

Collective excitations of even-even open-shell nuclei via PGCM and QRPA calculations

Nuclear ab initio spectroscopy workshop

Espace de Structure Nucléaire Théorique

May 21st, 2024

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WITH

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Robert Roth

TUD

Vittorio Somà

CEA IRFU



Based on



[arXiv:2402.15901, 2024]



[arXiv:2402.02228, 2024]



[arXiv:2404.14154, 2024]

Fourth (and last) paper of the series coming soon

Outline

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1

Introduction

- Giant Resonances Physics
- The PGCM
- Link between PGCM and QRPA

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Systematic study

- Numerical details
- Uncertainty estimate

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- Shape coexistence
- Deformation

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- Proof of principle
- Realistic calculations

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From finite nuclei to Astrophysics

- Preliminary incompressibility results

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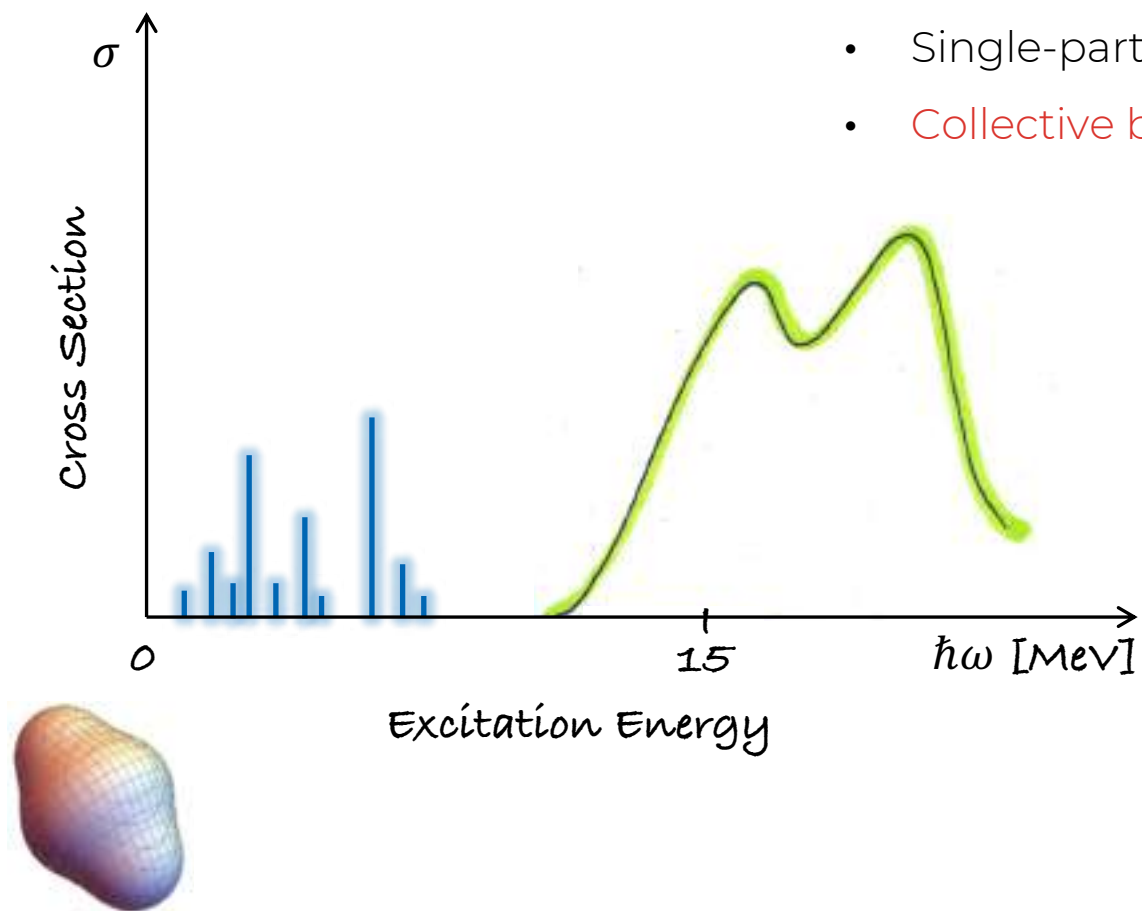
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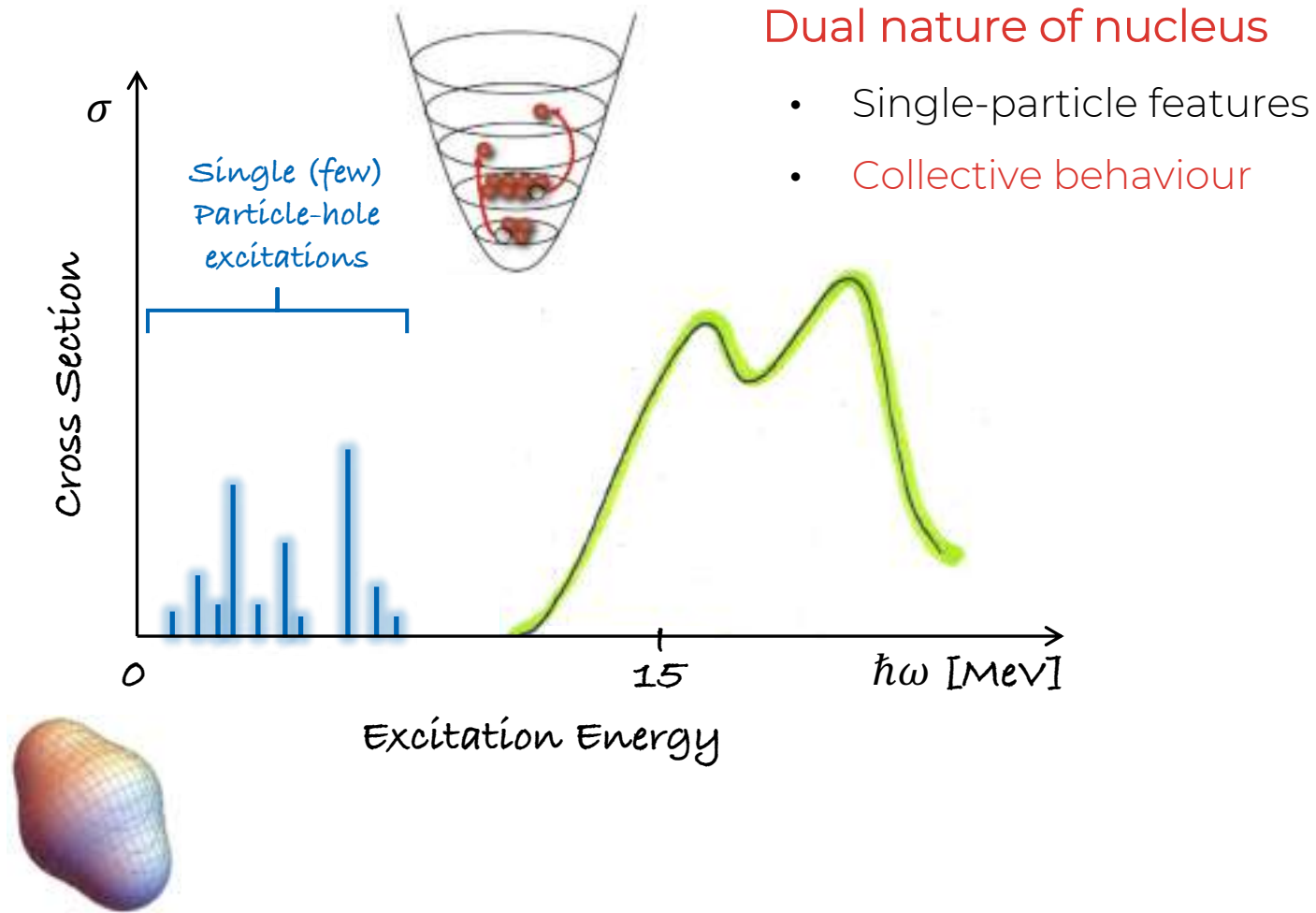
Giant Resonances

Dual nature of nucleus

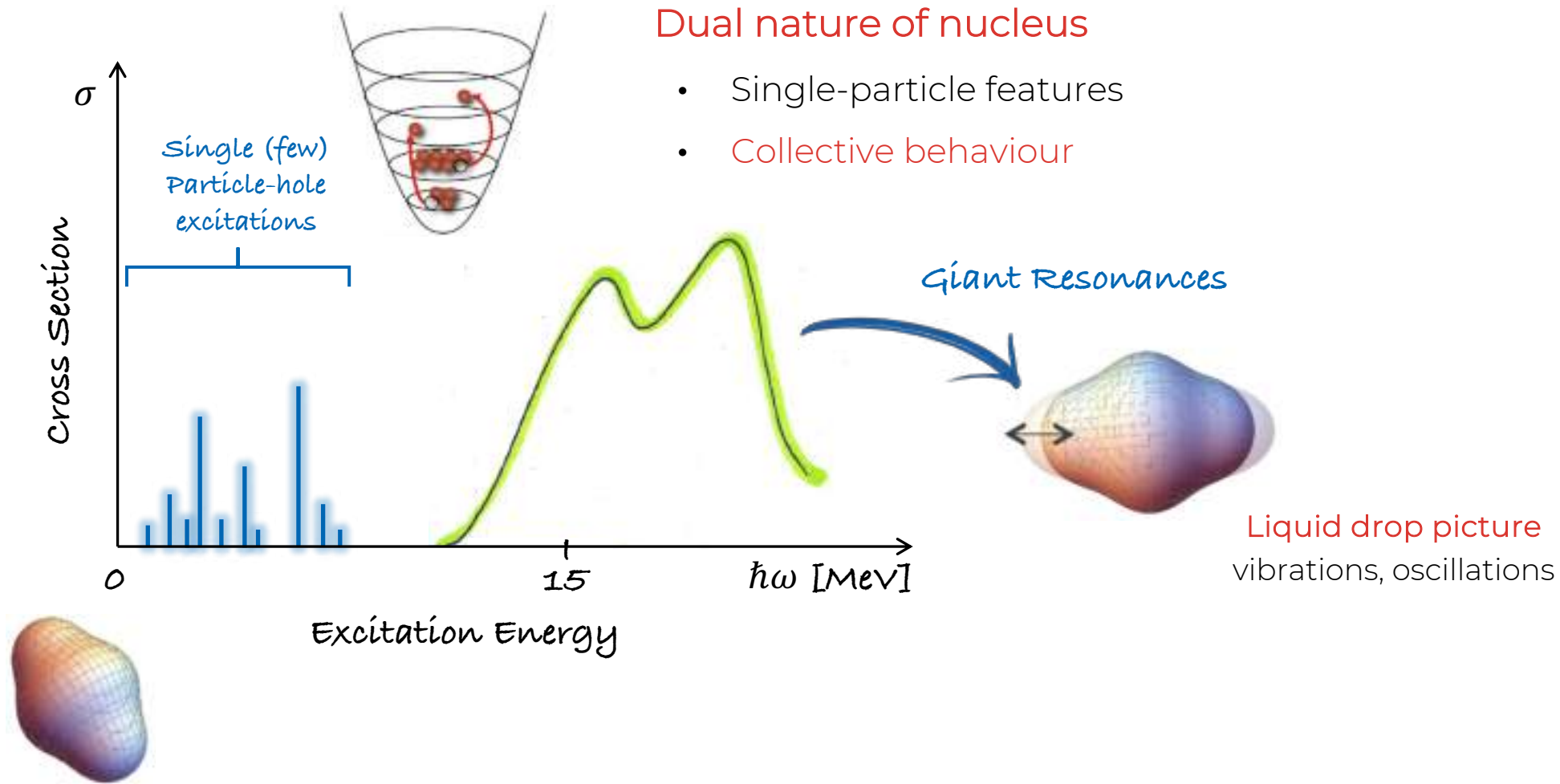
- Single-particle features
- Collective behaviour



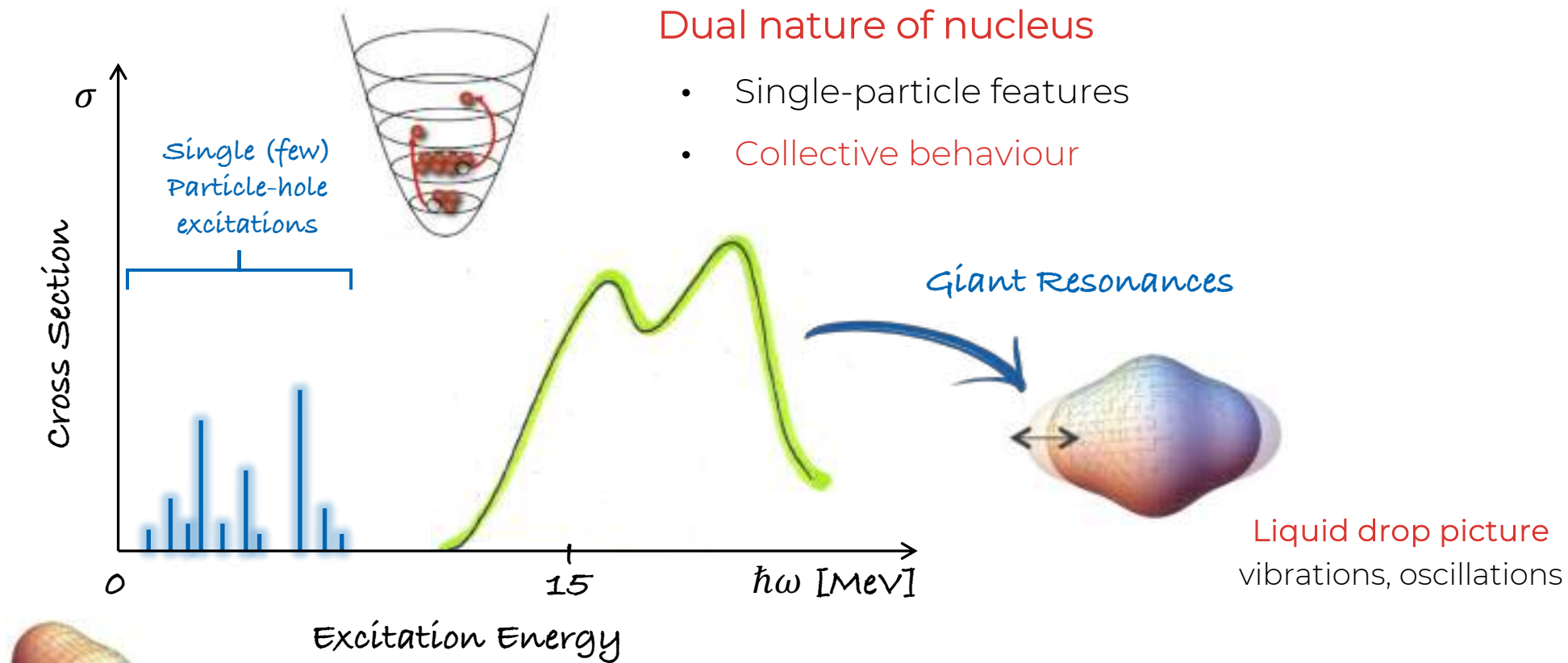
Giant Resonances



Giant Resonances



Giant Resonances



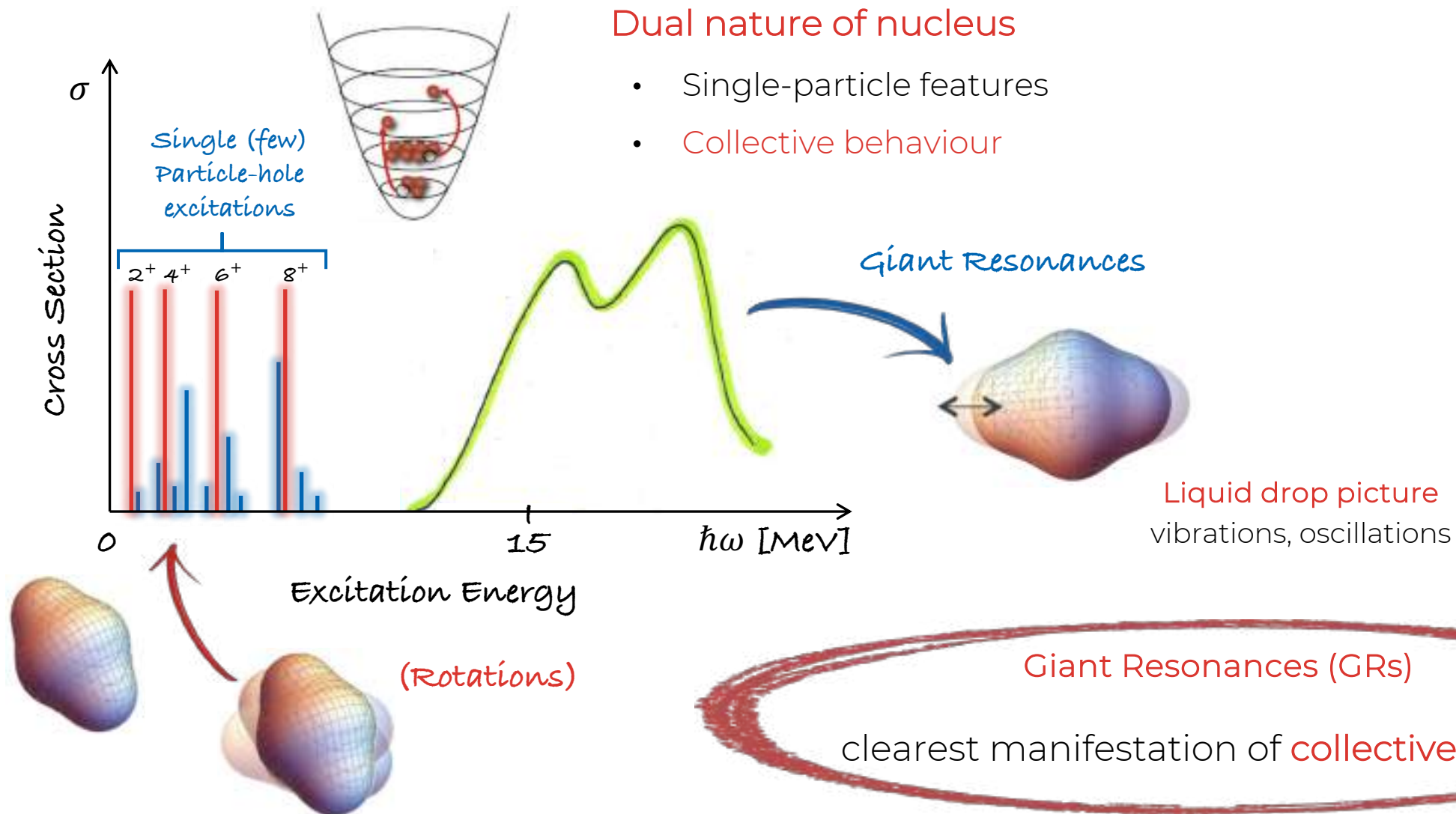
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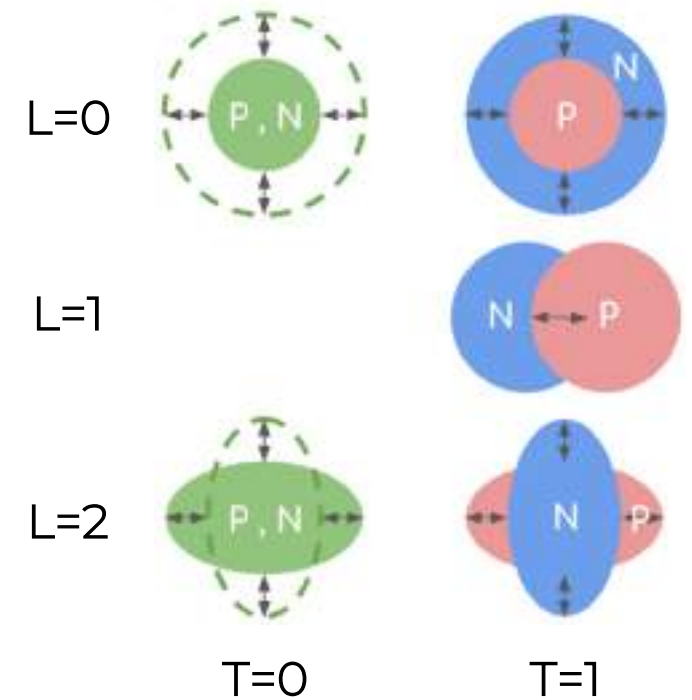
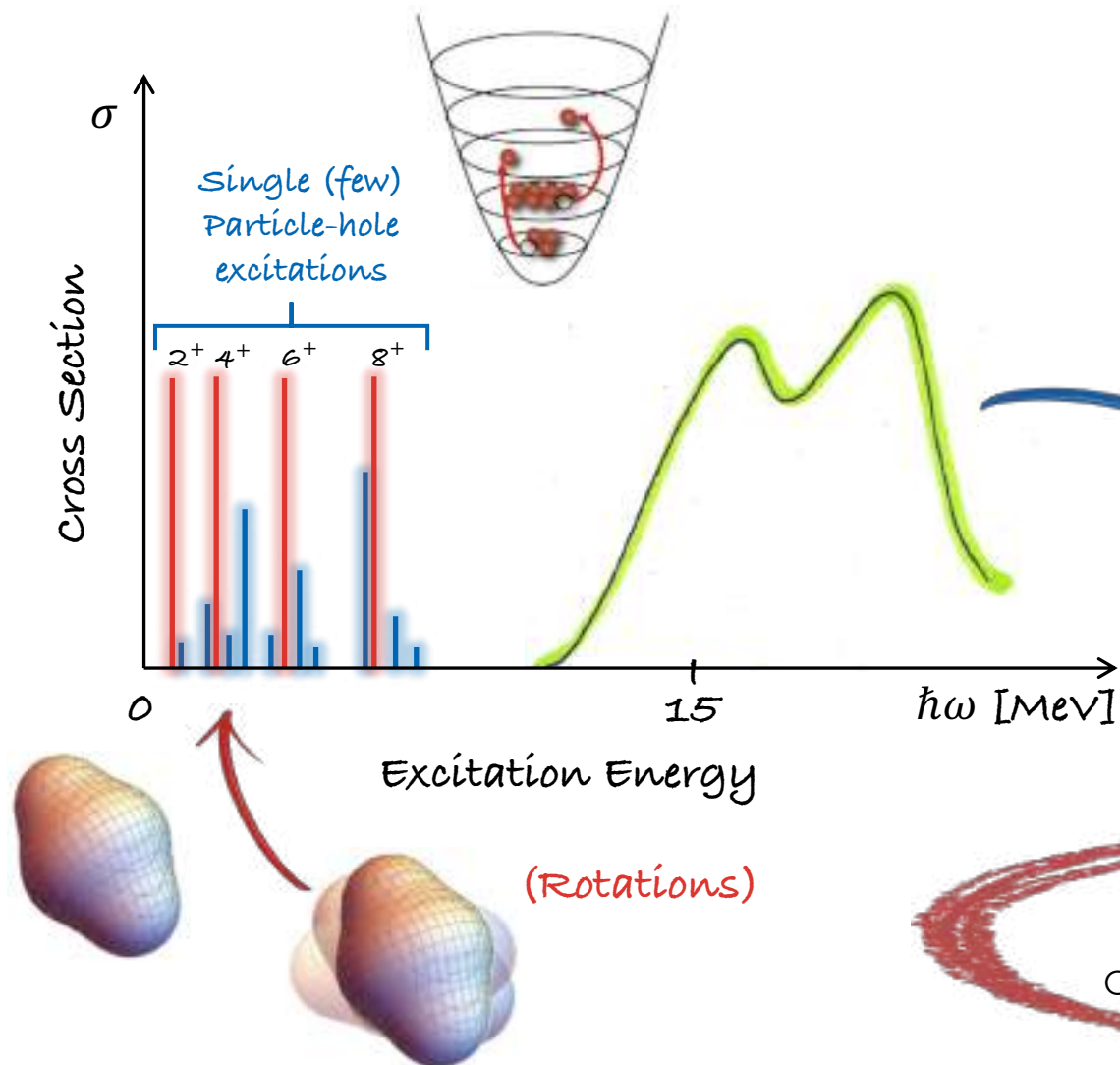
Giant Resonances (GRs)

clearest manifestation of collective motion

Giant Resonances



Giant Resonances

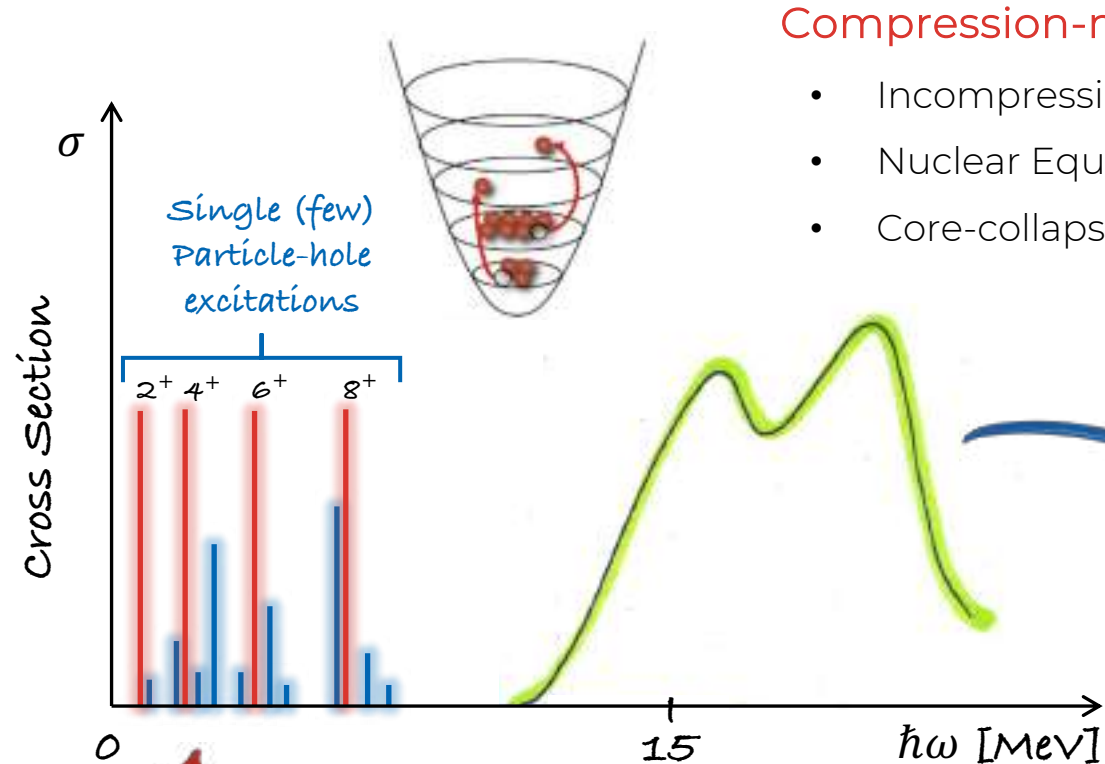


Liquid drop picture
vibrations, oscillations

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clearest manifestation of **collective motion**

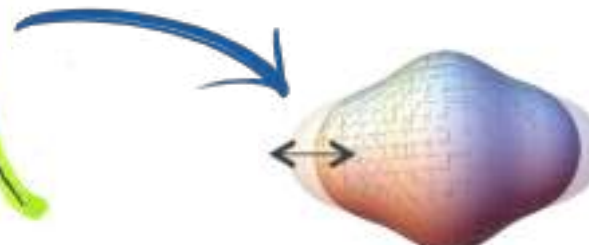
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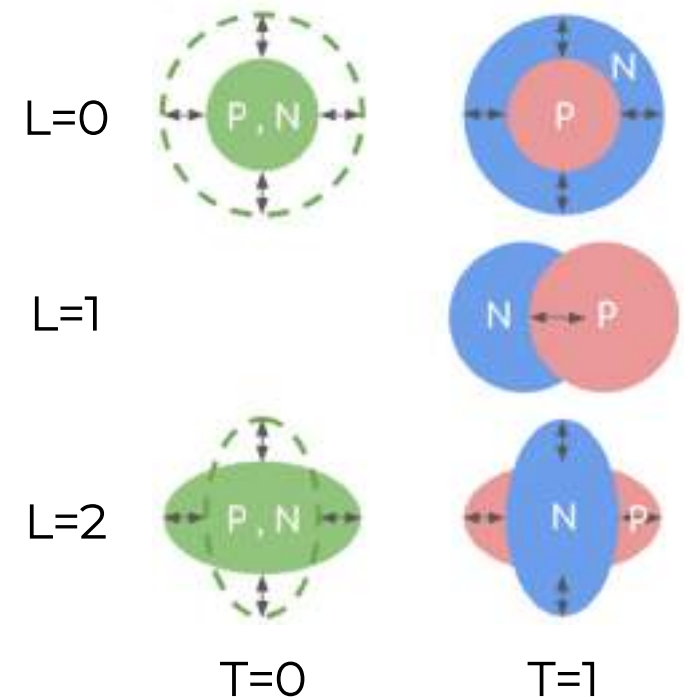
Compression-mode resonances

- Incompressibility of nuclear matter K_∞
- Nuclear Equation of State
- Core-collapse supernova explosion

Giant Resonances



Liquid drop picture
vibrations, oscillations



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clearest manifestation of **collective motion**

Projected Generator Coordinate Method

Schrödinger equation $H |\Psi_n\rangle = E_n |\Psi_n\rangle$

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Open-shell systems

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Strong **static correlations**

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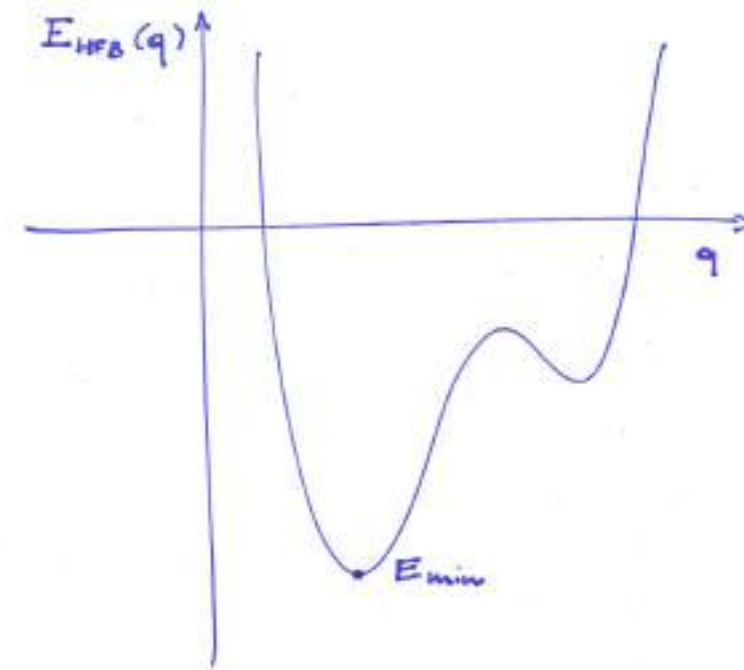


Strong static correlations



1 Constrained HFB solutions

$|\Phi(q)\rangle$



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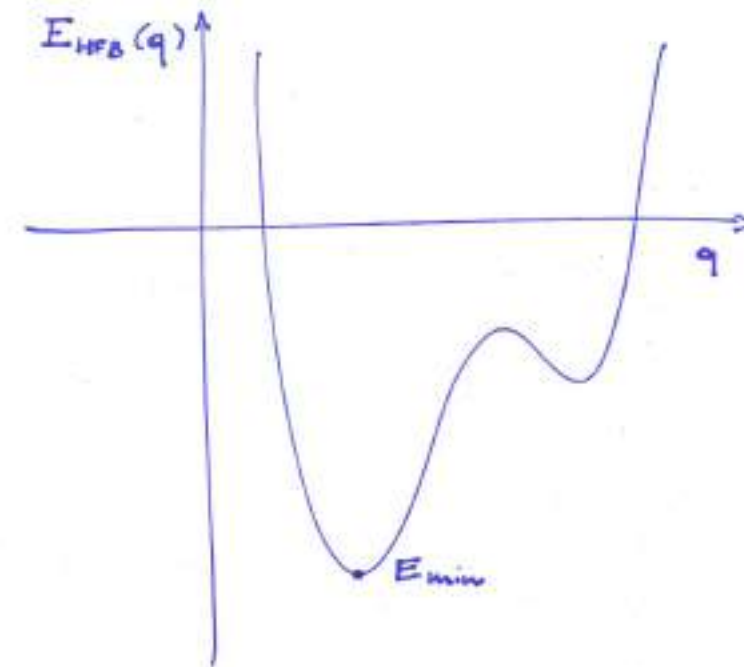


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Generator coordinates
(q can be any coordinate)



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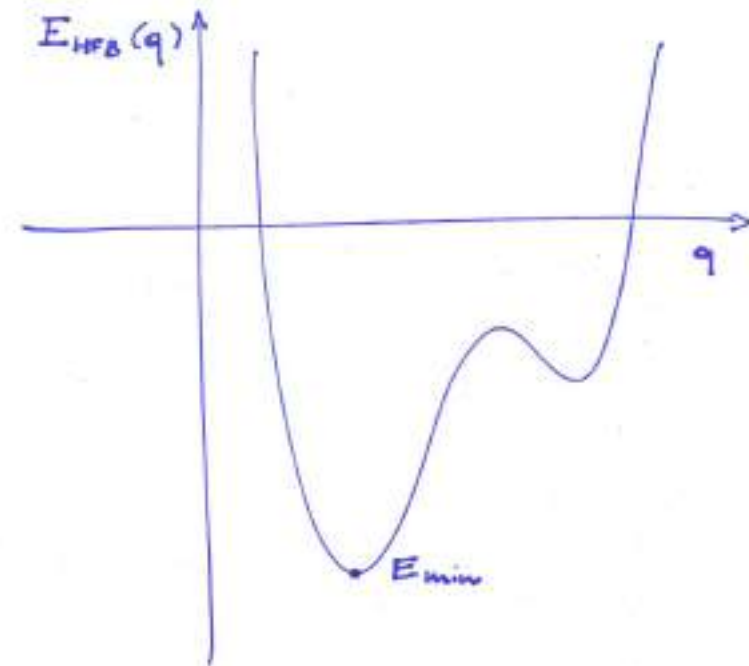
$$|\Phi(q)\rangle$$



Generator coordinates
(q can be any coordinate)

2 PGCM Ansatz

$$|\Psi_n\rangle = \int dq f_n(q) |\Phi(q)\rangle$$



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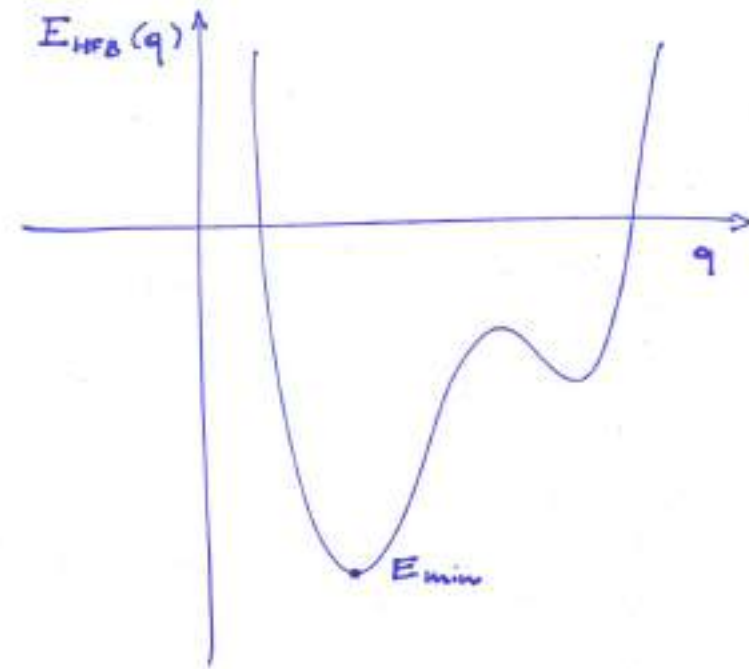
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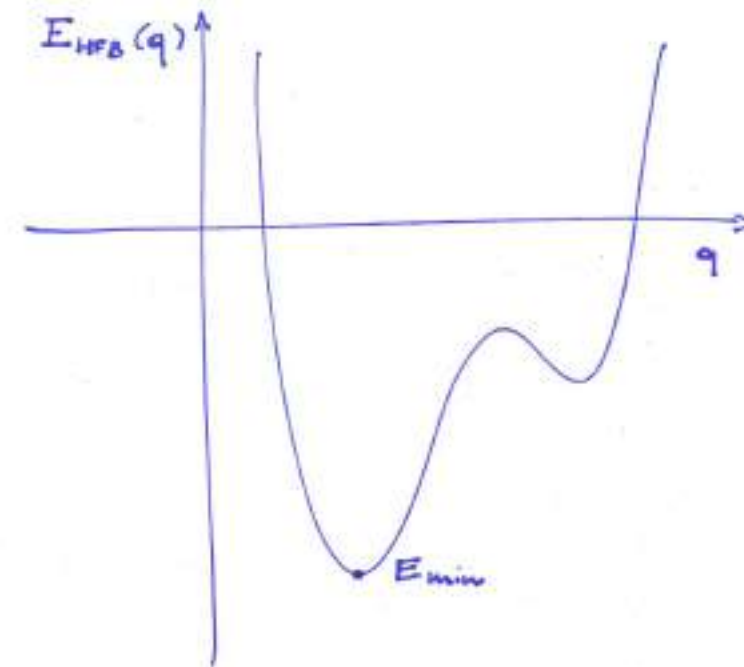


Linear coefficients

3 HWG Equation

Variational method

$$\delta \frac{\langle \Psi_n | H | \Psi_n \rangle}{\langle \Psi_n | \Psi_n \rangle} = 0$$



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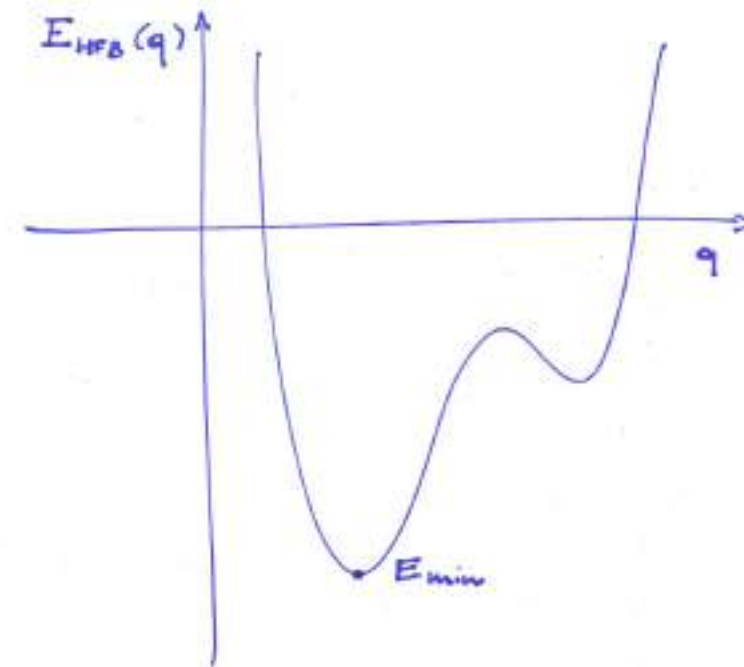
Schrödinger-like equation

$$\int [\mathcal{H}(p, q) - E_n \mathcal{N}(p, q)] f_n(q) dq = 0$$

Kernels evaluation

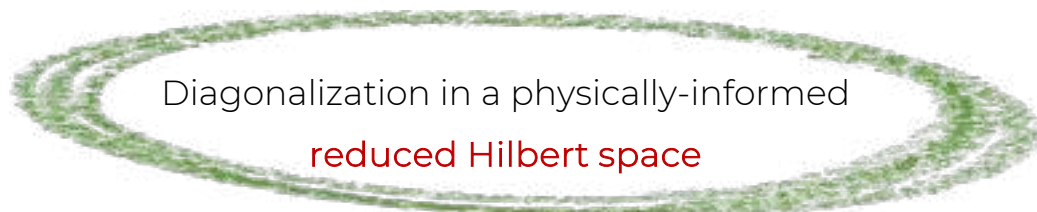
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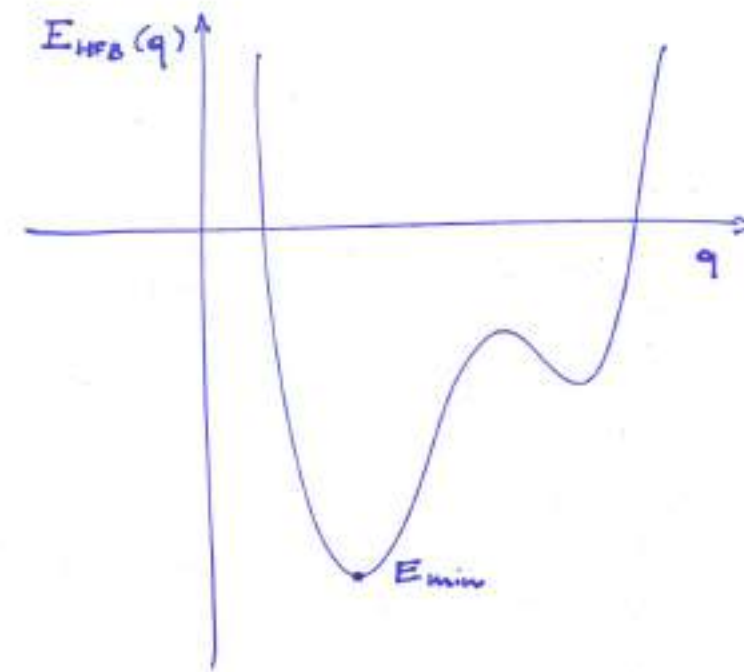
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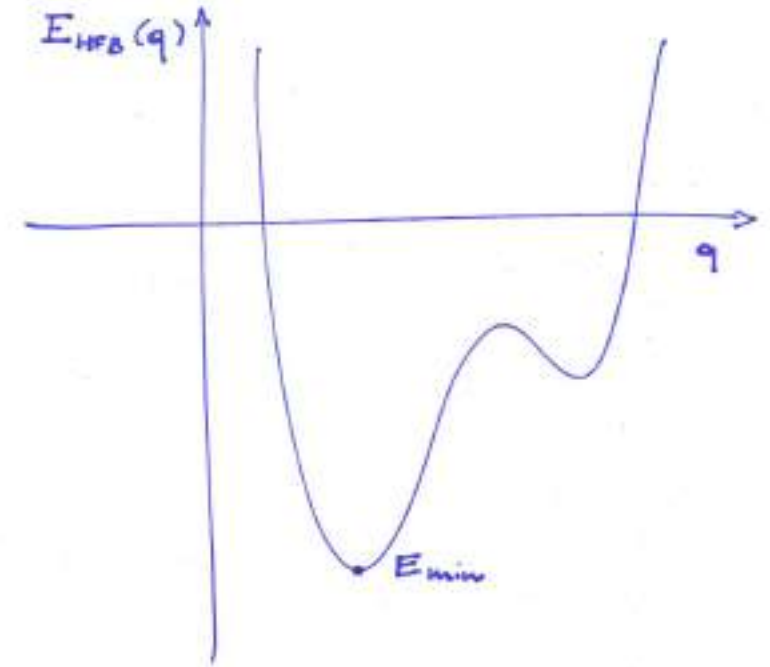
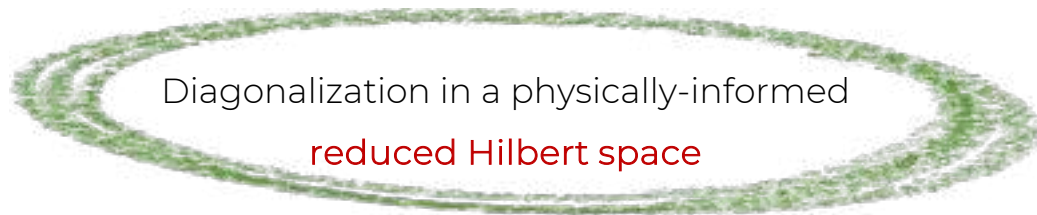
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+ Projection

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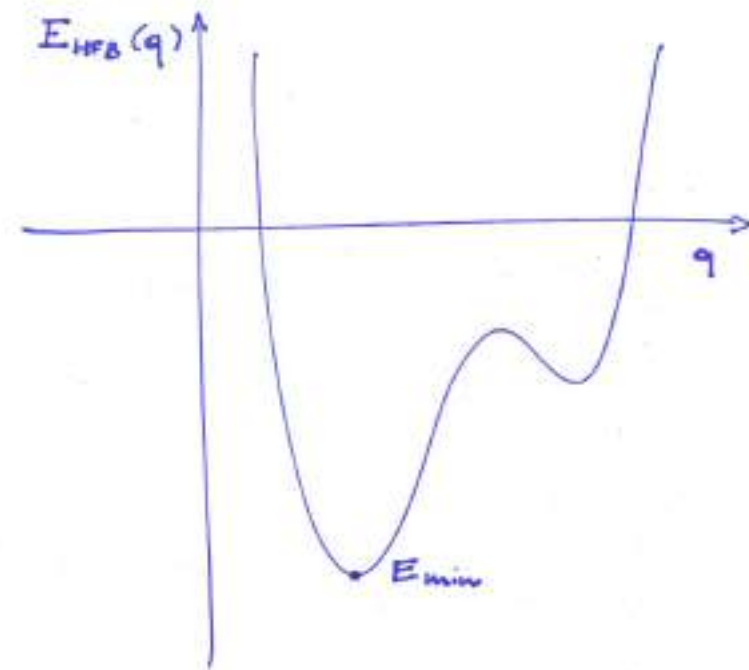
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$$\begin{aligned} \mathcal{H}(p, q) &\equiv \langle \Phi(p) | H | \Phi(q) \rangle \\ \mathcal{N}(p, q) &\equiv \langle \Phi(p) | \Phi(q) \rangle \end{aligned}$$

GCM and (Q)RPA

Thouless theorem

$$|\Phi(q)\rangle = \langle \Phi(q_{min}) | \Phi(q) \rangle e^{\mathbf{Z}(q, q_{min})} |\Phi(q_{min})\rangle$$

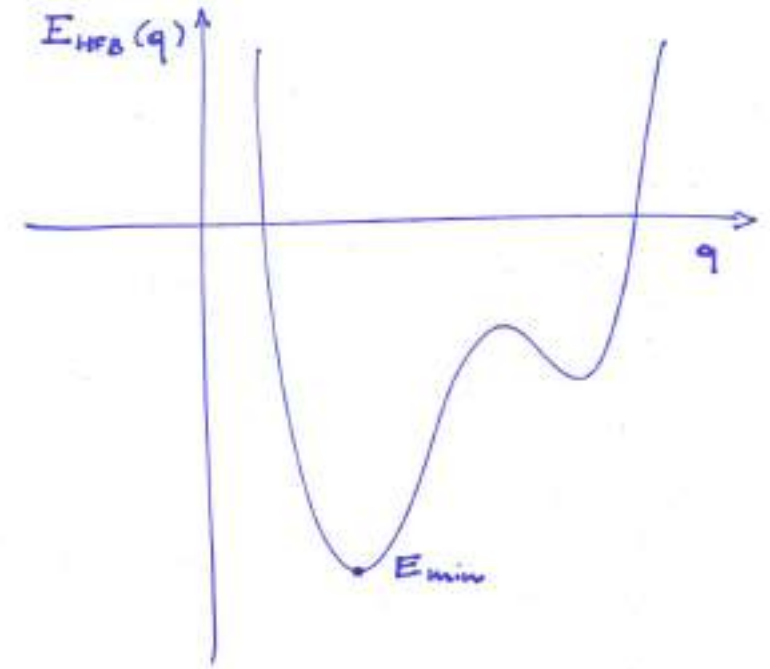


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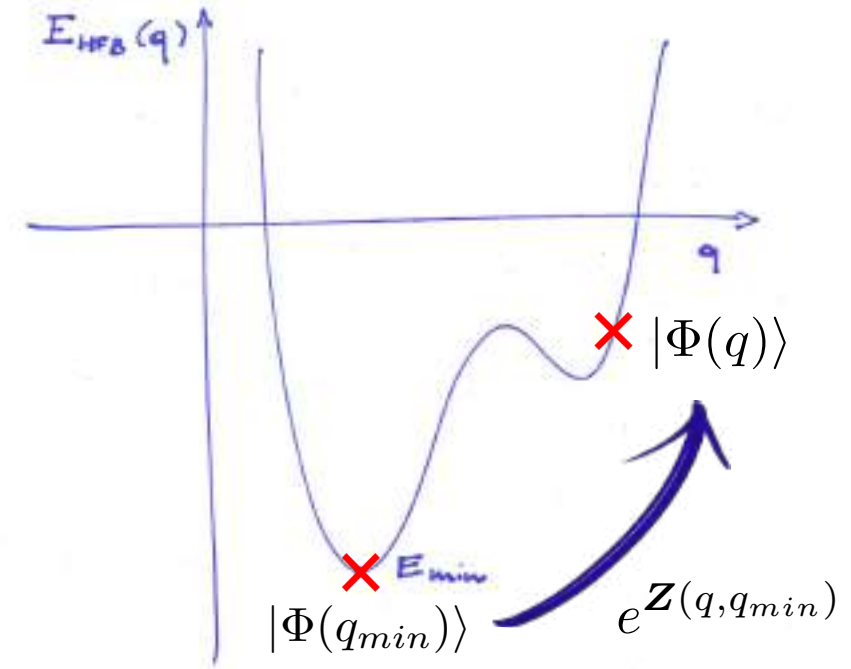


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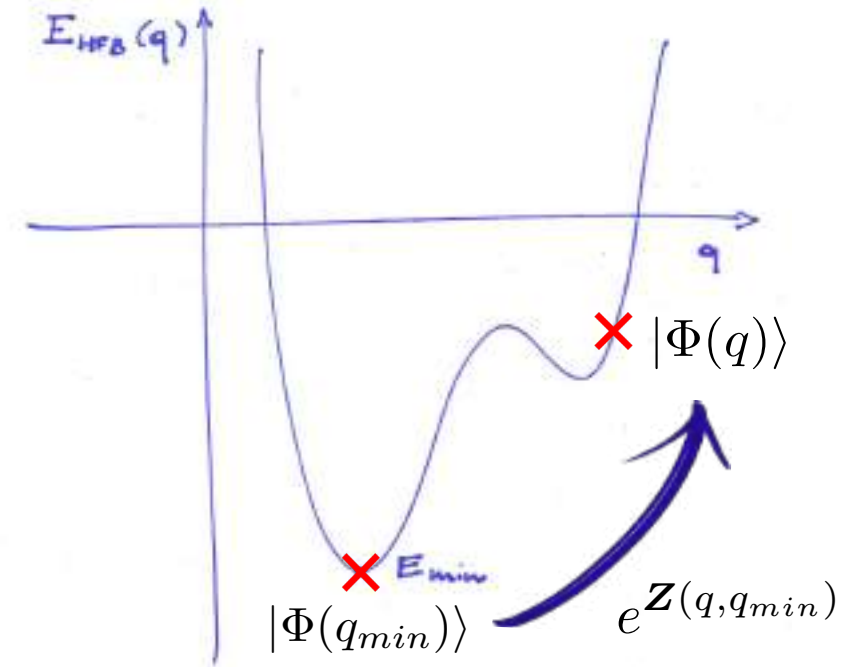
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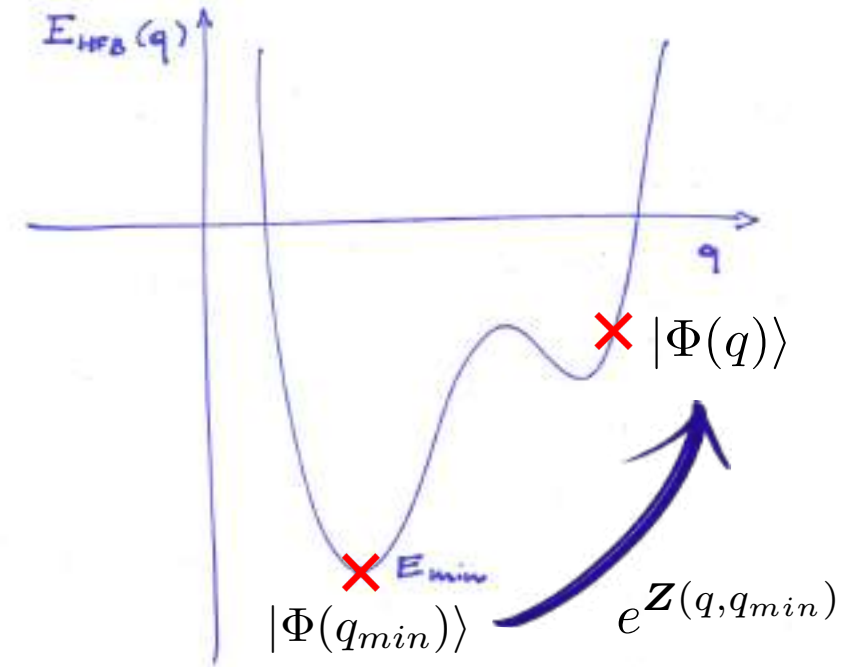
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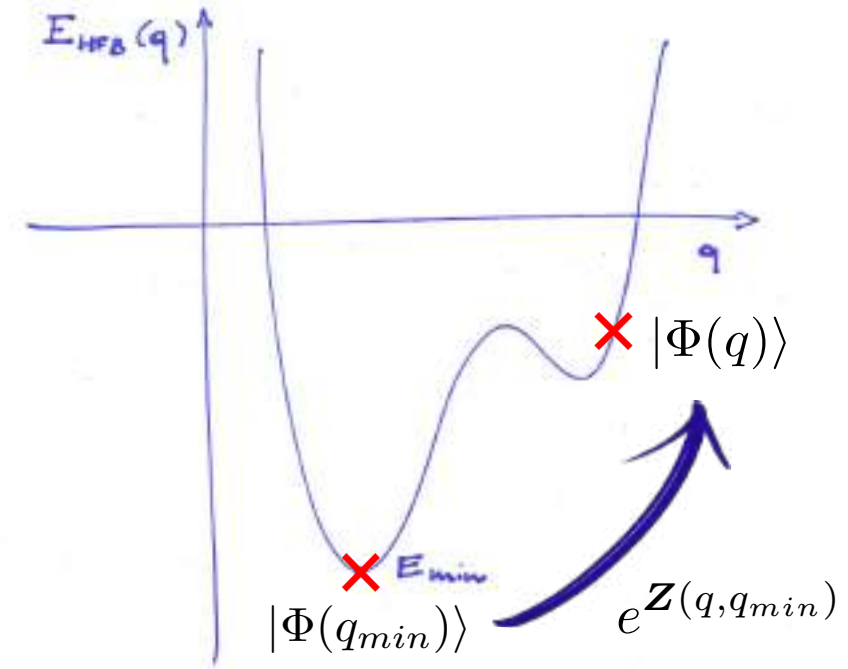
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Expand to the quadratic level in $\mathbf{Z}(q, q_{min})$



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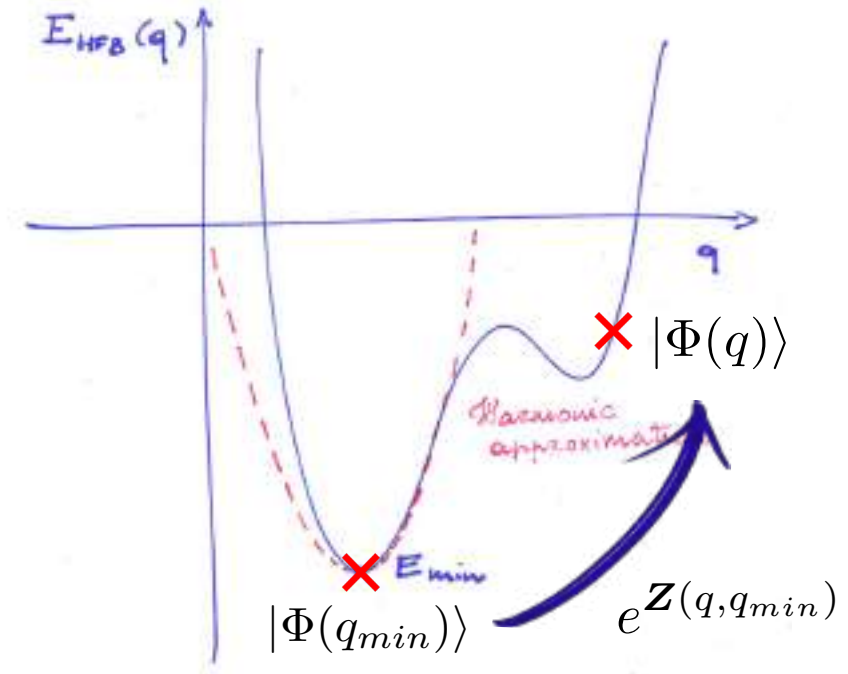
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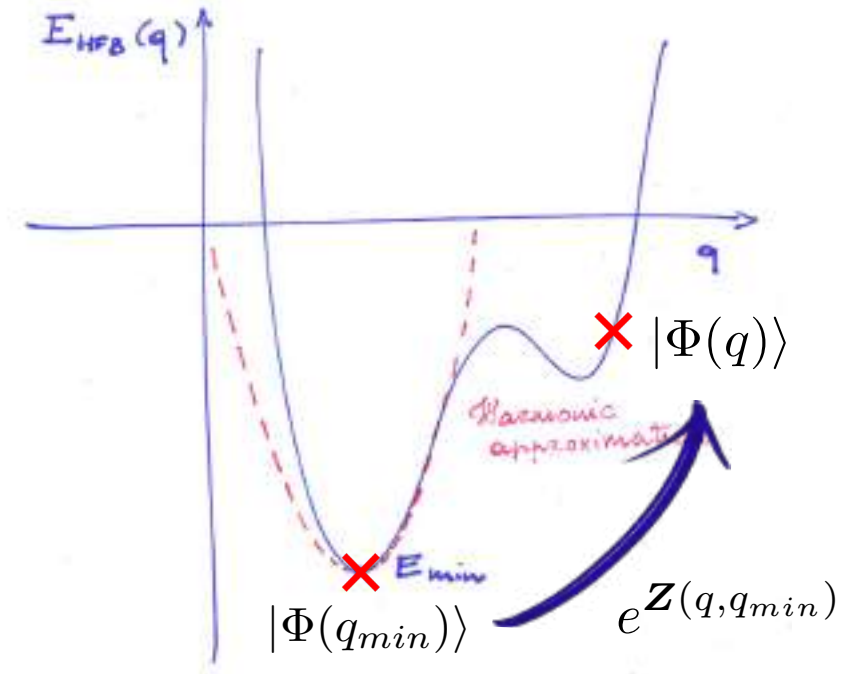
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No coordinates dependency !

All coordinates are explored
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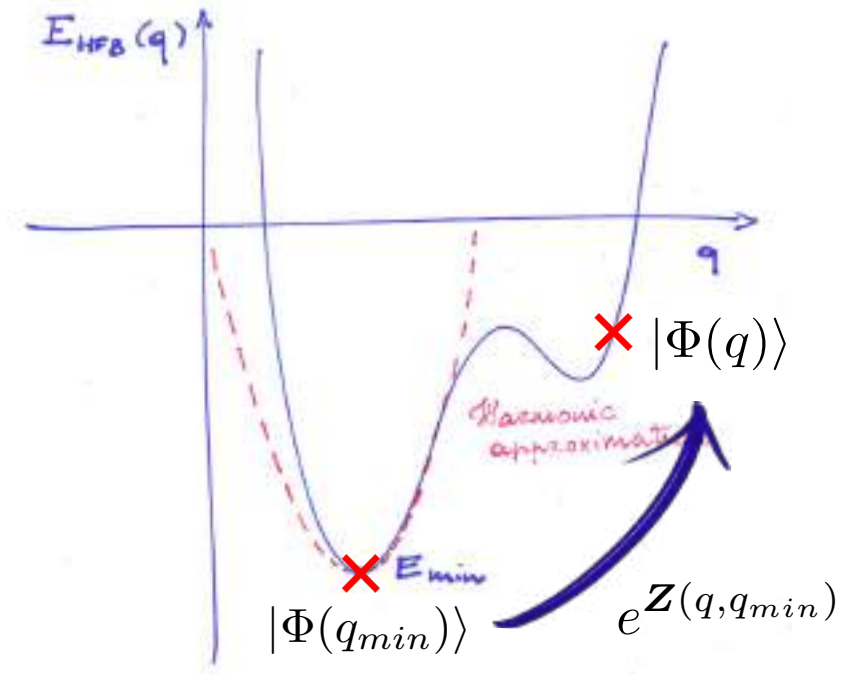
Expand to the quadratic level in $\mathbf{Z}(q, q_{min})$

Harmonic approximation

Eventually rewrites as (Q)RPA equations

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X^n \\ Y^n \end{pmatrix} = E_n \begin{pmatrix} X^n \\ Y^n \end{pmatrix}$$

[Jancovici, Schiff, 1964]



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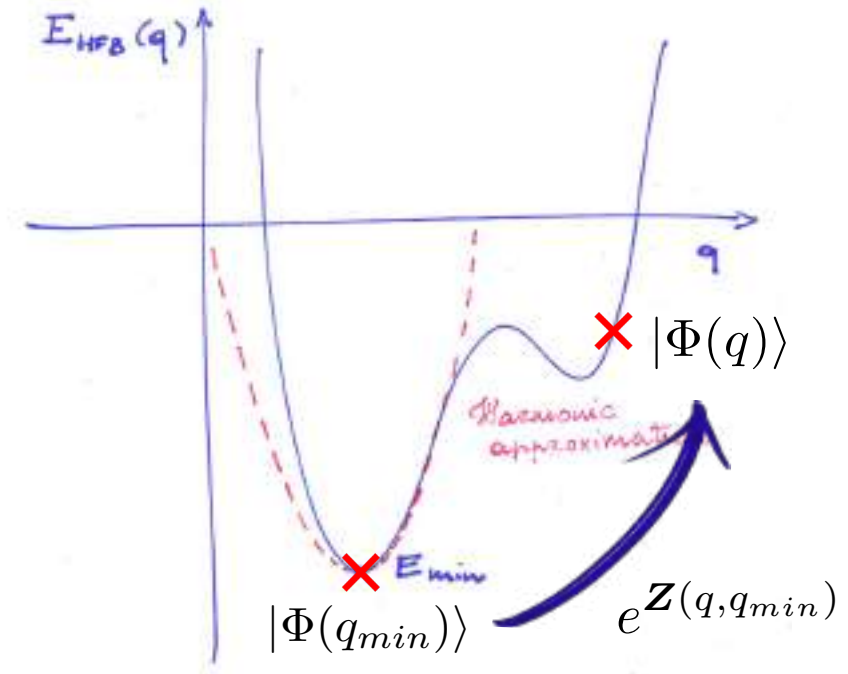
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Nuclei that are stiff against deformations
(anharmonic effects negligible)

[Jancovici, Schiff, 1964]



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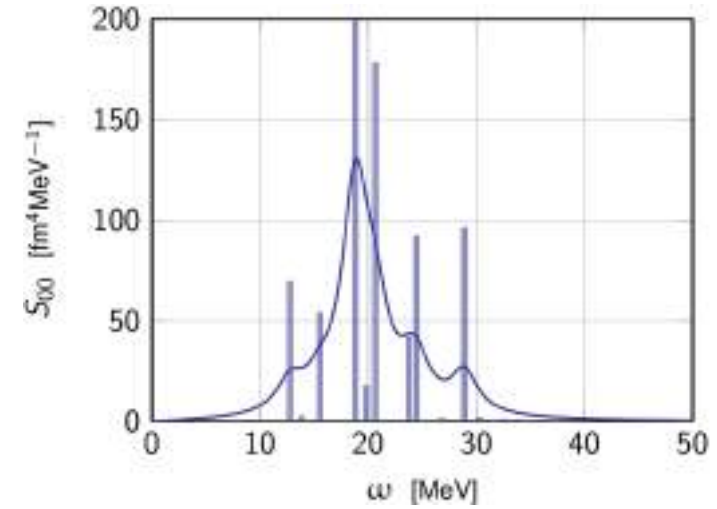
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Setting

Studied quantity: **monopole strength**

- Transition amplitudes: height of peaks
- Energy difference: position of peaks

$$S_{00}(\omega) \equiv \sum_v |\langle \Psi_v | r^2 | \Psi_0 \rangle|^2 \delta(E_v - E_0 - \omega)$$



Setting

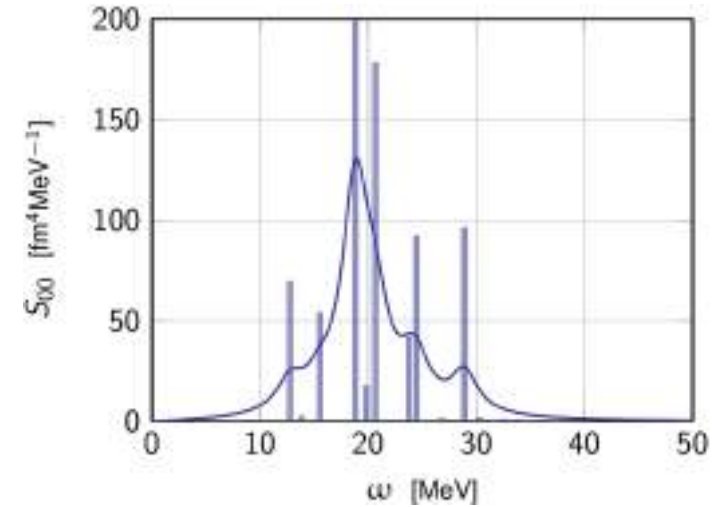
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JM=00



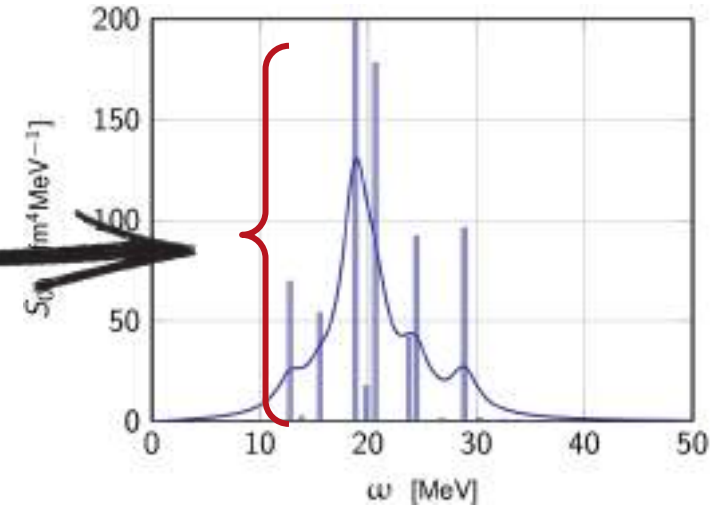


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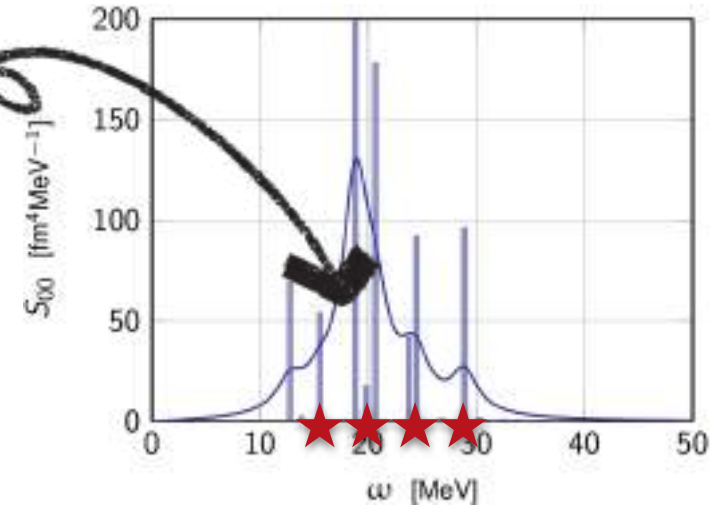


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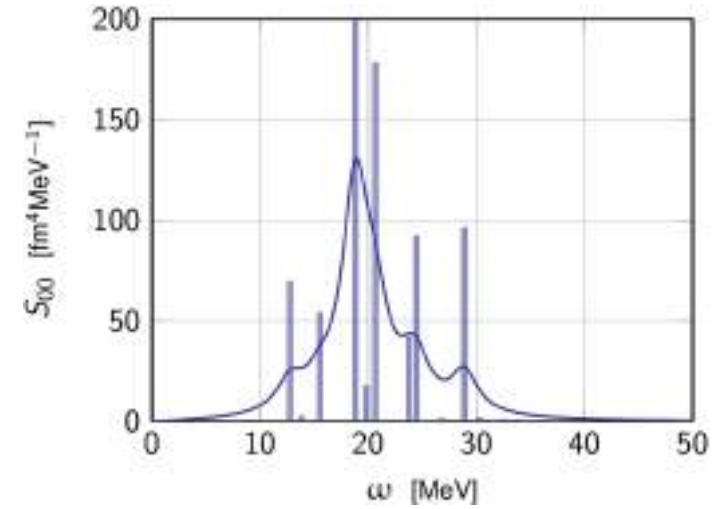


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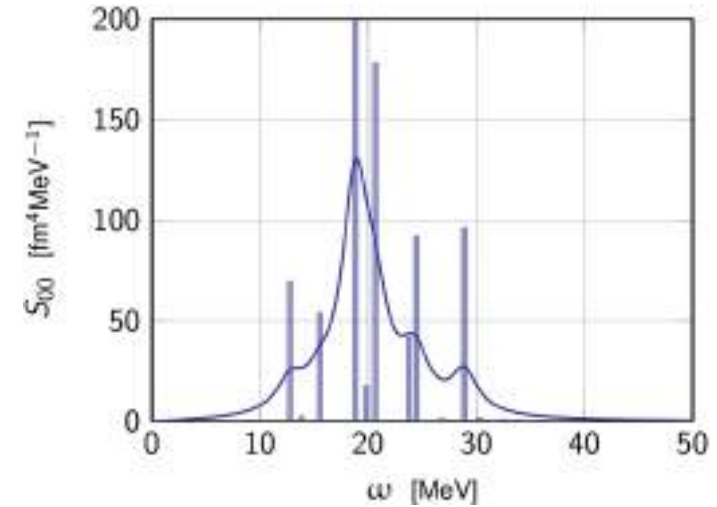
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Related moments

$$\begin{aligned} m_k &\equiv \int_0^\infty S_{00}(\omega) \omega^k d\omega \\ &= \sum_v (E_v - E_0)^k |\langle \Psi_v | r^2 | \Psi_0 \rangle|^2 \end{aligned}$$



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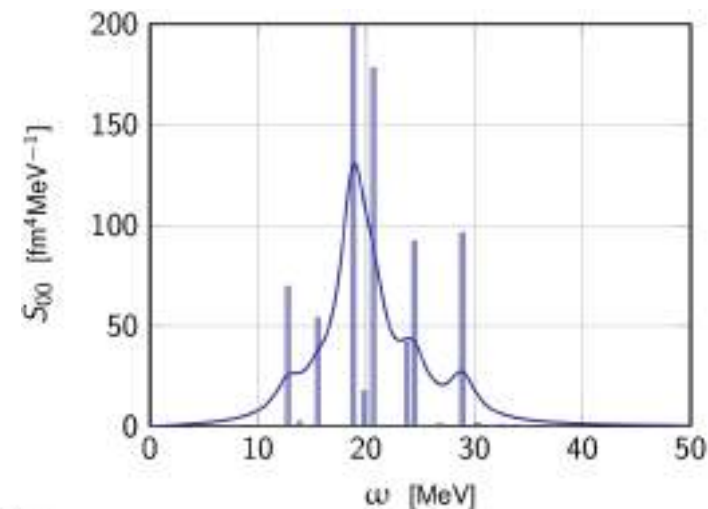
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Quantify the **most relevant features** of the strength

$$\bar{E}_1 = \frac{m_1}{m_0} \quad \sigma^2 = \frac{m_2}{m_0} - \left(\frac{m_1}{m_0} \right)^2 \geq 0$$



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$$S_{00}(\omega) \equiv \sum_v |\langle \Psi_v | r^2 | \Psi_0 \rangle|^2 \delta(E_v - E_0 - \omega)$$

Related moments

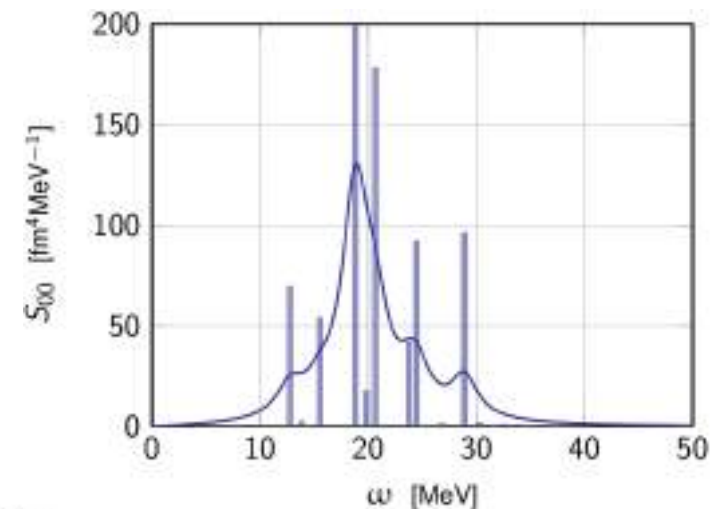
$$\begin{aligned} m_k &\equiv \int_0^\infty S_{00}(\omega) \omega^k d\omega \\ &= \sum_v (E_v - E_0)^k |\langle \Psi_v | r^2 | \Psi_0 \rangle|^2 \end{aligned}$$

Quantify the **most relevant features** of the strength

$$\bar{E}_1 = \frac{m_1}{m_0} \quad \sigma^2 = \frac{m_2}{m_0} - \left(\frac{m_1}{m_0} \right)^2 \geq 0$$

Ab-initio PGCM and QRPA **consistent numerical settings** (systematic study in ^{46}Ti)

- Quantities expanded on harmonic oscillator basis (characterised by $\hbar\omega$, e_{\max} , $e_{3\max}$)



Setting

Studied quantity: **monopole strength**

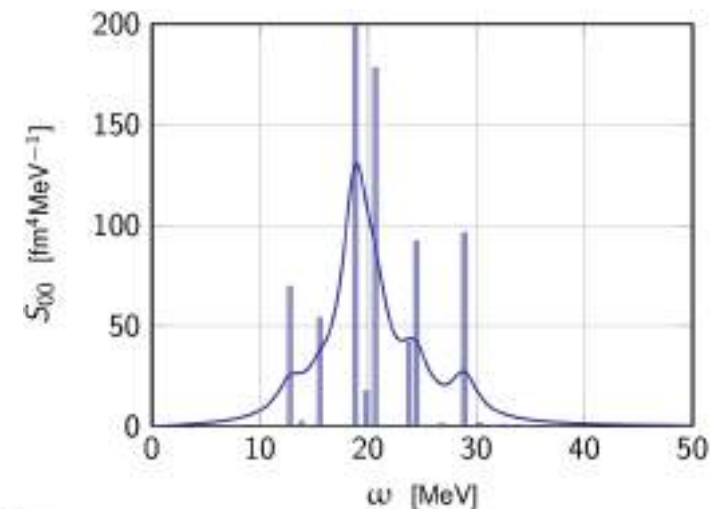
- Transition amplitudes: height of peaks
- Energy difference: position of peaks

$$S_{00}(\omega) \equiv \sum_v |\langle \Psi_v | r^2 | \Psi_0 \rangle|^2 \delta(E_v - E_0 - \omega)$$

Related moments

$$m_k \equiv \int_0^\infty S_{00}(\omega) \omega^k d\omega$$
$$= \sum_v (E_v - E_0)^k |\langle \Psi_v | r^2 | \Psi_0 \rangle|^2$$

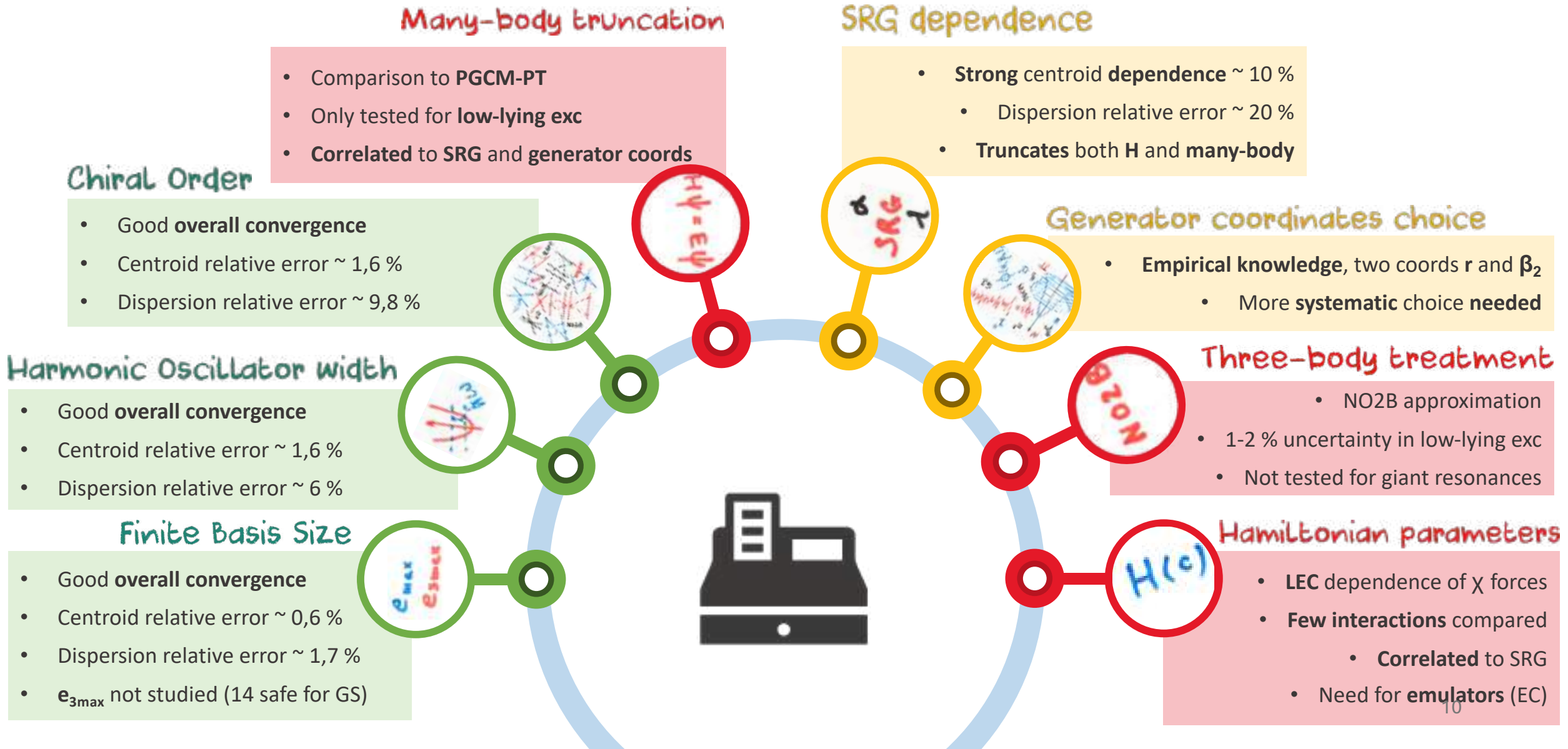
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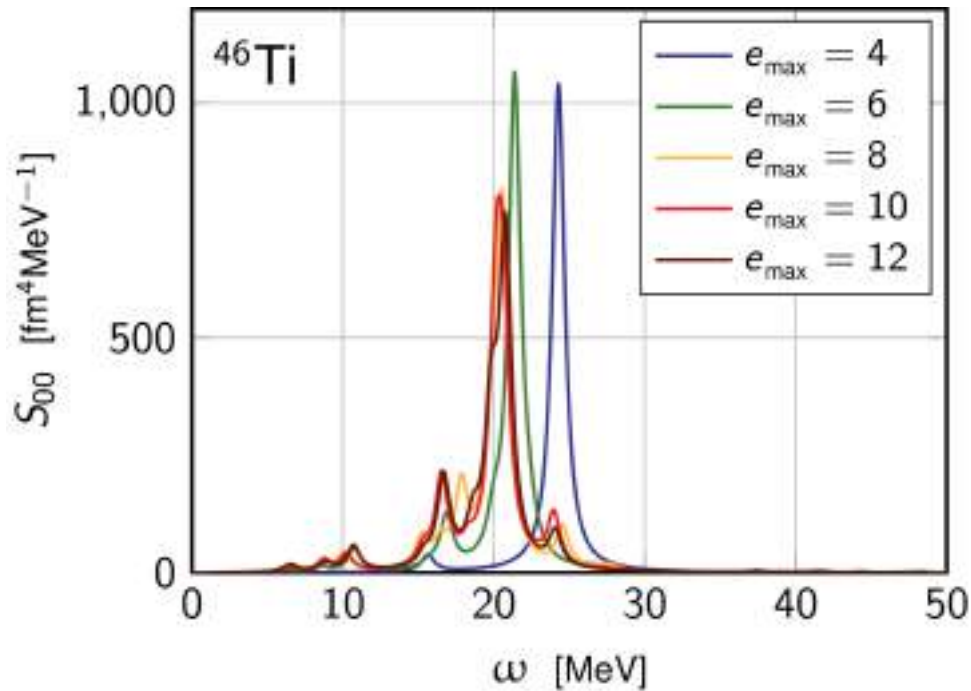
Ab-initio PGCM and QRPA **consistent numerical settings** (systematic study in ^{46}Ti)

- Quantities expanded on harmonic oscillator basis (characterised by $\hbar\omega$, e_{max} , $e_{3\text{max}}$)
- Family of chiral NN + in-medium 3N interactions (NLO, N2LO and N3LO)
 - T. H  ther, K. Vobig, K. Hebeler, R. Machleidt and R. Roth, "Family of chiral two-plus three-nucleon interactions for accurate nuclear structure studies", *Phys. Lett. B*, 808, 2020
 - In-vacuum SRG evolution ($\alpha=0.04 \text{ fm}^4$, $\alpha=0.08 \text{ fm}^4$)
 - M. Frosini, T. Duguet, B. Bally, Y. Beaujeault-Taudi  re, J.-P. Ebran and V. Som  , "In-medium k-body reduction of n-body operators", *The European Physical Journal A*, 57(4), 2021

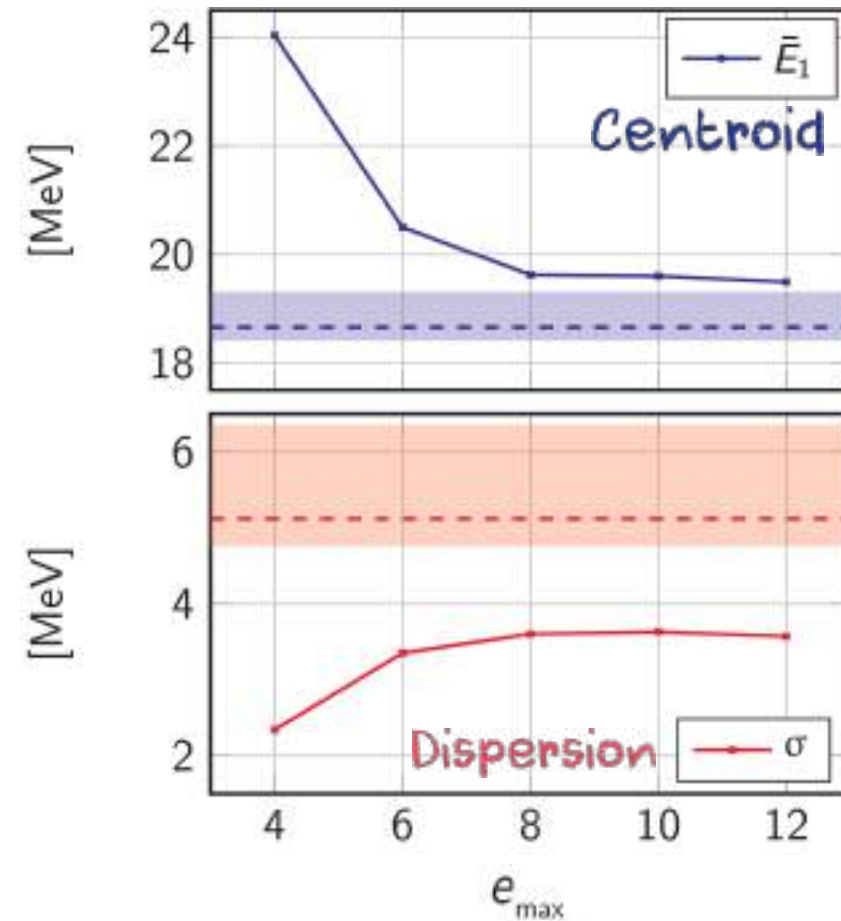
Uncertainty budget



Finite Basis Size



- Good overall convergence
- Centroid relative error $\sim 0,6 \%$
- Dispersion relative error $\sim 1,7 \%$
- $e_{3\text{max}}$ not studied (14 safe for GS)



[Myiagi et al., PRC, 2022]

Many-body truncation

Schrödinger equation $H|\Psi_k^A\rangle = E_k^A|\Psi_k^A\rangle$

A-body Hilbert space

\mathcal{H}_A

Exact solution



Many-body truncation

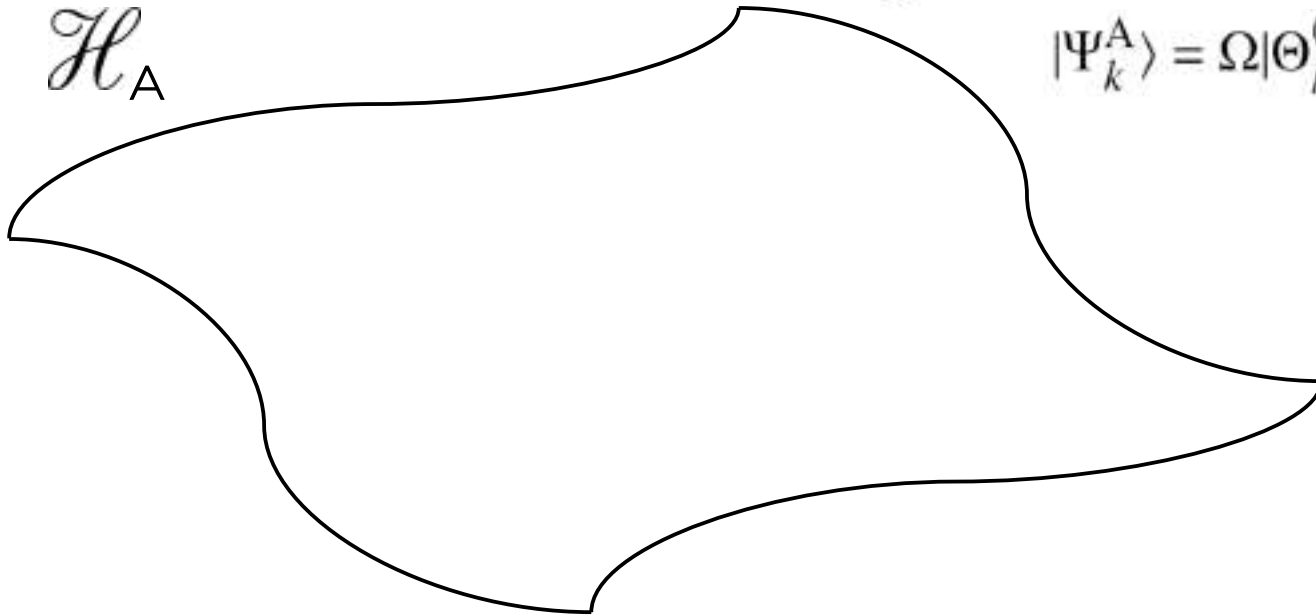
Schrödinger equation $H|\Psi_k^A\rangle = E_k^A|\Psi_k^A\rangle$

A-body Hilbert space

\mathcal{H}_A

Exact solution

$$|\Psi_k^A\rangle = \Omega|\Theta_k^{(0)}\rangle$$

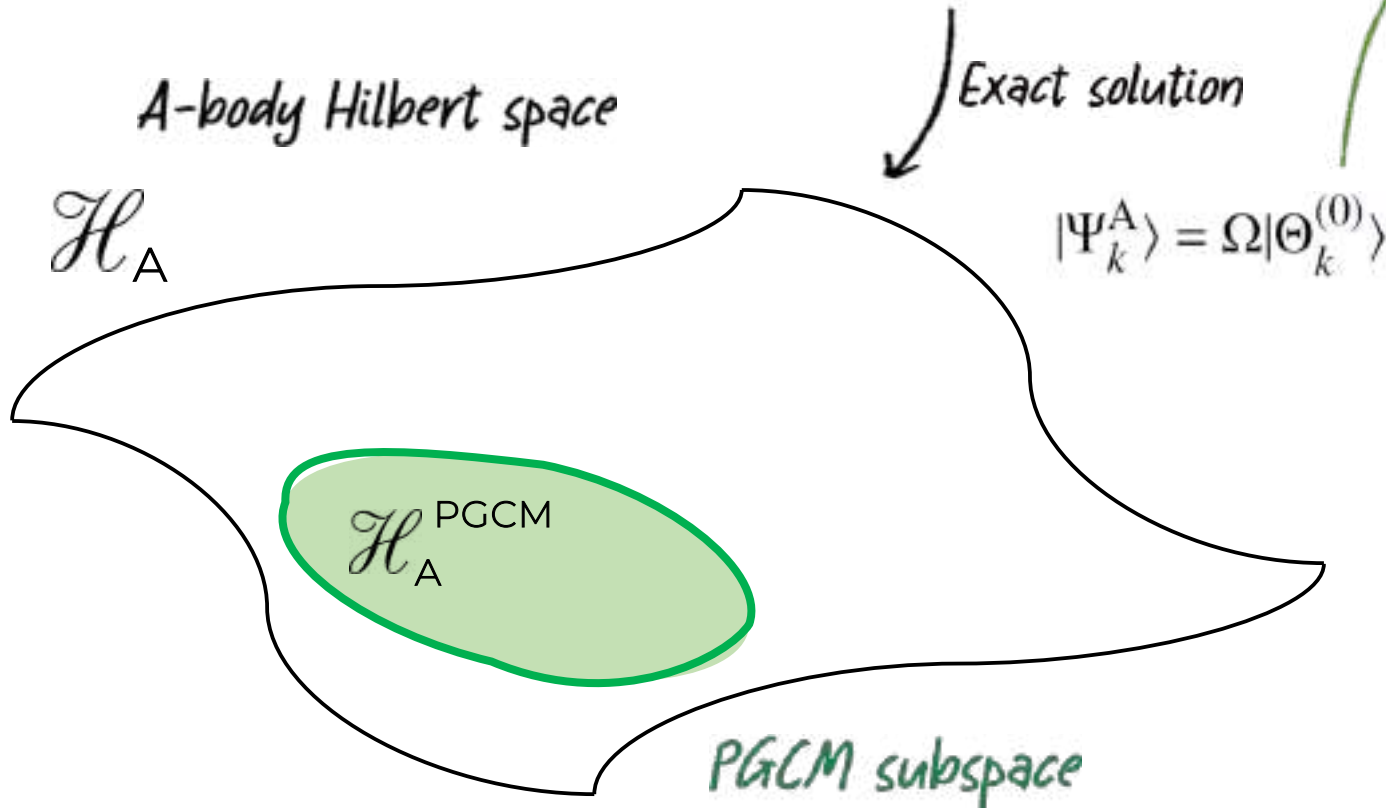


Many-body truncation

Schrödinger equation

$$H|\Psi_k^A\rangle = E_k^A|\Psi_k^A\rangle$$

PGCM : multi-reference unperturbed state

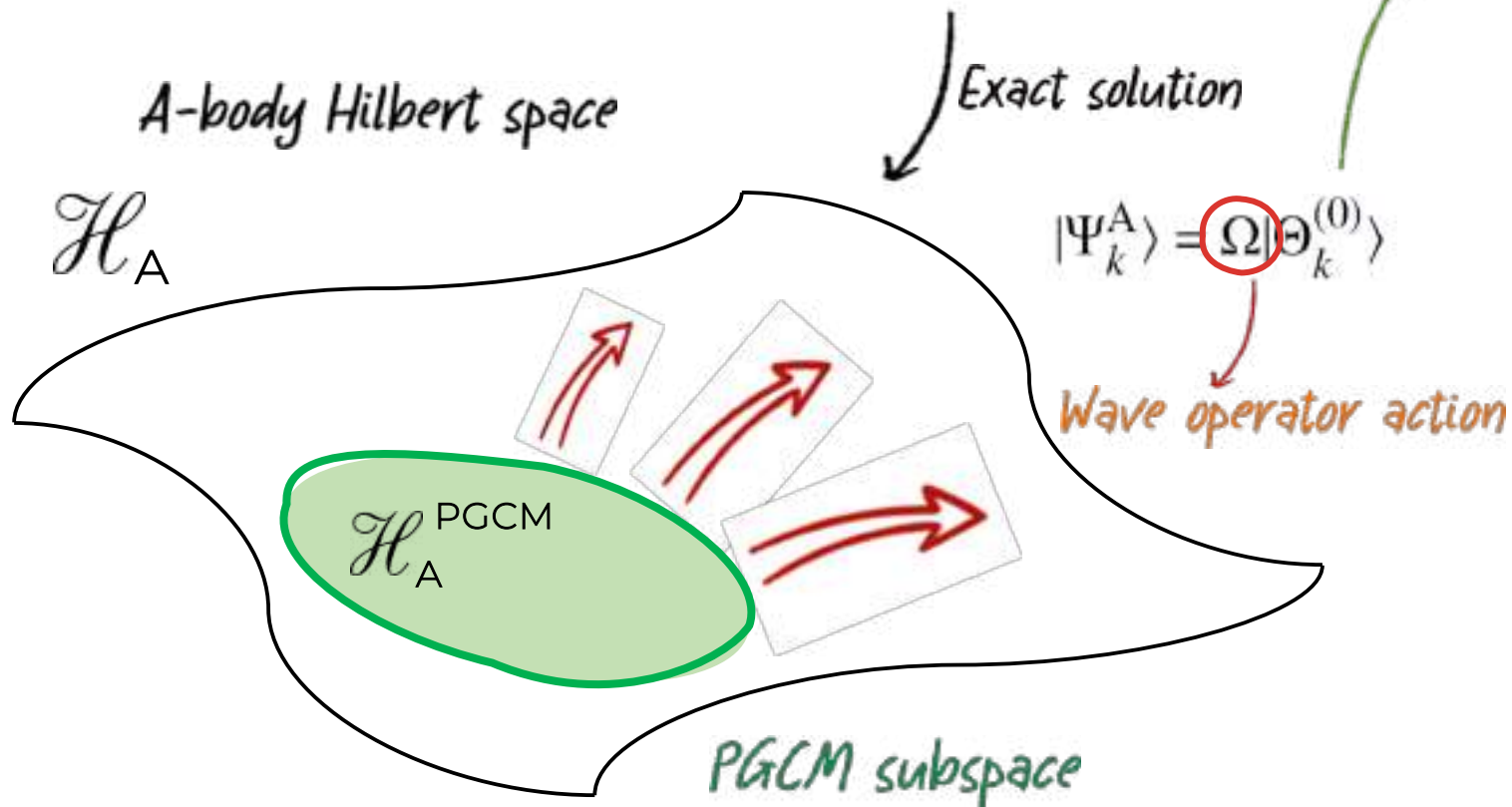


Many-body truncation

Schrödinger equation

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PGCM : multi-reference unperturbed state



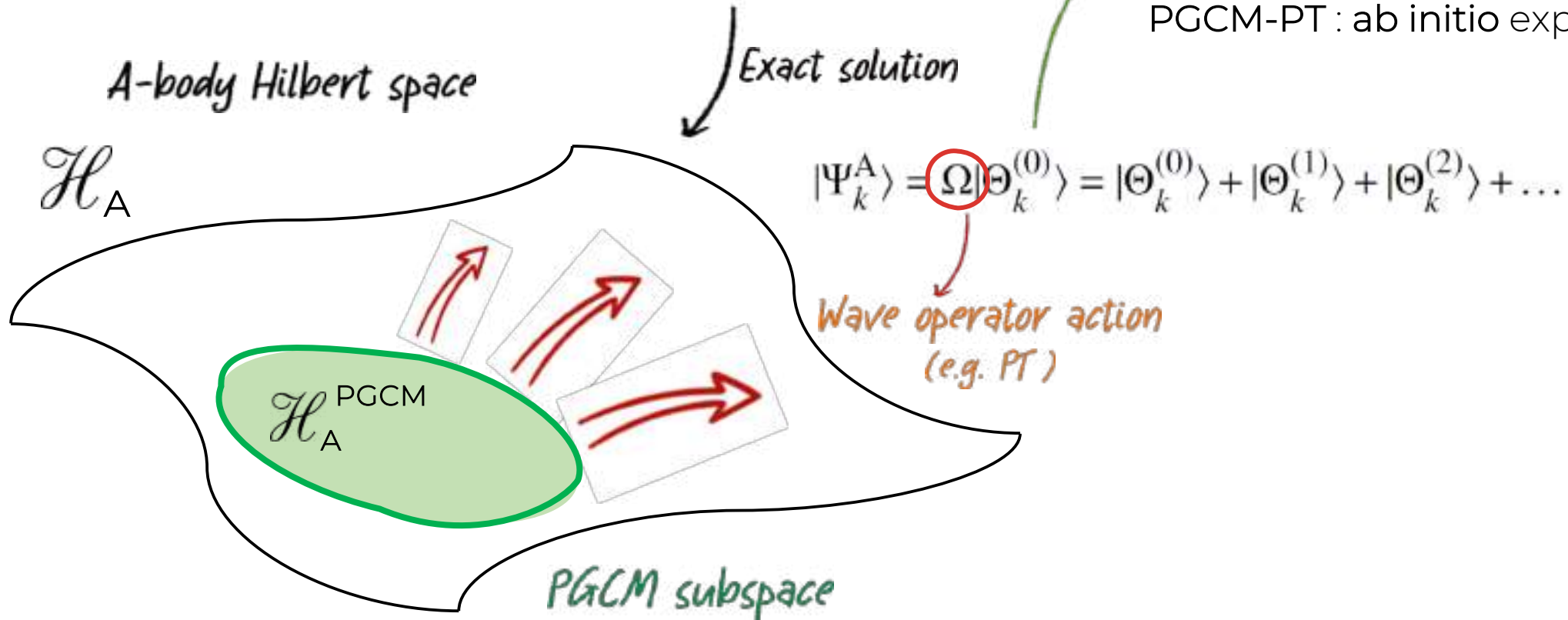
Many-body truncation

Schrödinger equation

$$H|\Psi_k^A\rangle = E_k^A|\Psi_k^A\rangle$$

PGCM : multi-reference unperturbed state

PGCM-PT : ab initio expansion method⁽¹⁾



⁽¹⁾ [Frosini, Duguet, Ebran and Somà, EPJA 58(62), 2022]

Many-body truncation

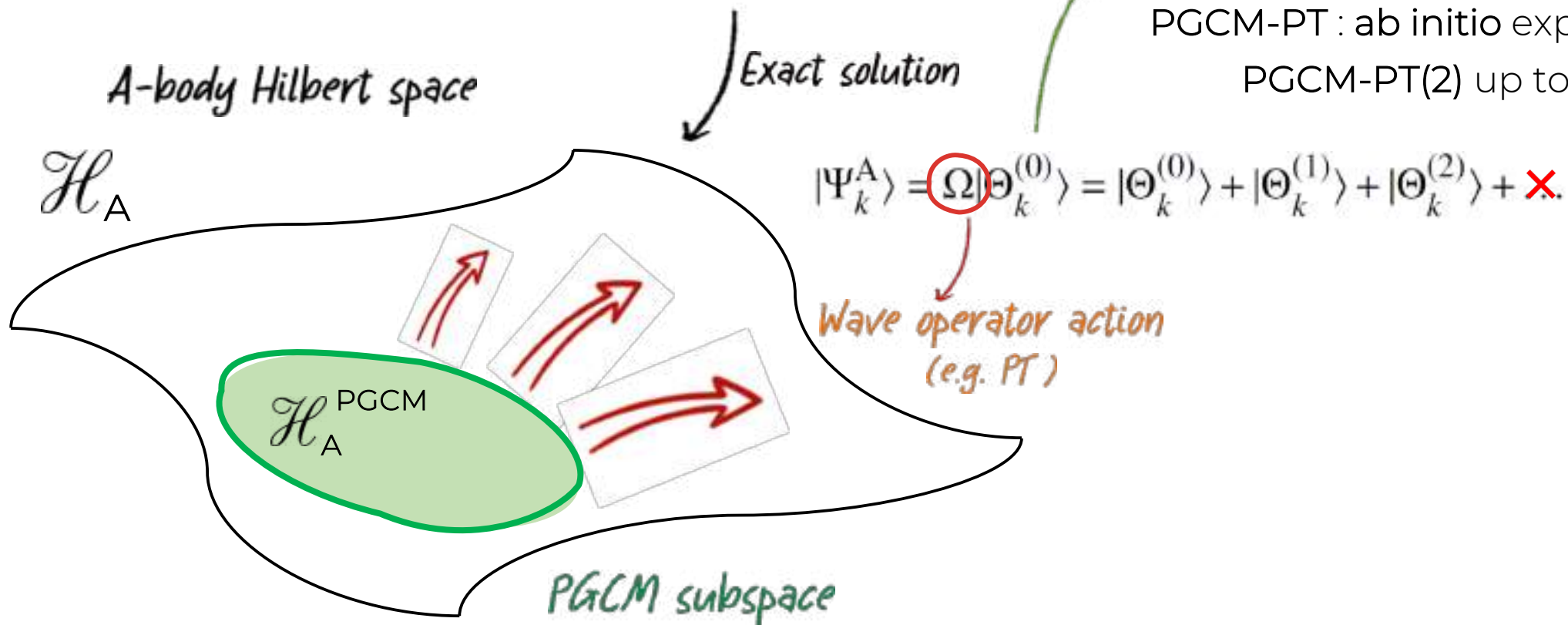
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PGCM : multi-reference unperturbed state

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PGCM-PT(2) up to 2nd order so far⁽²⁾



(1) [Frosini, Duguet, Ebran and Somà, EPJA 58(62), 2022]

(2) [Frosini, Duguet, Ebran, Bally, Hergert, Rodriguez, Roth, Yao and Somà, EPJA 58(64), 2022]

Many-body truncation

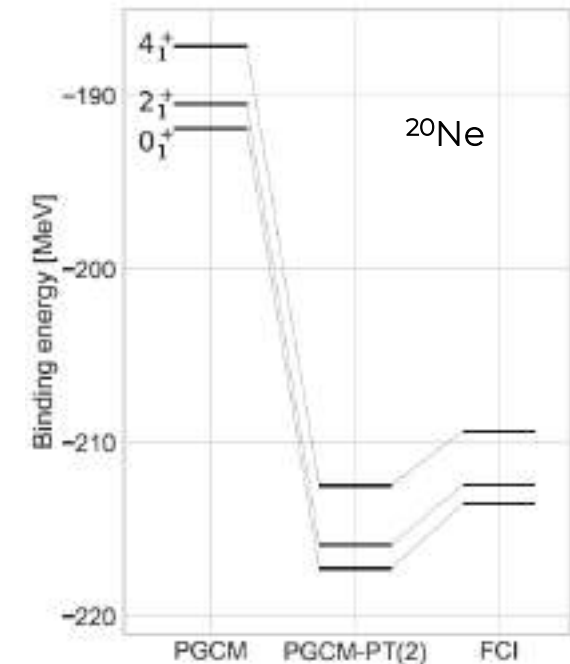
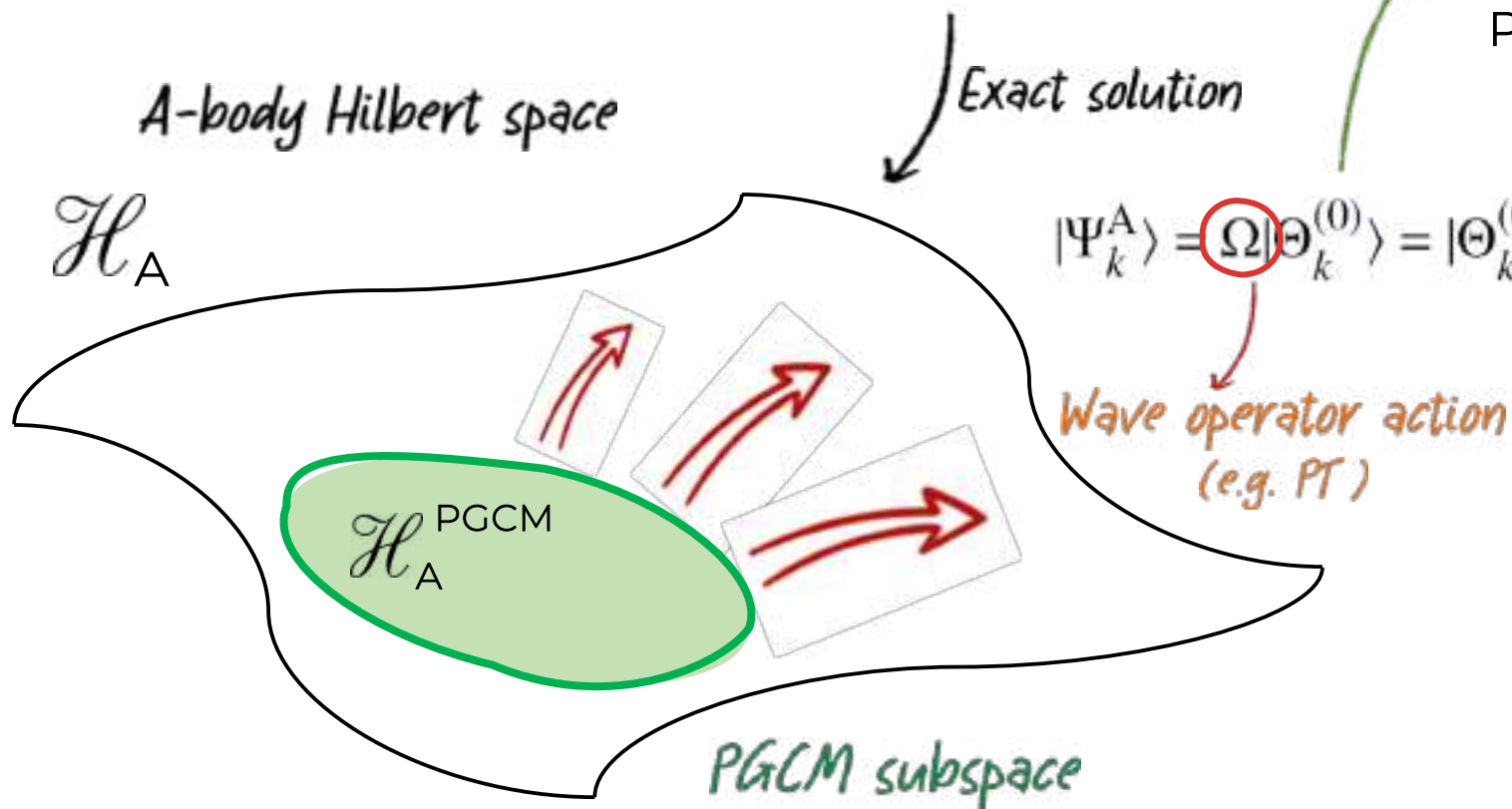
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Many-body truncation

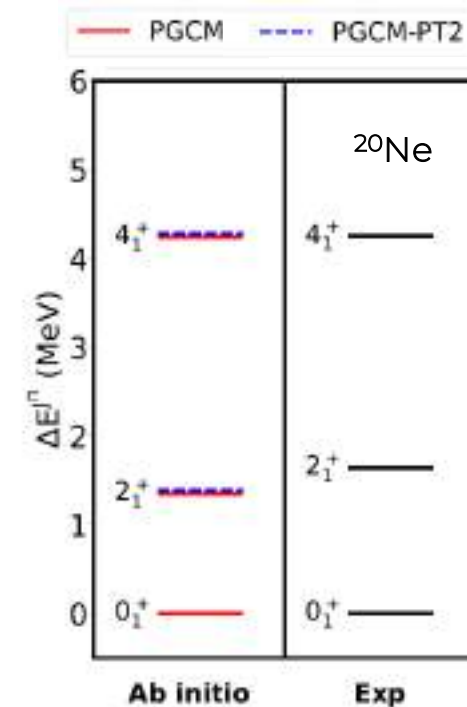
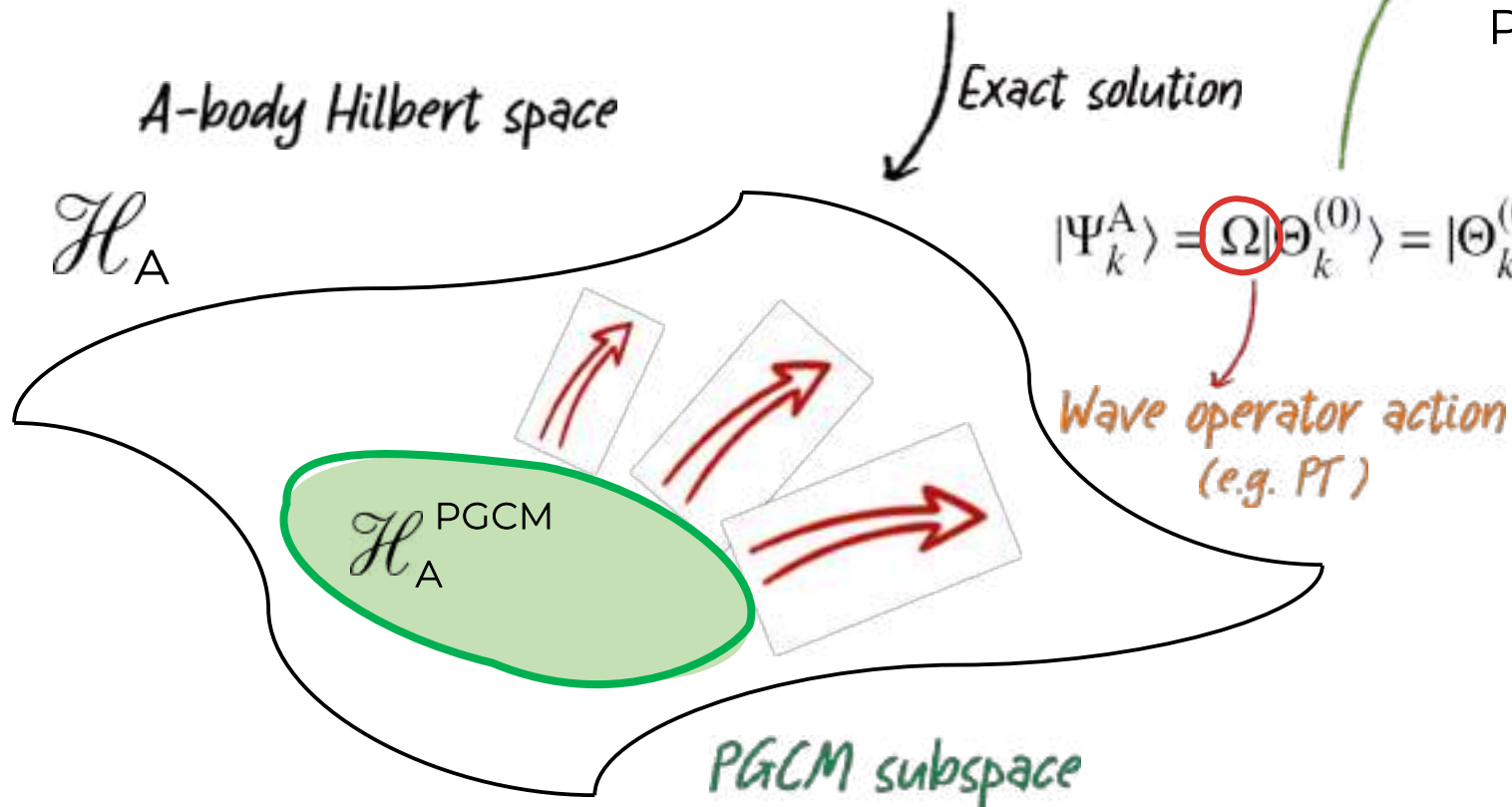
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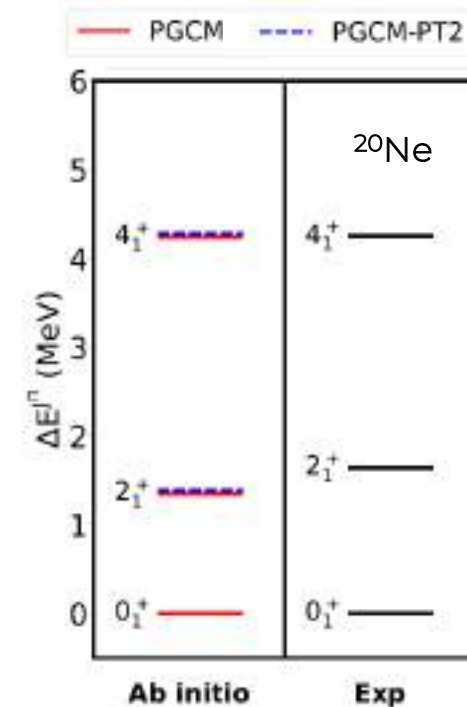
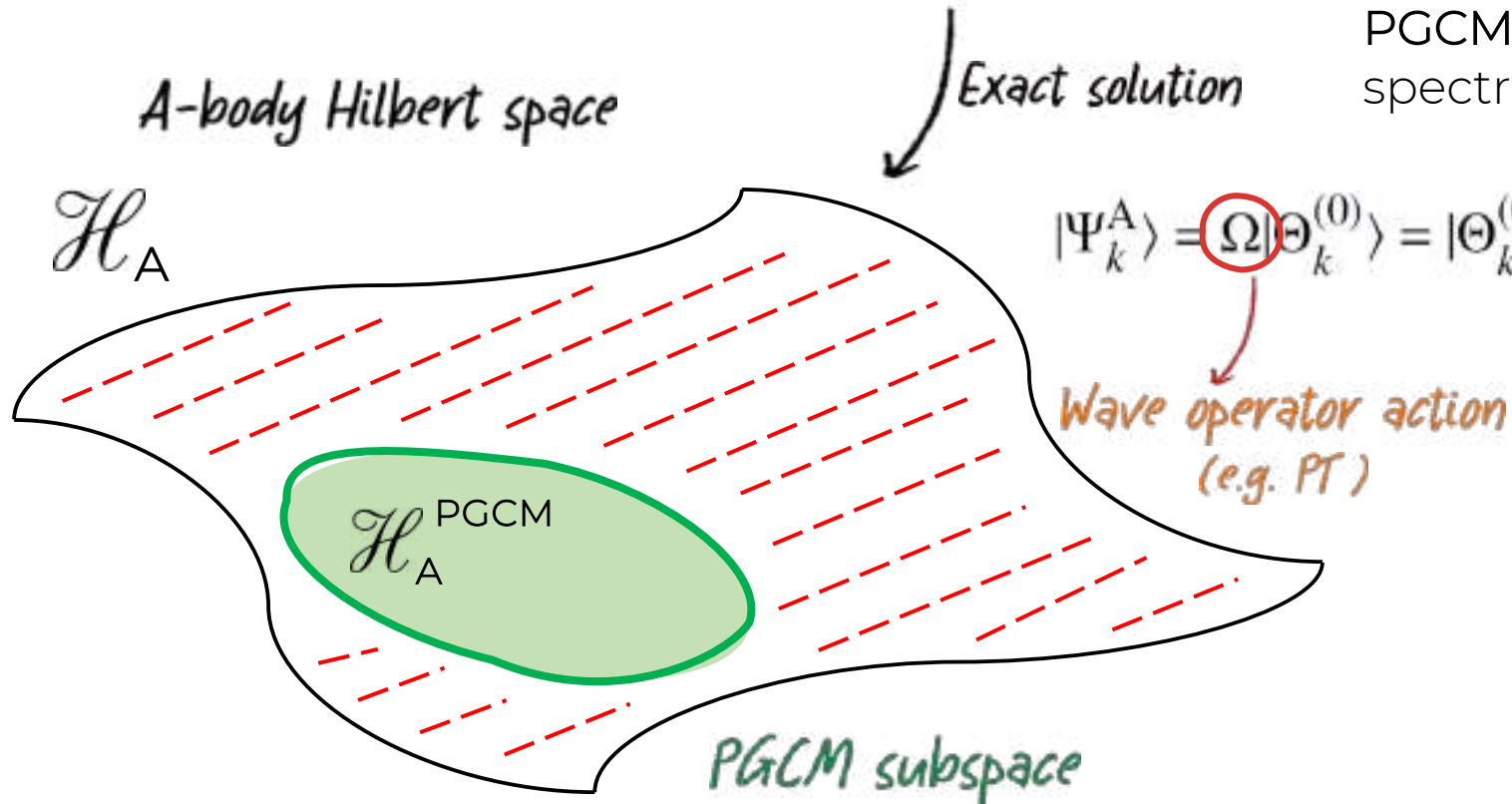
Many-body truncation

Schrödinger equation

$$H|\Psi_k^A\rangle = E_k^A|\Psi_k^A\rangle$$

Dynamical correlations mostly cancel out

PGCM reliable for **low-lying collective** spectroscopy



(1) [Frosini, Duguet, Ebran and Somà, EPJA 58(62), 2022]

(2) [Frosini, Duguet, Ebran, Bally, Hergert, Rodriguez, Roth, Yao and Somà, EPJA 58(64), 2022]

Outline

1 Introduction

- Giant Resonances Physics
- The PGCM
- Link between PGCM and QRPA

2 Systematic study

- Numerical details
- Uncertainty estimate

Conclusions and perspectives

Results

3

Selected applications

- Shape coexistence
- Deformation

Multi-phonon states

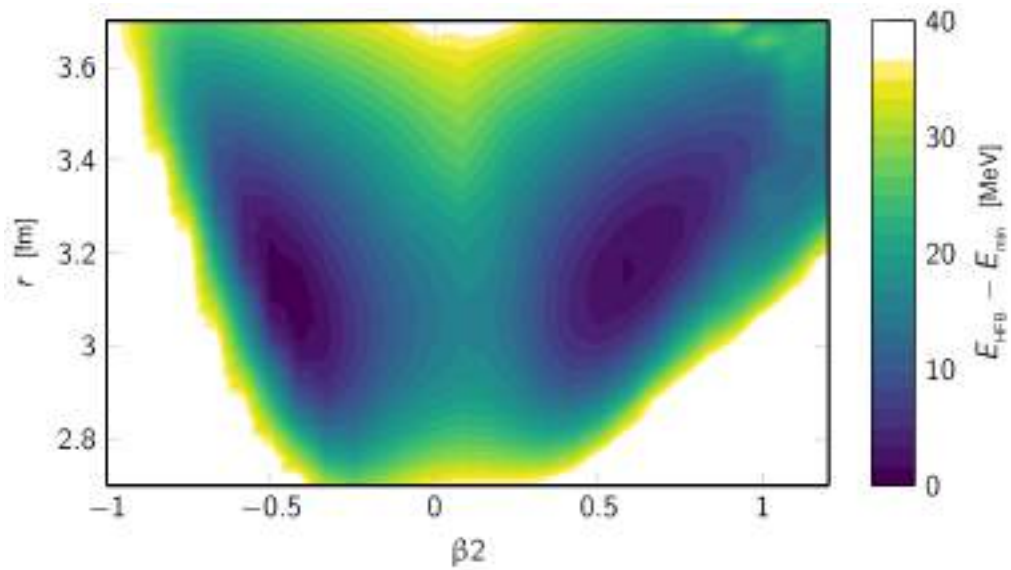
- Proof of principle
- Realistic calculations

From finite nuclei to Astrophysics

- Preliminary incompressibility results

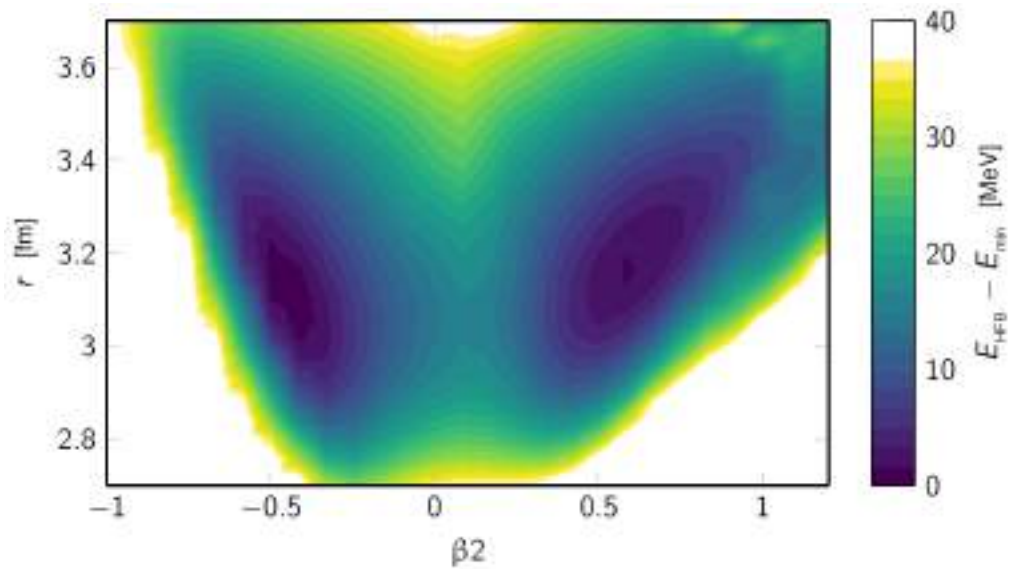
Shape coexistence effects in ^{28}Si

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



Shape coexistence effects in ^{28}Si

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$

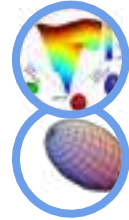
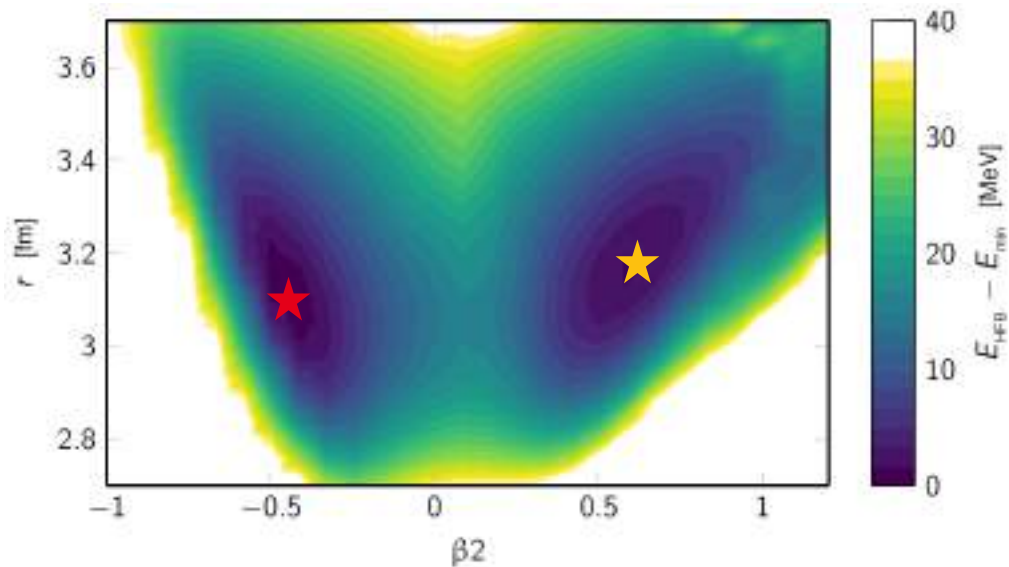


Shape coexistence [Jenkins et al., 2012]

Deformation

Shape coexistence effects in ^{28}Si

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



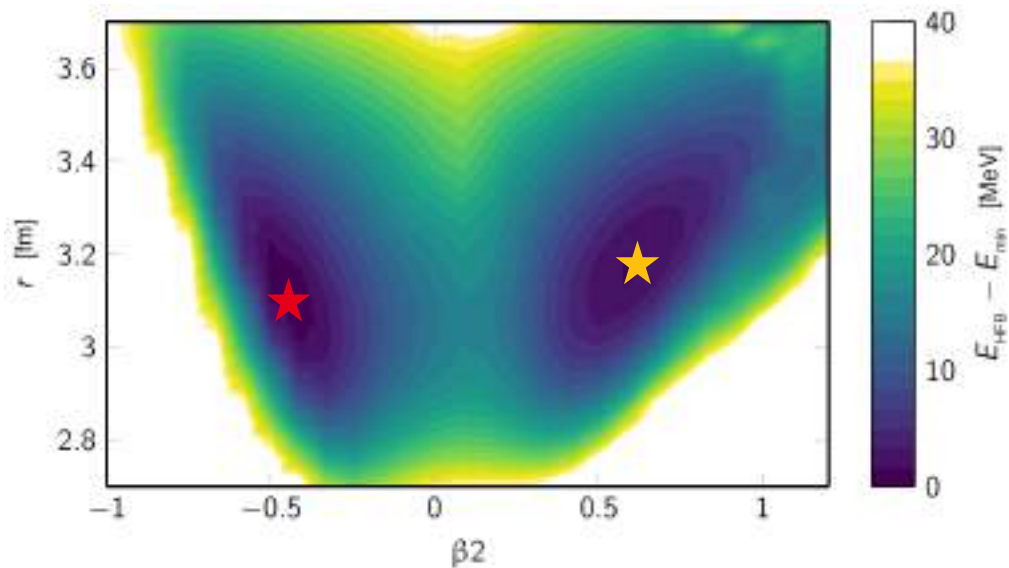
Shape coexistence [Jenkins et al., 2012]

Deformation

- Oblate GS and prolate-shape isomer

Shape coexistence effects in ^{28}Si

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



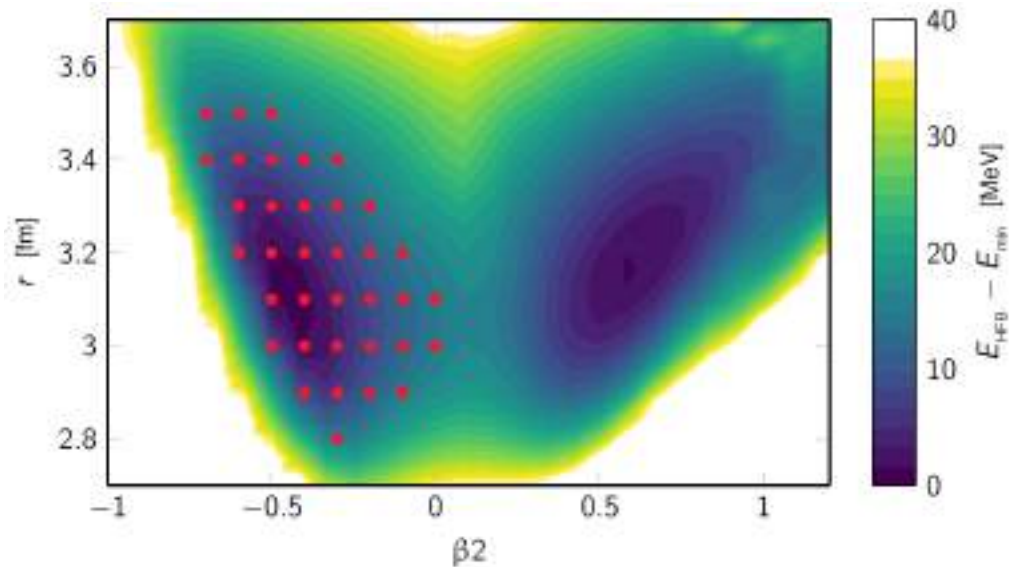
Shape coexistence [Jenkins et al., 2012]

Deformation

- Oblate GS and prolate-shape isomer
- Proper study of shape coexistence in PGCM

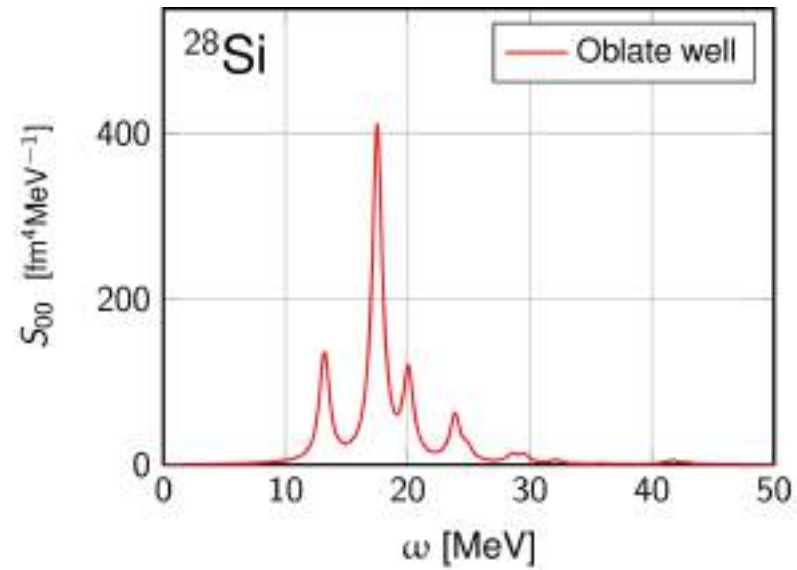
Shape coexistence effects in ^{28}Si

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



Shape coexistence [Jenkins et al., 2012]

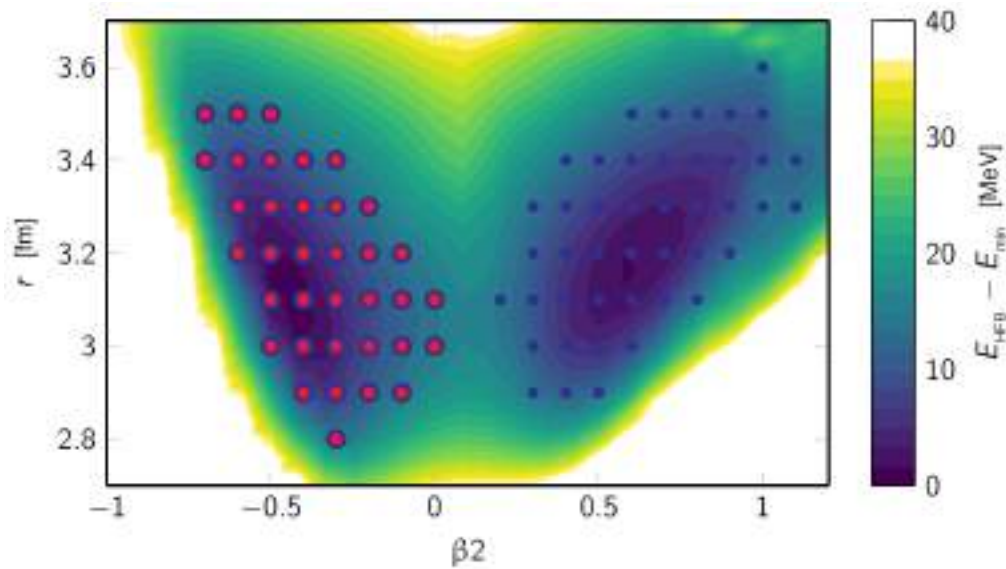
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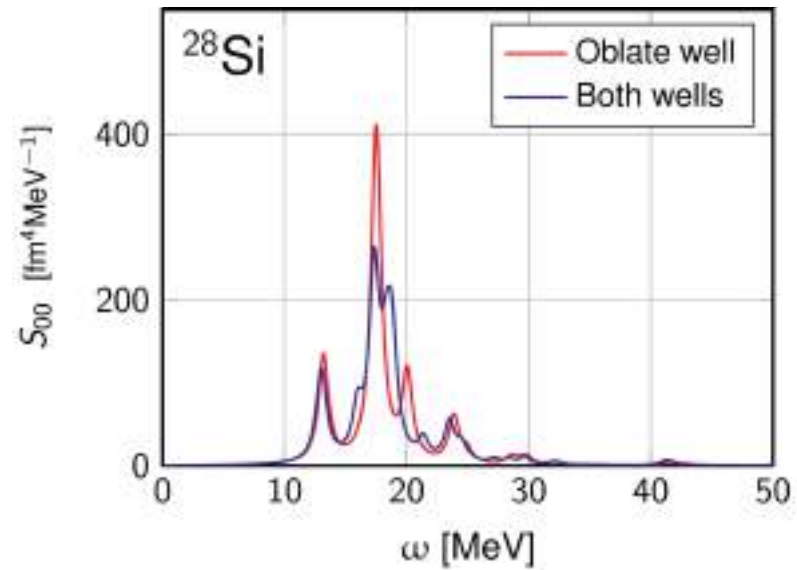
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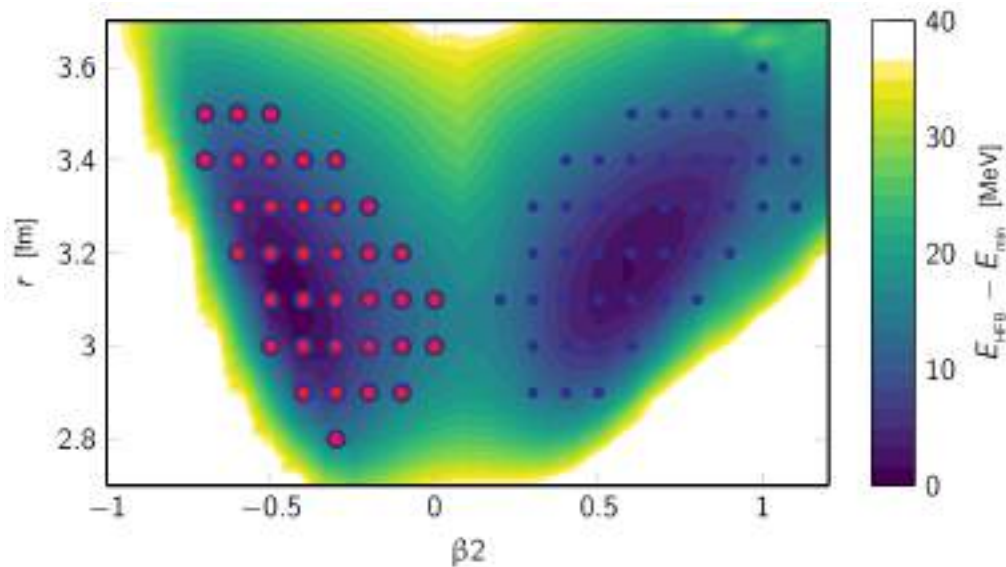
Deformation



- Oblate GS and prolate-shape isomer
- Proper study of shape coexistence in PGCM
 - Shape coexistence but weak mixing

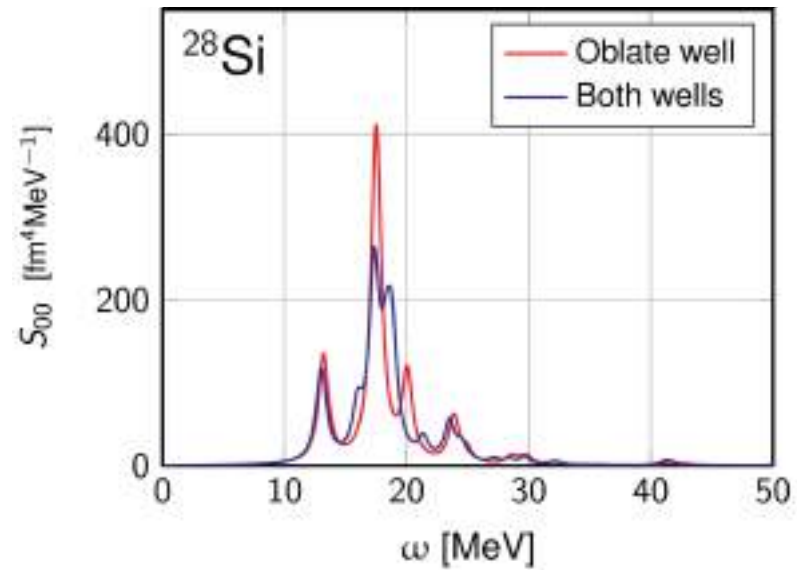
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Shape coexistence [Jenkins et al., 2012]

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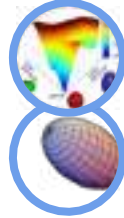
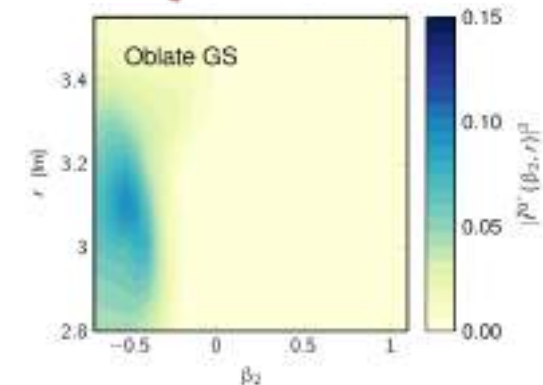
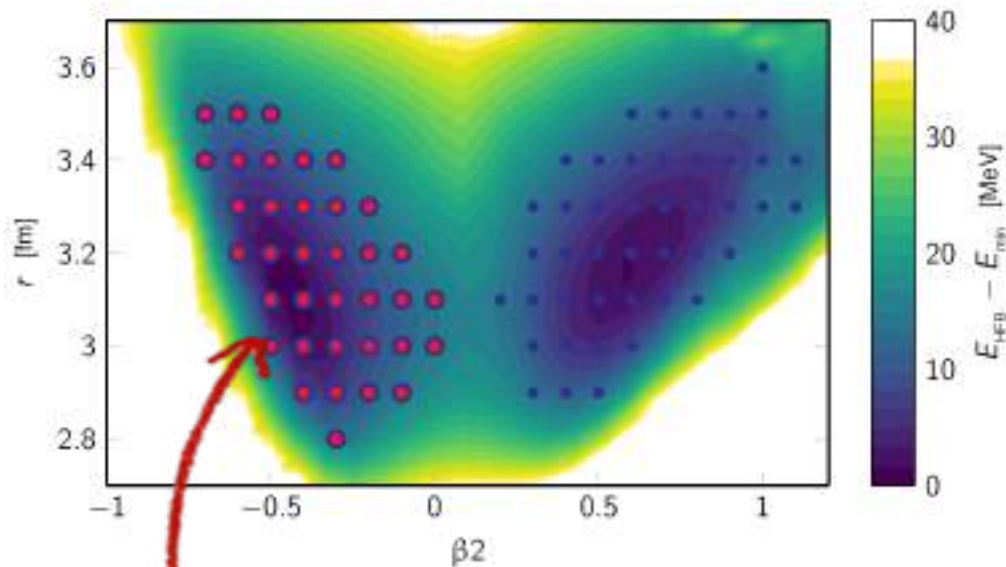


- Oblate GS and prolate-shape isomer
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Nuclei with stronger signature ? 14

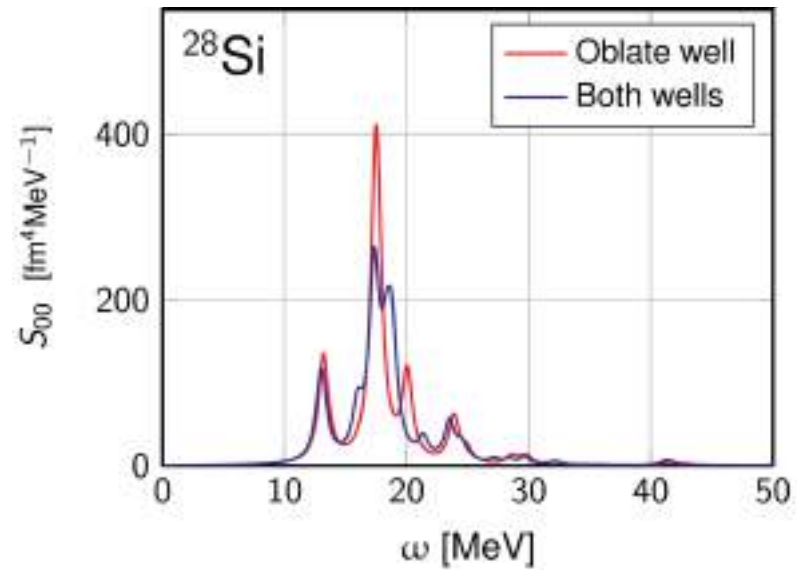
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Shape coexistence [Jenkins et al., 2012]

Deformation

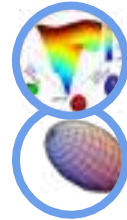
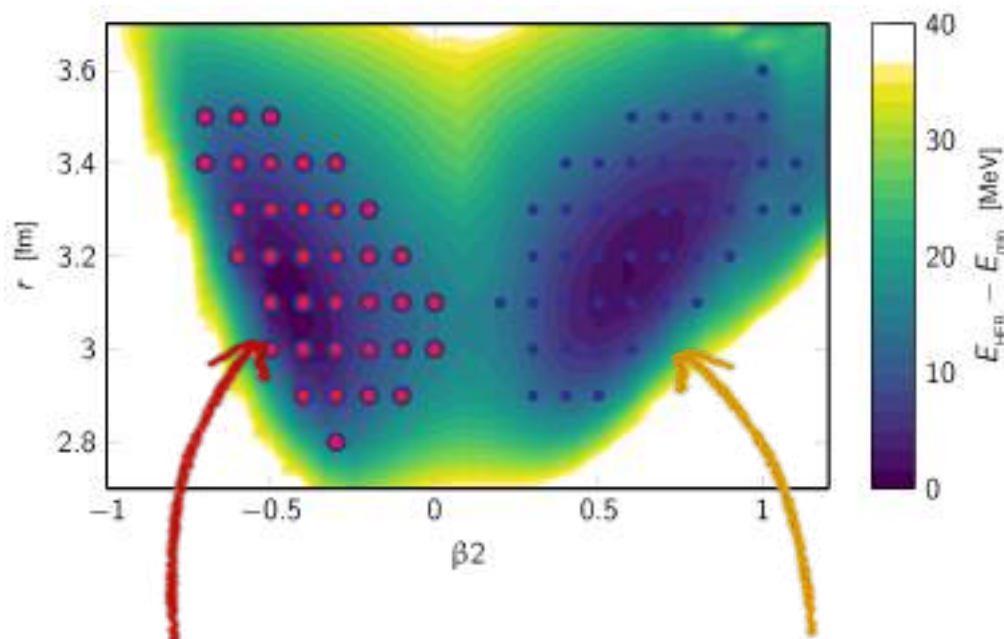


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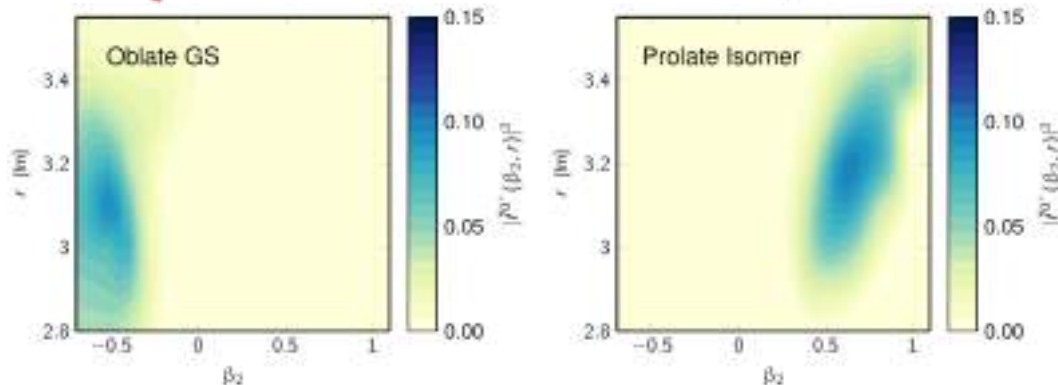
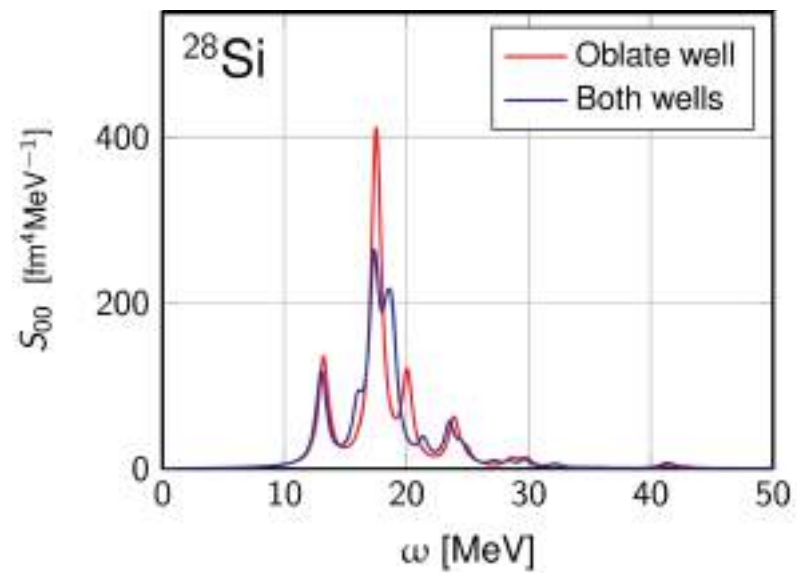
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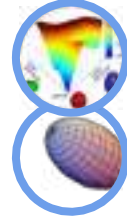
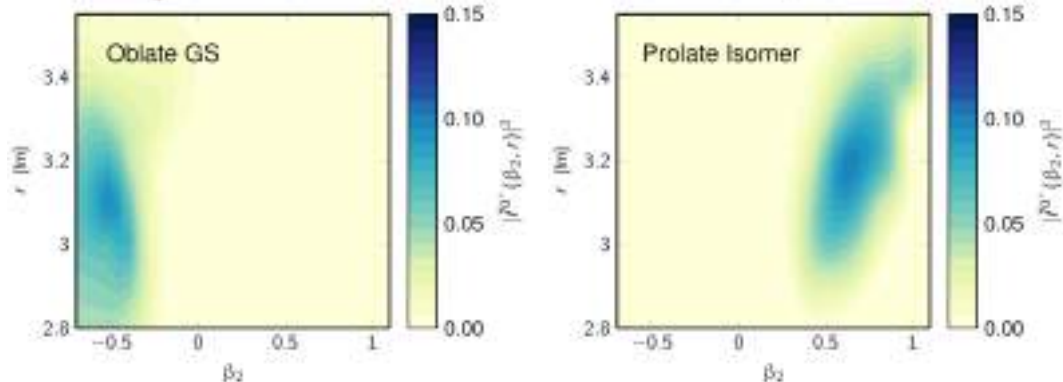
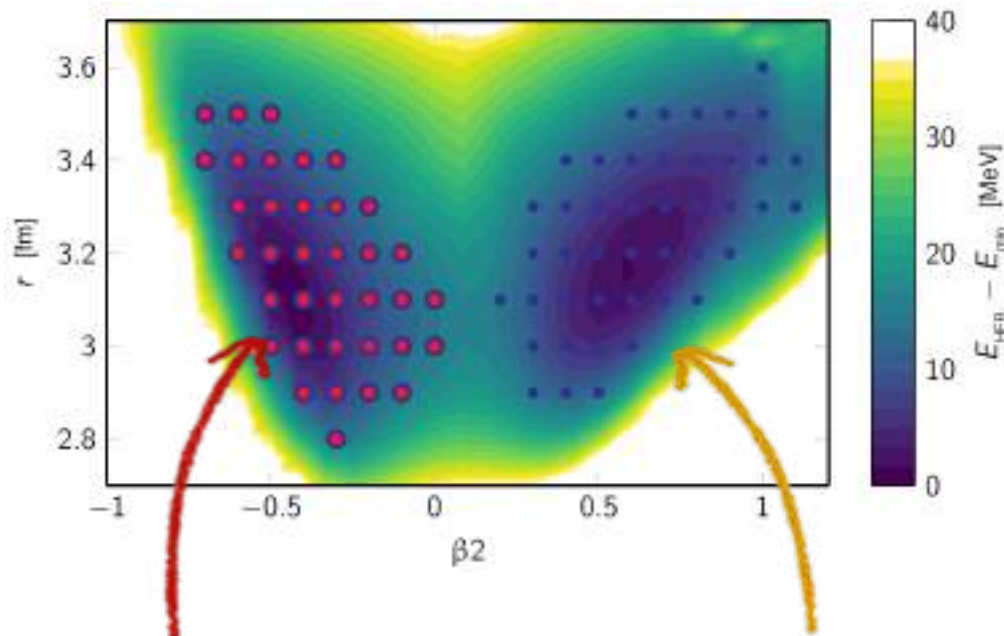


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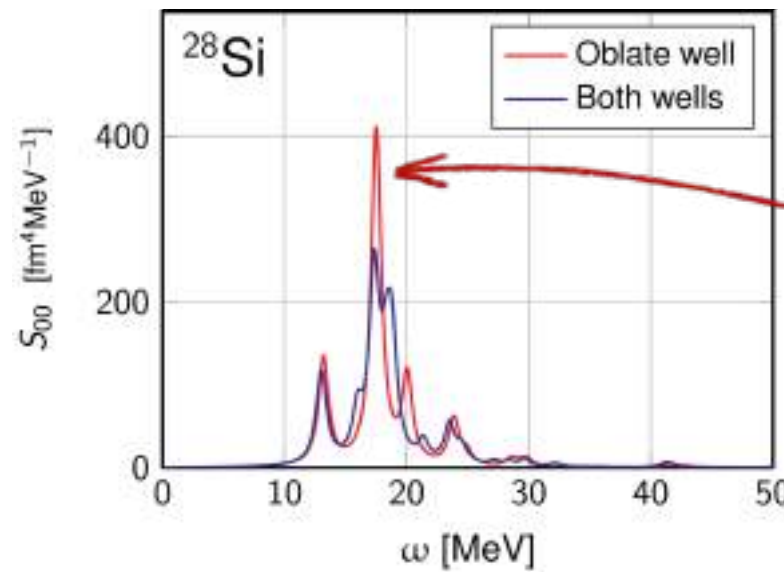
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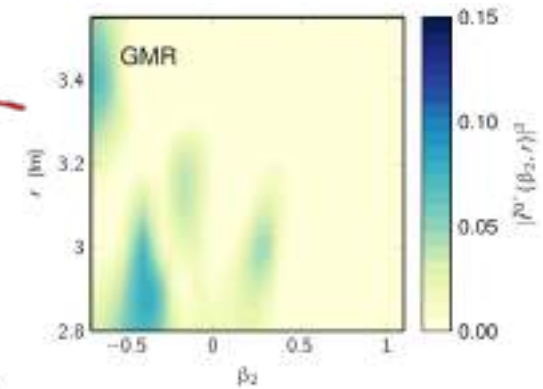


Shape coexistence [Jenkins et al., 2012]

Deformation



Radial vibration on oblate GS

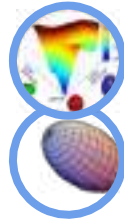
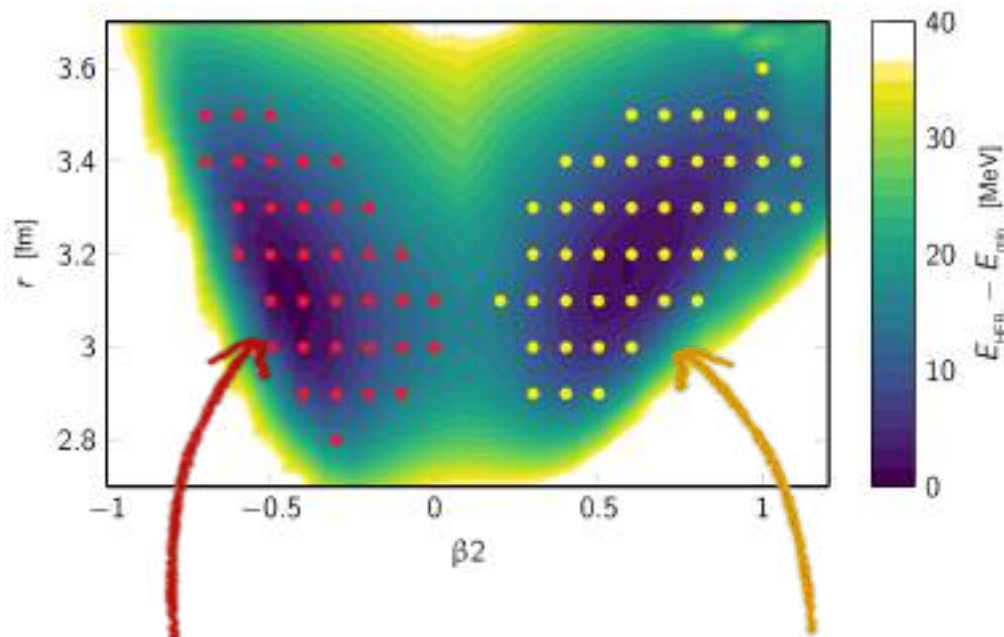


- Oblate GS and prolate-shape isomer
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Nuclei with stronger signature ? 14

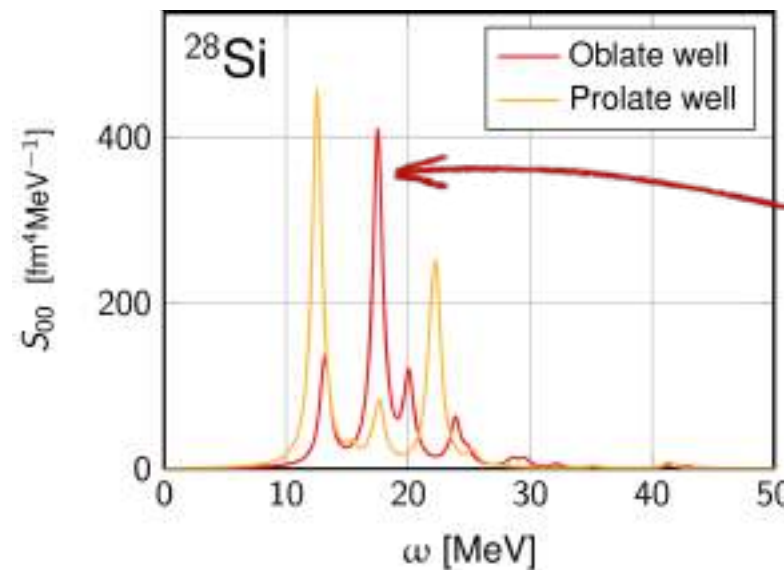
Shape coexistence effects in ^{28}Si

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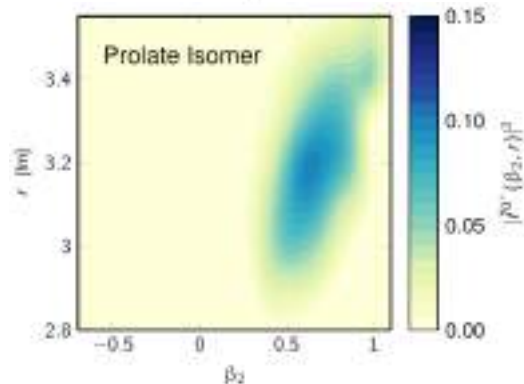
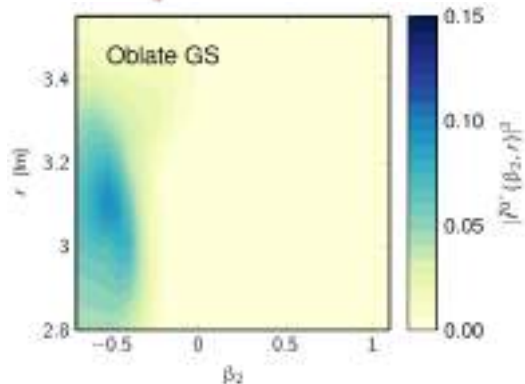
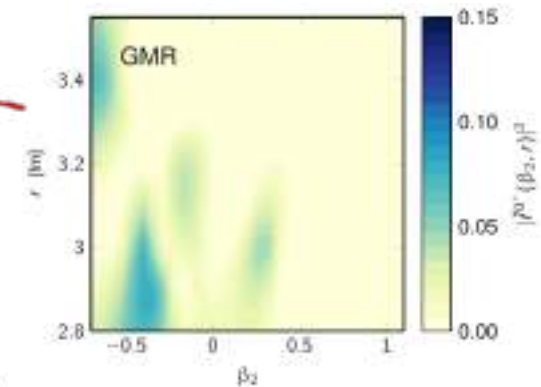


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Deformation



Radial vibration on oblate GS

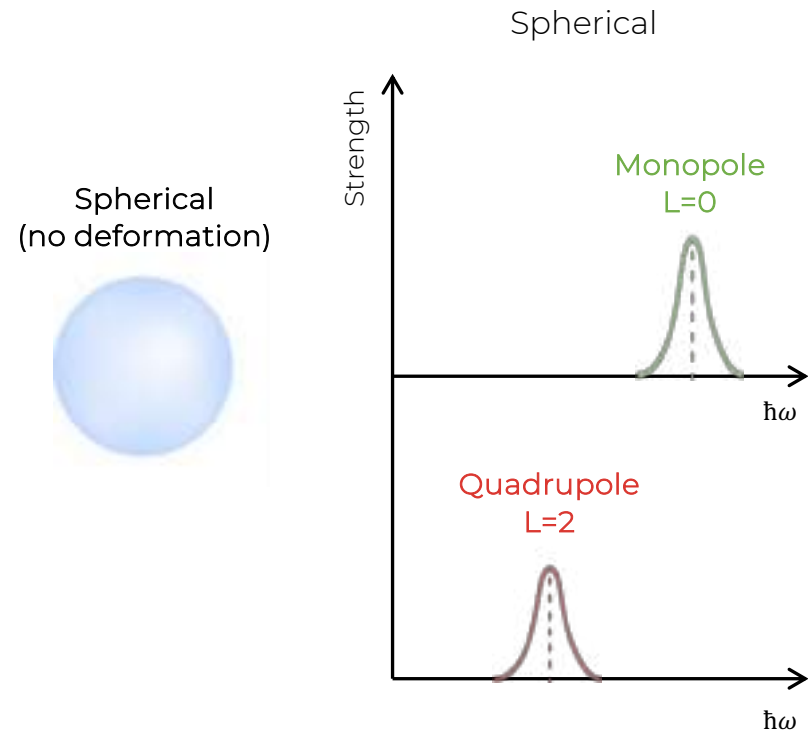


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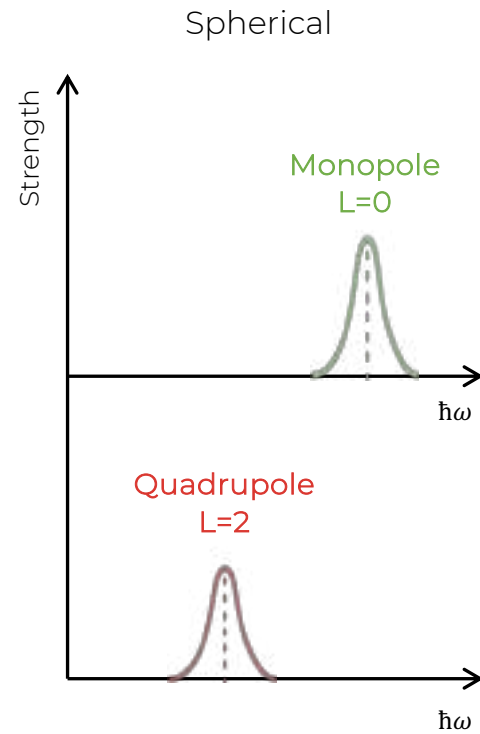
Deformation effects

Deformation effects



Deformation effects

Spherical
(no deformation)

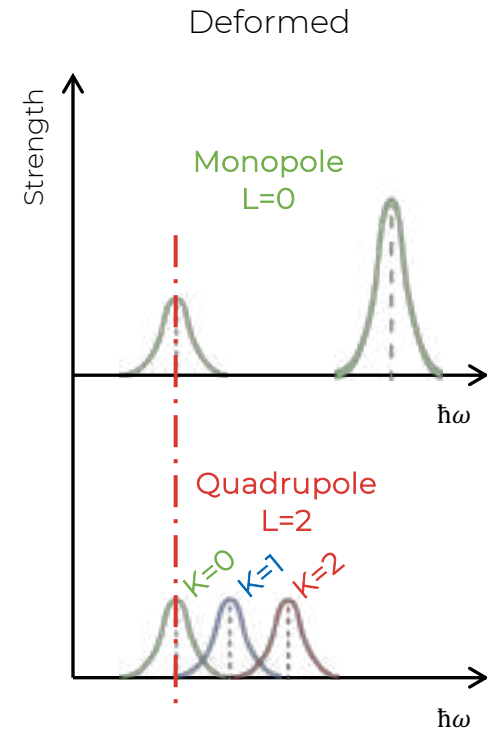


Spheroidal
(deformed)



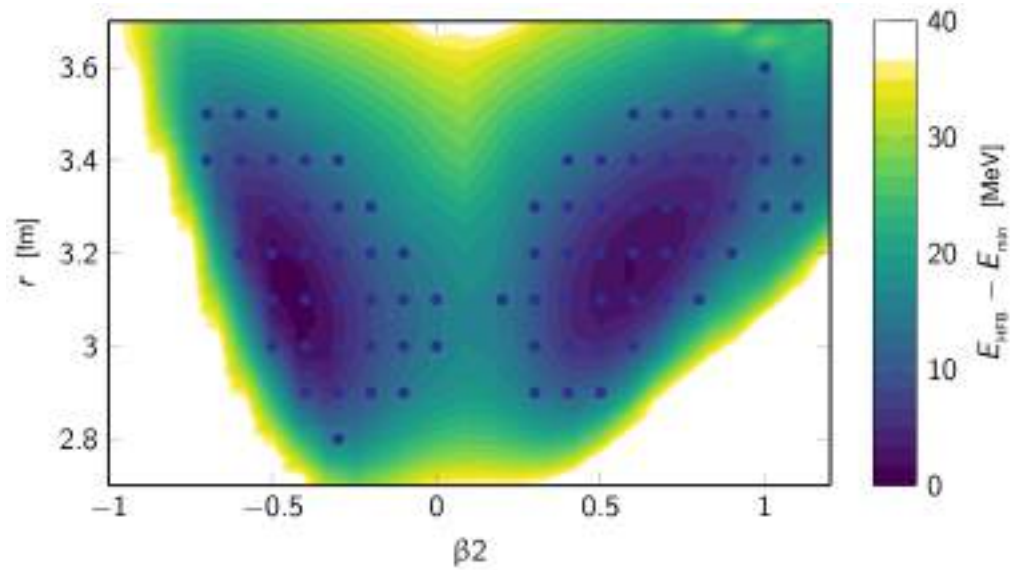
Prolate
(cigar type)

Oblate
(pancake type)



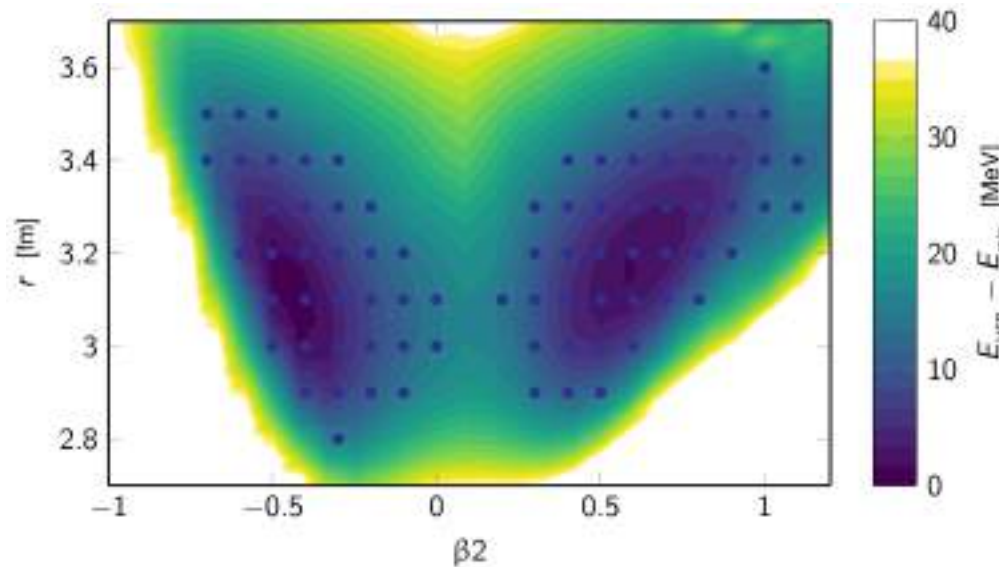
Deformation effects in prolate ^{28}Si

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$

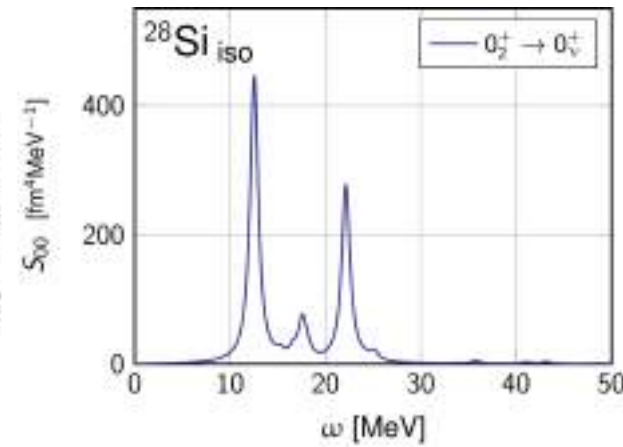


Deformation effects in prolate ^{28}Si

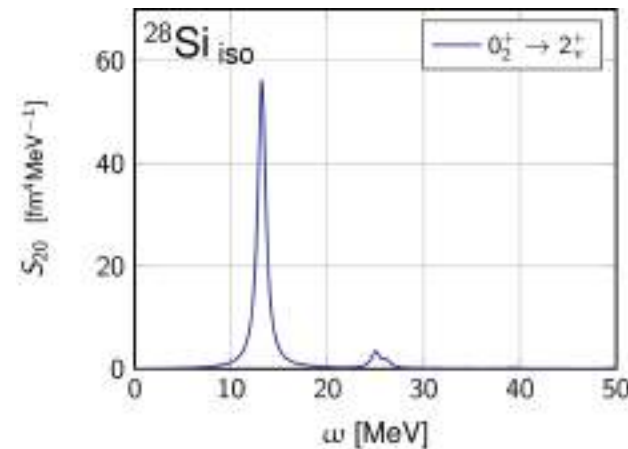
Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



Monopole Strength



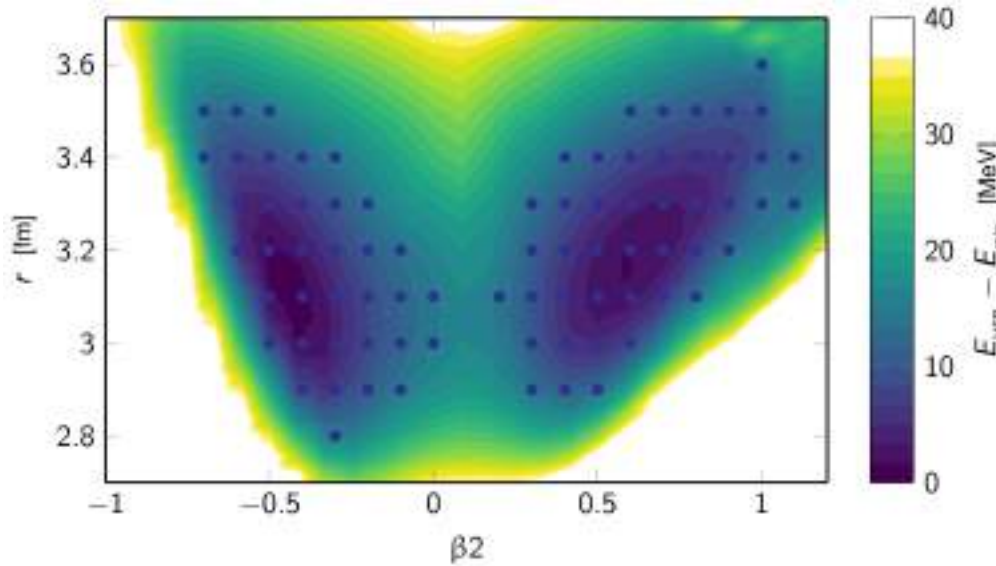
K=0 Quadrupole Strength



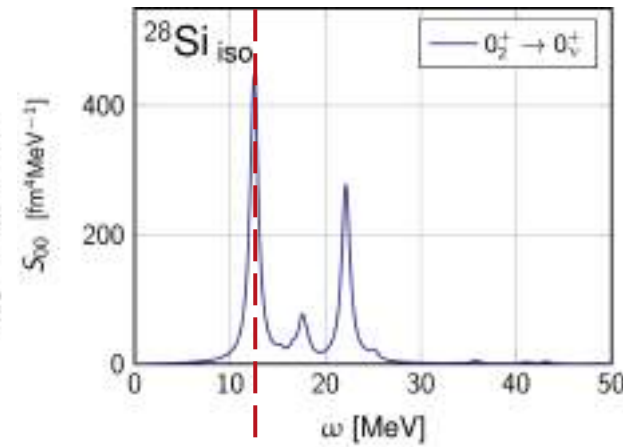
- Focus on the prolate-shape isomer
- Coupling to GQR generates **splitting**
 - ✗ High peak = shifted “spherical” breathing mode
 - ✗ Low peak = induced by coupling to GQR (K=0)
- Two-peak GMR on the prolate shape isomer

Deformation effects in prolate ^{28}Si

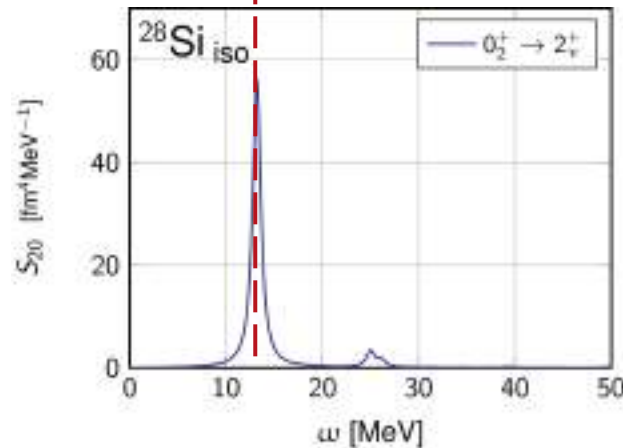
Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



Monopole Strength



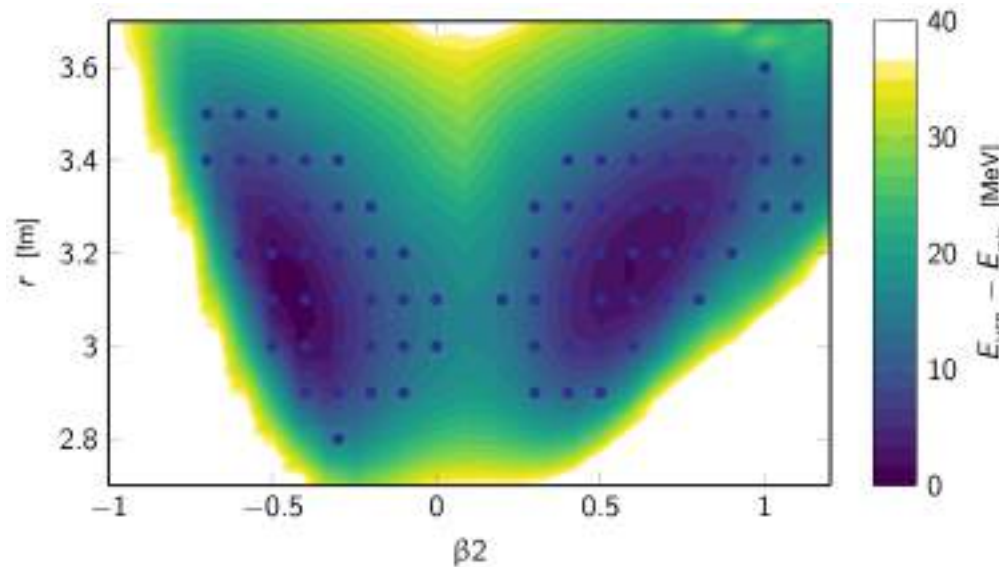
K=0 Quadrupole Strength



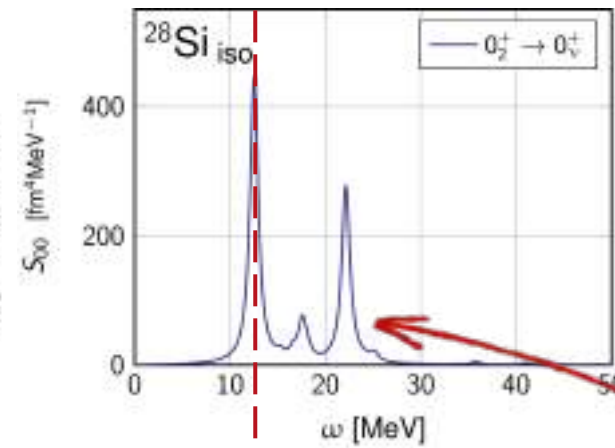
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Deformation effects in prolate ^{28}Si

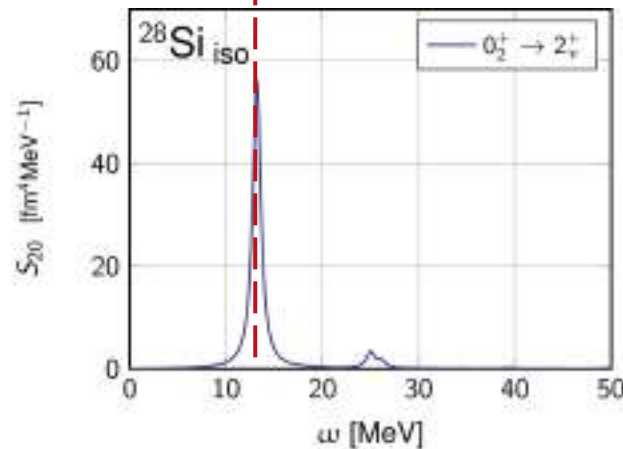
Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



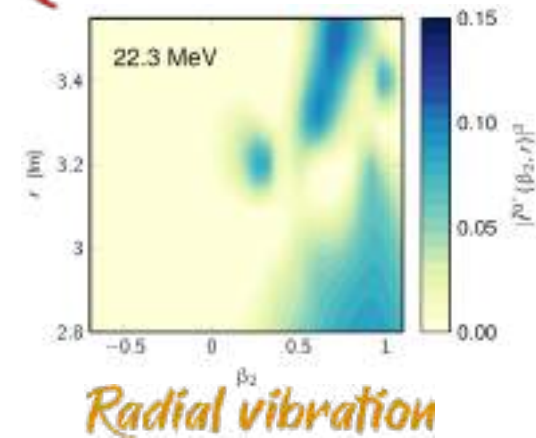
Monopole Strength



K=0 Quadrupole Strength

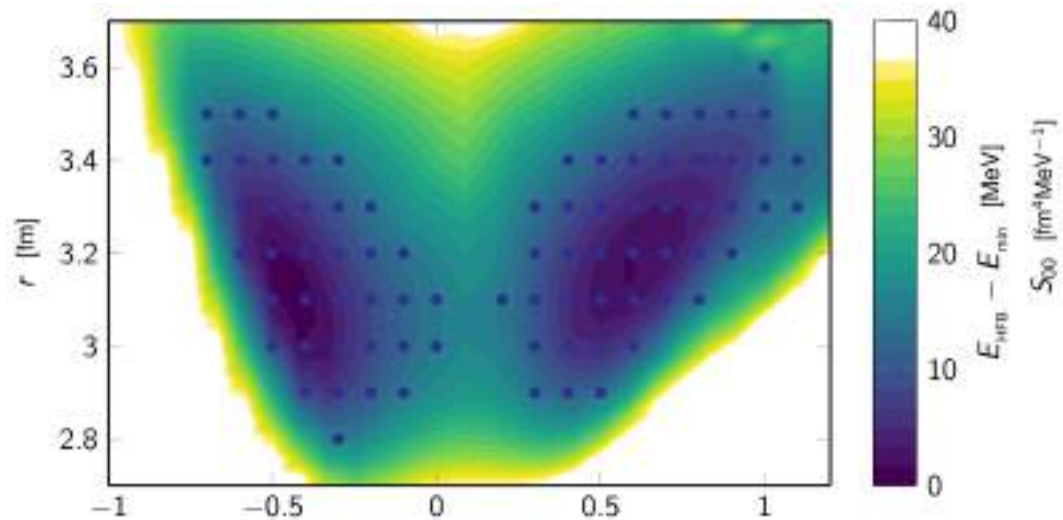


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 - ✗ Low peak = induced by coupling to GQR (K=0)
- Two-peak GMR on the prolate shape isomer

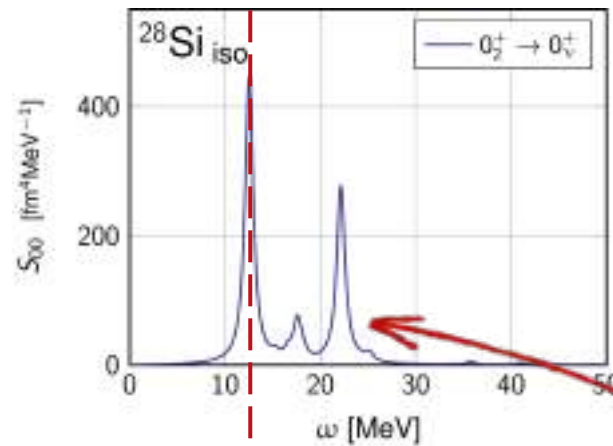


Deformation effects in prolate ^{28}Si

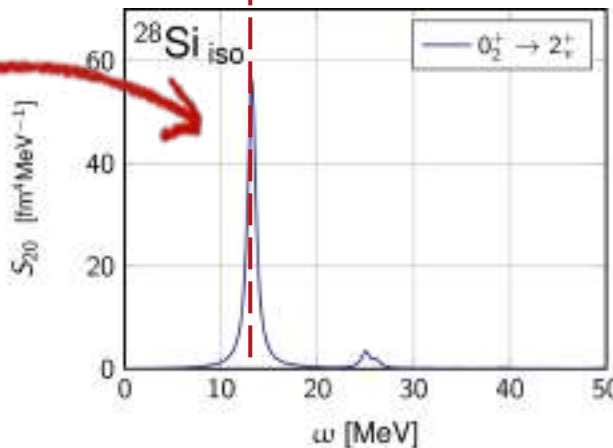
Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



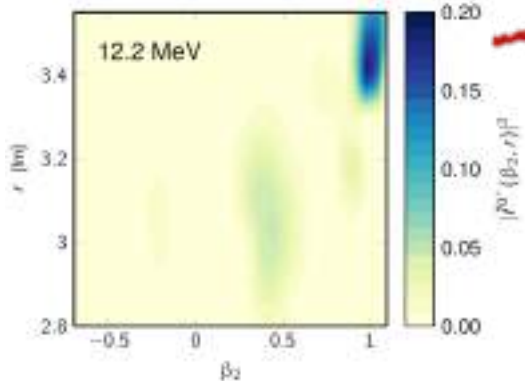
Monopole Strength



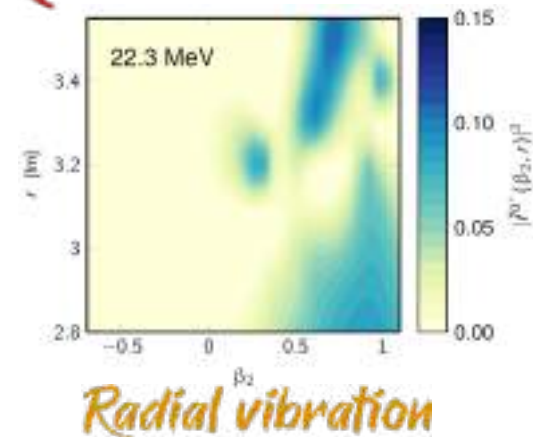
K=0 Quadrupole Strength



Radial + β_2 vibration



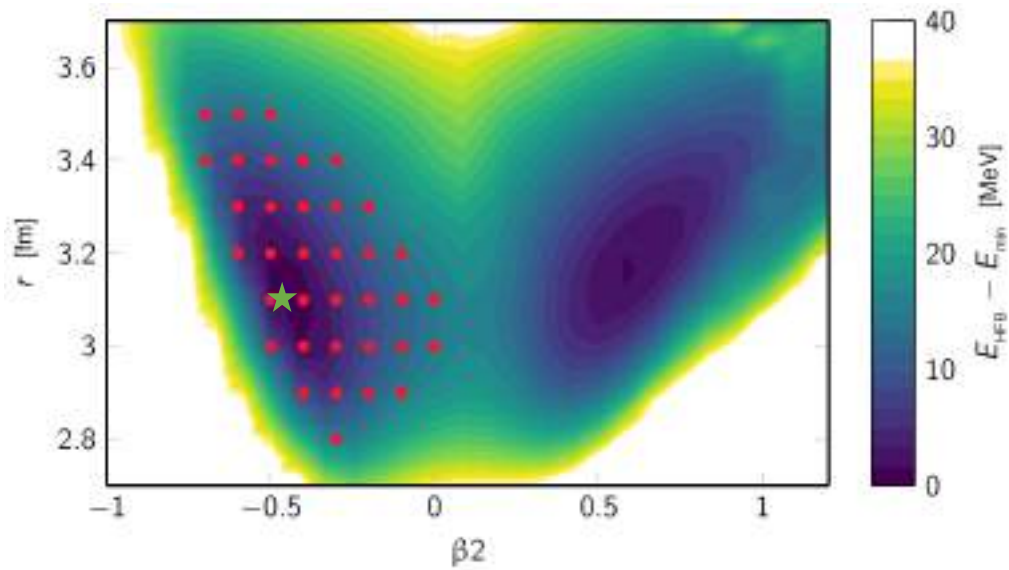
- Focus on the prolate-shape isomer
- Coupling to GQR generates **splitting**
 - ✗ High peak = shifted “spherical” breathing mode
 - ✗ Low peak = induced by coupling to GQR (K=0)
- Two-peak GMR on the prolate shape isomer



Radial vibration

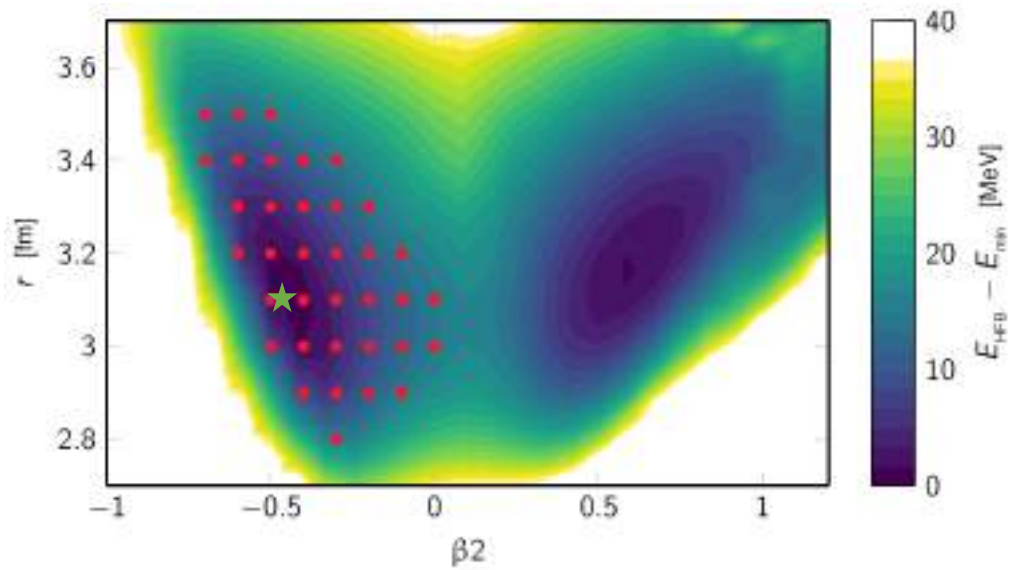
Anharmonic effects

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



Anharmonic effects

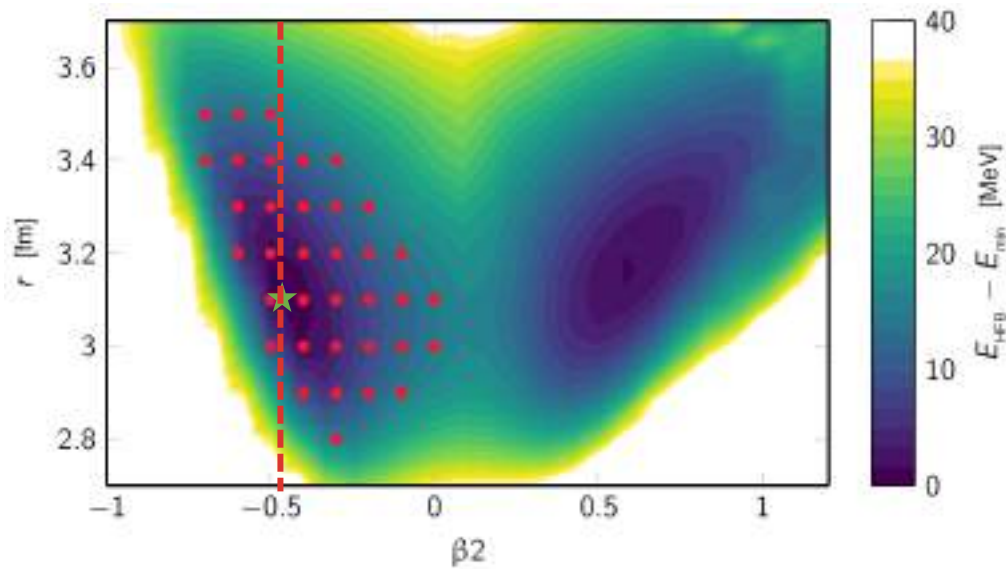
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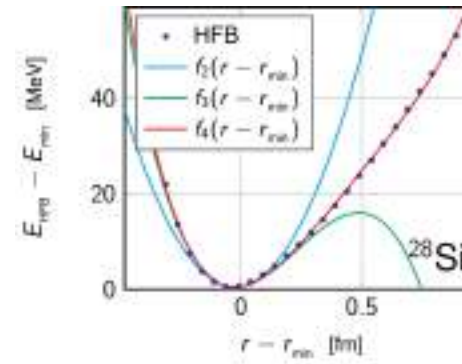
- Quantitative anharmonicities analysis

Anharmonic effects

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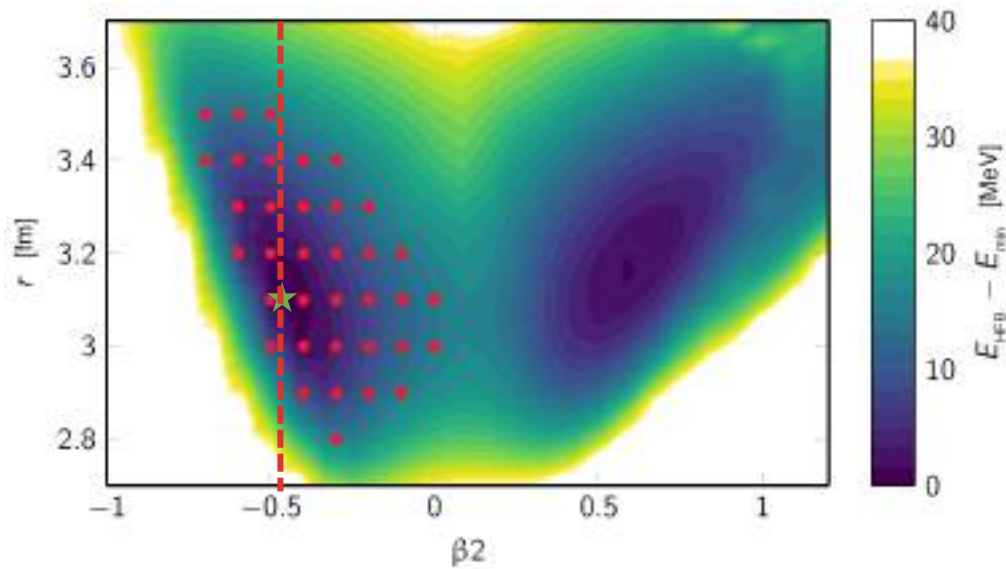
One-dimensional cut



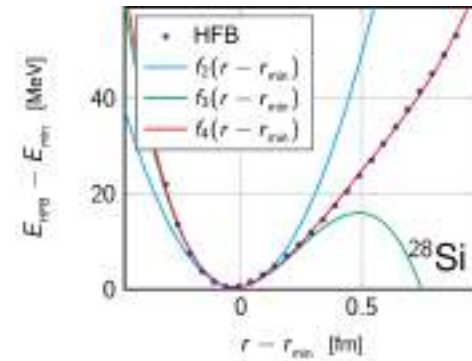
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Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



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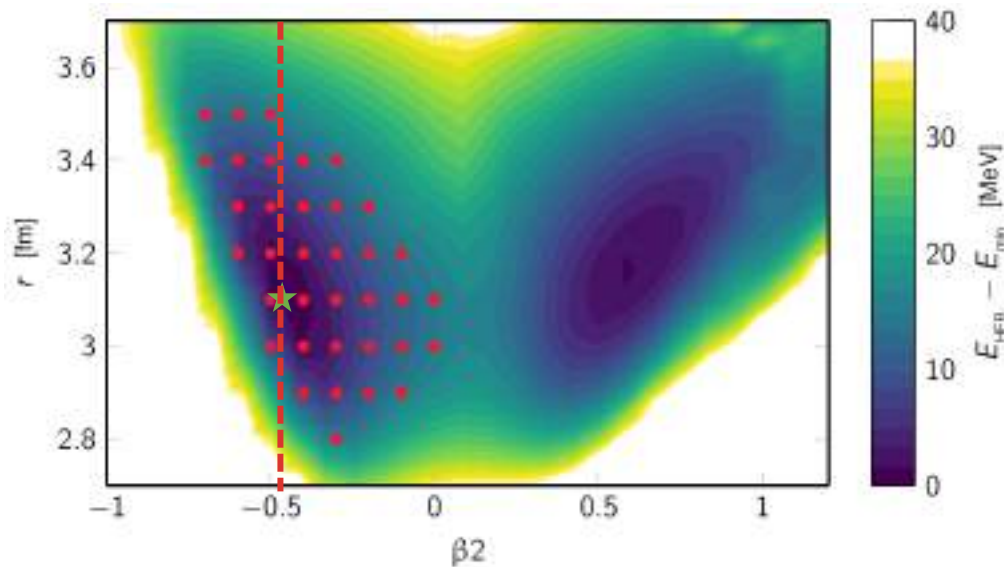
Polynomial fit

$$f(x) = a_2x^2 + a_3x^3 + a_4x^4$$

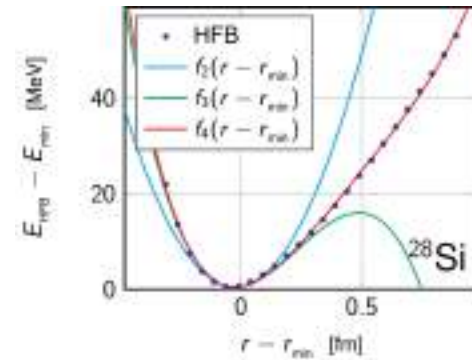
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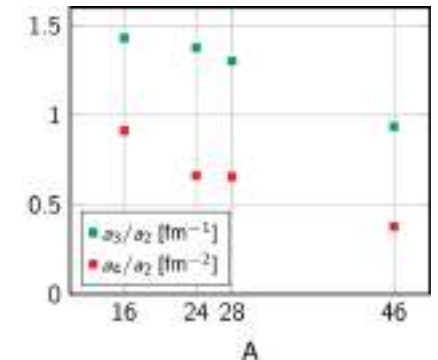


One-dimensional cut



Polynomial fit

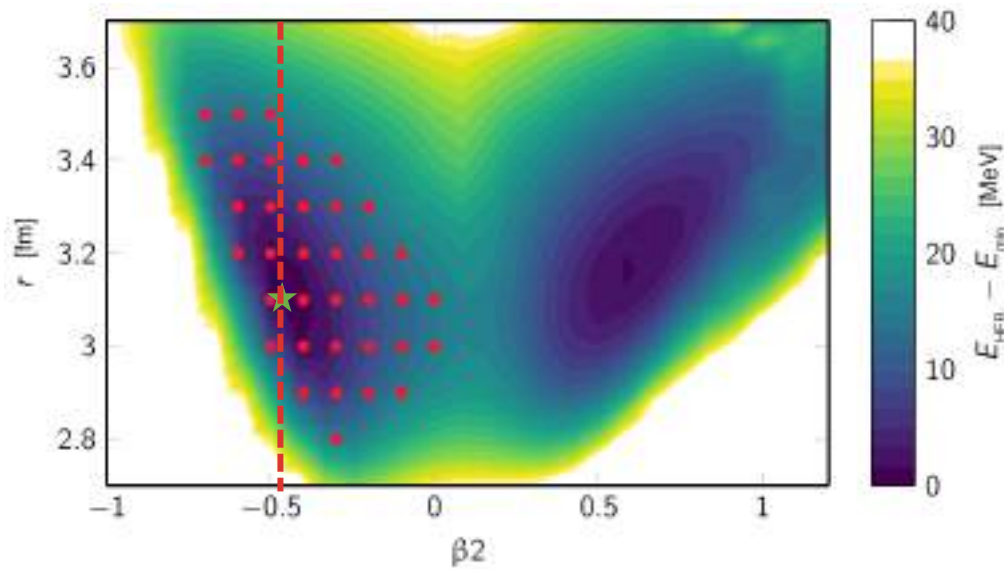
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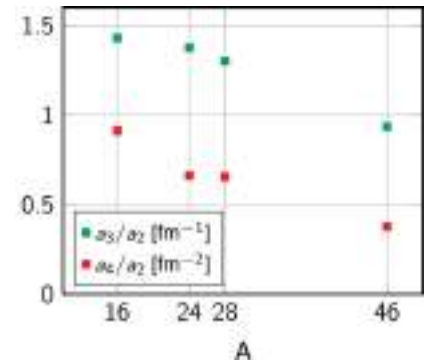
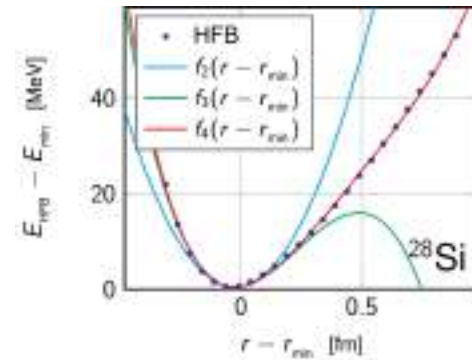
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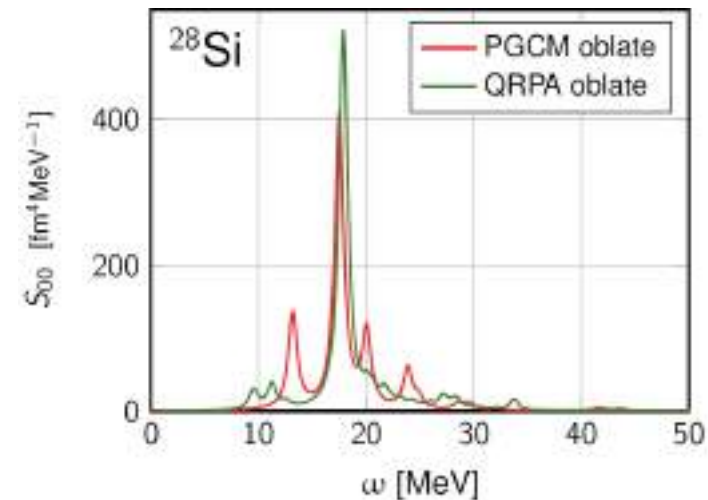
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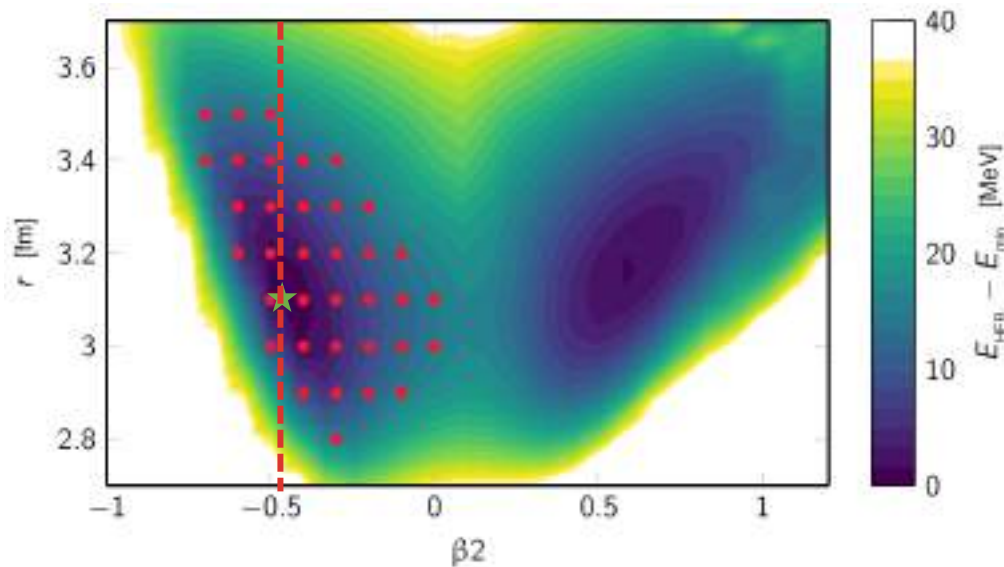
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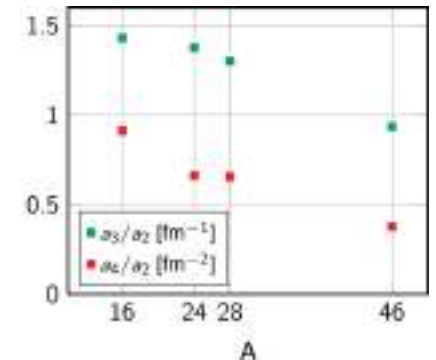
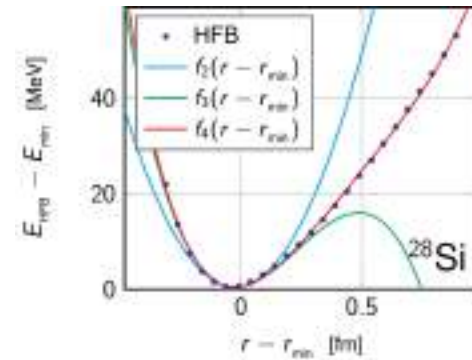


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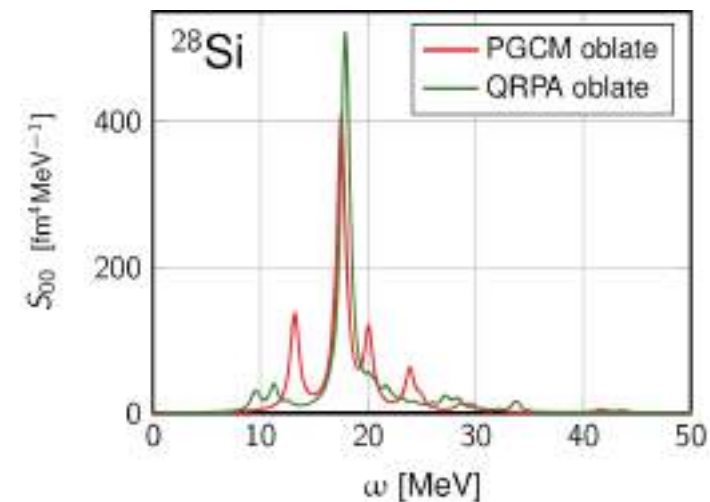
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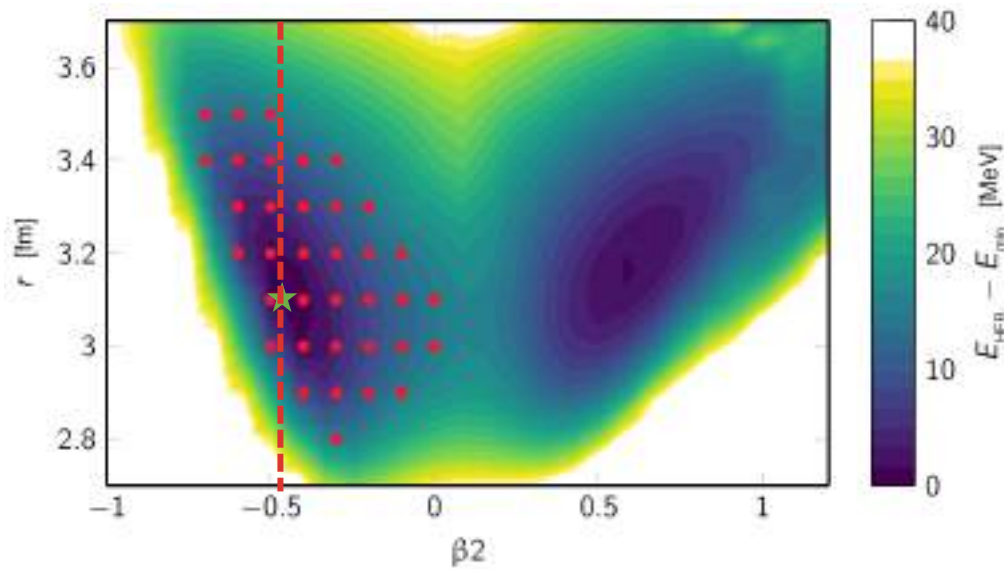
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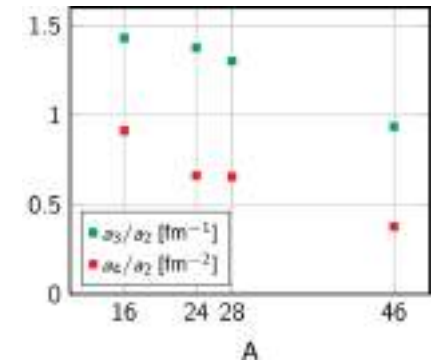
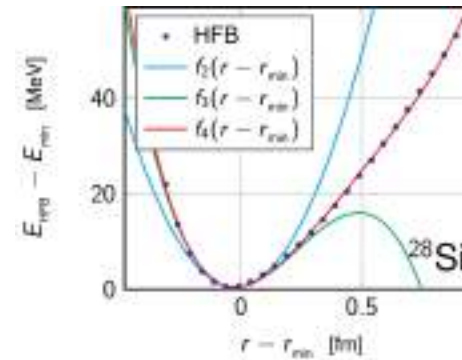


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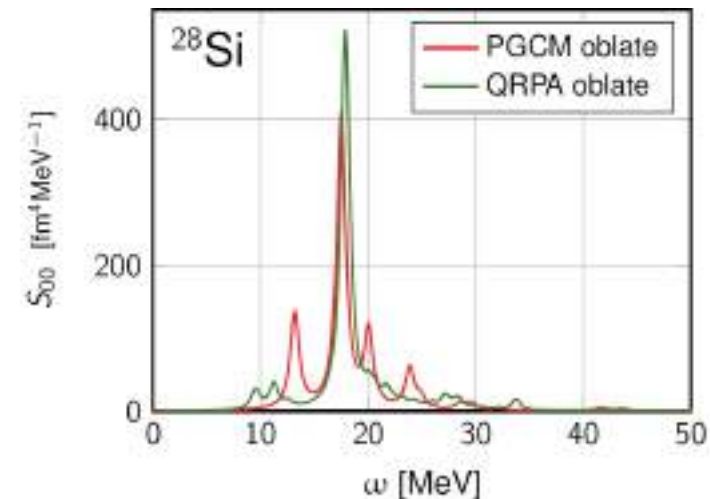
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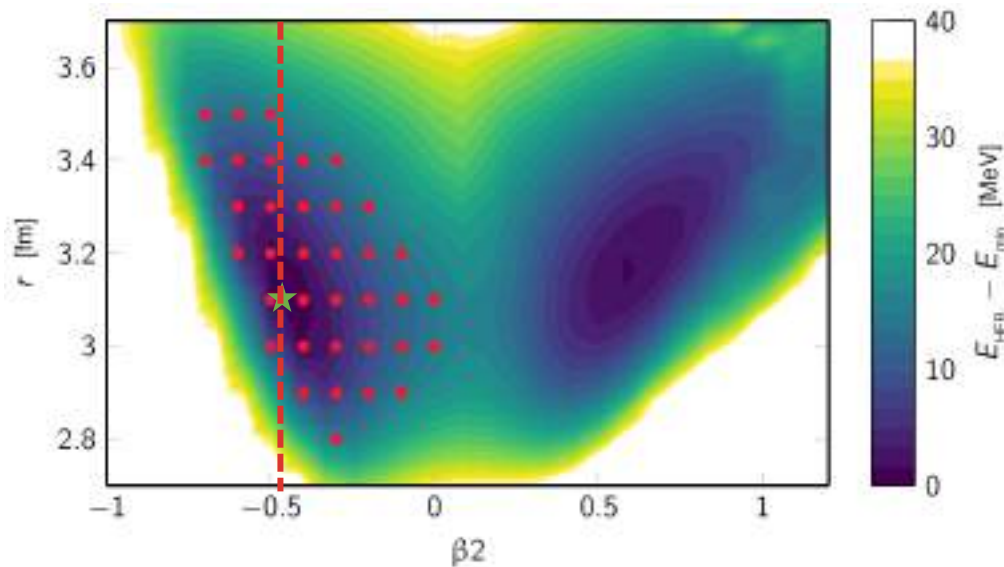
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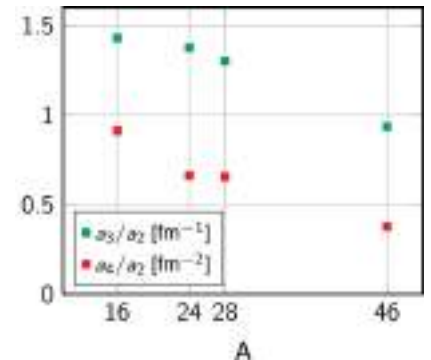
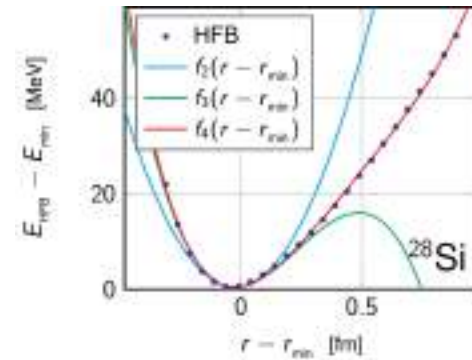


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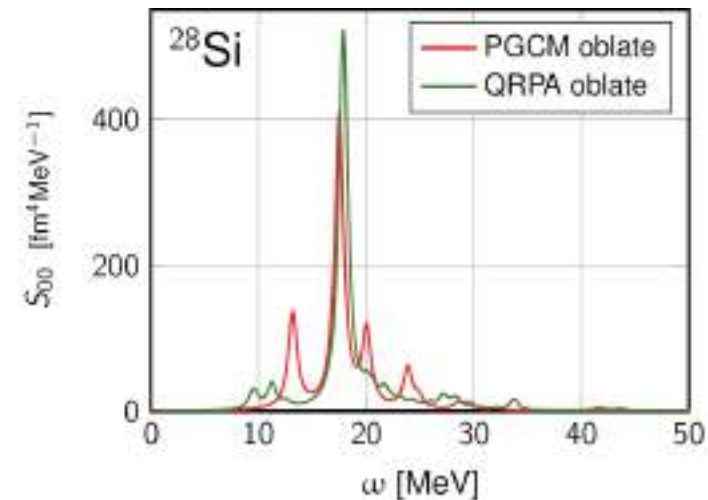
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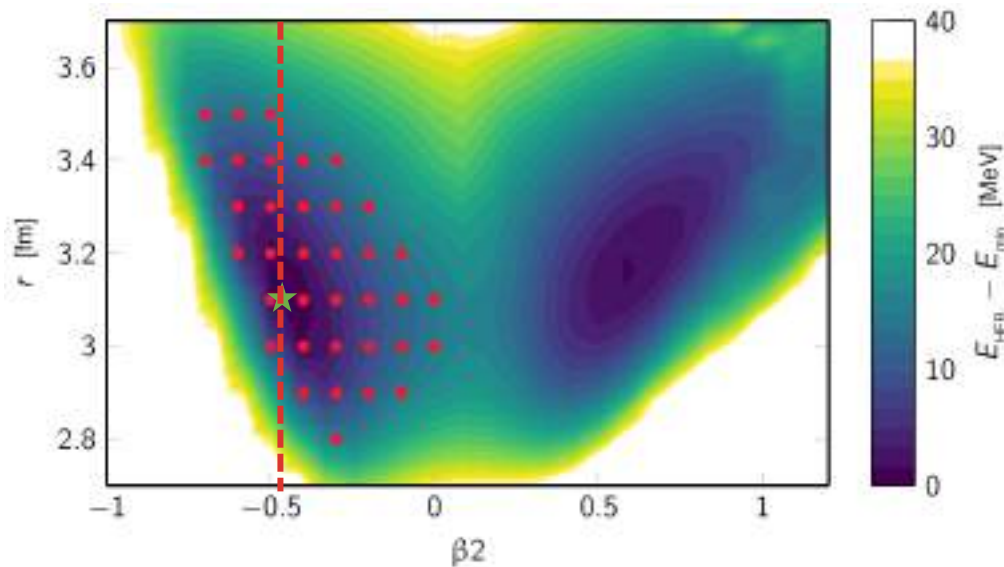
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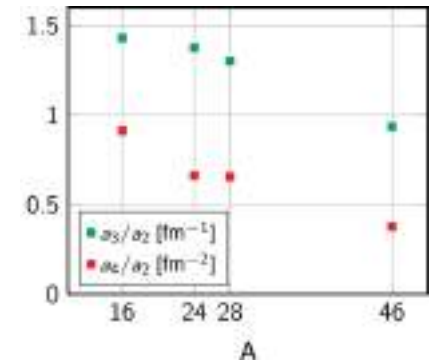
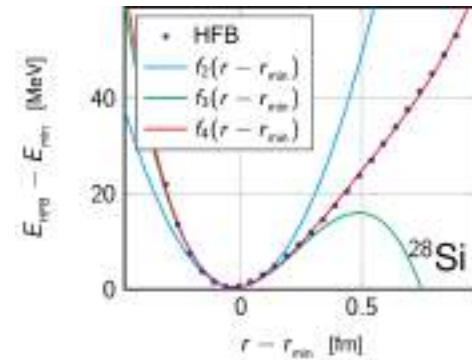


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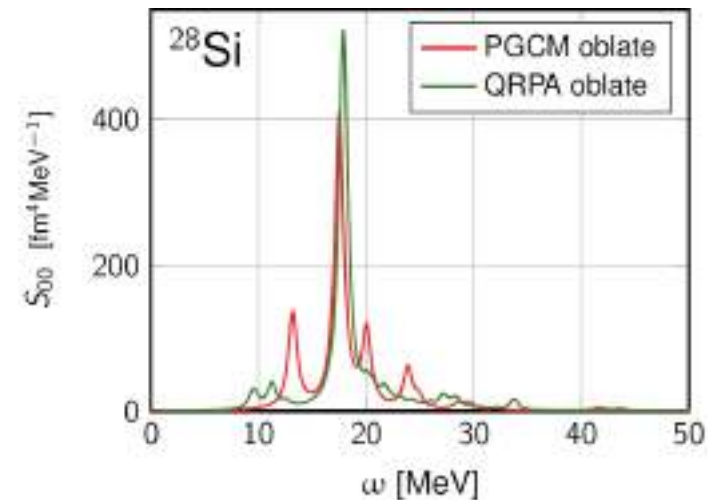
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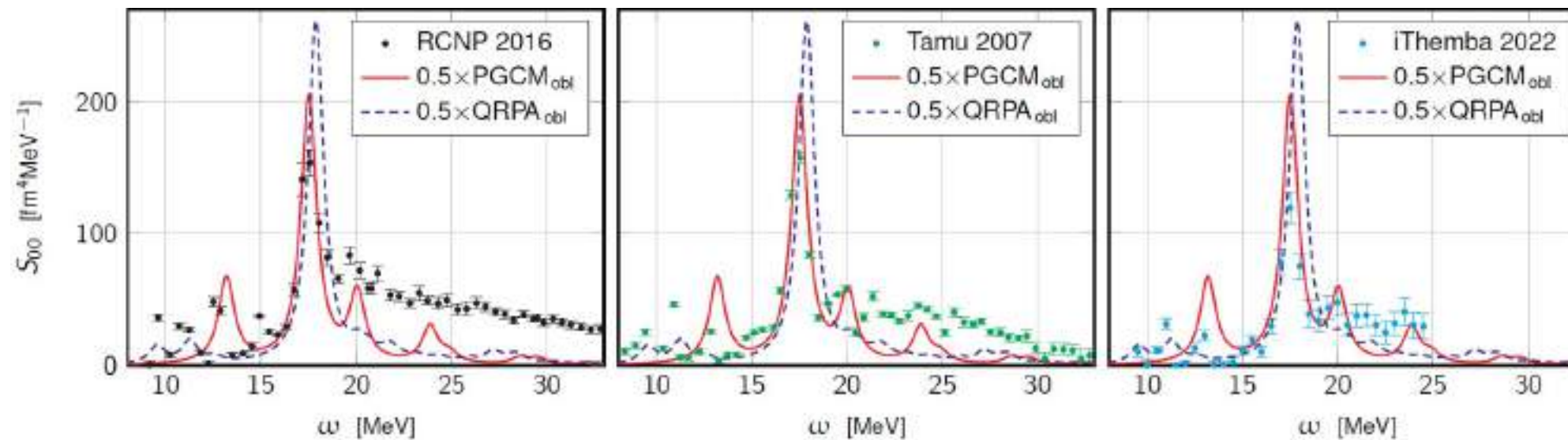
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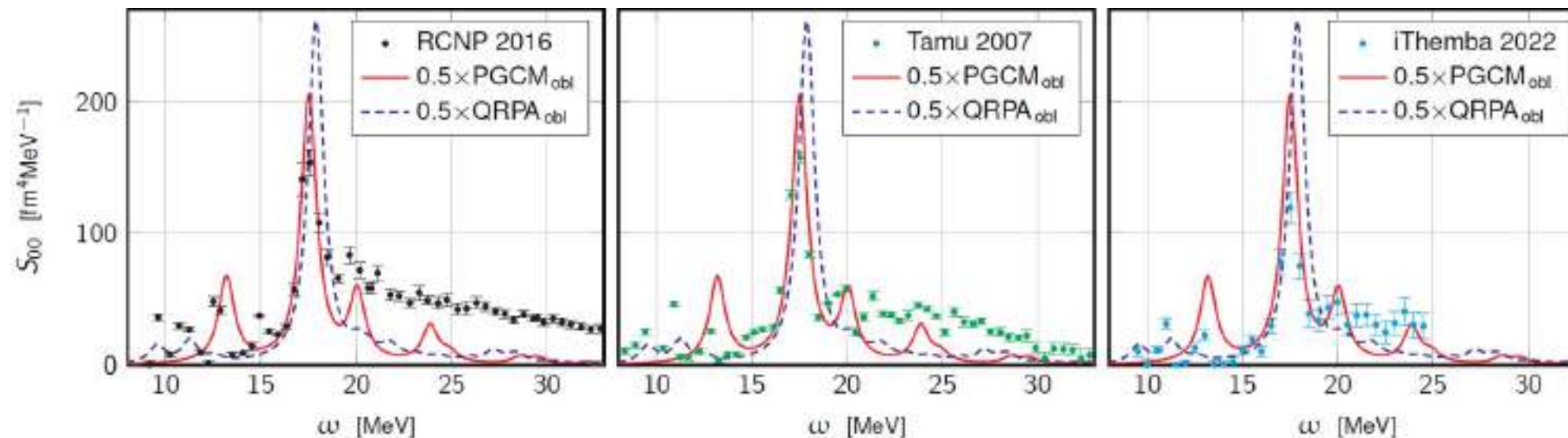
- Quantitative anharmonicities analysis
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- Case specific fragmentation, no quantitative correlation
- Projection contribution to fragmentation



Comparison to experimental data



Comparison to experimental data



Ab initio PGCM nicely reproduces the experimental data

- Nicer description of the main resonance and fragmentation

Experimental data are useful and promising to **test different many-body methods**

Data are not unambiguous, i.e. **higher resolution** would be beneficial

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- Shape coexistence
- Deformation

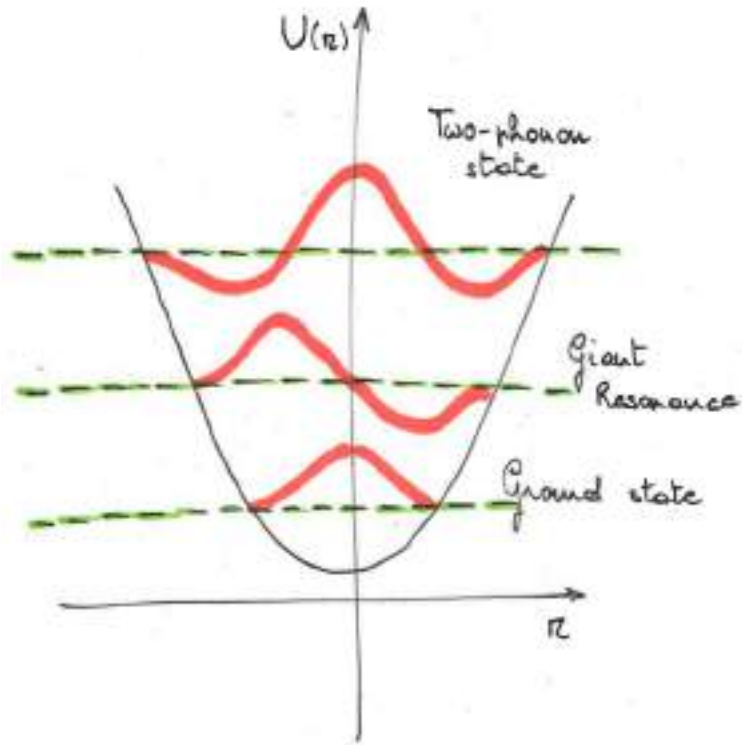
Multi-phonon states

- Proof of principle
- Realistic calculations

From finite nuclei to Astrophysics

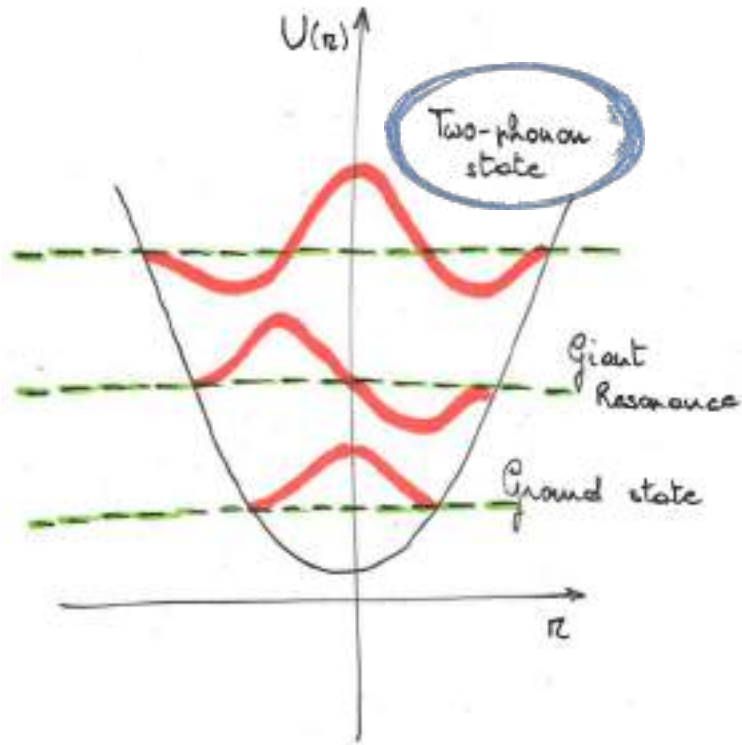
- Preliminary incompressibility results

Multi-phonon states in ^{46}Ti



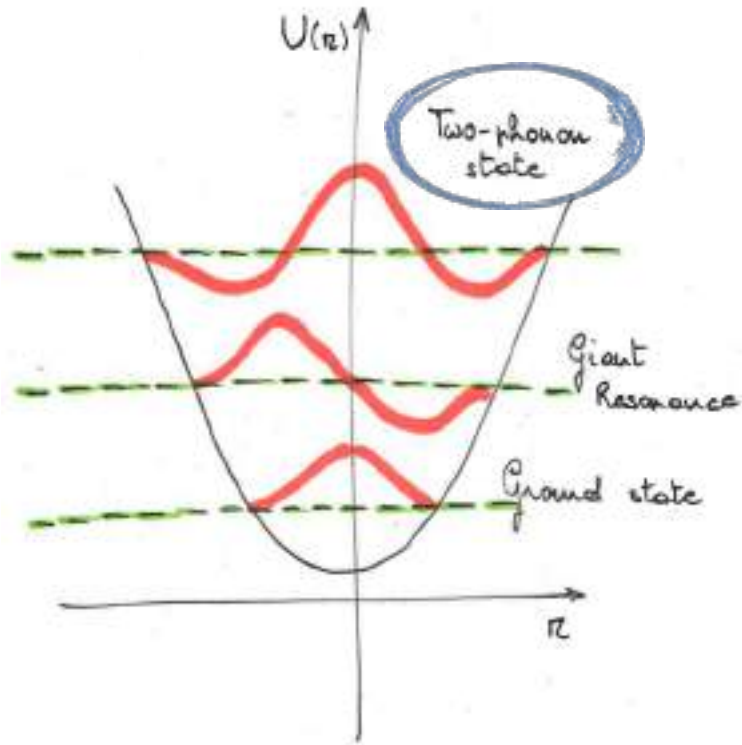
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Multi-phonon states in ^{46}Ti



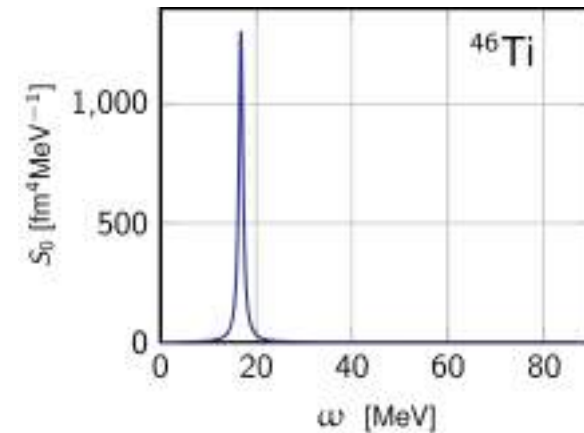
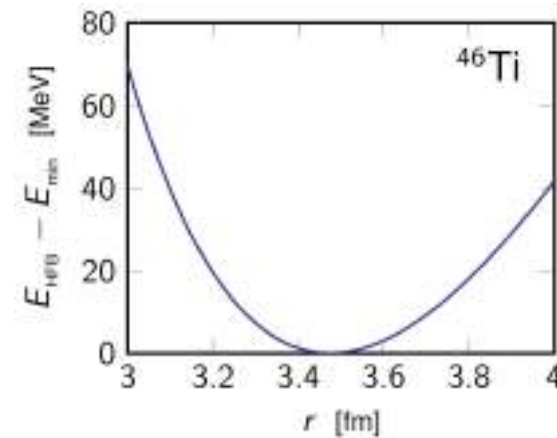
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- Higher phonons also exist **Multi-phonon states**

Multi-phonon states in ^{46}Ti

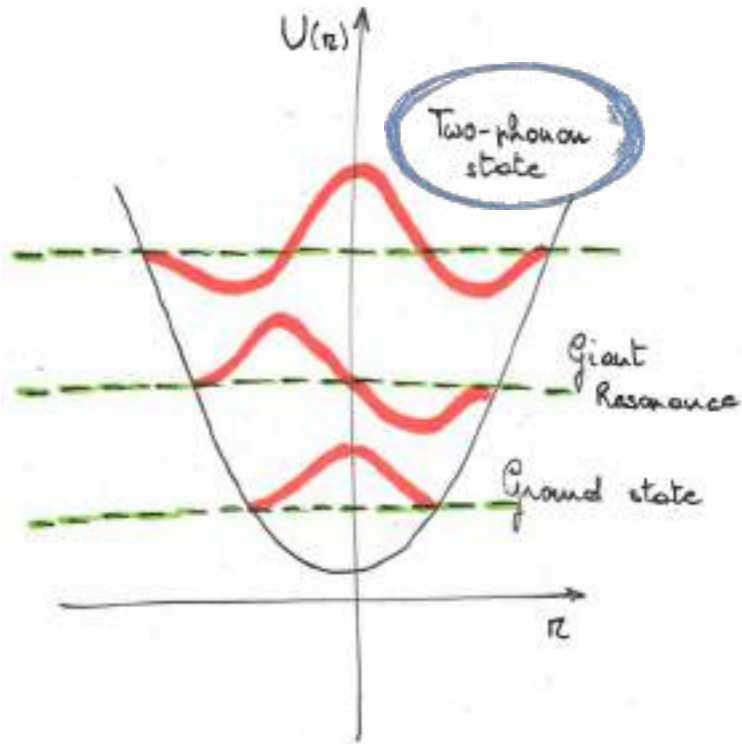


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One-dimensional PGCM calculation

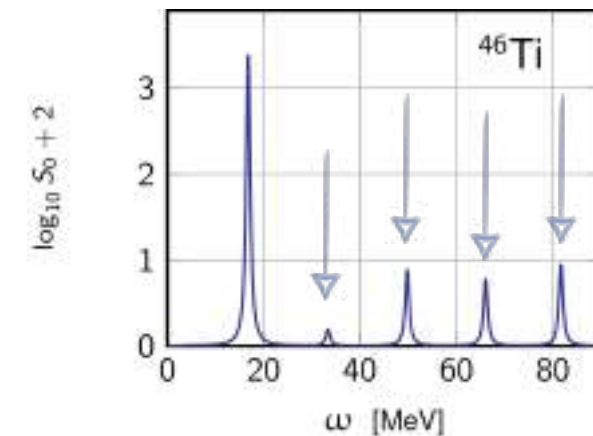
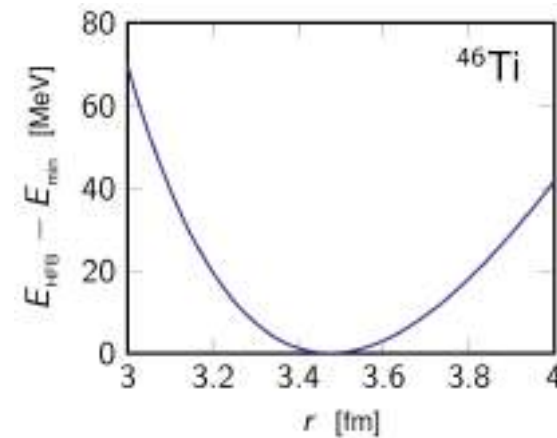


Multi-phonon states in ^{46}Ti



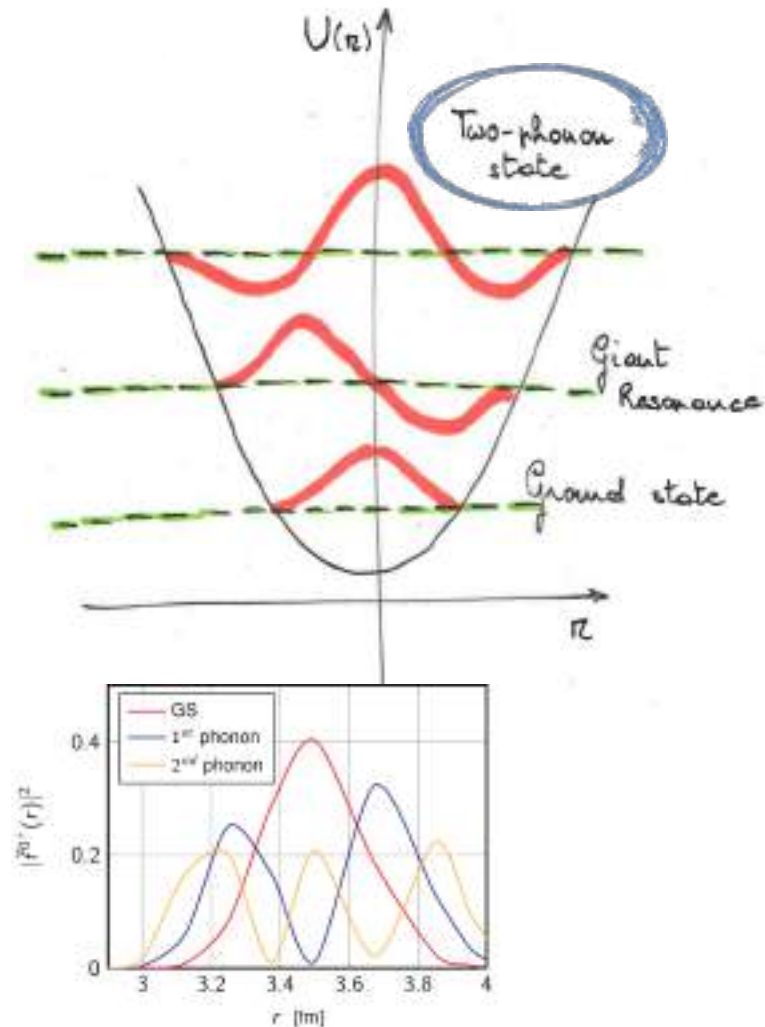
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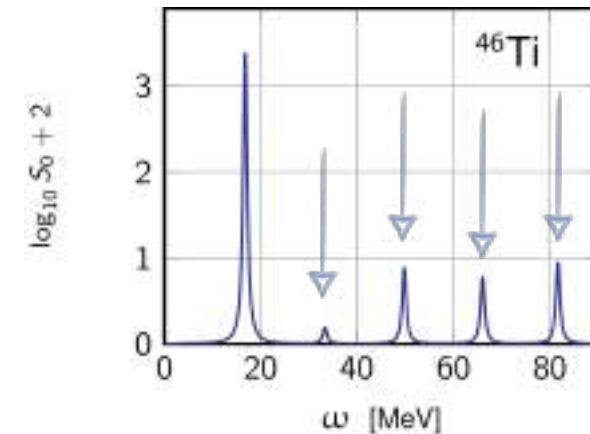
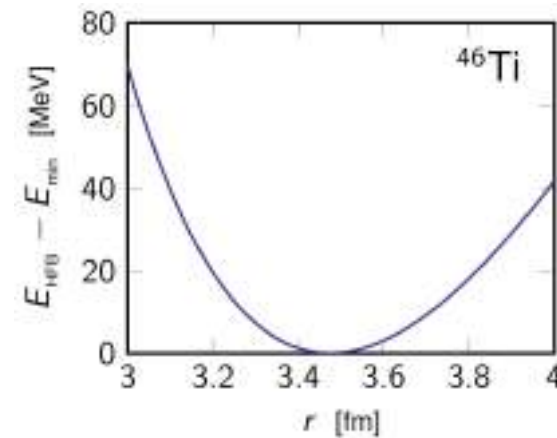
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Multi-phonon states in ^{46}Ti



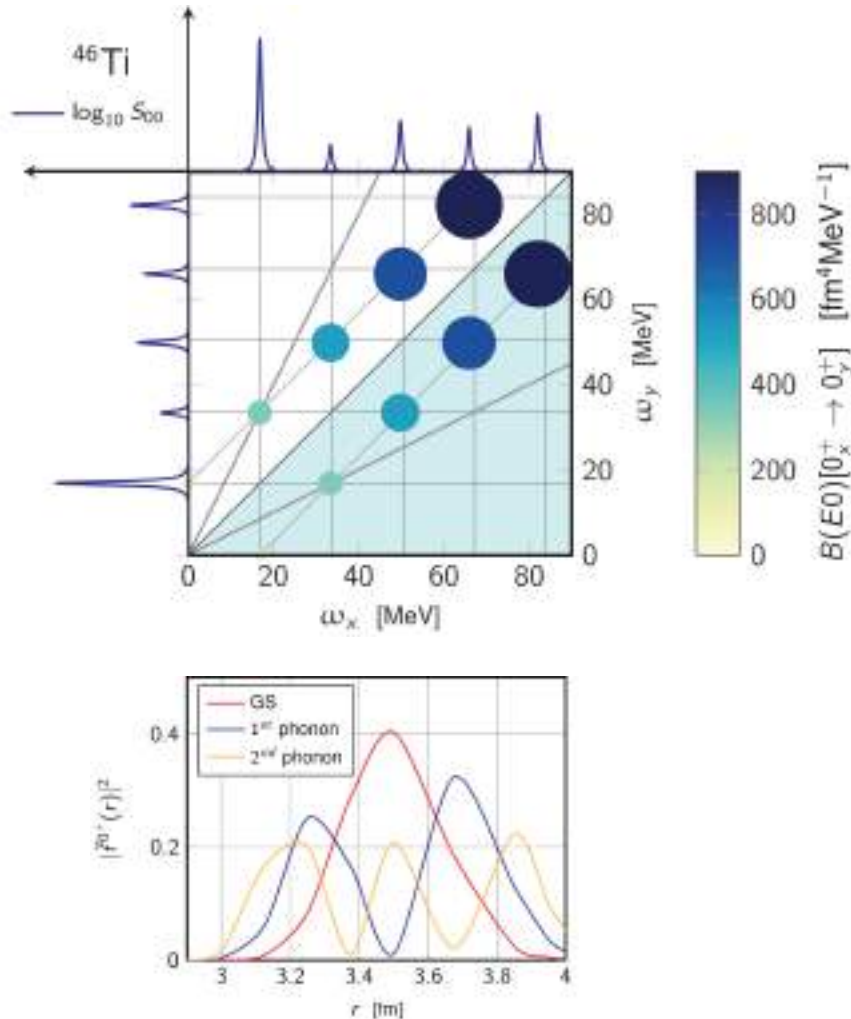
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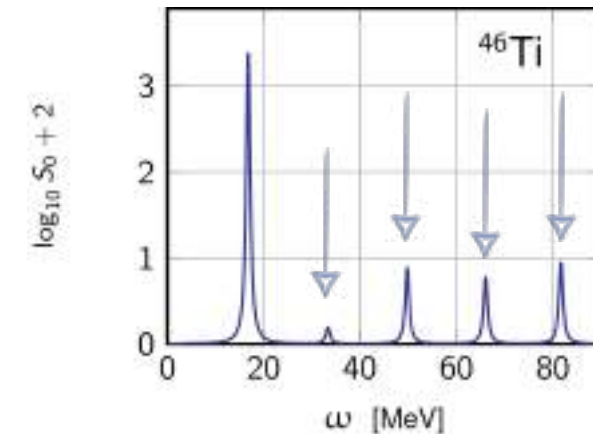
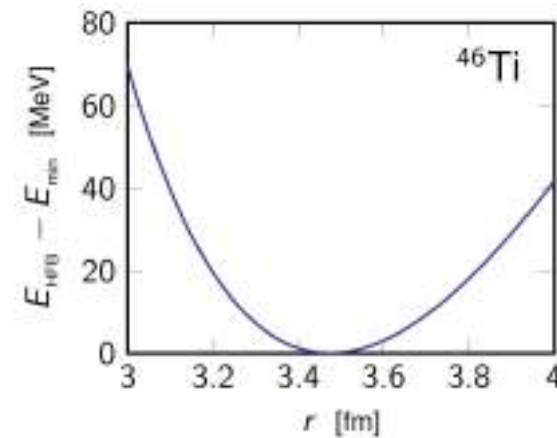
- PGCM predicts high-lying states**
- Close to the harmonic oscillator eigen-solutions

Multi-phonon states in ^{46}Ti



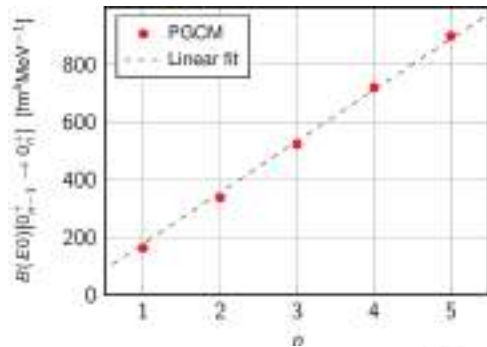
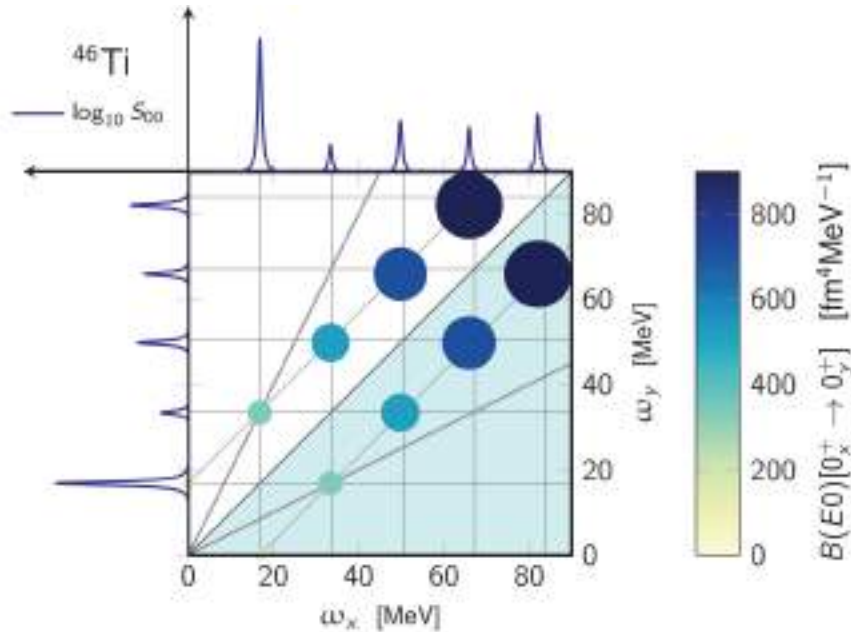
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One-dimensional PGCM calculation



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- Close to the harmonic oscillator eigen-solutions
- Transitions maximised between neighbouring phonons

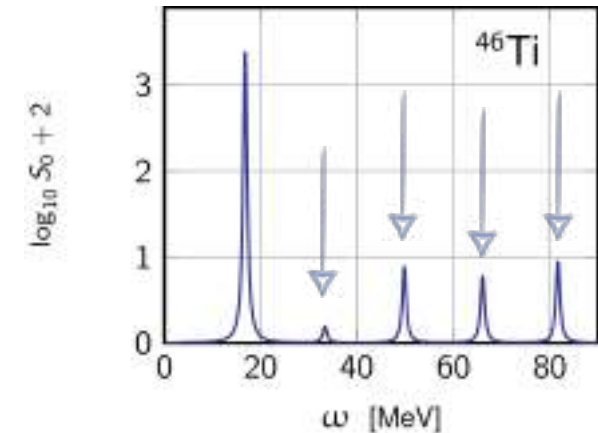
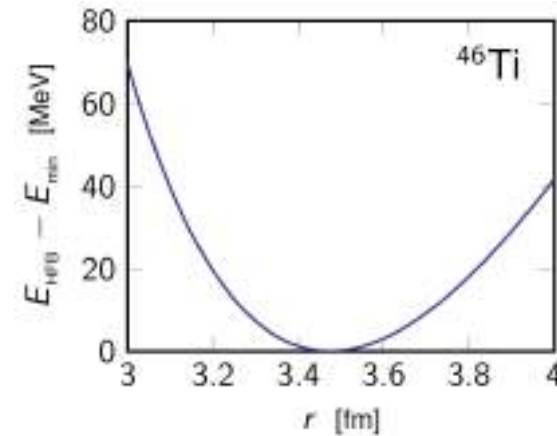
Multi-phonon states in ^{46}Ti



$$|\langle n-1 | r^2 | n \rangle|^2 = \frac{\hbar}{2m\omega} n$$

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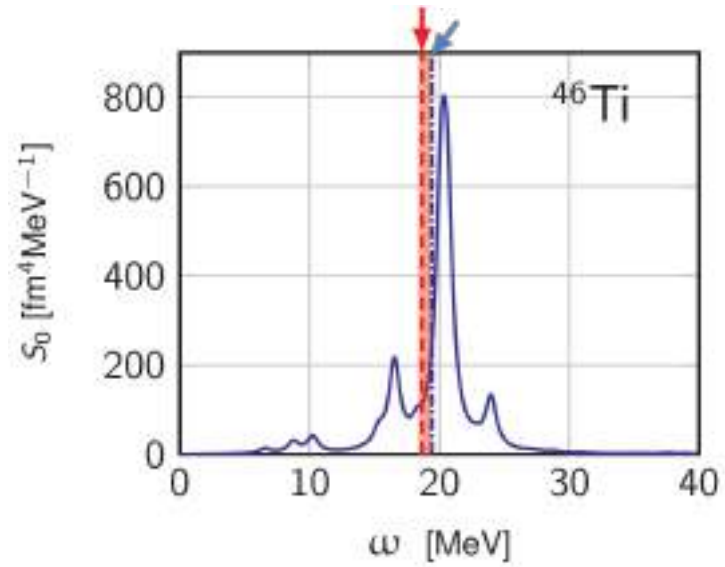
One-dimensional PGCN calculation



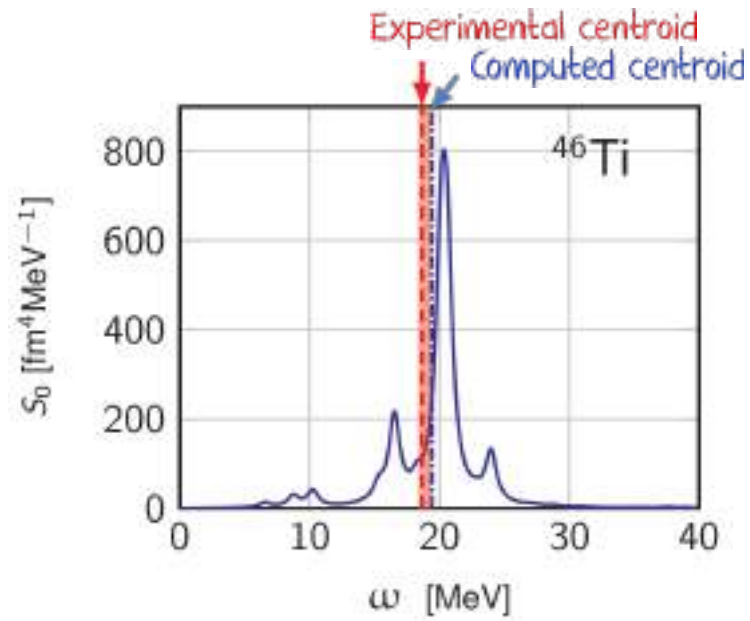
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- x** Linear trend in the transition strength

Two-dimensional calculations

- 2-D PGCM in the (r, β_2) plane

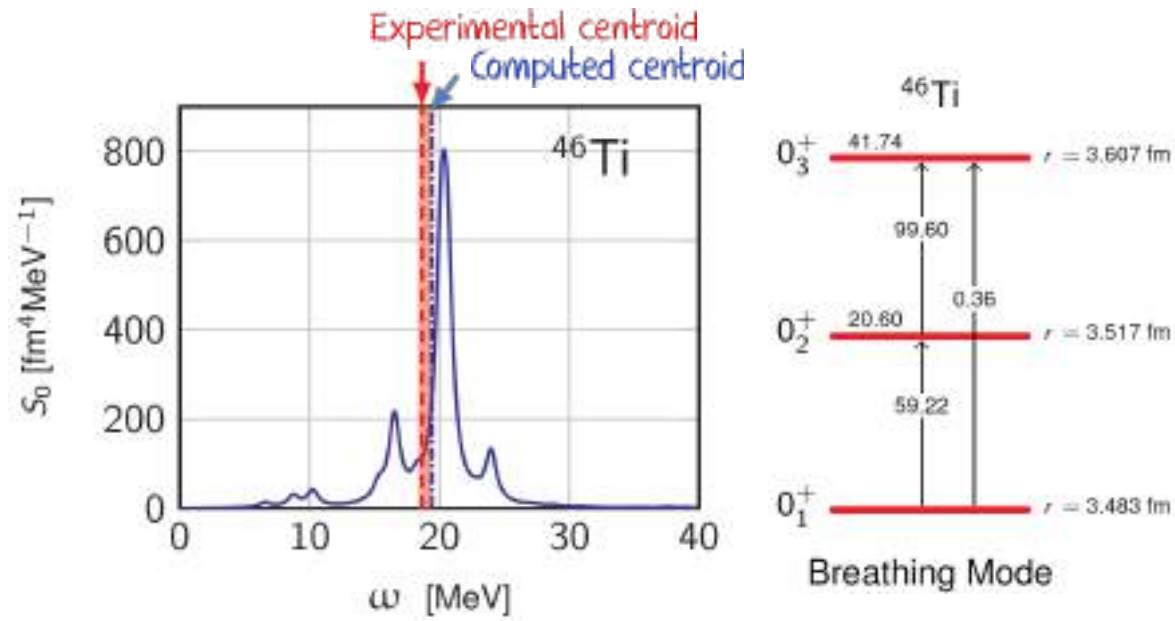


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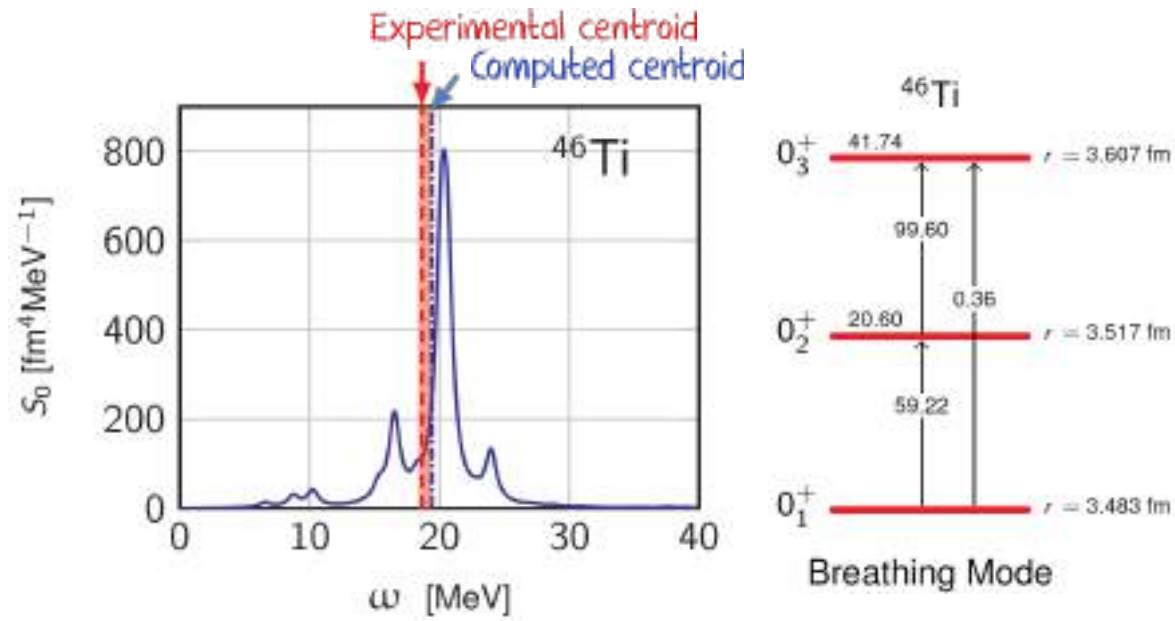
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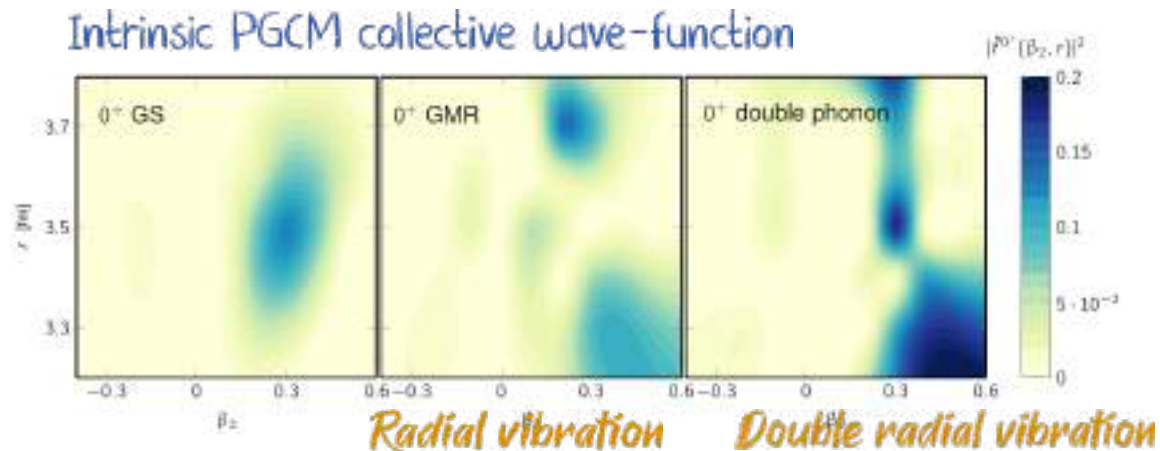


- 2-D PGCM in the (r, β_2) plane
- Good agreement with experiment
- Multi-phonon states observed

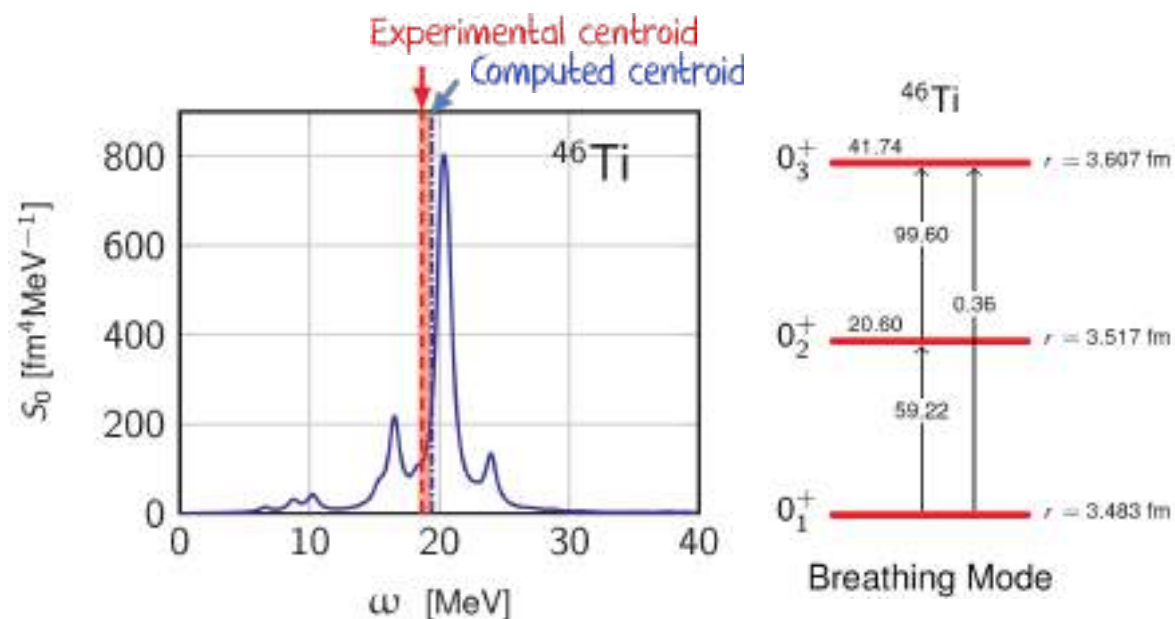
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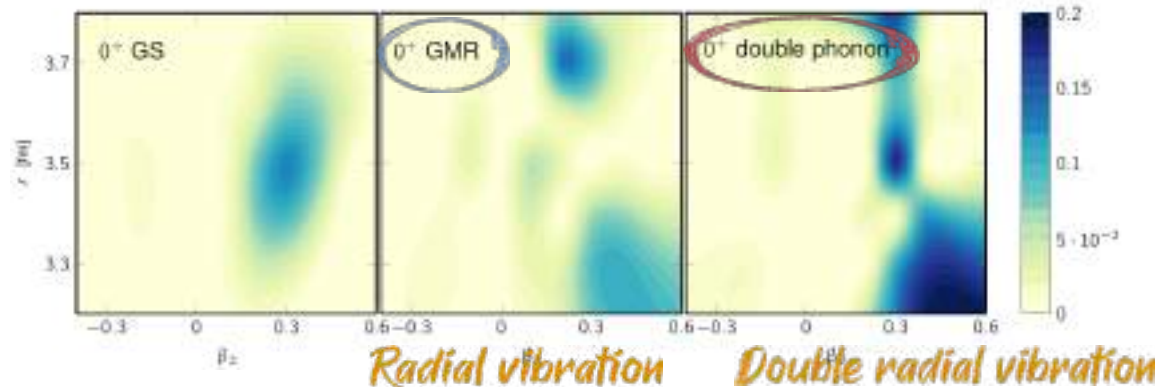
- 2-D PGCM in the (r, β_2) plane
- Good agreement with experiment
- Multi-phonon states observed
- Harmonicity well confirmed



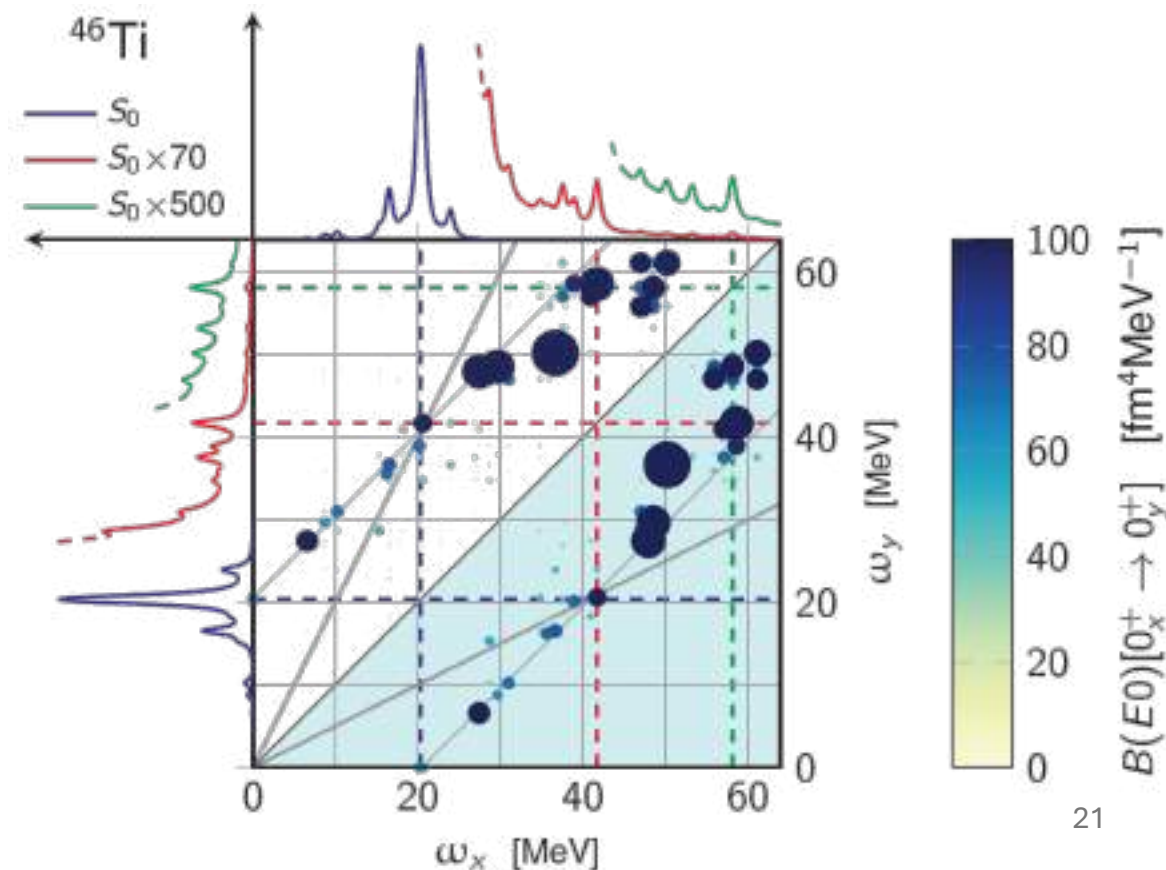
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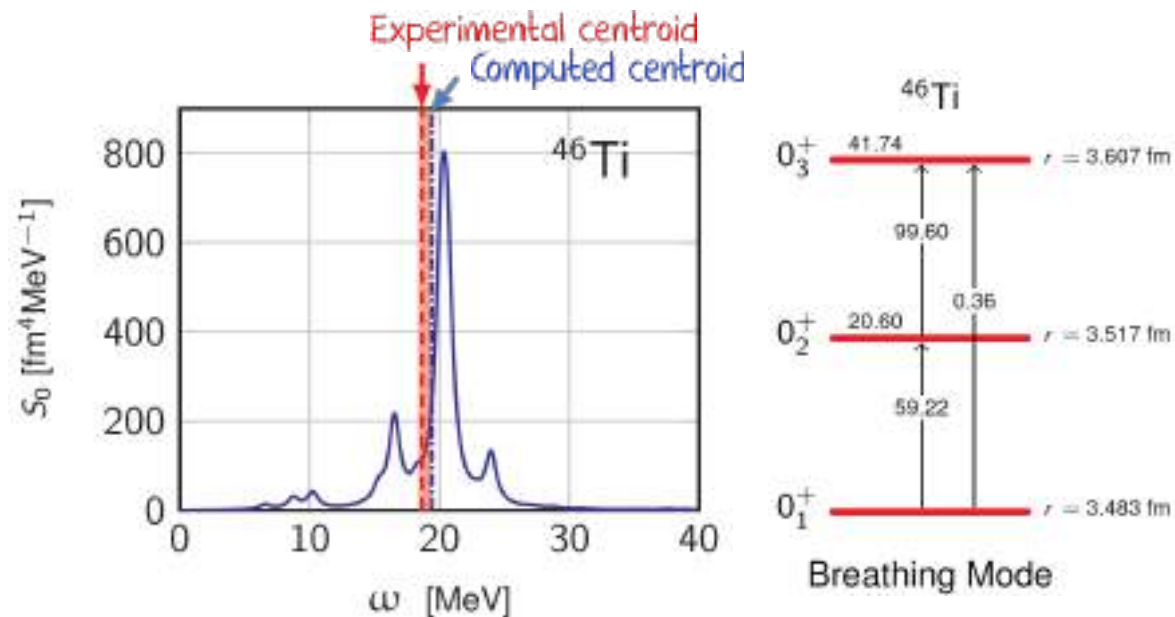
Intrinsic PGCM collective wave-function



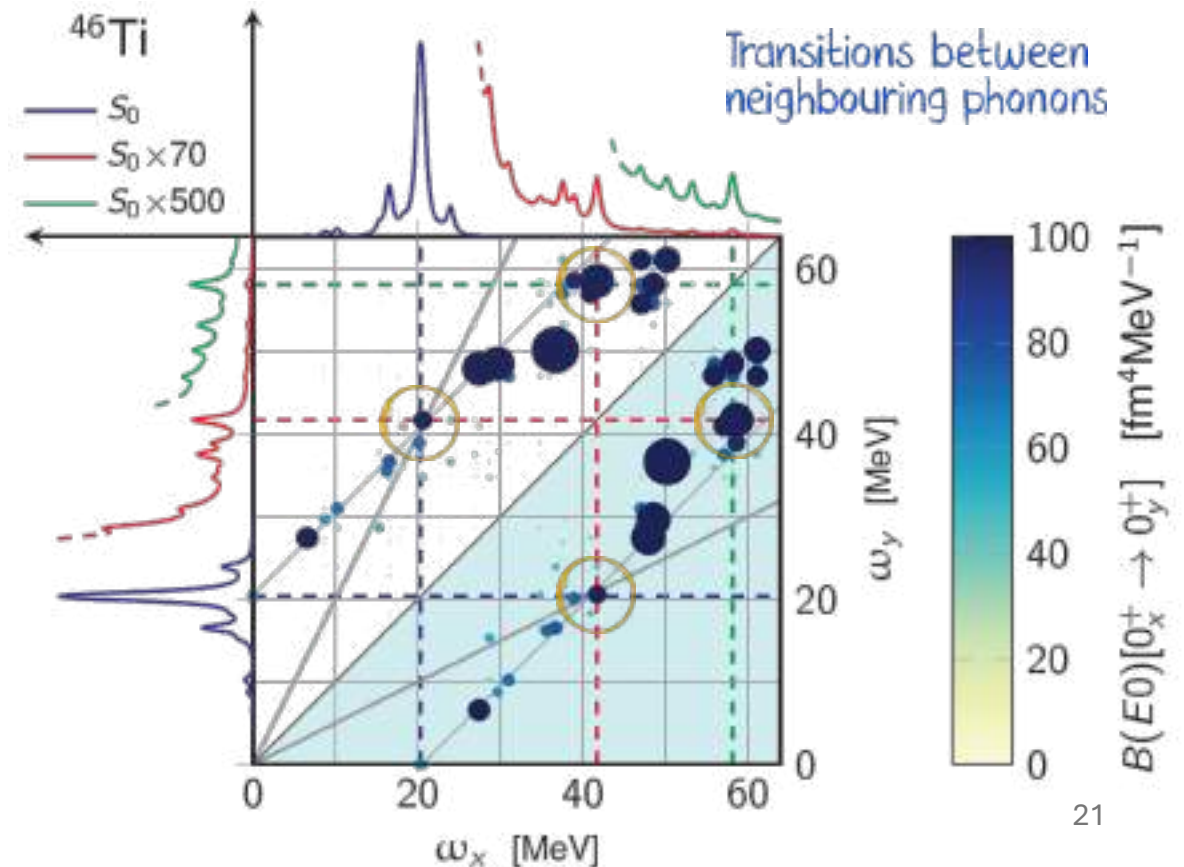
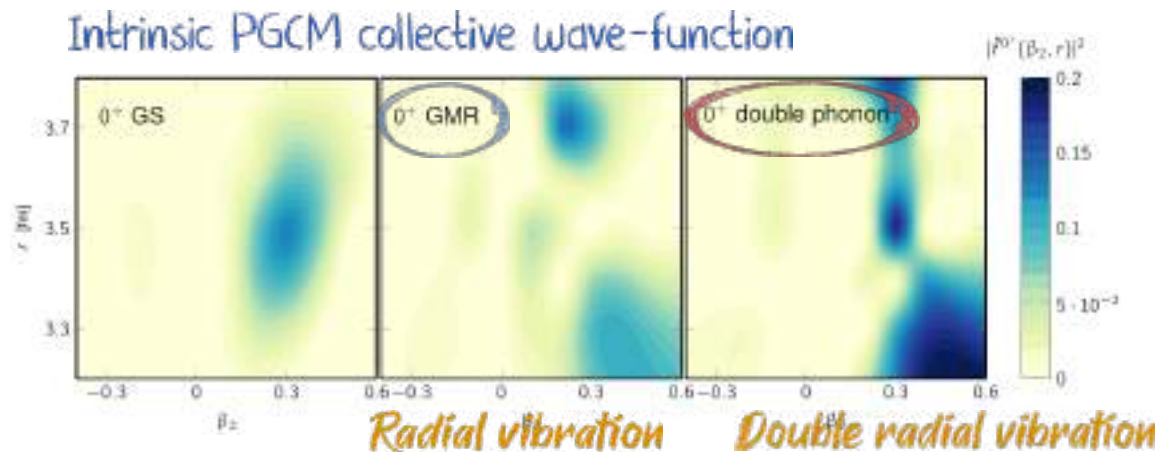
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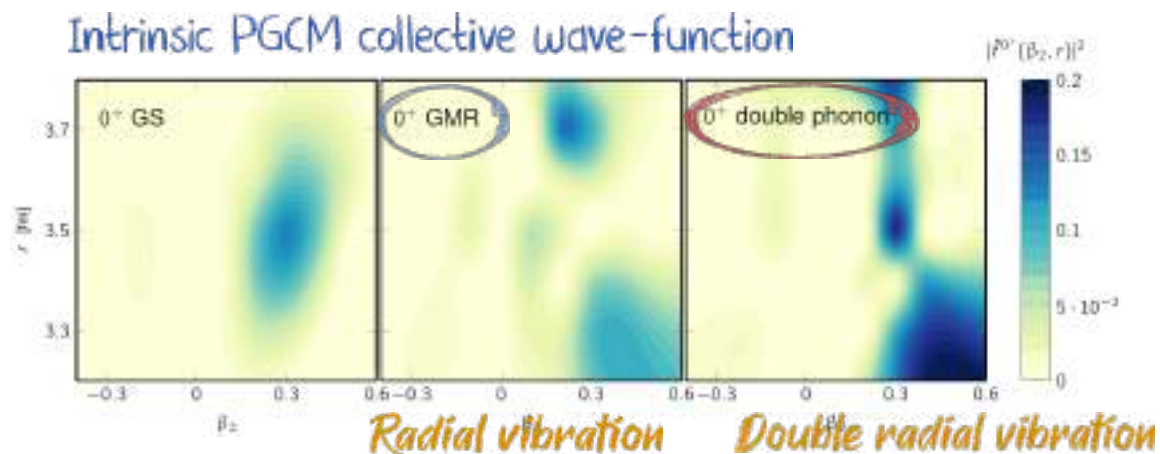
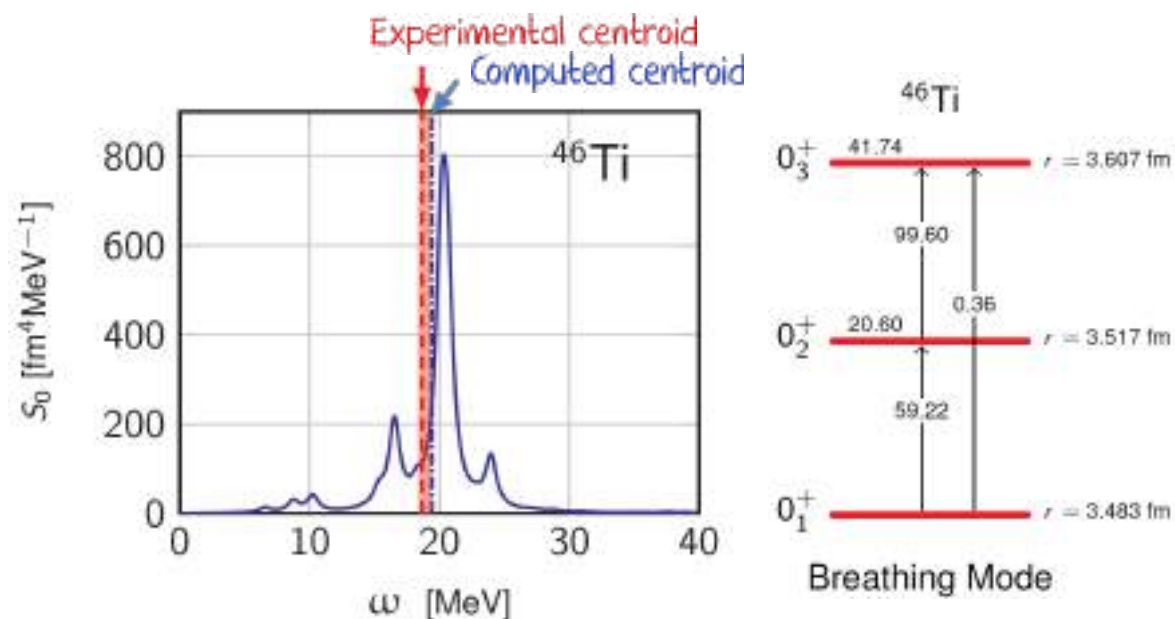
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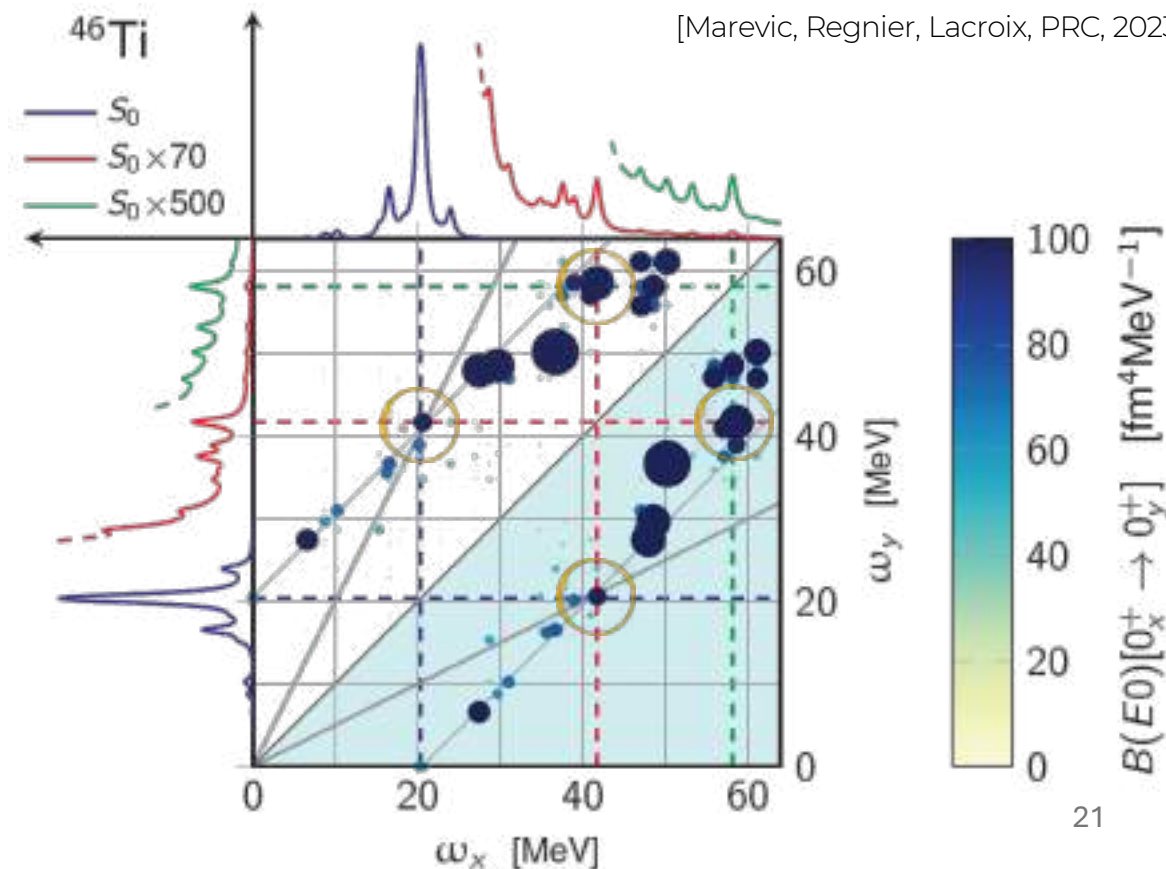


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[Marevic, Regnier, Lacroix, PRC, 2023]



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From finite nuclei to Astrophysics

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From finite nuclei to Astrophysics

Nuclear compressibility

- GMR

$$K_A = (M/\hbar^2)\langle r^2 \rangle E_{\text{GMR}}^2$$

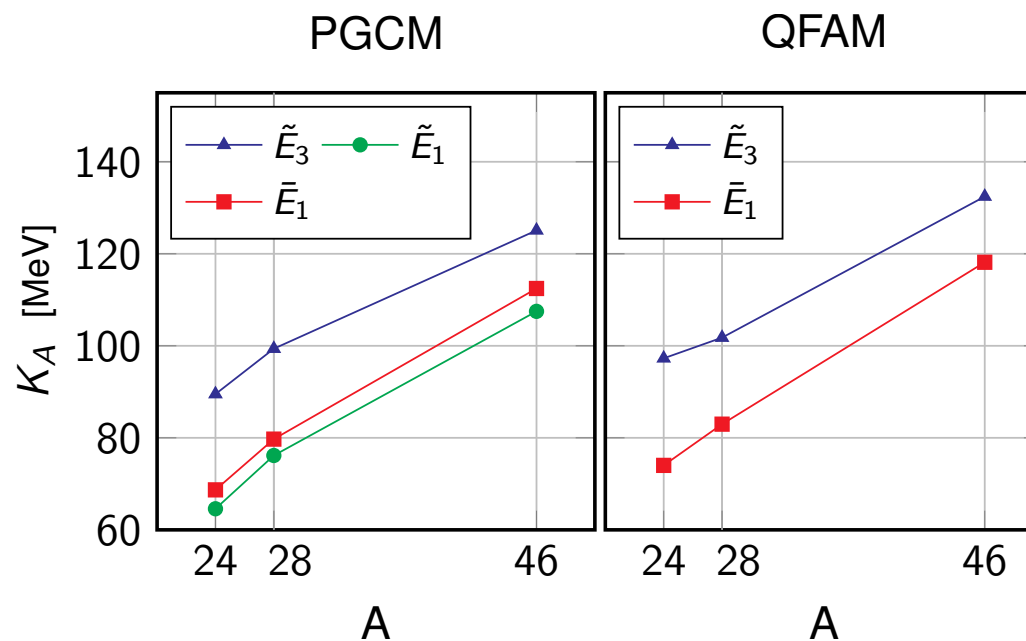
From finite nuclei to Astrophysics

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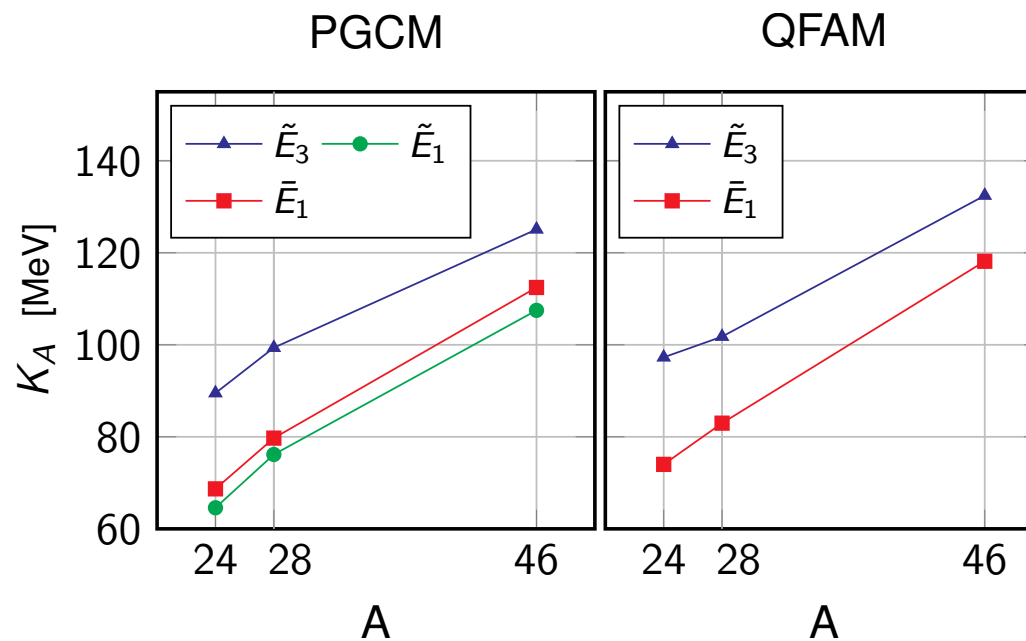
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Extrapolation to infinite matter



From finite nuclei to Astrophysics

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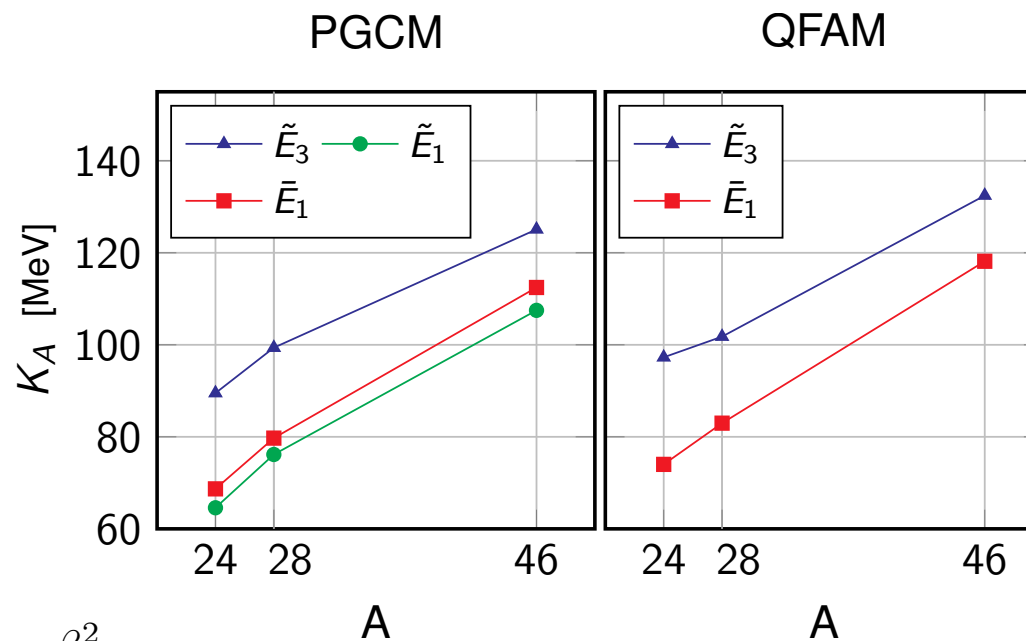
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Extrapolation to infinite matter

$$K_A = K_{\text{vol}} + K_{\text{surf}}A^{-1/3} + K_{\text{Coul}}Z^2A^{-4/3} + K_{\text{sym}}\beta^2$$

$$\beta \equiv \frac{N - Z}{N + Z}$$



From finite nuclei to Astrophysics

Nuclear compressibility

- GMR

$$K_A = (M/\hbar^2)\langle r^2 \rangle E_{\text{GMR}}^2$$

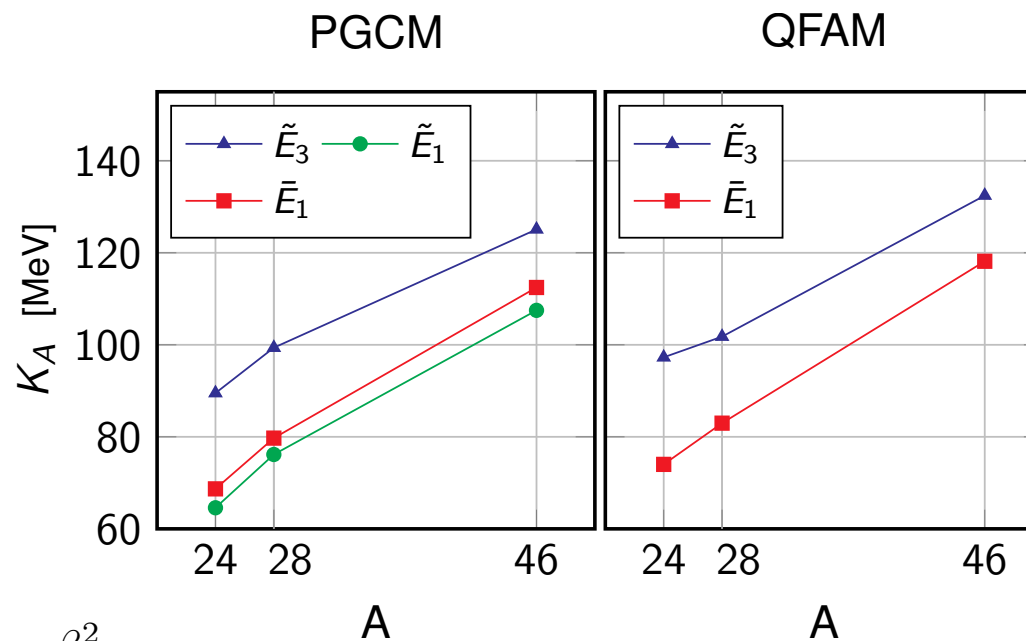
$$\tilde{E}_k = \sqrt{\frac{m_k}{m_{k-2}}} \quad \bar{E}_1 = \frac{m_1}{m_0}$$

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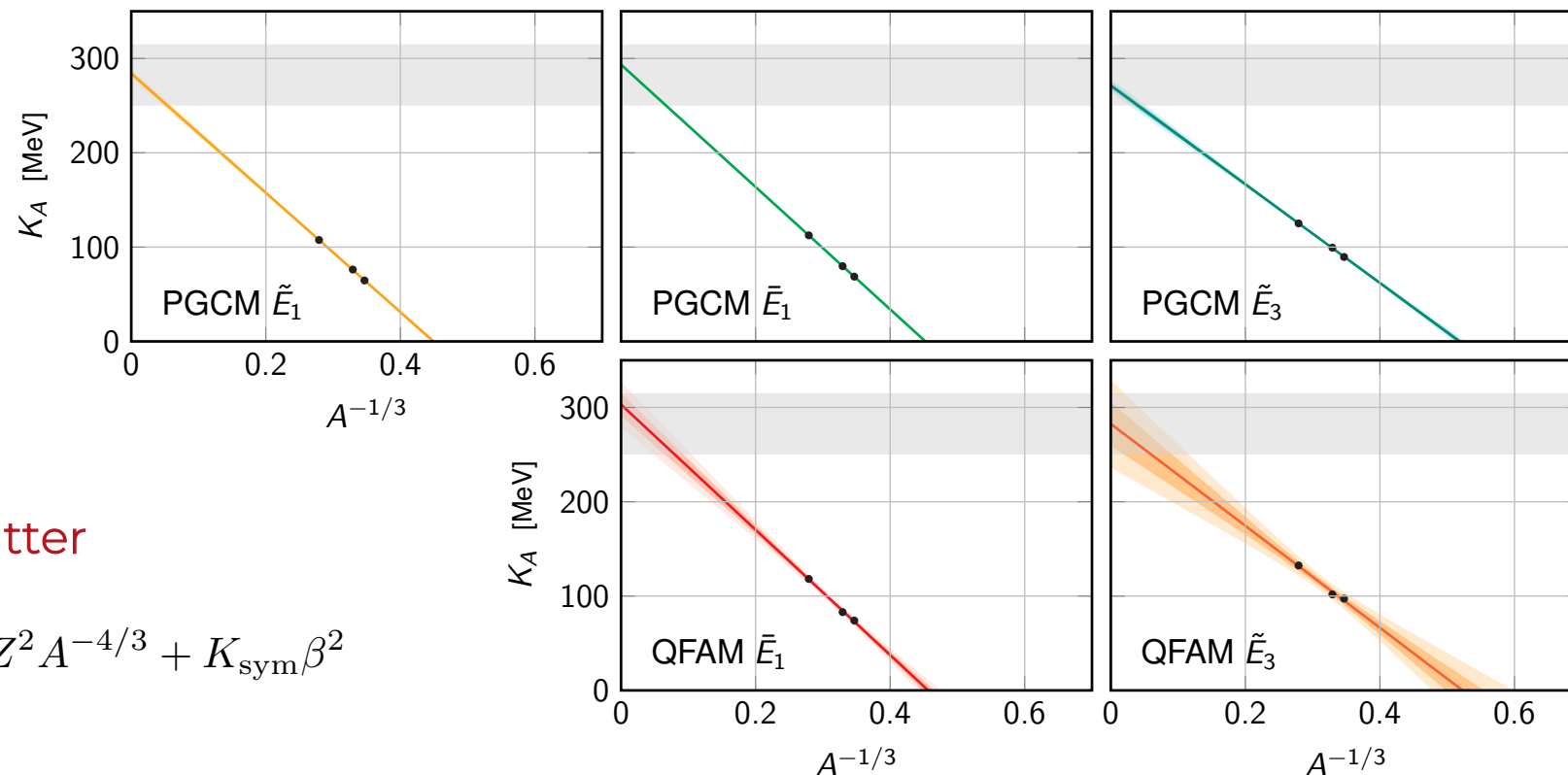
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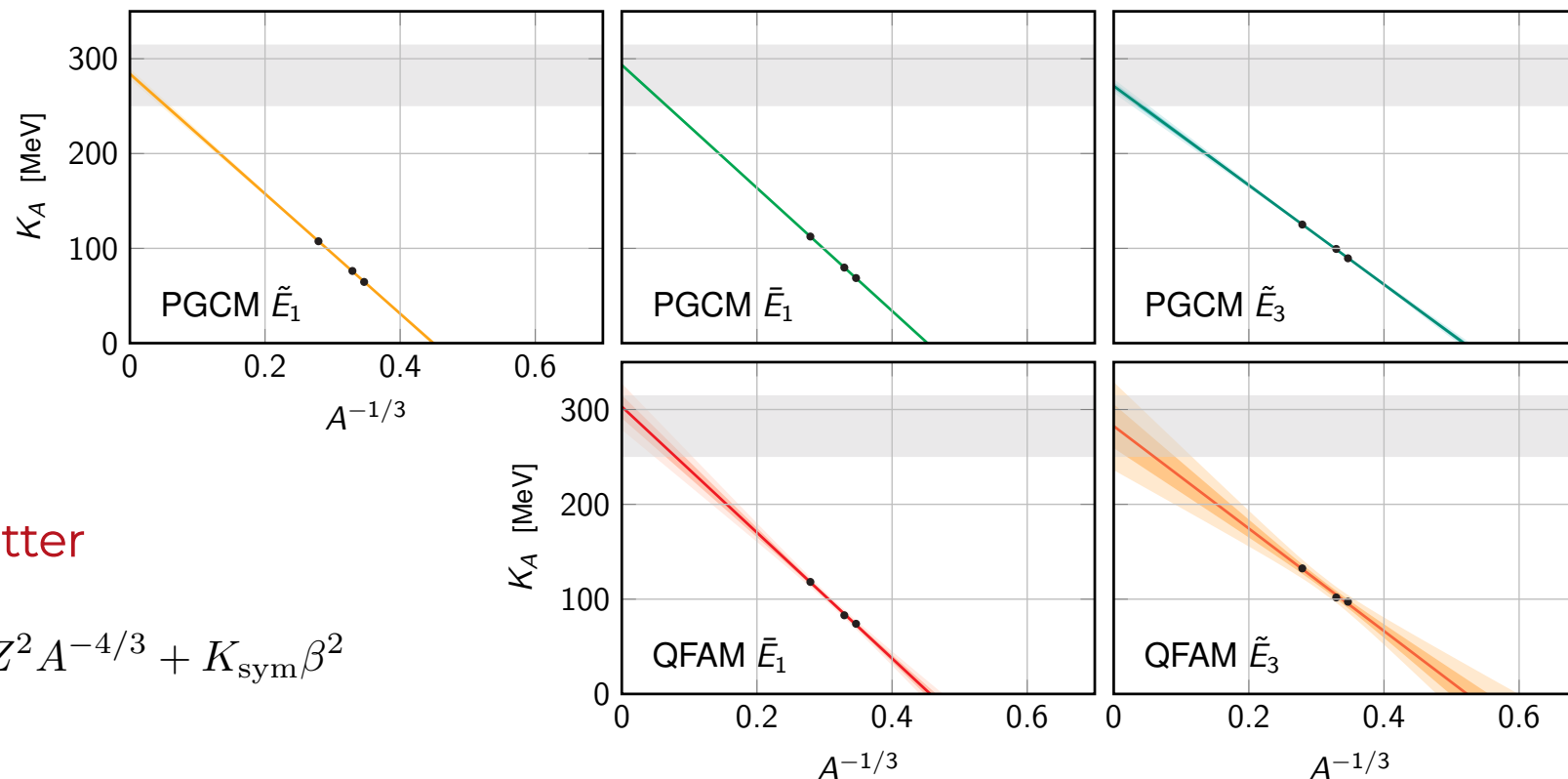
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Preliminary evaluation of K_{∞}

- Starting from **deformed** systems
- Extrapolation in **agreement** with commonly accepted values
- **Systematic** investigation in **heavier** systems (Sn, Mo isotopic chains, neutron rich)

Outline

1 Introduction

- Giant Resonances Physics
- The PGCM
- Link between PGCM and QRPA

2 Systematic study

- Numerical details
- Uncertainty estimate

Conclusions and perspectives

Results

3

Selected applications

- Shape coexistence
- Deformation

Multi-phonon states

- Proof of principle
- Realistic calculations

From finite nuclei to Astrophysics

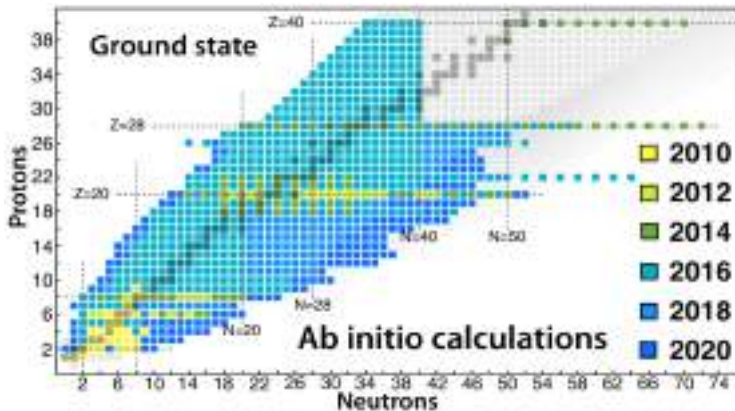
- Preliminary incompressibility results

Current frontiers

SPECTROSCOPY

- Single-particle
- Collective excitations

OPEN-SHELL



[Hergert, Front. Phys, 2020]

HEAVY-MASS SYSTEMS

ACCURACY

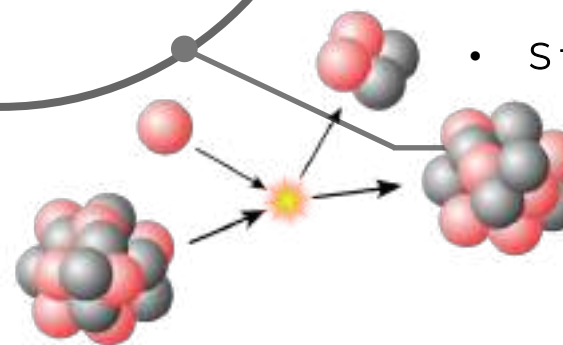
$$H = T + V_{\text{LO}} + V_{\text{NLO}} + V_{\text{N}^2\text{LO}} + \dots$$

$$|\Psi_k^A\rangle = \Omega|\Theta_k^{(0)}\rangle = |\Theta_k^{(0)}\rangle + |\Theta_k^{(1)}\rangle + |\Theta_k^{(2)}\rangle + \dots$$

UNCERTAINTIES

- Systematic uncertainties
 - Hamiltonian
 - A-body solution
 - Basis representation
- Statistical uncertainties

REACTIONS



Conclusions and perspectives

Perspectives

SPECTROSCOPY

OPEN-SHELL

UNCERTAINTIES

Take-away messages

Systematic comparison to new and existing **exp data**

Deeper **uncertainty quantification** (EC)

Develop full symmetry-conserving QRPA

More systematic choice of the **GC**

PGCM **reliable** tool for *ab initio** spectroscopy

Access to **new observables** and phenomena in *ab initio*



How is this even *ab initio*?

How do you choose the collective coordinates?

Thanks for the attention



Benjamin Bally

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