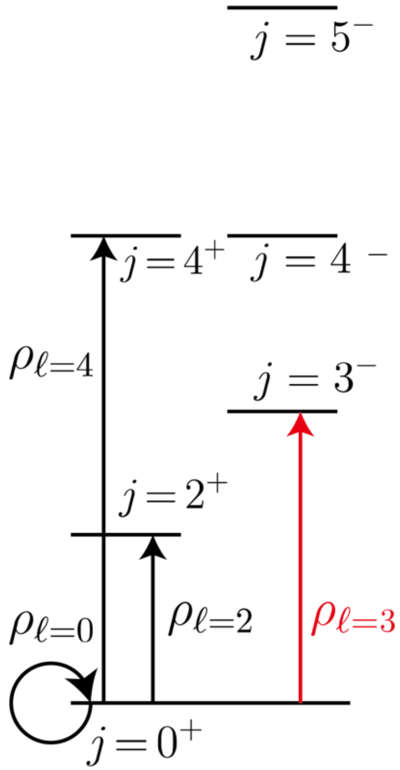
$$\underline{\rho^{\mathrm{rigid}}(\boldsymbol{r})} = \frac{1}{2}\sum_{lm}\underline{\rho^{\mathrm{rigid}}_{lm}(r)}\left\{Y_{lm}(\hat{r}) + (-)^m Y_{l-m}(\hat{r})\right\}$$

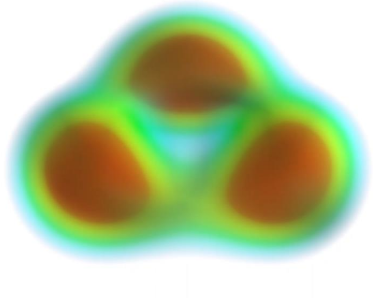
$$\ell = 0^+ + 2^+ + 4^+ + 3^-$$

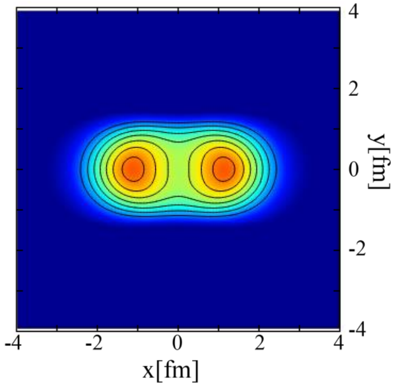
**rigid-body density (unobservable)**

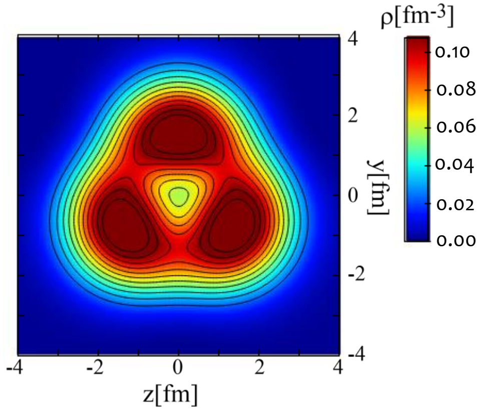
**multipole decomp. rigid-body density (unobservable)**

- With only the $\ell = 0$ density, it is spherical
- The $\ell = 2$ density makes it toroidal shape
- The $\ell = 4$ density emphasizes toroidal shape
- The $\ell = 3$ density makes it very exotic.
  - Emphasized triangular shape
  - Three prominent peaks of density



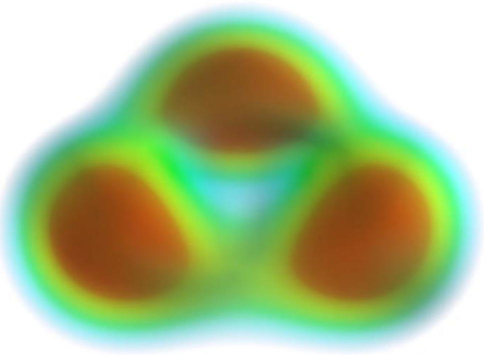


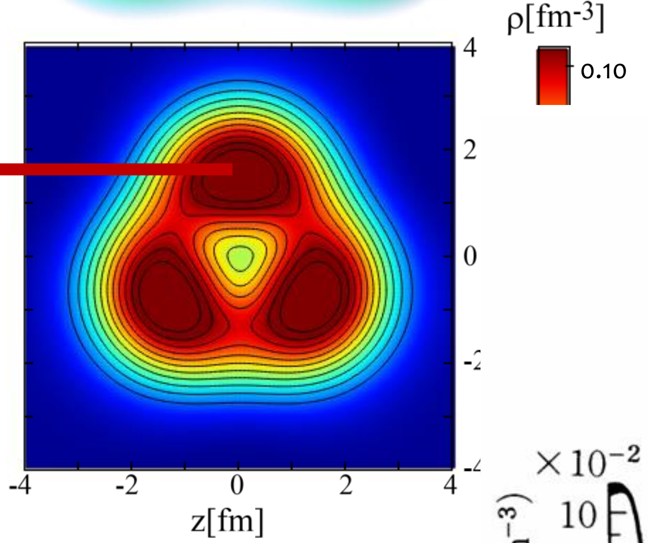



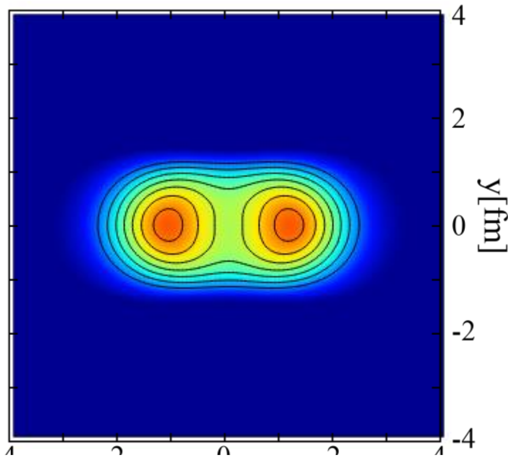
# Summary for the shape of $^{12}$C

Assumption: The $0^+$, $2^+$, $3^-$ and $4^+$ states constitute "the ground band" sharing the same intrinsic state

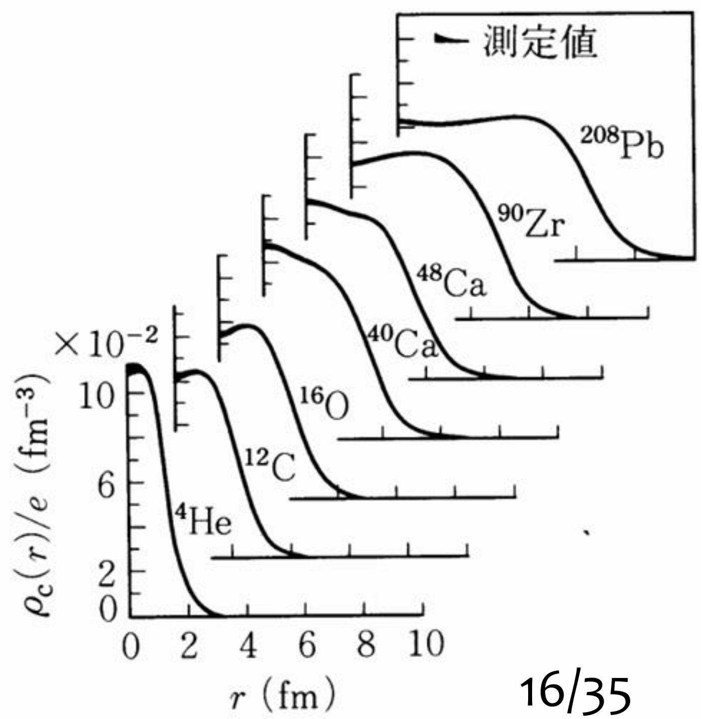
An exotic triangular shape of $^{12}$C shows up
(3 alphas?)






Density at the vertex is as high as 0.11 fm$^{-3}$ which is much larger than typical density (∼ 0.08 fm$^{-3}$).
Is this unrealistic?




Among all nuclei, there is only one nucleus which has such high central density.
The central density of alpha particle is 0.11 fm$^{-3}$.
Is this an accidental coincidence !?

# Theoretical check of the rigidity

Transition densities divided by Clebsch-Gordan coffeficient should be independent of initial and final states, but dependent on the transferred angular momentum

$$\underline{\rho_\ell^{int}(r)} = \boxed{(-)^{j_i+j_f}\rho_\ell^{j_i\to j_f}(r)/C_{j_f0\lambda0}^{j_i0}}$$

$j_i, j_f$ indep.

This should be $j_i, j_f$ indep.

# Real-Time evolution method

◎ Hamiltonian $H = \sum_{i=1}^{A} t(i) - t_{cm} + \sum_{i<j}^{A} v_{Volkov}(ij) + \sum_{ij} v_{Coulomb}(ij)$

The same Hamiltonian with other cluster models (Volkov No.2)

RGM: M. Kamimura, NPA351, 456 (1981). THSR: Y. Funaki PRC92 021302 (2015).

◎ Model wave function (time-dependent Brink wave func.)

nucleon: $\phi(\boldsymbol{r}, \boldsymbol{Z}(t)) = \left(\frac{2\nu}{\pi}\right)^{3/4} \exp\left\{-\nu\left(\boldsymbol{r} - \frac{\boldsymbol{Z}(t)}{\sqrt{\nu}}\right)^2 + \frac{1}{2}Z(t)^2 x\right\}$

αcluster: $\Phi_\alpha(\boldsymbol{Z}(t)) = \mathcal{A}\{\phi(\boldsymbol{r}_1, \boldsymbol{Z}(t))\chi_{p\uparrow}, \ldots, \phi(\boldsymbol{r}_4, \boldsymbol{Z}(t))\chi_{n\downarrow}\}$

3αsystem: $\Phi(\boldsymbol{Z}_1(t), \boldsymbol{Z}_2(t), \boldsymbol{Z}_3(t)) = \mathcal{A}\{\Phi_\alpha(\boldsymbol{Z}_1(t))\Phi_\alpha(\boldsymbol{Z}_2(t))\Phi_\alpha(\boldsymbol{Z}_3(t))\}$

○ $\boldsymbol{Z}_1(t), \boldsymbol{Z}_2(t), \boldsymbol{Z}_3(t)$ represent the positions (real part) and momenta (imaginary part) of alpha clusters

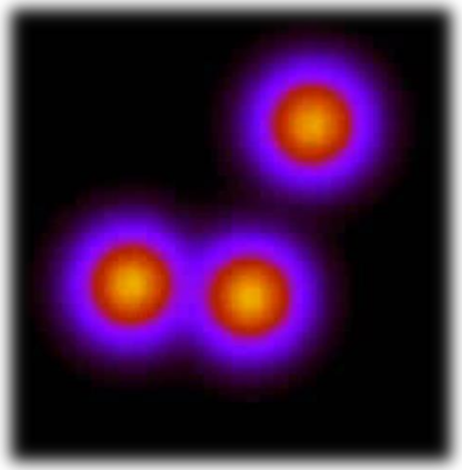
○ They are time-dependent dynamical variables

# Real-Time evolution method

## ◎ Equation of Motion

$$\delta \int dt \frac{\langle \Phi | i\hbar\, d/dt - H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0$$

$$\Rightarrow \quad i\hbar \frac{d\boldsymbol{Z}_i(t)}{dt} = \sum_j C^{-1}_{ij} \frac{\partial \mathcal{H}}{\partial \boldsymbol{Z}^*_j(t)}$$



- ○ By solving EOM, we obtain time-dependent wf.

## ◎ The ensemble of the time-dependent wave functions has very nice feature

- ○ It has ergodic nature
- ○ It follows quantum statistics (micro canonical ensemble)

J. Schnack and H. Feldmeier, NPA601, 181 (1996).
A. Ono and H. Horiuchi, PRC53, 845 (1996), PRC53, 2341 (1996).

# Real-Time evolution method

◎ This means that the superposition of the time-dependent wave functions describes the quantum state very well

○ All possible quantum states will appear after long-time propagation

○ More important states appear more frequently, if the excitation energy is properly chosen

Time dependent wave function must be a good basis for the generator coordinate method (GCM)

$$\Psi_M^{J\pi}(T) = \int_0^T dt \sum_{K=-J}^{J} \hat{P}_{MK}^{J\pi} \, f_K(t)\Phi(\boldsymbol{Z}_1(t), ..., \boldsymbol{Z}_N(t))$$

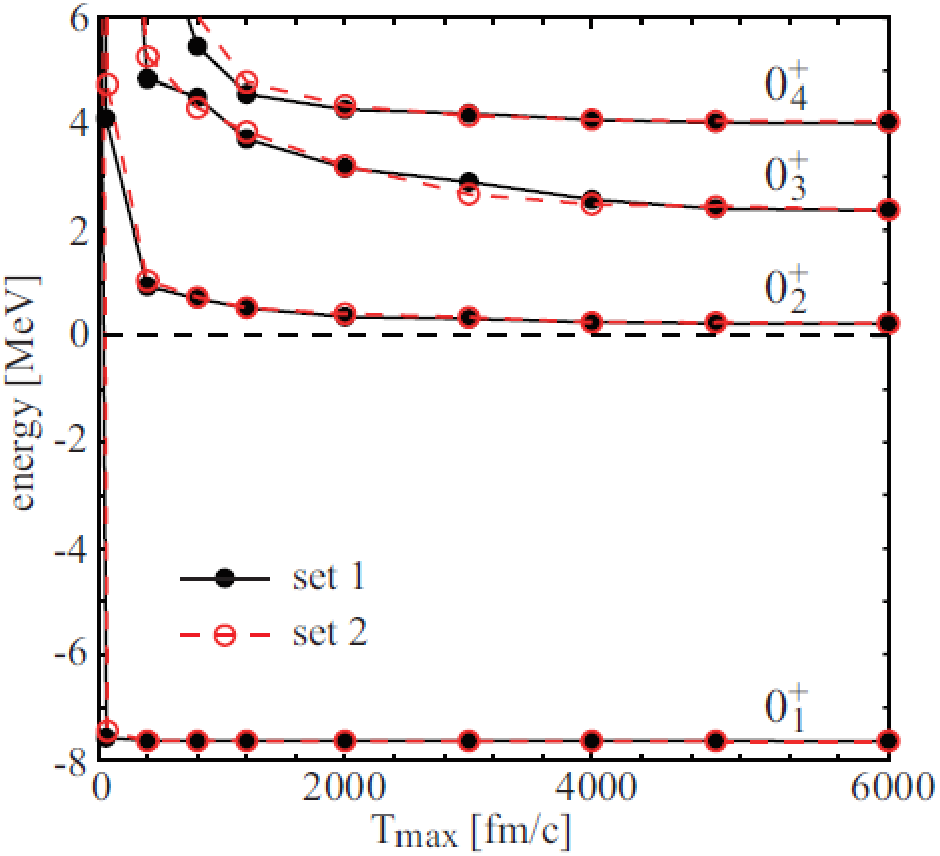
○ The result should be converged after the long-time propagation

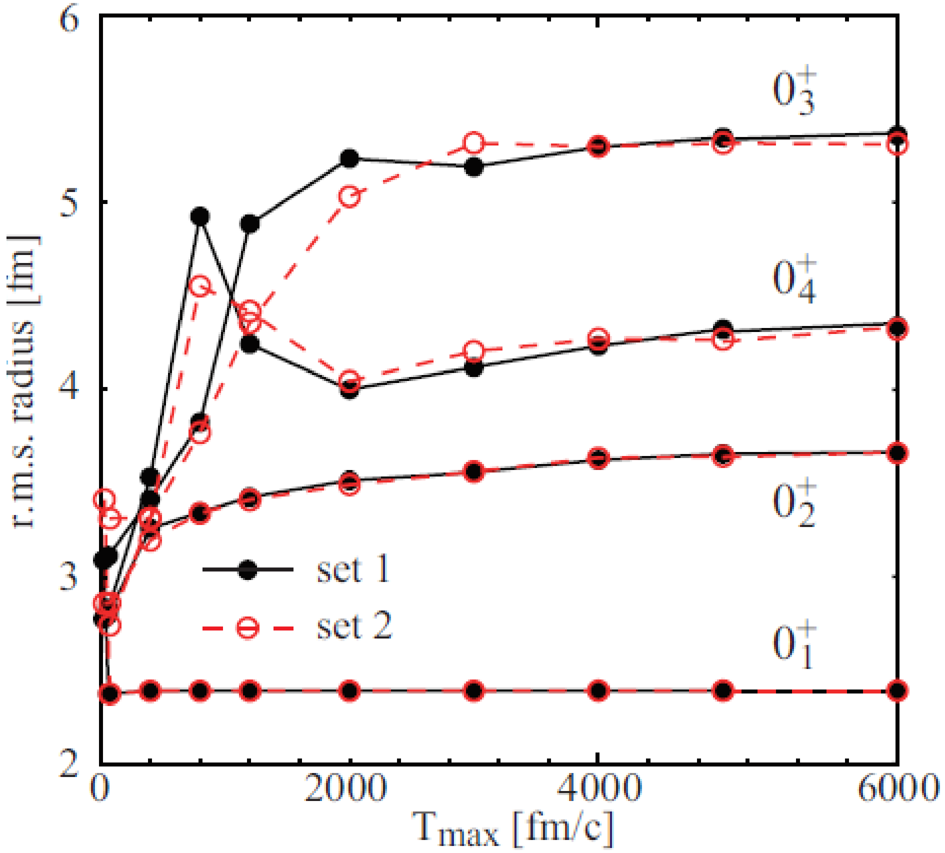
○ The results should not depend on the initial condition

# Real-Time evolution method

○ The result is converged after the long-time propagation
○ The results independent of the initial condition

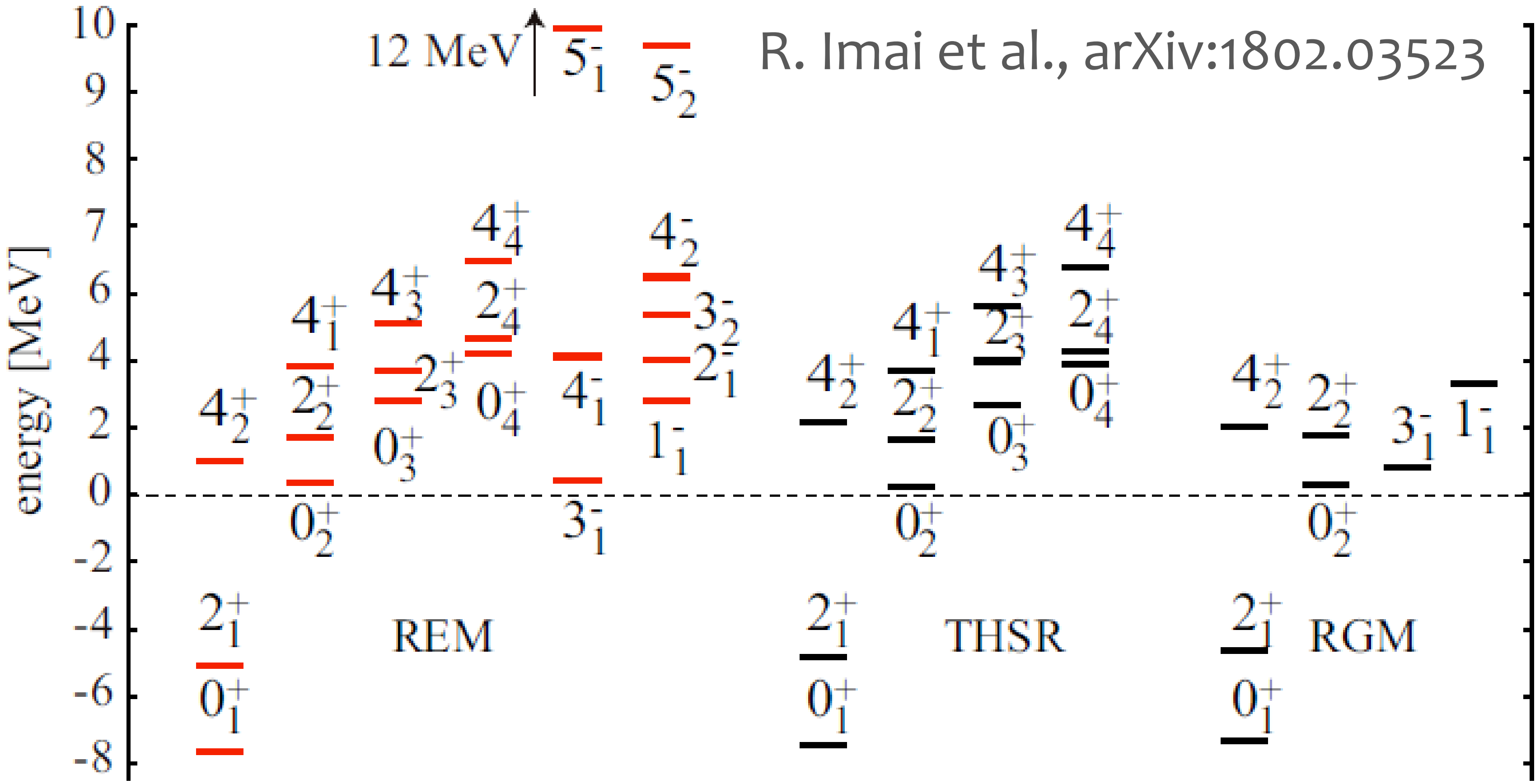
R. Imai et al., arXiv:1802.03523



# Real-Time evolution method

- The result is identical to the THSR (positive-parity)
- Many negative-parity states
- Everything comes from only single ensemble (high-performance)



# Theoretical check of the rigidity

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$j_i\,,j_f$ indep.

This should be
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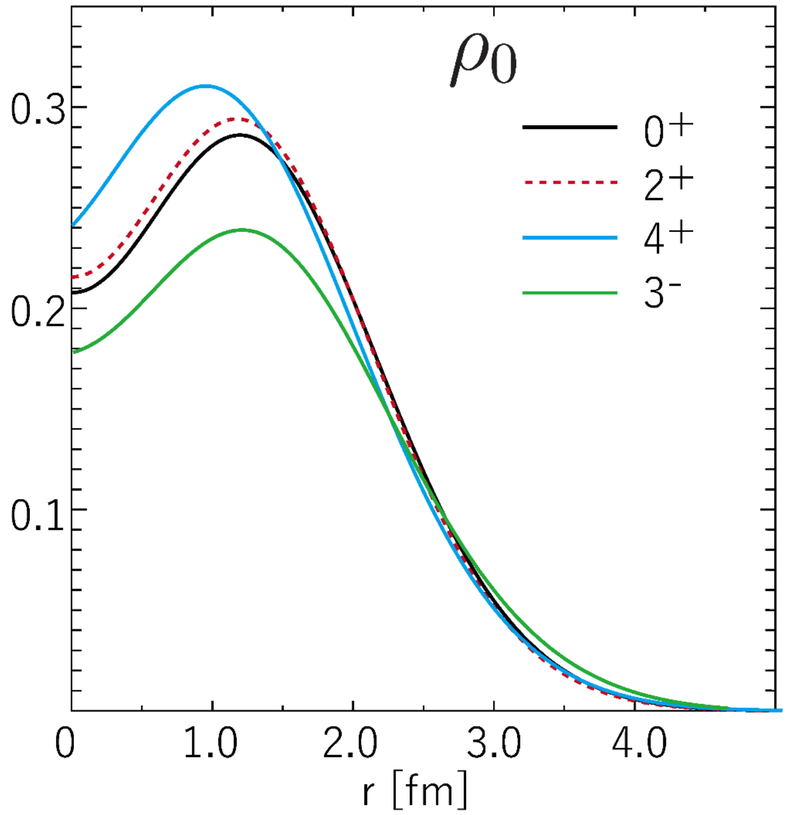
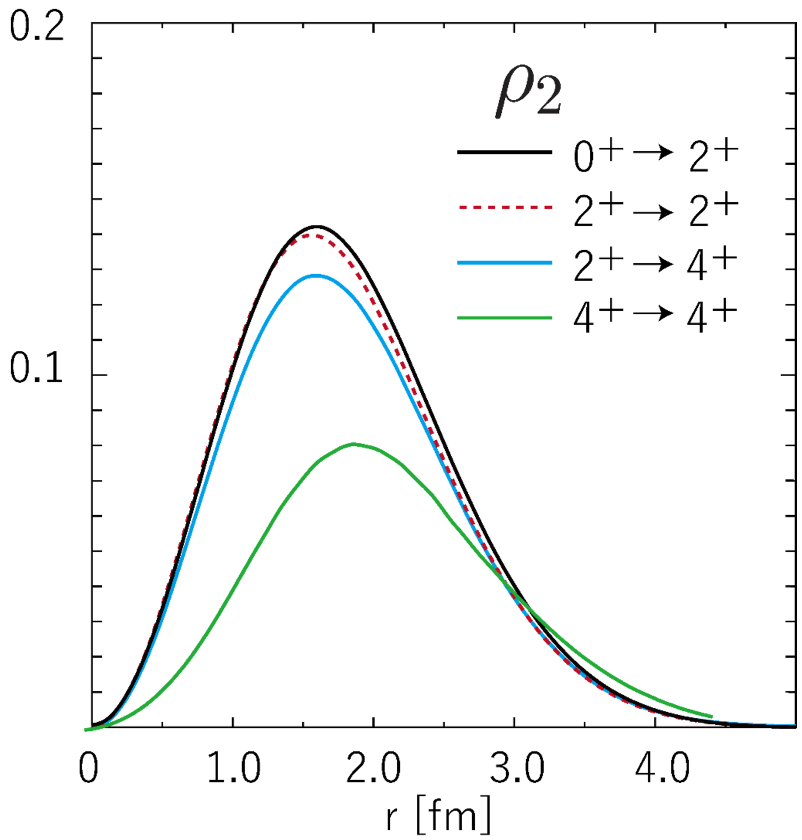
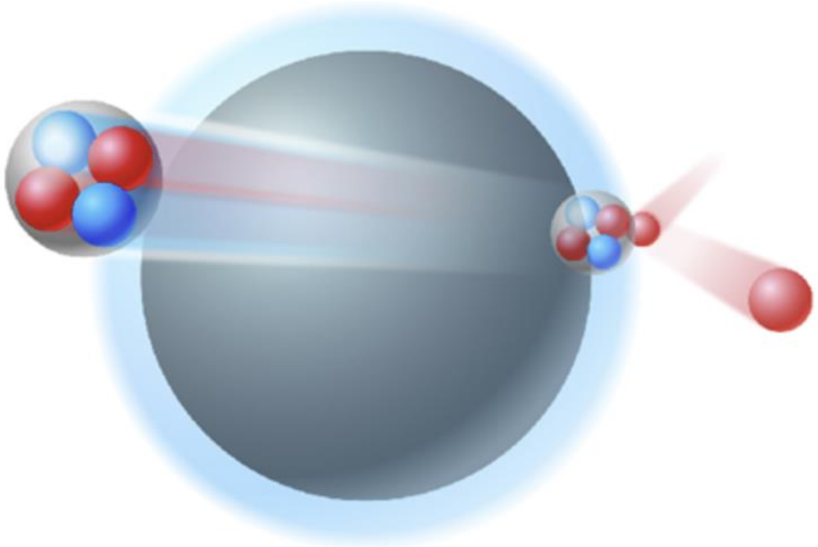
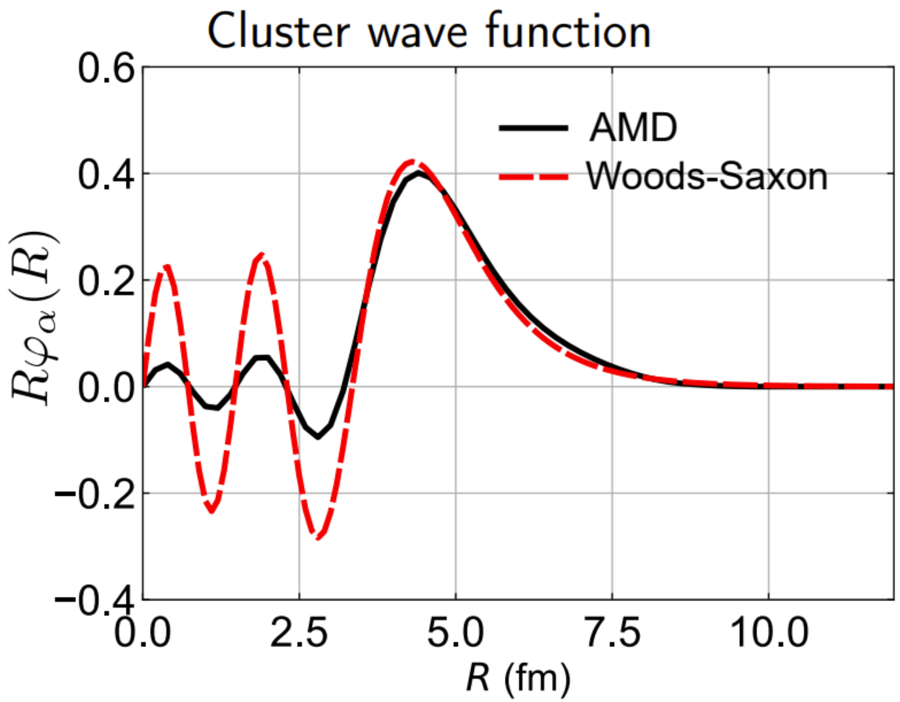
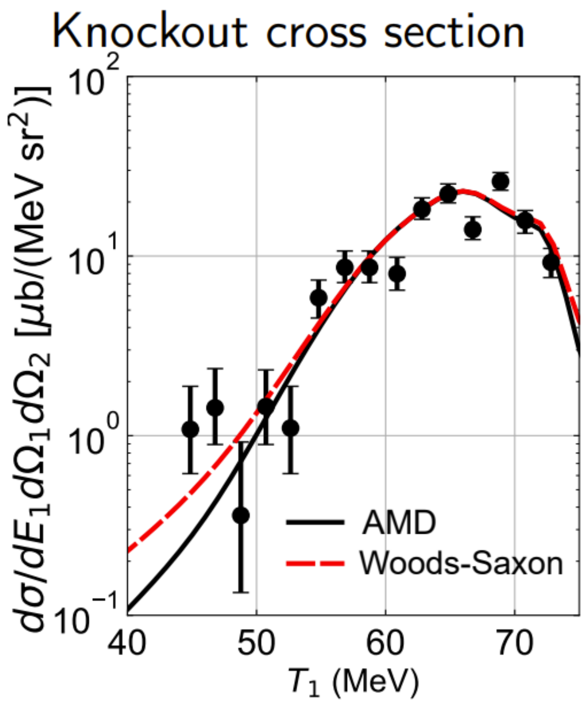


# Reason #2: Alpha knockout reactions (ongoing)






$^{20}$Ne(p,p$\alpha$) : K. Yoshida et al., PRC 100, 044601 (2019)

$^{48}$Ti(p,p$\alpha$): Y. Taniguchi, K. Yoshida et al., PRC 103, L031305 (2021).

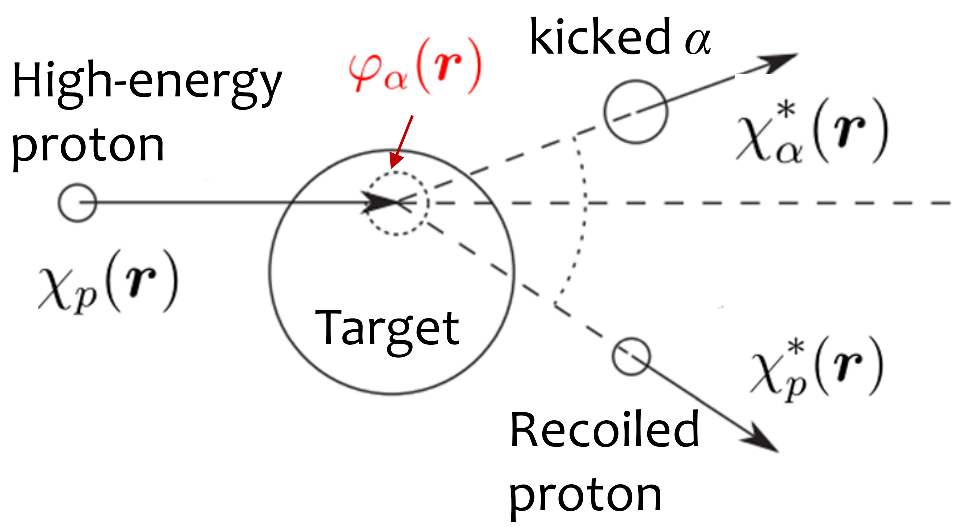
$^{A}$Be(p,p$\alpha$), $^{A}$B (p,p$\alpha$), : H.Motoki et al., PTEP2022, 113D01 (2022)

Q. Zhao et al., PRC 106, 054313 (2022)

# $(p, p\alpha)$ reaction as a probe for $\alpha$ particle preformed in nuclei

High-energy proton kicks out an alpha particle preformed in a target nucleus

Due to the high energy, the reaction mechanism is rather clean




## DWIA for $(p, p\alpha)$ reaction

K. Yoshida et al., PRC 94, 044604 (2016)

$$\frac{d^3\sigma}{dt_p d\Omega_p d\Omega_\alpha} = F_{\mathrm{kin}} \frac{d\sigma_{p\alpha}}{d\Omega_{p\alpha}} |T|^2$$

Transition matrix

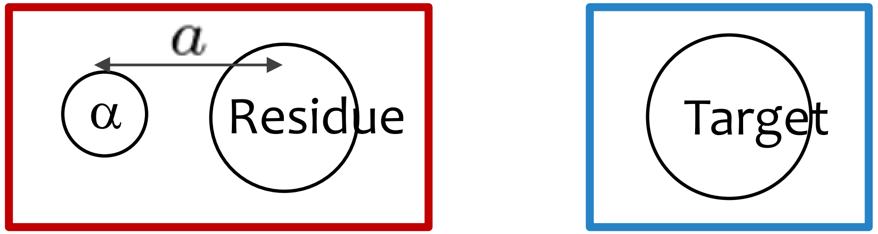
$$T = \int \chi^*_p(\boldsymbol{r}) \chi^*_\alpha(\boldsymbol{r}) \chi_p(\boldsymbol{r}) \varphi_\alpha(\boldsymbol{r}) e^{-i\boldsymbol{k}_0 \cdot \boldsymbol{r}}$$

$\alpha$ particle wave function inside nucleus

# $(p, p\alpha)$ reaction as a probe for $\alpha$ particle preformed in nuclei

The $\alpha$ particle wave function inside a target nucleus is defined as the overlap between the target and the $\alpha$ + Residue system

$$\varphi_\alpha(\boldsymbol{a}) = \sqrt{{}_A\mathrm{C}_4}\,\langle \delta(\boldsymbol{r}' - \boldsymbol{a})\Phi_\alpha \Phi_{\mathrm{Residue}} | \Phi_{\mathrm{Target}} \rangle$$




- Square of $\varphi_\alpha(\boldsymbol{a})$ means the probability to find alpha particle at distance $a$.

- Square integral of $\varphi_\alpha(\boldsymbol{a})$ is alpha formation probability

# Numerical method: Antisymmetrized Molecular Dynamics (AMD)

**Hamiltonian** Gogny D1S and D1M*density functionals

$$\hat{H} = \sum_{i}^{A} \hat{t}_i - \hat{t}_{c.m.} + \sum_{i<j}^{A} \hat{v}_{\mathrm{Gogny}}$$

J. F. Berger et al., CPC 63, 365 (1991)

C. Gonzalez-Boquera et al., PLB779, 195 (2018).

**Model wave function** Antisymmetrized product of nucleon wave packets

$$\Psi^{\pi} = \frac{1+\pi\hat{P}_r}{2}\mathcal{A}\{\varphi_1, \varphi_2, \dots, \varphi_A\}\,, \qquad \varphi_i(\boldsymbol{r}) = \exp\left\{-\nu(\boldsymbol{r}-\boldsymbol{Z}_i)^2\right\}\cdot(a_i\,|\!\uparrow\rangle + b_i\,|\!\downarrow\rangle)$$

Centroid of Gaussians (position and momentum of nucleon) and spins are the variational parameters

**Energy variation with constraint**

Energy of the system is minimized under the constraint on generator coordinate

$$E(\beta,\gamma) = \frac{\langle\Psi^{\pi}(\beta,\gamma)|H|\Psi^{\pi}(\beta,\gamma)\rangle}{\langle\Psi^{\pi}(\beta,\gamma)|\Psi^{\pi}(\beta,\gamma)\rangle}$$

**Angular momentum projection + Generator coordinate method (GCM)**

$$\Psi^{J\pi}_{M\alpha} = \sum_{iK} \underline{g_{iK\alpha}} P^{J}_{MK} \underline{\Phi^{\pi}(\beta_i,\gamma_i)},$$

# A test case ($^{20}$Ne)

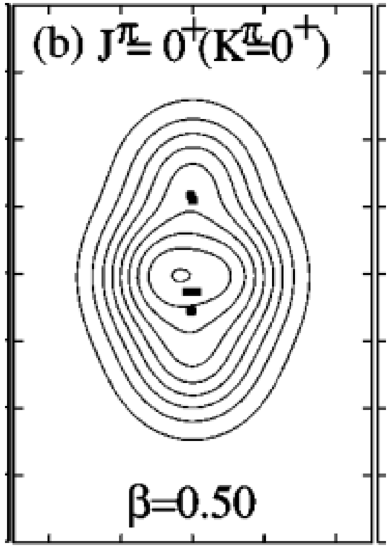
K. Yoshida et al., PRC 100, 044601 (2019)

We already had a good wave function of $^{20}$Ne which reproduces

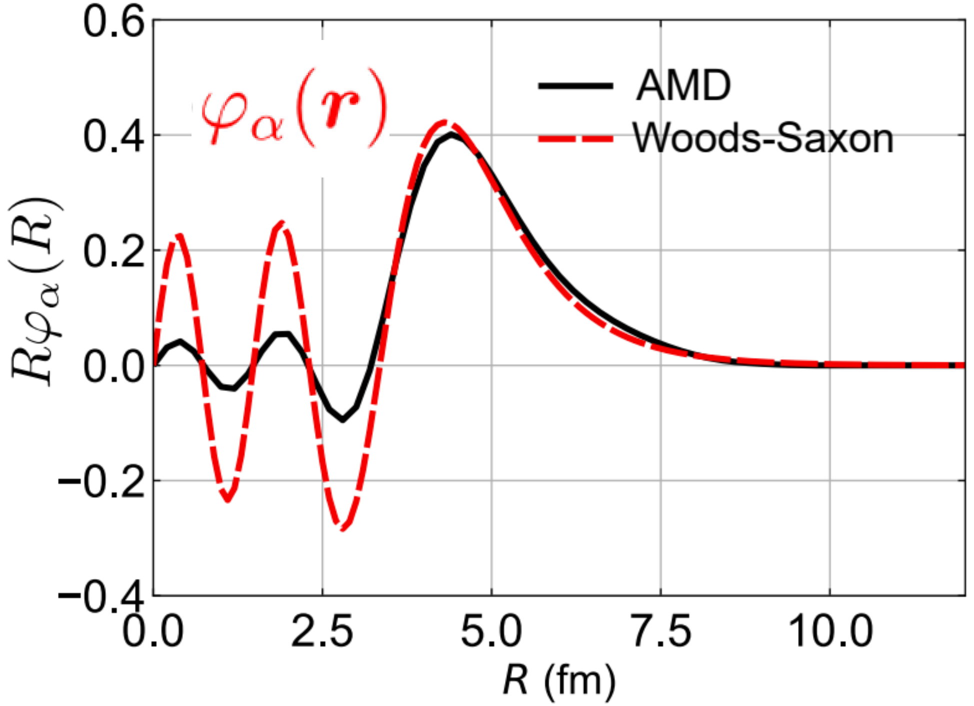
- Binding energy and charge radius
- Excitation spectrum
- $\alpha$ decay lifetime of the excited states
- $\alpha$ transfer reaction on $^{16}$O

$$\varphi_\alpha(\boldsymbol{r}) = \sqrt{{}_{20}\mathrm{C}_4}\,\langle \delta(\boldsymbol{r}' - \boldsymbol{r})\Phi_\alpha \Phi_{^{16}\mathrm{O}} | \Phi_{^{20}\mathrm{Ne}} \rangle$$

M. Kimura, PRC69, 044319 (2004)

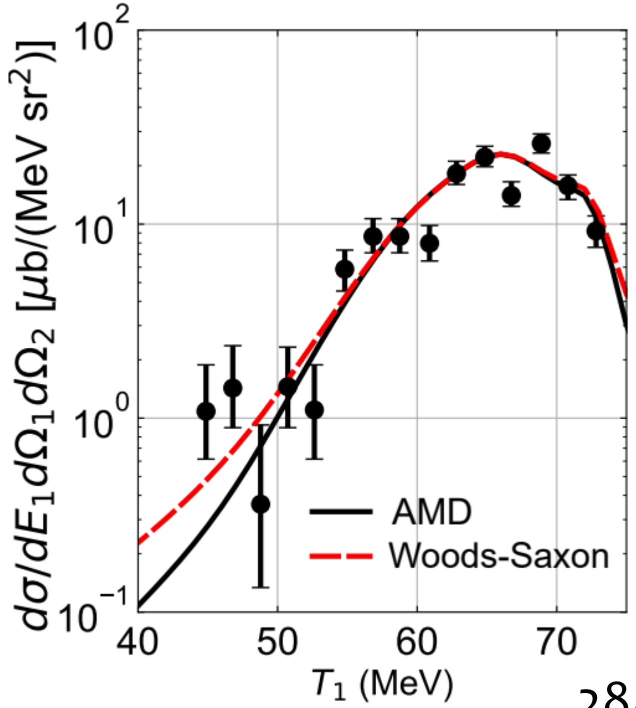



Cluster wave function




T. A. Carey et al., PRC29, 1273 (1984)

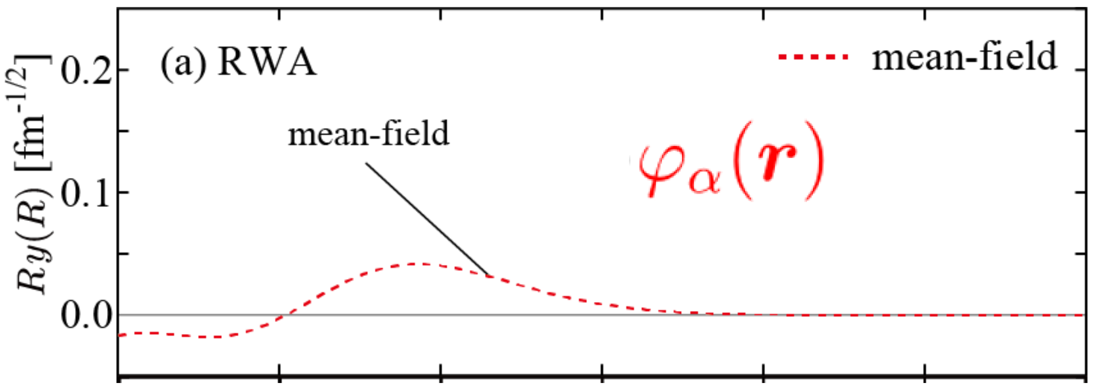
Knockout cross section

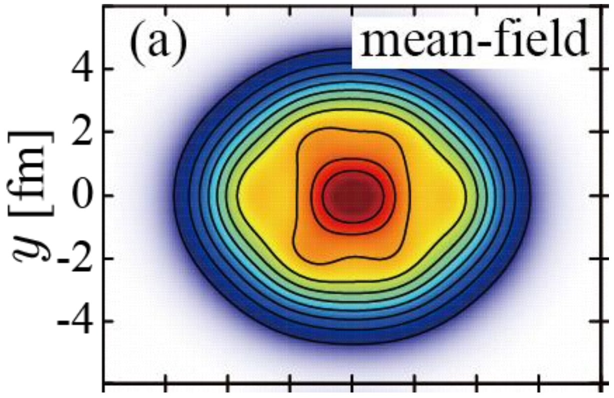


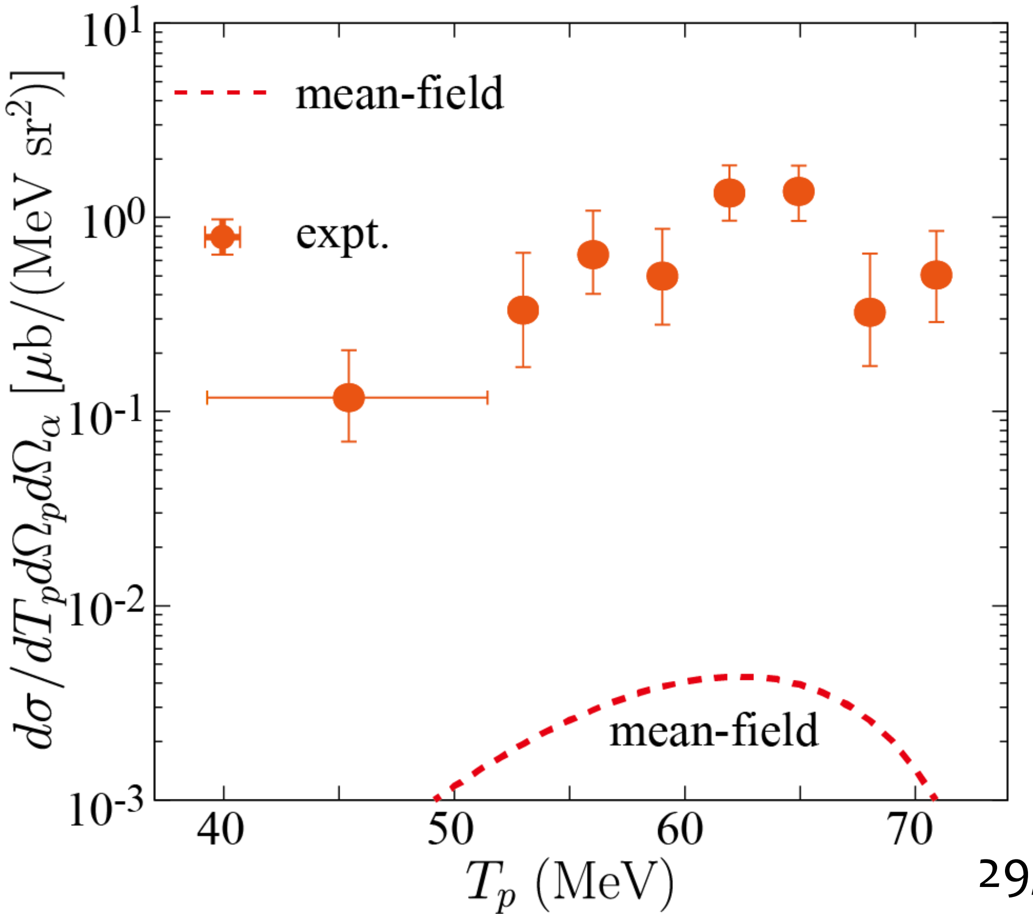
# Another test case（$^{48}$Ti）

Y. Taniguchi, K. Yoshida et al., PRC 103, L031305 (2021).

MF solution underestimates cross section two orders of magnitudes.
Something is completely missing in MF approx.



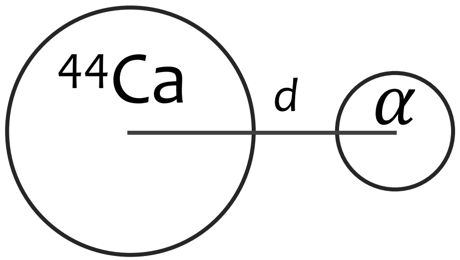



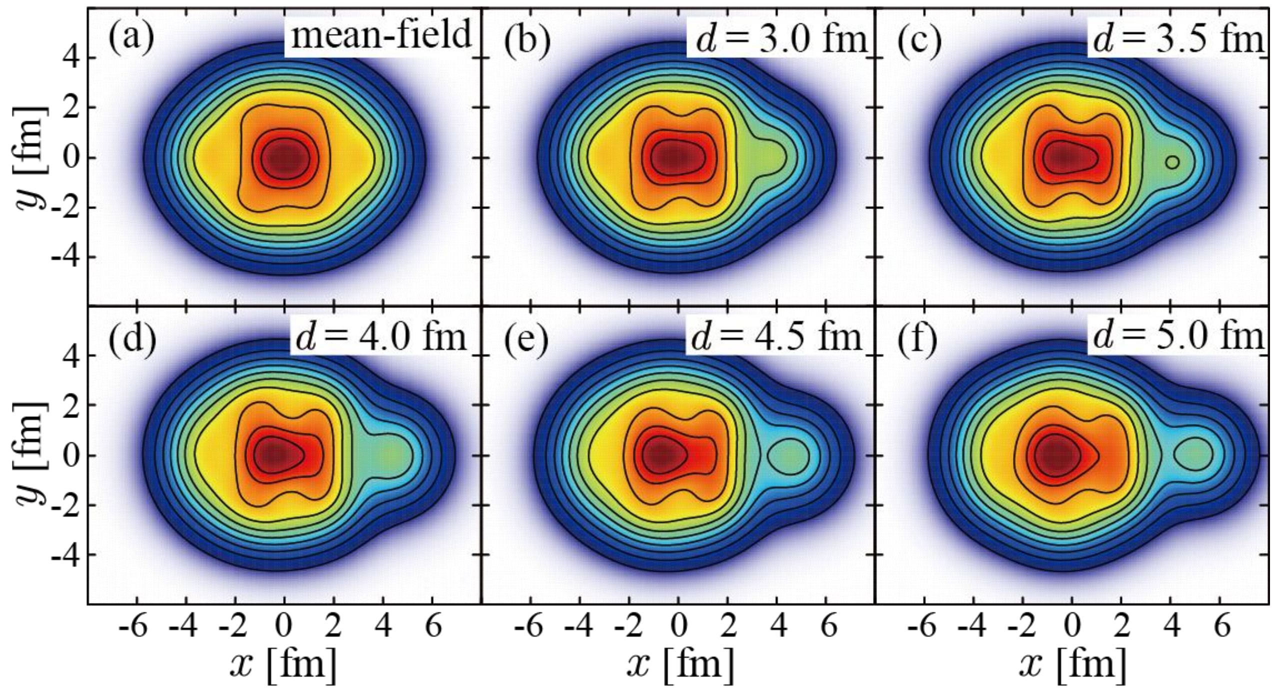
# Another test case（$^{48}$Ti）

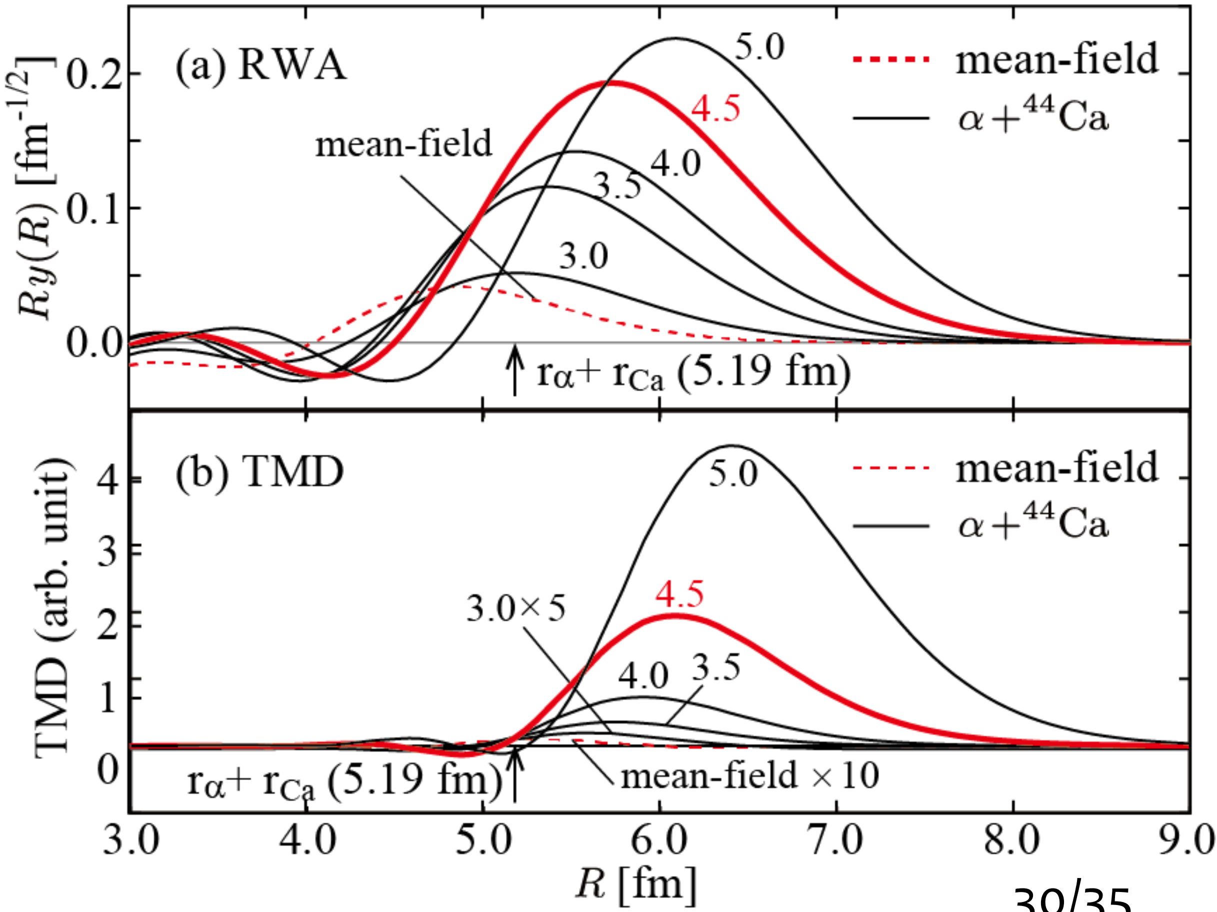
Y. Taniguchi, K. Yoshida et al., PRC 103, L031305 (2021).

ad-hoc wave functions composed of $^{44}$Ca and $\alpha$

Y. Taniguchi, K. Yoshida et al., PRC 103, L031305 (2021).



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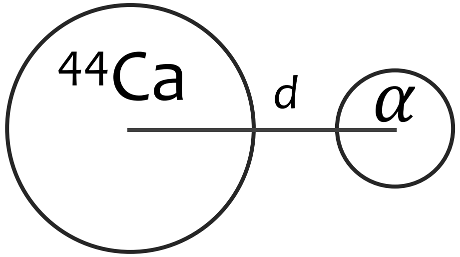
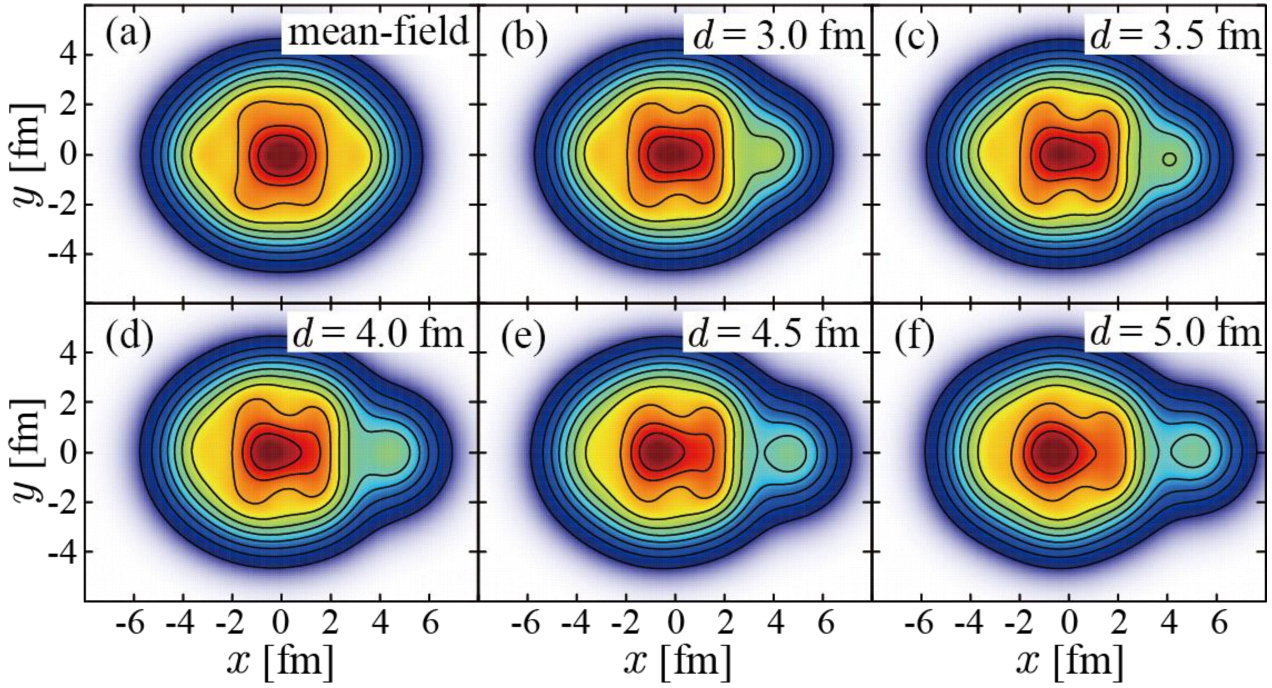
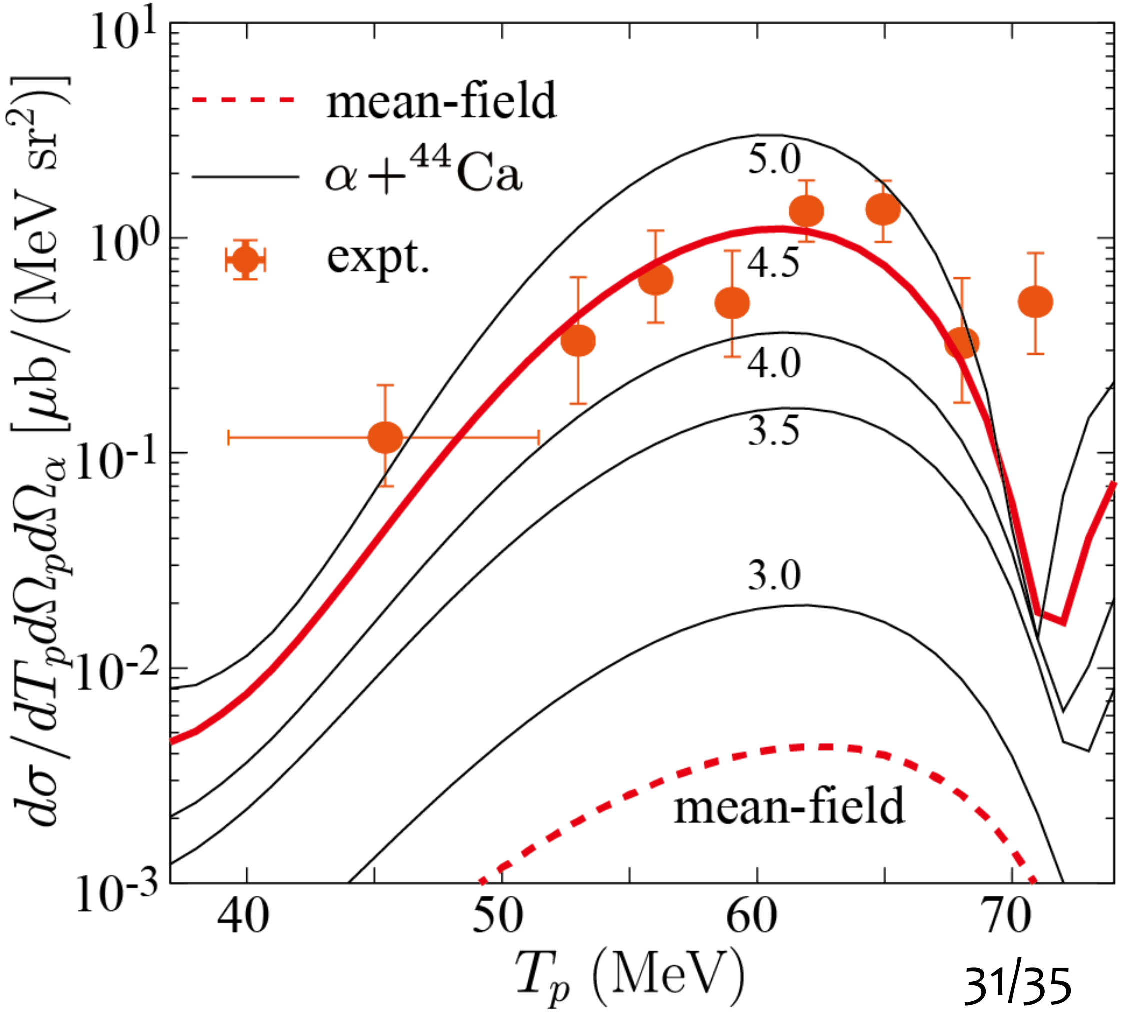



Pronounced cluster formation is required to explain data



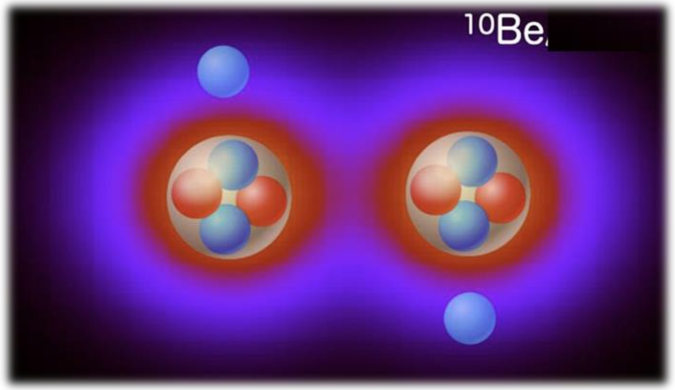



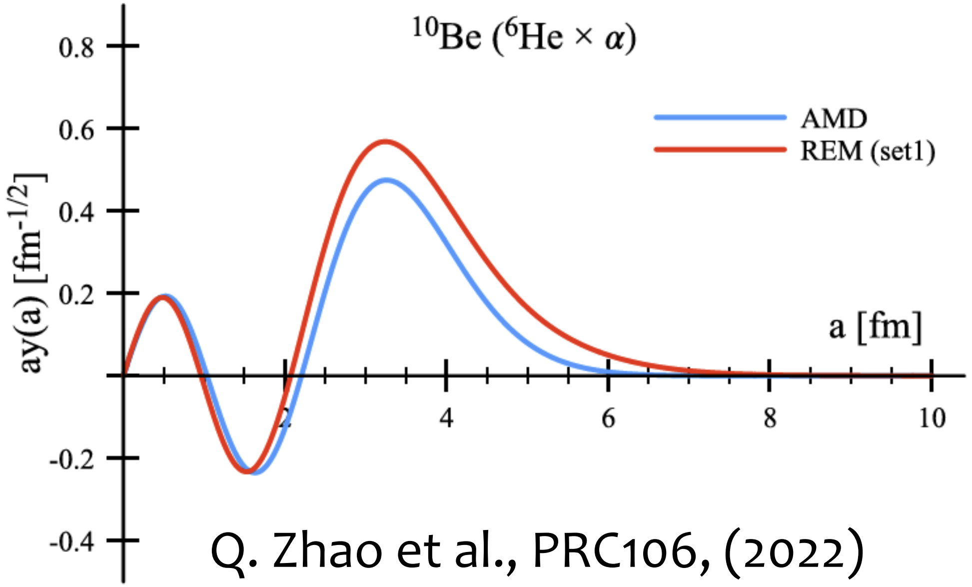
# More challenging case $^{10}$Be

P.J. Li et al., PRL131, 212501 (2023)

Be isotopes have been expected to have strong clustering

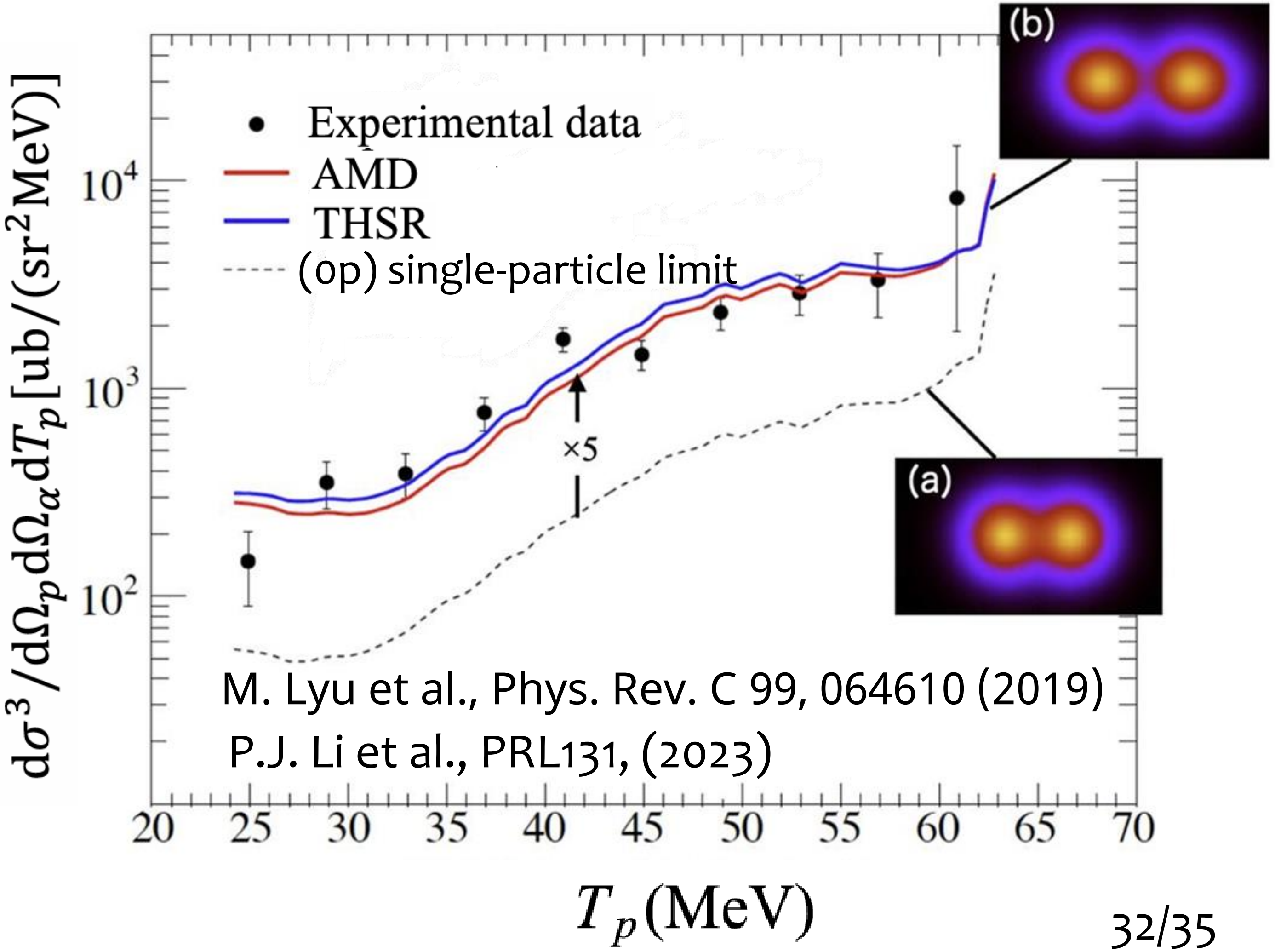
This may be the first direct observation




Q. Zhao et al., PRC106, (2022)

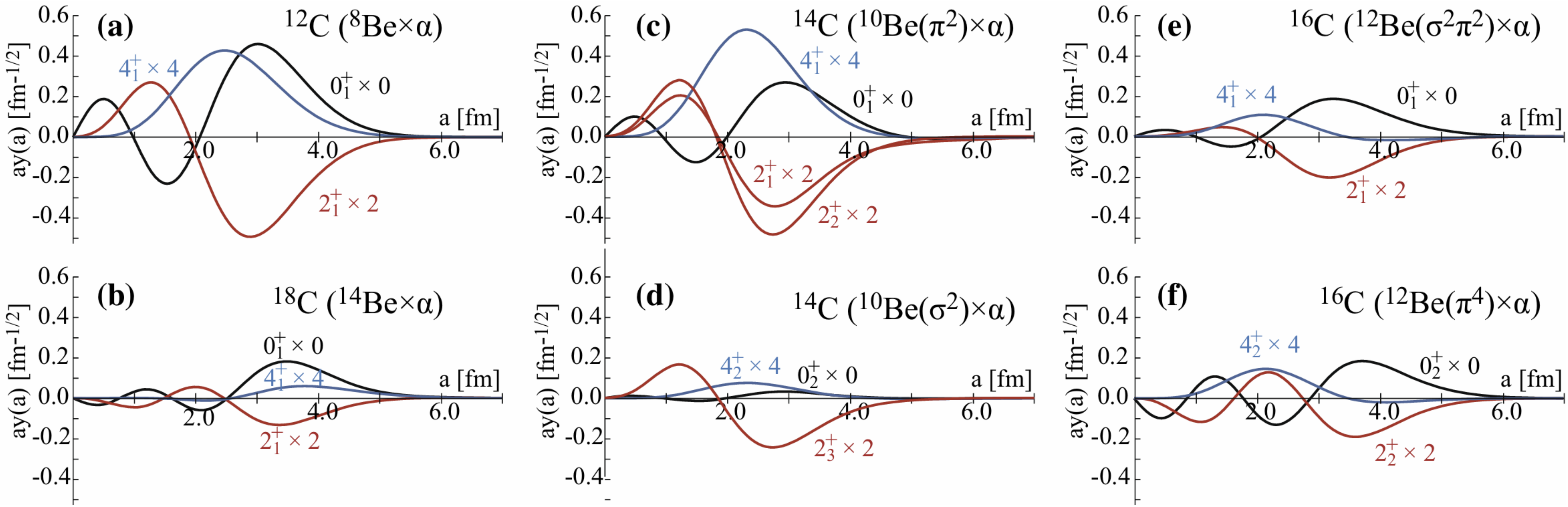
FIG. 1. The calculated RWA of the ground state of $^{10}$Be in the $^{6}$He + $^{4}$He channel. All the nuclei are in the ground state.




M. Lyu et al., Phys. Rev. C 99, 064610 (2019)

P.J. Li et al., PRL131, (2023)

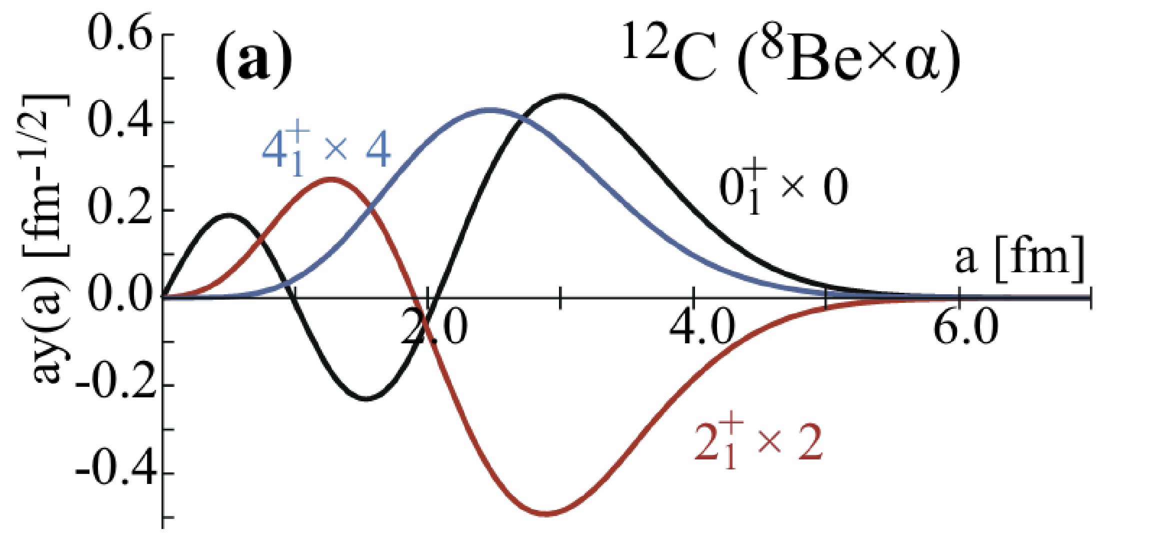
# On going project: alpha knockout from Carbon isotopes

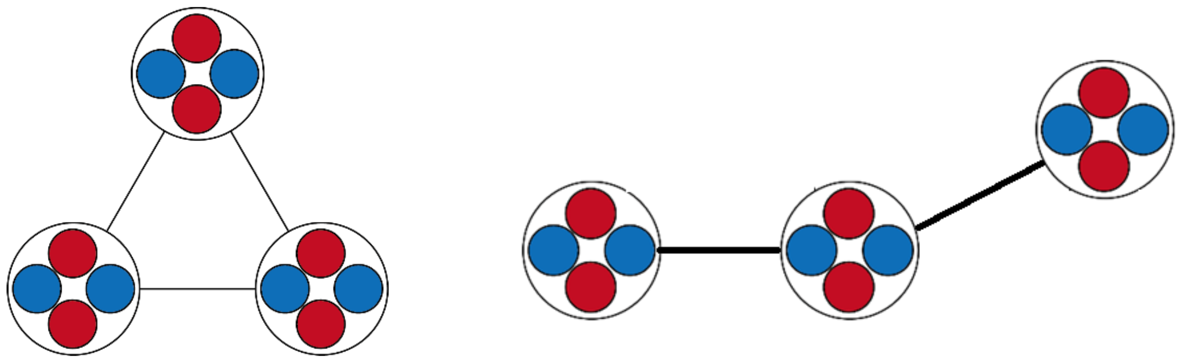



- Theory predicts the decrease of the cluster formation probability towards neutron dripline

# On going project: alpha knockout from Carbon isotopes

Q. Zhao et al., EPJA57, 157 (2021)



- Theory predicts the almost equal amplitude for the $^{8}Be(0+)$ and $^{8}Be(2+)$ channels
- $^{12}C(p, p\alpha)^{8}Be(2+)$ should be comparable with $^{12}C(p, p\alpha)^{8}Be(0^{+})$
- This indicates the angular correlation of alpha particles (geometrical arrangement)

# Summary

## (1) Shape of $^{12}C$ constructed from electron scattering data

Assumption: The $0^+$, $2^+$, $3^-$ and $4^+$ states constitute
"the ground band" sharing the same intrinsic state

⇒ An exotic triangular shape of $^{12}C$ shows up

## (2) Alpha knockout from the ground states (ongoing project)

- Quantitative description of several stable nuclei ($^{20}Ne$, $^{48}Ti$, … )
- First direct evidence of alpha clustering in $^{10}Be$
- Hindrance of clustering toward drip-line C isotopes
- More detailed analysis of $^{12}C$ ground state