Quartetting, clustering and related decay modes

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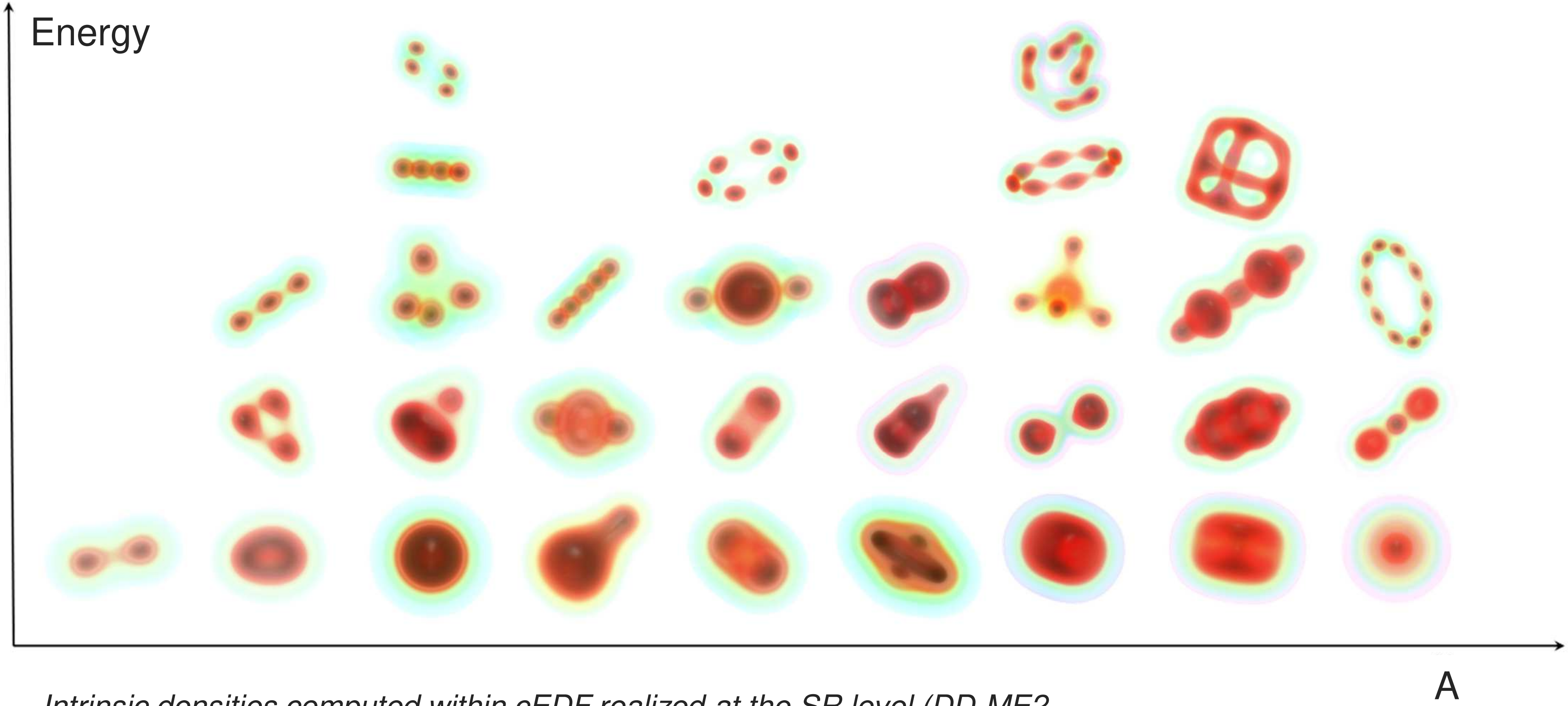
Outline

1. Generalities
2. Microscopic description of nuclear clustering
3. What are the consequences of the nuclear clustering phenomenon ?

1. Generalities

Nuclear clustering

- Nuclear clustering = nucleons clumping together into sub-groups within the nucleus



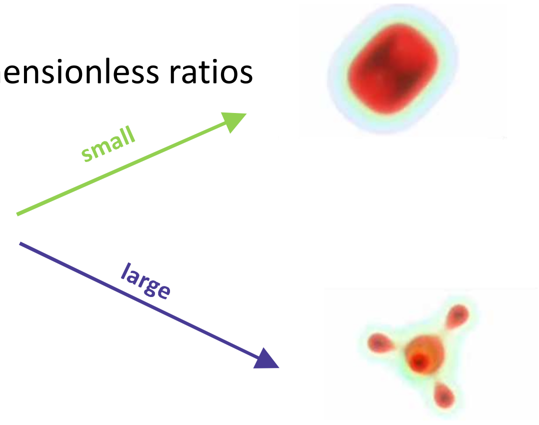
Intrinsic densities computed within cEDF realized at the SR level (DD-ME2 parametrization)

Strength of correlations

Ebran, Khan, Niksic & Vretenar Nature 2012
Ebran, Khan, Niksic & Vretenar PRC 2013

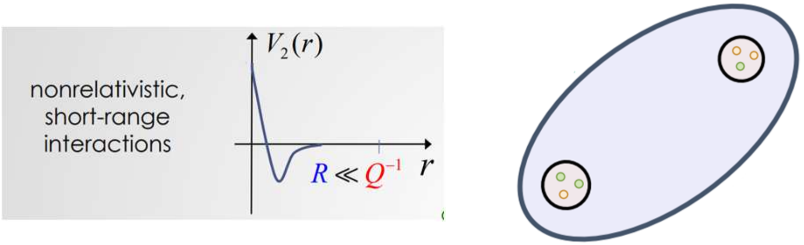
◉ Strength of correlations measured by dimensionless ratios

$$\sqrt{\Lambda} \equiv \sqrt{\frac{\langle \mathrm{V} \rangle}{\langle \mathrm{T} \rangle}} \sim \alpha_{\mathrm{loc}}$$



◉ Quantum systems near unitarity

A=2

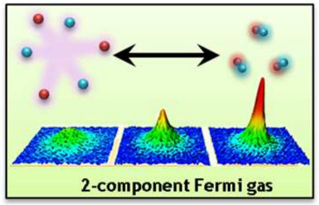


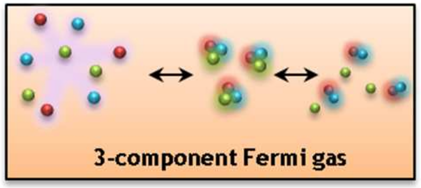
More bodies

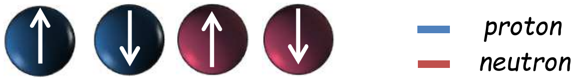
⇢ Bosons saturate
⇢ Multi-component fermions tend to clusterize

U. van Kolck et al

◉ Richness of correlations in a multi-component fermion system







Group theory considerations

- Schematic Hamiltonian $H = H_0 + \mathcal{V}_{\text{res}}$

$$H_0 = \int d^3r \sum_{\alpha} \varepsilon_\alpha \psi^\dagger_\alpha(\boldsymbol{r}) \psi_\alpha(\boldsymbol{r})$$

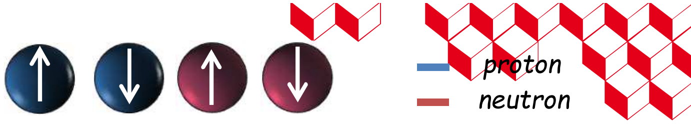
$$\mathcal{V}_{\text{res}} \sim V_{\text{pair}} = -\int d^3r \left[g^{\text{T}=1} \sum_{\nu=\pm1,0} P^\dagger_\nu(\boldsymbol{r}) P_\nu(\boldsymbol{r}) + g^{\text{T}=0} \sum_{\mu=\pm1,0} Q^\dagger_\mu(\boldsymbol{r}) Q_\mu(\boldsymbol{r}) \right]$$

Correlated pair operators

$$P^\dagger_\nu(\boldsymbol{r}) \equiv \sqrt{\frac{1}{2}} \sum_l \sqrt{2l+1} \left\{ \psi^\dagger_{l,s=\frac{1}{2},t=\frac{1}{2}}(\boldsymbol{r}) \psi^\dagger_{l,s=\frac{1}{2},t=\frac{1}{2}}(\boldsymbol{r}) \right\}^{(L=0,S=0,T=1)}_{M_L=0,M_S=0,M_T=\nu}$$

$$Q^\dagger_\mu(\boldsymbol{r}) \equiv \sqrt{\frac{1}{2}} \sum_l \sqrt{2l+1} \left\{ \psi^\dagger_{l,\frac{1}{2},\frac{1}{2}}(\boldsymbol{r}) \psi^\dagger_{l,\frac{1}{2},\frac{1}{2}}(\boldsymbol{r}) \right\}^{(L=0,S=1,T=0)}_{M_L=0,M_S=\mu,M_T=0}$$

Group theory considerations



◉ One-to-one correspondence with a system of spin-3/2 fermions with the Hamiltonian

$$H = \int d^3r \left\{ \sum_{\alpha} \varepsilon_\alpha \varphi_\alpha^\dagger(\boldsymbol{r}) \varphi_\alpha(\boldsymbol{r}) - g_0 S_{0,0}^\dagger(\boldsymbol{r}) S_{0,0}(\boldsymbol{r}) - \sum_{m=\pm2,\pm1,0} g_{2,m} D_{2,m}^\dagger(\boldsymbol{r}) D_{2,m}(\boldsymbol{r}) \right\}$$

Singlet (S=0) pairing operator $S_{0,0}^\dagger = \sum_{\alpha\beta} \langle \tfrac{3}{2}\tfrac{3}{2}; 00 | \tfrac{3}{2}\tfrac{3}{2}\alpha\beta \rangle \varphi_\alpha^\dagger \varphi_\beta^\dagger$

Quintet (S=2) pairing operator $D_{2,m}^\dagger = \sum_{\alpha\beta} \langle \tfrac{3}{2}\tfrac{3}{2}; 2m | \tfrac{3}{2}\tfrac{3}{2}\alpha\beta \rangle \varphi_\alpha^\dagger \varphi_\beta^\dagger$

with $S_{0,0}^\dagger = P_0^\dagger,\ D_{2,0}^\dagger = Q_0^\dagger,\ D_{2,\pm1}^\dagger = P_{\pm1}^\dagger \text{ and } D_{2,\pm2}^\dagger = Q_{\pm1}^\dagger$

Group theory considerations

- ◉ Sp(4) ~ SO(5) symmetry without fine tuning the coupling constants
- ◉ Generators of $\mathfrak{so}(5)$ $\quad \Gamma^{ab} \equiv -\frac{i}{2}\left[\Gamma^a, \Gamma^b\right] \quad (1 \leq a, b \leq 5)$ $\quad \Gamma^1 = \begin{pmatrix} 0 & -iI \\ iI & 0 \end{pmatrix}, \quad \Gamma^{2,3,4} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & -\boldsymbol{\sigma} \end{pmatrix}, \quad \Gamma^1 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$
- ◉ Bilinears of fermions can be classified according to their behavior under SO(5)

Particle-hole channel

$$\begin{aligned} n(\boldsymbol{r}) &= \sum_{\alpha} \varphi^{\dagger}_{\alpha}(\boldsymbol{r})\varphi_{\alpha}(\boldsymbol{r}), \\ n_a(\boldsymbol{r}) &= \frac{1}{2}\sum_{\alpha\beta} \varphi^{\dagger}_{\alpha}(\boldsymbol{r})\Gamma^{a}_{\alpha\beta}\varphi_{\beta}(\boldsymbol{r}), \\ L_{ab}(\boldsymbol{r}) &= -\frac{1}{2}\sum_{\alpha\beta} \varphi^{\dagger}_{\alpha}(\boldsymbol{r})\Gamma^{ab}_{\alpha\beta}\varphi_{\beta}(\boldsymbol{r}). \end{aligned}$$

Particle-particle channel

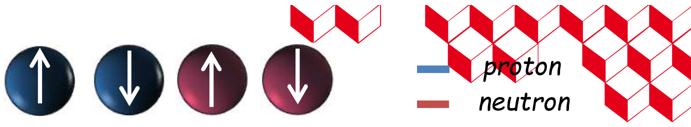
$$\begin{aligned} \eta^{\dagger}(\boldsymbol{r}) &= \frac{1}{2}\sum_{\alpha\beta} \varphi^{\dagger}_{\alpha}(\boldsymbol{r})C_{\alpha\beta}\varphi^{\dagger}_{\beta}(\boldsymbol{r}), \\ \xi^{\dagger}_{a}(\boldsymbol{r}) &= -\frac{i}{2}\sum_{\alpha\beta} \varphi^{\dagger}_{\alpha}(\boldsymbol{r})\left(\Gamma^{a}C\right)_{\alpha\beta}\varphi^{\dagger}_{\beta}(\boldsymbol{r}), \end{aligned}$$

$$C = \Gamma^1\Gamma^3$$

$$S^{\dagger}_{0,0} = -\frac{\eta^{\dagger}}{\sqrt{2}} \quad D^{\dagger}_{2,0} = -i\frac{\xi^{\dagger}_4}{\sqrt{2}}, \quad D^{\dagger}_{2,\pm 1} = -\frac{\xi^{\dagger}_3 \mp i\xi^{\dagger}_2}{\sqrt{2}}, \quad D^{\dagger}_{2,\pm 2} = \frac{\mp\xi^{\dagger}_1 + i\xi^{\dagger}_5}{\sqrt{2}}$$

C. Wu PRL 2005

Group theory considerations



$$H = \int d^3r \left\{ \sum_\alpha \varepsilon_\alpha \varphi^\dagger_\alpha(\boldsymbol{r})\varphi_\alpha(\boldsymbol{r}) - g_0 S^\dagger_{0,0}(\boldsymbol{r}) S_{0,0}(\boldsymbol{r}) - \sum_{m=\pm2,\pm1,0} g_{2,m} D^\dagger_{2,m}(\boldsymbol{r}) D_{2,m}(\boldsymbol{r}) \right\}$$

- If $g_0 = g_2 \equiv g$, singlet and quintet pairing states are degenerate and can be recast into a sextet pairing state ⇒ SU(4) symmetry

- 2 different superfluid orders :
  - i) Sp(4)-singlet BCS pairing phase : $\eta^\dagger(\boldsymbol{r})$
  - ii) SU(4) molecular superfluid phase formed from bound states of 4 fermions: $A^\dagger(\boldsymbol{r}) \equiv \varphi^\dagger_{\frac{3}{2}}(\boldsymbol{r})\varphi^\dagger_{\frac{1}{2}}(\boldsymbol{r})\varphi^\dagger_{-\frac{1}{2}}(\boldsymbol{r})\varphi^\dagger_{-\frac{3}{2}}(\boldsymbol{r})$

- Competition manifested by a $\mathbb{Z}_2$ discrete symmetry (coset between the center of SU(4) and the center of Sp(4)) $\mathcal{U}_n = e^{in_4\pi}$

$$\begin{aligned} \eta^\dagger &\mapsto \mathcal{U}_n \eta^\dagger \mathcal{U}_n^{-1} = -\eta^\dagger, \\ A^\dagger &\mapsto \mathcal{U}_n A^\dagger \mathcal{U}_n^{-1} = A^\dagger. \end{aligned}$$

$\mathbb{Z}_2$ needs to be spontaneously broken to stabilize the BCS quasi-long range order.

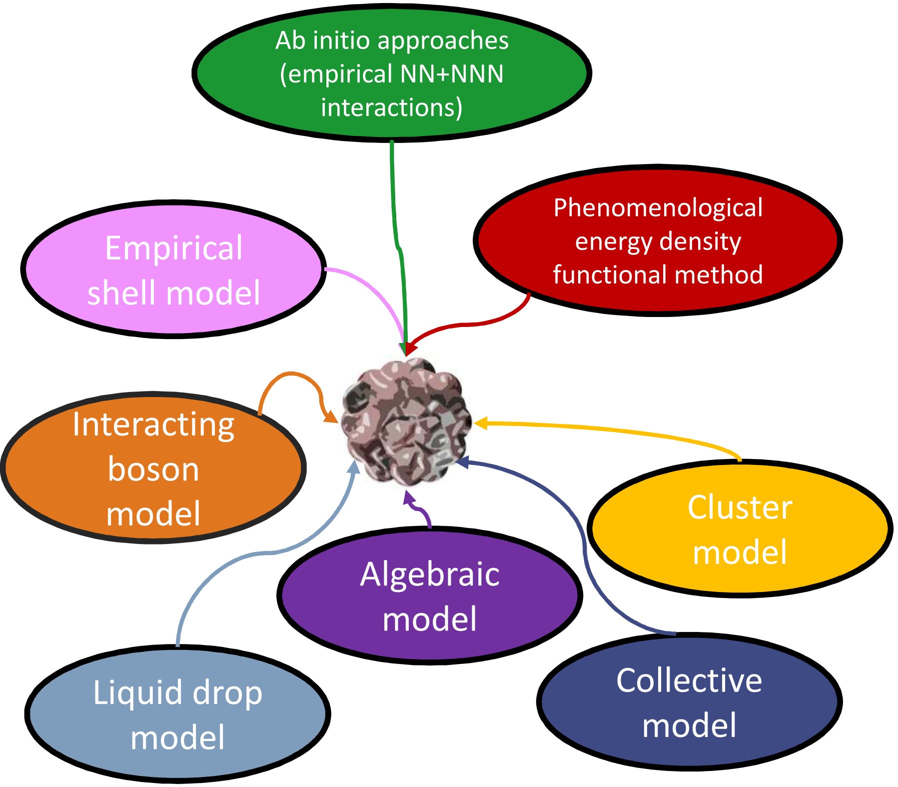
$\mathbb{Z}_2$ remaining unbroken ⇒ strong quantum fluctuations in the spin channel suppressing Cooper pairing (2 fermions can't form a $\mathbb{Z}_2$ singlet) ⇒ leading superfluid instability = quartetting

2. Microscopic viewpoint

Types of description

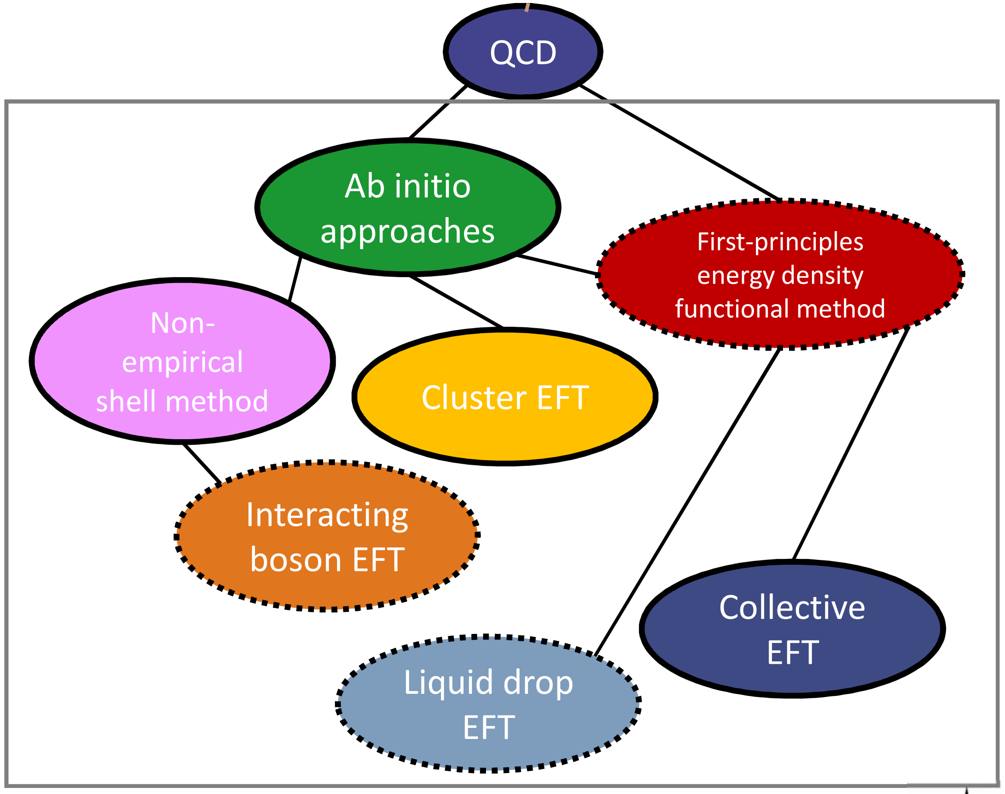
● Achieve a
accurate
predictive
computationally affordable
description ?

Era of models



☑ Gives insight about relevant scales/dofs
☑ Ready to be used
☒ Lack of control
⇒ double counting issues, error compensation, no error assessment

Era of effective (field) theories



☑ Full control ⇒ systematically improvable, no error compensation, no double counting, possibility of error estimation, …
☑ ☒ Force you to step back and rethink

Nuclear structure from a microscopic viewpoint

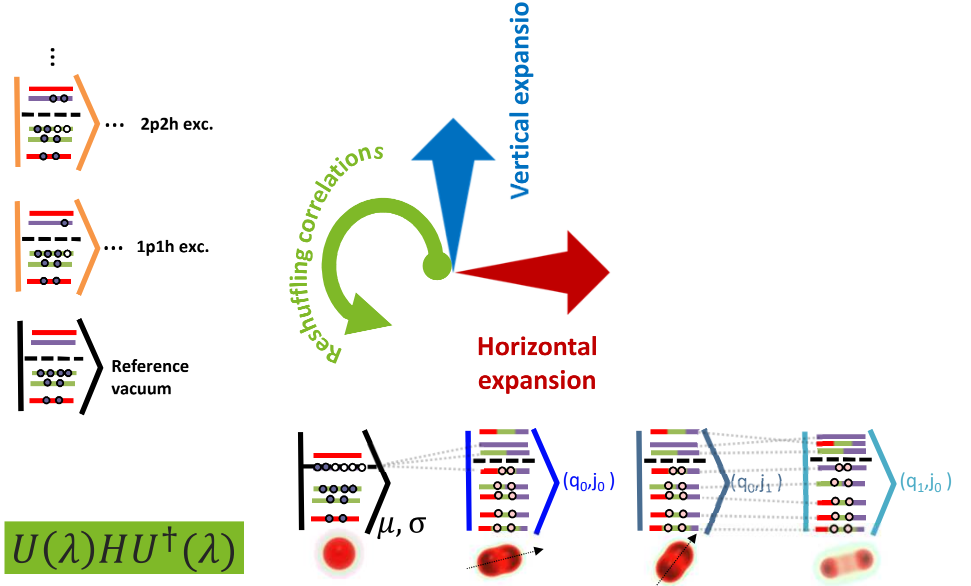
1) Nucleus: *A* interacting, structure-less nucleons
2) Structure & dynamic encoded in Hamiltonian, Functional, …
3) Solve *A*-nucleon Schrödinger/Dirac equation to desired accuracy

$$H(\ , \ , \ldots)|\Psi_{\mu,\sigma}\rangle = E_{\mu\tilde{\sigma}}\,|\Psi_{\mu,\sigma}\rangle \qquad N_{FCI} \propto \binom{L}{A}$$

Strongly correlated WF

$$|\Psi_{gs}\rangle = \sum_{i_1<\cdots<i_A}^{L} C_{i_1\cdots i_A}\,|\phi_{i_1}\cdots\phi_{i_A}\rangle \equiv \sum_{I}^{N_{FCI}} C_I\,|\Phi_I\rangle$$

Rationale for grasping nucleon correlations



$U(\lambda)HU^{\dagger}(\lambda)$

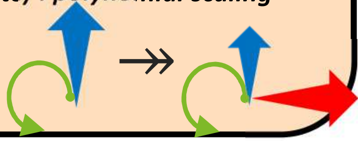
Ab initio

◉ **Systematically improvable free-space Hamiltonian in χEFT**

◉ **Solving Schrödinger equation**

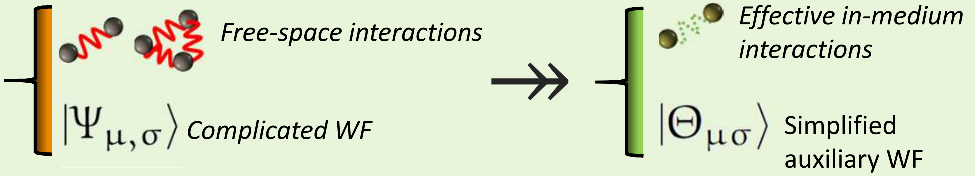
- ◈ Pre-processing H
- ◈ Refined many-body schemes with controlled uncertainties
  - --> *CI (full space diag.) : exponential scaling*
  - --> *Hybrids (valence space diag.) : mixed scaling*
  - --> ***Expansion methods (partition, expand and truncate) : polynomial scaling***

⊗ How to challenge ab initio frontiers



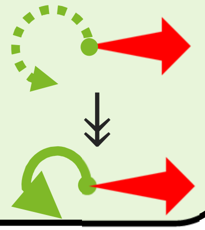
EDF

◉ **Effective pseudo-Hamiltonian**



◉ **Various levels of realization**

- --> *Hartree-Fock-Bogoliubov (HFB)*
- --> *Projected Generator Coordinate Method (PGCM)*
- --> *Quasiparticule Random Phase Approximation (QRPA)*



⊗ How to improve current EDFs
⊗ How to turn EDF in EFT ?

How to account for correlations causing clustering ?

i) Explicitly treat 4-nucleon correlations

$$\psi_T^A = f_J\left[\Phi^{\mathrm{I,II}}_{BCS}\Phi^{\mathrm{III,IV}}_{BCS} + \Phi^{\mathrm{I,III}}_{BCS}\Phi^{\mathrm{II,IV}}_{BCS} + \Phi^{\mathrm{I,IV}}_{BCS}\Phi^{\mathrm{II,III}}_{BCS}\right]$$

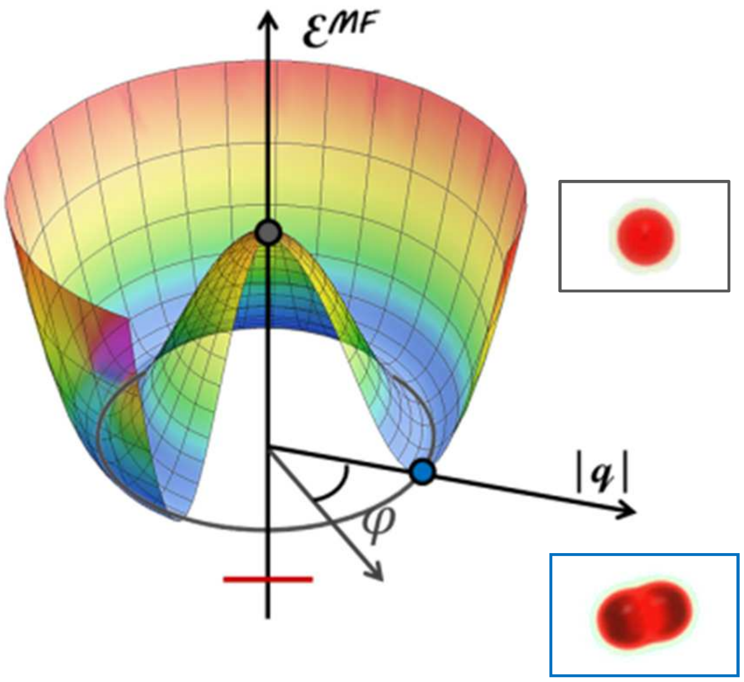
$$\begin{aligned}\psi_T^B =& \mathcal{A}\Big[e^{-\beta_0\sum_{i=1,3,5,7}(\mathbf{r}_i-\mathbf{r}_{CM}^{1,3,5,7})^2}\times\\ & e^{-\beta_0\sum_{j=2,4,6,8}(\mathbf{r}_j-\mathbf{r}_{CM}^{2,4,6,8})^2}\times\\ & e^{-\beta_1(\mathbf{r}_{CM}^{1,3,5,7}-\mathbf{r}_{CM}^{2,4,6,8})^2}(\mathbf{r}_{CM}^{1,3,5,7}-\mathbf{r}_{CM}^{2,4,6,8})^n\Big]\end{aligned}$$

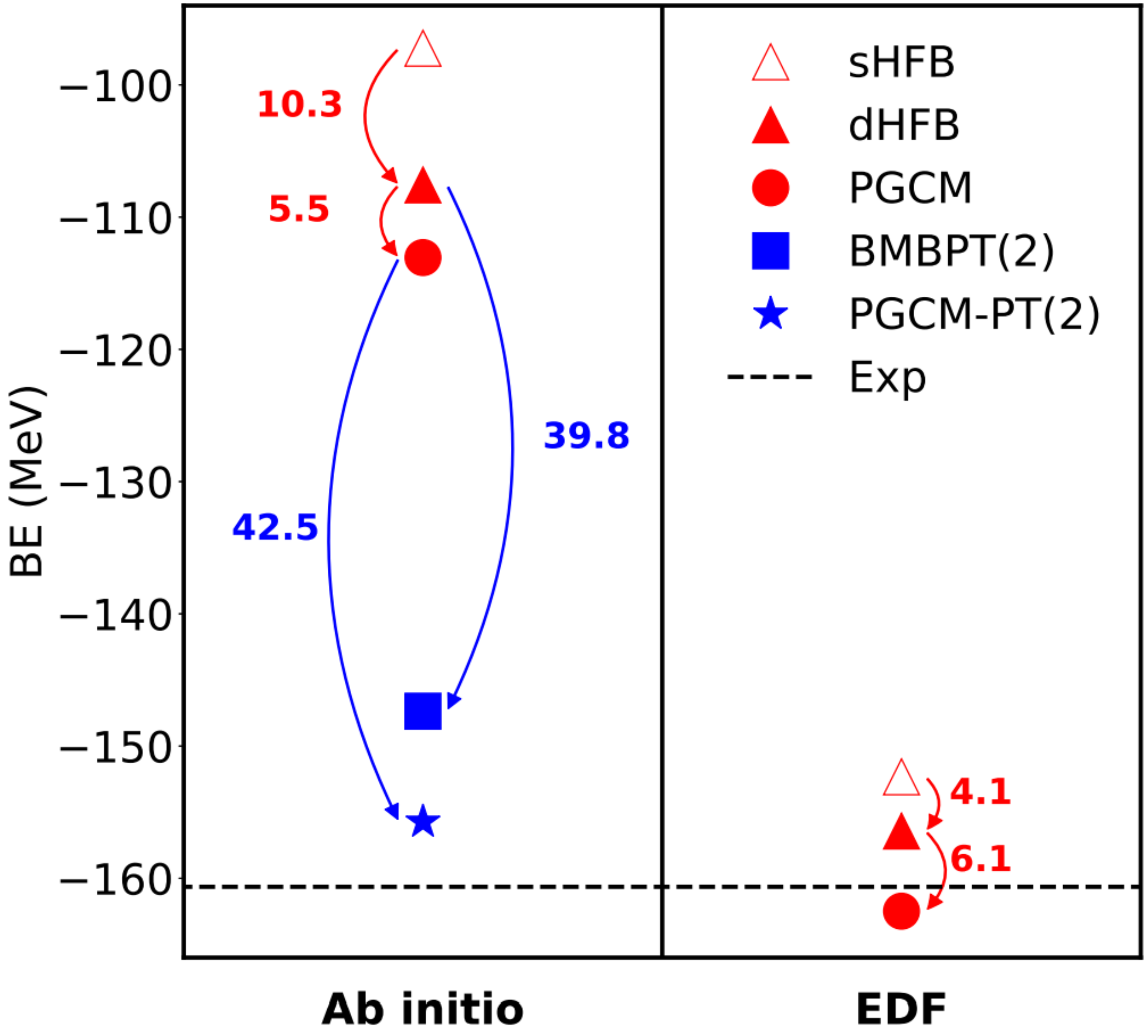
Dawkins et al PRL **124**, 143402 (2020)

$$\begin{aligned}\psi_T^C = & \mathcal{A}[F(\mathbf{r}_{CM}^{1,3,5,7}-\mathbf{r}_{CM}^{2,4,6,8})\times f_J(\mathbf{r}_1,\mathbf{r}_3,\mathbf{r}_5,\mathbf{r}_7)\times\\ & f_J(\mathbf{r}_2,\mathbf{r}_4,\mathbf{r}_6,\mathbf{r}_8)\times\prod_{\substack{n=1,3,5,7\\ m=2,4,6,8}} g(r_{nm})]\end{aligned}$$

ii) Look for a collective field whose fluctuations cause nucleon to aggregate into α dofs

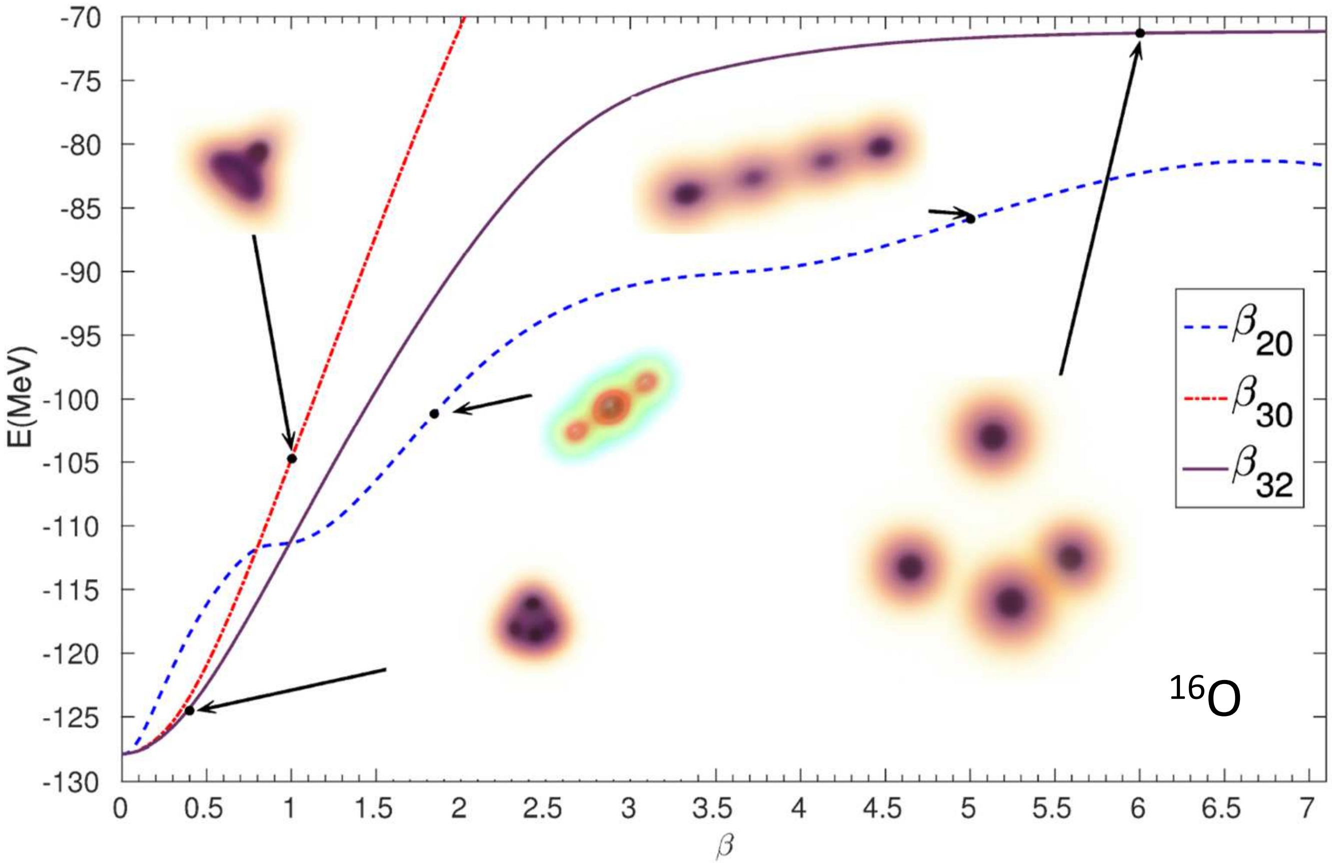
Dynamical and static correlations





Static correlations & Nuclear clustering

(Mott) transition from delocalized to totally localized nucleons takes the form of a transition from O(3) (or continuous subgroup) to a discrete point-group

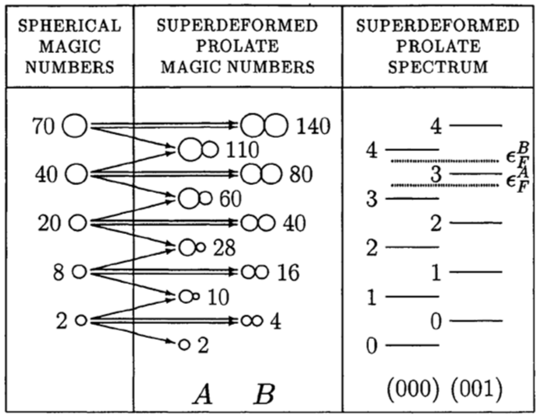


Deformation & Nuclear clustering

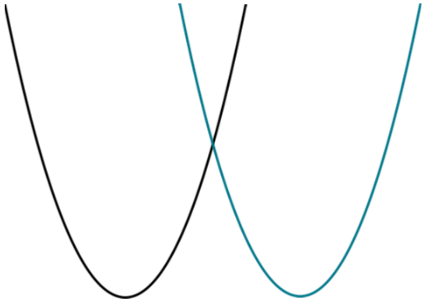
◉ Role of deformation

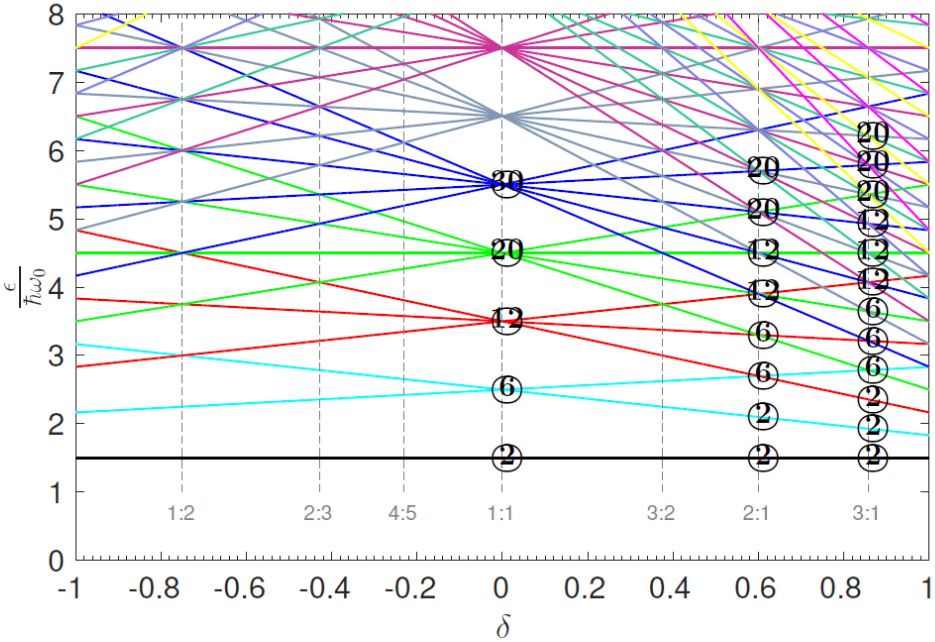
N-dimensional anisotropic HO with commensurate frequencies enjoys dynamical symmetries involving multiple independent copies of SU(N) irreps

Susceptibility of nucleons in deformed nuclei to arrange into multiple spherical fragments



Nazarewicz & *Dobaczewski*, PRL 1992





Deformation = necessary condition, but not a sufficient one

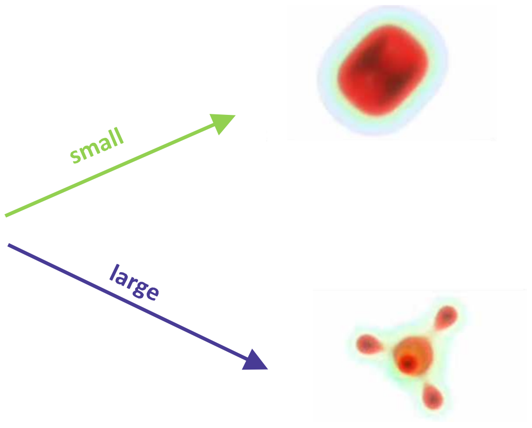
Strength of correlations

- Strength of correlations measured by dimensionless ratios

Nucleon mass **Number of nucleons**

$$\sqrt{\Lambda} \equiv \sqrt{\frac{\langle V\rangle}{\langle T\rangle}} = \sqrt{2}\left(\frac{3}{4\pi}\right)^{\frac{1}{6}} (2MU)^{\frac{1}{4}} \, (An)^{-\frac{1}{6}} \sim \alpha_{loc}$$

Depth of the confining potential **Mean density**



Ebran, Khan, Niksic & Vretenar Nature 2012
Ebran, Khan, Niksic & Vretenar PRC 2013

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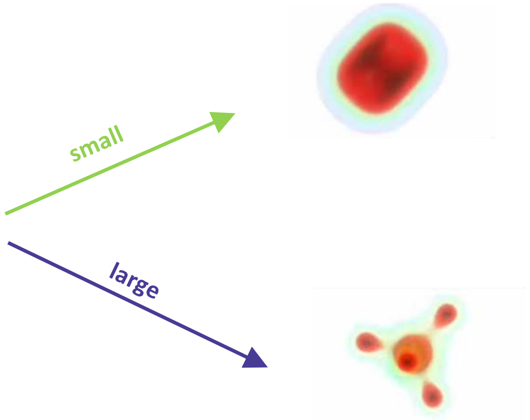
Depth of the confining potential

Mean density



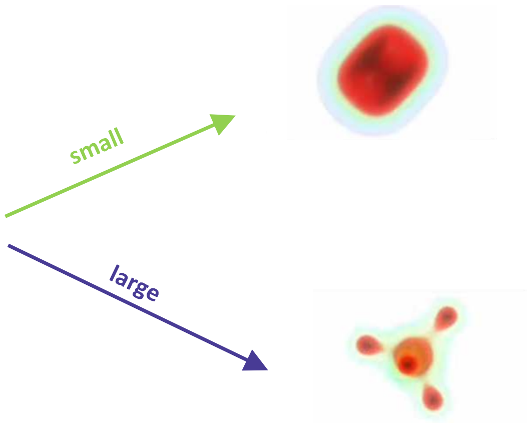
Clustering favored ⇢ For light nuclei
⇢ In regions at low-density

Ebran, Khan, Niksic & Vretenar Nature 2012
Ebran, Khan, Niksic & Vretenar PRC 2013

Strength of correlations

- Strength of correlations measured by dimensionless ratios

$$\sqrt{\Lambda} \equiv \sqrt{\frac{\langle V\rangle}{\langle T\rangle}} = \sqrt{2}\left(\frac{3}{4\pi}\right)^{\frac{1}{6}} (2\overset{\text{Nucleon mass}}{M}\underset{\text{Depth of the confining potential}}{U})^{\frac{1}{4}} (\overset{\text{Number of nucleons}}{A}\underset{\text{Mean density}}{n})^{-\frac{1}{6}} \sim \alpha_{loc}$$



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- Formation/dissolution of clusters : Mott parameter

$$\frac{\overset{\text{Size of the nucleus X}}{R_X}}{\underset{\text{inter-nucleon average distance}}{d^X_{Mott}}} \sim 1 \Rightarrow n^X_{Mott} \sim \frac{\rho_{sat}}{A_X}$$

$n^{\alpha}_{Mott} \sim \quad 0.25\rho_{sat}$ Size of an α in free-space

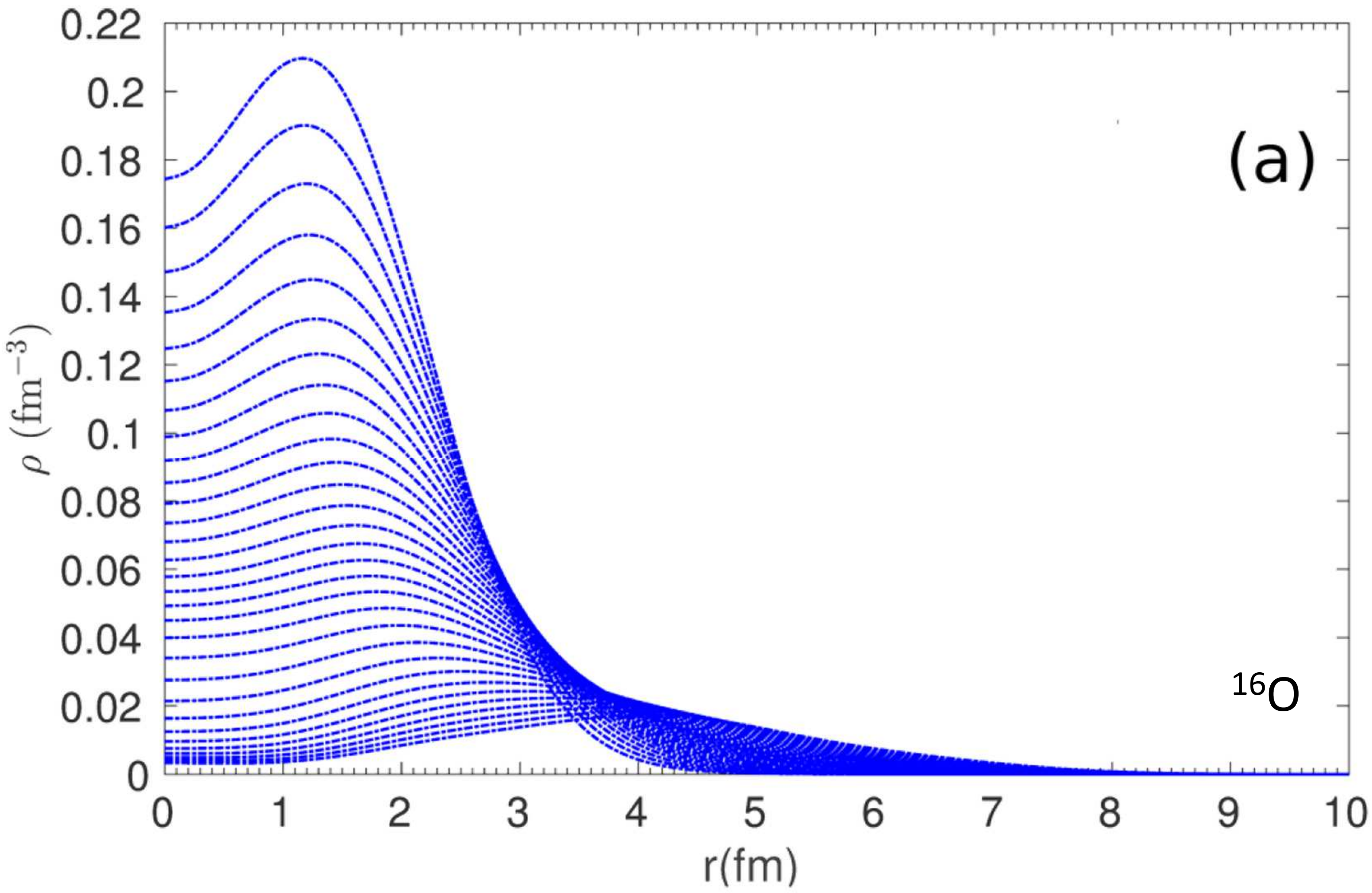
$\sim \frac{\rho_{sat}}{3}$ 0.9 size of an α in free-space

Ebran, Girod, Khan, Lasseri, Schuck, PRC 2020
Ebran, Khan, Niksic, Vretenar, PRC 2014

3. « Mean field » approximation

Effect of the density

- ◉ Isotropically inflate ^{16}O by constraining its r.m.s. radius while imposing a global quadrupole moment to be zero



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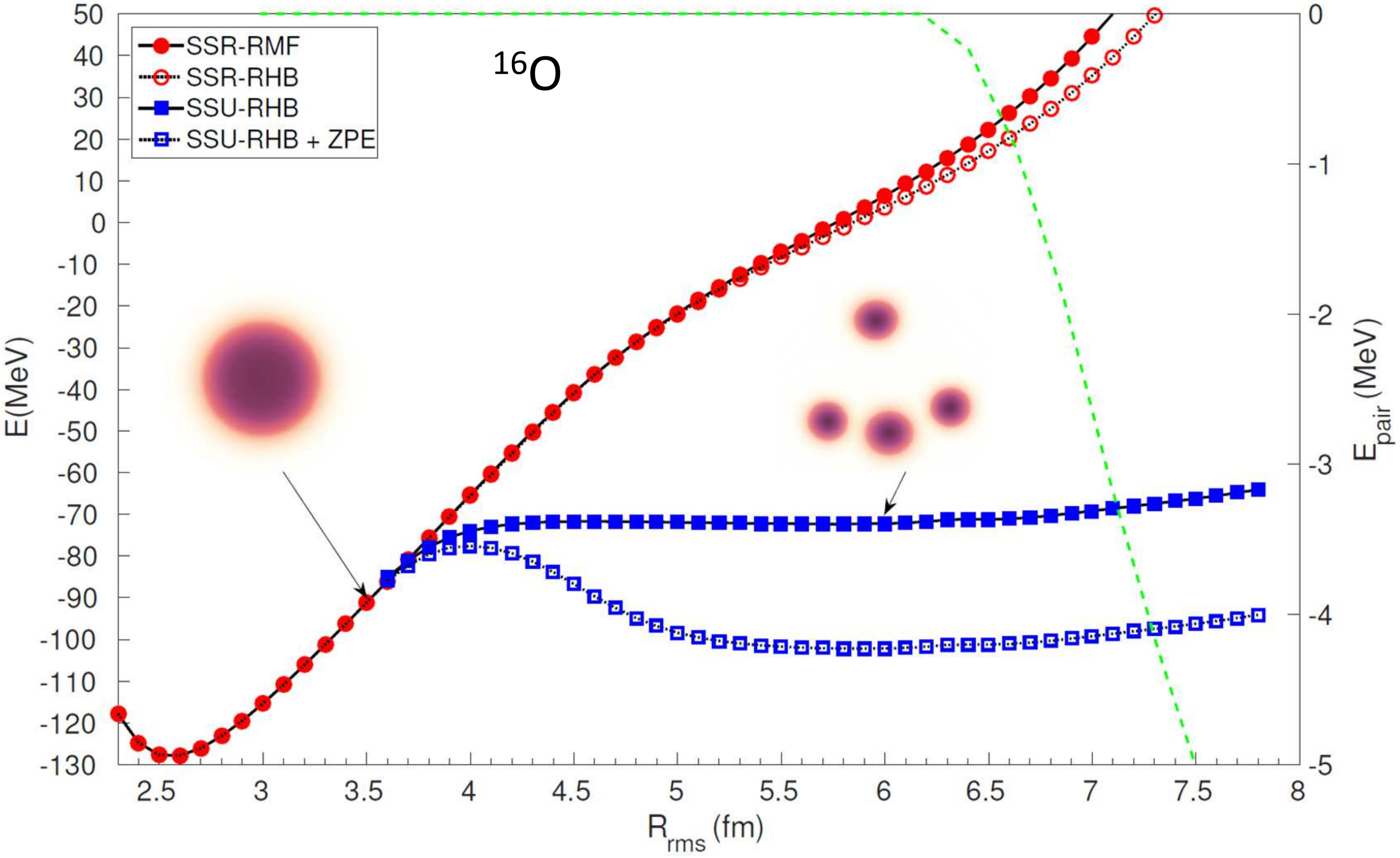

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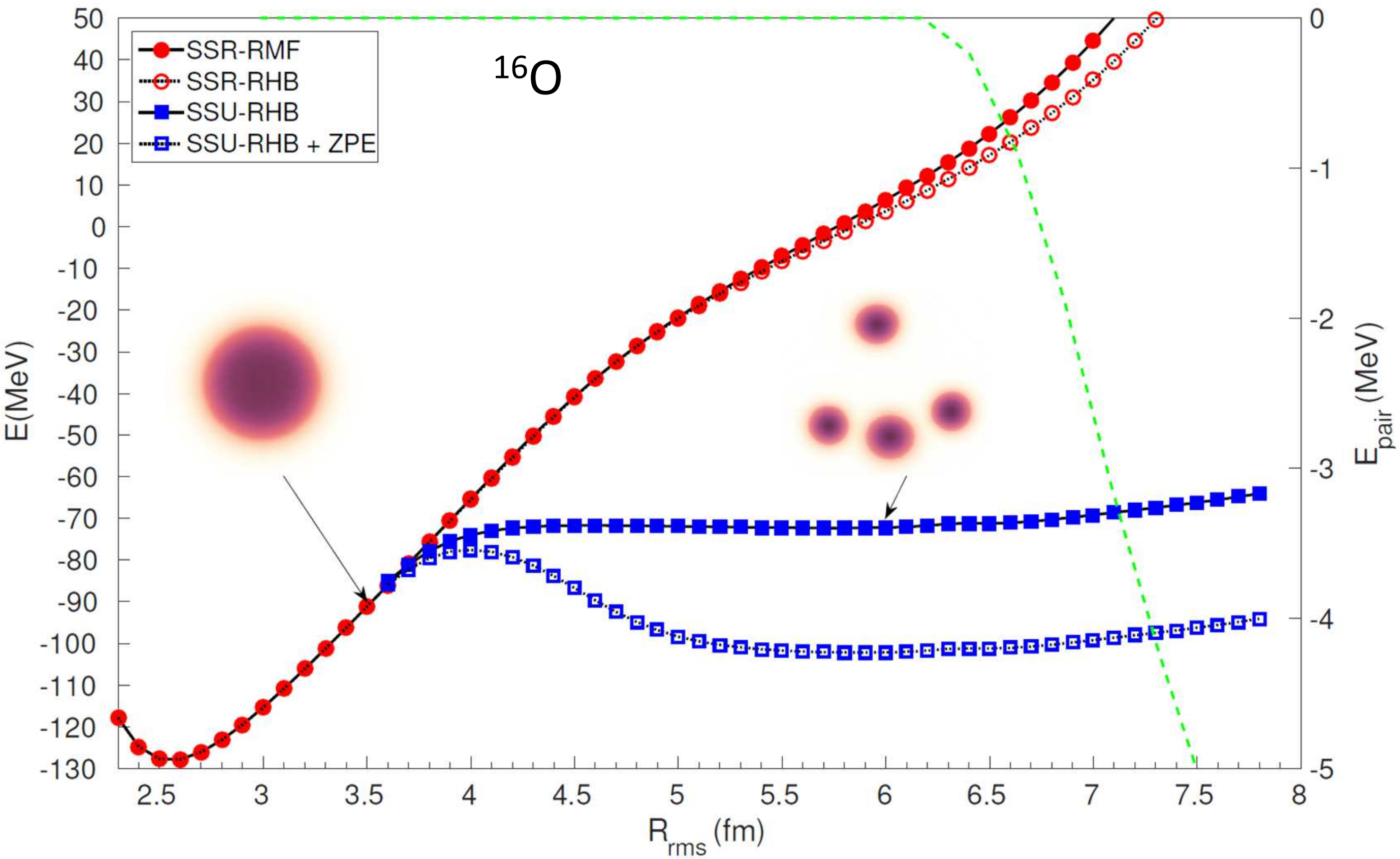

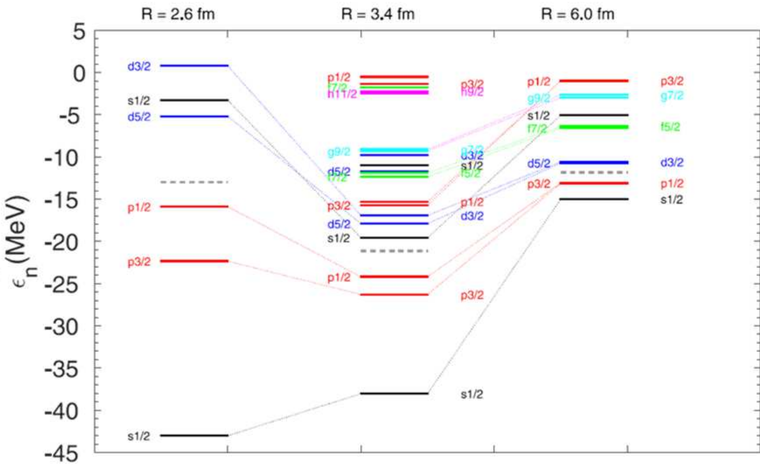

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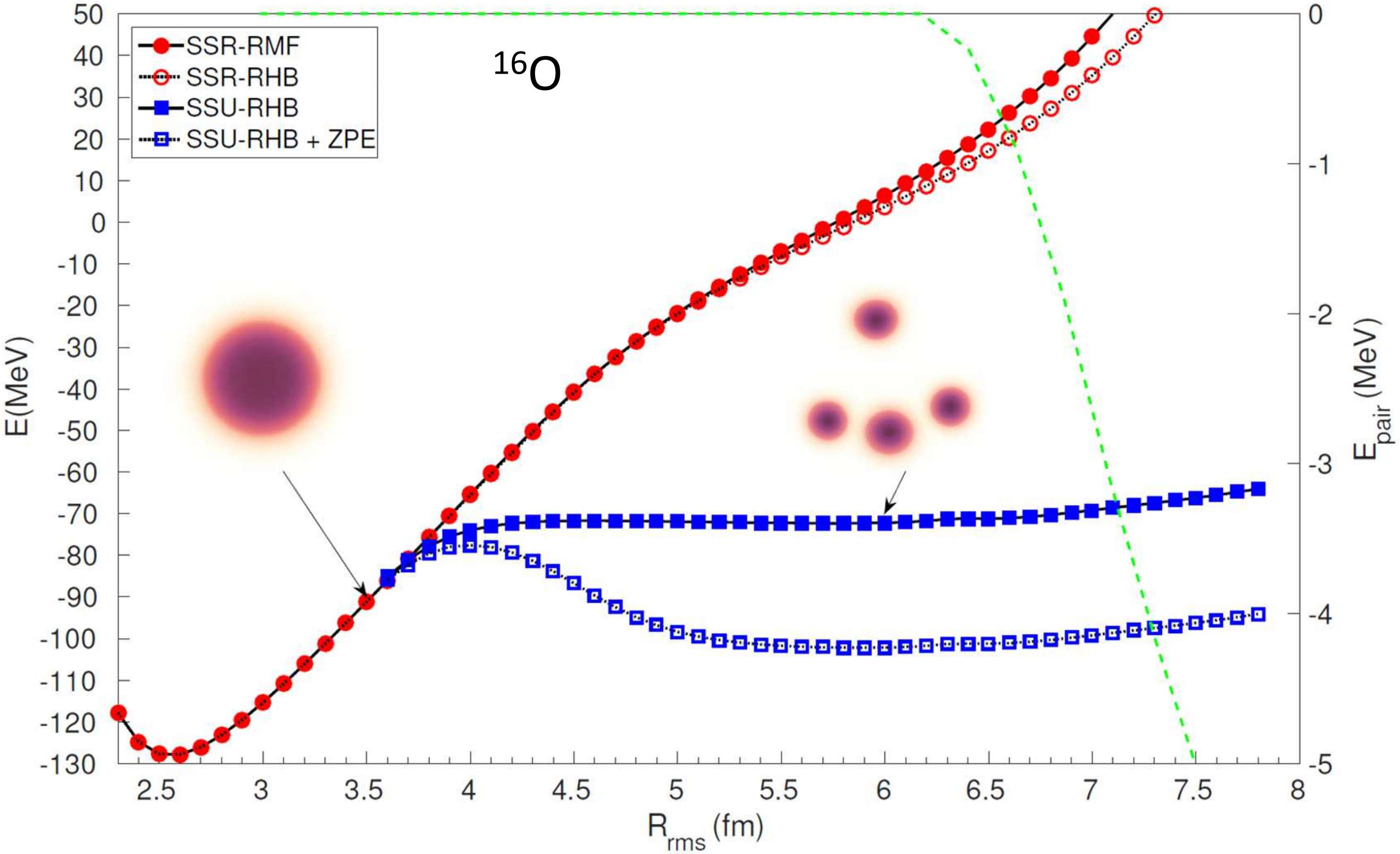

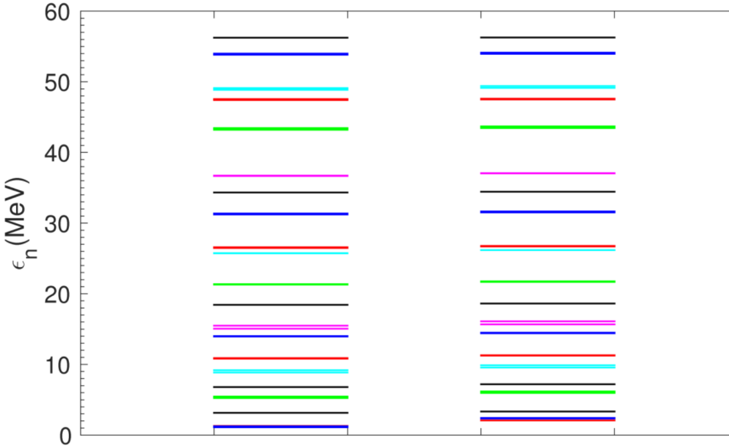

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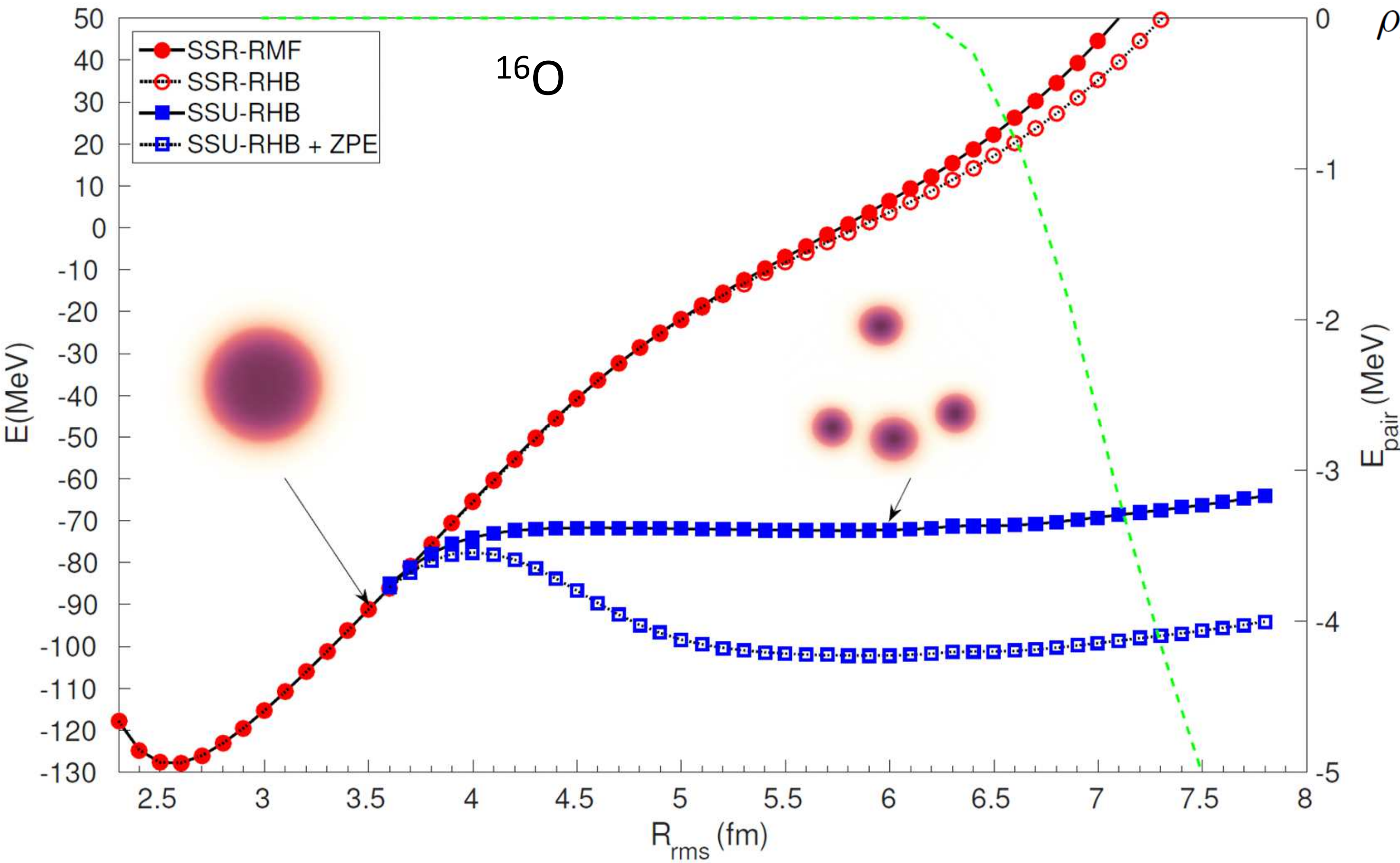
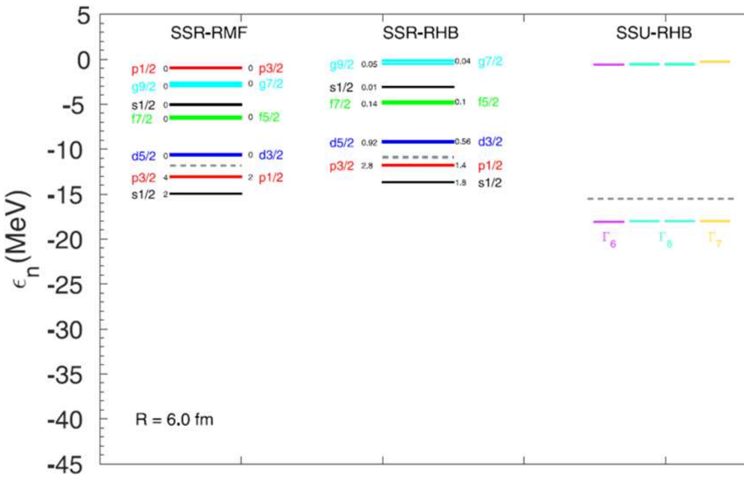
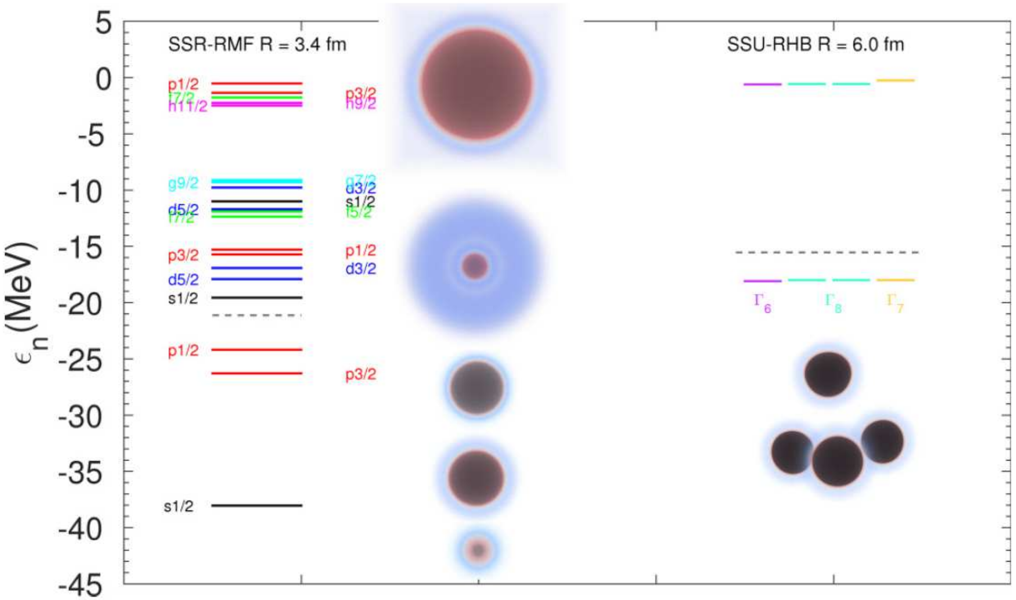


$$\rho_{\rm Mott}/\rho_0 = (R_{\rm eq}/R_c)^3 \approx \rho_0/3$$

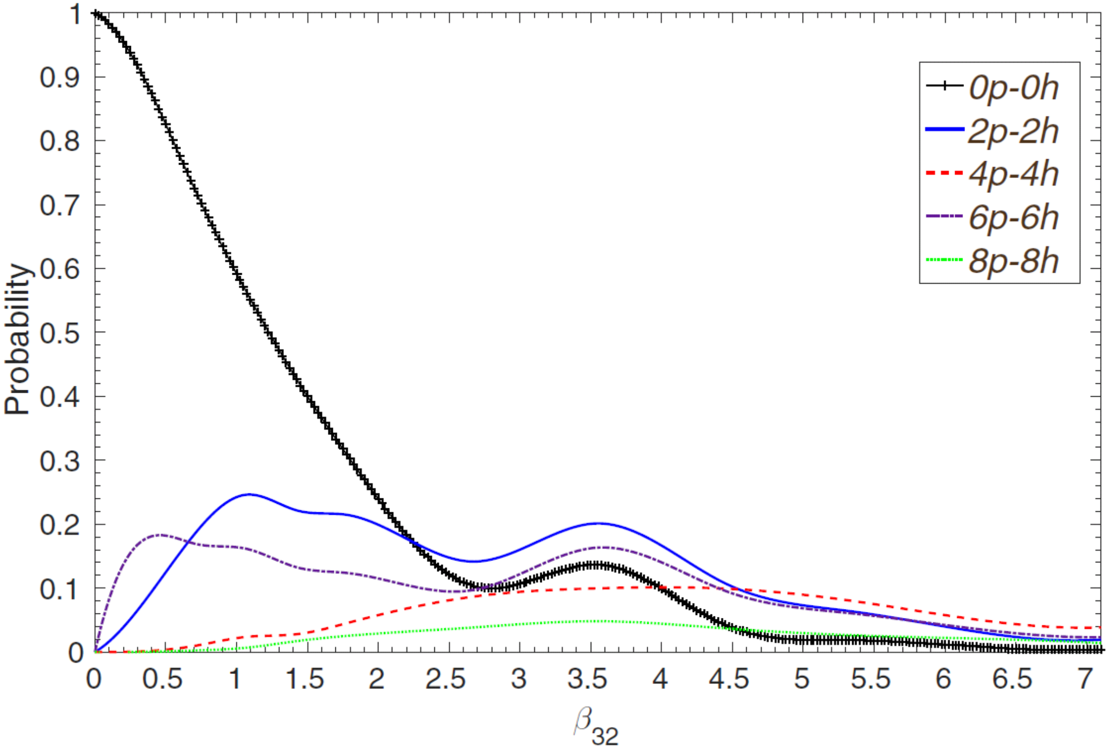


Effect of the density

- mp-mh content of a tetrahedrally-deformed Slater determinant



Symmetry breaking : O(3) → Td



LCAO-MO

- ◉ Borrowing the LCAO-MO language, on can think of the 16O thetrahedrally-deformed SD as a MO built from 4 1s α AOs

$$\psi_i = \sum_{j=1}^{4} f_j^i \phi_j$$

- ◉ Find the unknowns f in the Hückel approximation :

$\mathcal{N}_{ij} = 0 \forall i, j$

$\epsilon \equiv \mathcal{H}_{ii}$; $-\mu \equiv \mathcal{H}_{ij}$ for adjacent i,j ; $\mathcal{H}_{ij} = 0$ otherwise

$$\begin{pmatrix} \epsilon & -\mu & -\mu & -\mu \\ -\mu & \epsilon & -\mu & -\mu \\ -\mu & -\mu & \epsilon & -\mu \\ -\mu & -\mu & -\mu & \epsilon \end{pmatrix} \begin{pmatrix} f_1^i \\ f_2^i \\ f_3^i \\ f_4^i \end{pmatrix} = E_i \begin{pmatrix} f_1^i \\ f_2^i \\ f_3^i \\ f_4^i \end{pmatrix}$$

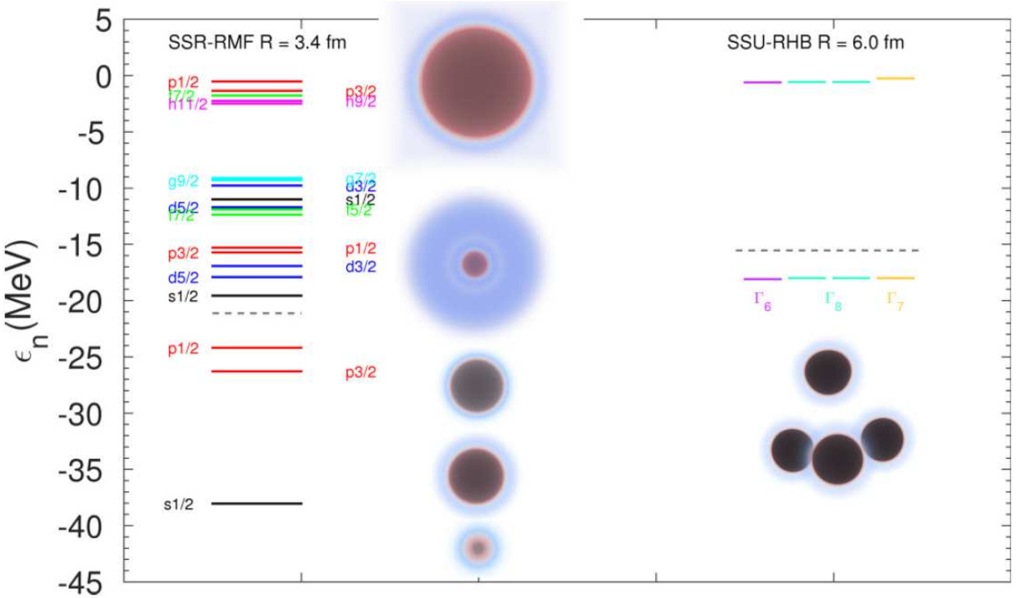
$$\psi_1 = \frac{1}{2}(\phi_1 + \phi_2 + \phi_3 + \phi_4) \qquad E_1 = \epsilon - 3\mu$$

$$\psi_2 = \frac{1}{\sqrt{2}}(-\phi_1 + \phi_2) \qquad E_2 = \epsilon + \mu$$

$$\psi_3 = \frac{1}{\sqrt{2}}(-\phi_1 + \phi_3) \qquad E_3 = E_2$$

$$\psi_4 = \frac{1}{\sqrt{2}}(-\phi_1 + \phi_4) \qquad E_4 = E_3 = E_2$$

$$\begin{aligned} \psi'_2 &= \frac{1}{2}(\phi_1 - \phi_2 - \phi_3 + \phi_4), \\ \psi'_3 &= \frac{1}{2}(\phi_1 + \phi_2 - \phi_3 - \phi_4), \\ \psi'_4 &= \frac{1}{2}(-\phi_1 + \phi_2 - \phi_3 + \phi_4). \end{aligned}$$

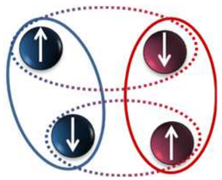


4. « Beyond-Mean field » approximation

EDF & Quartet condensation model

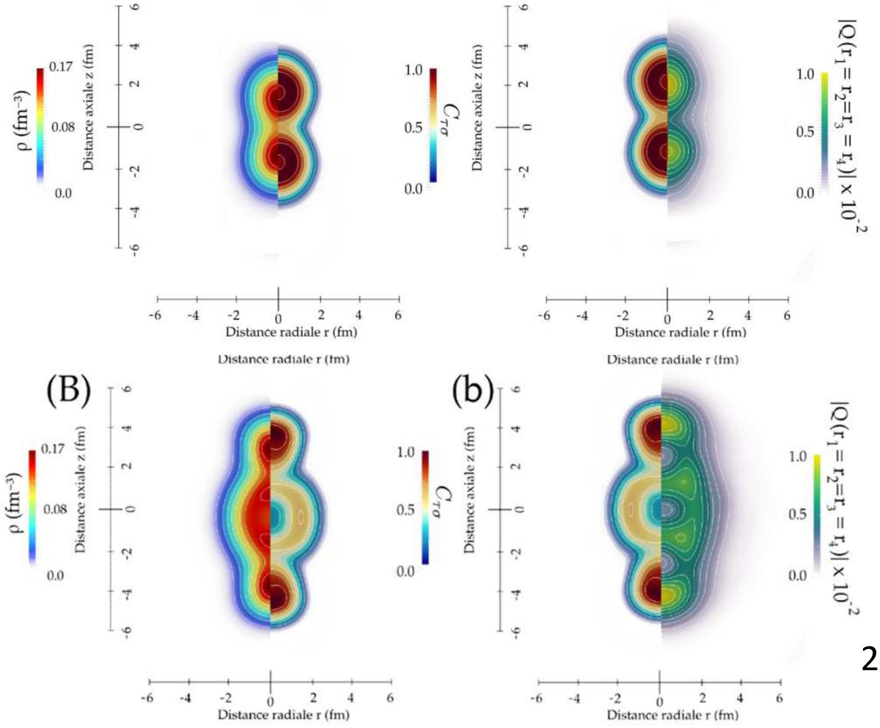
- How to account for correlations underpinning α-clustering ?

  i) Explicitly treat 4-nucleon correlations : RMF + QCM



$$|\Psi\rangle = \left(\mathcal{Q}^\dagger\right)^{n_q}|0\rangle \qquad \mathcal{Q}^\dagger = 2\Gamma_1^\dagger\Gamma_{-1}^\dagger - \left(\Gamma_0^\dagger\right)^2 \qquad \Gamma_t^\dagger = \sum_k x_k P_{k,t}^\dagger$$

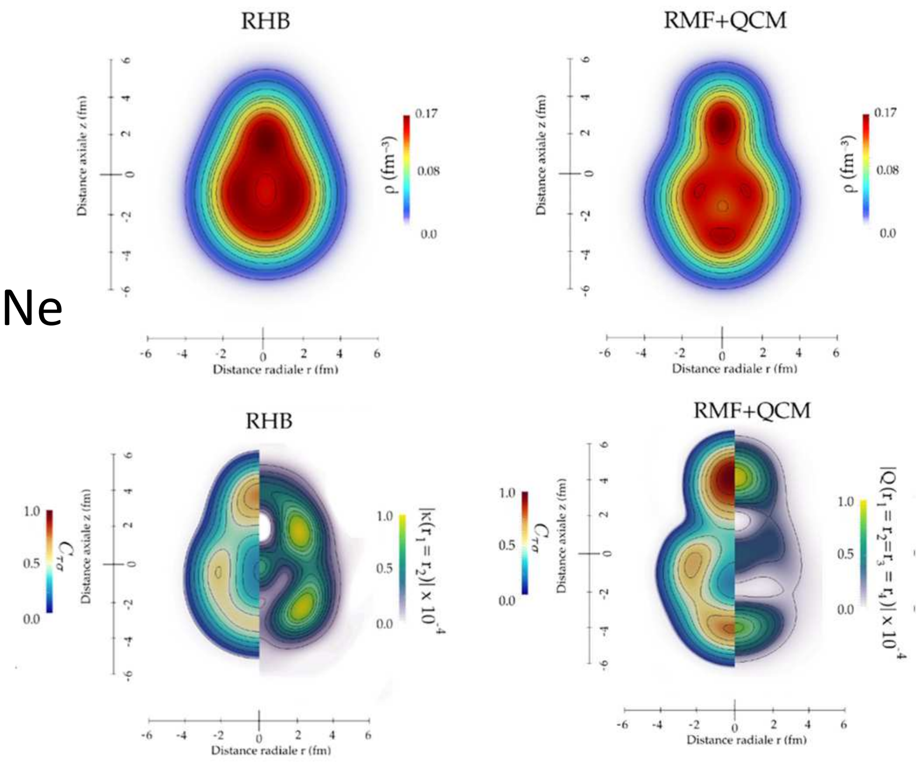
Lasseri, Ebran, Khan, Sandulescu



^{8}Be

^{24}Mg

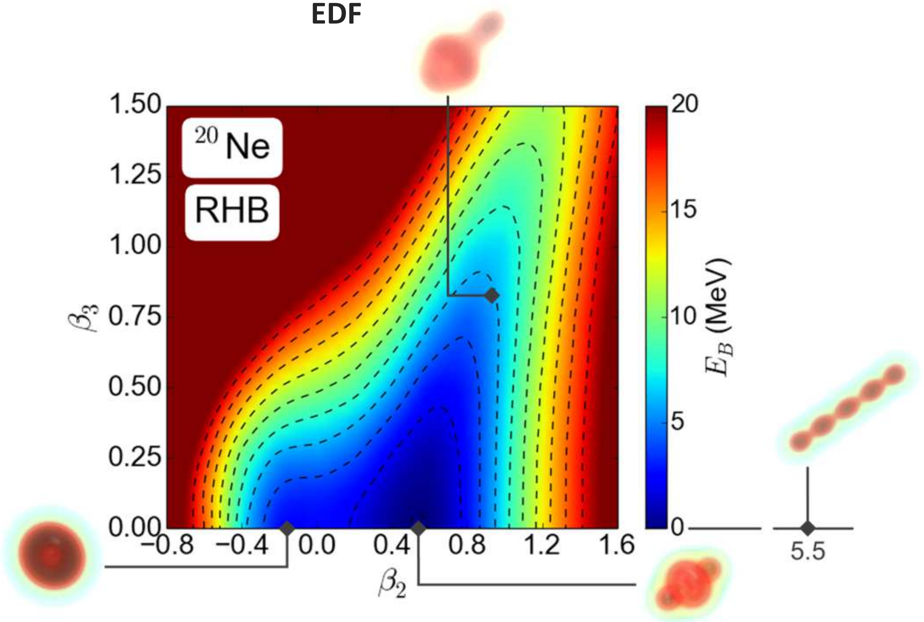
^{20}Ne



Nuclear clustering & PGCM

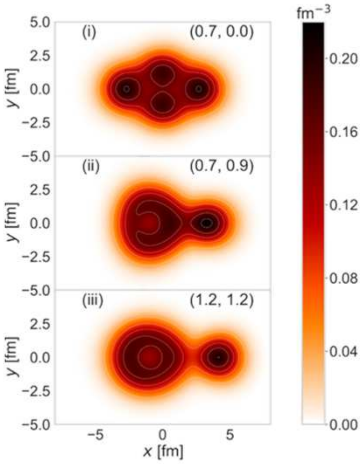
◉ Correlated GS

EDF

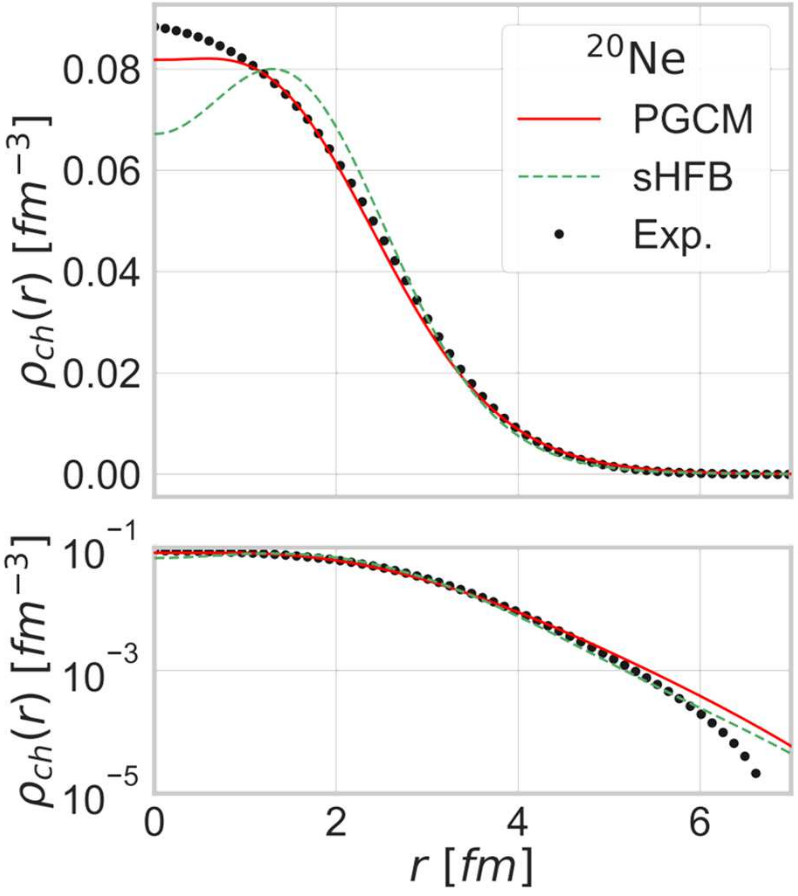


Marevic, Ebran, Khan, Niksic, Vretenar, PRC 97 (2018)

Ab initio



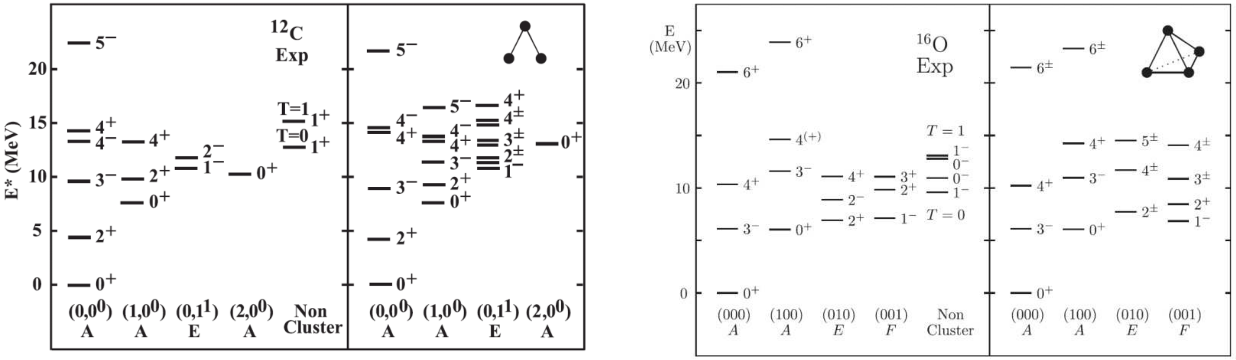
Frosini, Duguet, Ebran, Somà

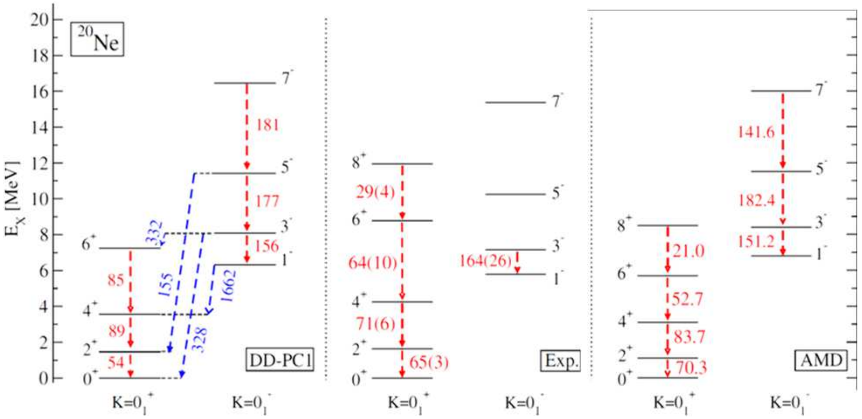


Nuclear clustering & PGCM

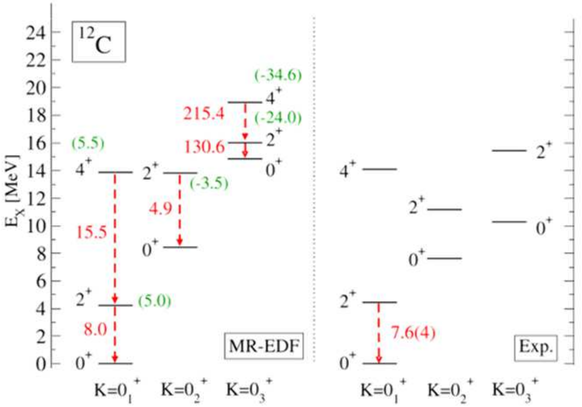
- ◉ Spectroscopy

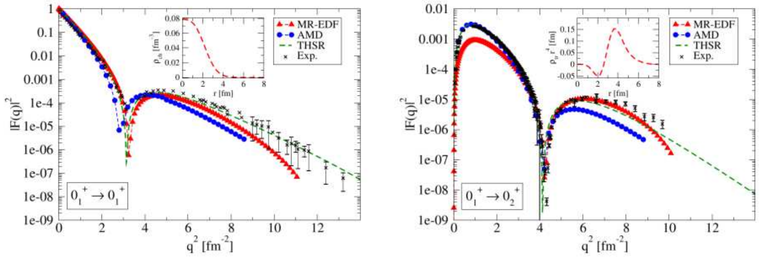
Bijker (2016)



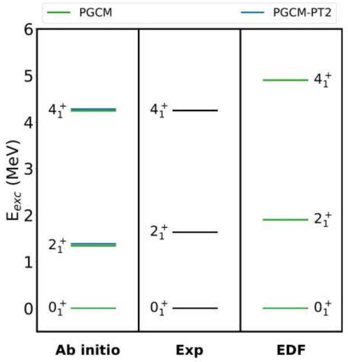


Marevic, Ebran, Khan, Niksic, Vretenar, PRC 2018





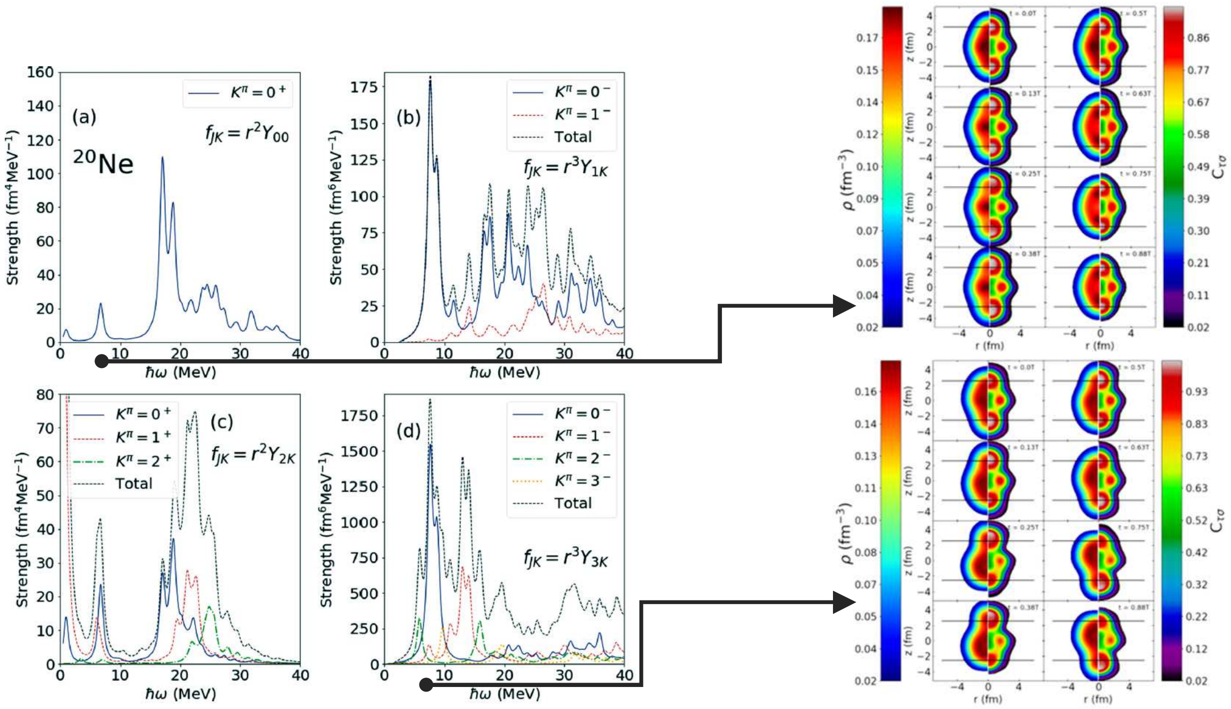
Marević, Ebran, Khan, Nikšić, and Vretenar PRC 2019



Frosini, Duguet, Ebran, Somà, EPJA 2022

Nuclear clustering & QRPA

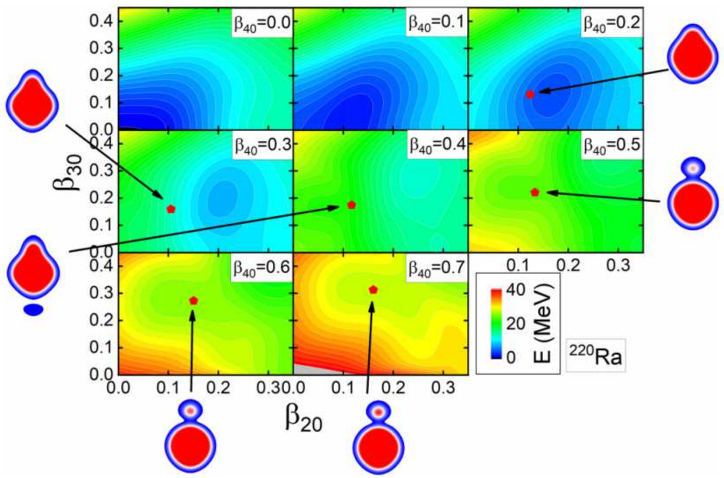
- Cluster vibration

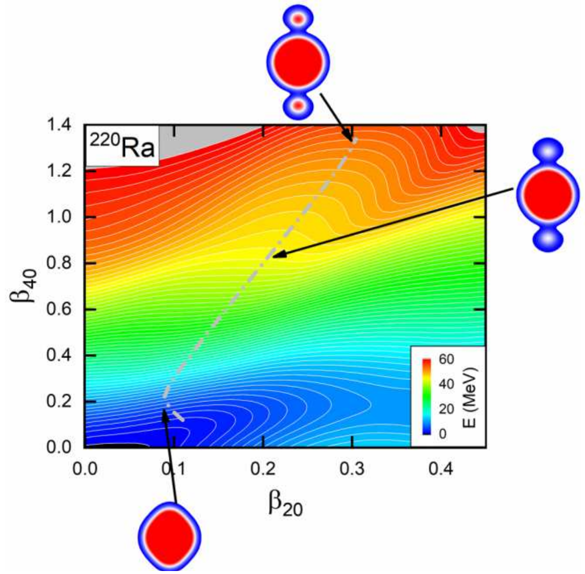


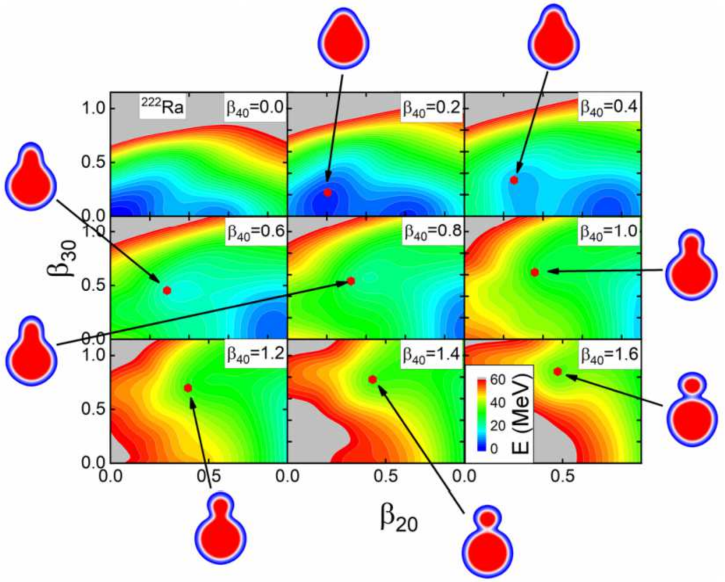
Mercier, Bjelčić, Nikšić, Ebran, Khan,Vretenar PRC 2021
Mercier, Ebran, Khan PRC 2022

4. Decay modes

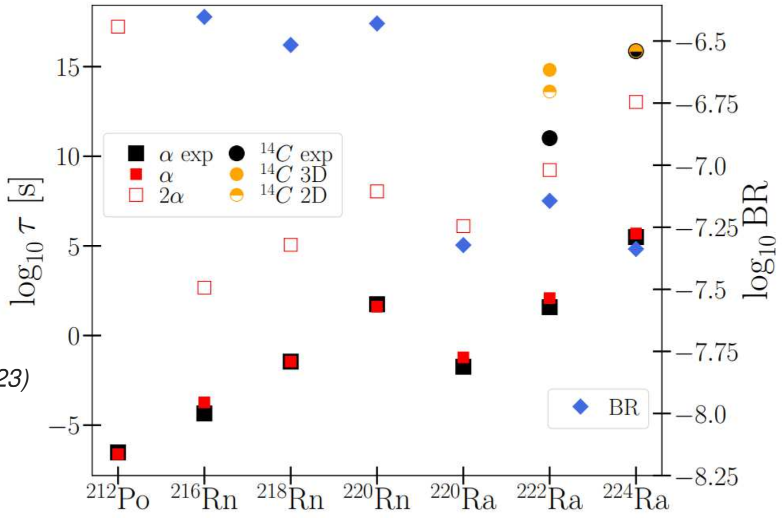
Cluster, α and 2α radioactivities







Zhao , Ebran, Heitz , Khan , Mercier, Nikšic,Vretenar, PRC (2023)

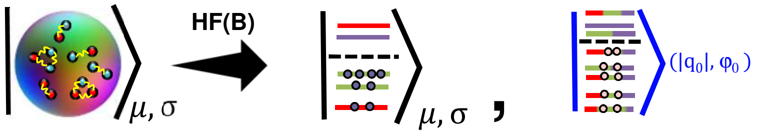


Thank you for your attention

The Energy Density Functional Method

◉ HFB treatment

--> *A*-nucleon problem ⟶ *A* 1-nucleon problems



Correlated A-nucleon wavefunction

Symmetry-conserving A independent nucleons wavefunction

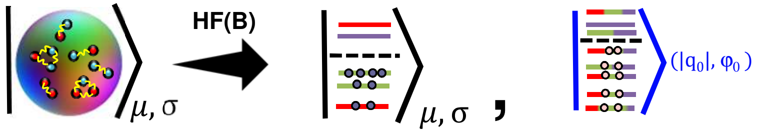
Symmetry-breaking A independent nucleons wavefunction

--> SSB : Efficient way for capturing so-called static correlations

The Energy Density Functional Method

◉ HFB treatment

--> *A*-nucleon problem ⟶ *A* 1-nucleon problems

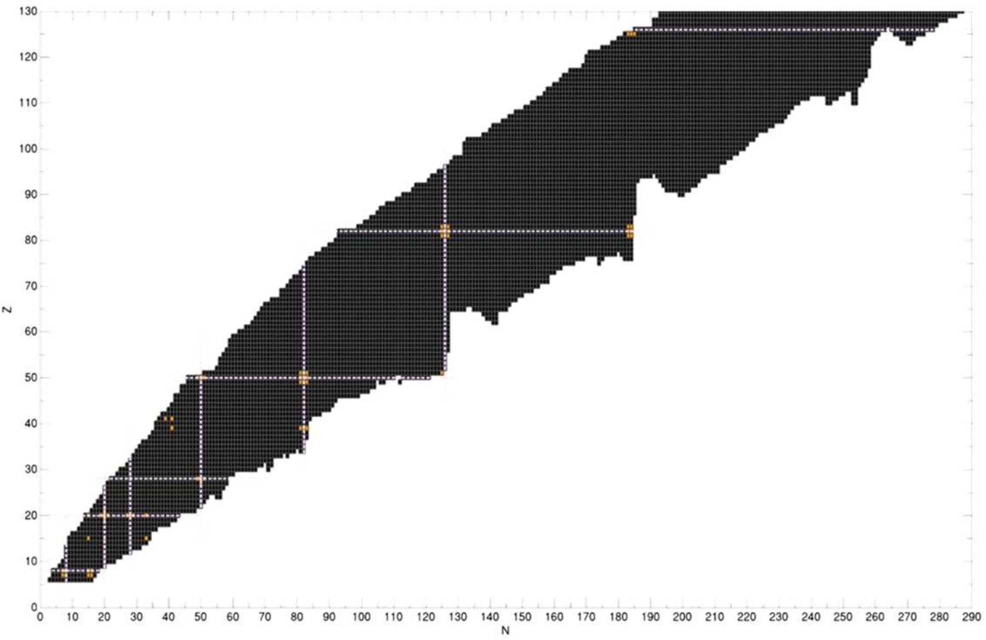


Correlated A-nucleon wavefunction

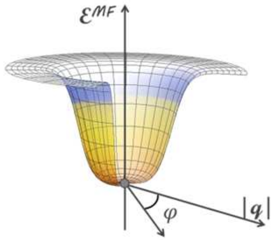
Symmetry-conserving A independent nucleons wavefunction

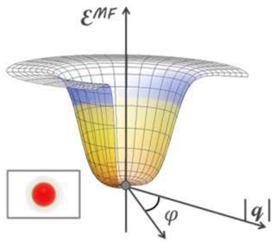
Symmetry-breaking A independent nucleons wavefunction

--> SSB : Efficient way for capturing so-called static correlations



Symmetry-restriced HF : good description of GS of doubly closed-shell nuclei & neighbors (~30 nuclei)

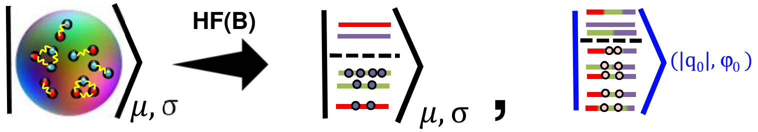




The Energy Density Functional Method

◉ HFB treatment

--> *A*-nucleon problem ⟶ *A* 1-nucleon problems

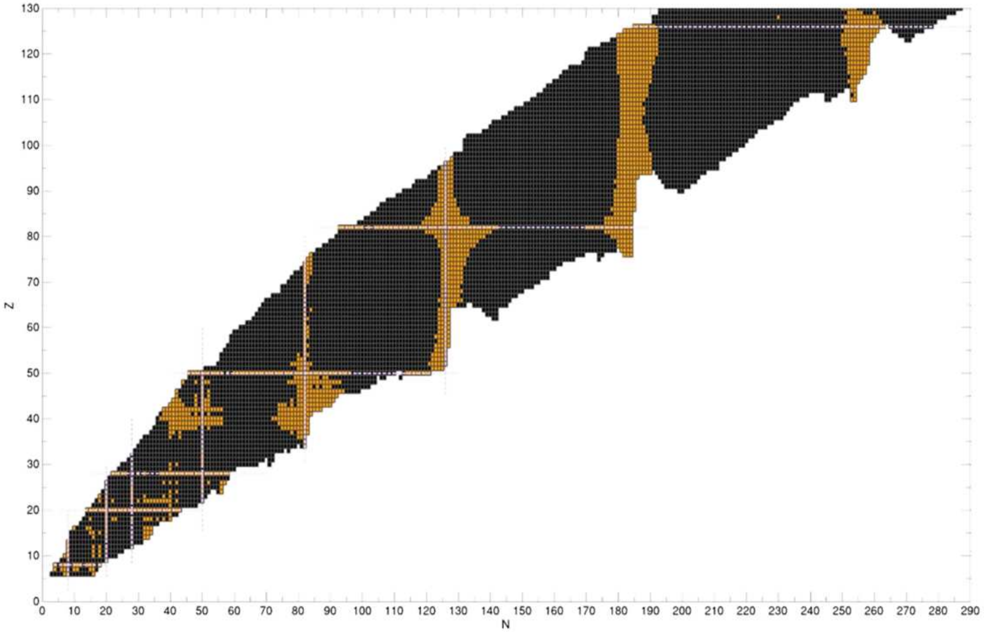


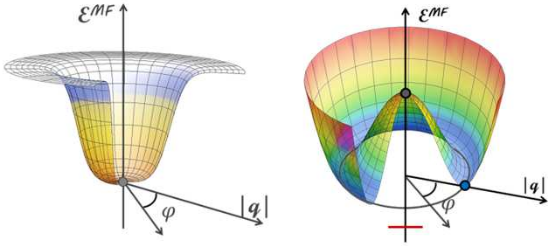
Correlated A-nucleon wavefunction

Symmetry-conserving A independent nucleons wavefunction

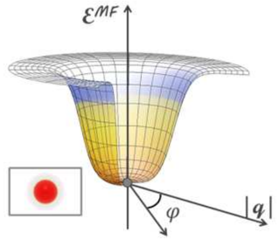
Symmetry-breaking A independent nucleons wavefunction

--> SSB : Efficient way for capturing so-called static correlations





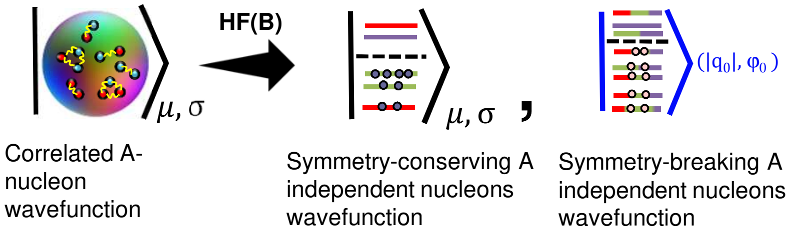
Spatial symmetry-restricted HFB: good description of GS of doubly and singly closed-shell nuclei & neighbors (~300 nuclei)



The Energy Density Functional Method

◉ HFB treatment

--> *A*-nucleon problem ⟶ *A* 1-nucleon problems

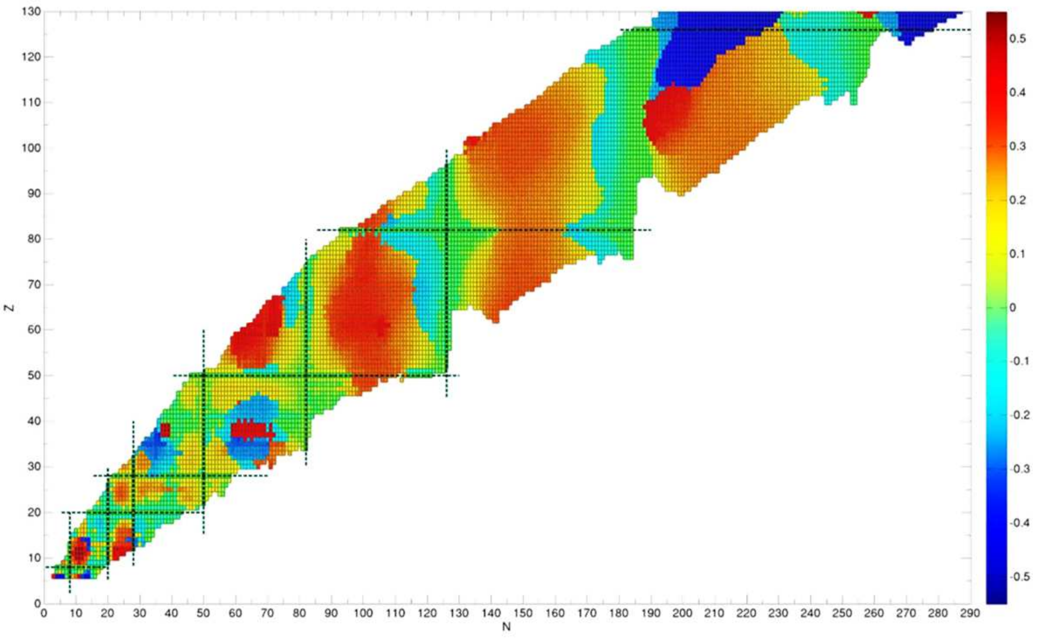


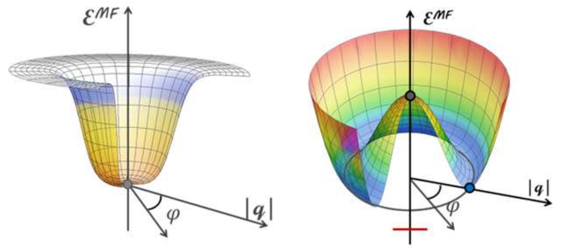
Correlated A-nucleon wavefunction

Symmetry-conserving A independent nucleons wavefunction

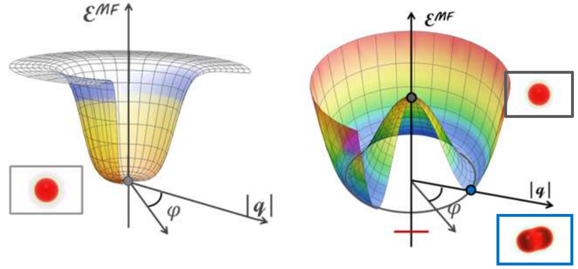
Symmetry-breaking A independent nucleons wavefunction

--> SSB : Efficient way for capturing so-called static correlations





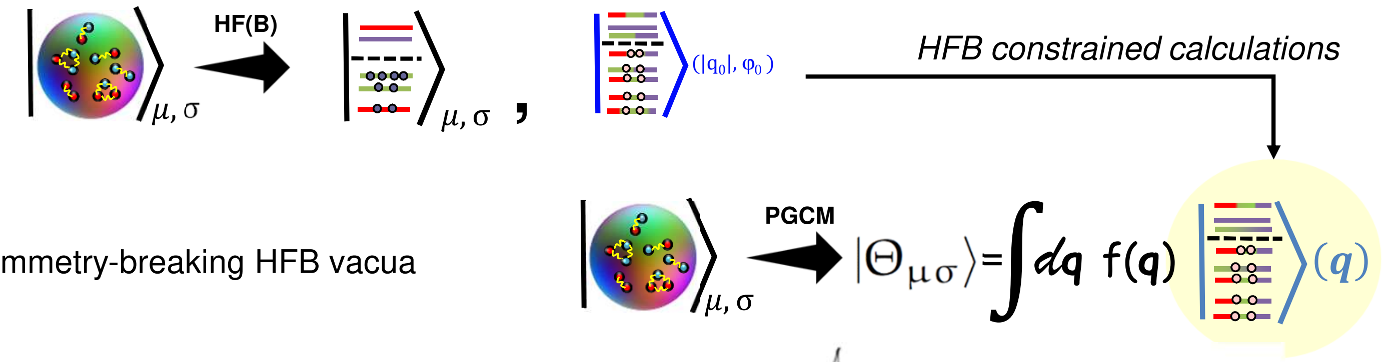
Symmetry-unrestricted HFB: good description of GS of all nuclei



The Energy Density Functional Method

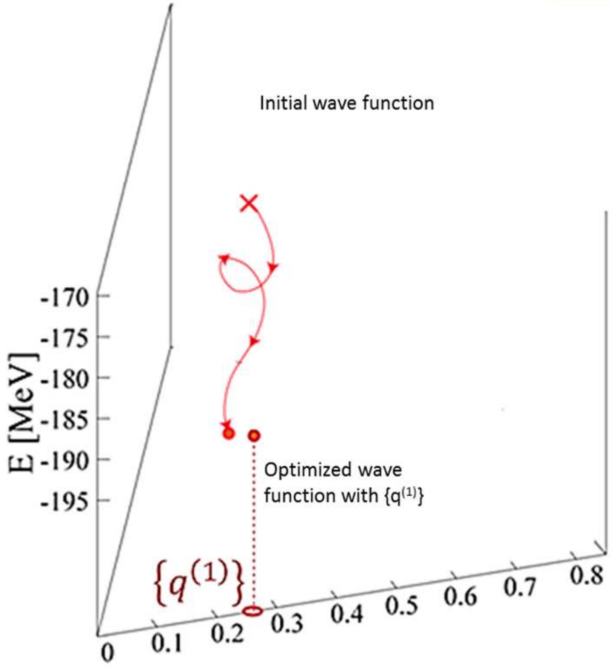
◉ HFB treatment

⇢ *A*-nucleon problem ⟶ *A* 1-nucleon problems



◉ Post-HFB treatment : PGCM

⇢ Symmetry-conserving (non orthogonal) mixture of symmetry-breaking HFB vacua



The Energy Density Functional Method

◉ HFB treatment

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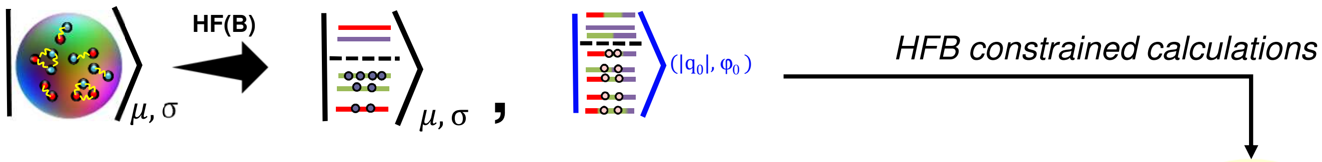

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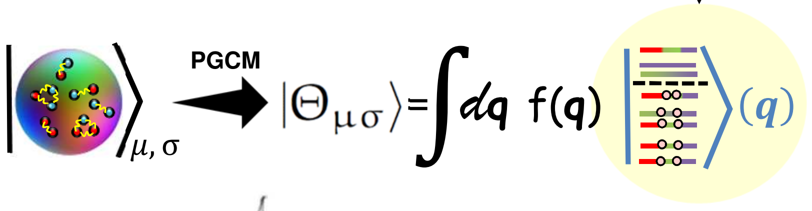

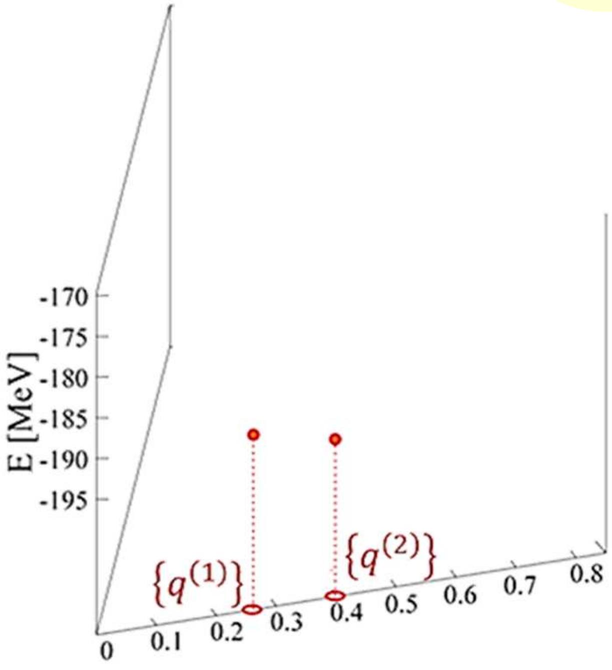

The Energy Density Functional Method

◉ HFB treatment

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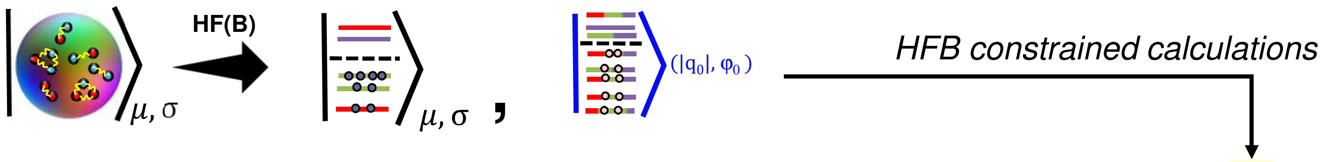

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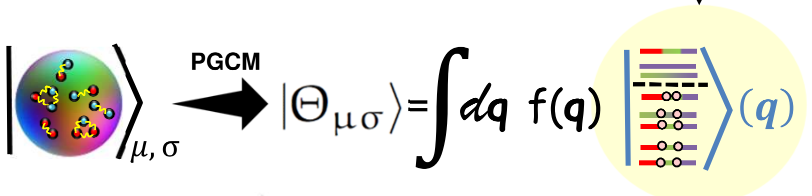

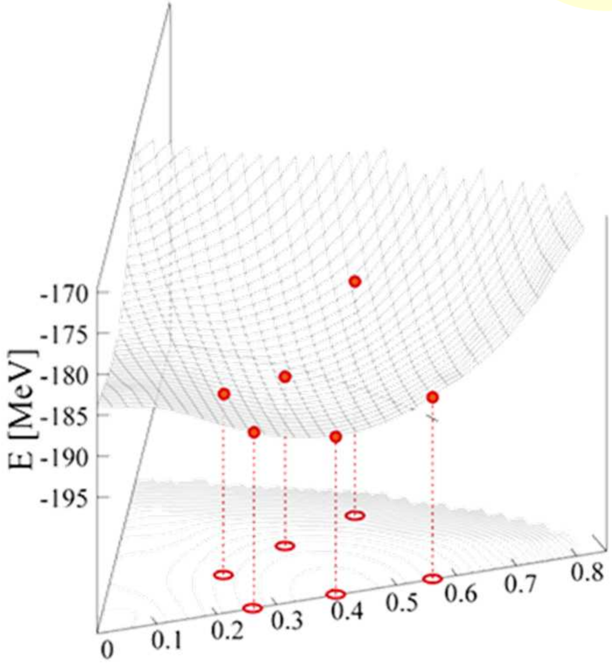

The Energy Density Functional Method

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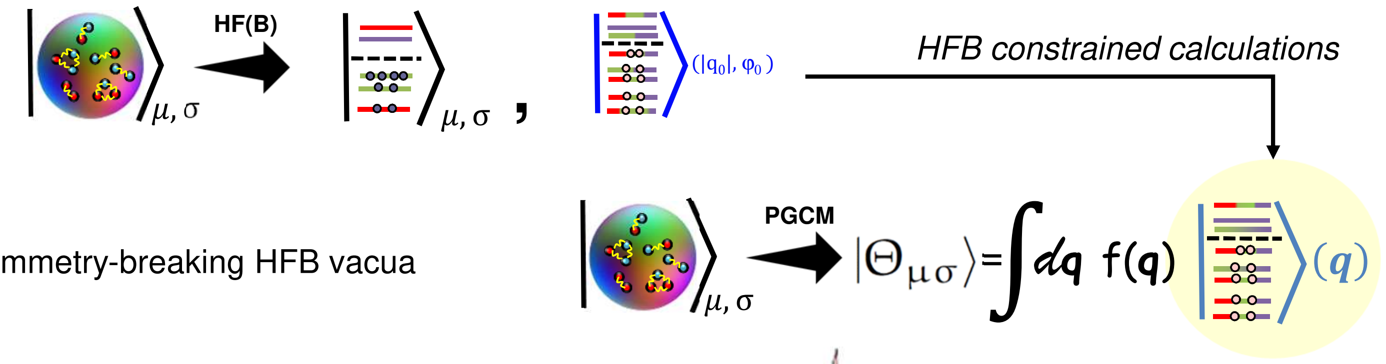

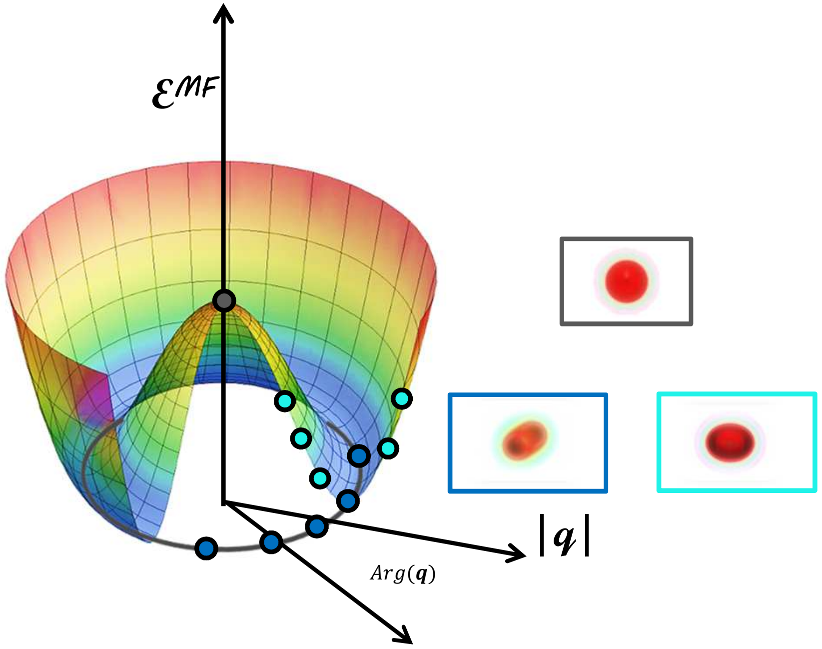

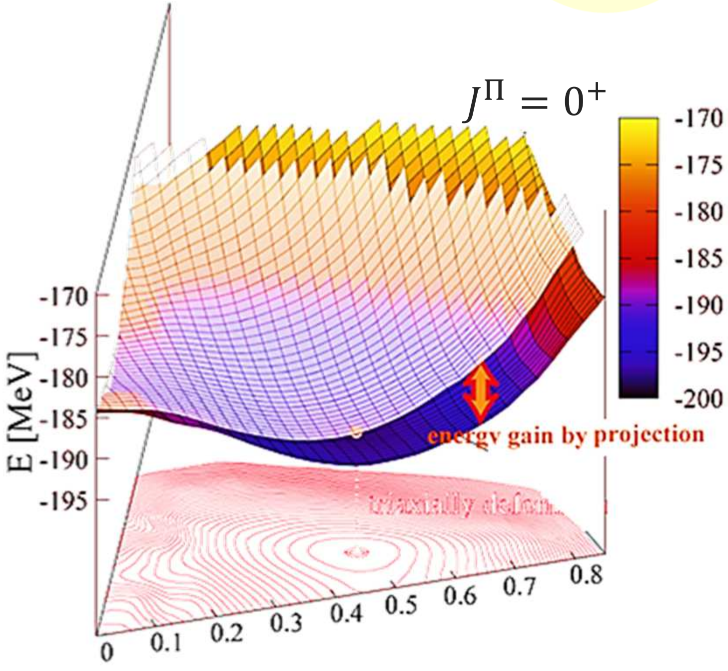

The Energy Density Functional Method

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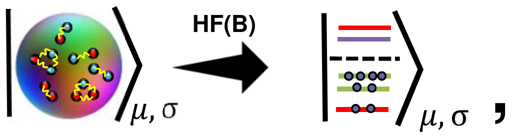
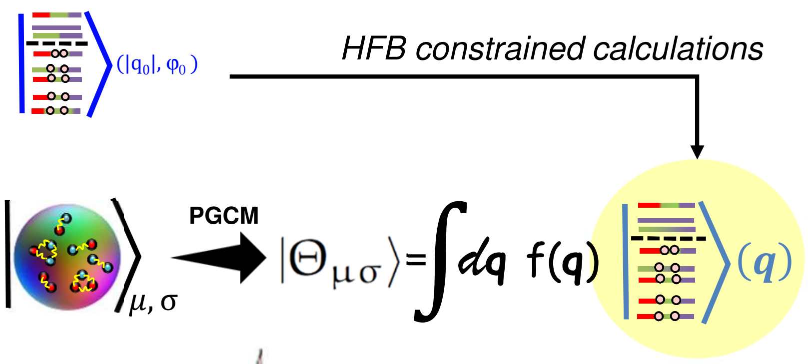
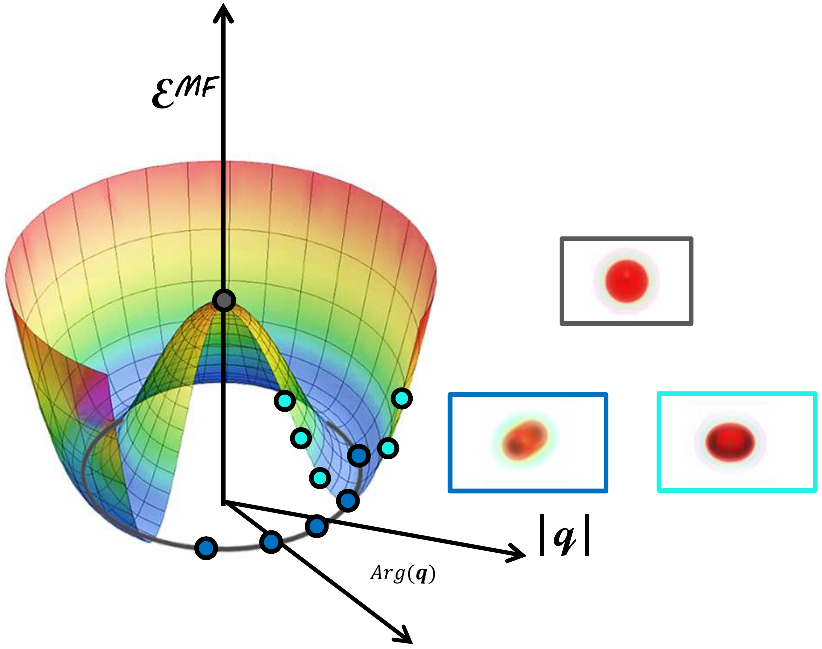
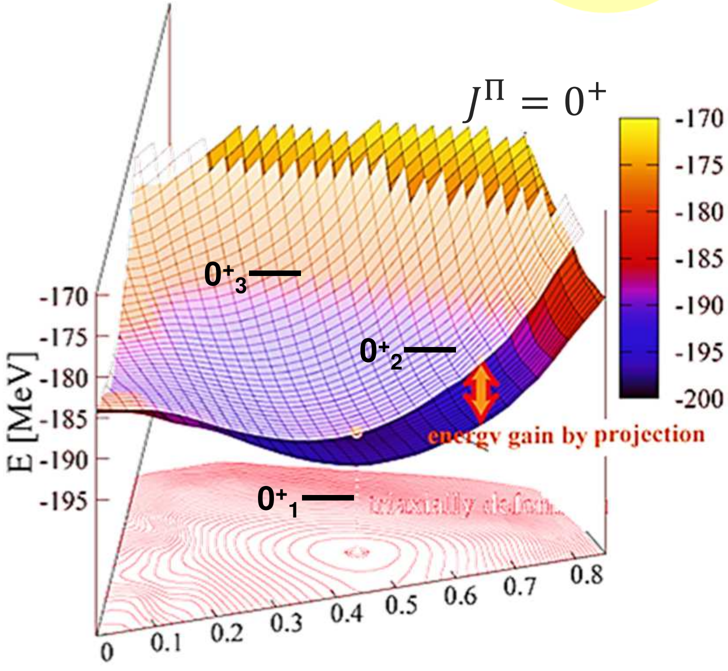




◉ Post-HFB treatment : PGCM

--> Symmetry-conserving (non orthogonal) mixture of symmetry-breaking HFB vacua





The Energy Density Functional Method

◉ HFB treatment

⇢ *A*-nucleon problem ⟶ *A* 1-nucleon problems

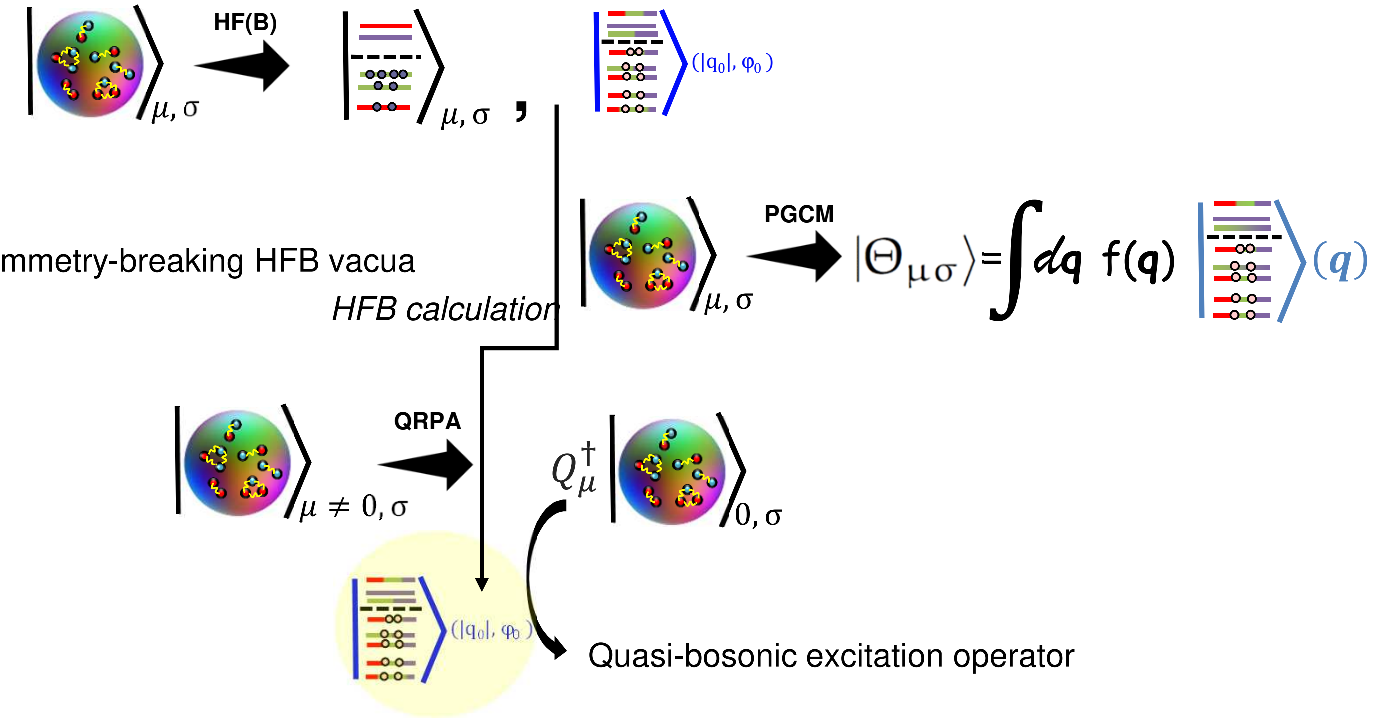
◉ Post-HFB treatment : PGCM

⇢ Symmetry-conserving (non orthogonal) mixture of symmetry-breaking HFB vacua

◉ Post-HFB : QRPA

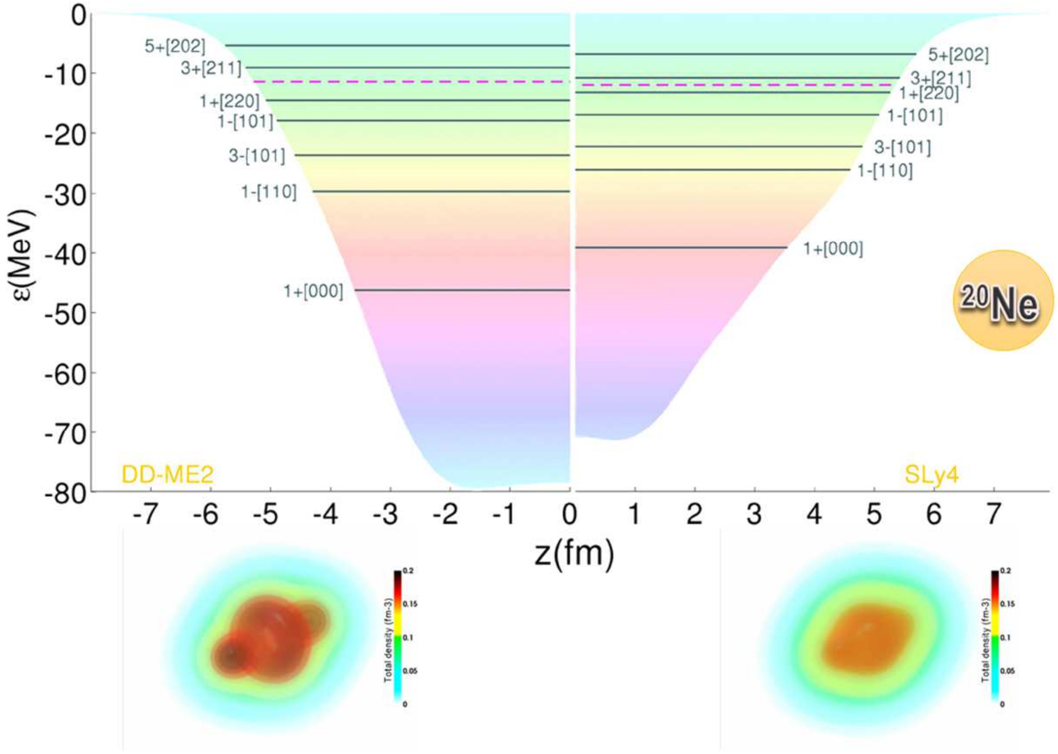
⇢ Excitations = coherent mixture of 2-qp excitations

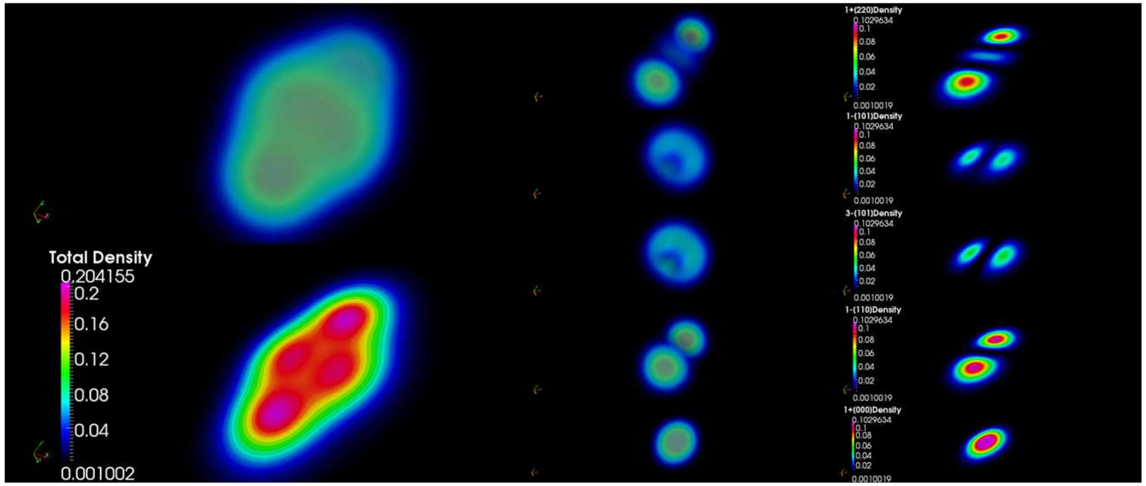
⇢ Harmonic limit of the GCM



Effect of the depth of the confining potential

- ⦿ Deeper potential yielding the same nuclear radii ⇒ more localized single-nucleon orbitals





- ⦿ When Coulomb effects are not too important and owing to Kramers degeneracy, proton ↑, proton ↓, neutron ↑, neutron ↓ share the same spatial properties