DE LA RECHERCHE À L'INDUSTRIE

cea

ESNT

Espace de Structure Nucléaire Théorique
DSM - DAM

Effective Field theory and Strong interaction with accurate error estimation

What is renormalizability and why is important?

What are the challenges and the future of these theories?

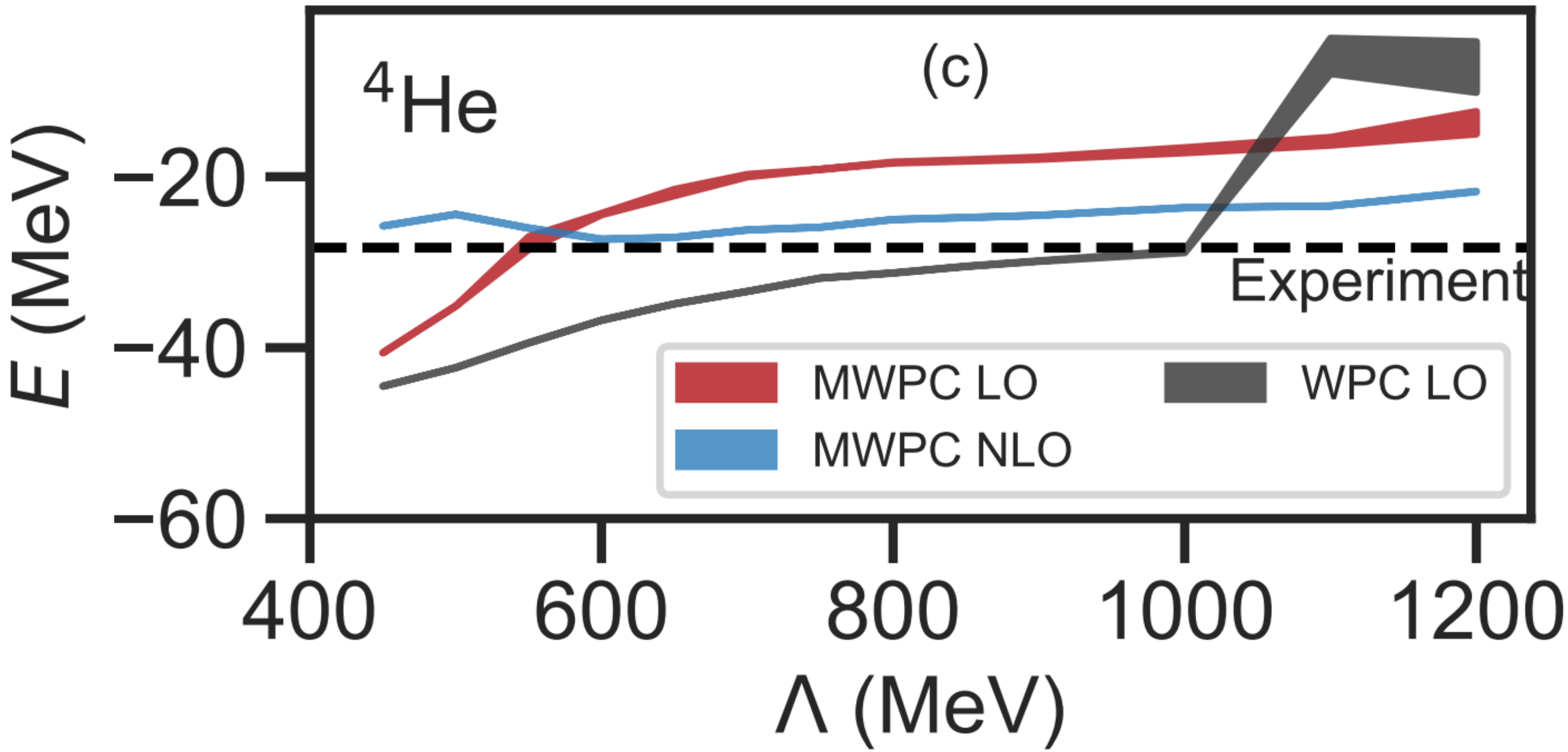
Pionless
Chiral
Cluster/Halo
…

Chiral EFT

Weinberg power counting it is not renormalizable

Alternative Chiral power counting solves this problem…

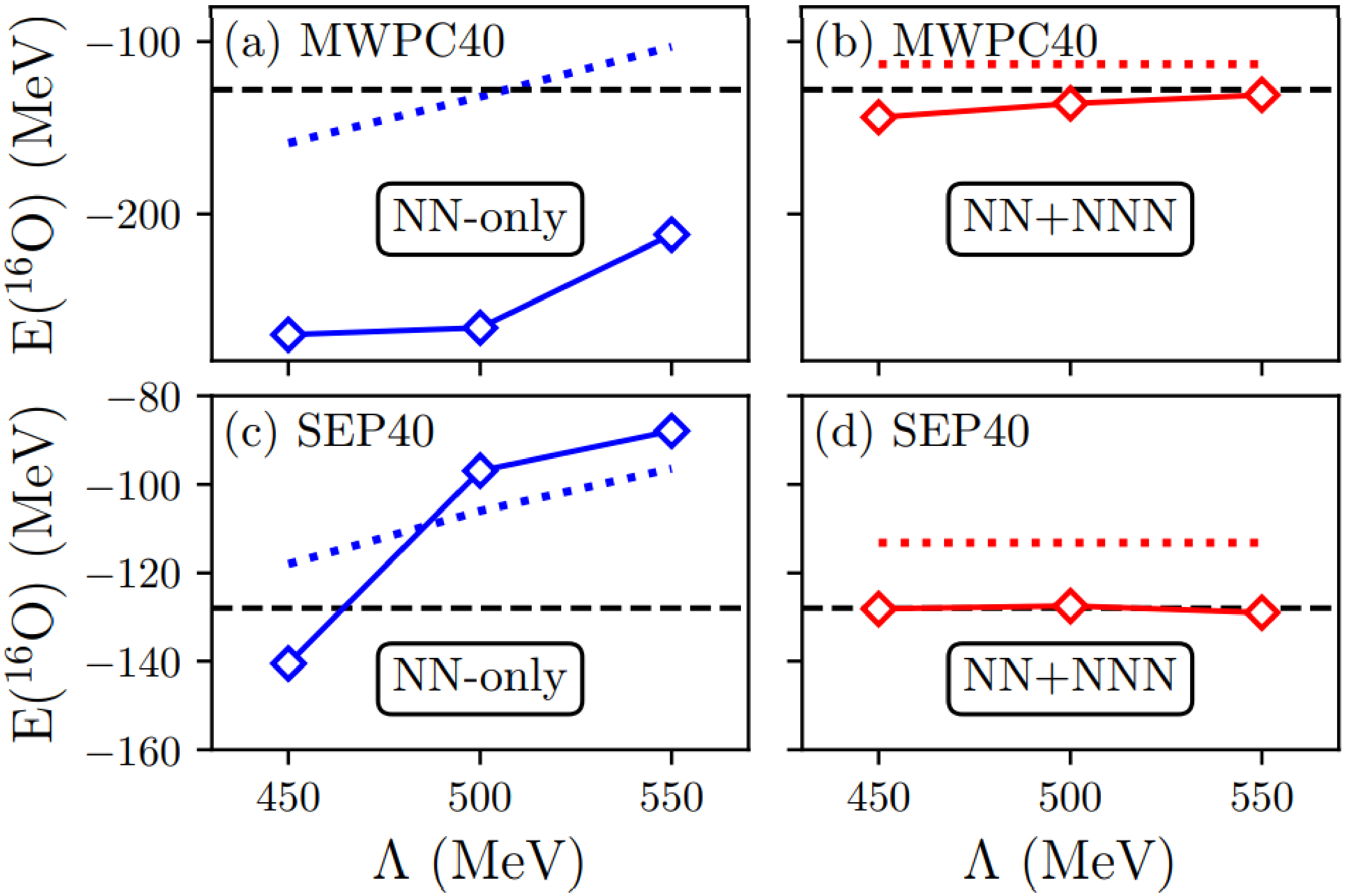
Phys. Rev. C C.-J. Yang, A. Ekström, C. Forssén, G. Hagen (2021)



Chiral EFT

… but it reveals other issues

C.-J. Yang, A. Ekström, C. Forssén, G. Hagen, G. Rupak, U. van Kolck (2021)



Instability problem

Appearance of unphysical states

Added a three-body force to solve instability

- ➢ Change power counting

Contact EFT: instability for fermions

5He   6He

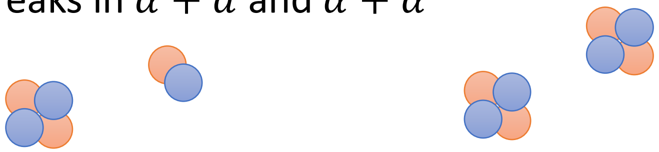
J. Kirscher, H. W. Grießhammer, D. Shukla,
H. M. Hofmann: arXiv:0909.5606

Breaks in $\alpha + n$ and $\alpha + n + n$

7Li   8Be

Our calculations in SU(4) symmetry

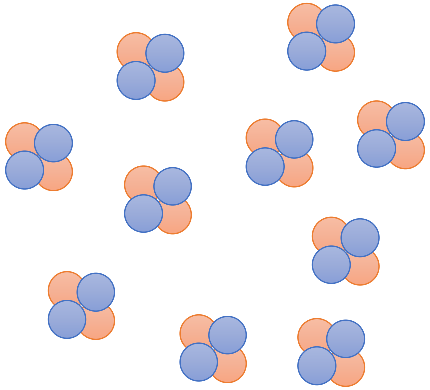
Breaks in $\alpha + d$ and $\alpha + \alpha$

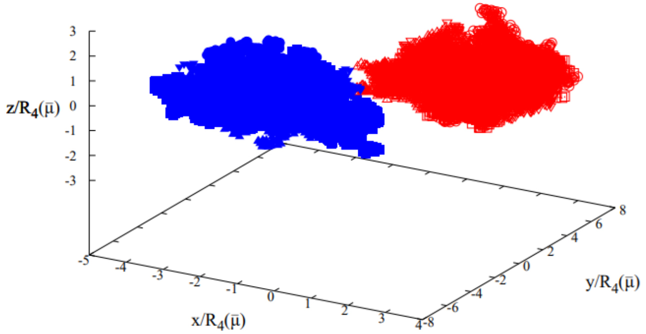


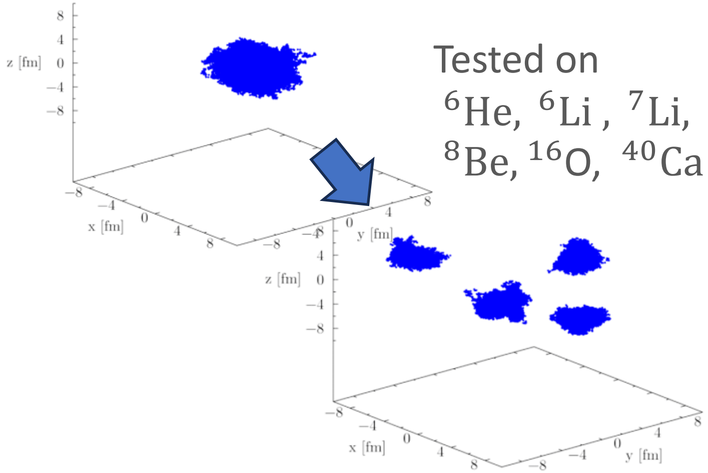
40Ca

QMC calculation suggests the breaking in:

Breaks in $\alpha + \alpha + \alpha +$ …



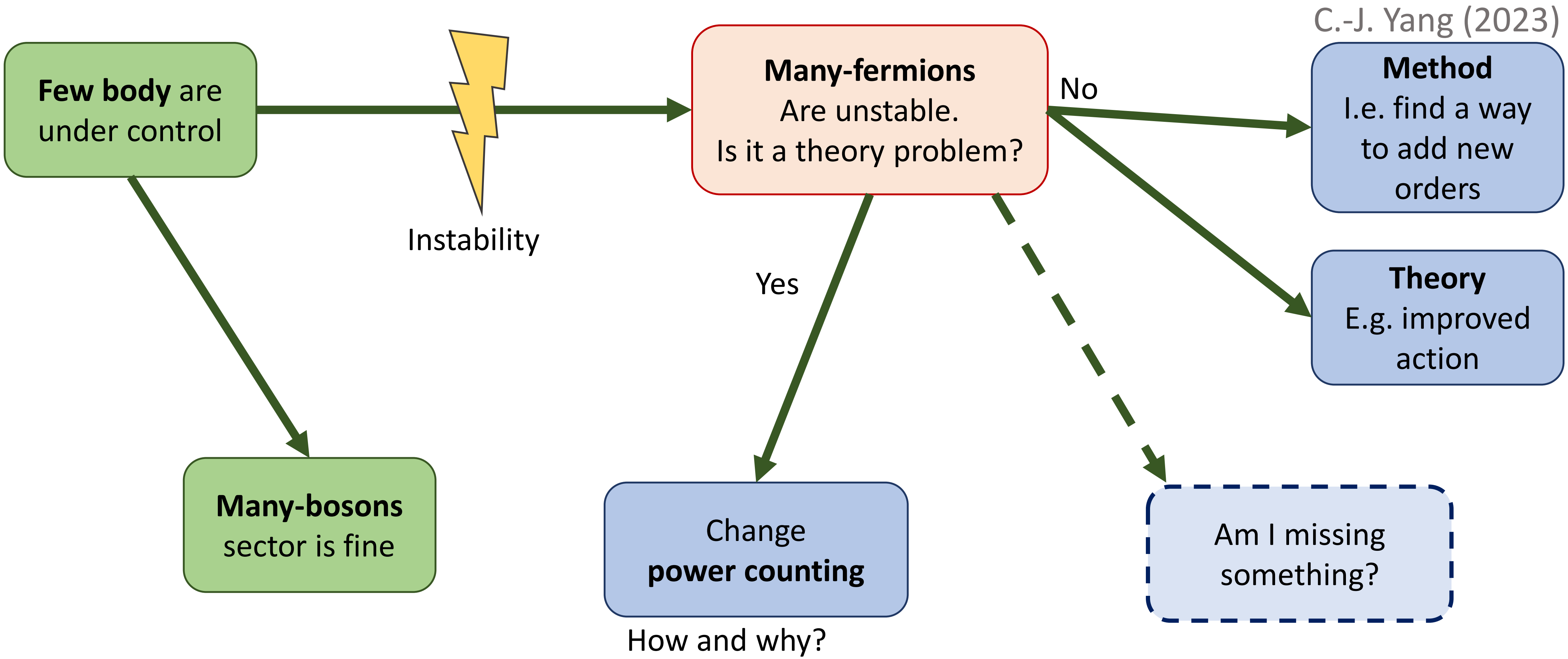




Tested on ^{6}He, ^{6}Li , ^{7}Li, ^{8}Be, ^{16}O, ^{40}Ca

Stetcu, B. R. Barrett, U. van Kolck, Phys.Lett.B653:358-362 (2007)
W. G. Dawkins, J. Carlson, U. van Kolck, A. Gezerlis (2020)
M. Schäfer, L. Contessi, J. Kirscher, J. Mareš PLB 816 (2021)

Contact EFT: instability for fermions

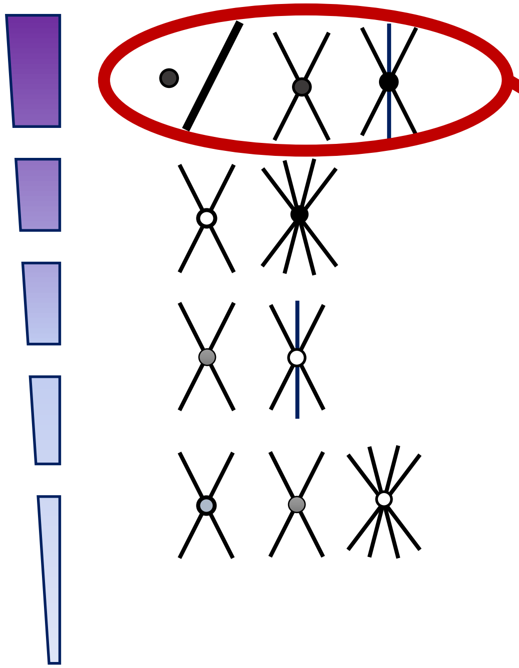


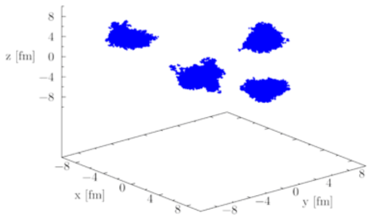
Improved action – general questions

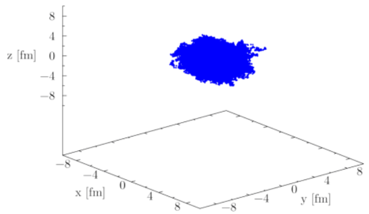
Nuc. Phys. B K. Symanzik (1983)
L.C., M. Schäfer, A. Gnech, A. Lovato, U. van Kolck (in preparation)

Phys. Rev. A L.C., M. Schäfer, U. van Kolck (2024)
arXiv L.C., M. Pavòn Valderrama, U. van Kolck (2024)

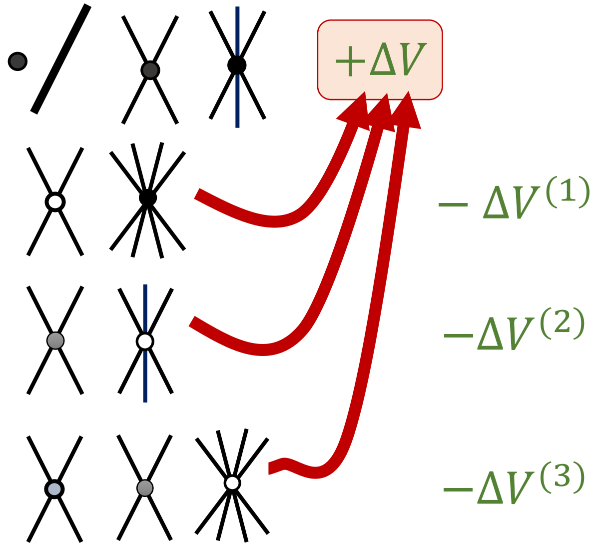
Standard theory





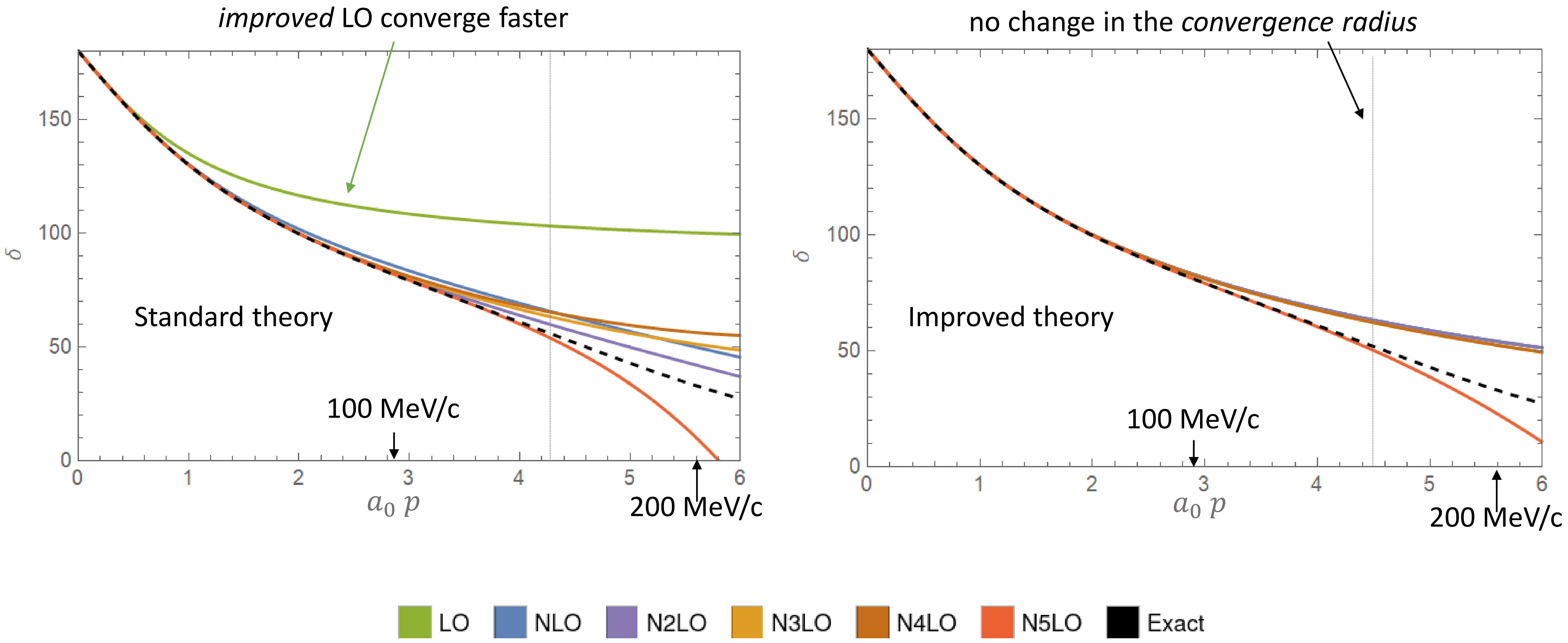


Improved



A NEW WAY OF DOING EFTS: 2-BODY PHASESHIFT

Phase shifts of n-p (deuteron channel):

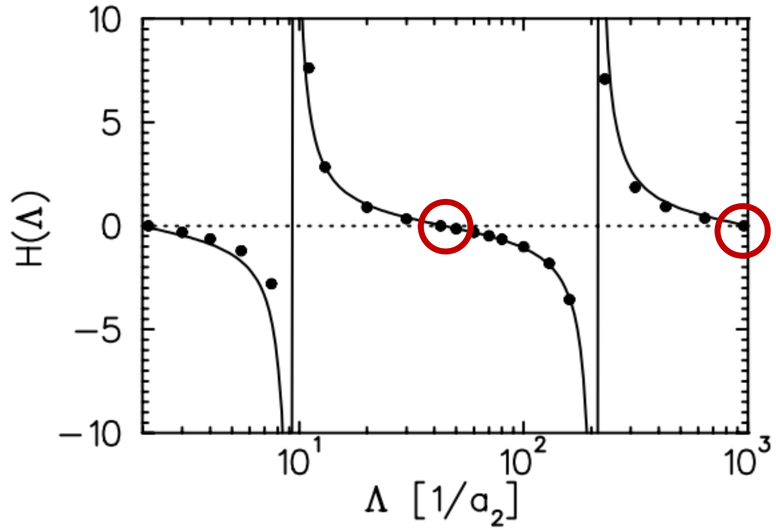


Improved action – general questions

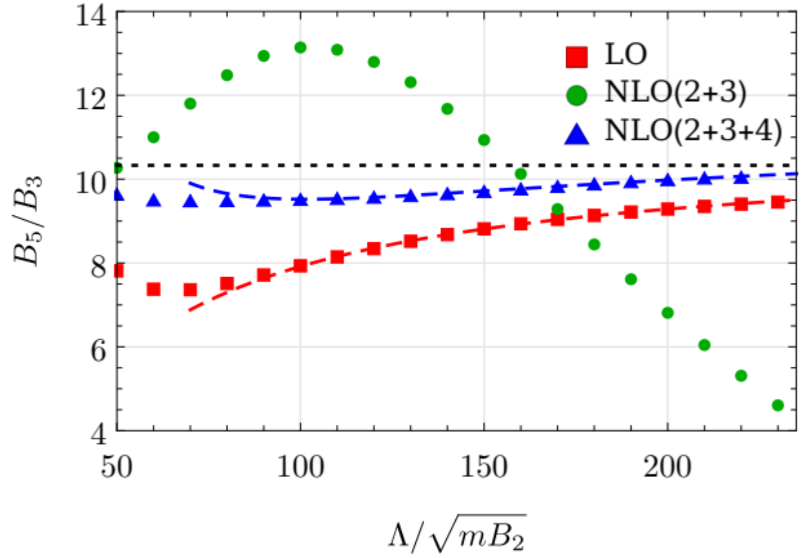
Improvement limits (without changing renormalizability at higher orders):

- Increase the theory precision including **full-orders nonperturbatively**

- Use **lower-dimensionality** operators to include larger-dimensionality scales
  e.g. remove LO 3-body operator by choosing a two-body potential
  use a three-body instead of a four-body force?



P.F. Bedaque, H.-W. Hammer, U. van Kolck (2008)



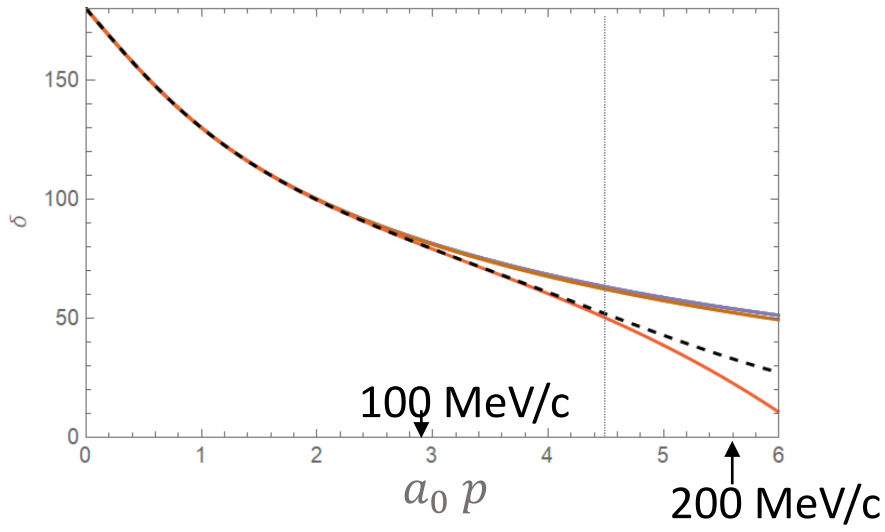
B. Bazak, J. Kirscher, S. König, M. Pavón Valderrama, N. Barnea, U. van Kolck

- Circumvent non-renormalizabity?

Positive and large effective range

A system where $a_0 \sim r_0 \gg w_n$

$$p\cot(\delta) = -\frac{1}{a_0} + \frac{1}{2} r_0 p^2 + w_1 + \cdots$$



Energy dependent formalism

Dibaryon, transvestite …

No apparent problem with positive r_0

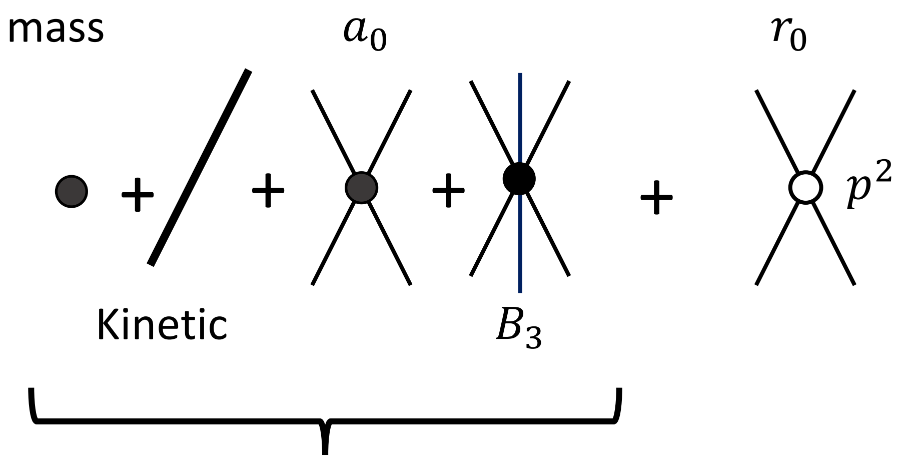
What is the relation between the two?

No energy dependent /

Numerical digestible

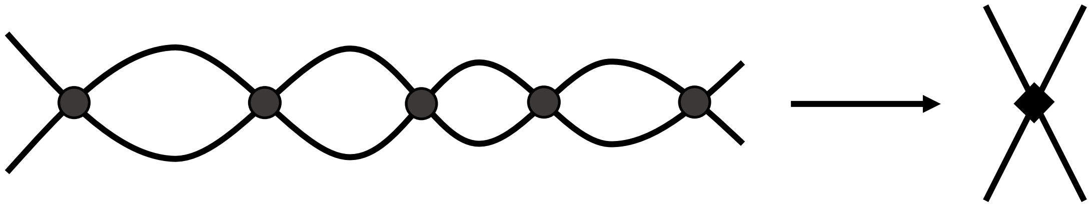
Cant use short range interactions and have r_0 Wigner-bound

Positive and large effective range - the problem (hand weaving)

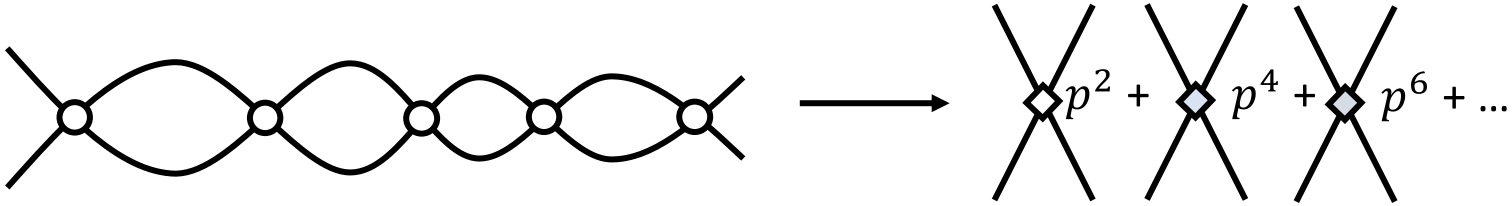


Can be iterated

This would be the interaction
I want to iterate (non perturbative)

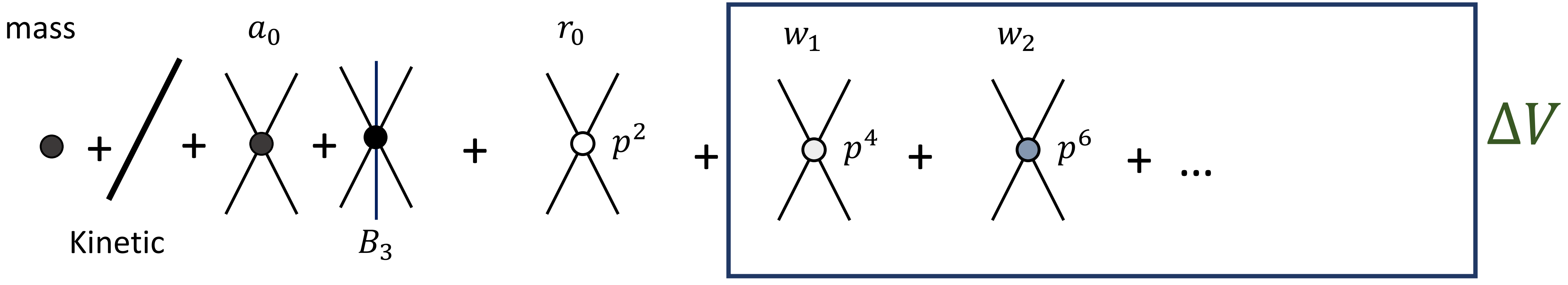


momentum independent operators is ok
→ cutoff can be reabsorbed in one constant.



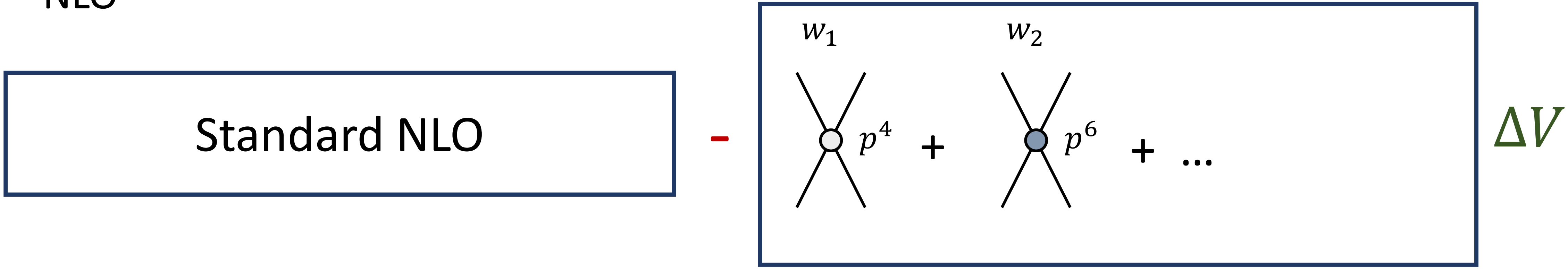
momentum dependent operators
are problematic:
need infinite constants

Positive and large effective range - the problem (hand weaving)



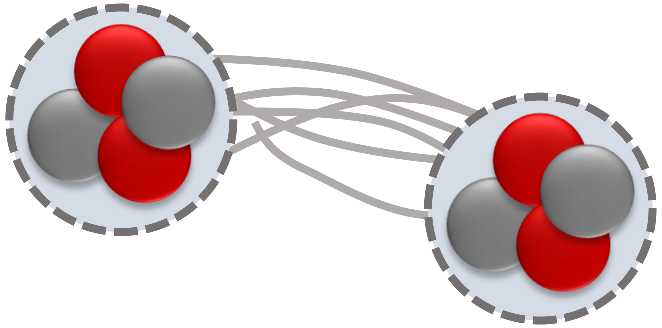
Can you add "small" perturbative sub-leading contributes to make the interaction renormalizable improved action mechanism … Or "finite cutoff" approach.

- -NLO - -

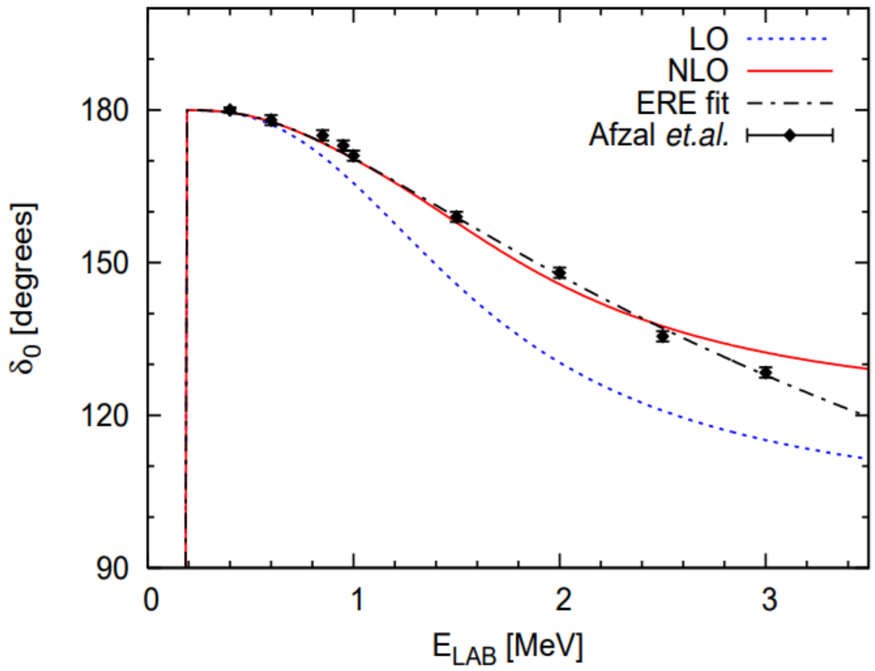


Coulomb systems and alphas

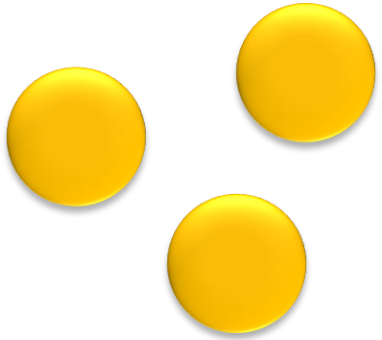
Large r_0 example:



In $\alpha - \alpha$ systems you need a p^2 term in addiction to contact and Coulomb



R. Higa, H.-W. Hammer, U. van Kolck (2008)



Need to iterate p^2:

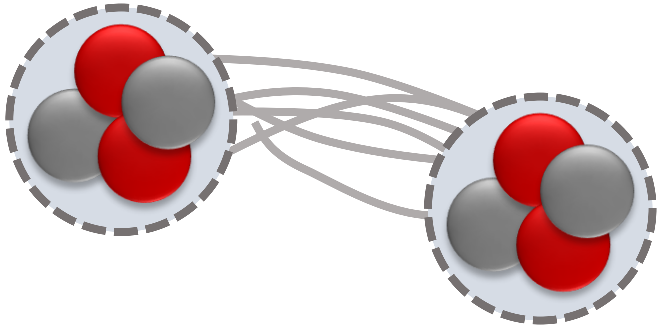
- Use dibaryon but not do many-body
- Find a way around Wigner bound

How much of the nuclear chart can be described with alphas?

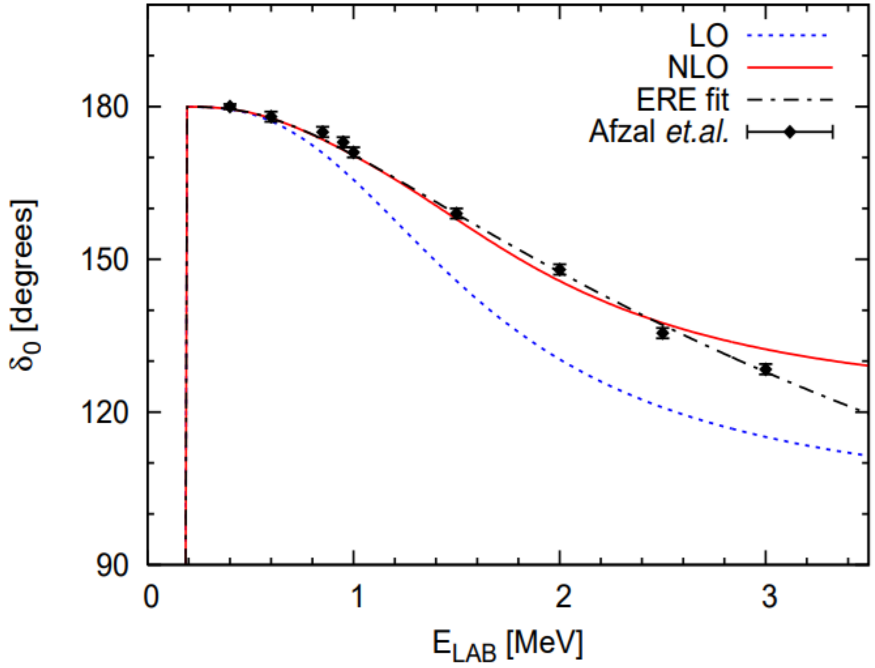
and alphas + single nucleons?

Coulomb systems and alphas

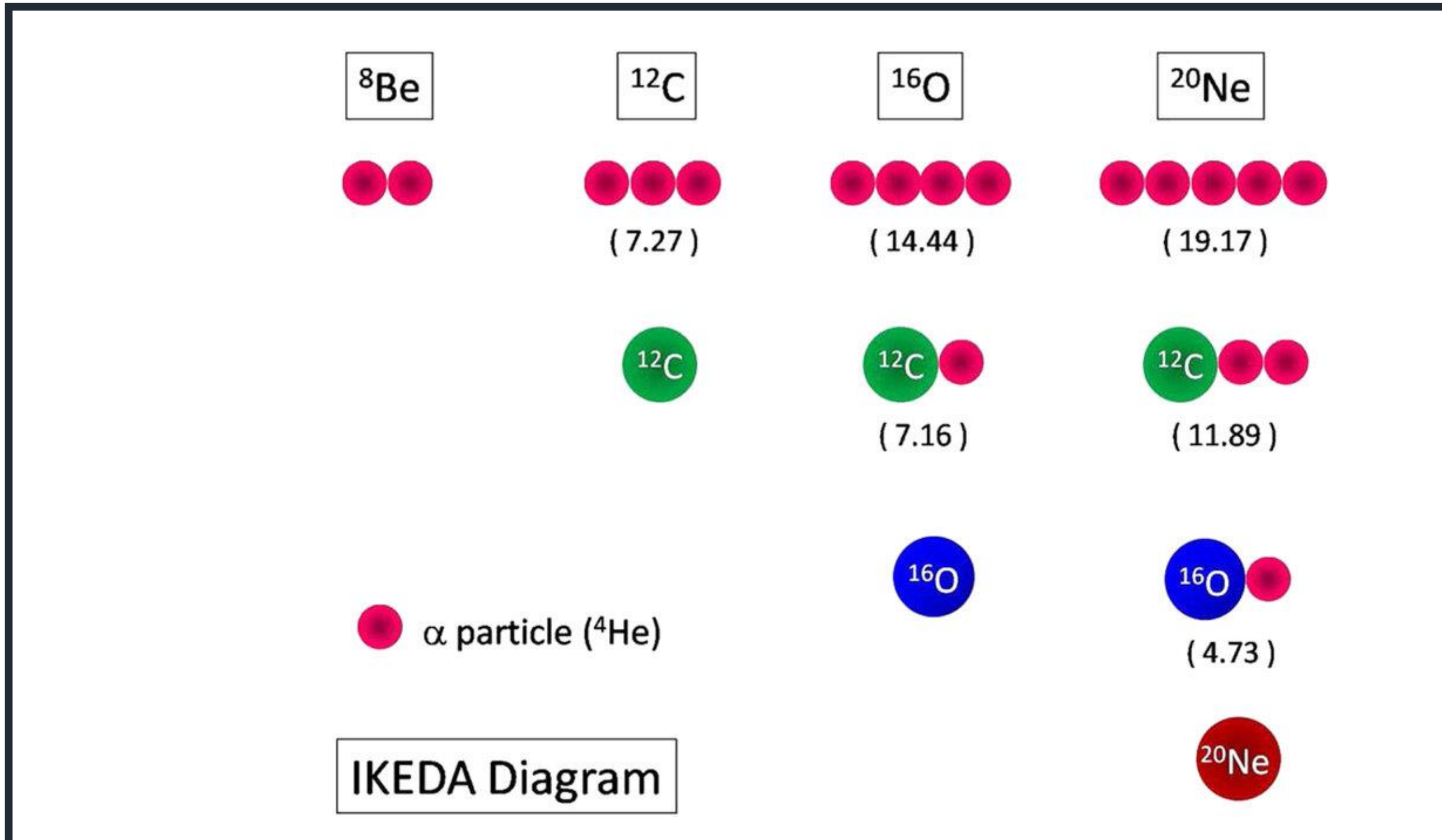
Large r_0 example:



In $\alpha - \alpha$ systems you need a p^2 term in addiction to contact and Coulomb



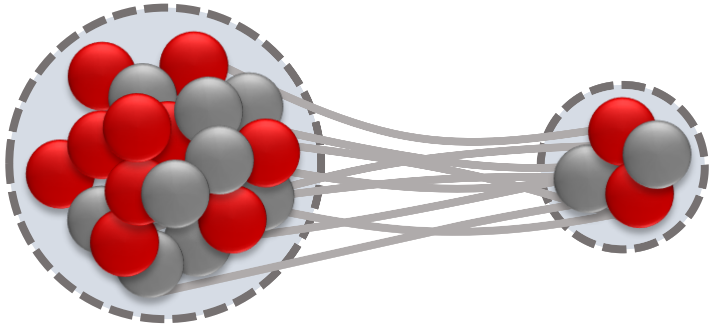
R. Higa, H.-W. Hammer, U. van Kolck (2008)



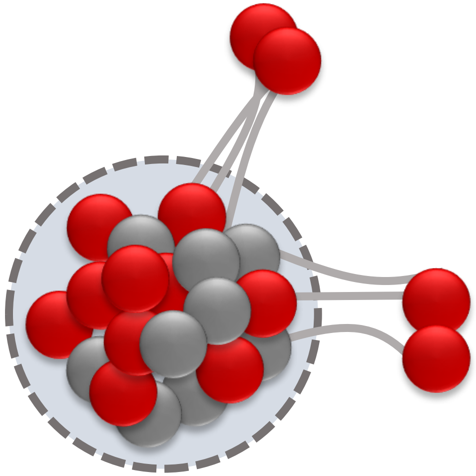
Up to ^{40}Ca

Other EFT formulations? – Increase the number of particles

New degrees of freedom



Clusters/halos

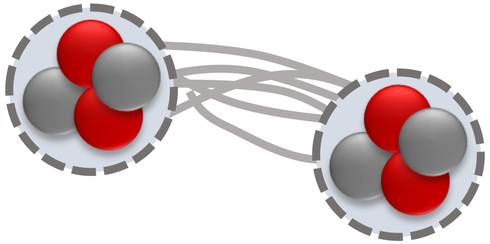


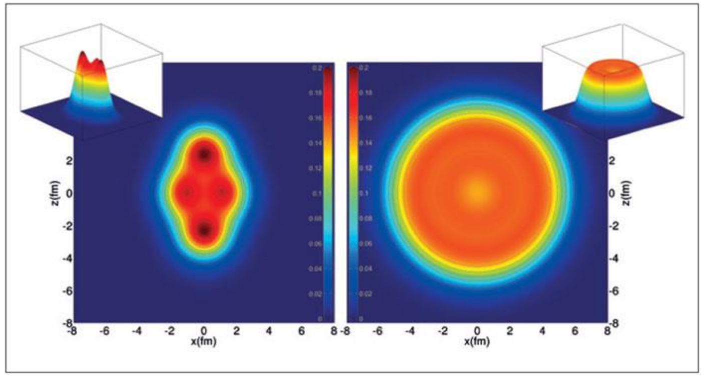
Is it similar to a **contact theory**? What about **Coulomb**?

Is there a way to connect “more microscopic” and cluster theories?

DFT effective theory

Alphas





Is there a way to connect “more microscopic” and cluster theories? RGM?

Power counting is unknown

Powercounting changes with new scales

A - Number of particles

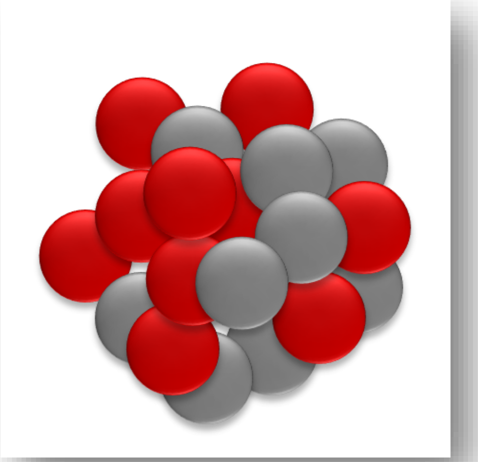
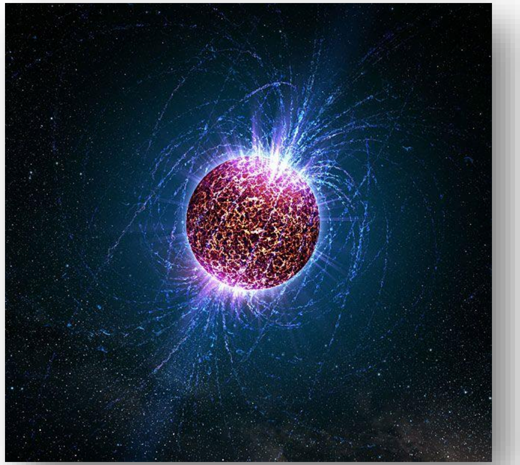
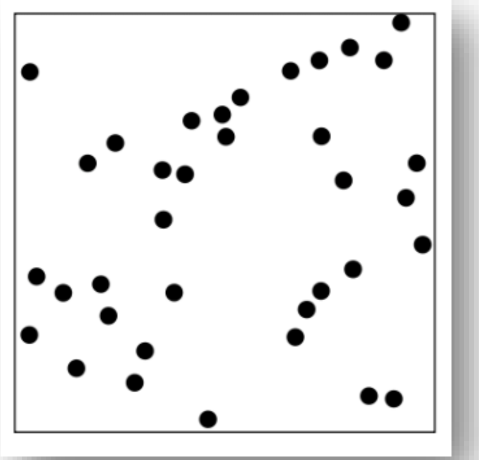
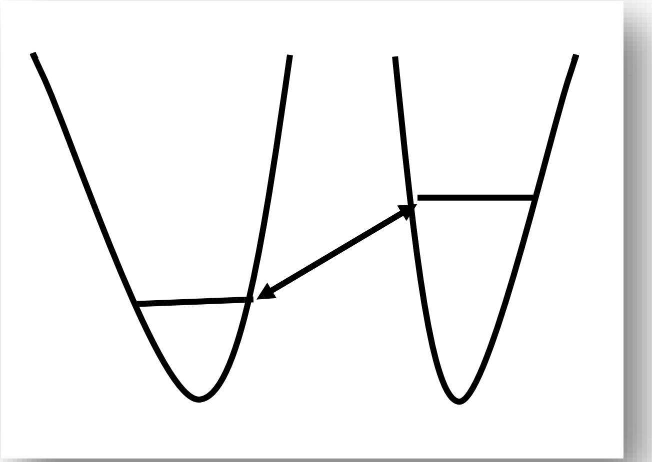
ρ - Density

L - Systems in traps

C - Multiple channels with large coupling

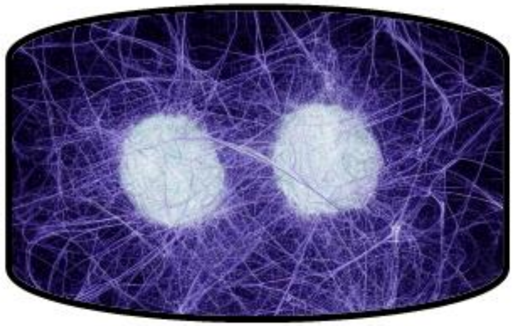
...

New scales that can change the power counting
Can we predict this a priori via scale analysis?

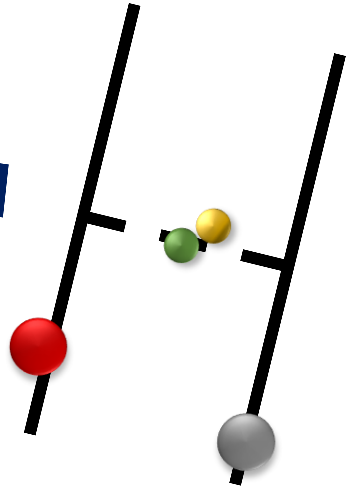





From one EFT to the other

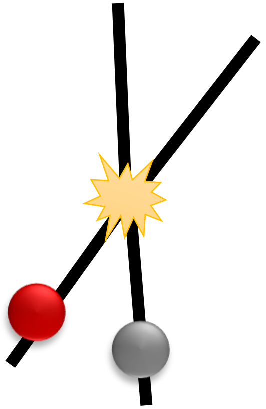
Do we have to use observables
Or can we derive one from the other
in a different way?



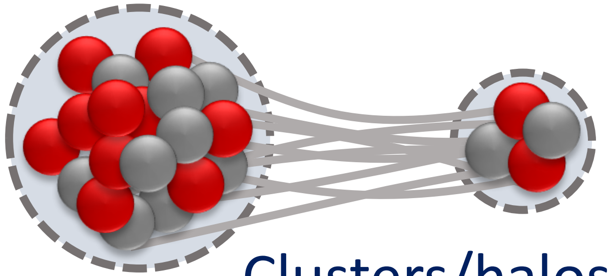
QCD
Non perturbative
lattice calculations



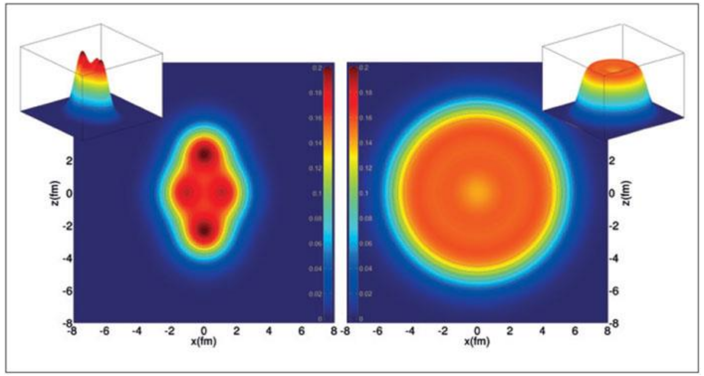
Mesons + nucleons
(χEFT)



Nucleons
(πEFT)



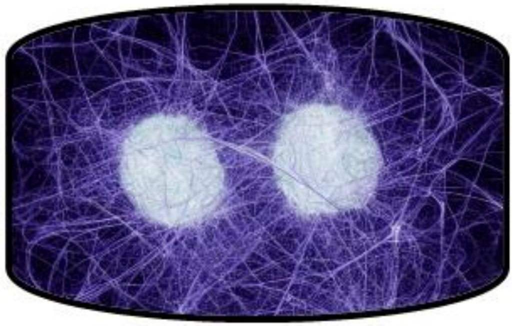
Clusters/halos
(cluster/halo EFT)



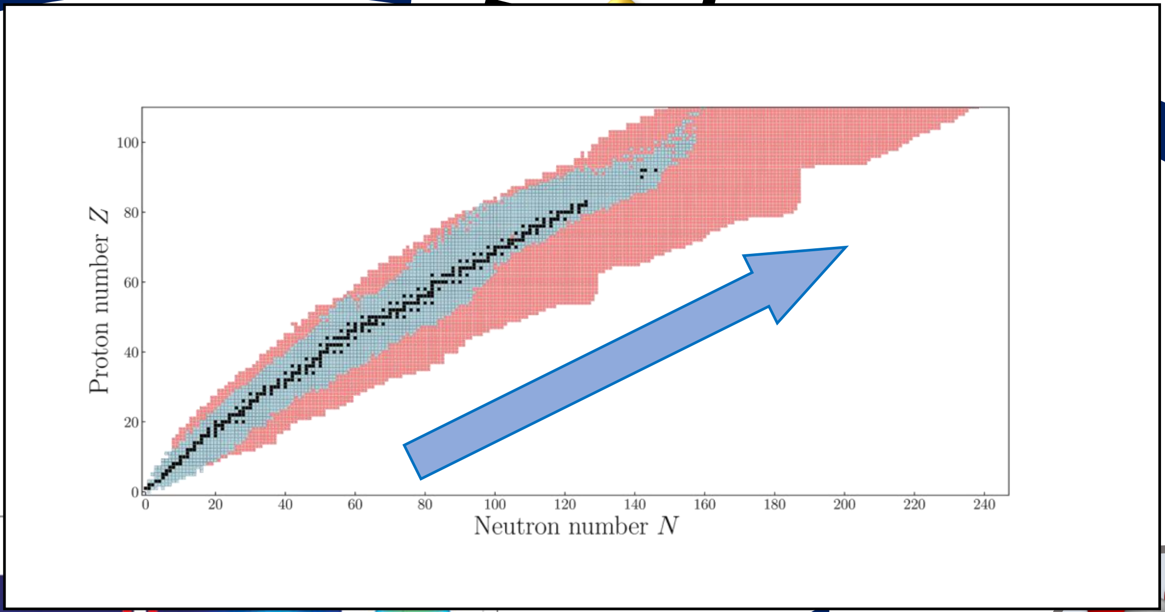
DFT
effective theory
(- ??? -)

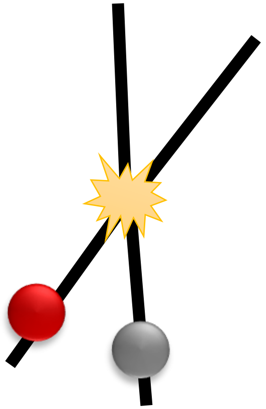
From one EFT to the other

Do we have to use observables
Or can we derive one from the other
in a different way?



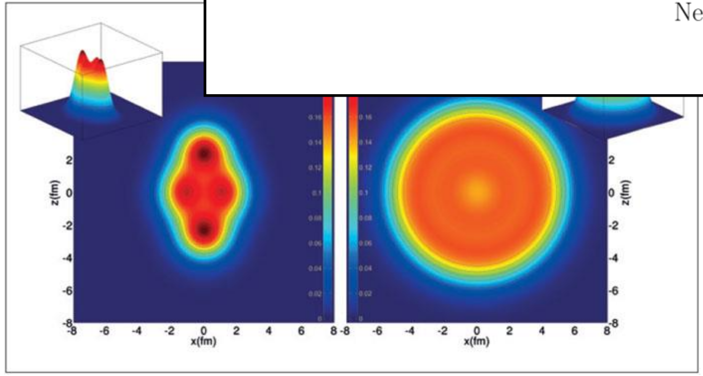
QCD
Non perturbative
lattice calculations

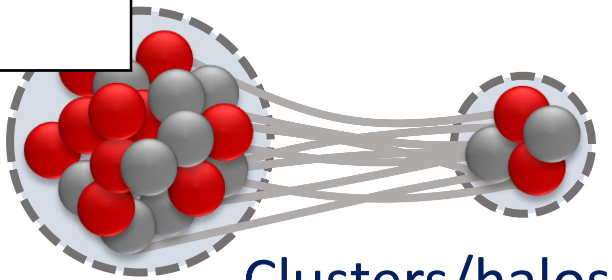




Nucleons
(πEFT)

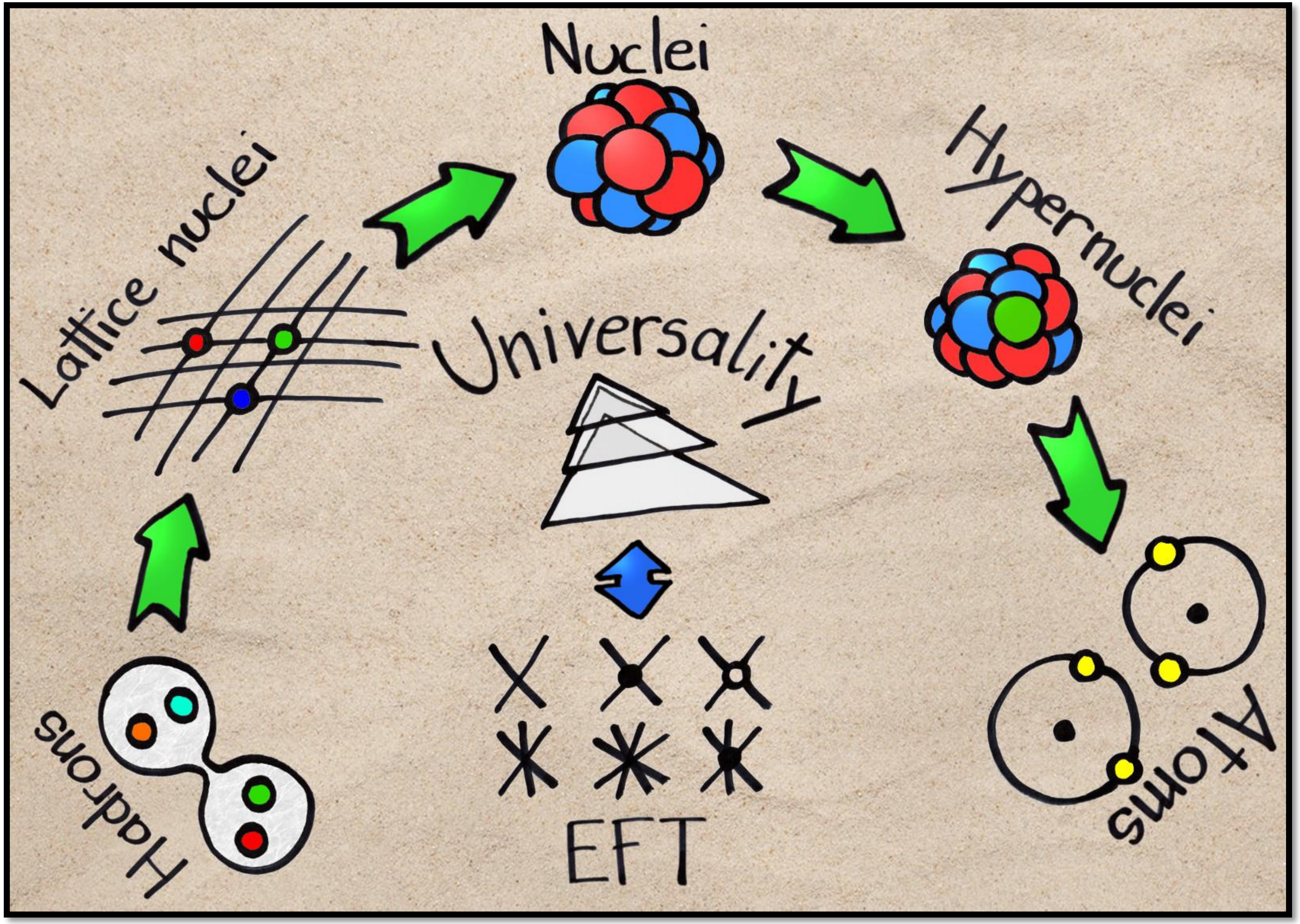
DFT
effective theory
(- ??? -)





Clusters/halos
(cluster/halo EFT)

Interdisciplinarity: transfer knowledge from and to other fields



Open questions:

Nucleons:

- Chiral renormalizable power counting
- Instability problem (also contact)
  - improve our treatment of the theory
  - power counting modification

Non-renormalizable theories:

E.g. Unnatural and positive effective range / alpha particles

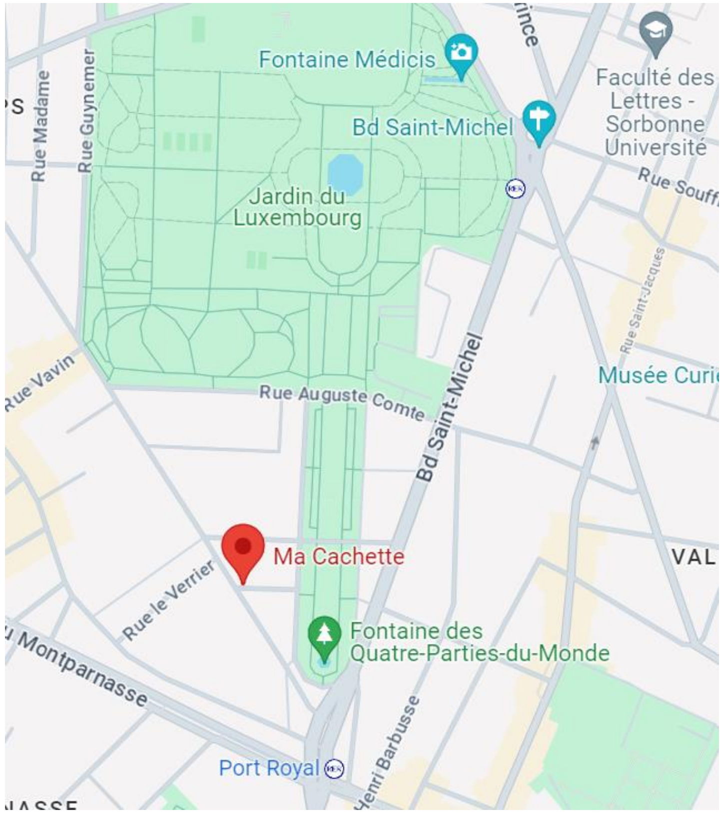
Other EFTs: different degrees of freedom

- Clusters, other mesons, densities, …
- Interdisciplinarity: atoms, hadrons, hypernuclei, …
- From one EFT to the next (towards the halo-cluster EFT)

How power counting changes introducing new degrees of freedom

- In a box, number of particles, coupled channels, …

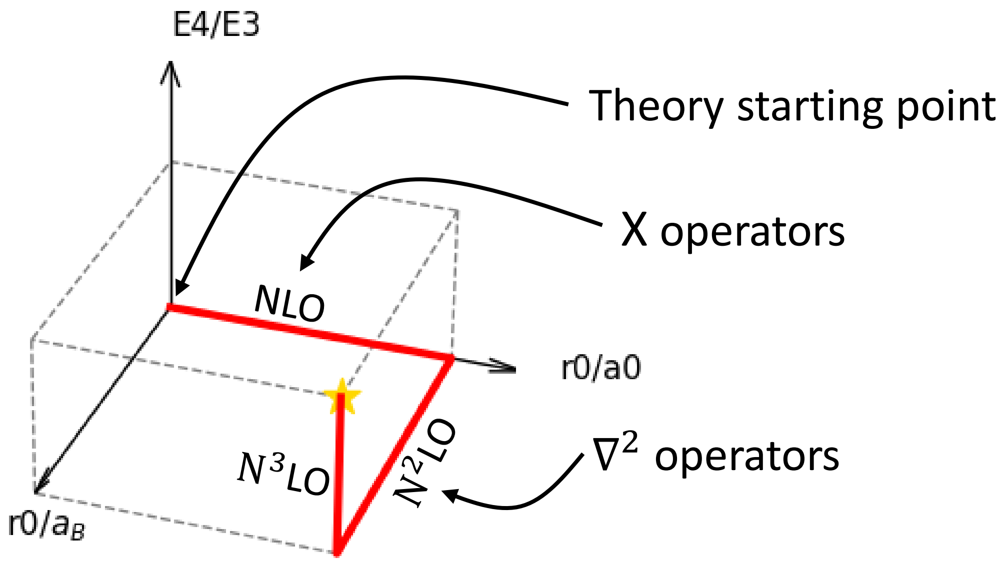
This evening:



Ma Cachette 8 pm
(Leave from Orsay station at 18.45
Or 18.30 from CEA)

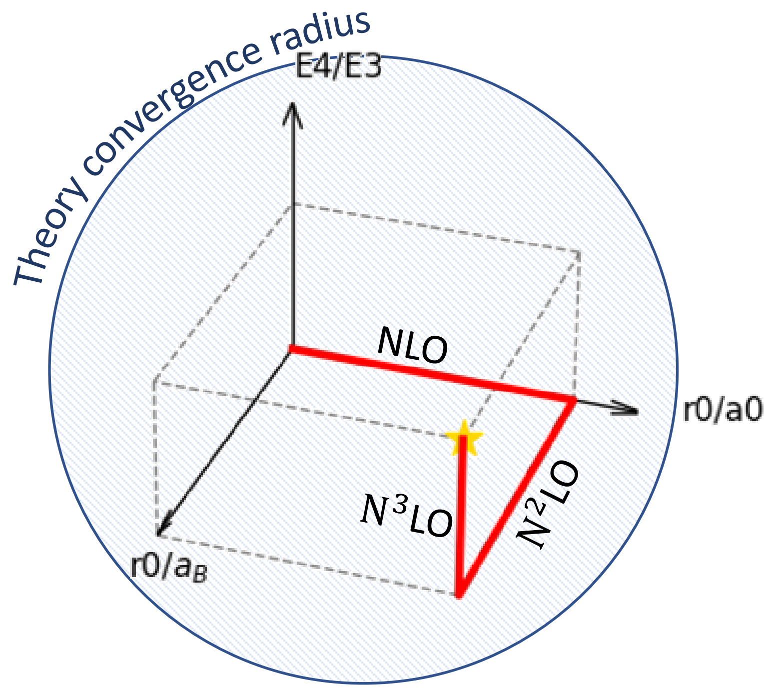
bus 9 and then RERB

Stability problem in renormalizable theories (just contact EFT for simplicity)



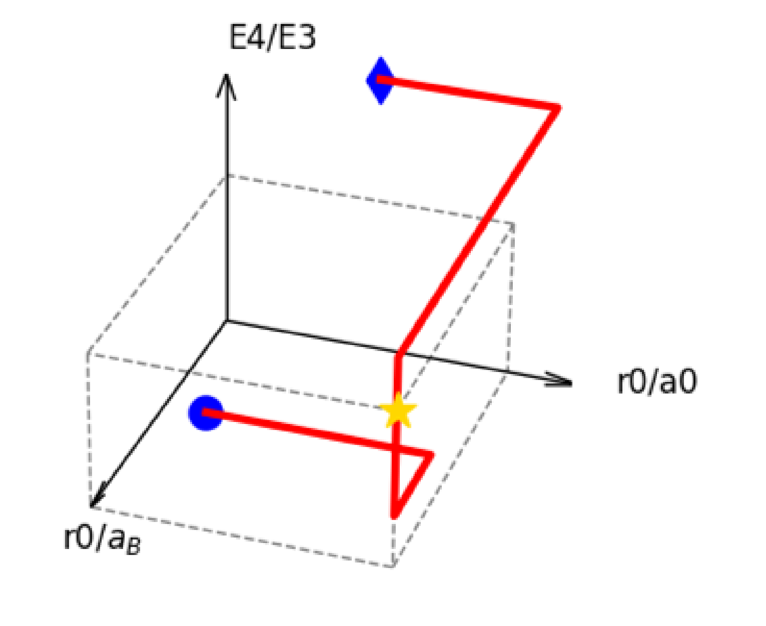
Starting from the **universal point** one can reach the **physical point** with perturbative inclusions.

Contact operators make these lines as perpendicular as possible (Other expansions are possible)



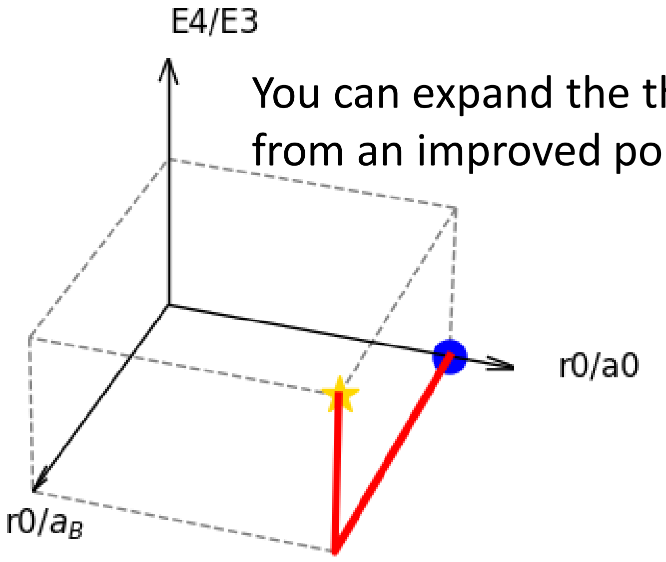
The radius of convergence of the theory: **points reachable in this way.**

Stability problem in renormalizable theories (just contact EFT for simplicity)



No needed to start from the universal point.

(the expansion should not necessarily modified)



You can expand the theory from an improved point.

Standard improvement:
treat **finite scattering length** as starting point.

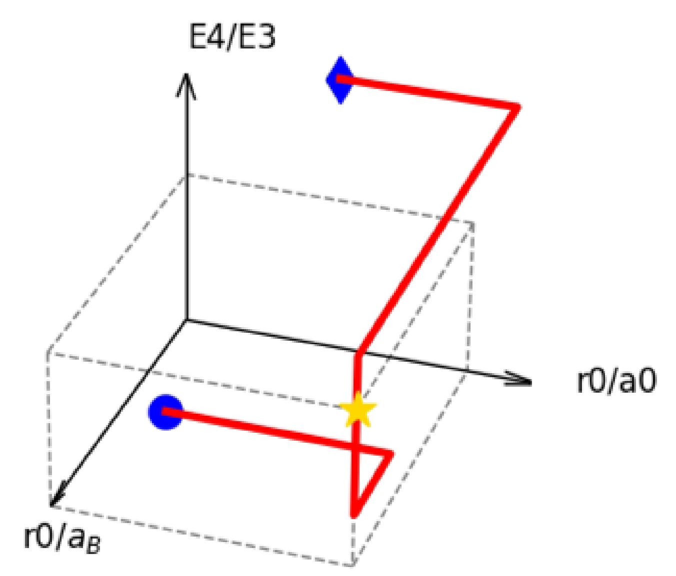
LO	$a_0 \to \infty$ **3B**
NLO	$a_0 < \infty$
N2LO	r_0 , **4B**
N3LO	a_1, **3B**p^2
N4LO	v_2

This effectively treat (resumm) **subleading** already **at LO**

doesn't change the power counting:

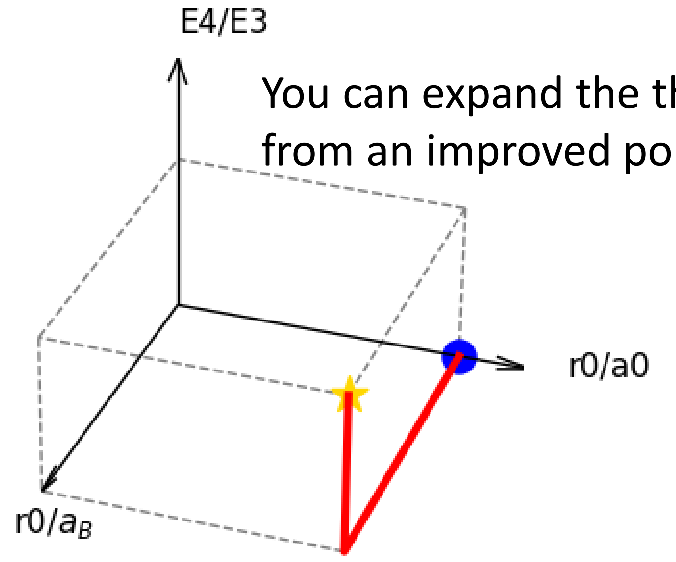
- The correction remain small
- The rest of the power counting is not perturbed

Stability problem in renormalizable theories (just contact EFT for simplicity)



No needed to start from the universal point.

(the expansion should not necessarily modified)



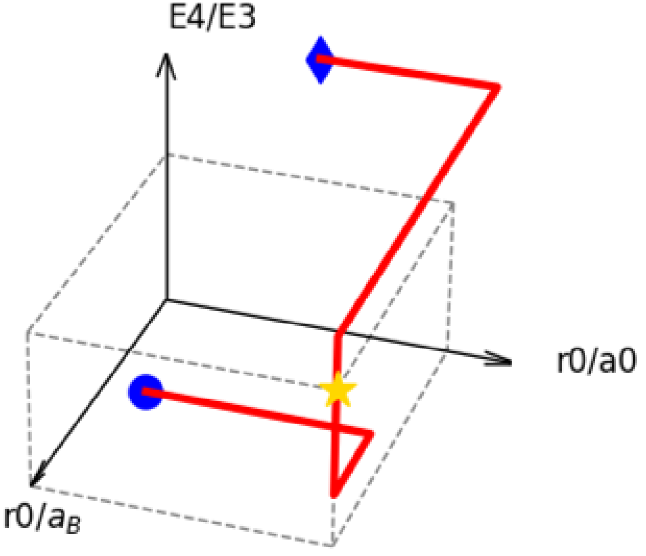
Standard improvement:
treat **finite scattering length** as starting point.

LO	$a_0 < \infty$ **3B**
~~**NLO**~~	–
N2LO 1	r_0 , **4B**
N3LO 2	a_1, **3B**p^2
N4LO 3	v_2

This effectively treat (resumm) **subleading** already **at LO**

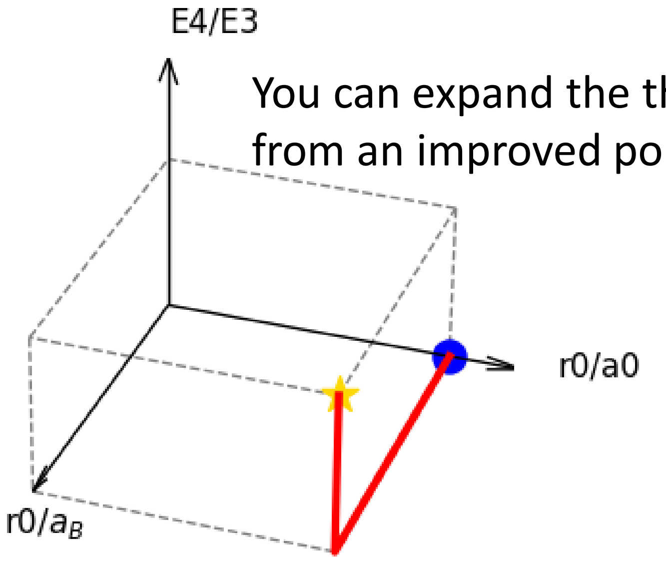
doesn't change the power counting:
- The correction remain small
- The rest of the power counting is not perturbed

Stability problem in renormalizabl… i… (just contact EFT for simplicity)



No needed to start from the universal point.

(the expansion should not necessarily modified)



You can expand the theory from an improved point.

Standard improvement: treat **finite scattering length** as starting point.

LO	a_0, r^* **3B**
~~**NLO**~~	–
N2LO	r_0 , **4B**
N3LO	a_1, **3B**p^2
N4LO	v_2

This effectively treat (resumm) **subleading** already **at LO**

doesn't change the power counting:

- The correction remain small
- The rest of the power counting is not perturbed

Designing an improved action (2B)

	Potential	T-matrix	Observable described
Non perturbative	$C_0(a_0 \to \infty)\, \delta(r_{ij})$	$\frac{1}{-ik}$	Universality
Perturbative	$C_1(a_0)\delta(r_{ij})$	$\frac{1}{-ik}\left(1+\frac{\alpha}{a_0}\right)$	a_0
Perturbative	$C_2(a_0, r_0)\, \nabla^2\left(\, \delta(r_{ij})\right)$	$\frac{1}{-ik}\left(1+\frac{\beta}{a_0}-\gamma\, r_0\right)$	r_0

Designing an improved action (2B)

	Potential	T-matrix	Observable described
Non perturbative	$C_0(a_0)\,\delta(r_{ij})$	$\dfrac{1}{-\frac{1}{a_0} - ik}$	a_0
Perturbative	$C_2(a_0, r_0)\,\nabla^2\left(\delta(r_{ij})\right)$	$\dfrac{1}{-\frac{1}{a_0} - ik}\left(1 + \dfrac{k^2 r_0}{2\left(k - i\frac{1}{a_0}\right)^2}\right)$	r_0
Perturbative	$C_2(a_0, r_0)\,\nabla^4\left(\delta(r_{ij})\right)$	$\dfrac{1}{-\frac{1}{a_0} - ik}(1 + \alpha_1\, r_0 + \beta_1\, \omega)$	ω_0

Designing an improved action (2B)

	Potential	T-matrix	Observable described
Non perturbative	$C_0(a_0)\,\delta(r_{ij})$	$\dfrac{1}{-\frac{1}{a_0}+\frac{1}{2}r_0k^2-ik}$	a_0, r_0
Perturbative	$C_2(a_0,r_0)\,\nabla^2\left(\delta(r_{ij})\right)$	It is not possible to include a contact interaction to correct the effective range (Wigner bound!)	
Perturbative	$C_2(a_0,r_0)\,\nabla^4\left(\delta(r_{ij})\right)$	$\dfrac{1}{-\frac{1}{a_0}-ik}(1+\alpha_1\,r_0+\beta_1\,\omega)$	

Designing an improved action (2B)

	Potential	T-matrix	Observable described
Non perturbative	$C_0(a_0)\,\delta(r_{ij}) + \Delta V$	$\dfrac{1}{-\frac{1}{a_0}+\frac{1}{2}r_0^* k^2 + \Delta\omega\, k^4 + \Delta\omega\, k^6 + \cdots - ik}$	a_0, r_0^*, (+ spurious components)
Perturbative	$C_2(a_0, r_0)\,\nabla^2\left(\delta(r_{ij})\right)$	$T_0\left(1+\alpha_1(r_0 - r_0^*)\right)$	r_0
Perturbative	$C_2(a_0, r_0)\,\nabla^4\left(\delta(r_{ij})\right)$	$\dfrac{1}{-\frac{1}{a_0} - ik}\left(1+\alpha_1(r_0 - r_0^*) + \beta_1\,\omega_0\right)$	ω_0

Hamiltonian formulation

$$\delta_\Lambda(\vec{r_i},\vec{r_j}) = e^{-\frac{\lambda^2 r_{ij}^2}{4}}$$

$$\delta\left(\vec{r_i},\vec{r_j},\vec{r_k}\right) = \sum_{cyc}\left[e^{-\frac{\lambda^2\left(r_{ij}^2+r_{ik}^2\right)}{4}}\right]$$

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0\,\delta_\Lambda(\vec{r_i},\vec{r_j}) + \sum_{ijk} D_0\,\delta_\Lambda\left(\vec{r_i},\vec{r_j},\vec{r_k}\right)$$

$$H^{NLO} = \sum_{ij} C_2\,\delta\left(\vec{r_{ij}}\right)\left(\overrightarrow{\nabla}^2+\overleftarrow{\nabla}^2\right) + \sum_{ijkl} E_0\,\delta\left(\vec{r_i},\vec{r_j},\vec{r_k},\vec{r_l}\right)$$

Fit to $a_0, r_0 = 0$
And B_3

Fit to r_0
And B_4

U. Van Kolck (1999)
B. Bazak (2018)

Hamiltonian formulation

Improve action mechanism:
K. Symanzik (1983)

$$\delta_\Lambda(\vec{r_i},\vec{r_j}) = e^{-\frac{\lambda^2 r_{ij}^2}{4}}$$

$$\delta(\vec{r_i},\vec{r_j},\vec{r_k}) = \sum_{cyc}\left[e^{-\frac{\lambda^2 (r_{ij}^2+r_{ik}^2)}{4}}\right]$$

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0\, \delta_\Lambda(\vec{r_i},\vec{r_j}) + \sum_{ijk} D_0\, \delta_\Lambda(\vec{r_i},\vec{r_j},\vec{r_k}) + \Delta V$$

Small (perturbative) auxiliary interaction

Reproduces $(a_0, r^*, \delta\omega, \delta\omega_2, \dots)$

$$H^{NLO} = \sum_{ij} C_2^*\, \delta(\vec{r_{ij}})\,(\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2) + \sum_{ijkl} E_0^*\, \delta(\vec{r_i},\vec{r_j},\vec{r_k},\vec{r_l})$$

Corrects $\mathrm{r}^* \to r_0$ and fit B_4

$$H^{N^{\geq 2}LO}$$

Corrects $\delta\omega, \delta\omega_2$

Hamiltonian formulation

$$\delta_\Lambda(\vec{r_i},\vec{r_j}) = e^{-\frac{\lambda^2 r_{ij}^2}{4}}$$

$$\delta\,(\vec{r_i},\vec{r_j},\vec{r_k}) = \sum_{cyc}\left[e^{-\frac{\lambda^2 (r_{ij}^2+r_{ik}^2)}{4}}\right]$$

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0\,\delta_\Lambda(\vec{r_i},\vec{r_j}) + \sum_{ijk} D_0\,\delta_\Lambda\,(\vec{r_i},\vec{r_j},\vec{r_k}) + \Delta V$$

$$H^{NLO} = \sum_{ij} C_2^*\,\delta\,(\overrightarrow{r_{ij}})\,(\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2) + \sum_{ijkl} E_0^*\,\delta\,(\vec{r_i},\vec{r_j},\vec{r_k},\vec{r_l})$$

$$\Delta V_2 = \sum_{ij}\left(C_0^*(\bar{R}^{-1})\delta_{\bar{R}^{-1}}(\vec{r_i},\vec{r_j}) - C_0(\Lambda)\,\delta_\Lambda(\vec{r_i},\vec{r_j})\right)$$

$$\Delta V_3 = \sum_{ijk}\left(D_0^*(\bar{R}^{-1})\,\delta_{\bar{R}^{-1}}\,(\vec{r_i},\vec{r_j},\vec{r_k}) - D_0(\Lambda)\delta_\Lambda\,(\vec{r_i},\vec{r_j},\vec{r_k})\right)$$

Hamiltonian formulation

$$\delta_\Lambda(\vec{r_i},\vec{r_j}) = e^{-\frac{\lambda^2 r_{ij}^2}{4}}$$

$$\delta\,(\vec{r_i},\vec{r_j},\vec{r_k}) = \sum_{cyc}\left[e^{-\frac{\lambda^2 (r_{ij}^2+r_{ik}^2)}{4}}\right]$$

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0\,\delta_\Lambda(\vec{r_i},\vec{r_j}) + \sum_{ijk} D_0\,\delta_\Lambda\,(\vec{r_i},\vec{r_j},\vec{r_k}) + \Delta V$$

$$H^{NLO} = \sum_{ij} C_2^*\,\delta\,(\overrightarrow{r_{ij}})\,(\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2) + \sum_{ijkl} E_0^*\,\delta\,(\vec{r_i},\vec{r_j},\vec{r_k},\vec{r_l})$$

Auxiliary interaction contains a lot of contributions but has no renormalizability problems

Can also be a phenomenological interaction!

$$\Delta V_2 = \sum_{ij}\left(C_0^*(\bar{R}^{-1})\delta_{\bar{R}^{-1}}(\vec{r_i},\vec{r_j}) - C_0(\Lambda)\,\delta_\Lambda(\vec{r_i},\vec{r_j})\right)$$

$$\Delta V_3 = \sum_{ijk}\left(D_0^*(\bar{R}^{-1})\,\delta_{\bar{R}^{-1}}\,(\vec{r_i},\vec{r_j},\vec{r_k}) - D_0(\Lambda)\delta_\Lambda\,(\vec{r_i},\vec{r_j},\vec{r_k})\right)$$

Hamiltonian formulation

$$\delta_\Lambda(\vec{r_i},\vec{r_j}) = e^{-\frac{\lambda^2 r_{ij}^2}{4}}$$

$$\delta\left(\vec{r_i},\vec{r_j},\vec{r_k}\right) = \sum_{cyc}\left[e^{-\frac{\lambda^2\left(r_{ij}^2+r_{ik}^2\right)}{4}}\right]$$

Option 1:

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0^*(\bar{R}^{-1})\delta_{\bar{R}^{-1}}(\vec{r_i},\vec{r_j}) + \sum_{ijk} D_0\,\delta_\Lambda\left(\vec{r_i},\vec{r_j},\vec{r_k}\right)$$

Option 2:

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0^*(\bar{R}^{-1})\delta_{\bar{R}^{-1}}(\vec{r_i},\vec{r_j}) + \sum_{ijk} D_0^*(\bar{R}^{-1})\,\delta_{\bar{R}^{-1}}\left(\vec{r_i},\vec{r_j},\vec{r_k}\right)$$

See also:
P. Recchia 2022

Subleading orders remain untouched:

$$H^{NLO} = \sum_{ij} C_2^*\,\delta\left(\vec{r_{ij}}\right)\left(\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2\right) + \sum_{ijkl} E_0^*\,\delta\left(\vec{r_i},\vec{r_j},\vec{r_k},\vec{r_l}\right)$$

Do not forget the four-body force!

Hamiltonian formulation

$$\delta_\Lambda(\vec{r_i},\vec{r_j}) = e^{-\frac{\lambda^2 r_{ij}^2}{4}}$$

$$\delta\left(\vec{r_i},\vec{r_j},\vec{r_k}\right) = \sum_{cyc}\left[e^{-\frac{\lambda^2\left(r_{ij}^2+r_{ik}^2\right)}{4}}\right]$$

Option 1:

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0^*(\bar{R}^{-1})\, e^{-\frac{r_{ij}^2}{4\bar{R}^2}} + \sum_{ijk} D_0\, \delta_\Lambda\left(\vec{r_i},\vec{r_j},\vec{r_k}\right)$$

Option 2:

$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0^*(\bar{R}^{-1})\, e^{-\frac{r_{ij}^2}{4\bar{R}^2}} + \sum_{ijk} D_0^*(\bar{R}^{-1}) \sum_{cyc}\left[e^{-\frac{\left(r_{ij}^2+r_{ik}^2\right)}{4\bar{R}^2}}\right]$$

See similarities with: R. Schiavilla (2021)

(but also notice that **the effective range is still subleading**!)

Subleading orders remain untouched:

$$H^{NLO} = \sum_{ij} C_2^*\, \delta\left(\vec{r_{ij}}\right)\left(\overrightarrow{\nabla}^2 + \overleftarrow{\nabla}^2\right) + \sum_{ijkl} E_0^*\, \delta\left(\vec{r_i},\vec{r_j},\vec{r_k},\vec{r_l}\right)$$

Test:

4He atoms up to 5 particles

D. Blume and C. H. Greene, Monte carlo hyperspherical description of helium cluster excited states, The Journal of Chemical Physics **112**, 8053 (2000), https://doi.org/10.1063/1.481404.

A. R. Janzen and R. A. Aziz, Modern he–he potentials: Another look at binding energy, effective range theory, retardation, and efimov states, The Journal of Chemical Physics **103**, 9626 (1995), https://doi.org/10.1063/1.469978.

E. A. Kolganova, A. K. Motovilov, and W. Sandhas, Scattering length of the helium-atom–helium-dimer collision, Phys. Rev. A **70**, 052711 (2004).

R. Lazauskas and J. Carbonell, Description of ^{4}He tetramer bound and scattering states, Phys. Rev. A **73**, 062717 (2006).

E. Hiyama and M. Kamimura, Variational calculation of 4He tetramer ground and excited states using a realistic pair potential, Phys. Rev. A **85**, 022502 (2012), arXiv:1111.4370 [physics.atom-ph].

	PCKLJS	LM2M2
a_2 [Å]	90.42(92)	100.23
r_2 [Å]	7.27	7.326
B_2 [mK]	1.3094	1.6154
B_3 [mK]	131.84	126.50
B_3^* [mK]	2.6502	2.2779
B_4 [mK]	573.90	559.22
B_5 [mK]	-	1306.7

R. A. Aziz and M. J. Slaman, An examination of ab-initio results for the helium potential energy curve, The Journal of Chemical Physics **94**, 8047 (1991), https://pubs.aip.org/aip/jcp/article-pdf/94/12/8047/9734055/8047_1_online.pdf.

M. Przybytek, W. Cencek, J. Komasa, G. Łach, B. Jeziorski, and K. Szalewicz, Relativistic and quantum electrodynamics effects in the helium pair potential, Phys. Rev. Lett. **104**, 183003 (2010).

R. E. Grisenti, W. Schollkopf, J. P. Toennies, G. C. Hegerfeldt, T. Kohler, and M. Stoll, Determination of the Bond Length and Binding Energy of the Helium Dimer by Diffraction from a Transmission Grating, Phys. Rev. Lett. **85**, 2284 (2000).

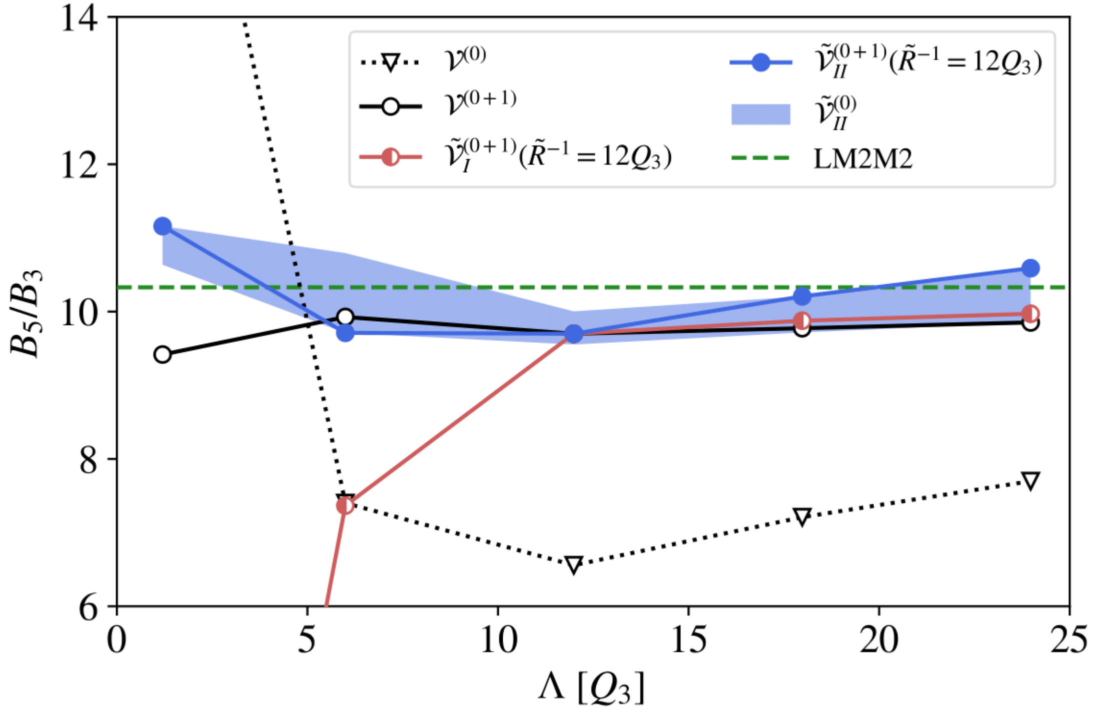
M. Kunitski *et al.*, Observation of the Efimov state of the helium trimer, Science **348**, 551 (2015), arXiv:1512.02036 [physics.atm-clus].

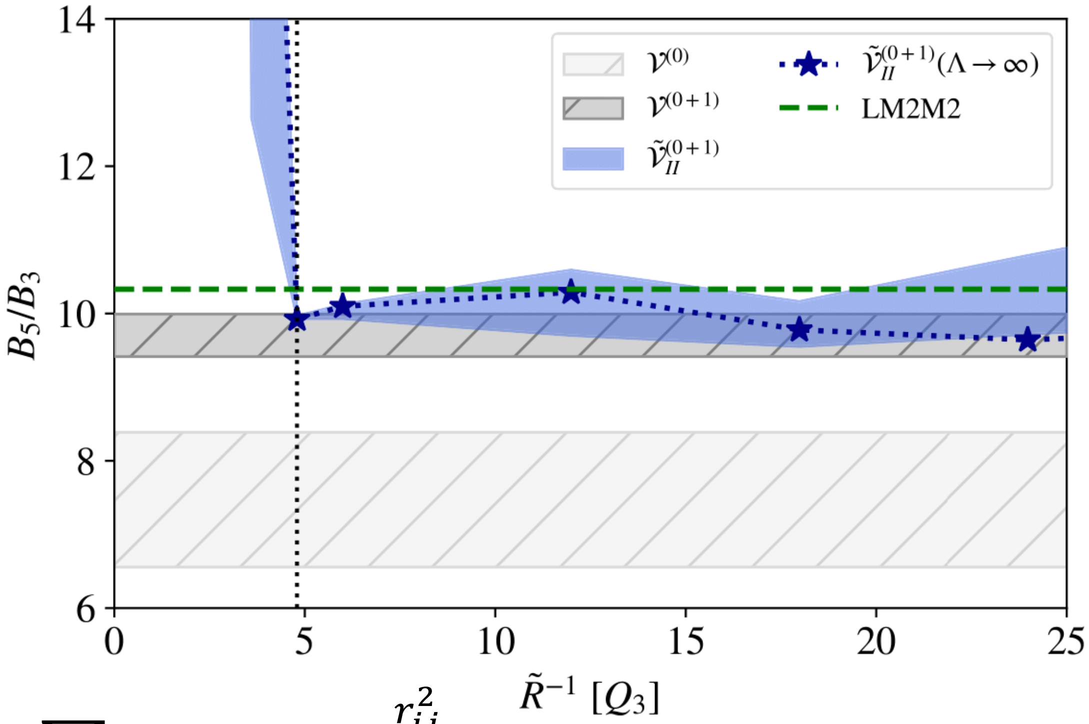
S. Zeller *et al.*, Imaging the He_2 quantum halo state using a free electron laser, Proc. Nat. Acad. Sci. **113**, 4651 (2016), arXiv:1601.03247 [physics.atom-ph].

Few-body sector (NLO)

$\bar{R}^{-1}$ is the parameter that controls the resummation
Λ is the theory cutoff that should go to "infinity"

LO	$\tilde{V}_I = \delta_{\bar{R}^{-1}}(r_{ij}) + \delta_\Lambda(r_{ijk})$
LO	$\tilde{V}_{II} = \delta_{\bar{R}^{-1}}(r_{ij}) + \delta_{\bar{R}^{-1}}(r_{ijk})$
NLO	$\nabla^2\delta_\Lambda(r_{ij})$ (the same using $r^2\delta_\Lambda(r_{ij})$)





$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0^*(\bar{R}^{-1})\, e^{-\frac{r_{ij}^2}{4\bar{R}^2}} + \sum_{ijk} D_0^*(\bar{R}^{-1}) \sum_{cyc}\left[e^{-\frac{(r_{ij}^2+r_{ik}^2)}{4\bar{R}^2}}\right]$$

5B ground state

Relevant fake ranges $\bar{R}$: $\bar{R} > 6\ Q_3$

$Q_3 = \sqrt{\frac{2}{3} m\, B_3}$

Few-body sector (LO)

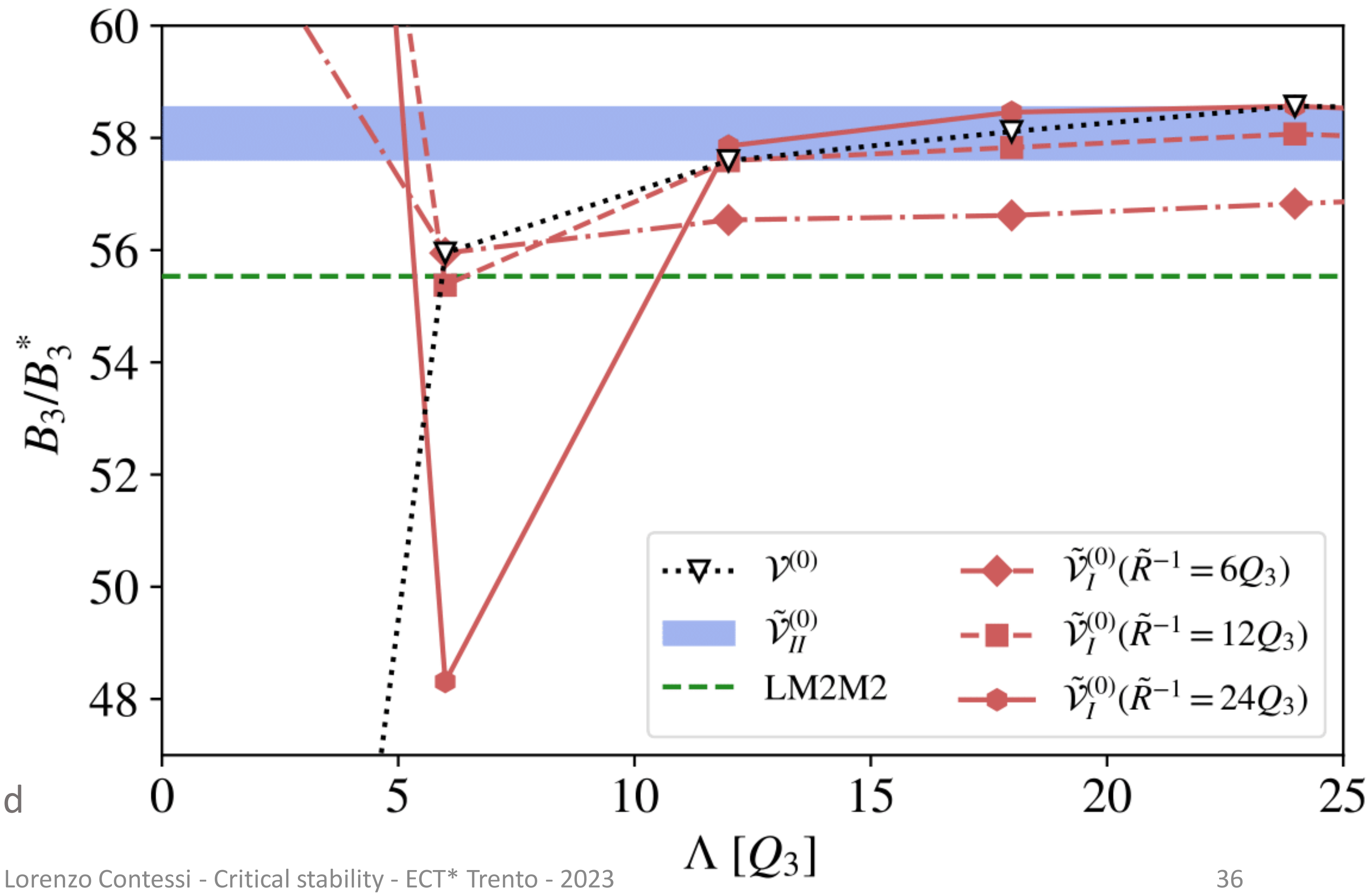
LO	$\tilde{V}_I = \delta_{\bar{R}^{-1}}(r_{ij}) + \delta_{\Lambda}(r_{ijk})$
LO	$\tilde{V}_{II} = \delta_{\bar{R}^{-1}}(r_{ij}) + \delta_{\bar{R}^{-1}}(r_{ijk})$
NLO	$\nabla^2 \delta_{\Lambda}(r_{ij})$

$\bar{R}^{-1}$ is the parameter that controls the resummation
Λ is the theory cutoff that should go to "infinity"

3B excited state

Relevant fake ranges $\bar{R}$: $\bar{R} > 6\ Q_3$

$$Q_3 = \sqrt{\frac{2}{3} m\ B_3}$$



Legend:
White triangles: the regular LO
Red lines: improved LO w/ 2Body
Blue band: improved LO 2+3 Body
Green Line (white circle): physical value
Vertical dashed line represents the r_0 treshold

Few-body sector (LO)

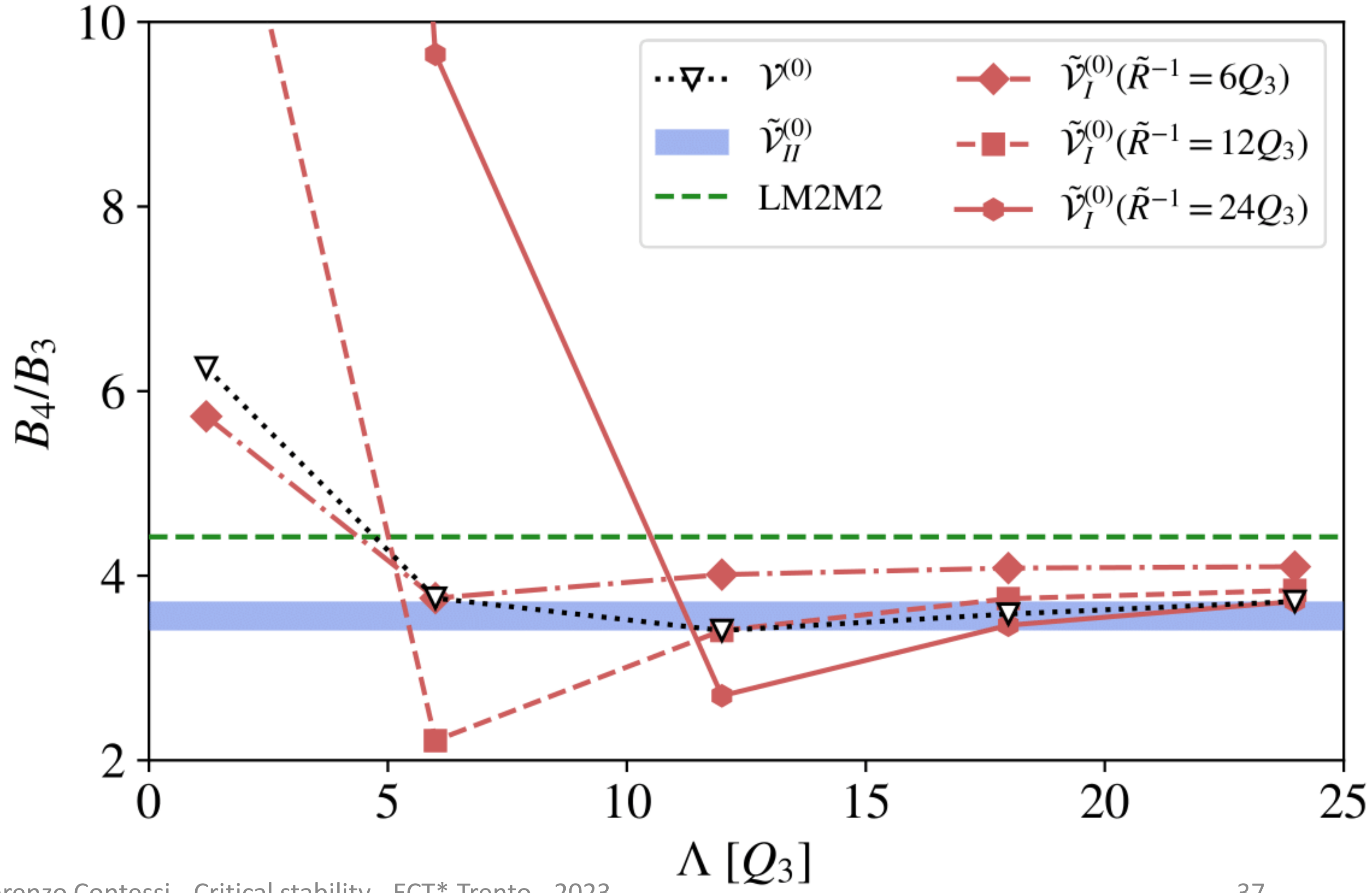
LO	$\tilde{V}_I = \delta_{\bar{R}^{-1}}(r_{ij}) + \delta_\Lambda(r_{ijk})$
LO	$\tilde{V}_{II} = \delta_{\bar{R}^{-1}}(r_{ij}) + \delta_{\bar{R}^{-1}}(r_{ijk})$
NLO	$\nabla^2 \delta_\Lambda(r_{ij})$

$\bar{R}^{-1}$ is the parameter that controls the resummation
Λ is the theory cutoff that should go to "infinity"

4B ground state

Relevant fake ranges $\bar{R}$: $\bar{R} > 6\ Q_3$

$$Q_3 = \sqrt{\frac{2}{3} m\ B_3}$$



Legend:
White triangles: the regular LO
Red lines: improved LO w/ 2Body
Blue band: improved LO 2+3 Body
Green Line (white circle): physical value
Vertical dashed line represents the r_0 treshold

Few-body sector (LO)

$\bar{R}^{-1}$ is the parameter that controls the resummation
Λ is the theory cutoff that should go to "infinity"

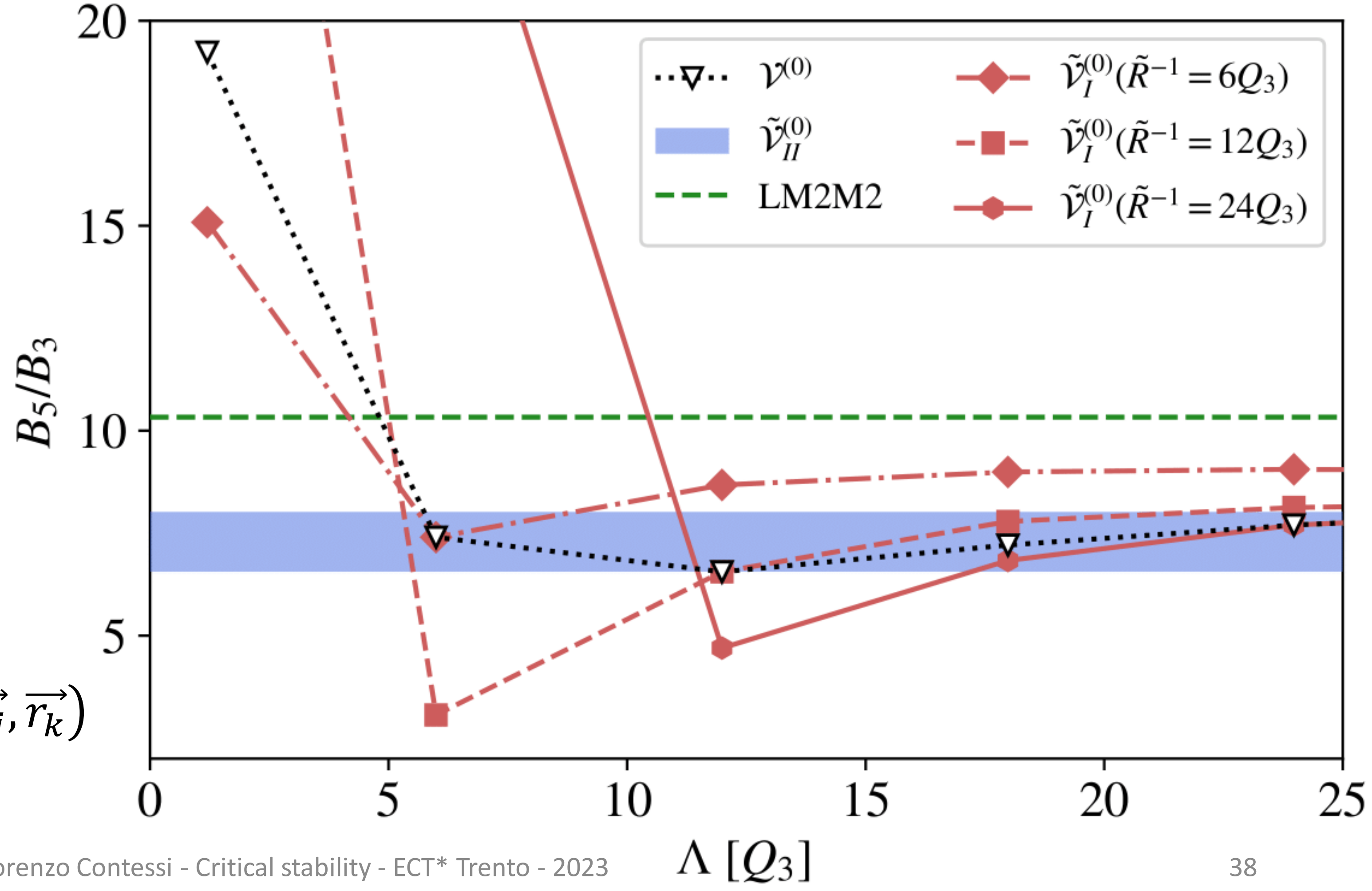
LO	$\tilde{V}_I = \delta_{\bar{R}^{-1}}(r_{ij}) + \delta_{\Lambda}(r_{ijk})$
LO	$\tilde{V}_{II} = \delta_{\bar{R}^{-1}}(r_{ij}) + \delta_{\bar{R}^{-1}}(r_{ijk})$
NLO	$\nabla^2 \delta_{\Lambda}(r_{ij})$

5B ground state

Relevant fake ranges $\bar{R}$: $\bar{R} > 6\ Q_3$

$$Q_3 = \sqrt{\frac{2}{3} m\, B_3}$$

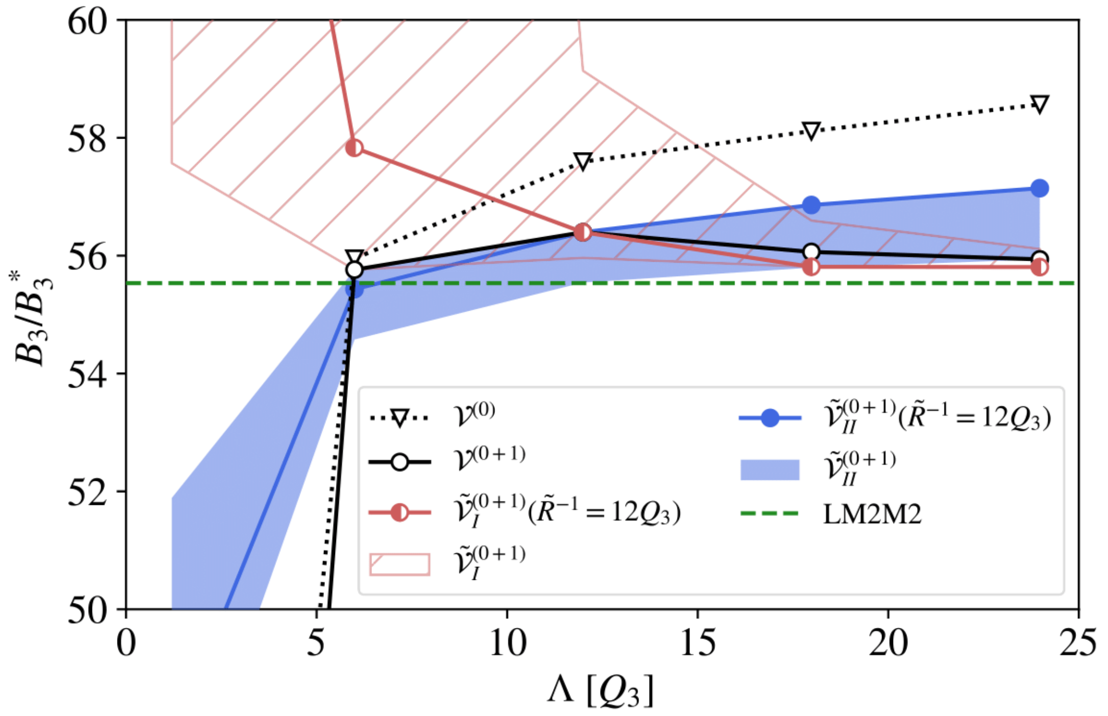
$$H^{LO} = -\frac{\hbar^2}{2m}\sum_i \nabla^2 + \sum_{ij} C_0^*(\bar{R}^{-1})\, e^{-\frac{r_{ij}^2}{4\bar{R}^2}} + \sum_{ijk} D_0\, \delta_\Lambda\left(\vec{r_i}, \vec{r_j}, \vec{r_k}\right)$$

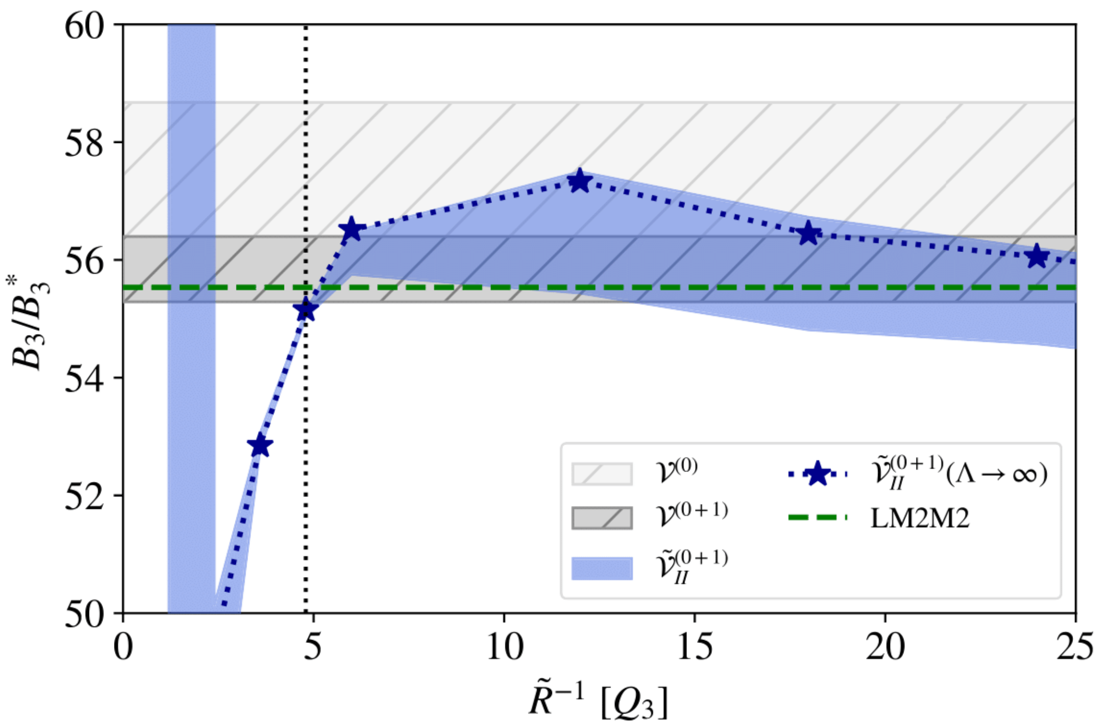


Few-body sector (NLO)

$\bar{R}^{-1}$ is the parameter that controls the resummation
Λ is the theory cutoff that should go to "infinity"

LO	$\tilde{V}_I = \delta_{\bar{R}^{-1}}(r_{ij}) + \delta_\Lambda(r_{ijk})$
LO	$\tilde{V}_{II} = \delta_{\bar{R}^{-1}}(r_{ij}) + \delta_{\bar{R}^{-1}}(r_{ijk})$
NLO	$\nabla^2\delta_\Lambda(r_{ij})$ (the same using $r^2\delta_\Lambda(r_{ij})$)





3B excited state

Legend:

White triangles:	the regular LO
Red lines:	improved LO w/ 2Body
Blue band:	improved LO 2+3 Body

Green Line (white circle): physical value
Vertical dashed line represents the r_0 treshold

Relevant fake ranges $\bar{R}$: $\bar{R} > 6\ Q_3$

$$Q_3 = \sqrt{\frac{2}{3} m\, B_3}$$