Rearrangement collisions
and
inter-cluster potentials
in the zer0-range limit

''Effective Field theory and Strong interaction with accurate error estimation''

12.4.2024 – CEA-ESNT

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$\otimes$

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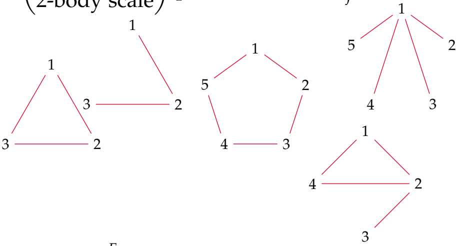
The problem: What $\begin{pmatrix}\text{few-body}\\ \text{non-relativistic}\\ \text{quantum}\end{pmatrix}$ - complexity can a

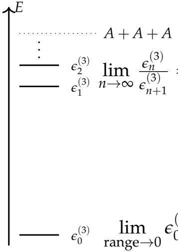
"The mind is localized and extends forever to infinity. The body is extended and remains localized." (R. Descartes)

universal/most versatile/elemental $\hat{=}$ $\begin{pmatrix}\text{no}\\ \text{2-body scale}\end{pmatrix}$ pair interaction v_{ij} evoke?

$$H = \sum_{i=1}^{A} t_i + \sum_{\{i,j\}\in\mathcal{I}} v_{ij}$$



$$v_{ij} = \begin{cases} c_0 \cdot \delta(|\boldsymbol{r}_{\rm ij}|) \\ c_0 \cdot e^{-c_1 \boldsymbol{r}_{\rm ij}^2} \\ c_0 \cdot |\boldsymbol{r}_{\rm ij}|^{-1} \cdot e^{-c_1|\boldsymbol{r}_{\rm ij}|} - c_3 \cdot |\boldsymbol{r}_{\rm ij}|^{-6} \\ c_0 \cdot |\boldsymbol{r}_{\rm ij}|^{-12} + c_1 \cdot |\boldsymbol{r}_{\rm ij}|^{-6} \\ \vdots \end{cases} \Rightarrow$$



$$\epsilon_2^{(3)},\ \epsilon_1^{(3)}: \quad \lim_{n\to\infty} \frac{\epsilon_n^{(3)}}{\epsilon_{n+1}^{(3)}} = \text{constant}(\mathcal{I})$$ *(V. Efimov, '70)*

$$\epsilon_0^{(3)}: \quad \lim_{\text{range}\to 0} \epsilon_0^{(3)} = \infty$$ *(L.H. Thomas, '35)*

A nuclear few-body theory.[1]

$$\text{EFT}(\not\pi) \;=\; \begin{matrix}\text{Degrees}\\ \text{of freedom}\end{matrix} \;\oplus\; \text{Symmetries} \;\oplus\; \begin{matrix}\text{Naïve dim.}\\ \text{analysis}\end{matrix} \;\oplus\; \begin{matrix}\text{“Constructive”}\\ \text{scales}\end{matrix}$$

[1] P. F. Bedaque, J.-W. Chen, H. W. Hammer, D. B. Kaplan, U. van Kolck, G. Rupak, M. J. Savage (199x-201y)

A UNIVERSAL FEW-BODY THEORY.[1]

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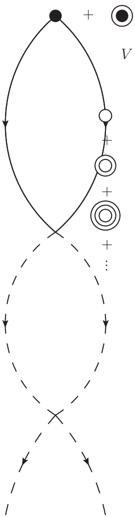
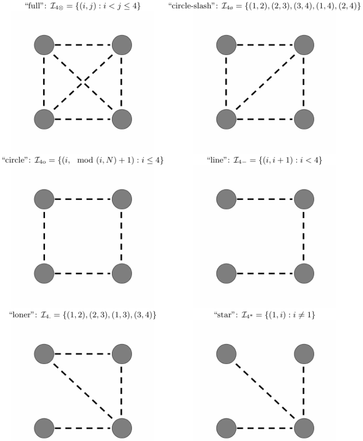


$V = C_0 + C_{0,0} + C_2 q^2 + \dots$

$\leftrightarrow \frac{\vec{\nabla}^2}{2m} + \frac{\vec{\nabla}^4}{8m^3} + \dots$



Unitary interaction graphs

M. P. Valderrama, L. Contessi, JK (2021)

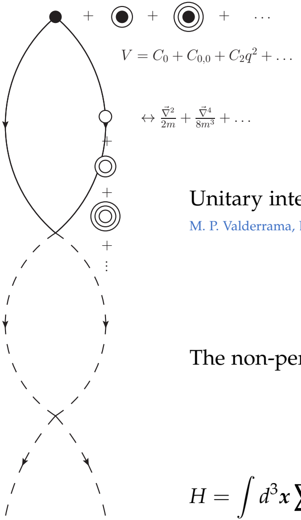
The non-perturbative leading order:

$j \in \{V_{ij} : |a_{ij}| \to \infty\}$

$$H = \int d^3\boldsymbol{x} \sum_i \left[\psi_i^\dagger \left(-\frac{\boldsymbol{\nabla}^2}{2m} \right) \psi_i + C \cdot \sum \psi_i^\dagger \psi_j^\dagger \psi_i \psi_j + D \cdot \sum \psi_i^\dagger \psi_j^\dagger \psi_k^\dagger \psi_i \psi_j \psi_k \right]$$

$j, k \in \{\geq 2 \text{ scattering lengths} \to \infty\}$

A universal few-body theory.



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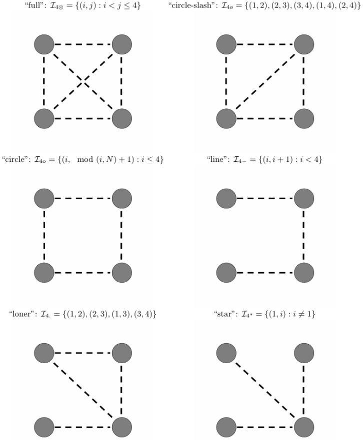

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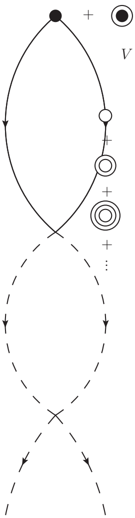

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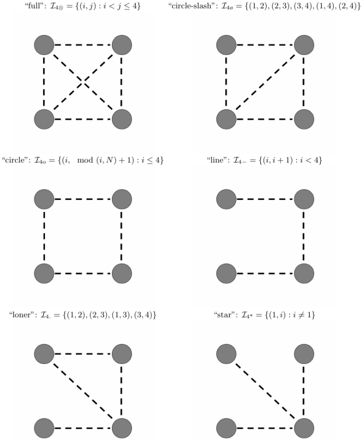

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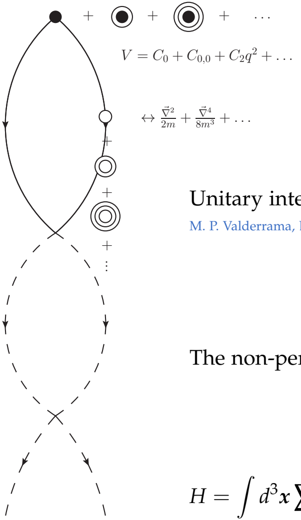

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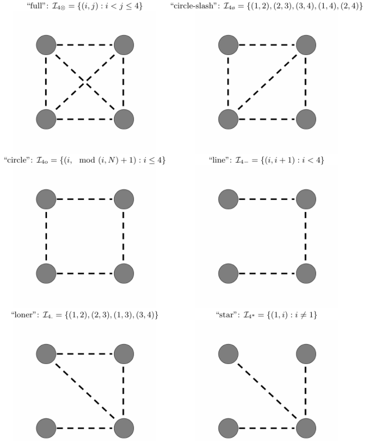

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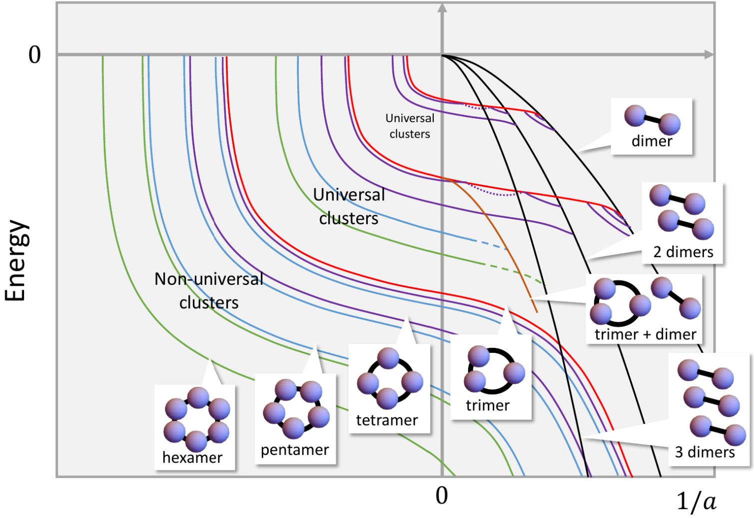
An open problem for few-/many-body theories:

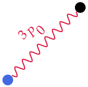
3+n spectra near/in the unitary limit.



P. Naidon and S. Endo, Efimov Physics: a review (2017)

The Refined[1] Resonating Group Method[2]

THE ATOMIC NUCLEUS AS A SET OF MOLECULES.



$$\Psi = \hat{\mathcal{A}} \left\{ \sum_i \phi(A_i)\phi(B_i)F_i(\boldsymbol{R}_i) + \sum_j \phi(A_j)\phi(B_j)\phi(C_j)F_j(\boldsymbol{R}_{1j}, \boldsymbol{R}_{2j}) \right.$$

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$$\left. + \ldots + \sum_m c_m \chi_m \right\} Z(\boldsymbol{R}_{c.m.})$$

$$\delta\Psi \stackrel{!}{=} \delta F$$

$$\Rightarrow \quad \left(-\frac{\hbar^2}{2\mu} \nabla^2_{\boldsymbol{R}'} + V_{\text{direct}}(\boldsymbol{R}') - E \right) F(\boldsymbol{R}') + \int K(\boldsymbol{R}', \boldsymbol{R}'') F(\boldsymbol{R}'') d\boldsymbol{R}'' = 0$$

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[2] J. A. Wheeler PR 52 (1937)

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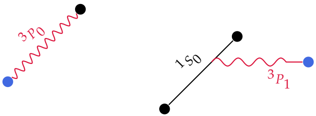

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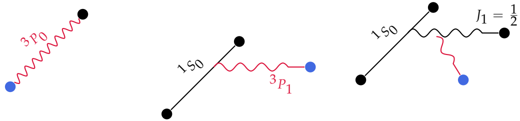

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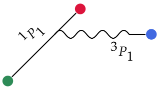

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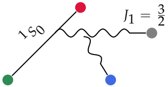

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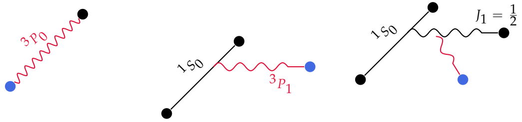

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The Refined Resonating Group Method

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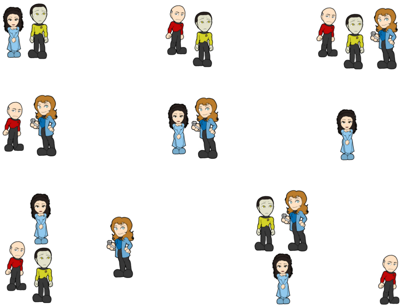
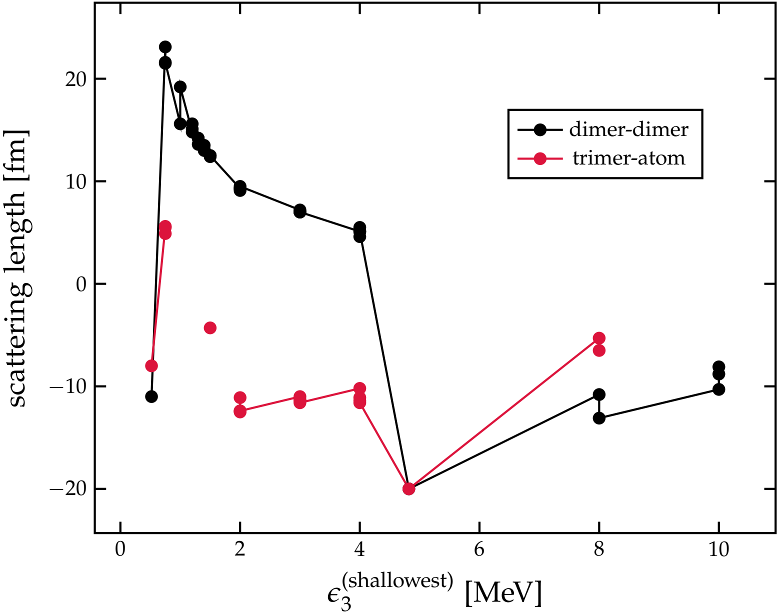


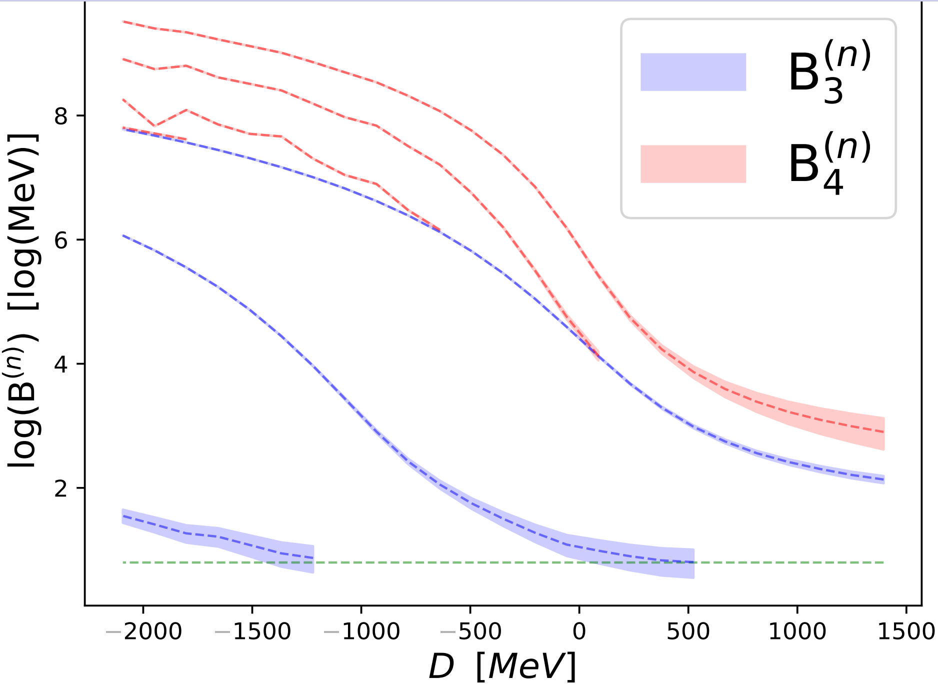
John Wheeler's idea (1937):

[...] It was as if, at a party, all the tall people clustered together at one moment, with all the short people in another cluster; then at the next moment [...] four groups formed, consisting of guests from the north, east, west, and south parts of the city; and so on, [...]

4-DISTINGUISHABLE-PARTICLE SCATTERING ($J^\pi = 0^+$)

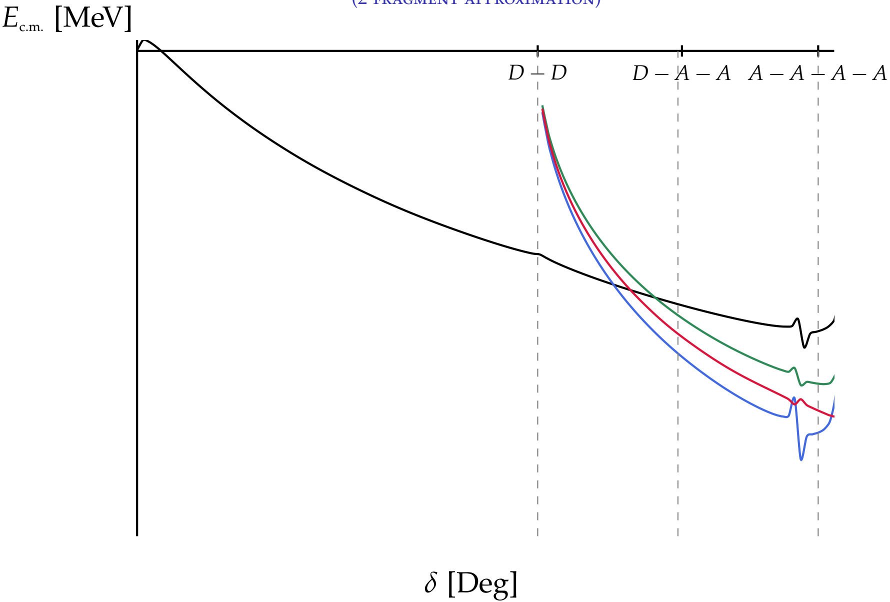
FOR (ISO)SPIN INDEPENDENT V_{ij} AND $B(2) = 0.5$ MEV AND $m \approx 1$ GEV AND $4\ \text{fm}^{-2} \lesssim \lambda \lesssim 10\ \text{fm}^{-2}$




$B_3^{(n)}$
$B_4^{(n)}$
8
6
4
2
$\log(B^{(n)})$ [log(MeV)]
−2000
−1500
−1000
−500
0
500
1000
1500
D [MeV]

The unitary 4-body scattering problem

(2-fragment approximation)



INTER-CLUSTER POTENTIALS FROM CONSTITUENTS.

$\overset{?}{=}$ MATCHING EFT'S WITH DIFFERENT DOF'S

cluster degree(s) of freedom

$\equiv$

element of the spectrum of the EFT at the desired order:

$$\hat{H}_{\mathrm{EFT}}\phi_A^{(n)} = e_A^{(n)}\phi_A^{(n)}$$

e.g. contact leading order:

$$\hat{V} = C_0(\Lambda)\sum_{i<j}^{X}\delta_\Lambda^{(3)}(\boldsymbol{r}_\mathrm{i}-\boldsymbol{r}_\mathrm{j}) + D_1(\Lambda)\sum_{\substack{i<j<k\\ \text{cyclic}}}^{X}\delta_\Lambda^{(3)}(\boldsymbol{r}_\mathrm{i}-\boldsymbol{r}_\mathrm{j})\,\delta_\Lambda^{(3)}(\boldsymbol{r}_\mathrm{j}-\boldsymbol{r}_\mathrm{k})$$

retain in(ter) cluster indistinguishability:

$$\hat{\mathcal{A}} = \hat{\mathcal{A}}_{AB}\hat{\mathcal{A}}_A\hat{\mathcal{A}}_B = \mathbb{1} + \sum_{P\in S_{A+B}}\mathrm{sign}(P)\hat{P}$$

variation in the relative motion **between** the composites:

$$\left(\hat{T}_{\boldsymbol{R}} - E_{\mathrm{rel}} + \mathbb{N}^{-1}\langle\phi_A\phi_B|\hat{V}|\phi_A\phi_B\rangle\right)\chi(\boldsymbol{R})$$
$$-\mathbb{N}^{-1}\int d\boldsymbol{R}'\left[\langle\phi_A\phi_B|\,(\hat{T}_{\boldsymbol{R}} - E_{\mathrm{rel}} + \hat{V})\,\hat{P}\{|\phi_A\phi_B\rangle\delta_\Lambda^{(3)}(\boldsymbol{R}-\boldsymbol{R}')\right]\chi(\boldsymbol{R}') = 0$$

Inter-cluster potentials from constituents.

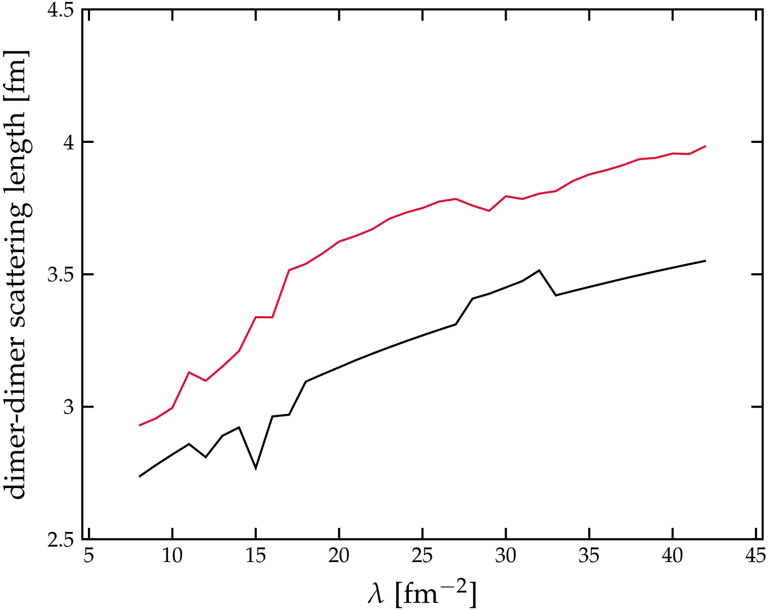
$\stackrel{?}{=}$ matching EFT's with different DoF's

$$\sum_{n=1}^{N_{\text{loc}}} \hat{\eta}_n \, e^{-w_n \boldsymbol{R}^2} \chi(\boldsymbol{R}) - \sum_{n=1}^{N_{\text{n-loc}}} \int \left\{ \hat{\zeta}_n \, e^{-a_n \boldsymbol{R}^2 - b_n \boldsymbol{R}\cdot\boldsymbol{R}' - c_n \boldsymbol{R}'^2} \right\} \chi(\boldsymbol{R}') d\boldsymbol{R}' = 0$$

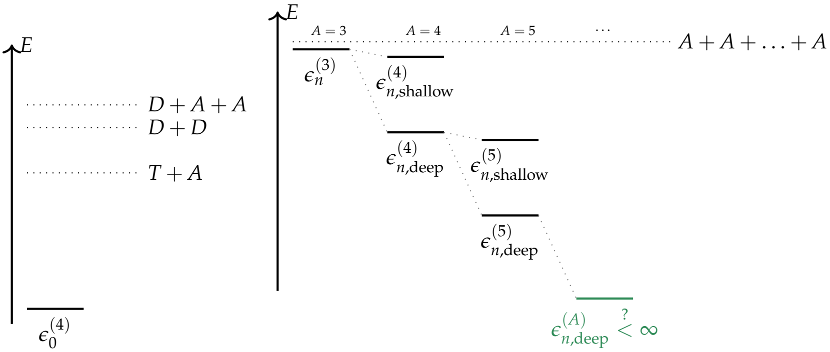
with $\hat{\eta}_n, \hat{\zeta}_n, w_n, a_n, b_n, c_n$ dependent upon $C(\lambda), D(\lambda), \alpha, \lambda, E, A, B$.

Dimer-Dimer scattering – DoF: dimer

universal/regulator-independent wave function $B(2) \approx 2.2$ MeV



Understanding correlations between **few-** and **(few+n)**-body systems



Some replied that happiness was a complex artifact and that man's aim lay not in happiness but in the zealous fulfillment of historical laws. And others said that happiness was a matter of out-and-out struggle, which would last eternally.

(Andrei Platonov, "Chevengur")