

# Five-body calculation of $n$ - $^4\text{He}$ scattering at next-to-leading order $\neq$ EFT

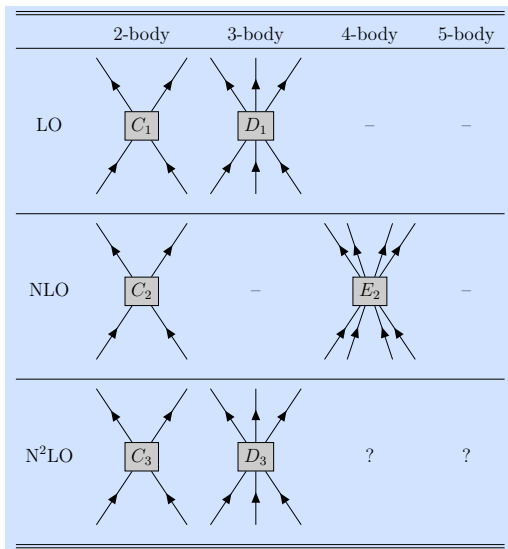
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- **Quantum Chromodynamics (QCD)** is the established fundamental theory of the strong interaction. At low energies ( $E \lesssim 200$  MeV) it is **not perturbative**.
- Two approaches:
  - Solve the Lagrangian by **brute force**, regardless of the cost ) **Lattice QCD (LQCD)**
  - Work with more appropriate **low-energy degrees of freedom** ) **Effective Field Theory (EFT)**
- We employ **Pionless EFT (=EFT)**, where the the degrees of freedom are the **nucleons** and the **pions** are integrated out



- Without pions, our Leading Order (LO) interaction is a **contact interaction**:

$$V_{\text{LO}, 2\text{B}}(F) = C_{S;l}(F)$$

$$V_{\text{LO}, 3\text{B}}(F_{ij}; F_{jk}) = D_{S;l}(F_{ij})(F_{jk})$$

- The **two body force** is projected to **two channels**,  $(S;l) = (1;0); (0;1)$ , while the **three body force** is projected to  $(S;l) = \frac{1}{2}; \frac{1}{2}$ , for a total of three LO LECs
- The promotion of a repulsive three body force at LO prevents the **Thomas collapse**
- In order to numerically solve Schrödinger's equation, we have to **smear** the Dirac delta, introducing the **cutoff  $\Lambda$**

$$\Lambda(F) = e^{-\frac{\Lambda^2 r^2}{4}}$$

- Our Next to LO (NLO) interaction has **momentum dependent two-body terms**, three **counterterms** and a **four-body force**:

$$\begin{aligned}
 V_{\text{NLO}, 2\text{B}} &= C_{S;l} r^2 (F) \\
 V_{\text{NLO}, \text{counter}} &= C_{S;l} \Lambda(F) + D_{S;l} (F_{ij}) (F_{jk}) \\
 V_{\text{NLO}, 4 \text{ Body}} &= E_{S;l} \Lambda(F_{ab}) \\
 &\qquad\qquad\qquad ab \text{ 2pairs}
 \end{aligned}$$

- The momentum dependent terms **introduce an effective range** to the interaction
- The counterterms have the **same form** of the LO terms and serve to keep **the LO observables reproduced at NLO**

- We have **three additional terms** if the angular momentum is  $L = 1$ :

$$V_{\text{NLO}, p p^0} = C_{S;1} r^{-1} (F) r^{-1}$$

$$V_{\text{NLO}, L S} = C_{S;1} (F) L \cdot S$$

$$V_{\text{NLO}, \text{Tensor}} = C_{S;1} (F) S_{12}(F)$$

- Their matrix element depends on  $J$ , allowing us to **distinguish different  $(L; S)$  couplings**
- They introduce a total of **five LECs**, but one is related to channel mixing ( $^3S_1 - ^3D_1$ )

- Between LO and NLO, our model has **six** parameters fixed to **few** body observables:

$$\begin{array}{lll} \text{LO:} & a_{nn}^0 & = 18.95 \text{ fm} \\ & B(^2\text{H}) & = 2.2246 \text{ MeV} \\ & B(^3\text{H}) & = 8.482 \text{ MeV} \\ \text{NLO:} & r_{nn}^0 & = 2.75 \text{ fm} \\ & r_{np}^1 & = 1.753 \text{ fm} \\ & B(^4\text{He}) & = 28.3 \text{ MeV} \end{array}$$

- NLO interaction is included **perturbatively** to circumvent the **Wigner bound**
- Four  $L$  1 new LECs are fixable with **NN**  $a_v$  of channels  $^1P_1$  and  $^3P_0$ ,  $^3P_1$ ,  $^3P_2$ , and one with a channel mixing angle

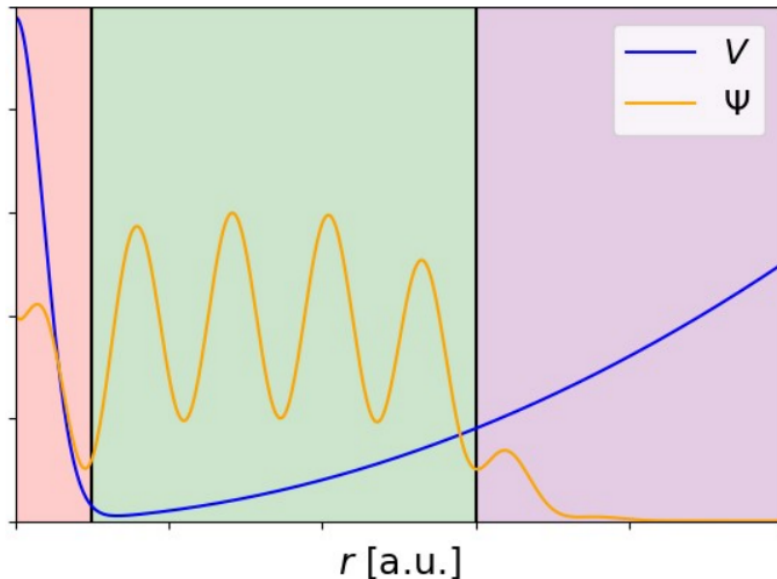
- We applied our interaction to  ${}^4\text{He}+n$  scattering in the  ${}^2S_{\frac{1}{2}+}$  channel
- We confined our system in an harmonic potential and used the Busch formula to extract the scattering parameters,  $a_0$  and  $r_{\text{eff}}$
- We solved the Schrödinger equation with the Stochastic Variational Method (SVM)



# Busch formula's idea

$^4\text{He}+n$  at NLO

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- We apply the **Busch formula** in order to extract the free space scattering parameters (scattering length  $a_0$  and effective range  $r_{\text{eff}}$ )

$$k \cot \delta_0 = 2^{\rho-1} \frac{\Gamma\left(\frac{3}{4} - \frac{E}{2-c}\right)}{\Gamma\left(\frac{1}{4} - \frac{E}{2-c}\right)}$$

- The effective range expansion (ERE) gives us the scattering parameters

$$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2$$

- The Busch formula relates **trapped** energies (solvable with bound state methods like SVM) with **free space, untrapped** scattering parameters

- The Busch formula includes a Gamma ratio: it needs very high **energy accuracy** to give reliable results, **below  $10^{-2}$  MeV**
- The harmonic constant  $\Gamma$  has to be **as low as possible**, in order to well separate the scales of the system
- The typical scale of  $^4\text{He}+n$  is estimated as the **scattering length  $a \approx 2.5$  fm**

$$\sim \Gamma = \frac{\tilde{a}^2}{L^2} < 8 \text{ MeV}$$

- In practice, it has to be **at most 2 MeV** in order to have negligible trap effects
- This formula does **not** take into account **the Coulomb interaction**, but **recently a generalized form has been derived**

- Method to solve the Schrödinger equation standing on the **variational principle**, proposed by Suzuki and Varga in 1996
- The wave function is expanded as

$$|j\rangle = \sum_{k=1}^M |k\rangle$$

- Each  $|k\rangle$  depends on some **parameters**, which **are chosen randomly**
- More and more states are added until **convergence**

- The single basis state is expressed as a **correlated Gaussian** and a **orbital, spin and isospin part**

$$|j\rangle_i = A(G(A)|j\rangle_i)$$

$$\langle \mathbf{x} | G(A) | \mathbf{x} \rangle = G(\mathbf{x}; A) = e^{-\frac{1}{2} \mathbf{x}^T A \mathbf{x}}$$

$$\langle \mathbf{x}; s; t | j \rangle_i = \langle \mathbf{x}; s; t | (LS) J M_J I M_I | j \rangle_i = [ \begin{matrix} L & S \\ J; M_J \end{matrix} ] [ \begin{matrix} I & M_I \end{matrix} ]$$

- The **Gaussian form** of the wave function **allows analytical calculation** of matrix elements
- The spin and isospin parts are just coupling of the single spins

$$|s; M_s\rangle = |j\rangle_i [ [ [s_1 \quad s_2]_{s_{12}} \quad s_3 ] \quad \dots \quad s_N ]_{s; M_s} | j \rangle_i$$

In presence of multiple configurations, they are **chosen randomly** as well!

- The stochastic selection process eventually becomes too slow when the basis is big enough
- In order to reach the desired accuracy ad-hoc designed states can be generated
- We generated states that capture the  $^4\text{He}$  core -n dynamic as follows

$$A = \begin{pmatrix} 3 & 3 \\ 0 & \frac{0}{(n-1)^2} \end{pmatrix} !$$

$$\exp \left[ \frac{1}{2} \mathbf{x}^T A \mathbf{x} \right] = \exp \left[ ^4\text{He core} \exp \left[ \frac{1}{2} \frac{x_4^2}{(n-1)^2} \right] \right]$$

- The  $3 \times 3$  matrices are generated for  $^4\text{He}$  with SVM,  $\alpha$  is an optimized parameter and  $n$  runs from 1 to 10

# Convergence example

# Phase shifts

Results

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# Scattering parameters

# $a_0$ in the literature

Results

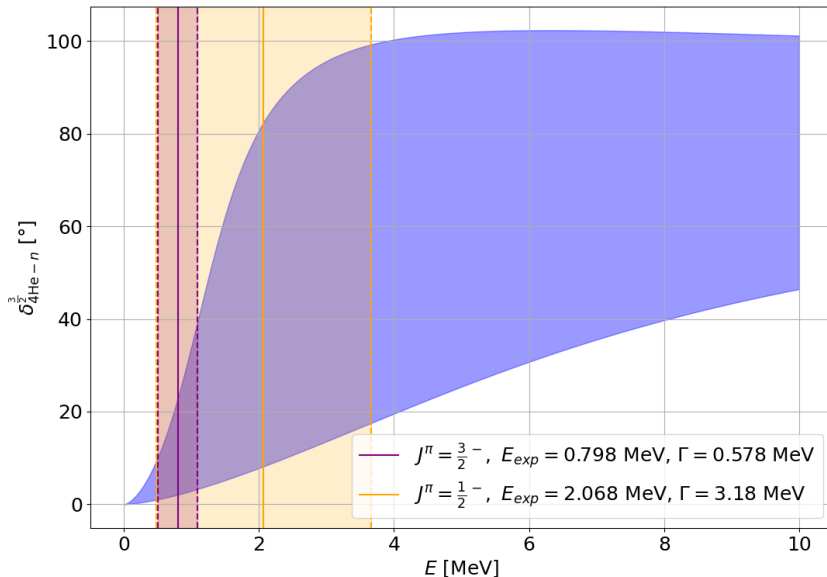
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# $r_0$ in the literature

Results

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- We can predict the **scattering parameters** for
  - $d + n$
  - $t + n$
  - ${}^3\text{He} + n$
  - ${}^4\text{He} + n$
- Possibly, **core-proton scattering** can be reachable taking into account the **Coulomb force** in the Busch formula
- We can distinguish **different  $J$  channels**
- The most thriving application is  ${}^4\text{He} + n$  in the channels  ${}^2P_{\frac{1}{2}}$  and  ${}^2P_{\frac{3}{2}}$ , where there are **two resonances**
- We computed the **LO phase shifts span** of  ${}^4\text{He} + n$  from **cutoffs 1.25 to 6 fm** <sup>1</sup>



- We presented the **pionless Effective Field Theory** potential up to **NLO**
- We extracted the **scattering parameters**  $a_0$  and  $r_{\text{eff}}$  with the **Busch formula**
- We got **amazing results** compared to the literature and to other more sophisticated models!
- We hinted on future application to  $L = 1$  **systems**

