

Neutrino-nucleus cross sections for neutrino oscillation experiments: status and challenges

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Preamble:
neutrino oscillations physics

Neutrino Oscillations

Flavor neutrinos ν_e, ν_μ, ν_τ produced in Weak interactions

Massive neutrinos ν_1, ν_2, ν_3 propagate from source to detector

A flavor neutrino is a superposition of massive neutrinos

$$\nu_\alpha = U_{\alpha i} \nu_i \quad \begin{array}{l} \alpha = e, \mu, \tau \\ i = 1, 2, 3 \end{array}$$

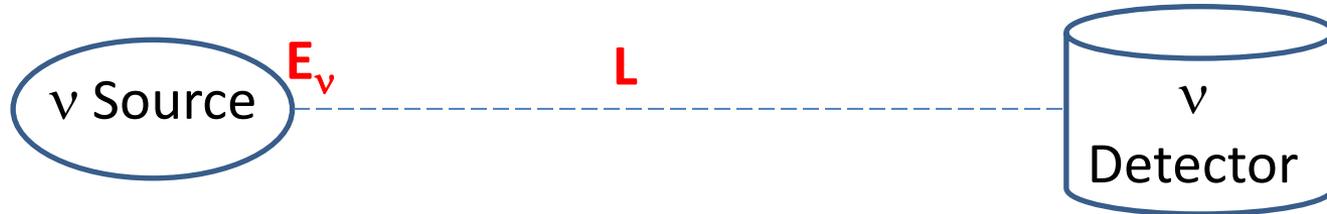
↑
Mixing Matrix
(PMNS)

Neutrino oscillations are flavor transitions

$$\begin{array}{cccc} \nu_e \rightarrow \nu_\mu & \nu_e \rightarrow \nu_\tau & \nu_\mu \rightarrow \nu_e & \nu_\mu \rightarrow \nu_\tau \\ \bar{\nu}_e \rightarrow \bar{\nu}_\mu & \bar{\nu}_e \rightarrow \bar{\nu}_\tau & \bar{\nu}_\mu \rightarrow \bar{\nu}_e & \bar{\nu}_\mu \rightarrow \bar{\nu}_\tau \end{array}$$

Neutrino physics

Perform appearance and/or disappearance experiments using different neutrino sources and baselines



ν oscillation probability

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{ij} U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} e^{-i \frac{(m_i^2 - m_j^2)L}{2E}}$$

Reactor ν

$E \sim 5$ MeV ; $L \sim 1$ -100 km

e.g. Double Chooz $E \sim 5$ MeV ; $L \sim 1$ km



Accelerator ν

$E \sim 0.6$ - 20 GeV ; $L \sim 300$ -1300 km

e.g. T2K: $E \sim 0.6$ GeV ; $L \sim 300$ km



Super-Kamiokande
(ICRR, Univ. Tokyo)



J-PARC Main Ring
(KEK-JAEA, Tokai)

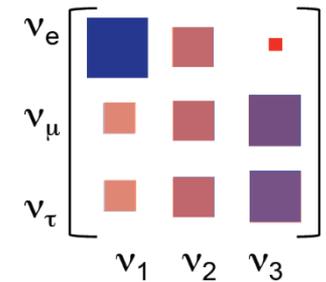


The Pontecorvo-Maki-Nakagawa-Sakata Mixing Matrix

$$U_{\text{PMNS}} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\substack{\text{Atmospheric } \nu \\ \text{Accelerator } \nu}} \underbrace{\begin{pmatrix} c_{13} & 0 & e^{-i\delta_{\text{CP}}} s_{13} \\ 0 & 1 & 0 \\ -e^{i\delta_{\text{CP}}} s_{13} & 0 & c_{13} \end{pmatrix}}_{\substack{\text{SBL reactor } \nu \\ \text{Accelerator } \nu}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\text{solar } \nu \\ \text{LBL reactor } \nu}}$$

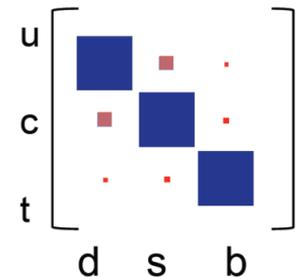
$(\theta_{12}, \theta_{23}, \theta_{13})$
 3 mixing angles
 δ_{CP}
 CP-violating phase

$$|U_{\text{PMNS}}| \sim \begin{pmatrix} 0.8 & 0.5 & 0.1 \\ 0.5 & 0.6 & 0.7 \\ 0.3 & 0.6 & 0.7 \end{pmatrix}$$



The Cabibbo-Kobayashi-Maskawa Quark Mixing Matrix

$$|V_{\text{CKM}}| \sim \begin{pmatrix} 1 & 0.2 & 0.004 \\ 0.2 & 1 & 0.04 \\ 0.008 & 0.04 & 1 \end{pmatrix}$$



Present and future of neutrino oscillation physics

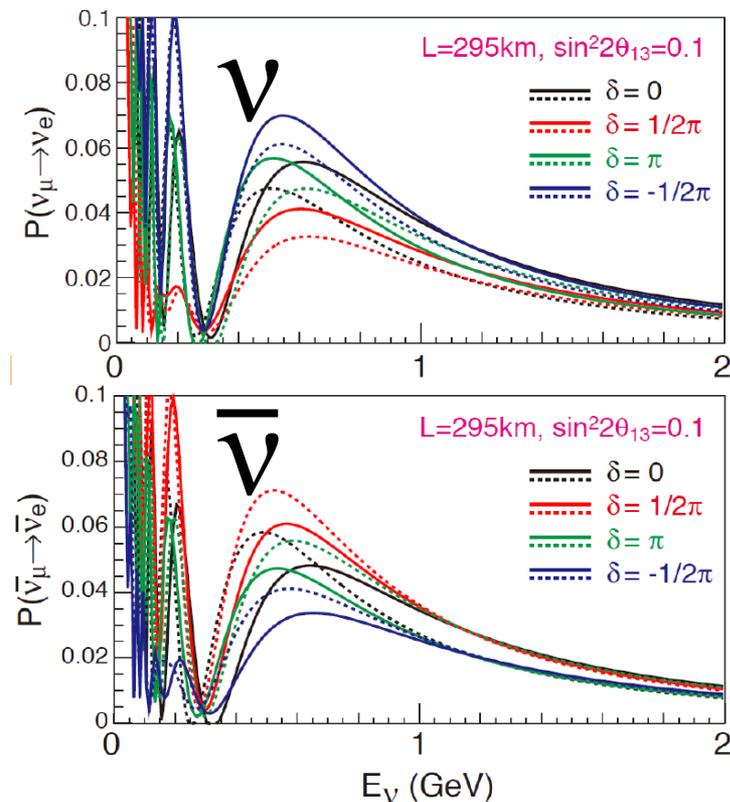
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

sun reactors → U_{e1} U_{e2} U_{e3}
Atmosph., Accel. ↑ $U_{\tau3}$

- The mixing parameters not known to the same precision as those in the quark sector

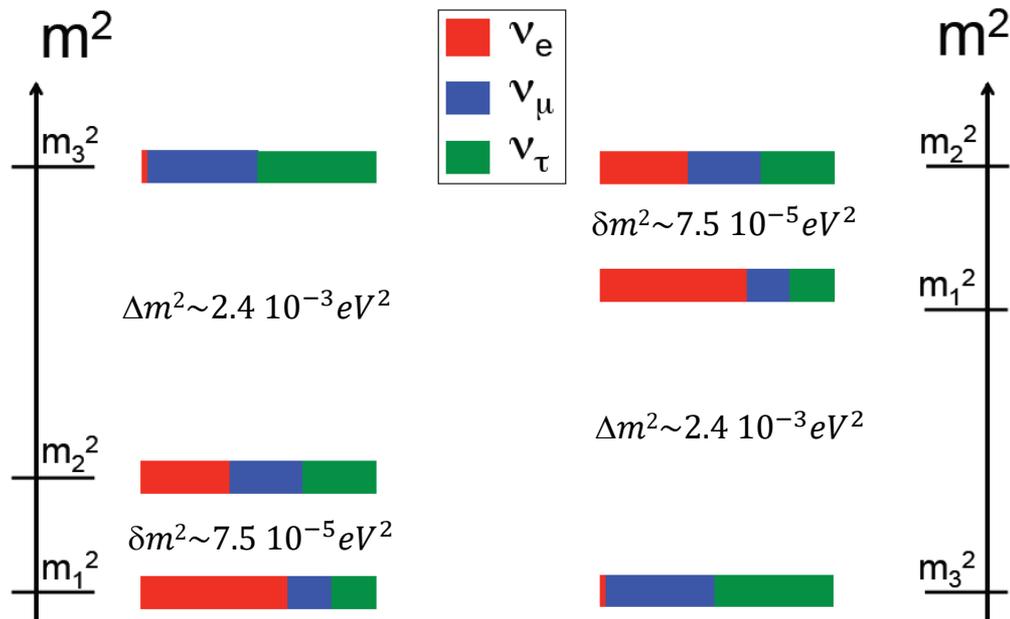
- The value of δ_{CP} is undetermined

- The ordering of the mass states i.e. the **neutrino mass hierarchy** is undetermined



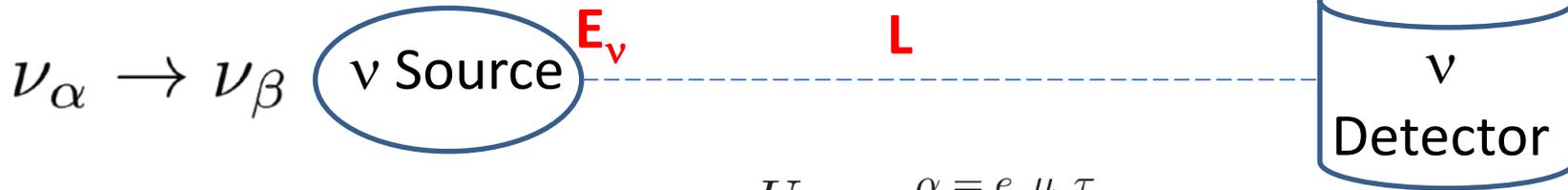
Normal Hierarchy

Inverted Hierarchy

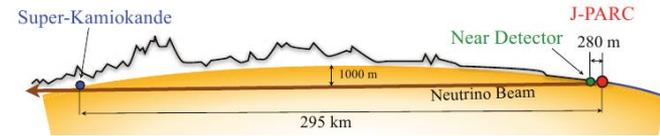


Neutrino cross sections generalities and models

Neutrino oscillation experiments



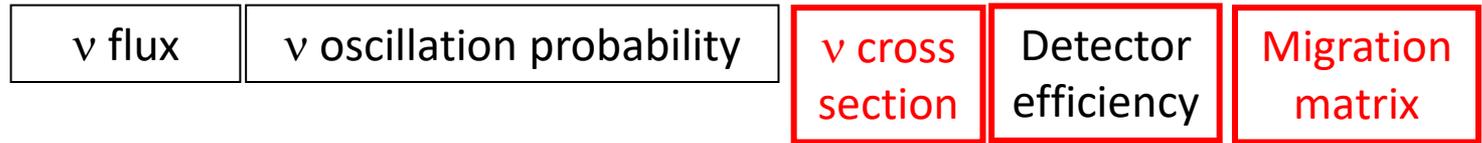
$$\nu_\alpha = U_{\alpha i} \nu_i \quad \begin{matrix} \alpha = e, \mu, \tau \\ i = 1, 2, 3 \end{matrix}$$



$$N_{\nu_\beta}(\overline{E_\nu}) \sim \int \Phi_{\nu_\alpha}(E_\nu) P_{\nu_\alpha \rightarrow \nu_\beta}(E_\nu, L, \{\Theta\}) \sigma_{\nu_\beta}(E_\nu) \epsilon_{\text{det.}} d(E_\nu, \overline{E_\nu}) dE_\nu$$

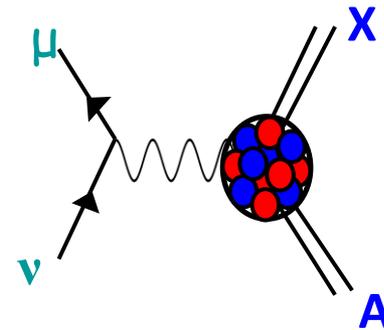
Reconstructed ν energy $\overline{E_\nu}$ (blue box)
 True ν energy E_ν (pink box)

Number of detected events



Modern accelerator-based neutrino oscillation experiments:

- The neutrino energy is reconstructed from the final states
- Nuclear targets (C, O, Ar, Fe...)



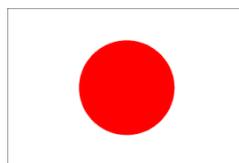
→ the knowledge of the neutrino-nucleus cross section is crucial

Present and future LBL oscillation experiments

$$P(\nu_{\mu} \rightarrow \nu_e) \stackrel{?}{\neq} P(\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e)$$

$$\begin{array}{c} \updownarrow \\ \delta_{CP} \end{array}$$

Present



Future



Nuclear targets:

Carbon: T2K(ND) and NOvA

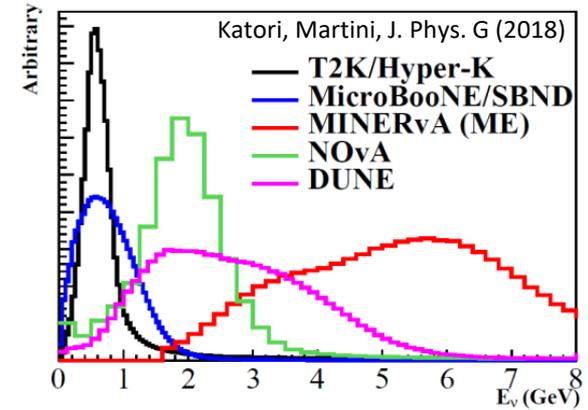
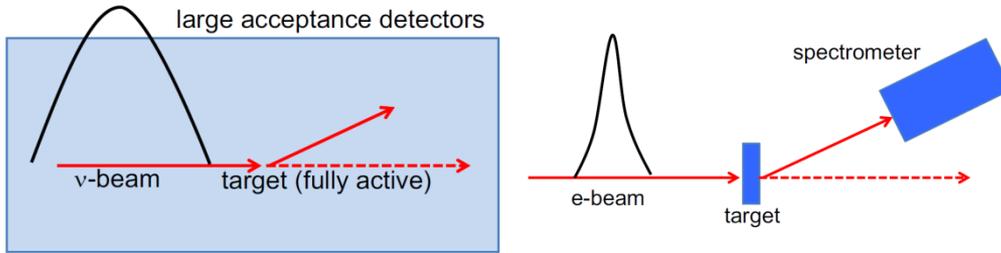
Oxygen (water): T2K (SuperK) and Hyper-K

Argon: DUNE

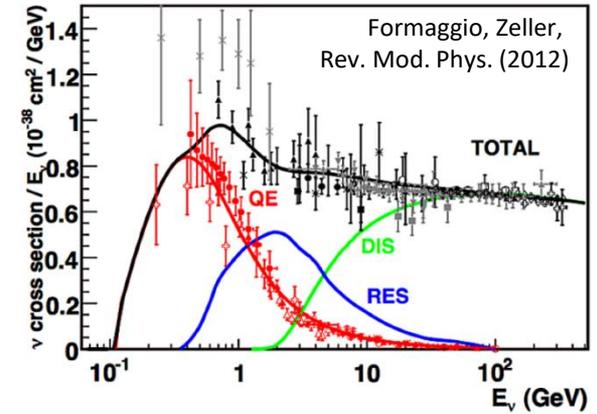
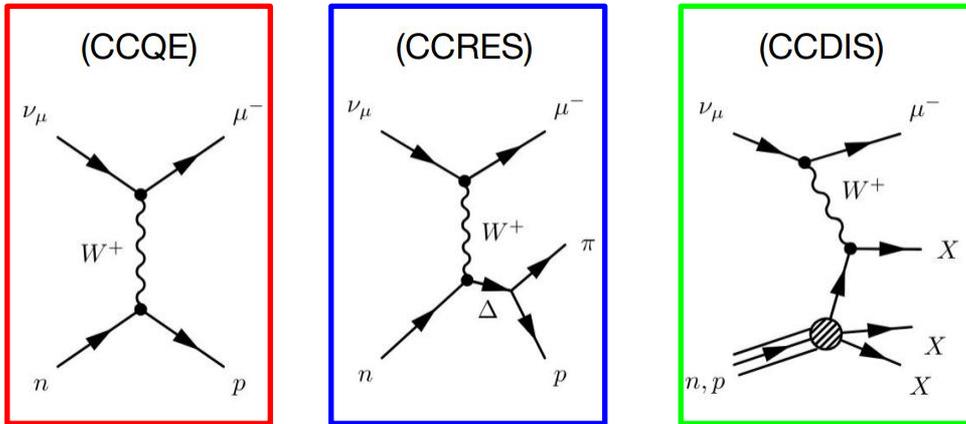
In the last 15 years many cross sections measurements and theoretical studies have been performed for Carbon (^{12}C). Less for Oxygen (^{16}O) and Argon (^{40}Ar)

Some important points of the accelerator-based ν experiment

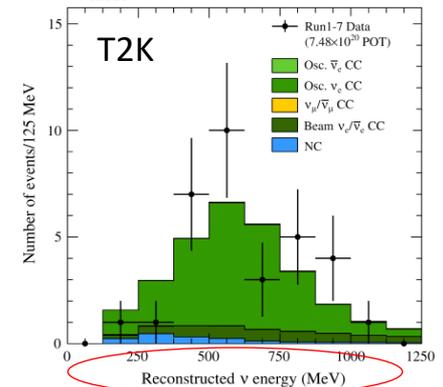
- Neutrino beams are not monochromatic (at difference with respect to electron beams)



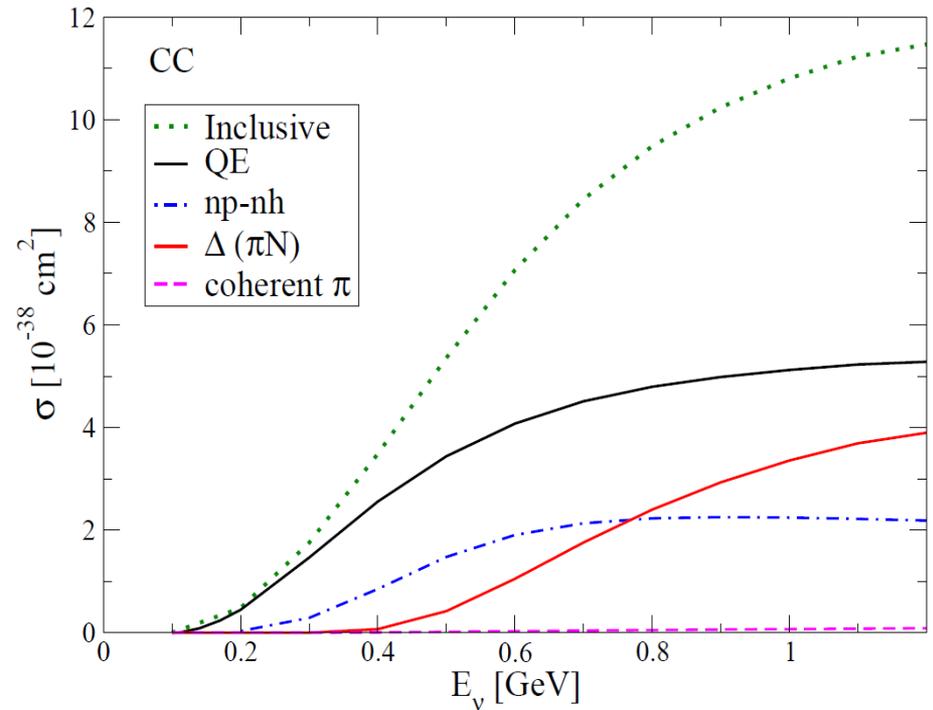
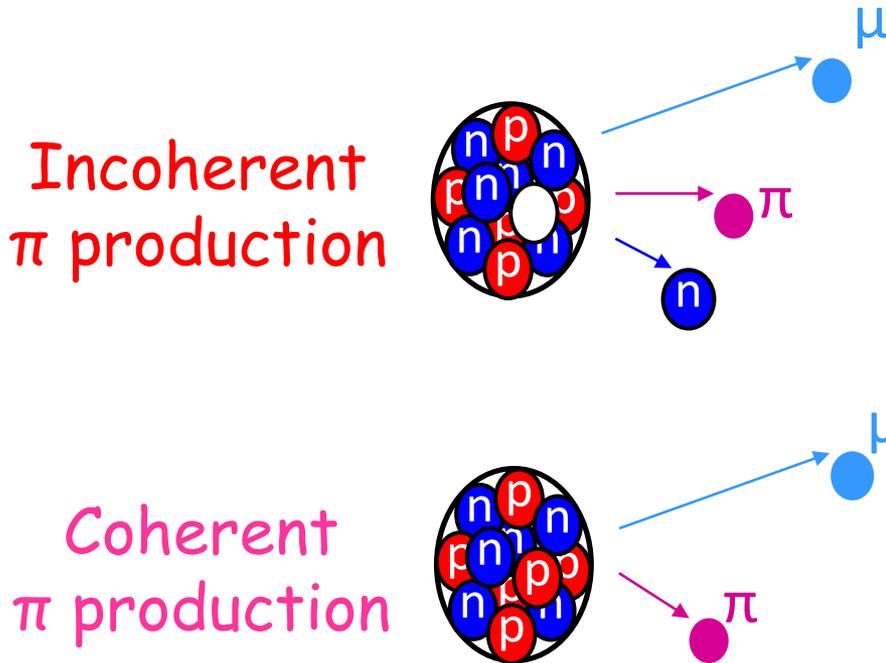
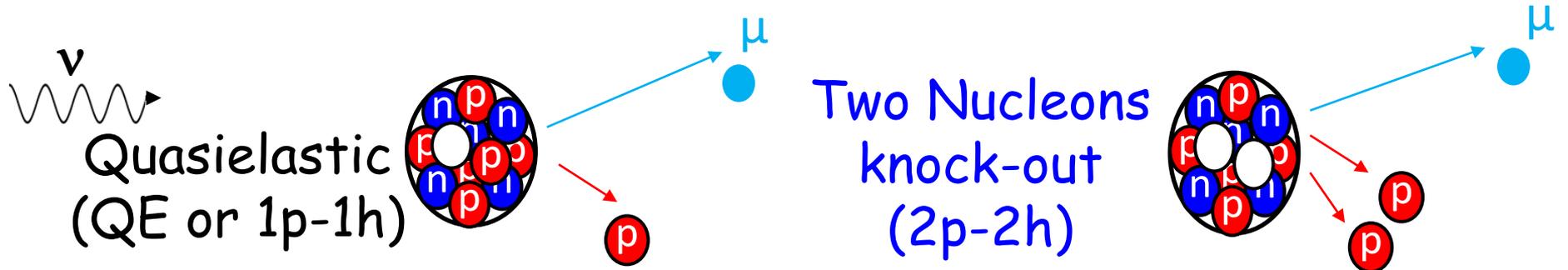
- Different reaction mechanisms contribute



- The neutrino energy is reconstructed from the final states of the reaction (often from CCQE events)

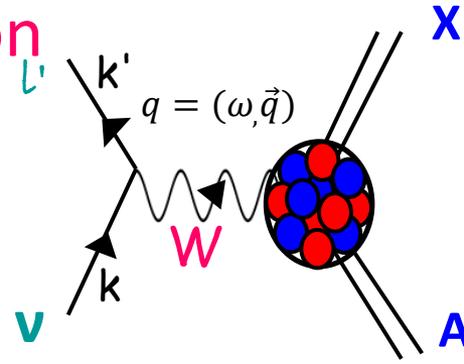
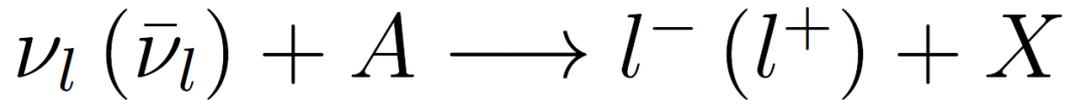


In this talk: Neutrino - nucleus interaction @ $E_\nu \sim O(1 \text{ GeV})$



Different processes are entangled

Charged current neutrino-nucleus cross section



$$\mathcal{L}_W = \frac{G_F}{\sqrt{2}} \cos \theta_C l_\mu J^\mu$$

From weak Lagrangian to cross section in terms of

Leptonic and **Hadronic** tensors

Lab frame

$$\frac{d^2\sigma}{d\Omega_{k'} d\omega} = \frac{G_F^2 \cos^2 \theta_C}{4\pi^2} \frac{|\mathbf{k}'|}{|\mathbf{k}|} L_{\mu\nu} W^{\mu\nu}(\mathbf{q}, \omega)$$

$d\Omega_{k'}$ differential solid angle in the direction specified by the charged-lepton momentum \mathbf{k}'

$$k \equiv (E_\nu, \mathbf{k}) \quad k' \equiv (E'_l, \mathbf{k}') \quad q = k - k' \equiv (\omega, \mathbf{q}) \quad \omega = E_\nu - E'_l$$

initial and final lepton 4-momenta

four-momentum transfer

energy transfer

$$L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k \cdot k' \pm i \varepsilon_{\mu\nu\kappa\lambda} k^\kappa k'^\lambda \quad W^{\mu\nu} = \sum_f \langle 0 | J^{\mu\dagger}(q) | f \rangle \langle f | J^\nu(q) | 0 \rangle \delta^{(4)}(p_0 + q - p_f)$$

Leptonic tensor

Hadronic tensor

The “inclusive” charged current cross section is a linear combination of five contributions

$$\frac{d^2\sigma}{d\Omega_{k'} d\omega} = \sigma_0 [L_{00}W^{00} + L_{33}W^{33} + (L_{03} + L_{30})W^{03} + (L_{11} + L_{22})W^{11} \pm (L_{12} - L_{21})W^{12}]$$

A simplified expressions particularly useful for illustration

- Final lepton mass contributions ignored ($m_l=0$)
- Obtained by keeping only the leading terms for the hadronic tensor in the development of the hadronic current in p/M_N

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2 \cos^2\theta_c}{\pi} |k'| E_l' \cos^2\frac{\theta}{2} \left[\frac{(\mathbf{q}^2 - \omega^2)^2}{\mathbf{q}^4} G_E^2 R_T(\mathbf{q}, \omega) + \frac{\omega^2}{\mathbf{q}^2} G_A^2 R_{\sigma\tau(L)}(\mathbf{q}, \omega) \right] + 2 \left(\tan^2\frac{\theta}{2} + \frac{\mathbf{q}^2 - \omega^2}{2\mathbf{q}^2} \right) \left(G_M^2 \frac{\mathbf{q}^2}{4M_N^2} + G_A^2 \right) R_{\sigma\tau(T)}(\mathbf{q}, \omega) \pm 2 \frac{E_\nu + E_l'}{M_N} \tan^2\frac{\theta}{2} G_A G_M R_{\sigma\tau(T)}(\mathbf{q}, \omega)$$

Explicitly appear:

- The different **kinematic variables** (related to the leptonic tensor)
- The nucleon Electric, Magnetic, and Axial **form factors** (\leftrightarrow nucleon properties)
- The **nuclear response functions** (\leftrightarrow nuclear dynamics)

Nuclear response functions $R(\mathbf{q}, \omega)$:

$$R_\alpha^{PP'}(\mathbf{q}, \omega) = \sum_n \langle n | \sum_{j=1}^A O_\alpha^P(j) e^{i\mathbf{q}\cdot\mathbf{x}_j} | 0 \rangle \langle n | \sum_{k=1}^A O_\alpha^{P'}(k) e^{i\mathbf{q}\cdot\mathbf{x}_k} | 0 \rangle^* \delta(\omega - E_n + E_0),$$

Isovector R_τ

$$O_\alpha^N(j) = \tau_j^\pm$$

Isospin Spin-Longitudinal $R_{\sigma\tau(L)}$

$$(\boldsymbol{\sigma}_j \cdot \hat{\mathbf{q}}) \tau_j^\pm$$

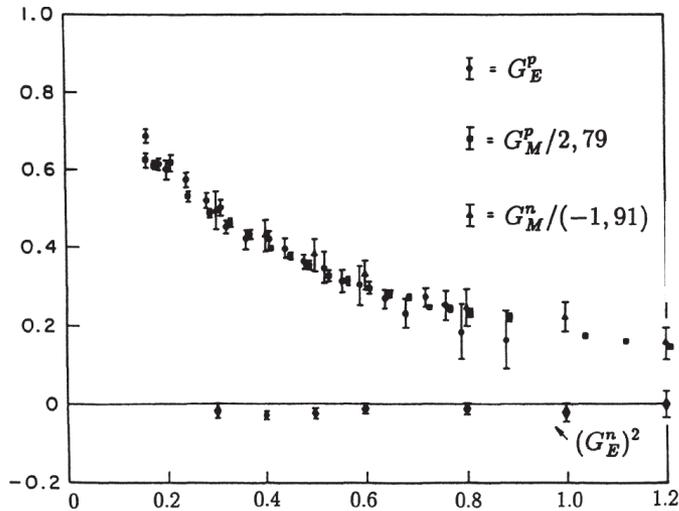
Isospin Spin-Transverse $R_{\sigma\tau(T)}$

$$(\boldsymbol{\sigma}_j \times \hat{\mathbf{q}})^i \tau_j^\pm$$

The Form Factors

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2 \cos^2\theta_c}{\pi} |\mathbf{k}'| E_l' \cos^2\frac{\theta}{2} \left[\frac{(q^2 - \omega^2)^2}{q^4} G_E^2 R_\tau + \frac{\omega^2}{q^2} G_A^2 R_{\sigma\tau(L)} + 2 \left(\tan^2\frac{\theta}{2} + \frac{q^2 - \omega^2}{2q^2} \right) \left(G_M^2 \frac{\omega^2}{q^2} + G_A^2 \right) R_{\sigma\tau(T)} \pm 2 \frac{\epsilon + \epsilon'}{M_N} \tan^2\frac{\theta}{2} G_A G_M R_{\sigma\tau(T)} \right]$$

Vector form factors



$$G_E^p(Q^2) = \frac{G_M^p(Q^2)}{2.79} = \frac{G_M^n(Q^2)}{-1.91} = G^{\text{dipole}}(Q^2)$$

$$G^{\text{dipole}}(Q^2) = \left(1 + \frac{Q^2}{0.71 (\text{GeV}/c)^2} \right)^{-2}$$

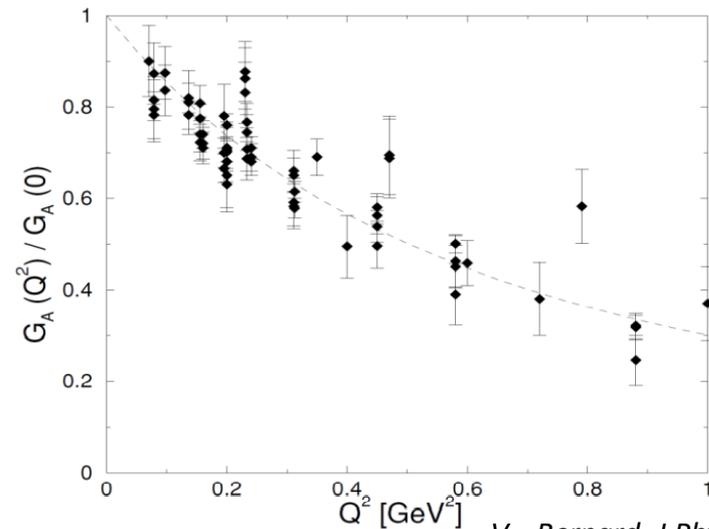
$$Q^2 = q^2 - \omega^2$$

Global dipole-like behavior

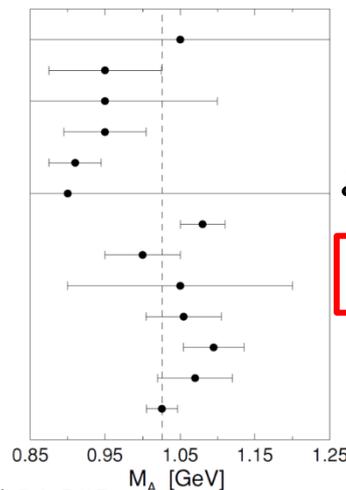
Weak **vector form factors** are well constrained by electron scattering experiments (CVC)

Q^2 evolution of the **axial form factor** is less well-known, mainly based on old bubble chamber data

Axial form factor



V. Bernard, J.Phys. G28 (2002) R1-R35



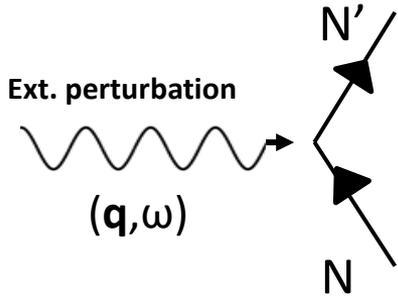
$$G_A(Q^2) = g_A (1 + Q^2 / M_A^2)^{-2}$$

$$g_A = 1.26 \text{ from neutron } \beta \text{ decay}$$

$$M_A = (1.026 \pm 0.021) \text{ GeV}/c^2$$

from ν - ^2H (bubble-chamber) CCQE
and
from π electroproduction

Free (or bare) nuclear response function



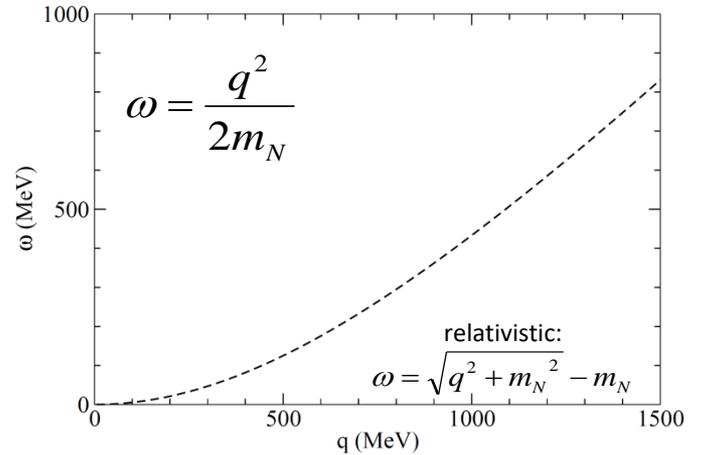
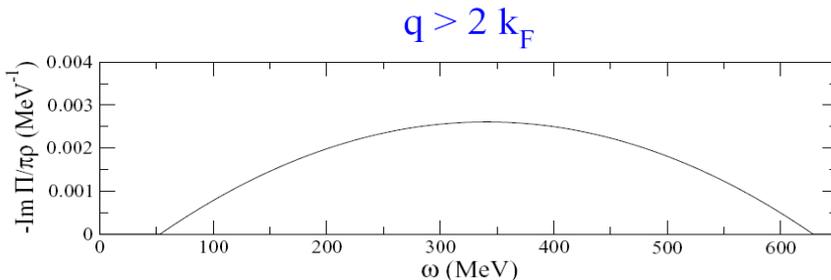
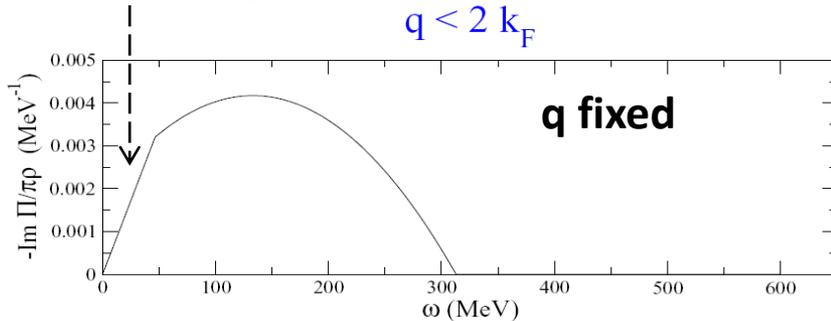
- Free nucleon at rest:
Response functions $\propto \delta(\omega - q^2/2m_N)$

- Nucleon inside the nucleus:

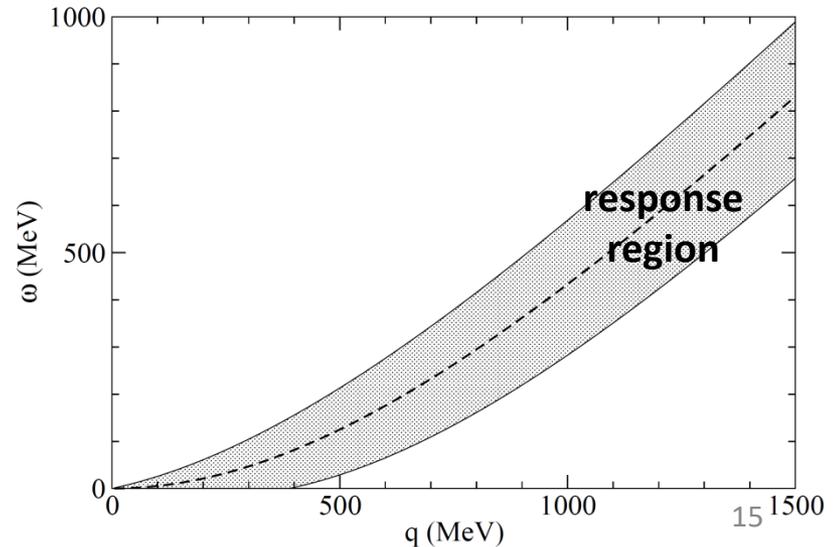
Nucleon-Nucleon interaction switched off \leftrightarrow Nucleons respond individually

Fermi Gas Quasielastic Response

- Fermi motion spreads δ distribution
- Pauli blocking cuts part of the low q and ω response



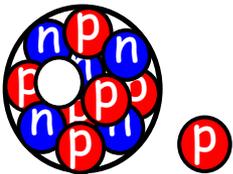
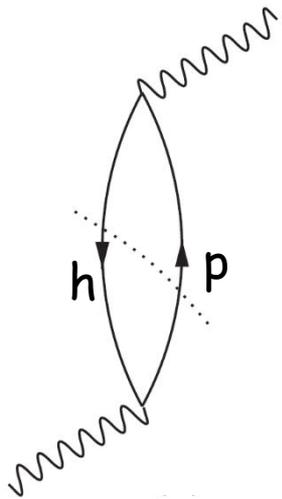
$$\frac{q^2}{2m_N} - \frac{qk_F}{m_N} \leq \omega \leq \frac{q^2}{2m_N} + \frac{qk_F}{m_N}$$



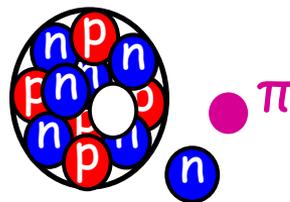
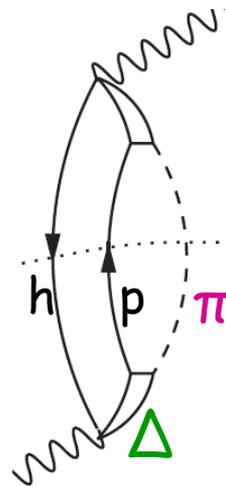
Nuclear Responses for different excitations

$$R_\alpha = \sum_{n \neq 0} |\langle n | \hat{O}_{(\alpha)} | 0 \rangle|^2 \delta[\omega - (E_n - E_0)]$$

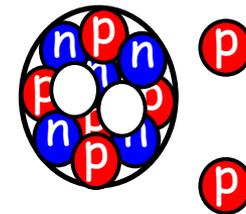
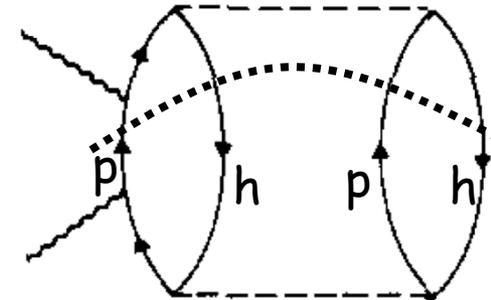
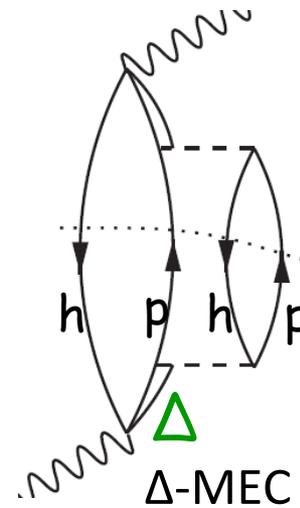
1p-1h
Quasielastic



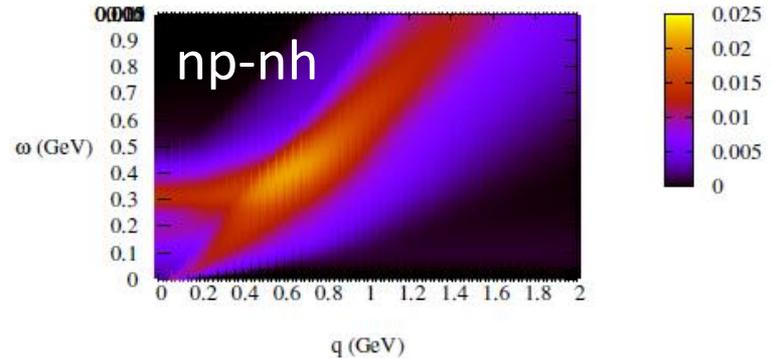
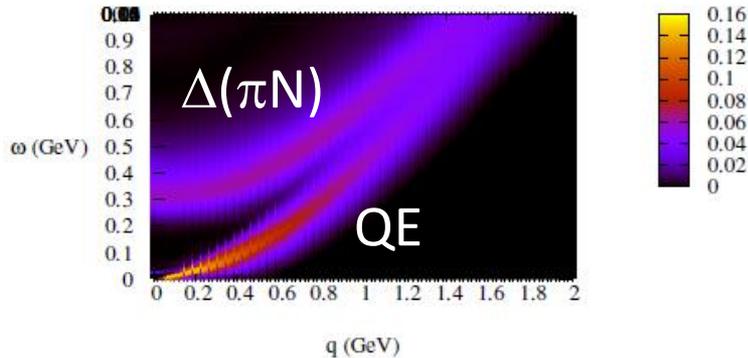
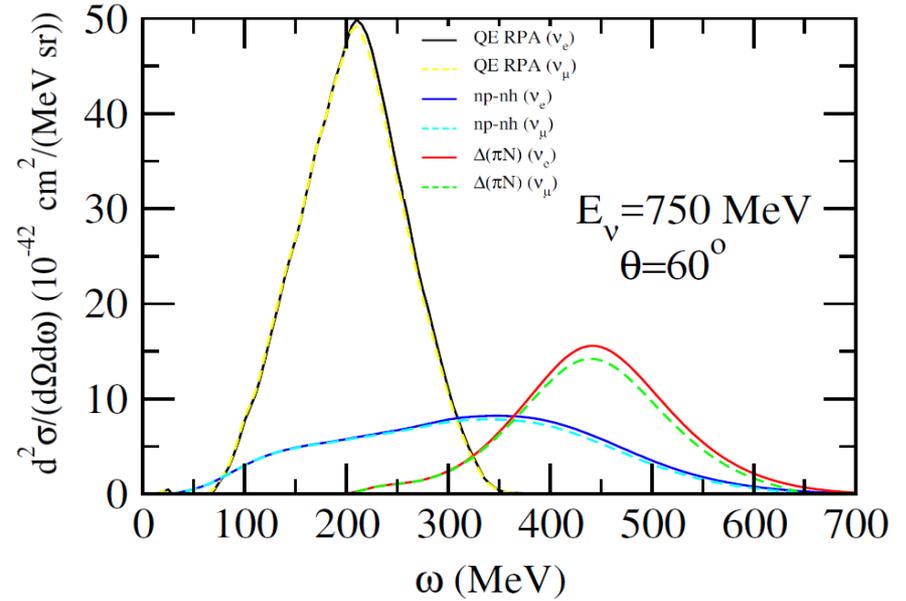
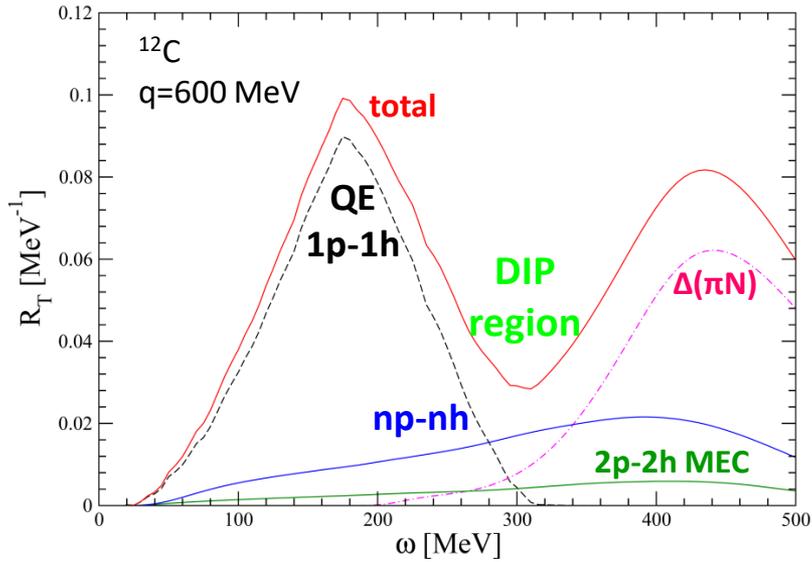
1p-1h
($\Delta \rightarrow \pi N$) 1π production



2p-2h:
two examples



Nuclear responses and neutrino cross sections at fixed kinematics



QE peak:

$$\omega = \sqrt{q^2 + M_N^2} - M_N = \frac{Q^2}{2M_N} = \frac{q^2 - \omega^2}{2M_N}$$

Δ peak:

$$\omega = \sqrt{q^2 + M_\Delta^2} - M_N = \frac{Q^2}{2M_N} + \frac{M_\Delta^2 - M_N^2}{2M_N}$$

np-nh excitations fill the DIP region

np-nh enlarges the region of response to the whole (ω, q) plane

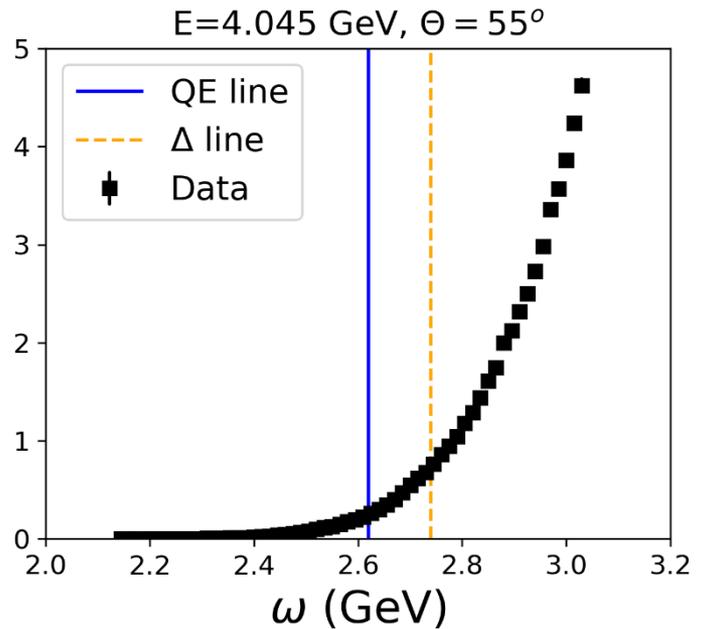
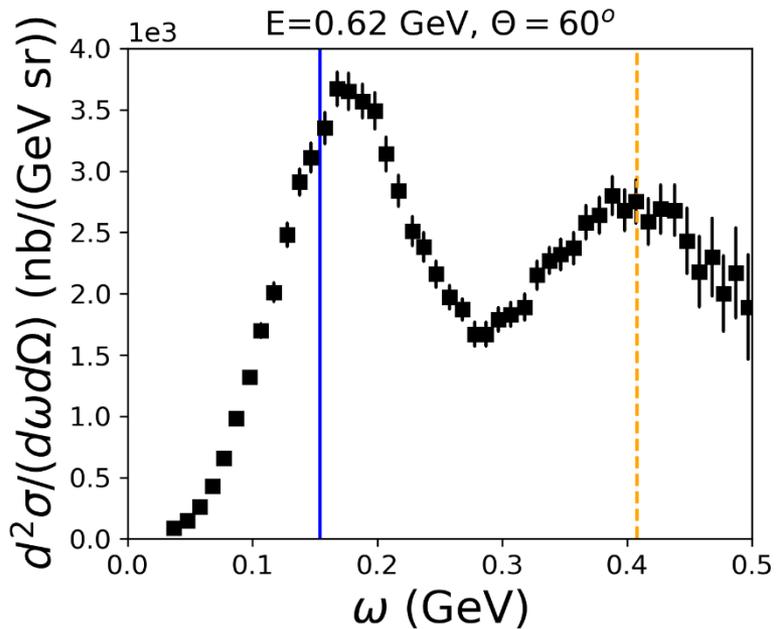
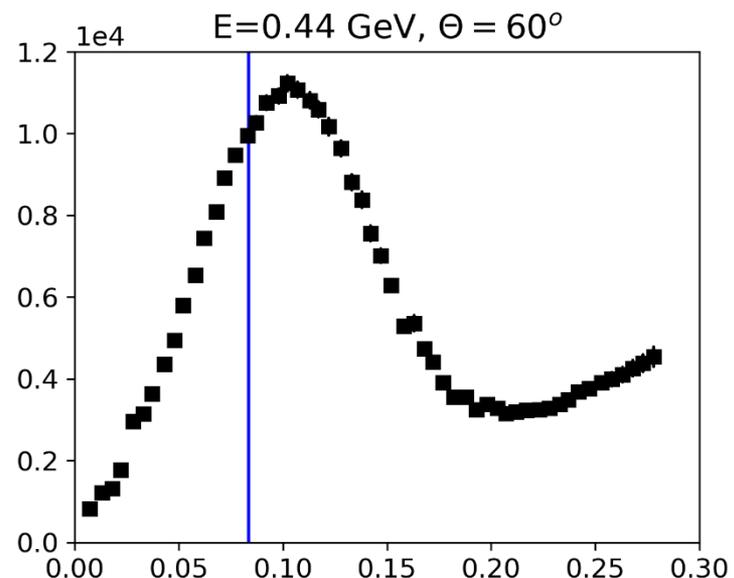
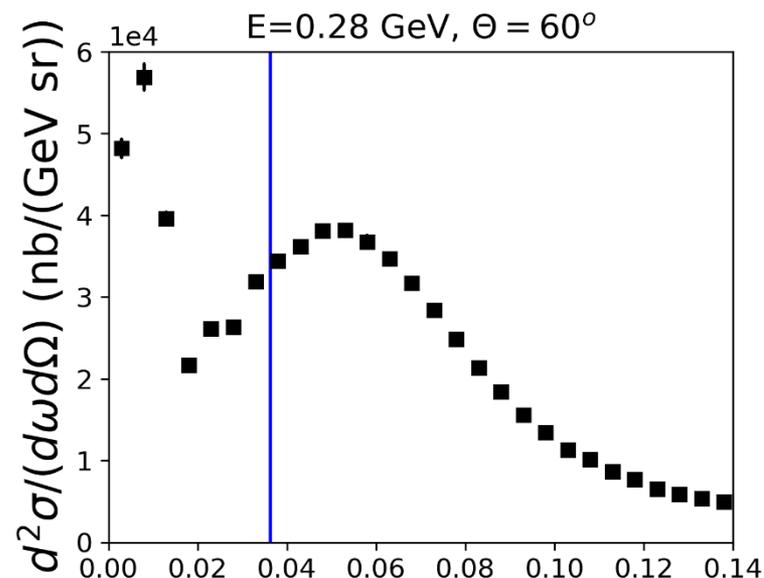
N.B. The responses can be tested in other processes (scattering of e, π ...)

Examples of electron scattering cross section on ^{12}C

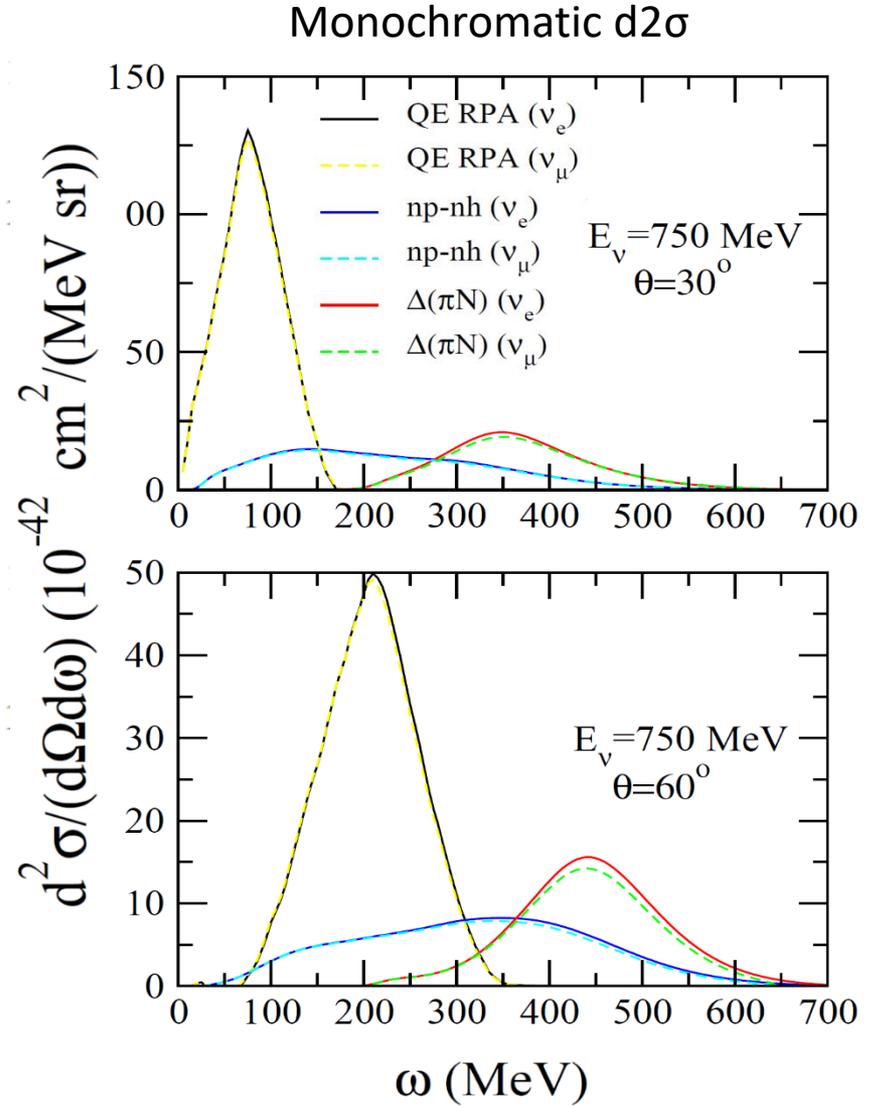
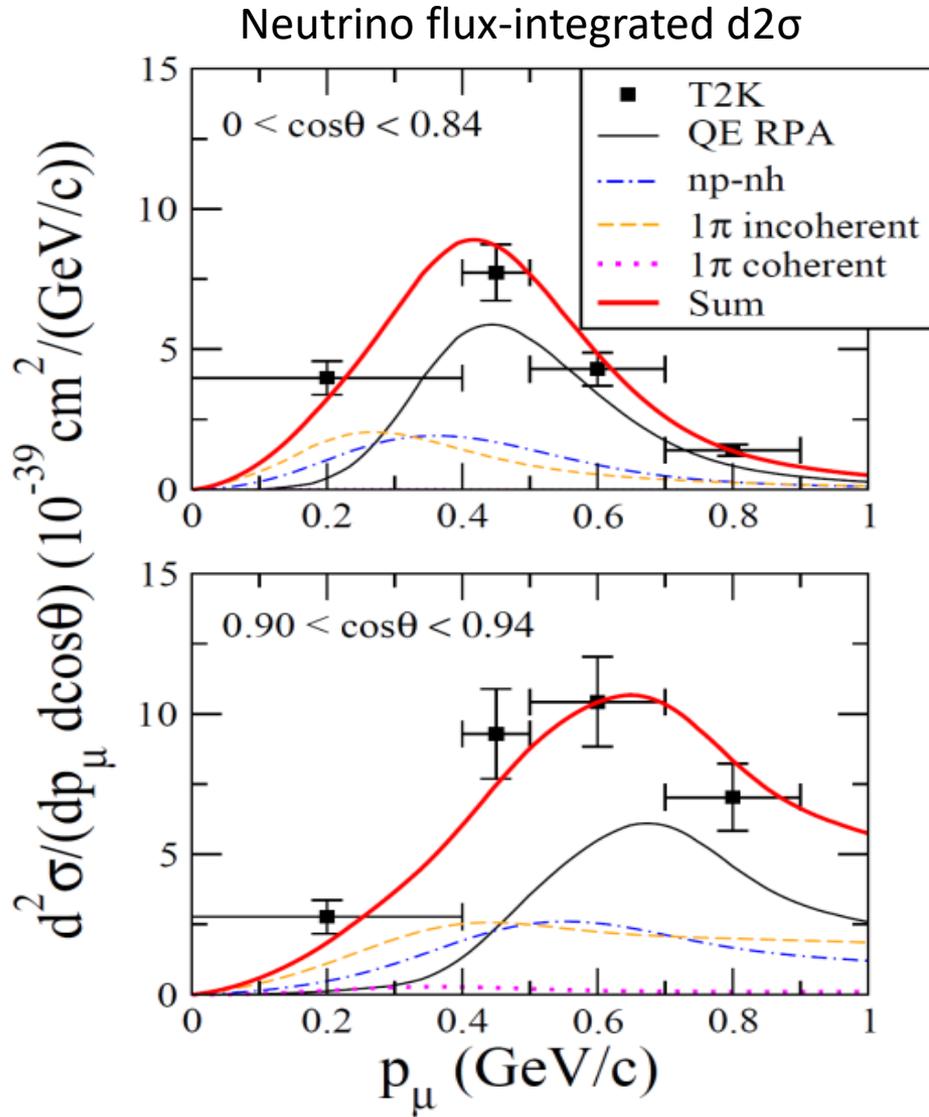
Remind: monochromatic beam

$$\omega_{QE} = \frac{E^2(1 - \cos\theta)}{M_N + E(1 - \cos\theta)}$$

$$\omega_{\Delta} = \frac{M_N\Delta M + E^2(1 - \cos\theta)}{M_N + E(1 - \cos\theta)}$$



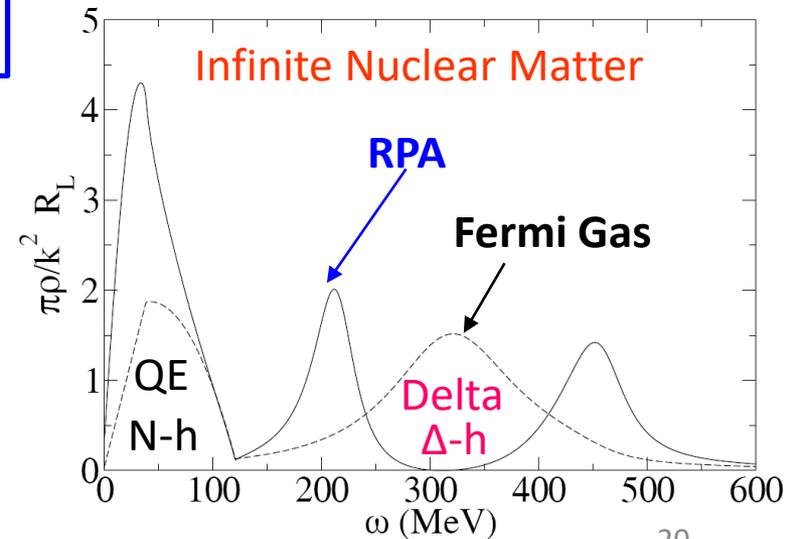
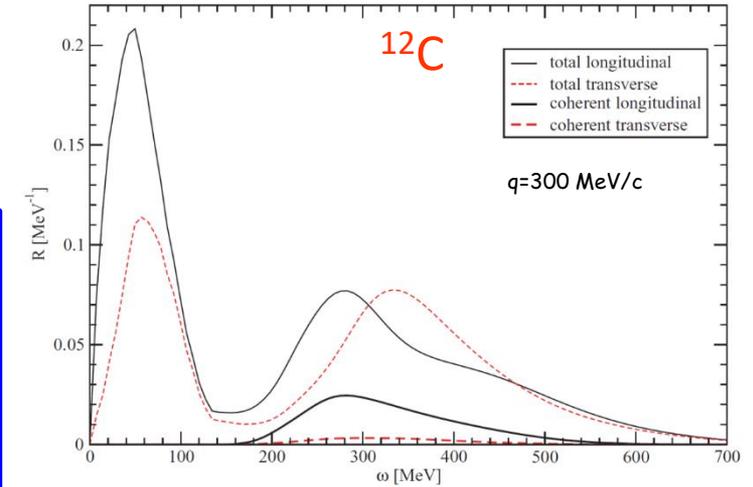
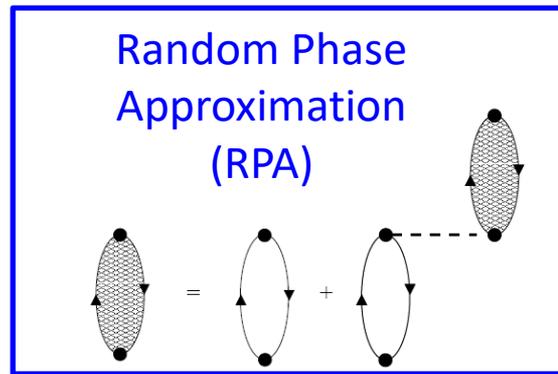
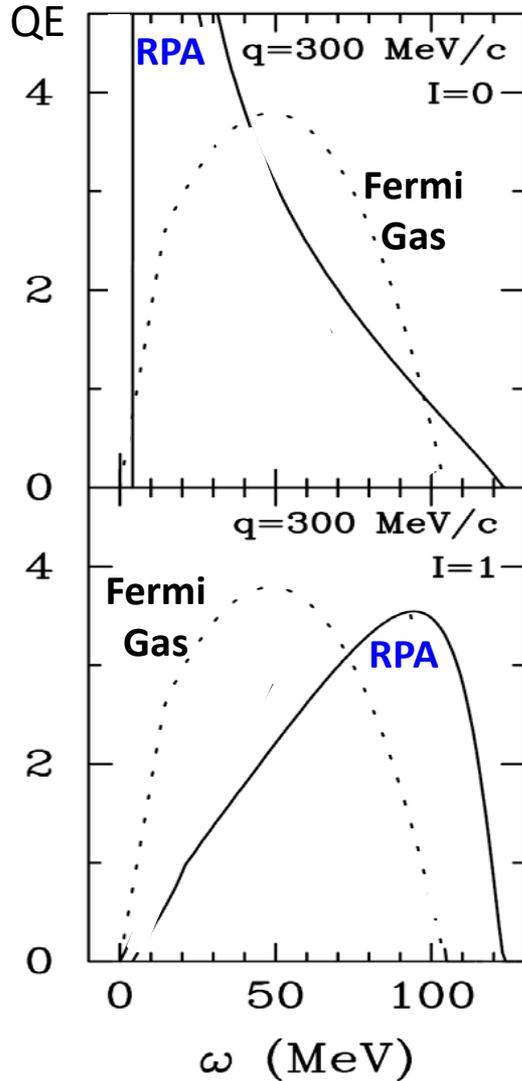
Remark: flux-integrated .vs. monochromatic beam cross sections



In the flux-integrated cross sections the different channels are entangled

The Random Phase Approximation

- External force acting on one nucleon is transmitted to the neighbors by the nucleon interaction – **Long Range Correlations**
- The nuclear response becomes collective
- Shift of the peak with respect to Fermi Gas, decrease, increase depending on the channels of excitation



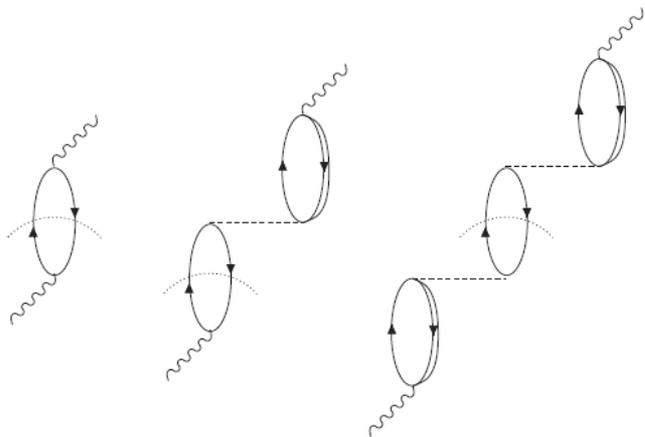
Neutrino scattering - Effects of the RPA in the genuine quasielastic channel

QE totally dominated by isospin spin-transverse response $R_{\sigma\tau(T)}$

RPA reduction

- expected from the repulsive character of p-h interaction in T channel
- also due to interference term $R^{N\Delta} < 0$
(Lorentz-Lorenz or Ericson-Ericson effect [*M.Ericson, T. Ericson, Ann. Phys. 36, 323 (1966)*])

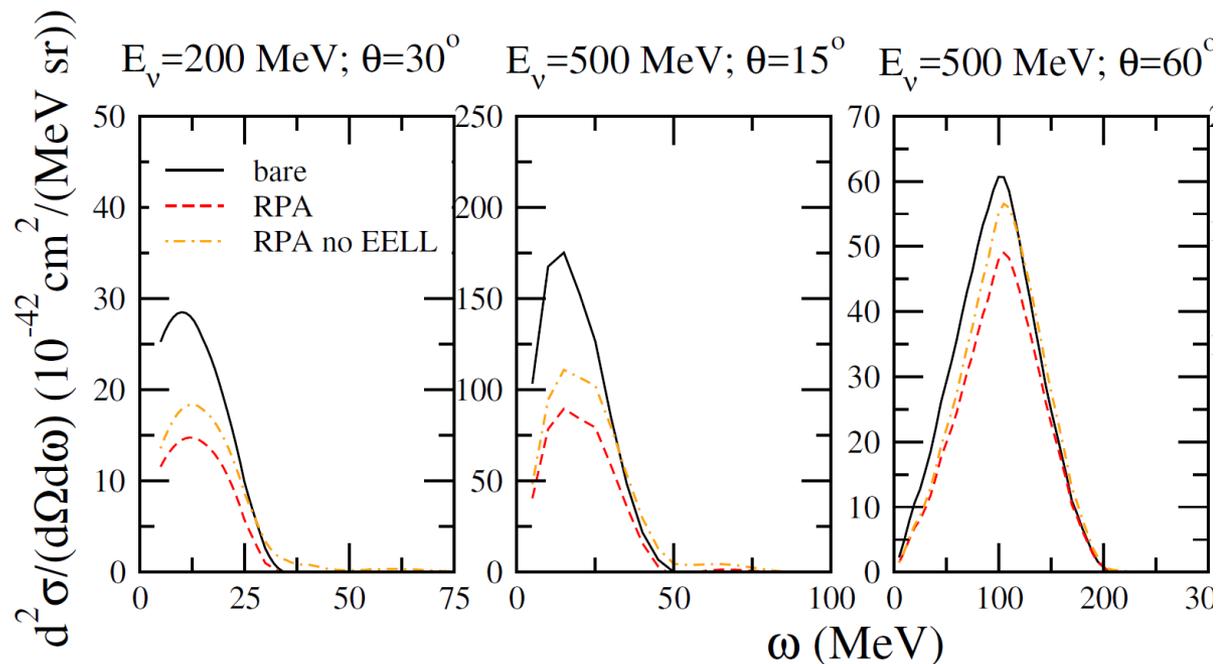
Lowest order contribution to QE:



R_{QE}^{NN}

$R_{QE}^{N\Delta}$

$R_{QE}^{\Delta\Delta}$

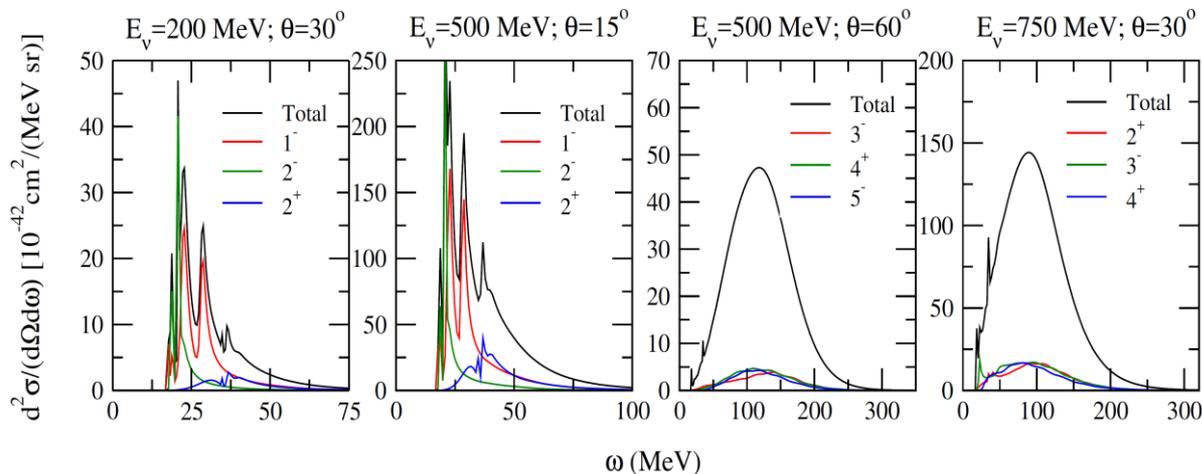


The Hartree Fock + Continuum RPA for giant resonances and QE

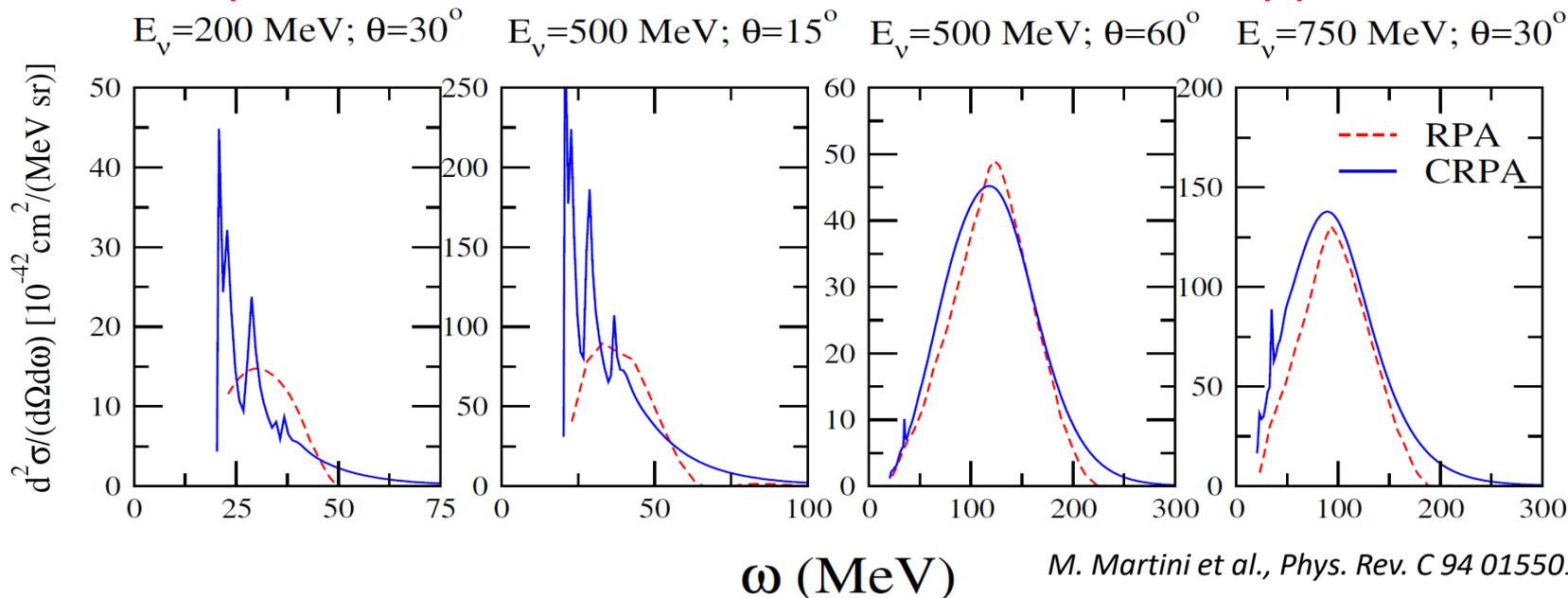
HF+CRPA (Ghent)

Pandey et al. Phys.Rev. C94 054609 (2016)

- Shell effects and giant resonances
- Different multipolar excitations



Comparison between LFG+RPA and HF+CRPA approaches



M. Martini et al., Phys. Rev. C 94 015501 (2016)

- The two approaches are essentially in agreement
- In the low energy part the LFG+RPA results represent the average of the HF+CRPA ones

Several models to calculate the responses and the ν cross sections

- Local Fermi Gas + Random Phase Approximation

Lyon M. Martini, M. Ericson, G. Chanfray, J. Marteau, *Phys. Rev. C* 80 065501 (2009)

Valencia J. Nieves, I. Ruiz Simo, M.J. Vicente Vacas, *Phys. Rev. C* 83 045501 (2011)

- Hartree-Fock + (Continuum) Random Phase Approximation

Ghent V. Pandey, N. Jachowicz, T. Van Cuyck, J. Ryckebusch, M. Martini, *Phys. Rev. C* 92 024606 (2015)

Other groups focused on giant resonances and below Kolbe et al. ; Volpe et al.; Co' et al.; ...

- SuSAv2 superscaling/relativistic mean field

Granada, Madrid, MIT, Sevilla, Torino

G.D. Megias, J.E. Amaro, M.B. Barbaro, J.A. Caballero, T.W. Donnelly, I. Ruiz Simo, *PRD* 94 093004 (2016)

I. Ruiz Simo, J. E. Amaro, M. B. Barbaro, A. De Pace, J. A. Caballero, T. W. Donnelly, *JPG44* 065105 (2017)

- Spectral function approach

Roma N. Rocco, C. Barbieri, O. Benhar, A. De Pace, A. Lovato, *Phys. Rev. C* 99 025502 (2019)

- Relativistic Green's function

Pavia A. Meucci, C. Giusti, F. D. Pacati, *Nucl.Phys.A* 739 277-290 (2004)

- Green's function Monte Carlo ("ab initio")

Argonne, Los Alamos A. Lovato, J. Carlson, S. Gandolfi, N. Rocco, R. Schiavilla, *PRX* 10 031068 (2020)

- GiBUU transport theory

Giessen O. Buss, T. Gaitanos, K. Gallmeister, H. van Hees, M. Kaskulov, O. Lalakulich, A.B. Larionov, T. Leitner, J. Weil, U. Mosel, *Phys.Rept.* 512 1-124 (2012)

p.s. only one representative reference for each approach (not necessarily the founding paper)

For discussions and comparisons of different models see for example:

- G.T. Garvey, D.A. Harris, H.A. Tanaka, R. Tayloe, G.P. Zeller, *Phys.Rept.* 580 (2015) 1-45
- T. Katori, M. Martini, *J.Phys.G* 45 (2018) 1, 013001
- M. Sajjad Athar, A. Fatima, S. K. Singh, *Prog.Part.Nucl.Phys.* 129 (2023) 104019

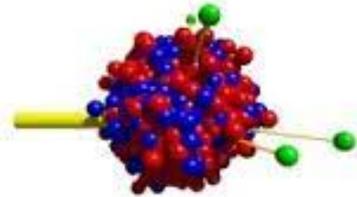
Monte Carlo Event Generators

Monte Carlo event generators connects theoretical models to experimental measurements

Main Event Generators for neutrino interactions:



GiBUU



NEUT



L. Alvarez-Ruso et al.,
EPJ Spec. Top. 230, 4449 (2021)

O. Buss et al.,
Phys.Rept. 512 1-124 (2012)

Y. Hayato and L. Pickering,
EPJ Spec. Top. 230, 4469 (2021)

T. Golan et al.,
NPB 229–232, 499 (2012)

PHYSICAL REVIEW D **105**, 092004 (2022)

Comparisons and challenges of modern neutrino-scattering experiments

M. Buizza Avanzini¹, M. Betancourt², D. Cherdack³, M. Del Tutto^{2,4}, S. Dytman⁵, A. P. Furmanski^{6,7},
S. Gardiner², Y. Hayato⁸, L. Koch⁹, K. Mahn¹⁰, A. Mastbaum¹¹, B. Messerly^{5,7}, C. Riccio^{12,13},
D. Ruterbories¹⁴, J. Sobczyk¹⁵, C. Wilkinson¹⁶ and C. Wret¹⁴

Main models implemented for the quasielastic (and 2p-2h):

- Relativistic global and local Fermi Gas
- RPA
- SuperScaling (SuSAv2)
- Spectral Function

SuperScaling

- The basic idea of the approach [J.E. Amaro et al., PRC71 (2005) 015501] is to exploit electron scattering in order to predict the neutrino scattering cross section based on the “superscaling” properties of inclusive electron scattering data, extensively analysed in the 90s [Day et al., Ann.Rev.Nucl.Part.Sci.40 (1990); Donnelly and Sick, PRL82; PRC60 (1999)]

- Extract a **SuperScaling function** from electron scattering inclusive data

$$f(q, \omega; k_F) = k_F \times \frac{[d^2\sigma/d\omega d\Omega]_{exp}^{(e,e')}}{\bar{\sigma}_{eN}}$$

- Plot it as function of a **Scaling variable** which is a combination of q and ω

$$\psi \equiv \psi(q, \omega; k_F)$$

- SuperScaling** is realized if:

$$\psi(q, \omega; k_F) \longrightarrow f(\psi)$$

- f is independent of the kinematics (q) for a given nucleus (scaling of first kind)
- f is independent of the nucleus (k_F) for given kinematics (scaling of second kind)

The SuperScaling function f is a universal function encoding the nuclear dynamics.

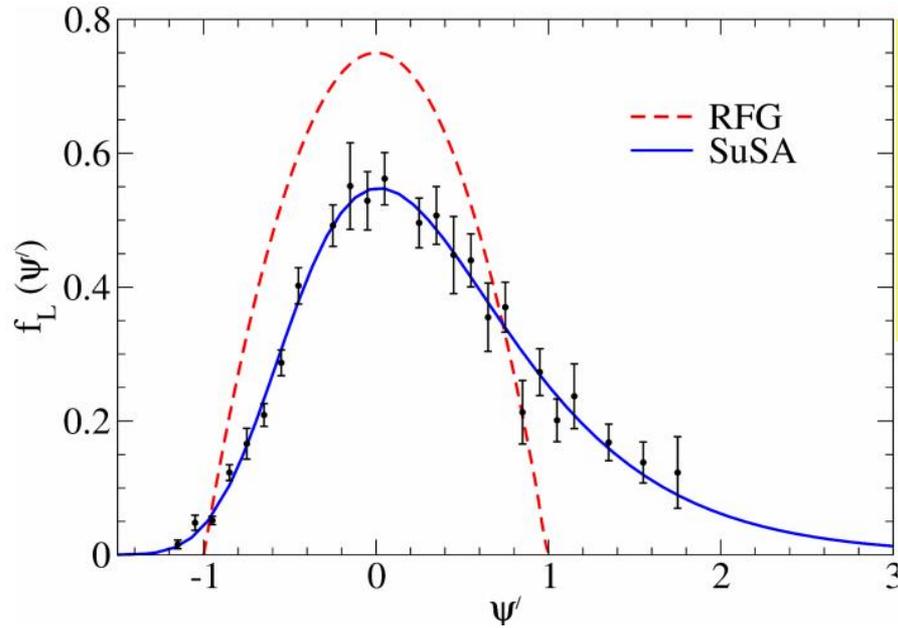
It can be extracted from electron scattering experiment or calculated within a model.

- Final step: Use the SuperScaling function to predict the **neutrino cross sections**

$$[d^2\sigma/d\omega d\Omega]^{(\nu,l)} = \frac{1}{k_F} \bar{\sigma}_{\nu N} f(\psi)$$

The SuSA and SuSAv2 models in the quasielastic region

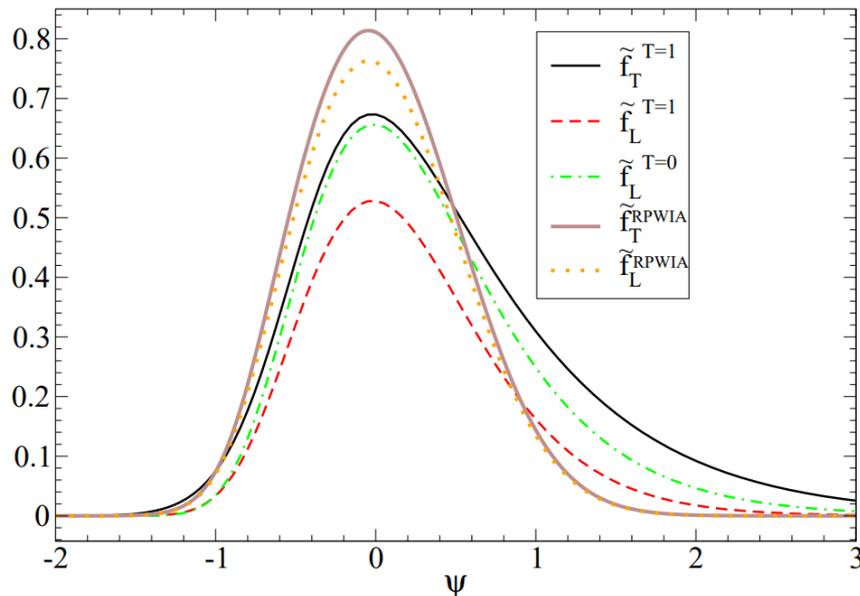
The scaling function(s) are used to describe simultaneously electron and neutrino scattering



SuSA model - phenomenological

J.E. Amaro et al., PRC71 (2005) 015501

- One scaling function extracted from longitudinal inclusive (e,e') data



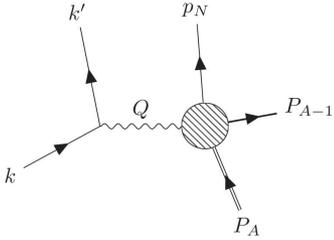
SuSAv2 model - microscopic

R. Gonzalez-Jimenez et al., PRC90 (2014) 035501

- Based on Relativistic Mean Field calculation
- A set of scaling functions in L,T and isospin channels

The Spectral Function

- The spectral function $S(E_m, \mathbf{p}_m)$ represents the joint probability of removing a nucleon of given momentum \mathbf{p}_m from the nuclear ground state A leaving the residual nucleus A-1 in a state characterized by missing energy E_m



Missing Energy

$$E_m = \omega - T_N - T_{A-1}$$

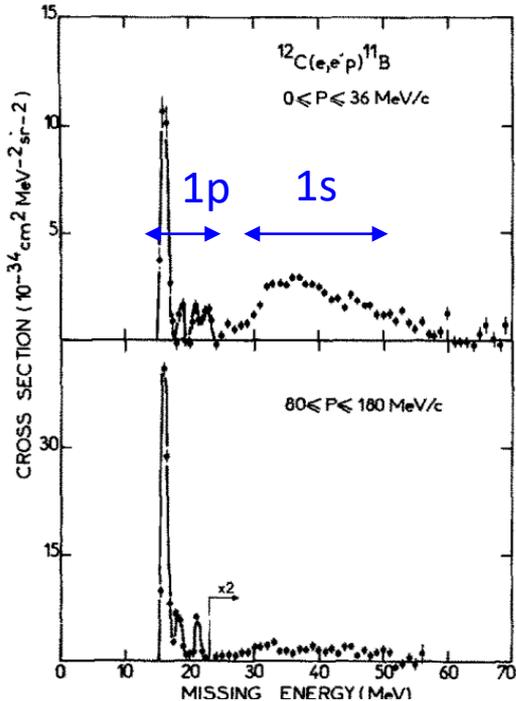
Missing momentum

$$\mathbf{p}_m = \mathbf{q} - \mathbf{p}_N = \mathbf{p}_{A-1} \quad \text{recoil momentum}$$

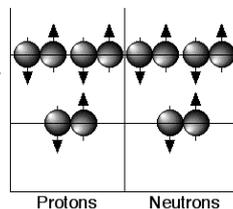
p.s. Often in literature the sign is opposite: $\mathbf{p}_m = \mathbf{p}_N - \mathbf{q} = -\mathbf{p}_{A-1}$

$$\frac{d\sigma}{d\epsilon' d\Omega_e dT' d\Omega_p} = K \left(\frac{d\sigma}{d\Omega_e} \right)_{ep} S(E, \mathbf{P})$$

J. Mougey et al, Nucl. Phys. A 262 (1976)



$1p_{3/2}$
 $1s_{1/2}$



Protons Neutrons

470

J. MOUGEY et al.

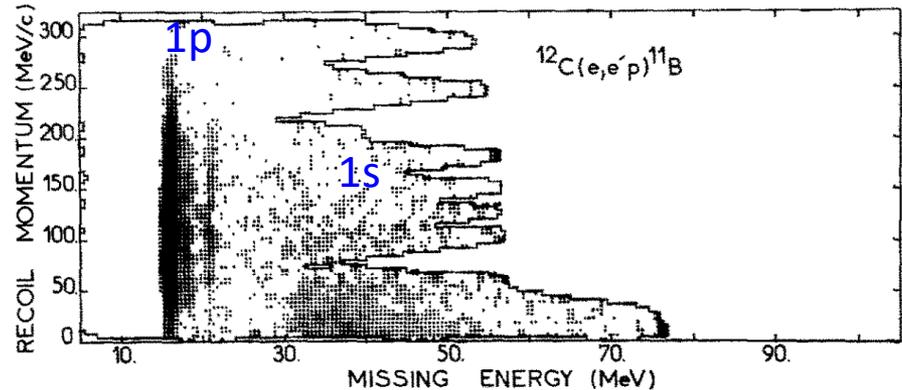
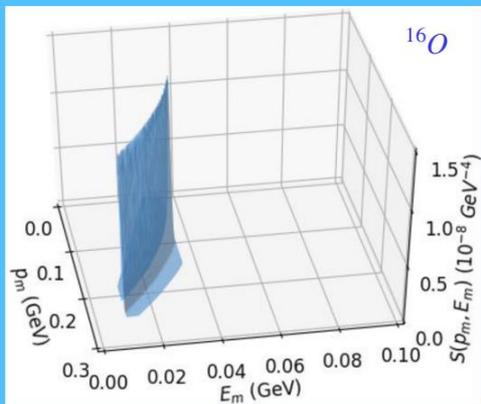


Fig. 8. Cross section $\sigma(E, P)$ for ^{12}C , showing the 1p shell at 17 MeV and the 1s shell around 38 MeV.

Different ^{16}O Theoretical Spectral Functions

RFG

$$S_{RFG}(p_m, E_m) = \theta(p_F - p_m) \delta\left(E_m - \sqrt{p_m^2 + m_N^2}\right)$$



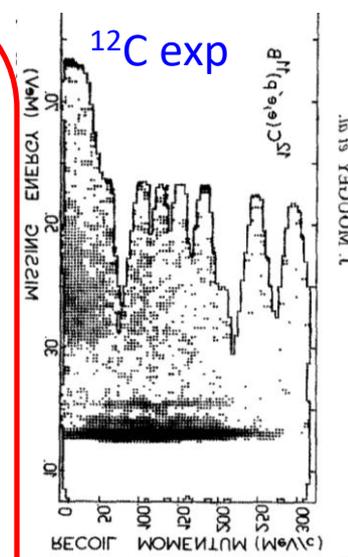
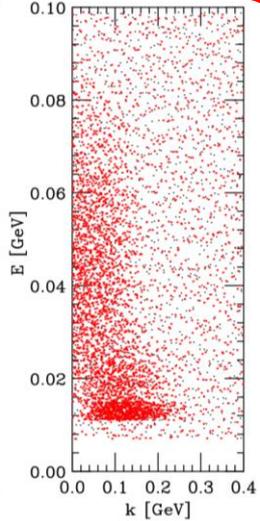
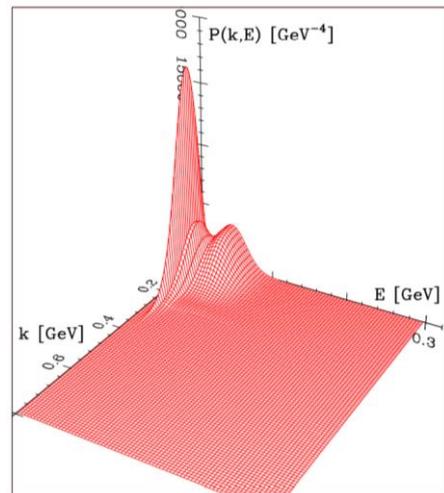
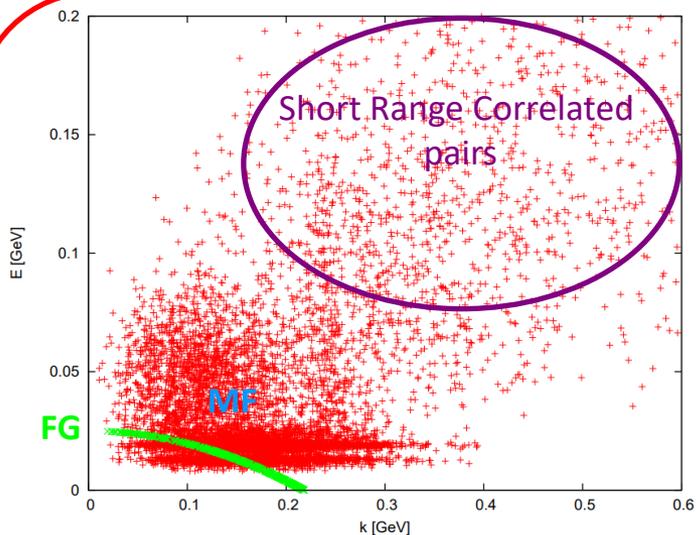
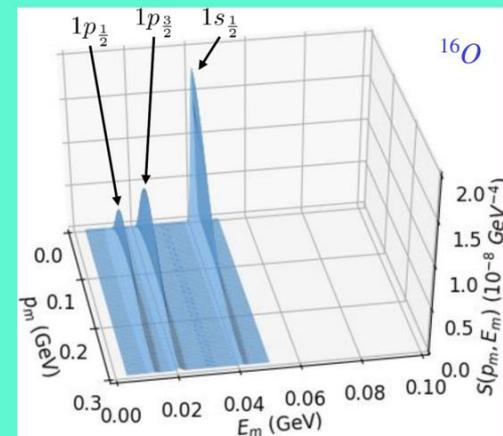
Independent-particle models

J. M. Franco Patino et al, PRC 102 064626 (2020)

Figures from M. B. Barbaro talk @NUFACT 2021

IPSM/RMF

$$S_{IPSM}(p_m, E_m) = \sum_{nlj} (2j+1) n_{nlj}(p_m) \delta(E_m - E_{nlj})$$



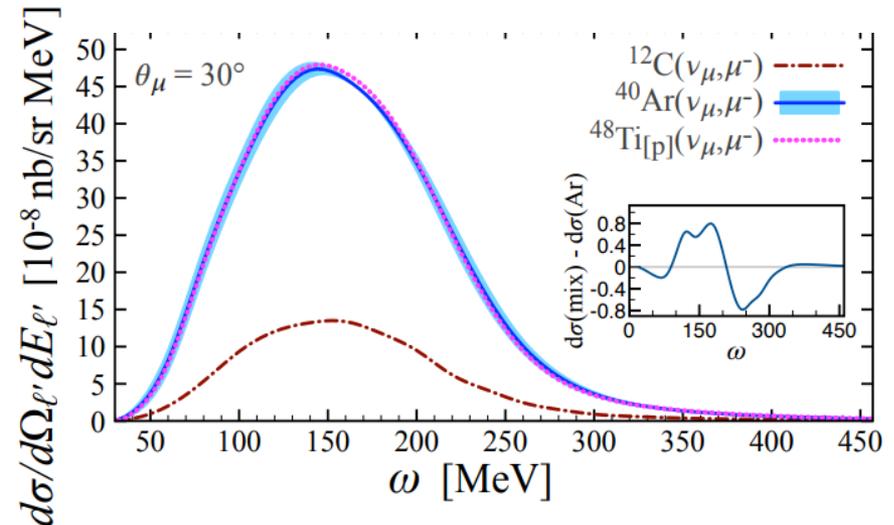
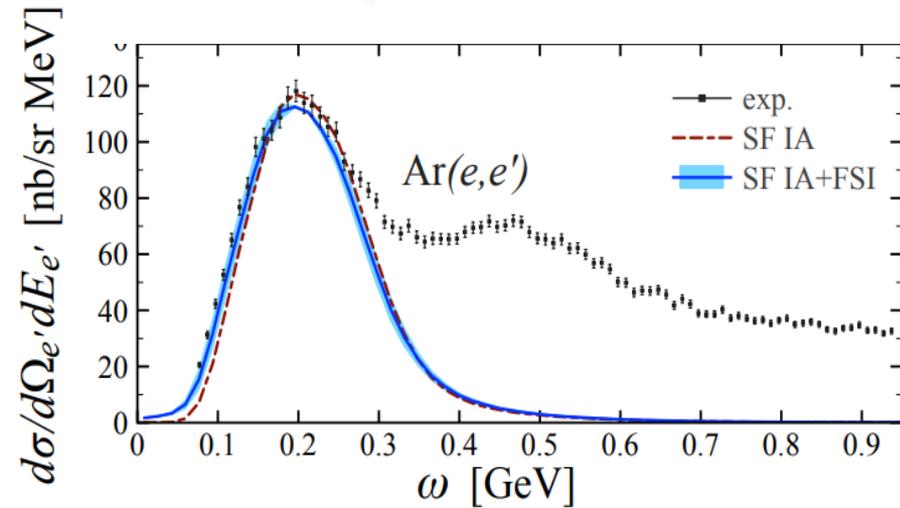
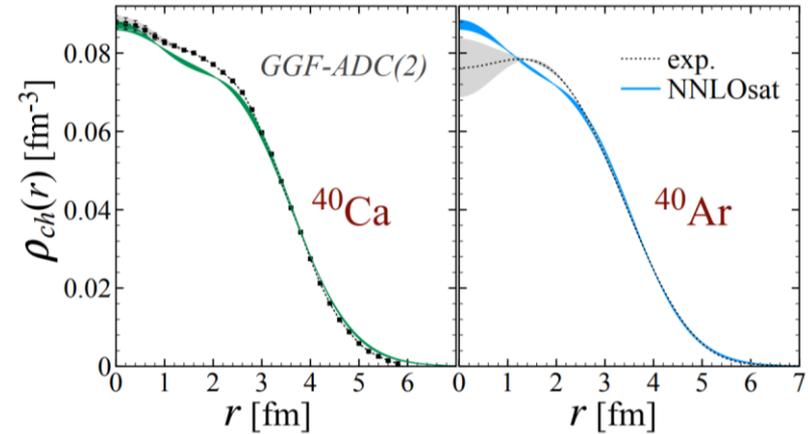
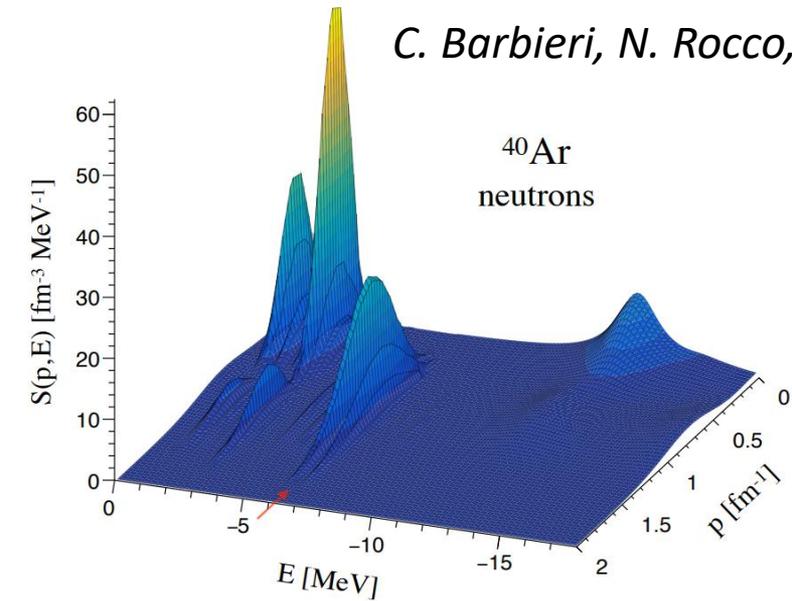
Correlated Basis Functions $S_{CBF}(p_m, E_m) = S_{MF}(p_m, E_m) + S_{corr}(p_m, E_m)$

O. Benhar, A. Fabrocini, and S. Fantoni, Nucl. Phys. A505, 267 (1989)

Figures from N. Rocco talk @ESNT-CEA workshop 2016

Ab-initio self-consistent Green's function calculation of ^{40}Ar Spectral function

C. Barbieri, N. Rocco, V. Somà, *Phys.Rev.C* 100 (2019) 6, 062501



Extension of the calculation including two-body spectral function and two-body current contributions would be very important

Models .vs. Data: CCQE, CCQE-like and $CC0\pi$

MiniBooNE CC Quasielastic cross section on Carbon and the M_A puzzle

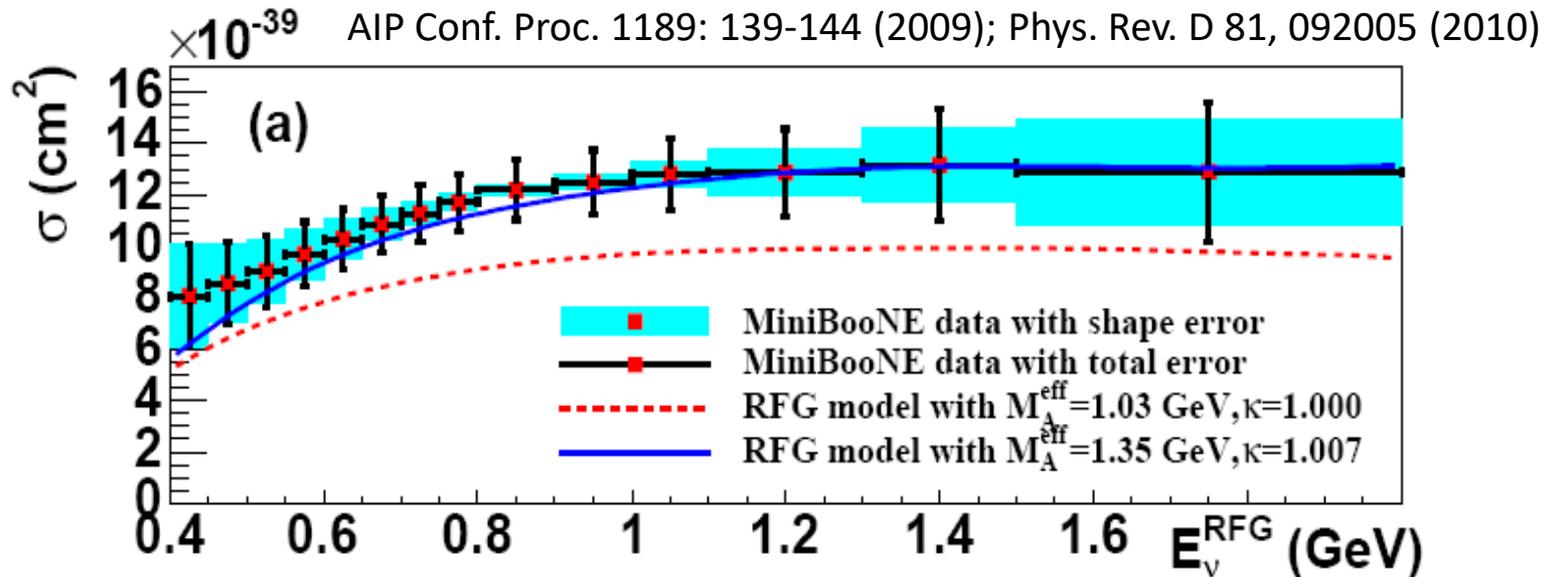
First Measurement of Muon Neutrino Charged Current Quasielastic (CCQE) Double Differential Cross Section

Cite as: AIP Conference Proceedings 1189, 139 (2009); <https://doi.org/10.1063/1.3274144>
Published Online: 02 December 2009

Tepei Katori and MiniBooNE collaboration

PHYSICAL REVIEW D 81, 092005 (2010)

First measurement of the muon neutrino charged current quasielastic double differential cross section

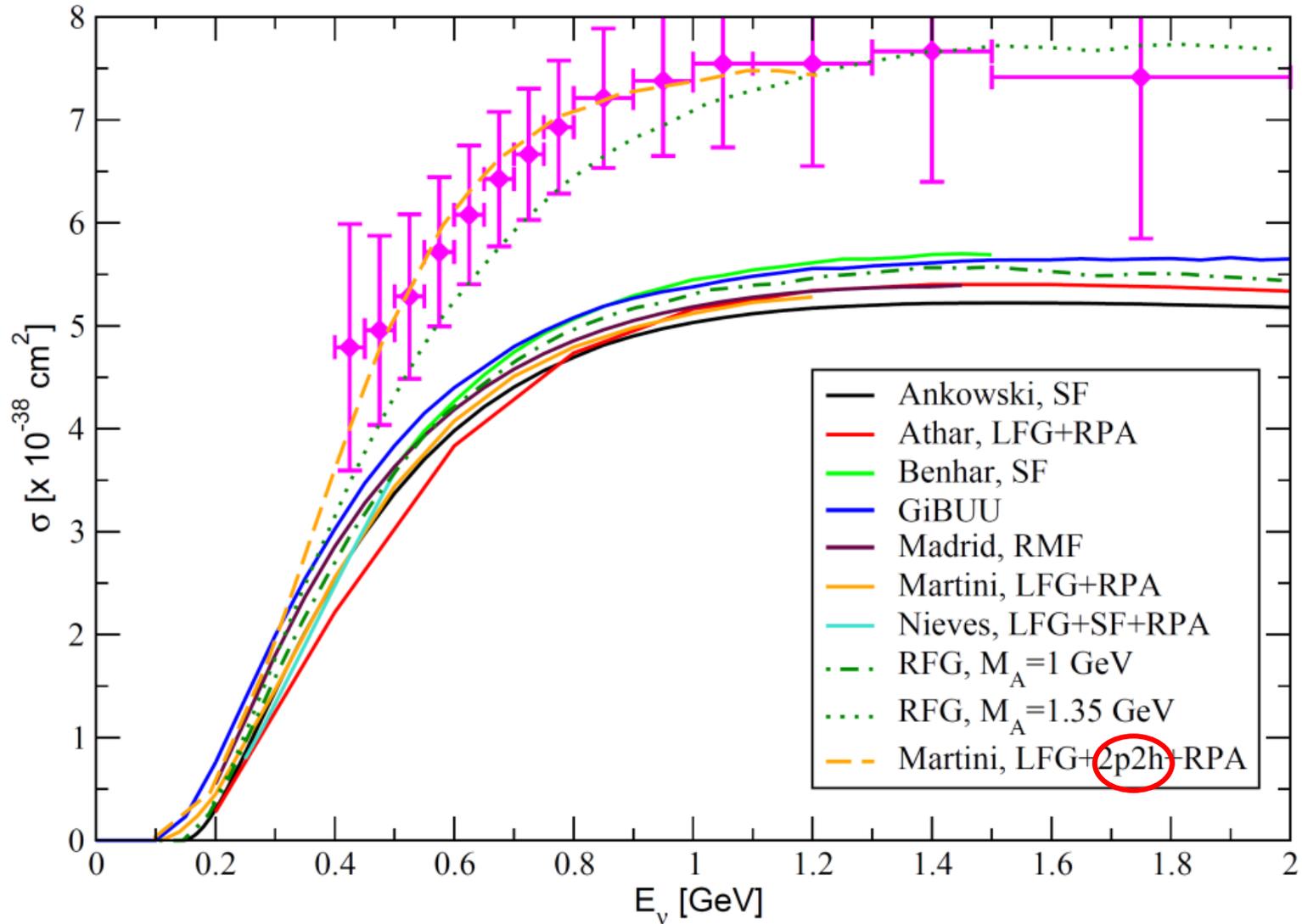


Comparison with a prediction based on Relativistic Fermi Gas (RFG) using $M_A=1.03$ GeV (standard value) reveals a discrepancy

In the Relativistic Fermi Gas (RFG) model an axial mass of **1.35 GeV** is needed to account for data **puzzle??**

Comparison of different theoretical models for Quasielastic

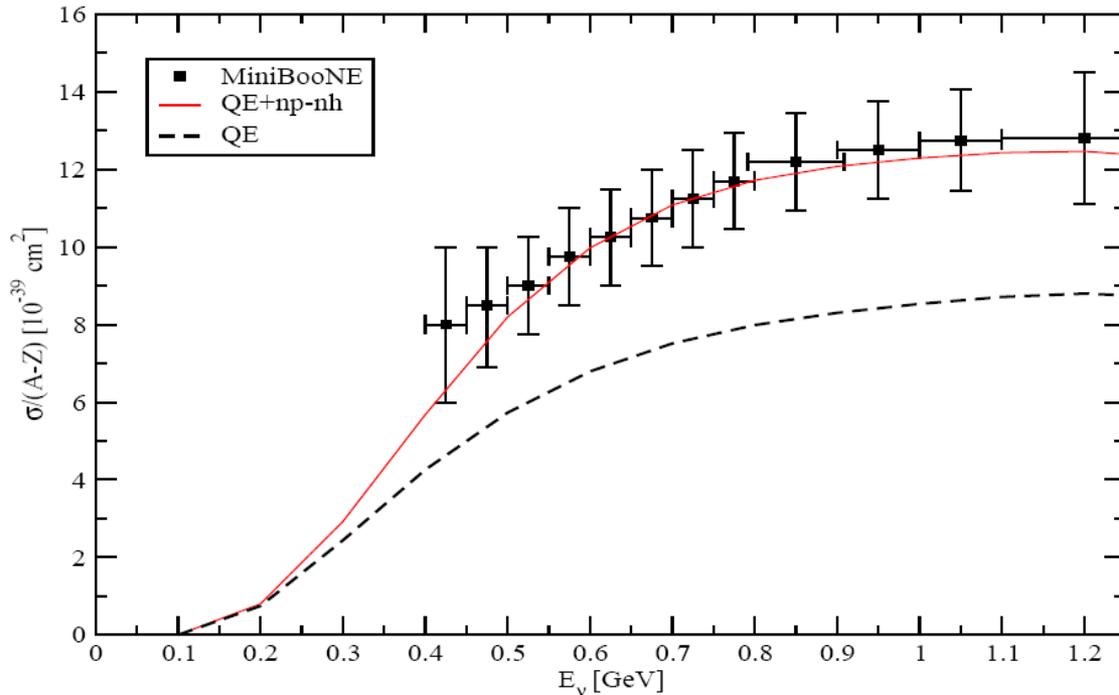
L. Alvarez-Ruso , arXiv:1012.3871 (Neutrino 2010)



puzzle??

An explanation of this puzzle

Inclusion of the multinucleon emission channel (np-nh = 2p-2h + 3p-3h)

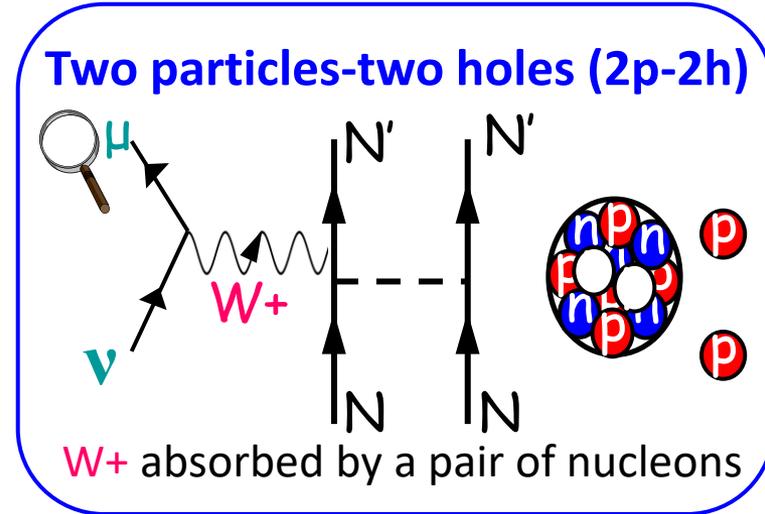
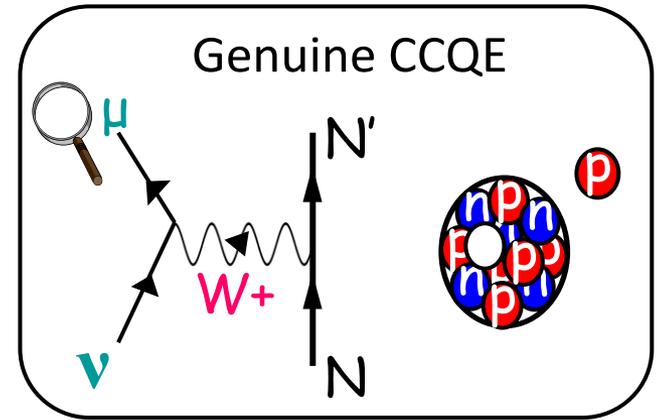


CCQE-like = Genuine CCQE + np-nh

M. Martini, M. Ericson, G. Chanfray, J. Marteau, Phys. Rev. C 80 065501 (2009)

Agreement with MiniBooNE without increasing M_A

➔ MiniBooNE measured CCQE-like, not genuine CCQE

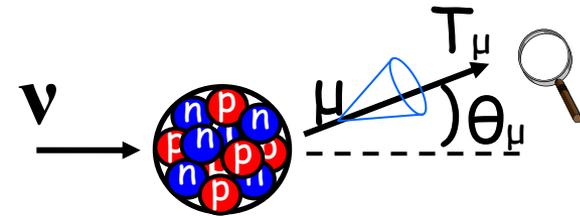
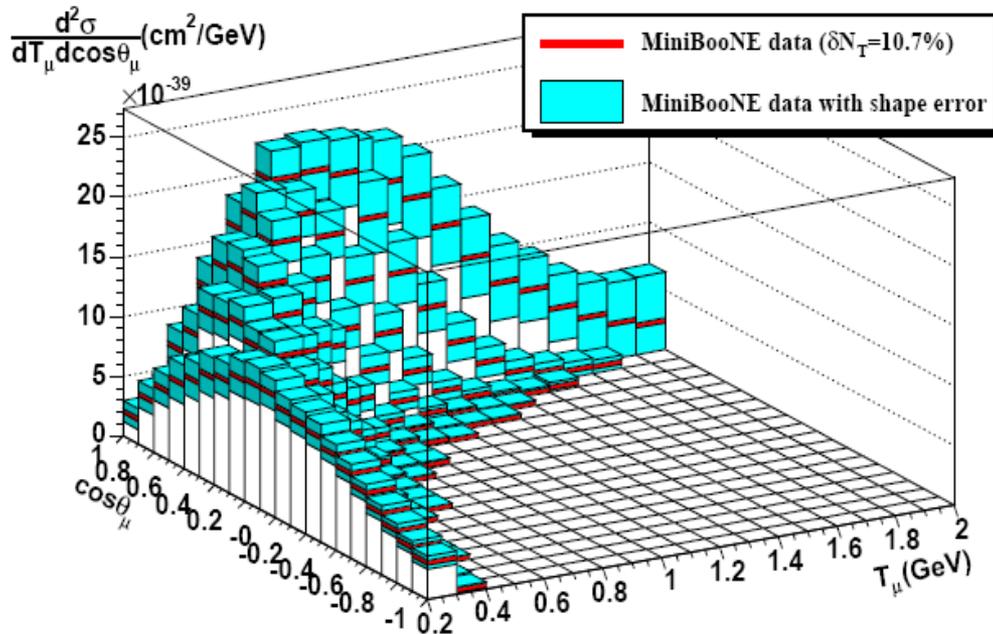


Flux-integrated double differential cross section

$$\left(\frac{d^2\sigma}{dT_l \cos\theta} \right)_i = \frac{\sum_j U_{ij}(d_j - b_j)}{\Phi \cdot T \cdot \epsilon_i \cdot (\Delta T_l, \Delta \cos\theta)_i}$$

PHYSICAL REVIEW D **81**, 092005 (2010)

First measurement of the muon neutrino charged current quasielastic double differential cross section

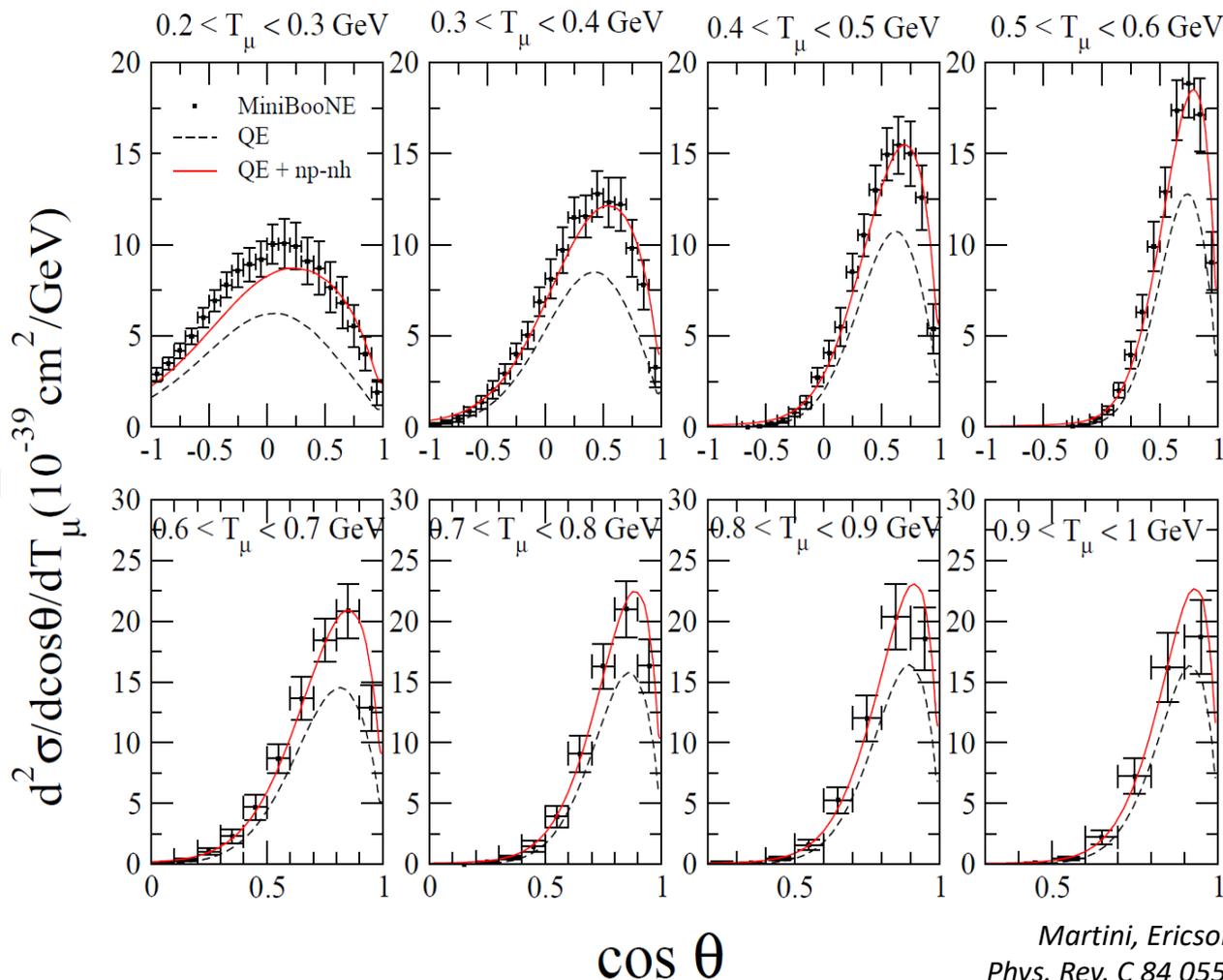


- Function of two measured variables
- Less model dependent than $\sigma(E_\nu)$: free from the neutrino energy reconstruction problem (see later)
- Flux dependent

Flux-integrated differential cross section is where theorists and experimentalists meet for ν interaction

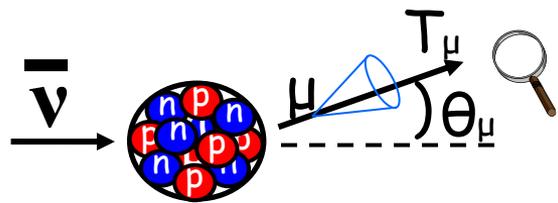
MiniBooNE CCQE-like flux-integrated double differential cross section

$$\frac{d^2\sigma}{dT_l d\cos\theta} = \frac{1}{\int \Phi(E_\nu) dE_\nu} \int dE_\nu \left[\frac{d^2\sigma}{d\omega d\cos\theta} \right]_{\omega=E_\nu-E_l} \Phi(E_\nu)$$

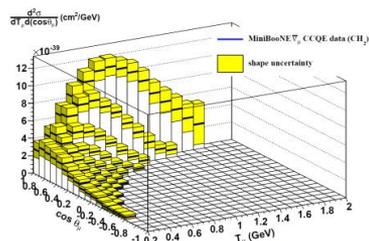


- Good agreement with data once multinucleon contributions are included
- Similar conclusions obtained by different theoretical calculations (see later)

MiniBooNE CCQE-like flux-integrated double differential cross section

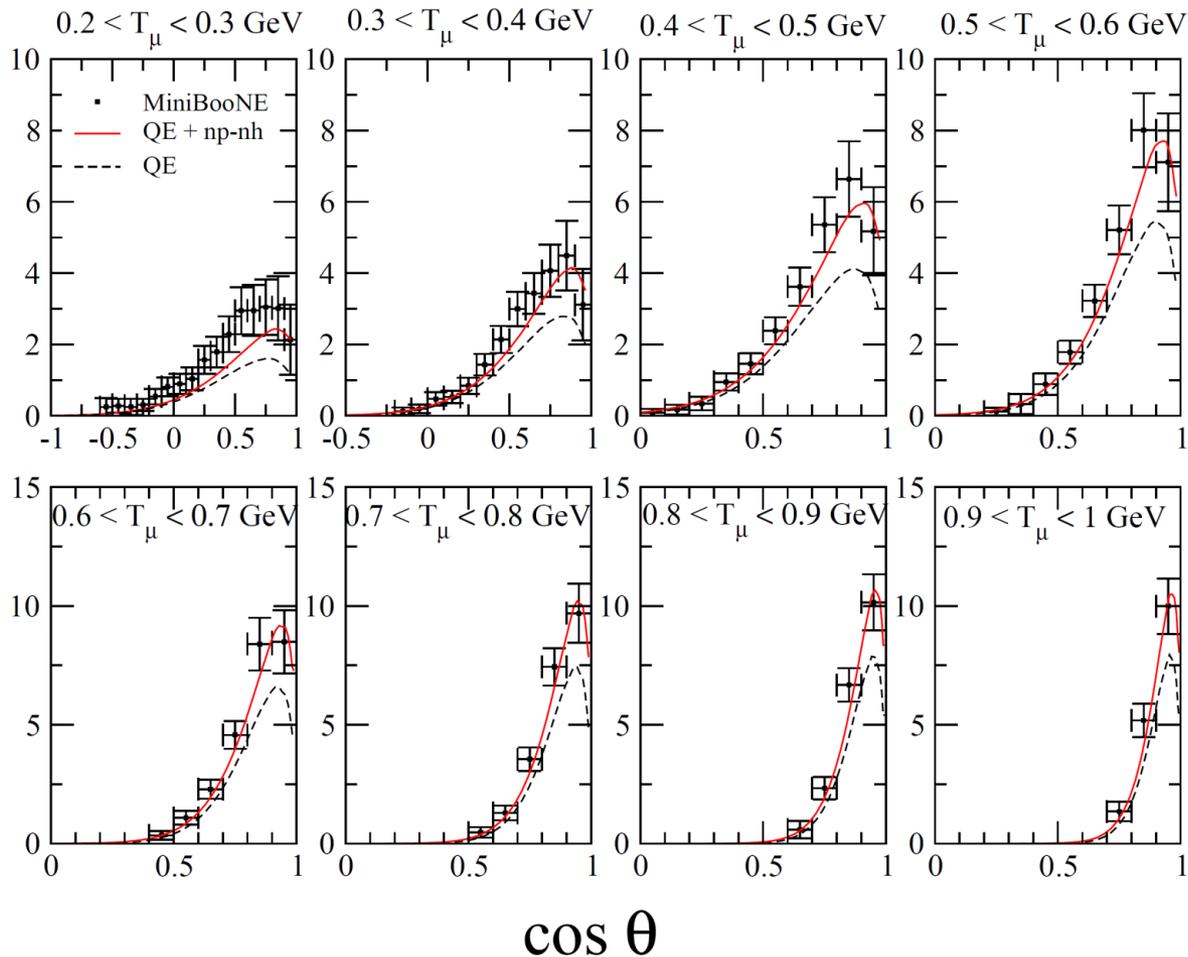


$\bar{\nu}$



MiniBooNE, *Phys. Rev. D* 88 032001 (2013)

$d^2 \sigma / d \cos \theta / d T_{\mu} (10^{-39} \text{ cm}^2 / \text{GeV})$



Martini, Ericson, *Phys. Rev. C* 87 065501 (2013)

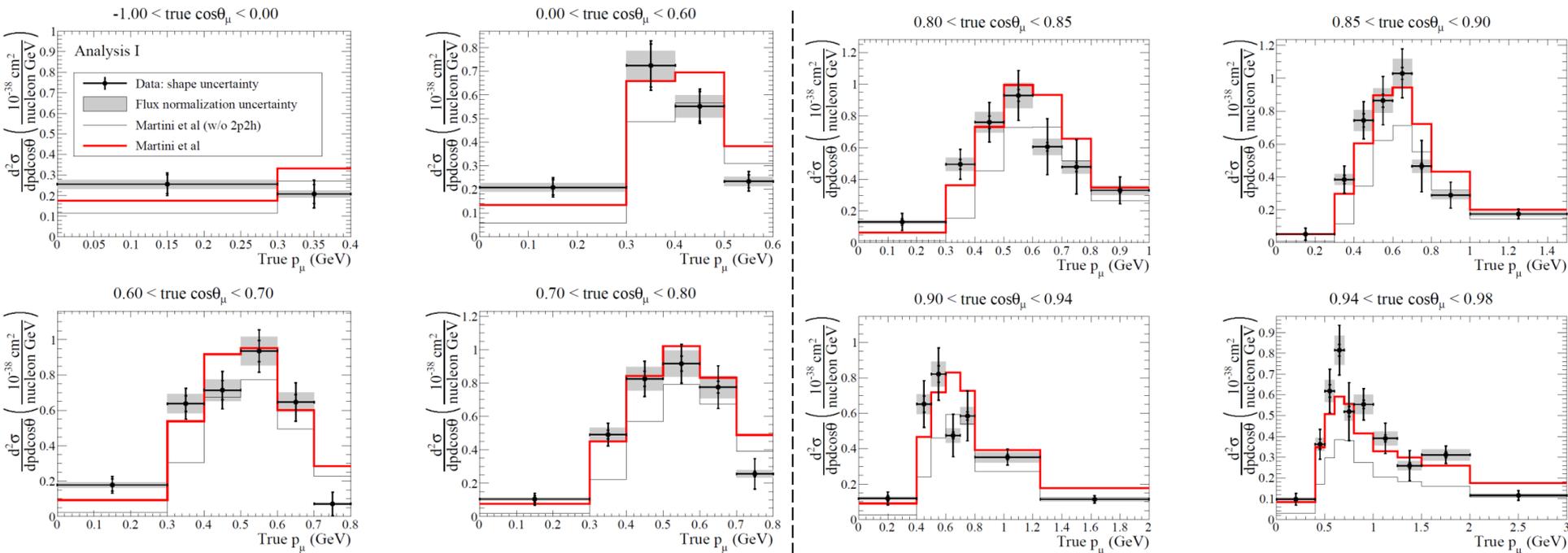
Similar conclusion also for the MiniBooNE CCQE-like antineutrino cross sections

The $CC0\pi$ measurement

After MiniBooNE, it has become more popular to present the data in terms of **final state** particles
 $CC0\pi = CCQE$ -like without subtraction of π absorption background ($CC0\pi \geq CCQE$ -like)

PHYSICAL REVIEW D **93**, 112012 (2016)

Measurement of double-differential muon neutrino charged-current interactions on C_8H_8 without pions in the final state using the T2K off-axis beam



— Including np - nh
 — Without np - nh

Better agreement including np - nh

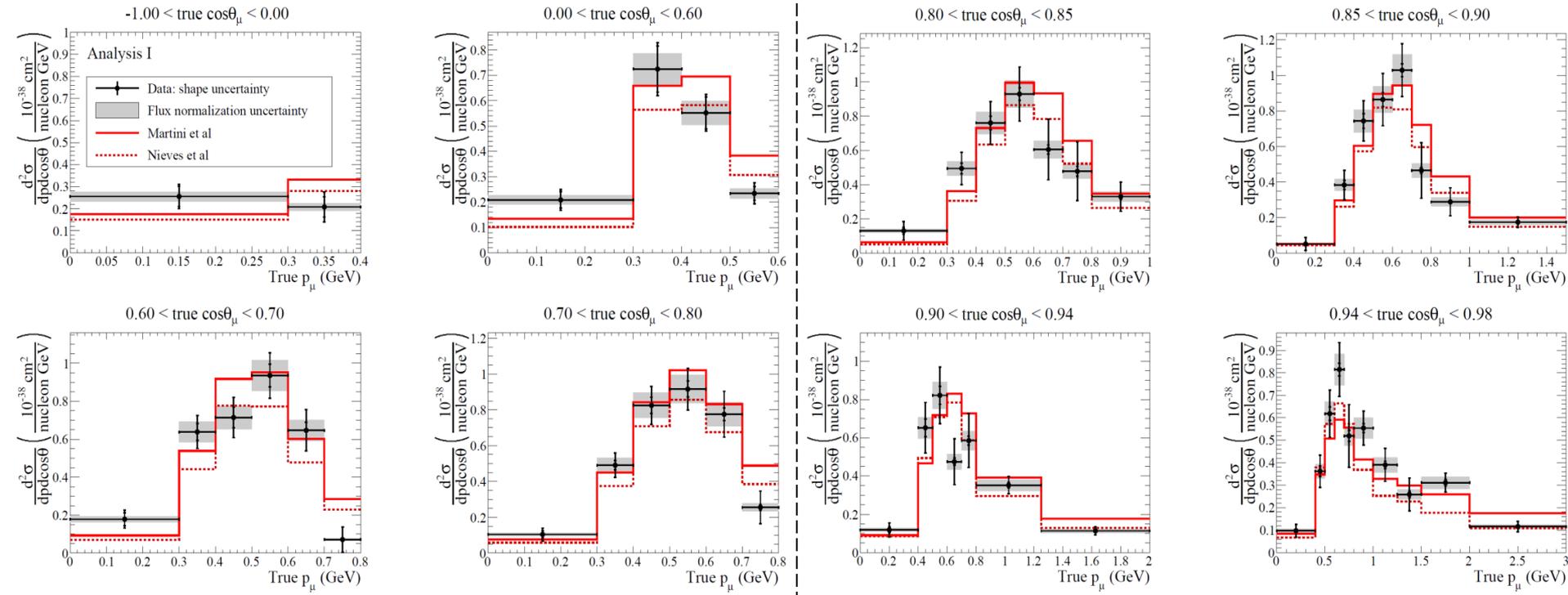
The $CC0\pi$ measurement

After MiniBooNE, it has become more popular to present the data in terms of final state particles

$CC0\pi = CCQE$ -like without subtraction of π absorption background

PHYSICAL REVIEW D **93**, 112012 (2016)

Measurement of double-differential muon neutrino charged-current interactions on C_8H_8 without pions in the final state using the T2K off-axis beam

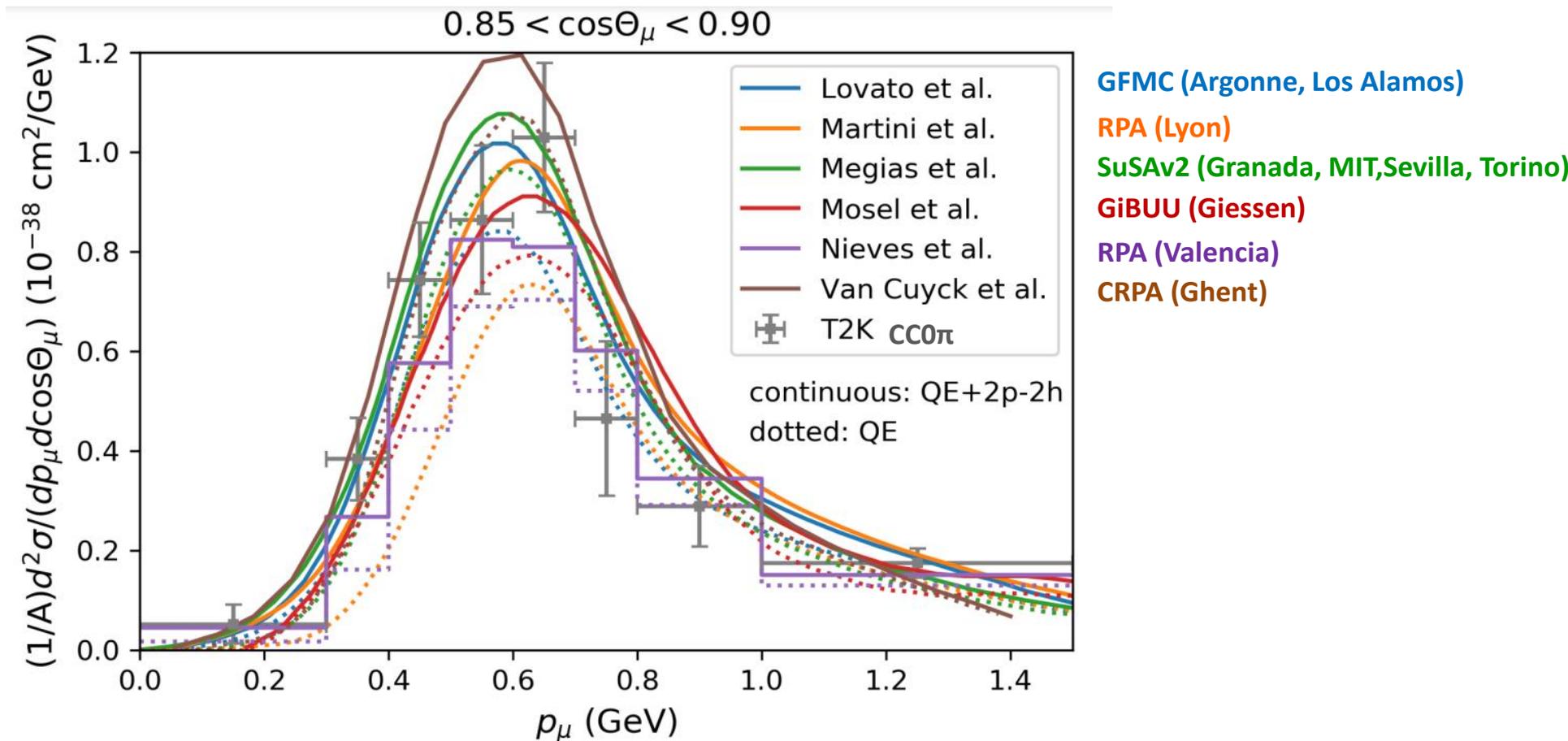


— Martini et al.
..... Nieves et al.

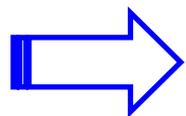
Differences between models' predictions

Comparison between different CCQE+2p-2h theoretical predictions

A. Branca et al. *Symmetry* 13 (2021) 9, 1625



Several theoretical calculations agree on the crucial role of 2p-2h to reproduce data but there are discrepancies between the different models' predictions



2p-2h are one of the most important source of the cross section uncertainties (systematic errors in oscillation experiments)

Some details on 2p-2h

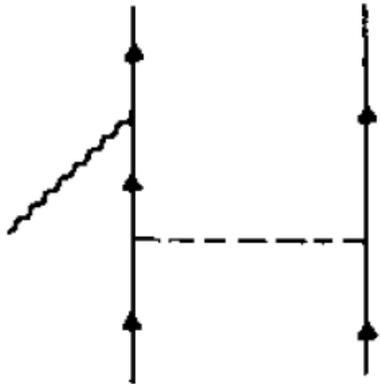
See also 2016 ESNT workshop

[Two-body current contributions in neutrino-nucleus scattering \(cea.fr\)](#)

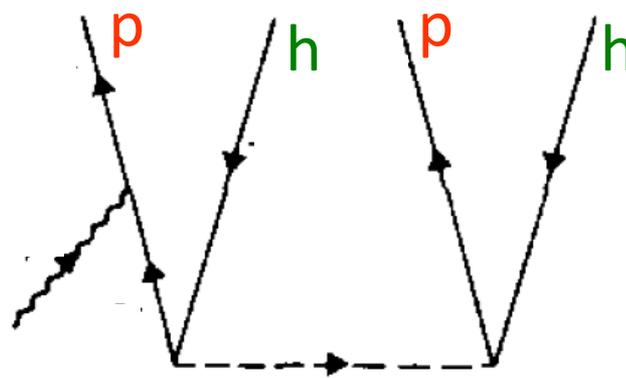
Two particle-two hole sector (2p-2h)

Three equivalent representations of the same process

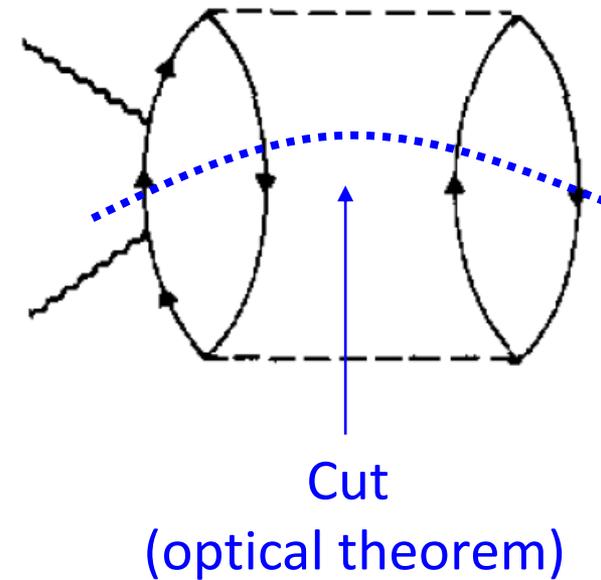
2 body current



2p-2h matrix element



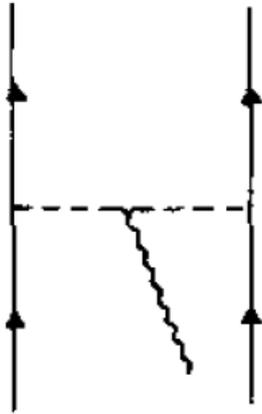
2p-2h response



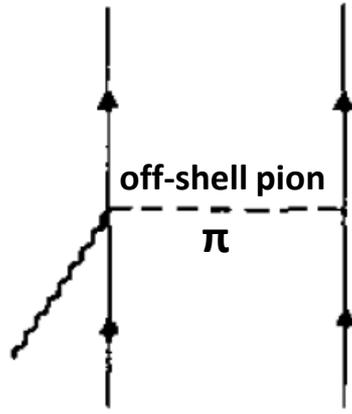
Final state: two particles-two holes

Diagrams for 2 body currents

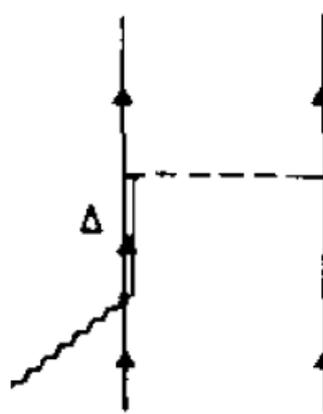
Meson Exchange Currents (MEC) J^{MEC}



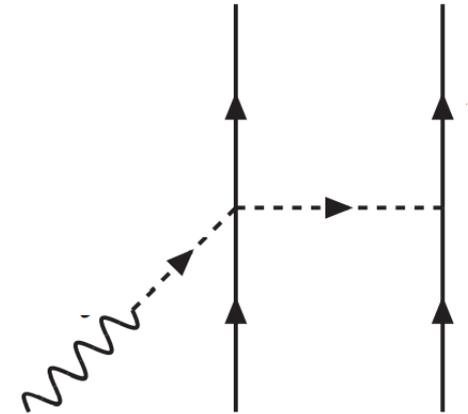
Pion in flight



Seagull or
Contact

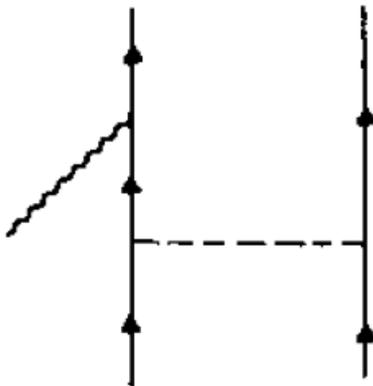


Delta



Pion pole
(purely axial)

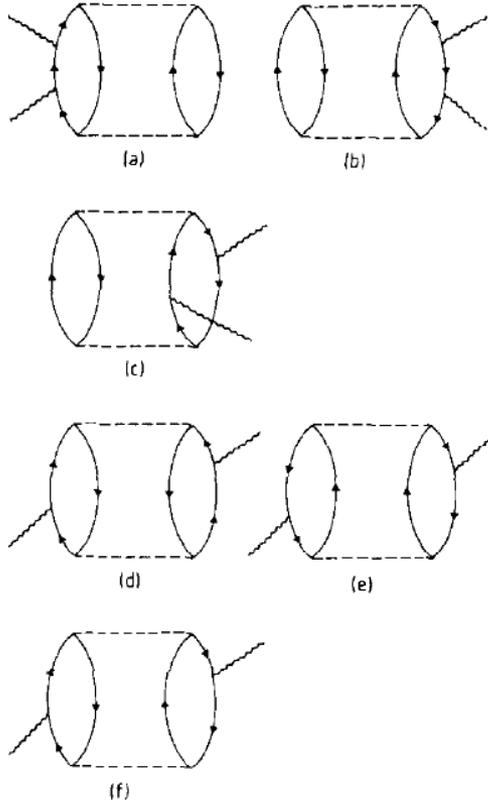
Nucleon-Nucleon Correlations (SRC) J^{corr}



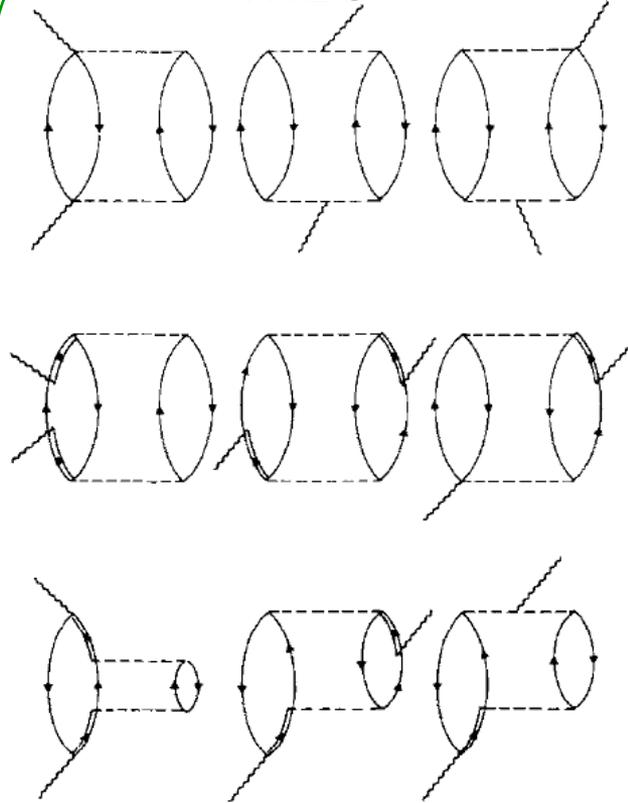
- An additional two-body current to be included in the framework of independent particle models for QE such as the Fermi Gas or Hartree-Fock.
- Absent in the approaches which start from the description of the nucleus in terms of correlated wave functions (such as CBF spectral function or GFMC) since the hadronic tensor of the one body current already includes this contribution.
- **There is a risk of a double counting of SRC in the Monte Carlo if different contributions to the neutrino cross sections are taken from different models.**

Some diagrams for 2p-2h responses

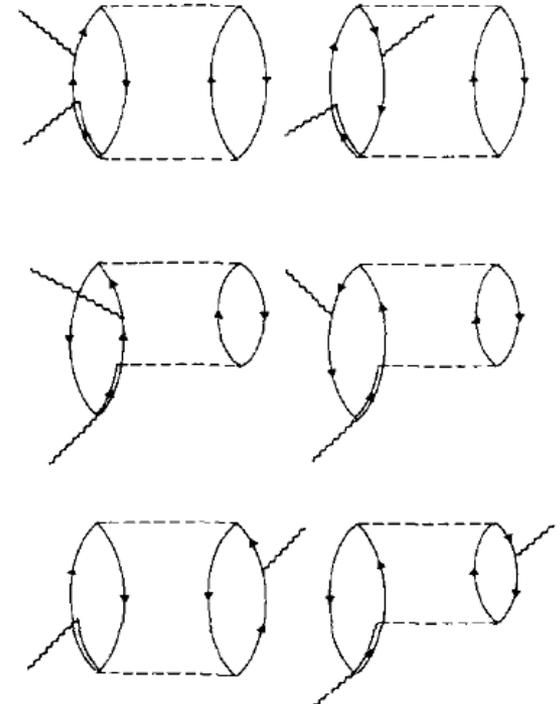
NN correlations



MEC



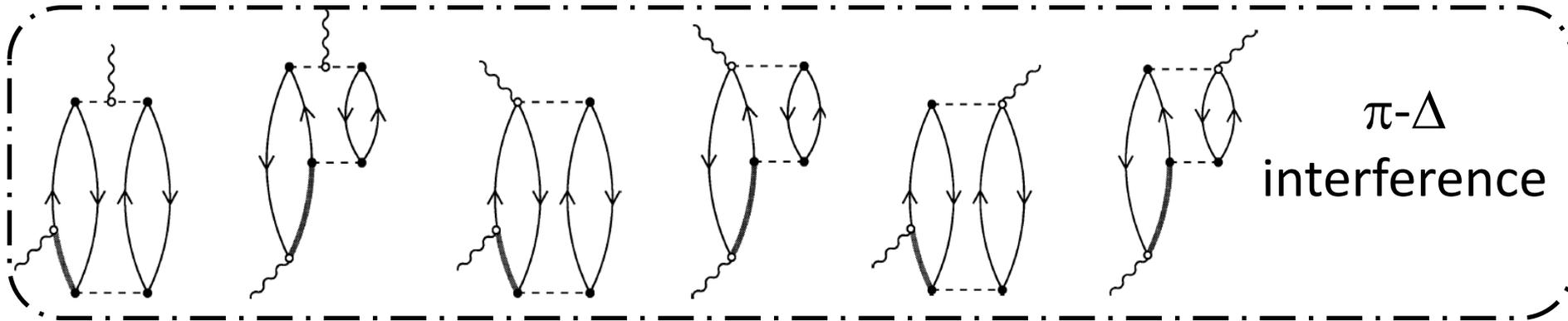
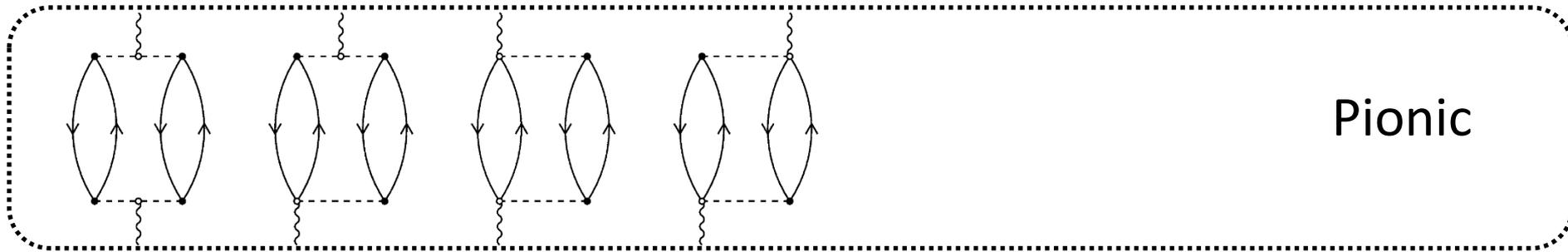
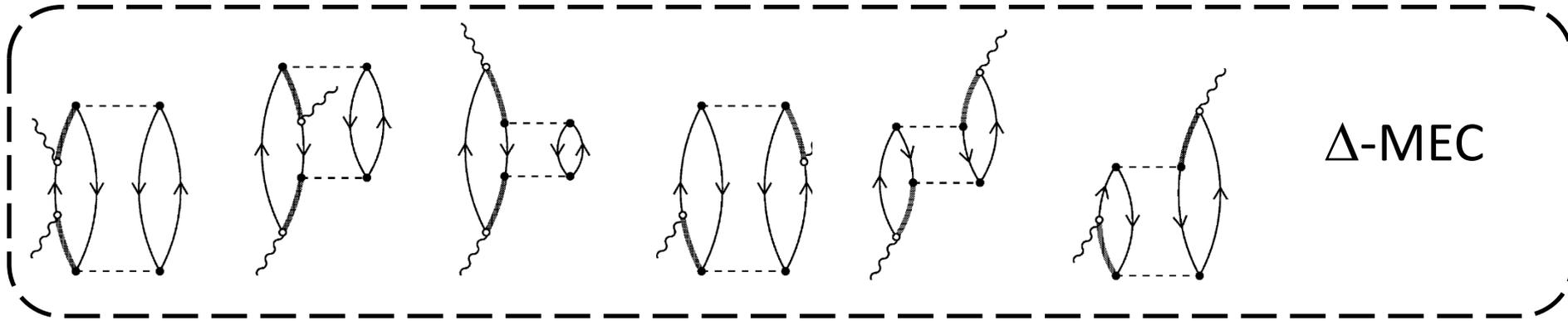
NN correlation-MEC interference



also called
1-body—2-body interference

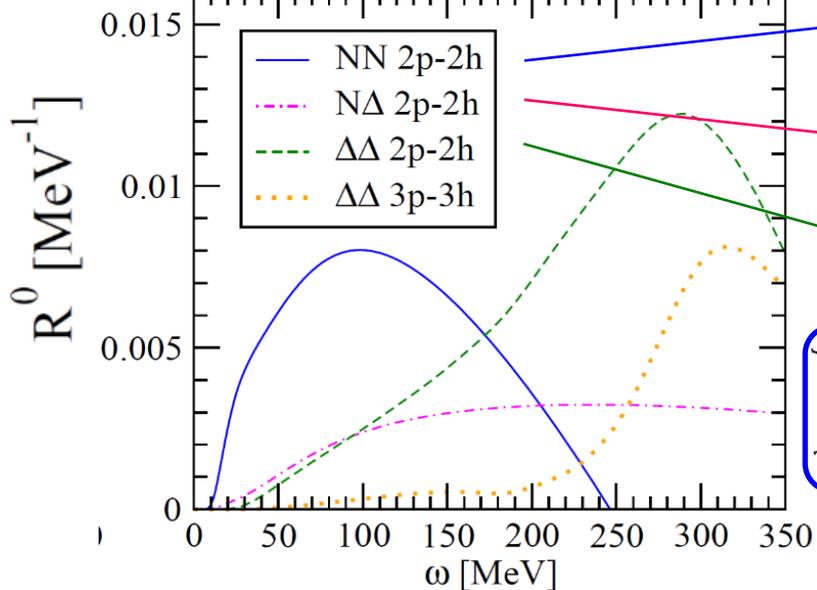
Alberico, Ericson, Molinari, Ann. Phys. 154, 356 (1984)

MEC contributions



Separation of np-nh contributions in the nuclear responses

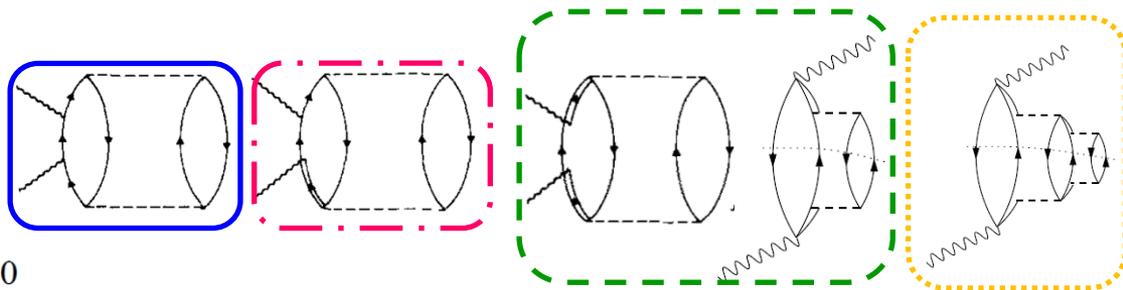
M. Martini, M. Ericson, G. Chanfray, J. Marteau, PRC 80 065501 (2009)



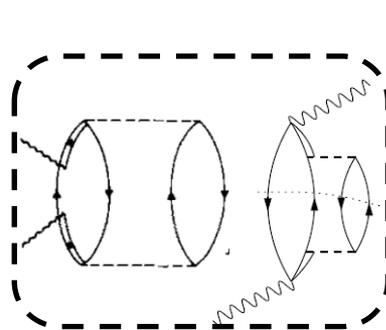
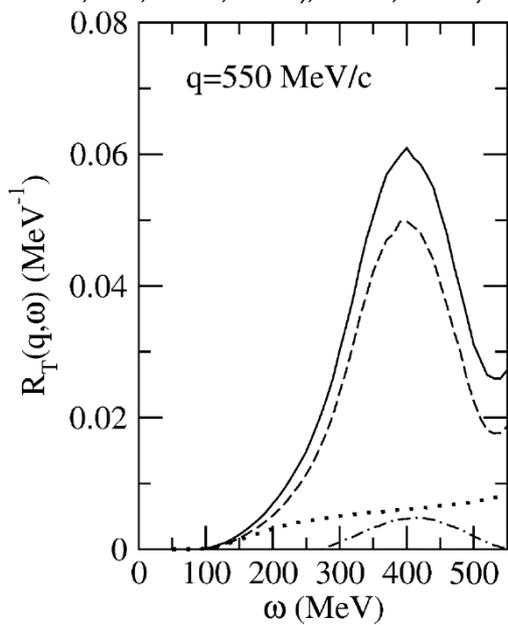
also called NN SRC; part of 1-body current contribution in correlated nuclear wave functions approaches, like SF or GFMC

$N\Delta$ interference, also called NN correlation- Δ MEC interference or 1-body-2-body interference

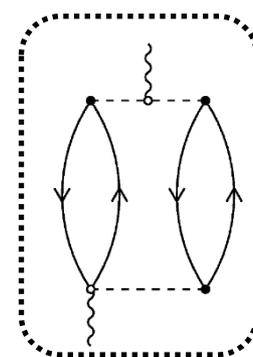
Δ mediated MEC



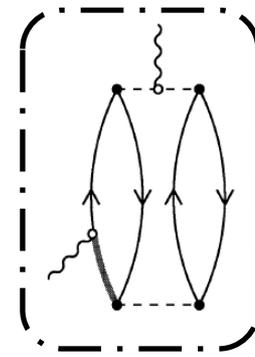
De Pace, Nardi, Alberico, Donnelly, Molinari, Nucl. Phys. A741, 249 (2004)



Δ



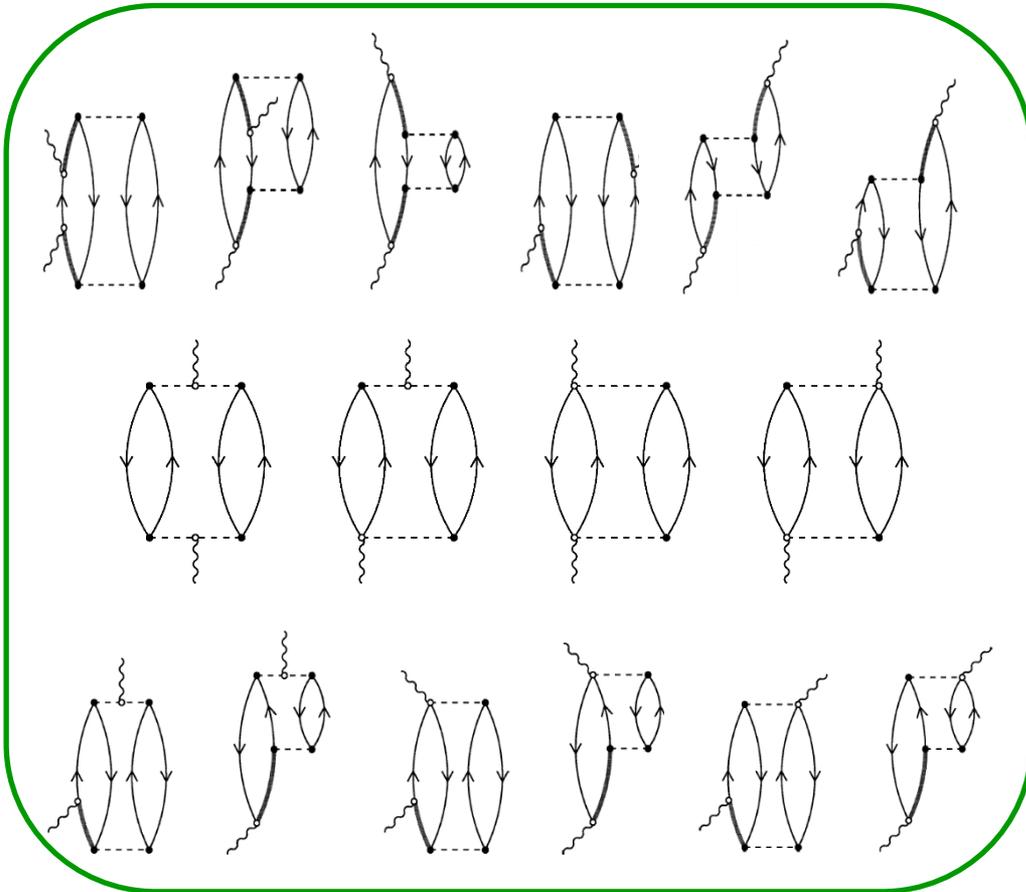
π



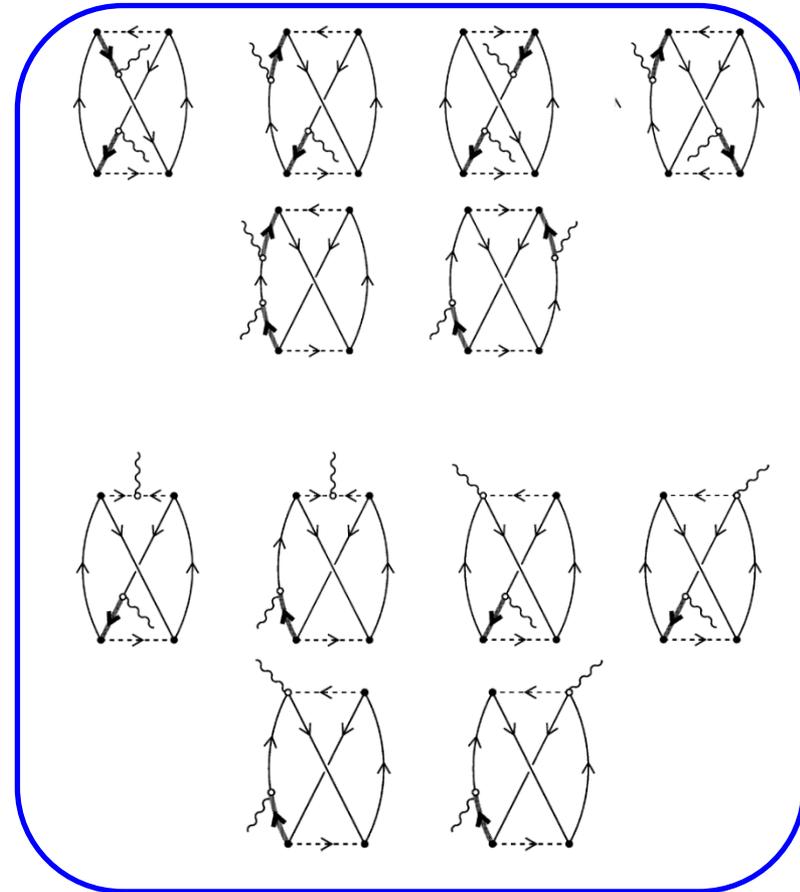
π - Δ intf.

Direct and exchange MEC contributions

Direct



Exchange



Fully relativistic calculation of *De Pace, Nardi, Alberico, Donnelly, Molinari, NPA741 (2004)*:

3000 direct terms

More than **100 000** exchange terms

Main difficulties in the np-nh sector

$$W^{\mu\nu}(\mathbf{q}, \omega) = W_{1p1h}^{\mu\nu}(\mathbf{q}, \omega) + W_{2p2h}^{\mu\nu}(\mathbf{q}, \omega) + \dots$$

$$W_{2p-2h}^{\mu\nu}(\mathbf{q}, \omega) = \frac{V}{(2\pi)^9} \int d^3p'_1 d^3p'_2 d^3h_1 d^3h_2 \frac{m_N^4}{E_1 E_2 E'_1 E'_2} \theta(p'_2 - k_F) \theta(p'_1 - k_F) \theta(k_F - h_1) \theta(k_F - h_2) \\ \underbrace{\langle 0 | J^\mu | \mathbf{h}_1 \mathbf{h}_2 \mathbf{p}'_1 \mathbf{p}'_2 \rangle \langle \mathbf{h}_1 \mathbf{h}_2 \mathbf{p}'_1 \mathbf{p}'_2 | J^\nu | 0 \rangle}_{\text{matrix elements}} \delta(E'_1 + E'_2 - E_1 - E_2 - \omega) \delta(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{h}_1 - \mathbf{h}_2 - \mathbf{q})$$

- 7-dimensional integrals $\int d^3h_1 d^3h_2 d\theta'_1$ of thousands of terms
- Huge number of diagrams and terms
- Divergences (NN correlations contributions)
- Calculations for all the kinematics compatible with the experimental neutrino flux

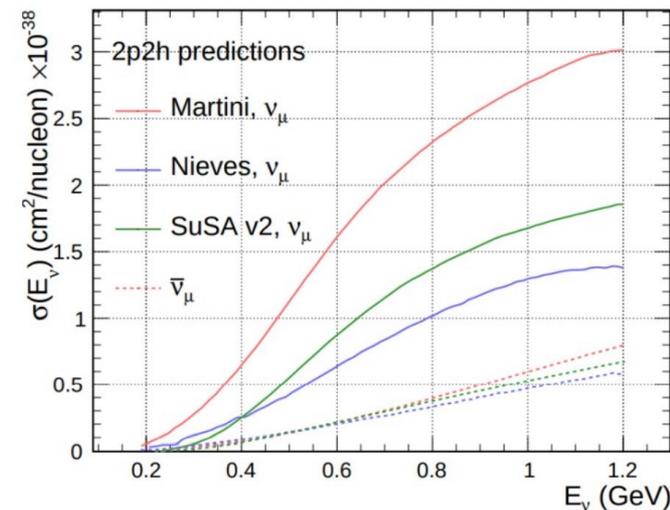
Computing very demanding

Hence different approximations by different groups:

- choice of subset of diagrams and terms;
- different prescriptions to regularize the divergences;
- reduce the dimension of the integrals
(7D --> 2D if non relativistic; 7D --> 1D if $h_1 = h_2 = 0$)

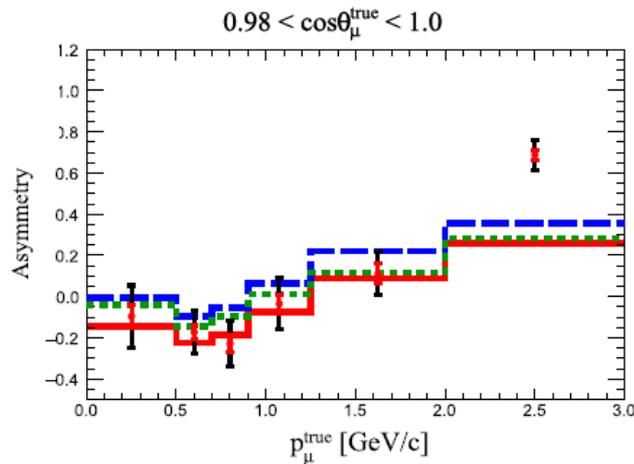
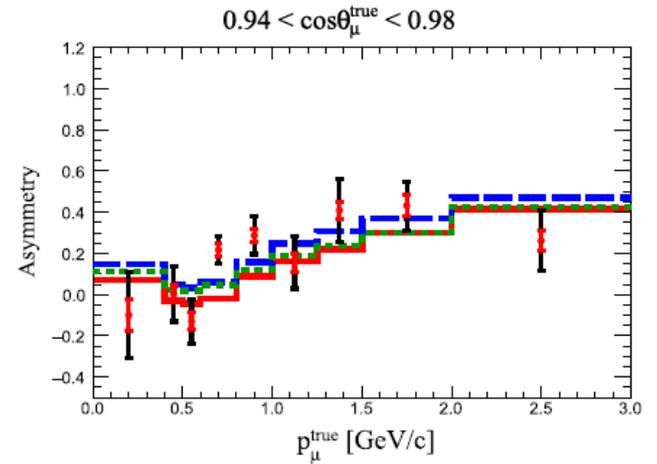
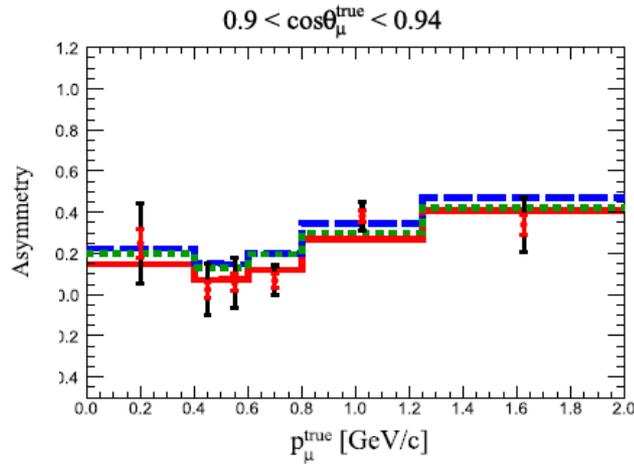
⇒ **Different final results by different groups**

- **The relative role of np-nh for neutrinos and antineutrinos is different in different approaches**



First combined measurement of the muon neutrino and antineutrino charged-current cross section without pions in the final state at T2K

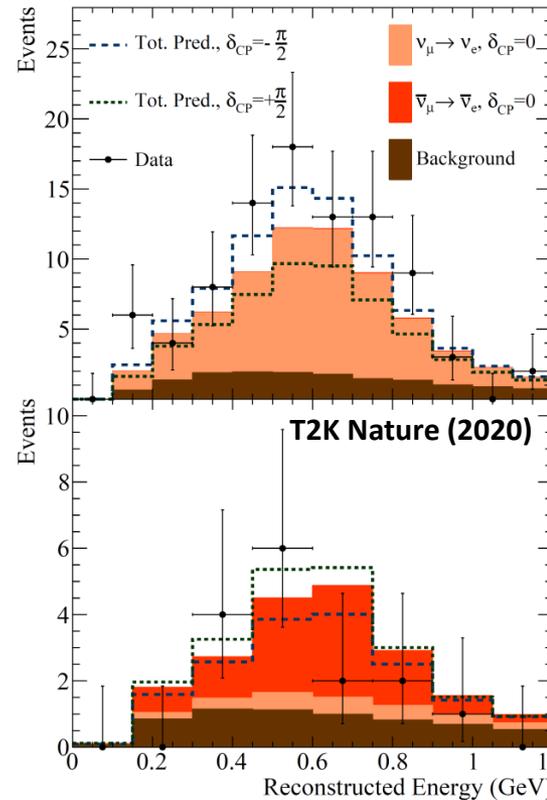
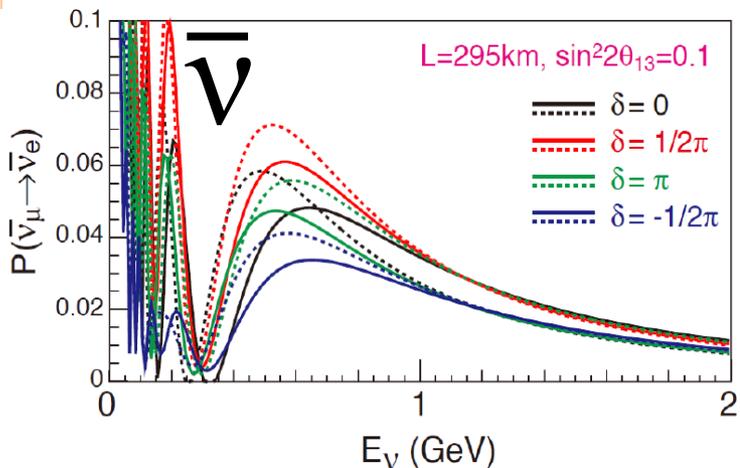
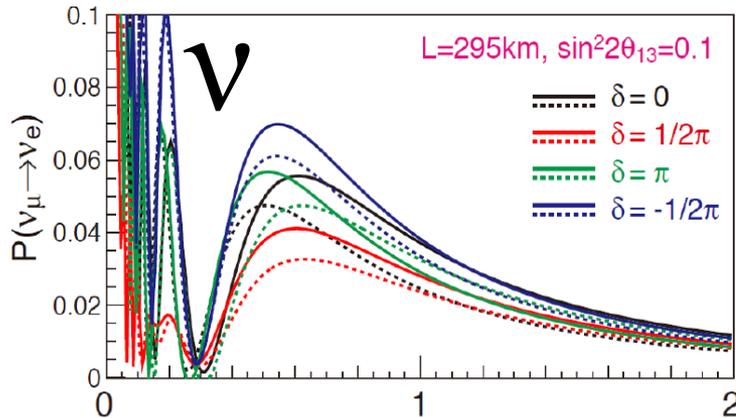
$$\frac{\nu - \bar{\nu}}{\nu + \bar{\nu}}$$



-  Total Uncertainty (stat+syst)
-  Systematic Uncertainty
-  NEUT LFG+2p2h $\chi^2 = 150.5(147.8)/58$
-  Martini et al. $\chi^2 = 93.9(131.2)/48$
-  SuSAv2 $\chi^2 = 152.6(146.3)/58$

What about ν vs $\bar{\nu}$ interaction? And ν_μ vs ν_e ?

$$P(\nu_\mu \rightarrow \nu_e) \stackrel{?}{\neq} P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$$



$\nu_\mu \rightarrow \nu_e$

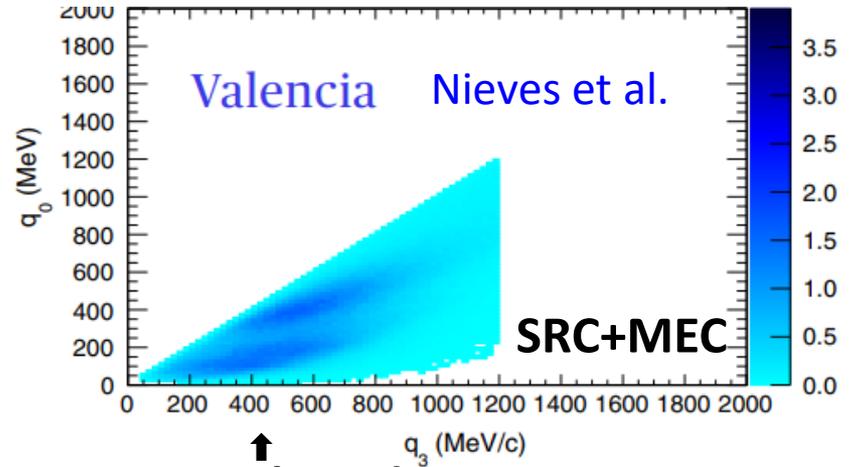
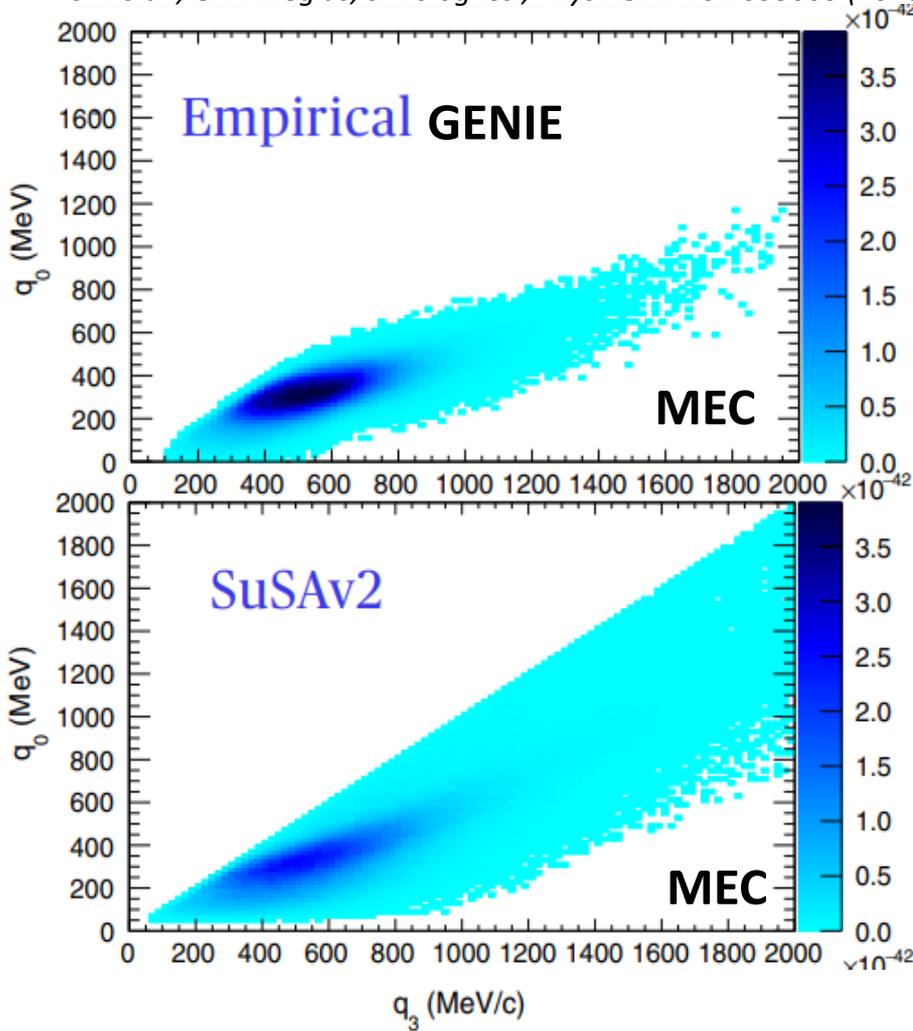
$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

A precise and simultaneous knowledge of the four cross sections is important in connection to the oscillation experiments aiming at the search for CP violation in the lepton sector (T2K, NOvA, Hyper-K, DUNE).

Non-trivial differences in the cross sections (see Appendix I)

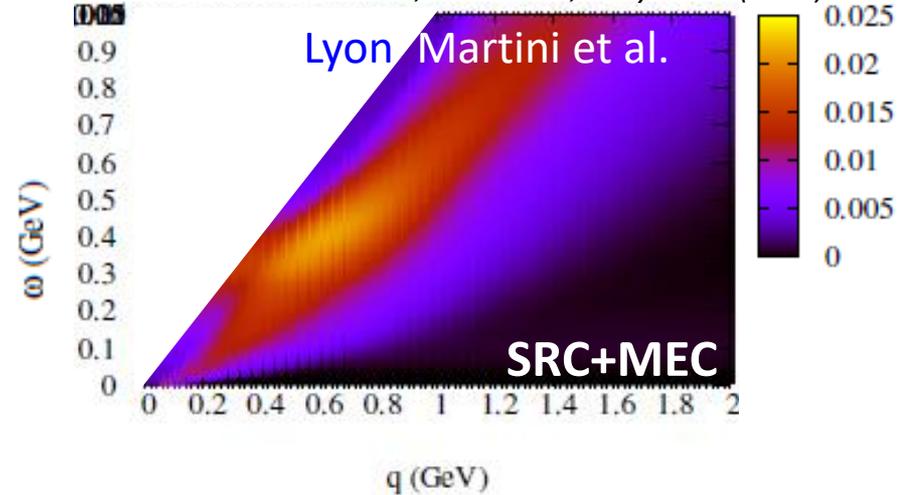
Example of different results for 2p-2h in the (q,ω) or (q_0,q_3) plane

S. Dolan, G.D. Megias, S. Bolognesi, *Phys.Rev.D* 101 033003 (2020)



RPA-based

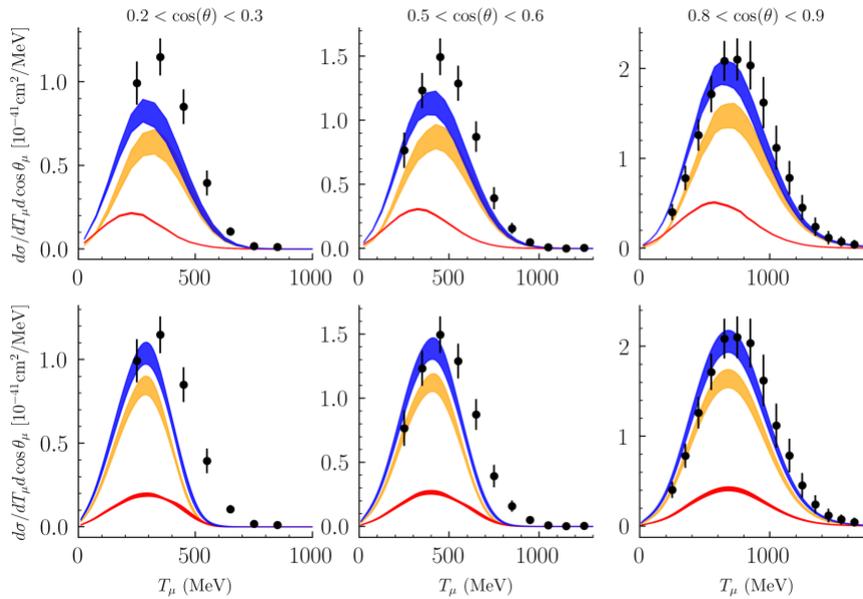
T. Katori, M. Martini, *J.Phys.G* 45 (2018)



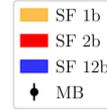
N.B. A one-to one correspondence between different exclusive channel's contributions can be misleading [e.g. NN SRC contributions are part of the 2p-2h channel in RPA-based approaches while they are included in QE in SuSA.]

Example of different results in recent Spectral Function and Green's Function Monte Carlo (ab-initio) calculations

MiniBoONE



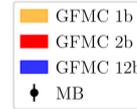
SF



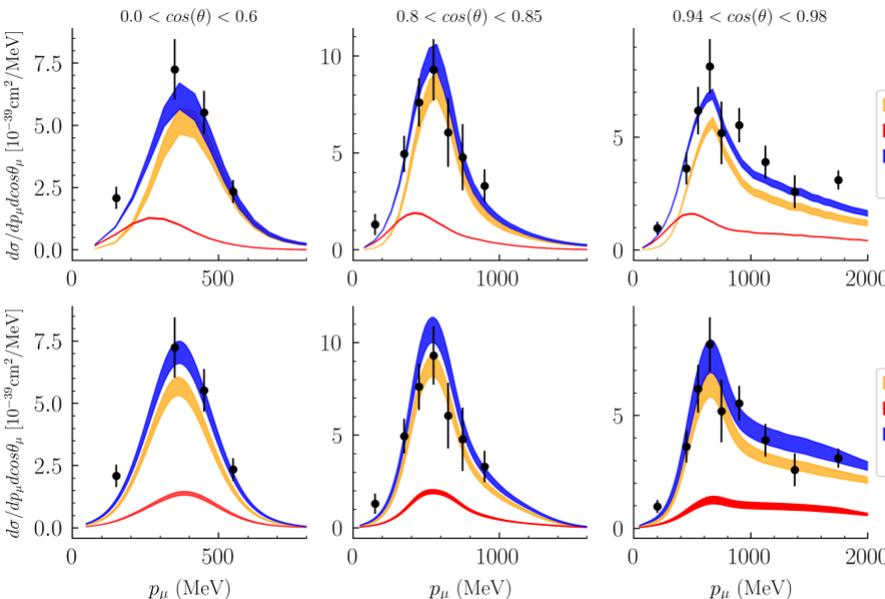
D. Simons et al. 2210.02455

N. Steinberg talk @ NUINT 2022

GFMC



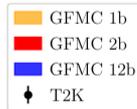
T2K



SF



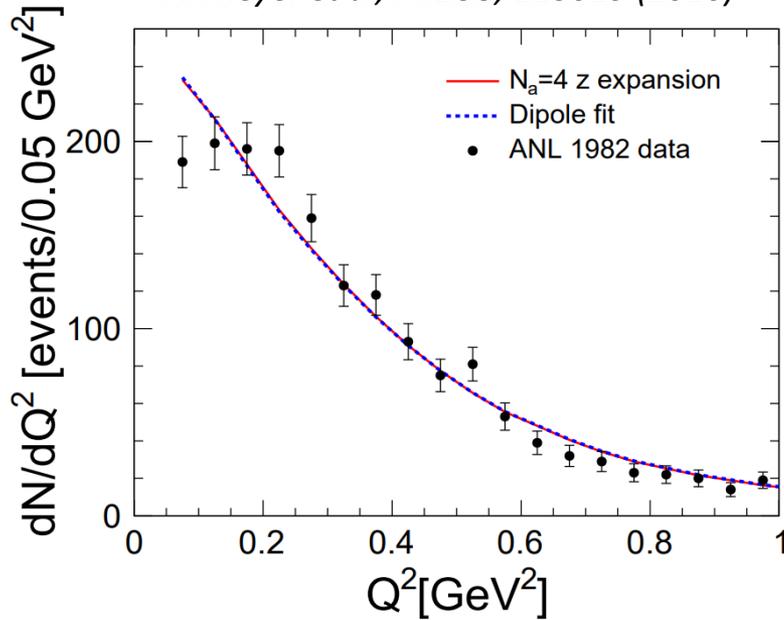
GFMC



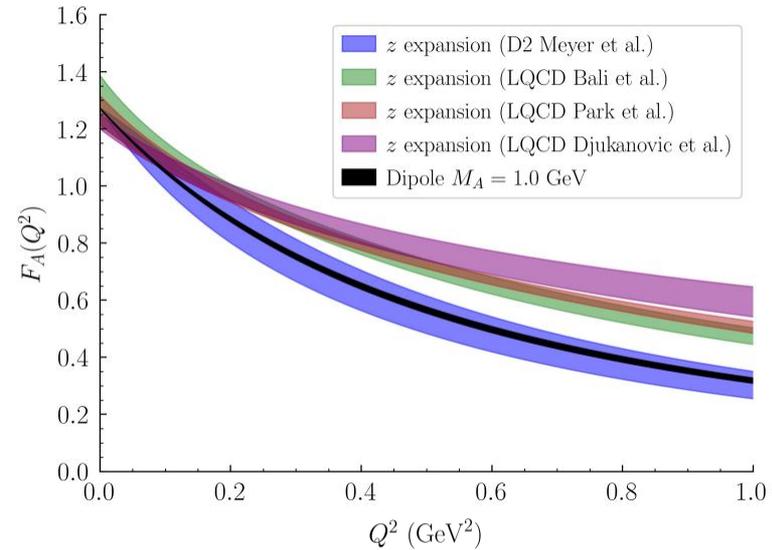
SF and GFMC 2-body contributions shifted because of different 1 body – 2 body interference effects

Axial Form factor and Lattice QCD predictions

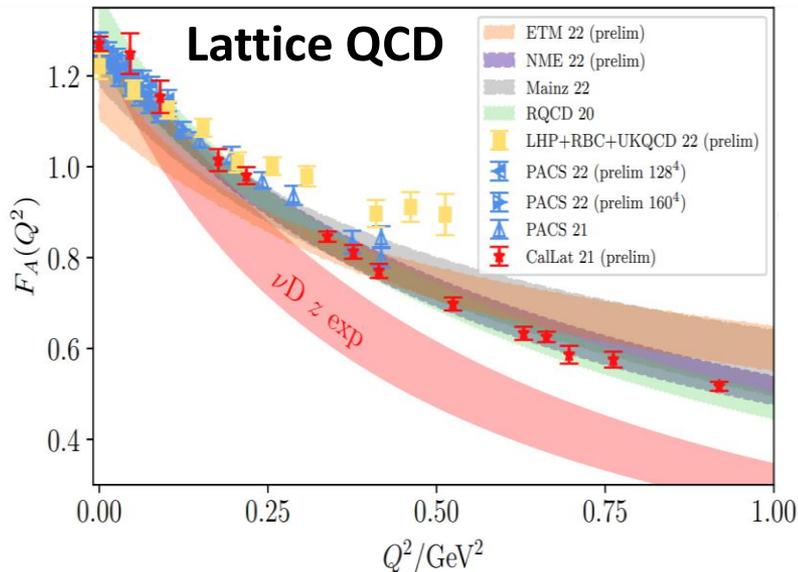
A. Meyer et al, PRD93, 113015 (2016)



D. Simons et al. 2210.02455



A. Meyer talk @ NUINT 2022; Ann.Rev.Nucl.Part.Sci. 72 (2022)

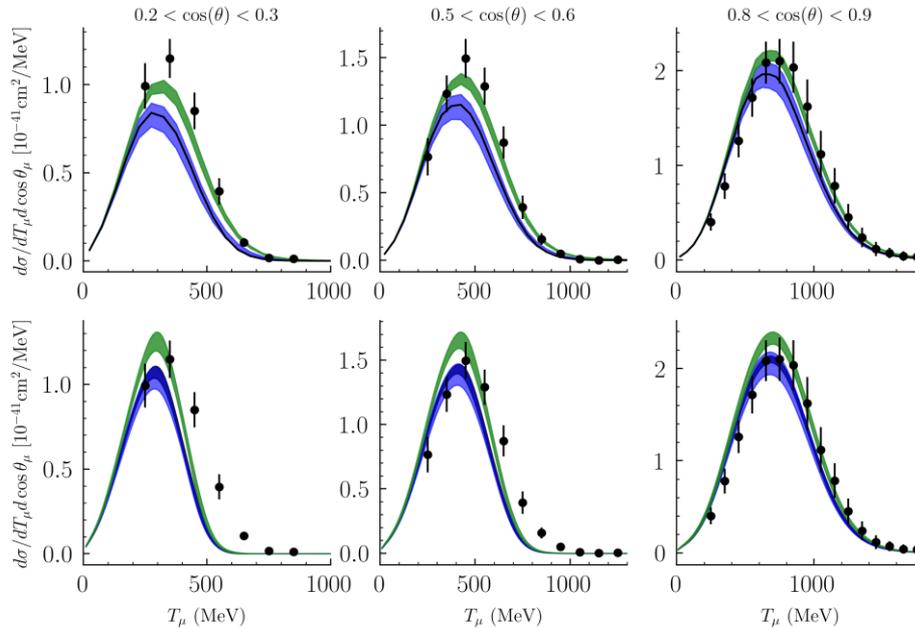


- Dipole parameterization underestimates uncertainties
- Meyer et al. z-expansion: similar to dipole parameterization but larger errors
- Lattice QCD calculations show evidence of slow Q^2 falloff
- LQCD: much larger normalization at $Q^2 > 0.3$ GeV²

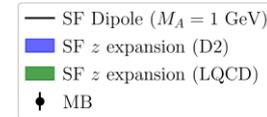
Impact of enhanced axial form factor from LQCD

D. Simons et al. 2210.02455

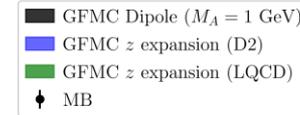
MiniBooNE



SF

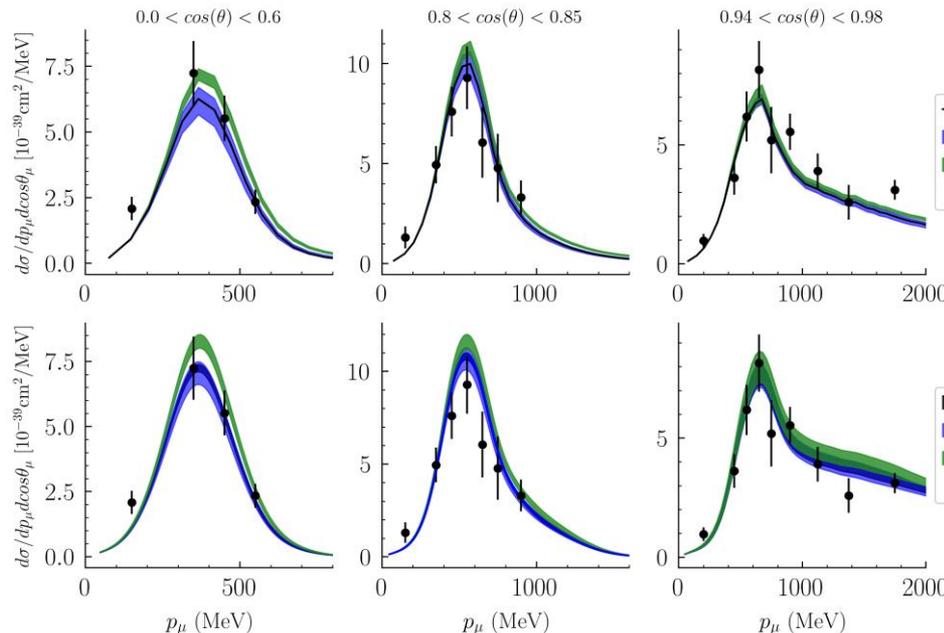


GFMC

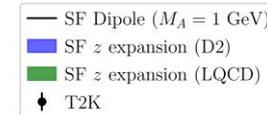


MiniBooNE:
 Universal 10-20% increase
 in normalization with LQCD

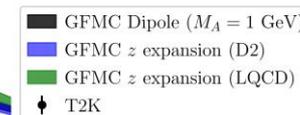
T2K



SF



GFMC



T2K:
 Results fairly independent of
 parameterization
 Mostly due to T2K's lower
 beam energy hence lower Q^2
 where form factors agree

Data have room for both 2p-2h and
 enhanced axial form factor for LQCD

Neutrino energy reconstruction

Energy reconstruction in neutrino oscillation experiments

$$N_{\nu\beta}(\overline{E_\nu}) \sim \int \Phi_{\nu\alpha}(E_\nu) P_{\nu\alpha \rightarrow \nu\beta}(E_\nu, L, \{\Theta\}) \sigma_{\nu\beta}(E_\nu) \epsilon_{\text{det.}} d(E_\nu, \overline{E_\nu}) dE_\nu$$

Reconstructed ν energy
True ν energy

Number of detected events

ν flux	ν oscillation probability	ν cross section	Detector efficiency	Migration matrix
------------	-------------------------------	---------------------	---------------------	-------------------------

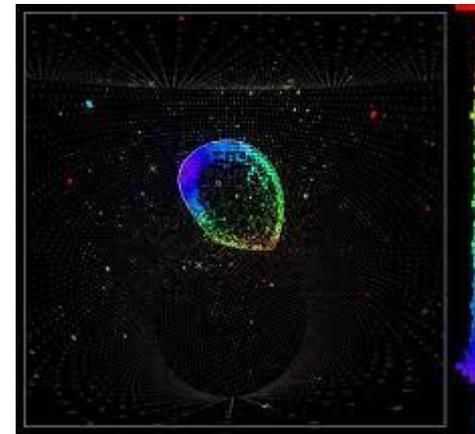
Two methods for ν energy reconstruction

Tracking detectors

- Use all the detected particles
- Calorimetric method

Cherenkov detectors

- Use only lepton (1 ring signal)
- Quasielastic-based method

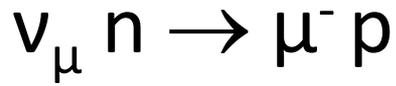
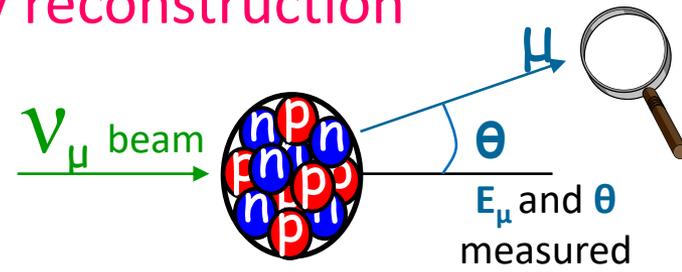


[For details see the cross section lectures at the GIF school]

Quasielastic-based neutrino energy reconstruction

Reconstructed neutrino energy

$$\overline{E}_\nu = \frac{m_p^2 - (m_n - E_b)^2 - m_\mu^2 + 2(m_n - E_b)E_\mu}{2(m_n - E_b - E_\mu + p_\mu \cos \theta_\mu)}$$

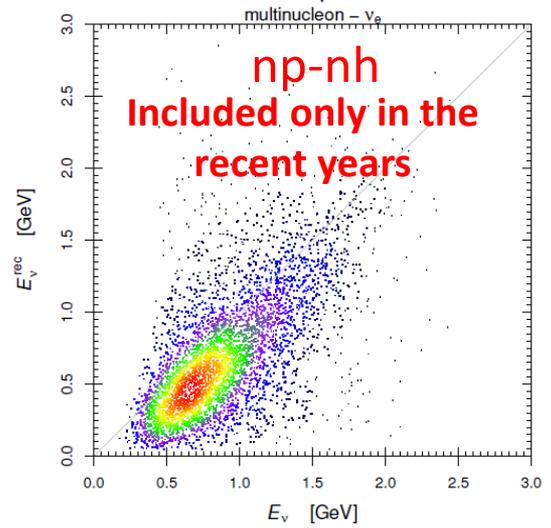
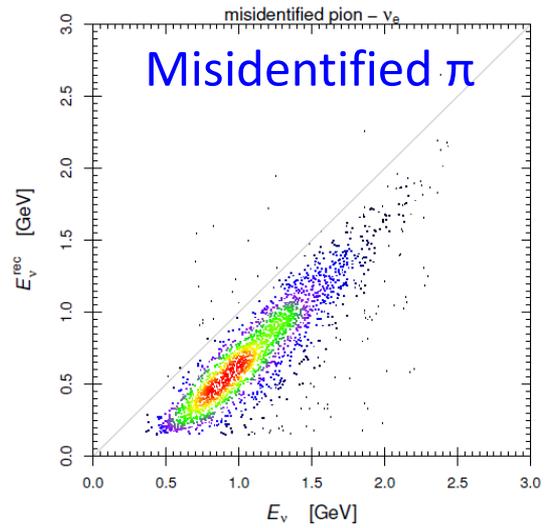
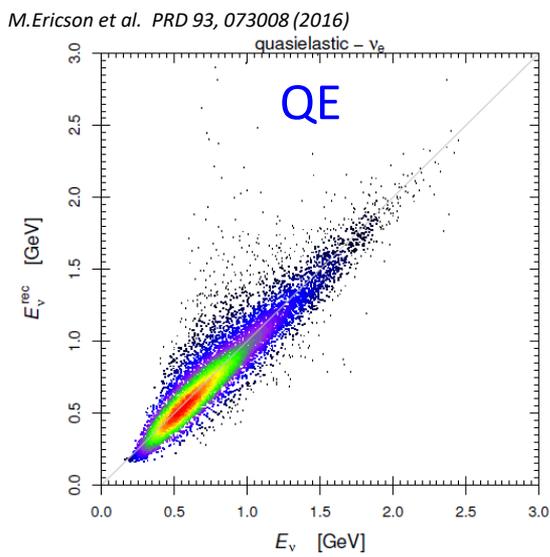
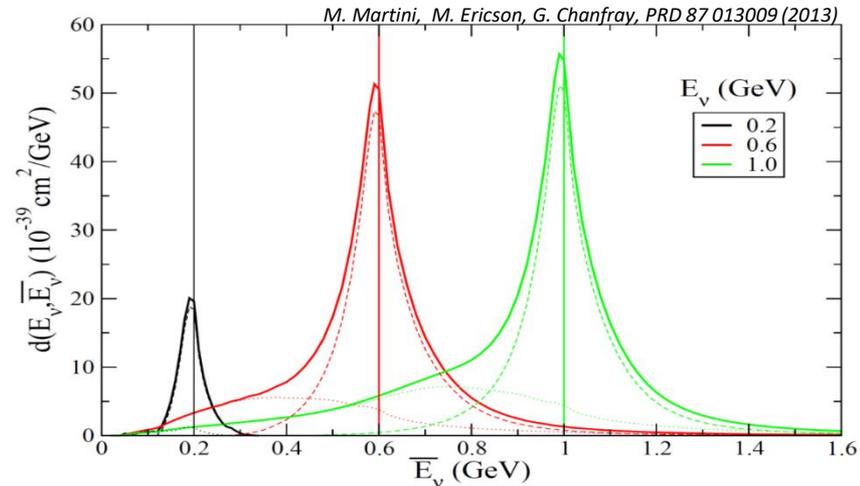


via two-body kinematics

$\overline{E}_\nu = E_\nu$ exact only for CCQE with free nucleon

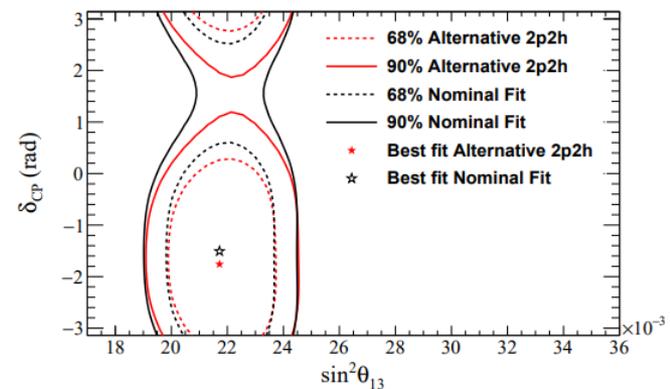
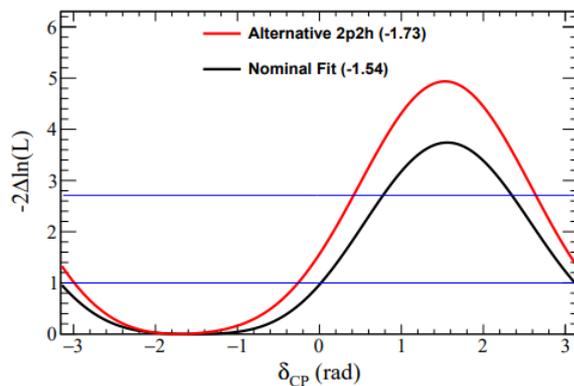
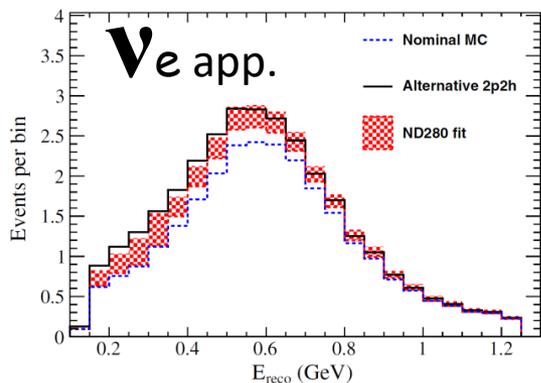
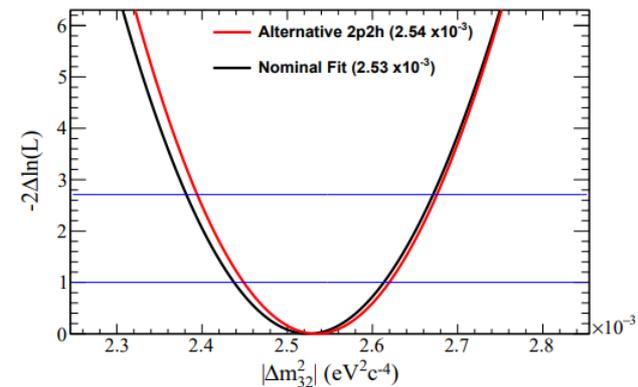
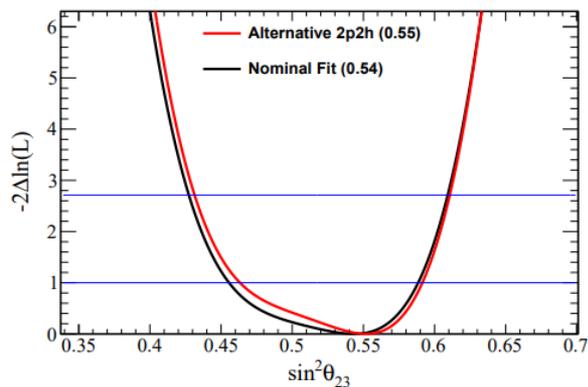
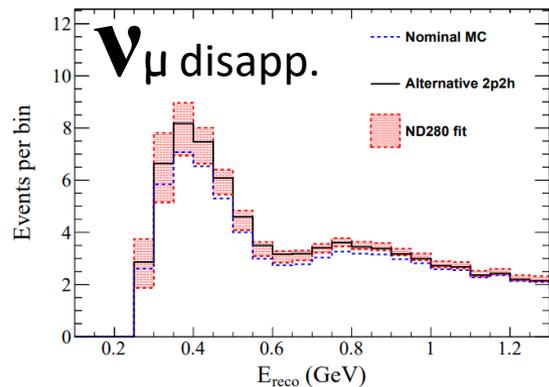
$$d(E_\nu, \overline{E}_\nu)$$

Migration matrix:
to take into account
nuclear effects



Impact of 2p-2h modeling on T2K oscillation analysis

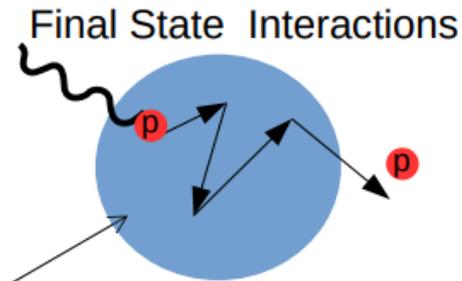
T2K Phys.Rev.D 96 (2017) 9, 092006



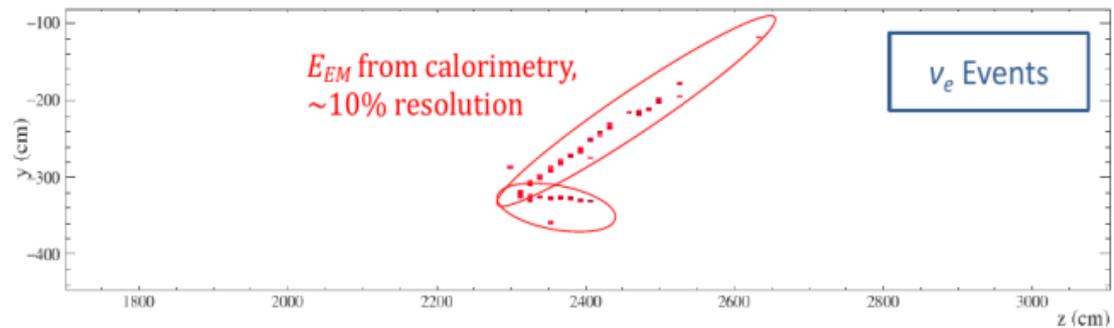
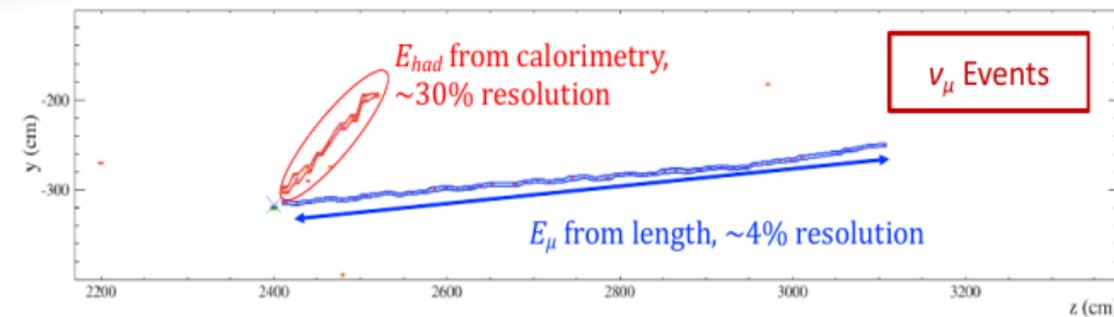
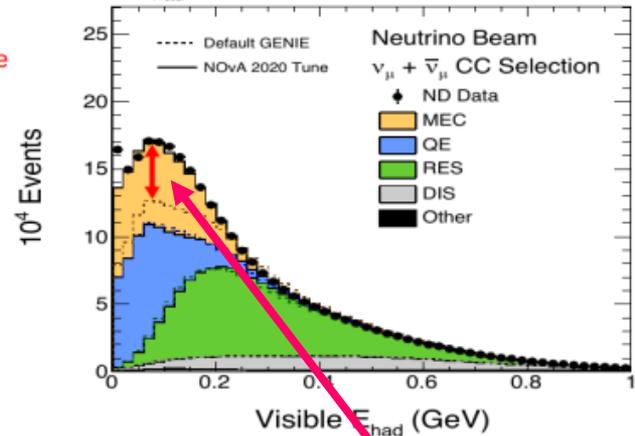
E_ν reconstruction NOvA

Calorimetric method

- E_ν reconstructed with hadronic deposits:
 - important difference $\nu - \bar{\nu}$: proton vs neutron (~undetected)
 - proton/pion energy smeared by Final State Interactions
- Different reconstruction and energy resolution for ν_μ and ν_e



Important to tune model predictions for E_{had} **NOvA Preliminary**



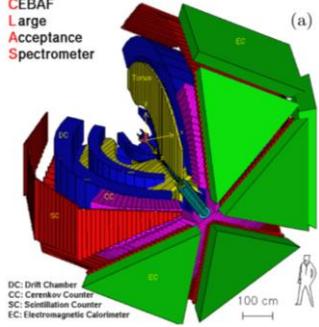
S. Bolognesi @ GIF school 119

Electron-beam energy reconstruction for ν oscillation measurements

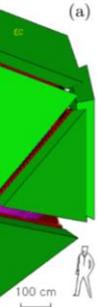
$e4\nu$



CEBAF
Large
Acceptance
Spectrometer

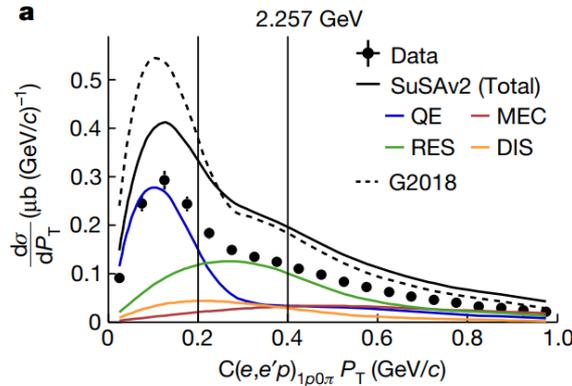


DC: Drift Chamber
CC: Cerenkov Counter
SC: Scintillation Counter
EC: Electromagnetic Calorimeter

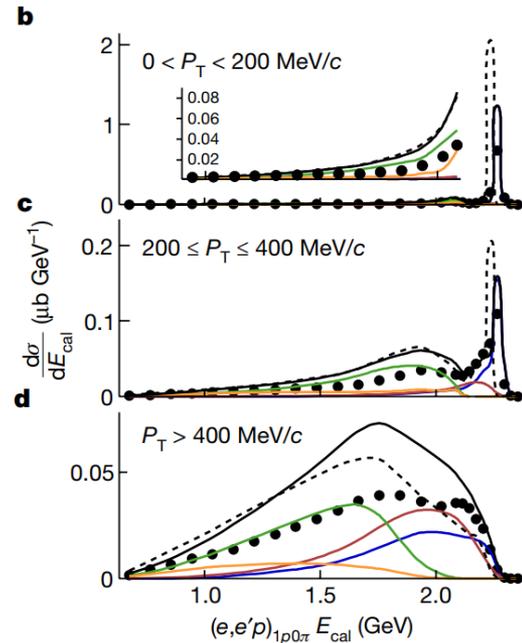


Nature 599 (2021) 7886, 565-570

QE-based
(e, e')



$$\mathbf{P}_T = \mathbf{P}_T^{e'} + \mathbf{P}_T^p$$

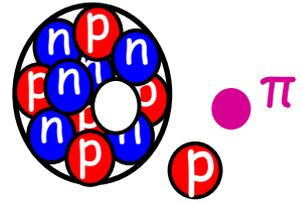


Calorimetric
-based
($e, e'p$)

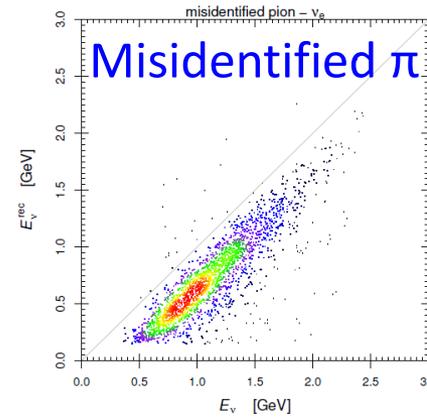
1π production

The one pion production channel

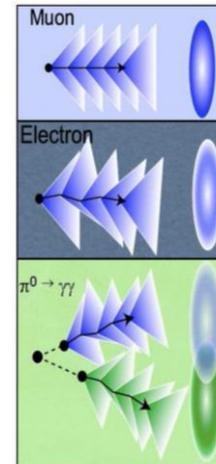
Important for several reasons:



- Misidentified π is part of the ν energy migration matrix in QE-based method



- In Cherenkov detectors NC1 π^0 can mimic electron-like signal in $\nu_\mu \rightarrow \nu_e$ oscillation search

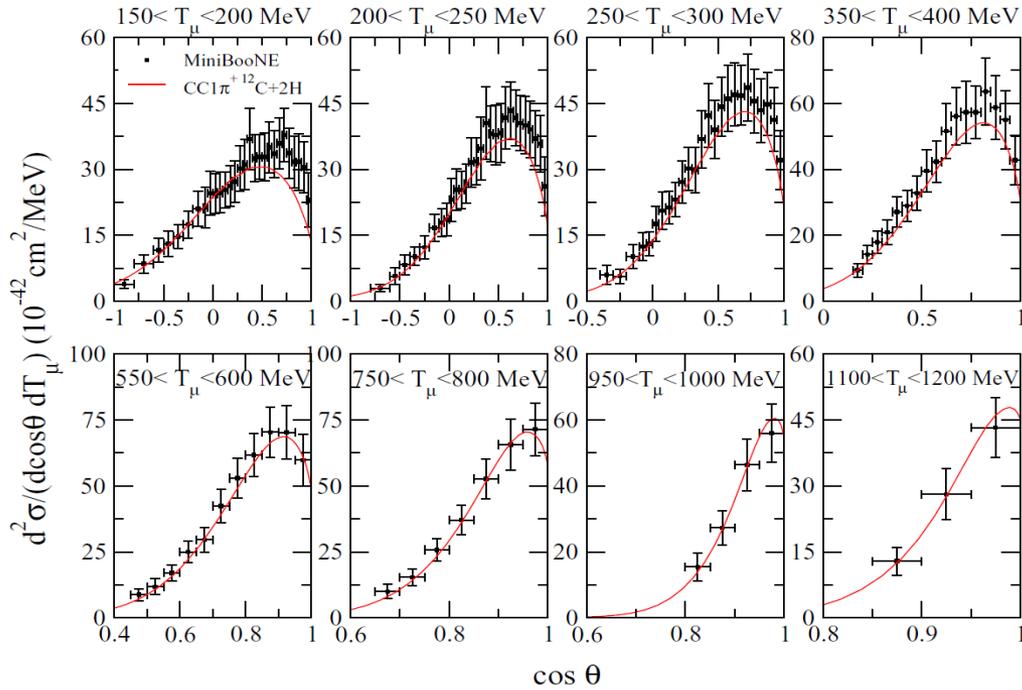


- There is an increasing interest on CC 2-ring signal (charged lepton and π) at SK
- It is one of the dominant channels in DUNE

CC1 π + flux-integrated differential cross sections on carbon

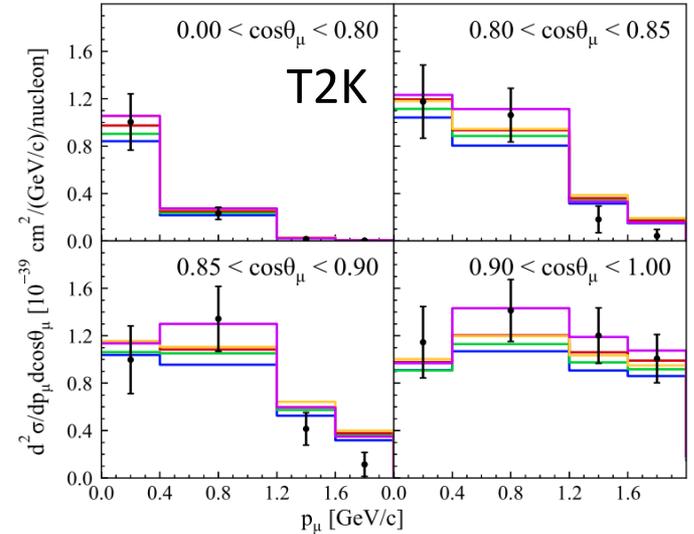
Results in terms of muon variables

MiniBooNE

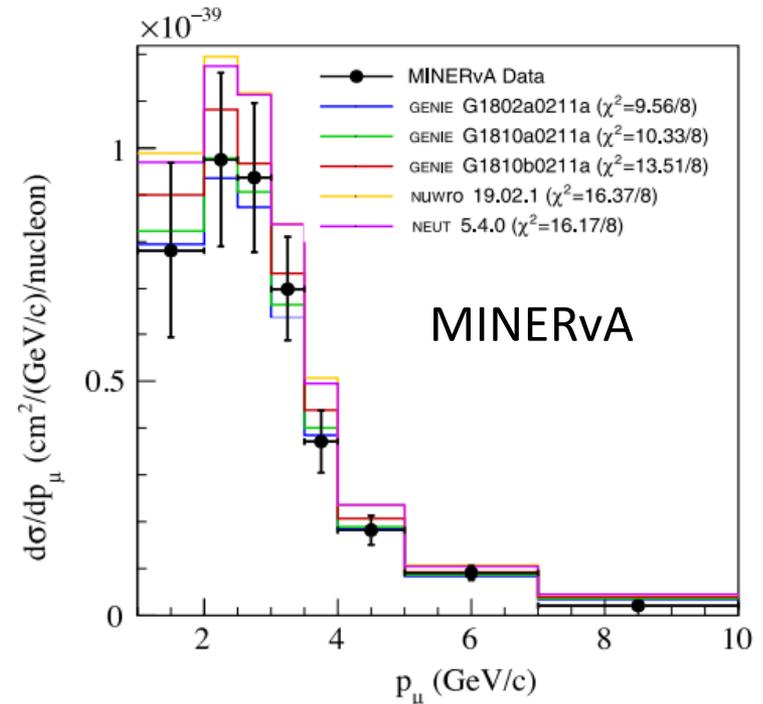


M. Martini, M. Ericson, *Phys. Rev. C* 90 025501 (2014)

Reasonable agreement between models and data, in particular at MiniBooNE and T2K energies



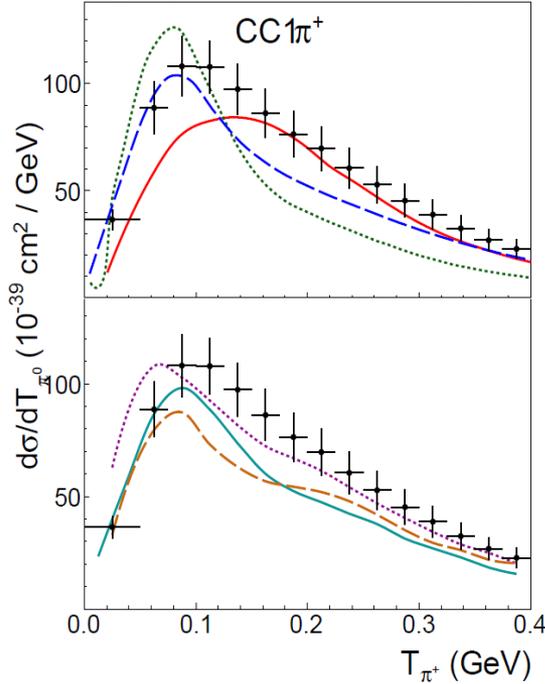
M. Buizza Avanzini et al. *PRD* 105, 092004 (2022)



CC1 π results in terms of pion variables

MiniBooNE

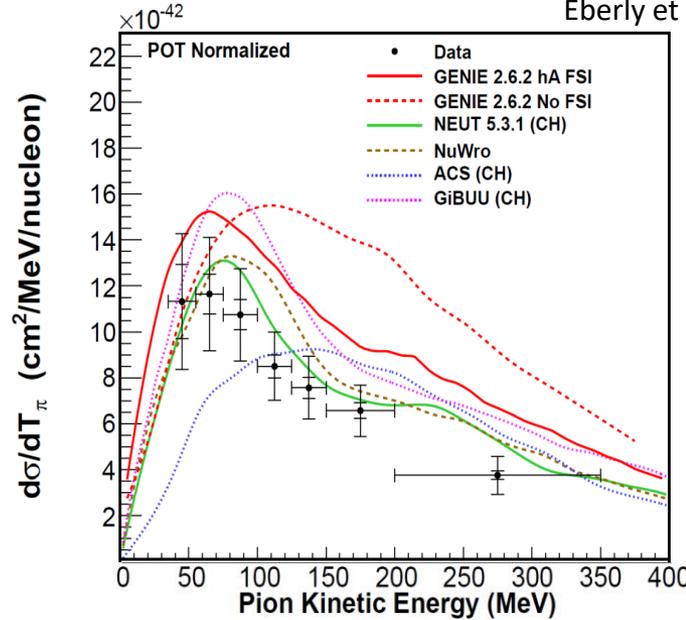
Rodrigues, AIP Conf. Proc. 1663 (2015)



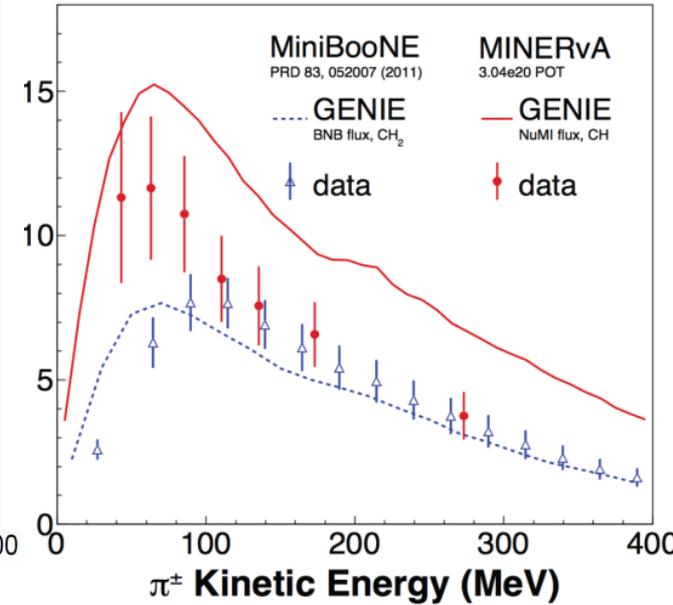
— Athar *et al.* - - - Nieves *et al.* - - - GiBUU — NuWro
 - - - GENIE — NEUT — + MB data

MINERvA

Eberly *et al.*, PRD 92 (2015)



MiniBooNE - MINERvA

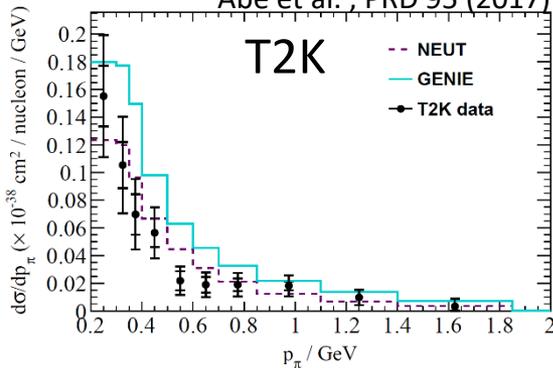


Historically many tensions

- models .vs. data ??
- models .vs. models??
- data .vs. data (through models)??

the 1 π puzzle

Abe *et al.*, PRD 95 (2017)



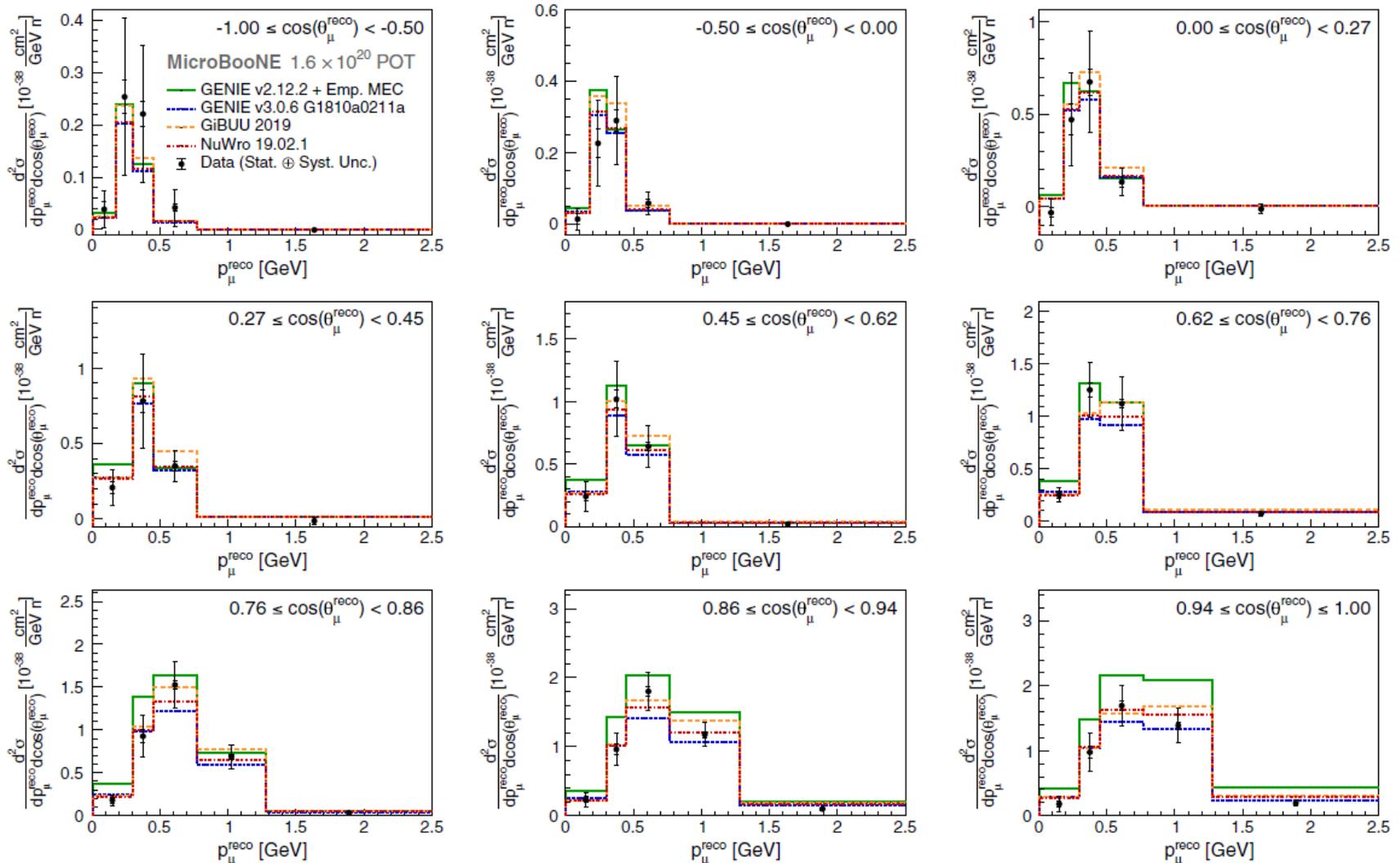
Recent hot topics:

- Argon cross sections
- Semi-inclusive processes

First MicroBooNE measurement on Argon: inclusive $d^2\sigma/dp_\mu d\cos\theta_\mu$

- CC Inclusive: only the charged lepton is detected. All reaction mechanisms contribute

PHYSICAL REVIEW LETTERS **123**, 131801 (2019)

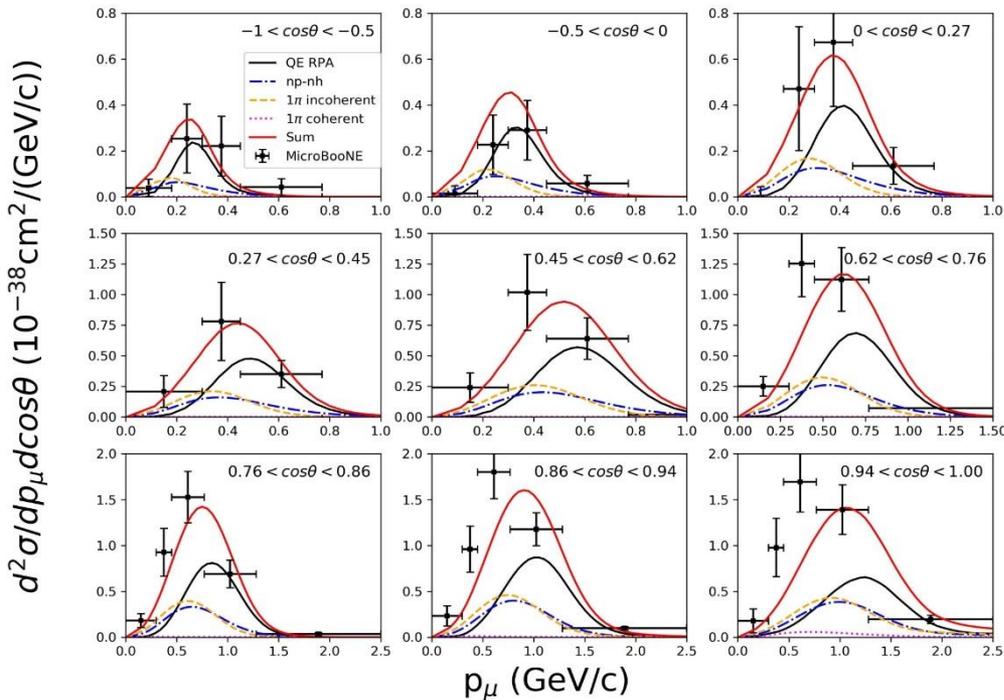


RPA and SuSAv2 calculations of MicroBooNE inclusive $d^2\sigma$ on argon

RPA

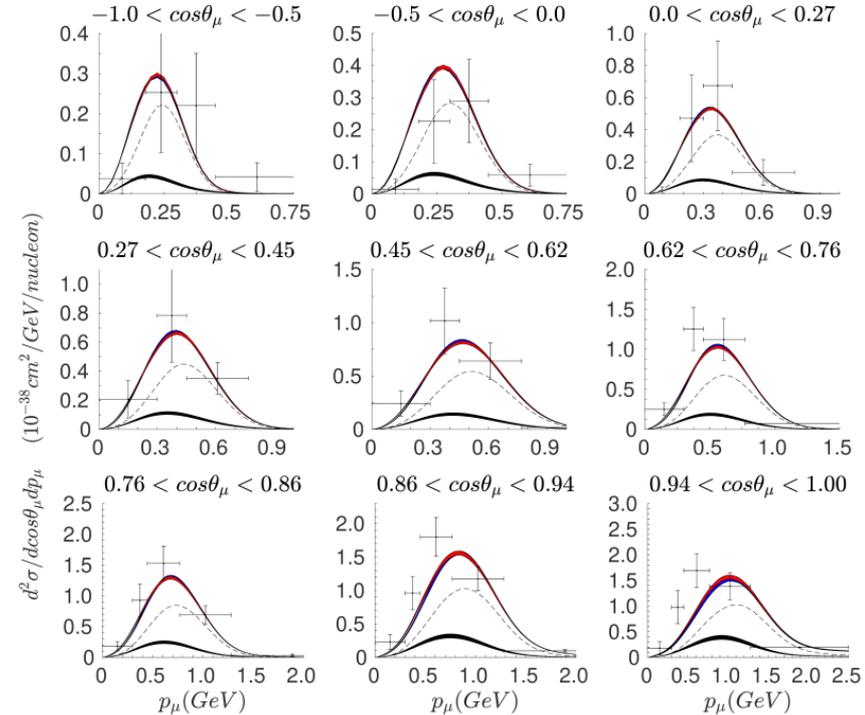
Total = QE + np-nh + 1π inc. + 1π coh.

M. Martini, M. Ericson, G. Chanfray, PRC 106 (2022)



SuSAv2

Gonzalez-Rosa et al. PRD 105 (2022)



Results also with SuSA

Barbaro et al. Universe 7 (2021)

- Reasonable overall **agreement**, though **not as good as in the ^{12}C T2K** inclusive case (see next slide)
- At backward angles the predictions of the different models are slightly shifted to lower values of p_μ , whereas the reverse occurs at forward angles

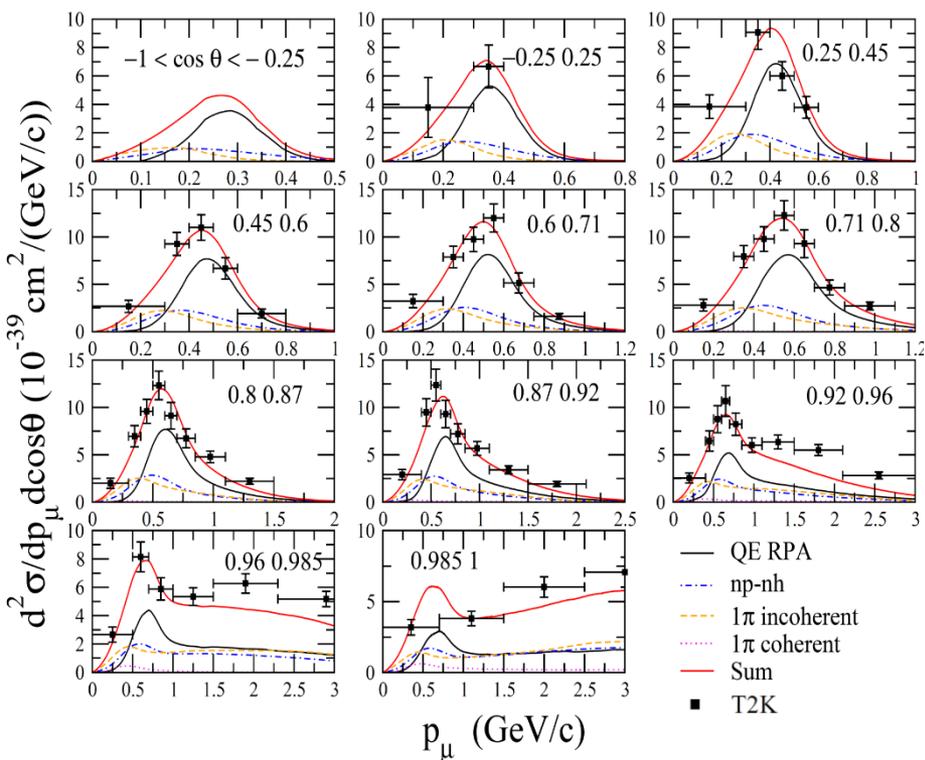
RPA and Monte Carlo calculations of T2K inclusive $d^2\sigma$ on carbon

PHYSICAL REVIEW D **98**, 012004 (2018)

Measurement of inclusive double-differential ν_μ charged-current cross section with improved acceptance in the T2K off-axis near detector

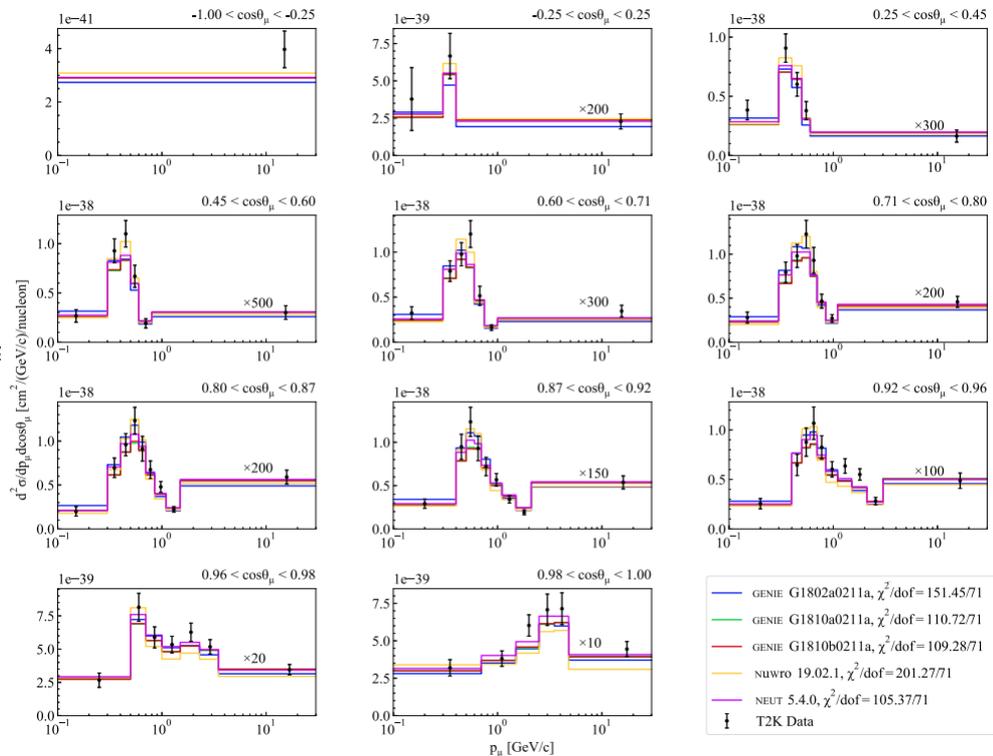
RPA

M. Martini, M. Ericson, G. Chanfray, PRC 106, 015503 (2022)



Monte Carlo

M. Buizza Avanzini et al. PRD 105, 092004 (2022)



Remarkable agreement

Semi-inclusive processes:
muon + proton(s) are detected

This week ESNT workshop

[Meson Exchange Current contributions in semi inclusive lepton nucleus scattering \(cea.fr\)](#)

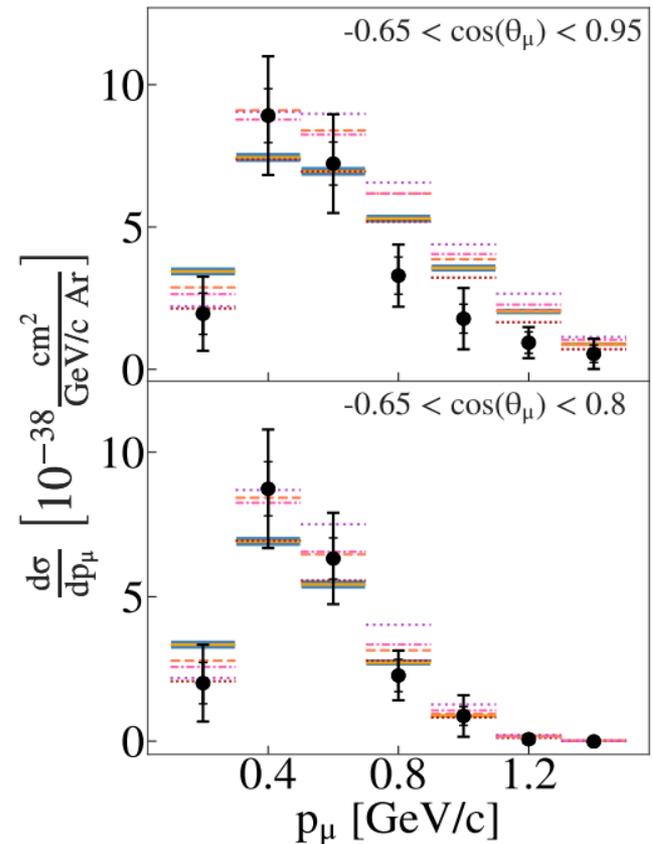
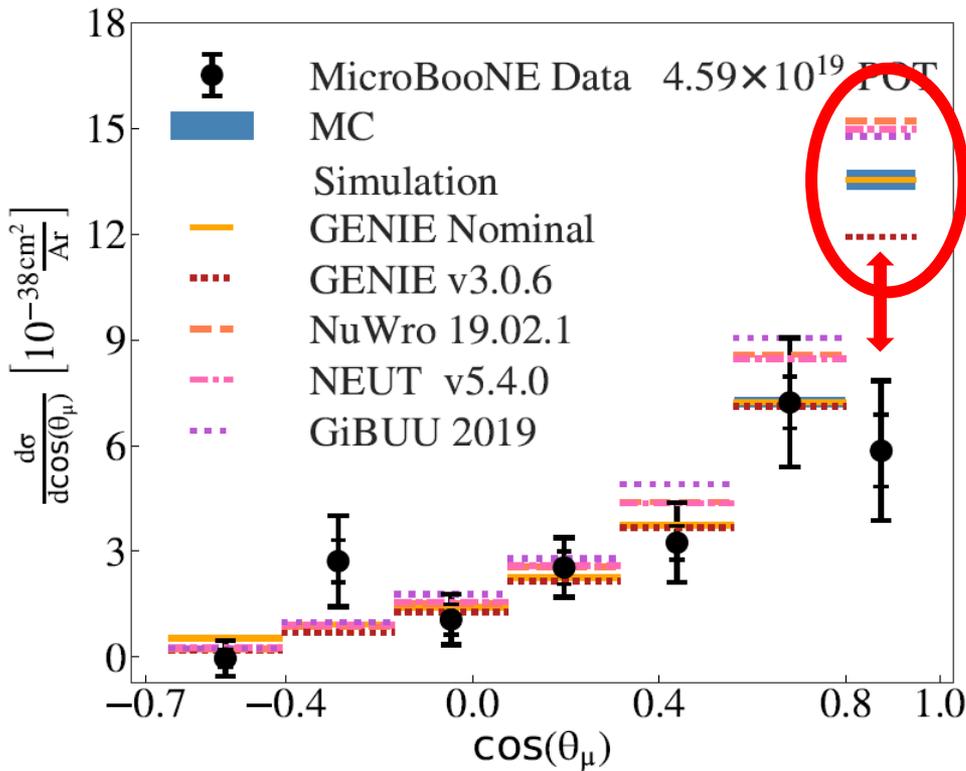
MicroBooNE semi-inclusive CC0 π 1p on argon

PHYSICAL REVIEW LETTERS 125, 201803 (2020)

First Measurement of Differential Charged Current Quasielasticlike ν_μ -Argon Scattering Cross Sections with the MicroBooNE Detector

?! CCQE-like with another meaning than in the past

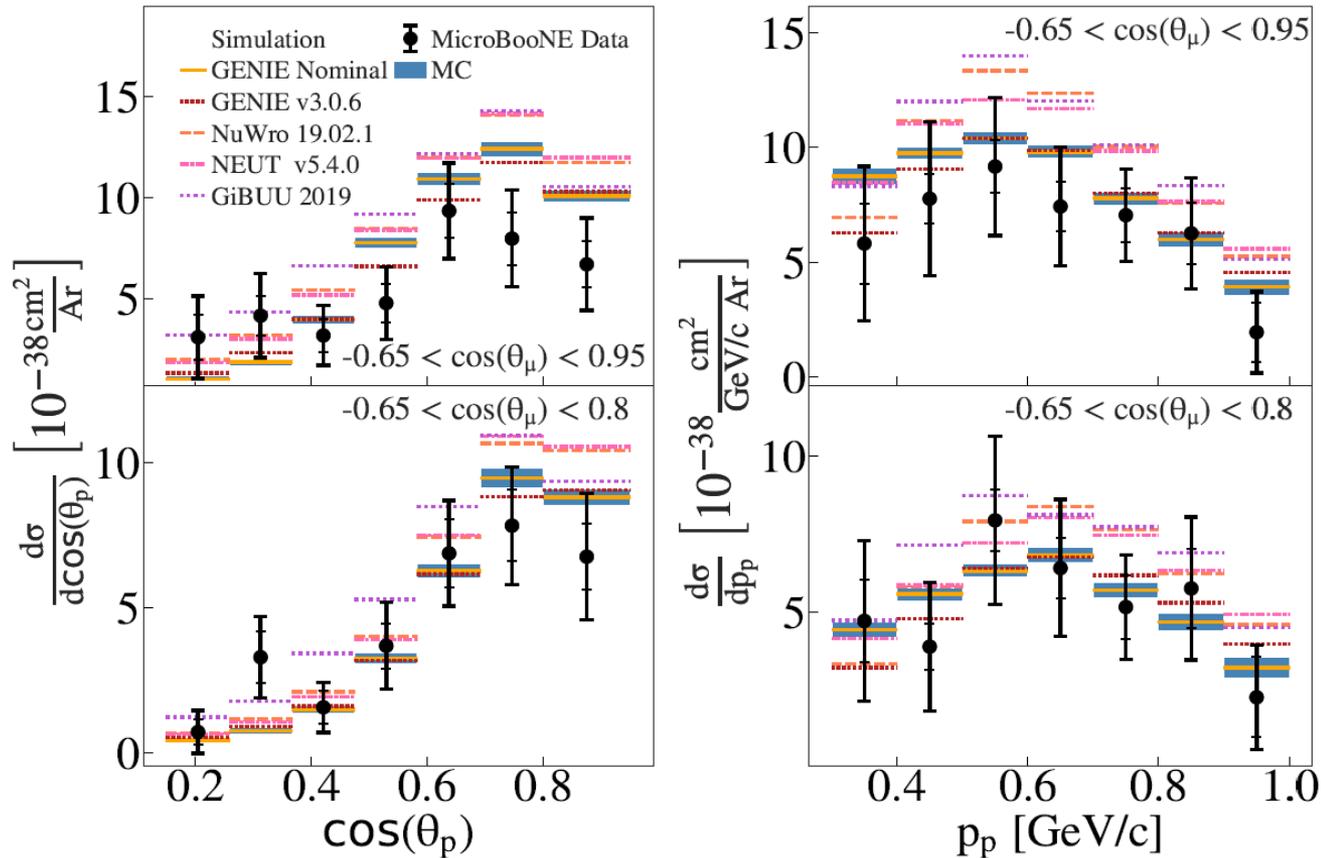
Results versus muon variables



Overestimation of Monte Carlo predictions in the muon forward direction

MicroBooNE semi-inclusive CC0 π 1p on argon versus proton variables

MicroBooNE PRL 125(2020)

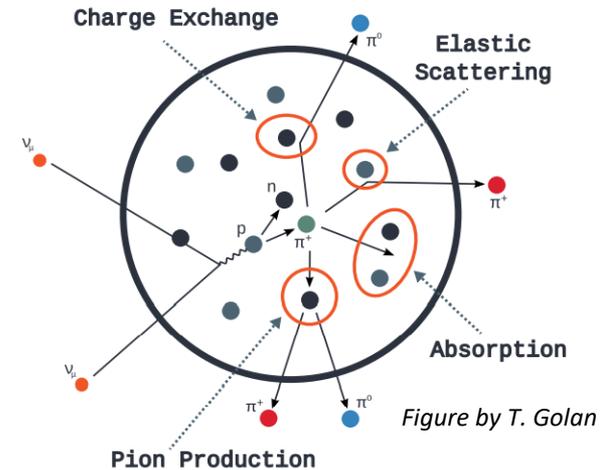
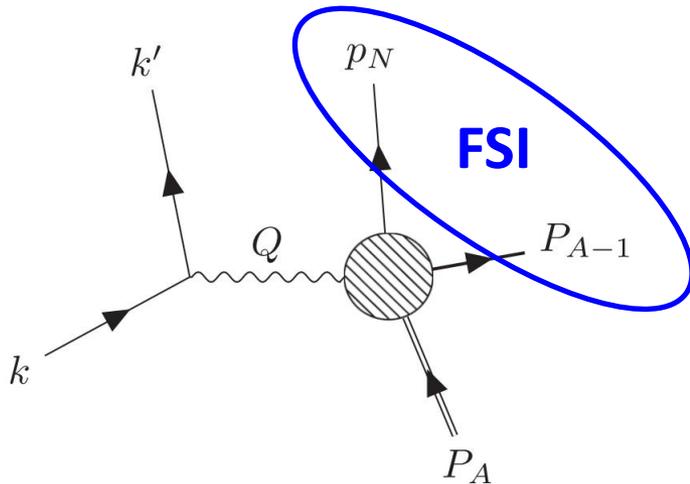


- Poor Monte Carlo – data agreement
- Spread of Monte Carlo predictions

How good are the current approximations (use “inclusive” models, factorization,...) of the Monte Carlos for the semi-inclusive processes?

Final State Interactions

FSI between the knocked-out particle(s) and the residual nucleus



- Monte Carlo event generators include different models of intra-nuclear cascades: particles are assumed to be classical and move along a straight line
- FSI between the knocked-out nucleon and the residual nucleus can be microscopically treated using different approaches: Optical Potential, RMF, Energy-Dependent RMF

The inclusion of FSI effects is extremely important for the description of semi-inclusive data

Some recent references:

R. Gonzalez-Jimenez et al., PRC 101, 015503 (2020) ;

J. Isaacson et al., PRC 103 015502 (2021);

A. Nikolakopoulos et al. PRC 105, 054603 (2022);

A. Ershova et al., PRD 106 032009 (2022); PRD 108 112008 (2023) [PhD thesis @DPhN - DPhP](#)

The semi-inclusive neutrino cross section

There is an increasing interest on semi-inclusive cross sections

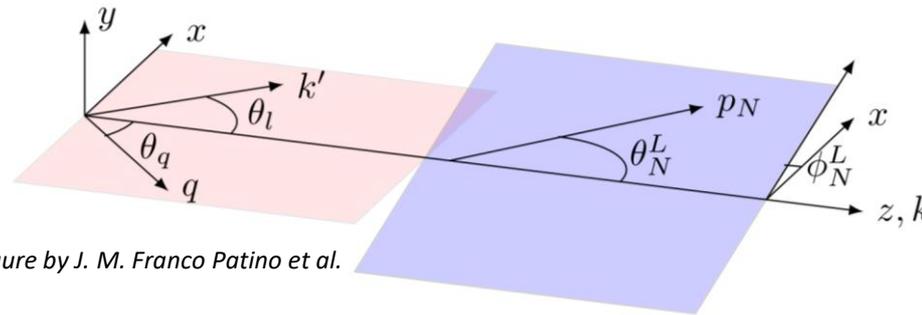


Figure by J. M. Franco Patino et al.

M. B. Barbaro talk @NUFACT 2021

$$\begin{aligned} \mathcal{F}_\chi^2 &= L_{\mu\nu} W^{\mu\nu} \\ &= V_{CC} R^{CC} + 2V_{CL} R^{CL} + V_{LL} R^{LL} + V_T R^T + V_{TT} R^{TT} + V_{TC} R^{TC} + V_{TL} R^{TL} + \chi (V_T R^T + V_{TC} R^{TC} + V_{TL} R^{TL}) \end{aligned}$$

The $(\nu_\mu, \mu p)$ cross section is decomposed in **10 independent response functions** of **5 variables** $(\omega, q, \mathbf{p}_N)$.

More complex structure than in the **inclusive** (ν_μ, μ) case: **5 new responses**, which vanish after integration over the final nucleon variables

$$R^{TT,TC,TL,TC',TL'} \propto \cos(\phi), \cos(2\phi) \quad \phi \text{ outgoing nucleon azimuthal angle}$$

Semi-inclusive \rightarrow Inclusive (but not viceversa!)

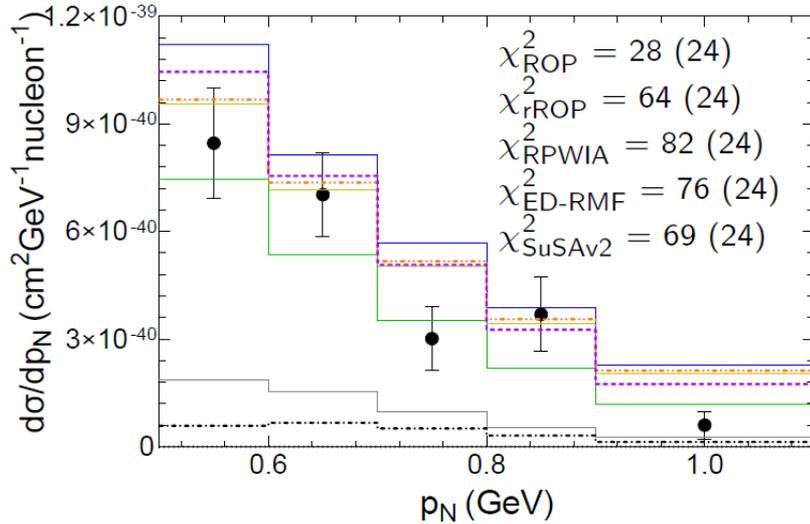
Theoretical situation:

- Few models and papers for genuine CCQE [J. M. Franco Patino et al, PRC 102 (2020); PRD 104 (2021); PRD 106 (2022); 2304.01916; A. V. Butkevich PRC 105 (2022)]
- 1 (incomplete due to the absence of Δ -MEC) published result for 2p-2h [T. Van Cuyck et al. (Ghent) PRC 94 (2016); PRC 95 (2017)] + PhD thesis of Kajetan Niewczas (inclusion of Δ -MEC)
- **PhD thesis of Valerio Belocchi (Torino) – talk tomorrow** [V. Belocchi et al. arXiv: 2401.13640]
- 1 very recent work on two-nucleon emission: V.L. Martinez-Cosentino et al. (Granada) PRC 109 (2024)

Semi-inclusive CC0 π 1p cross section: role of proton FSI

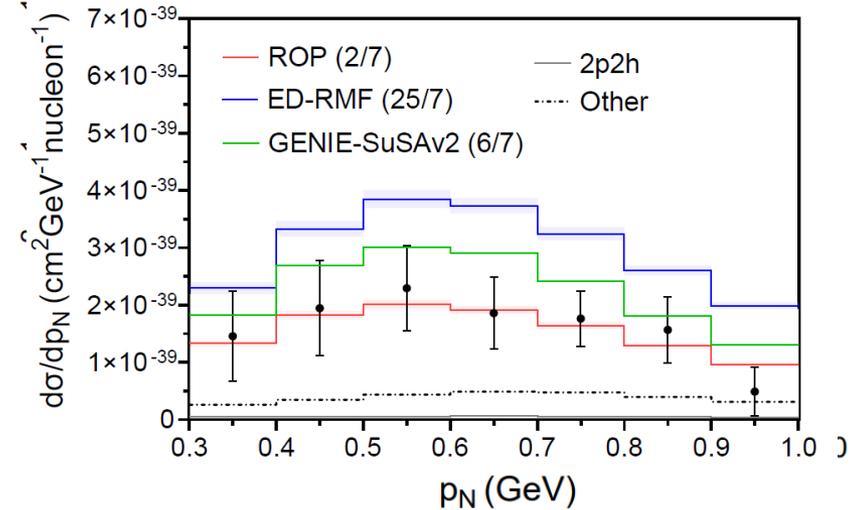
T2K data

$0.8 < \cos(\theta_l) < 1.0$; $0.3 < \cos(\theta_N^L) < 0.8$



MicroBooNE data

$-0.65 < \cos(\theta_l) < 0.95$

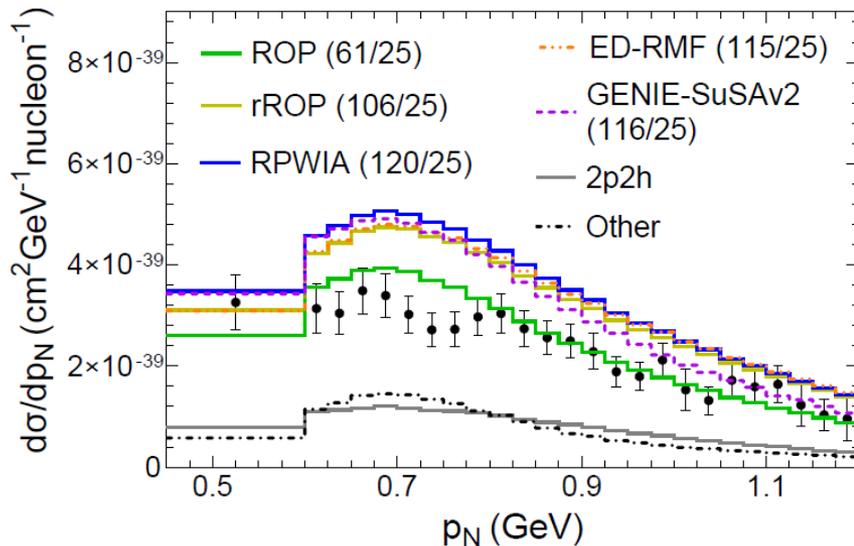


RPWIA: no FSI

GENIE-SuSAv2: include FSI but from inclusive model (factorization)

ED-RMF, rROP, ROP: different theoretical approaches for FSI

MINERvA data



J. M. Franco Patino et al, PRD 106 (2022); 2304.01916

- FSI improve the agreement with data with respect to the RPWIA (no FSI) prediction
- Large differences between different FSI models

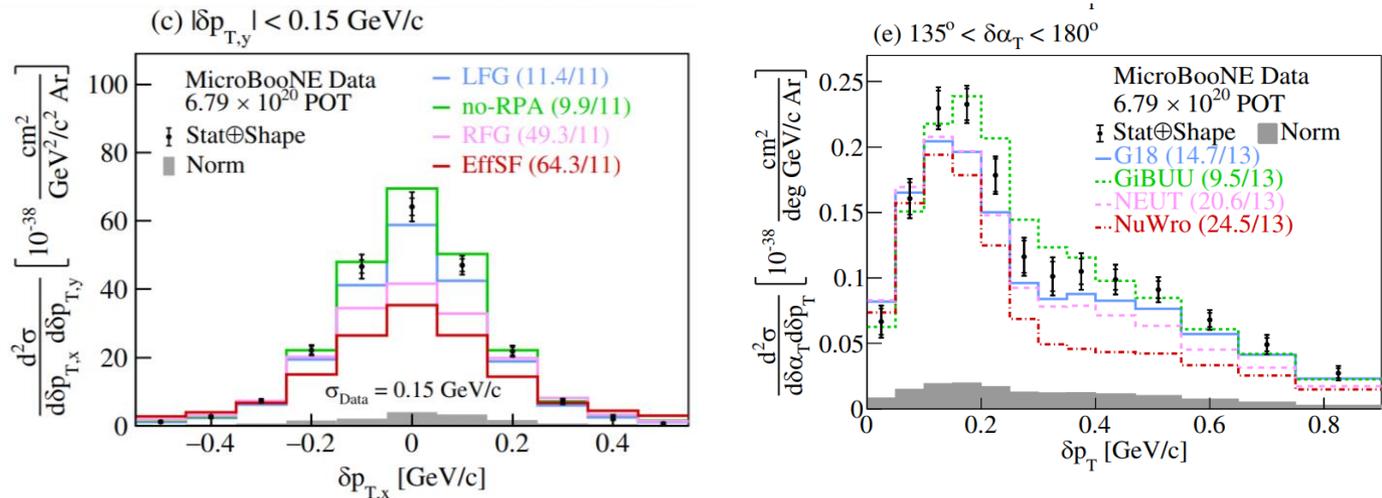
Several recent MicroBooNE studies using Kinematic Imbalance Variables

PHYSICAL REVIEW LETTERS **131**, 101802 (2023)

First Double-Differential Measurement of Kinematic Imbalance in Neutrino Interactions with the MicroBooNE Detector

PHYSICAL REVIEW D **108**, 053002 (2023)

Multidifferential cross section measurements of ν_μ -argon quasielasticlike reactions with the MicroBooNE detector



[2310.06082](#)

Measurement of nuclear effects in neutrino-argon interactions using generalized kinematic imbalance variables with the MicroBooNE detector

“These measurements allow us to demonstrate that the treatment of CCQE interactions in GENIEv2 is inadequate to describe data. Further, they reveal tensions with more modern generator predictions particularly in regions of phase space where FSI are important.”

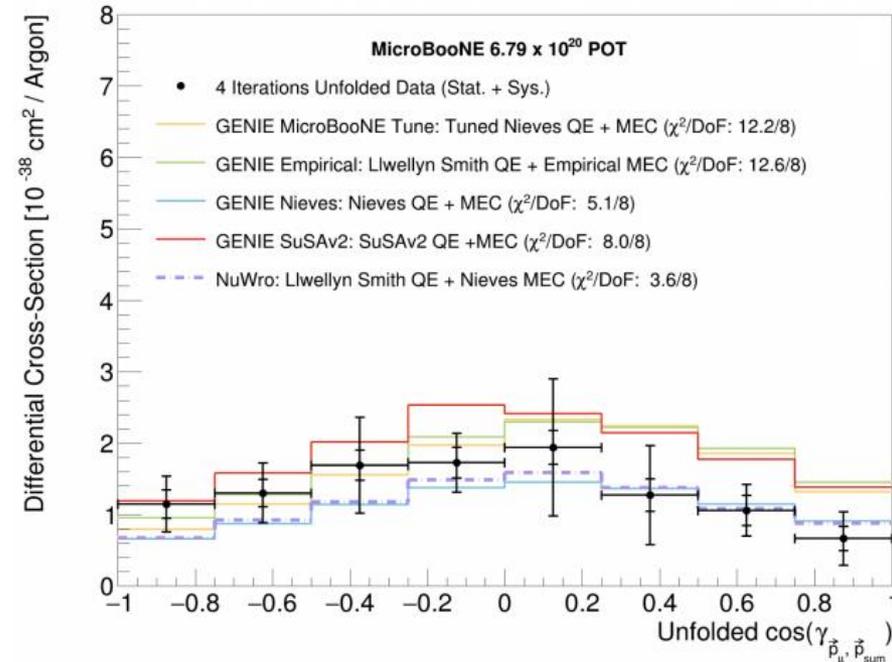
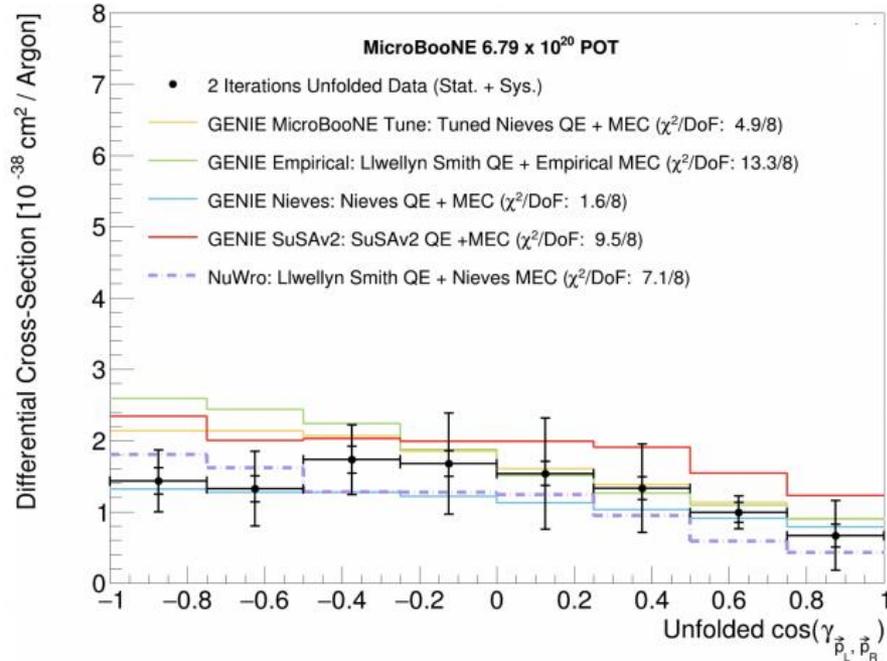
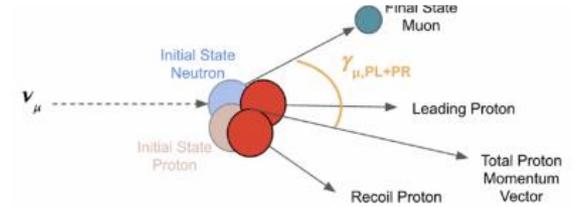
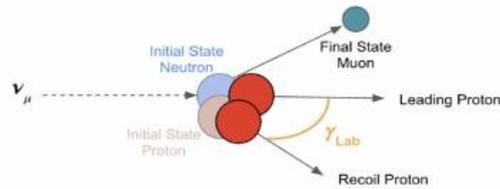
This is not a surprise since the generators implement “inclusive” models

What we learn by comparing semi-inclusive measurement as a function of hadronic variables with Monte Carlo predictions based on inclusive models?

MicroBooNE semi-inclusive $CC0\pi2p$ on argon

First Measurement of Differential Cross Sections for Muon Neutrino Charged Current Interactions on Argon with a Two-proton Final State in the MicroBooNE Detector

[2211.03734](#)

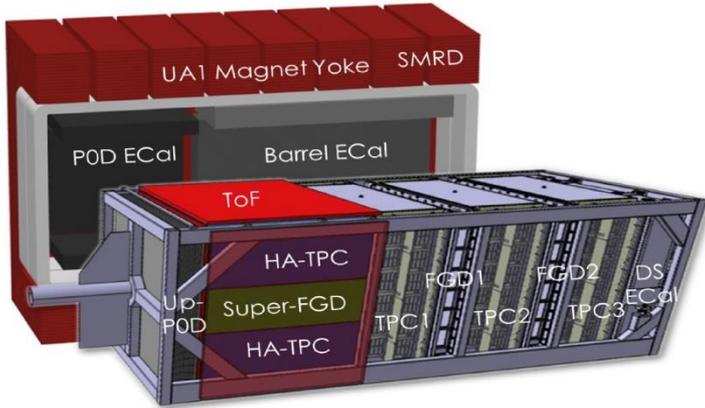


- Spread of Monte Carlo predictions
- How good are the current approximations of the MC for the semi-inclusive processes?

Complete semi-inclusive fully microscopic calculations of 2p-2h are not yet available

T2K News – December 2023: Started data taking with ND280 Upgrade

T2K experiment enters a new phase with significantly improved sensitivity for its world leading neutrino oscillation research – KEK | 高エネルギー加速器研究機構

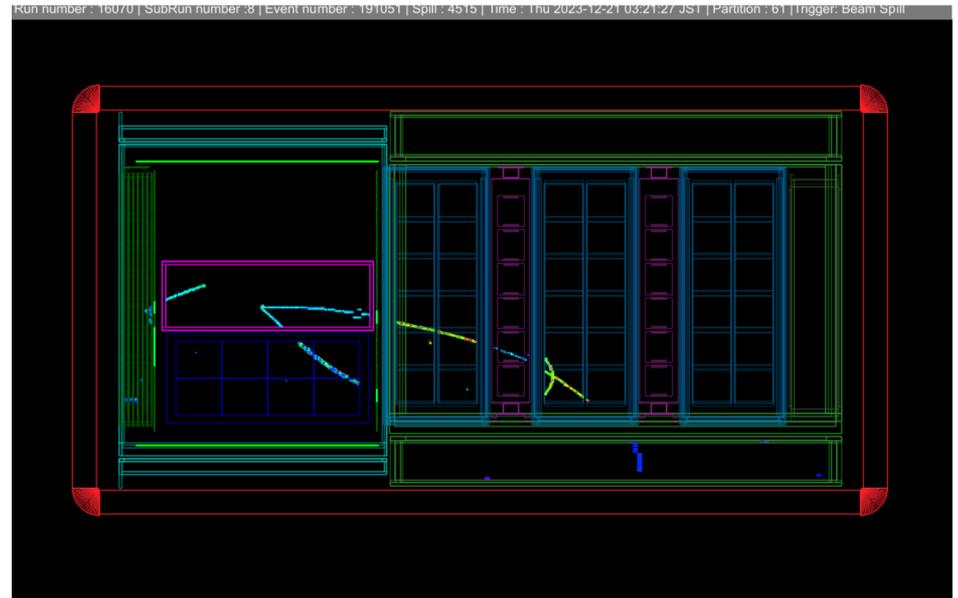


PRESS RELEASE



T2K experiment enters a new phase with significantly improved sensitivity for its world leading neutrino oscillation research

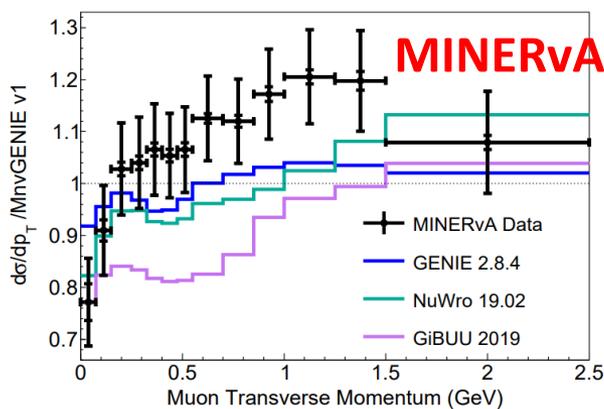
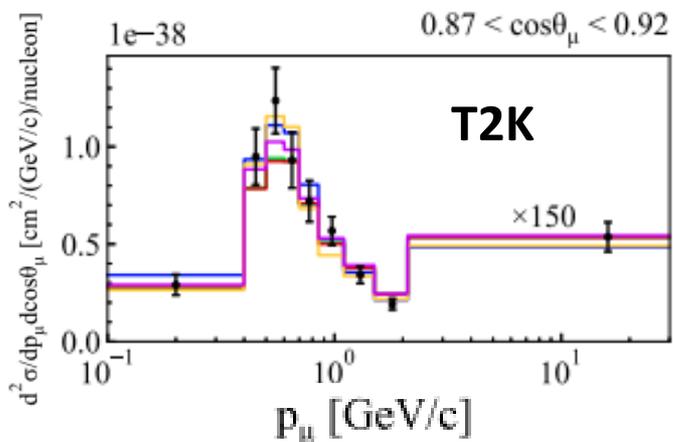
- Started data taking with upgraded accelerator neutrino beam and new detectors -



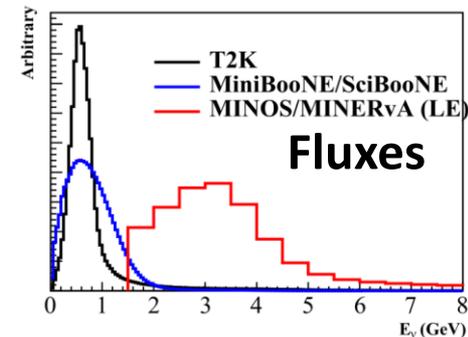
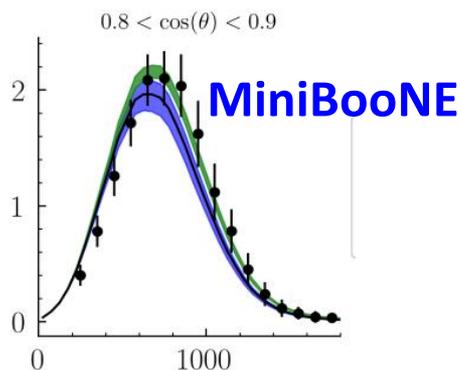
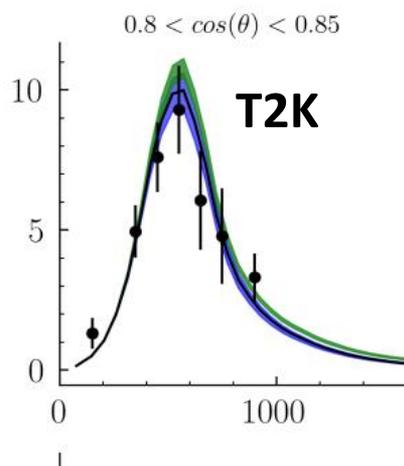
General considerations

1) The spread of the models increases with the neutrino energy

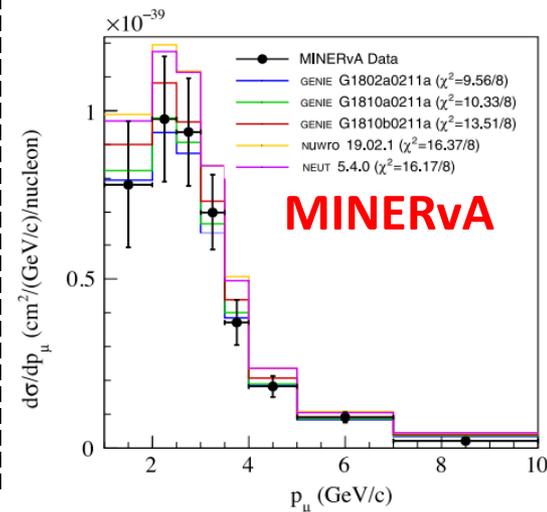
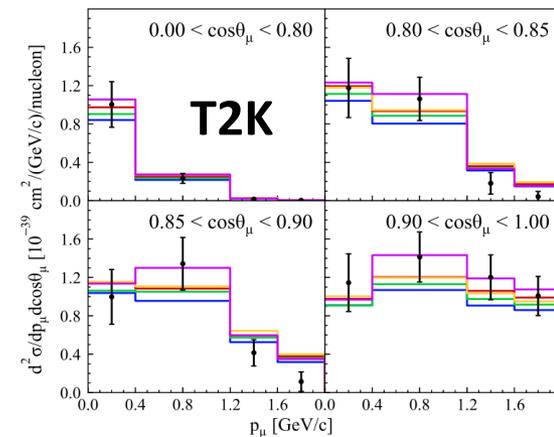
CC Inclusive



CC0 π

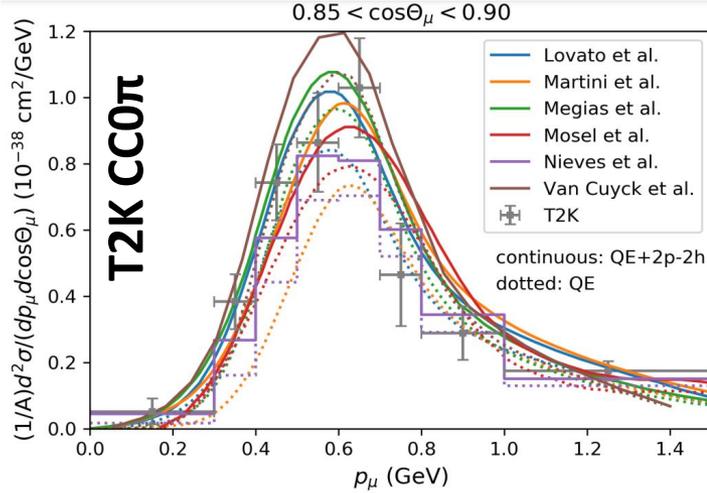


CC1 π

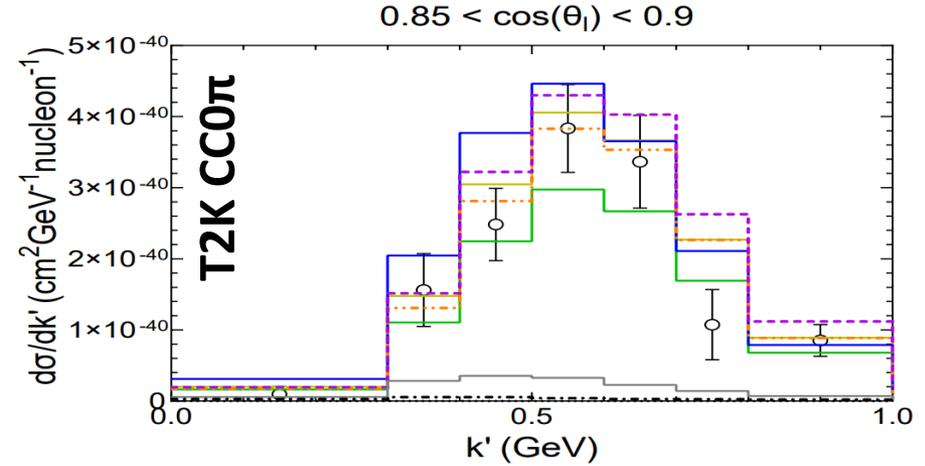


2) The spread of the models is larger in semi-inclusive processes

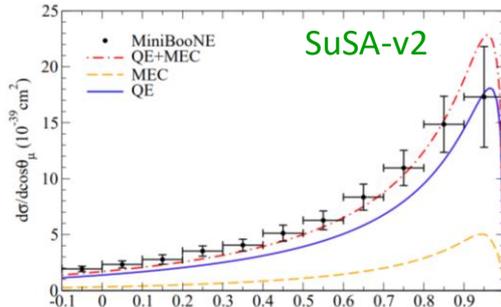
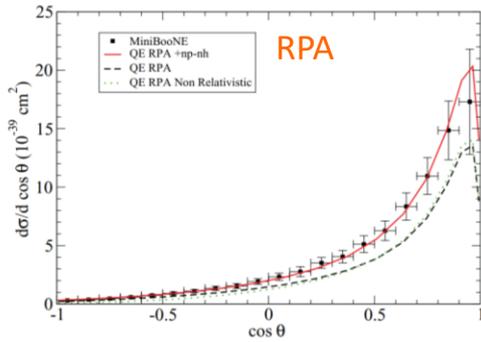
“Inclusive”



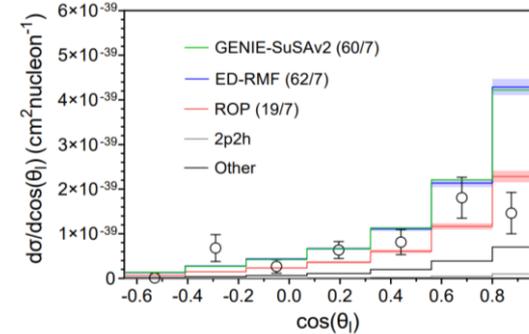
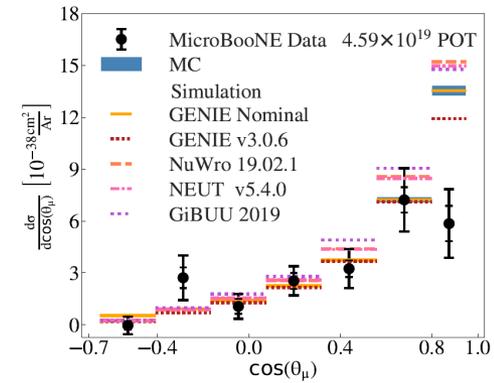
Semi-inclusive



MiniBooNE “CCQE-like”

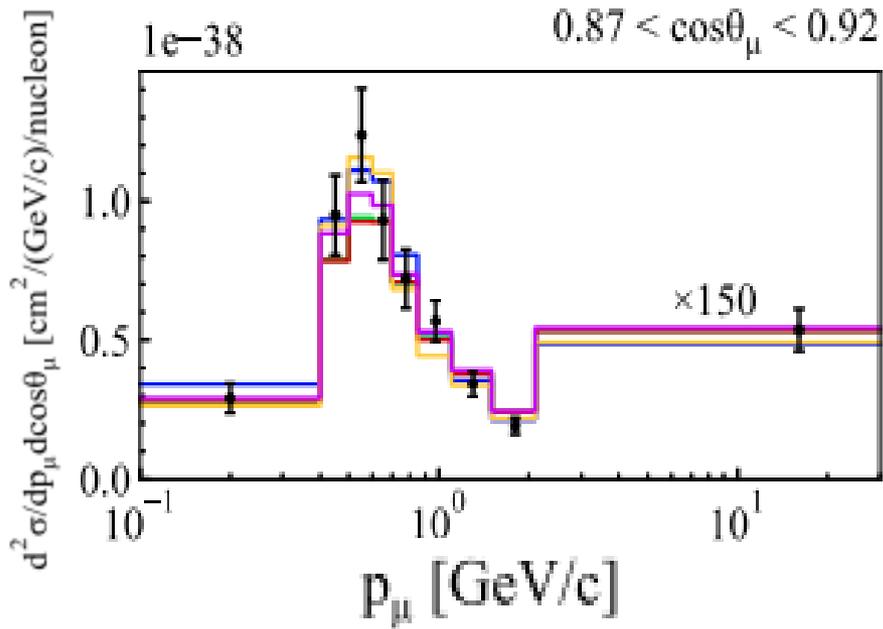


MicroBooNE CC0 π 1p

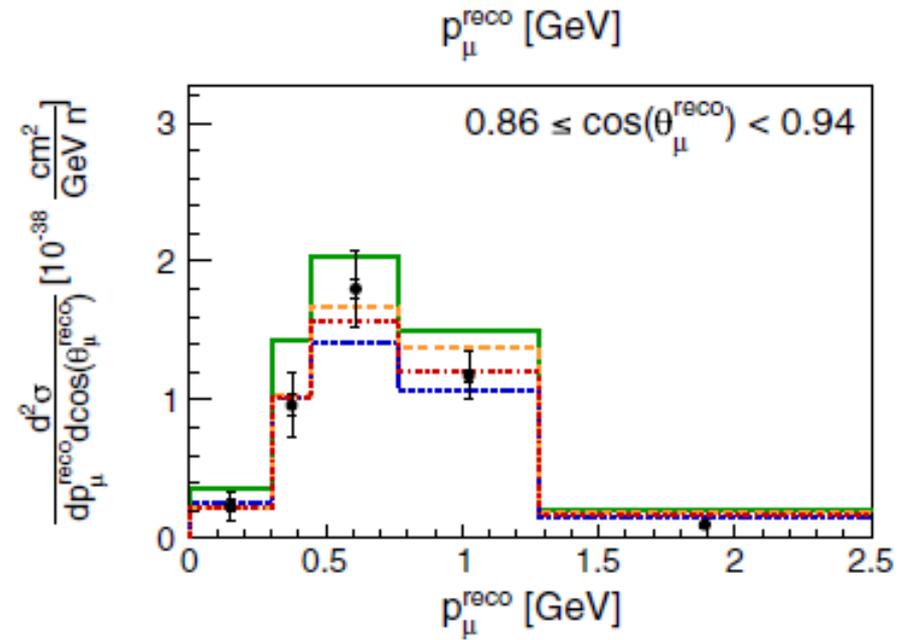


3) The spread of the models is larger for Argon than for Carbon

T2K Carbon



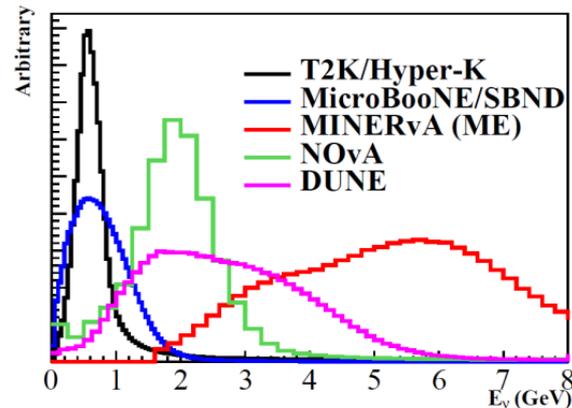
CC Inclusive



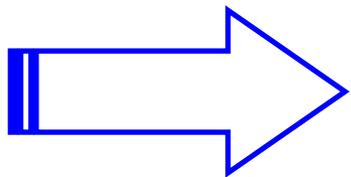
General comments

- 1) The spread of the models increases with the neutrino energy
- 2) The spread of the models is larger in semi-inclusive processes
- 3) The spread of the models is larger for Argon than for Carbon

This is not surprising since in the last 15 years the neutrino community focused on Carbon, on “inclusive” measurements as a function of the leptonic variables (Cherenkov detectors) and on “low” neutrino energy (MiniBooNE and T2K)



DUNE will be at larger energies, will use Argon detectors, will exploit semi-inclusive measurements as a function of leptonic and hadronic variables



Many studies are needed!

Neutrino cross sections: summary of status and perspectives

A) Cross sections in terms of muon variables (CC inclusive, $CC0\pi$)

Significant progress in the last 15 years

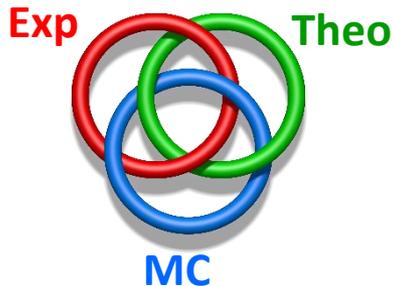
- Many experimental and theoretical results
- Still we have to tackle currently existing degeneracies:
 1. between cross sections and flux uncertainties
 2. between nucleon uncertainties and nuclear effects
 3. between different nuclear models and approximations

B) Cross sections in terms of hadronic variables ($CC0\pi1p$, $CC0\pi Np$, $CC1\pi$, $CCOther$)

We are only at the beginning!

- Few experimental and theoretical results
- **Theoretical models and Monte Carlo implementation of semi-inclusive processes are needed**
- The one pion puzzle is still there

New generation experiments open important perspectives for neutrino cross sections



Close collaboration between **theorists**,
experimentalists and **generator developers** is crucial

For the moment the community (at least theorists and
generator developers) is not so large

In the precision era of neutrino physics new intriguing results, like CP violation, necessary
passes through a precise knowledge of neutrino-nucleus cross sections

Some Review papers

[1305.7513.pdf \(arxiv.org\)](#)

REVIEWS OF MODERN PHYSICS, VOLUME 84, JULY–SEPTEMBER 2012

From eV to EeV: Neutrino cross sections across energy scales

J. A. Formaggio*

Laboratory for Nuclear Science Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

G. P. Zeller†

Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA

[1611.07770.pdf \(arxiv.org\)](#)

IOP Publishing

Journal of Physics G: Nuclear and Particle Physics

J. Phys. G: Nucl. Part. Phys. **45** (2018) 013001 (98pp)

<https://doi.org/10.1088/1361-6471/aa8bf7>

Topical Review

Neutrino–nucleus cross sections for oscillation experiments

Teppei Katori^{1,4,5} and Marco Martini^{2,3,4,5}

[2108.12212.pdf \(arxiv.org\)](#)

 **symmetry**



Review

A New Generation of Neutrino Cross Section Experiments: Challenges and Opportunities

Antonio Branca^{1,*}, Giulia Brunetti¹, Andrea Longhin^{2,3}, Marco Martini^{4,5}, Fabio Pupilli³ and Francesco Terranova¹

[1706.03621.pdf \(arxiv.org\)](#)

Progress in Particle and Nuclear Physics 100 (2018) 1–68



Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp



Review

NuSTEC¹ White Paper: Status and challenges of neutrino–nucleus scattering

L. Alvarez-Ruso^a, M. Sajjad Athar^b, M.B. Barbaro^c, D. Cherdack^d, M.E. Christy^e, P. Coloma^f, T.W. Donnelly^g, S. Dytman^h, A. de Gouvêaⁱ, R.J. Hill^{j,f}, P. Huber^k, N. Jachowicz^l, T. Katori^m, A.S. Kronfeld^f, K. Mahnⁿ, M. Martini^o, J.G. Morfin^{f,*}, J. Nieves^a, G.N. Perdue^f, R. Petti^p, D.G. Richards^q, F. Sánchez^r, T. Sato^{s,t}, J.T. Sobczyk^u, G.P. Zeller^f



[2206.13792.pdf \(arxiv.org\)](#)



Progress in Particle and Nuclear
Physics

Volume 129, March 2023, 104019



Review

Neutrinos and their interactions with matter

M. Sajjad Athar  , A. Fatima, S.K. Singh

Further details

- My lectures at Ecole de GIF 2022

[Ecole de Gif 2022: La Physique des Neutrinos \(5-9 septembre 2022\): Sections efficaces d'interaction de neutrinos](#)

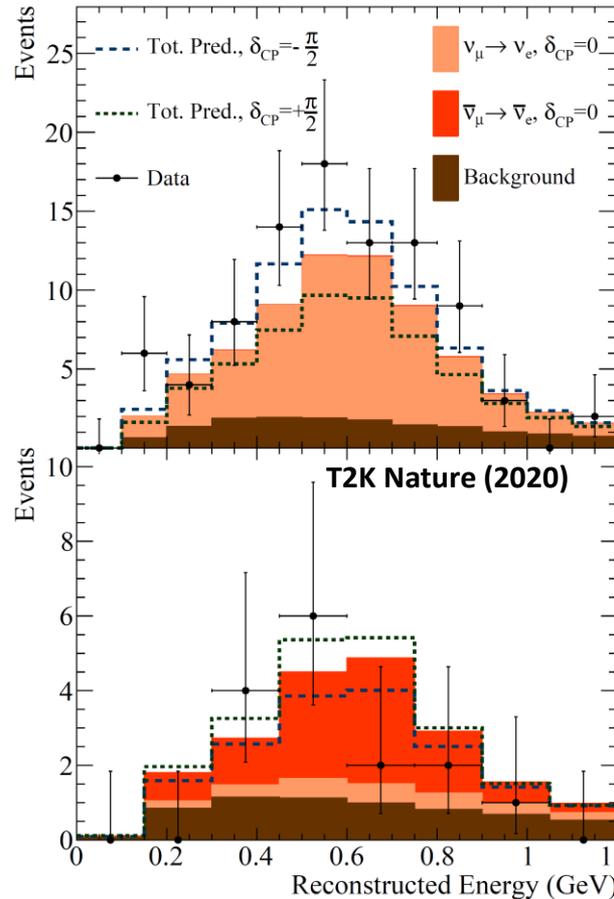
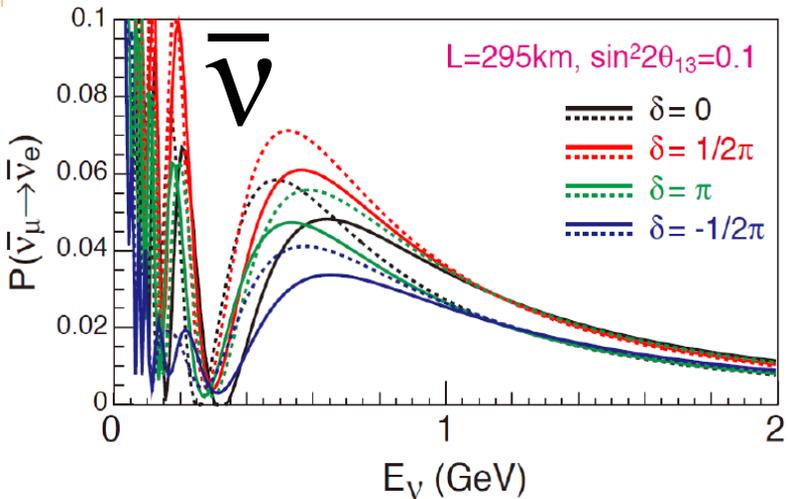
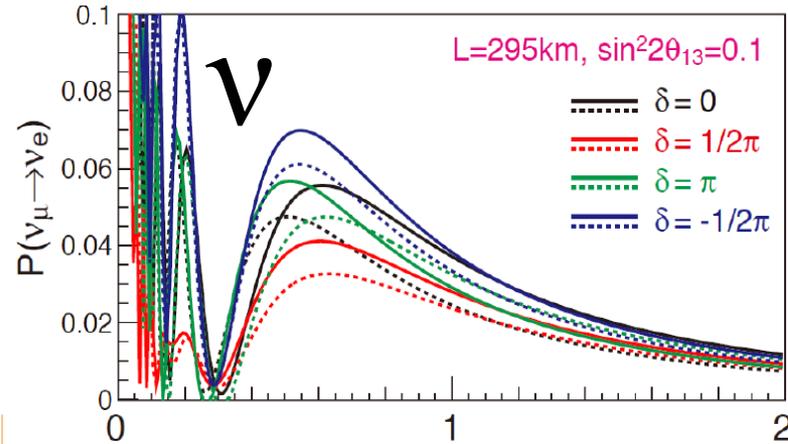


APPENDIX

ν .vs. $\bar{\nu}$ and ν_{μ} .vs. ν_e

ν oscillation and CP violation

$$P(\nu_\mu \rightarrow \nu_e) \stackrel{?}{\neq} P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \iff \delta_{\text{CP}}$$



$\nu_\mu \rightarrow \nu_e$

$\bar{\nu}_\mu \rightarrow \bar{\nu}_e$

A precise and simultaneous knowledge of the four cross sections is important in connection to the oscillation experiments aiming at the search for CP violation in the lepton sector.

Neutrino vs Antineutrino interactions

The ν and anti ν cross sections differ by the sign of the V-A interference term

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2 \cos^2\theta_c |\mathbf{k}'| E_l' \cos^2\frac{\theta}{2}}{\pi} \left[\frac{(\mathbf{q}^2 - \omega^2)^2}{\mathbf{q}^4} G_E^2 R_\tau(\mathbf{q}, \omega) + \frac{\omega^2}{\mathbf{q}^2} G_A^2 R_{\sigma\tau(L)}(\mathbf{q}, \omega) \right. \\ \left. + 2 \left(\tan^2\frac{\theta}{2} + \frac{\mathbf{q}^2 - \omega^2}{2\mathbf{q}^2} \right) \left(G_M^2 \frac{\mathbf{q}^2}{4M_N^2} + G_A^2 \right) R_{\sigma\tau(T)}(\mathbf{q}, \omega) \pm 2 \frac{E_\nu + E_l'}{M_N} \tan^2\frac{\theta}{2} G_A G_M R_{\sigma\tau(T)}(\mathbf{q}, \omega) \right]$$

Vector-Axial interference

$$\begin{cases} + & (\nu) \\ - & (\bar{\nu}) \end{cases}$$

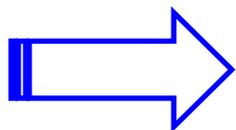
Vector-Axial interference:

basic asymmetry from weak interaction theory

different sign in the Leptonic tensor

$$L_{\mu\nu} = k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k \cdot k' \mp i\varepsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta$$

ν
 $\bar{\nu}$

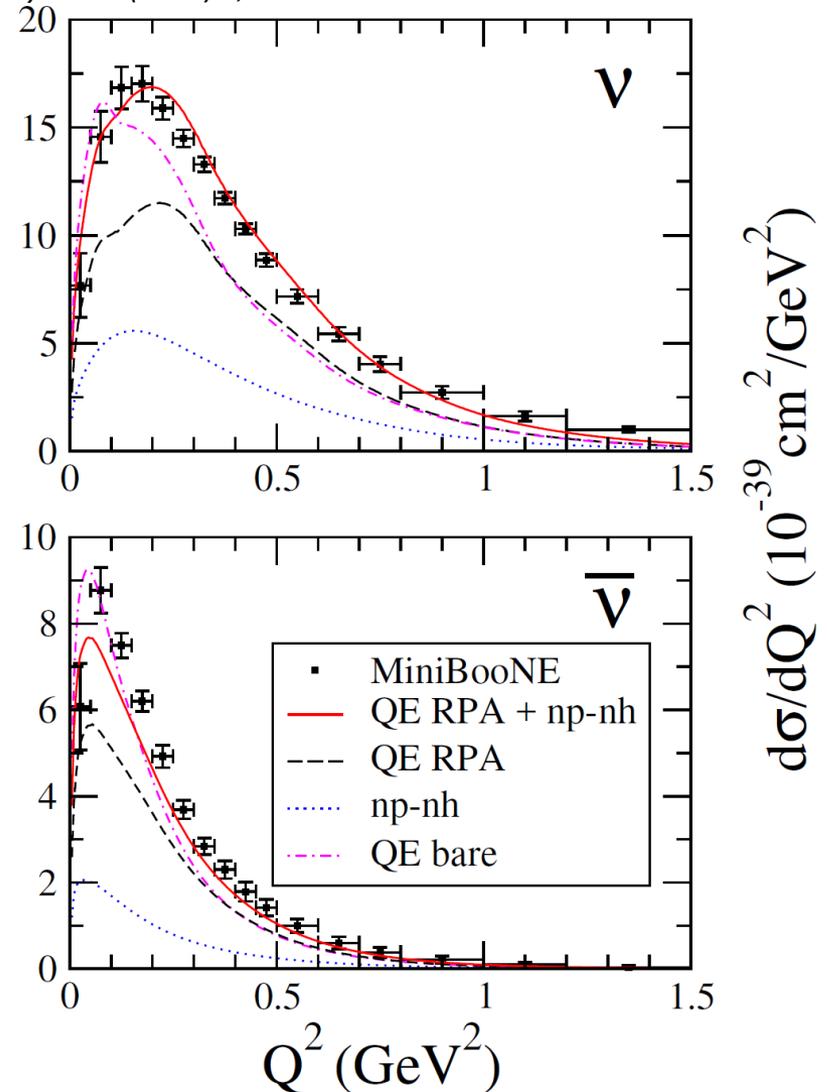
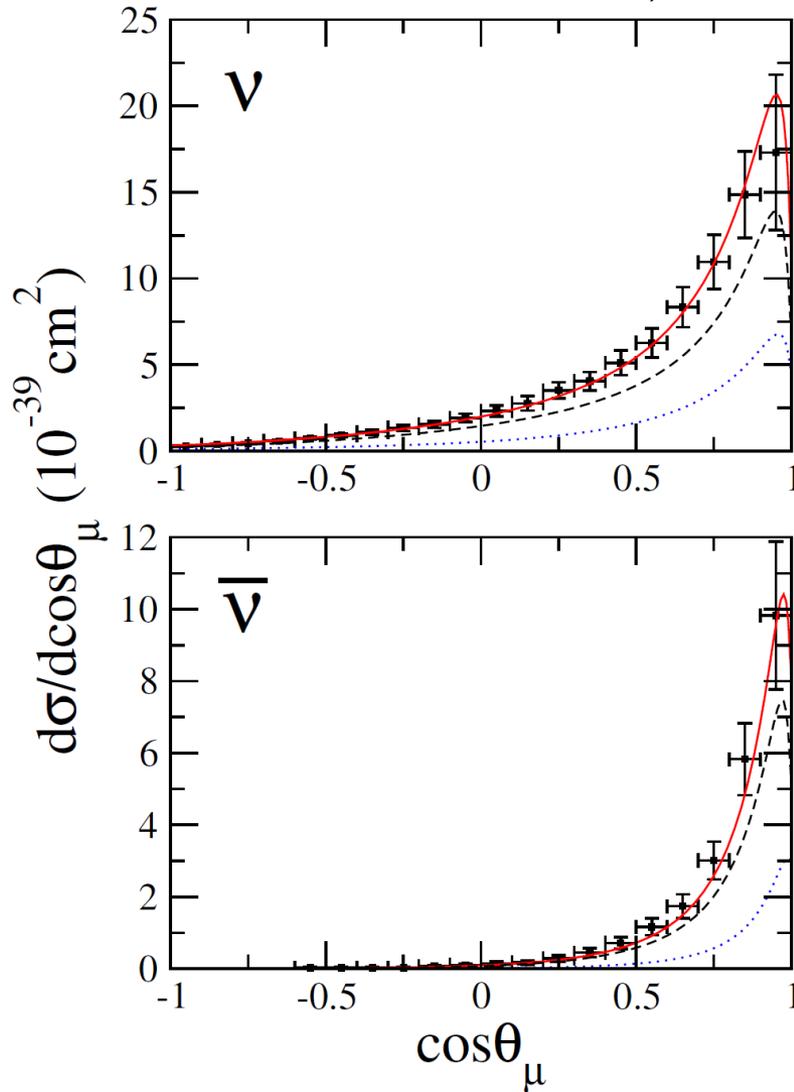


Even neglecting nuclear effects, the absolute value and the kinematic behavior of neutrino and antineutrino cross sections are different

$d\sigma/d\cos\theta$

Q^2 distribution

T. Katori, M. Martini, *J.Phys.G* 45 (2018) 1, 013001



- Antineutrino cross section falls more rapidly than the neutrino one
- Antineutrino Q^2 distribution peaks at smaller Q^2 values than the neutrino one

Neutrino vs Antineutrino interactions and nuclear effects

$$\frac{d^2\sigma}{d\cos\theta d\omega} = \frac{G_F^2 \cos^2\theta_c}{\pi} |\mathbf{k}'| E_l' \cos^2\frac{\theta}{2} \left[\frac{(\mathbf{q}^2 - \omega^2)^2}{\mathbf{q}^4} G_E^2 R_\tau(\mathbf{q}, \omega) + \frac{\omega^2}{\mathbf{q}^2} G_A^2 R_{\sigma\tau(L)}(\mathbf{q}, \omega) \right]$$

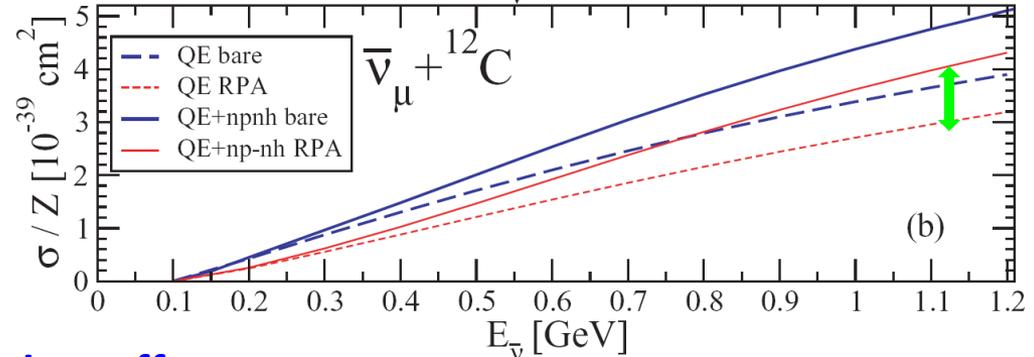
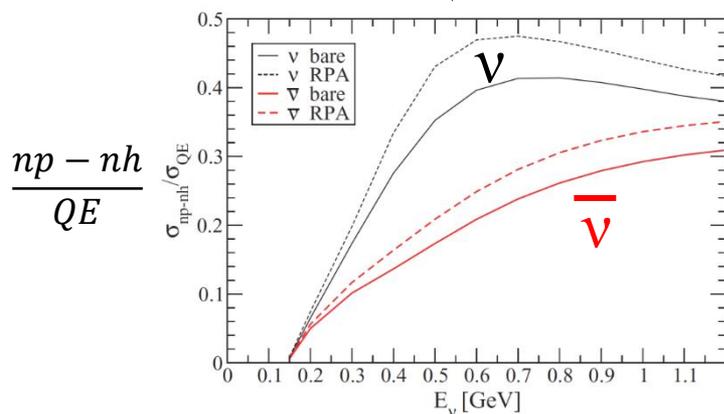
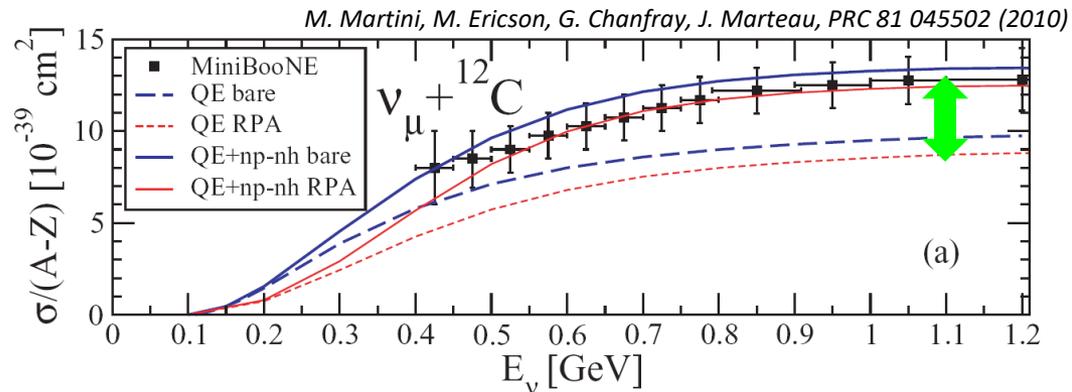
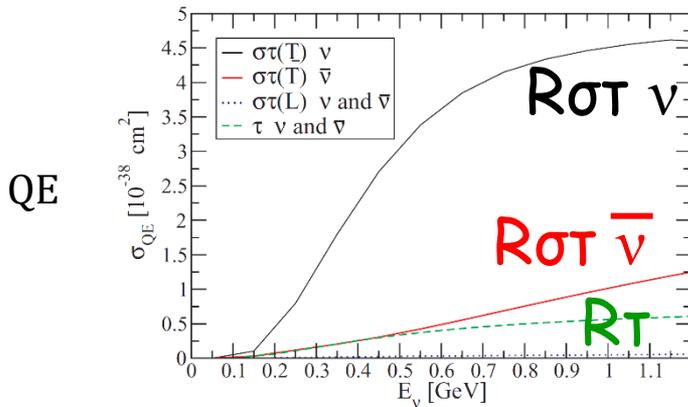
$$+ 2 \left(\tan^2\frac{\theta}{2} + \frac{\mathbf{q}^2 - \omega^2}{2\mathbf{q}^2} \right) \left(G_M^2 \frac{\mathbf{q}^2}{4M_N^2} + G_A^2 \right) R_{\sigma\tau(T)}(\mathbf{q}, \omega) \pm 2 \frac{E_\nu + E_l'}{M_N} \tan^2\frac{\theta}{2} G_A G_M R_{\sigma\tau(T)}(\mathbf{q}, \omega)$$

Vector-Axial interference

The ν and anti ν interactions differ by the sign of the V-A interference term

→ the relative weight of the different nuclear responses is different for neutrinos and antineutrinos

→ the relative role of np-nh contributions is different for neutrinos and antineutrinos



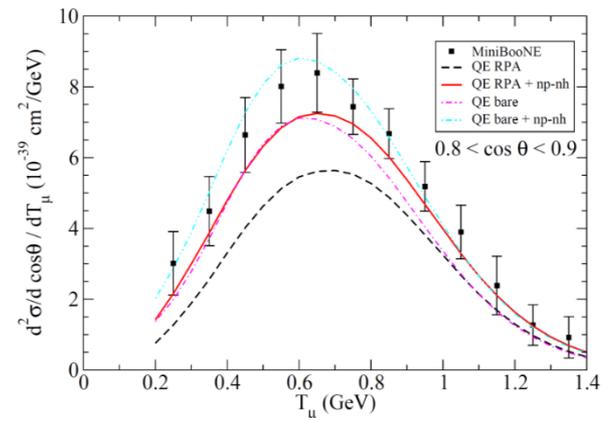
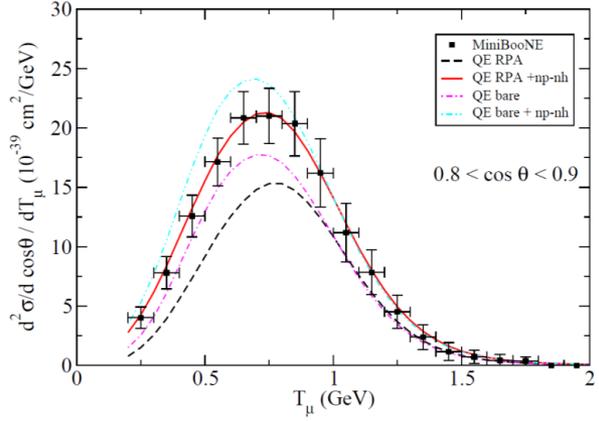
Nuclear effects generate an asymmetry unrelated to CP violation

The relative role of np-nh for neutrinos and antineutrinos is different in different approaches



Lyon RPA
Martini et al.

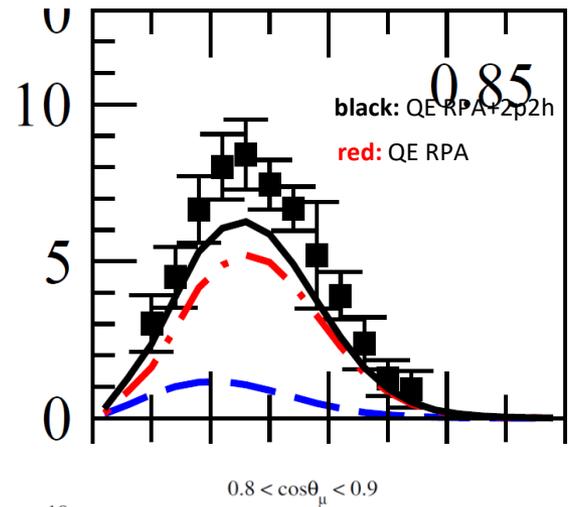
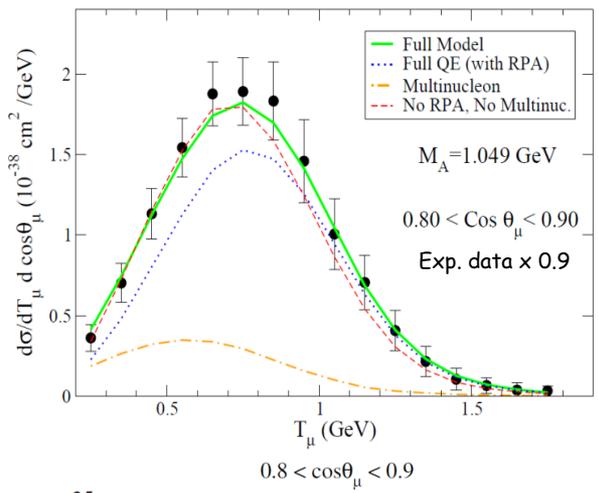
PRC 84 (2011)



PRC 87 (2013)

Valencia RPA
Nieves et al.

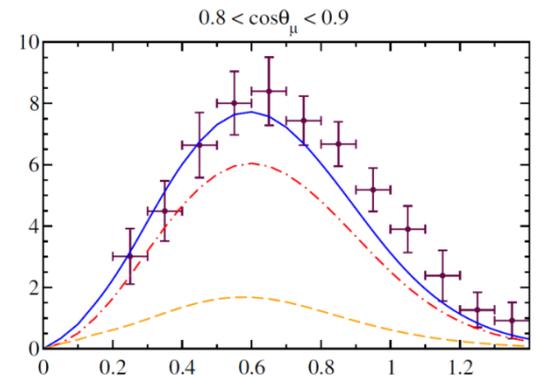
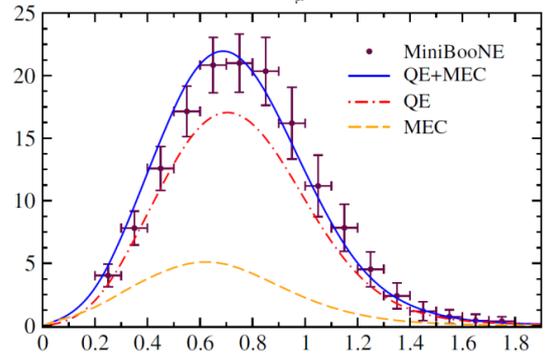
PLB 707 (2012)



PLB 721 (2013)

SuSAv2

PRD 94 (2016)



PRD 94 (2016)

Difference of ν and $\bar{\nu}$ cross sections and the VA interference term

$$d\sigma \sim d\sigma_L + d\sigma_T \pm d\sigma_{VA}$$

$$d\sigma_\nu - d\sigma_{\bar{\nu}} \overset{?}{\leftrightarrow} 2d\sigma_{VA}$$

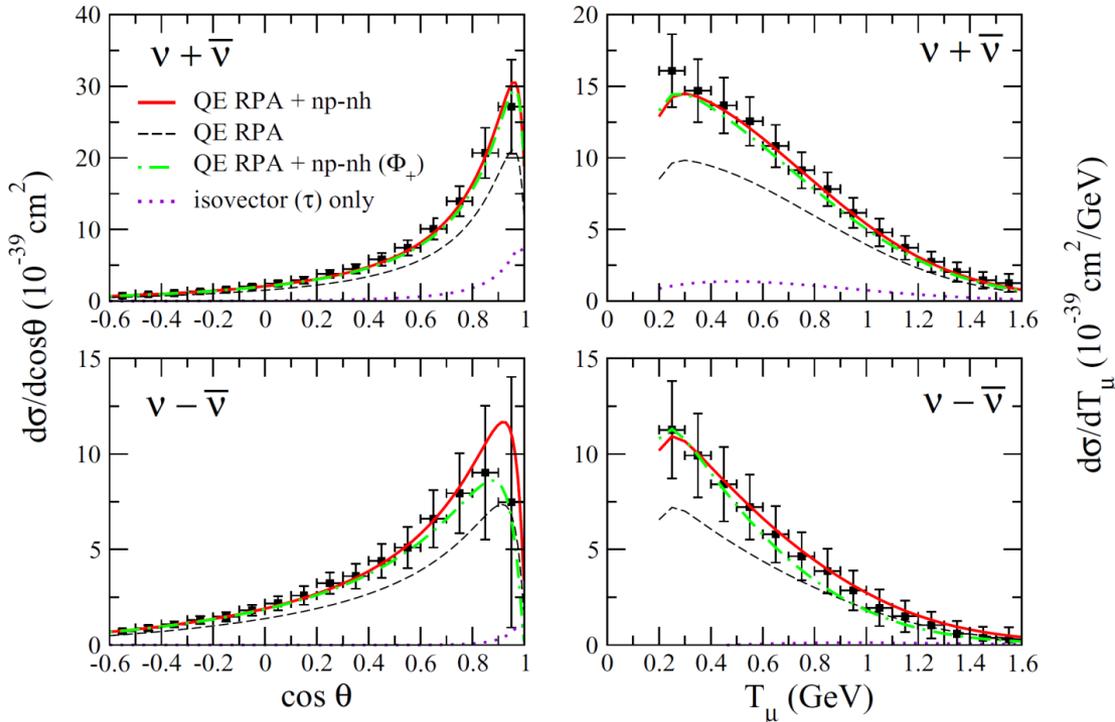
Difference gives only the VA term for identical ν and $\bar{\nu}$ flux

Problem: flux dependence of $d\sigma$ $\frac{d^2\sigma}{dE_\mu d\cos\theta} = \int dE_\nu \left[\frac{d^2\sigma}{d\omega d\cos\theta} \right]_{\omega=E_\nu-E_\mu} \Phi(E_\nu)$

We introduce the **mean flux** $\Phi_+ = 1/2[\Phi_\nu + \Phi_{\bar{\nu}}]$

We calculate the sum and the difference using **real** and **mean** MiniBooNE fluxes results

M. Ericson, M. Martini Phys. Rev. C 91 035501 (2015)



The mean flux contribution is dominant



The VA interference term is experimentally accessible in MBdata

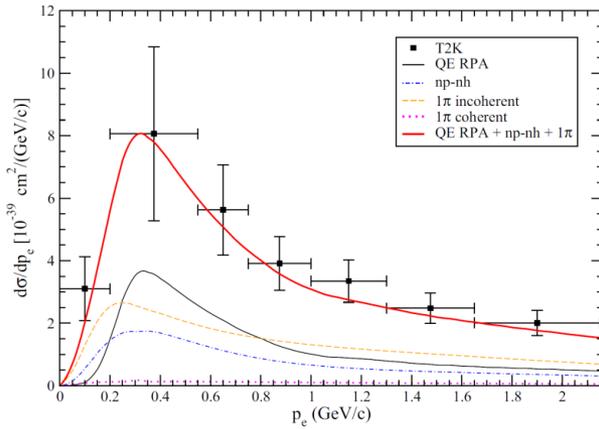


Need for the multinucleon component in the VA interference

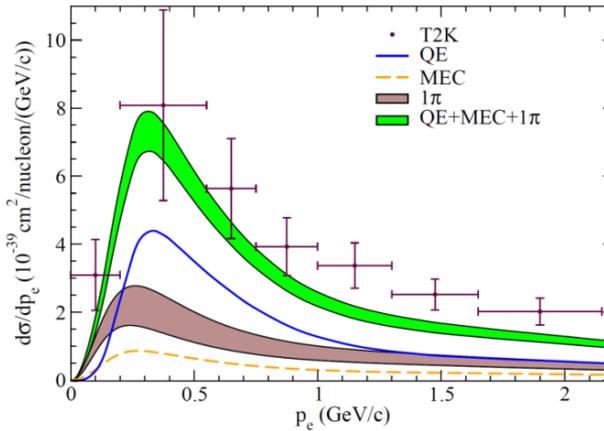
ν_e cross sections

- There are few published results on ν_e cross sections. This is essentially due the relatively small component of ν_e fluxes with respect to the ν_μ ones hence to small statistics.
- The ν_e experimental published results essentially concern inclusive cross sections
T2K flux-integrated ν_e CC inclusive differential cross sections on carbon

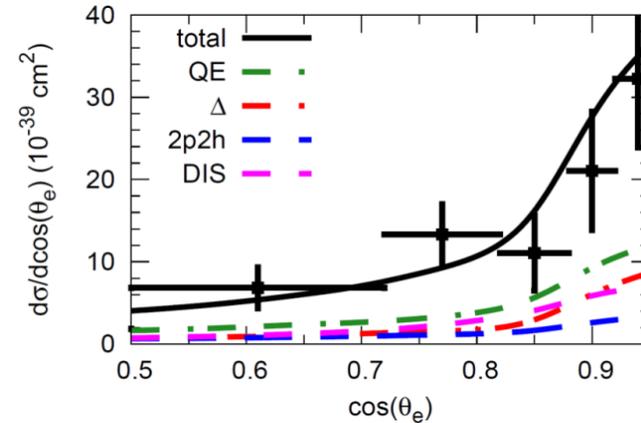
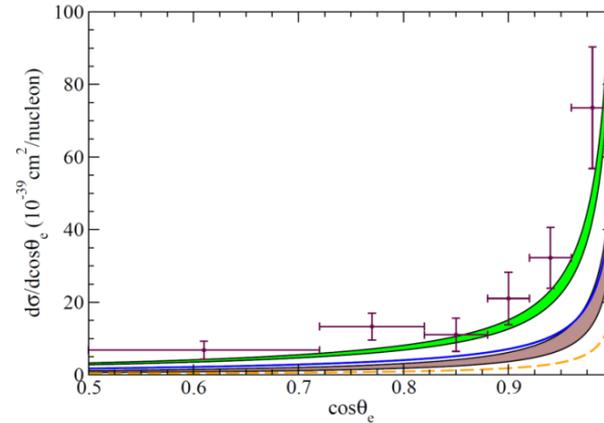
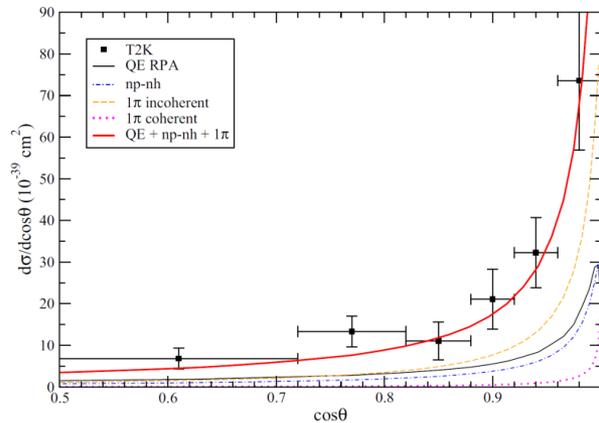
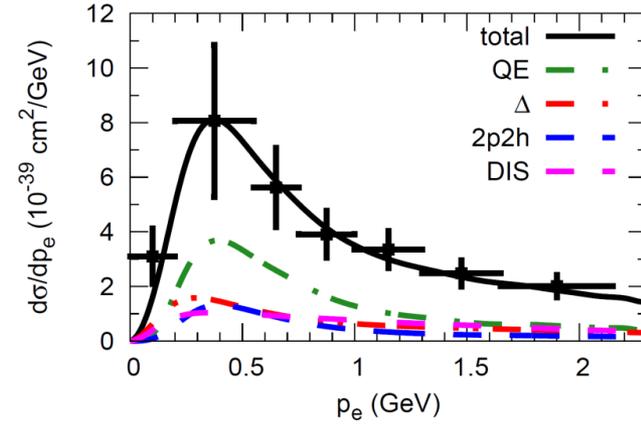
Martini et al., PRC 94 (2016)



Megias et al., PRD 94 (2016)

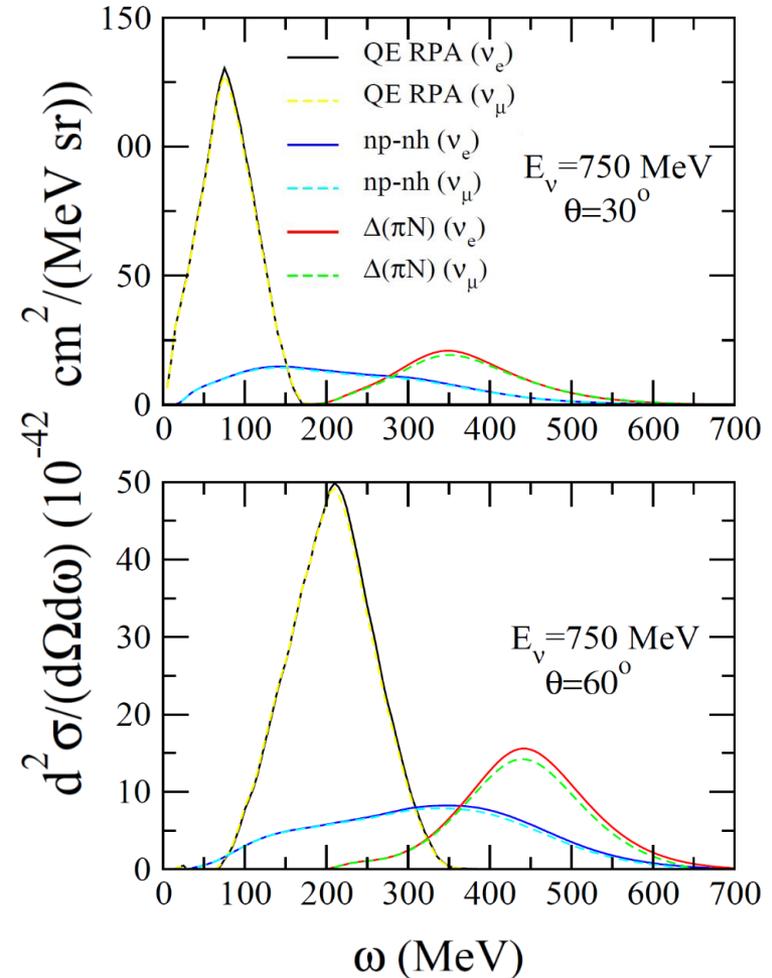
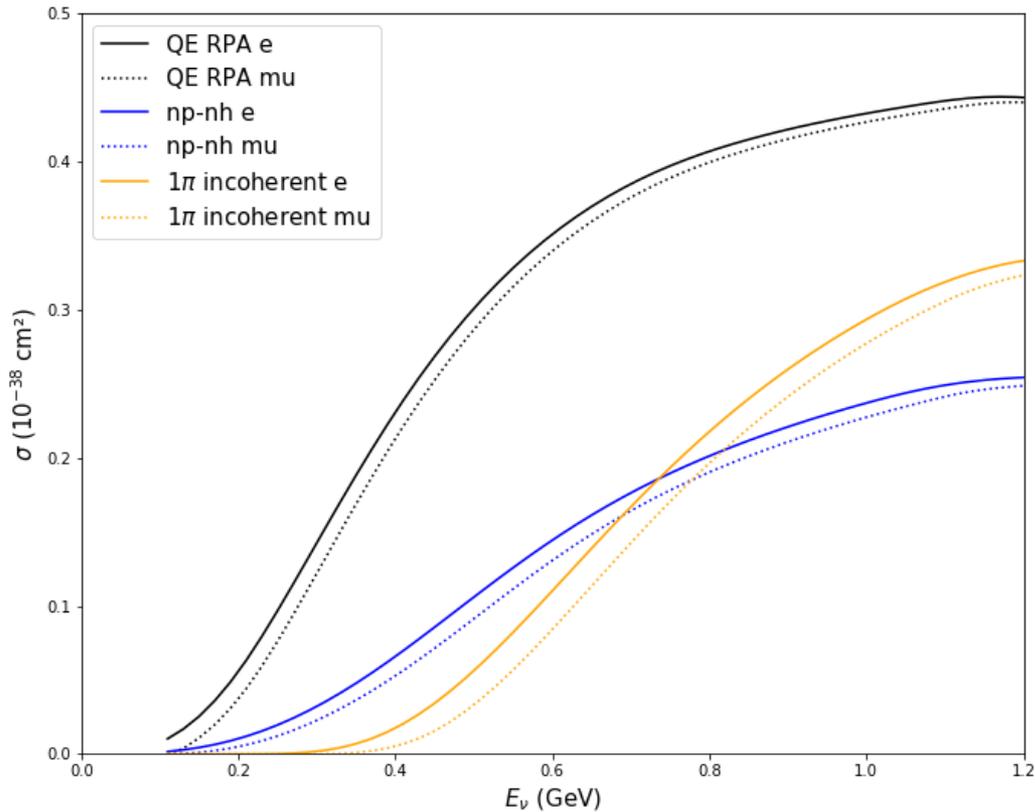


Gallmeister et al. PRC 94(2016)



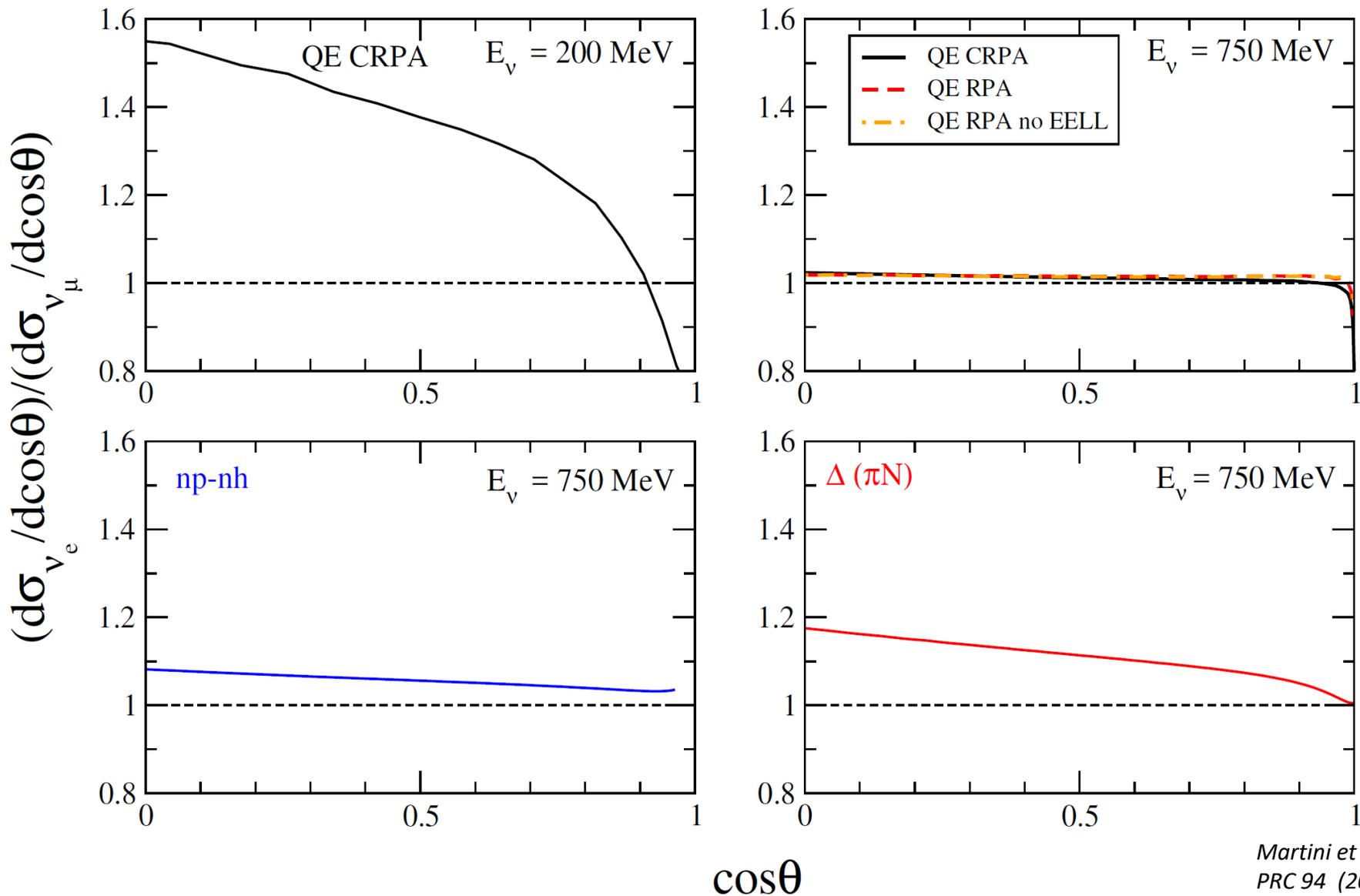
- Theoretical results agree with data
- Similarity of the theoretical results for the inclusive $d\sigma$

ν_e and ν_μ total and double differential cross sections



Due to the different kinematic limits, the ν_e cross sections are expected to be larger than the ν_μ ones

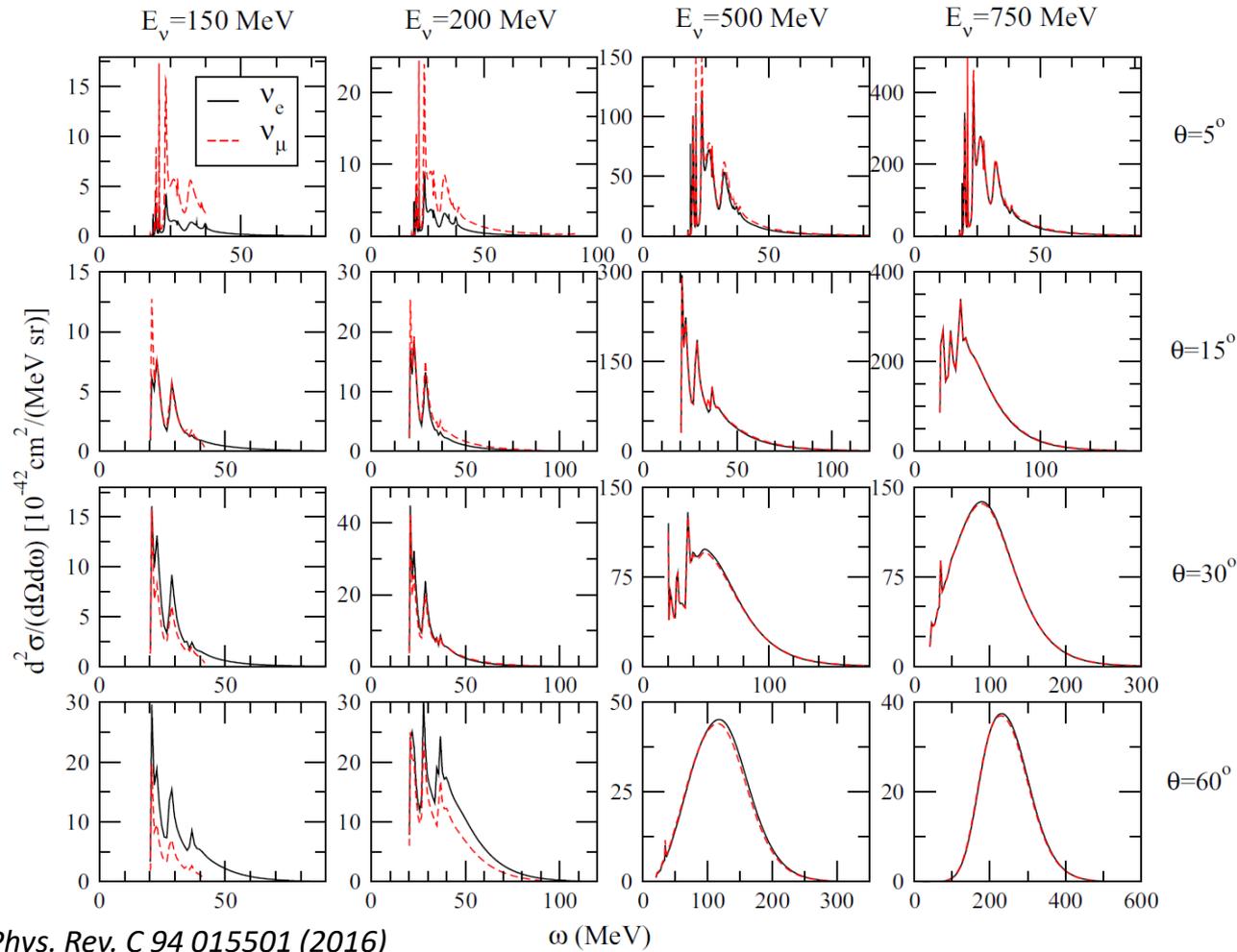
Ratio v_e/v_μ for $d\sigma/d\cos\theta$ in different channels



Martini et al.,
PRC 94 (2016)

Due to the different kinematic limits, the ν_e cross sections are expected to be larger than the ν_μ ones. However for forward scattering angles this hierarchy is opposite in the QE channel.

A theoretical study (HF+CRPA Ghent) of the ν_μ and ν_e $d^2\sigma$



M. Martini et al., Phys. Rev. C 94 015501 (2016)

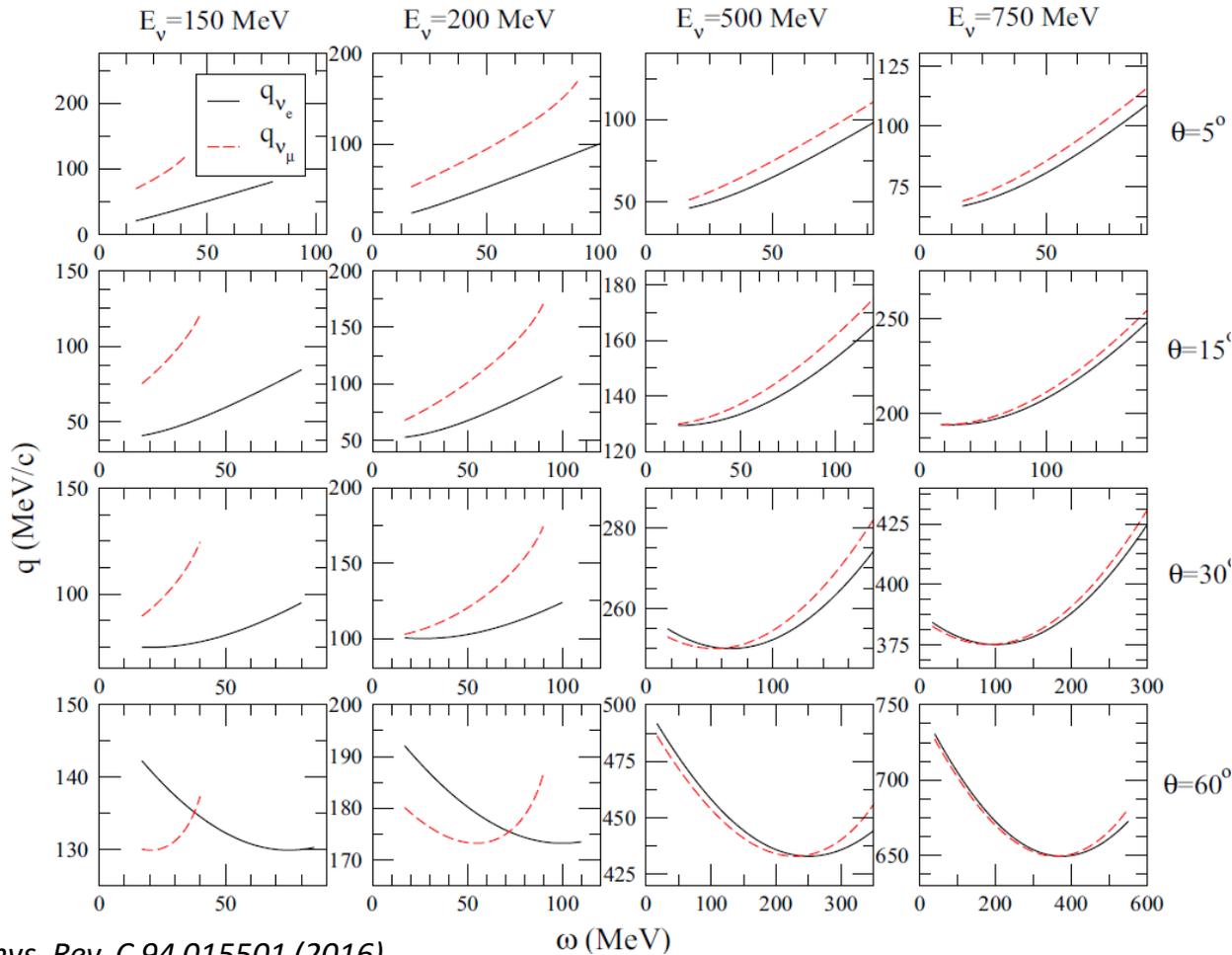
Due to the different kinematic limits, the ν_e cross sections are expected to be larger than the ν_μ ones. However for forward scattering angles this hierarchy is opposite.

The only difference between ν_μ and ν_e cross sections is the mass of the outgoing lepton.

But the mass affects the three momentum transfer which enters into the kinematics as well as the dynamics of the nuclear model

Momentum transfer q versus transferred energy ω for ν_μ and ν_e $d^2\sigma$

Kinematical conditions of the previous slide



M. Martini et al., Phys. Rev. C 94 015501 (2016)

$$q^2 = E_\nu^2 + p_l^2 - 2E_\nu p_l \cos \theta$$

$$p_l^2 = E_l^2 - m_l^2 = (E_\nu - \omega)^2 - m_l^2$$

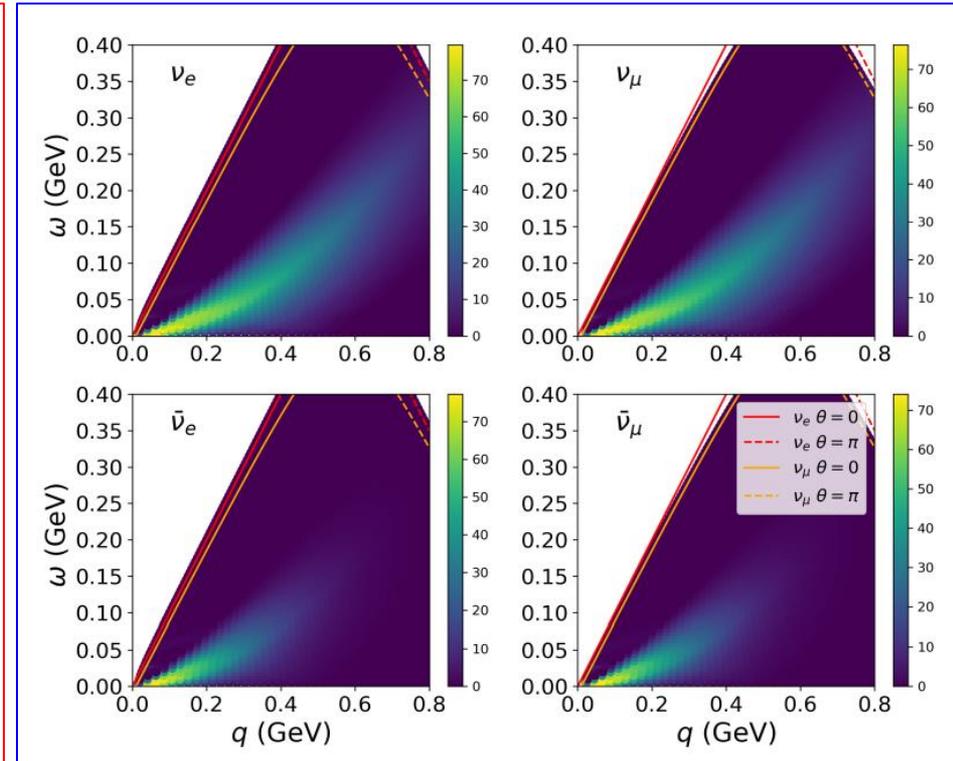
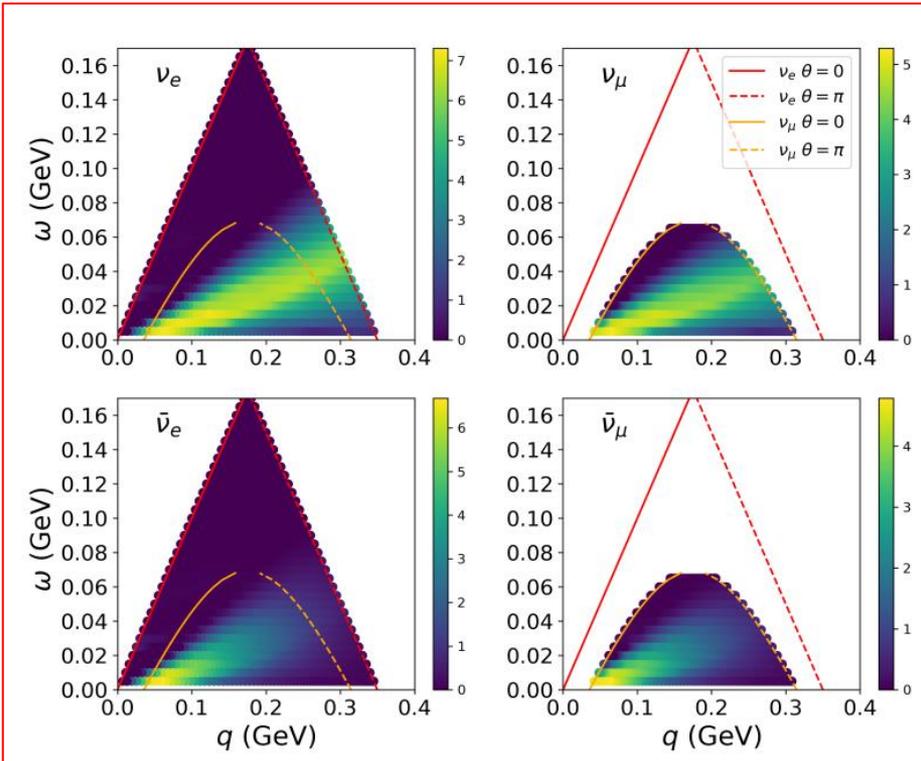
The only difference between ν_μ and ν_e cross sections is the mass of the outgoing lepton. But the mass affects the three-momentum transfer which enters into the kinematics as well as the dynamics of the nuclear model

Projection of ν_μ and ν_e $d^2\sigma$ on (q, ω) plane

Martini, Ericson, Chanfray [2310.06388](#)

Ev = 175 MeV

Ev = 575 MeV



For neutrino and antineutrino scattering the $\theta = 0$ muon and electron lines explore in the (q, ω) plane two different regions, the muon one corresponding to larger quasielastic cross sections

By increasing the neutrino energies the difference between the muon and electron $\theta = 0$ lines decreases and the two curves explore more and more similar region in the (q, ω) plane