Inclusive and semi-inclusive lepton nucleus scattering in quasielastic region and beyond

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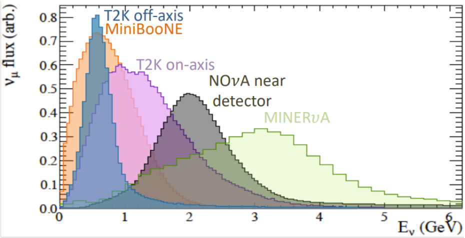
Outline

- Introduction
- Quasi-Elastic
  - Nuclear models: SF, SuSA, HF, RPA
  - Comparison between models *V.Belocchi et al., arXiv 2310.02007 [nucl-th]*
- Meson exchange currents
  - Inclusive process
  - Semi-inclusive process *V.Belocchi et al., arXiv 2401.13640 [nucl-th]*
- Future developments

Neutrinos:

- Light fermions that interact through Weak interaction only $\Rightarrow$ Very low signals, heavy target needed: nuclei!
- Flavour Oscillation: mass eigenstate $\neq$ flavour eigenstate

Neutrino experiments want to study the properties of this particle, and extract information on the **Oscillation Matrix**, especially on the **CP violating phase**.

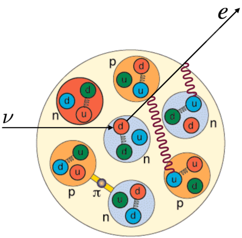


Incident neutrino fluxes distribution for several experiments

Experiments measure the number of events. In a neutrino oscillation experiment:

$$N_{\nu_\beta}(\overline{E_\nu}) \sim \int dE_\nu \Phi_{\nu_\alpha}(E_\nu) P_{\nu_\alpha \to \nu_\beta}(E_\nu) \sigma(E_\nu) \epsilon_{det} d(E_\nu, \overline{E_\nu})$$

Reconstructed energy $\overline{E_\nu} \Leftrightarrow E_\nu$ True neutrino energy



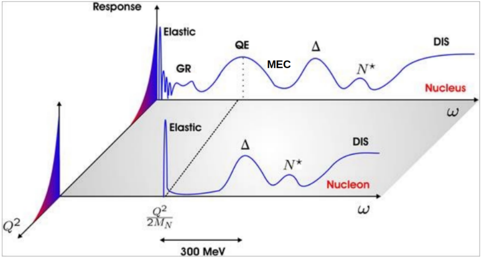
Nucleus is a very rich and complex target, composed by

- Nucleons, not elementary particles
- Mesons, that can be considered as the mediators of nuclear interaction

Lepton-Nucleus interaction: several processes

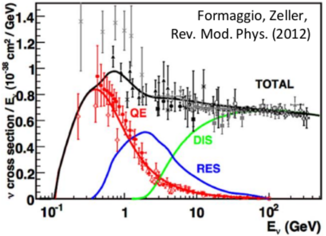
Nuclear Effects:
Free Nucleon $\rightarrow$ Nucleus

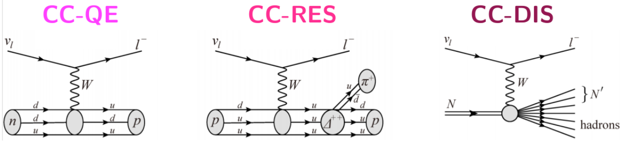
- Broadening of QE, Fermi motion $\rightarrow$ Initial hadronic state from nuclear model
- Pauli Blocking PB, Final State Interactions FSI
- Multinucleon excitations: **2p-2h**



Channels Overlap

At a given E_ν, several channels are active



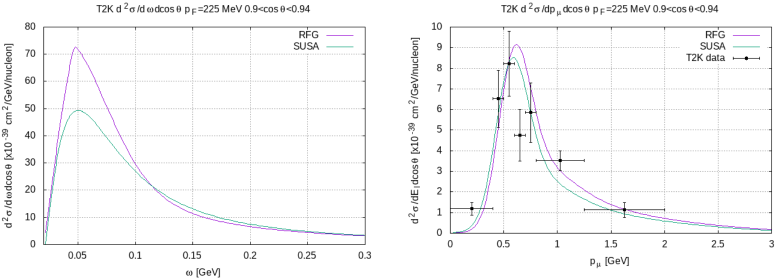


It's possible to determine the channel and to reconstruct the interaction vertex? NO!

- We don't know the initial hadronic state $\rightarrow$ nuclear model
- Incident flux wide in energy: we don't know initial E_ν
- Outgoing hadron particles are affected by final state interactions

Importance of electron scattering for neutrino processes

Here flux integrated CCQE neutrino-carbon cross-section is reported, with the T2K near detector ν_μ flux.



Flux integration hides very different behaviour of adopted nuclear model

$\Downarrow$

Electron-nucleus scattering

- No incident energy reconstruction problem
- Easy four-momentum transfer reconstruction
- Numerous high precision data available

$\Rightarrow$ **Channels separation**

Purpose

Motivation:

- Deeper comprehension of QE and 2p2h processes starting from electron-nucleus scattering
- Inputs and theoretical computations to improve the accuracy for neutrino experiments

  Neutrino physics community is recently focusing on semi-inclusive processes (hadrons detected in the final state), a lot of data available and coming soon

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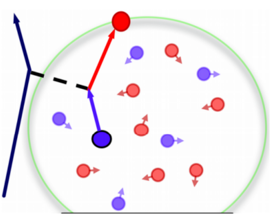
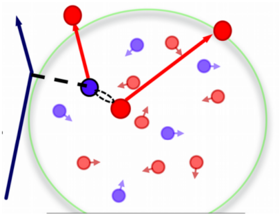
- Build a general framework capable to compute Inclusive and Semi-Inclusive cross sections starting from microscopic computations, for QE and 2p2h channels





EM Cross-Section

The general EM cross-section formula is:

$$\frac{\mathrm{d}\sigma}{\mathrm{dE_{k'}\,d\Omega_{k'}}} = \underbrace{\frac{\alpha^2}{Q^2}\frac{|\mathbf{k}'|}{E_k}}_{\sigma_{Mott}} \nu_0 (2\pi)^3 \frac{L_{\mu\nu}}{\nu_0} W_A^{\mu\nu}$$

Nuclear Hadronic Tensor: described using a Nuclear Model

$$W_A^{\mu\nu} := \sum_X \langle A| \left(J_A^\mu\right)^\dagger |X\rangle \langle X| J_A^\nu |A\rangle \, \delta^4 \left(M_A + q - P_X\right)$$

If the sum is performed over every possible final state X, W_A describes an inclusive process

QE

QE: relatively easy to describe, relies on lepton-nucleon interaction

Comparing nuclear models: why?

- Discrepancies between models
- ν physics requires better accuracy: too high uncertainties also in QE cross-section

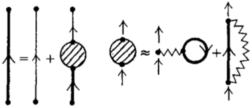
$\Rightarrow$ **Comparison between models can be extremely useful**

Studied models:

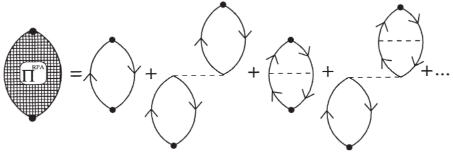
- RFG: very simple but relativistic model
- Spectral Function SF: MF with short range NN correlations (Rome SF)
- SuperScaling Approach SuSA: extracts nuclear features from electron data via a fit
- Hartree-Fock HF: MF to describe the nucleus, based on quantum theory of many body systems
- Random Phase Approximation RPA: describe the probe propagation inside the nuclear medium

Developed RPA scheme:

- Based on HF, nuclear matter approximations, non relativistic meson potentials *M.B. Barbaro et al, Nucl. Phys. A 596 (1996)* , *R. Machleidt et al, Phys. Rep. 149 (1987)*
- Computations of **Polarization Tensor**,
  p-h pairs propagation under an interaction potential



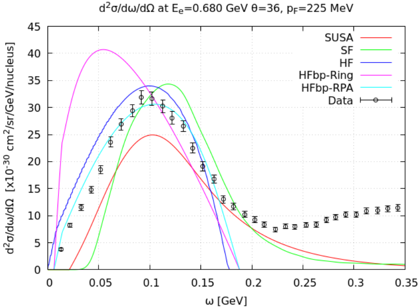
HF Dyson equation



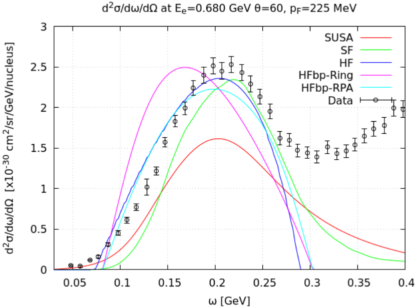
Two approaches:

- Ring approximation: resummation of direct diagrams only
- Antisymmetyc (aRPA): direct and exchange diagrams $\rightarrow$ **continued fraction expansion**
  A. De Pace, Nucl.Phys. A 635 (1998)

Some electron-carbon cross-sections are shown, at several kinematics. Data from archive *nucl-ex/0603032*



$q_{\mathrm{QEP}} = 400$ MeV

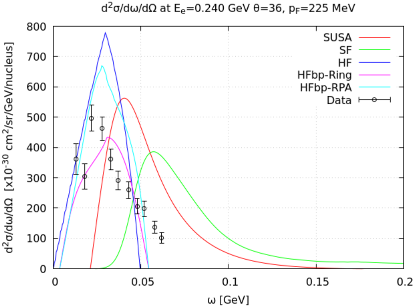


$q_{\mathrm{QEP}} = 600$ MeV

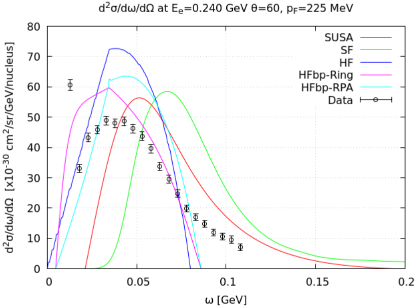
- SuSA underestimates the cross-section, but has a better shape
- RPA is in good agreement with data
- SF overestimates the cross-section, the peak is shifted at larger energy
- Ring is not reproducing the data: antisymmetrization is needed
- **At $\omega > 0.2 GeV$ missing strength: MEC start to contribute!**

Results and comparisons

Low kinematics, PB and FSI effects



$q_{\mathrm{QEP}} = 140$ MeV

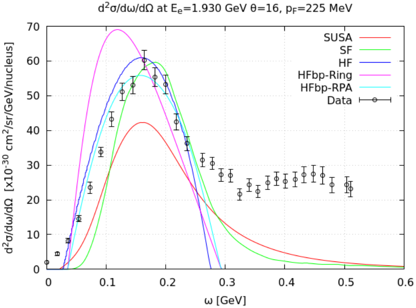


$q_{\mathrm{QEP}} = 220$ MeV

- HF and HF-based RPA, Ring reproduces peak position
- SuSA and SF are not adequate at $q < 300$ MeV
- HF, HF-RPA overestimate the cross-section
- HF-ring seems to partially account for giant resonances

Results and comparisons

Low angles, forward scattering



$q_{\text{QEP}} = 540$ MeV

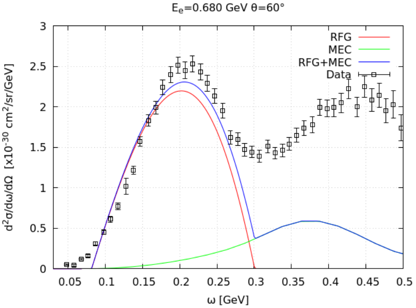
- General agreement in the peak position
- SuSA underestimates cross-section (corrected in SuSAv2)
- General lack of strength at very low ω

Summary

Summary

- Several nuclear models for QE are analyzed: the aim is not perfect agreement with data -at least for the moment!-, but the possibility to compare models in the same framework (form factors, nuclear tensor description...)
- HF, RPA are more appropriate at low momentum transfer, where IA-based approches are not
- This RPA calculation is among the few that include explicitly exchange diagrams. Others account for them by adding effective terms to the ring approach

Increasing the transferred energy, two nucleons can be emitted: **2p-2h**



Features:

- Leptonic probe interacts with a nucleon in a correlated pair of nucleons exchanging mediator (III order process, not FSI)
- Leptonic probe interacts directly with the exchanged virtual meson between two nucleons, exciting them both

$$\Downarrow$$

Meson Exchange Current: two-body current

Nuclear EW Lagrangian

Effective Field Theory to describe nucleons-mesons interaction:
E. Hernandez et al, Mod.Phys.Lett.A 23 (2008)

Non-Linear σ-model

$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - M)\Psi + \frac{1}{2}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - \frac{1}{2}m_\pi^2\vec{\phi}^2 + \frac{g_A}{f_\pi}\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}(\partial_\mu\vec{\phi})\Psi + \mathcal{O}(1/f_\pi^2)$$

- Provides a description of a system composed by nucleons and pions
- Nucleon-pion vertex is dominated by pseudo-vector interaction
- πNN coupling is $\frac{g_{\pi NN}}{2m_N} = \frac{g_A}{2f_\pi}$ using the Goldberger-Treiman relation
- Ψ: nucleon, isospin doublet
- $\vec{\phi}$: pionic field, scalar isospin triplet, $\pi^\pm = \frac{1}{\sqrt{2}}(\phi_1 \pm i\phi_2)$
- $\vec{\tau}$: isospin Pauli matrices, $\tau^\pm = \tau_1 \pm i\tau_2$

Physical pions in the Lagrangian:

$$\frac{\vec{\tau}}{2}\vec{\phi} = \frac{1}{\sqrt{2}}(\tau^+\pi^- + \tau^-\pi^+) + \frac{\tau_3}{2}\pi_0$$

EW coupling

To 'switch on' the EW interaction in the previous Lagrangian, the standard $SU(2)_L \times U(1)_Y$ local symmetry procedure is performed, with associated gauge bosons $\vec{W}^\mu$ and B^μ, followed by the physical separation between weak and EM interactions

Covariant derivatives

$$\text{Fermions:} \quad D^\mu = \partial^\mu + ig\frac{\vec{\tau}}{2}\vec{W}^\mu + ig' YB^\mu$$

$$\text{Pions:} \quad \mathcal{D}^\mu \phi_i = \partial^\mu \phi_i - g\epsilon_{ijk}\phi_j W_k^\mu$$

with

$$\frac{\vec{\tau}}{2}\vec{W}_\mu = \frac{1}{\sqrt{2}}(\tau^+ W_\mu^- + \tau^- W_\mu^+) + \frac{\tau_3}{2}W_{3\mu} \quad W_\mu^\pm = \frac{1}{\sqrt{2}}(W_{1\mu} \pm iW_{2\mu})$$

EM interaction

$$ig\frac{\vec{\tau}}{2}\vec{W}^\mu + ig' YB^\mu \quad \rightarrow \quad ieA^\mu$$

$$g\epsilon_{ij3}W_3^\mu \quad \rightarrow \quad e\epsilon_{ij3}A^\mu$$

Δ-MEC Lagrangian

W^μ / A^μ $N\,\Delta$ transition

$$\mathcal{L} = g\bar{\Psi}_\mu \vec{T}^\dagger \vec{W}^\mu \Psi + h.c. \quad \rightarrow \quad \mathcal{L}_{EM} = e\bar{\Psi}_\mu T_3^\dagger A^\mu \Psi + h.c.$$

$N\,\Delta\,\pi$ transition

$$\mathcal{L} = \sqrt{\frac{3}{2}} \frac{f^*}{m_\pi} \bar{\Psi}_\mu \vec{T}^\dagger \partial^\mu \vec{\phi} \Psi + h.c.$$

- Ψ_μ : Rarita-Schwinger $\frac{3}{2}$-spinor, with four isospin indices (μ) and four Dirac indices -omitted-
- $T^\dagger$: $\frac{1}{2} \rightarrow \frac{3}{2}$ isospin transition 4x2 operator

$$T_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{3} & 0 & 1 & 0 \\ 0 & -1 & 0 & \sqrt{3} \end{pmatrix} \; T_2 = -\frac{i}{\sqrt{6}} \begin{pmatrix} \sqrt{3} & 0 & 1 & 0 \\ 0 & 1 & 0 & \sqrt{3} \end{pmatrix} \; T_3 = \sqrt{\frac{2}{3}} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$T_i T_j^\dagger = \frac{2}{3}\delta_{ij} - \frac{i}{3}\epsilon_{ijk}\tau_k$$

Δ Propagator

In the MEC currents, Δ appears always as a virtual particle

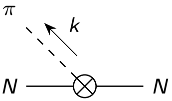
Δ propagator

$$G_{\alpha\beta}(p) = \frac{\mathcal{P}_{\alpha\beta}(p)}{p^2 - M_\Delta^2 + iM_\Delta\Gamma_\Delta}$$

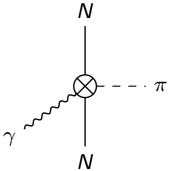
Where $\mathcal{P}_{\alpha\beta}$ is the projector over the physical states

$$\sum_{spin} u_\alpha(p)\overline{u}_\beta(p) = \mathcal{P}_{\alpha\beta}(p) = -(\not{p} + M_\Delta)\left[g_{\alpha\beta} - \frac{1}{3}\gamma_\alpha\gamma_\beta - \frac{2}{3}\frac{p_\alpha p_\beta}{M_\Delta} + \frac{p_\alpha\gamma_\beta - p_\beta\gamma_\alpha}{3M_\Delta}\right]$$

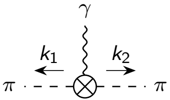
In the following the imaginary contribution to the responses arising from the Δ propagator is not included



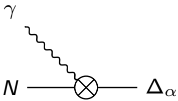
$i\frac{g_A}{2f_\pi}F_{\pi NN}(k^2)\vec{\tau}k^\mu$



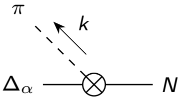
$$\frac{ie}{\sqrt{2}}\frac{g_A}{2f_\pi}F_1^V(q^2)\gamma^\mu\gamma_5(\tau_-\pi^+ - \tau_+\pi^-)$$



$eF_1^V(q^2)(k_1^\mu - k_2^\mu)(\pi^+\pi^- - \pi^-\pi^+)$



$eT_3^\dagger\Gamma^{\alpha\mu}$



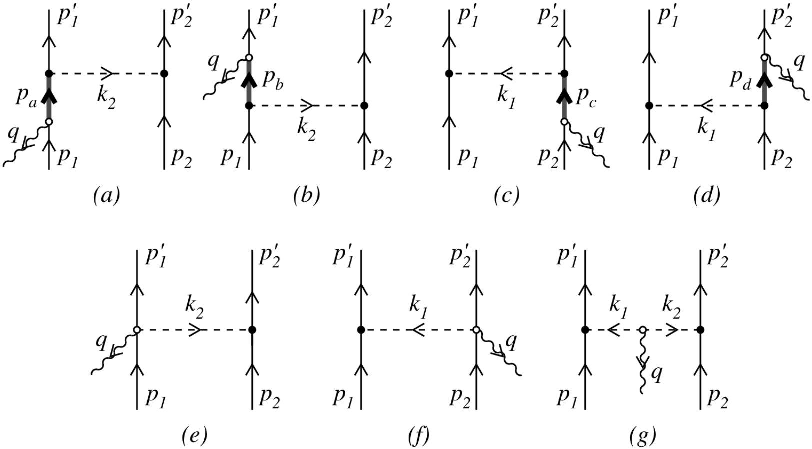
$i\sqrt{\frac{3}{2}}\frac{f^*}{m_\pi}F_{\pi N\Delta}(k^2)\vec{T}k^\alpha$

$\Gamma^{\alpha\mu}$ contains vector and axial form factors. EM case:

$$\Gamma_V^{\alpha\mu} = \Big[\overbrace{\frac{C_{3V}}{M}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu)}^{\text{dominant FF}} + \frac{C_{4V}}{M^2}(g^{\alpha\mu}qp_\Delta - q^\alpha p_\Delta^\mu) + \frac{C_{5V}}{M^2}(g^{\alpha\mu}qp - q^\alpha p^\mu)\Big]\gamma_5$$

Describe the possibility that a probe excites two holes into two particles: *De Pace et al, Nucl.Phys.A 726 (2003)*

- Two-body ElectroMagnetic Meson Exchange Currents J^{μ}_{2p2h}



RFG 2p2h

Inclusive hadronic tensor, no hadronic particles detected:

$$W^{\mu\nu}_{2p2h} = (2\pi)^3 \frac{V}{4} \int_{|\mathbf{p}|\leq p_F} \frac{m_N \mathrm{d}\mathbf{p_1}}{(2\pi)^3 E_{p1}} \frac{m_N \mathrm{d}\mathbf{p_2}}{(2\pi^3) E_{p2}} \frac{m_N \mathrm{d}\mathbf{p'_1}}{(2\pi)^3 E_{p1'}} \frac{m_N \mathrm{d}\mathbf{p'_2}}{(2\pi^3) E_{p2'}} \tilde{W}^{\mu\nu}_{2p2h} \delta^4 \{\theta_{PB}\}$$

Semi-inclusive hadronic tensor, one final proton detected (p'_1):

$$W^{\mu\nu}_{2p2h}(N'_1) = (2\pi)^3 \frac{V}{4} \int_{|\mathbf{p}|\leq p_F} \frac{m_N \mathrm{d}\mathbf{p_1}}{(2\pi)^3 E_{p1}} \frac{m_N \mathrm{d}\mathbf{p_2}}{(2\pi^3) E_{p2}} \frac{m_N \mathrm{d}\mathbf{p'_2}}{(2\pi^3) E_{p2'}} \tilde{W}^{\mu\nu}_{2p2h} \delta^4 \{\theta_{PB}\}$$

$$\tilde{W}^{\mu\nu}_{2p2h} = \sum_{\substack{spin \\ isospin}} \langle 2p2h| J^{\mu}_{2p2h} |F\rangle \langle F| J^{\nu\dagger}_{2p2h} |2p2h\rangle \qquad |2p2h\rangle = b^{\dagger}_{p'_2} b^{\dagger}_{p'_1} b_{p_1} b_{p_2} |F\rangle$$

- J^{μ}_{2p2h} is a two-body operator, acting on the spin, isospin and momentum space
- Is possible to invert the two particles, obtaining another current that must be included

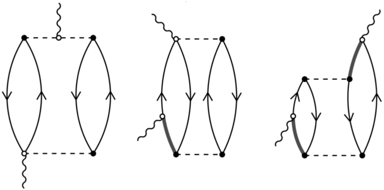
$$\downarrow$$

$$J^{\mu}_{2p2h} = J^{\mu}_{2p2h}(p_1, p_2, p'_1, p'_2) - J^{\mu}_{2p2h}(p_1, p_2, p'_2, p'_1)$$

MEC Polarization Tensors

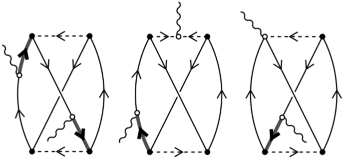
Combining two meson exchange currents → **Two possibilities!**

- Direct term: $J^{\mu} J^{\nu\dagger}(p_1, p_2, p_{1'}, p_{2'}) + J^{\mu} J^{\nu\dagger}(p_1, p_2, p_{2'}, p_{1'})$



3 examples of the 16 EM many body direct diagrams

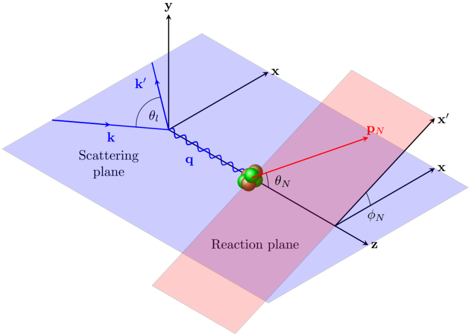
- Exchange term: $J^{\mu}(p_1, p_2, p_{1'}, p_{2'}) J^{\nu\dagger}(p_1, p_2, p_{2'}, p_{1'}) + \mu \leftrightarrow \nu$



3 examples of the 12 EM many body exchange diagrams

Inclusive 2p2h process

Inclusive calculation, fixing ω, q



- Integration over two particles and two holes momenta
- Four-momentum conservation
- q-system: nucleus symmetry $\rightarrow$ azimuthal invariance

$\Rightarrow$ 7 dimension integration

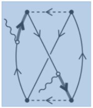
Non-vanishing responses

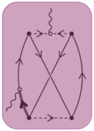
$$R_L = W^{00}_{2p2h} \qquad R_T = W^{11}_{2p2h} + W^{22}_{2p2h}$$

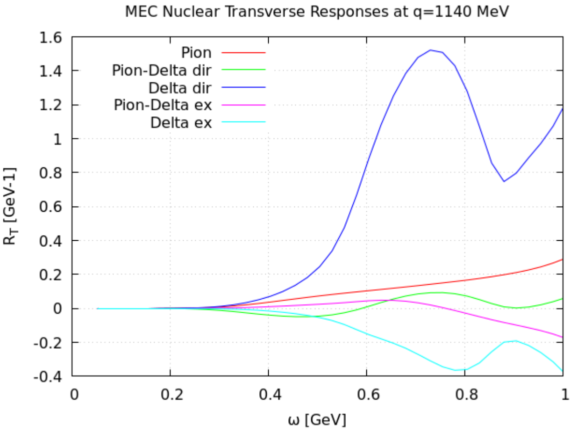
Electric charge conservation:
R_L includes contribution from W^{00}_{2p2h}, W^{03}_{2p2h}, W^{33}_{2p2h}

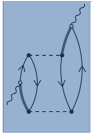
MEC Responses

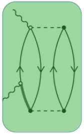
- RFG model in nuclear matter for Carbon target, $p_F = 228$ MeV
- Energy shift $E_s = 20$ MeV for each particle $\quad \rightarrow \quad E_s^{2p2h} = 2E_s$

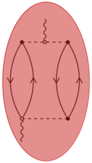






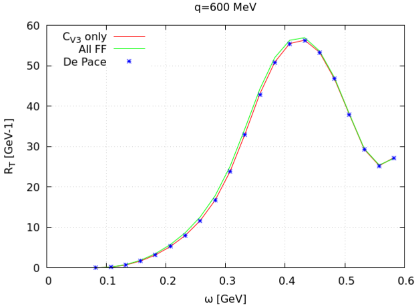


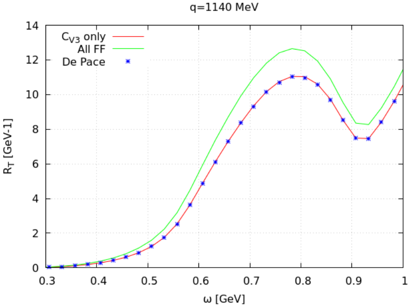




MEC Responses

We tested the results with previous work, for iron *De Pace et al, Nucl.Phys.A 726 (2003)*

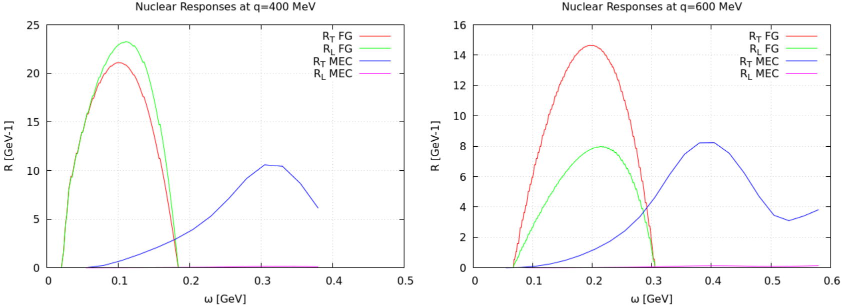




Delta Form Factors impact

- In *De Pace et al* only C_{3V} was included. Here all Δ form factors are included: responses increase, more relevant at high q-values

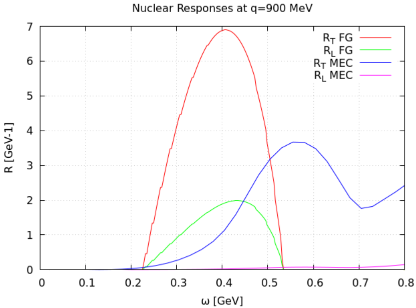
MEC Transverse and Longitudinal Nuclear Responses at several q-values:

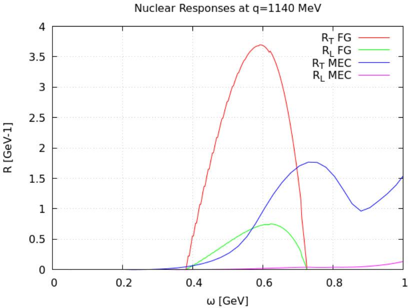


- EM MEC contribution is almost totally transverse with respect to q^μ
- MEC responses show a wide but defined peak, due to Δ role. Strenght is about a half of the transverse QE responses
- At low q-values QE peak is well separated from MEC contributions. As q increases the two peaks overlap
- MEC responses are truncated when the exchanged q^μ becomes time-like

MEC Responses: Results

Higher q-values:

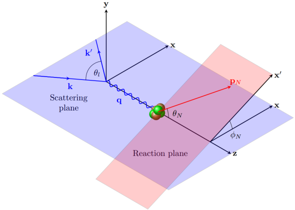




- Same considerations as before
- Ratio between RFG and MEC transverse responses is still the same

Semi-inclusive 2p2h process

Semi-inclusive calculation, fixing
ω, q, $\mathbf{p}'_1(p'_1, \theta_{p_{1'}}, \phi_{p_{1'}})$



- Integration over one particle and two holes momenta
- Four-momentum conservation
- q-system: nucleus symmetry
  $\rightarrow$ NO MORE azimuthal invariance

$\Rightarrow$ 5 dimension integration

Non-vanishing responses

$$R_L = W^{00}_{2p2h} \qquad R_T = W^{11}_{2p2h} + W^{22}_{2p2h}$$

$$R_{TT} = W^{22}_{2p2h} - W^{11}_{2p2h} \qquad R_{TL} = \frac{1}{2}(W^{10}_{2p2h} + W^{01}_{2p2h})$$

Electric charge conservation:
R_{TL} includes contributions from W^{10}_{2p2h}, W^{13}_{2p2h}

MEC Semi-Inclusive Responses

We tested our models with data showed in *J. Ryckebusch et al, Phys. Lett. B 333, 310 (1994), arXiv:nucl-th/9406015.*

Experimental settings, q-system

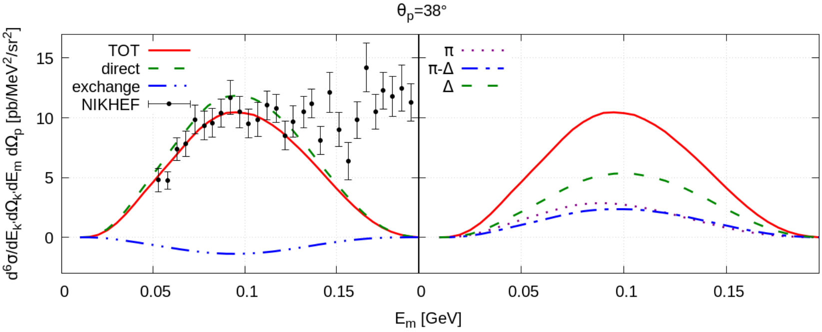
- Fixed incident and final lepton energy E_k, $E_{k'}$, scattering angle and $\mathbf{p}'_1$
- Scattering plane $x - z \quad \rightarrow \quad$ no contribution from $W^{\mu 2}$, $\mu \neq 2$
- Proton detected in the scattering-plane $\quad \rightarrow \quad \phi_{p_{1'}} = 0, \pi$
  Note that $\phi_{p_{1'}}$ value affects the sign of R_{TL} contribution
- 6^{th} differential cross-section

$$\frac{d\sigma}{d\omega d\Omega_{k'} dE_m d\Omega_{p_{1'}}}$$

$$E_m = \omega - T_{p_{1'}} \qquad T_{p_{1'}} = E_{p_{1'}} - m_N$$

Semi-Inclusive Results

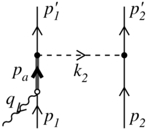
R_T and R_{TT} contributions included only
Kinematics: $E_k = 470$ MeV, $\omega = 263$ MeV, $q = 303$ MeV



- Defined peak
- Direct and exchange contribution included (exchange $\sim$ 12% of direct, $\sim$ 13% of total)
- Δ most important contribution ($\sim$ 50%), π and $\pi - \Delta$ similar
- **Very good agreement with data below $E_m \simeq 130$ MeV**
- For $E_m > 130$ MeV other process starts to contributes: π production via Δ excitation

Isospin channel separation

Example: Δ forward current



- Isospin operator: $$I_{\Delta F} = 2\tau_3^{(1)}\mathbb{1}^{(2)} - I_{V_3}$$

$$I_{V_3} = \frac{1}{2}(\tau_-^{(1)}\tau_+^{(2)} - \tau_+^{(1)}\tau_-^{(2)}) \qquad I_{V_3}^\dagger \left|pp\right\rangle = 0$$

- Δ current is the only term contributing to pp channel (pionic current has I_{V_3} only)

$$I_{\Delta F}^\dagger \left|pp\right\rangle = 2\left|pp\right\rangle \qquad I_{\Delta F}^\dagger \left|pn\right\rangle = 2\left|pn\right\rangle - 2\left|np\right\rangle$$

In the semi-inclusive channel, a proton is detected in the final state

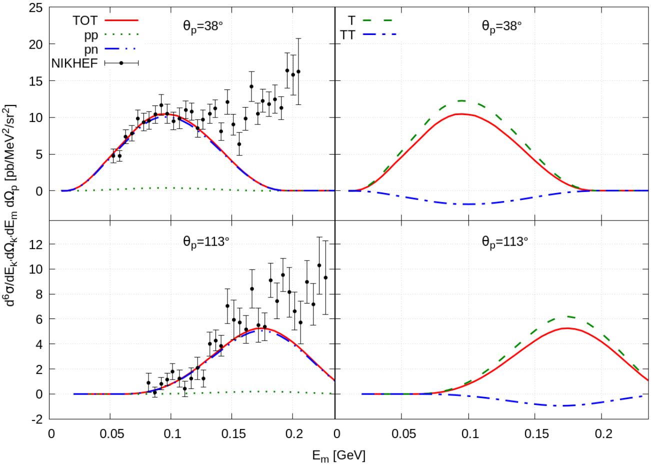
$\Downarrow$

$\left|pn\right\rangle$, $\left|np\right\rangle$ both contribute

$\Downarrow$

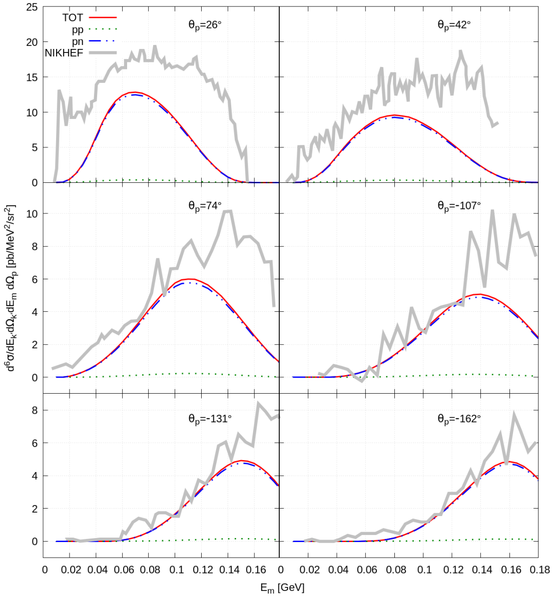
pn channel four times bigger than pp

Kinematics: $E_k = 470$ MeV, $\omega = 263$ MeV, $q = 303$ MeV

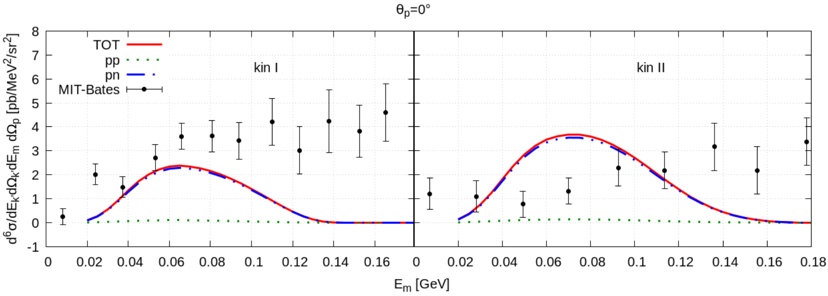


- *pn* dominates with to respect *pp* channel
- *T* contribution is the most important, *TT* reduces the strength for $\simeq 14\%$ (of T)

Kinematics: $E_k = 475$ MeV, $\omega = 212$ MeV, $q = 270$ MeV
data from *L. J. H. M. Kester et al., Phys. Lett. B 344, 79 (1995)*

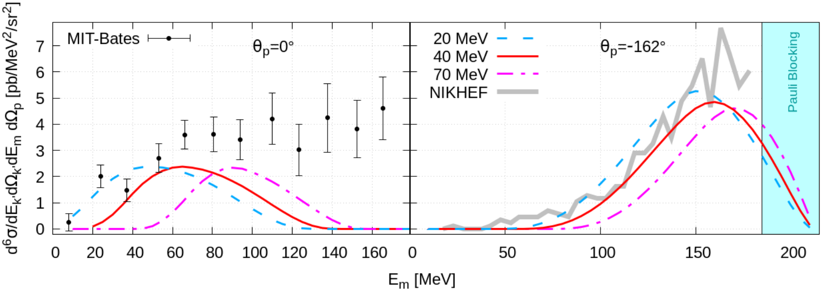


Higher Q^2 values, parallel kinematics ($\theta_p = 0$)
data from *H. Baghaei et al., Phys. Rev. C 39, 177 (1989).*



- kinI: $E_k = 460$ MeV, $\omega = 275$ MeV, $q = 401$ MeV
- kinII: $E_k = 647$ MeV, $\omega = 382$ MeV, $q = 473$ MeV
  Discrepancies:
- Higher Q^2 $\rightarrow$ RFG unable to describe very off-shell probe interaction
- Higher q values $\rightarrow$ $E_s^{2p2h} = 40$ MeV probably not enough, FSI effects reduced

E_s^{2p2h} dependence and Pauli Blocking effect



- Increasing E_s^{2p2h} , shift toward higher E_m
- Effect more relevant in parallel kinematics, response accumulated and localized at lower E_m
- Best agreement with data in the range $E_s^{2p2h} = 20 - 40$ MeV
- Pauli Blocking, included via step function, truncates the responses at $E_m = \omega - T_F$

Summary

Summary

- MEC contributions are computed in a microscopic way in the RFG model, and account for the first contribution at higher energy transfer values beyond the QE channel
- Semi-inclusive 2p2h theoretical predictions are in a very good agreement with available data, providing a first direct proof of the importance of this process and of the validity of the MEC model.
- The only free parameter E_s^{2p2h} accounts for binding energy and other effects not included in the model, as FSI. We adopted $E_s^{2p2h} = 40$ MeV being the 'natural' choice coherent with QE approach.

Work in Progress

- Further investigation over the other TL and L nuclear responses
- Obtain the **Weak Nuclear Responses** in the 2p2h channel
- Adapt our model to describe semi-inclusive process using the TKI variables, that mix leptonic and hadronic momentum variables
- Extend this computational tool to describe 1body-2body **inclusive interference**

Thanks for the attention!

Backup

E_ν Reconstruction

We only know, in a reliable way, outgoing lepton kinematics. Modelling the nuclear initial state, it's possible to reconstruct the $\overline{E_\nu}$, using the inclusive CCQE formula:

$$\overline{E_\nu} = \frac{m_p^2 - (m_n - E_b)^2 - m_\mu^2 + 2(m_n - E_b)E_\mu}{2(m_n - E_b - E_\mu + p_\mu \cos\theta)}$$

- There are biases due to Fermi motion inside the nucleus
- This formula doesn't work for other channels

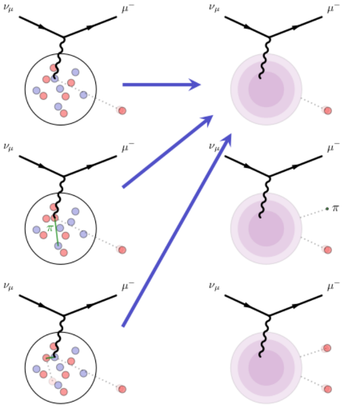
But what do we see? Event topology

$\Downarrow$

Classification based on detected final particles

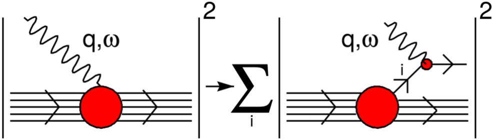
$\nu_\mu + A \to \mu + X$ no π

CC0π event!



Basic ideas:

- Impulse Approximation: incoherent scattering on single nucleons



- Spectral function as a way to describe nuclear dynamics

$$P(\mathbf{p}, E) = \sum_R |\langle A|N, \mathbf{p}; R, -\mathbf{p}\rangle|^2 \delta\left(E - m + M_A - E_R\right)$$

R is the residual nucleus
The angle bracket is the amplitude associated to factorization

- Taking into account the energy absorbed by the nucleus, $\tilde{\omega}$ is the effective exchanged energy $\tilde{\omega} := \omega + m - E - E_p$

$P(\mathbf{p}, E)$ is the probability density function to find a nucleon with momentum p and removal energy E

$l - A$ interaction: Spectral Function FS

Factorization! Hence the hadronic tensor:

$$W_A^{\mu\nu}(q) = \frac{1}{(2\pi)^3}\sum_i \int \mathrm{d}\mathbf{p}\mathrm{d}E P(E,\mathbf{p}) \frac{1}{4E_{\mathbf{p}}E_{|\mathbf{p}+\mathbf{q}|}} W_i^{\mu\nu}(p,\tilde{q})$$

and the cross section

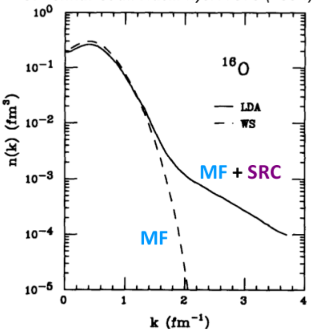
$$\frac{\mathrm{d}^2\sigma_{\mathrm{IA}}}{\mathrm{d}\Omega_{k'}\mathrm{d}E_{k'}} = \int \mathrm{d}\mathbf{p}\mathrm{d}E P(E,\mathbf{p}) \sum_i \frac{\mathrm{d}^2\sigma_i}{\mathrm{d}\Omega_{k'}\mathrm{d}E_{k'}}(q,\tilde{q})\delta(\omega - E + m - E_{|\mathbf{p}+\mathbf{q}|})$$

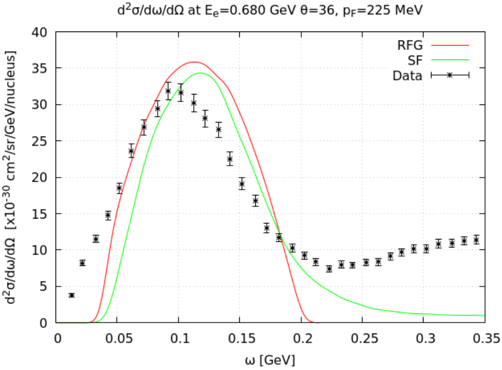
- $\tilde{q} = (\tilde{\omega}, \mathrm{q})$
- $\mathrm{d}^2\sigma_i$ single nucleon cross section

In this work the Rome SF is used

- Based on a mean-field model, contributes for the 80%
- NN short range correlations, account for 20%, for high nucleon momentum

O. Benhar et al. Nucl.Phys. A 579 (1994)



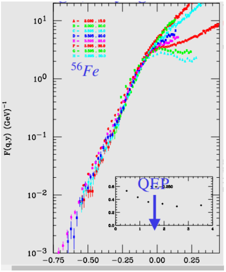


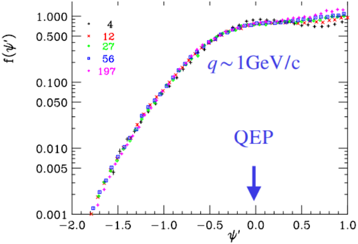
- From scattering data: $E_b = 35$ MeV, $k_f = 225$ MeV
- Pauli blocking included with step function $\Theta(|\mathbf{p}'| - k_f)$
- High energy tail provided by NN short range correlations
- CCQE peak affected by FSI: shift to lower energy and reduction + broadening

$l - A$ interaction: SuperScaling Approach SuSA

Main features:

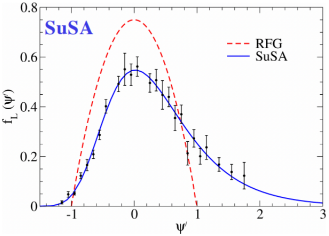
- Generalization based on the relativistic Fermi gas RFG
- From electron scattering data, reduced cross-section depends on ω and q not independentely, but on the scaling variable $\psi(\omega, q)$. This dependency is expressed by a function $f(\psi)$
- Extract from scattering data a realistic scaling function
  Day et al., Ann.Rev.Nucl.Part.Sci.40 (1990); Donnelly and Sick, PRL82; PRC60 (1999)





- Left panel: scaling behaviour at different kinematics. Broken above the Quasi-Elastic Peak due to resonances, 2p-2h effects
- Right panel: scaling behaviour with several nuclei targets

$l-A$ interaction: SuperScaling Approach SuSA



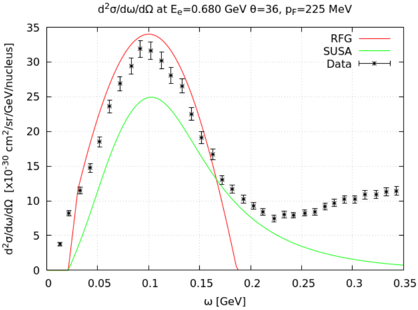
- From $e-A$ scattering data:

$$f(q,\omega;k_F) = k_F \times \frac{\left[d^2\sigma/d\omega d\Omega\right]^{(e,e')}_{exp}}{\overline{\sigma}_{eN}}$$

$$\Downarrow$$

$$f(q,\omega;k_f) \Rightarrow f(\psi)$$

- $E_{shift} = 20$ MeV



$l - A$ interaction: Hartree-Fock HF

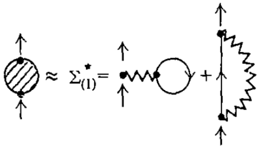
Fundamental basis:

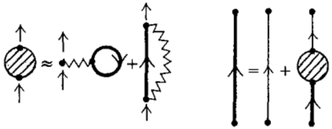
- Averaged NN potential: Mean Field MF approach, each particle moves in a single-particle potential that comes from its average interactions with all of other particles

- Definition of particle proper self-energy
- Propagator dressed with self-energy corrections via Dyson equation

$$\Downarrow$$

Self consistent equations for proper self-energy and propagator



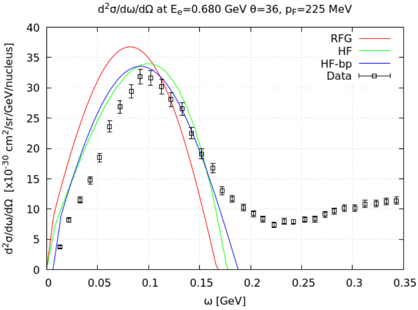


In this work:

- Infinite nuclear matter
- Not Relativistic meson exchange Bonn potential (σ, ω, π, ρ)
  M.B. Barbaro et al, Nucl. Phys. A 596 (1996) , *R. Machleidt et al, Phys. Rep. 149 (1987)*

Comparison between Hartree-Fock self-energy computations:

- HF: exact results
- HF-bp: biparabolic approximation in the self energy $\Rightarrow$ analytical result
  M.B. Barbaro et al, Nucl. Phys. A 596 (1996)



Bonn Potentials

$$V = V^{\pi} + V^{\rho} + V^{\sigma} + V^{\omega}$$

$$V^{\pi} = (V_S^{\pi} \sigma_{\mathbf{1}} \cdot \sigma_{\mathbf{2}} + V_T^{\pi} S_{12}) \tau_{\mathbf{1}} \cdot \tau_{\mathbf{2}}$$

$$V^{\rho} = (V_0^{\rho} + V_S^{\rho} \sigma_{\mathbf{1}} \cdot \sigma_{\mathbf{2}} + V_T^{\rho} S_{12}) \tau_{\mathbf{1}} \cdot \tau_{\mathbf{2}}$$

$$V^{\sigma} = V_0^{\sigma}$$

$$V^{\omega} = V_0^{\omega} + V_S^{\omega} \sigma_{\mathbf{1}} \cdot \sigma_{\mathbf{2}} + V_T^{\omega} S_{12}$$

$$\Downarrow$$

$$V(\mathbf{k}) = V_0(k) + V_{\tau}(k) \tau_{\mathbf{1}} \cdot \tau_{\mathbf{2}} + V_{\sigma}(k) \sigma_{\mathbf{1}} \cdot \sigma_{\mathbf{2}} + V_{\sigma\tau} \sigma_{\mathbf{1}} \cdot \sigma_{\mathbf{2}} \tau_{\mathbf{1}} \cdot \tau_{\mathbf{2}}$$

Dipole Interaction: $S_{12}(q) = 3(\sigma_{\mathbf{1}} \cdot \hat{\mathbf{q}})(\sigma_{\mathbf{2}} \cdot \hat{\mathbf{q}}) - \sigma_{\mathbf{1}} \cdot \sigma_{\mathbf{2}}$

Bonn Potentials

$$V_S^{\pi}(q) = V_T^{\pi}(q) = -\frac{g_{\pi}^2}{12m_N^2}\Gamma_{\pi}^2(q)\frac{q^2}{q^2+m_{\pi}^2}$$

$$V_0^{\rho}(q,P) = \left[g_{\rho}^2\left(1+\frac{3}{2}\frac{P^2}{m_N^2}\right) - \frac{g_{\rho}(g_{\rho}+4f_{\rho})}{8m_N^2}q^2\right]\Gamma_{\rho}^2(q)\frac{1}{q^2+m_{\rho}^2}$$

$$V_S^{\rho}(q) = -2V_T^{\rho}(q) = -\frac{(g_{\rho}+f_{\rho})^2}{6m_N^2}\Gamma_{\rho}^2(q)\frac{q^2}{q^2+m_{\rho}^2}$$

$$V_0^{\sigma}(q,P) = g_{\sigma}^2\left[-1+\frac{1}{2}\frac{P^2}{m_N^2} - \frac{q^2}{8m_N^2}\right]\Gamma_{\sigma}^2(q)\frac{1}{q^2+m_{\sigma}^2}$$

$$V_0^{\omega}(q,P) = g_{\omega}^2\left[1+\frac{3}{2}\frac{P^2}{m_N^2} - \frac{q^2}{8m_N^2}\right]\Gamma_{\omega}^2(q)\frac{1}{q^2+m_{\omega}^2}$$

$$V_S^{\omega}(q) = -2V_T^{\omega}(q) = -\frac{g_{\omega}^2}{6m_N^2}\Gamma_{\omega}^2(q)\frac{1}{q^2+m_{\omega}^2}$$

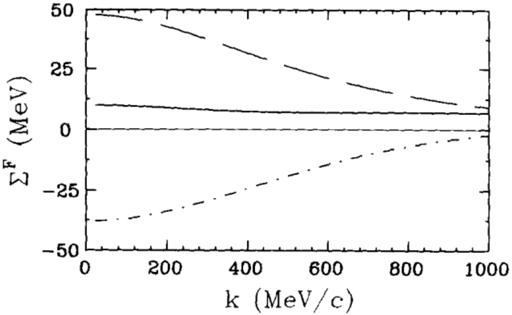
Cut-off: $\Gamma^2(q)_{\alpha} = \frac{\Lambda_{\alpha}^2 - m_{\alpha}^2}{\Lambda_{\alpha}^2 + q^2}$

Non-local interaction: $\mathbf{P} = \frac{1}{4}(\mathbf{k_1} - \mathbf{k_2} + \mathbf{k_1'} - \mathbf{k_2'})$

Meson	T	J^π	m (MeV/c)	Λ (MeV)	$g^2/4\pi$	$f^2/4\pi$
π	1	0^-	138.03	1300	14.9	–
ρ	1	1^-	769	1300	0.95	$0.95 \times (6.1)^2$
σ	0	0^+	550	2000	7.7823	–
ω	0	1^-	782.6	1500	20	–

HF Self-Energy

- Tadpoles $\Rightarrow$ Hartree Self-Energy Σ^H: parabolic in k^2
- Oysters $\Rightarrow$ Fock Self-Energy Σ^F: cumbersome dependency on k^2



Fock self-energy of σ (dashed) and of ω (dot-dashed) meson as functions of nucleon momentum k, with $k_f = 225$ MeV. Total contribution is the solid line

HF Self-Energy

Biparabolic approximation

$$\Sigma^{HF} = \overline{A} + \overline{B}\frac{k^2}{2m_N^2}, \qquad k < k_f$$

$$\Sigma^{HF} = A + B\frac{k^2}{2m_N^2}, \qquad max(q - k_f, k_f) < k < q + k_f$$

q (MeV/c)	$\bar{A}$ (10^{-2})	$\bar{B}$	A (10^{-2})	B	$m_N^*(p)/m_N$
300	−1.671	0.246	−0.961	0.146	0.77
500	−1.671	0.246	−0.424	0.116	0.81
800	−1.671	0.246	1.034	0.085	0.85
1000	−1.671	0.246	1.466	0.081	0.86

Effective Field Theory to describe nucleons-mesons interaction:
E. Hernandez et al, Mod.Phys.Lett.A 23 (2008)

Non-Linear σ-model
Non-Linear: chirality is not like other simmetries, i.e. isospin
Lagrangian invariant under chiral transformation, constructed starting from pionic fields and isospin operators instead γ_5 and isospin operators

$$\mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - M)\Psi + \frac{1}{2}\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - \frac{1}{2}m_\pi^2\vec{\phi}^2 + \mathcal{L}^\sigma_{\rm int}$$

$$\mathcal{L}^\sigma_{\rm int} = \frac{g_A}{f_\pi}\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}(\partial_\mu\vec{\phi})\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma^\mu\vec{\tau}(\vec{\phi}\times\partial^\mu\vec{\phi})\Psi - \frac{1}{6f_\pi^2}\left[\vec{\phi}^2\partial_\mu\vec{\phi}\partial^\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})(\vec{\phi}\partial^\mu\vec{\phi})\right]$$
$$+\frac{m_\pi^2}{24f_\pi^2}(\vec{\phi}^2)^2 - \frac{g_A}{6f_\pi^3}\bar{\Psi}\gamma^\mu\gamma_5\left[\vec{\phi}^2\frac{\vec{\tau}}{2}\partial_\mu\vec{\phi} - (\vec{\phi}\partial_\mu\vec{\phi})\frac{\vec{\tau}}{2}\vec{\phi}\right]\Psi + \mathcal{O}(\frac{1}{f_\pi^4})$$