How to reveal alphas in a nuclear state ?

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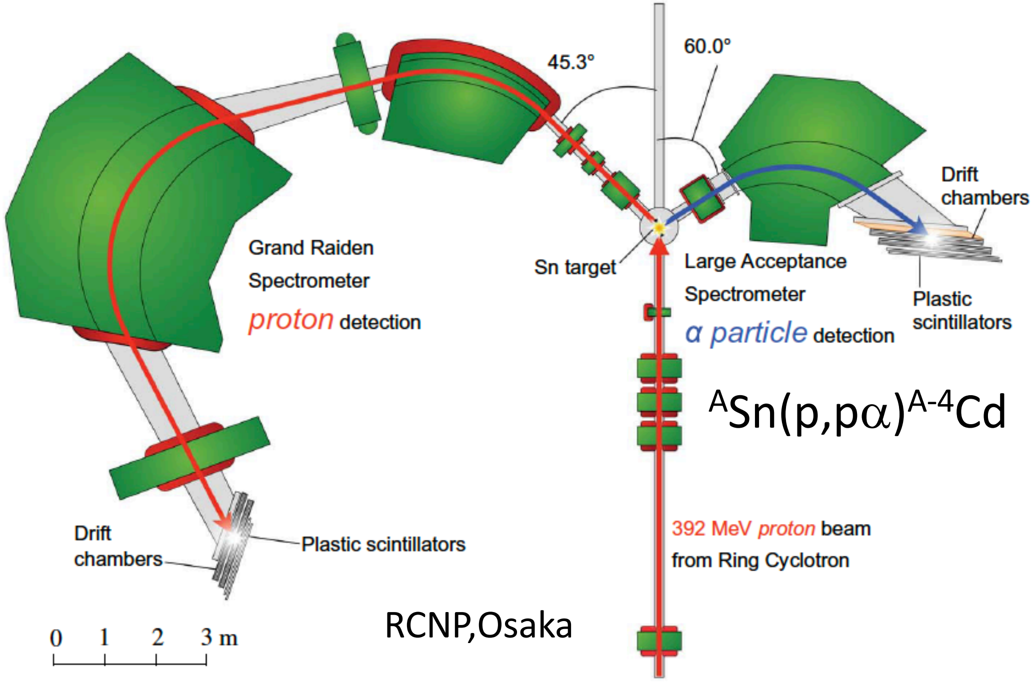
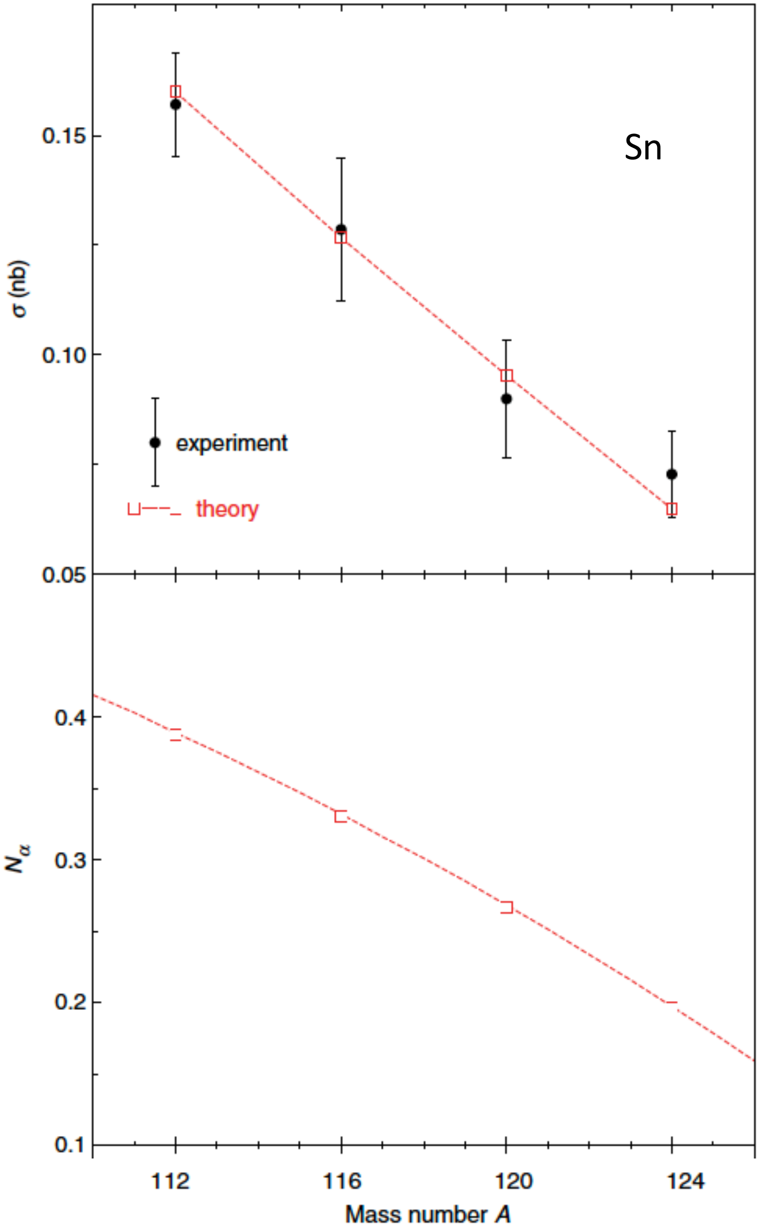
Experimental indication



$^{A}Sn(p,p\alpha)^{A-4}Cd$

RCNP,Osaka

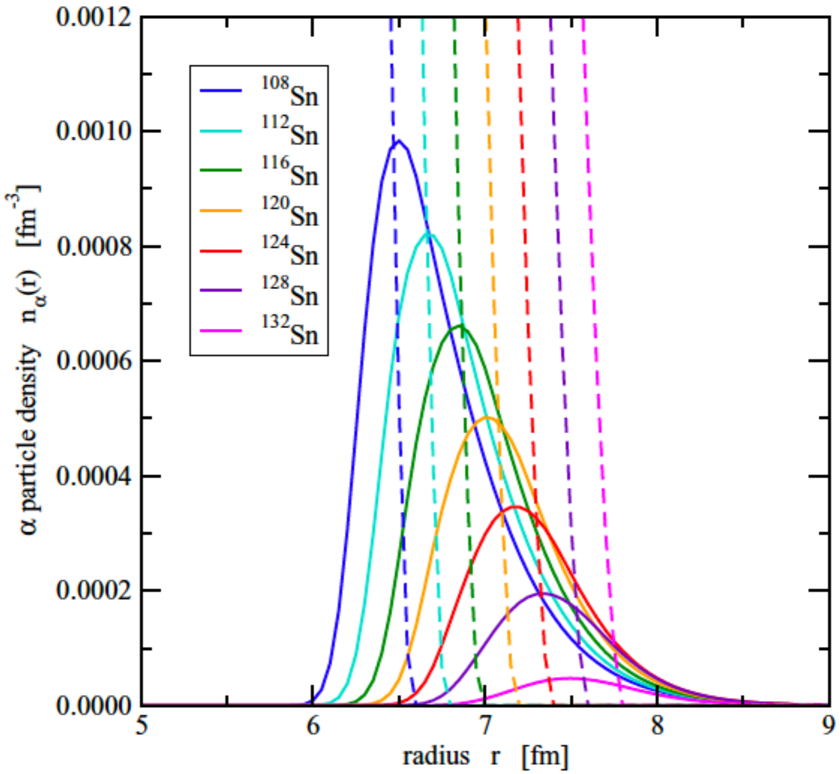


J. Tanaka et al., Science 371(2021)260

Interpretation

$$\begin{aligned}\mathcal{L} = \sum_{i=n,p,t,h} & \bar{\psi}_i\left(\gamma_\mu i D_i^\mu - M_i\right)\psi_i + \frac{1}{2}\left(i D_\alpha^\mu \varphi_\alpha\right)^* \\ & \times \left(i D_{\alpha\mu}\varphi_\alpha\right) - \frac{1}{2}\varphi_\alpha^* M_\alpha^2 \varphi_\alpha + \frac{1}{4}\left(i D_d^\mu \varphi_d^\nu - i D_d^\nu \varphi_d^\mu\right)^* \\ & \times \left(i D_{d\mu}\varphi_{d\nu} - i D_{d\nu}\varphi_{d\mu}\right) - \frac{1}{2}\varphi_d^{\mu *} M_d^2 \varphi_{d\mu} \\ & + \frac{1}{2}\left(\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2 - \frac{1}{2} G^{\mu\nu} G_{\mu\nu} + m_\omega^2 \omega^\mu \omega_\mu \right. \\ & \left. - \frac{1}{2}\vec{H}^{\mu\nu}\cdot\vec{H}_{\mu\nu} + m_\rho^2 \vec{\rho}^\mu \cdot \vec{\rho}_\mu\right), \qquad (3\end{aligned}$$

Explicit inclusion of t,^{3}He,α,d in the Lagrangian



α density appears in the surface of nuclei
(low density-Mott effect)

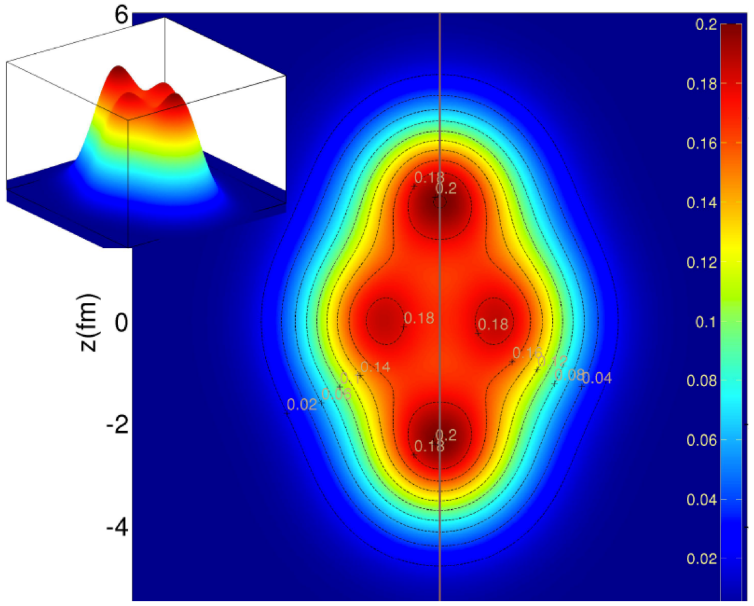
S. Typel, PRC 89(2014) 064321

Indicators for alpha particles in nuclear states

(local quantities in the EDF framework)

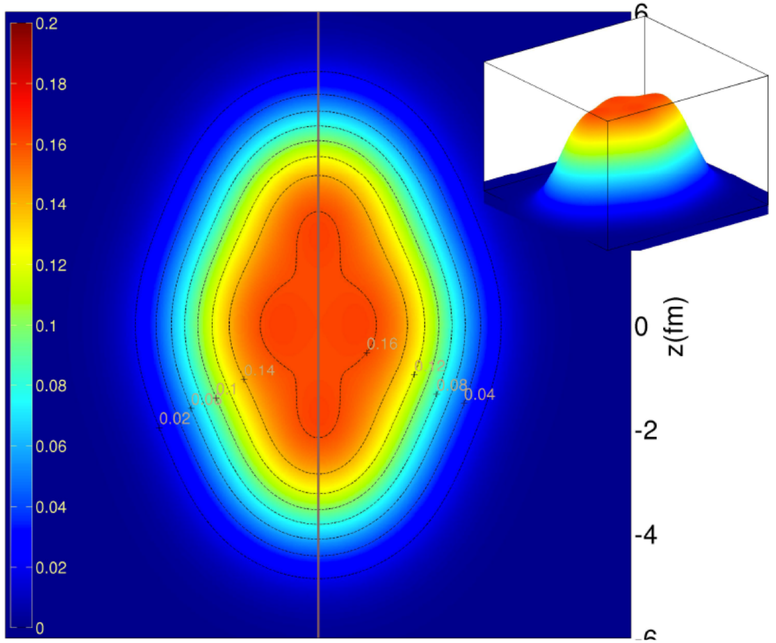
• Nucleonic density	
• α particle density	S. Typel, PRC 89(2014) 064321
• Nucleonic Localization function (NLF)	P.-G. Reinhard et al., PRC 83(2011) 034312
• Saturation + compactness + purity	E. Khan et al., PRC 106(2022) 064330
• Local α removal strength	T. Nakatsukasa et al., PRC 108(2023) 014318
• Localization parameter	J.P. Ebran et al., Nature 487(2012)341

Nucleonic density: covariant EDF clusterizes



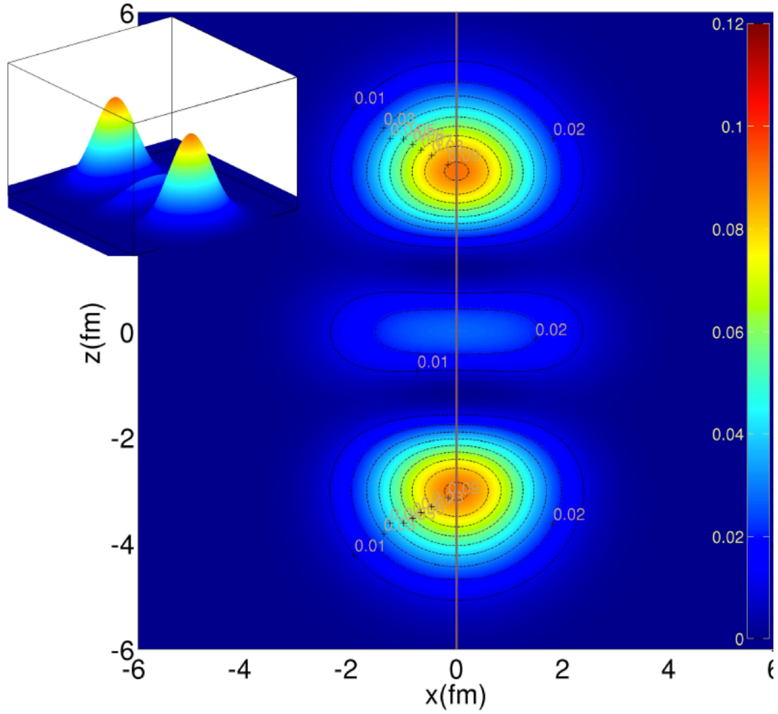
^{20}Ne

Total density

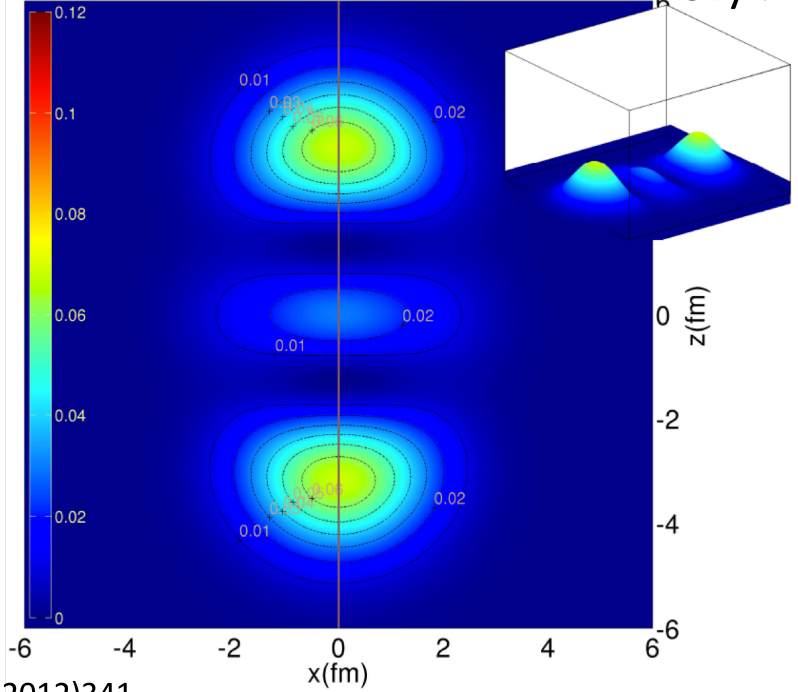


DDME2

SLy4

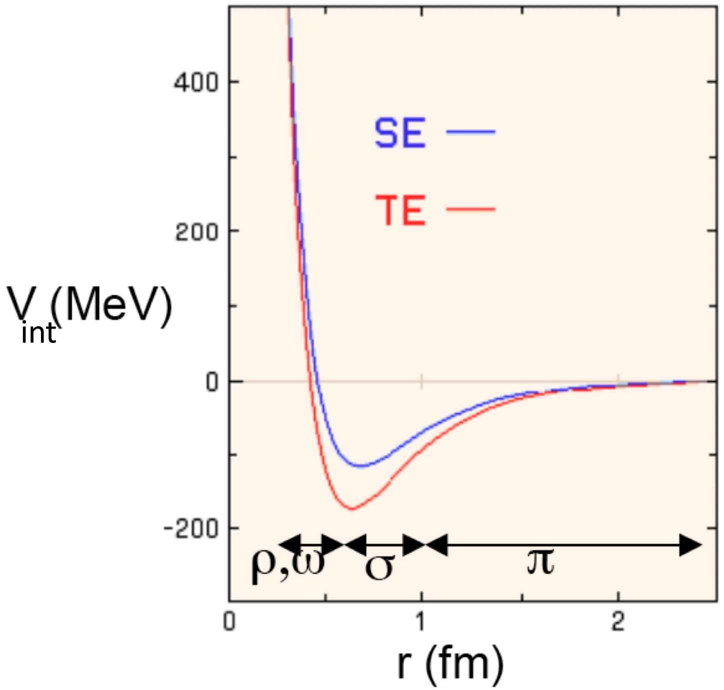


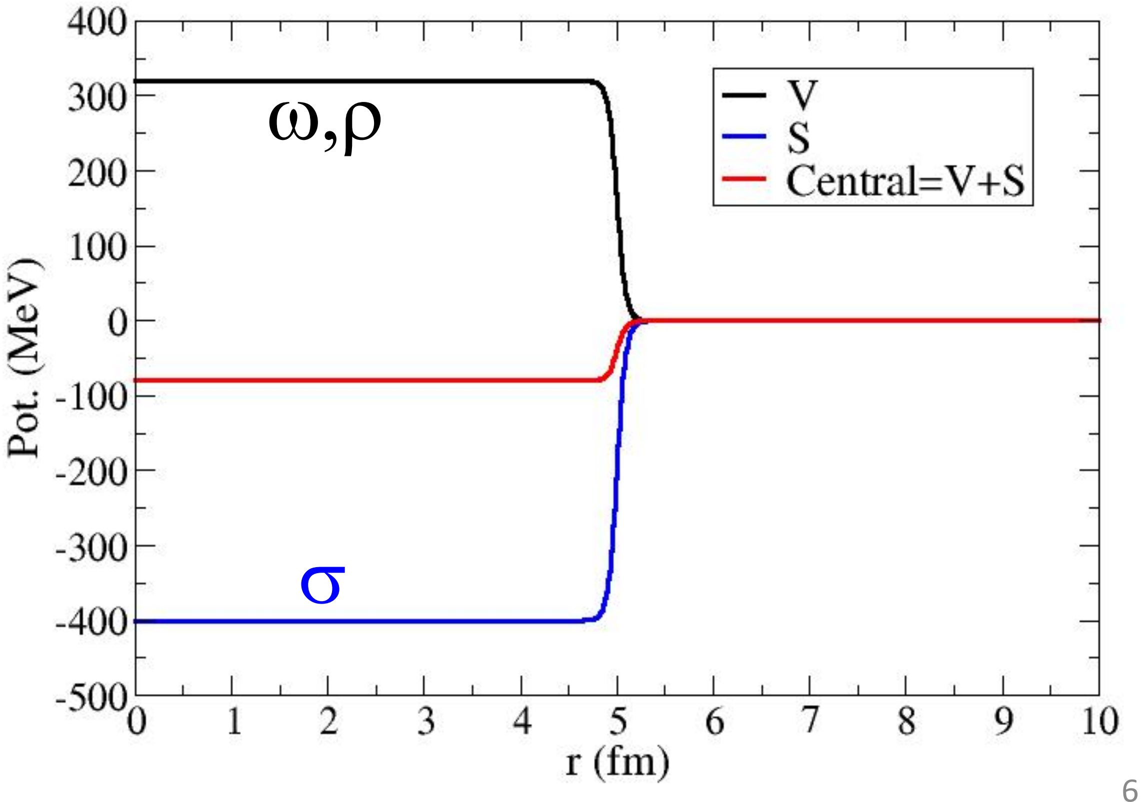
Last occupied state



J.P. Ebran et al., Nature 487(2012)341

V and S potentials

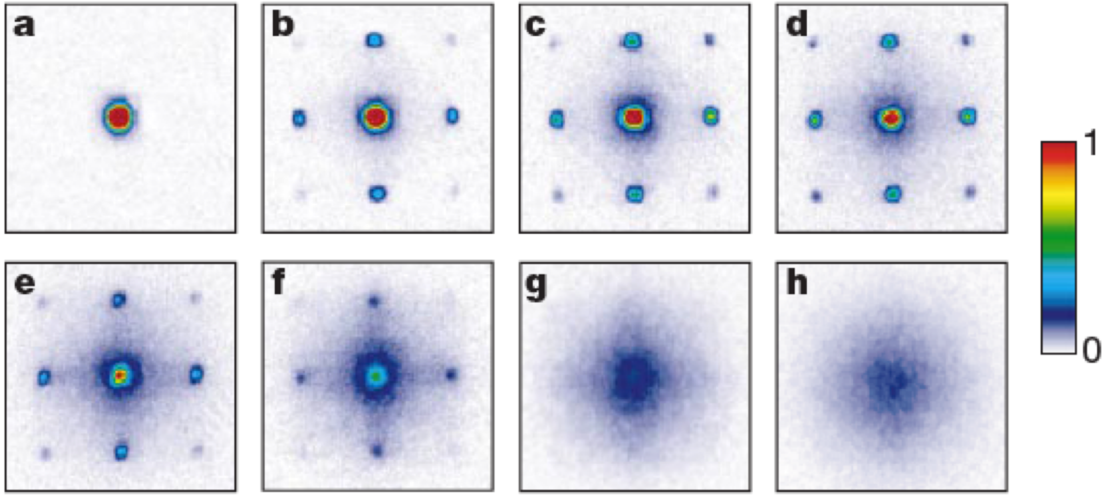




The depth of the potential

- **Ultracold atoms** : optical trap of variable depth V_0

  M. Greiner at al., Nature 415 (2002) 39



- **Nuclei** : depth of the potential **consistently** determined (relativisitic)

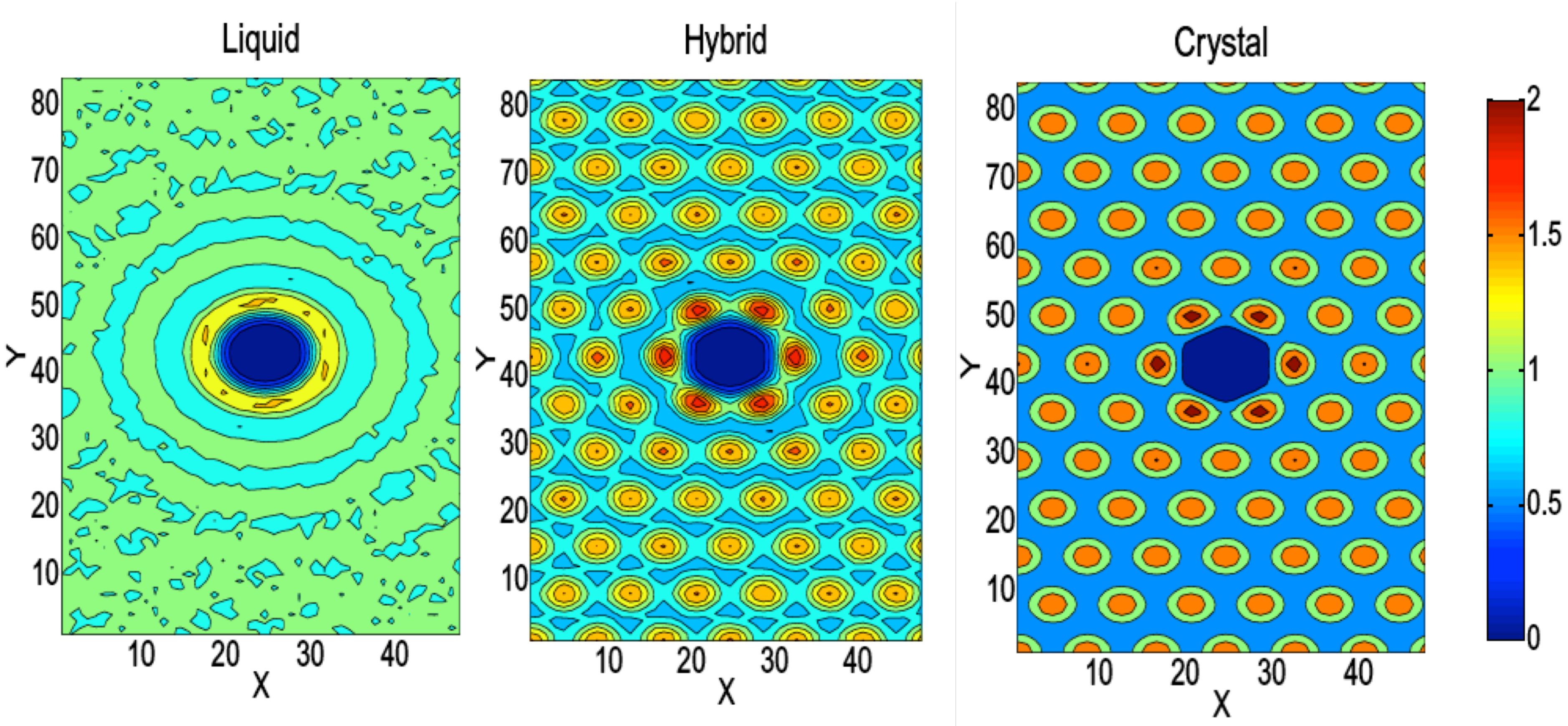
$$\left\{ p\frac{1}{2\tilde{M}(r)}p + W(r) + V_{ls}(r)l.s \right\} \varphi_i = \varepsilon_i \varphi_i$$

S≈ - 400 MeV
V≈ 320 MeV $\longrightarrow$ V_0 ≈ 80 MeV

$$W(r) = [V + S](r)$$

$$V_{ls}(r) = \frac{1}{2\tilde{M}^2(r)}\frac{1}{r}\frac{d}{dr}(V - S)$$

Analogies



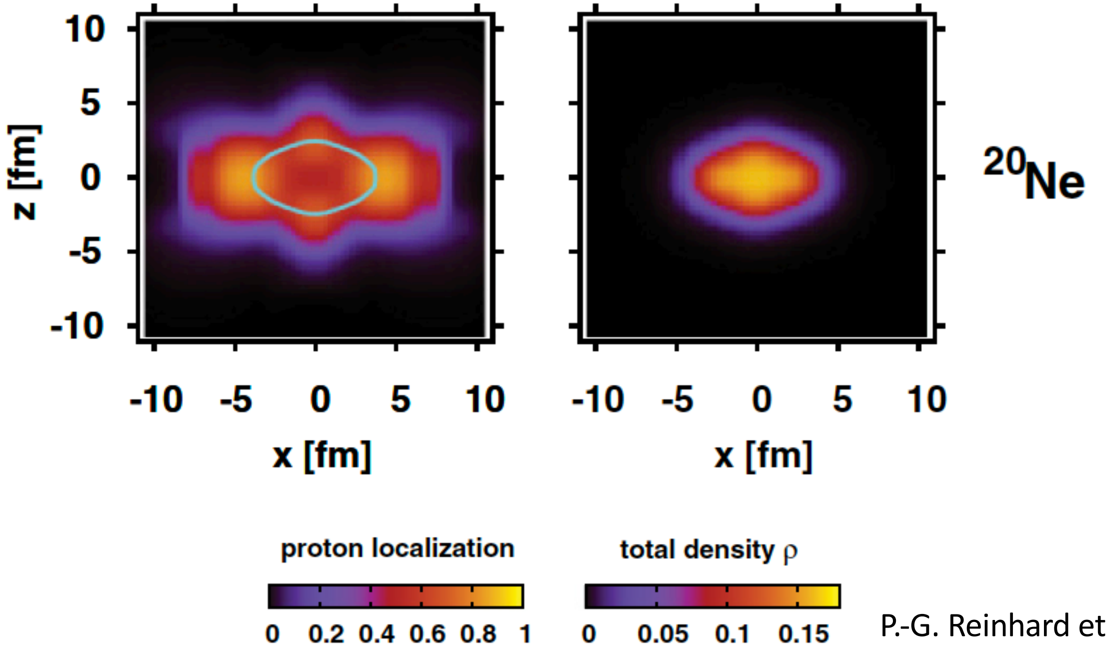
2D electronic system

H. Flakshahi and X. Waintal Phys. Rev. Lett 94 (2005) 046801

The Nucleonic Localization Function (NLF)

- Probability to have a single nuclear state at a given position:

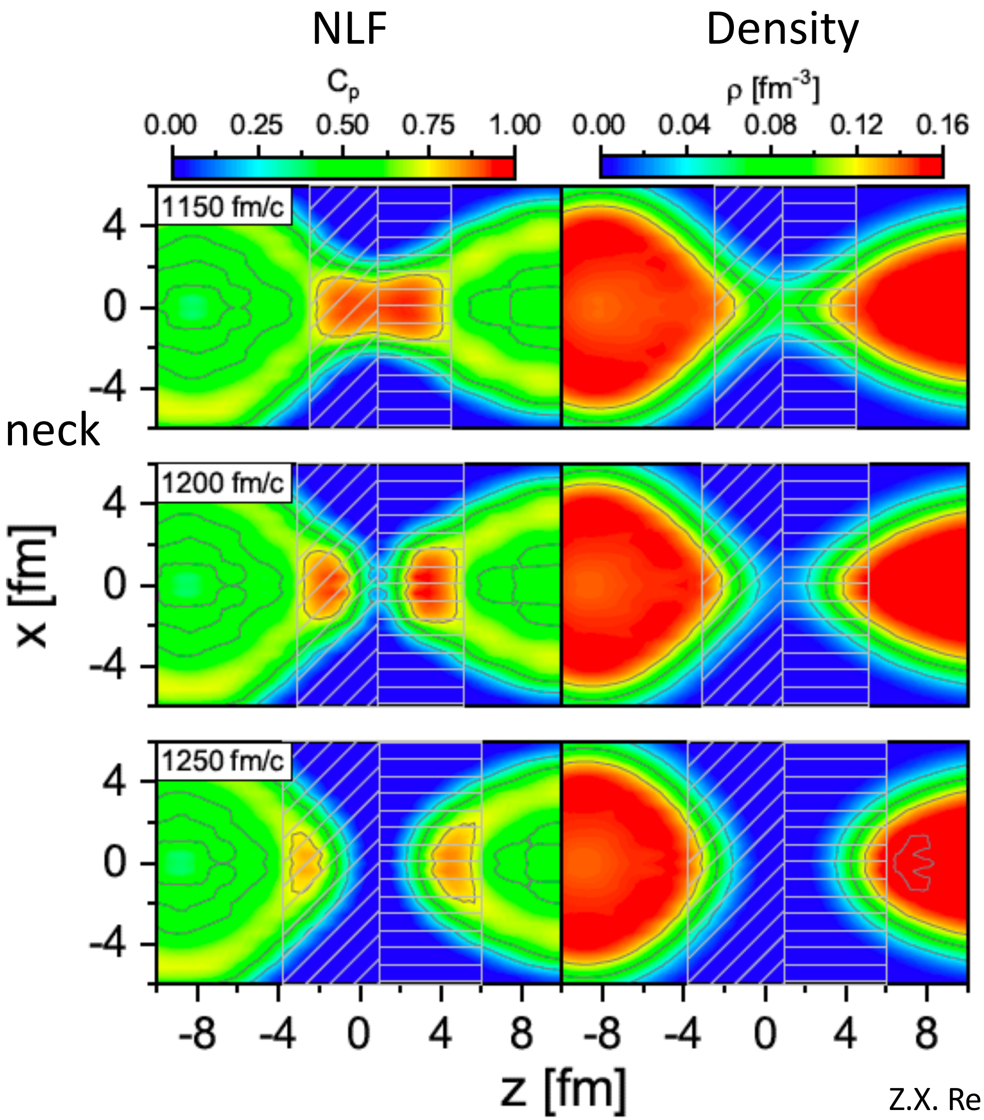
$$\mathcal{C}_{q\sigma}(\mathbf{r}) = \left[1 + \left(\frac{\tau_{q\sigma}\rho_{q\sigma} - \frac{1}{4}[\nabla\rho_{q\sigma}]^2 - \mathbf{j}^2_{q\sigma}}{\rho_{q\sigma}\tau^{\mathrm{TF}}_{q\sigma}} \right)^2 \right]^{-1}$$



P.-G. Reinhard et al., PRC 83(2011) 034312

- In N=Z nuclei, this implies a pure (n↑+n↓+p↑+p↓) wave function
- No indication on saturation or compactness of a possible α cluster
- Should be renamed as the « purity function »

Application: α particles in fission ?



^{240}Pu fission

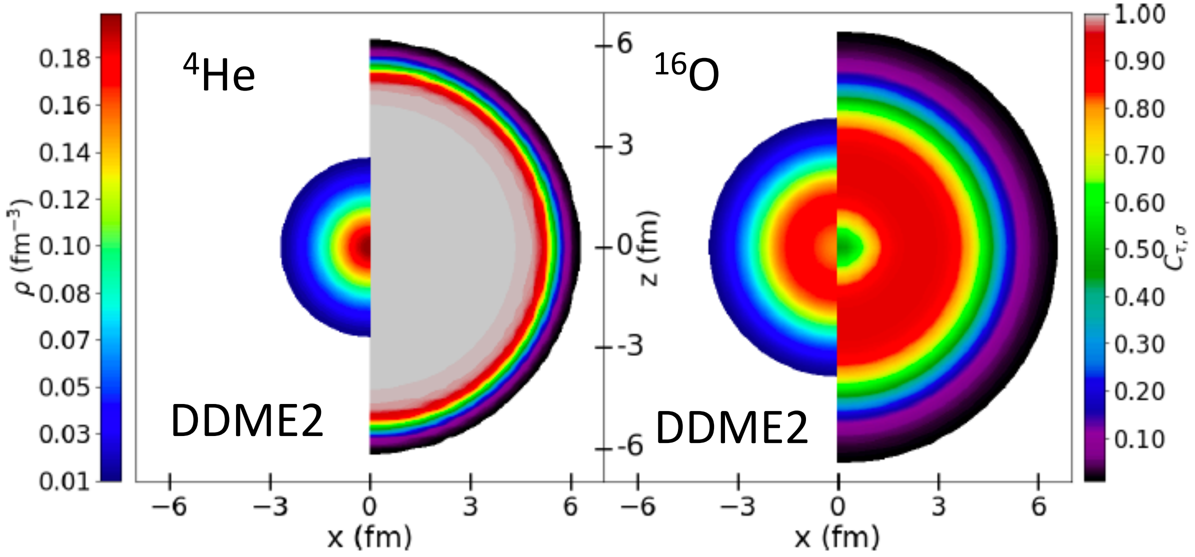
PC-PK1, TD-DFT

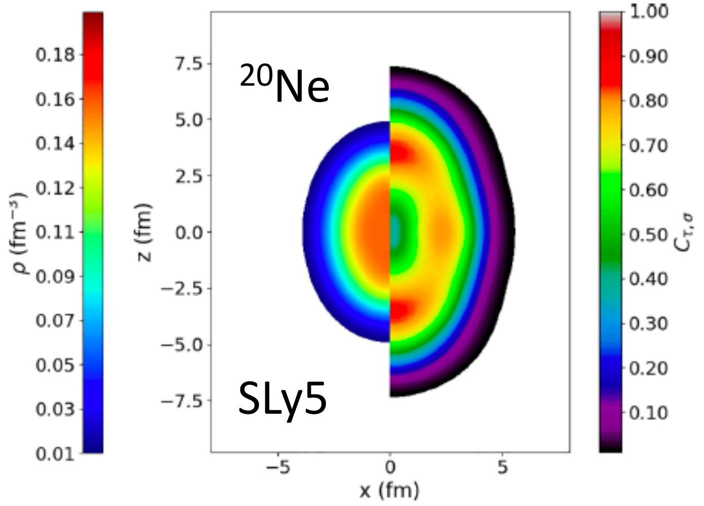
NLF close to 1 in the neck

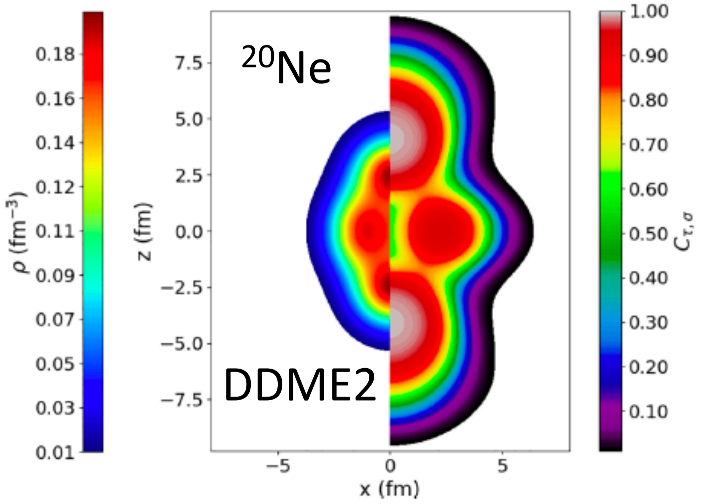
Z.X. Ren et al., PRL 128(2022) 172501

α formation or only pure (non-localized wave function) in the neck ?

Criteria for alpha clusterization in nuclei







	$C_{q\sigma}$	$\rho/\rho_{\max}$	$F(R_\alpha)_\parallel$	$F(R_\alpha)_\perp$
Criteria for cluster	>0.9	>0.8	>0.6	>0.6
^{4}He (DD-ME2)	1	1	0.82	0.82
^{20}Ne (DD-ME2)	0.95	1	0.75	0.70
^{20}Ne (SLy5)	0.6	1	0.45	0.25
^{16}O (DD-ME2)	0.9	1	0.79	0.41

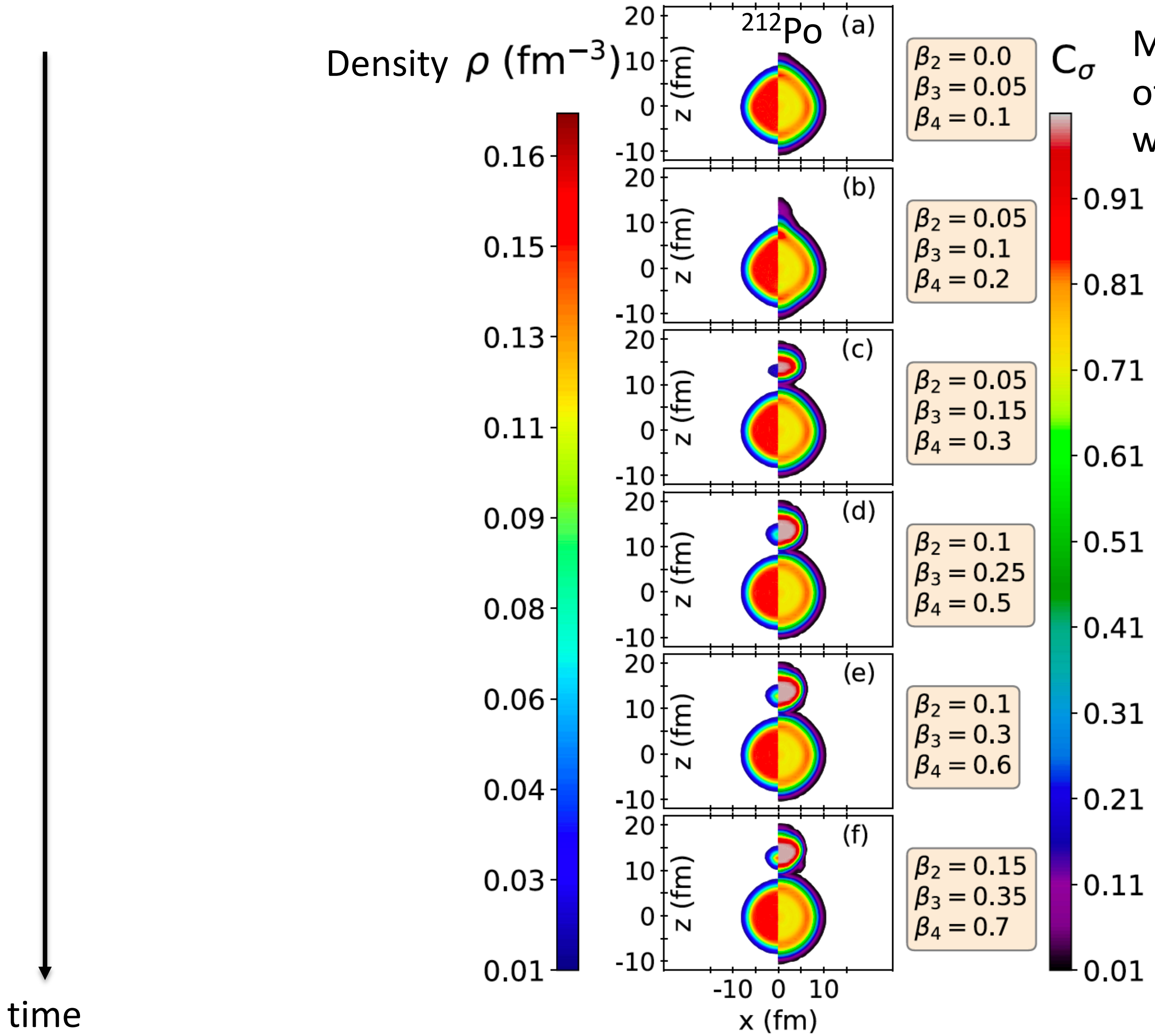
Purity Saturation Compactness

with $F(r) = \dfrac{\rho_{\max} - \rho(r)}{\rho_{\max}}$

(see Bubble nuclei)

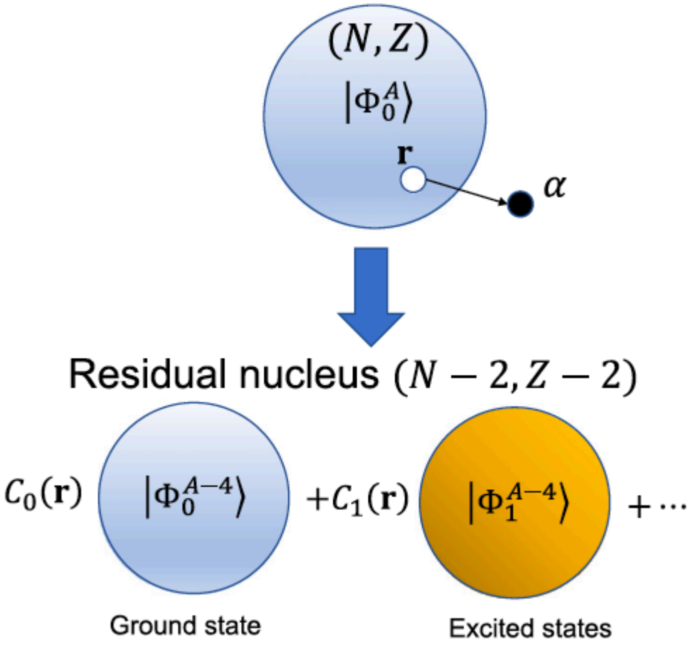
E. Khan et al., PRC 106(2022) 064330

Microscopic insight on the origin of the α decay mechanism



Measure of the purity
of the (2n+2p)
wave function

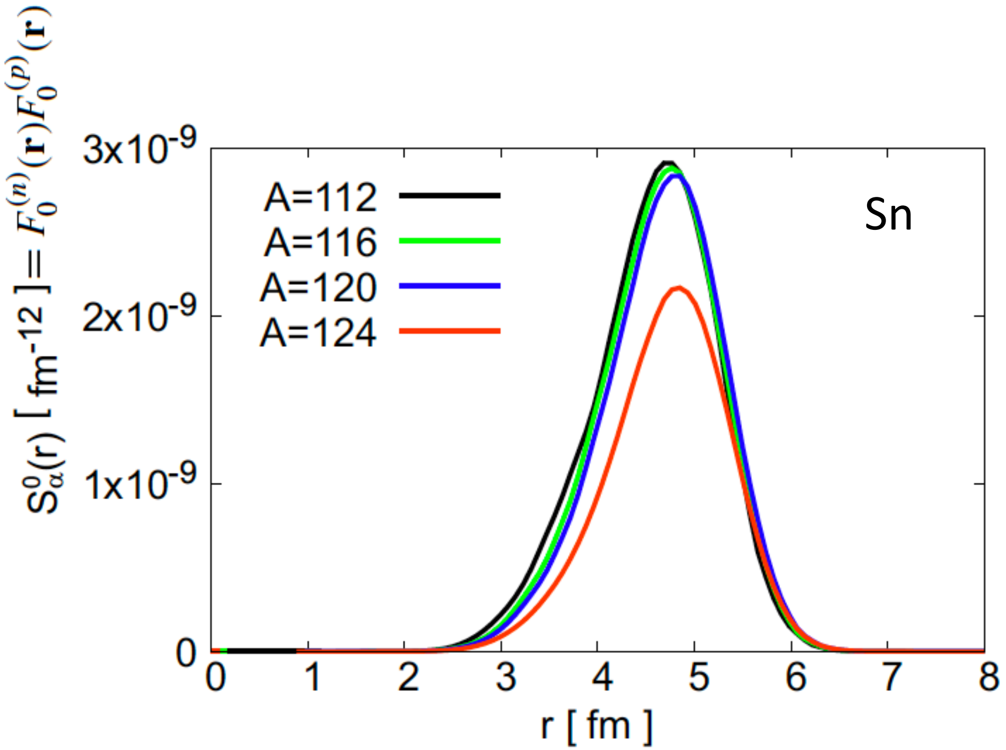
The local α removal strength



$$F_k^{(q)}(\mathbf{r}) = \left|\langle \Phi_k^{N_q-2} | \hat{\psi}_\uparrow^{(q)}(\mathbf{r}) \hat{\psi}_\downarrow^{(q)}(\mathbf{r}) | \Phi_0^{N_q} \rangle\right|^2$$

towards the g.s.: $F_0(\mathbf{r}) = |\kappa(\mathbf{r})|^2$

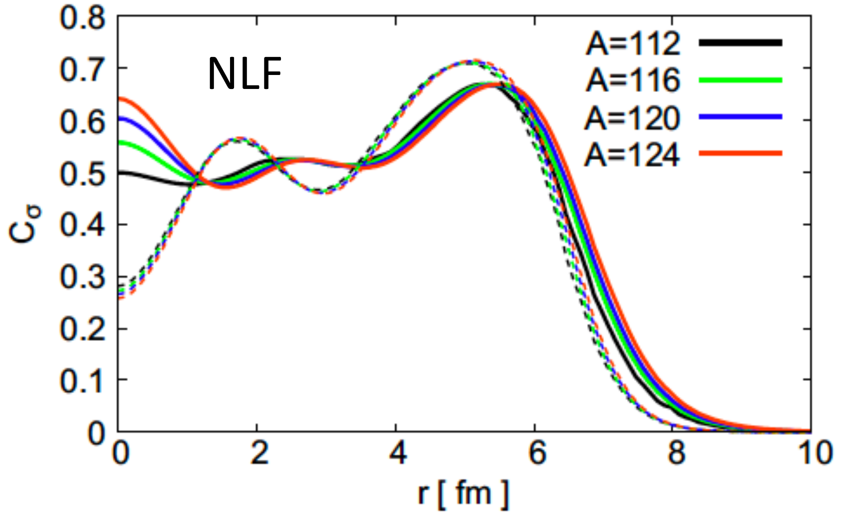
If no pairing: removal of 2 particles

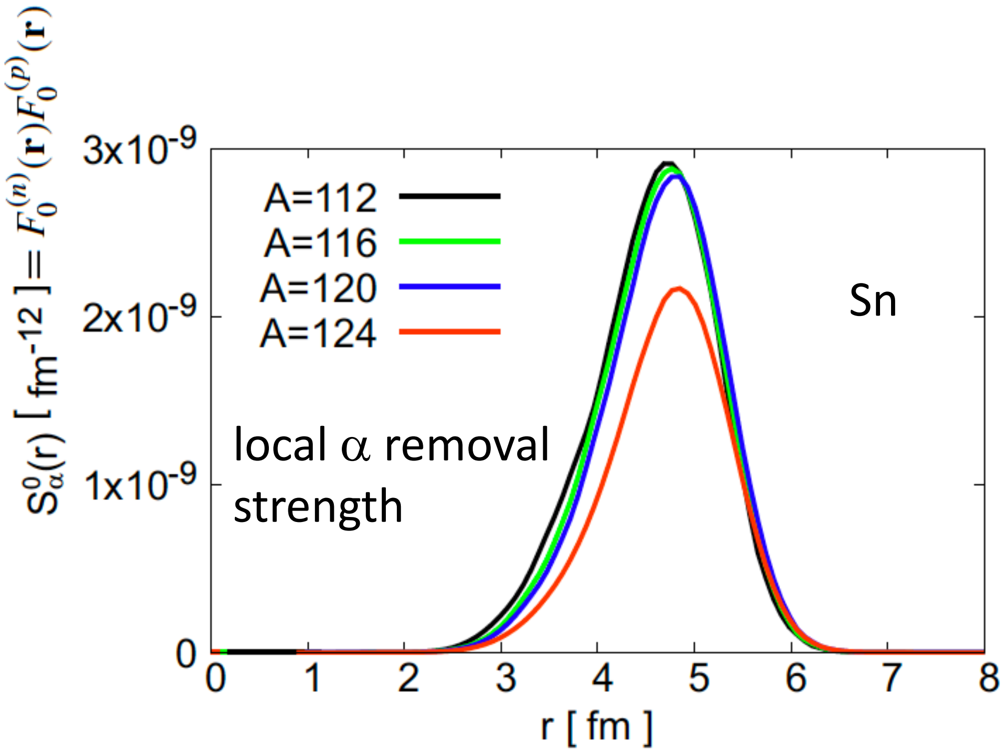


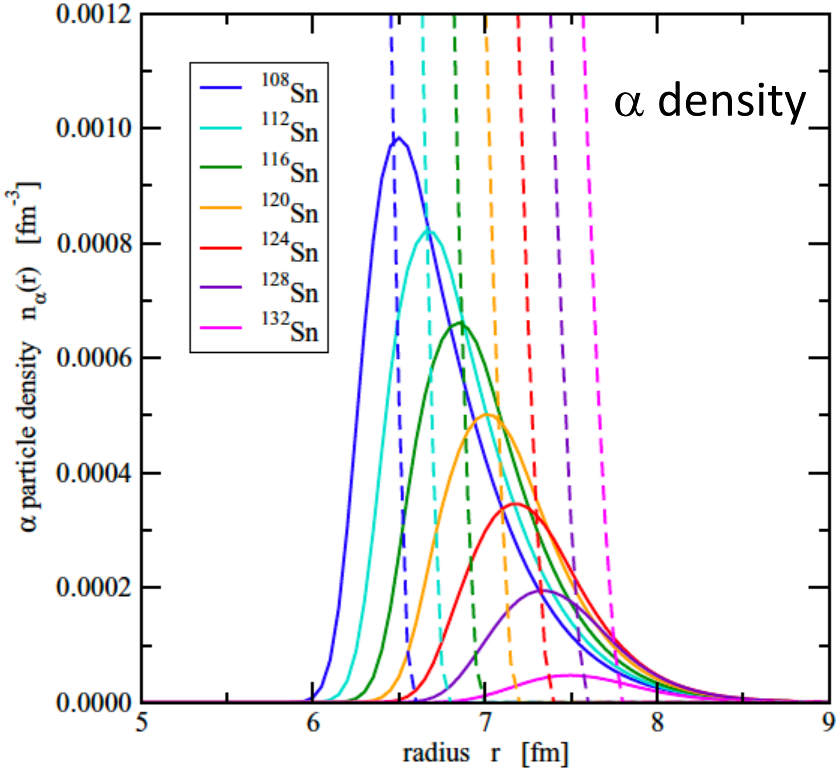
Peaked at the surface

T. Nakatsukasa et al., PRC 108(2023) 014318

Comparison with the NLF and the α density

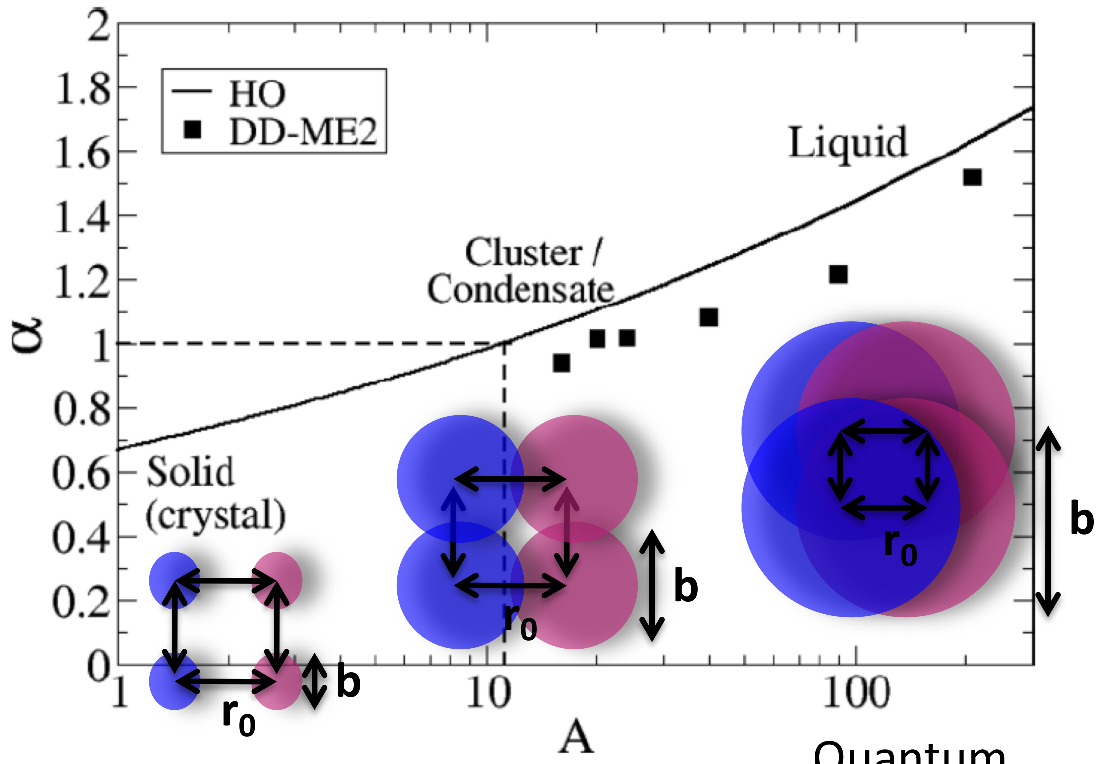






- NLF has a different profile inside the nuclei
- α density peaked at larger values (ρ_0/10)

A global indicator for localization



Crystal
α_{loc} <1

Cluster
$\alpha_{\text{loc}} \sim 1$

Quantum
liquid
$\alpha_{\text{loc}} > 1$

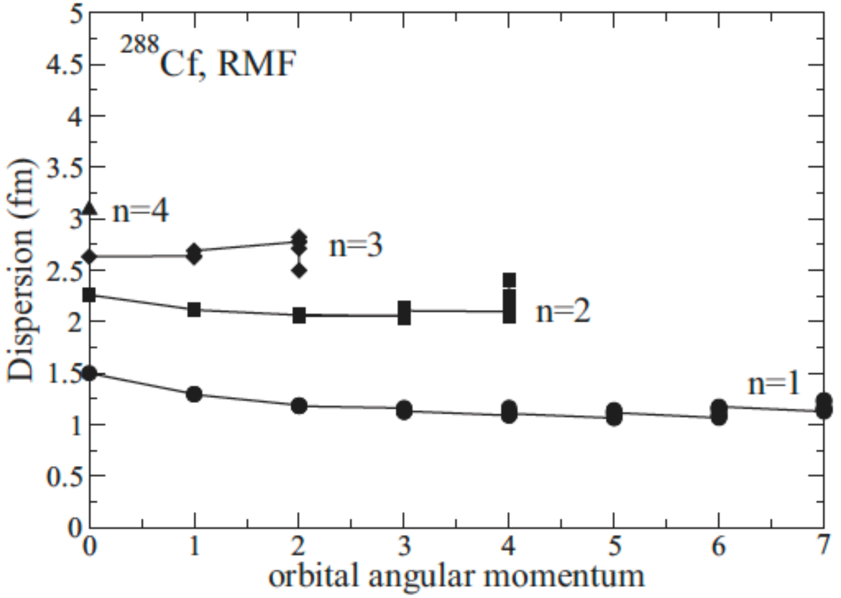
J.-P. Ebran et al., PRC 87(2013)044307

$$\alpha_{\text{loc}} = \frac{2\Delta r}{r_0} \simeq \frac{b}{r_0} = \frac{\sqrt{\hbar} A^{1/6}}{\left(2m V_0 r_0^2\right)^{1/4}}$$

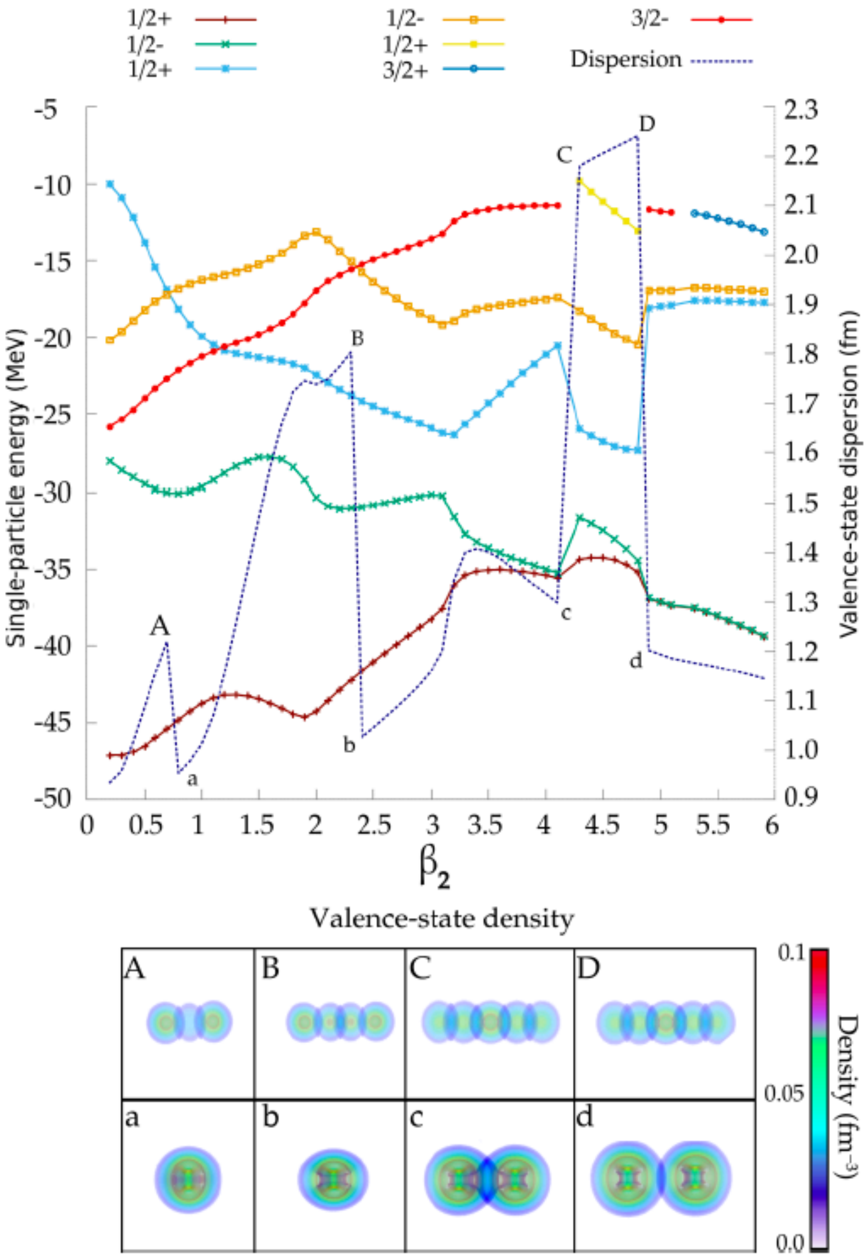
whereas in the NLF, $\rho_{q\sigma} = \frac{1}{4}\left(\frac{4}{3}\pi r_0^3\right)^{-1}$

(only depends on r_0)

n=1 states favor localization



$$\alpha_{\rm loc} = \frac{2\Delta r}{r_0} \simeq \frac{b}{r_0}\sqrt{2n-1} = \frac{\sqrt{\hbar(2n-1)}}{\left(2mV_0r_0^2\right)^{1/4}}A^{1/6}$$



J.-P. Ebran et al., PRC 97(2018)061301

To be discussed

- Indicator for alpha clustering in spherical nuclei ?
- Role of pairing ?
- Links with quarteting ?
- Are there convenient non-local functions ?

Thank you !