

Collective excitations of even-even open-shell nuclei via PGCM and QRPA calculations

Nuclear ab initio spectroscopy workshop

Espace de Structure Nucléaire Théorique

May 21st, 2024

Andrea Porro

Technische Universität Darmstadt

WITH

Thomas Duguet

CEA IRFU

Jean-Paul Ebran

CEA DAM

Mikael Frosini

CEA Cadarache

Robert Roth

TUD

Vittorio Somà

CEA IRFU



Based on



arXiv > nucl-th > arXiv:2402.02228

Nuclear Theory

[Submitted on 7 Feb 2024]

Ab initio description of monopole resonances in light- and medium-mass nuclei: I. Technical aspects and uncertainties of ab initio PGCM calculations

Andrea Porro, Thomas Duguet, Jean-Paul Ebran, Mikael Frosini, Robert Roth, Vittorio Somà

[arXiv:2402.15901, 2024]



arXiv > nucl-th > arXiv:2402.15901

Nuclear Theory

[Submitted on 24 Feb 2024]

Ab initio description of monopole resonances in light- and medium-mass nuclei: II. Ab initio PGCM calculations in ^{46}Ti , ^{28}Si and ^{24}Mg

Andrea Porro, Thomas Duguet, Jean-Paul Ebran, Mikael Frosini, Robert Roth, Vittorio Somà

[arXiv:2402.02228, 2024]



arXiv > nucl-th > arXiv:2404.14154

Nuclear Theory

[Submitted on 22 Apr 2024]

Ab initio description of monopole resonances in light- and medium-mass nuclei: III. Moments evaluation in ab initio PGCM calculations

Andrea Porro, Thomas Duguet, Jean-Paul Ebran, Mikael Frosini, Robert Roth, Vittorio Somà

Acces
• View
• HTML
• TeX
• Other
How to
Current I

[arXiv:2404.14154, 2024]

Fourth (and last) paper of the series coming soon

Outline

Outline

1

Introduction

- Giant Resonances Physics
- The PGCM
- Link between PGCM and QRPA

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2 Systematic study

- Numerical details
- Uncertainty estimate

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Selected applications

- Shape coexistence
- Deformation

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- Proof of principle
- Realistic calculations

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From finite nuclei to Astrophysics

- Preliminary incompressibility results

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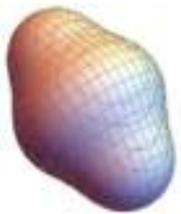
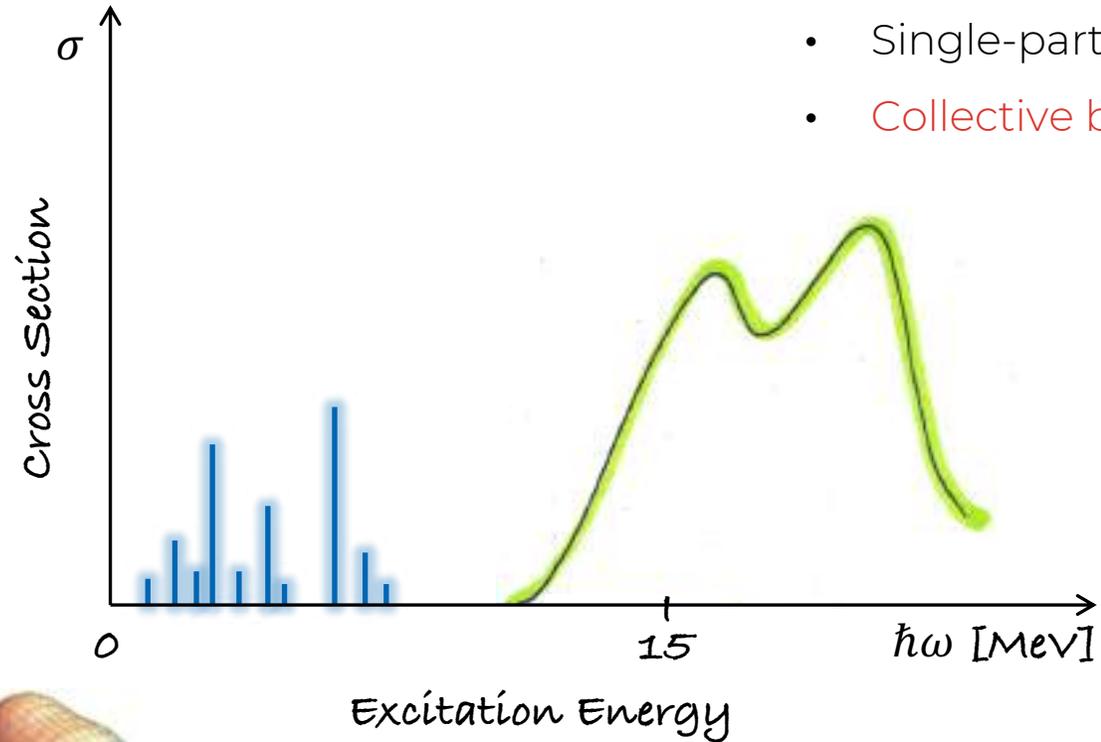
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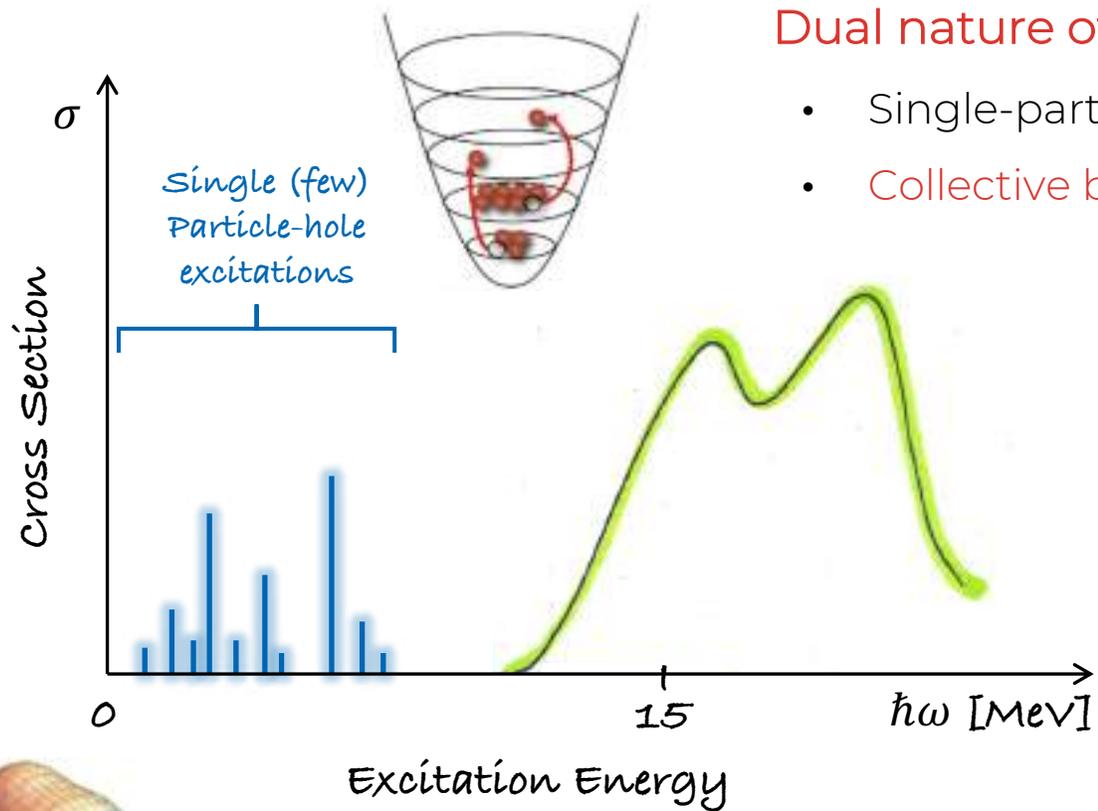
Giant Resonances

Dual nature of nucleus

- Single-particle features
- Collective behaviour

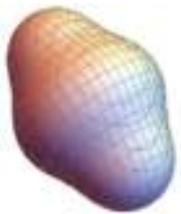


Giant Resonances

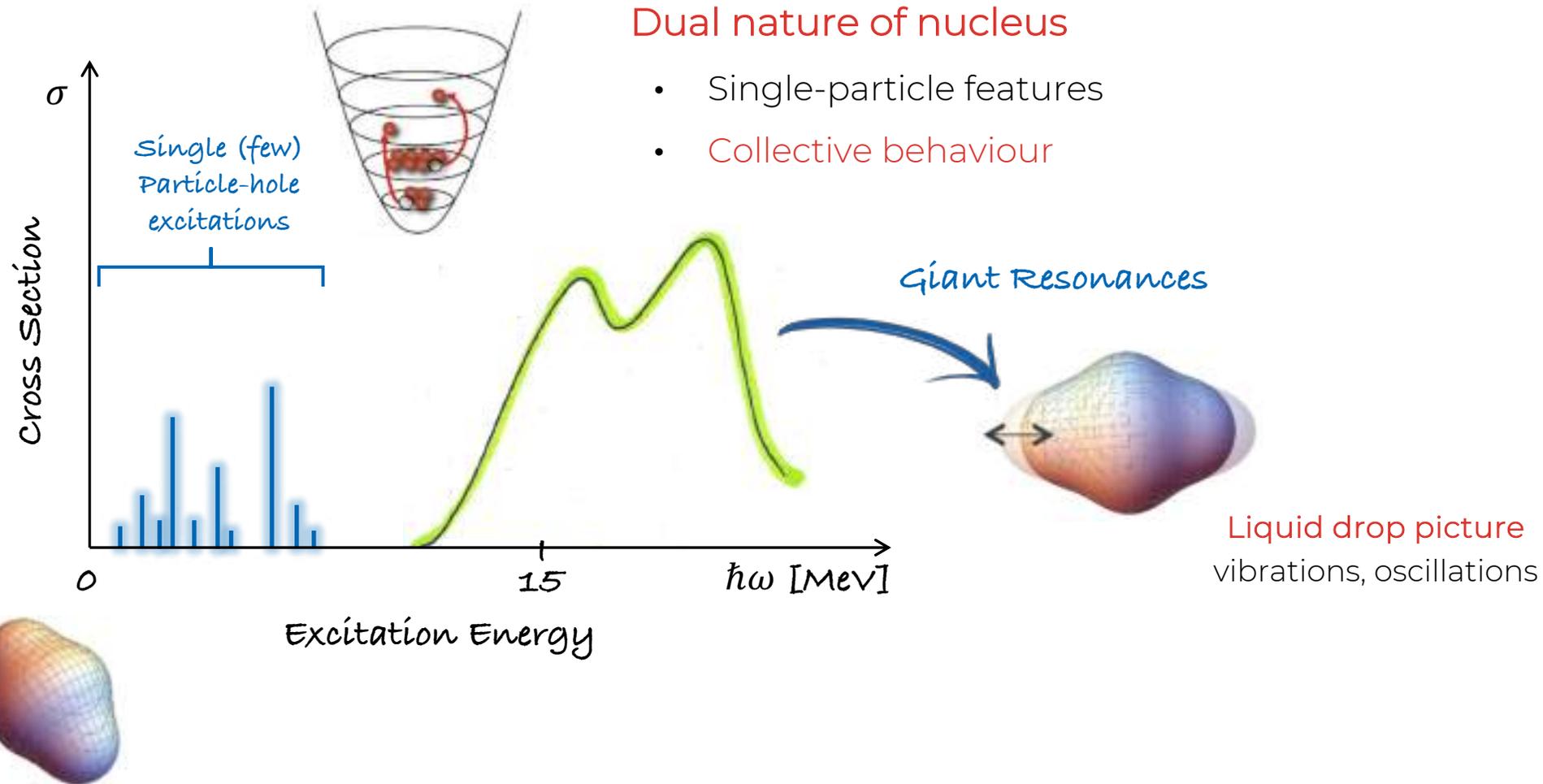


Dual nature of nucleus

- Single-particle features
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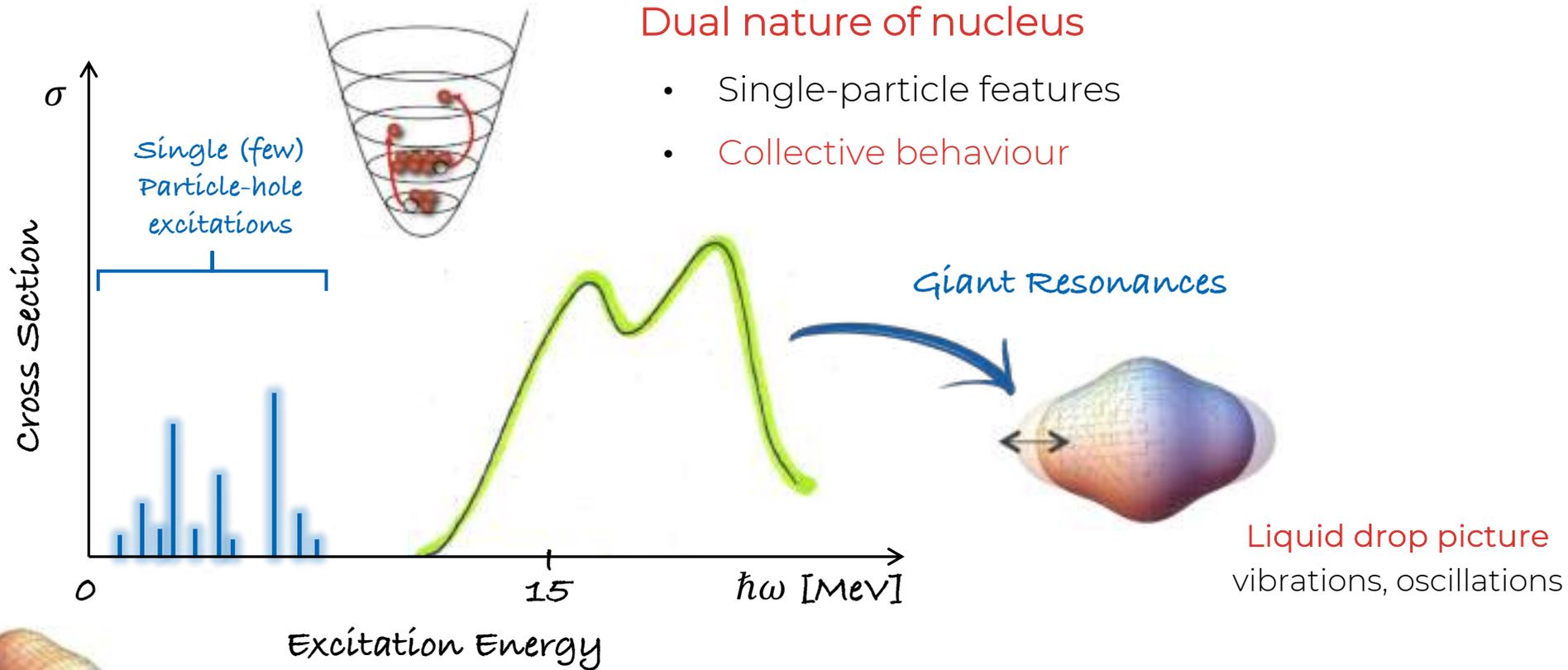
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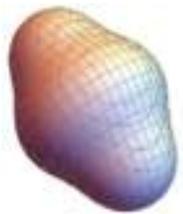
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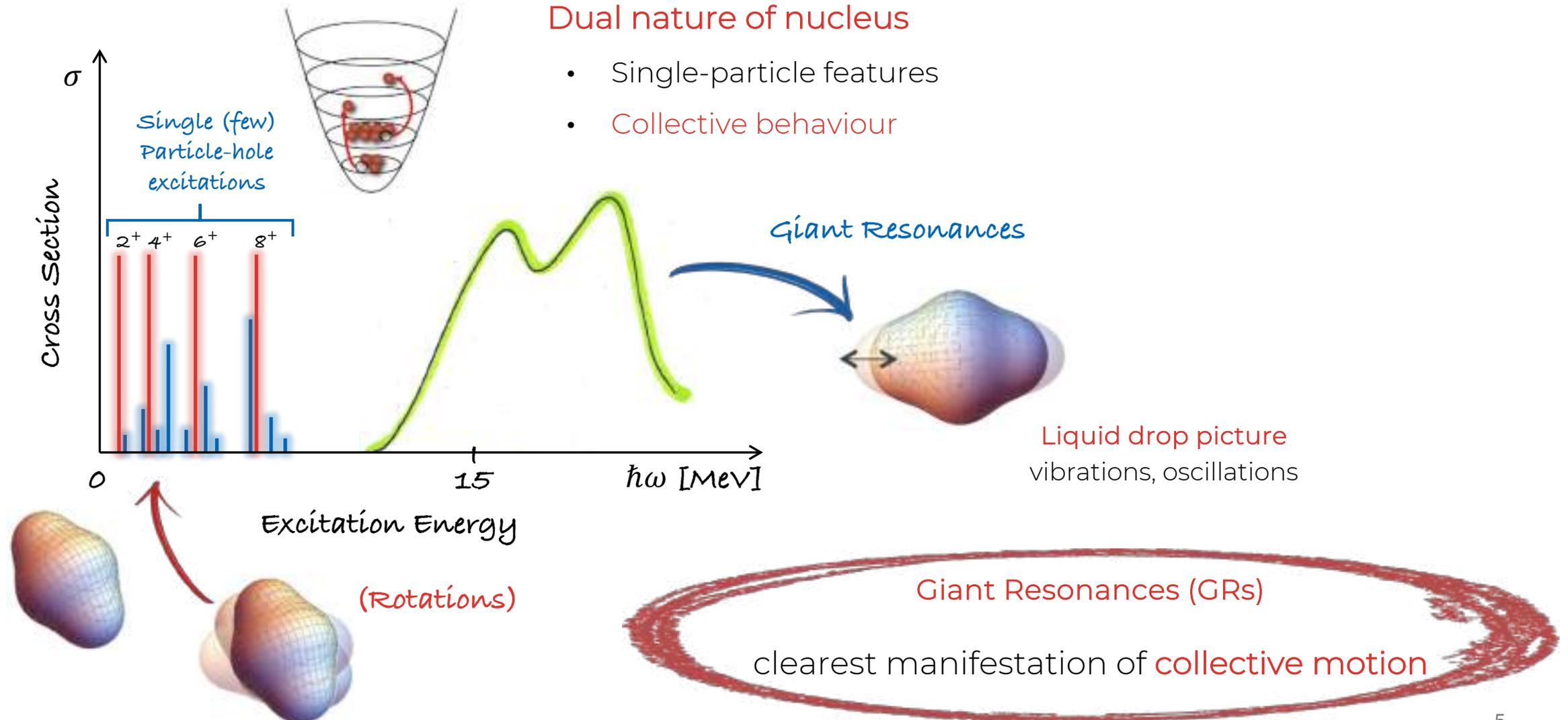
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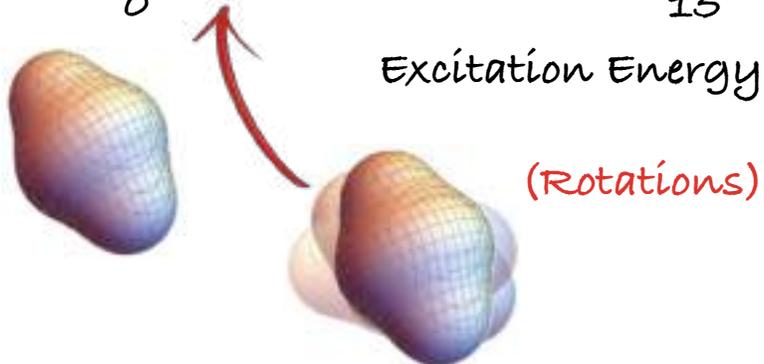
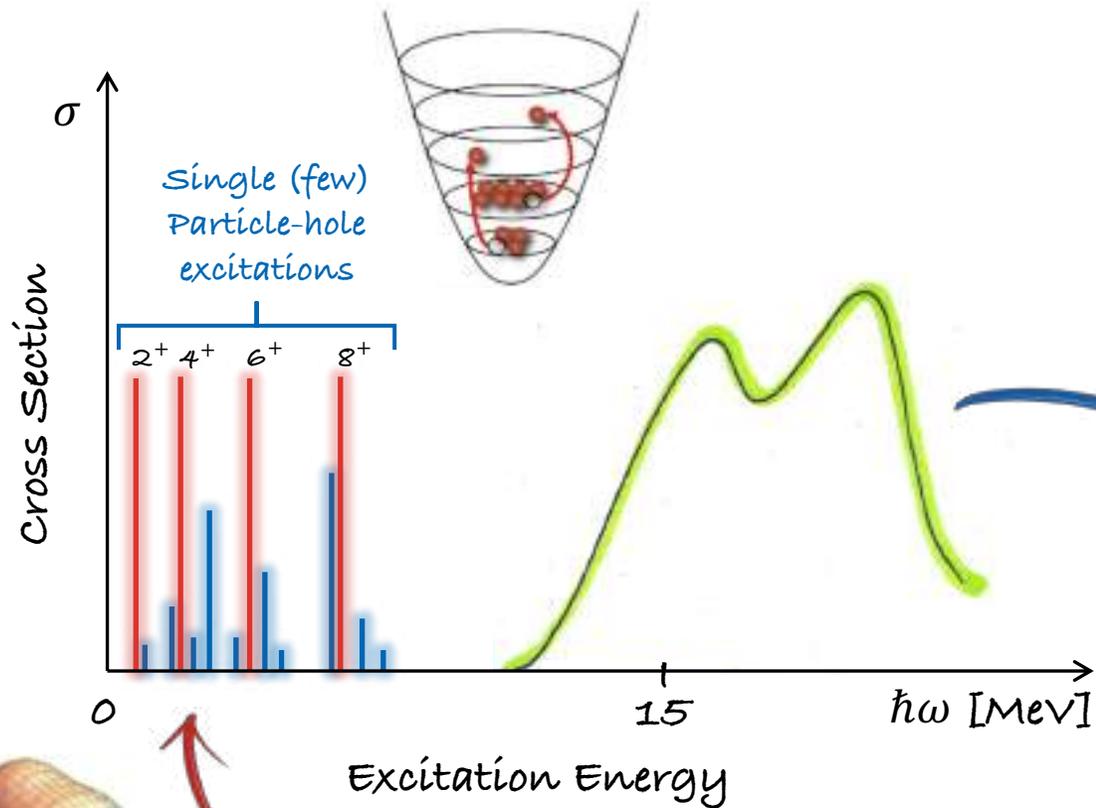


Giant Resonances (GRs)
clearest manifestation of collective motion

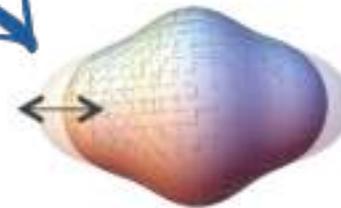
Giant Resonances



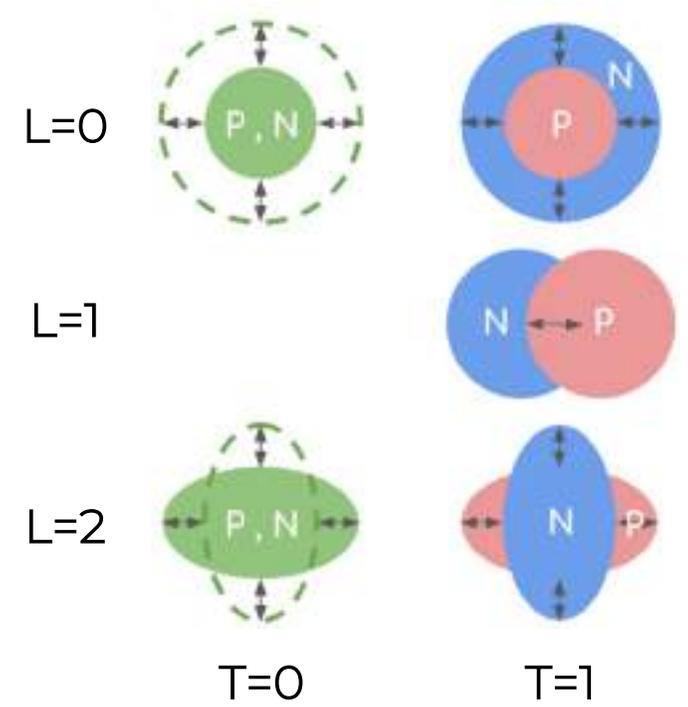
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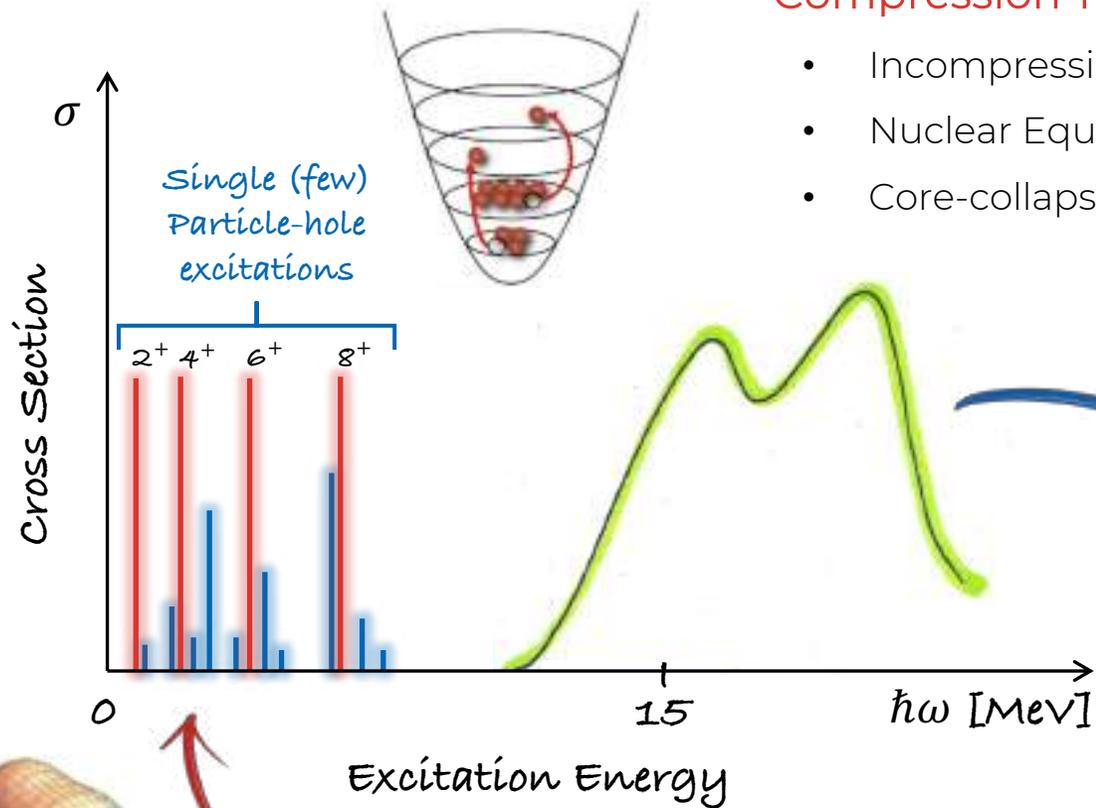


Liquid drop picture
vibrations, oscillations



Giant Resonances (GRs)
clearest manifestation of collective motion

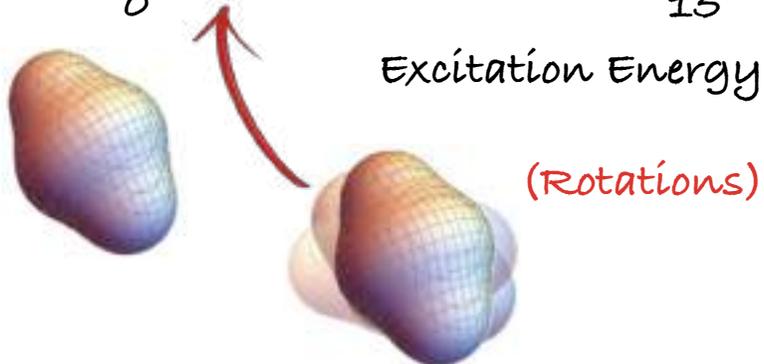
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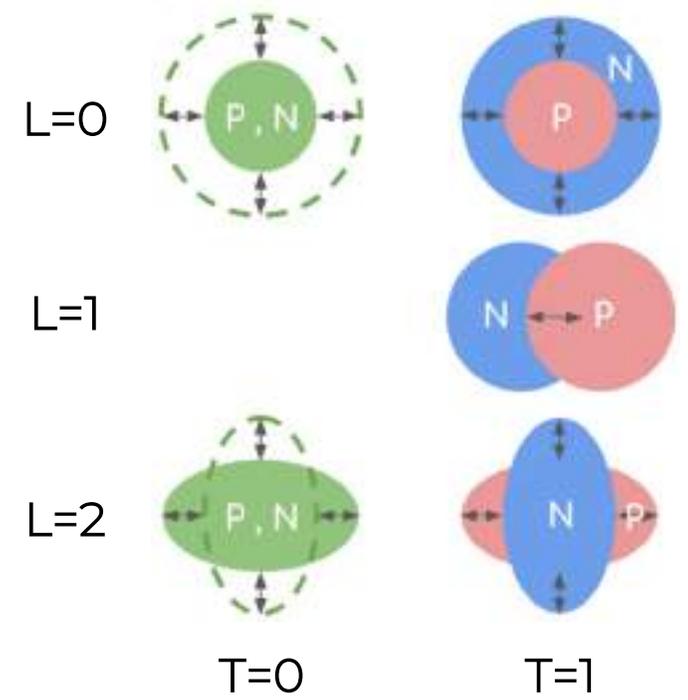
Compression-mode resonances

- Incompressibility of nuclear matter K_∞
- Nuclear Equation of State
- Core-collapse supernova explosion

Giant Resonances



Liquid drop picture
vibrations, oscillations



Giant Resonances (GRs)
clearest manifestation of collective motion

Projected Generator Coordinate Method

Schrödinger equation $H |\Psi_n\rangle = E_n |\Psi_n\rangle$

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Open-shell systems

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Open-shell systems



Strong **static correlations**

Projected Generator Coordinate Method

Schrödinger equation $H |\Psi_n\rangle = E_n |\Psi_n\rangle$

Open-shell systems

Symmetry-breaking reference states



Strong static correlations



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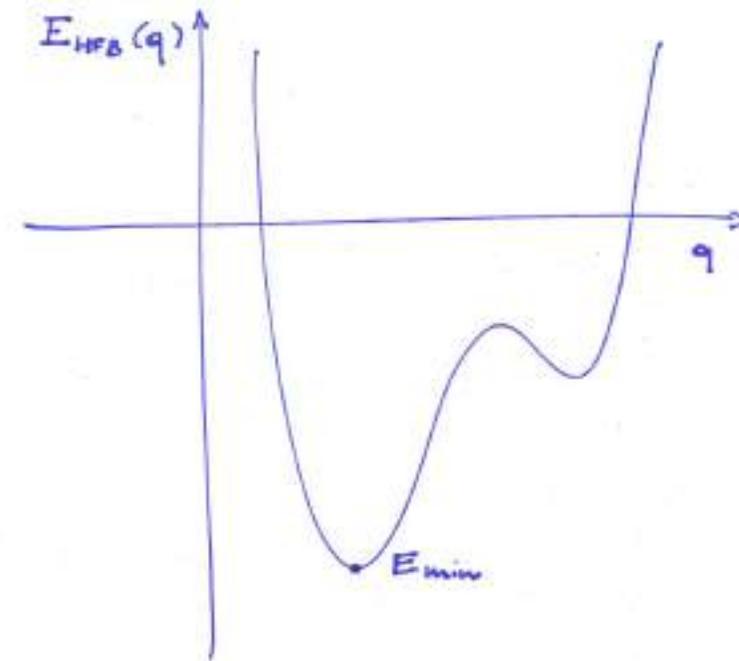


Strong static correlations



1 Constrained HF solutions

$|\Phi(q)\rangle$



Projected Generator Coordinate Method

Schrödinger equation $H |\Psi_n\rangle = E_n |\Psi_n\rangle$

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Strong static correlations

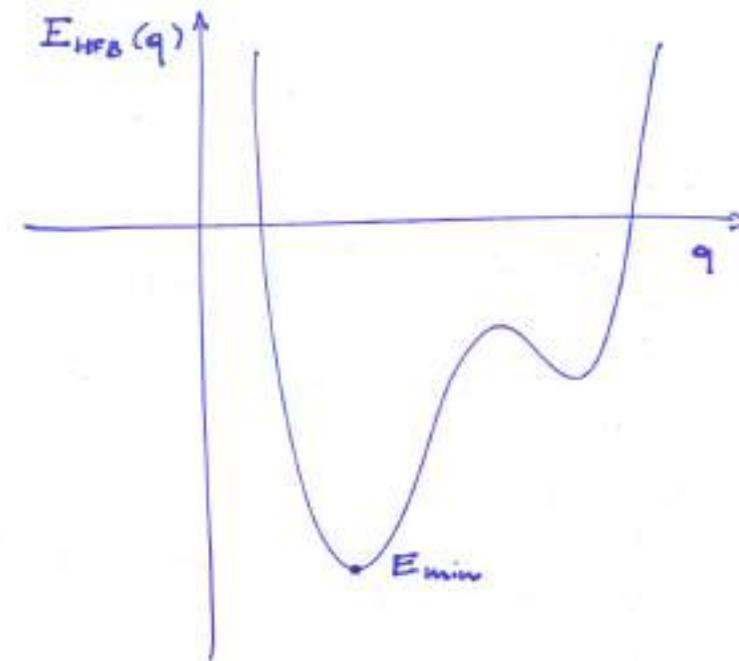


1 Constrained HF solutions

$|\Phi(q)\rangle$



Generator coordinates
(q can be any coordinate)



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1 Constrained HF solutions

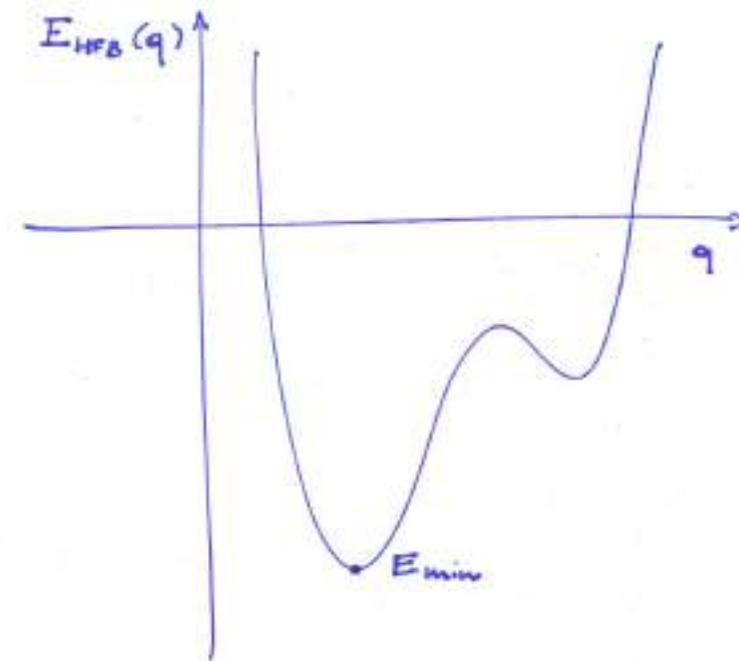
$|\Phi(q)\rangle$



Generator coordinates
(q can be any coordinate)

2 PGCM Ansatz

$$|\Psi_n\rangle = \int dq f_n(q) |\Phi(q)\rangle$$



Projected Generator Coordinate Method

Schrödinger equation $H |\Psi_n\rangle = E_n |\Psi_n\rangle$

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Strong static correlations



1 Constrained HF solutions

$|\Phi(q)\rangle$



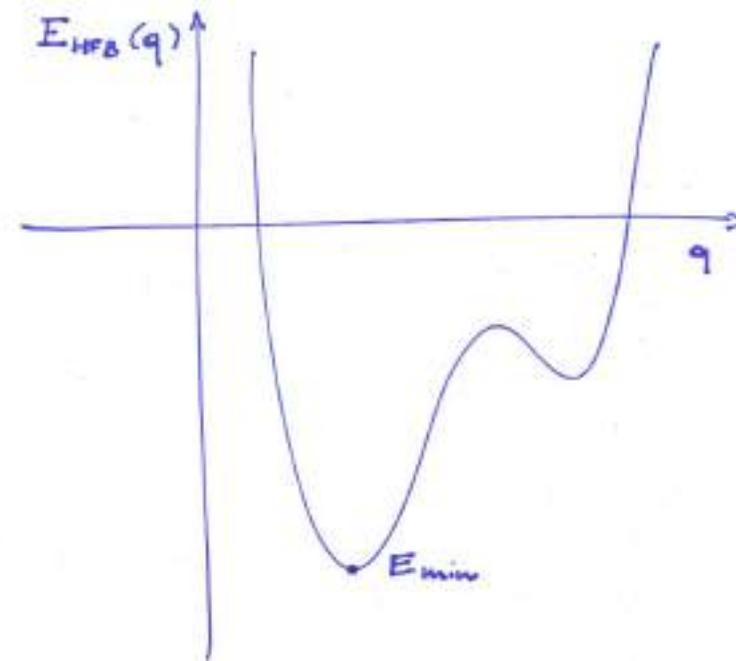
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Linear coefficients



Projected Generator Coordinate Method

Schrödinger equation

$$H |\Psi_n\rangle = E_n |\Psi_n\rangle$$

Open-shell systems

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Strong static correlations



1 Constrained HFB solutions

$$|\Phi(q)\rangle$$



Generator coordinates
(q can be any coordinate)

2 PGCM Ansatz

$$|\Psi_n\rangle = \int dq f_n(q) |\Phi(q)\rangle$$

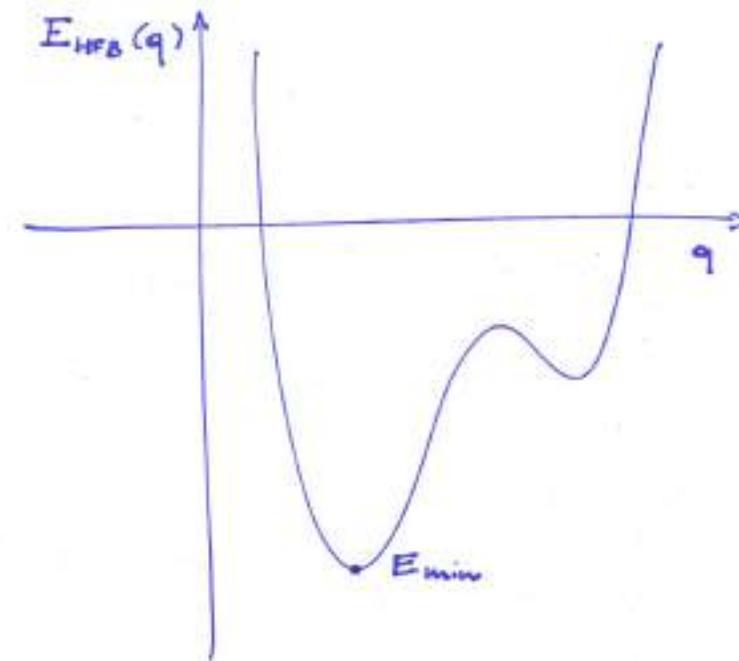


Linear coefficients

3 HWG Equation

Variational method

$$\delta \frac{\langle \Psi_n | H | \Psi_n \rangle}{\langle \Psi_n | \Psi_n \rangle} = 0$$



Projected Generator Coordinate Method

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1 Constrained HF solutions

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Linear coefficients

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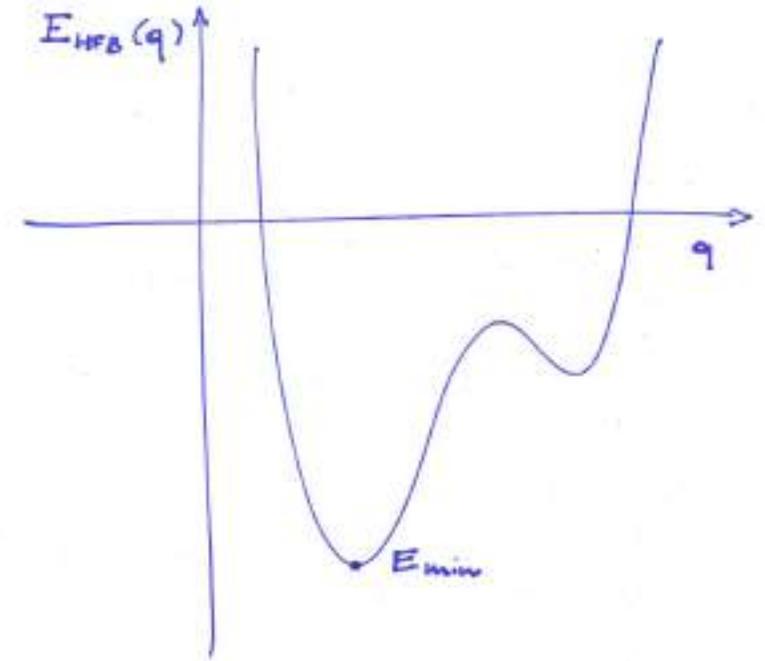
$$\delta \frac{\langle \Psi_n | H | \Psi_n \rangle}{\langle \Psi_n | \Psi_n \rangle} = 0$$

Schrödinger-like equation

$$\int [\mathcal{H}(p, q) - E_n \mathcal{N}(p, q)] f_n(q) dq = 0$$

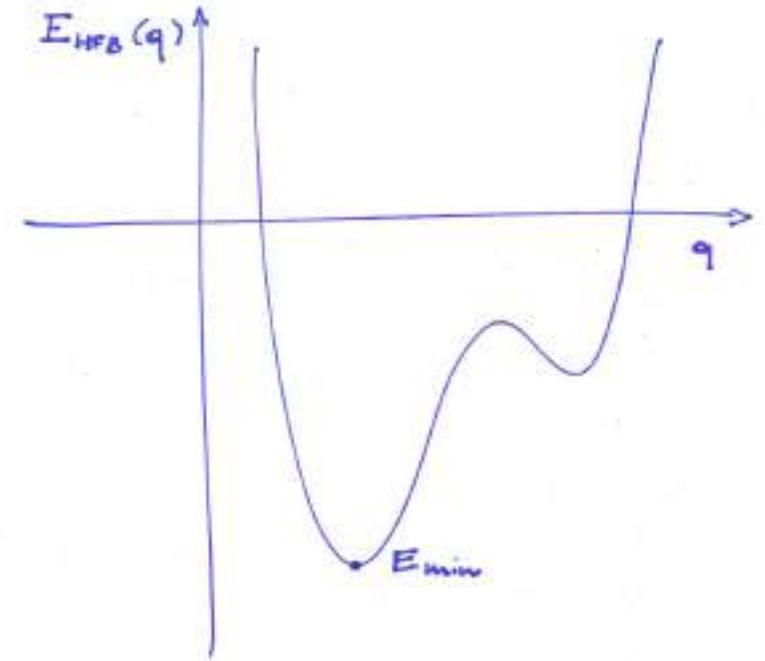
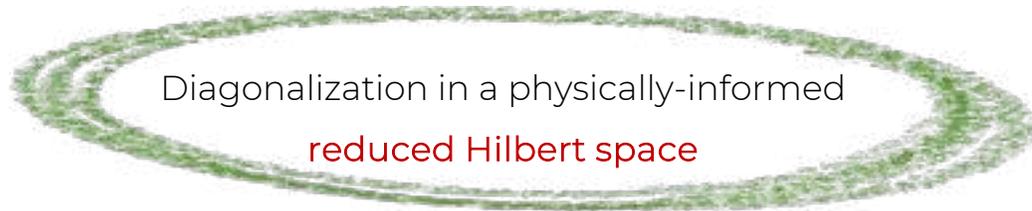
Kernels evaluation

$$\begin{aligned} \mathcal{H}(p, q) &\equiv \langle \Phi(p) | H | \Phi(q) \rangle \\ \mathcal{N}(p, q) &\equiv \langle \Phi(p) | \Phi(q) \rangle \end{aligned}$$



Projected Generator Coordinate Method

Schrödinger equation $H |\Psi_n\rangle = E_n |\Psi_n\rangle$



1 Constrained HFB solutions

$$|\Phi(q)\rangle$$

Generator coordinates
(q can be any coordinate)

2 PGCN Ansatz

$$|\Psi_n\rangle = \int dq f_n(q) |\Phi(q)\rangle$$

Linear coefficients

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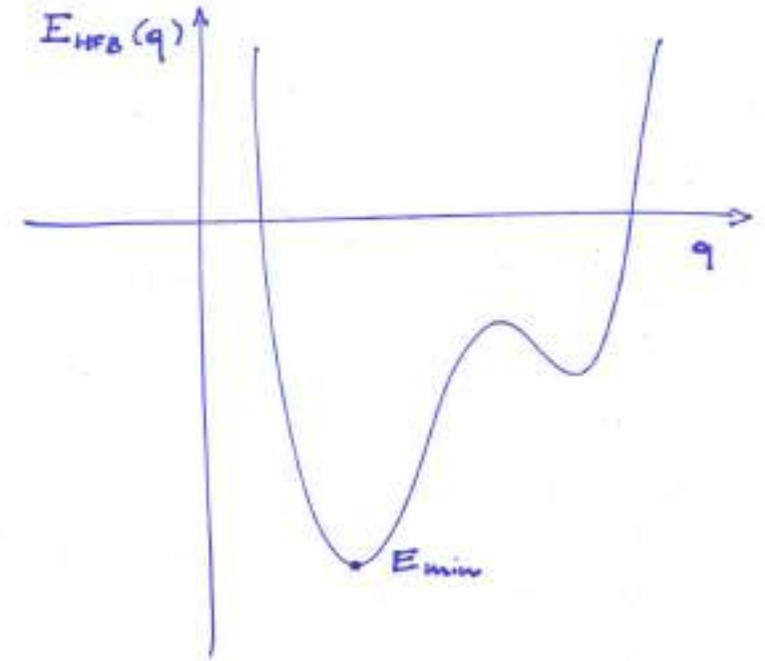
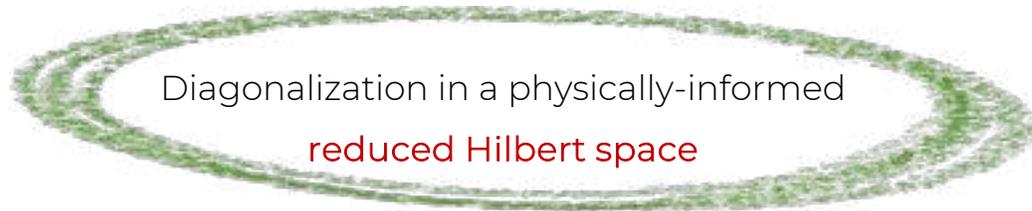
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Schrödinger equation $H |\Psi_n\rangle = E_n |\Psi_n\rangle$



1 Constrained HFB solutions

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+ Projection

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Kernels evaluation

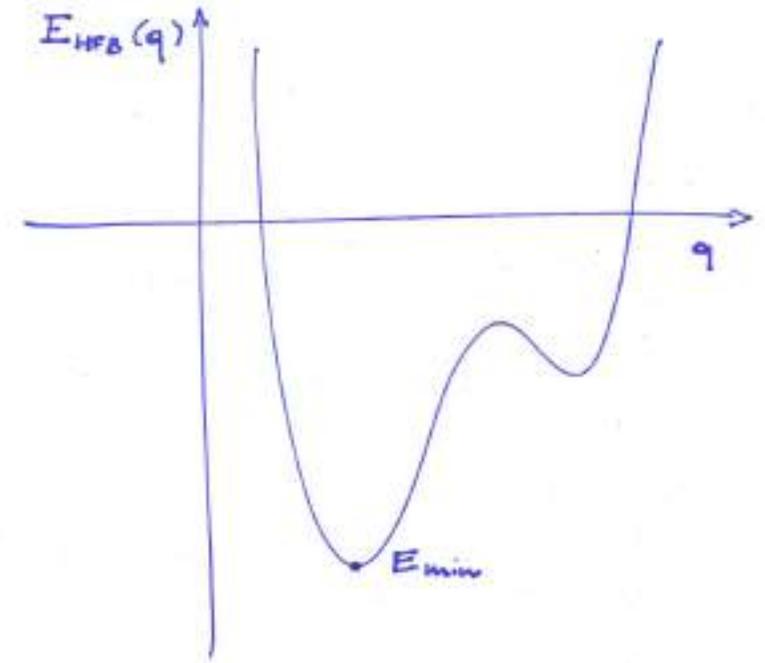
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GCM and (Q)RPA

Thouless theorem

$$|\Phi(q)\rangle = \langle \Phi(q_{min}) | \Phi(q) \rangle e^{\mathbf{Z}(q, q_{min})} |\Phi(q_{min})\rangle$$

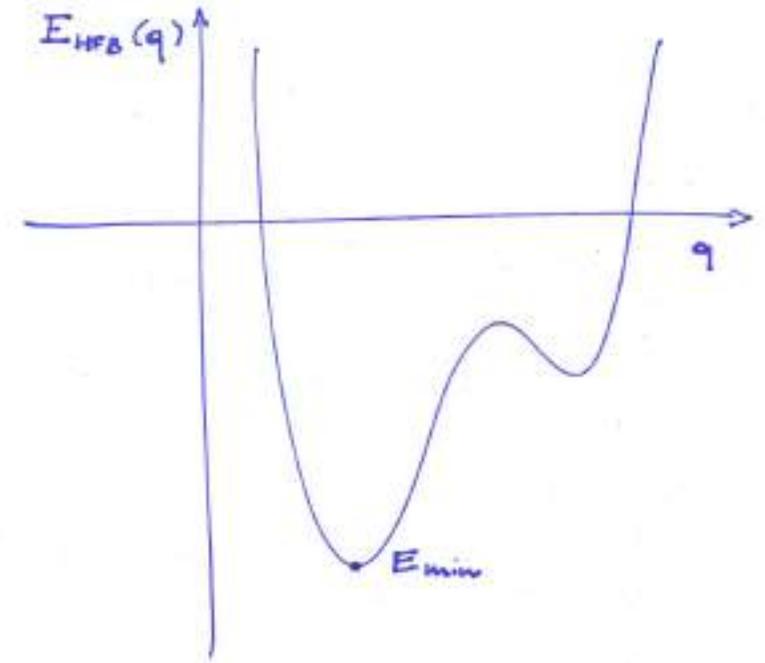


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Non-unitary transformation

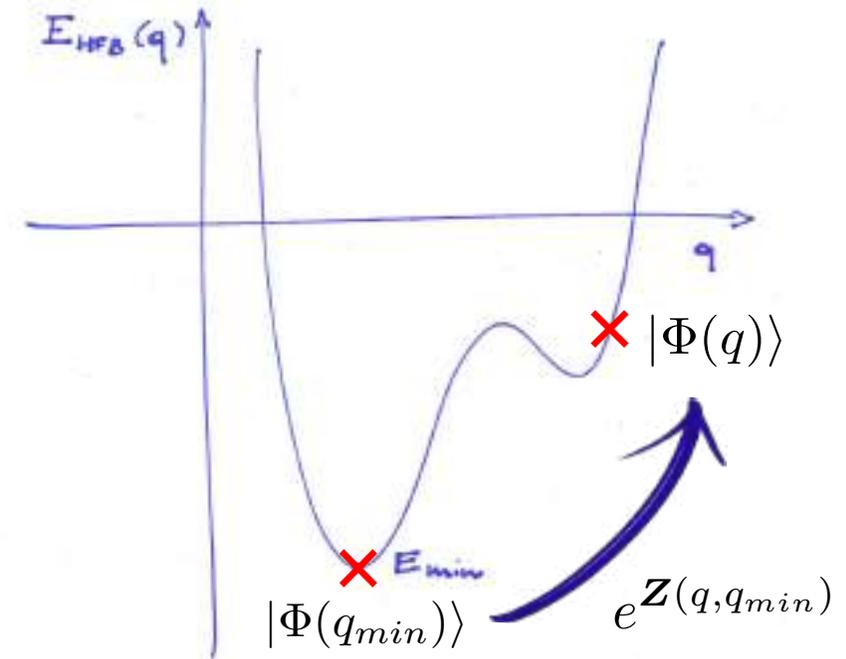


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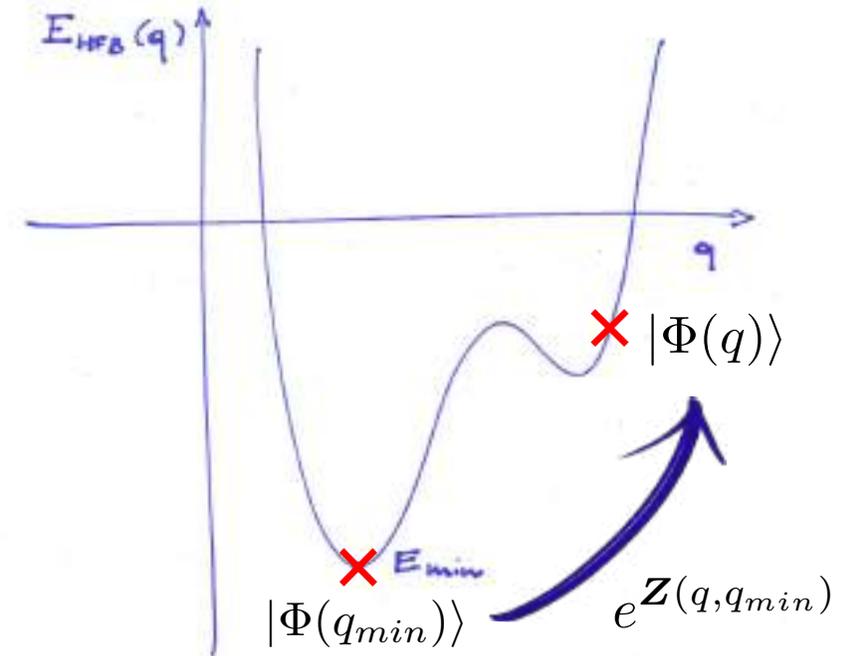
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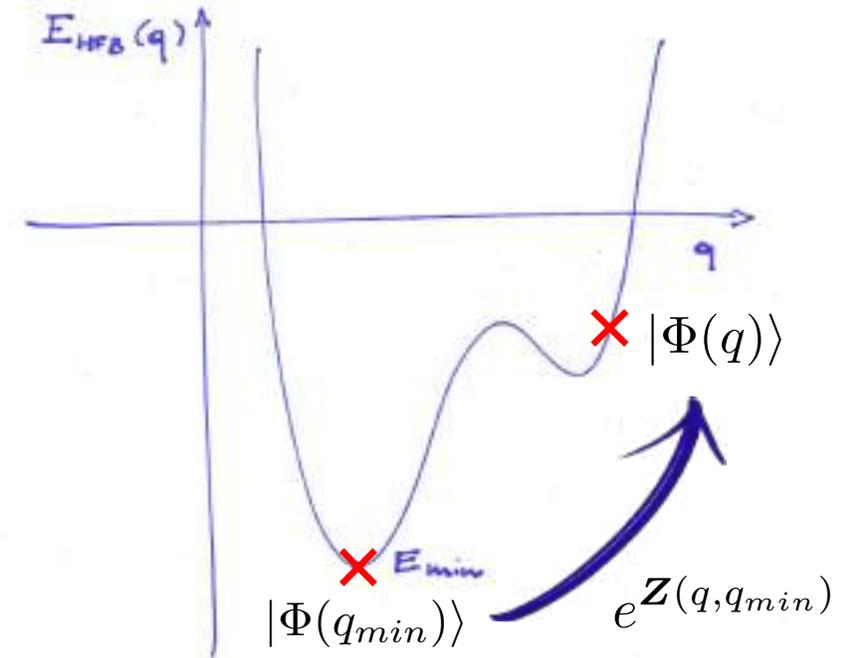
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GCM and (Q)RPA

Thouless theorem

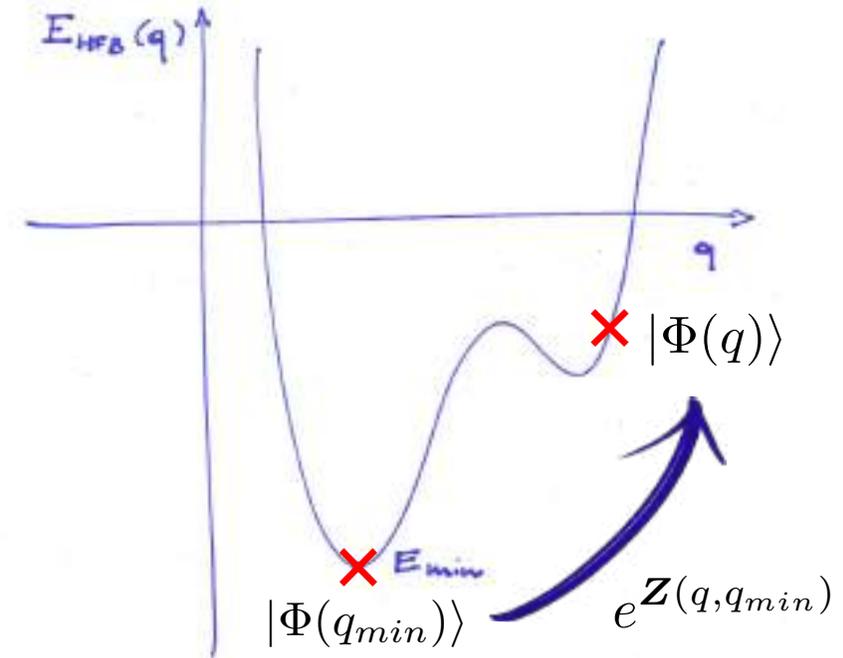
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Expand to the quadratic level in $\mathbf{Z}(q, q_{min})$



GCM and (Q)RPA

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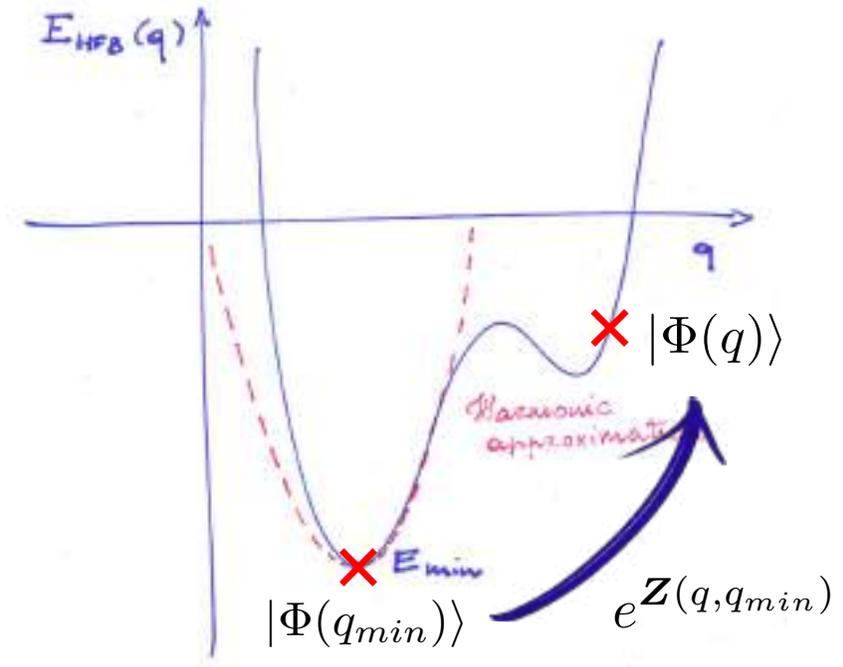
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Harmonic approximation



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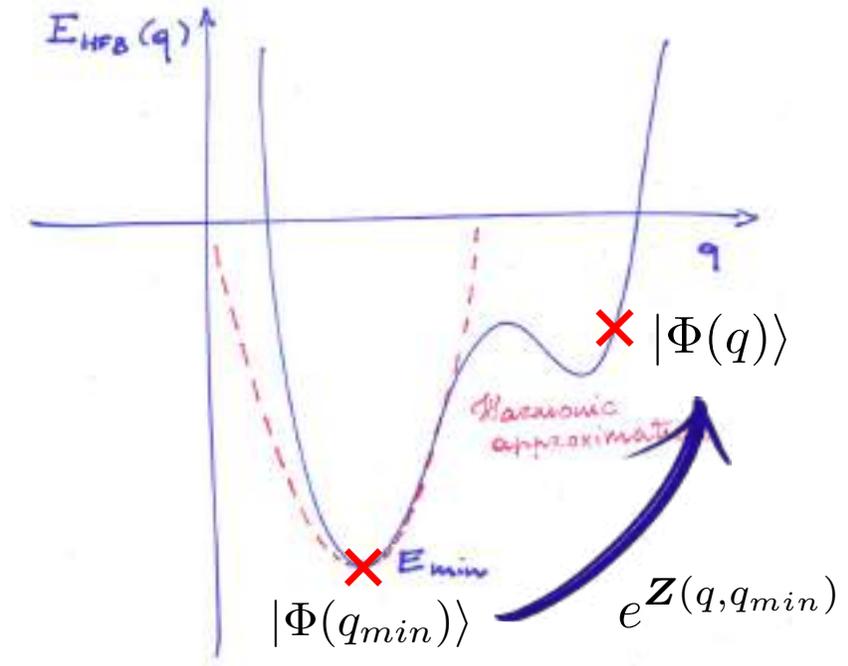
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Expand to the quadratic level in $\mathbf{Z}(q, q_{min})$

Harmonic approximation



No coordinates dependency!

All coordinates are explored
(differently from GCM)

GCM and (Q)RPA

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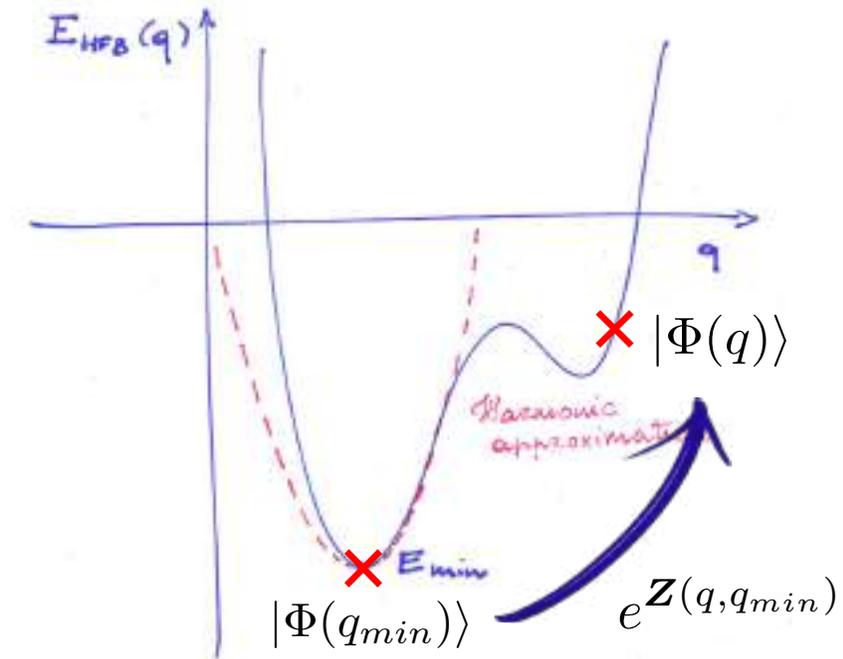
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Eventually rewrites as (Q)RPA equations

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X^n \\ Y^n \end{pmatrix} = E_n \begin{pmatrix} X^n \\ Y^n \end{pmatrix}$$

GCM and (Q)RPA

Thouless theorem

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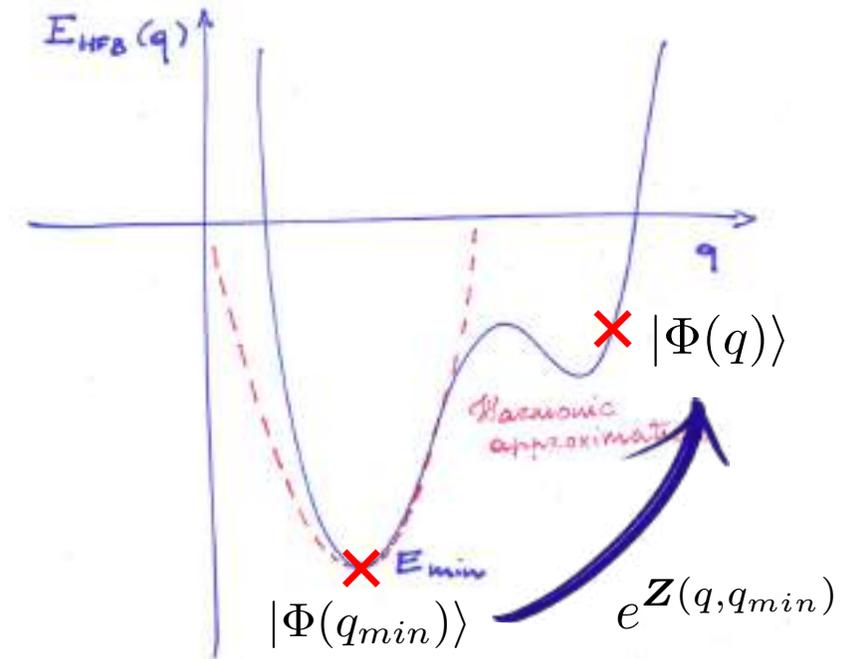
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Nuclei that are stiff against deformations
(anharmonic effects negligible)



All coordinates are explored
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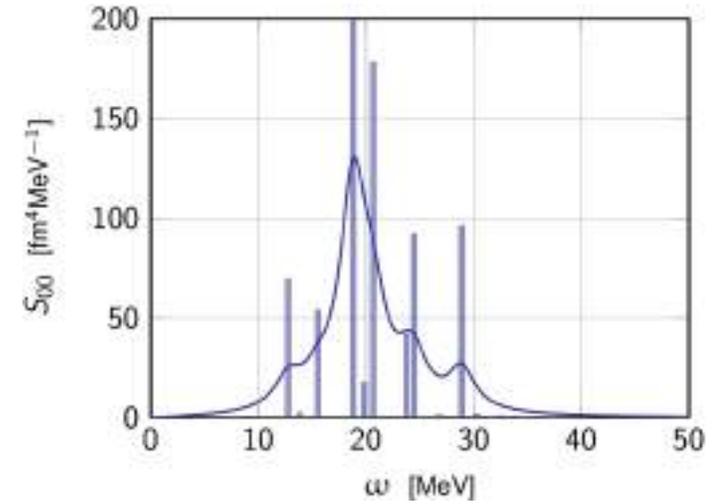
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Setting

Studied quantity: **monopole strength**

- Transition amplitudes: height of peaks
- Energy difference: position of peaks

$$S_{00}(\omega) \equiv \sum_{\nu} |\langle \Psi_{\nu} | r^2 | \Psi_0 \rangle|^2 \delta(E_{\nu} - E_0 - \omega)$$



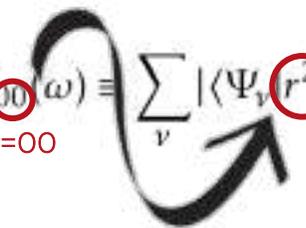
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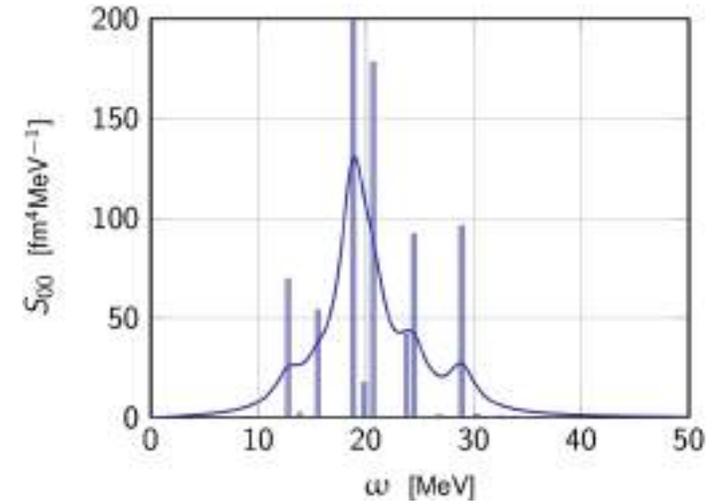
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JM=00



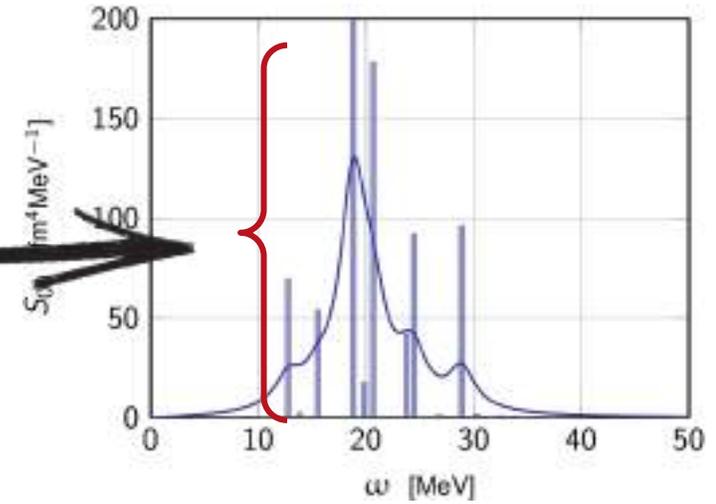


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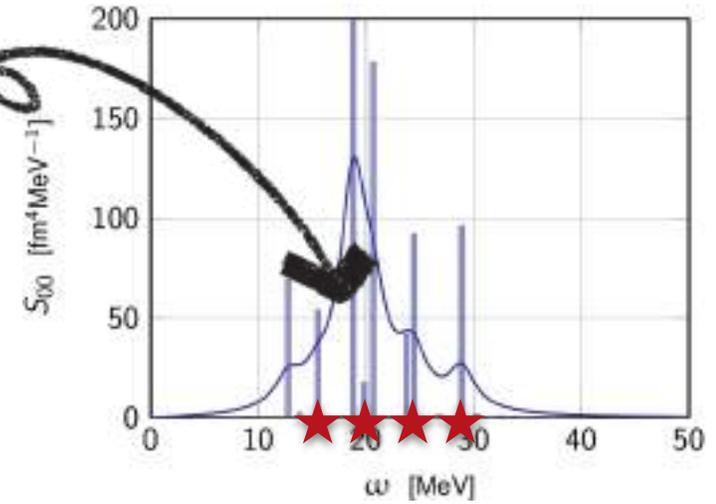


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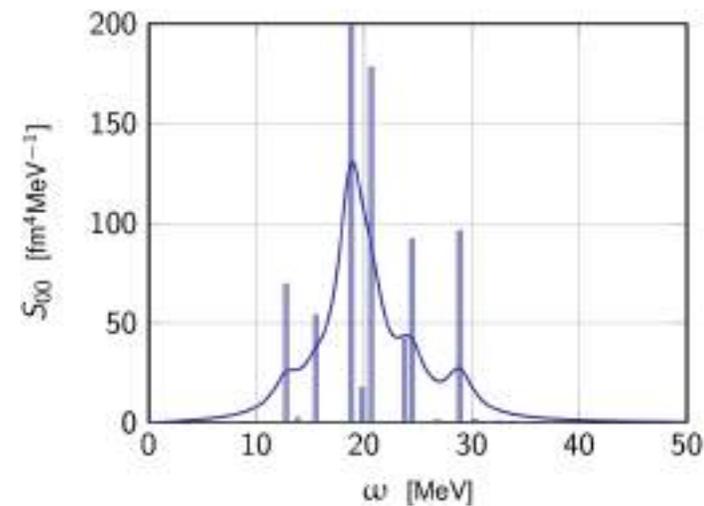


Setting

Studied quantity: **monopole strength**

- Transition amplitudes: height of peaks
- Energy difference: position of peaks

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Setting

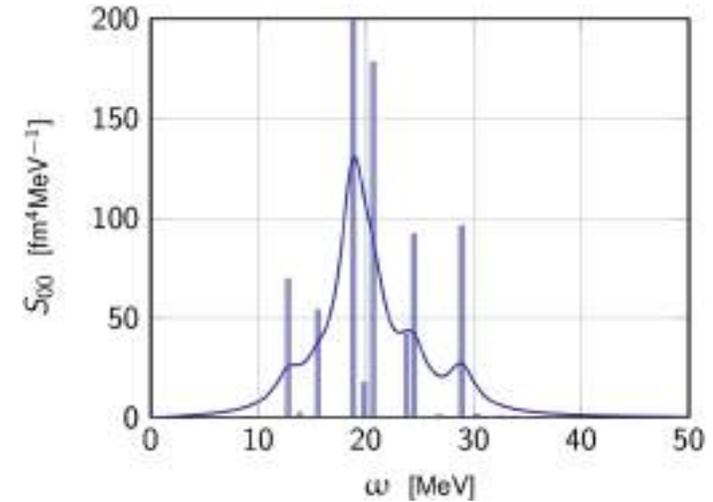
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Related moments

$$\begin{aligned} m_k &\equiv \int_0^{\infty} S_{00}(\omega) \omega^k d\omega \\ &= \sum_{\nu} (E_{\nu} - E_0)^k |\langle \Psi_{\nu} | r^2 | \Psi_0 \rangle|^2 \end{aligned}$$



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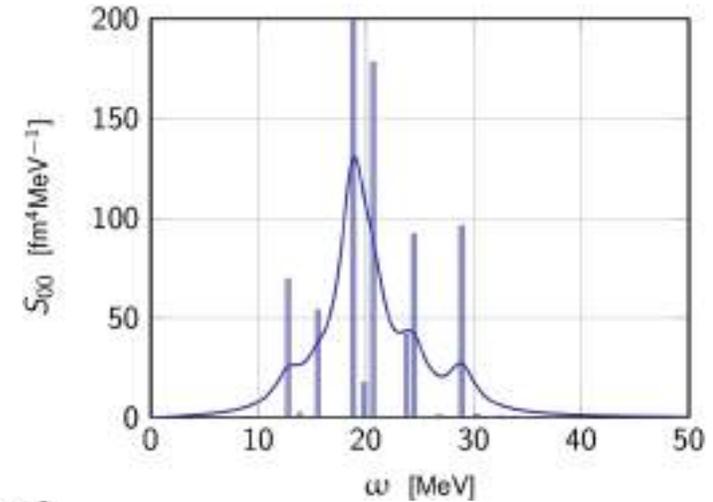
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Quantify the **most relevant features** of the strength

$$\bar{E}_1 = \frac{m_1}{m_0} \quad \sigma^2 = \frac{m_2}{m_0} - \left(\frac{m_1}{m_0}\right)^2 \geq 0$$


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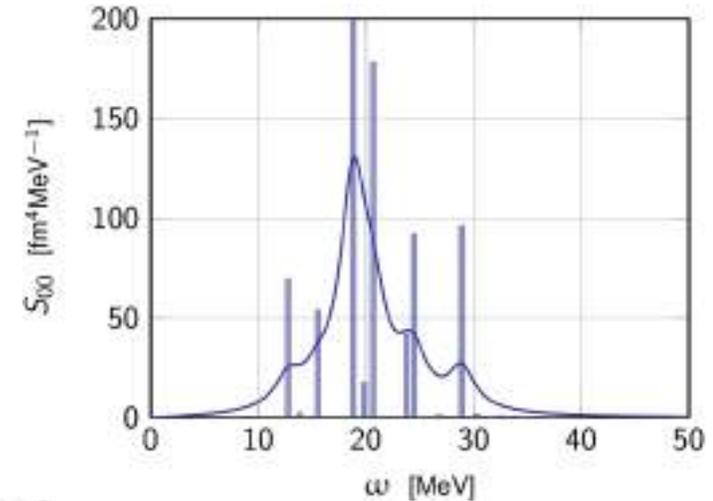
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Ab-initio PGCM and QRPA **consistent numerical settings** (systematic study in ^{46}Ti)

- Quantities expanded on harmonic oscillator basis (characterised by $\hbar\omega$, e_{\max} , $e_{3\max}$)



Setting

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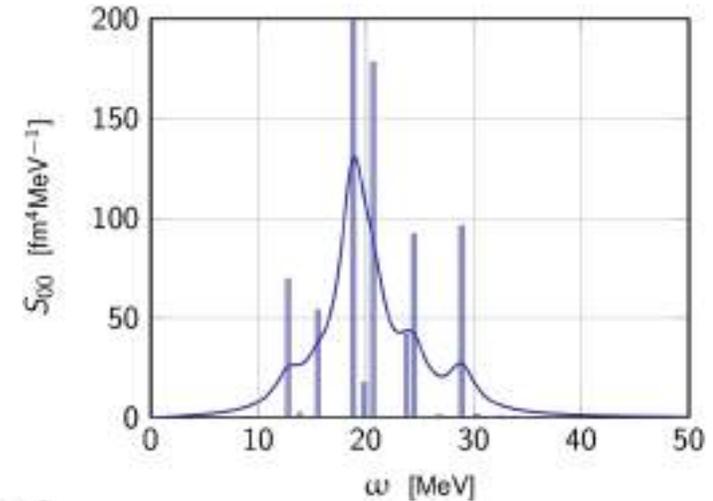
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Ab-initio PGCM and QRPA **consistent numerical settings** (systematic study in ^{46}Ti)

- Quantities expanded on harmonic oscillator basis (characterised by $\hbar\omega$, e_{\max} , $e_{3\max}$)
- Family of chiral NN + in-medium 3N interactions (NLO, N2LO and N3LO)
 - T. Hüther, K. Vobig, K. Hebeler, R. Machleidt and R. Roth, "Family of chiral two-plus three-nucleon interactions for accurate nuclear structure studies", *Phys. Lett. B*, 808, 2020
 - In-vacuum SRG evolution ($\alpha=0.04 \text{ fm}^4$, $\alpha=0.08 \text{ fm}^4$)
 - M. Frosini, T. Duguet, B. Bally, Y. Beaujeault-Taudière, J.-P. Ebran and V. Somà, "In-medium k-body reduction of n-body operators", *The European Physical Journal A*, 57(4), 2021

Uncertainty budget

Many-body truncation

- Comparison to PGCM-PT
- Only tested for **low-lying exc**
- **Correlated to SRG and generator coords**

SRG dependence

- **Strong centroid dependence** $\sim 10\%$
- Dispersion relative error $\sim 20\%$
- **Truncates both H and many-body**

Chiral Order

- Good **overall convergence**
- Centroid relative error $\sim 1,6\%$
- Dispersion relative error $\sim 9,8\%$

Generator coordinates choice

- **Empirical knowledge**, two coords r and β_2
- More **systematic choice needed**

Three-body treatment

- NO2B approximation
- 1-2 % uncertainty in low-lying exc
- Not tested for giant resonances

Hamiltonian parameters

- LEC dependence of χ forces
- **Few interactions** compared
 - **Correlated to SRG**
- Need for **emulators** (EC)

Harmonic Oscillator width

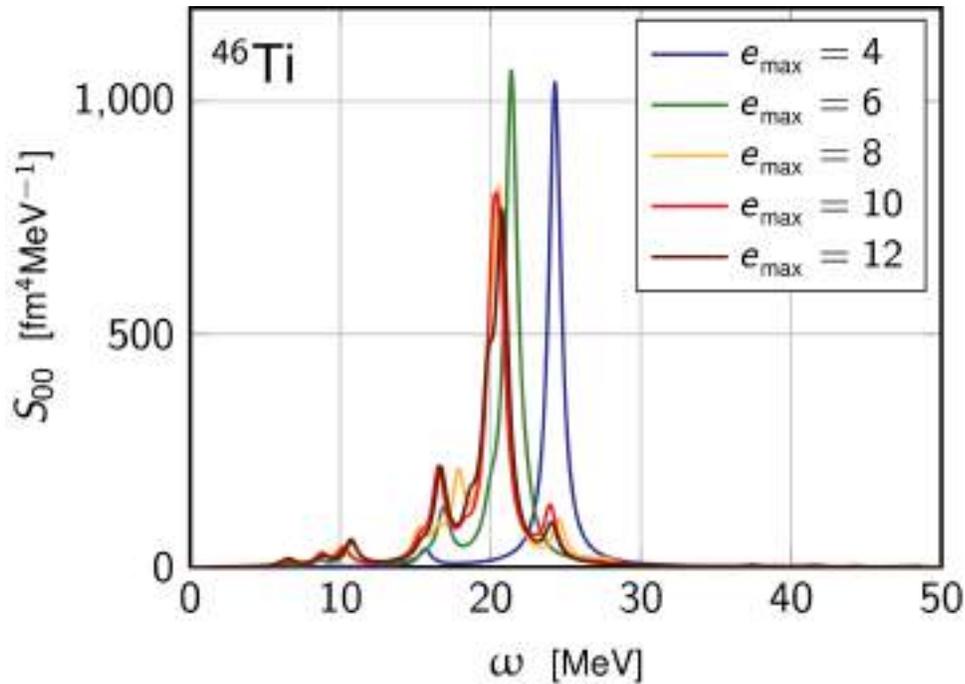
- Good **overall convergence**
- Centroid relative error $\sim 1,6\%$
- Dispersion relative error $\sim 6\%$

Finite Basis Size

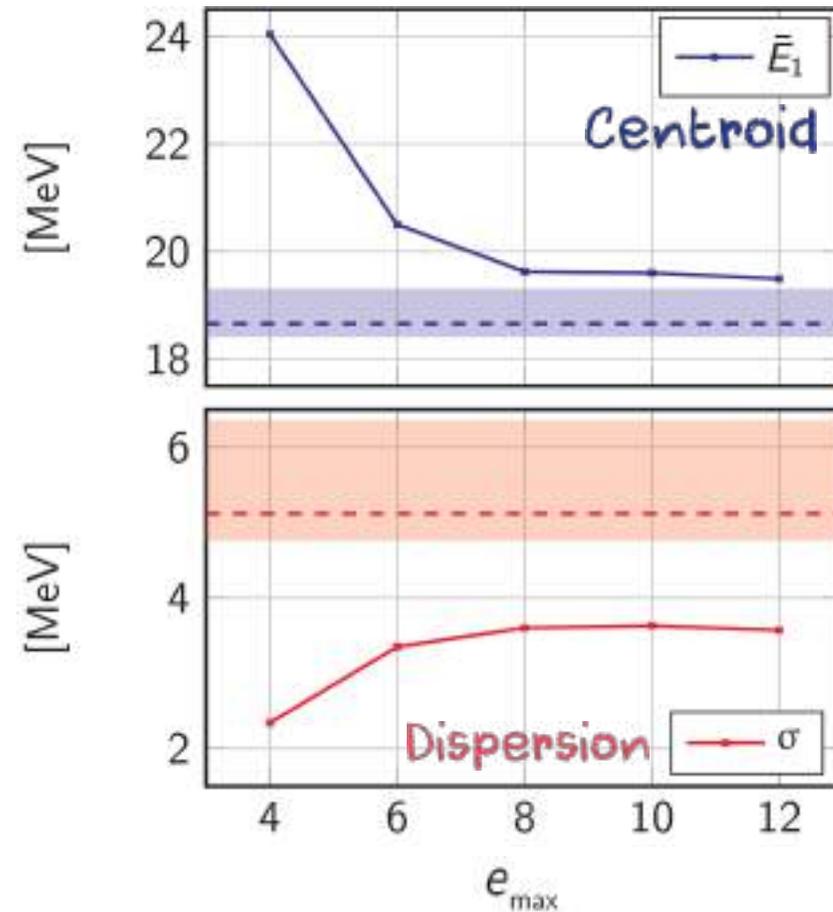
- Good **overall convergence**
- Centroid relative error $\sim 0,6\%$
- Dispersion relative error $\sim 1,7\%$
- $e_{3\max}$ not studied (14 safe for GS)



Finite Basis Size



- Good overall convergence
- Centroid relative error $\sim 0,6\%$
- Dispersion relative error $\sim 1,7\%$
- $e_{3\text{max}}$ not studied (14 safe for GS)



[Myiagi et al., PRC, 2022]

Many-body truncation

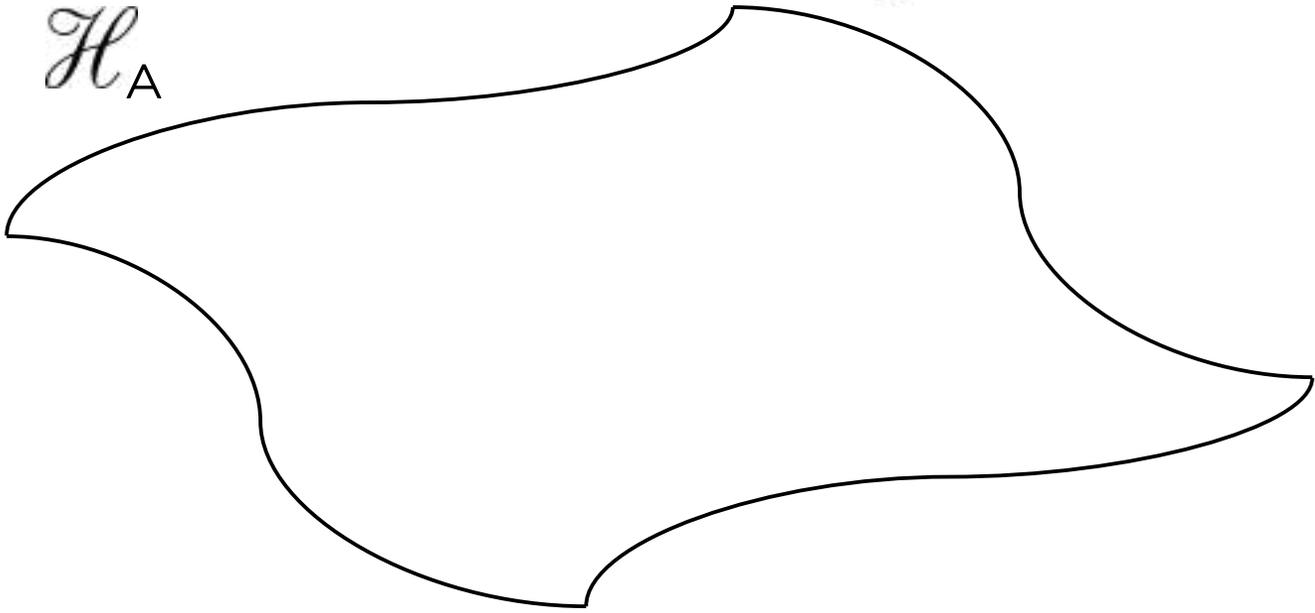
Schrödinger equation

$$H|\Psi_k^A\rangle = E_k^A|\Psi_k^A\rangle$$

A-body Hilbert space

\mathcal{H}_A

Exact solution



Many-body truncation

Schrödinger equation

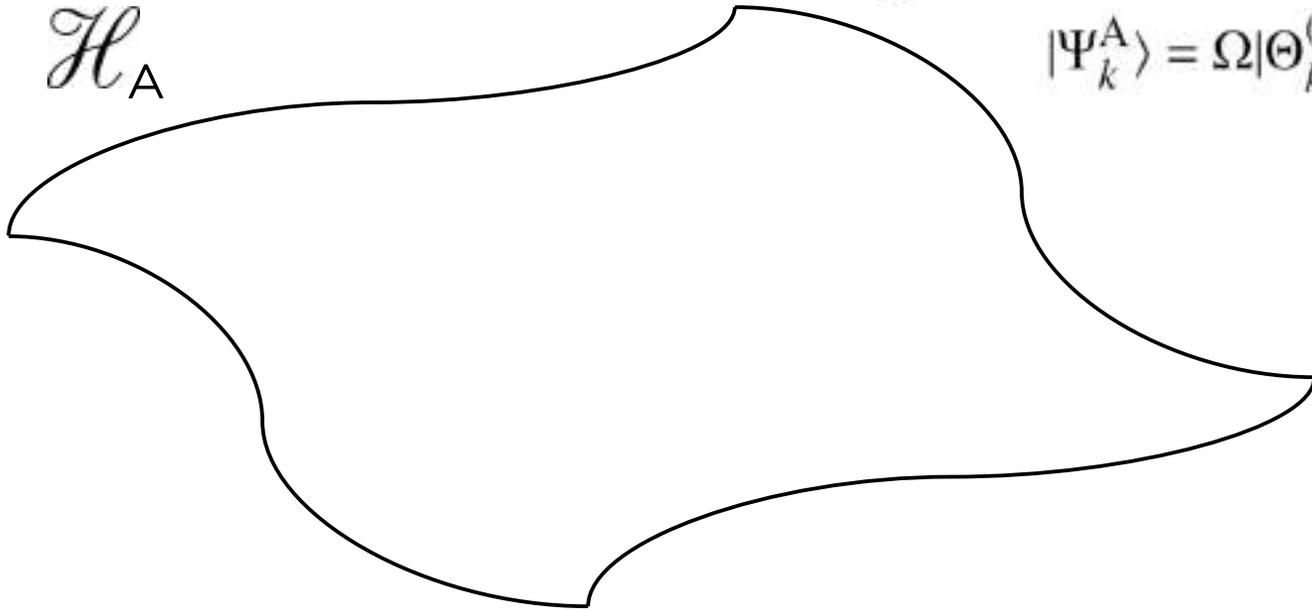
$$H|\Psi_k^A\rangle = E_k^A|\Psi_k^A\rangle$$

A-body Hilbert space

\mathcal{H}_A

Exact solution

$$|\Psi_k^A\rangle = \Omega|\Theta_k^{(0)}\rangle$$

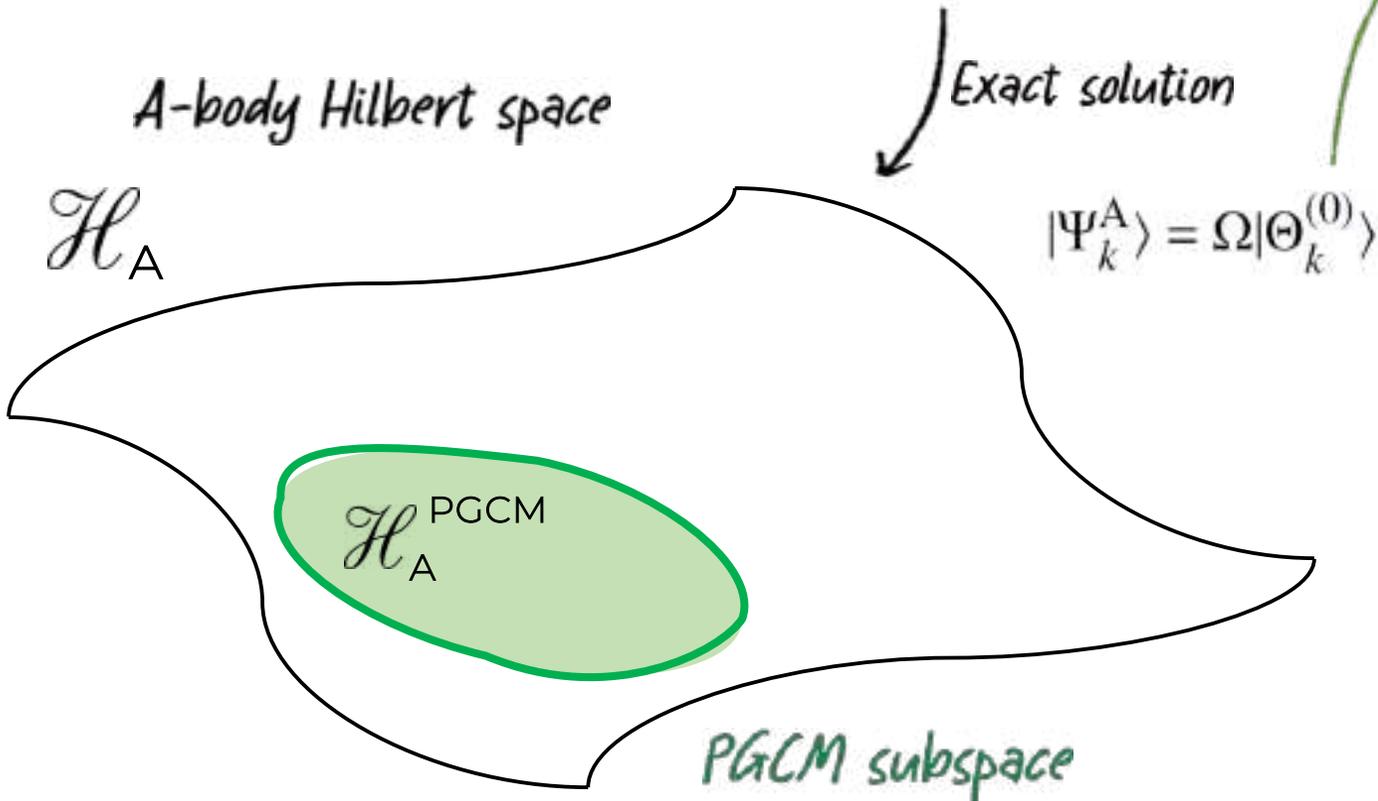


Many-body truncation

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PGCM : multi-reference unperturbed state

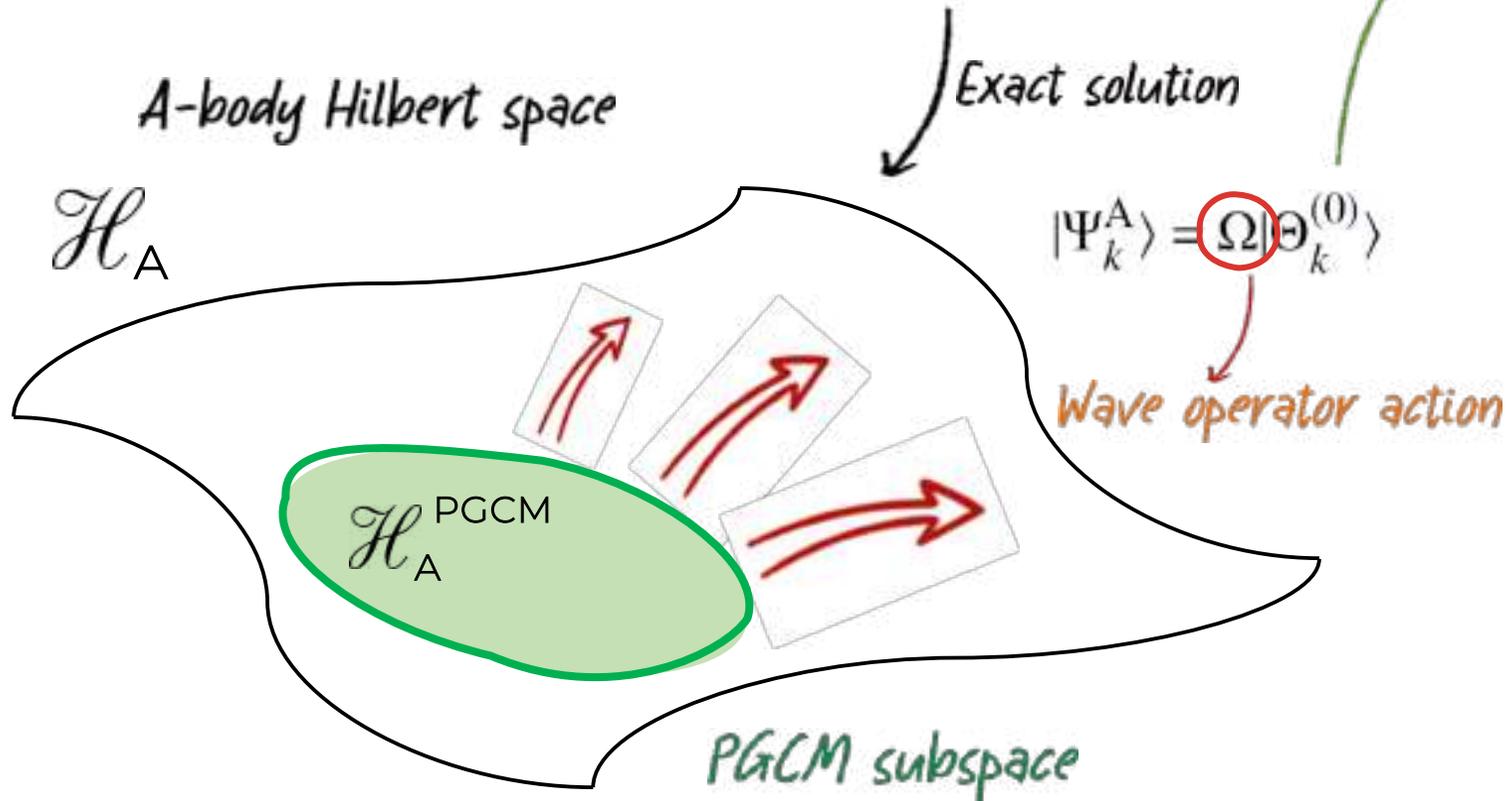


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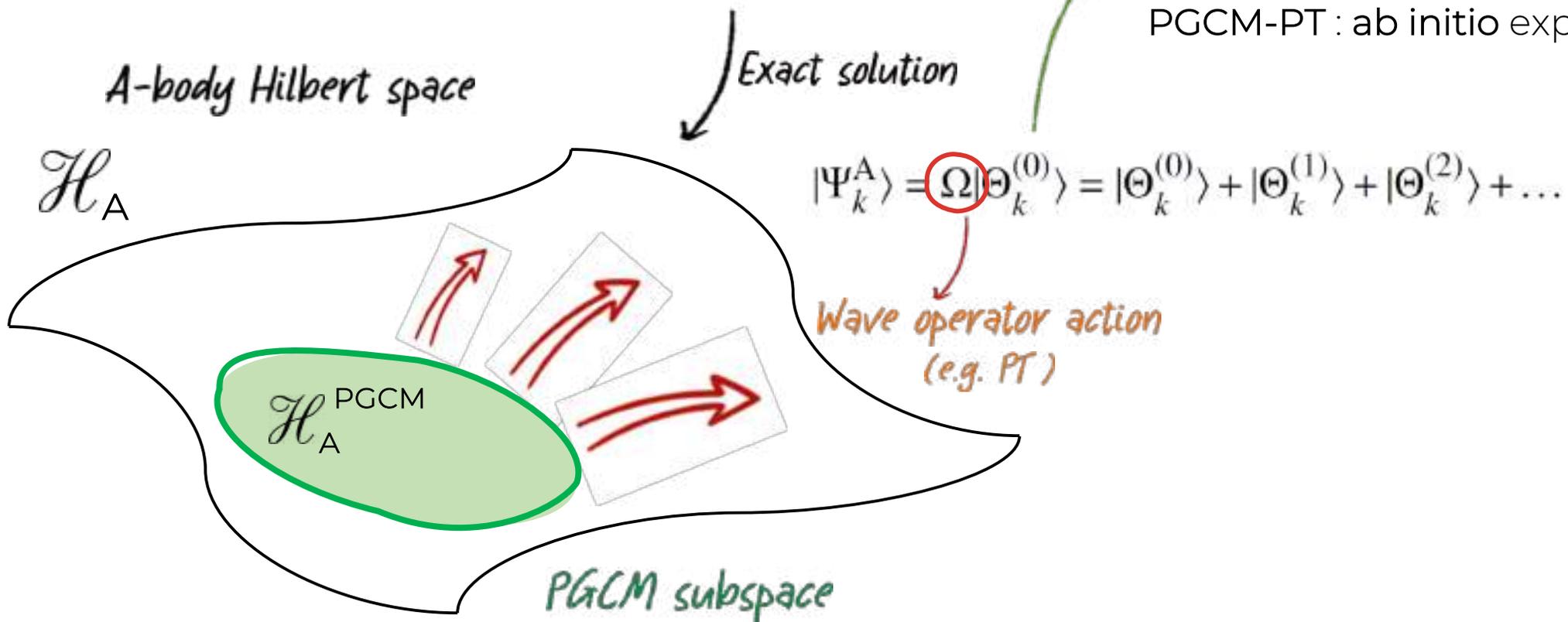
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PGCM : multi-reference unperturbed state

PGCM-PT : ab initio expansion method⁽¹⁾



(1) [Frosini, Duguet, Ebran and Somà, EPJA 58(62), 2022]

Many-body truncation

Schrödinger equation

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PGCM : multi-reference unperturbed state

PGCM-PT : ab initio expansion method⁽¹⁾

PGCM-PT(2) up to 2nd order so far⁽²⁾

A-body Hilbert space

Exact solution

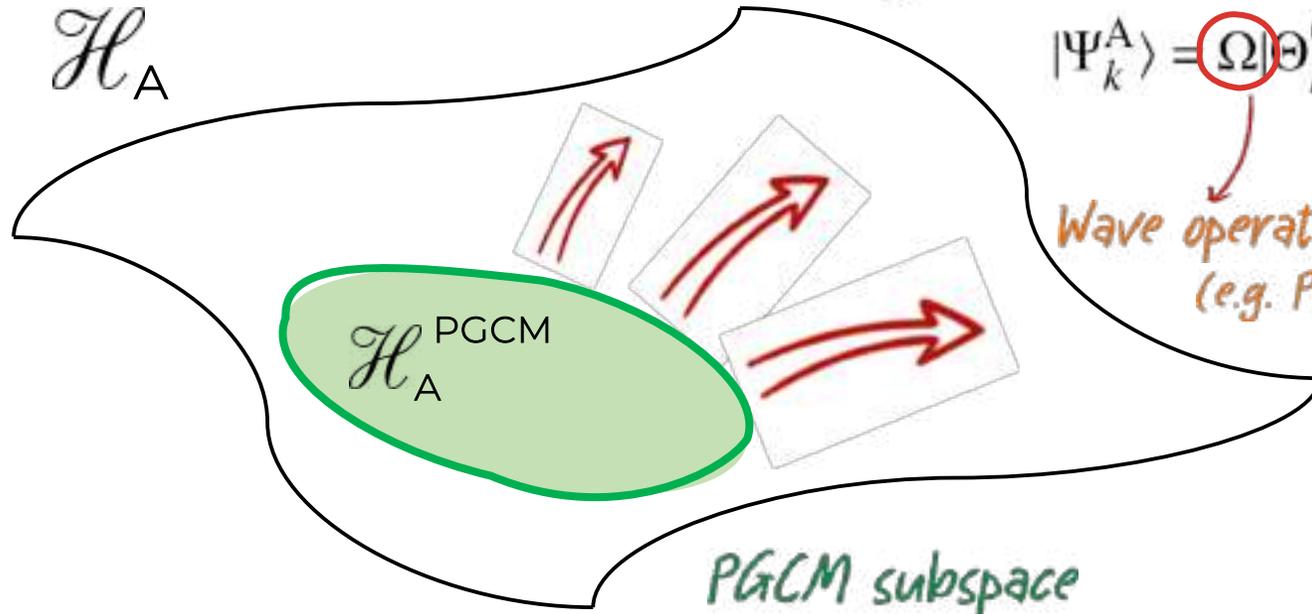
\mathcal{H}_A

$$|\Psi_k^A\rangle = \Omega|\Theta_k^{(0)}\rangle = |\Theta_k^{(0)}\rangle + |\Theta_k^{(1)}\rangle + |\Theta_k^{(2)}\rangle + \dots$$

Wave operator action
(e.g. PT)

PGCM subspace

$\mathcal{H}_A^{\text{PGCM}}$



(1) [Frosini, Duguet, Ebran and Somà, EPJA 58(62), 2022]

(2) [Frosini, Duguet, Ebran, Bally, Hergert, Rodriguez, Roth, Yao and Somà, EPJA 58(64), 2022]

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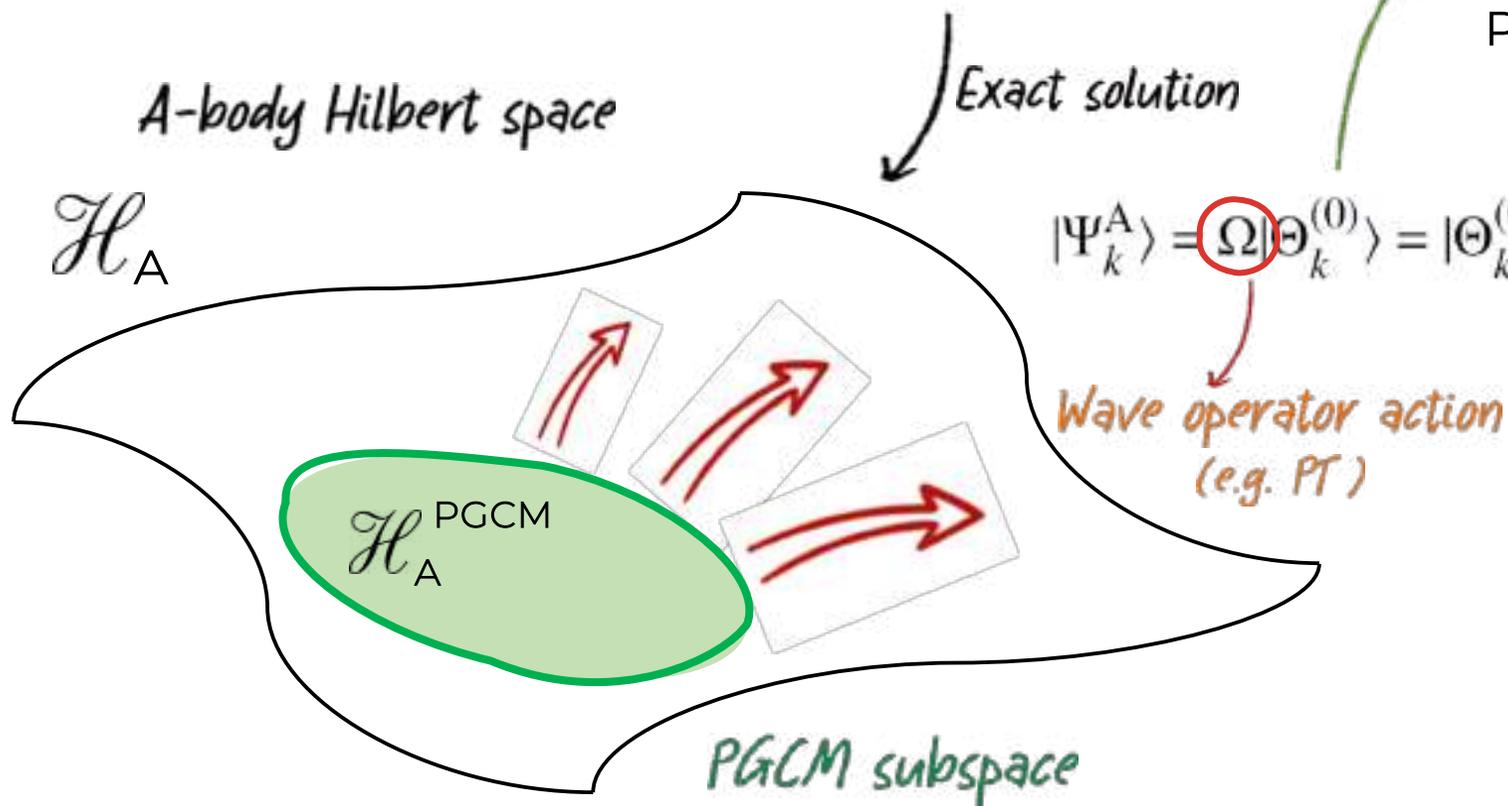
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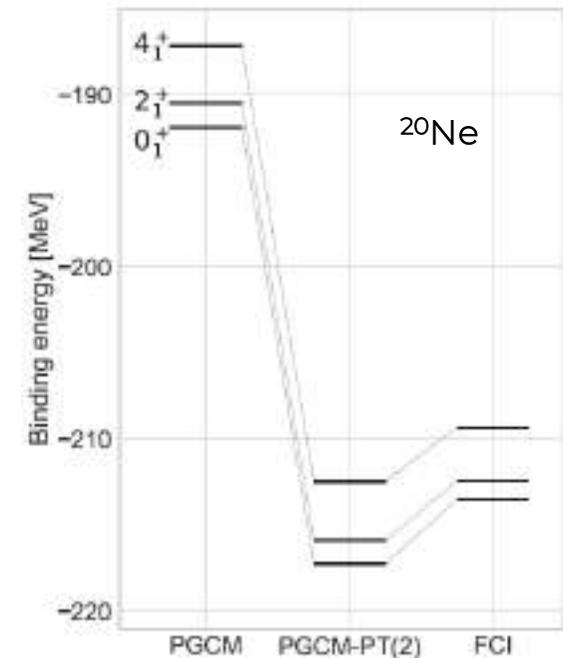
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Many-body truncation

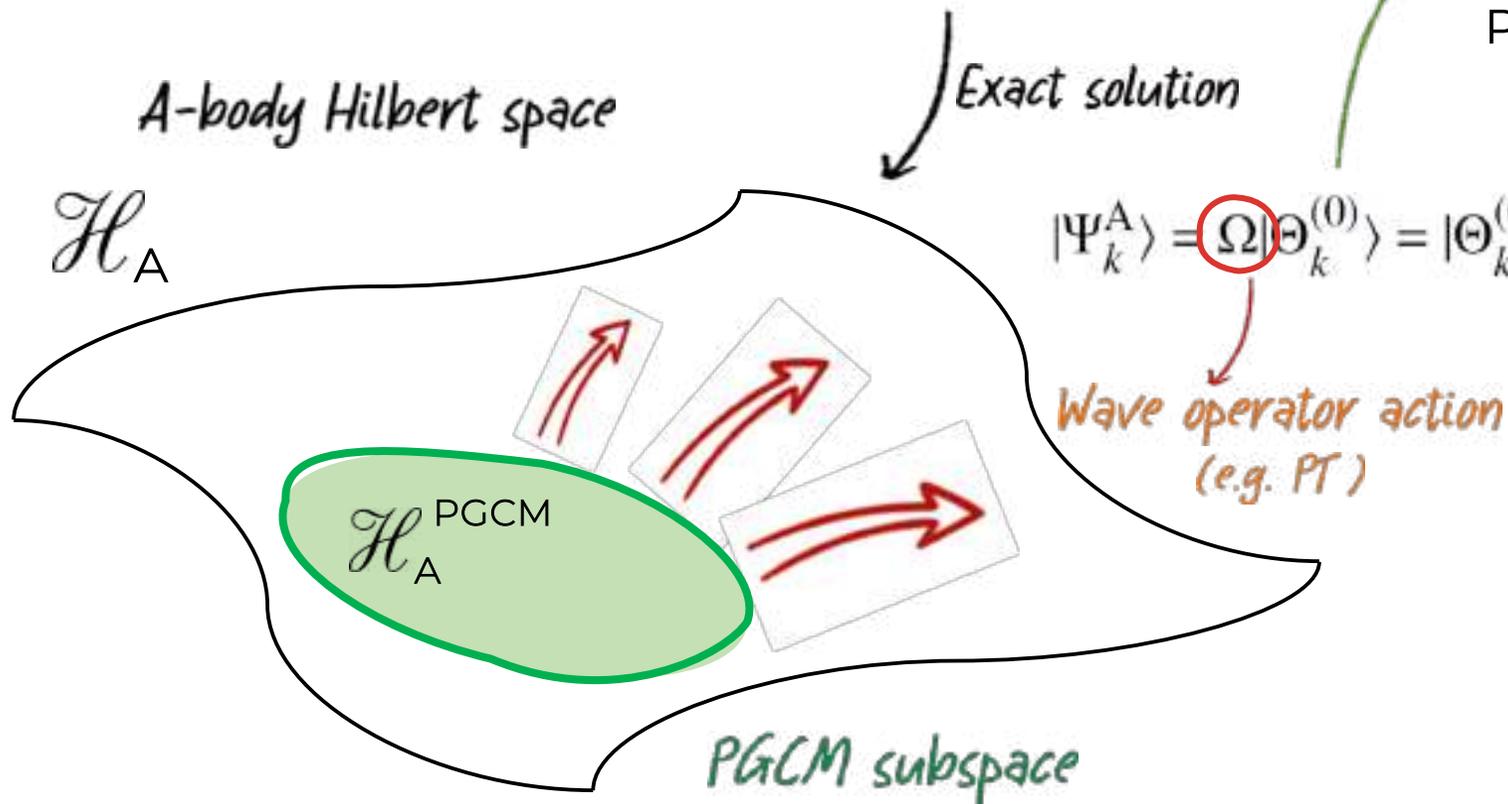
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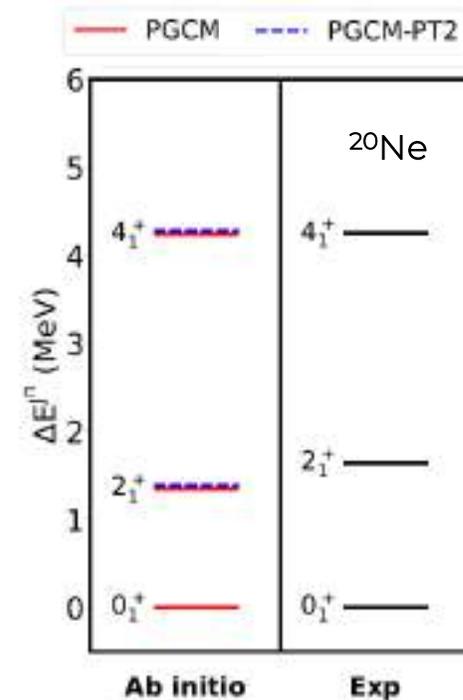
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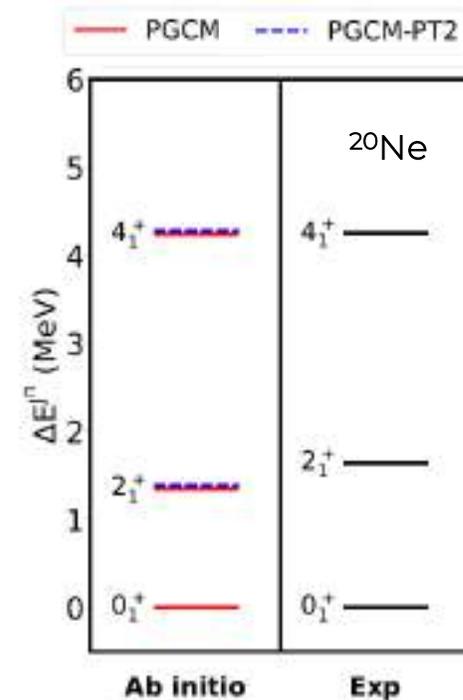
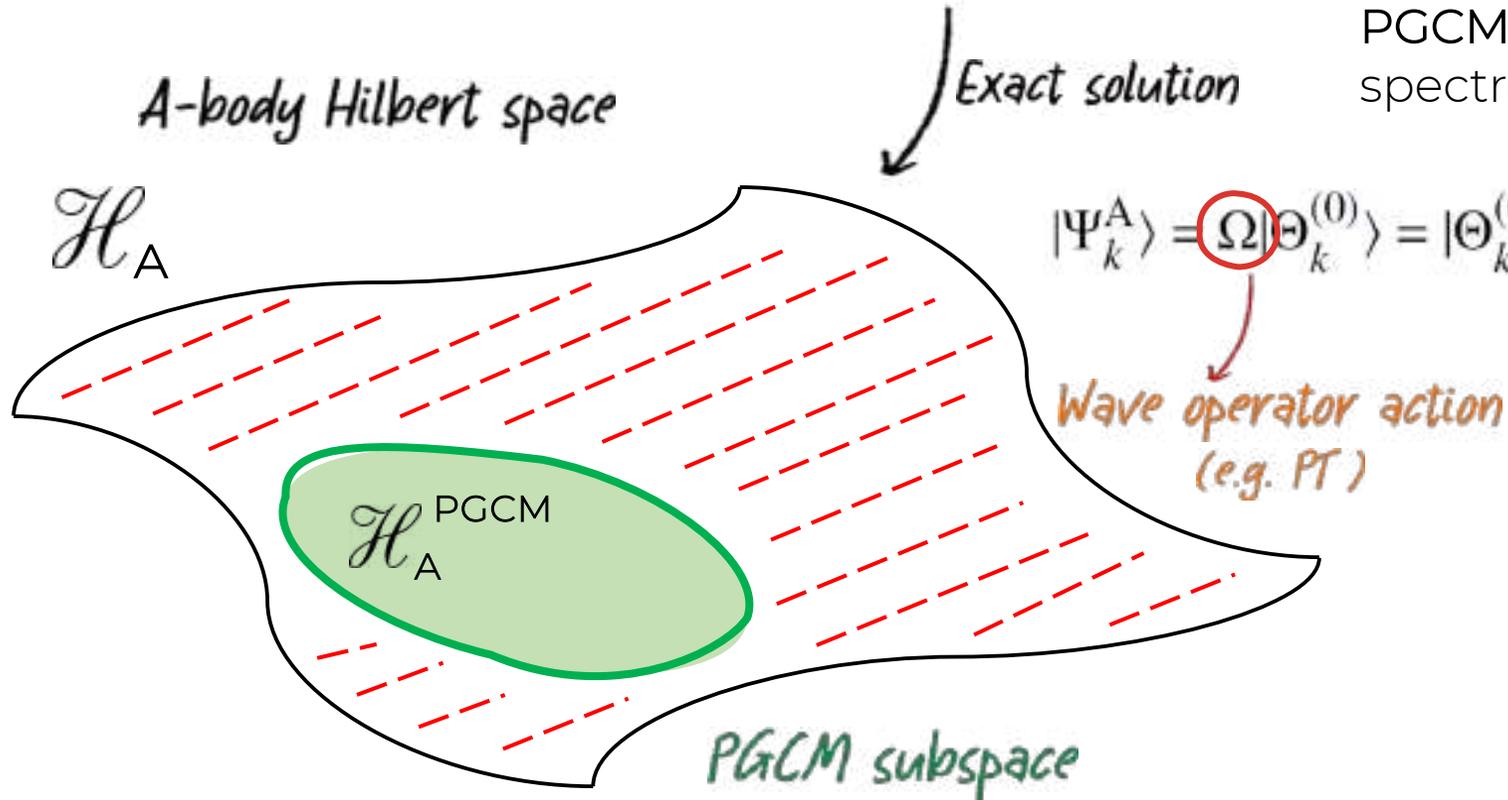
Many-body truncation

Schrödinger equation

$$H|\Psi_k^A\rangle = E_k^A|\Psi_k^A\rangle$$

Dynamical correlations mostly cancel out

PGCM reliable for **low-lying collective** spectroscopy



(1) [Frosini, Duguet, Ebran and Somà, EPJA 58(62), 2022]

(2) [Frosini, Duguet, Ebran, Bally, Hergert, Rodriguez, Roth, Yao and Somà, EPJA 58(64), 2022]

Outline

1 Introduction

- Giant Resonances Physics
- The PGCM
- Link between PGCM and QRPA

2 Systematic study

- Numerical details
- Uncertainty estimate

Conclusions and perspectives

Results

3

Selected applications

- Shape coexistence
- Deformation

Multi-phonon states

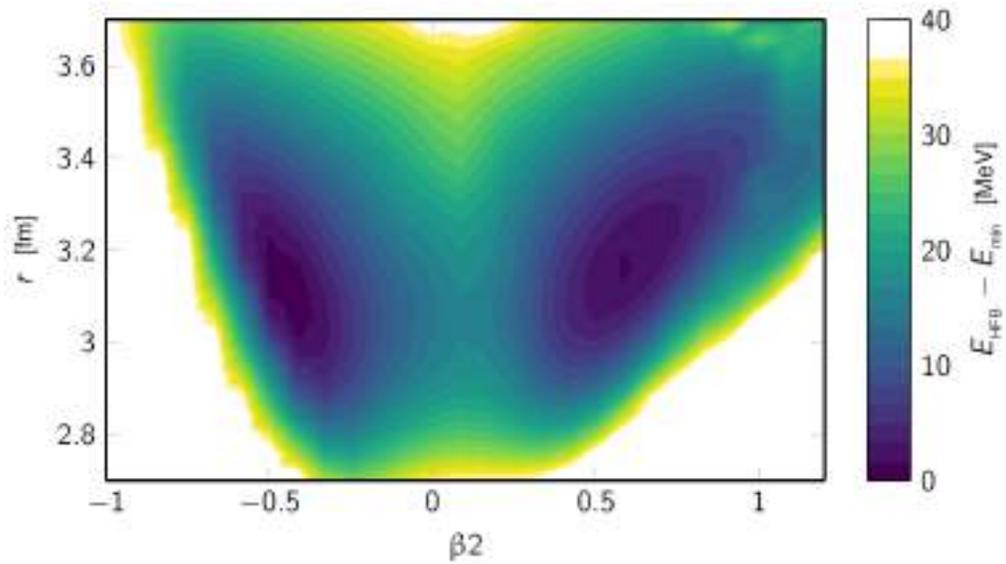
- Proof of principle
- Realistic calculations

From finite nuclei to Astrophysics

- Preliminary incompressibility results

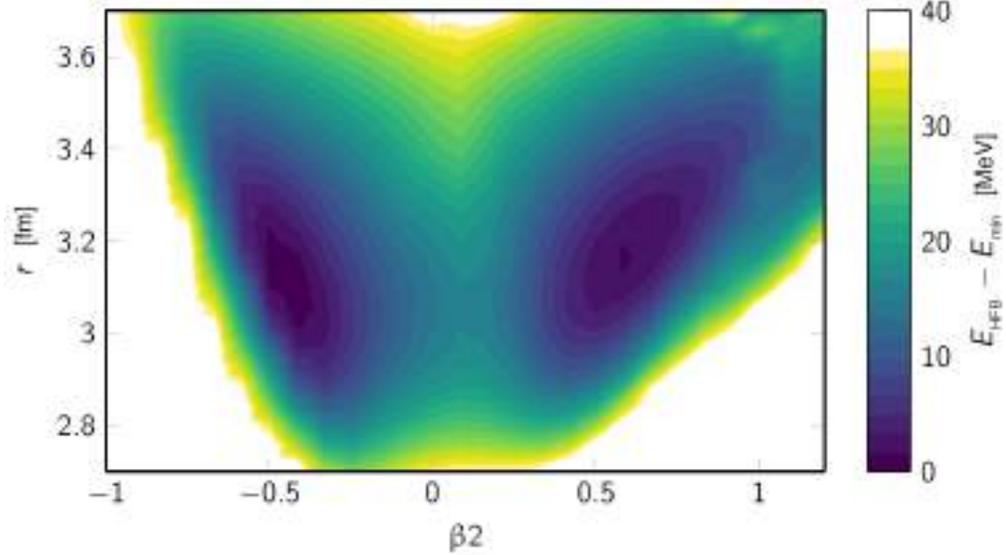
Shape coexistence effects in ^{28}Si

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



Shape coexistence effects in ^{28}Si

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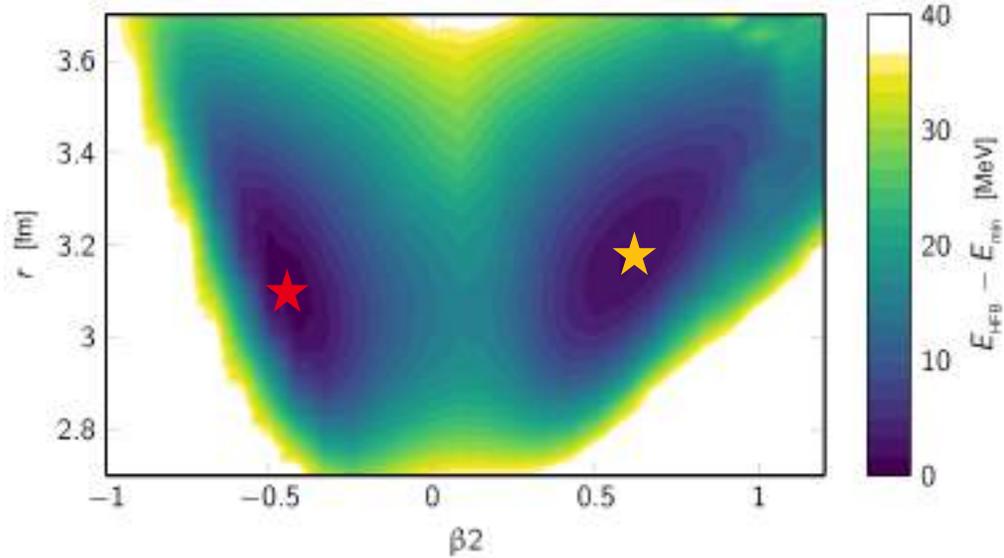


Shape coexistence [Jenkins et al., 2012]

Deformation

Shape coexistence effects in ^{28}Si

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



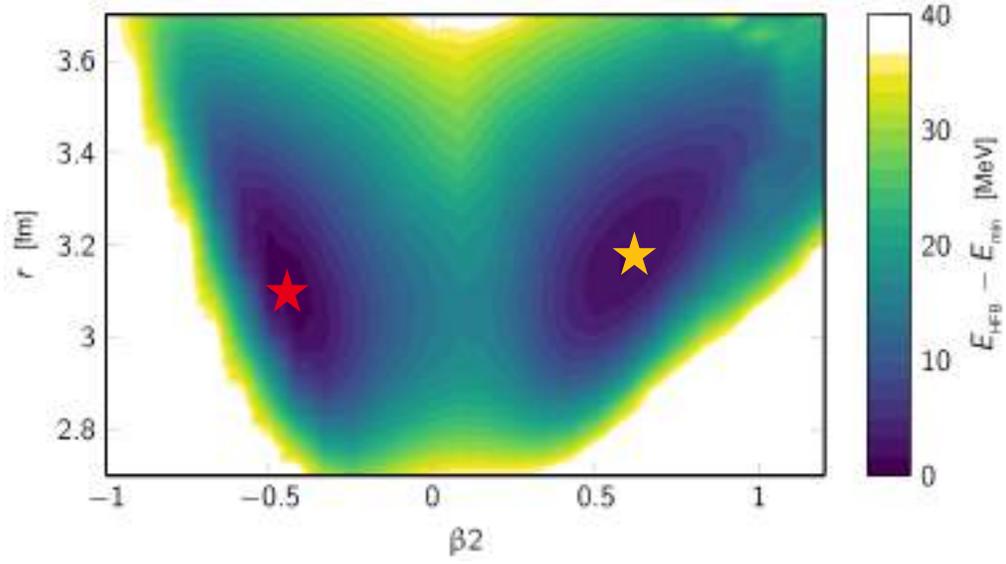
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- Oblate GS and prolate-shape isomer

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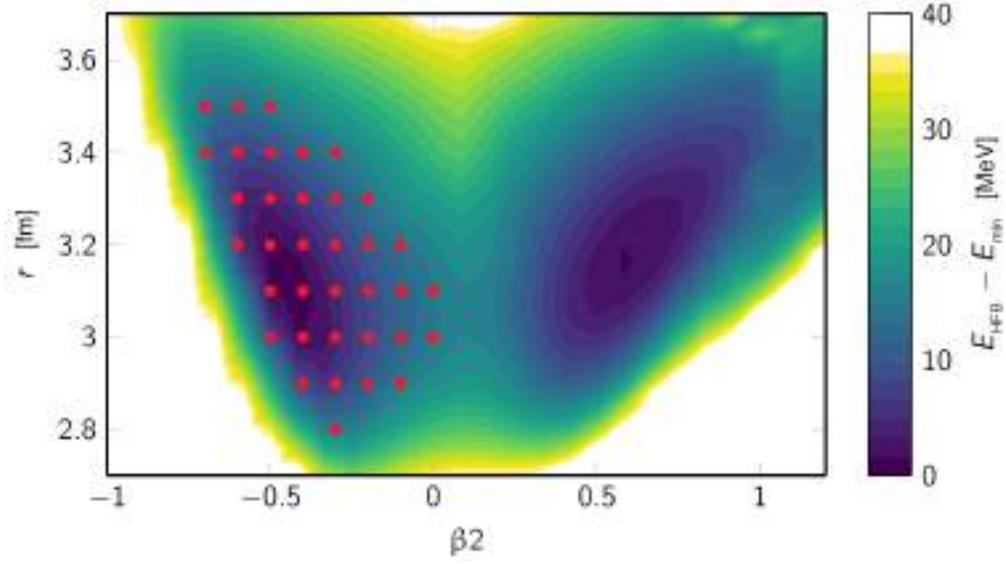
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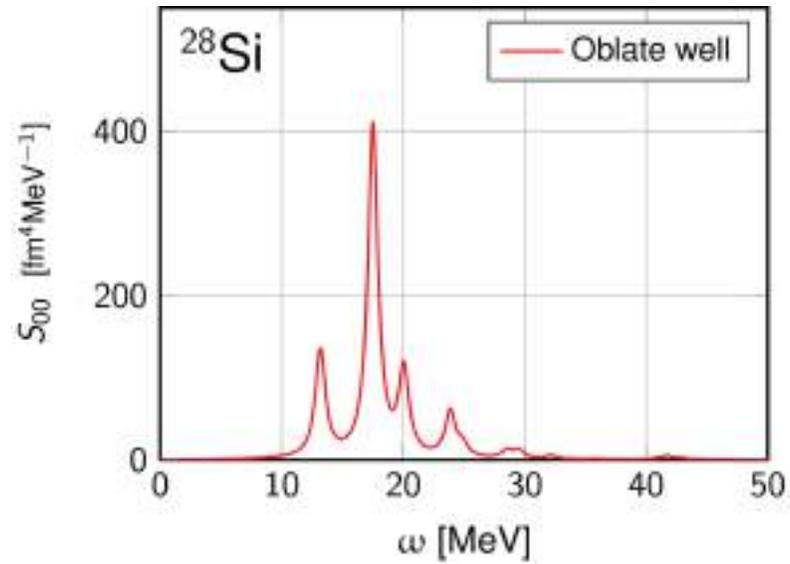
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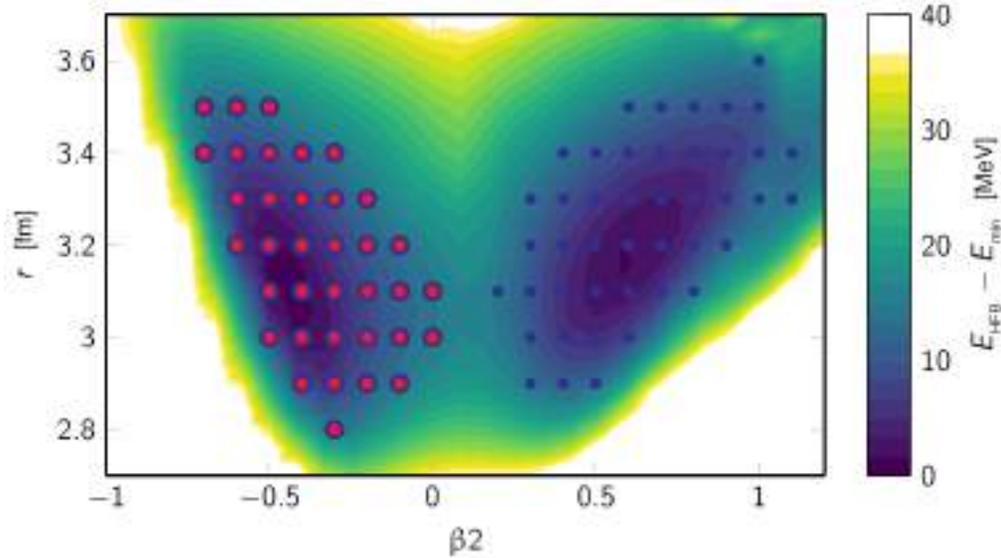
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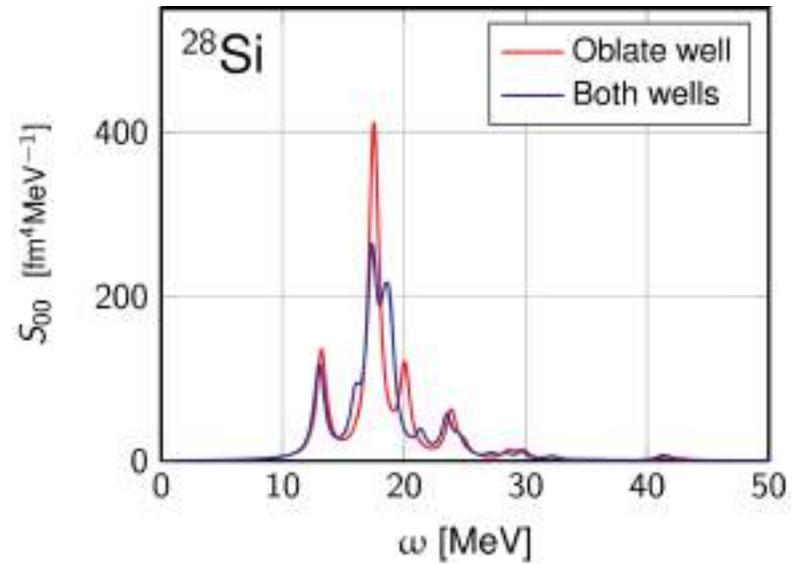
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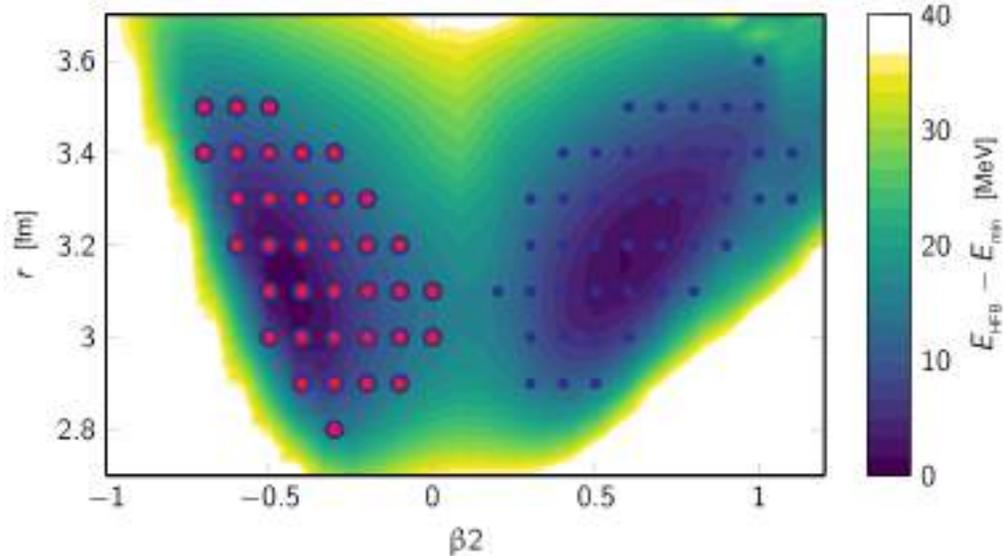
Deformation



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 - Shape coexistence but weak mixing

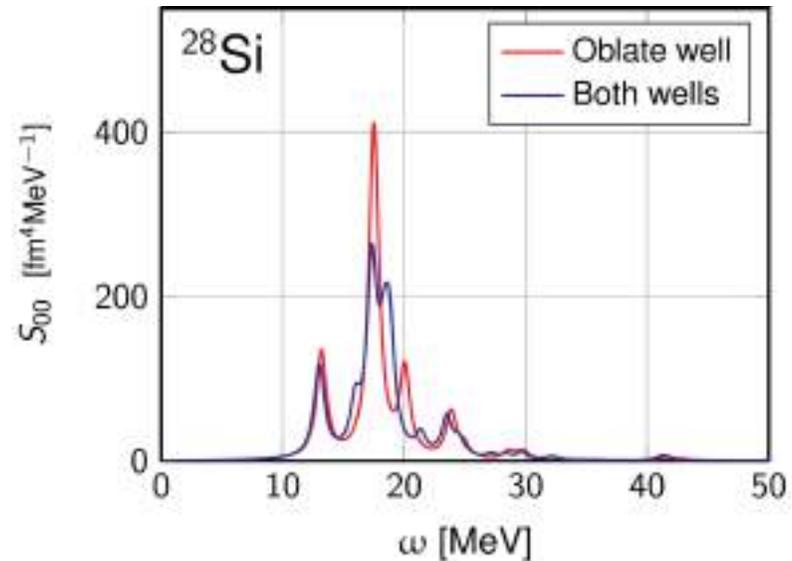
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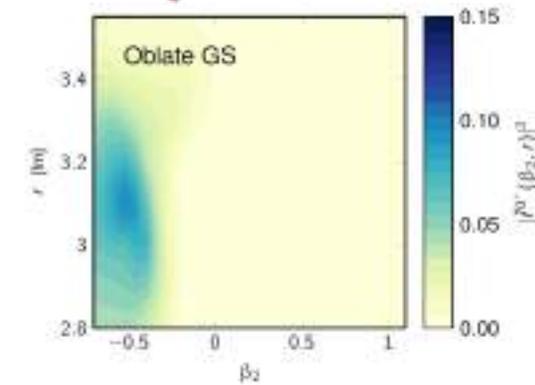
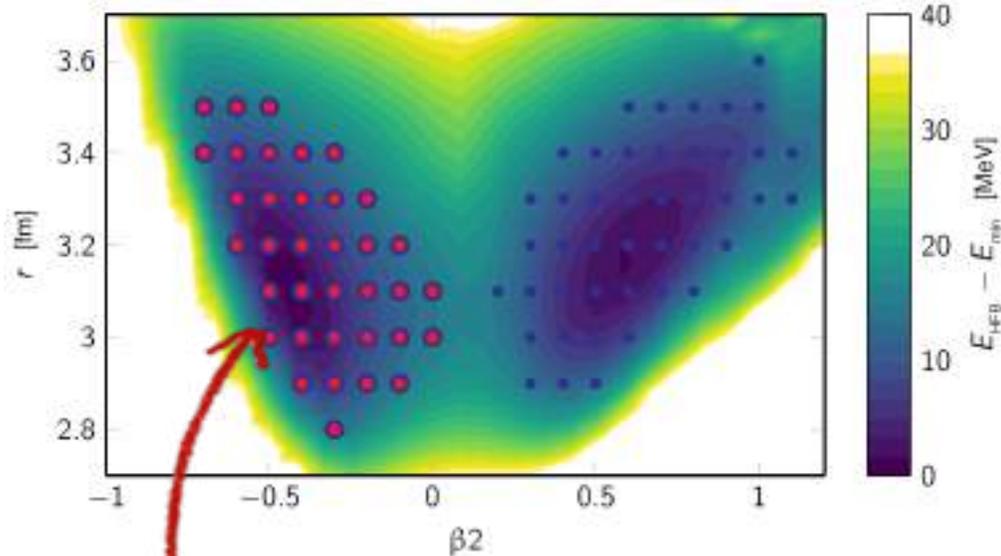


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Nuclei with stronger signature? 14

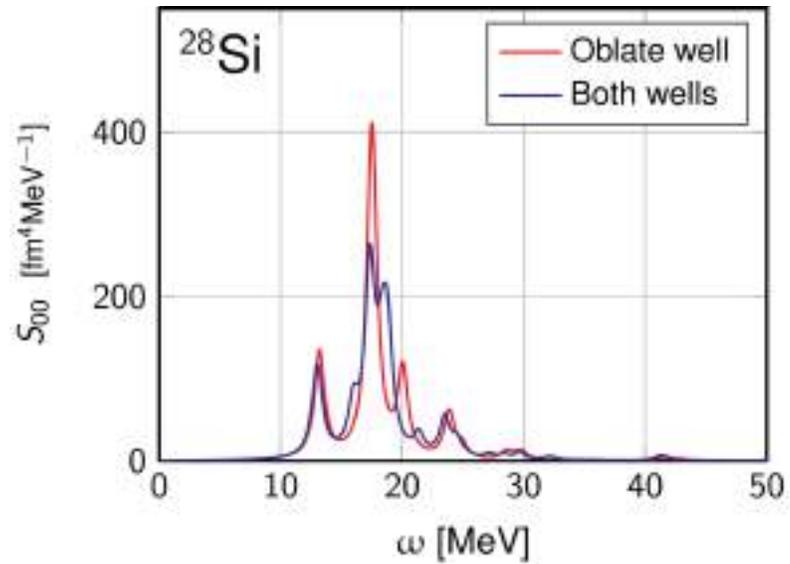
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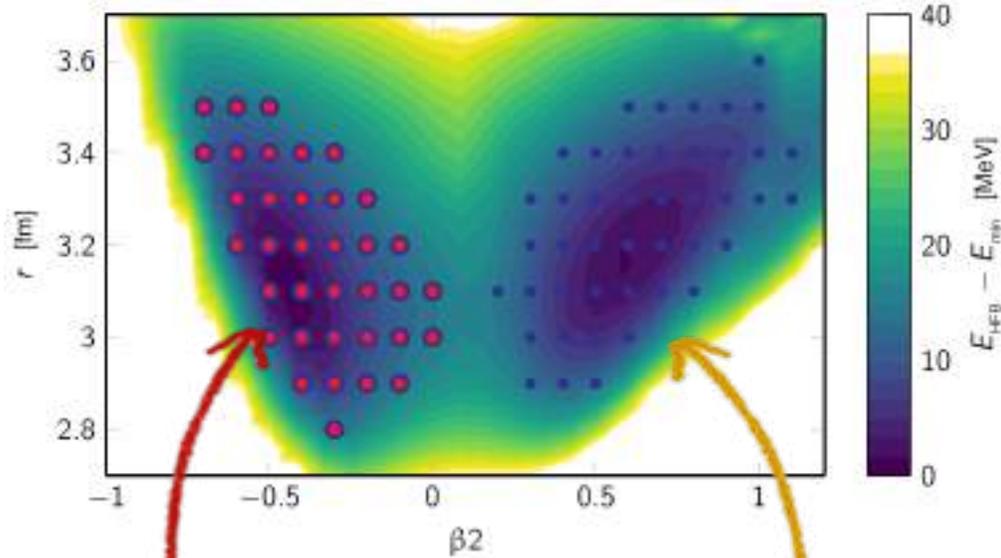


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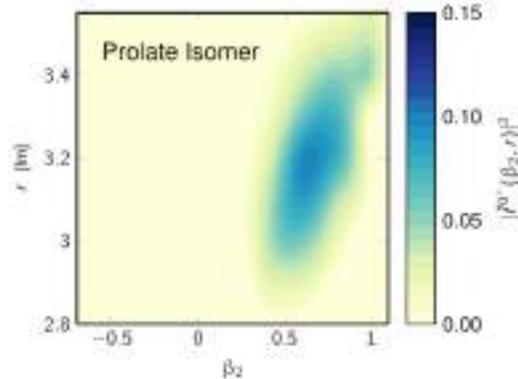
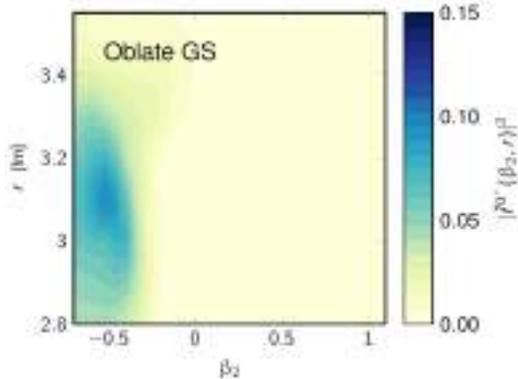
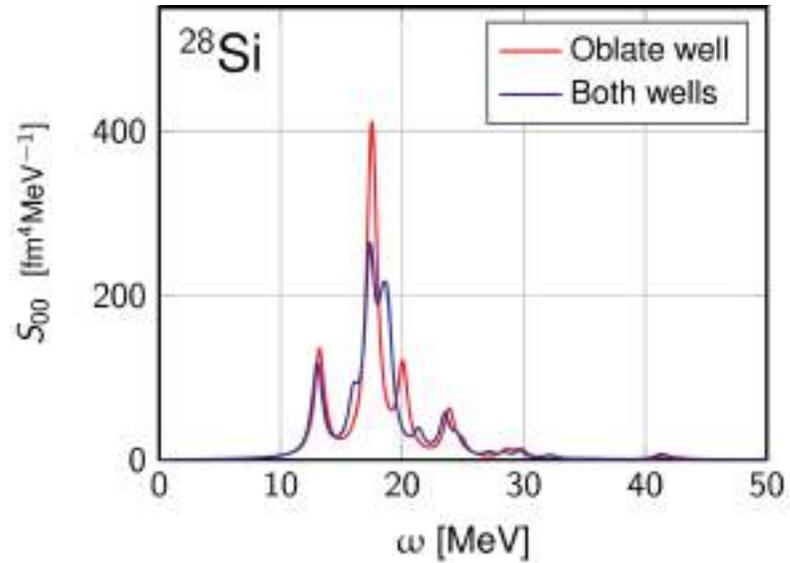
Shape coexistence effects in ^{28}Si

Total Energy Surface $E_{\text{HF}}(\beta_2, r)$



Shape coexistence [Jenkins et al., 2012]

Deformation

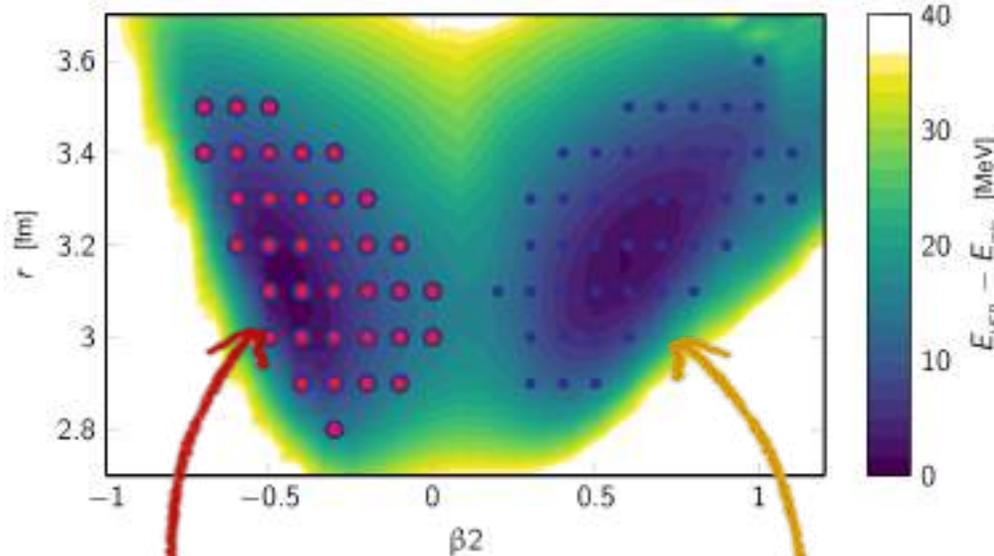


- Oblate GS and prolate-shape isomer
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Nuclei with stronger signature? 14

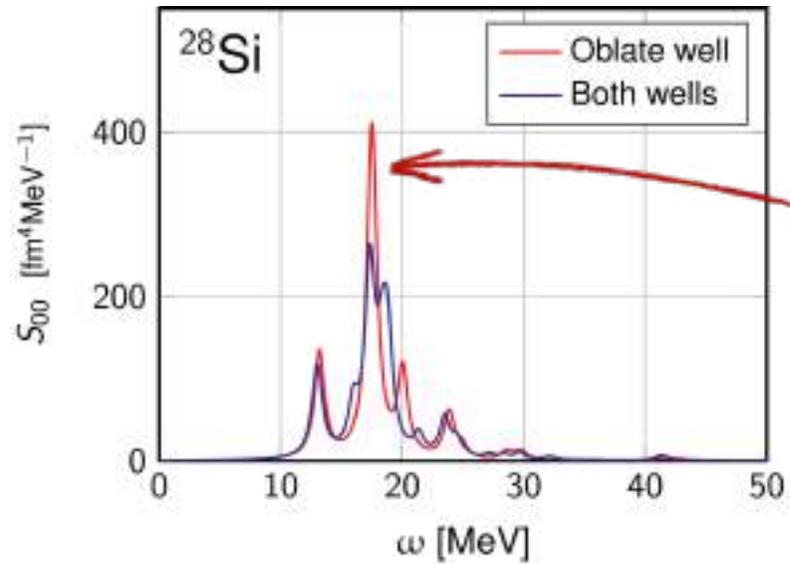
Shape coexistence effects in ^{28}Si

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$

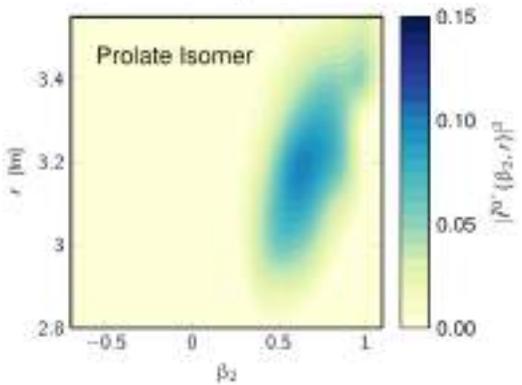
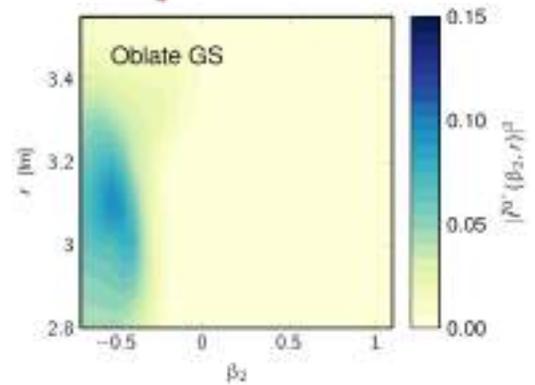
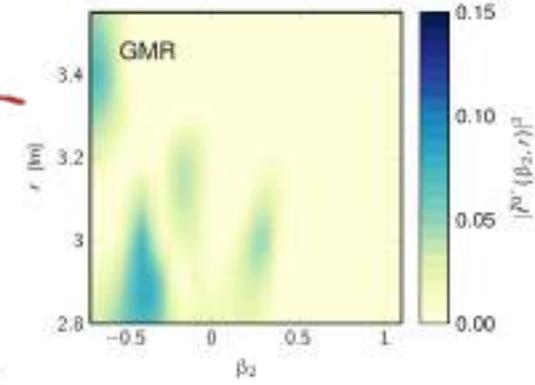


Shape coexistence [Jenkins et al., 2012]

Deformation



Radial vibration on oblate GS

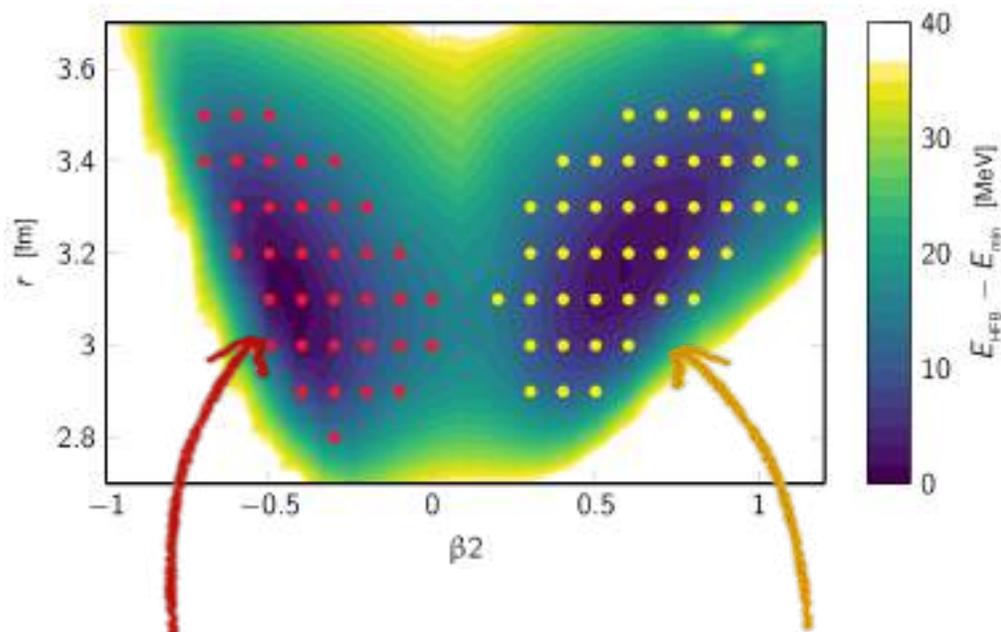


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Nuclei with stronger signature? 14

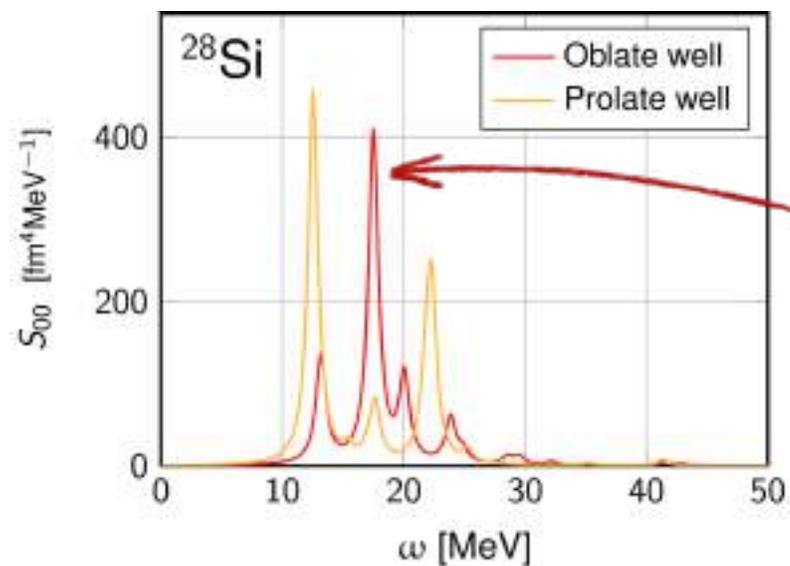
Shape coexistence effects in ^{28}Si

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$

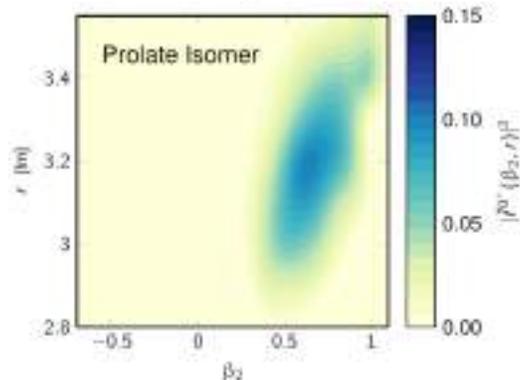
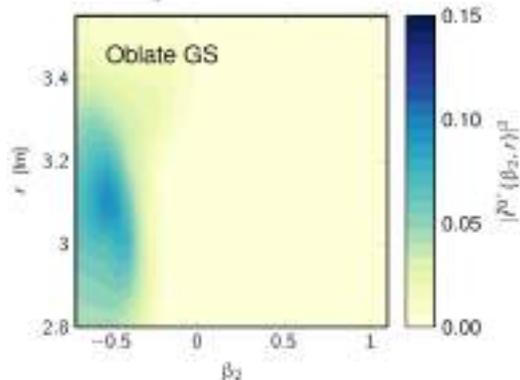
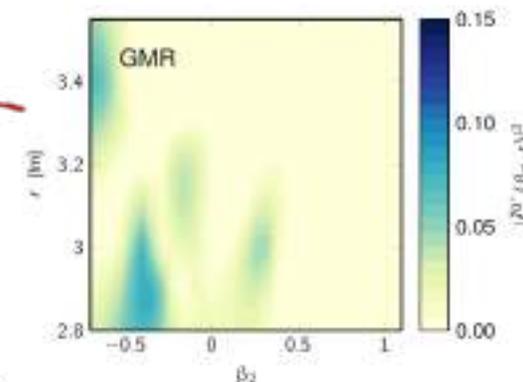


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Deformation



Radial vibration on oblate GS



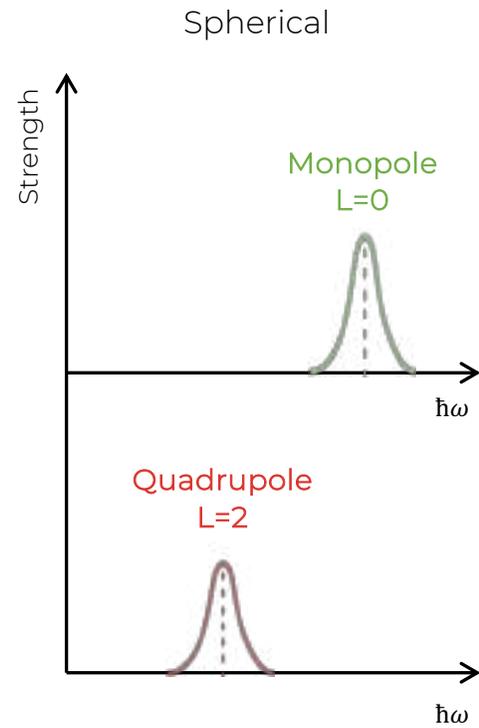
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Deformation effects

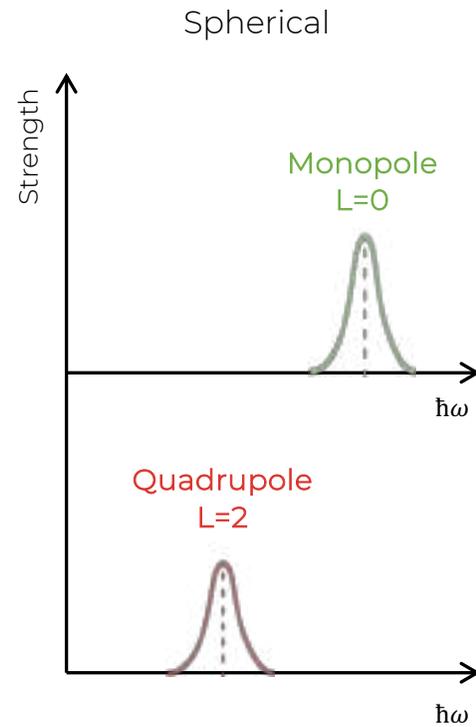
Deformation effects

Spherical
(no deformation)



Deformation effects

Spherical
(no deformation)

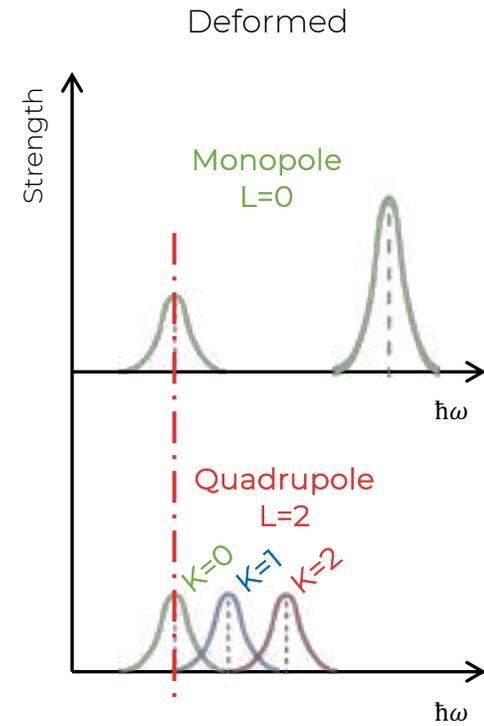


Spheroidal
(deformed)



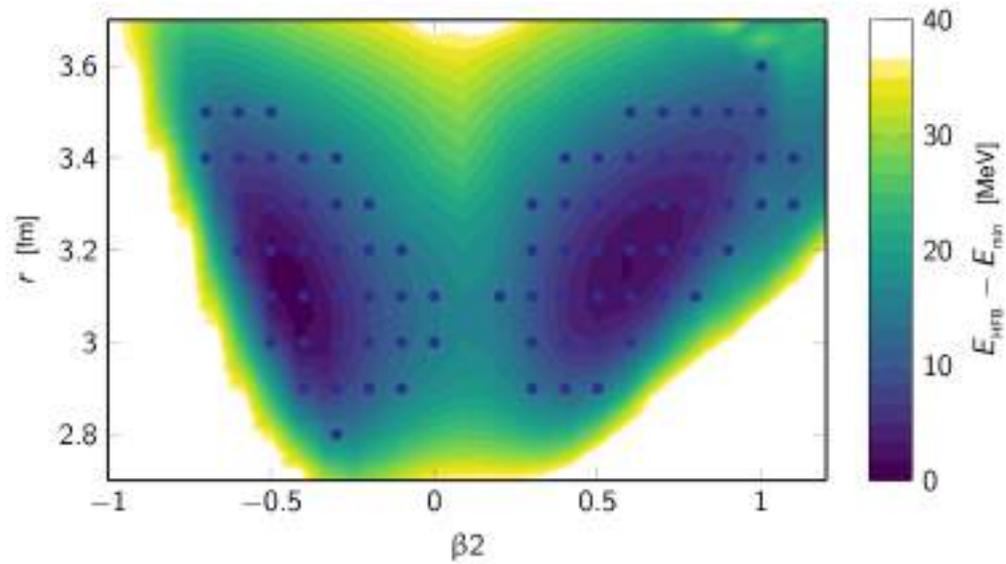
Prolate
(cigar type)

Oblate
(pancake type)



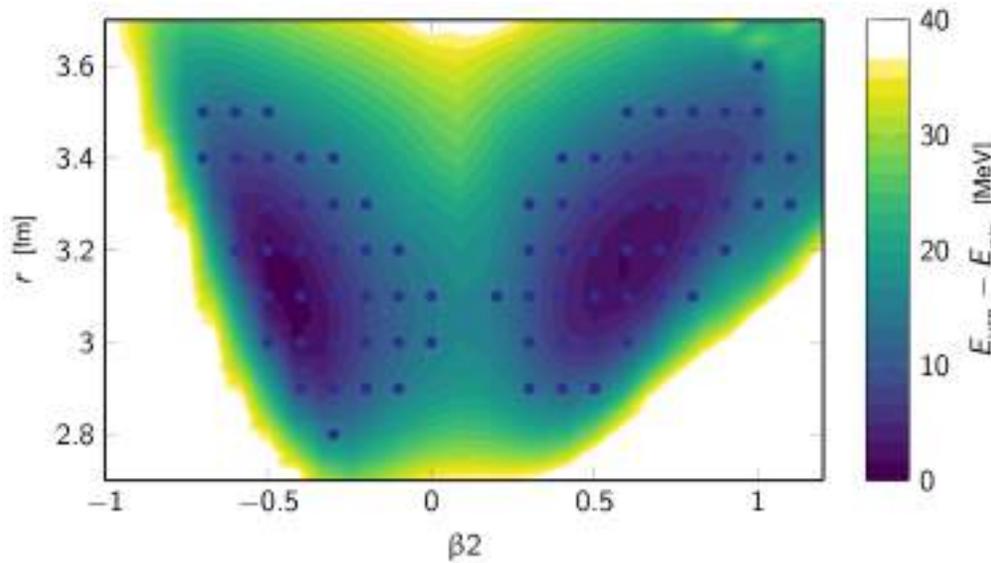
Deformation effects in prolate ^{28}Si

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$

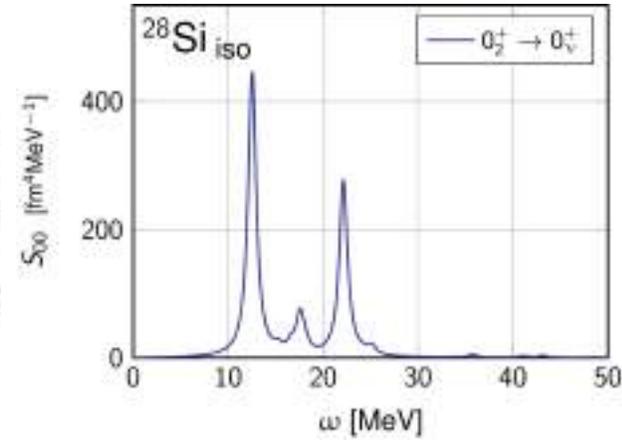


Deformation effects in prolate ^{28}Si

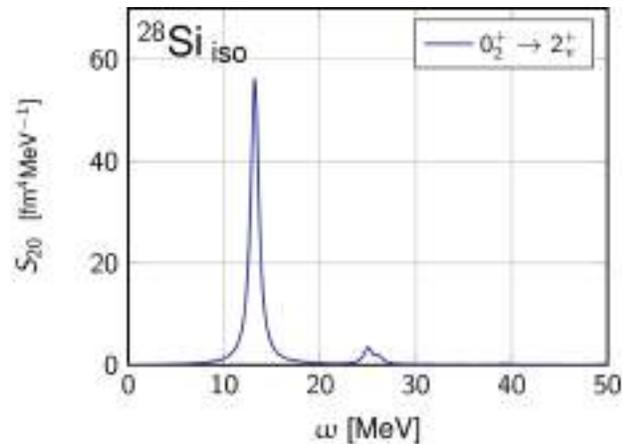
Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



Monopole Strength



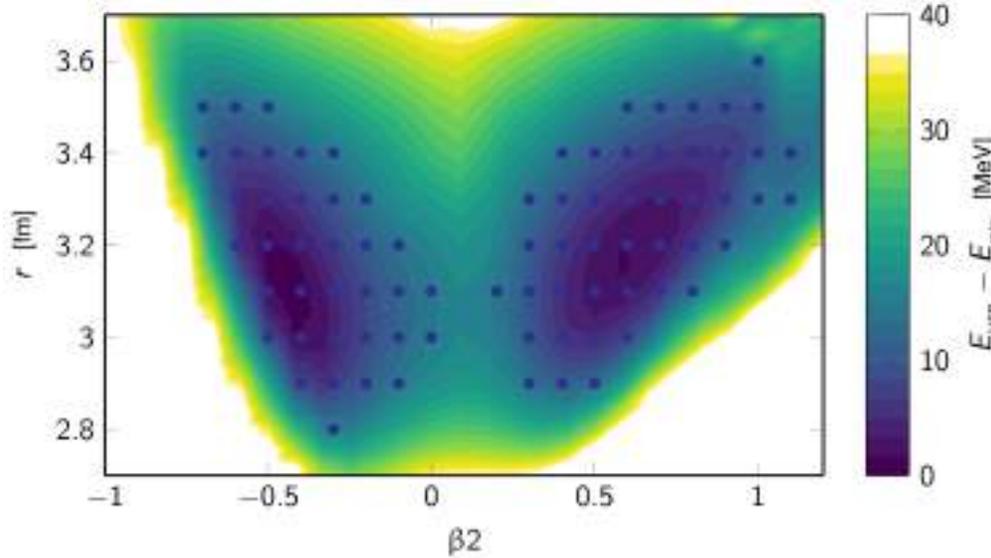
K=0 Quadrupole Strength



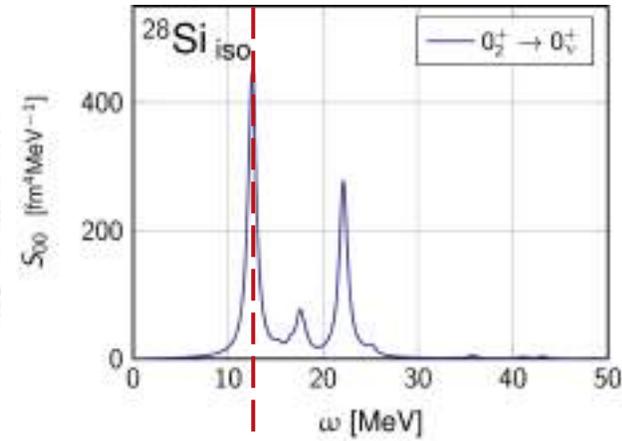
- Focus on the prolate-shape isomer
- Coupling to GQR generates **splitting**
 - × High peak = shifted “spherical” breathing mode
 - × Low peak = induced by coupling to GQR (K=0)
- Two-peak GMR on the prolate shape isomer

Deformation effects in prolate ^{28}Si

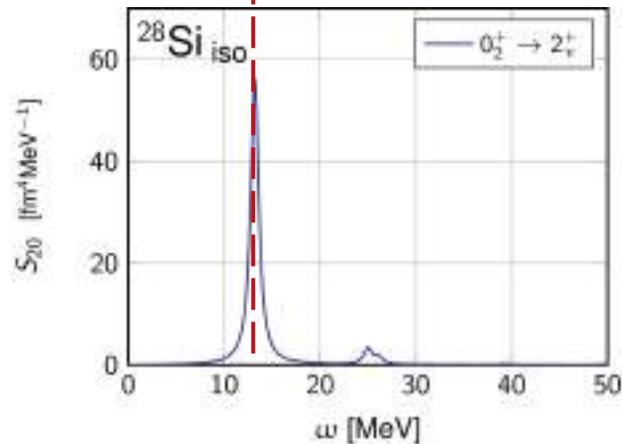
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Monopole Strength



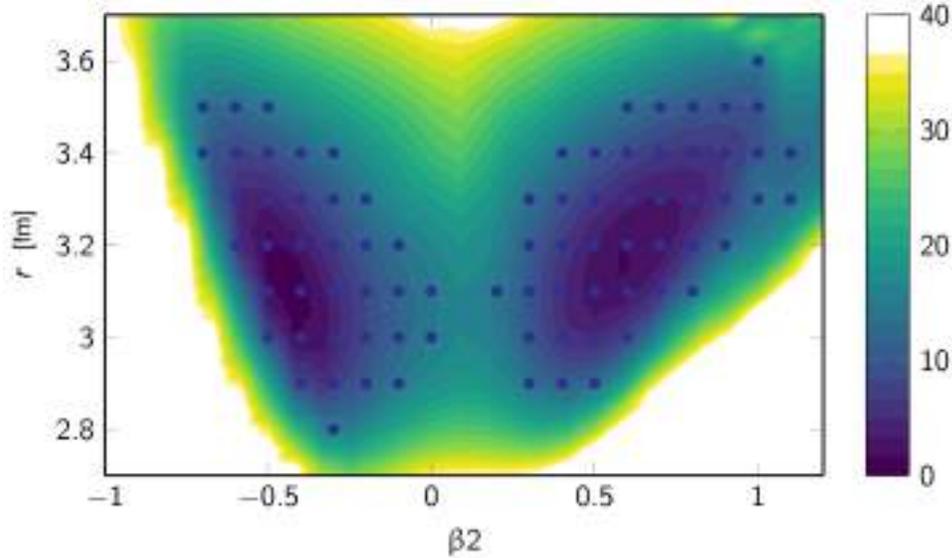
K=0 Quadrupole Strength



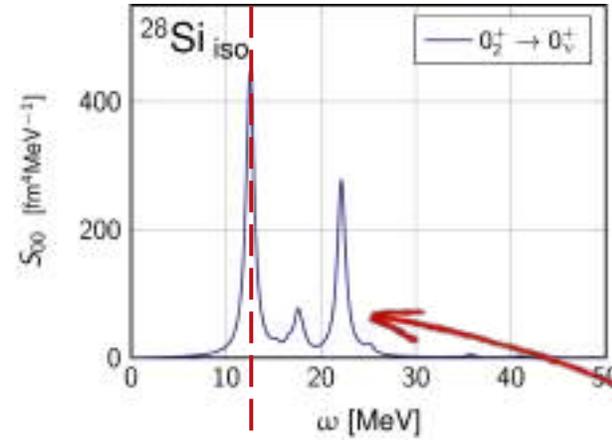
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Deformation effects in prolate ^{28}Si

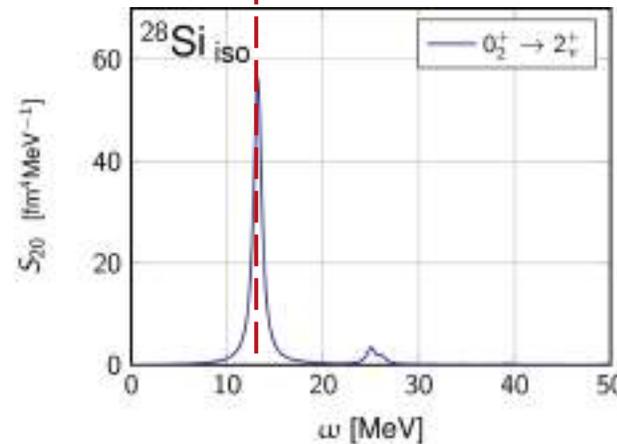
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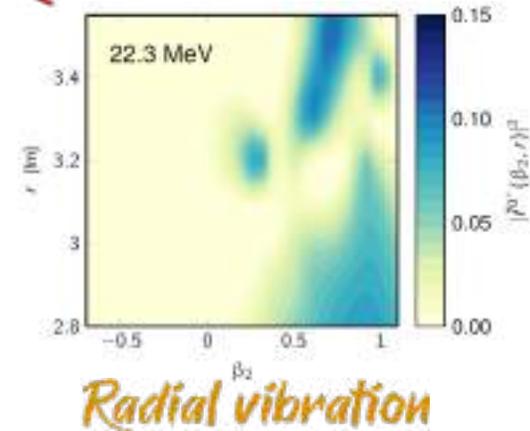
Monopole Strength



K=0 Quadrupole Strength

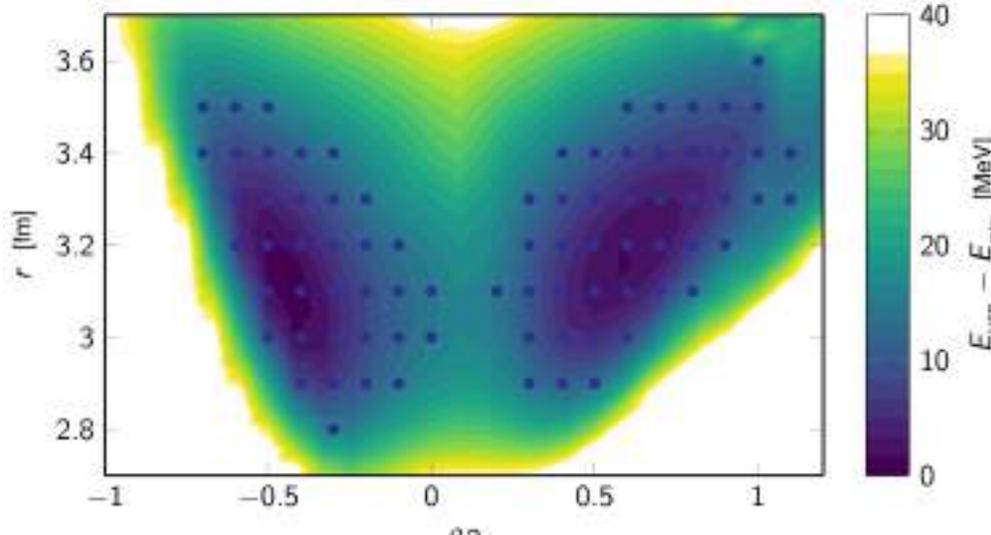


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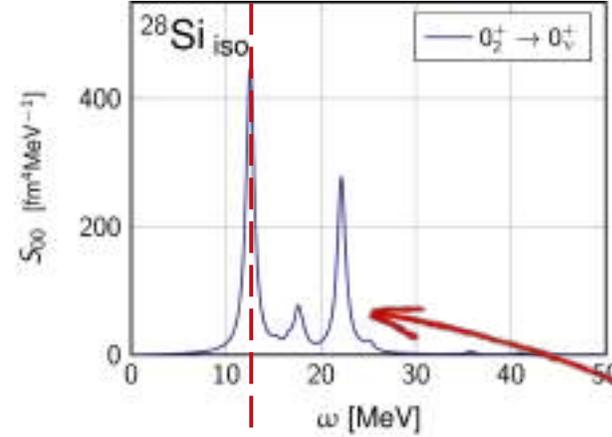


Deformation effects in prolate ^{28}Si

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$

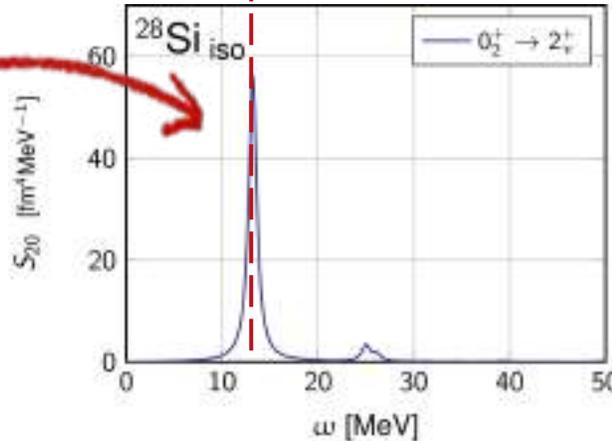


Monopole Strength

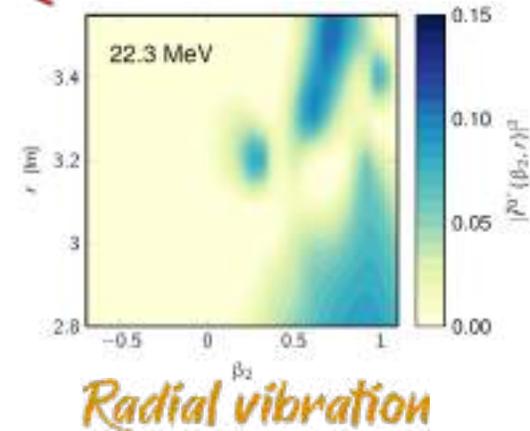
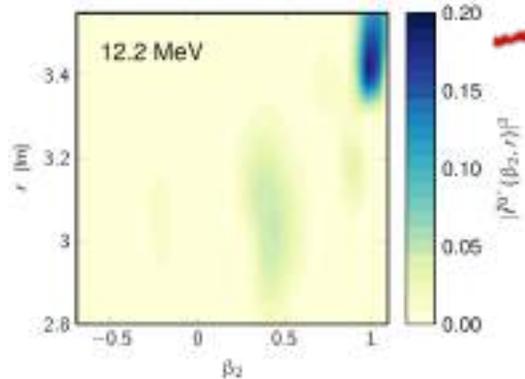


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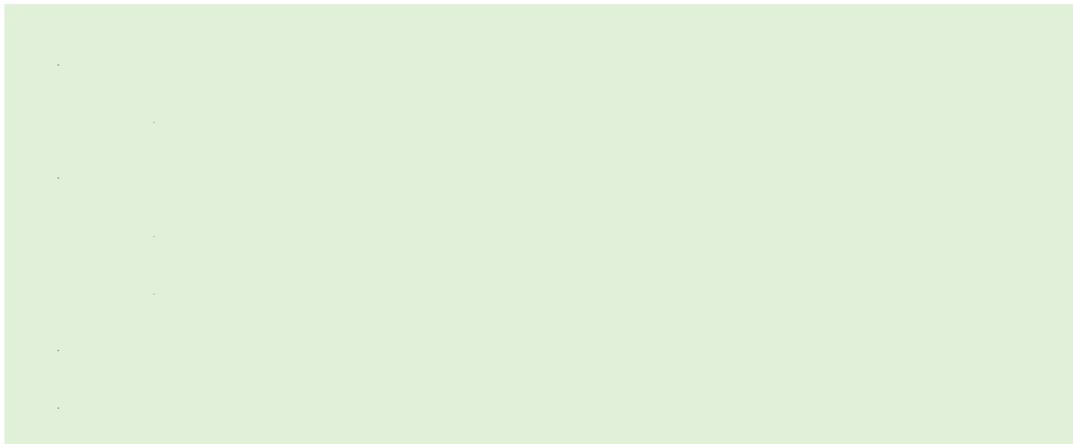
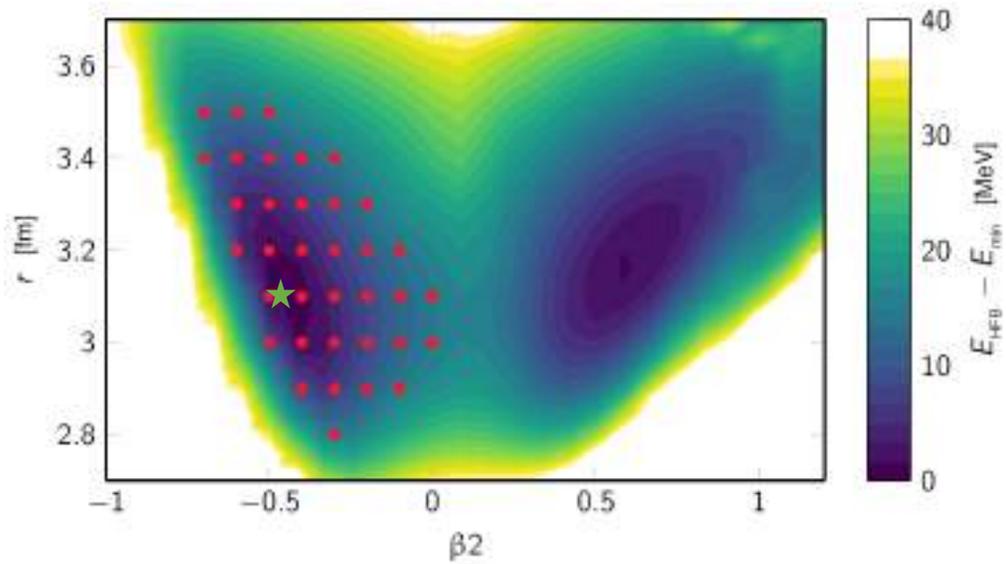


Radial + β_2 vibration



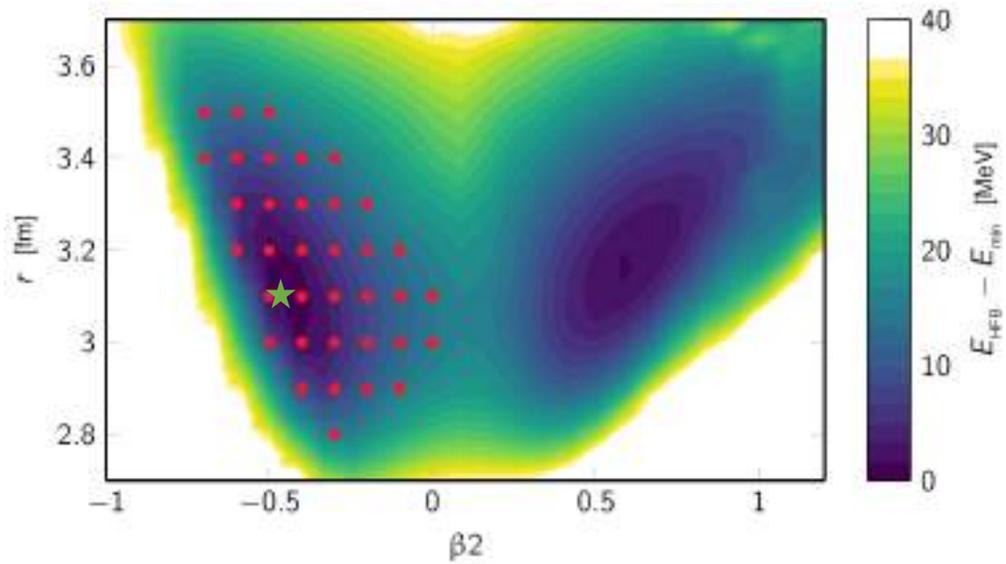
Anharmonic effects

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



Anharmonic effects

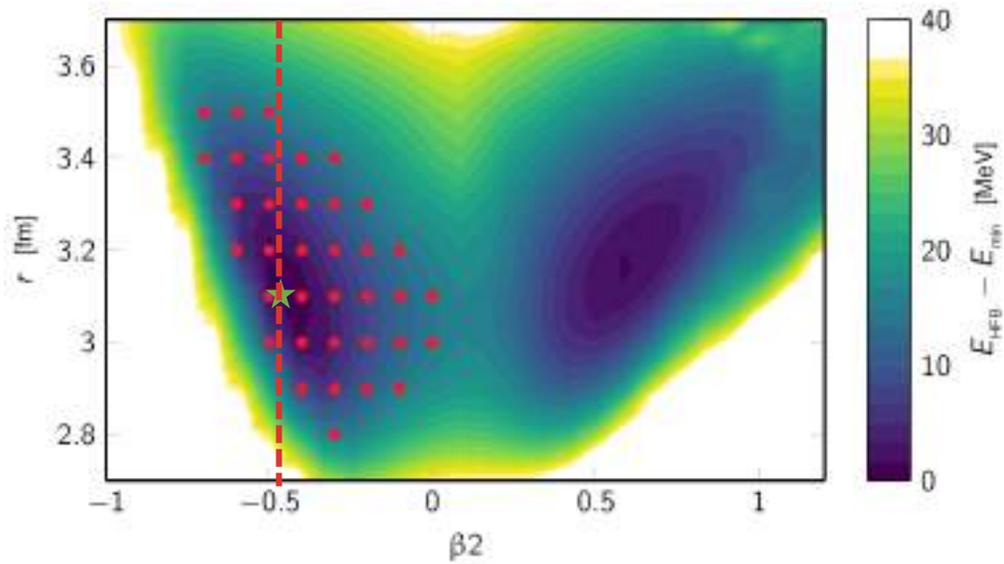
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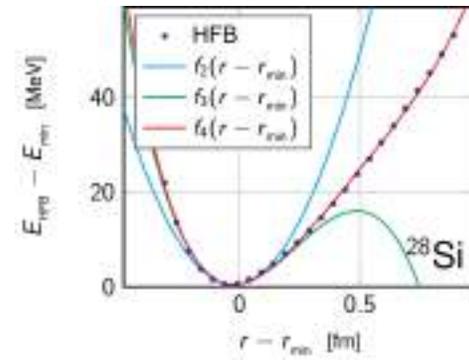
- Quantitative anharmonicities analysis

Anharmonic effects

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



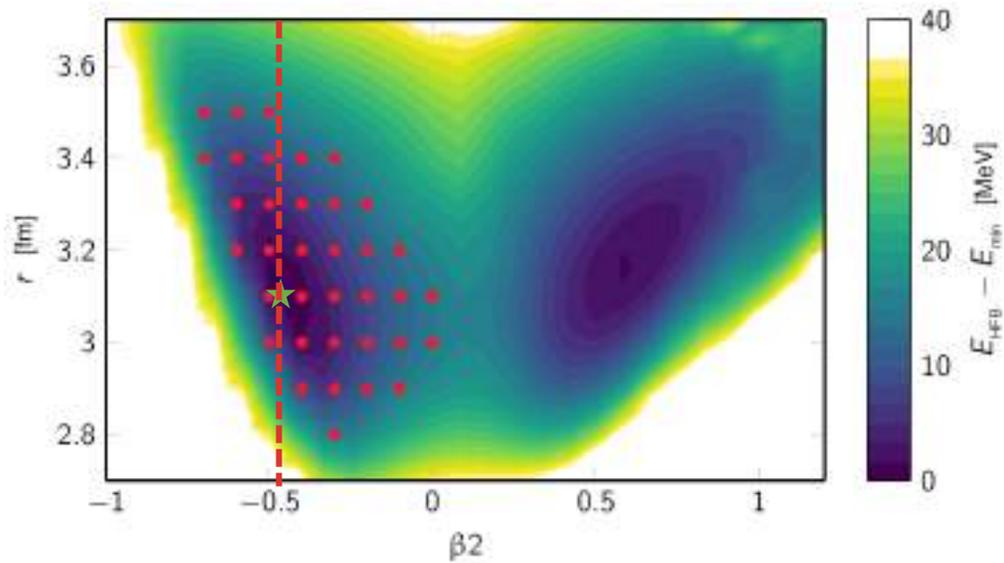
One-dimensional cut



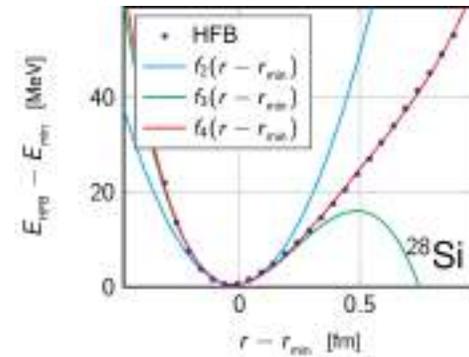
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Anharmonic effects

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



One-dimensional cut



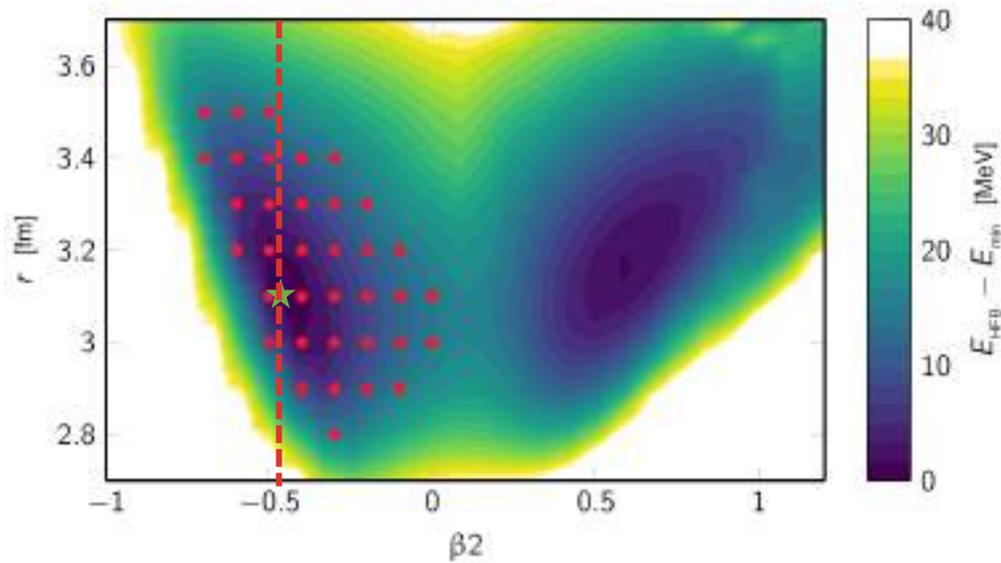
Polynomial fit

$$f(x) = a_2x^2 + a_3x^3 + a_4x^4$$

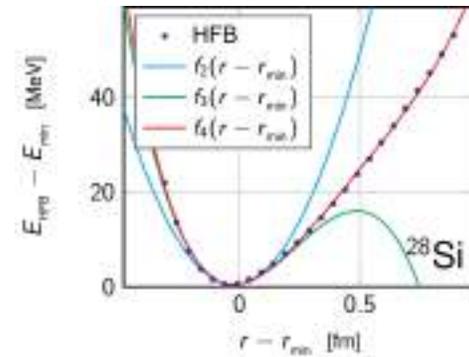
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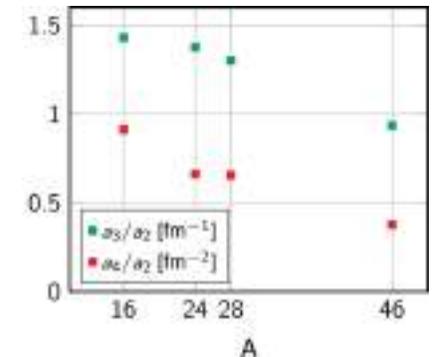


One-dimensional cut



Polynomial fit

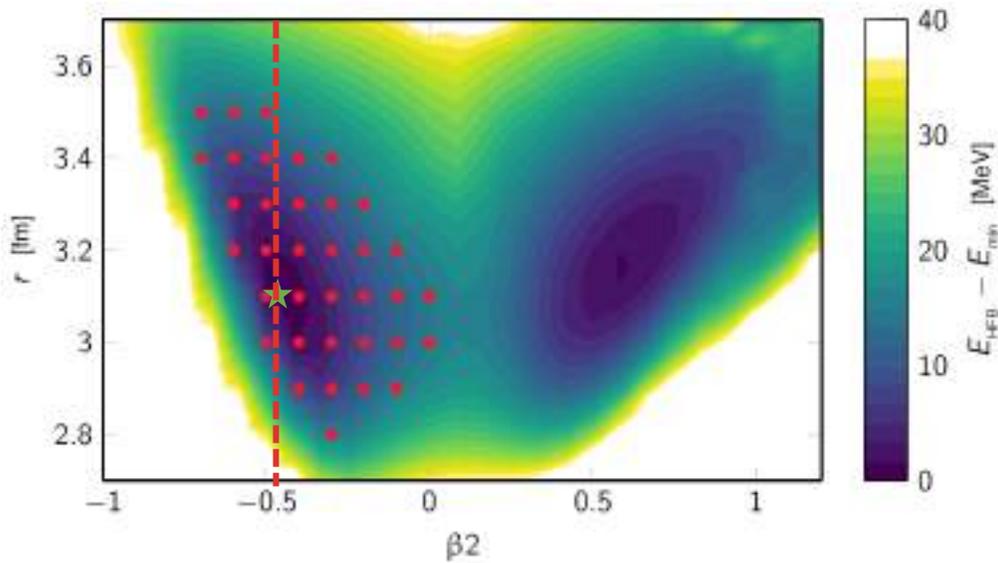
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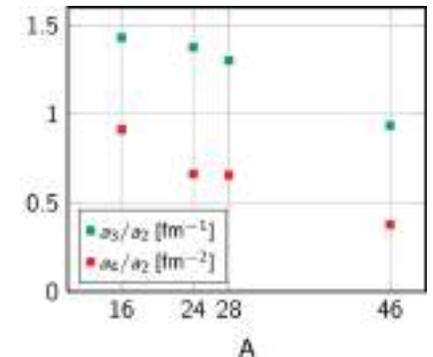
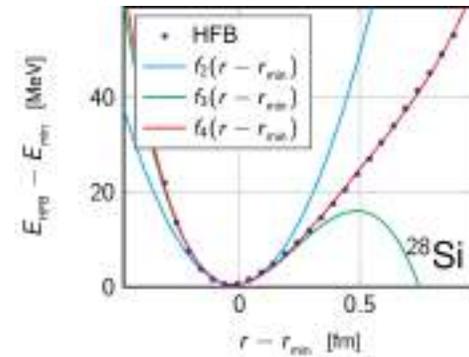
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Anharmonic effects

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



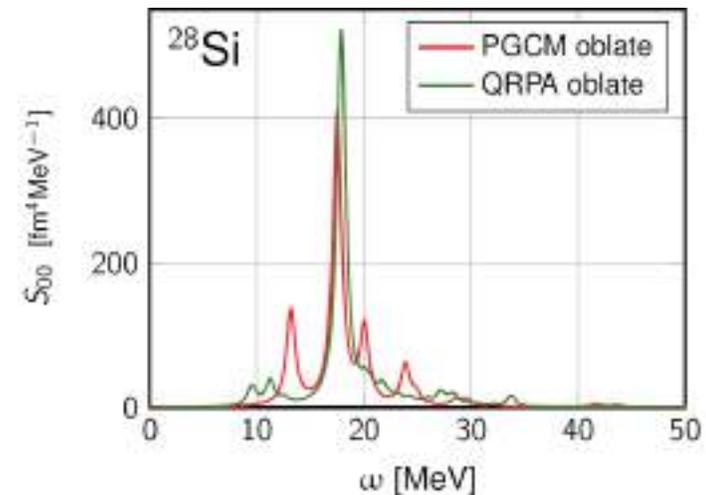
One-dimensional cut



Polynomial fit

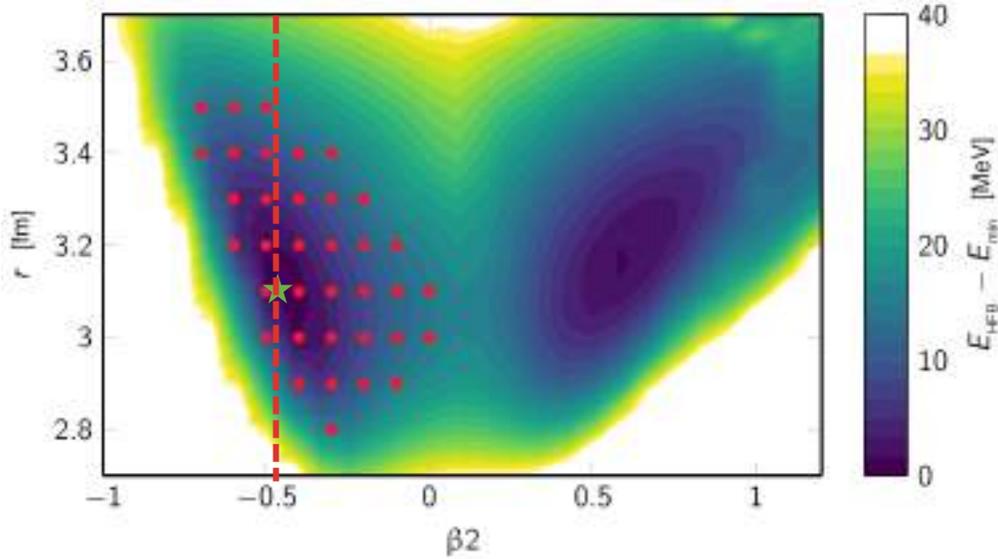
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- Quantitative anharmonicities analysis
 - Light nuclei are less harmonic
- Qualitatively similar results QRPA/PGCM

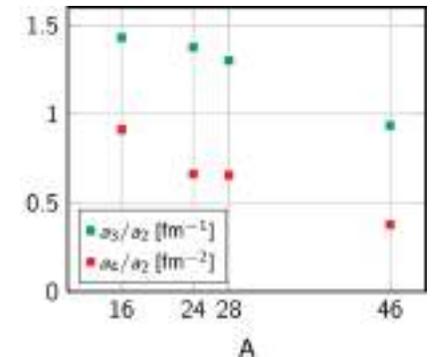
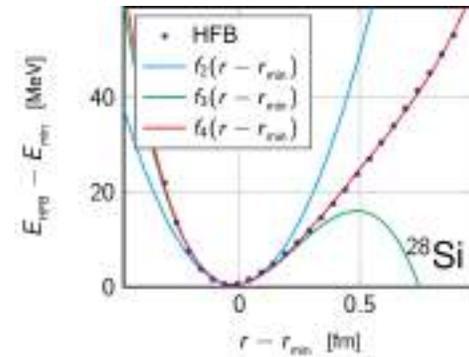


Anharmonic effects

Total Energy Surface $E_{\text{HFB}}(\beta_2, r)$



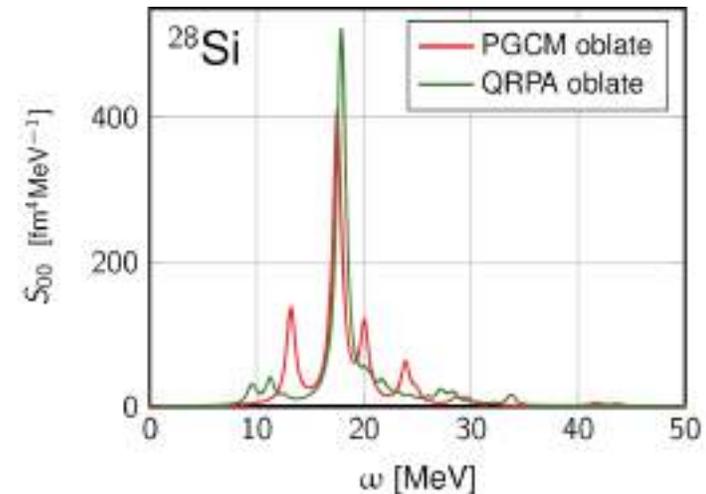
One-dimensional cut



Polynomial fit

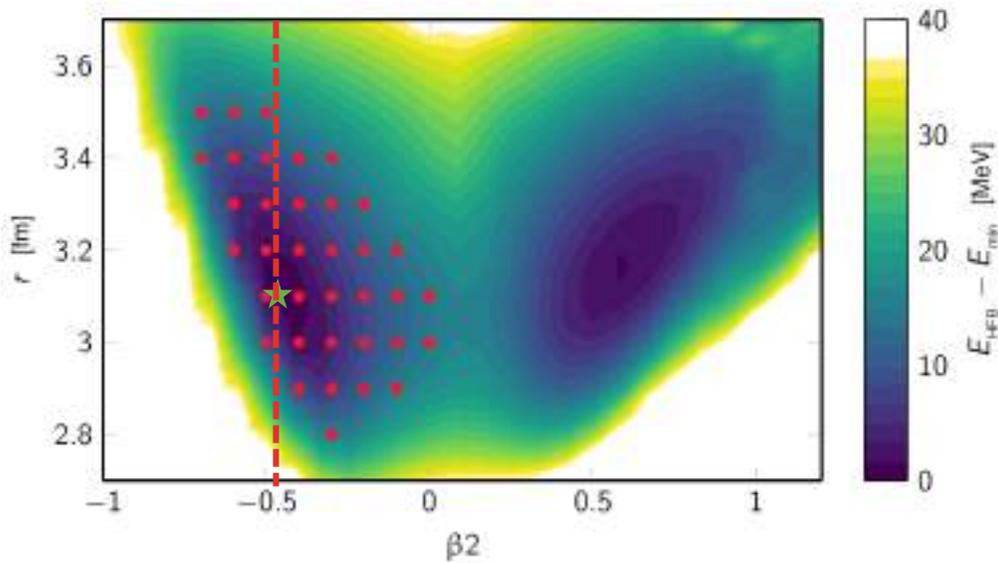
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- Quantitative anharmonicities analysis
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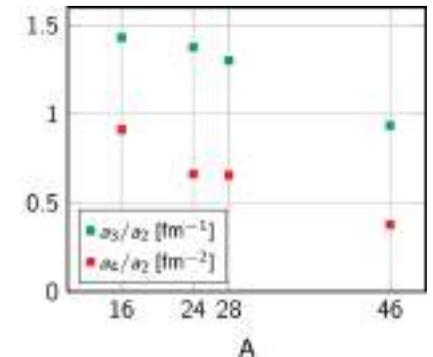
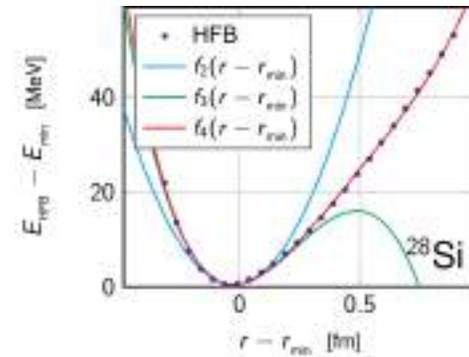


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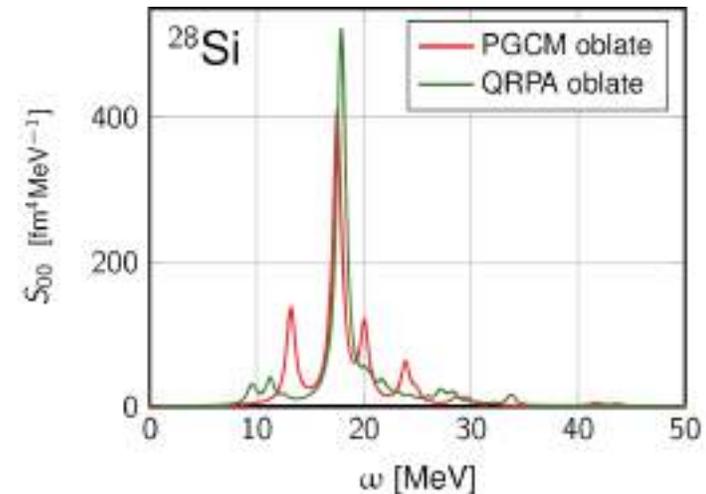
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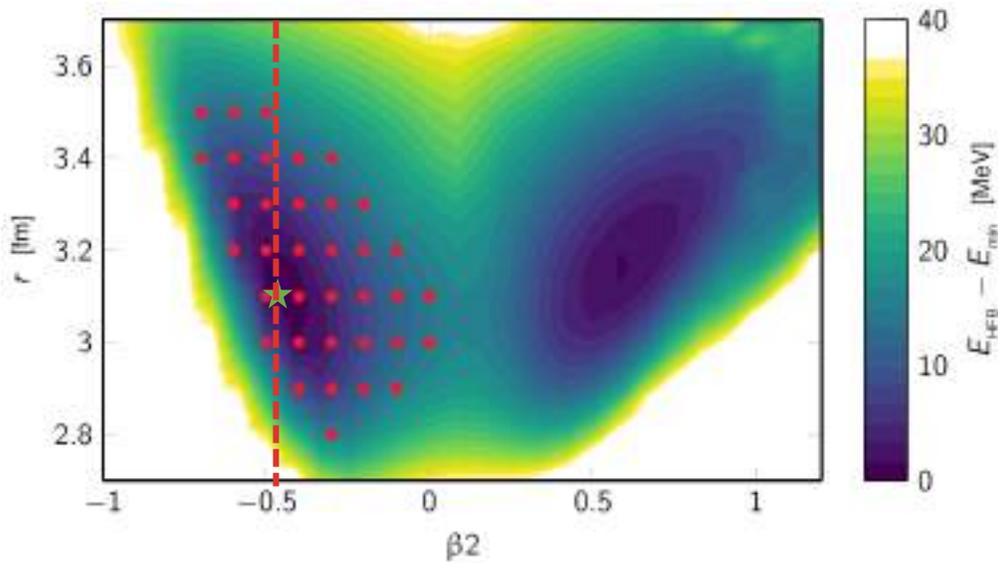
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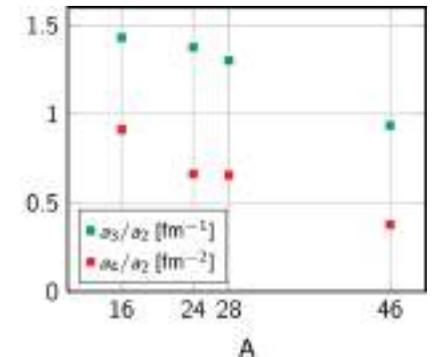
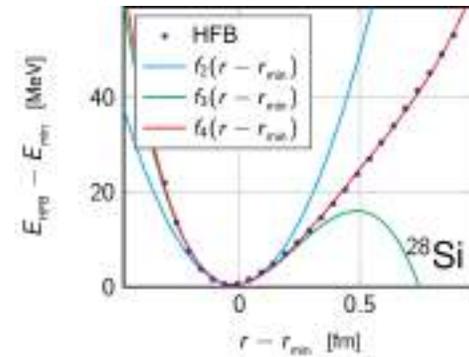


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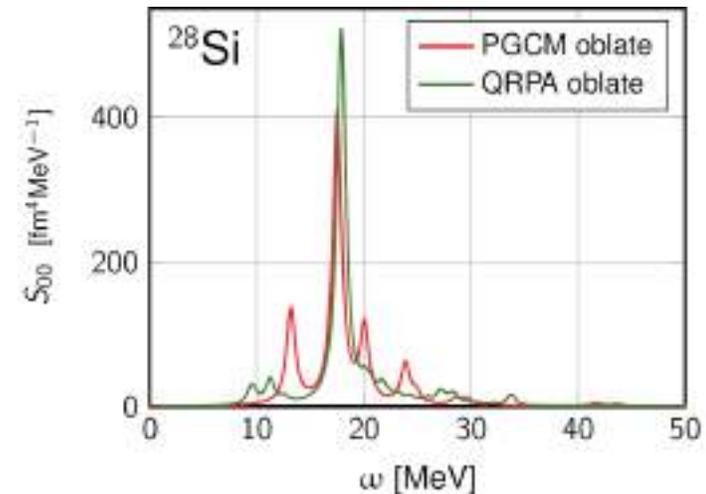
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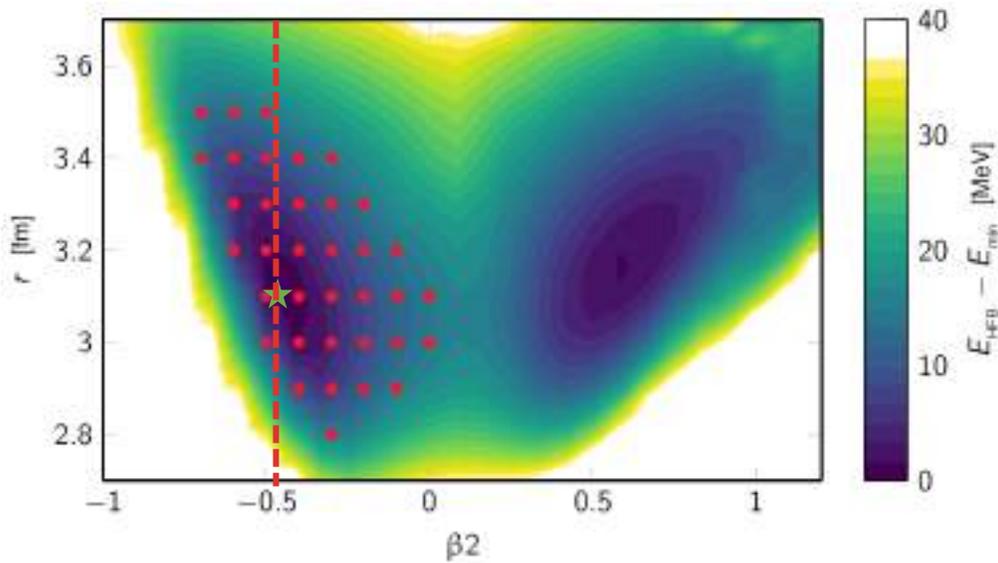
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 - QRPA response less fragmented
- Case specific fragmentation, no quantitative correlation

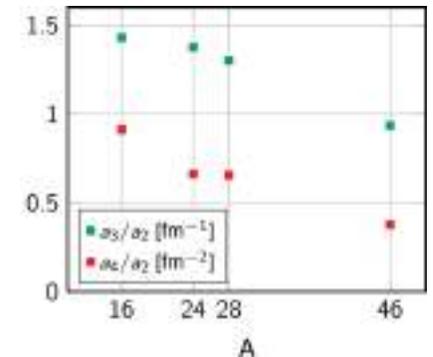
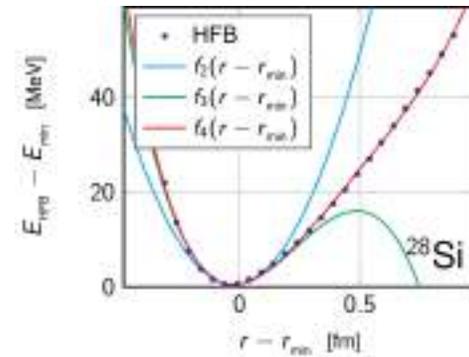


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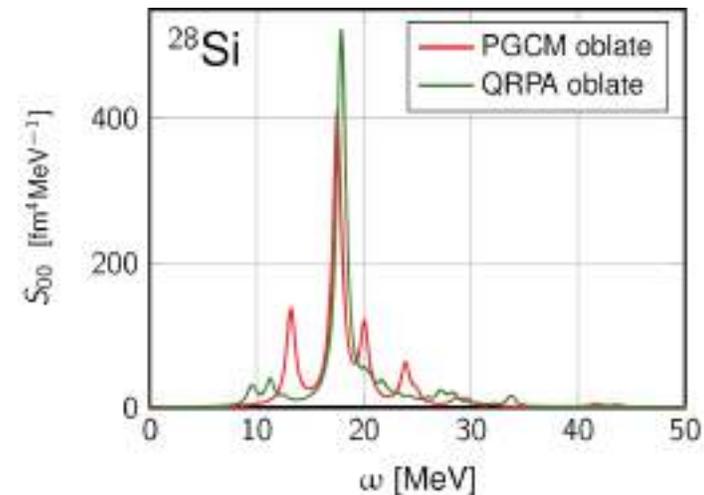
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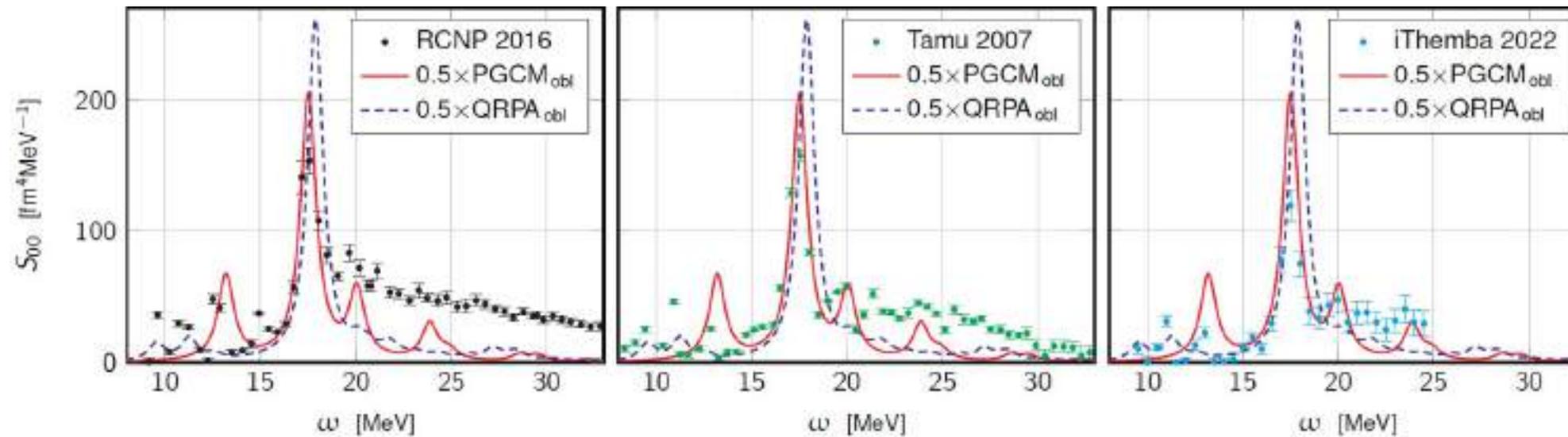
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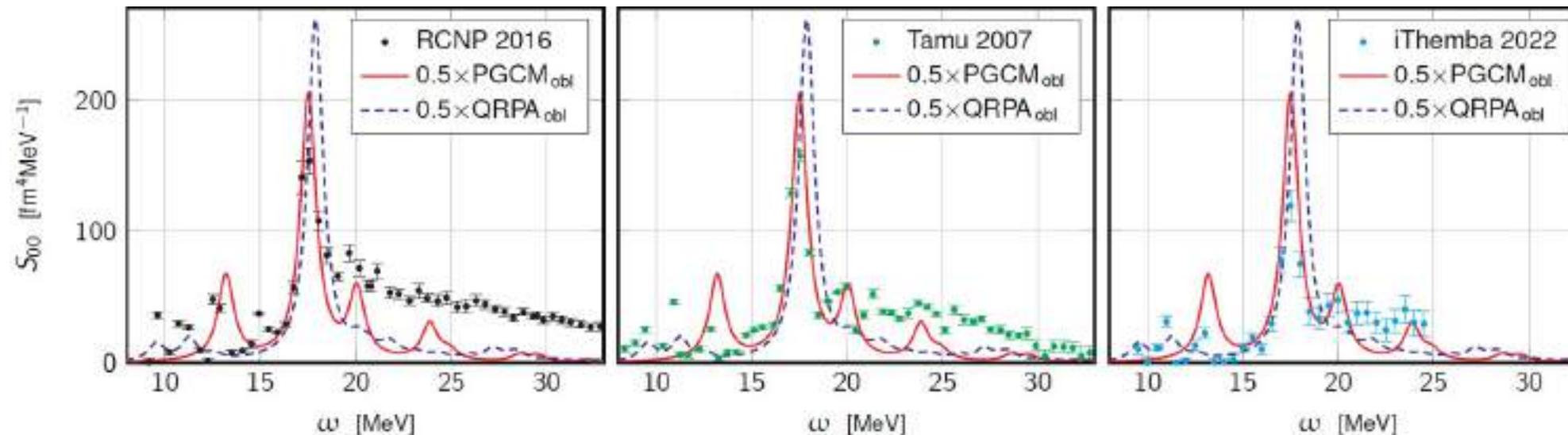
- Quantitative anharmonicities analysis
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- Qualitatively similar results QRPA/PGCM
 - Consistent monopole response
 - QRPA response less fragmented
- Case specific fragmentation, no quantitative correlation
- Projection contribution to fragmentation



Comparison to experimental data



Comparison to experimental data



Ab initio PGCM nicely reproduces the experimental data

- Nicer description of the main resonance and fragmentation

Experimental data are useful and promising to **test different many-body methods**

Data are not unambiguous, i.e. **higher resolution** would be beneficial

Outline

1 Introduction

- Giant Resonances Physics
- The PGCM
- Link between PGCM and QRPA

2 Systematic study

- Numerical details
- Uncertainty estimate

Conclusions and perspectives

Results

3

Selected applications

- Shape coexistence
- Deformation

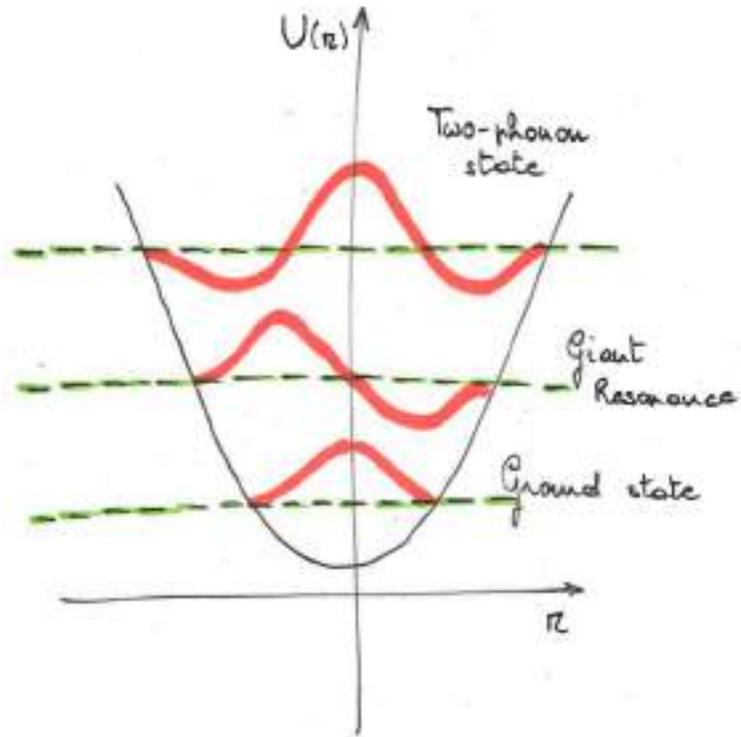
Multi-phonon states

- Proof of principle
- Realistic calculations

From finite nuclei to Astrophysics

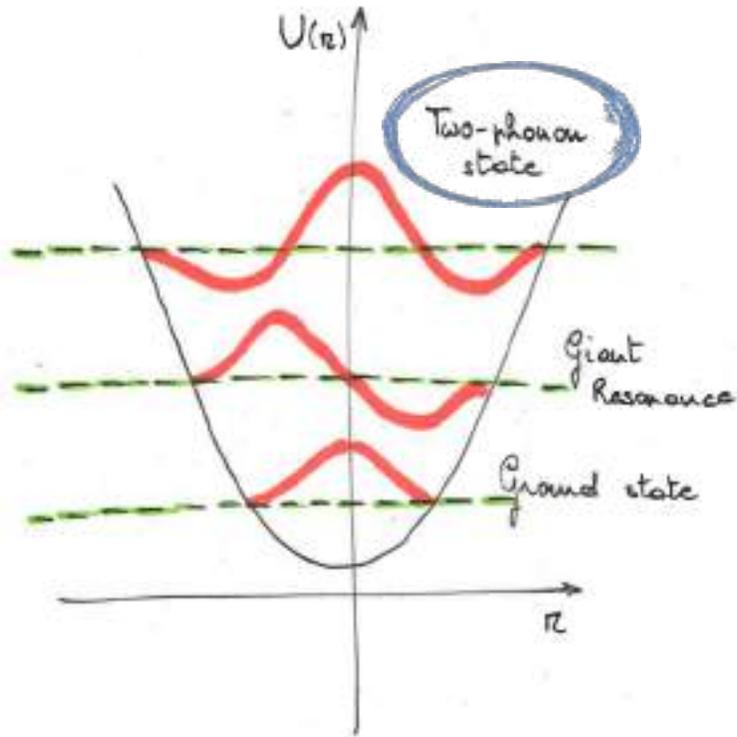
- Preliminary incompressibility results

Multi-phonon states in ^{46}Ti



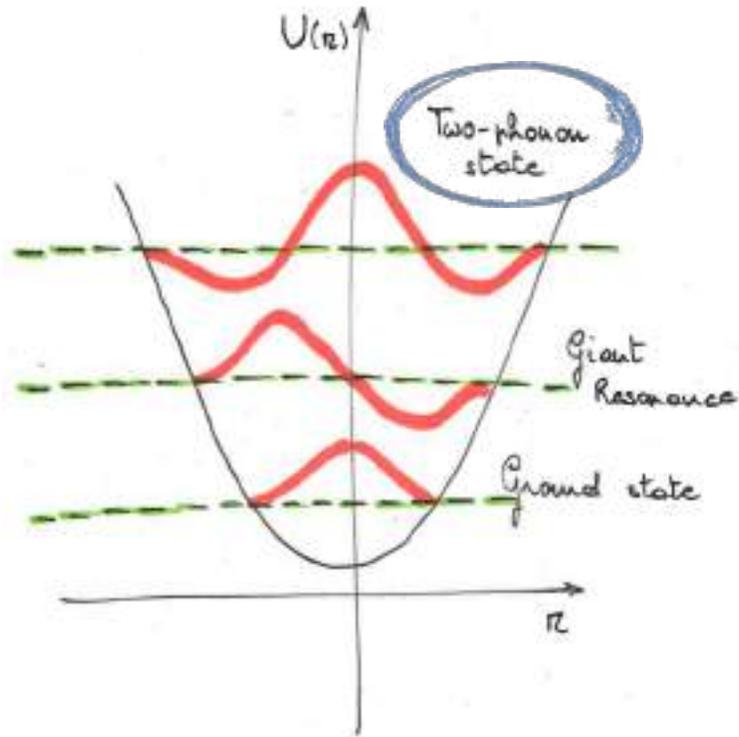
- GRs can be interpreted as the **first phonon** of a collective excitation

Multi-phonon states in ^{46}Ti



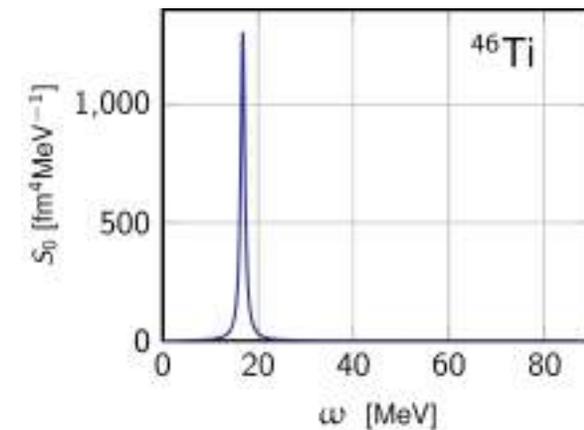
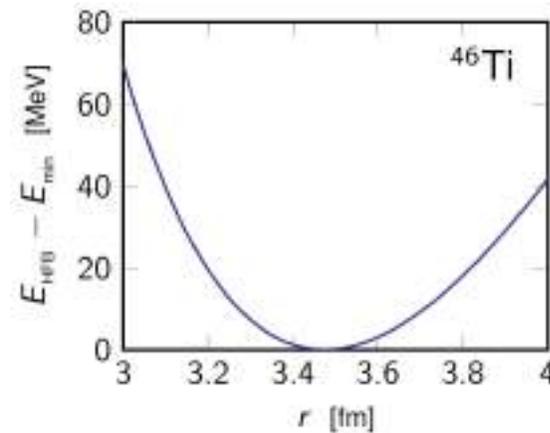
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- **Higher phonons also exist** **Multi-phonon states**

Multi-phonon states in ^{46}Ti

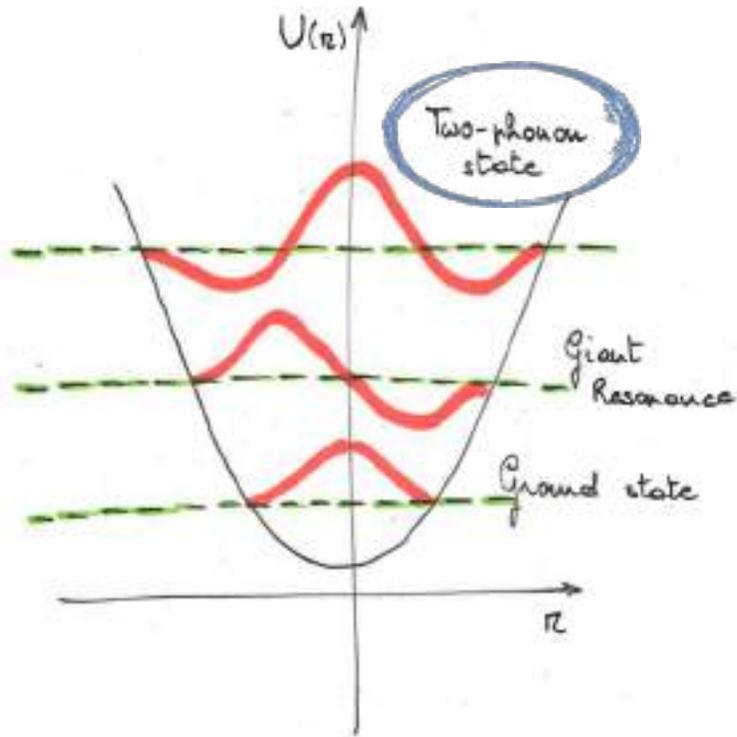


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One-dimensional PGCM calculation

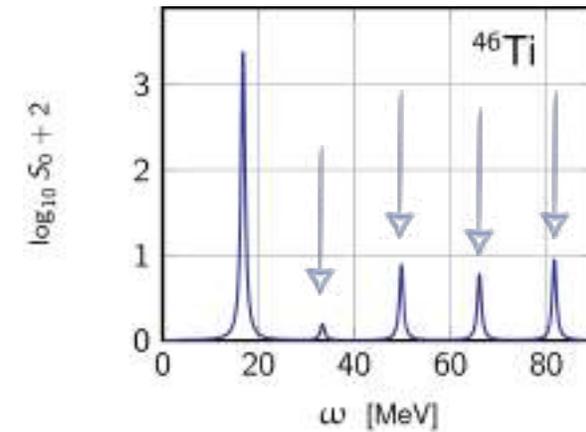
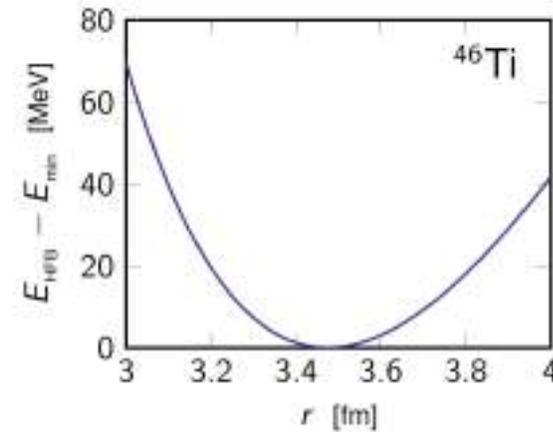


Multi-phonon states in ^{46}Ti



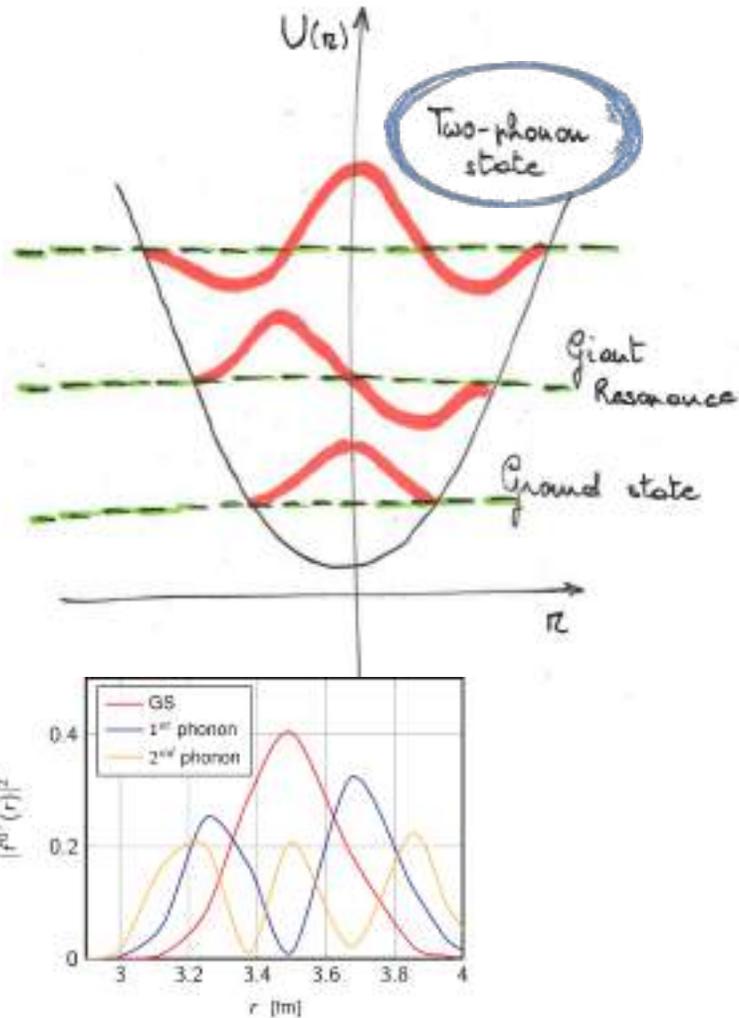
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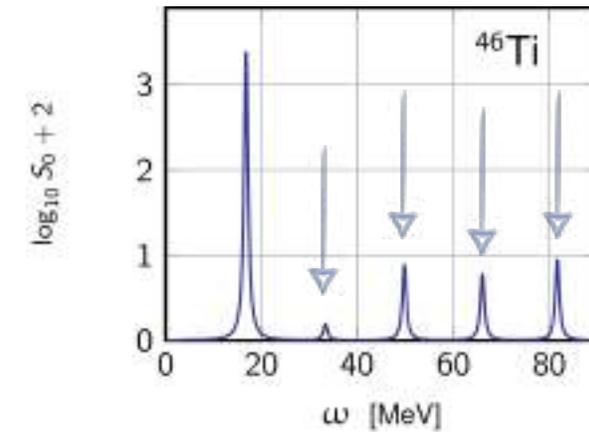
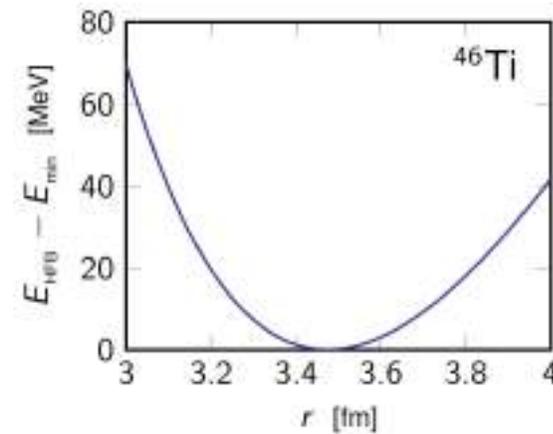
- PGCM predicts high-lying states

Multi-phonon states in ^{46}Ti



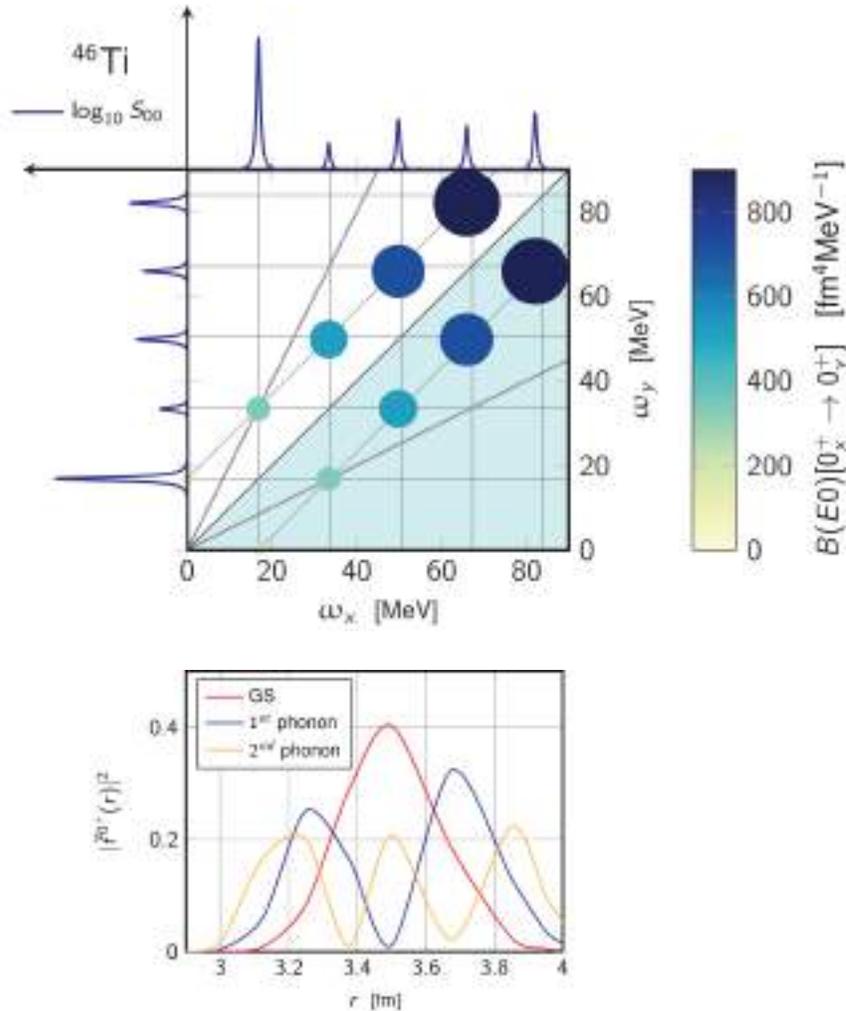
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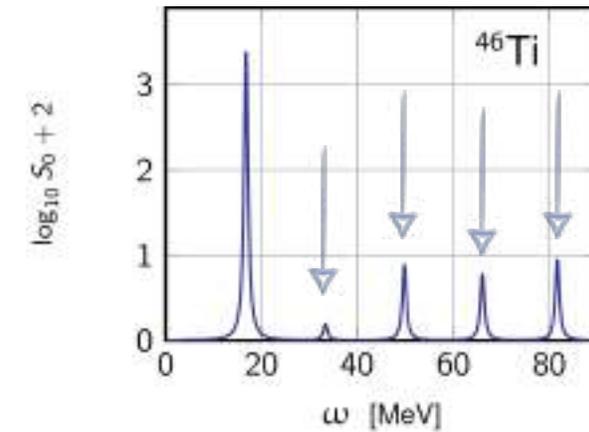
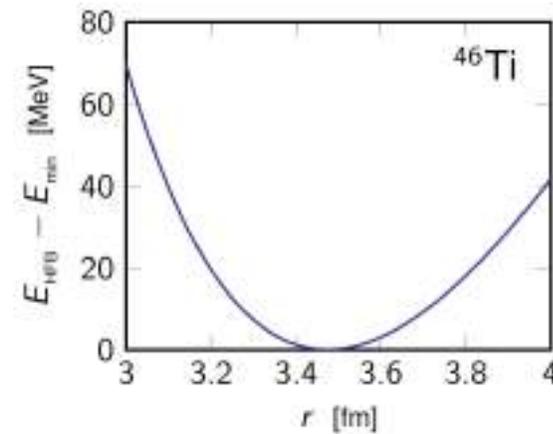
- PGCM predicts high-lying states
- Close to the harmonic oscillator eigen-solutions

Multi-phonon states in ^{46}Ti



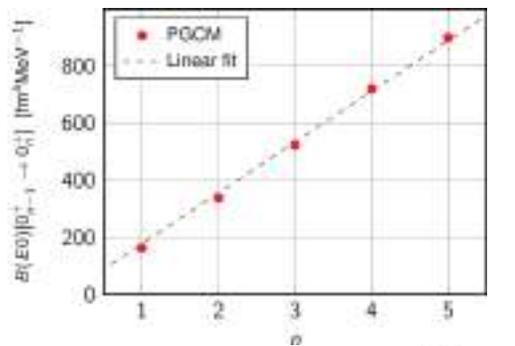
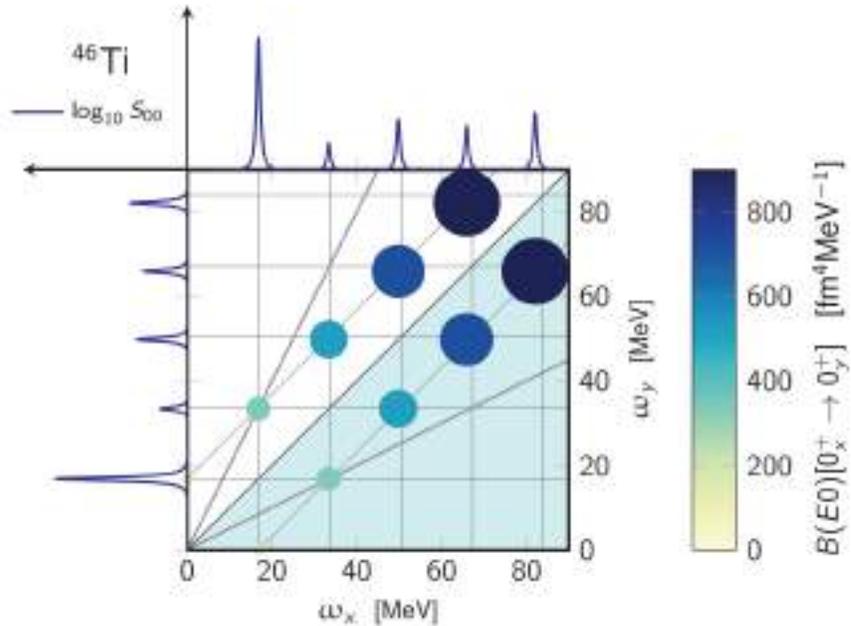
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One-dimensional PGCM calculation



- **PGCM predicts high-lying states**
- Close to the harmonic oscillator eigen-solutions
- Transitions maximised between neighbouring phonons

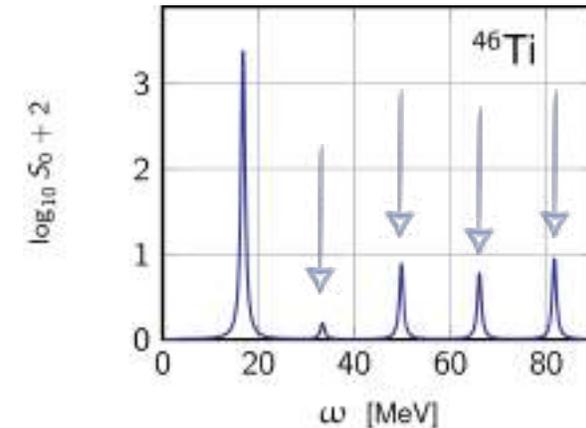
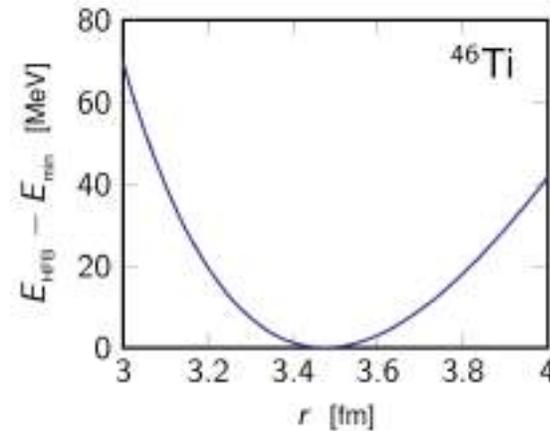
Multi-phonon states in ^{46}Ti



$$|\langle n-1 | r^2 | n \rangle|^2 = \frac{\hbar}{2m\omega} n$$

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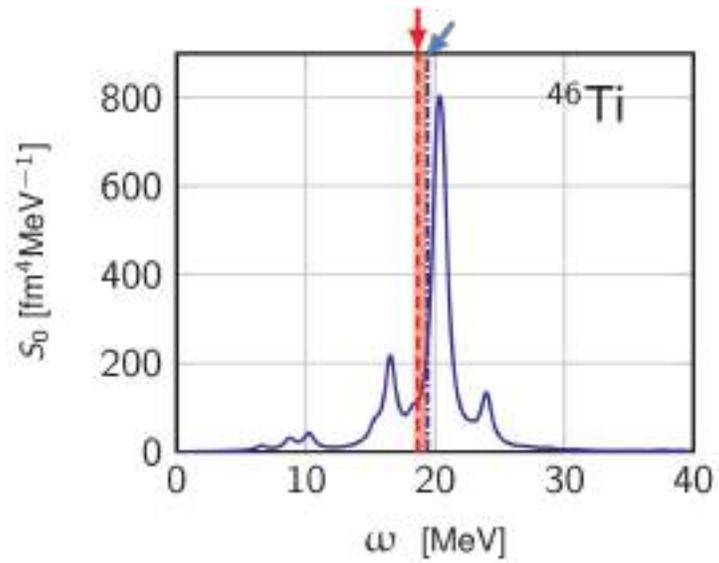
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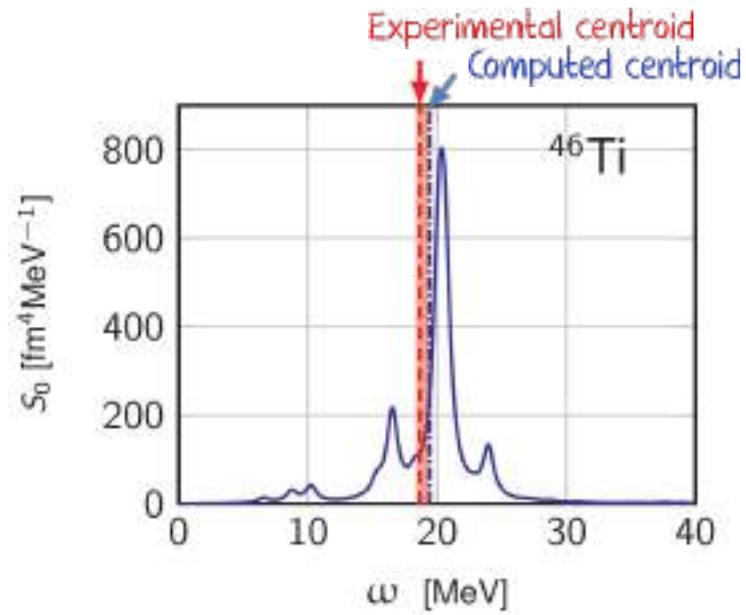
- PGCM predicts high-lying states**
- Close to the harmonic oscillator eigen-solutions
- Transitions maximised between neighbouring phonons
- ✗ Linear trend in the transition strength

Two-dimensional calculations

- 2-D PGCM in the (r, β_2) plane

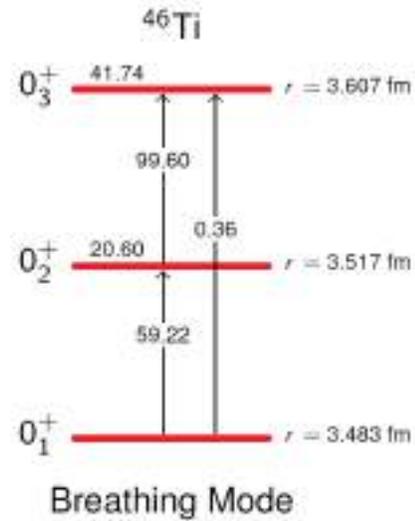
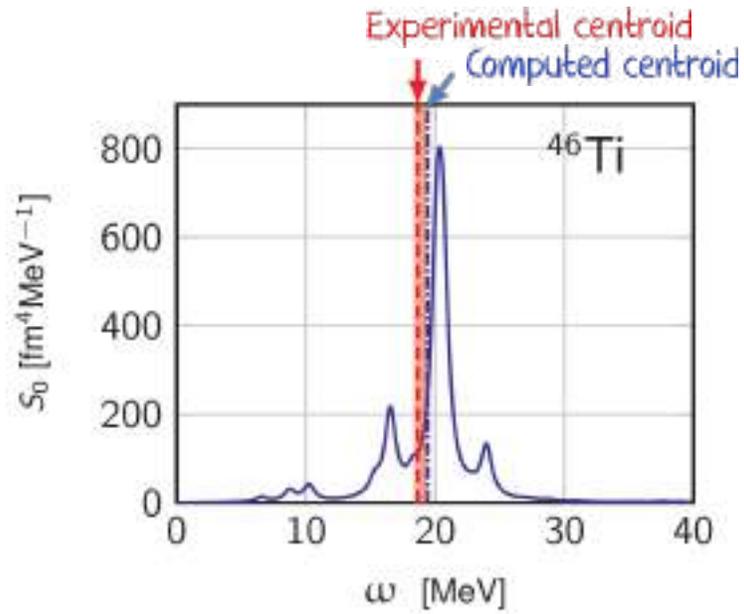


Two-dimensional calculations



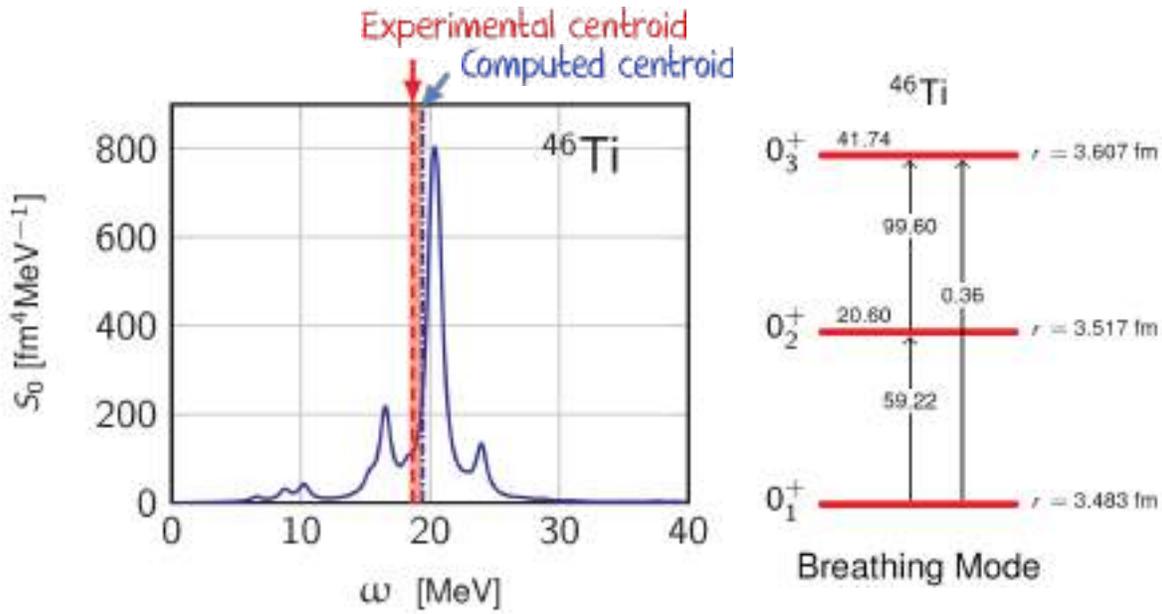
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Two-dimensional calculations



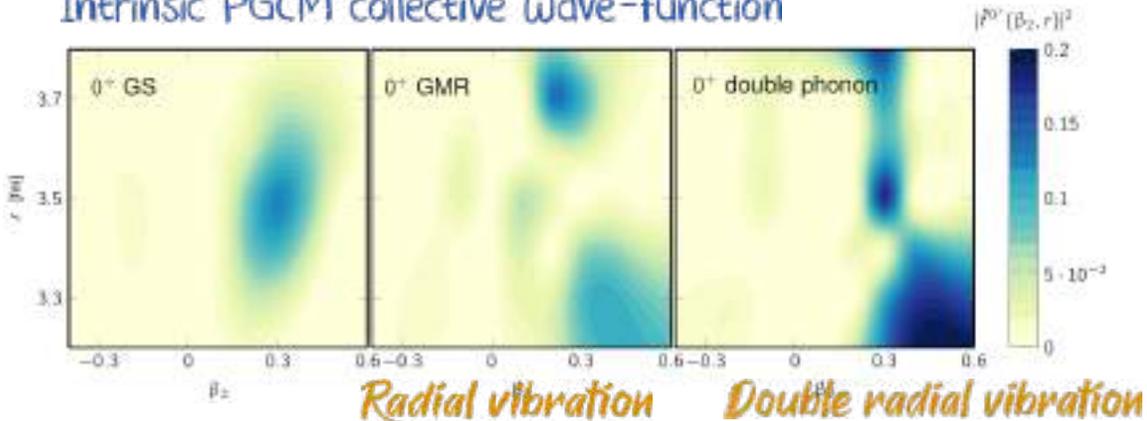
- 2-D PGCM in the (r, β_2) plane
- Good agreement with experiment
- Multi-phonon states observed

Two-dimensional calculations

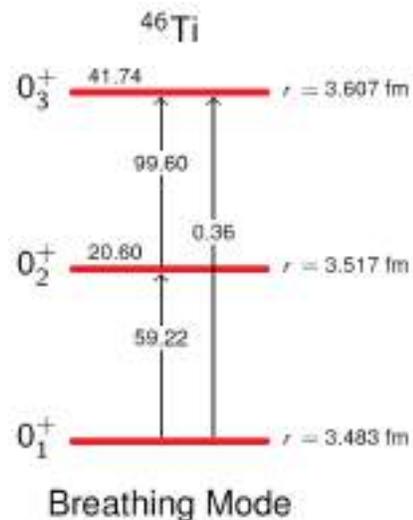
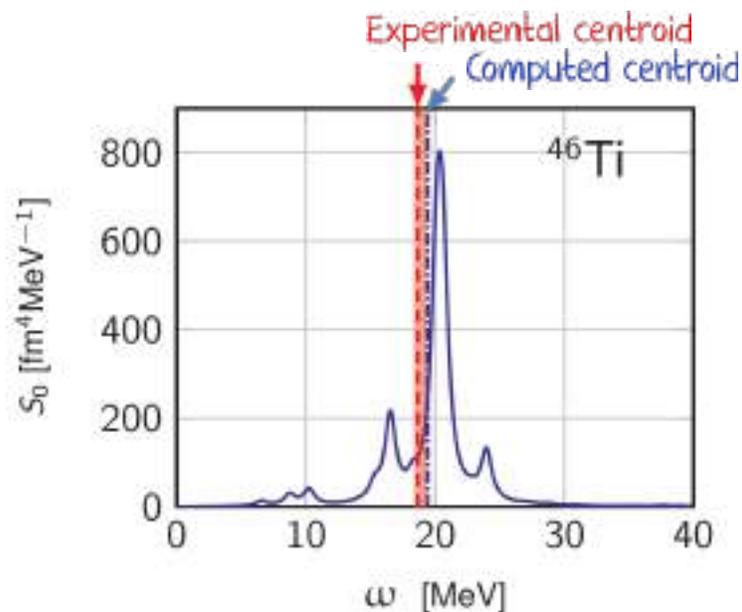


- 2-D PGCM in the (r, β_2) plane
- Good agreement with experiment
- Multi-phonon states observed
- Harmonicity well confirmed

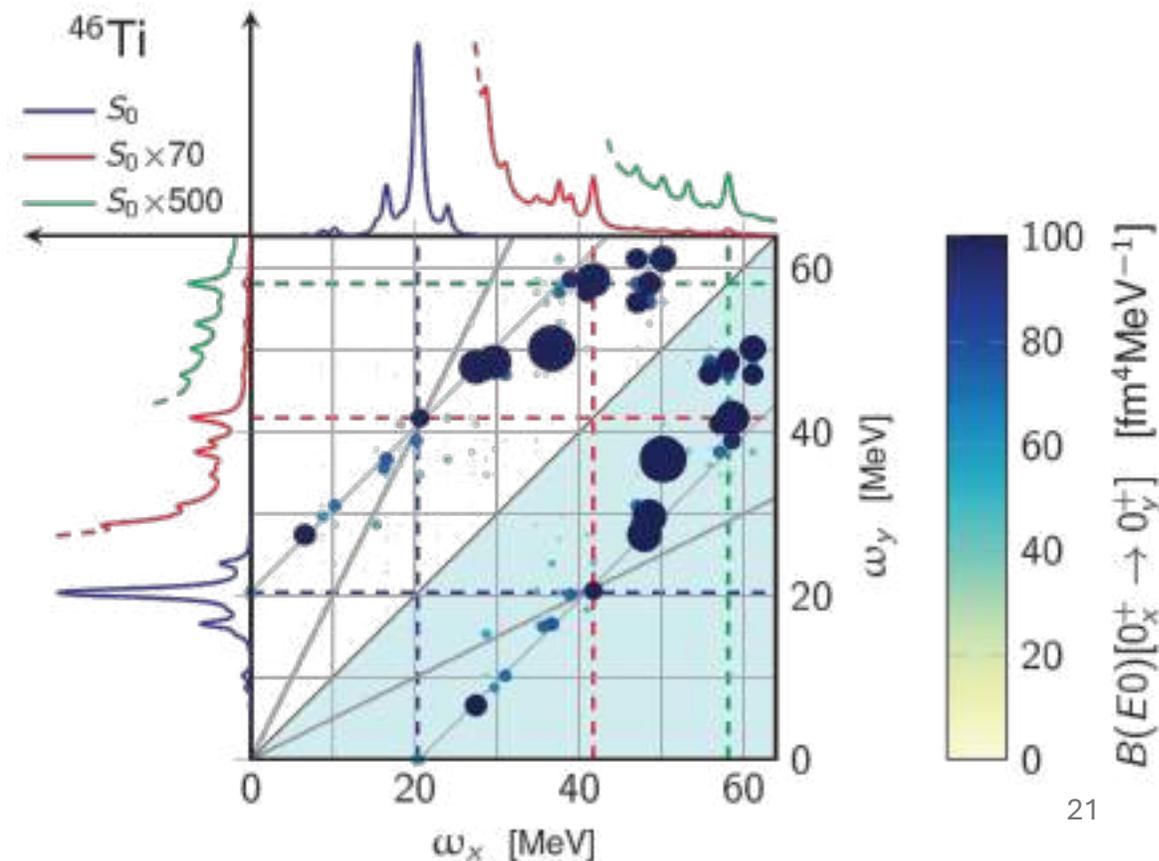
Intrinsic PGCM collective wave-function



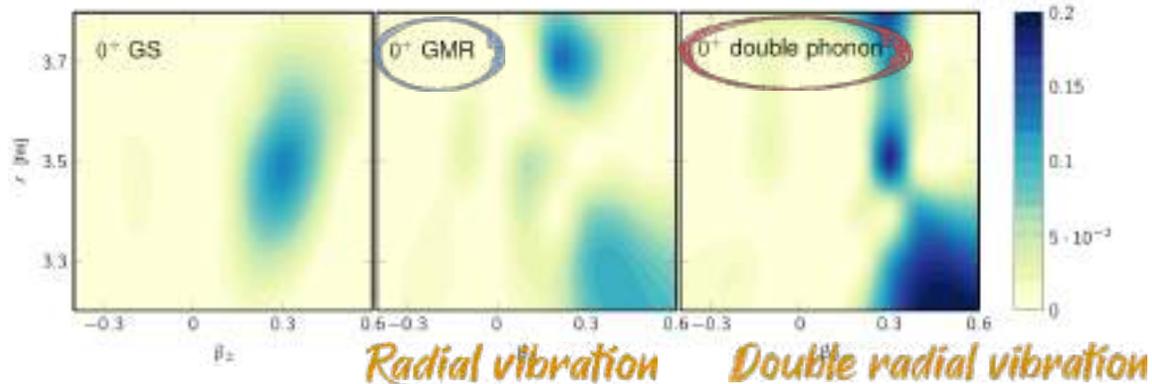
Two-dimensional calculations



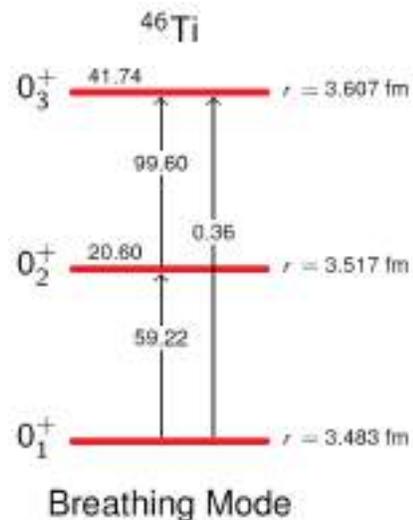
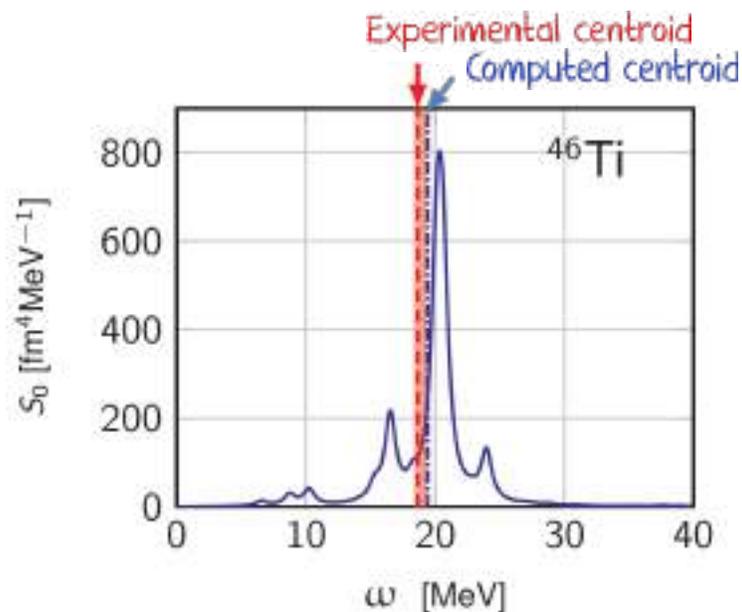
- 2-D PGCM in the (r, β_2) plane
- Good agreement with experiment
- Multi-phonon states observed
- Harmonicity well confirmed



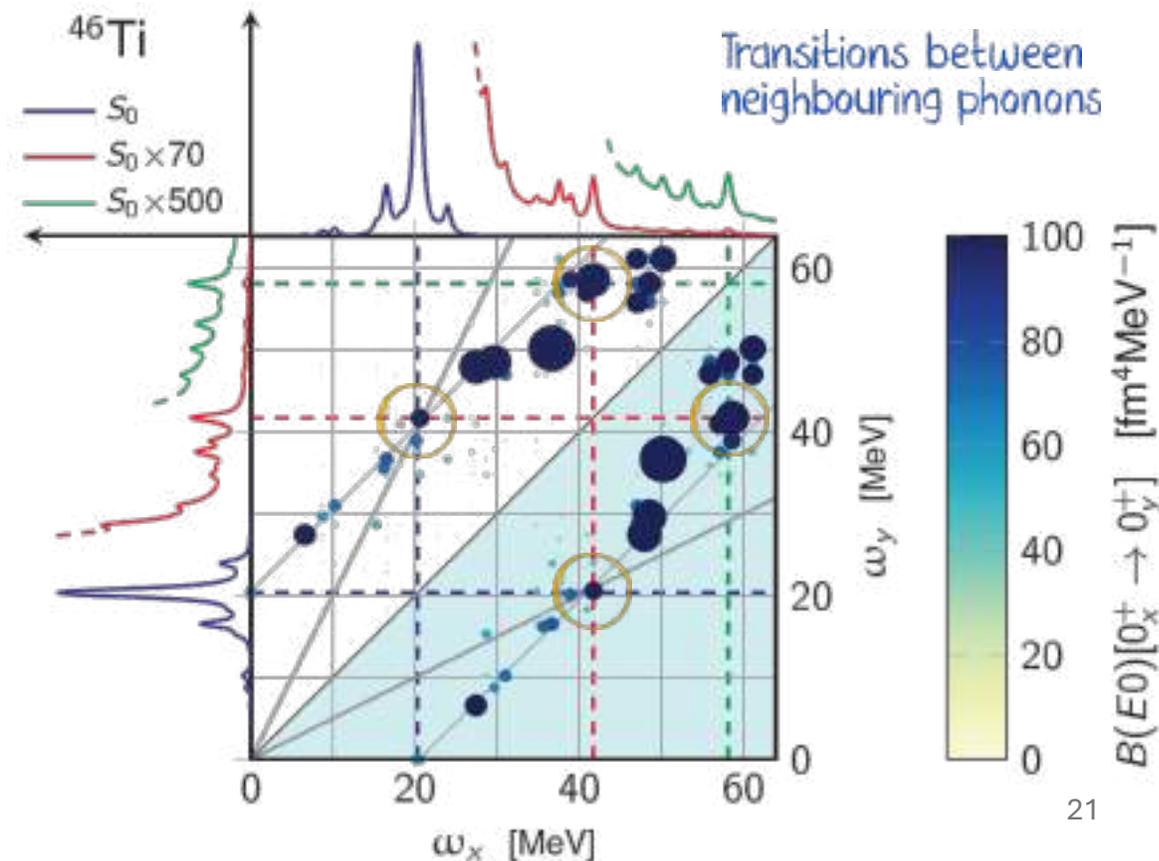
Intrinsic PGCM collective wave-function



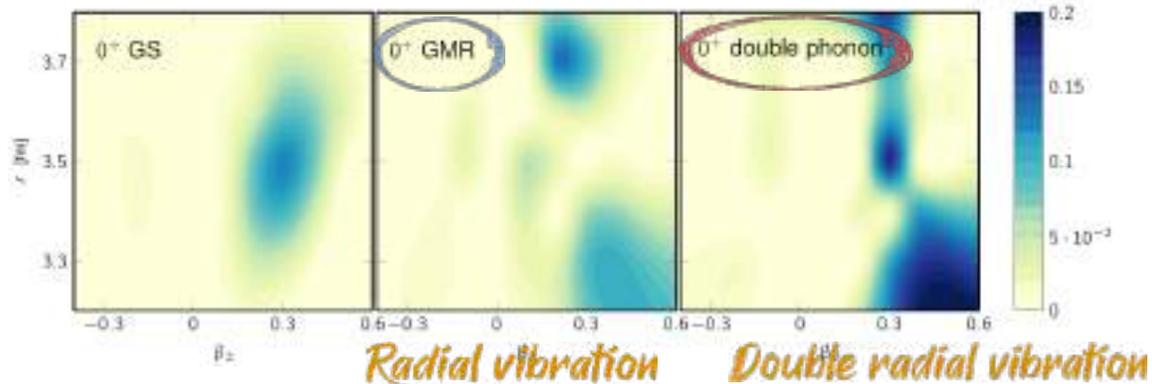
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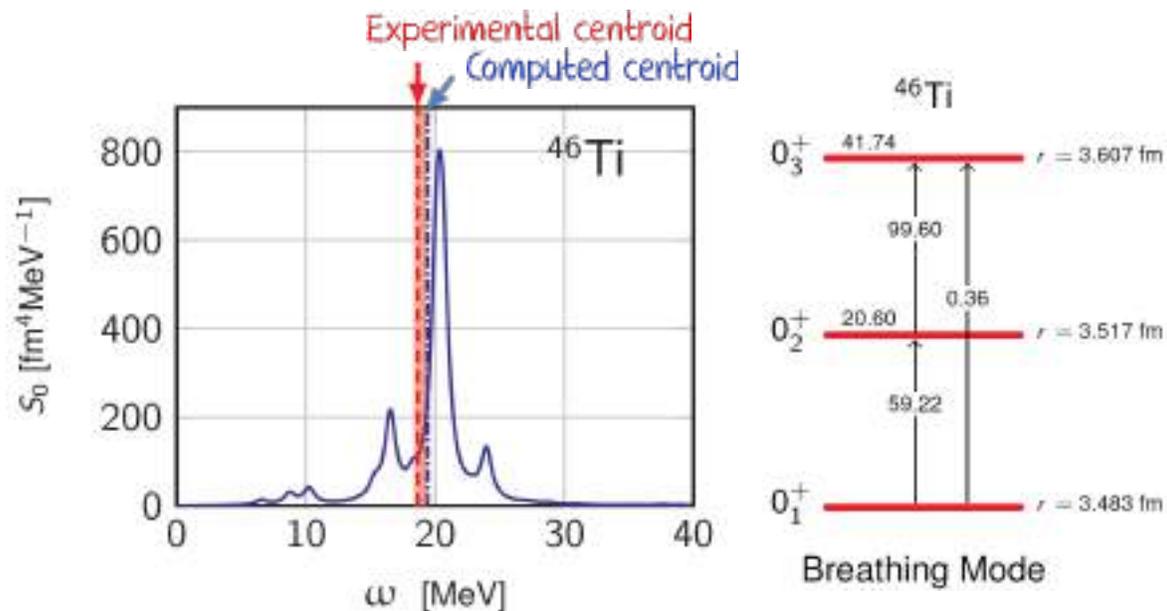
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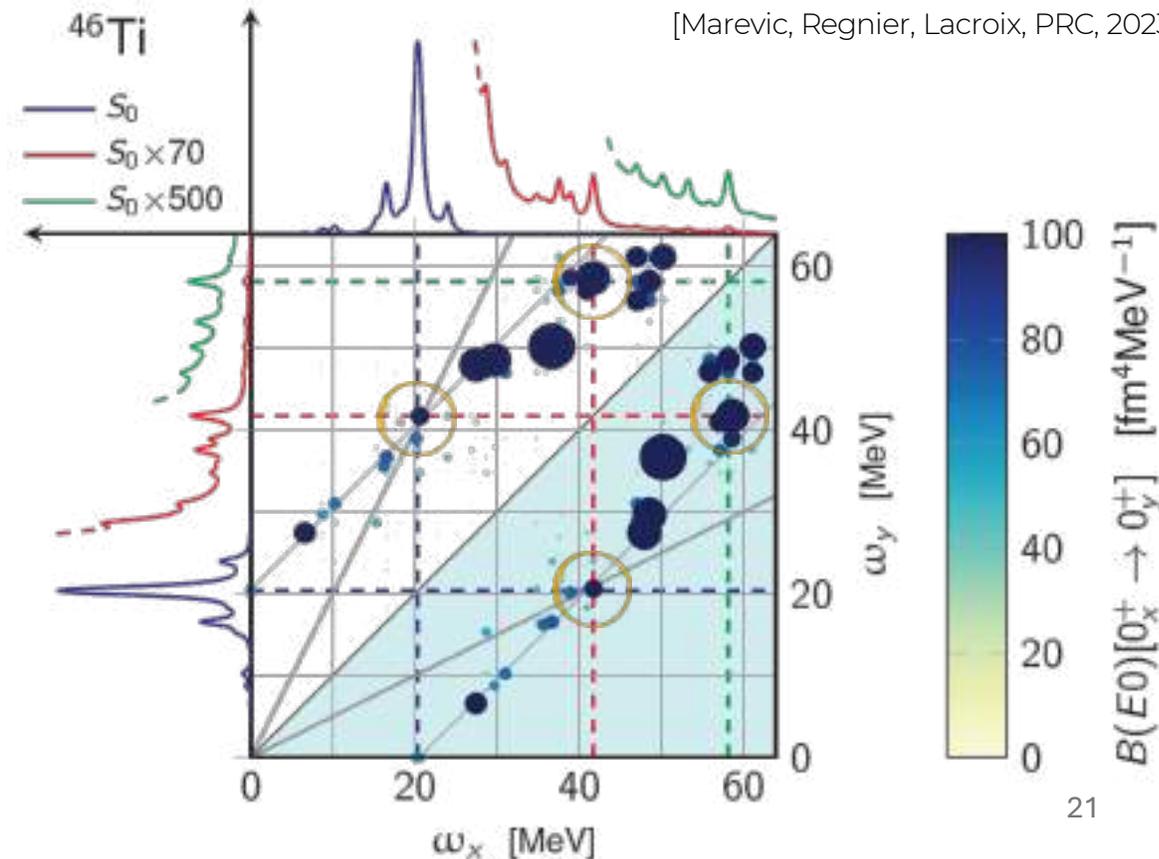
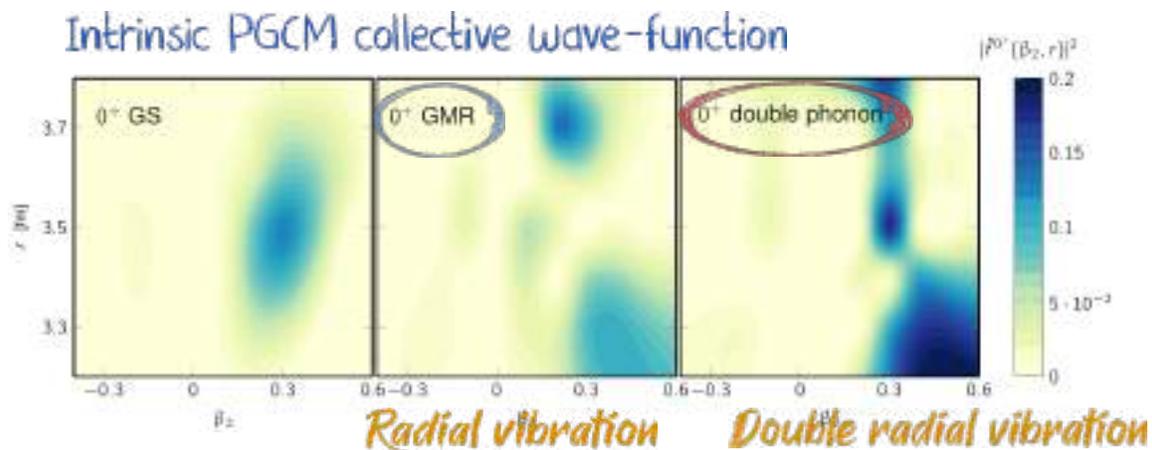


Two-dimensional calculations



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[Marevic, Regnier, Lacroix, PRC, 2023]



Outline

1 Introduction

- Giant Resonances Physics
- The PGCM
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- Numerical details
- Uncertainty estimate

Conclusions and perspectives

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Selected applications

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From finite nuclei to Astrophysics

- Preliminary incompressibility results

From finite nuclei to Astrophysics

Nuclear compressibility

- GMR

$$K_A = (M/\hbar^2)\langle r^2 \rangle E_{\text{GMR}}^2$$

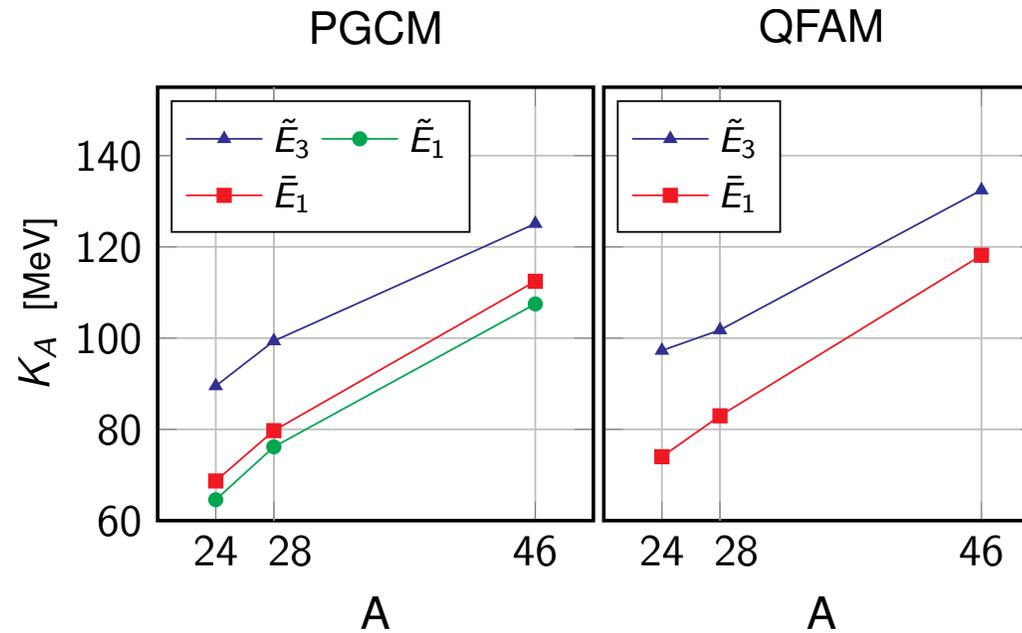
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$$\tilde{E}_k = \sqrt{\frac{m_k}{m_{k-2}}} \quad \bar{E}_1 = \frac{m_1}{m_0}$$



From finite nuclei to Astrophysics

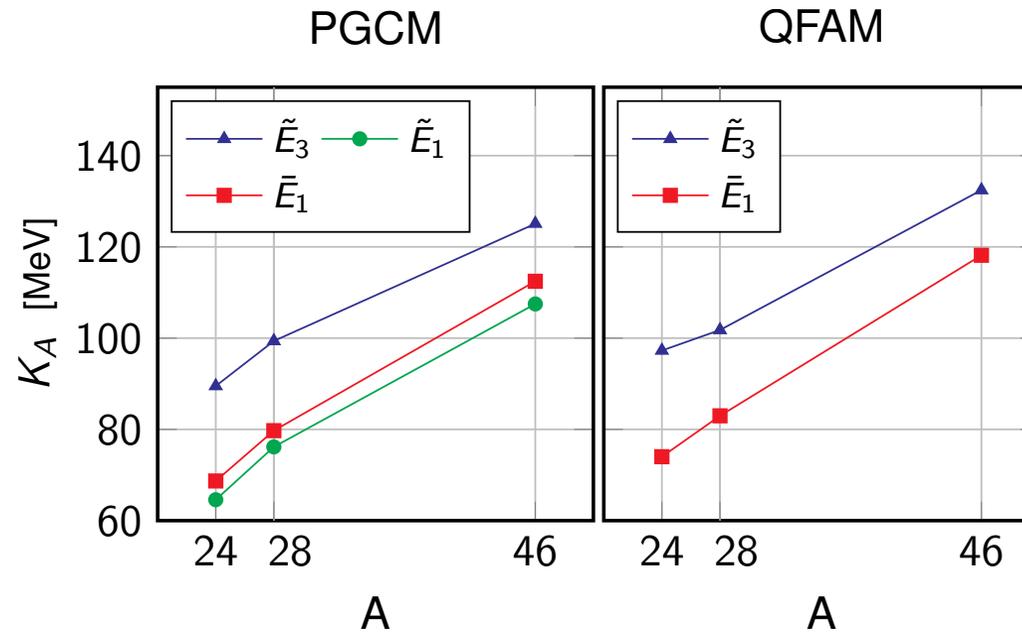
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From finite nuclei to Astrophysics

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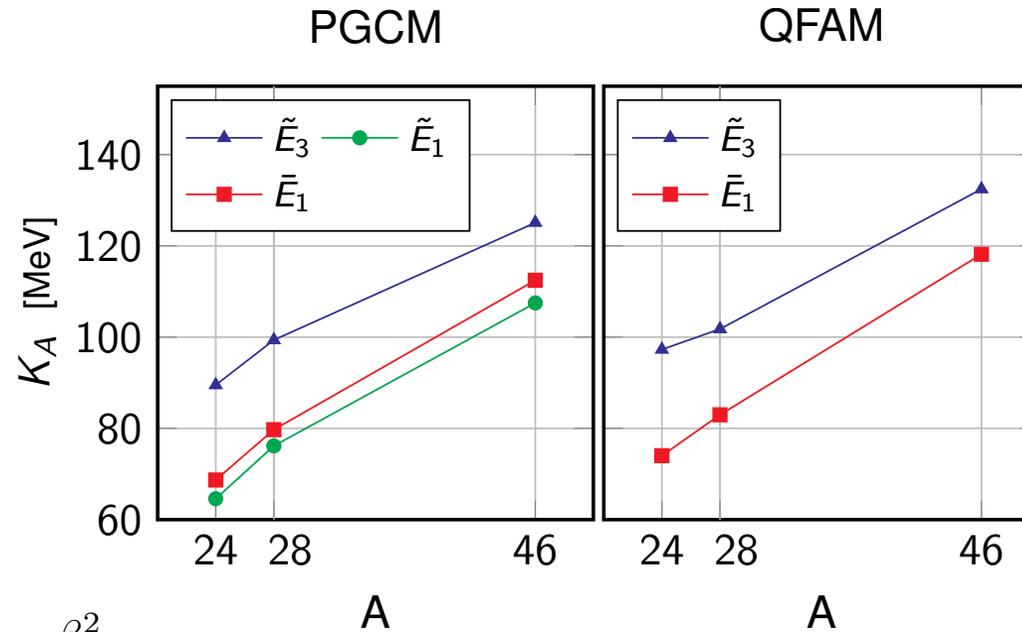
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Extrapolation to infinite matter

$$K_A = K_{\text{vol}} + K_{\text{surf}}A^{-1/3} + K_{\text{Coul}}Z^2A^{-4/3} + K_{\text{sym}}\beta^2$$

$$\beta \equiv \frac{N - Z}{N + Z}$$



From finite nuclei to Astrophysics

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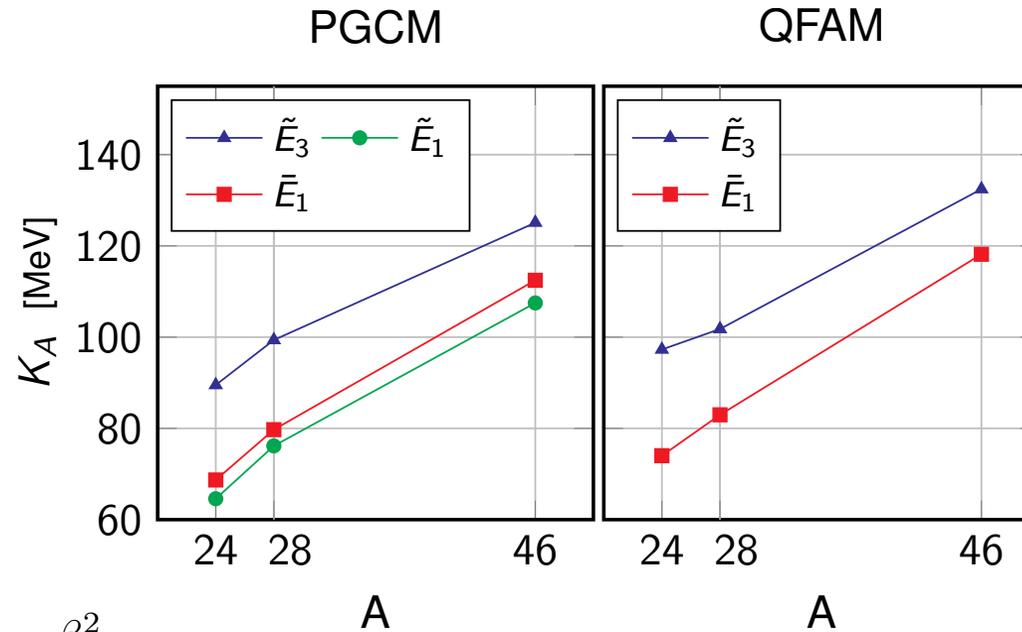
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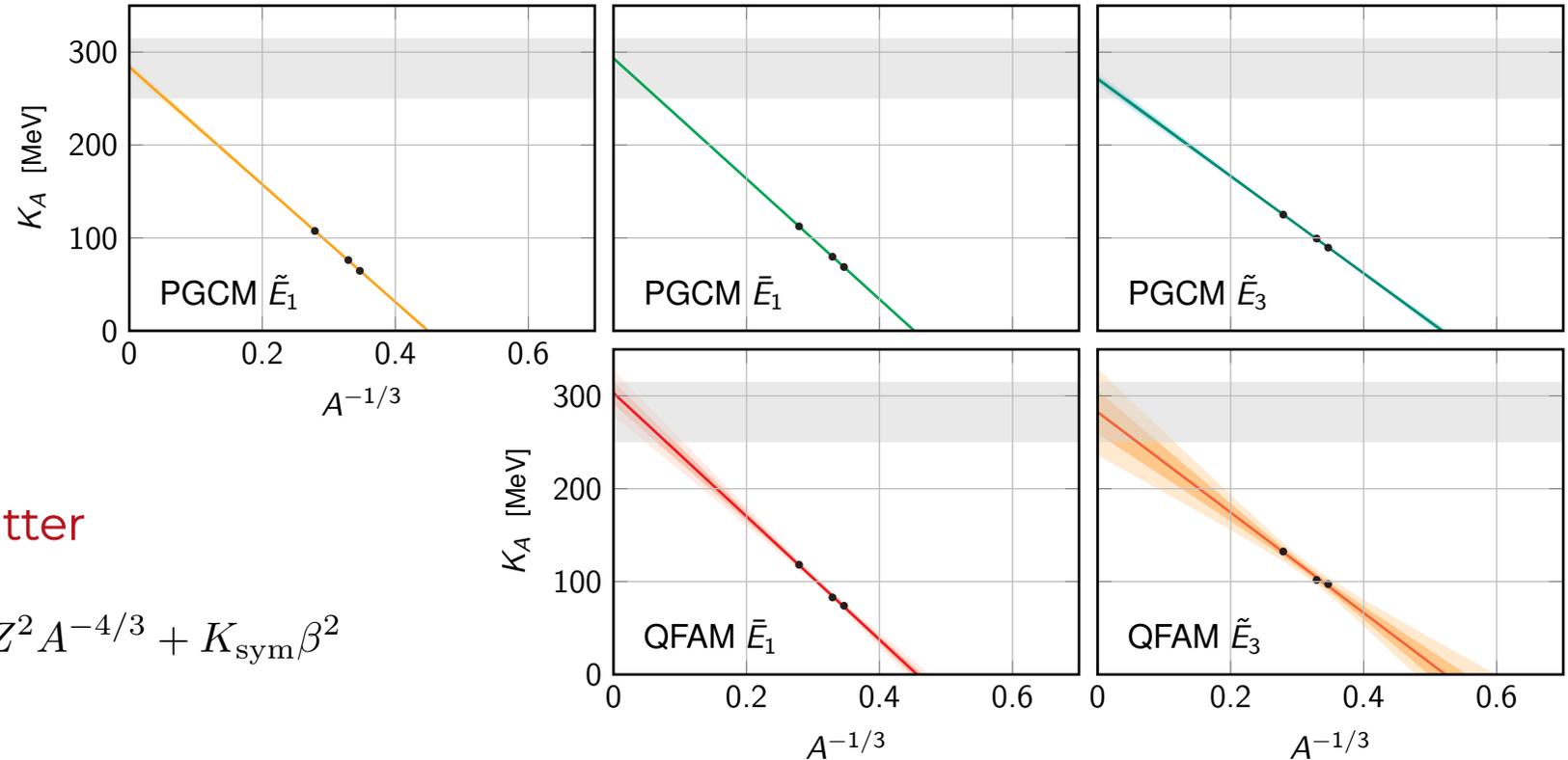
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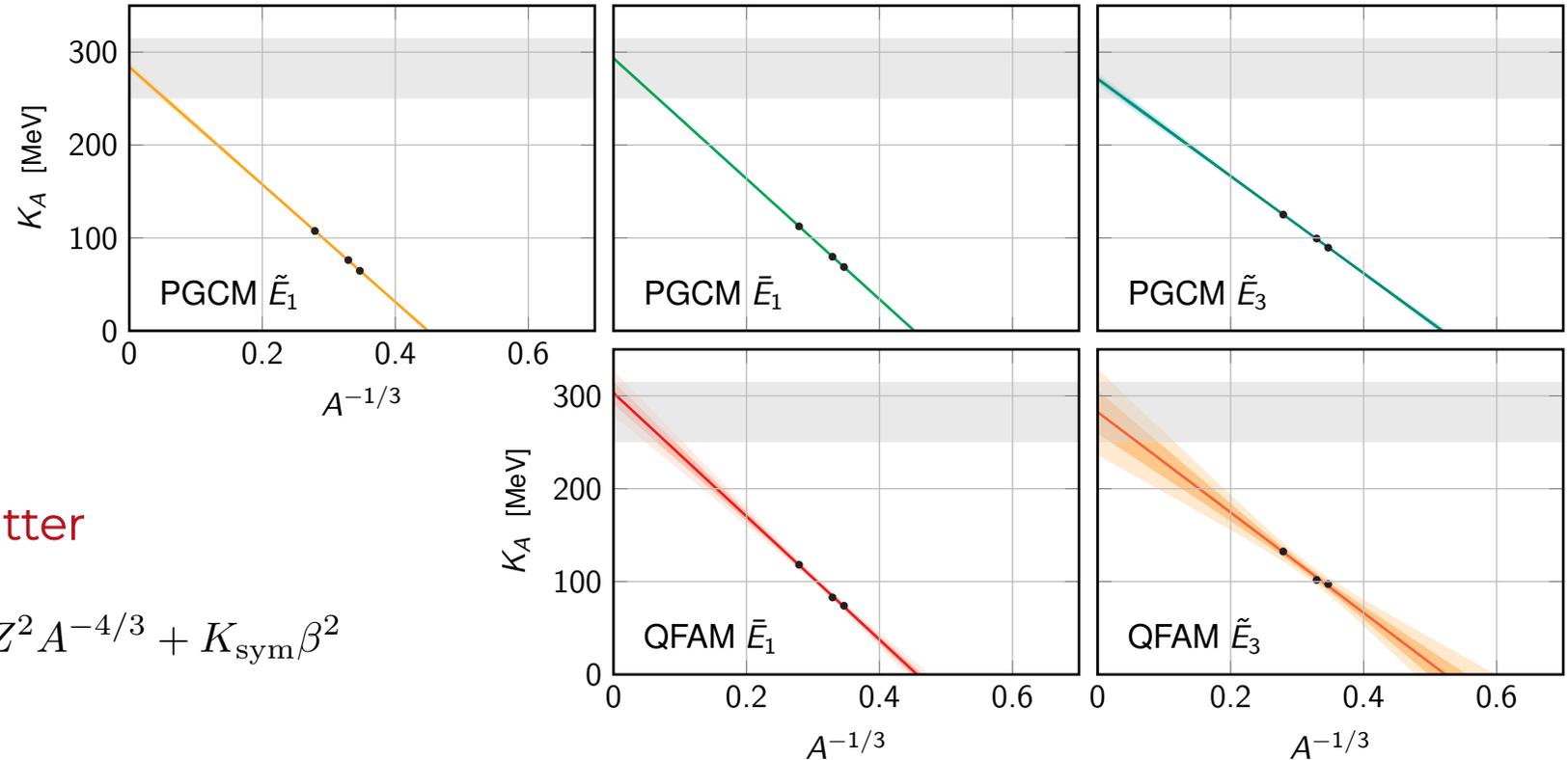
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$$K_A = K_{\infty} + K_{\text{surf}} x$$



Preliminary evaluation of K_{∞}

- Starting from **deformed** systems
- Extrapolation in **agreement** with commonly accepted values
- **Systematic** investigation in **heavier** systems (Sn, Mo isotopic chains, neutron rich)

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Current frontiers

SPECTROSCOPY

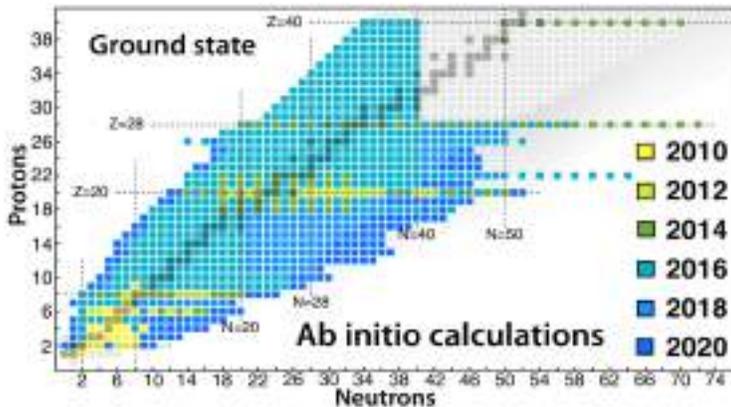
- Single-particle
- Collective excitations

ACCURACY

$$H = T + V_{\text{LO}} + V_{\text{NLO}} + V_{\text{N}^2\text{LO}} + \dots$$

$$|\Psi_k^A\rangle = \Omega|\Theta_k^{(0)}\rangle = |\Theta_k^{(0)}\rangle + |\Theta_k^{(1)}\rangle + |\Theta_k^{(2)}\rangle + \dots$$

OPEN-SHELL



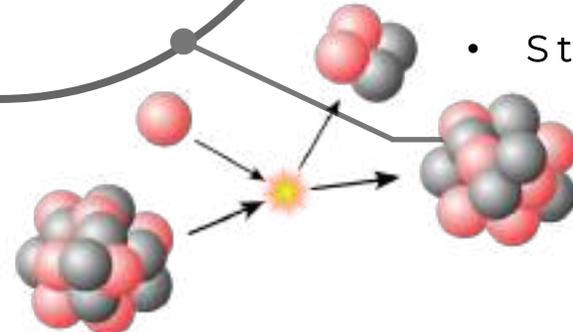
[Hergert, Front. Phys, 2020]

HEAVY-MASS SYSTEMS

UNCERTAINTIES

- Systematic uncertainties
 - Hamiltonian
 - A-body solution
 - Basis representation
- Statistical uncertainties

REACTIONS



Conclusions and perspectives

Perspectives

SPECTROSCOPY

OPEN-SHELL

UNCERTAINTIES

Take-away messages

Systematic comparison to new and existing **exp data**

Deeper **uncertainty quantification** (EC)

Develop full symmetry-conserving QRPA

More systematic choice of the **GC**

PGCM **reliable** tool for *ab initio** spectroscopy

Access to **new observables** and phenomena in *ab initio*



How is this even *ab initio*?

How do you choose the collective coordinates?

Thanks for the attention



Benjamin Bally

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