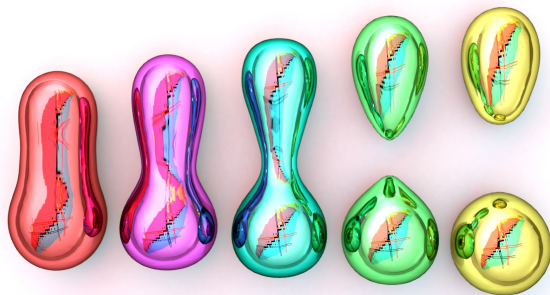


ESNT Workshop : Dynamics of Nuclear Fission

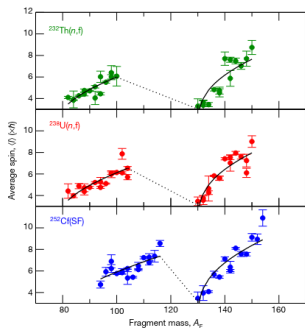
Open Angle Between the Spins of Fission Fragments

Guillaume SCAMPS

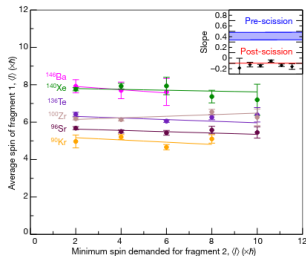
Laboratoire des deux infinis de Toulouse



Spin of the fragments

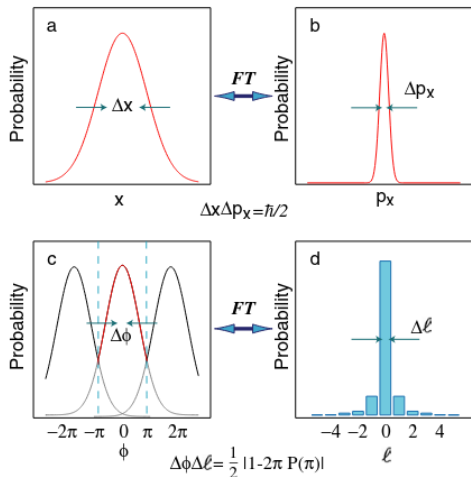


Correlations

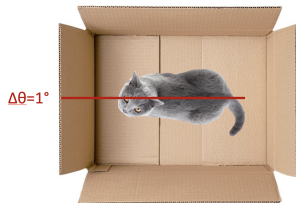
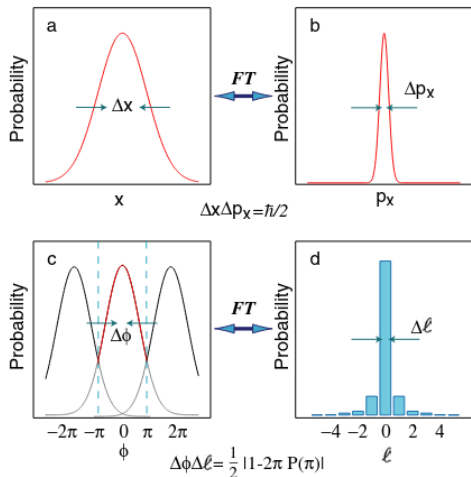


J. N. Wilson, Nature, 590, 566 (2021)

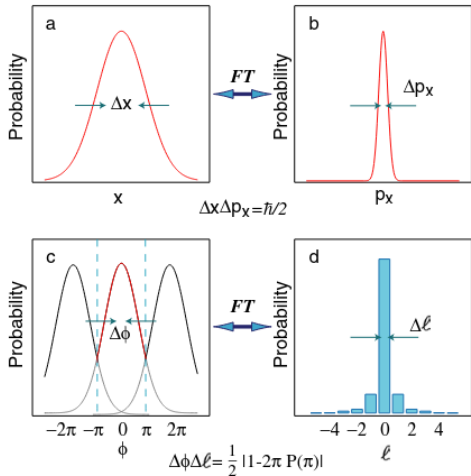
- The average spin follows a sawtooth shape
- No correlations between the spins of the fragments



S. Franke-Arnold, et al. New Journal of Physics 6, 103 (2004)

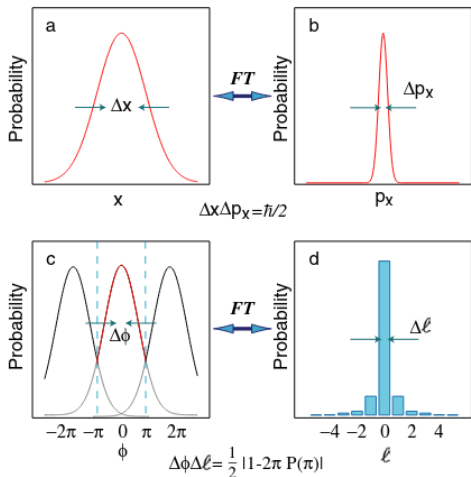


S. Franke-Arnold, et al. New Journal of Physics 6, 103 (2004)



For $\Delta\Theta = 1^\circ$, $\Delta L = 56\hbar$.
 For a cat, angular velocity
 $\omega = 10^{-33} \text{ s}^{-1}$

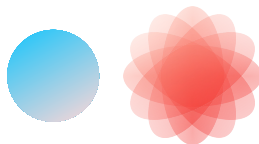
S. Franke-Arnold, et al. New Journal of Physics 6, 103 (2004)



S. Franke-Arnold, et al. New Journal of Physics 6, 103 (2004)

Orientation pumping mechanism

Isotropic potential at scission



Confining potential at scission



I.N Mikhailov, P Quentin, PLB 462 (1999).

For $\Delta\Theta = 1^\circ$, $\Delta L = 56\hbar$.

For a nucleus, angular velocity
 $\omega = 10^{20} \text{s}^{-1}$

$^{144}\text{Ba} + ^{96}\text{Sr}$ at 16 Fm, $\Theta_{ini}=25$ deg, Functional : Skyrme Sly4d

$J_y(x, z)[\hbar \text{ fm}^{-3}]$

G. Scamps, PRC 106, 054614 (2022).

One body-evolution - One body-observable

$^{144}\text{Ba} + ^{96}\text{Sr}$ at 16 Fm, $\Theta_{ini}=25$ deg, Functional : Skyrme Sly4d

$J_y(x, z)[\hbar \text{ fm}^{-3}]$

G. Scamps, PRC 106, 054614 (2022).

One body-evolution - One body-observable

$^{144}\text{Ba} + ^{96}\text{Sr}$ at 16 Fm, $\Theta_{ini} = 25$ deg, Functional : Skyrme Sly4d

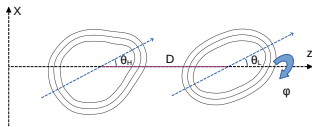
$J_y(x, z)[\hbar \text{ fm}^{-3}]$

G. Scamps, PRC 106, 054614 (2022).

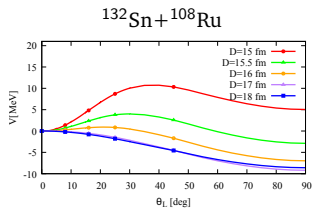
$^{144}\text{Ba} + ^{96}\text{Sr}$ at 16 Fm, $\Theta_{ini} = 25$ deg, Functional : Skyrme Sly4d

$J_y(x, z)[\hbar \text{ fm}^{-3}]$

G. Scamps, PRC 106, 054614 (2022).



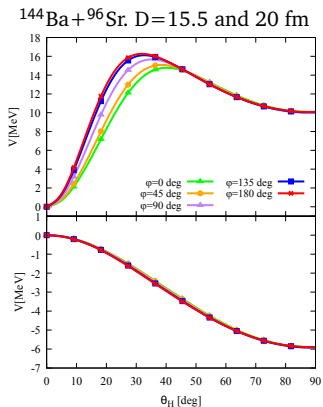
Potential as a function of the light fragment angle



Two torques :

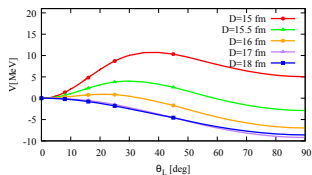
- attractive nucleus-nucleus torque
- repulsive Coulomb torque

Potential as a function of the light fragment angle



The azimuthal angle doesn't have an important role.

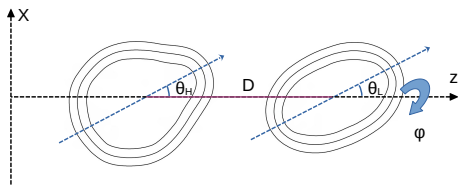
Frozen Hartree-Fock potential



Two torques :

- attractive nucleus-nucleus torque
- repulsive Coulomb torque

4 degrees of freedom



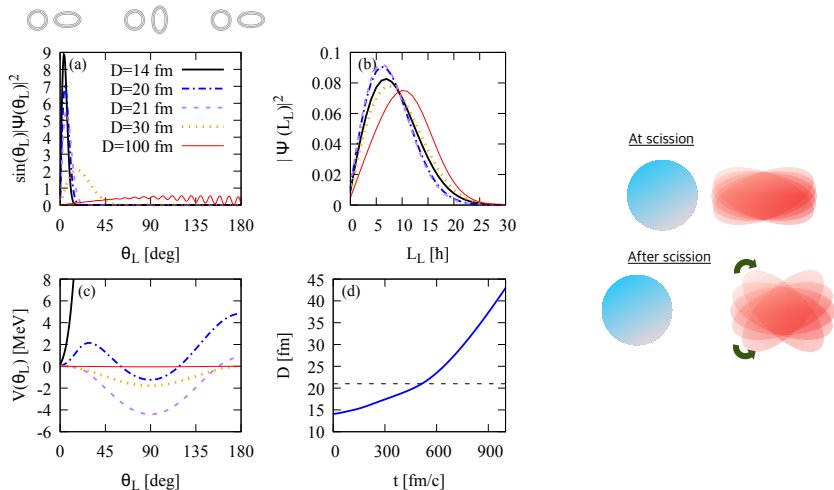
Hamiltonian

$$\hat{H}(D) = \frac{\hbar^2}{2I_H} \hat{L}_H^2 + \frac{\hbar^2}{2I_L} \hat{L}_L^2 + \frac{\hbar^2}{2I_\Lambda(D)} \hat{\Lambda}^2 + \hat{V}(\hat{\Theta}_H, \hat{\Theta}_L, \hat{\varphi}, D)$$

Solved in basis $|L_H, m, L_L, -m\rangle$

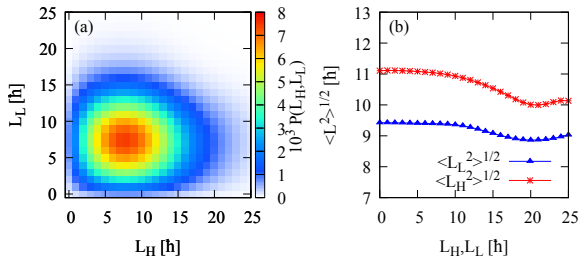
G. Scamps, G. Bertsch, Phys. Rev. C 108, 034616(2023).

Similar to the orientation pumping mechanism model Mikhailov, I. N., and Quentin, P. Physics Letters B, 462(1-2), 7-13 (1999)

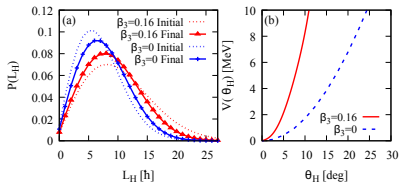
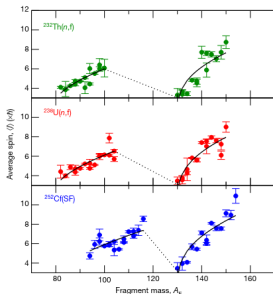


G. Scamps, G. Bertsch, Phys. Rev. C 108, 034616 (2023).

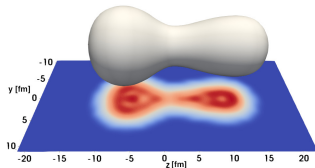
Correlation between the angular momentum

 $^{144}\text{Ba} + ^{96}\text{Sr}$

- No or small correlation observed in the magnitude of the angular momentum.
- More angular momentum for the heavy fragment



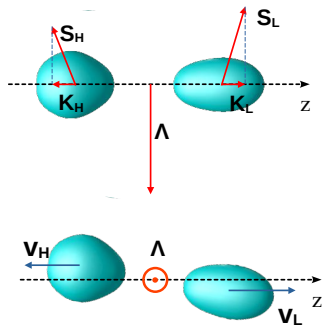
Mechanism



- Pear-shaped deformation plays an important role at scission. G. Scamps C. Simenel, Nature 564, pages 382–385 (2018)
- Octupole deformation makes the angular potential stiffer which increase the zero-point motion \rightarrow more angular momentum

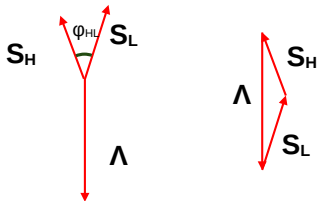
G. Scamps, G. Bertsch, Phys. Rev. C 108, 034616 (2023).

Orbital angular momentum

In spontaneous fission of a 0^+ state

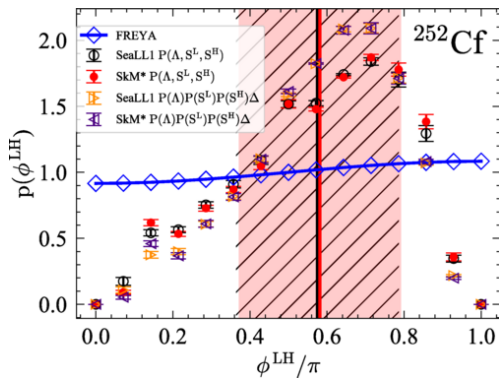
$$\mathbf{S}_H + \mathbf{S}_L + \mathbf{\Lambda} = 0,$$

Triangular rule :

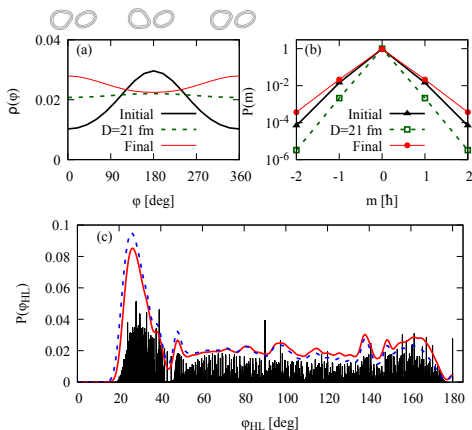


$$\cos(\varphi_{HL}) = \left(\frac{\Lambda(\Lambda + 1) - S_H(S_H + 1) - S_L(S_L + 1)}{2\sqrt{S_H(S_H + 1)S_L(S_L + 1)}} \right)$$

TDDFT (in 2022) vs Freya



A. Bulgac, I. Abdurrahman, K. Godbey, and I. Stetcu, Phys. Rev. Lett. 128, 022501(2022).

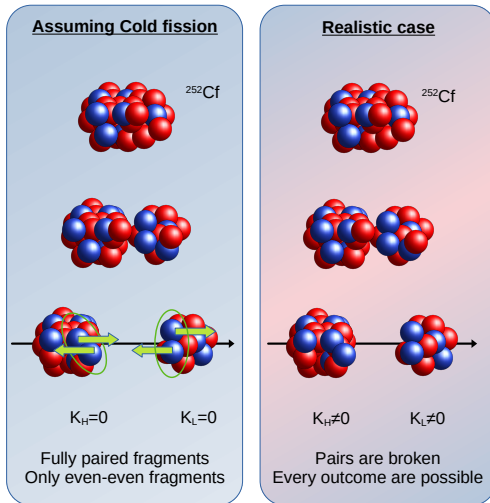


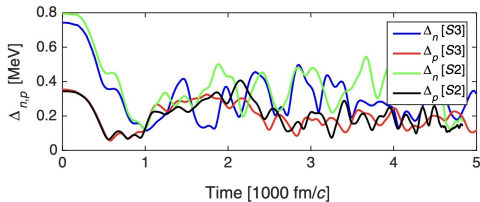
G. Scamps, G. Bertsch, Phys. Rev. C 108, 034616 (2023).

Geometry

- Small azimuthal correlation
- Spins are perpendicular to the fission axis
- Complex pattern in the opening angle, different from previous model

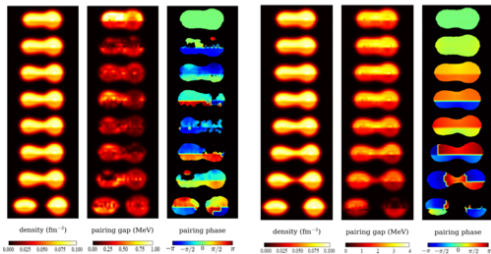
Open angle in the TDHFB model





²⁴⁰Pu fission with the normal pairing gap

²⁴⁰Pu fission with a larger pairing gap



- A. Bulgac, P. Magierski, Kenneth J. Roche, and I. Stetcu, PRL 116, 122504 (2016).
 A. Bulgac, S. Jin, K. J. Roche, N. Schunck, and I. Stetcu, PRC 100, 034615 (2019).

Projection method

Projection on the spin and K number (Projection of the spin on the fission axis)

$$\hat{P}_{MK}^S = \frac{(2S+1)}{16\pi^2} \int d\Omega \mathcal{D}_{MK}^{S*}(\Omega) e^{i\alpha\hat{S}_z} e^{i\beta\hat{S}_y} e^{i\gamma\hat{S}_z},$$

$$P(S_F, K_F) = \langle \Psi | \hat{P}_{K_F K_F}^{S_F} | \Psi \rangle,$$

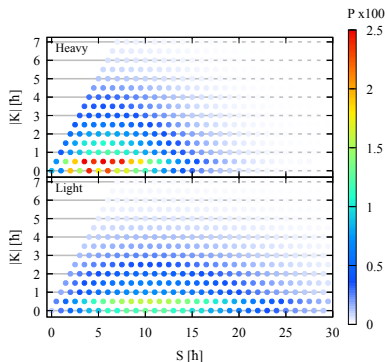
Calculation of the overlap : G. F. Bertsch and L. M. Robledo, PRL 108, 042505 (2012)

$$\langle \Psi | \hat{R} | \Psi \rangle = \frac{(-1)^n}{\prod_{\alpha}^n v_{\alpha}^2} \text{pf} \begin{bmatrix} V^T U & V^T R^T V^* \\ -V^{\dagger} R V & U^{\dagger} V^* \end{bmatrix}$$

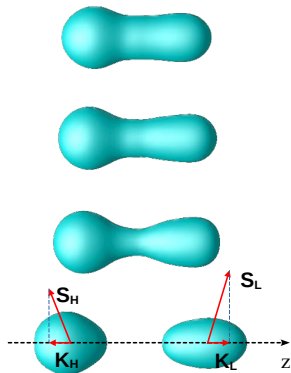
Optimized Pfaffian calculation : M. Wimmer, ACM Trans. Math Softw. 38, 30 (2012).

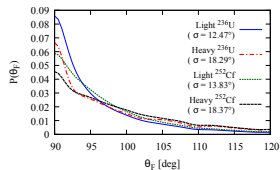
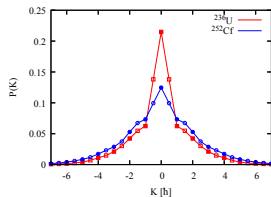
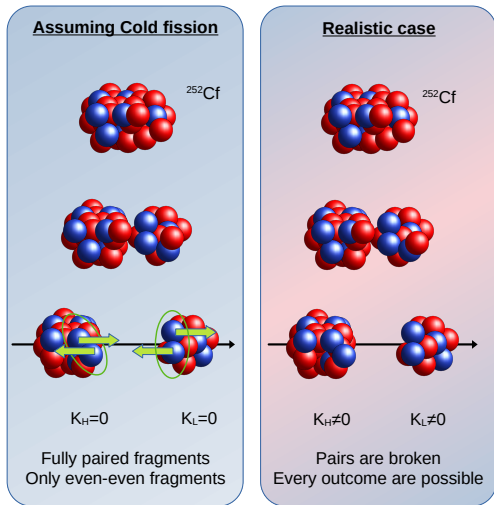
Spin distribution in the fragments

Obtained using 3-angle projection operator



Geometry of the reaction





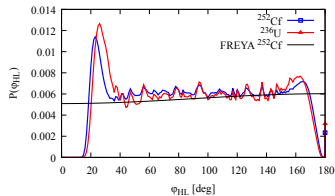
$$\cos \theta_F = \frac{K_F}{\sqrt{S_F(S_F + 1)}}$$

$$\varphi_{HL} = \arccos \left(\frac{\Lambda(\Lambda + 1) - S_H(S_H + 1) - S_L(S_L + 1)}{2\sqrt{S_H(S_H + 1)S_L(S_L + 1)}} \right)$$

$$P(\Lambda, S_H, S_L) = \sum_{k_H k_L} \langle \Psi | \hat{P}_{0,0}^\Lambda \hat{P}_{k_H k_H}^{S_H} \hat{P}_{k_L k_L}^{S_L} | \Psi \rangle.$$

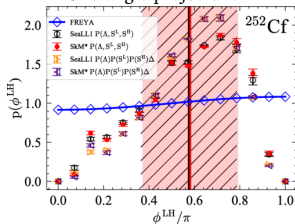
$$P(\Lambda, S_H, S_L) = \sum_{K_H K_L K'_H K'_L} (-1)^{K'_H - K_H + K'_L - K_L}$$

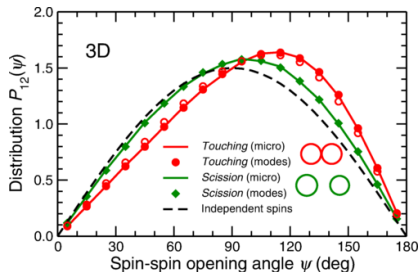
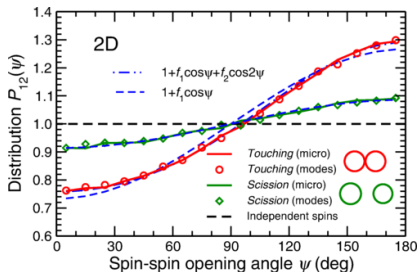
$$C_{S_H, -K_H, S_L, -K_L}^{\Lambda, 0} C_{S_H, -K'_H, S_L, -K'_L}^{\Lambda, 0} \langle \Psi | \hat{P}_{K_H K'_H}^{S_H} \hat{P}_{K_L K'_L}^{S_L} | \Psi \rangle$$



G.scamps, I. Abdurrahman, M. Kafker, A. Bulgac, and I. Stetcu, PRC 108 (6), L061602.

One angle projector :



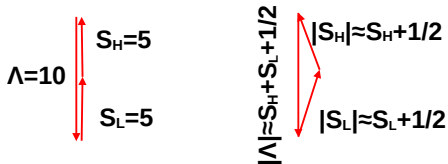


J. Randrup, Phys. Rev. C 106, L051601 (2022).

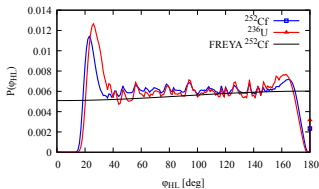
Question

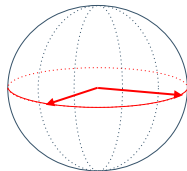
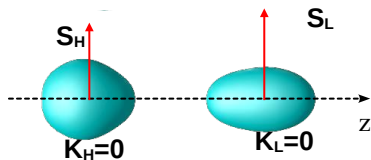
- How the quantal effects change this picture ?
- How the geometry change the opening angle distribution assuming no correlation ?

Non alignment of the spins



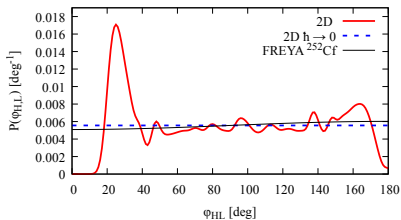
To get a 5 degrees angle between two spins require spins of $262 \hbar$ and $6565 \hbar$ for 1 degree

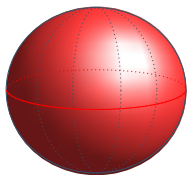
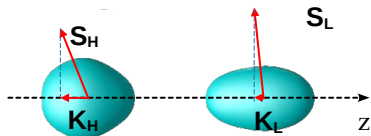




$$|\psi\rangle = \sum_{S_H, K_H, S_L, K_L} c_{S_H, K_H, S_L, K_L} |S_H, K_H, S_L, K_L\rangle,$$

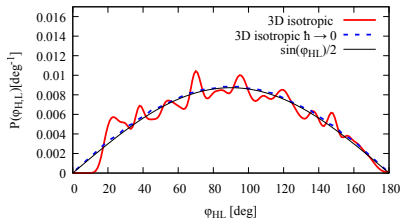
$$|c_{S_H, K_H, S_L, K_L}|^2 \propto \delta_{K_H, 0} \delta_{K_L, 0} (2S_H + 1) e^{\frac{-S_H(S_H+1)}{2\sigma_H^2}} \times (2S_L + 1) e^{\frac{-S_L(S_L+1)}{2\sigma_L^2}}.$$



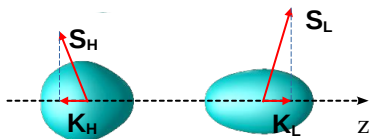


$$|\psi\rangle = \sum_{S_H, K_H, S_L, K_L} c_{S_H, K_H, S_L, K_L} |S_H, K_H, S_L, K_L\rangle,$$

$$|c_{S_H, K_H, S_L, K_L}|^2 \propto e^{\frac{-S_H(S_H+1)}{2\sigma_H^2}} e^{\frac{-S_L(S_L+1)}{2\sigma_L^2}}.$$

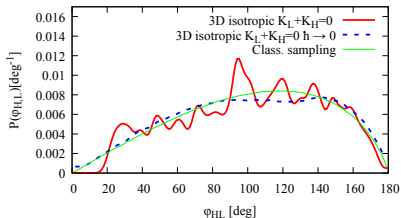
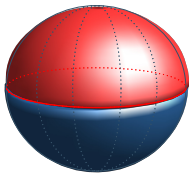


G. Scamps, PRC 109, L011602 (2024).

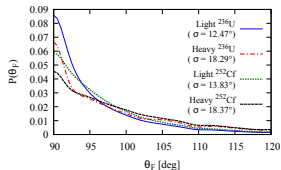
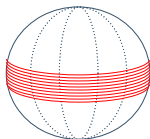
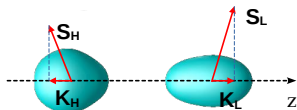


$$|\psi\rangle = \sum_{S_H, K_H, S_L, K_L} c_{S_H, K_H, S_L, K_L} |S_H, K_H, S_L, K_L\rangle,$$

$$|c_{S_H, K_H, S_L, K_L}|^2 \propto \delta_{K_H - K_L} e^{-\frac{S_H(S_H+1)}{2\sigma_H^2}} e^{-\frac{S_L(S_L+1)}{2\sigma_L^2}}.$$

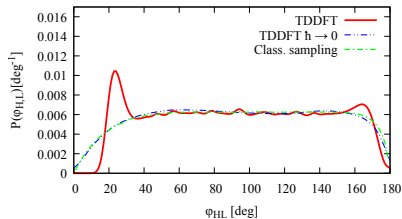


G. Scamps, PRC 109, L011602 (2024).



$$|\Psi\rangle = \sum_{S_H, K_H, S_L, K_L} c_{S_H, K_H, S_L, K_L} |S_H, K_H, S_L, K_L\rangle,$$

$|c_{S_H, K_H, S_L, K_L}|^2$ From TDDFT



TDDFT shows an intermediate case between 2D and 3D.

G. Scamps, PRC 109, L011602 (2024).

Main points

- Orientation-pumping (uncertainty principle) mechanism at scission
- Additional effect of the Coulomb torque
- Internal excitation (breaking of pairs)
- Spins are mainly perpendicular to the fission axis
- Uncorrelated magnitude and orientation of the spins
- Dependence of the mechanism with the deformation (quadrupole and octupole)

Outlook

- TD-GCM with rotated fragments
- Fission system with initial spin

Thank you

Temporary page!

\LaTeX was unable to guess the total number of pages correctly. As there was some unprocessed data that should have been added to the final page this extra page has been added to receive it.

If you rerun the document (without altering it) this surplus page will go away, because \LaTeX now knows how many pages to expect for this document.