Clustering in nuclei at finite temperature

Elias Khan with M. Davies, J.-P. Ebran, F. Mercier, P. Stevenson, E. Yüksel,

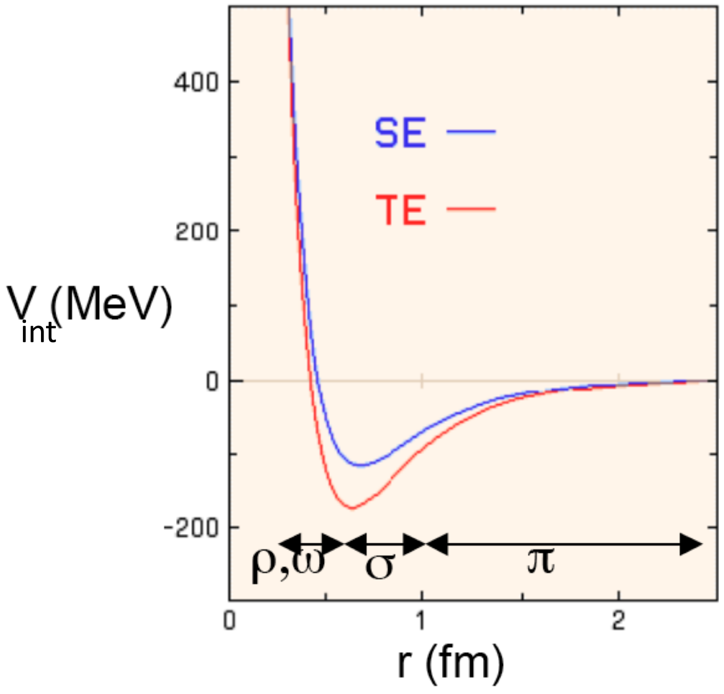
Questions addressed

What is the impact of temperature on nuclear clusterization ?

What is the (ρ,T) diagram for clusterization ?

Method: covariant EDF

V and S potentials



$$\left\{ p\frac{1}{2\tilde{M}(r)}p + W(r) + V_{ls}(r)\boldsymbol{l}.\boldsymbol{s} \right\} \varphi_i = \varepsilon_i^{NR}\varphi_i$$

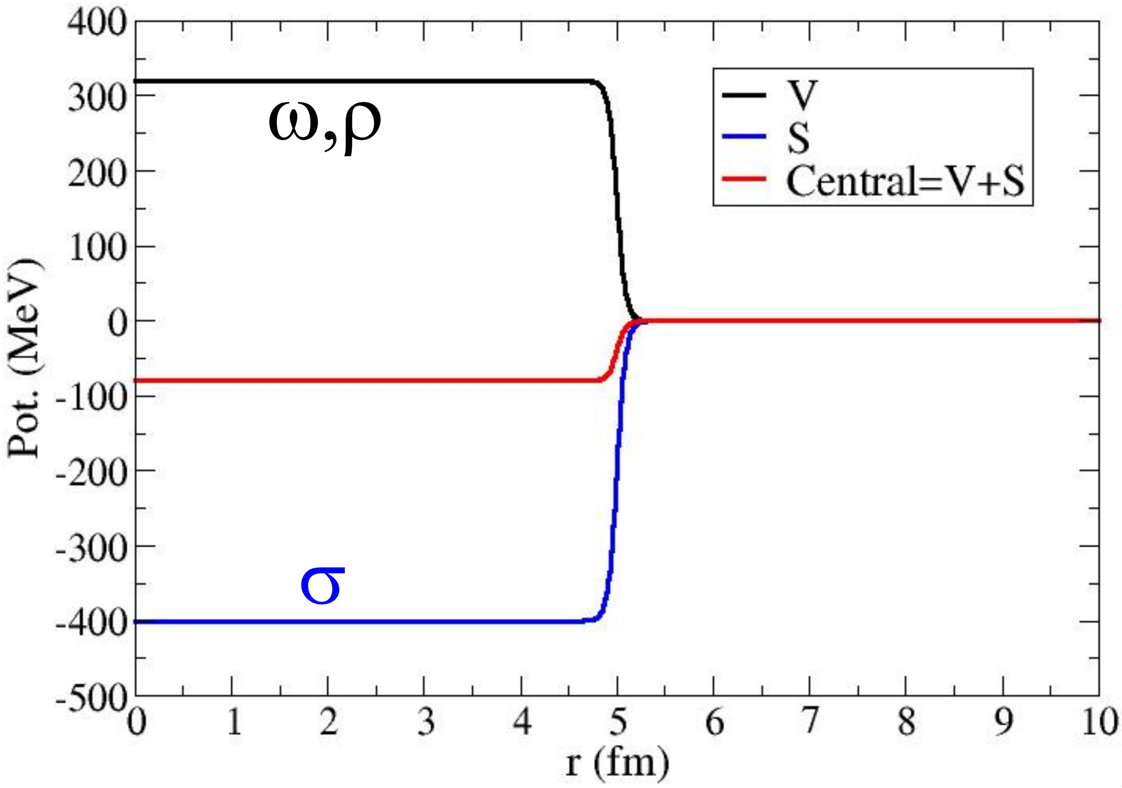
$$W(r) = [V+S](r)$$

$$V_{ls}(r) = \frac{1}{2\tilde{M}^2(r)}\frac{1}{r}\frac{d}{dr}(V-S)$$

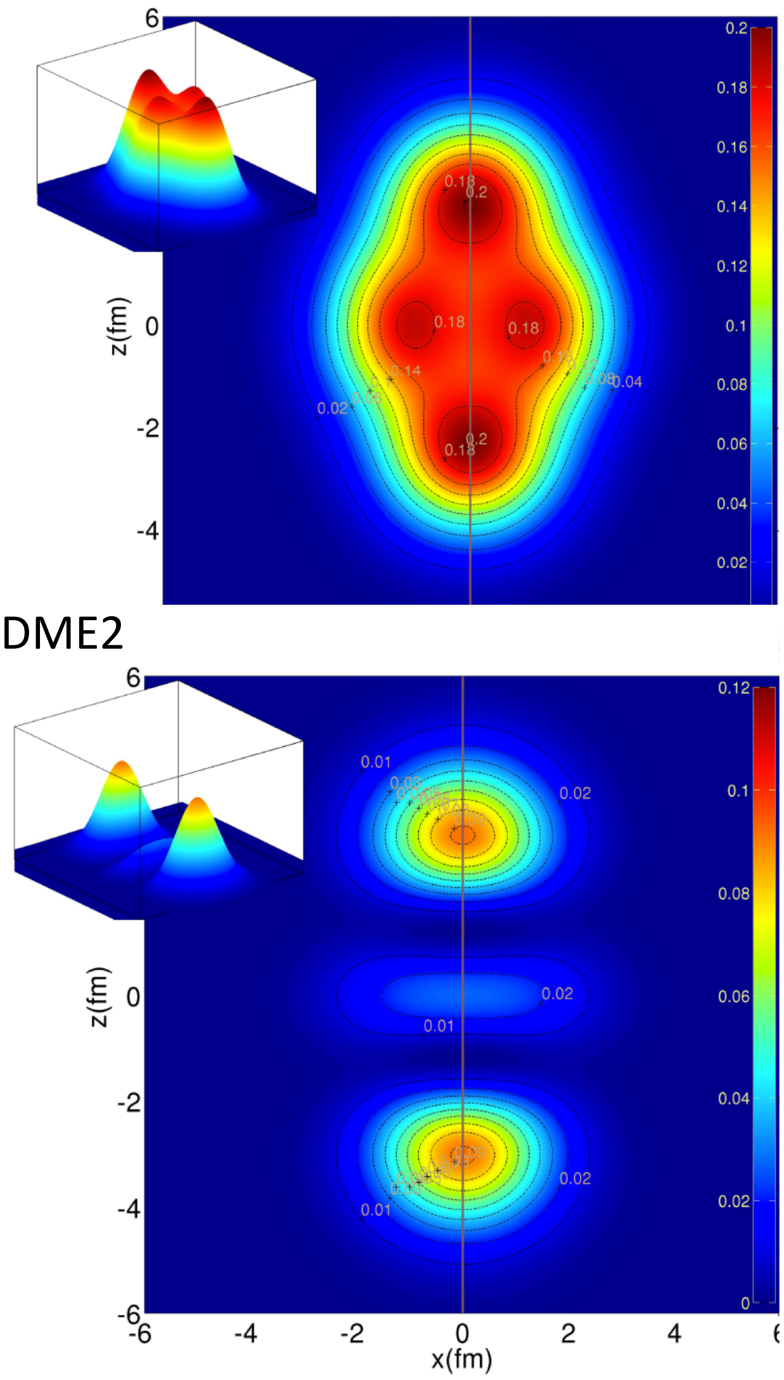
S≈ - 400 MeV

V≈ 320 MeV

$V_0 \approx$ 80 MeV



Nucleonic density: covariant EDF clusterizes

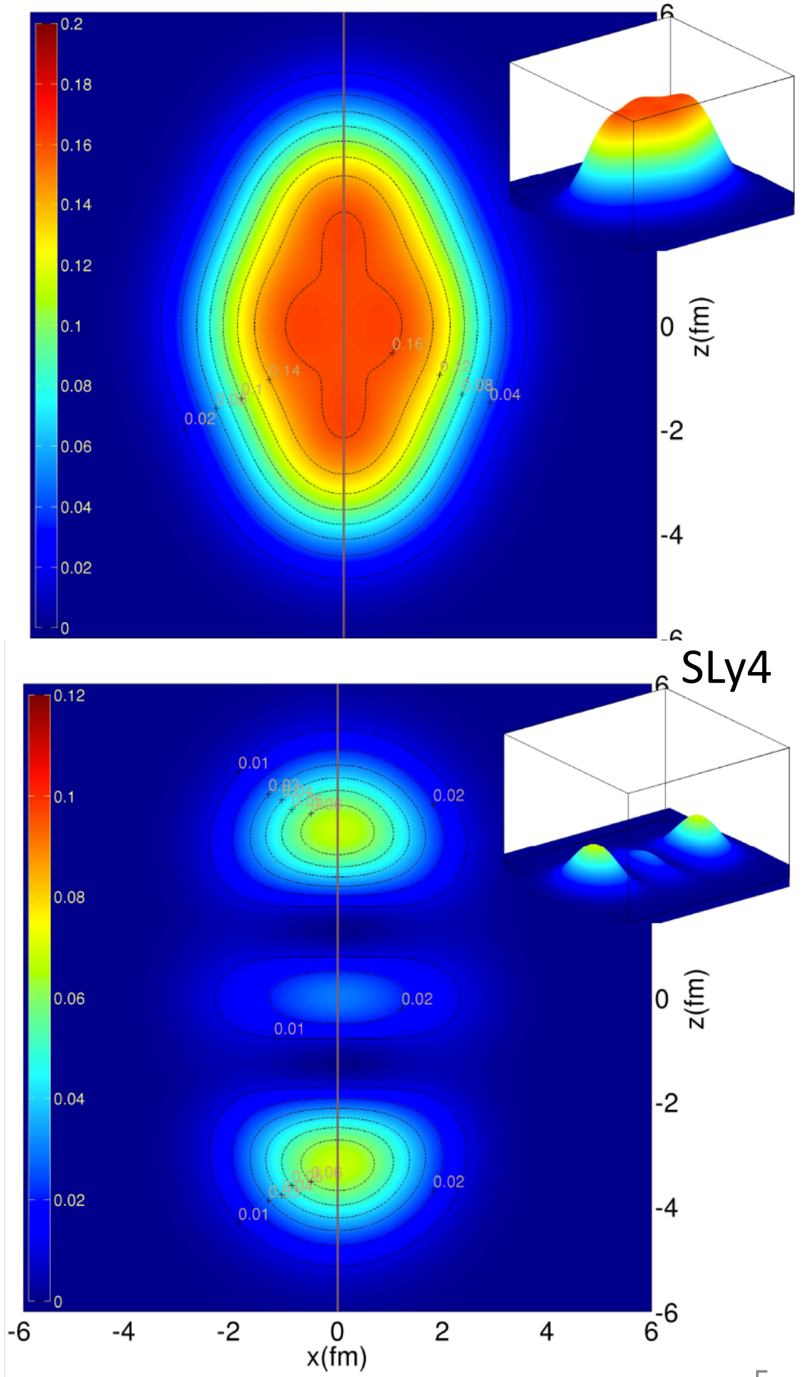


DDME2

^{20}Ne

Total density

Last occupied state

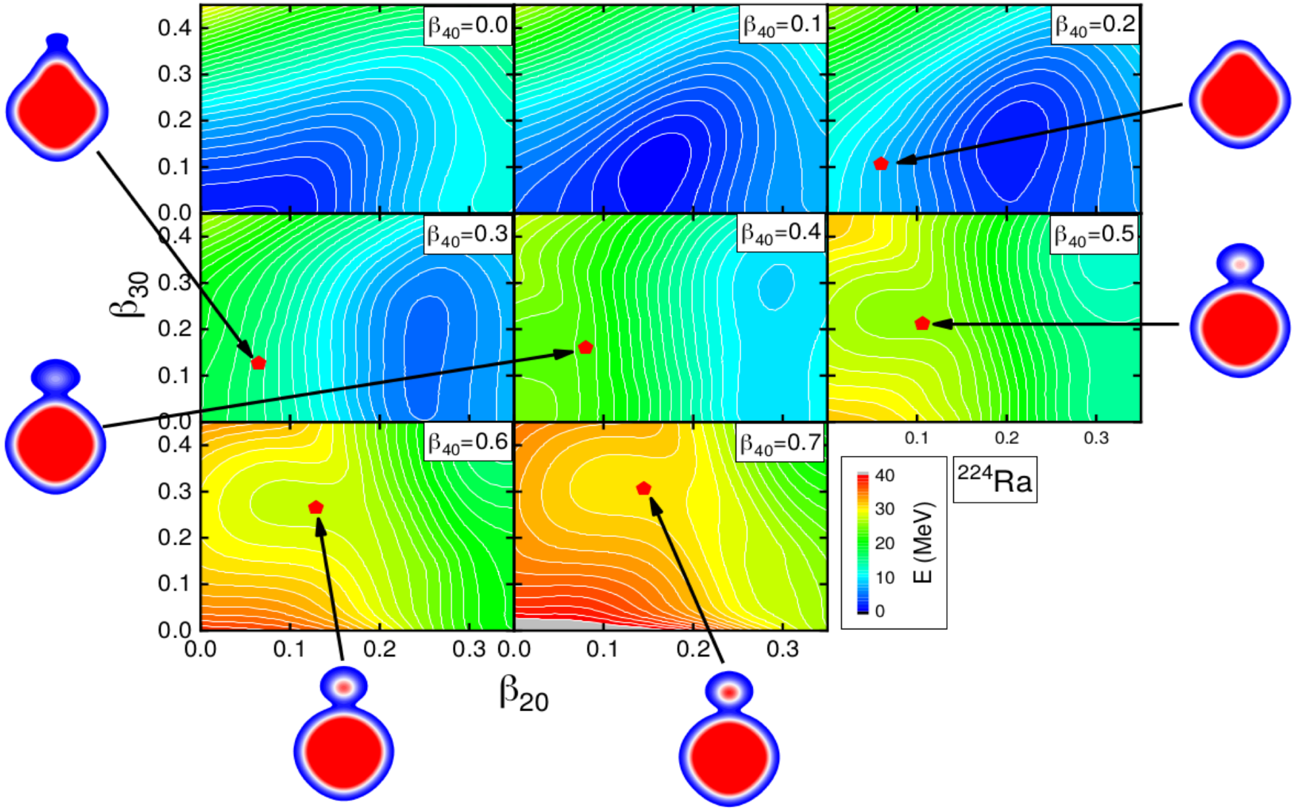


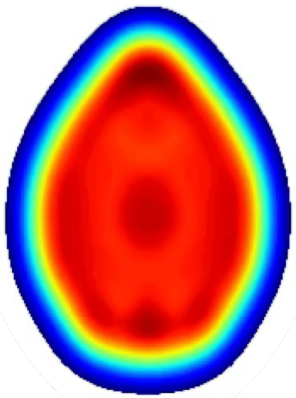
SLy4

J.P. Ebran, , E. Khan, T. Nikšić, and D. Vretenar, Nature 487, 341 (2012)

α decay in ^{224}Ra

^{224}Ra
T_{exp} 3.6 d
T_{theo} 5.7 d



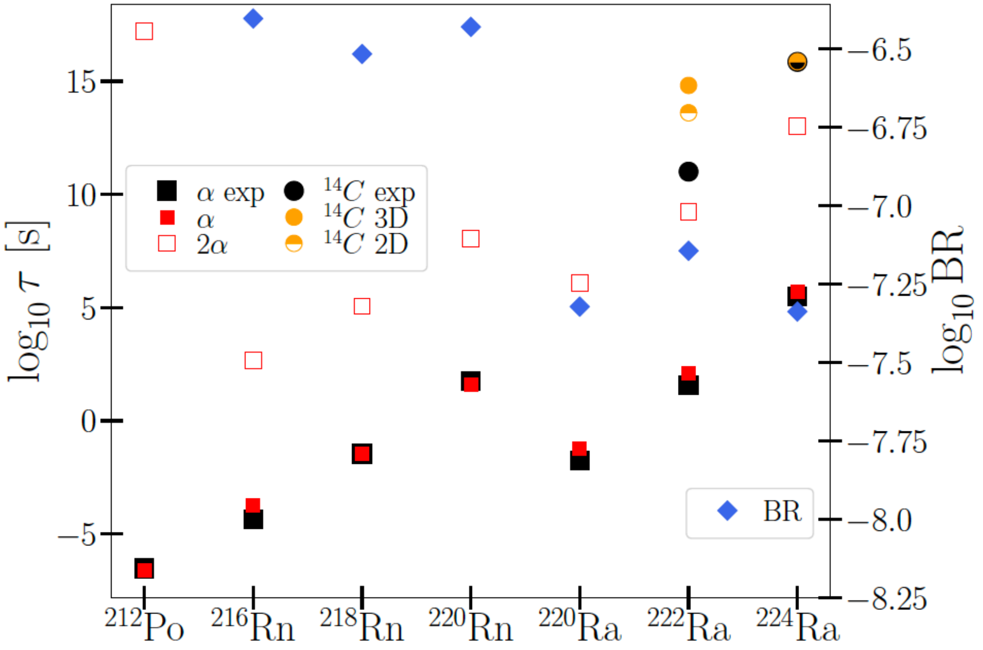


RHB density plots

For heavy nuclei, the hexadecapole constraint β_4 is necessary to form the neck for alpha-particle emission

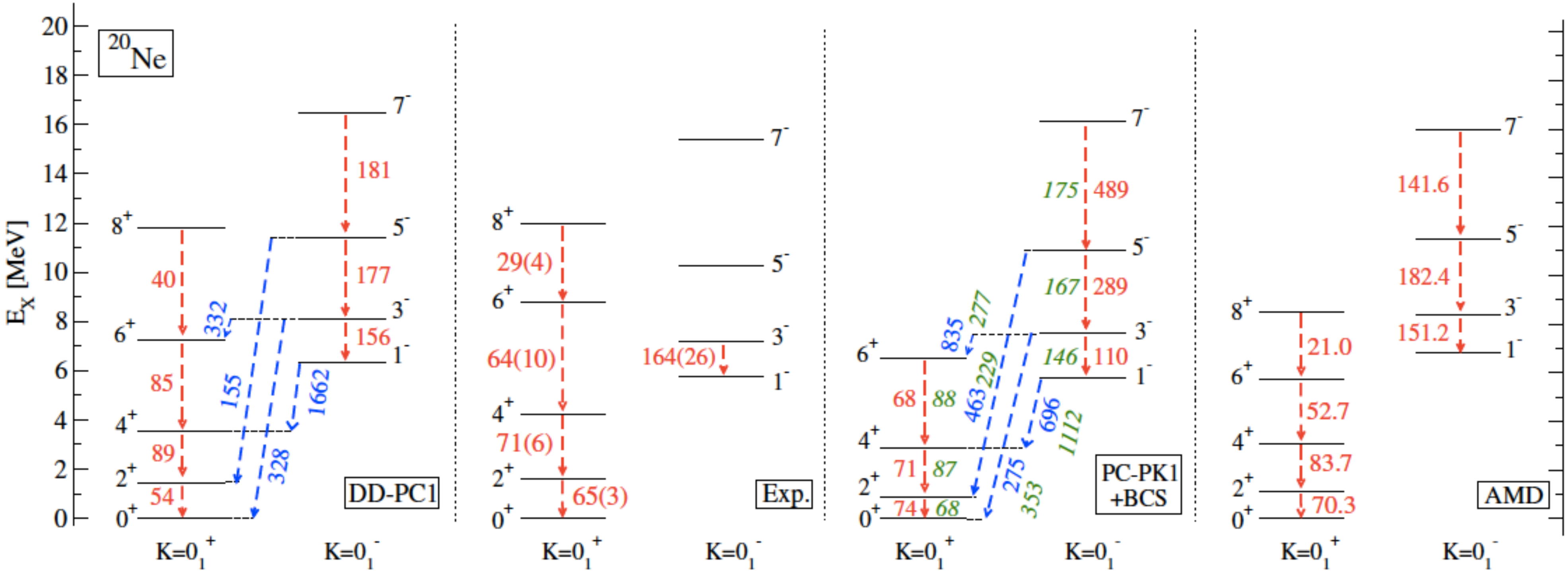
J. Zhao, J. P. Ebran, L. Heitz, E. Khan, F. Mercier, T. Nikšić, and D. Vretenar, Phys. Rev. C 107, 034311 (2023)

Relevant constraints



	Fission	Cluster	Alpha	Double alpha
β_{20}	X	X	X	X
β_{30}	X	X	X	
β_{40}			X	X

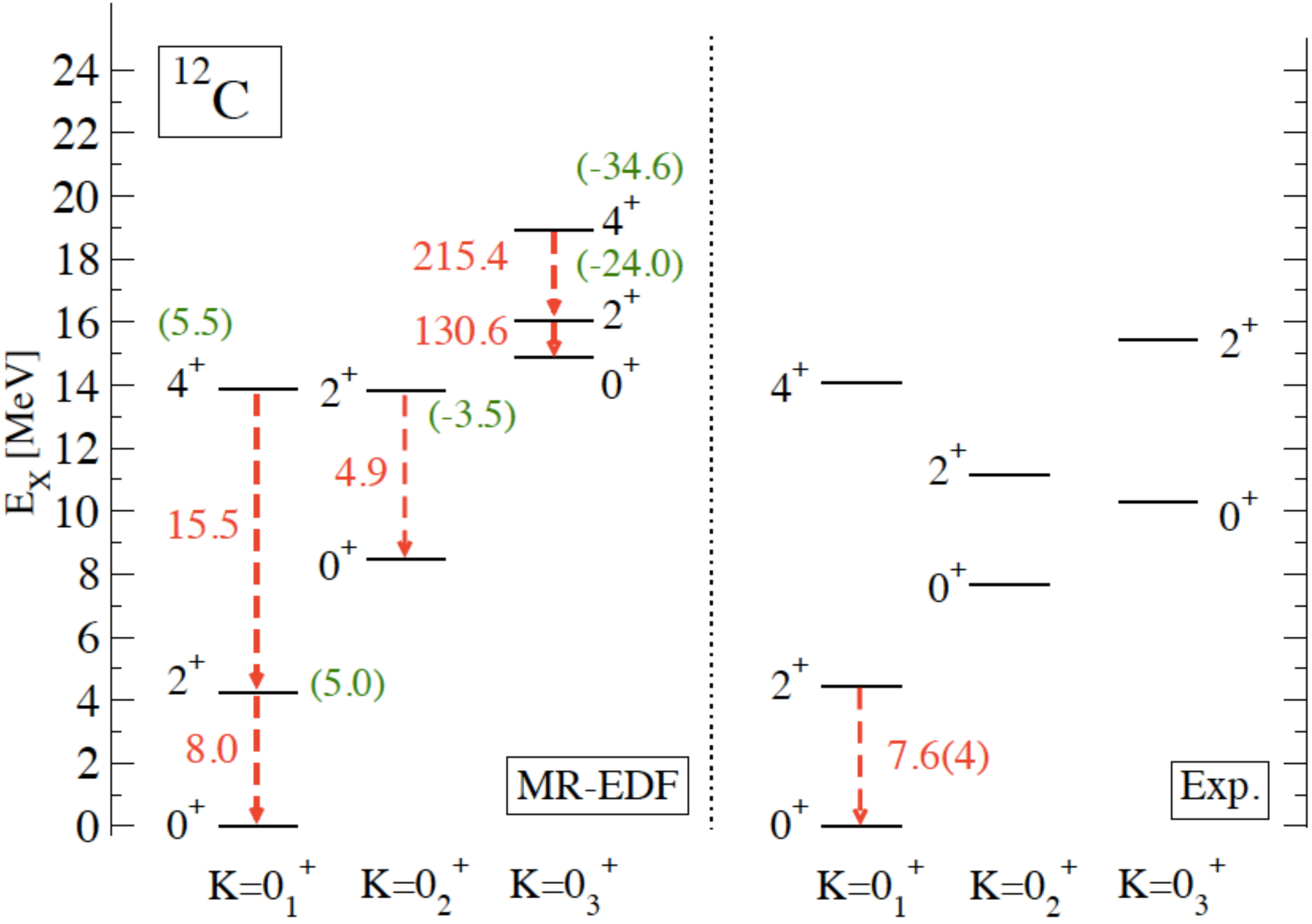
Comparison with the exp. spectrum on ^{20}Ne



- GCM on top of axially symmetric /reflection asymmetric RHB (DD-PC1) :
- Angular momentum, parity and particle number projections

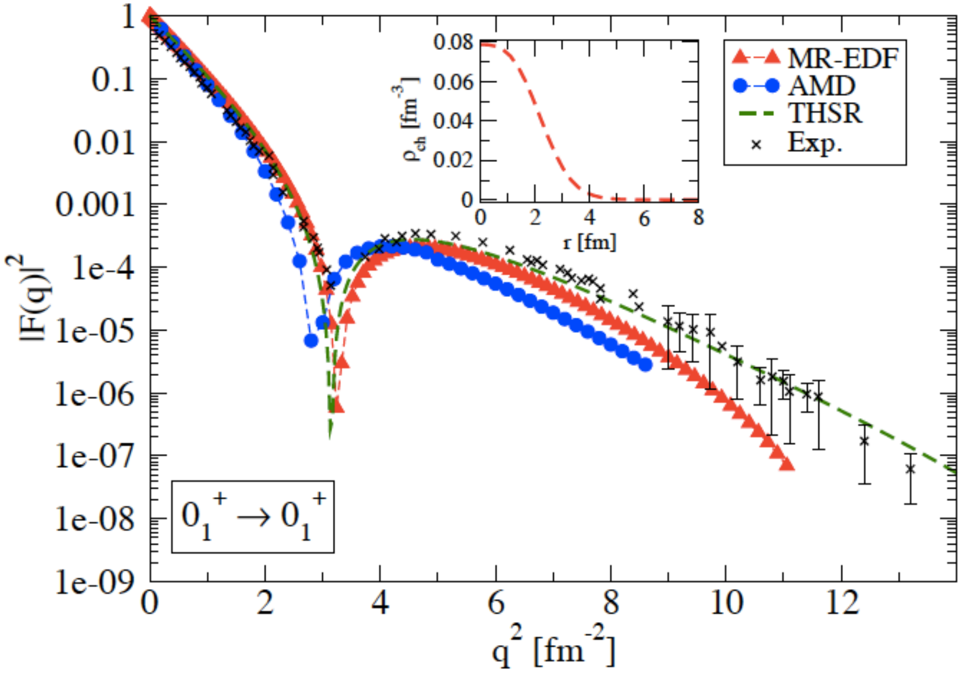
P. Marević , J.-P. Ebran, E. Khan, T. Nikšić, and D. Vretenar, PRC 97, 024334 (2018)

Comparison with exp. spectrum on ^{12}C

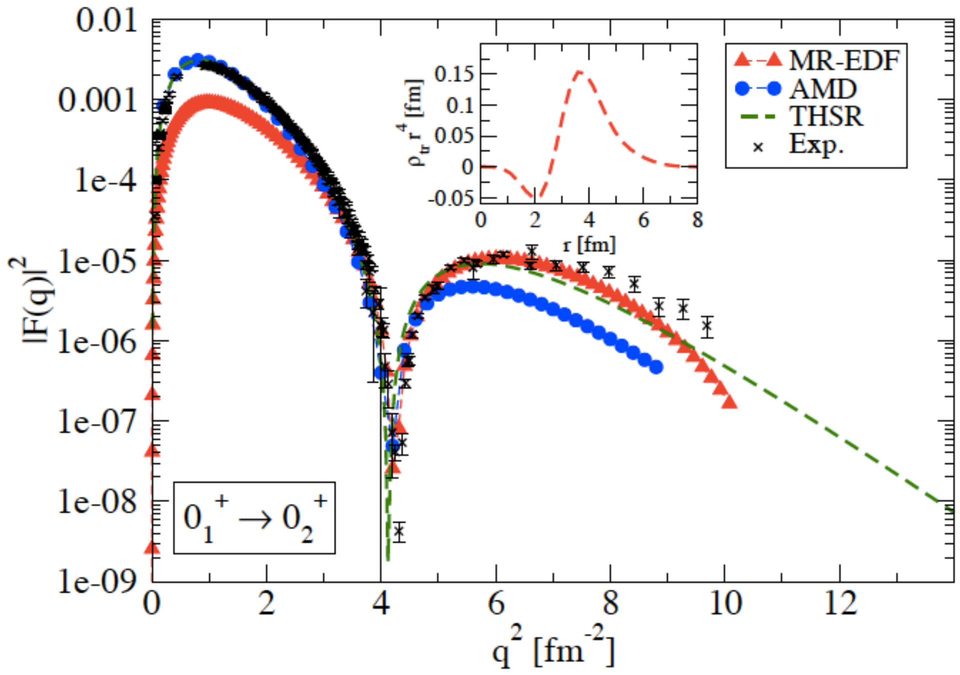


P. Marević , J.-P. Ebran, , E. Khan, T. Nikšić, and D. Vretenar, PRC 99, 034317 (2019)

Comparison with exp. form factors on ^{12}C

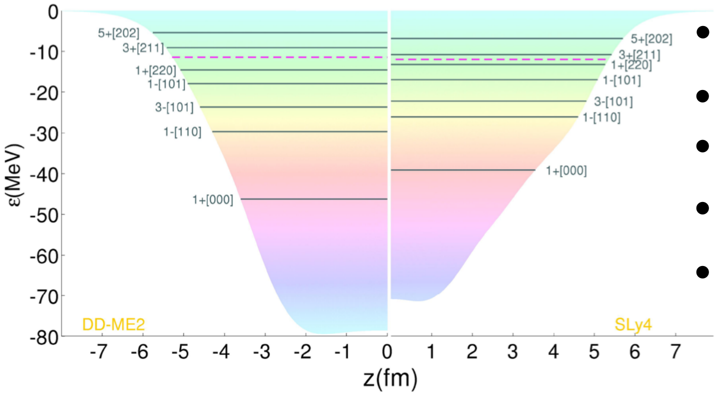


g.s.

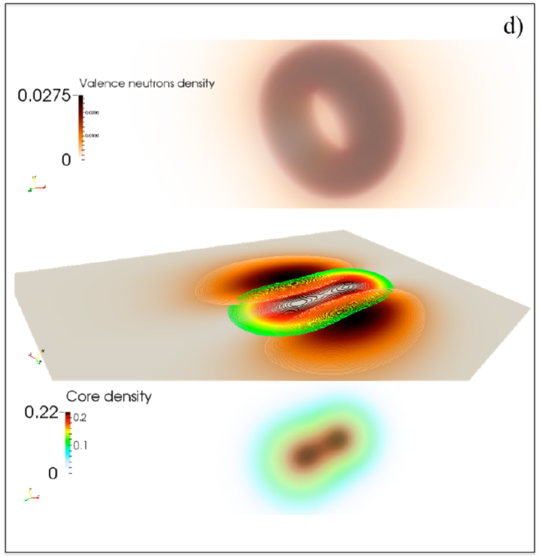


Hoyle

Origins of nuclear clustering: control parameters of a QPT



- Depth of the confining potential
- Heavy vs. Light nuclei
- Deformation / excitation energy
- Density
- Neutron excess

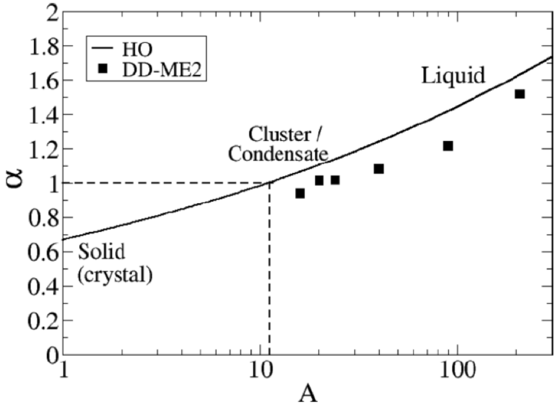


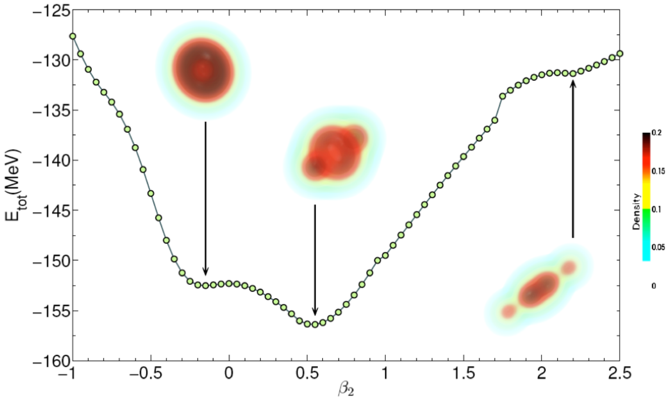
J.-P. Ebran, E. Khan, T. Niksic, D. Vretenar, Nature 487(2012)341

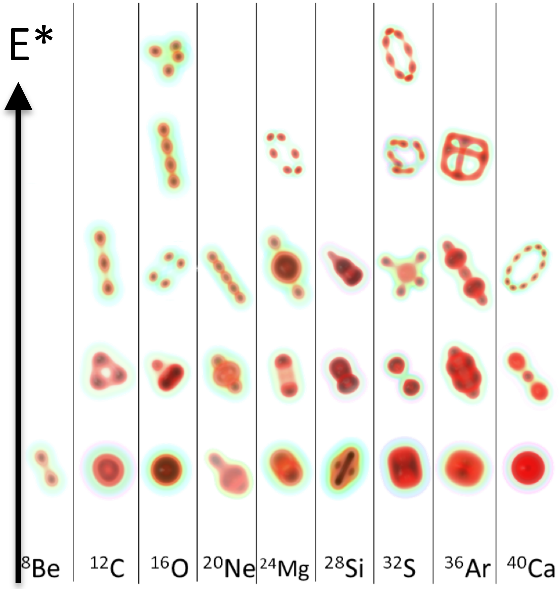
PRC87(2013)044307

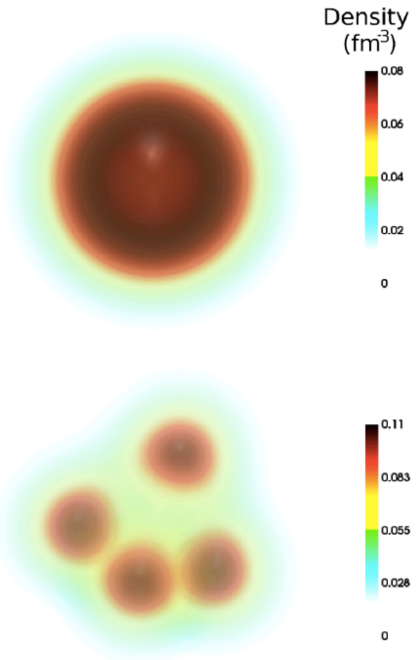
PRC90(2014)054329

PRC89(2014)031303(R)







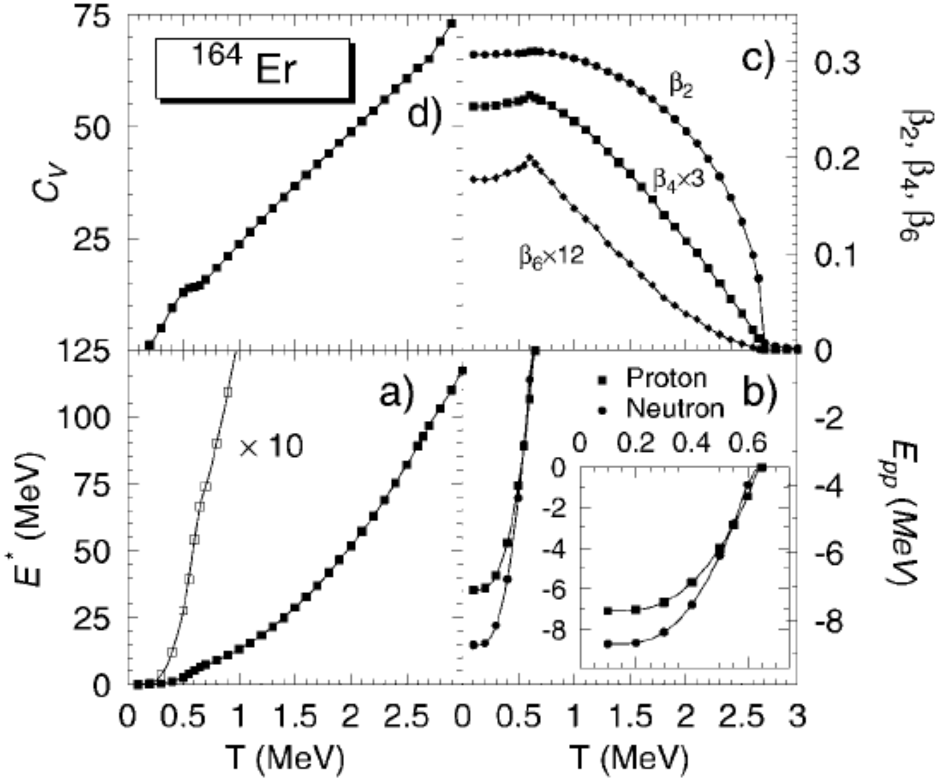


What is the impact of temperature on nuclear clustering ?

Effect of temperature on nuclei

Increasing the temperature :

- Critical temperature for pairing collapse
- Critical temperature for deformation collapse



J.L. Egido, L.M. Robledo, V. Martin, PRL 85, 27 (2000)

What happens for temperature effect on clustering: critical temperature ?

Finite-temperature-deformed-RHB

$$\Omega = E - TS - \lambda N$$

$$\begin{pmatrix} h_D - \lambda - m & \Delta \\ -\Delta^* & -h_D^* + \lambda + m \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

$$\rho_s = \sum_{E_k>0} V_k^\dagger \gamma^0 (1 - f_k) V_k + U_k^T \gamma^0 f_k U_k^*$$

$$\rho_v = \sum_{E_k>0} V_k^\dagger (1 - f_k) V_k + U_k^T f_k U_k^*,$$

$$\rho_{tv} = \sum_{E_k>0} V_k^\dagger \tau_3 (1 - f_k) V_k + U_k^T \tau_3 f_k U_k^*,$$

$$f_k = \frac{1}{1 + e^{\beta E_k}}$$

E. Yüksel, F. Mercier, J. P. Ebran, E. Khan, Phys. Rev. C 106, 054309 (2022)

Taking into account statistical fluctuations

- Quantum fluctuations: not considered here
- Statistical fluctuations:

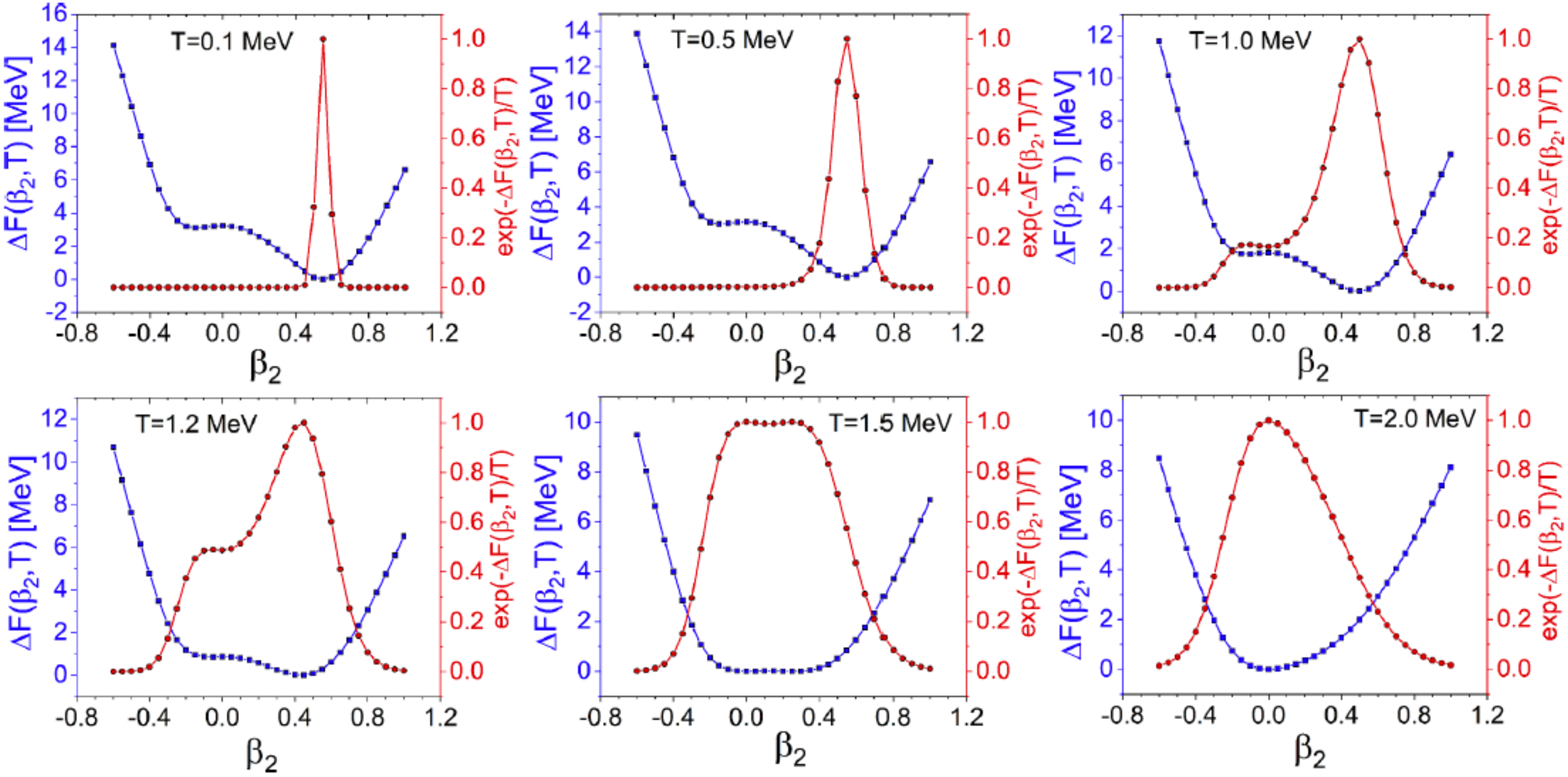
$$\overline{O} = \frac{\int d\beta_2 O(\beta_2, T) \exp(-\Delta F(\beta_2, T)/T)}{\int d\beta_2 \exp(-\Delta F(\beta_2, T)/T)}$$

with $\Delta F(\beta_2,T) = F(\beta_2,T) - F_{min}(T)$

$$F(\beta_2, T) = E(\beta_2, T) - TS(\beta_2, T)$$: free energy

$$S = -k_B \sum_k [f_k \ln f_k + (1 - f_k) \ln(1 - f_k)]$$: entropy

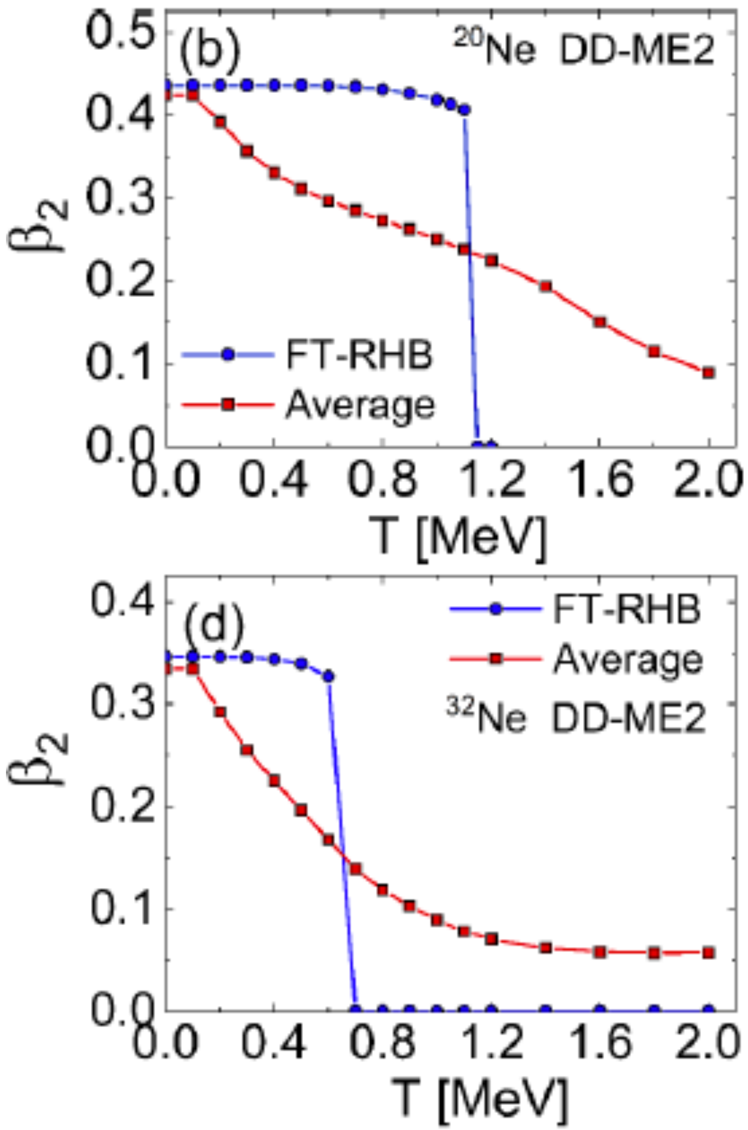
Taking into account statistical fluctuations



^{20}Ne

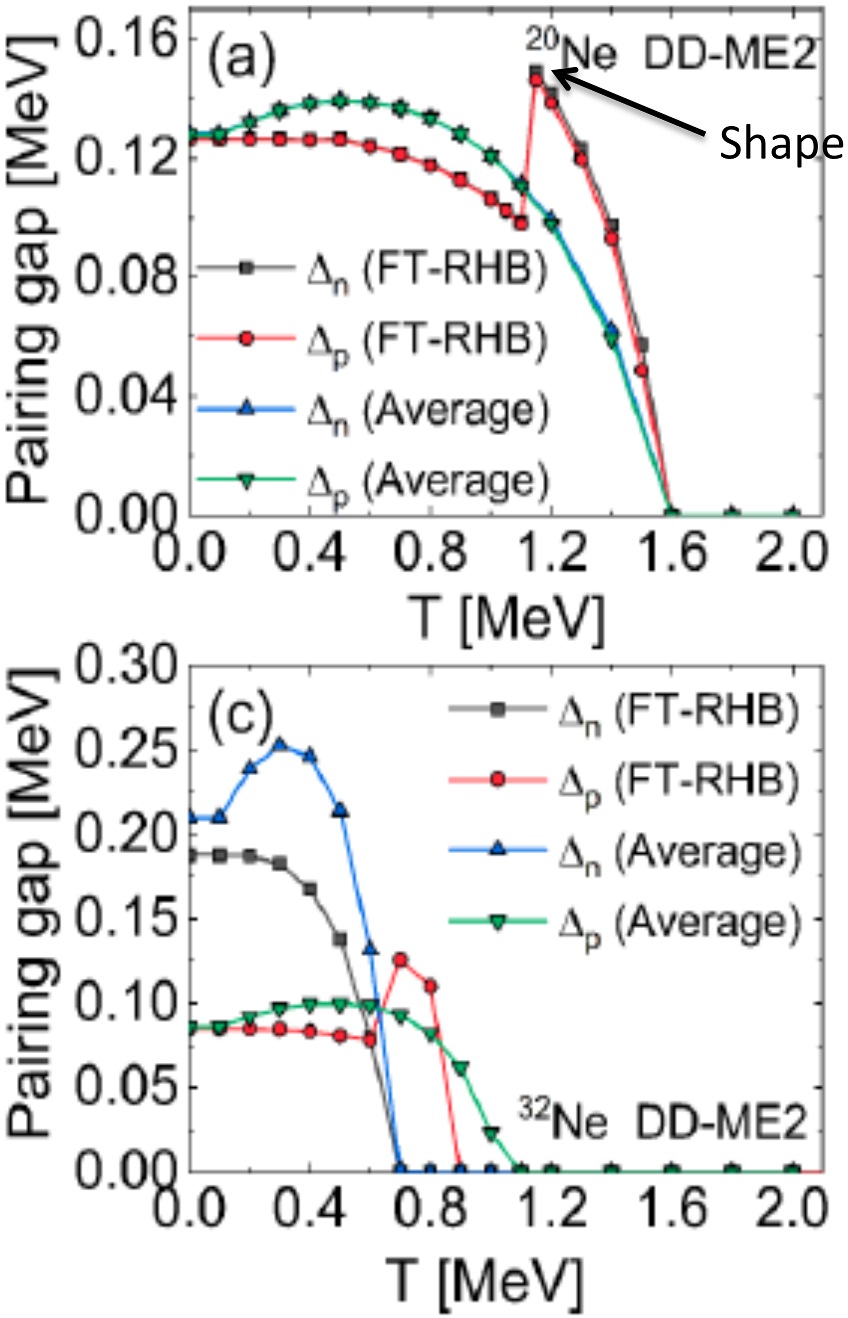
Mixing of the states increases with temperature

Shape phase transition



Smoothing of the transition due to the mixing

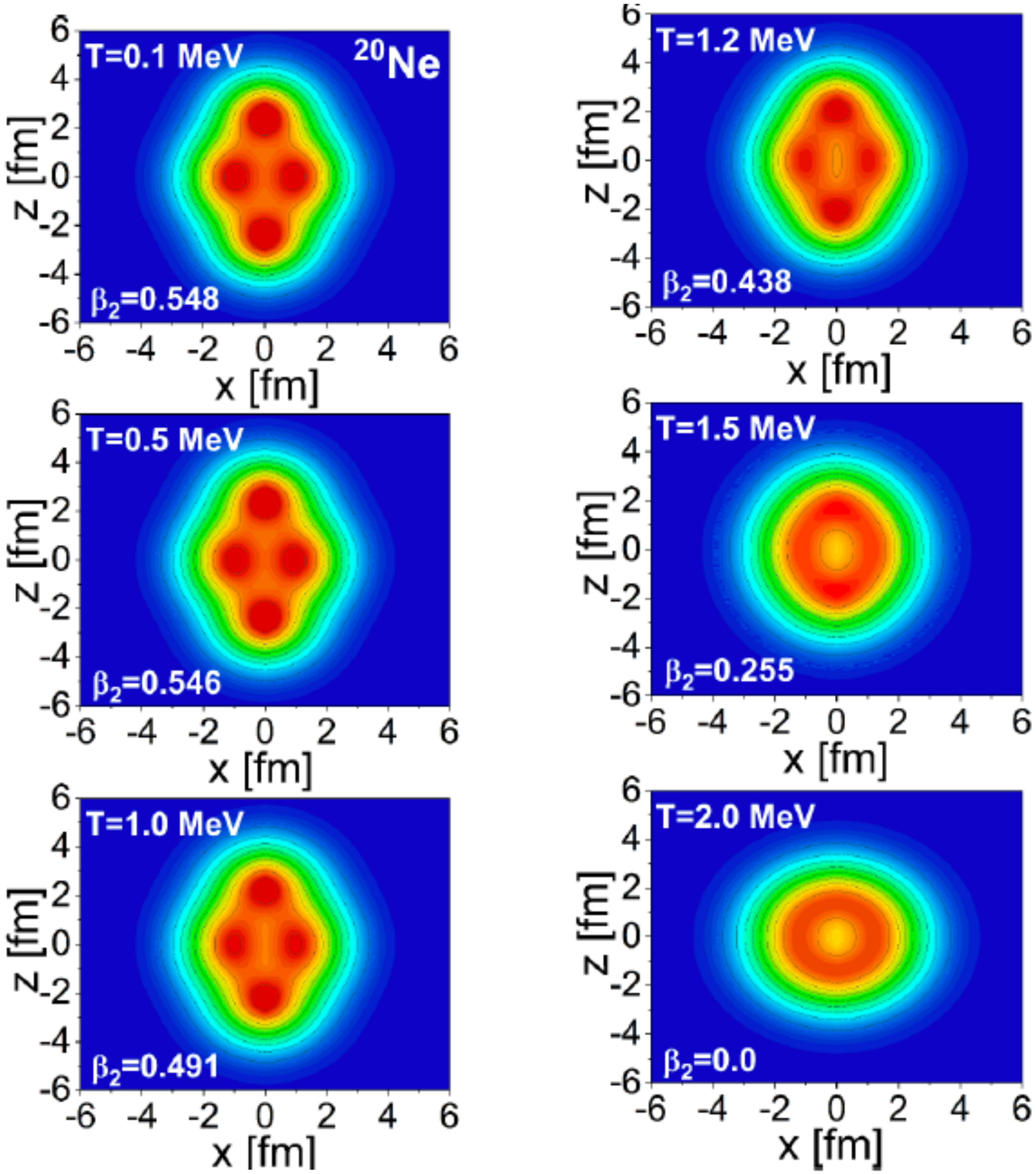
Pairing phase transition



Shape PT

Effect of mixing

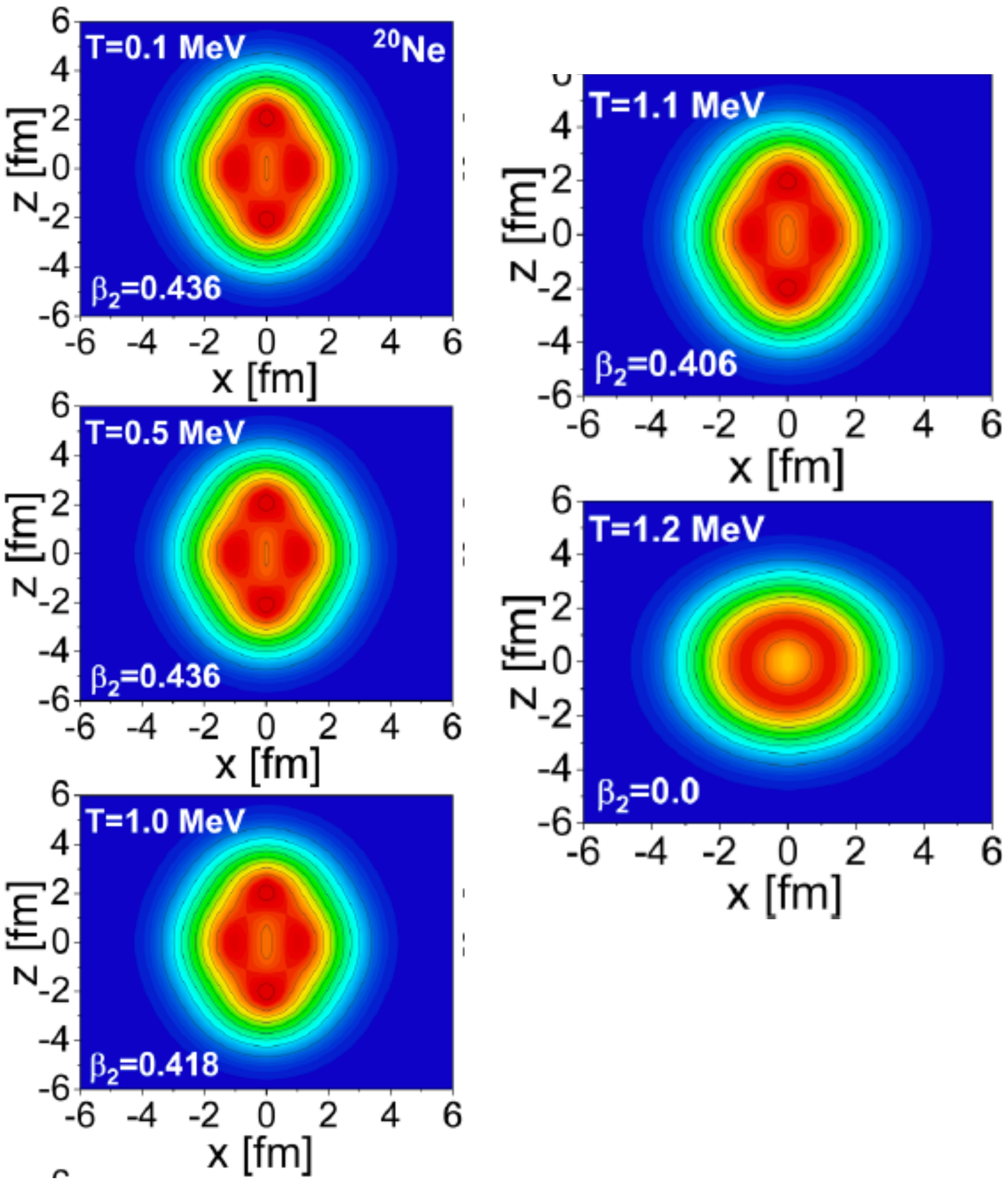
Temperature effect on clusterization in nuclei



Weak pairing: gradual vanishing of clusterization

E. Yüksel, F. Mercier, J. P. Ebran, E. Khan, Phys. Rev. C 106, 054309 (2022)

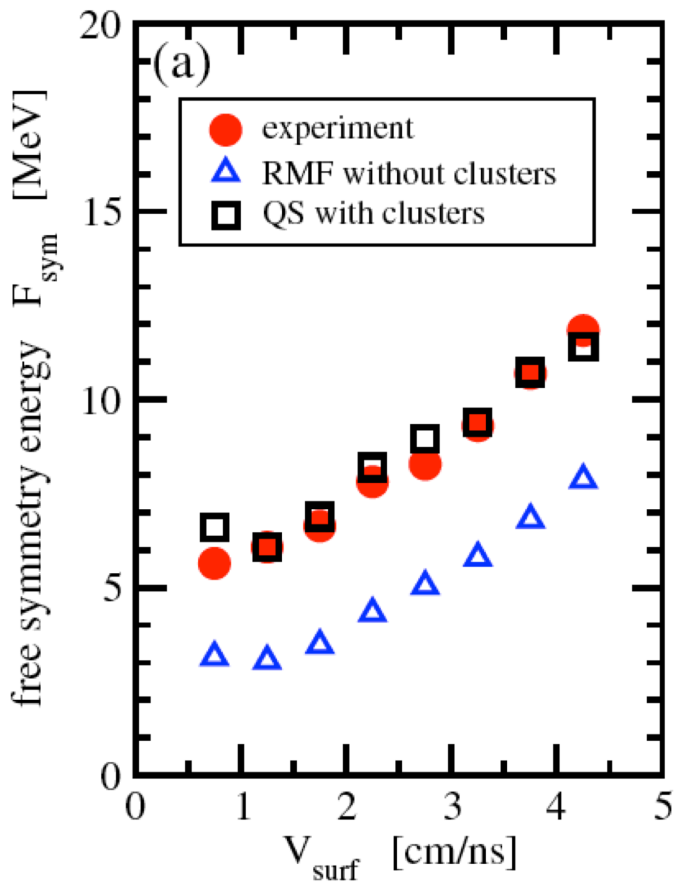
Temperature effect on clusterization in nuclei



Stronger pairing: pairing synchronizes the cluster and shape phase transitions

On the way to a (ρ,T) diagram: what is the impact of density on nuclear clustering ?

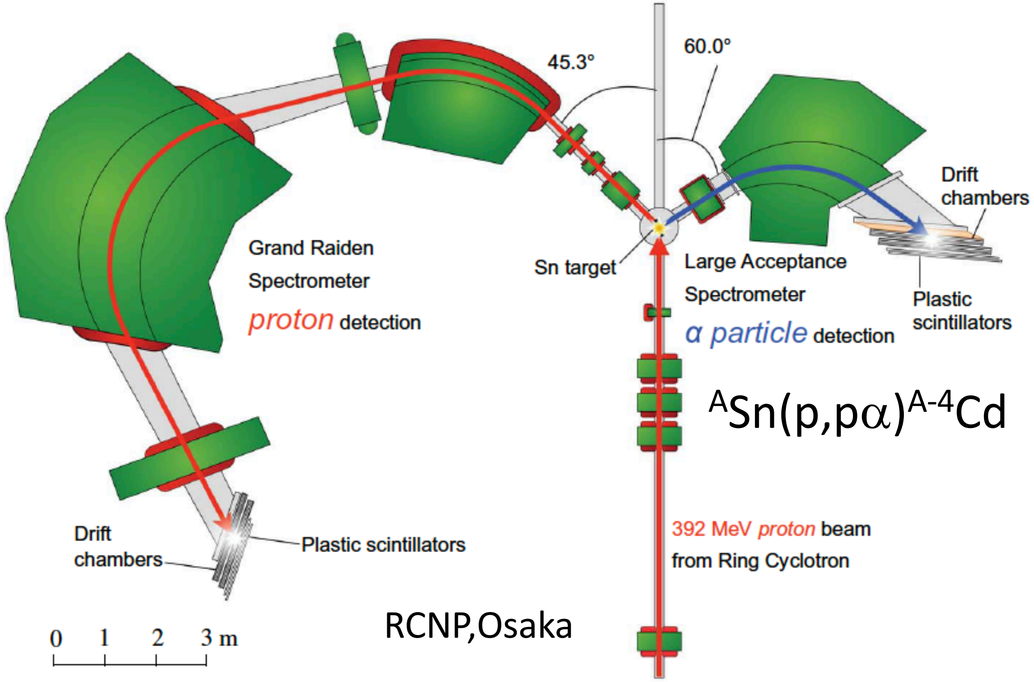
Clusters in low density nuclear matter



J.B. Natowicz et al. PRL104(2010)202501

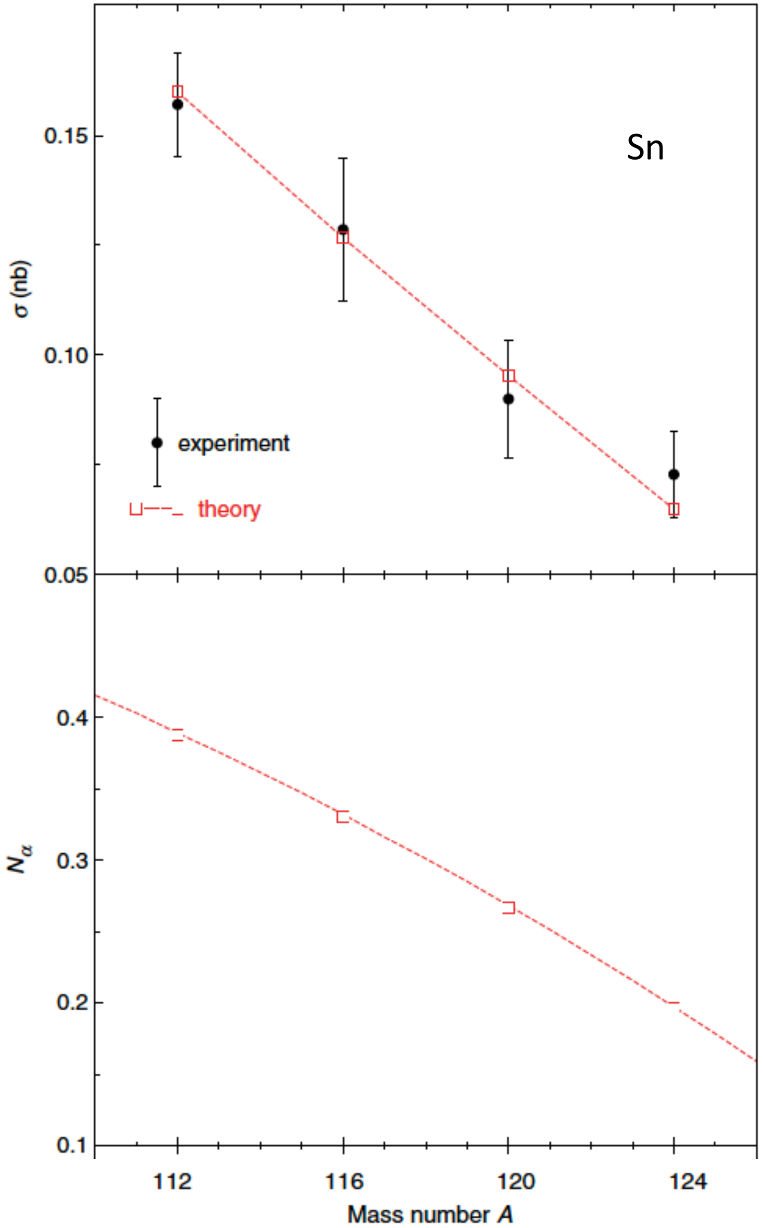
Clusters in EoS better describe experiment
Data from heavy ion collision

Clusters in low-density part of nuclei ?



^{A}Sn(p,pα)$^{A-4}$Cd

RCNP,Osaka

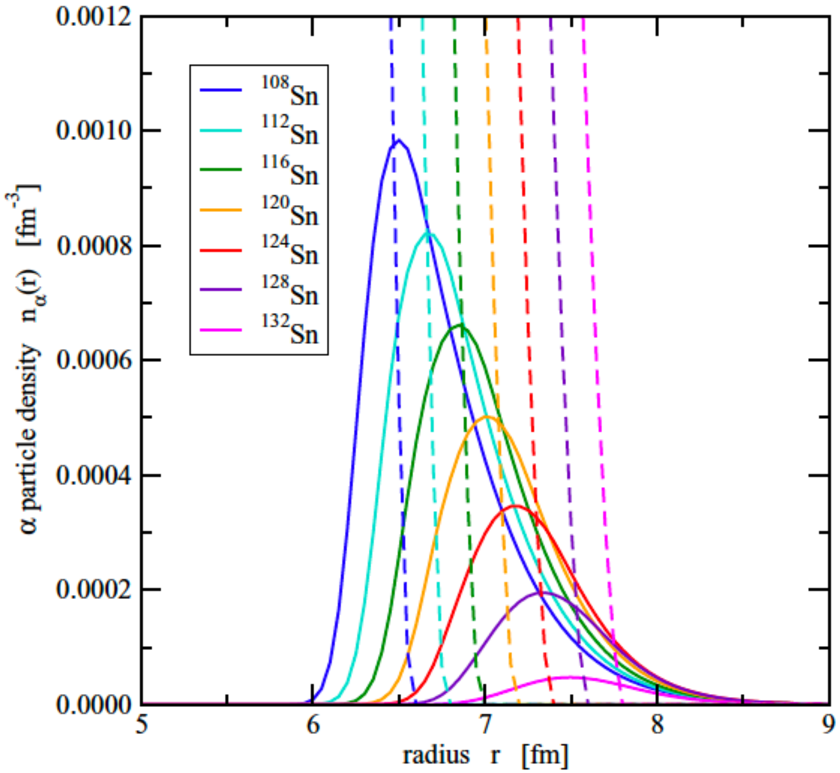


J. Tanaka et al., Science 371(2021)260

Interpretation

$$\begin{aligned}\mathcal{L} = \sum_{i=n,p,t,h} & \bar{\psi}_i\left(\gamma_\mu i D_i^\mu - M_i\right)\psi_i + \frac{1}{2}\left(i D_\alpha^\mu \varphi_\alpha\right)^* \\ & \times \left(i D_{\alpha\mu}\varphi_\alpha\right) - \frac{1}{2}\varphi_\alpha^* M_\alpha^2 \varphi_\alpha + \frac{1}{4}\left(i D_d^\mu \varphi_d^\nu - i D_d^\nu \varphi_d^\mu\right)^* \\ & \times \left(i D_{d\mu}\varphi_{d\nu} - i D_{d\nu}\varphi_{d\mu}\right) - \frac{1}{2}\varphi_d^{\mu *} M_d^2 \varphi_{d\mu} \\ & + \frac{1}{2}\left(\partial^\mu\sigma\,\partial_\mu\sigma - m_\sigma^2\sigma^2 - \frac{1}{2}G^{\mu\nu}G_{\mu\nu} + m_\omega^2\omega^\mu\omega_\mu\right. \\ & \left. - \frac{1}{2}\vec{H}^{\mu\nu}\cdot\vec{H}_{\mu\nu} + m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu\right), \qquad (3\end{aligned}$$

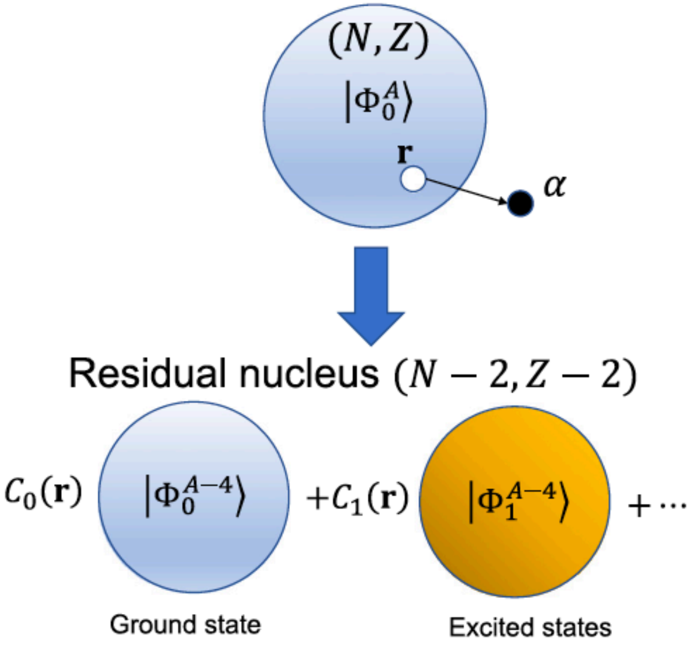
Explicit inclusion of t,^{3}He,α,d in the Lagrangian



α density appears in the surface of nuclei (low density-Mott effect)

S. Typel, PRC 89(2014) 064321

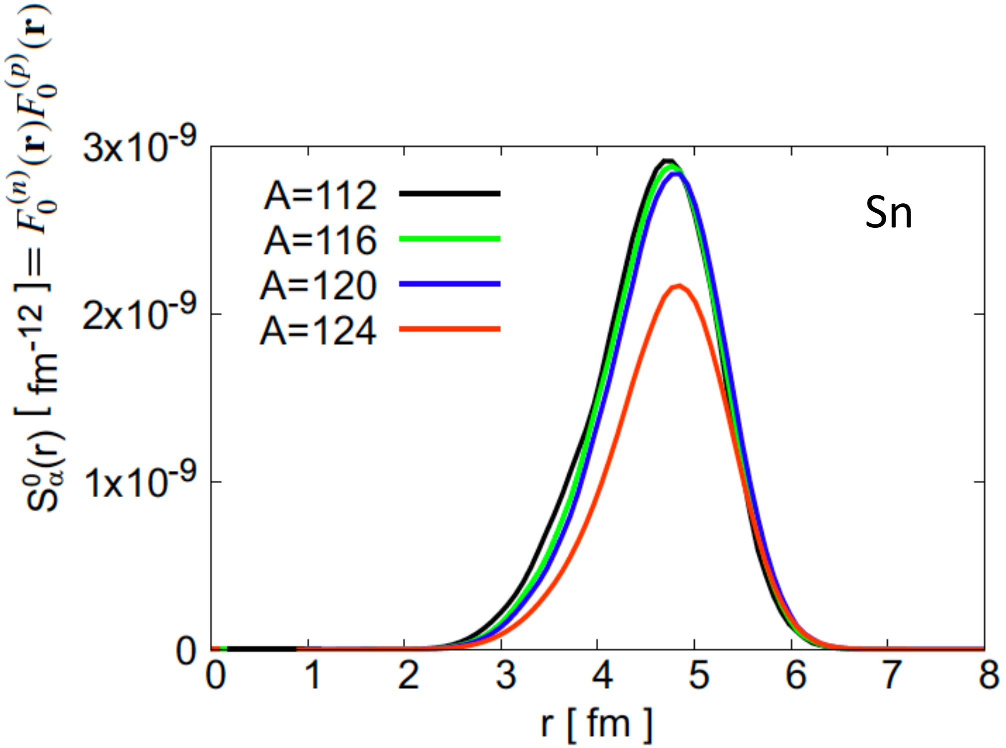
The local α removal strength



$$F_k^{(q)}(\mathbf{r}) = \left|\langle \Phi_k^{N_q-2} \middle| \hat{\psi}_\uparrow^{(q)}(\mathbf{r})\hat{\psi}_\downarrow^{(q)}(\mathbf{r}) \middle| \Phi_0^{N_q}\rangle\right|^2$$

towards the g.s.: $F_0(\mathbf{r}) = |\kappa(\mathbf{r})|^2$

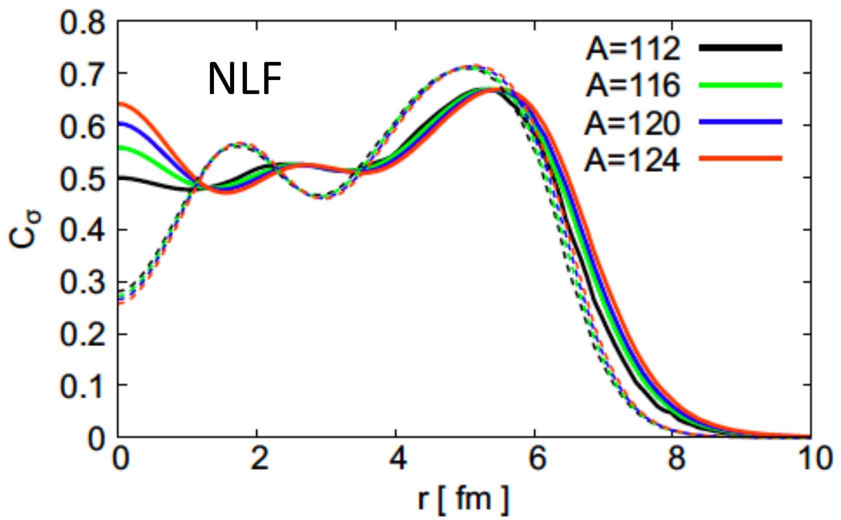
If no pairing: removal of 2 particles

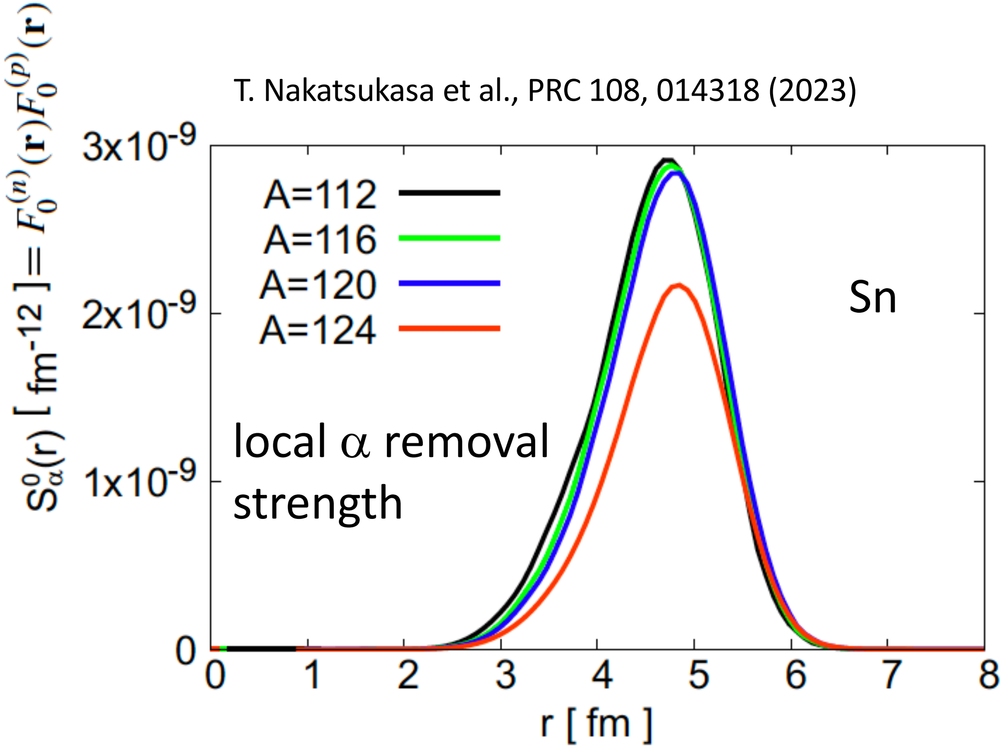


Peaked at the surface

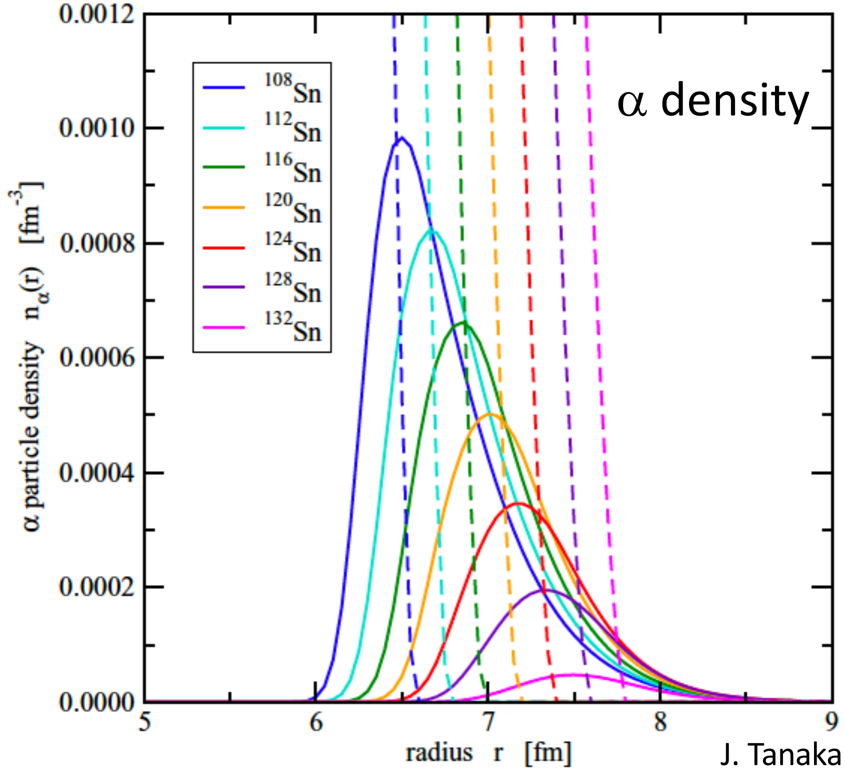
T. Nakatsukasa et al., PRC 108(2023) 014318

Comparison with the NLF and the α density





T. Nakatsukasa et al., PRC 108, 014318 (2023)

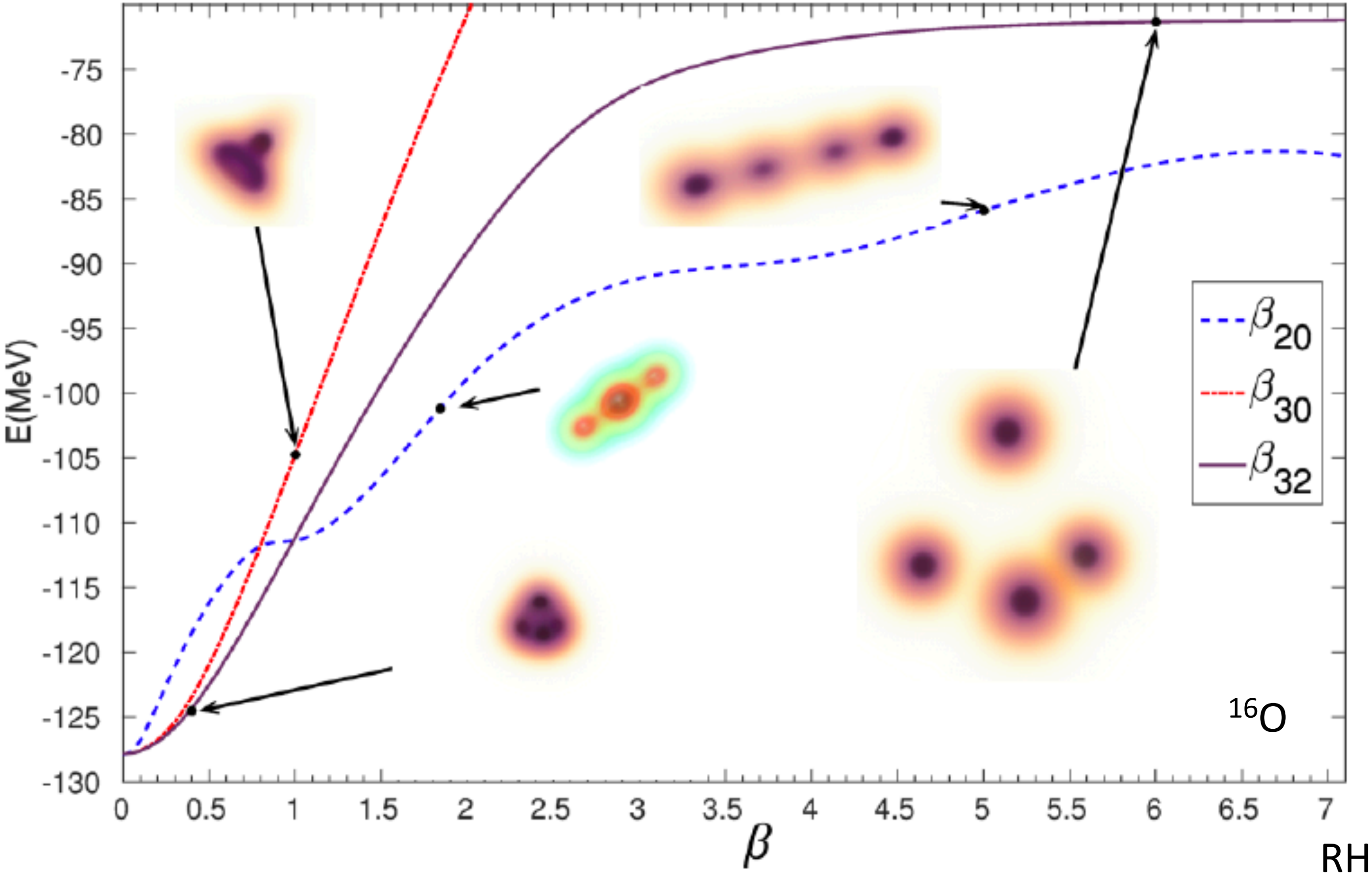


J. Tanaka et al., Science 371, 260 (2021)

- NLF has a different profile inside the nuclei
- α density peaked at larger r values (ρ_0/100)
- See a general discussion in

E. Khan, L. Heitz, F. Mercier, J.-P. Ebran, PRC 106 (2022) 064330

Large deformations lead to clusterization

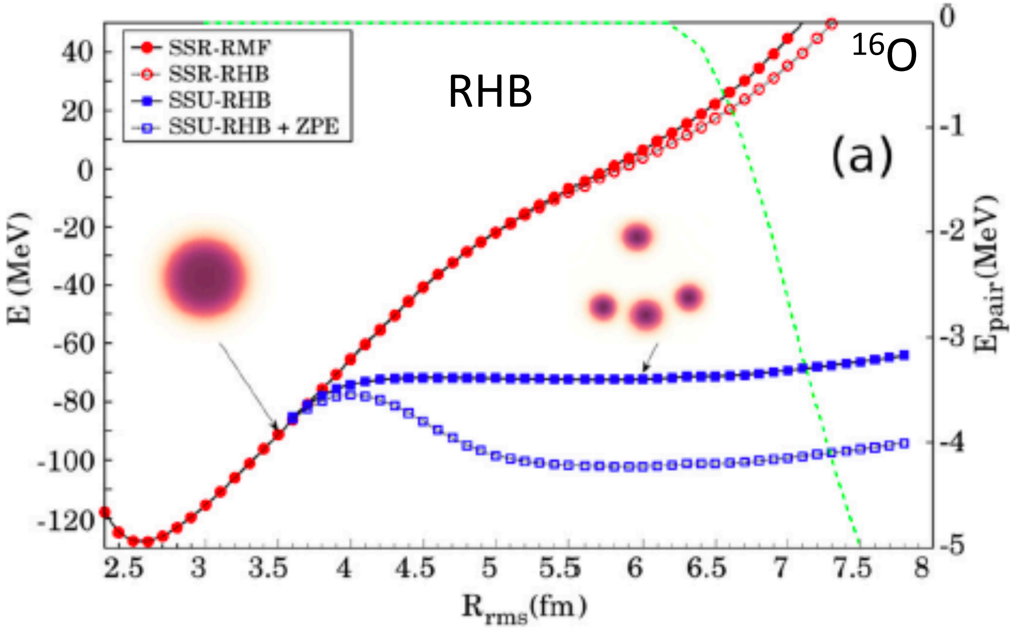


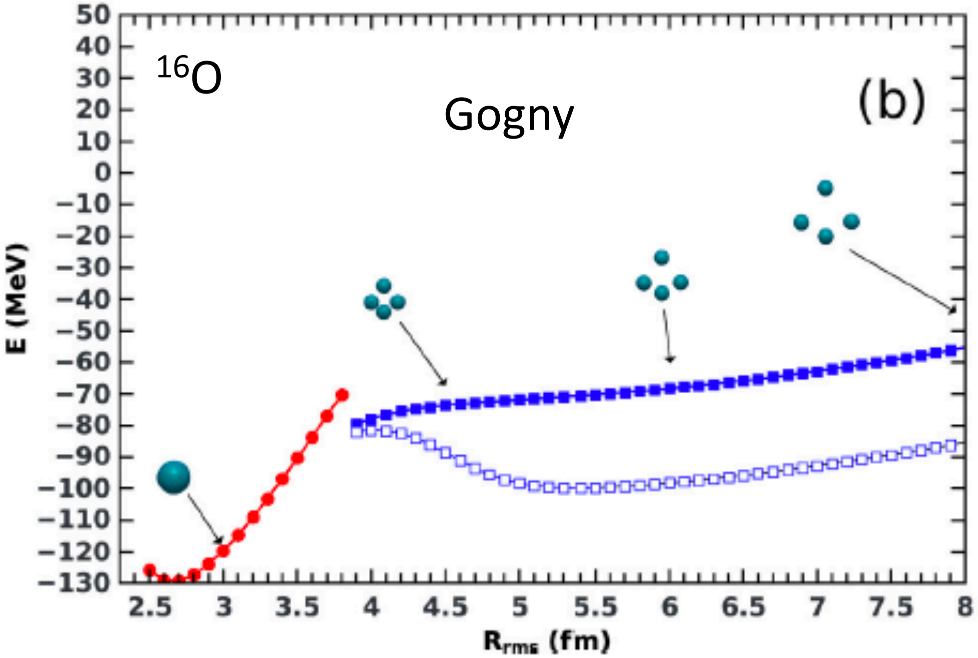
RHB-DDME2

J.-P. Ebran, M. Girod, E. Khan, R. D. Lasseri, and P. Schuck, Phys. Rev. C **102**, 014305 (2020)

Large deformation ⟶ lowering the density ?

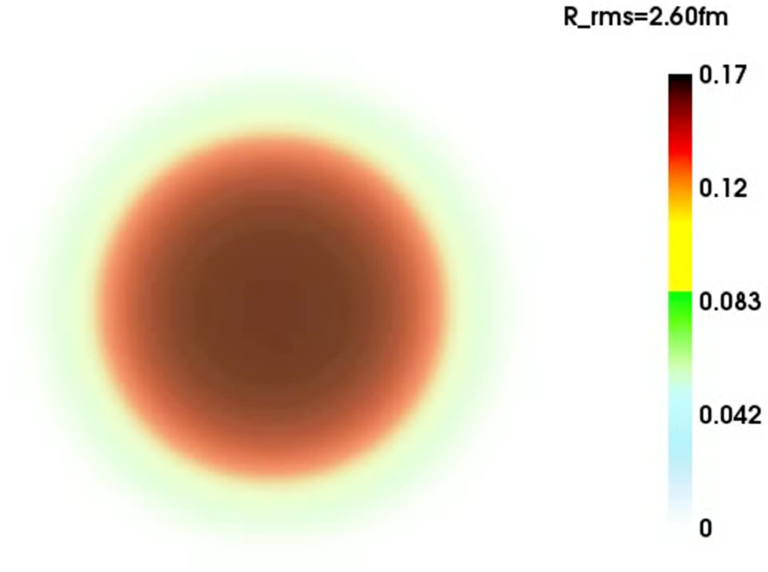
Clusters in low density nuclear matter





Clusters in low density nuclear matter

Dilution of ^{16}O

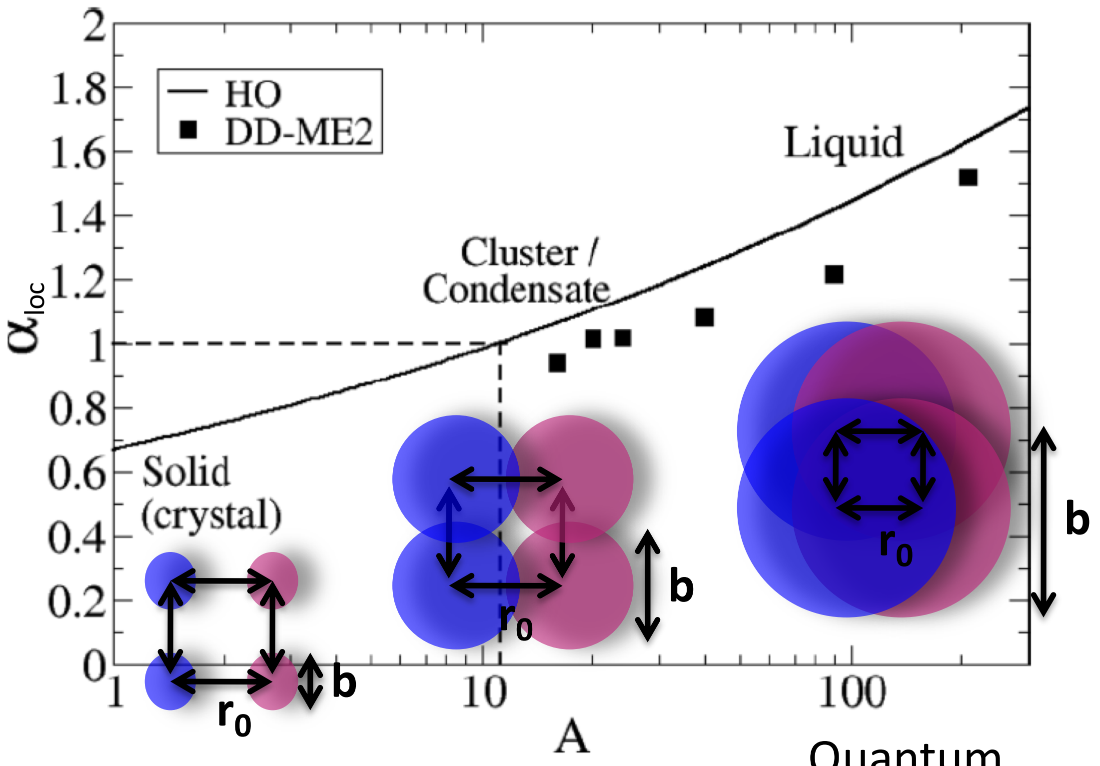


RHB-DDME2

See also: P. Schuck and M. Girod Phys. Rev. Lett. 111, 132503 (2013)

(ρ,A) phase diagram

A global indicator for localization



HO
and
RHB calc.

Crystal
α_{loc} <1

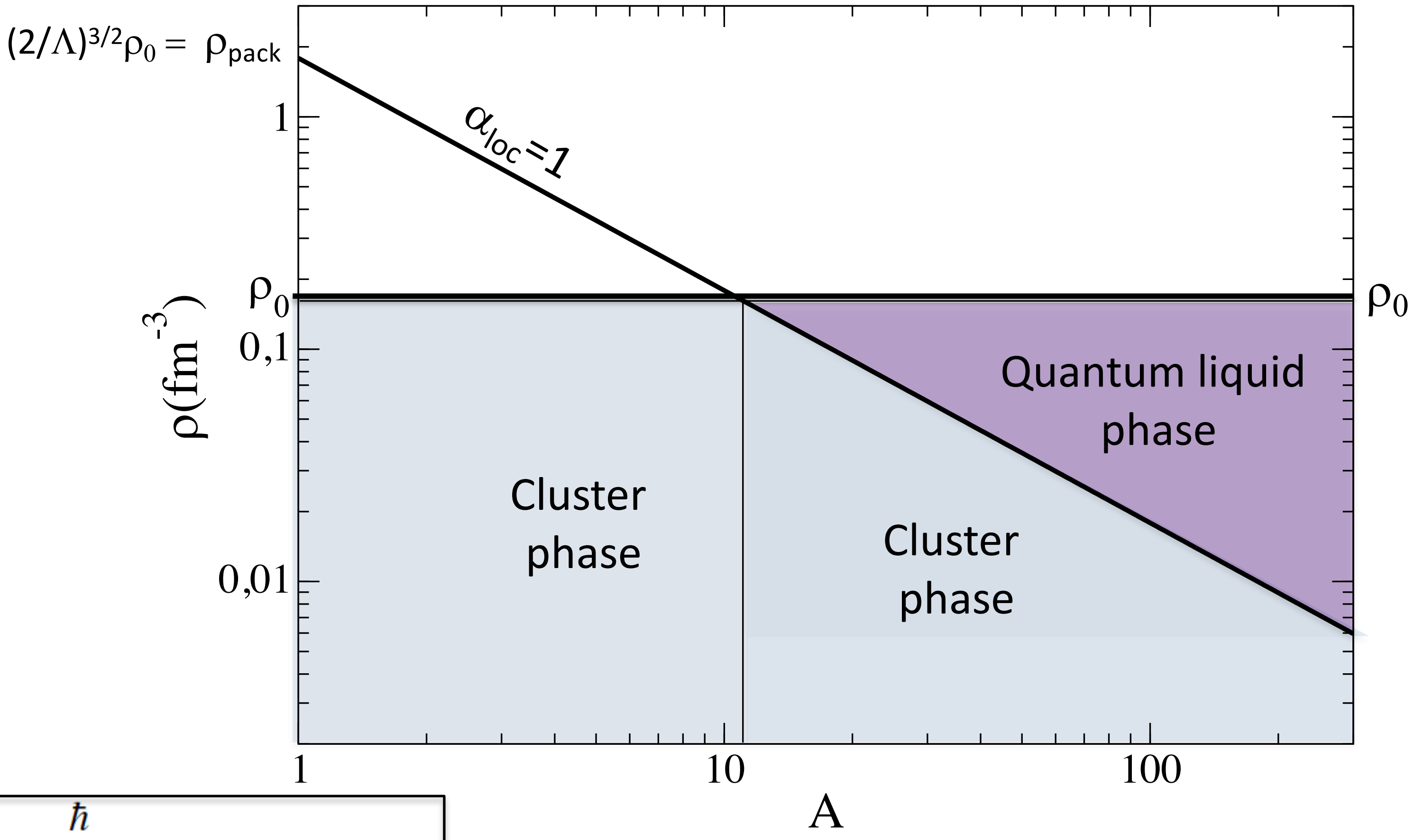
Cluster
$\alpha_{loc} \sim 1$

Quantum
liquid
$\alpha_{loc} > 1$

$$\alpha_{loc} = \frac{2\Delta r}{r_0} \simeq \frac{b}{r_0} = \frac{\sqrt{\hbar} A^{1/6}}{\left(2mV_0 r_0^2\right)^{1/4}}$$

J.-P. Ebran, E. Khan, T. Nikšić, and D. Vretenar, Phys. Rev. C **87**, 044307 (2013)
J.-P. Ebran, E. Khan, R. Lasseri, and D. Vretenar, Phys. Rev. C **97**, 061301(R) (2018)

Density vs. number of nucleons as control parameters of the cluster phase in nuclei



$$r_l = \frac{\hbar}{\sqrt{2mV_0}}$$

$$\alpha_{\rm loc} = \sqrt{\frac{r_l}{\bar{r}}}A^{1/6} = \left(\frac{\rho}{\rho_l}A\right)^{1/6}$$

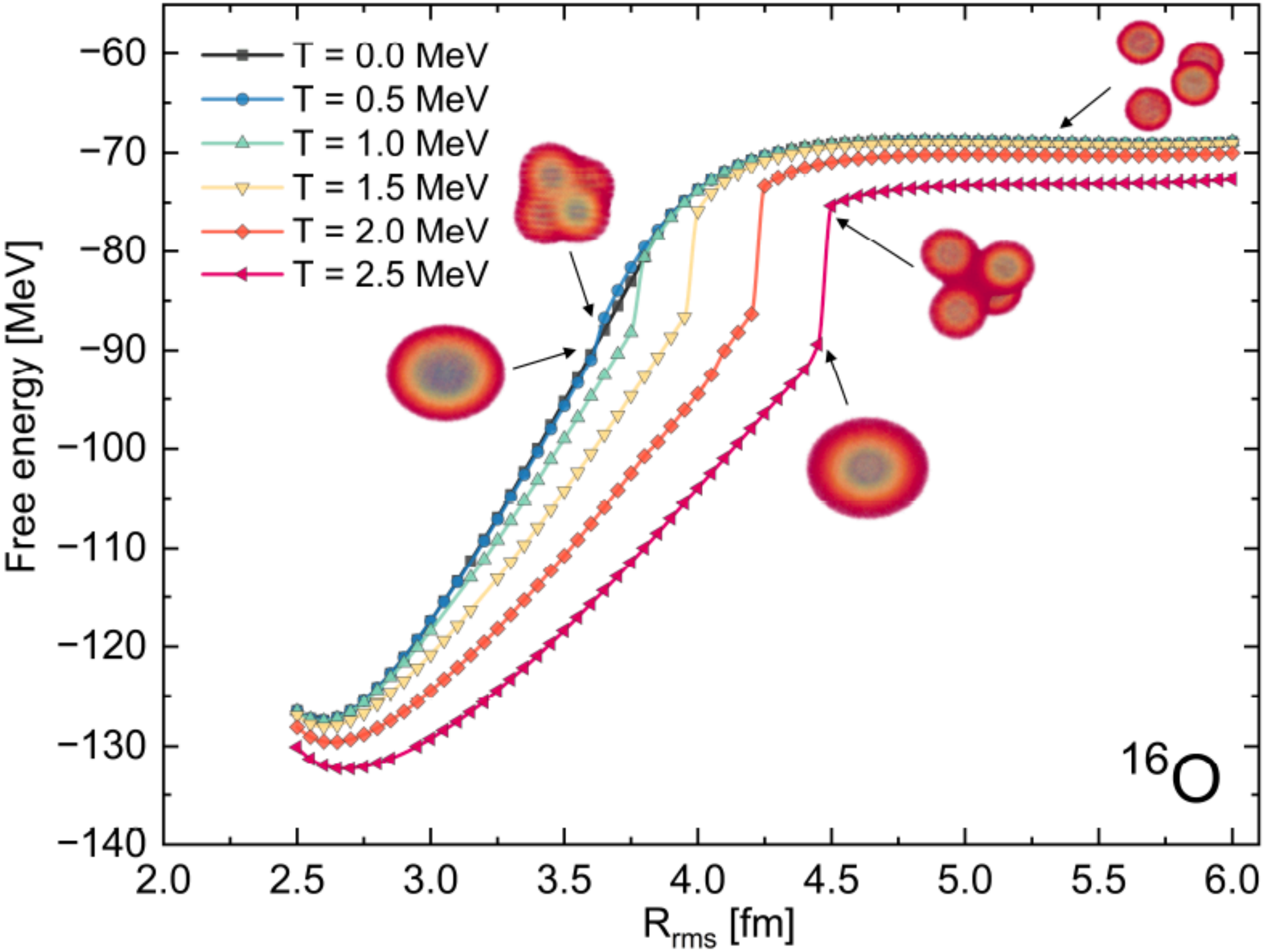
$$\rho A \lesssim \rho_l = \left(\frac{2}{\Lambda}\right)^{3/2}\rho_0 \simeq 10\rho_0$$

J.P. Ebran, L. Heitz, E. Khan, PRC 110(2024)044311

Driven by the quantality and related to the packing density

A (ρ,T) phase diagram ?

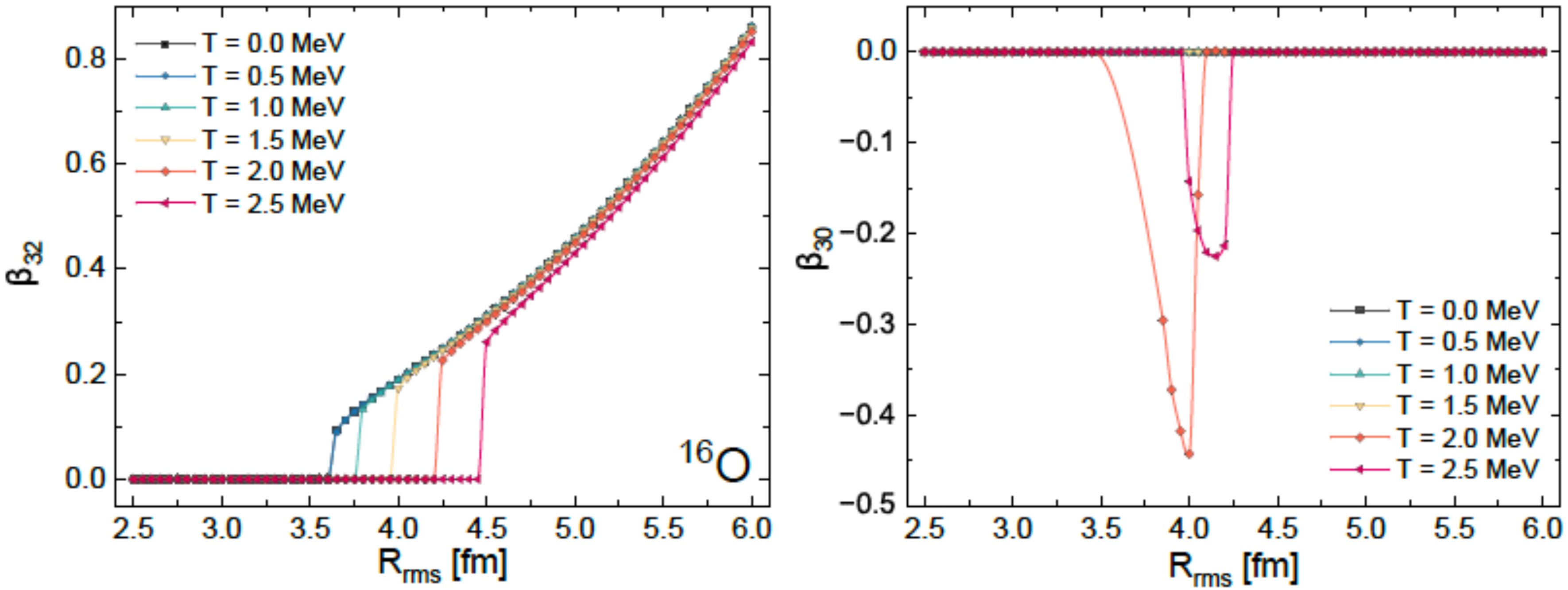
Dilution at finite temperature



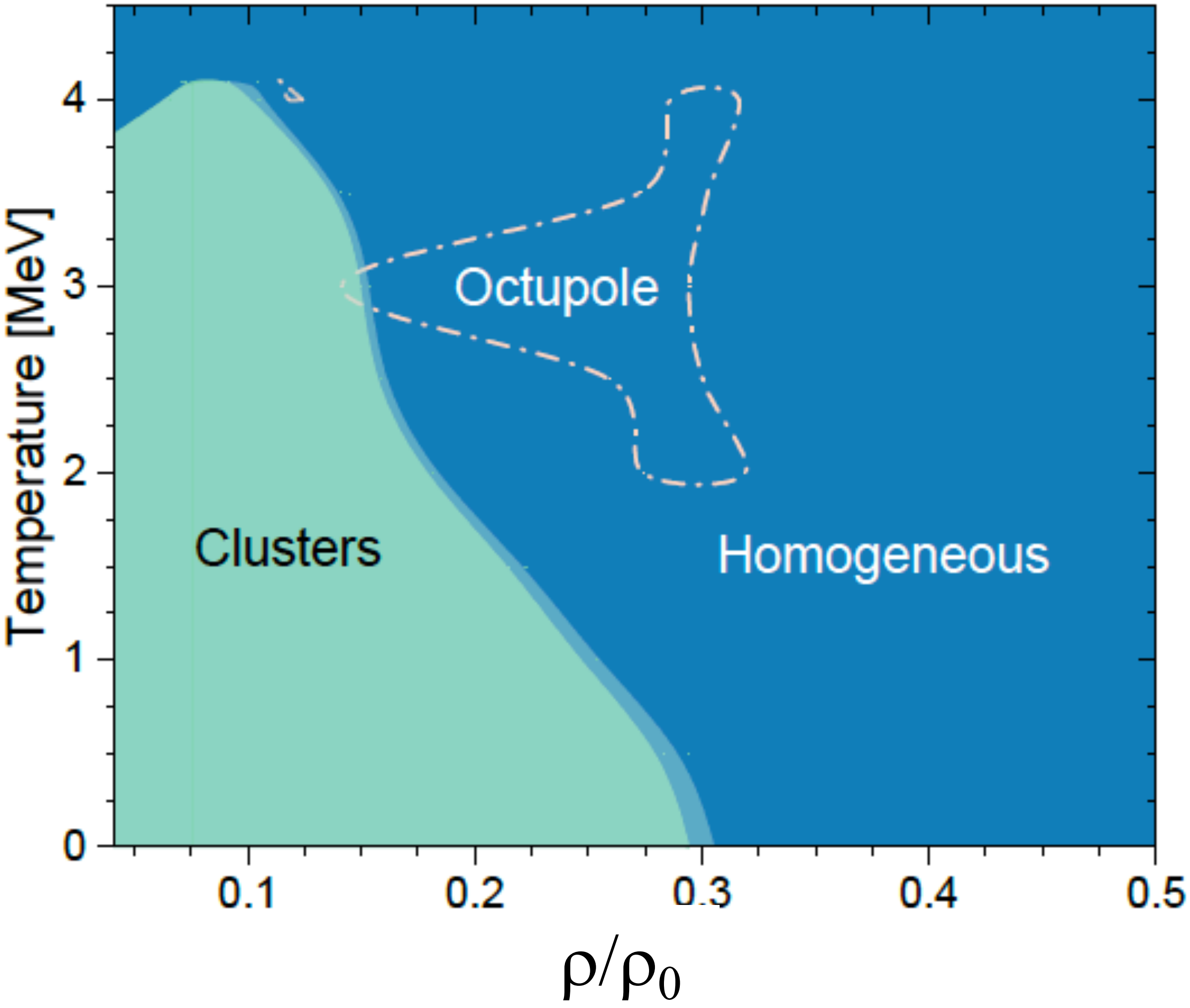
M. Davies et al, in Prep.

RHB-DDPC1

Occurrences of tetrahedral and octupole deformations



(ρ,T) diagram


Temperature [MeV]
0
1
2
3
4
0.1
0.2
0.3
0.4
0.5
ρ/ρ_0
Octupole
Clusters
Homogeneous

Summary and outlook

- Nuclear clusterization is a phase which can be controled by
  - deformation
  - density
  - temperature
  - number of nucleons
  - excitation energy

- Useful for clusterization in Heavy Ion Collisions: density and temperature

- Quantum fluctuations in addition to statistical ones ?

- Comparison (limit) with nuclear matter calculations ?