Semimicroscopic algebraic approach to quarteting and its relation to other models

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I. Defintions

II. Relations to other quartet models

III. Algebraic formalism

IV. Relation to shell, cluster, collective mod: MUSY

V. Features of MUSY

VI. Applications

VII. Summary and conclusions

I. Definitions (SAQM)

Quartet: 2p+2n shell configuration

with quartet symmetry

light: Ust(4) [1,1,1,1], S(4): [4]

heavy: S(4): [4]

Note: different major shells are allowed.

Algebraic U(3): both the basis and the operators.

Semimicroscopic: microsc. model space (no-core)

+ phenomenological oprators

II. Relation to other quartet models

1. 67-73: Danos, Gillet, Arima, Ginocchio, Satpathy

   Similar: 2p+2n shell configuration

   Different: spectrum from empirical data.

2. M. Harvey NPA 202, 191

   Similar: Quartet symmetry

   Different: spectrum from empirical data.

3. 80’s: Algebraic: Iachello, Jackson, Dukelsky…

   Similar: algebraic spectrumgeneration

   Different: no micriscopic quartet,

   U(6)

4. Quartet condensate ’83 Gambhir, Ring, Schuck

Similar: 2p+2n

Different: condansate

5. Recent (N. Sandulescu, M. Sambataro…)

Similar: quartet degrees of freedom

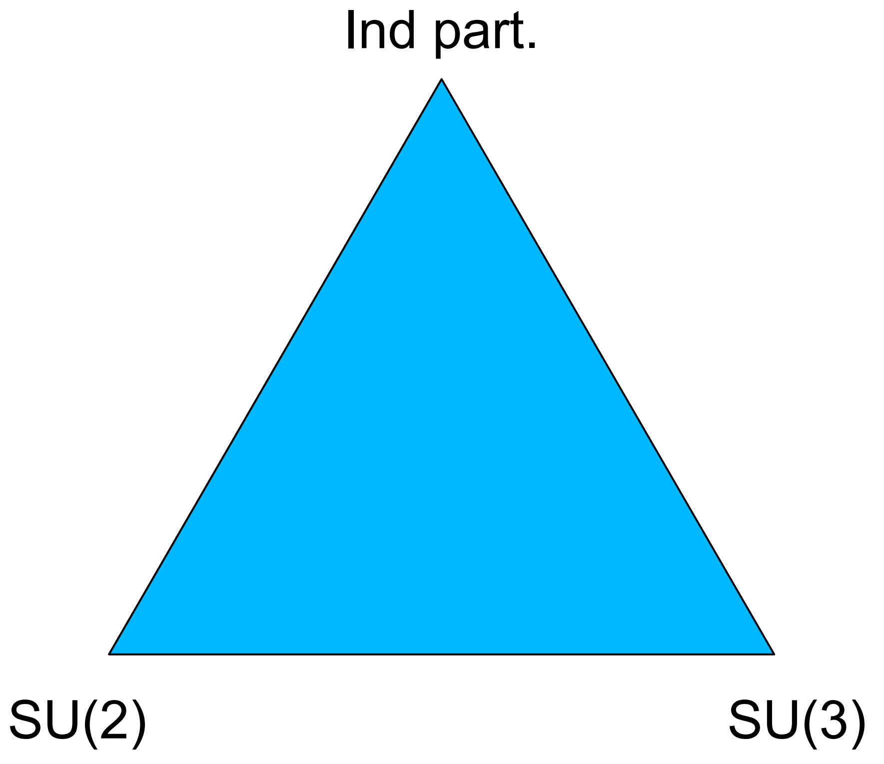
Different: condensate

single shell

formalism

group: U(6)

Shell Model



III. Algebraic formalism

III.1. Elliott-model

J.P. Elliott, Proc. R. Soc. 245 (1958) 128, 562.

Single major shell

Bilinear products of creation and annihil operators of oscillator quanta.

$$\begin{array}{cccc} & U(3) \supset SU(3) \supset SO(3) & & \\ |\,[n_1, n_2, n_3], & (\lambda, \mu) & K & L\rangle \end{array}$$

$$H = C^{(1)}_{U3} + aC^{(2)}_{SU3} + aC^{(2)}_{SO3}$$

Dynamical symmetry: basis + analytical solution.

III.2. Connection to collective and cluster mod.1958

Elliott: *Proc. Roy. Soc. A 245, 28,562 (1958)*

Spectra of light nuclei,
deformation + rotation from spherical shell model:
SU(3).

From shell model to cluster model:
Wildermuth-Kanellopoulos: *Nucl. Phys. 7, 150 (1958)*
Harm. osc. appr.

Cluster-shell connection: SU(3).
Bayman-Bohr: *Nucl. Phys. 9, 596 (1958/59).*

A quadrupole collective or a cluster band is picked up from the spherical shell model basis by their special SU(3) symmetry.

For a single major-shell problem the connection between the shell, collective and cluster models is provided by an SU(3) dynamical symmetry:

$$U(3) \supset SU(3) \supset SO(3)\,.$$

IV. Relation to shell, cluster and collective models: MUSY
Multiconfigurational dynamical symmetry

Extension of the U(3) connection from 1958
(J. Cseh, Phys. Rev. C 103 (2021) 064322.)

$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

Intersection of the
-(No-core) Symplectic shell model
-Contracted symplectic collective model
-Microscopic and semimicroscopic cluster models

Symplectic shell model

G. Rosensteel, D. Rowe, PRL 38 (1977) 10

Extension of Elliott, microscopic coll. model

$$\mathrm{Sp(6,R)} \supset \mathrm{U(3)} \supset \mathrm{SU(3)} \supset \mathrm{SO(3)}$$

$$| [n_1^s, n_2^s, n_3^s], [n_1^e, n_2^e, n_3^e], \rho, [n_1, n_2, n_3], (\lambda, \mu), \quad K, \quad L \rangle$$

Multi-shell extension of the Elliott model, microscopic version of the collective model.

Symmetry-adapted no-core shell model

T. Dytrych et al. J. Phys. G 35 (2008) 123101

Symmetry-adapted (no-core) quartet model

J. Cseh, Phys. Lett. B **743**, 213 (2015).

Contracted symplectic model

(D.J. Rowe, G. Rosensteel, Phys. Rev. C 25 (1982) 3236(R);

O. Castanos, J. P. Draayer, Nucl. Phys. A 491 (1989) 349.)

$$U_s(3) \otimes U_e(6) \supset U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

Simpler mathematical structure,
bosonized description,
algebraic collective model of the multi-shell problem.

Cluster model

Microscopic or semimicroscopic:
antisymmetrization.

Semimicroscopic algebraic cluster model
J. Cseh, G. Lévai, Ann. Phys. 230 (1994) 165.

$$U_{C_1}(3) \otimes U_{C_2}(3) \otimes U_R(4) \supset$$
$$U_c(3) \otimes U_R(3) \supset U(3) \supset SU(3) \supset SO(3)$$

Microscopic model space, algebraic operators.
Internal cluster structure: Elliott model,
relative motion: modified vibron.

Shell (quartet), cluster and collective states: representation labels of an algebra-chain

(J. Cseh, Phys. Rev. C 103 (2021) 064322.)

$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

Spin-isospin sector:
in symplectic
contarcted symplectic
semimicroscopic algebraic cluster models

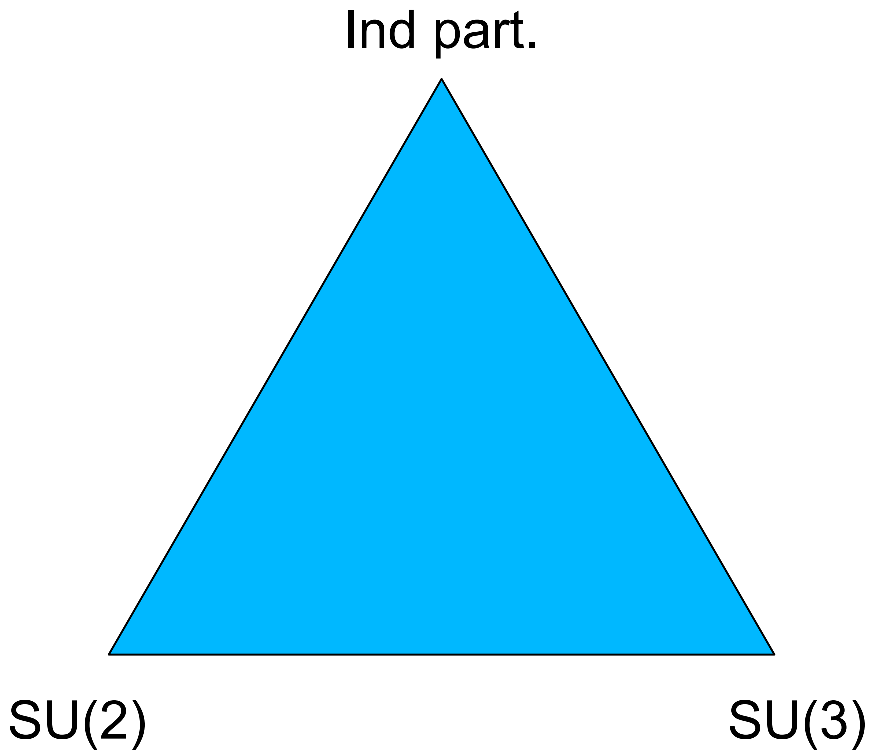
$$U^{ST}(4) \supset U^{S}(2) \otimes U^{T}(2)$$

Relation of the SAQM to the shell model:
it is a symmetry-governed truncation of the no-core shell model

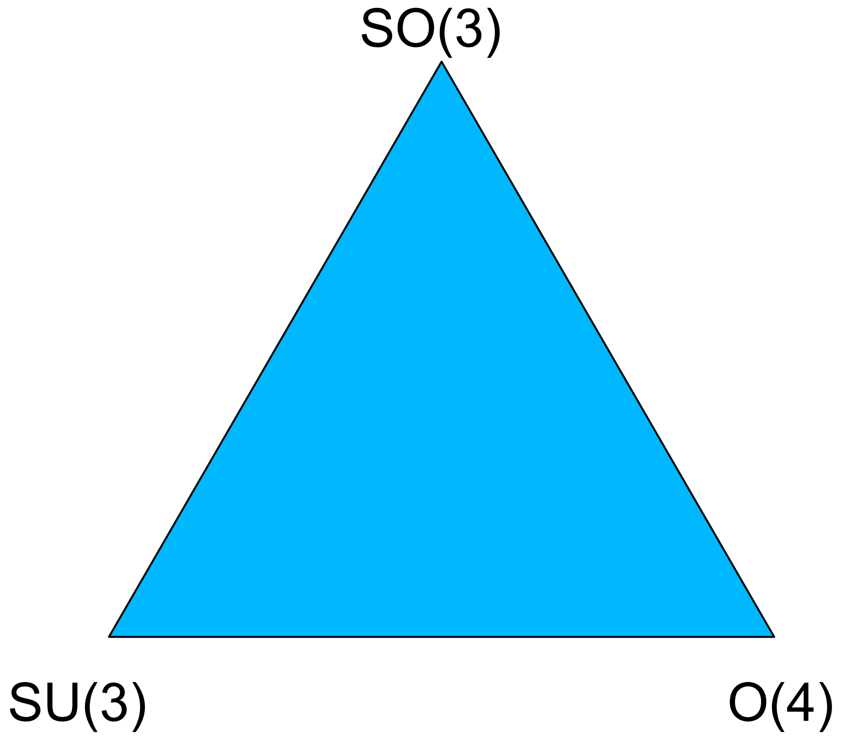
Relation of the cluster and collective model:
MUSY is the common intersection of them

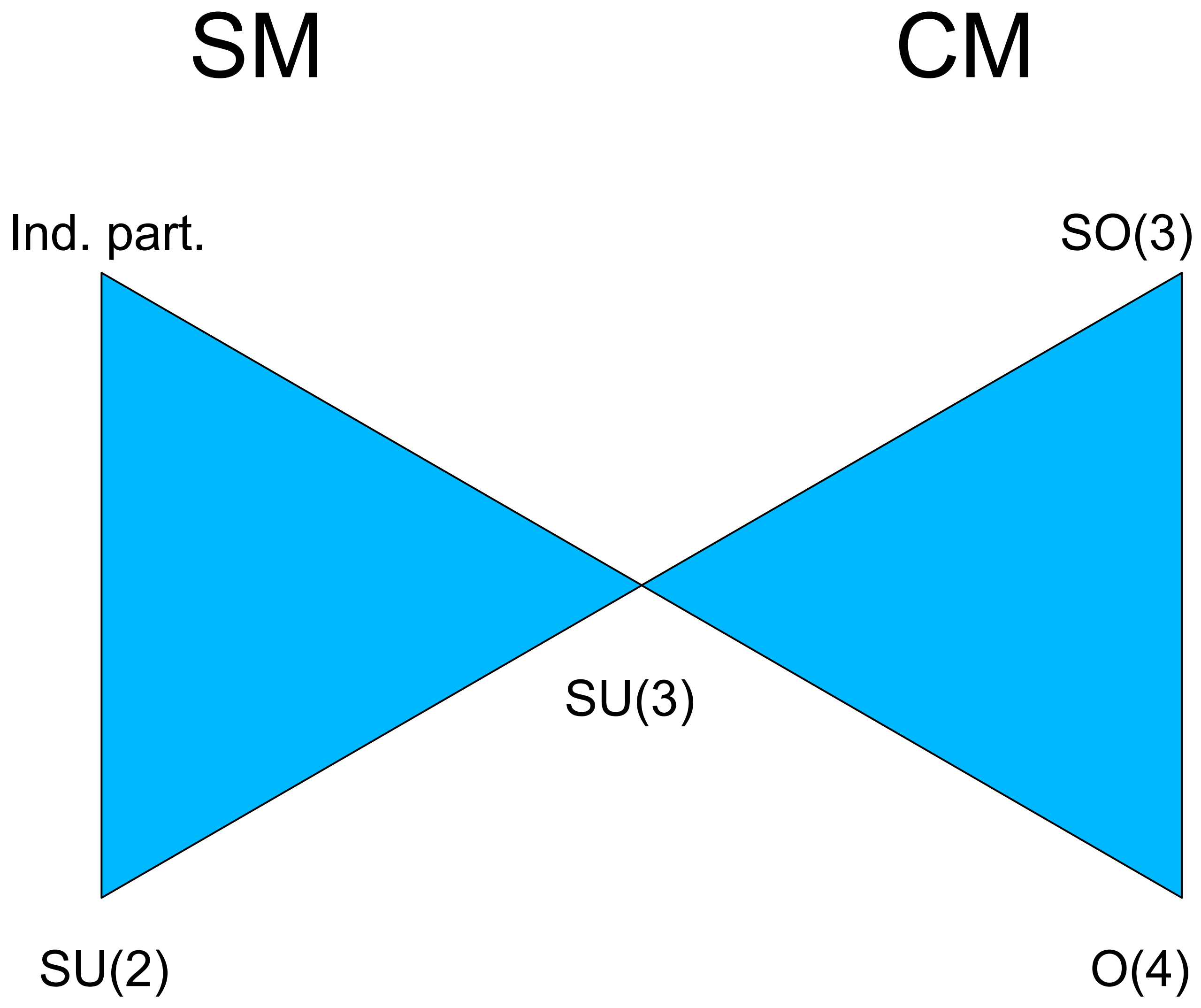
$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

Shell Model



Cluster Model




SM
CM
Ind. part.
SO(3)
SU(3)
SU(2)
O(4)

IV. Features of MUSY

A) Composite symmetry of a composite sytem:

1. U(3) dysy in each configuration,

$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

2. symmetry transforming the configurations
   (in the pseudo space of particle indeces)
   identical spectra
   invariant or transformed H-operator

Similar to SUSY models.

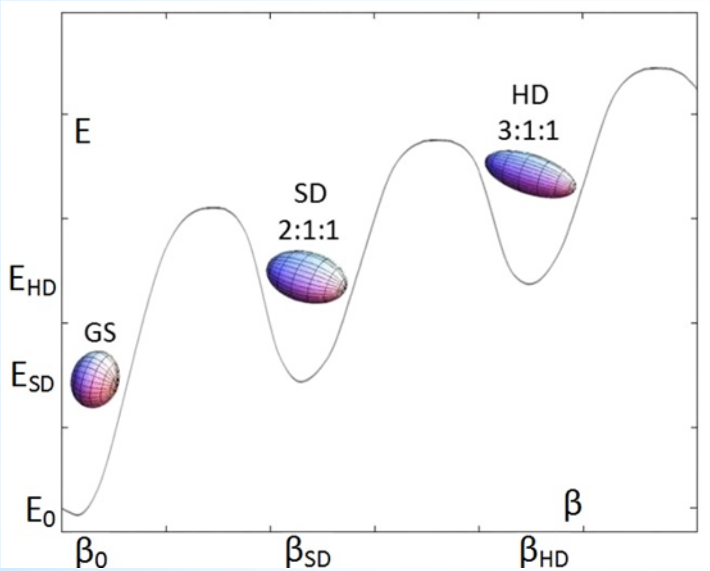
B) Can be realised with invariant and non-invariant interactions.

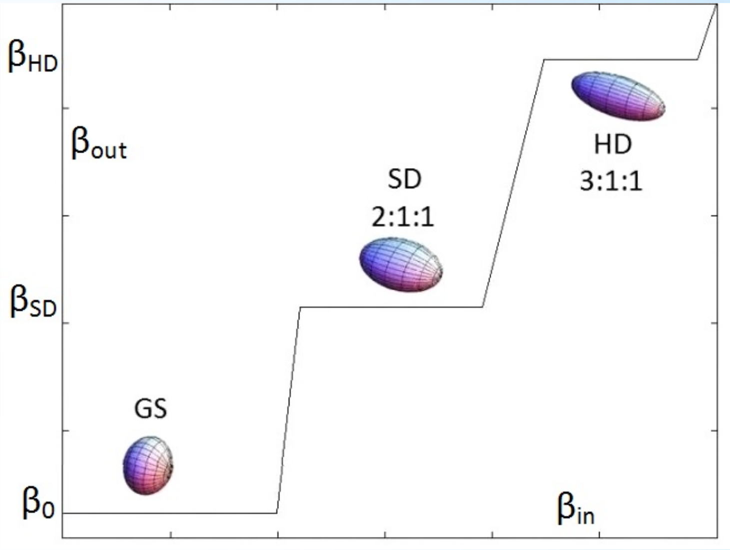
C) Dual (dynamic SU3) and spont. (SO3) breaking.

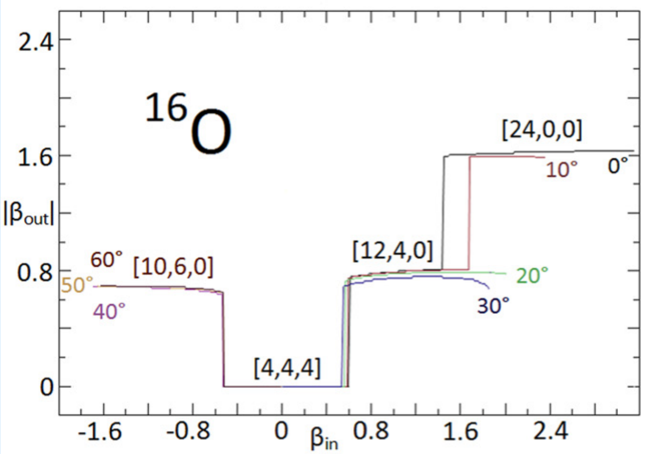
D) Shape isomers: stability and selfconsist. of SU3 emerging symmetry.

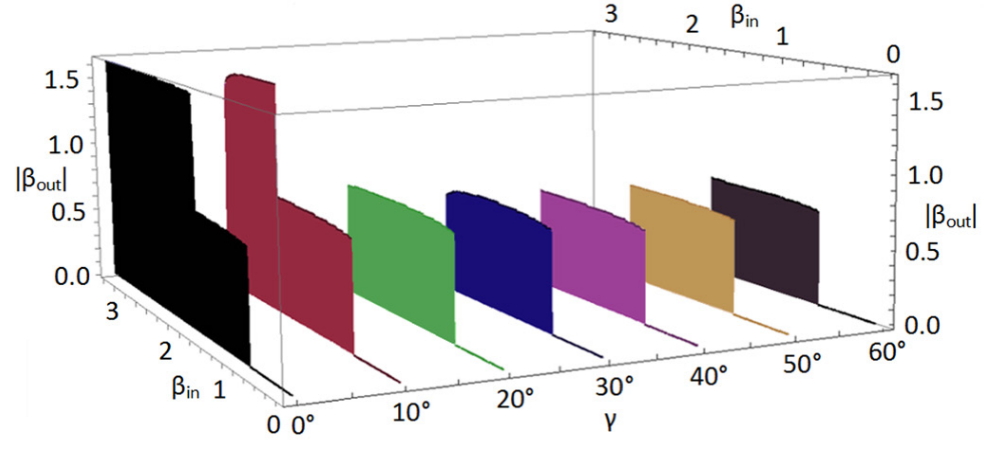
V. Applications

A) Shape isomers from the stability and selfconsistency of the (quasidynamiclal) U(3)

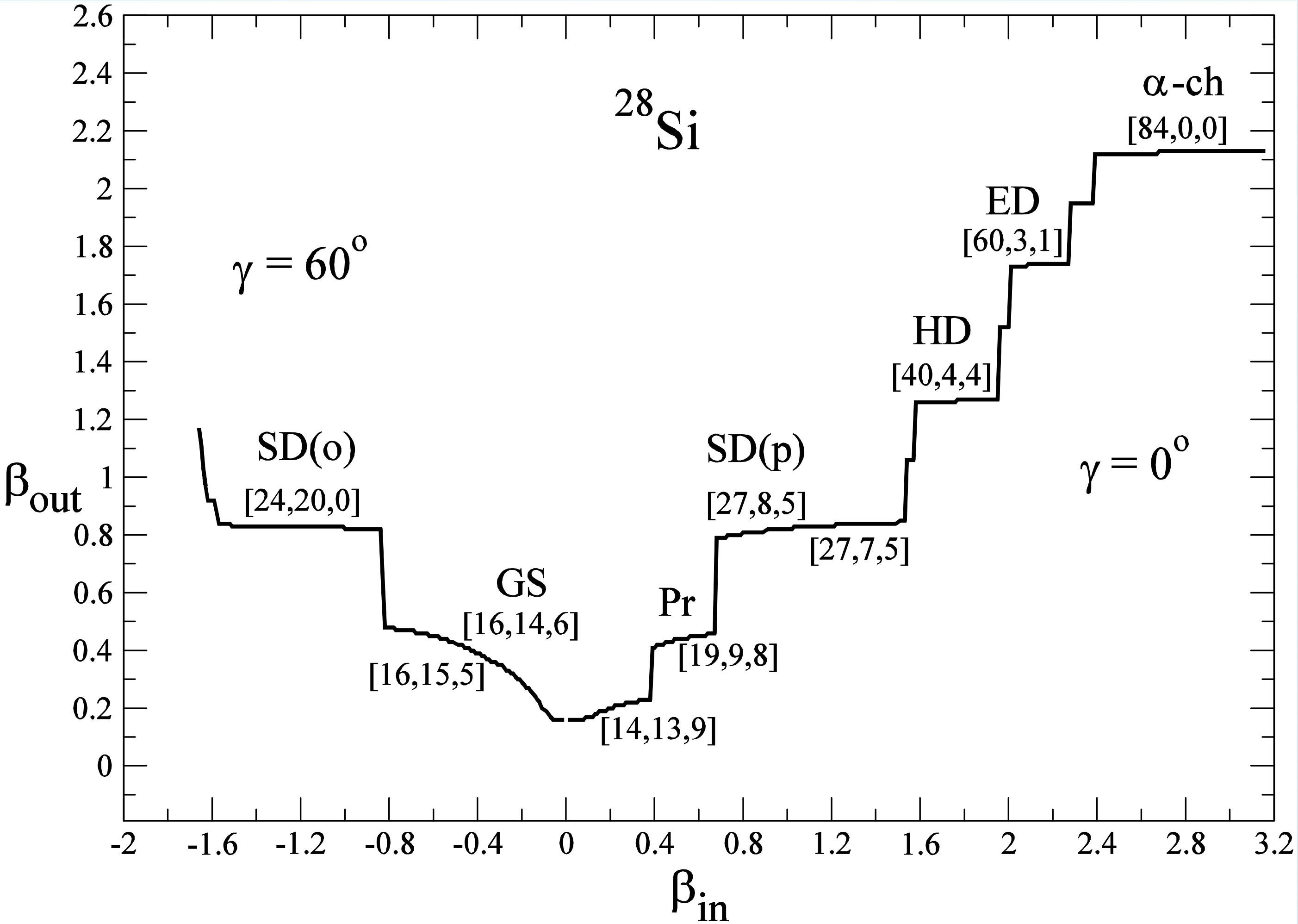




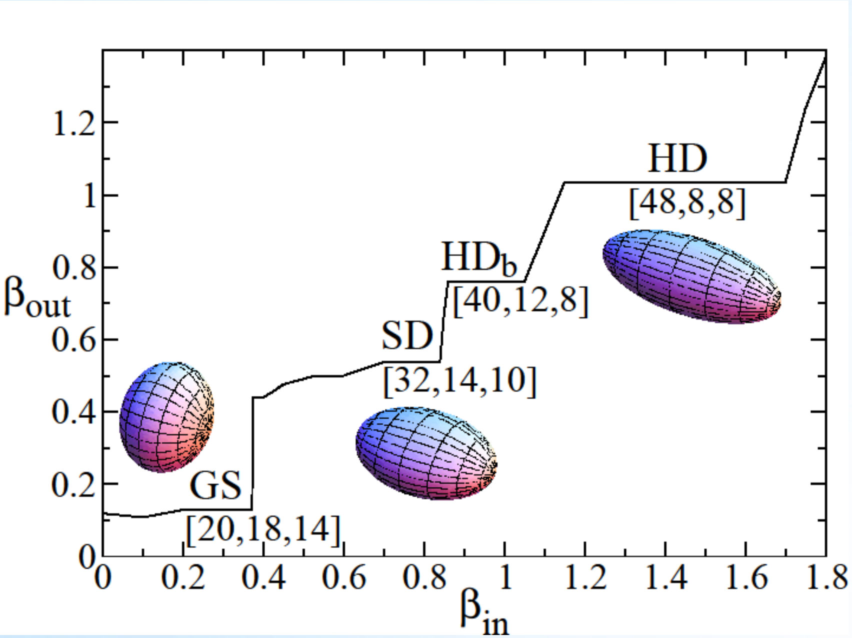


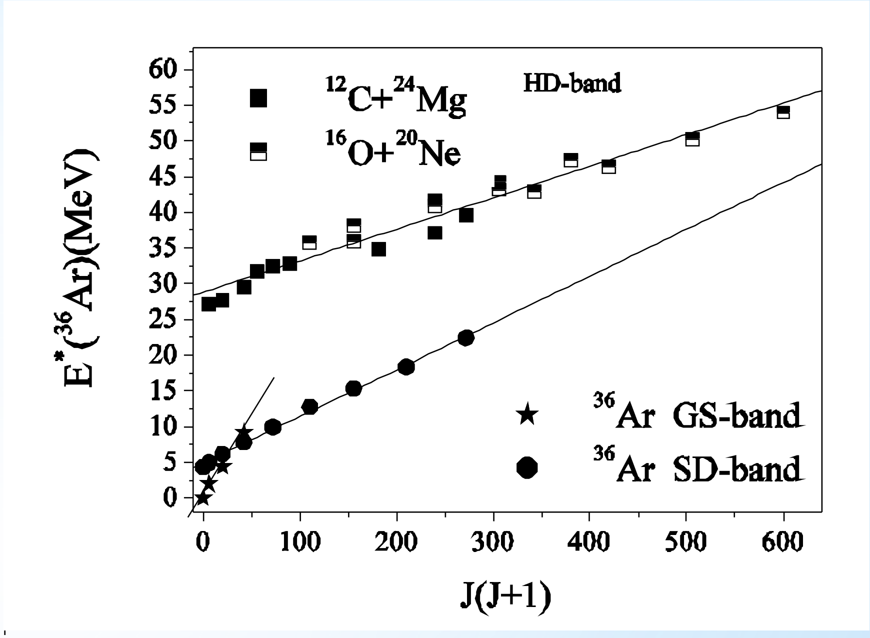


J. Cseh, G. Riczu, J. Darai, Phys. Lett. B 795 (2019) 160.



J. Darai, J. Cseh, D. Jenkins, Phys. Rev. C 86 (2012) 064309
D. Jenkins et al. Phys. Rev. C 86 (2012) 064308



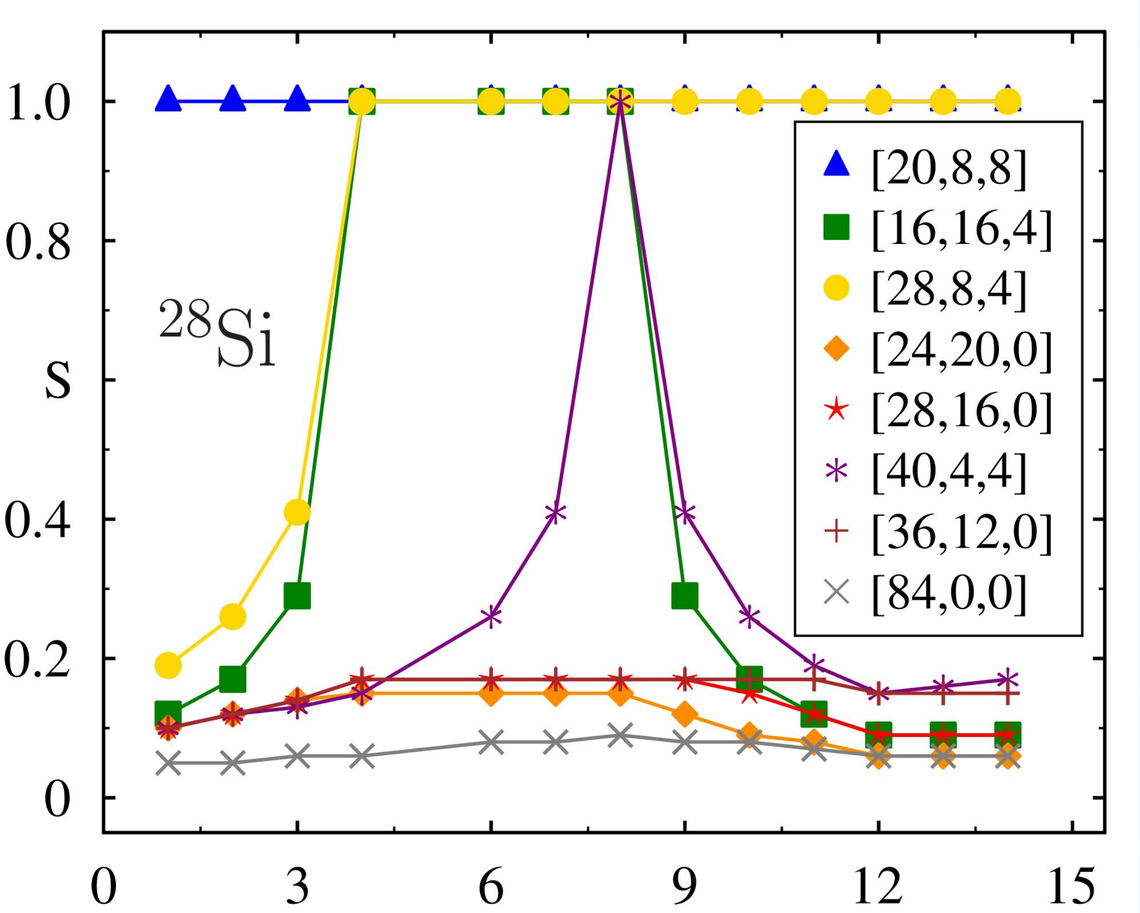


W. Sciani, Y. Otani, A. Lépine-Szily, et al, Phys. Rev. C 80 (2009) 034319
J. Cseh, J. Darai,et al. Phys Rev. C 80 (2009) 034320

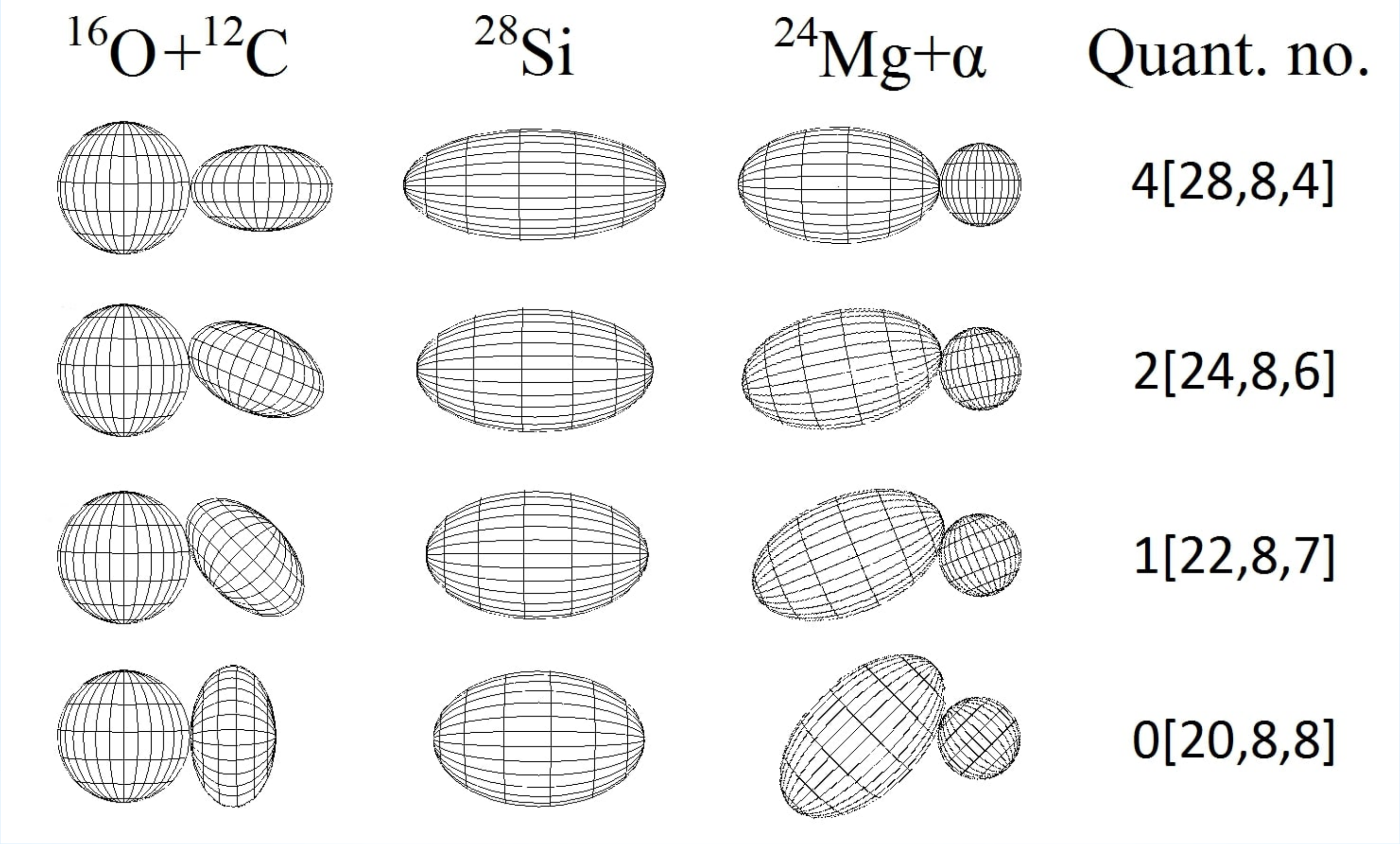
Applications C)

Allowed and forbidden cluster configurations
(reaction channels)

Cluster-shell duality



1.0
0.8
0.4
0.2
0
S
^{28}Si
0
3
6
9
12
15
[20,8,8]
[16,16,4]
[28,8,4]
[24,20,0]
[28,16,0]
[40,4,4]
[36,12,0]
[84,0,0]

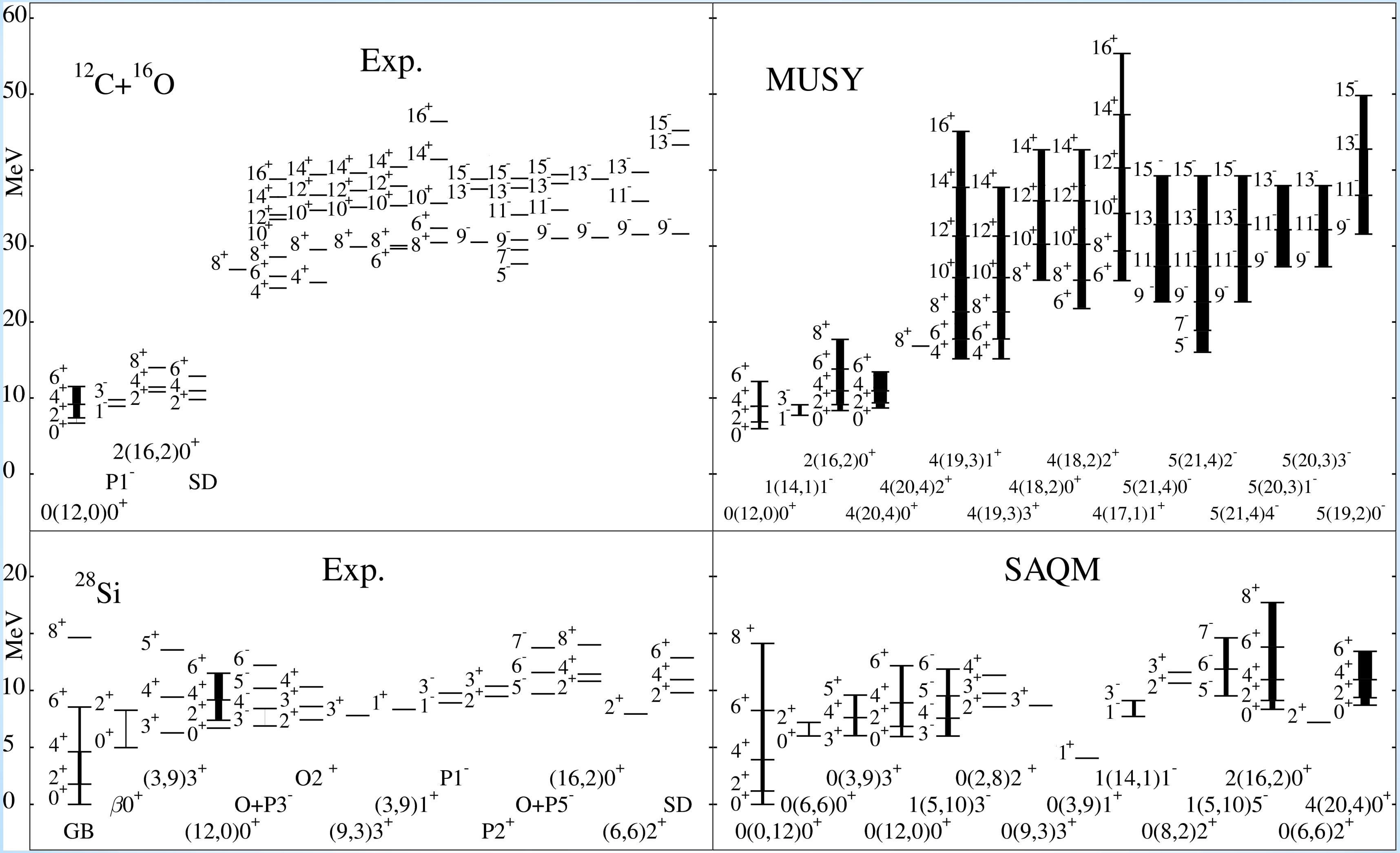


$^{16}O+^{12}C$
^{28}Si
$^{24}Mg+\alpha$
Quant. no.
4[28,8,4]
2[24,8,6]
1[22,8,7]
0[20,8,8]

Applications D)

Unified description of spectra of different configurations in different regions of energy and deformation (experimental spectra, shape isomers).

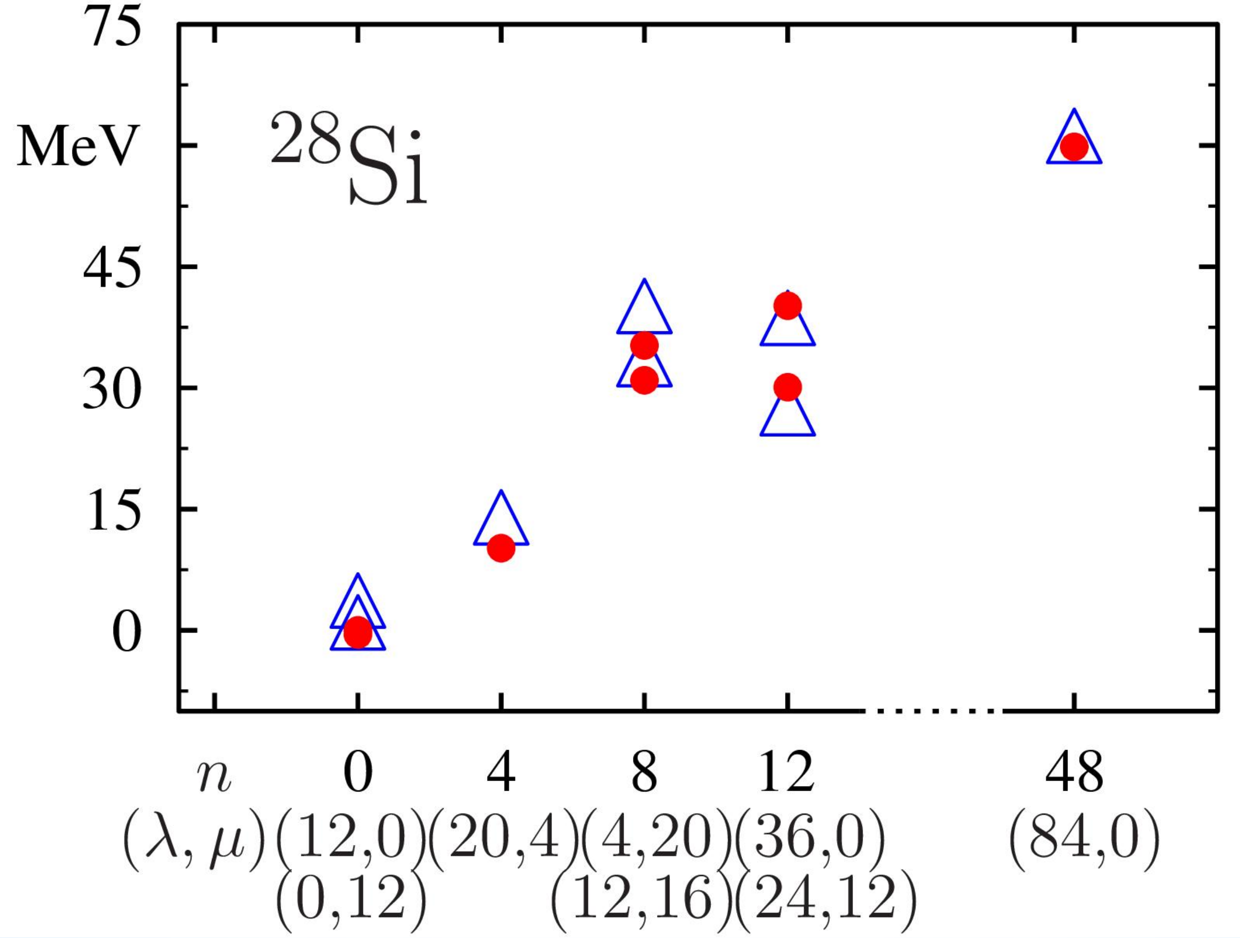
$$H = \epsilon n + \alpha C^2_{SU3} + \beta C^3_{SU3} + \frac{1}{2\theta} L^2$$



J. Cseh, G Riczu, Phys. Lett B **757**, 312 (2016)

Shape isomers of N=Z nuclei
PRC **107**, 044315 (2023)

Stable deformations
Possible clusterizations
Cluster-shell duality
Energy spectra


75
MeV
^{28}Si
45
30
15
0
n
0
4
8
12
48
(λ, μ)
(12,0)
(0,12)
(20,4)
(4,20)
(12,16)
(36,0)
(24,12)
(84,0)

V. Summary

SAQM:
2p+2n shell config. q-symm: [4],
multi-shell space,
algebraic U(3) formalism.

Friendly relations to other quartet models.

Well-defined connection to shell, cluster and collective models: MUSY.

$$U_s(3) \otimes U_e(3) \supset U(3) \supset SU(3) \supset SO(3)$$

dynamical symmetry in each configuration (SAQM central pillar).

+ transformations between them.

Comparison

	NS-MS	SAQM
quartet	4 nucl.	2p+2n
configuration	condensate	individual
symmetry	T=0	[1,1,1,1],[4]
mod. space	single shell	multi shell
formalism	coll.pair.approx	SA-NCSM
algebra	(U(6))	U(3)
dominent interact.	pairing	q.q

exact location on the shell model phase space?

Thank you for your attention!