Overview of pn pairing and $\alpha-$ like quarteting in N=Z nuclei

Nicolae Sandulescu

National Institute of Physics and Nuclear Engineering (IFIN-HH), Bucharest

Outlook

- Introduction: pn pairing in HFB
- Quartet condensation model versus BCS/HFB
- Pn pairing and binding energies of N=Z nuclei
- Probing α-like quarteting by α transfer ?

M. Sambataro and N. S, PLB820 (2021)136476
D. Negrea, N.S, D. Gambacurta, PRC105(2022) 034325
A. Volya, M. Sambataro, N.S, in preparation

Proton-neutron pairing in N=Z nuclei: main issues

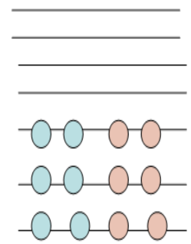
6 types of spin-isospin pairs

S=0, T=1

$\nu^+_\uparrow \nu^+_\downarrow$ $\quad$ $\pi^+_\uparrow \pi^+_\downarrow$ $\quad$ $\nu^+_\uparrow \pi^+_\downarrow + \pi^+_\uparrow \nu^+_\downarrow$

$\Gamma^+_{\nu\nu} = \sum_i x_i \nu^+_i \nu^+_{\bar{i}}$ $\quad$ pairs $\quad$ $\Gamma^+_{\pi\nu} = \sum_i x_i (\nu^+_i \pi^+_{\bar{i}} + \pi^+_i \nu^+_{\bar{i}})$

$(\Gamma^+_{\nu\nu})^{N/2}$ $\quad$ condensates $\quad$ $(\Gamma^+_{\nu\pi})^{N_{\pi\nu}/2}$



S=1, T=0

$\nu^+_\uparrow \pi^+_\downarrow - \pi^+_\uparrow \nu^+_\downarrow$ $\quad$ $\nu^+_\downarrow \pi^+_\downarrow$ $\quad$ $\nu^+_\uparrow \pi^+_\uparrow$

$\Delta^+_0 = \sum_i x_i (\nu^+_i \pi^+_{\bar{i}} - \pi^+_i \nu^+_{\bar{i}})$

$(\Delta^+_0)^{N_{\pi\nu}/2}$

<u>Long standing questions</u>

there is a "condensate" of pn pairs in nuclei ?

the fingerprints of a pn condensate ?

These questions are related to the BCS/HFB approximation of the pairing interactions !

$$\hat{H} = \sum_i \langle i|H_{\rm sp}|j\rangle a_i^\dagger a_j + \sum_{i>j,k>l} \langle ij|v|kl\rangle a_i^\dagger a_j^\dagger a_l a_k,$$

SOVIET PHYSICS JETP VOLUME 11, NUMBER 3 SEPTEMBER, 1960

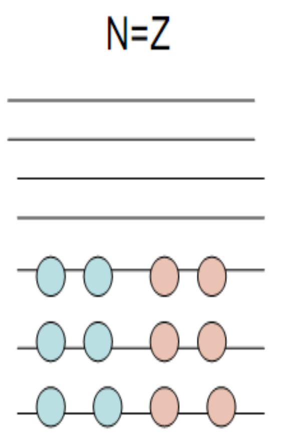
SUPERFLUIDITY OF LIGHT NUCLEI

V. B. BELYAEV, B. N. ZAKHAR'EV, and V. G. SOLOV'EV

Joint Institute of Nuclear Research

Submitted to JETP editor October 12, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 952-954 (March, 1960)



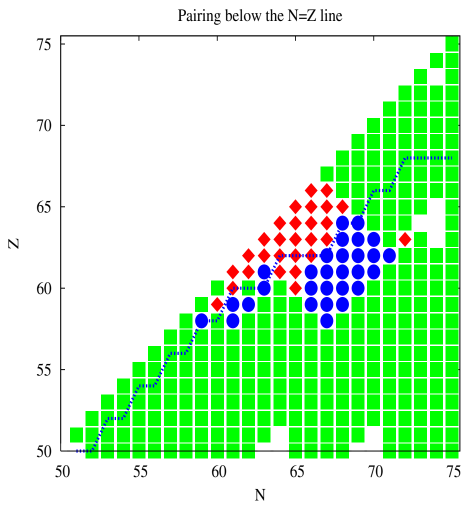
first HFB treatment of isovector pairing

PRL **106,** 252502 (2011) PHYSICAL REVIEW LETTERS week ending 24 JUNE 2011

Mixed-Spin Pairing Condensates in Heavy Nuclei

Alexandros Gezerlis,[1] G. F. Bertsch,[1,2] and Y. L. Luo[1]

$$|\Phi\rangle = \mathrm{pf}(U^\dagger V^*) \exp\left[\tfrac{1}{2}(VU^{-1})^*{}_{ij} c_i^\dagger c_j^\dagger\right]|0\rangle,$$



<u>search for condensate states</u>

spin-singlet spin-triplet mixed states

TABLE I. Spin-isospin channels for pairing condensates.

α	1	2	3	4	5	6
(S, S_z)	(0, 0)	(0, 0)	(0, 0)	(1, 1)	(1, 0)	(1, −1)
(T, T_z)	(1, 1)	(1, 0)	(1, −1)	(0, 0)	(0, 0)	(0, 0)

<u>Approximations</u>

- mean field: spherical WS (fixed !)
- particle number is not conserved
- not well-defined isospin
- not well-defined angular momentum

consequences for pn pairing ?

PHYSICAL REVIEW C **99**, 014321 (2019)

Symmetry restoration in mixed-spin paired heavy nuclei

Ermal Rrapaj,[1] A. O. Macchiavelli,[2] and Alexandros Gezerlis[1]
[1]*Department of Physics, University of Guelph, Guelph, Ontario N1G 2W1, Canada*
[2]*Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

particle-number and angular momentum projection AFTER variation

analysed the various components of w.f.

<u>open questions</u>

the effects on the type of correlations ?

isospin projection ?

Isovector (T=1) proton-neutron pairing

effect of particle-number and isospin projections

Isovector proton-neutron pairing in BCS-like models

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V^{T=1}_{J=0}(i,j) \sum_{t=-1,0,1} P^+_{it} P_{jt}$$

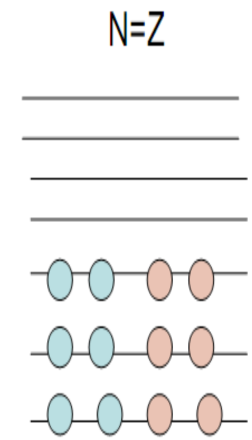
$$P^+_{i0} \propto \nu^+_i \pi^+_{\bar{i}} + \pi^+_i \nu^+_{\bar{i}} \qquad P^+_{i1} \propto \nu^+_i \nu^+_{\bar{i}} \qquad P^+_{i-1} \propto \pi^+_i \pi^+_{\bar{i}}$$

$$\Gamma^+_{\pi\nu} = \sum_i x_i (\nu^+_i \pi^+_{\bar{i}} + \pi^+_i \nu^+_{\bar{i}}) \qquad \Gamma^+_{\pi\pi} = \sum_i x_i \pi_i \pi_{\bar{i}} \qquad \Gamma^+_{\nu\nu} = \sum_i x_i \nu_i \nu_{\bar{i}}$$

BCS(pn)> degenerate with |BCS(nn)>|BCS(pp>

$$|PBCS0> \propto (\Gamma^+_{\nu\pi})^{\frac{N+Z}{2}} |->$$

$$|PBCS1> \propto (\Gamma^+_{\nu\upsilon})^{N/2} \Gamma^+_{\pi\pi})^{Z/2} |->$$



	SM	PBCS1	PBCS0
^{44}Ti	5.973	5.487 (8.134%)	4.912 (17.763%)
^{48}Cr	9.593	8.799 (8.277%)	7.885 (17.805%)
^{52}Fe	10.768	9.815 (8.850%)	8.585 (20.273%)

restoration of the isospin symmetry ?

Isospin projection

$$\mathcal{P}_{T;T_z=0} = \int_{S^2} \mathrm{d}\hat{n}\, D^{T*}_{00}(\hat{n})\, R(\hat{n})$$

$$\mathcal{P}_{T;T_z=0} \;\; (\Gamma^{+}_{\nu\pi})^{\frac{N+Z}{2}} \, |->\quad \Rightarrow \quad (2\Gamma^{+}_{\nu\nu}\Gamma^{+}_{\pi\pi} - \Gamma^{+}_{\nu\pi}\Gamma^{+}_{\nu\pi})^{n_q} \, |->$$

$$Q^{+} = 2\Gamma^{+}_{\nu\nu}\Gamma^{+}_{\pi\pi} - \Gamma^{+}_{\nu\pi}\Gamma^{+}_{\nu\pi}$$ has T=0 (and J=0)

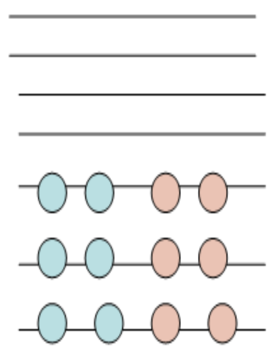
correlated 4-body structure !

V. V. Baran, D. Nichita, D. Negrea, D. Delion, N. Sandulescu, P. Schuck, PRC 102, 061301®(2020)

Isospin conservation and quarteting

$$H = \sum_i \varepsilon_i (N_i^{(\nu)} + N_i^{(\pi)}) + \sum_{ij,\tau} V(i,j) P_{i,\tau}^{+} P_{j,\tau}$$

$$P_{i1}^{+} \propto \nu_i^{+} \nu_{\bar{i}}^{+} \qquad P_{i-1}^{+} \propto \pi_i^{+} \pi_{\bar{i}}^{+} \qquad P_{i0}^{+} \propto \nu_i^{+} \pi_{\bar{i}}^{+} + \pi_i^{+} \nu_{\bar{i}}^{+}$$



non-collective quartets

$$Q_{ij}^{+} = [P_{i\tau}^{+} P_{j\tau'}^{+}]^{T=0} \propto P_{\nu\nu,i}^{+} P_{\pi\pi,j}^{+} + P_{\pi\pi,i}^{+} P_{\nu\nu,j}^{+} - P_{\nu\pi,i}^{+} P_{\nu\pi,j}^{+}$$

collective quartet

$$Q^{+} = \sum_{ij} x_{ij} [P_{i\tau}^{+} P_{j\tau'}^{+}]^{T=0}$$

quartet condensate

$$|QCM> = Q^{+n_q} |->$$ (has T=0, J=0)

N. S, D. Negrea, J. Dukelsky, C.W. Johnson, PRC85, 061303(R) (2012)

Quartet condensation and isospin projection

$$|QCM> = Q^{+n_q}|->$$

$$Q^+ = \sum_{ij} x_{ij}[P^+_{i\tau}P^+_{j\tau'}]^{T=0}$$

separability condition: $x_{ij} = x_i\, x_j$

$$Q^+ = 2\Gamma^+_{\nu\nu}\Gamma^+_{\pi\pi} - \Gamma^+_{\nu\pi}\Gamma^+_{\nu\pi}$$

$$\Gamma^+_\tau = \sum_i x_i P^+_{i,\tau}$$

$$\boxed{|QCM> = (2\Gamma^+_{\nu\nu}\Gamma^+_{\pi\pi} - \Gamma^+_{\nu\pi}\Gamma^+_{\nu\pi})^{n_q}|->}$$

'coherent' mixing of condenstates formed by nn, pp and pn pairs

$(\Gamma^{+2}_{\nu\pi})^{n_q}$ and $(\Gamma^+_{\nu\upsilon}\Gamma^+_{\pi\pi})^{n_q}$ are only two terms of the binomial expansion !

calculations

$$\delta_x <QCM|H|QCM> = 0$$

isospin projection before the variations !

Quartet condensation versus pair condensation

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_t P_{it}^+ P_{jt}$$

pairing forces extracted from shell model interactions

$$|QCM> \equiv (Q^+)^{n_q} |->$$ $$|PBCS1> \propto (\Gamma_{\nu\nu}^+ \Gamma_{\pi\pi}^+)^{n_q} |->$$ $$|PBCS0> \propto (\Gamma_{\nu\pi}^{+2})^{n_q} |->$$

	SM	QCM	PBCS1	PBCS0
^{20}Ne	9.173	9.170 (0.033%)	8.385 (8.590%)	7.413 (19.187%)
^{24}Mg	14.460	14.436 (0.166%)	13.250 (8.368%)	11.801 (18.389%)
^{28}Si	15.787	15.728 (0.374%)	14.531 (7.956%)	13.102 (17.008%)
^{32}S	15.844	15.795 (0.309%)	14.908 (5.908%)	13.881 (12.389%)
^{44}Ti	5.973	5.964 (0.151%)	5.487 (8.134%)	4.912 (17.763%)
^{48}Cr	9.593	9.569 (0.250%)	8.799 (8.277%)	7.885 (17.805%)
^{52}Fe	10.768	10.710 (0.539%)	9.815 (8.850%)	8.585 (20.273%)
^{104}Te	3.831	3.829 (0.052%)	3.607 (5.847%)	3.356 (12.399%)
^{108}Xe	6.752	6.696 (0.829%)	6.311 (6.531%)	5.877 (12.959%)
^{112}Ba	8.680	8.593 (1.002%)	8.101 (6.670%)	13.064 (13.064%)

Conclusions

- *T=1 pairing is accurately described by quartets, not by pairs*
- *there is not a pure condensate of isovector pn pairs in N=Z nuclei*

N. S, D. Negrea, J. Dukelsky, C.W. Johnson, PRC85, 061303(R) (2012)

Conclusions on T=1 pairing

Isospin projection generate 4-body quartet correlations !

$$\mathcal{P}_{T=0}^{\mathcal{N}=4n_q}|BCS\rangle = (Q^{\dagger})^{n_q}|0\rangle = |QCM\rangle$$

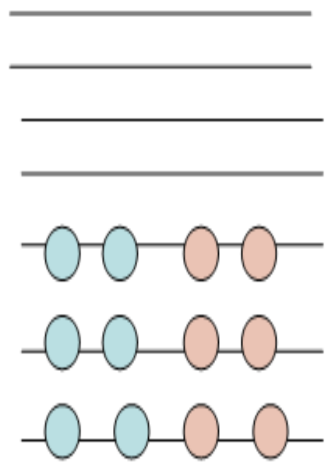
QCM *with the separability condition* is **echivalent** with isospin projected -BCS

QCM *without the separability condition* includes additional 4-body correlations

Isoscalar and isovector pairing in N=Z nuclei

N=Z

$$P^{+}_{i,T_z} = [a^{+}_{i} a^{+}_{i}]^{T=1,J=0}_{T_z}$$



$$D^{+}_{ij,J_z} = [a^{+}_{i} a^{+}_{j}]^{J=1,T=0}_{J_z}$$

Quartetting for isovector (J=0) and isoscalar (J=1) pairing

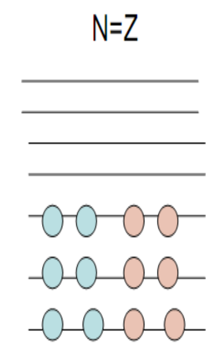
$$H = \sum \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{\tau} P_{i\tau}^{+} P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i,j) \sum_{\sigma} D_{i\sigma}^{+} D_{j\sigma}$$

isovector

$$P_{i,T_z}^{+} = [a_i^{+} a_i^{+}]_{T_z}^{T=1,J=0}$$

isoscalar

$$D_{ij,J_z}^{+} = [a_i^{+} a_j^{+}]_{J_z}^{J=1,T=0}$$



collective quartets

$$Q_{\nu}^{+(iv)} = \sum_{i,j} x_{ij}^{(\nu)} [P_i^{+} P_j^{+}]^{T=0}$$

$$Q_{\nu}^{+(is)} = \sum_{ij,kl} y_{ij,kl}^{(\nu)} [D_{ij}^{+} D_{kl}^{+}]^{J=0}$$

generalised quartet

$$Q_{\nu}^{+} = Q_{\nu}^{+(iv)} + Q_{\nu}^{+(is)}$$

ground state

$$| QCM >= Q^{+n_q} | - >$$

superposition of T=0 and T=1 quartets

M. Sambataro,, N. S, C. W. Johnson, Phys. Lett. B740 (2015)137

Quartet condensation versus pair condensation for isovector & isoscalar pairing

$$H = \sum \varepsilon_i N_i + \sum_{ij} V^{T=1}_{J=0}(i,j) \sum_{\tau} P^{+}_{i\tau} P_{j\tau} + \sum_{ij} V^{T=0}_{J=1}(i,j) \sum_{\sigma} D^{+}_{i\sigma} D_{j\sigma}$$

$$(Q^{+})^{n_q} |-> \qquad (\Gamma^{+}_{\nu\nu}\Gamma^{+}_{\pi\pi})^{n_q} |-> \qquad (\Gamma^{+}_{\nu\pi})^{2n_q} |-> \qquad (\Delta_0^{+})^{2n_q} |0\rangle$$

	QCM	PBC1	$PBCS0_{iv}$	$PBCS0_{is}$
^{20}Ne	15.985 (-)	14.011 (12.35%)	13.664 (14.52%)	13.909 (12.99%)
^{24}Mg	28.595 (0.24%)	21.993 (23.35%)	20.516 (28.50%)	23.179 (19.22%)
^{28}Si	35.288 (0.57%)	27.206 (23.58%)	25.293 (28.95%)	27.740 (22.19%)
^{44}Ti	7.019 (-)	5.712 (18.62%)	5.036 (28.25%)	4.196 (40.22%)
^{48}Cr	11.614 (0.21%)	9.686 (16.85%)	8.624 (25.97%)	6.196 (46.81%)
^{52}Fe	13.799 (0.42%)	11.774 (15.21%)	10.591 (23.73%)	6.673 (51.95%)
^{104}Te	3.147 (-)	2.814 (10.58%)	2.544 (19.16%)	1.473 (53.19%)
^{108}Xe	5.489 (0.20%)	4.866 (11.61%)	4.432 (19.49%)	2.432 (55.82%)
^{112}Ba	7.017 (0.34%)	6.154 (12.82%)	5.635 (20.17%)	3.026 (57.13%)

- T=1 and T=0 pairing correlations always coexist in quartets
- a pure isoscalar pairing condensation is not a good approximation !

M. Sambataro and N. S , Phys. Rev C93, 054320 (2016)

QCM and isospin-spin projected - HFB

$$|\Psi_{\rm g.s.}\rangle = (Q^+)^{n_q}|0\rangle \qquad Q^+ = Q_1^+ + Q_0^+,$$

$$Q_1^+ = \sum_{j_1 j_2} x_{j_1 j_2}\left[P_{j_1}^+ P_{j_2}^+\right]^{T=0}, \qquad Q_0^+ = \sum_{j_1 j_2 j_3 j_4} y_{j_1 j_2 j_3 j_4}\left[D_{j_1 j_2}^+ D_{j_3 j_4}^+\right]^{J=0}.$$

$$x_{j_1 j_2} = \bar{x}_{j_1}\bar{x}_{j_2} \qquad y_{j_1 j_2 j_3 j_4} = \bar{y}_{j_1 j_2}\bar{y}_{j_3 j_4}$$

$$\Gamma_{T_z}^+ = \sum_j \bar{x}_j P_{j,T_z}^+, \qquad \Delta_{J_z}^+ = \sum_{j_1 j_2} \bar{y}_{j_1 j_2} D_{j_1 j_2 J_z}^+.$$

$$\bar{Q}_1^+ = 2\Gamma_1^+\Gamma_{-1}^+ - (\Gamma_0^+)^2, \qquad \bar{Q}_0^+ = 2\Delta_1^+\Delta_{-1}^+ - \Delta_0^{+2}.$$

$$|\overline{\rm QCM}_{T=1}\rangle = (\bar{Q}_1^+)^{n_q}|0\rangle \qquad |\overline{\rm QCM}_{T=0}\rangle = (\bar{Q}_0^+)^{n_q}|0\rangle.$$

equivalent to isospin projection

equivalent to spin projection

$$|\overline{\rm QCM}\rangle = (\bar{Q}_1^+ + \bar{Q}_0^+)^{n_q}|0\rangle$$

isospin- spin projection from a mixed state

TABLE II. Correlation energies (19) relative to various calculations for $N = Z$ nuclei described by the Hamiltonian (17). We show the results for the QCM the state (4) as well as for the QCM approximations relative to the quartets (10) and (11), i.e., $|\overline{\mathrm{QCM}}\rangle = (\bar{Q}_1^+ + \bar{Q}_0^+)^{n_q}|0\rangle$, $|\overline{\mathrm{QCM}}_{T=1}\rangle = (\bar{Q}_1^+)^{n_q}|0\rangle$, and $|\overline{\mathrm{QCM}}_{T=0}\rangle = (\bar{Q}_0^+)^{n_q}|0\rangle$. The QM results refer to the state (18) and are taken from Ref. [17]. In brackets we show the relative errors with respect to the exact results obtained by diagonalization. All energies are in MeV.

	Exact	QM	QCM	$\overline{\mathrm{QCM}}$	$\overline{\mathrm{QCM}}_{T=1}$	$\overline{\mathrm{QCM}}_{T=0}$
^{20}Ne	15.985	15.985 (-)	15.985 (-)	15.510 (2.97%)	14.373 (10.08%)	14.930 (6.60%)
^{24}Mg	28.694	28.626 (0.24%)	28.595 (0.34%)	27.764 (3.24%	23.229 (19.04%)	26.299 (8.35%)
^{28}Si	35.600	35.396 (0.57%)	35.288 (0.88%)	33.913 (4.74%)	28.830 (19.02%)	32.067 (9.92%)
^{44}Ti	7.019	7.019 (-)	7.019 (-)	6.302 (10.21%)	6.273 (10.63%)	4.825 (31.26%)
^{48}Cr	11.649	11.624 (0.21%)	11.614 (0.30%)	10.674 (8.37%)	10.582 (10.67%)	7.075 (39.26%)
^{52}Fe	13.887	13.828 (0.42%)	13.799 (0.63%)	12.971 (6.60%)	12.795 (7.92%)	9.589 (30.95%)
^{104}Te	3.147	3.147 (-)	3.147 (-)	3.052 (3.02%)	3.041 (3.37%)	1.512 (51.95%)
^{108}Xe	5.505	5.495 (0.20%)	5.489 (0.29%)	5.279 (4.10%)	5.239 (4.83%)	2.530 (54.04%)
^{112}Ba	7.059	7.035 (0.34%)	7.017 (0.59%)	6.691 (5.21%)	6.609 (6.37%)	4.391 (37.79%)

the isospin-spin projected state $|\overline{\mathrm{QCM}}\rangle$ is less accurate than full QCM

Conclusions on T=1 + T=0 pairing

BCS/HFB	projected-BCS/HFB
T=1 pn pair condensate	T=1 quartet condensate
T=0 pn pair condensate	T=0 quartet condensate
T=1&T=0 pn pair condensate	T=1 & T=0 quartet condensate

QCM with the separability condition, is equivalent with projected- BCS/HFB

QCM without the separability condition is more accurate than projected-BCS/HFB

in QCM the T=1 and T=0 correlations always coexist

Physics Letters B 820 (2021) 136476

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

α-Like quartetting in the excited states of proton-neutron pairing Hamiltonians

M. Sambataro [a], N. Sandulescu [b,*]

[a] Istituto Nazionale di Fisica Nucleare - Sezione di Catania, Via S. Sofia 64, I-95123 Catania, Italy
[b] National Institute of Physics and Nuclear Engineering, RO-077125 Măgurele, Romania

, **Excited states for the isovector pairing**

$$H=\sum_i \epsilon_i N_i+\sum_{i,j} V^{T=1}_{J=0}(i,j)\sum_{T_z} P^+_{i,T_z} P^+_{j,T_z}$$

$$P^+_{i,T_z}=\sqrt{\frac{2j_i+1}{2}}[a_i^+ a_i^+]^{T=1,J=0}_{T_z}.$$

<u>Ground state</u>

$$|QCM\rangle=(Q^+_{iv})^{n_q}|-\rangle$$

$$Q^+_{iv}=\sum_{ij} x_{ij}[P^\dagger_i P^\dagger_j]^{T=0}$$

<u>Excited states</u>

$$|\Phi_\nu\rangle=\tilde{Q}^+_\nu (Q^+_{iv})^{n_q-1}|-\rangle,$$

$$\tilde{Q}^+_{\nu,JJ_z}=\sum_{T'}\sum_{J_1(i_1 j_1)}\sum_{J_2(i_2 j_2)} Y^{(\nu)}_{JJ_z}(T',J_1(i_1 j_1),J_2(i_2 j_2)) \times [P^+_{J_1,T'}(i_1,j_1)P^+_{J_2,T'}(i_2,j_2)]^{J,T=0}_{J_z}.$$

$$P^+_{JJ_z,TT_z}(i,j)=[a_i^+ a_j^+]^{JT}_{J_zT_Z}$$

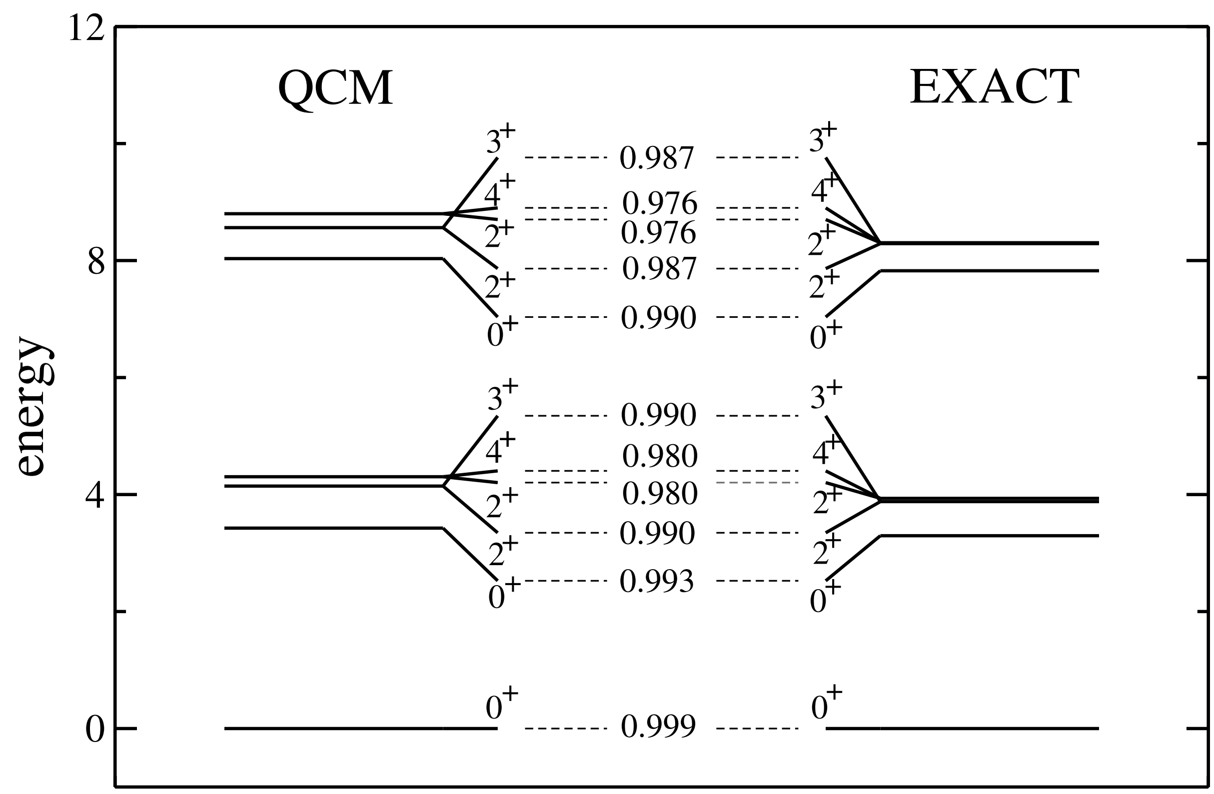


Fig. 3. The low-lying spectrum provided by the QCM approximation (17) for the valence nucleons of ^{28}Si interacting by an isovector pairing force extracted from the USDB interaction. The numbers are the overlaps between the QCM and the exact wave functions. Energies are in MeV.

Excited states for the isovector-isoscalar pairing

$$H=\sum_i \epsilon_i N_i+\sum_{i,j} V_{J=0}^{T=1}(i,j)\sum_{T_z} P^+_{i,T_z}P_{j,T_z} + \sum_{i\le j,k\le l} V_{J=1}^{T=0}(ij,kl)\sum_{J_z} D^+_{ij,J_z}D_{kl,J_z}.$$

$$D^+_{j_1 j_2 J_z}=\frac{1}{\sqrt{1+\delta_{j_1 j_2}}}[a^+_{j_1}a^+_{j_2}]^{J=1,T=0}_{J_z}$$

<u>Ground state</u>

$$|\Psi_{gs}\rangle=(Q^+_{ivs})^{n_q}|0\rangle.$$

$$Q^+_{ivs}=Q^+_{iv}+Q^+_{is},$$

$$Q^+_{is}=\sum_{j_1 j_2 j_3 j_4} y_{j_1 j_2 j_3 j_4}[D^+_{j_1 j_2}D^+_{j_3 j_4}]^{J=0}.$$

<u>Excited states</u>

$$|\Phi_{\nu,JJ_z}\rangle=\tilde{Q}_{\nu,JJ_z}(Q^+_{ivs})^{n_q-1}|-\rangle,$$

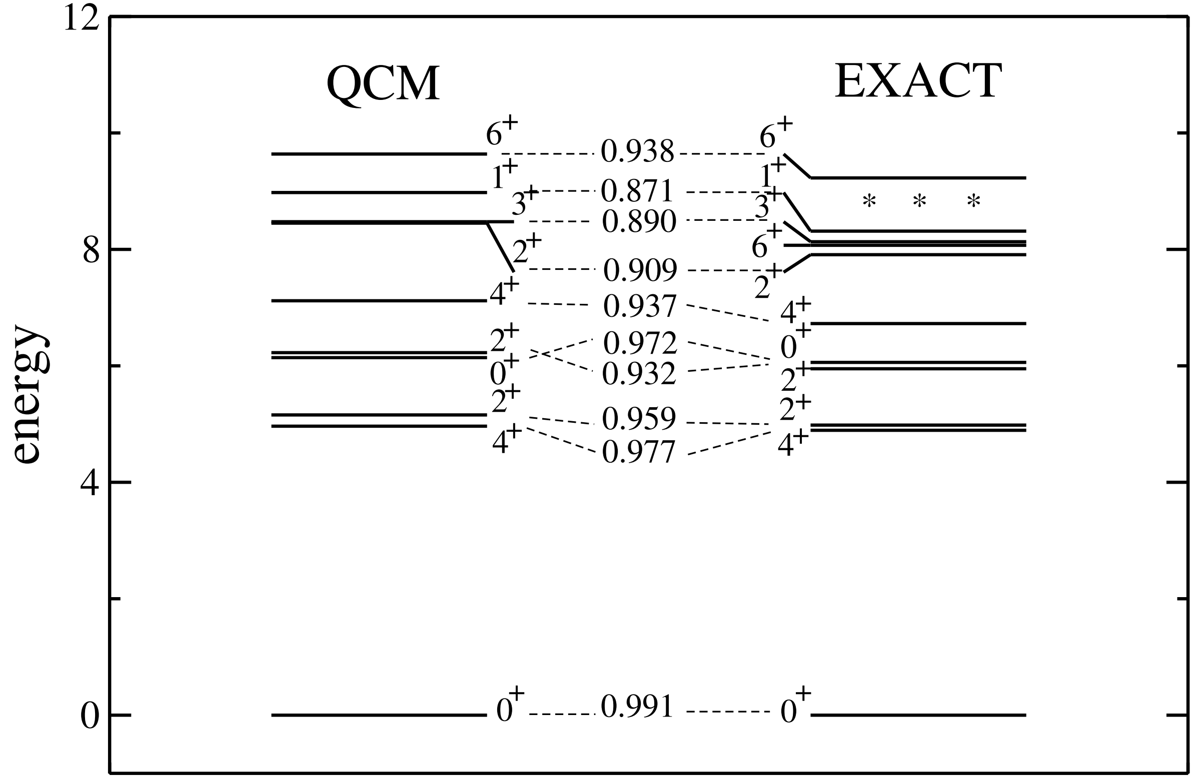


Fig. 4. The low-lying spectrum provided by the QCM approximation (24) for the valence nucleons of ^{28}Si interacting by an isovector-isoscalar pairing force extracted from the USDB interaction. The numbers are the overlaps between the QCM and the exact wave functions. Energies are in MeV.

Conclusions on excited states of pn pairing Hamiltonians

low-lying excitations are of one-broken-quartet type !

physical consequencies (e.g., on delayed alignement) ?

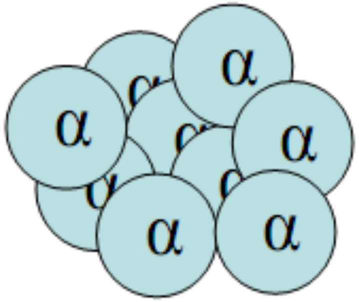
for the one-broken-quartet excitations related to shell model Hamiltonians, see the talk by Michelangelo Sambataro

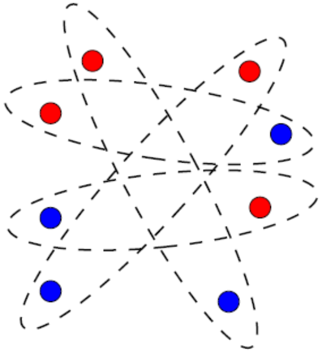
quartets : 4-body structures of two neutrons and two protons

correlated in configuration space (isospin, spin)

not necessary correlated in the real space !

- α-like quartets - analog to Cooper pairs





A bit of history . . .

SOVIET PHYSICS JETP VOLUME 11, NUMBER 3 SEPTEMBER, 1960

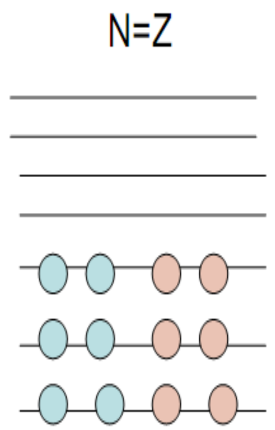
SUPERFLUIDITY OF LIGHT NUCLEI

V. B. BELYAEV, B. N. ZAKHAR'EV, and V. G. SOLOV'EV

Joint Institute of Nuclear Research

Submitted to JETP editor October 12, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 952-954 (March, 1960)



first HFB treatment of isovector pairing

"we must take into consideration the quadruple correlation of α-particle-like nucleons [...]; *these new correlations evidently play a very important role and somewhat* mask the effect of pair correlations"

EFFECT OF QUADRUPLE CORRELATIONS IN LIGHT NUCLEI

V G SOLOVIEV
Joint Institute of Nuclear Research, Dubna, USSR

Received 25 December 1959

“quadruple"= two interacting pn pairs

Fingerprints of alpha-like (quadruple) correlations

1) Extracting a pn pair from a even-even N=Z nucleus costs more energy than adding to it a pn pair

^{22}Na ← ^{24}Mg → ^{26}Al ← ^{28}Si

22.1 MeV 13.6 Mev 24.4 MeV

2) Extracting one neutron from a even-even N=Z nucleus costs more energy than from neighbouring nuclei

B(^{24}Mg)-B(^{23}Mg) = 16.6 MeV

B(^{25}Mg) -B(^{24}Mg) = 7.3 Mev

B(^{26}Mg)-B(^{25}Mg)= 11.3 Mev

to brake a quadruple (quartet) in pairs takes about 4-5 MeV

CHARGE-INDEPENDENT PAIRING CORRELATIONS

B. H. FLOWERS and M. VUJIČIĆ †
Department of Theoretical Physics, The University, Manchester, England

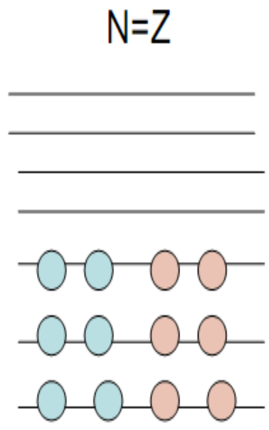
Received 26 June 1963

Abstract: It is shown that the generalization of the BCS method of treating correlations due to pairing forces when these forces are charge-independent leads inevitably to there being 4-body correlations whose symmetry properties are similar to those of an α-particle model. Rough estimates of the magnitude of the correlations indicate that it is probably strong enough to account for the energy gap found in light nuclei.

BCS-like state in terms of quartets

$$\Phi = \prod_{ll'mm'} \prod_{pqrs}{}' (U_l + V_{ll'} \epsilon_{pqrs} a^\dagger_{lmp} a^\dagger_{l-mq} a^\dagger_{l'm'r} a^\dagger_{l'-m's}) |0\rangle$$

difficult problem : not solved yet for realistic cases !



Theoretical studies on pn pairing & alpha correlations

B. H. Flowers and M. Vijicic,NPA49(1963)

$$\Phi = \prod_{ll'mm'} \prod_{pqrs}{}' (U_l + V_{ll'} \epsilon_{pqrs} a^{\dagger}_{lmp} a^{\dagger}_{l-mq} a^{\dagger}_{l'm'r} a^{\dagger}_{l'-m's}) |0\rangle$$

not actually solved !

B. Bremond and J. G. Valatin NP41(1963)

$$\prod_{\alpha} (S_{\alpha} + V_{\alpha p} a^{\dagger}_{\alpha p} a^{\dagger}_{\bar{\alpha} p} + V_{\alpha n} a^{\dagger}_{\alpha n} a^{\dagger}_{\bar{\alpha} n} + T_{\alpha} a^{\dagger}_{\alpha p} a^{\dagger}_{\bar{\alpha} p} a^{\dagger}_{\alpha n} a^{\dagger}_{\bar{\alpha} n}) |0\rangle$$

not included all relevant configurations !

J. Eichler and M. Yamamura, NPA182(1972)

non-collective quartets

R. Chasman, PLB577(2003)

J. Dobes and S. Pittel PRC57(1998)

quartetting for a degenerate state

R. A. Senkov and V. Zelevinski (2011)

....

almost all studies on pn pairing: in BCS/HFB approximation

Goodman , …, Bertsch & Gezerlis

EVIDENCE FOR QUARTET STRUCTURE IN MEDIUM AND HEAVY NUCLEI

M. DANOS‡ and V. GILLET
Service de Physique Théorique, Centre d Etudes Nucléaires de Saclay, B.P. no 2-91-Gif-sur-Yvette, France

Received 1 November 1970

The second differences of the nuclear masses keeping T constant are discussed for even-even nuclei throughout the mass table. They are shown to be consistent with the quartet picture of weakly interacting tight two-proton two-neutron structures

VOLUME 25, NUMBER 15 PHYSICAL REVIEW LETTERS 12 OCTOBER 1970

ENERGIES OF QUARTET STRUCTURES IN EVEN-EVEN $N=Z$ NUCLEI

Akito Arima,* Vincent Gillet, and Joseph Ginocchio†
Service de Physique Théorique, Centre d'Etudes Nucléaires de Saclay, 91, Gif-sur-Yvette, France
(Received 7 August 1970)

Mass relationships are used to compute the energy of quartet excited states in $N=Z$ even-even nuclei for ^{12}C up to ^{52}Fe. The states obtained are quasibound up to excitation energies of about 40 MeV and could account for the narrow structures recently observed in heavy-ion transfer experiments.

ANNALS OF PHYSICS: **66**, 117–136 (1971)

The Roton Model of Quartets in Nuclei

AKITO ARIMA* AND VINCENT GILLET
Service de Physique Théorique, Centre d'Études Nucléaires de Saclay, 91, Gif-sur-Yvette, France
Received October 1, 1970

A simple approximation of the dynamics of proton–neutron systems is constructed from the experimental evidence of quasiindependent two-proton two-neutron structures in nuclei. These strongly self-bound structures have limited angular momenta components, and their mutual weak correlations are responsible for deformations and rotations. A schematic calculation is discussed.

Quartet Effects in Rare-Earth Nuclei

H. J. Daley, M. A. Nagarajan, and N. Rowley
Science and Engineering Research Council Daresbury Laboratory, Daresbury, Warrington WA4 4AD, England

D. Morrison
University of Liverpool, Liverpool, England

and

A. D. May
University of Sheffield, Sheffield, England
(Received 5 March 1986)

Quartet effects in deformed rare-earth nuclei are confronted from a phenomenological point of view. Some very simple systematic trends are evident in the experimental data when plotted as a function of a quartet number. The interacting-boson model has been modified to include quartet effects explicitly and it is able to reproduce accurately the experimental trends with fixed parameters.

PACS numbers: 21.10.Re, 21.60.Fw, 23.20.Lv

Nuclei: A Superfluid Condensate of α Particles? A Study within the Interacting-Boson Model

Y. K. Gambhir[(a)] and P. Ring
Physik-Department, Technische Universität München, D-8046 Garching, West Germany

and

P. Schuck
Institut des Sciences Nucléaires, F-38042 Grenoble, France
(Received 11 July 1983)

The authors have studied the question of whether pairs of neutrons and pairs of protons of the usual superfluid phases form a bound state to give rise to a superfluid condensate of "α particles." They indeed find indications for this to be the case from a BCS-like study for bosons using the proton-neutron interacting-boson model as well as from an even-odd effect in the number of pairs using experimental binding energies.

Condensed structure of $J = T = 0$ α-like clusters in $f_{7/2}$-shell even–even nuclei with $N = Z$

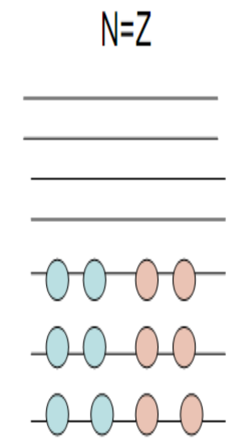
M. Hasegawa [a], S. Tazaki [b], R. Okamoto [c]

[a] *Laboratory of Physics, Fukuoka Dental College, Fukuoka 814-01, Japan*
[b] *Department of Applied Physics, Fukuoka University, Fukuoka 814-80, Japan*
[c] *Department of Physics, Kyushu Institute of Technology, Kitakyushu 804, Japan*

Received 15 February 1995; revised 25 April 1995

Isoscalar – isovector pairing in axially deformed mean-fields

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P^+_{i,t} P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D^+_{i,0} D_{j,0}$$



isovector

isoscalar

$$P^+_{i,0} = (\nu^+_i \pi^+_{\bar{i}} + \pi^+_i \nu^+_{\bar{i}})/\sqrt{2}.$$

$$D^+_{i,0} = (\nu^+_i \pi^+_{\bar{i}} - \pi^+_i \nu^+_{\bar{i}})/\sqrt{2}$$

$$P^+_{i1} = \nu^+_i \nu^+_{\bar{i}} \qquad P^+_{i-1} = \pi^+_i \pi^+_{\bar{i}}$$

$$Q^+ = \sum_{ij} x_i x_j [P^+_{i\tau} P^+_{j\tau'}]^{T=0}$$

$$\Delta^+_0 = \sum y_i D^+_{i,0}$$

ansatz for ground state

$$|\Psi> = (Q^+ + \Delta_0^{+2})^{n_q} |->$$

exact solution for degenerate levels !

N.S, D.Negrea, D. Gambacurta, Phys. Lett. B (2015)

Competition between isovector and isoscalar pairing

pairing on top of deformed Skyrme-HF

$$V_{paring}^{T=\{0,1\}} = \mathrm{v}_0^{T=\{0,1\}} \delta(r_1 - r_2) \hat{P}_{S=\{0,1\}} \qquad \mathrm{v}_0^{T=0} = 1.5\ \mathrm{v}_0^{T=1}$$

$$|\Psi> = (Q^+ + \Delta_0^{+2})^{n_q} |-> \qquad |iv> = (Q^+)^{n_q} |-> \qquad |is> = (\Delta_0^{+2})^{n_q} |->$$

	exact	$\mid \Psi\rangle$	$\mid iv\rangle$	$\mid is\rangle$	$\langle iv \mid is\rangle$
^{20}Ne	11.38	11.38 (0.00%)	11.31 (0.62%)	10.92 (4.00%)	0.976
^{24}Mg	19.32	19.31 (0.03%)	19.18 (0.74%)	18.93 (2.00%)	0.980
^{28}Si	18.74	18.74 (0.01%)	18.71 (0.14%)	18.54 (1.07%)	0.992
^{44}Ti	7.095	7.094 (0.02%)	7.08 (0.18%)	6.30 (10.78%)	0.928
^{48}Cr	12.78	12.76 (0.1%)	12.69 (0.67%)	12.22 (4.37%)	0.936
^{52}Fe	16.39	16.34 (0.26%)	16.19 (1.17%)	15.62 (4.65%)	0.946
^{104}Te	4.53	4.52 (0.06%)	4.49 (0.82%)	4.02 (11.26%)	0.955
^{108}Xe	8.08	8.03 (0.61%)	7.96 (1.45%)	6.75 (16.47%)	0.814
^{112}Ba	9.36	9.27 (0.93%)	9.22 (1.43 %)	7.50 (19.81%)	0.784

isovector and isoscalar pairing **always coexist** together

N.S, D.Negrea, D. Gambacurta, Phys. Lett. B (2015)

PHYSICAL REVIEW C **105**, 034325 (2022)

Proton-neutron pairing and binding energies of nuclei close to the $N = Z$ line

D. Negrea and N. Sandulescu*

National Institute of Physics and Nuclear Engineering, 077125 Măgurele, Romania

D. Gambacurta

INFN-LNS, Laboratori Nazionali del Sud, 95123 Catania, Italy

Isovector-isoscalar pairing in Skyrme-HF+QCM

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,0} D_{j,0}$$

from Skyrme-HF

treated in QCM

- Binding energy $E = E_{mf} + E_{p}$
- Pairing energy $E_{P} =$ interaction energy – self-energy
- Interaction energy $E_{int} = \langle QCM | V_{P} | QCM \rangle$
- self-energy

$$E^{mf}_{n(p)} = \sum_{i} V^{T=1}(i,i) v^{4}_{i,n(p)},$$

$$E^{mf}_{pn}(T) = \sum_{i} V^{T}(i,i) v^{2}_{i,p} v^{2}_{i,n}.$$

Calculation scheme

- Skyrme functional: UNE1
- Pairing interaction $V^T(r_1, r_2) = V_0^T \delta(r_1 - r_2) \hat{P}^T_{S,S_z},$

$V_0 = V_0^{T=1}$ $\qquad V_0 = \{300, 350, 400, 465\}$

$w = V_0^{T=0}/V_0^{T=1}.$ $\qquad w =$ [0, 1, 1.5, 2]

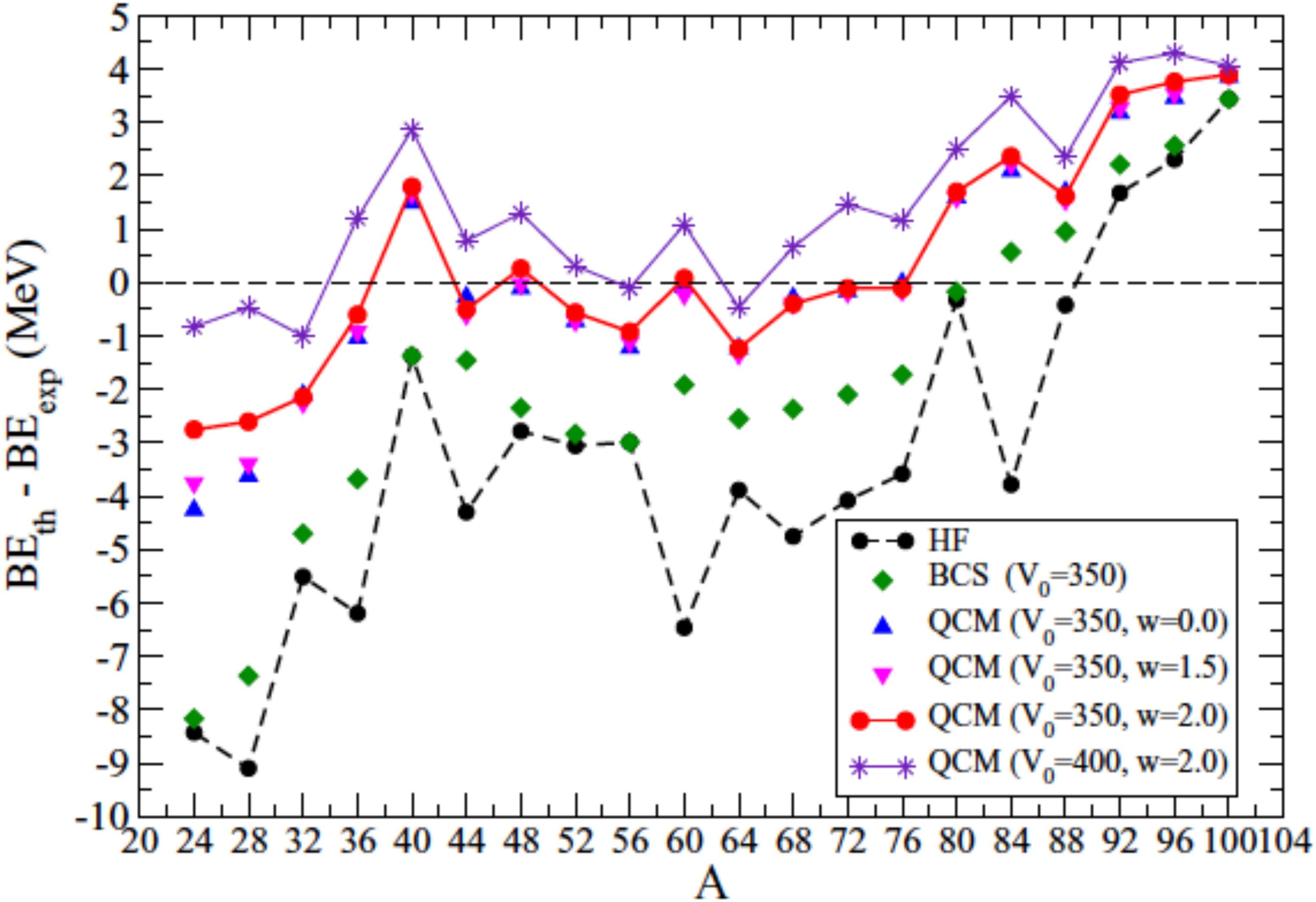


FIG. 1. Binding-energy residuals, in MeV, for even-even $N = Z$ nuclei as a function of $A = N + Z$. The results correspond to the pairing forces and the approximations indicated in the figure.

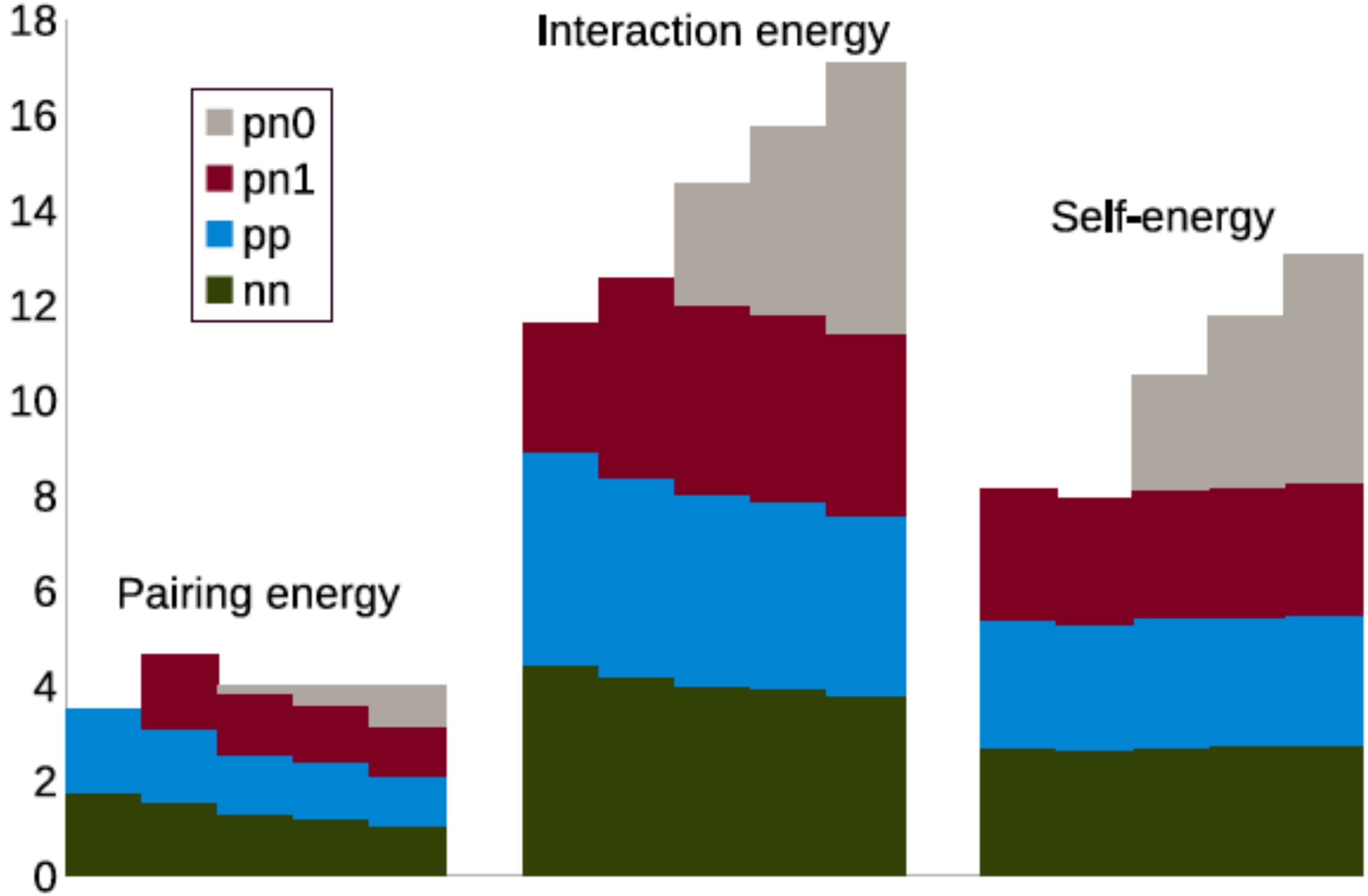


FIG. 2. Pairing energy, interaction energy and self-energy, in MeV, for ^{64}Ge. From the left to the right are shown, for each quantity, the PBCS result and the QCM results for $w = \{0.0, 1.0, 1.5, 2.0\}$. $pn0$ and $pn1$ indicate the $T = 0$ and $T = 1$ pn channels.

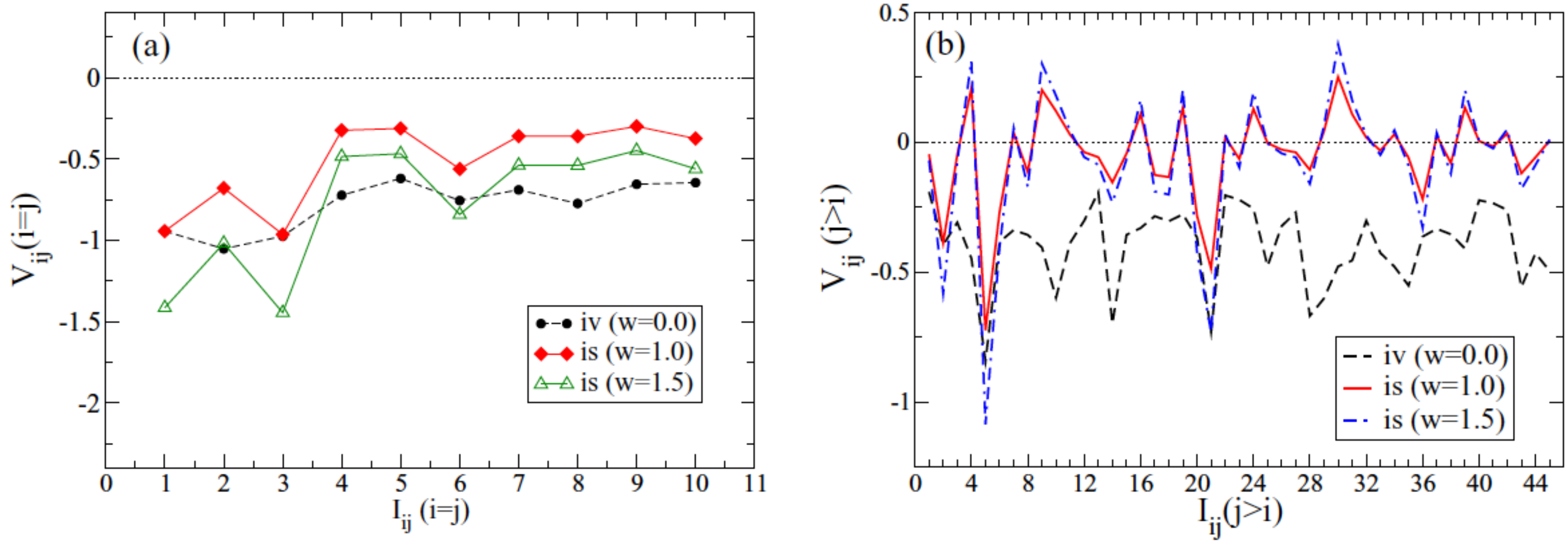


FIG. 3. (a) Diagonal and (b) nondiagonal matrix elements of the isovector and isoscalar pairing force for ^{64}Ge. The quantity I_{ij} enumerates the pair indices of V_{ij}.

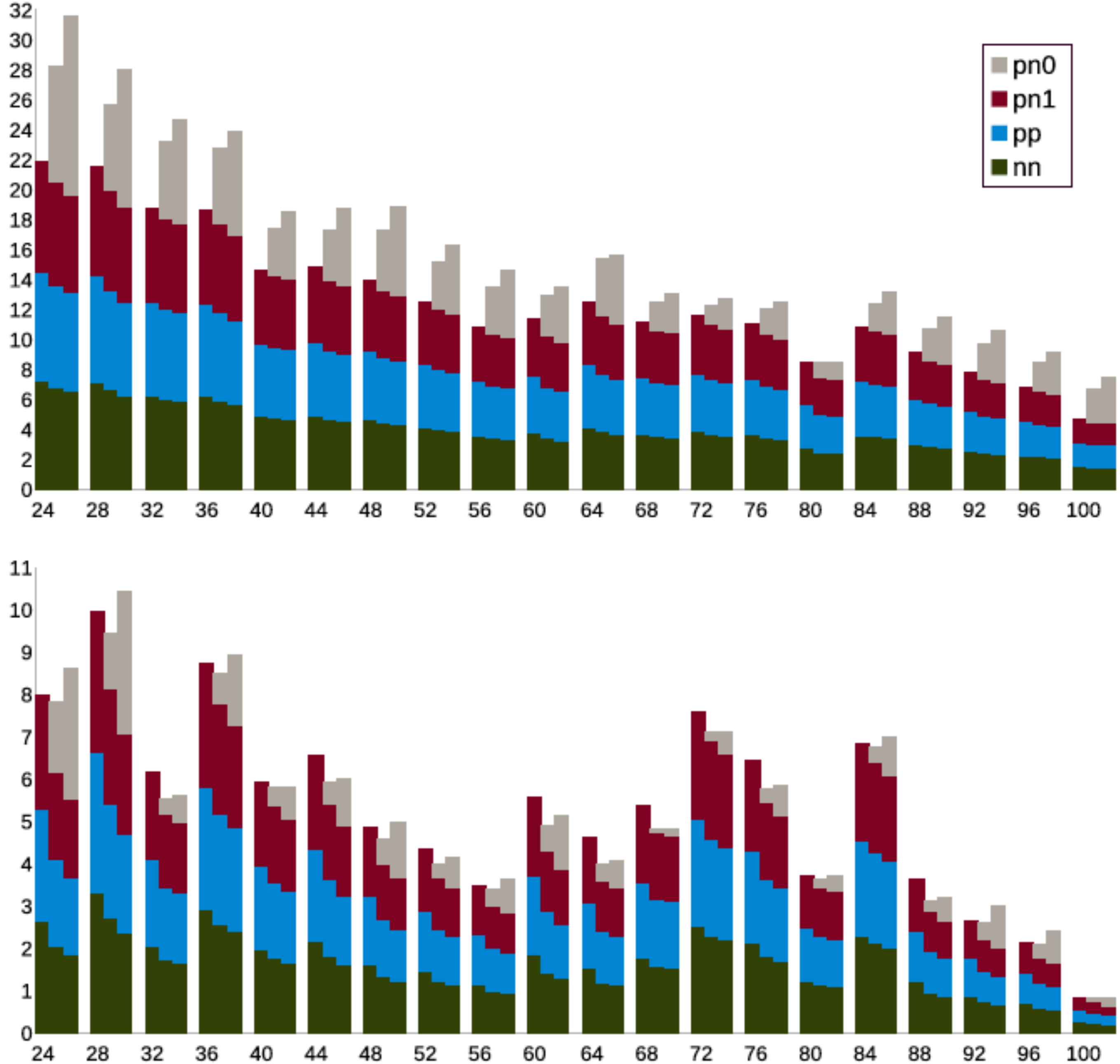


FIG. 4. Interaction energies (top) and pairing energies (bottom), in MeV, for $N = Z$ nuclei. For each nucleus are shown, from the left to the right, the results for $w = \{0.0, 1.5.2.0\}$.

Conclusions on binding energies

T=1 & T=0 pairing can correct the underbinding of N=Z nuclei

T=1 and T=0 always coexist and compete with each other

T=0 pairing contributes less than T=1pairing to binding energies

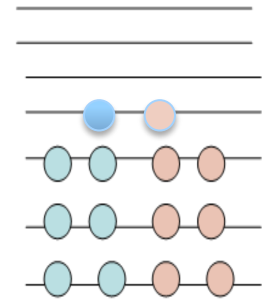
do to the smaller & repulsive off-diagonal matrix elements

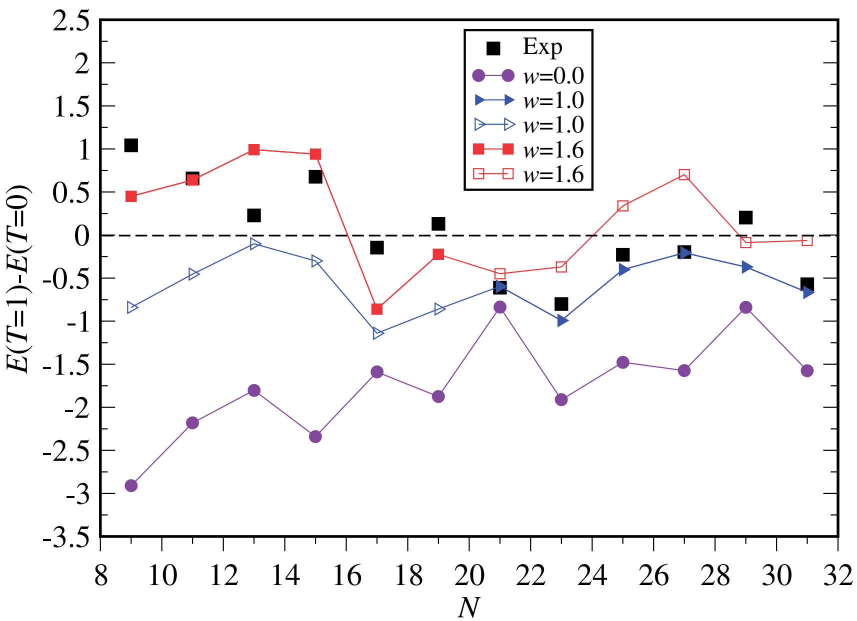
Isovector and isoscalar pairing in odd-odd N=Z

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,0} D_{j,0}$$

T=1 state $\quad | iv; QCM \rangle = \tilde{\Gamma}^{+}_{\nu\pi} (Q^{+}_{T=1} + \Delta^{+2}_{\nu\pi})^{n_q} | - \rangle$

T=0 state $\quad | is; QCM \rangle = \tilde{\Delta}^{+}_{\nu\pi} (Q^{+}_{T=1} + \Delta^{+2}_{\nu\pi})^{n_q} | - \rangle$





$$w = \frac{V_0^{T=0}}{V_0^{T=1}} \qquad V_{paring}^{T=\{0,1\}} = V_0^{T=\{0,1\}} \delta(r_1 - r_2) \hat{P}_{S=\{0,1\}}$$

calculations on top of Skyrme-HF spectrum

D. Negrea, N.S. and D. Gambacurta, Prog. Theor. Exp. Phys. 073D05 (2017)

The structure of lowest T=0 and T=1 states

T=0 ground state

		Exact	$\tilde{\Delta}^{+}_{\nu\pi}(Q^{+}_{T=1}+\Delta^{+2}_{\nu\pi})^{n_q}$	$\tilde{\Delta}^{+}_{\nu\pi}(Q^{+}_{T=1})^{n_q}$	$(\Delta^{+}_{\nu\pi})^{2n_q+1}$	$\tilde{\Delta}^{+}_{\nu\pi}(\Gamma^{+2}_{\nu\pi})^{n_q}$
^{30}P	T=0	12.66	12.60 (0.44%)	12.55 (0.86%)	11.96 (5.86%)	11.94 (5.95%)

T=1 ground state

		Exact	$\tilde{\Gamma}^{+}_{\nu\pi}(Q^{+}_{T=1}+\Delta^{+2}_{\nu\pi})^{n_q}$	$\tilde{\Gamma}^{+}_{\nu\pi}(Q^{+}_{T=1})^{n_q}$	$\tilde{\Gamma}^{+}_{\nu\pi}(\Delta^{+2}_{\nu\pi})^{n_q}$	$(\Gamma^{+}_{\nu\pi})^{2n_q+1}$
^{54}Co	T=1	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)

D. Negrea, N.S. and D. Gambacurta, Prog. Theor. Exp. Phys. 073D05 (2017)

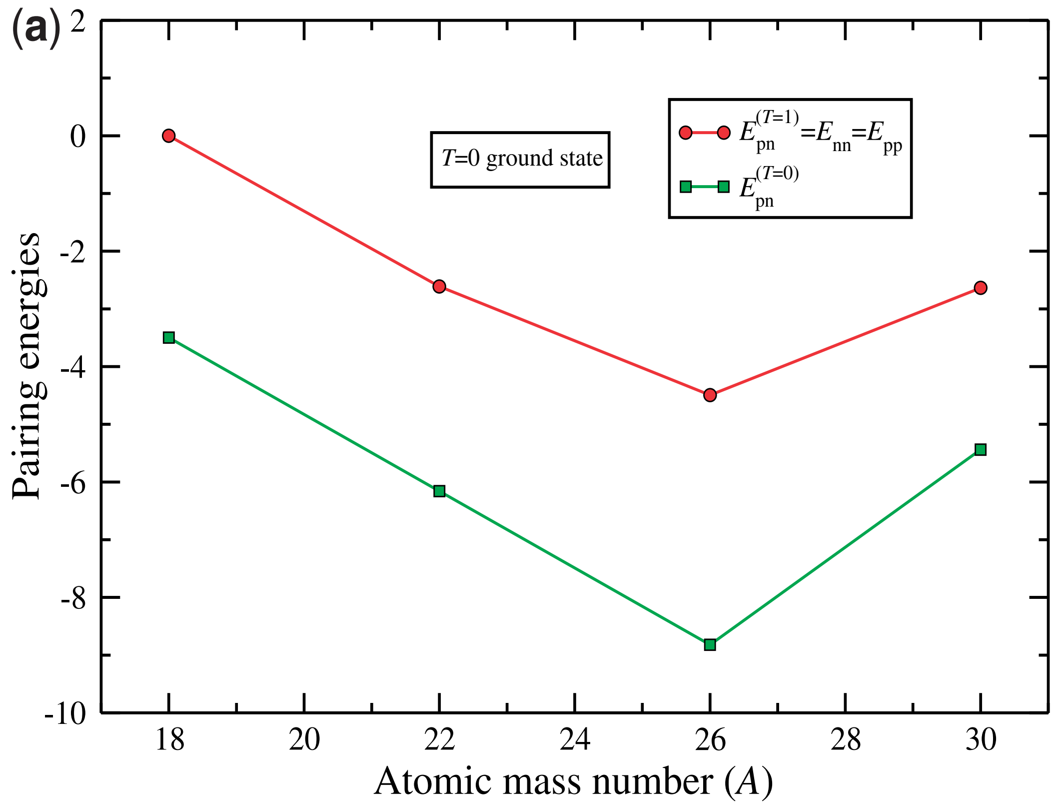


Fig. 2. Pairing energies, in MeV, for the odd–odd $N = Z$ nuclei as a function of the mass number A.

T=0 pairing energy in odd-odd N=Z is originating from the odd T=0 pair

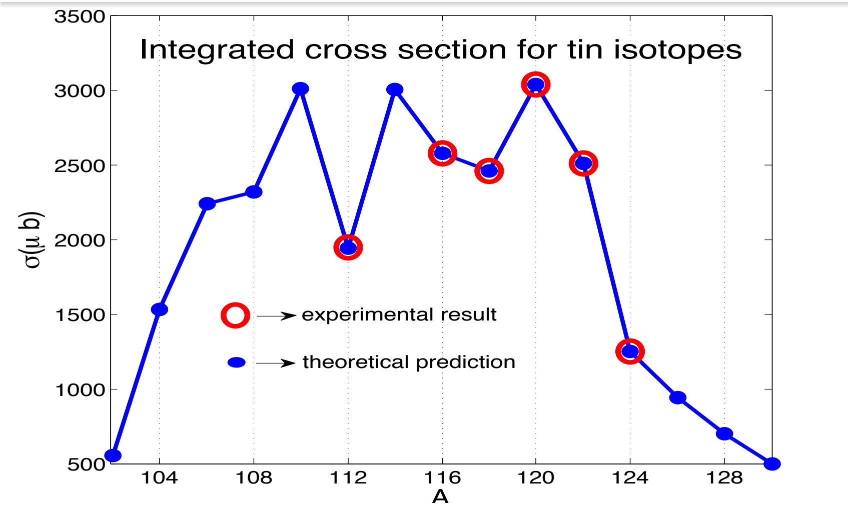
conclusions on odd-odd N=Z nuclei

from the isospin of the ground states of odd-odd N=Z nuclei one cannot draw conclusions on the pn condensates in these nuclei !

can we really probe the T=0 pn condensation by pn transfer ?

Like-particle pair transfer versus a transfer

- fingerprints of nn pair condensation: *pair transfer on a chain of isotopes*



G. Potel et al, PRL107 (2011)

can we extrapolate this BCS picture of nn pairing condensation to pn pairing ?

projected-HFB/BCS and QCM predict a quartet condensation, not a pn condensation !

- **fingerprints of quartet condensation :** *alpha particle transfer along N=Z line ?*

$$^{16}O \xrightarrow{\alpha} {}^{20}Ne \xrightarrow{\alpha} {}^{24}Mg \xrightarrow{\alpha} {}^{28}Si \xrightarrow{\alpha} {}^{32}S$$

Systematics of Ground-State α-Particle Spectroscopic Strengths for *sd*- and *fp*-Shell Nuclei*

N. Anantaraman, C. L. Bennett, J. P. Draayer,† H. W. Fulbright, H. E. Gove, and J. Tōke
Nuclear Structure Research Laboratory, University of Rochester, Rochester, New York 14627
(Received 13 August 1975)

We present systematics of the ground-state α-particle spectroscopic strengths for nuclei from ^{20}Ne to ^{66}Zn, measured in the (^{6}Li,d) reaction. An oscillatory decrease from ^{20}Ne to ^{32}S, which is in excellent agreement with SU(3) theory, is followed by a striking and unexplained increase at ^{36}Ar and ^{40}Ca and then a decrease up to ^{52}Ti, after which there is again a rise.

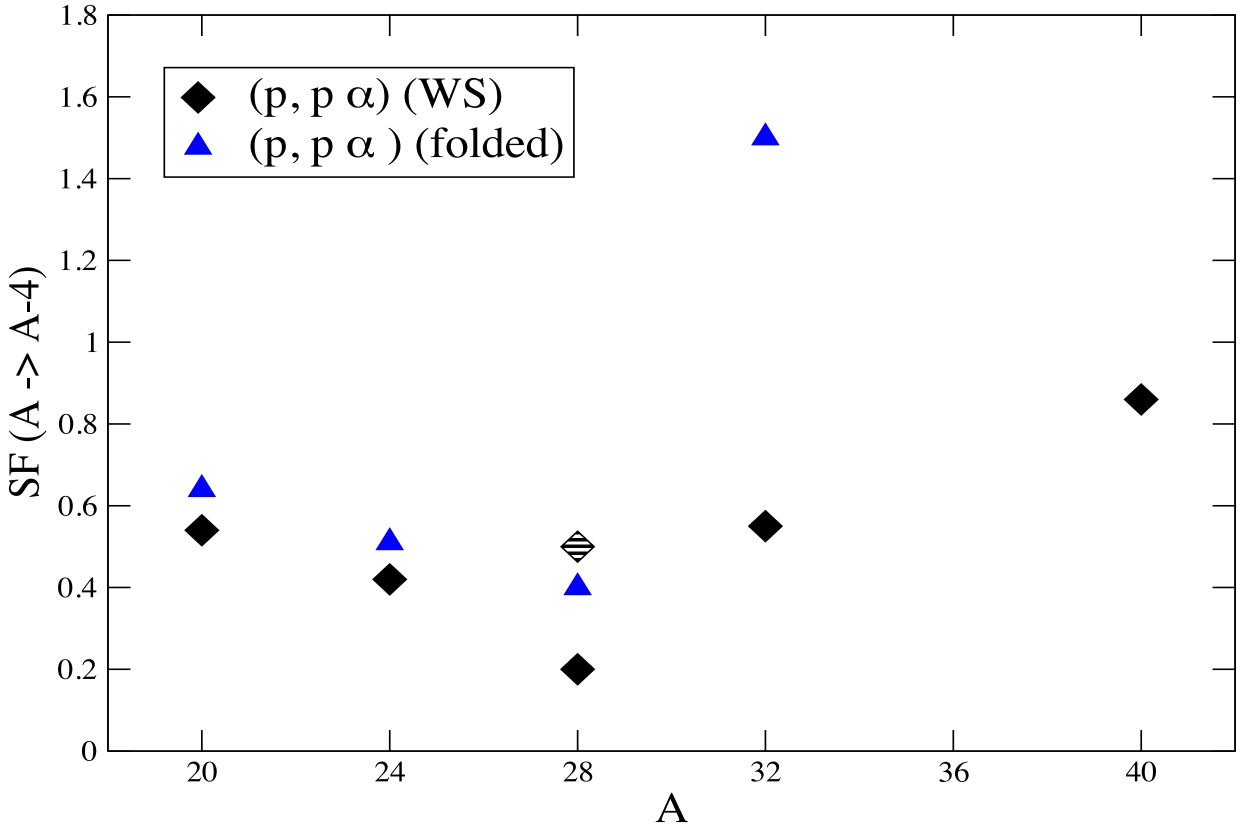
Alpha-clustering systematics from the quasifree (p,$p\alpha$) knockout reaction

T. A. Carey,* P. G. Roos, N. S. Chant, A. Nadasen, and H. L. Chen
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
(Received 19 August 1980)

Cross sections for the (p,$p\alpha$) reaction at 101.5 MeV have been measured for nine nuclei ranging from ^{16}O to ^{66}Zn. Distorted-wave impulse approximation analyses of the ground state transitions provide relative alpha-cluster spectroscopic factors in qualitative agreement with (^{6}Li,d) studies, although quantitative differences exist. The calculations are sensitive to the bound alpha-cluster parametrization, so that the experimental data suggest limits on the rms radius of the cluster-core wave function.

(^{6}Li,d) and (p,pα) data are consistent up to ^{32}S ->^{28}Si

Spectroscopic factors (SF) : experiment



^{28}Si: two possible values & large errors !

need for better data !

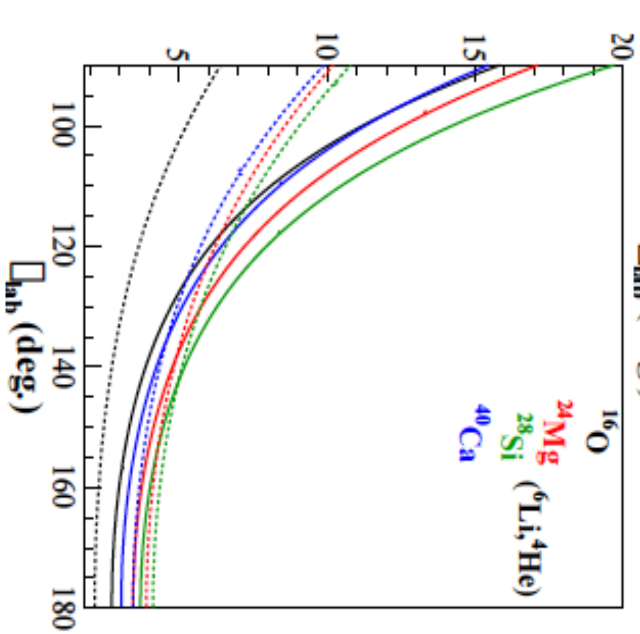
Accepted experiment @ALTO : (^{6}Li,d) & (^{6}Li,α) on sd-shell nuclei

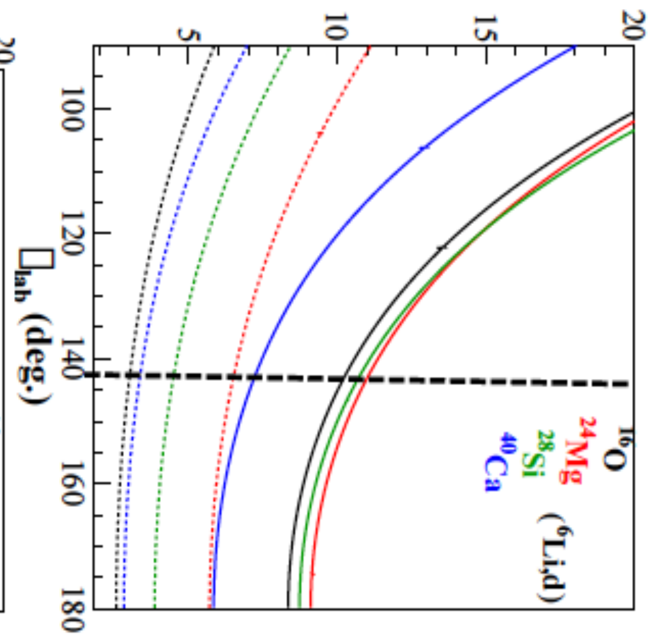
(^{6}Li,d) & (^{6}Li, α) on ^{16}O, ^{24}Mg, ^{28}Si, ^{40}Ca in inverse kinematics with MUGAST

Both reactions are selective in ΔT=0, so we will only populate T=0 states

➡ Aim for (^{6}Li,d) : study quartetting effects (ratios of cross-sections)

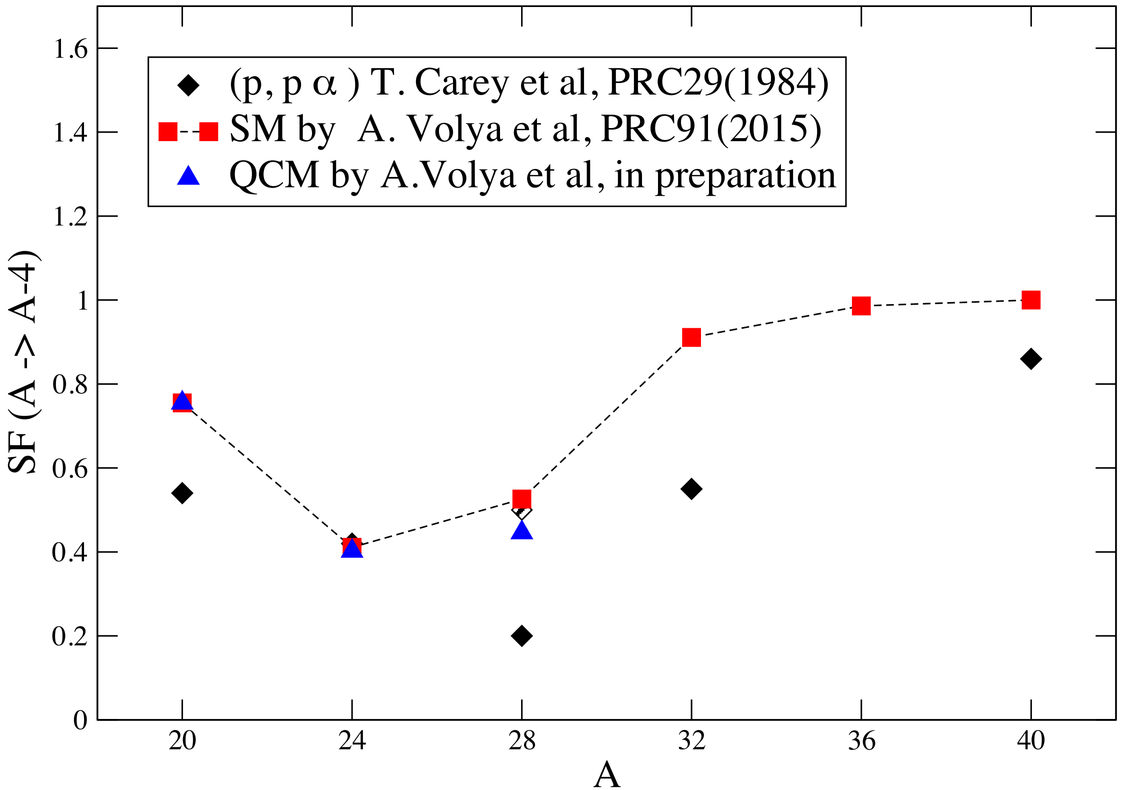
➡ Aim for (^{6}Li, α) :study the reaction mechanism for L=0, T=0 transfer on ^{40}Ca & ^{28}Si at energies from 1 to 4 MeV/u





E* resolution= 250 keV (from simulations with 0.5 mg/cm^{2} ^{6}LiF)

Spectroscopic factors : theoretical calculations



QCM (J=0,2,4) gives results close to SM

not a clear plateau region: ***sd- shell is too small***

A. Volya, M. Sambataro and N.S, in preparation

Conclusions on a transfer (SF)

SM and QCM gives similar results for SF

↓

the ground state correlations in N=Z nuclei are of quartet type

to probe the quarteting one needs a transfer data for a longer chain of N=Z nuclei !

α transfer in pf – shell nuclei ?

one step ahead ?

the Josephson effect ?

Nuclear Josephson-like γ-emission

R. A. Broglia
The Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen, Blegdamsvej 17, Denmark and
Dipartimento di Fisica, Università degli Studi di Milano, Via Celoria 16, I-20133 Milano, Italy

F. Barranco
Departamento de Física Aplicada III, Escuela Superior de Ingenieros,
Universidad de Sevilla, Camino de los Descubrimientos, Sevilla, Spain

L. Corradi
INFN, Laboratori Nazionali di Legnaro-35020 Legnaro, Italy

G. Potel
Lawrence Livermore National Laboratory, Livermore, California 94550, USA

S. Szilner
Ruđer Bošković Institute, HR-10 001 Zagreb, Croatia

E. Vigezzi
INFN Sezione di Milano, Via Celoria 16, I-20133 Milano, Italy
(Dated: June 22, 2022)

Systematics of the (d, ^{6}Li) Reaction and α Clustering in Heavy Nuclei*

F. D. Becchetti, L. T. Chua, J. Jänecke, and A. M. VanderMolen
Cyclotron Laboratory, Physics Department, The University of Michigan, Ann Arbor, Michigan 48105
(Received 5 August 1974)

Data for the α-particle pickup reaction (d, ^{6}Li) have been obtained at 35-MeV bombarding energy for even-even nuclei from ^{12}C to ^{238}U. The cross sections for the transitions to the ground states decrease approximately as $1/A_t^3$ where A_t is the target mass. α-particle transfer probabilities have been extracted from the data and are found to be substantially enhanced in heavy nuclei away from shell closures, particularly for deformed nuclei near $A \approx 150$. α-particle correlations appear to be related to two-nucleon pairing effects.

" It has been suggested that heavy-ion reactions involving transfer of two nucleons between superconducting nuclei [...] should exhibit enhancement phenomena similar to those observed in the Josephson effect in ordinary superconductors. Such an effect might also be observed in **the alpha-transfer between alpha-superconducting nuclei.**"

Thanks for your attention