Proton-Neutron Pairing within the EDF framework

Danilo Gambacurta
gambacurta@lns.infn.it
INFN-LNS Catania

Experimental and Theoretical Aspects of Neutron-Proton Pairing
ESNT workshop
In collaboration with: N. Sandulescu, D. Negrea, T. Popa

Background and Motivations:

- Energy Density Functional (EDF) approach
- Pairing treatment within the EDF: BCS and HFB
- Quartet Condensation Model
- Applications and Results
- Outlook and Future Work

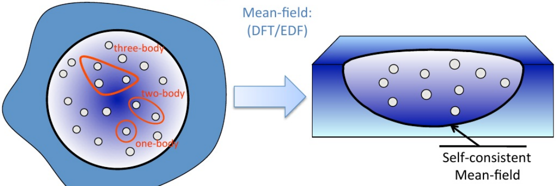
Energy Density Functional (EDF) approach:

- a mapping between the original many-body problem of interacting particles and a functional $\mathcal{E}$ that can be solved using an independent particle method

$$E = \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} \Longleftrightarrow \mathcal{E}_{EDF}[\rho] \Leftarrow \rho_{ij} = \langle \Phi | a_j^\dagger a_i | \Phi \rangle, \quad \text{where} \quad |\Phi\rangle = \prod a_i^\dagger | - \rangle$$

- Effective interaction between nucleons is represented by a functional
- All the information are encapsulated in the one-body densities and currents
- Variatonally solved (and iteratively)

The Energy Density Functional approach



Complex many-body states:

$\Psi(r_1, \cdots, r_{12}, \cdots, r_{123}, \cdots)$

Independent particles or quasi-particle states
Parameters of the functional are directly adjusted on data
Link to underlying bare Hamiltonian is lost

Energy Density Functional (EDF) approach

Advantages and applicability

- EDF can be applied through the entire nuclear chart
- Consistent approach to nuclear structure and excitations in nuclei
- Nuclear matter equation for astrophysically relevant processes
- Deformed and superfluid nuclei can be described through symmetry breaking approaches (rotational, particle number, ...)

Phenomenological or empirical EDF

- Skyrme (zero-range)
- Gogny (finte-range)
- Covariant (meson couplings)

The parameters of the interaction are directly adjusted to reproduce properties of finite and infinite nuclear systems

Energy Density Functional (EDF) approach

Particle-hole (p-h) and particle-particle (p-p) channels of the interaction

- Except the Gogny case, p-h and p-p channels are different
- p-p strength tuned to reproduce some observables (S_n, Δ, ...)
- No clear way to fix the isoscalar channel strength
- Gogny isoscalar channel not well constrained

Current challenges

- **Ab-initio** inspired/constrained functionals providing all the channels
  Quark-Meson Coupling Model see A. Thomas's Talk: couplings
- Restoring symmetries: N, T, J

EDF for superfluid systems

- Quasiparticle states via Bogoliubov transformation
- Slater Determinant in terms of quasi-particle $|\Phi\rangle = \prod \alpha_i^\dagger |-\rangle$
- BCS or HFB equations are solved
- Very efficient in treating particle-like pairing, e.g. T=1 p-p or n-n superfluid systems
- BCS and HFB not suited to treat p-n pairing (isospin and particle number violation)

Extensions to treat T=1 and T=0 pairing, beyond BCS/HFB

- (1) Solution of the pairing Hamiltonian via full diagonalization, (cumbersome and not easily intelligible)
- (2) Quartet Condensation Model (QCM)

Goals

- Full self-consistent frame based on EDF preserving N and T
- Treatment of all the pairing channels (and their possible coexistence)
- Deformation effects described microscopically
- Interplay between pairing(s) and deformation

References:

- D. Gambacurta, Denis Lacroix, and N. Sandulescu, Phys. Rev. C 88, 034324 (2013) and Phys. Rev. C 91, 014308 (2015): Skyrme-HF plus Exact Pairing solution (**Diagonalization**) in deformed nuclei
- *Proton-neutron pairing in* **N = Z** *nuclei:* **Quartetting versus pair condensation**, N. Sandulescu, D. Negrea and D. Gambacurta, PLB751, 348 (2015)
- *Isovector and isoscalar pairing in* **odd–odd N = Z** *nuclei within a quartet approach*, D. Negrea, N. Sandulescu, D. Gambacurta; PTEP, 2017, 073D05 (2017)
- *Isovector and isoscalar proton-neutron* **pairing in N>Z** *nuclei*,D. Negrea, P. Buganu, D. Gambacurta, and N. Sandulescu, PRC,98, 064319 (2018)
- *Proton-neutron pairing and binding energies of nuclei close to the N=Z line*, D. Negrea, N. Sandulescu, and D. Gambacurta,PRC 105, 034325 (2022): **First full-self consistent QCM-EDF calculations**

PHYSICAL REVIEW C **91**, 014308 (2015)

Effects of deformation on the coexistence between neutron-proton and particle-like pairing in $N = Z$ medium-mass nuclei

Danilo Gambacurta*
Istituto Nazionale di Fisica Nucleare, Sezione di Catania, Via S. Sofia 64, I-95123 Catania, Italy
and GANIL, CEA/DSM and CNRS/IN2P3, Boîte Postale 55027, 14076 Caen Cedex, France

Denis Lacroix†
Institut de Physique Nucléaire, IN2P3-CNRS, Université Paris-Sud, F-91406 Orsay Cedex, France
(Received 10 November 2014; revised manuscript received 9 December 2014; published 12 January 2015)

PHYSICAL REVIEW C **88**, 034324 (2013)

Pairing and specific heat in hot nuclei

Danilo Gambacurta*
GANIL, CEA, and CNRS/IN2P3, Boîte Postale 55027, 14076 Caen Cedex, France

Denis Lacroix†
Institut de Physique Nucléaire, IN2P3-CNRS, Université Paris-Sud, F-91406 Orsay Cedex, France
and GANIL, CEA/DSM, and CNRS/IN2P3, Boîte Postale 55027, 14076 Caen Cedex, France

N. Sandulescu‡
National Institute of Physics and Nuclear Engineering, P. O. Box MG-6, Magurele-Bucharest, Romania
(Received 16 July 2013; published 27 September 2013)

Combining mean field (MF) and shell-model (SM) techniques

MF calculations

- Self-Consistent Skyrme-HF + BCS calculations
  (ev8 code[a]: three-dimensional mesh, deformation accounted for)
- SLy4 Interaction + Pairing contact interaction
  $V(\mathbf{r},\mathbf{r'}) = -v_0 \left(1 - \eta \frac{\rho(\mathbf{r})}{\rho_0}\right) \delta(\mathbf{r}-\mathbf{r'})^b$
- MF calculations provide: s.p. basis and (T=1 and T=0) paring matrix elements (consistently calculated)

[a] P.Bonche, H: Flocard, P.H. Heenen
[b] parameters from: F. Bertsch *et al.*, Phys. Rev. C 79, 034306 (2009)

SM calculations, e.g. exact diagonalization

- SM calculations in a pairing window (5 MeV) around the Fermi energy
- $n = z = 8$ active neutrons and protons
- Applications for Z=N nuclei in pf shells[a]
- *Not self-consistent*, e.g. SM results do not feed MF calculations

[a] D. Gambacurta and D. Lacroix, Phys. Rev. C 91, 014308 (2015)

Matrix Elements

Matrix Elements (I)

$$|k\tau_k\rangle = \int d^3r \sum_{\sigma_k} \phi_k(\sigma_k, r)|r\sigma_k\tau_k\rangle$$

$$|k\tau_k.\bar{k}\tau_{\bar{k}}\rangle = \int d^3r_1 d^3r_2 \sum_{\sigma_k,\sigma_{\bar{k}}} \phi_k(\sigma_k, r_1)\phi_{\bar{k}}(\sigma_{\bar{k}}, r_2)|r_1\sigma_k\tau_k, r_2\sigma_{\bar{k}}\tau_{\bar{k}}\rangle$$

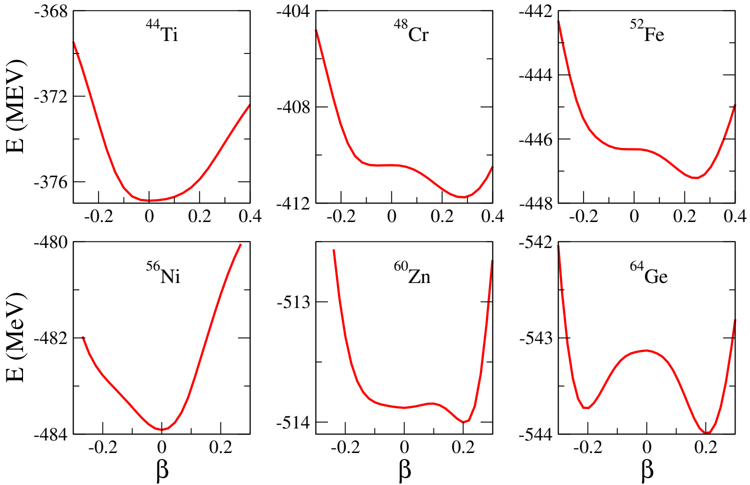
$$V_{ij}^{T,S} = \langle i\tau_i\bar{i}\tau_{\bar{i}}|V^{T,S}|j\tau_j\bar{j}\tau_{\bar{j}}\rangle = \langle i\tau_i\bar{i}\tau_{\bar{i}}|VP_SP_T|j\tau_j\bar{j}\tau_{\bar{j}}\rangle$$

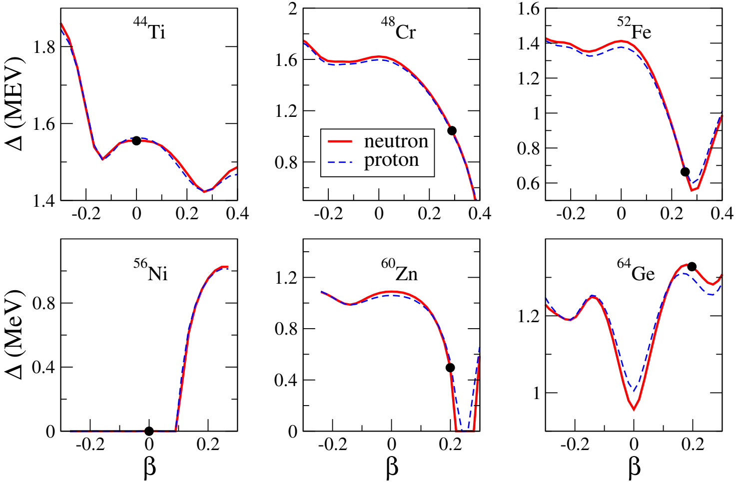
where P_S, P_T are the standard spin-isospin projection operators

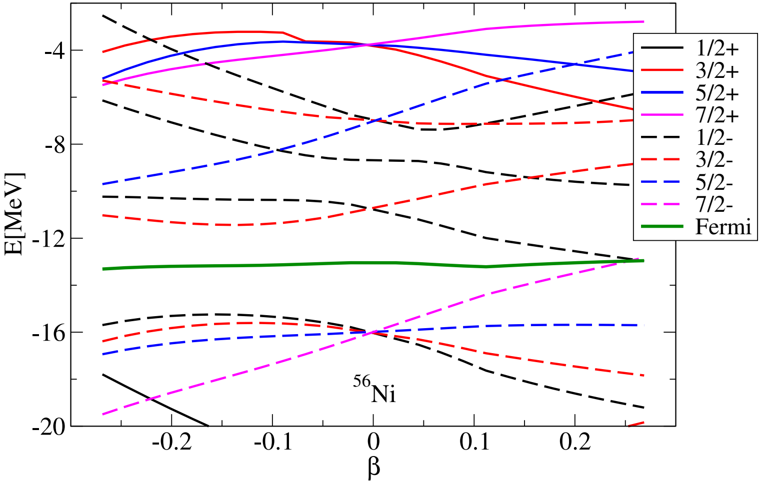
Matrix Elements (II)

- Matrix elements consistently calculated
- No G constant approximation

Pairing versus deformation, BCS versus diagonalization



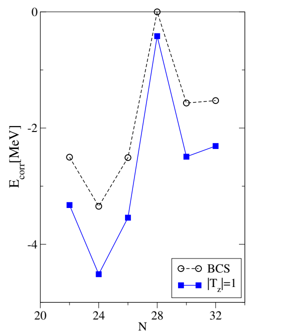

44Ti
48Cr
52Fe
56Ni
60Zn
64Ge
neutron
proton
Δ (MEV)
Δ (MeV)
β
β
β
1.8
1.6
1.4
2
1.6
1.2
0.8
1.6
1.4
1.2
1
0.8
0.6
0.8
0.4
0
1.2
0.8
0.4
0
1.2
1
-0.2
0
0.2
0.4



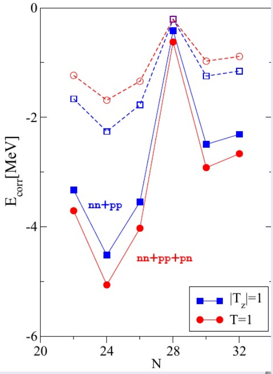
-4
-8
-12
-16
-20
E[MeV]
-0.2
-0.1
0
0.1
0.2
β
56Ni
1/2+
3/2+
5/2+
7/2+
1/2-
3/2-
5/2-
7/2-
Fermi

Schematic results

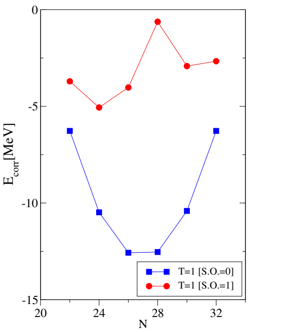
BCS vs SM



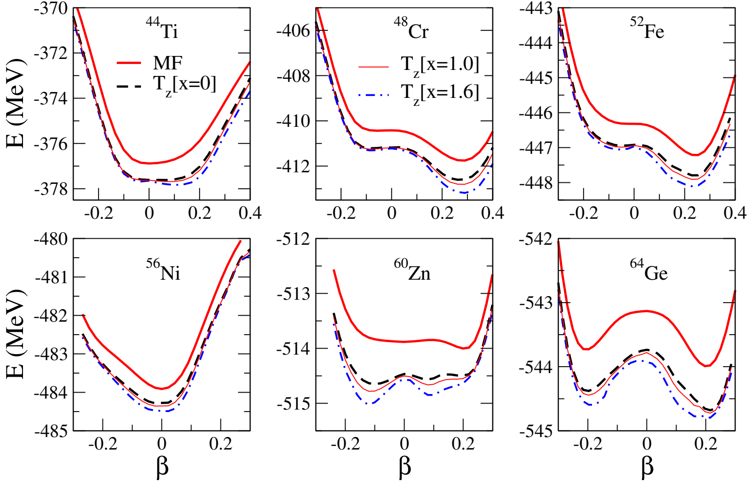
proton-neutron T=1 pairing



Spin Orbit suppression



Pairing versus deformation, BCS versus diagonalization



Exact pn T=1 and T=0 pairing

Pairing versus deformation, BCS versus diagonalization

$$E_{10} = \left\langle \sum_{i\neq j} V_{ij}^{10} P_0^\dagger(i) P_0(j) \right\rangle,$$

$$E_{01} = \left\langle \sum_{i\neq j} V_{ij}^{01} D_0^\dagger(i) D_0(j) \right\rangle.$$

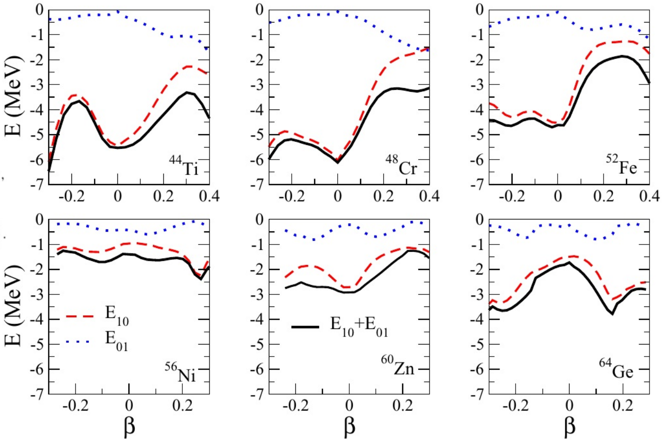


FIG. 14. (Color online) The isovector (red dashed line) and isoscalar (blue dotted line) energy contributions, defined by Eqs. (10) respectively, and the sum of them (black solid line) are plotted as a function of the deformation and corresponding to a ($|T_z| = 0$) c with $x = 1.6$.

$$\hat{Q}_{10} = \sum_{ij} \hat{P}_0^\dagger(i)\hat{P}_0(j),$$

$$\hat{R}_{01} = \sum_{ij} \hat{D}_0^\dagger(i)\hat{D}_0(j).$$

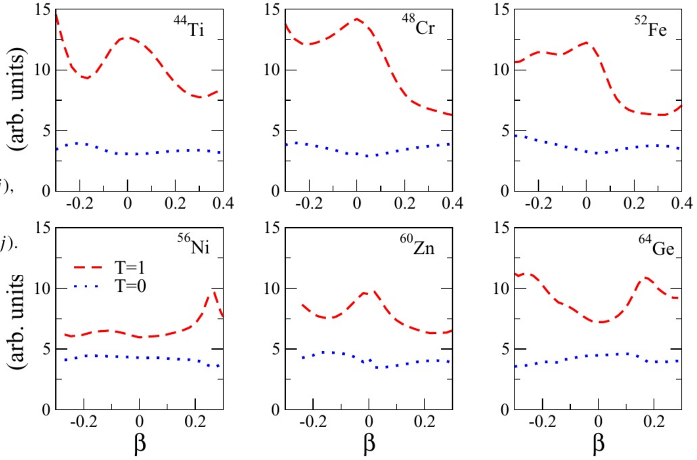


FIG. 15. (Color online) Isovector (red dashed lines) and isoscalar (blue dotted lines) total deuteron transfer probability obtained corresponding, respectively, to the Q and R quantities defined in the text.

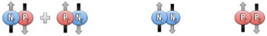
Take-Home messages

- We investigated proton-neutron correlations and deformation effects combining MF and SM approaches
- Deformation not imposed, (e.g. energy minimum)
- T=1 pairing correlations typically much stronger than T=0
- Strong interplay between different ingredients:
  - Pairing T=1 and T=0 channels,
  - Deformation,
  - Spin-Orbit,
  - s.p. gaps and pairings strength

QCM formalism

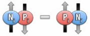
Pairing in even-even N=Z nuclei: axially deformed symmetry

Isovector T=1, J_z=0 pairs



$$P^+_{i,0} = \frac{1}{\sqrt{2}}(\nu^+_i \pi^+_{\bar{i}} + \pi^+_i \nu^+_{\bar{i}}) \qquad P^+_{i,1} = \nu^+_i \nu^+_{\bar{i}} \qquad P^+_{i,-1} = \pi^+_i \pi^+_{\bar{i}}$$

Isoscalar T=0, J_z=0 pairs



$$D^+_{i,0} = \frac{1}{\sqrt{2}}(\nu^+_i \pi^+_{\bar{i}} - \pi^+_i \nu^+_{\bar{i}})$$

Hamiltonian

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \underbrace{\sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^+_{i,t_z} P_{j,t_z}}_{\text{isovector}} + \underbrace{\sum_{i,j} V^{T=0}(i,j) D^+_{i,J_z=0} D_{j,J_z=0}}_{\text{isoscalar}}$$

Isovector quartets

$$A^+ = \underbrace{2\Gamma^+_1 \Gamma^+_{-1} - \Gamma^{+2}_0}$$
$$\Gamma^+_t = \sum_i x_i P^+_{i,t}$$

Isovector-isoscalar quartet

$$Q^+ = 2\Gamma^+_1 \Gamma^+_{-1} - \Gamma^{+2}_0 + \Delta^{+2}_0$$

Collective isoscalar pairs

$$\Delta^+_0 = \sum_i y_i D^+_{i,0}$$

⇩

Quartet condensate

$$|QCM\rangle = (Q^+)^{n_q}|0\rangle \qquad n_q = (N+Z)/4$$

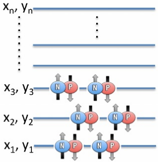
(exact solution for a set of degenerate states)

$$\Gamma^+_t = \sum_i x_i P^+_{i,t} \qquad \Delta^+_0 = \sum_i y_i D^+_{i,0}$$

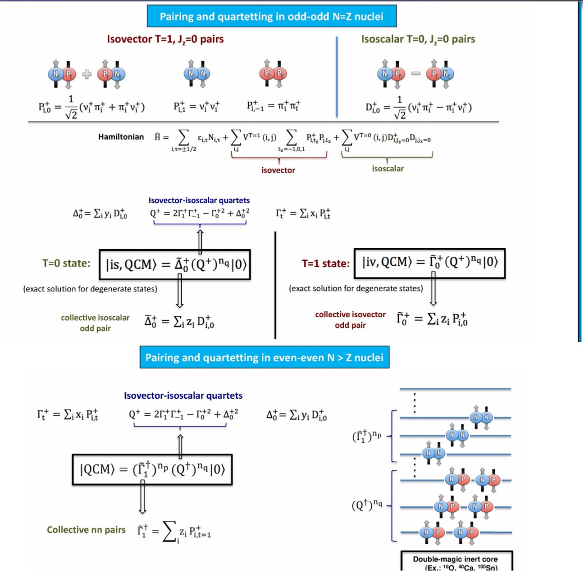
Unknown parameters: mixing amplitudes x_i and y_i

Minimization: $\delta_{x,y}\langle\Psi|\hat{H}|\Psi\rangle = 0$

Constraint: $\langle\Psi|\Psi\rangle = 1$



QCM formalism



PHYSICAL REVIEW C **98**, 064319 (2018)

Isovector and isoscalar proton-neutron pairing in $N > Z$ nuclei

D. Negrea,[1] P. Buganu,[1] D. Gambacurta,[2] and N. Sandulescu[1,*]

[1] *National Institute of Physics and Nuclear Engineering, 077125 Măgurele, Romania*

[2] *Extreme Light Infrastructure - Nuclear Physics (ELI-NP), National Institute of Physics and Nuclear Engineering, 077125 Măgurele, Romania*

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Strength of the pairing force in T=1 and T=0 channels

$$\hat{H} = \hat{H}_0 + \hat{H}_p$$

$$\hat{H}_0 = \sum_{i,\tau=\pm 1} \varepsilon_{i,\tau} N_{i,\tau}$$ - s.p. states given by Skyrme-HF calculations for axially deformed m.f.

Pairing interactions:

(I) State-independent pairing force

$$\hat{H}_p = \boxed{V^{(T=1)}} \sum_{\substack{i,j,\\ t=-1,0,1}} P^\dagger_{i,t} P_{j,t} + \boxed{V^{(T=0)}} \sum_{i,j} D^\dagger_{i,0} D_{j,0}$$

Strength of the isovector pairing force

$$V^{(T=1)} = -24/A$$

Strength of the isoscalar pairing force

$$V^{(T=0)} = \boxed{w \cdot} V^{(T=1)}$$

w = ? ⟹
- sd-shell nuclei: w=1.2
- heavier nuclei: w=0.8

(II) Zero range delta interaction

$$\hat{H}_p = \sum_{i,j} V^{(T=1)}_{i,j} \sum_{t=-1,0,1} P^\dagger_{i,t} P_{j,t} + \sum_{i,j} V^{(T=0)}_{i,j} D^\dagger_{i,0} D_{j,0}$$

$$V^{T=\{0,1\}}_{pairing}(\vec{r}_1 - \vec{r}_2) = \boxed{V^{T=\{0,1\}}_0} \delta(\vec{r}_1 - \vec{r}_2) \hat{P}_{S=\{0,1\}}$$

Strength of the isovector pairing force

$$V^{T=1}_0 = 465 \ \text{MeV fm}^{-3}$$

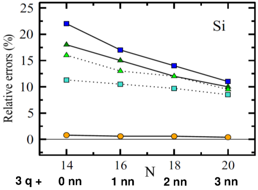
Strength of the isoscalar pairing force

$$V^{T=0}_0 = \boxed{w \cdot} V^{T=1}_0$$

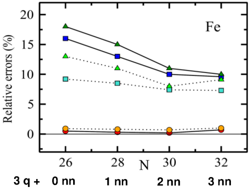
w = ? ⟹
- sd-shell nuclei: w=1.6
- heavier nuclei: w=1.0

Isovector and isoscalar pn pairing in N > Z nuclei: results (I)

sd-shell nucleus



pf-shell nucleus



Correlation energies (MeV): $E_{corr} = E_0 - E$

(I) State-independent pairing force

(II) Zero range delta interaction

$|C_{iv}\rangle = (\tilde{\Gamma}_1^\dagger)^{n_p}(\Gamma_0^{\dagger 2})^{n_q}|0\rangle$

$|C_{is}\rangle = (\tilde{\Gamma}_1^\dagger)^{n_p}(\Delta_0^{\dagger 2})^{n_q}|0\rangle$

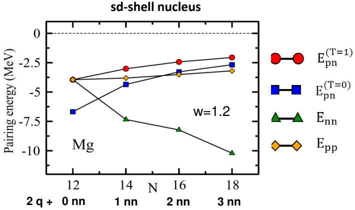
$|QCM\rangle = (\tilde{\Gamma}_1^\dagger)^{n_p}(Q^\dagger)^{n_q}|0\rangle$

|QCM> describes well the ground state pairing correlations (errors < 1%).

|Civ> and |Cis> (larger errors): not a fast transition to a pure condensate of iv/is pn pairs.

D. N., P. Buganu, D. Gambacurta, and N. Sandulescu, Phys. Rev. C98, 064319 (2018)

Isovector and isoscalar pn pairing in N > Z nuclei: results (II)

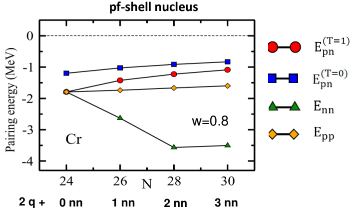


State-independent pairing force

Pairing energies (MeV):

$$E_t^{(T=1)} = V^{(T=1)} \sum_{i,j,t} \langle QCM | P_{i,t}^{\dagger} P_{j,t} | QCM \rangle$$

$$E_{pn}^{(T=0)} = V^{(T=0)} \sum_{i,j} \langle QCM | D_{i,0}^{\dagger} D_{j,0} | QCM \rangle$$



Pn pairing energies are decreasing, but remain significantly large even when 3 extra nn pairs are added.

Isovector and isoscalar pn pairing correlations coexist in both N = Z and N > Z nuclei.

D. N., P. Buganu, D. Gambacurta, and N. Sandulescu, Phys. Rev. C98, 064319 (2018)

Full Self-consistent Skyrme-QCM calculations

Proton-neutron pairing and binding energies of nuclei close to the $N = Z$ line

D. Negrea and N. Sandulescu*
National Institute of Physics and Nuclear Engineering, 077125 Măgurele, Romania

D. Gambacurta
INFN-LNS, Laboratori Nazionali del Sud, 95123 Catania, Italy

(Received 1 November 2021; accepted 18 February 2022; published 23 March 2022)

We analyze the contribution of isovector and isoscalar proton-neutron pairing to the binding energies of even-even nuclei with $N - Z = 0, 2, 4$ and atomic mass $20 < A < 100$. The binding energies are calculated in the mean-field approach by coupling a Skyrme-type functional to an isovector-isoscalar pairing force of zero range. The latter is treated in the framework of quartet condensation model (QCM), which conserves exactly the particle number and the isospin. The interdependence of pairing and deformation is taken into account by performing self-consistent Skyrme-HF + QCM calculations in the intrinsic system. It is shown that the binding energies are not changing much when the isoscalar pairing is switched on. This fact is related to the off-diagonal matrix elements of the pairing force, which are less attractive for the isoscalar force, and to the competition between the isoscalar and isovector pairing channels.

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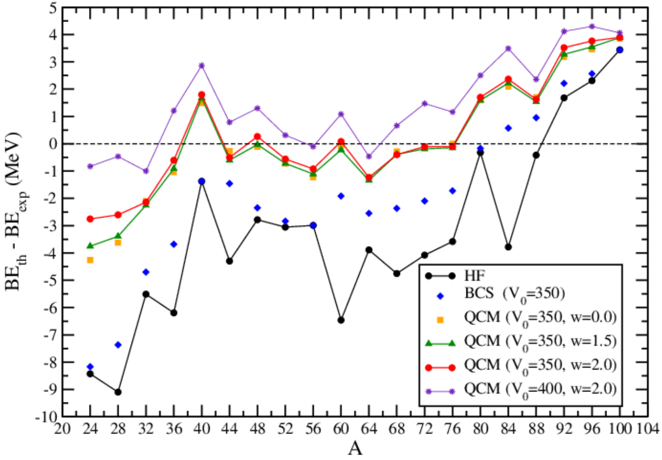
Calculations taking into account dynamically the competition between pairing and deformation conserving exactly both the particle number and the isospin.



Even-even N = Z nuclei as a function of A = N + Z.

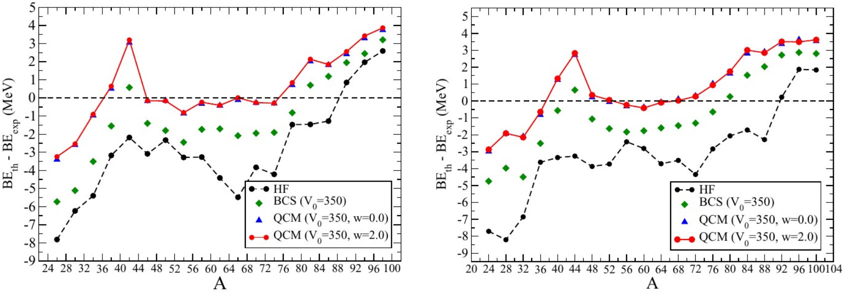
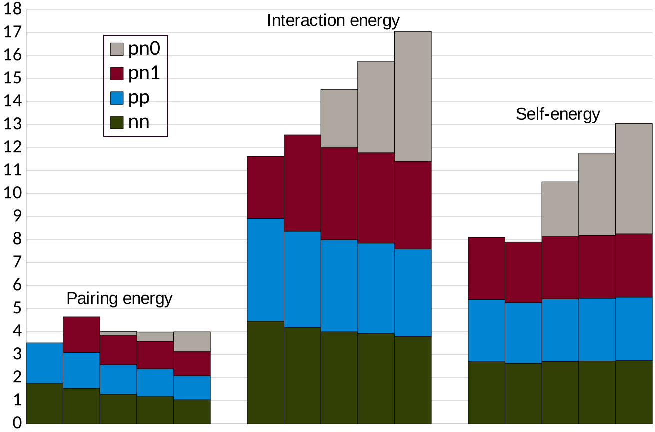


FIG. 5. The residuals, function of $A = N + Z$, for (left) $N = Z + 2$ and (right) $N = Z + 4$ nuclei.

PBCS and the QCM results for w={0.0, 1.0, 1.5, 2.0} in ^{64}Ge



$$\langle H_p \rangle - E^{Self} \quad , \quad \langle H_p \rangle \quad , E^{Self}_{n(p)} = \sum V^{T=1}(i,i) v^4_{i,n(p)}$$

Full Self-consistent Skyrme-QCM calculations

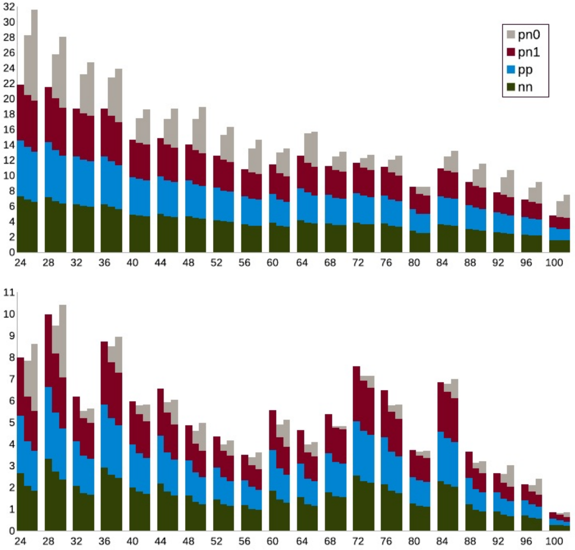


FIG. 4. Interaction energies (top) and pairing energies (bottom), in MeV, for $N = Z$ nuclei. For each nucleus are shown, from the left to the right, the results for $w = \{0.0, 1.5, 2.0\}$.

Full Self-consistent Skyrme-QCM calculations

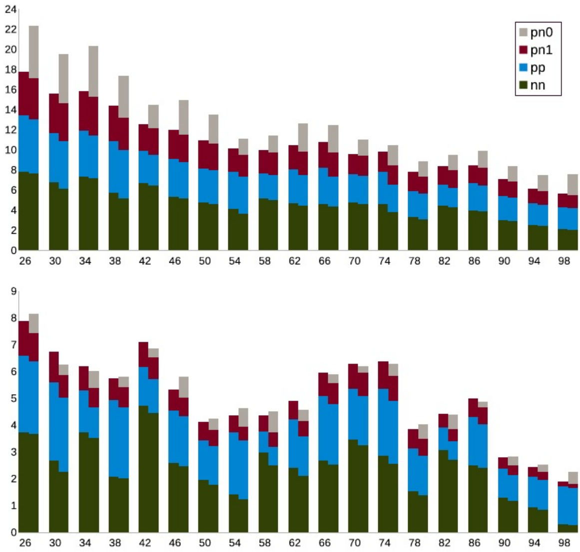


FIG. 6. Interaction energies (top) and pairing energies (bottom), in MeV, function of $A = N + Z$, for the nuclei with $N = Z + 2$. For each nucleus are shown, from left to right, the results for $w = \{0.0, 2.0\}$.

Full Self-consistent Skyrme-QCM calculations

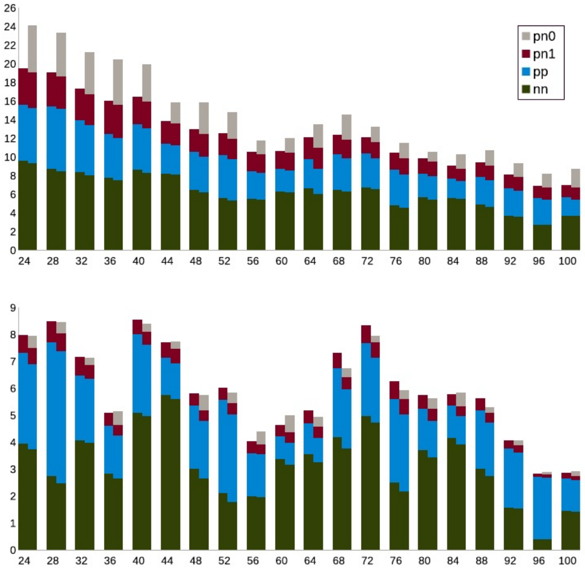


FIG. 7. The same as in Fig. 6 but for the nuclei with $N = Z + 4$.

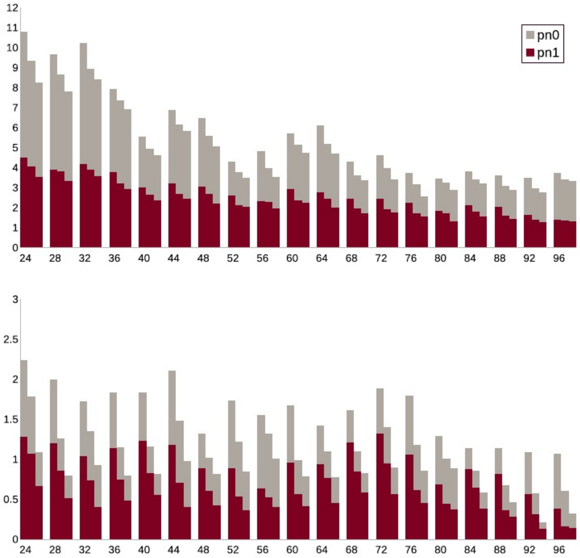


FIG. 8. Interaction energies (top) and pairing energies (bottom) for $T=0$ and $T=1$ pn pairing for $w=2.0$. The results, from left to right, are for the nuclei with $N=Z$, $N=Z+2$, and $N=Z+4$. On the x axis is shown the atomic mass of $N=Z$ nuclei.

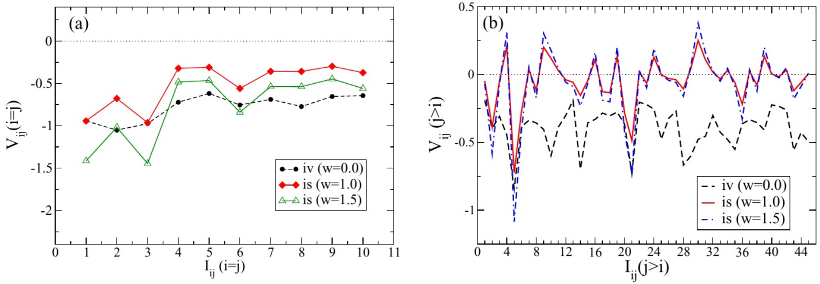


FIG. 3. (a) Diagonal and (b) nondiagonal matrix elements of the isovector and isoscalar pairing force for ^{64}Ge. The quantity I_{ij} enumerates the pair indices of V_{ij}.

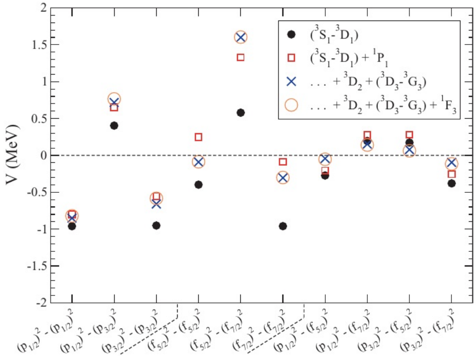


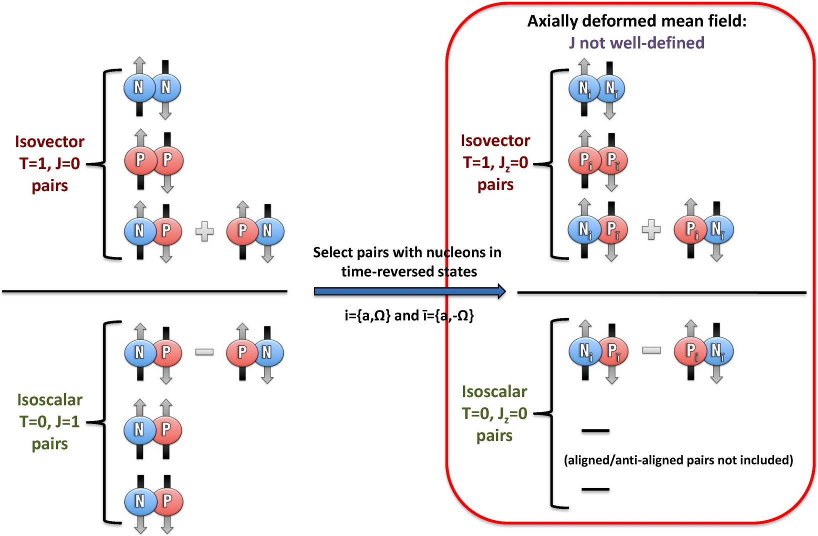
FIG. 13. (Color online) As Fig. 12 but for the pf shell, where partial waves up to the $^3G_3$ channel can contribute to $T = 0$, $J = 1$ pairing matrix elements.

From S. Baroni, A. Macchiavelli, and Achim Schwenk, PRC 81, 064308 (2010)

Take-Home messages

- T=1 and T=0 pairing impact on binding energies (B.E). of N = Z, N = Z + 2 and N = Z + 4 nuclei
- First ever self-consistent Skyrme-HF + QCM calculations
- All pairing channels and deformation are described consistently
- Main result: strong interdependence between all types of pairing correlations (e.g. pairing correlations are redistributed among all the pairing channels without changing significantly the total pairing energy)
- B.E are not much affected by T=0 pairing, but two channels cohesistence is always observed
- Cohesistence survives also moving away from N=Z

Isovector and isoscalar pairs in N=Z nuclei



Outlook and Future Work

- Only T=0 proton-neutron pairs in time-reversed states (common choice). Effect of proton-neutron pairs with $S = 1, S_z = \pm 1$ should be investigated
- Calculations are done in the intrinsic system, the ground states do not have a well-defined angular momentum (e.g. Projection on good J)
- Quark-Meson Coupling Model EDF calculations (p-h and p-p channels consistently provided) plus QCM (in progress, T. Popa)

Thanks For Your Attention !!!

Pairing and quartetting in odd-odd N=Z nuclei

Isovector T=1, J_z=0 pairs | **Isoscalar T=0, J_z=0 pairs**

$$P^+_{i,0} = \frac{1}{\sqrt{2}}(\nu^+_i \pi^+_{\bar i} + \pi^+_i \nu^+_{\bar i}) \qquad P^+_{i,1} = \nu^+_i \nu^+_{\bar i} \qquad P^+_{i,-1} = \pi^+_i \pi^+_{\bar i} \qquad D^+_{i,0} = \frac{1}{\sqrt{2}}(\nu^+_i \pi^+_{\bar i} - \pi^+_i \nu^+_{\bar i})$$

Hamiltonian
$$\hat H = \sum_{i,\tau=\pm1/2} \varepsilon_{i,\tau} N_{i,\tau} + \underbrace{\sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^+_{i,t_z} P_{j,t_z}}_{\text{isovector}} + \underbrace{\sum_{i,j} V^{T=0}(i,j) D^+_{i,J_z=0} D_{j,J_z=0}}_{\text{isoscalar}}$$

Isovector-isoscalar quartets

$$\Delta^+_0 = \textstyle\sum_i y_i D^+_{i,0} \qquad Q^+ = 2\Gamma^+_1\Gamma^+_{-1} - \Gamma^{+2}_0 + \Delta^{+2}_0 \qquad \Gamma^+_t = \textstyle\sum_i x_i P^+_{i,t}$$

T=0 state: $|is, QCM\rangle = \tilde\Delta^+_0 (Q^+)^{n_q}|0\rangle$

(exact solution for degenerate states)

collective isoscalar odd pair $\quad \tilde\Delta^+_0 = \sum_i z_i D^+_{i,0}$

T=1 state: $|iv, QCM\rangle = \tilde\Gamma^+_0 (Q^+)^{n_q}|0\rangle$

(exact solution for degenerate states)

collective isovector odd pair $\quad \tilde\Gamma^+_0 = \sum_i z_i P^+_{i,0}$

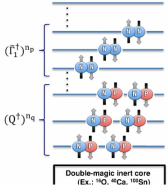
Pairing and quartetting in even-even N > Z nuclei

Isovector-isoscalar quartets

$$\Gamma^+_t = \textstyle\sum_i x_i P^+_{i,t} \qquad Q^+ = 2\Gamma^+_1\Gamma^+_{-1} - \Gamma^{+2}_0 + \Delta^{+2}_0 \qquad \Delta^+_0 = \textstyle\sum_i y_i D^+_{i,0}$$

$$|QCM\rangle = (\tilde\Gamma^\dagger_1)^{n_p}(Q^\dagger)^{n_q}|0\rangle$$

Collective nn pairs $\quad \tilde\Gamma^\dagger_1 = \sum_i z_i P^+_{i,t=1}$

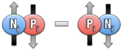


Pairing in even-even N=Z nuclei: axially deformed symmetry

Isovector T=1, J_z=0 pairs

$$P^+_{i,0} = \frac{1}{\sqrt{2}}(\nu^+_i \pi^+_{\bar{i}} + \pi^+_i \nu^+_{\bar{i}}) \qquad P^+_{i,1} = \nu^+_i \nu^+_{\bar{i}} \qquad P^+_{i,-1} = \pi^+_i \pi^+_{\bar{i}}$$

Isoscalar T=0, J_z=0 pairs



$$D^+_{i,0} = \frac{1}{\sqrt{2}}(\nu^+_i \pi^+_{\bar{i}} - \pi^+_i \nu^+_{\bar{i}})$$

Hamiltonian

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \underbrace{\sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^+_{i,t_z} P_{j,t_z}}_{\text{isovector}} + \underbrace{\sum_{i,j} V^{T=0}(i,j) D^+_{i,j_z=0} D_{j,j_z=0}}_{\text{isoscalar}}$$

Isovector quartets

$$A^+ = \underbrace{2\Gamma^+_1 \Gamma^+_{-1} - \Gamma^{+2}_0}$$

$$\Gamma^+_t = \sum_i x_i P^+_{i,t}$$

Isovector-isoscalar quartet

$$Q^+ = 2\Gamma^+_1 \Gamma^+_{-1} - \Gamma^{+2}_0 + \Delta^{+2}_0$$

$\Downarrow$

Collective isoscalar pairs

$$\Delta^+_0 = \sum_i y_i D^+_{i,0}$$

Quartet condensate

$$|QCM\rangle = (Q^+)^{n_q}|0\rangle$$

$$n_q = (N+Z)/4$$

(exact solution for a set of degenerate states)

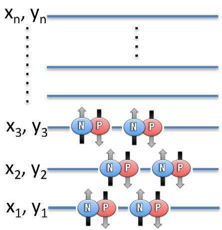
Calculation scheme

Hamiltonian: $\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^{+}_{i,t_z} P_{j,t_z} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,j_z=0} D_{j,j_z=0}$

Quartet condensate: $|QCM\rangle = (Q^{+})^{n_q}|0\rangle = (2\Gamma_1^{+}\Gamma_{-1}^{+} - \Gamma_0^{+2} + \Delta_0^{+2})^{n_q}|0\rangle$

$\Gamma_t^{+} = \sum_i x_i P^{+}_{i,t} \quad \Delta_0^{+} = \sum_i y_i D^{+}_{i,0}$

Unknown parameters: mixing amplitudes x_i and y_i



Minimization: $\delta_{x,y}\langle\Psi|\hat{H}|\Psi\rangle = 0$

Constraint: $\langle\Psi|\Psi\rangle = 1$

The method of recurrence relations

Auxiliary states: $|n_1 n_2 n_3 n_4\rangle = (\Gamma_1^{+})^{n_1}(\Gamma_{-1}^{+})^{n_2}(\Gamma_0^{+})^{n_3}(\Delta_0^{+})^{n_4}|0\rangle$

nn, pp, pn(T=1), pn(T=0)

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Proton–neutron pairing in $N = Z$ nuclei: Quartetting versus pair condensation

N. Sandulescu [a,*], D. Negrea [a], D. Gambacurta [b,c]

[a] National Institute of Physics and Nuclear Engineering, RO-077125 Măgurele, Romania
[b] Istituto Nazionale di Fisica Nucleare - Sezione di Catania, Via S. Sofia 64, I-95123 Catania, Italy
[c] Extreme Light Infrastructure - Nuclear Physics (ELI-NP), National Institute of Physics and Nuclear Engineering, RO-077125 Măgurele, Romania

QCM in N=Z nuclei

Competition between T=1 and T=0 pairing in realistic calculations

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,0} D_{j,0}$$

- s.p. states given by axially deformed Skyrme-HF calculations
- zero range delta interaction $V^{T=\{0,1\}}_{\text{pairing}}(\vec{r}_1 - \vec{r}_2) = V_0^{T=\{0,1\}} \delta(\vec{r}_1 - \vec{r}_2)\hat{P}_{S=\{0,1\}}$ $\left\{ \begin{array}{l} V_0^{T=1} = 465 \text{ MeV fm}^{-3} \\ V_0^{T=0}/V_0^{T=1} = 1.5 \end{array} \right.$ (Bertsch et al.)

$$|QCM\rangle = (A^+ + \Delta_0^{+2})^{n_q}|0\rangle \quad |iv\rangle = (A^+)^{n_q}|0\rangle \quad |is\rangle = (\Delta_0^{+2})^{n_q}|0\rangle$$

		Exact	|QCM>	$\|iv\rangle$	$\|is\rangle$	$\langle iv\|is\rangle$
^{16}O	^{20}Ne	11.38	11.38 (0.00%)	11.31 (0.62%)	10.92 (4.00%)	0.976
	^{24}Mg	19.32	19.31 (0.03%)	19.18 (0.74%)	18.93 (2.00%)	0.980
	^{28}Si	18.74	18.74 (0.01%)	18.71 (0.14%)	18.54 (1.07%)	0.992
^{40}Ca	^{44}Ti	7.095	7.094 (0.02%)	7.08 (0.18%)	6.30 (10.78%)	0.928
	^{48}Cr	12.78	12.76 (0.1%)	12.69 (0.67%)	12.22 (4.37%)	0.936
	^{52}Fe	16.39	16.34 (0.26%)	16.19 (1.17%)	15.62 (4.65%)	0.946
^{100}Sn	^{104}Te	4.53	4.52 (0.06%)	4.49 (0.82%)	4.02 (11.26%)	0.955
	^{108}Xe	8.08	8.03 (0.61%)	7.96 (1.45%)	6.75 (16.47%)	0.814
	^{112}Ba	9.36	9.27 (0.93%)	9.22 (1.43%)	7.50 (19.81%)	0.784

Correlation energies (MeV)

$E_{corr} = E_0 - E$

Conclusions:
- **QCM describes with very good precision the isoscalar-isovector pairing (errors under 1%);**
- **isovector pairing correlations are stronger than the isoscalar ones;**
- **isoscalar pairing coexist with the isovector pairing.**

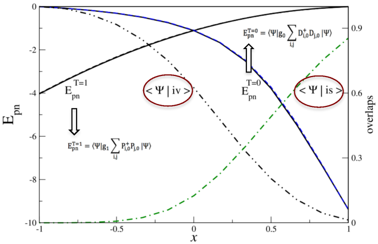
N. Sandulescu, D. N. and D. Gambacurta, Phys. Lett. B751 (2015), p. 348

Evolution of the isovector and isoscalar proton-neutron pairing correlations

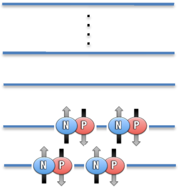
$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + g_1 \sum_{i,j} \sum_{t=-1,0,1} P^+_{i,t} P_{j,t} + g_0 \sum_{i,j} D^+_{i,0} D_{j,0}$$

$$\begin{cases} g_1 = g(1-x)/2 \\ g_0 = g(1+x)/2 \end{cases}$$

$$|\Psi\rangle = (A^+ + \Delta_0^{+2})^{n_q}|0\rangle \quad |iv\rangle = (A^+)^{n_q}|0\rangle \quad |is\rangle = (\Delta_0^{+2})^{n_q}|0\rangle$$



- 4 proton-neutron pairs;
- 10 equidistant levels.



Pairing energies:

- E(T=1) and E(T=0) follow very well the exact pairing energies (obtained by diagonalization);
- isovector and isoscalar pairing correlations coexist for any ratio between the strengths of the two pairing forces.

Overlaps:

- the overlaps show a smooth transition from a condensate of quartets to a condensate of pairs.

N. Sandulescu, D. N. and D. Gambacurta, Phys. Lett. B751 (2015), p. 348

PTEP

Prog. Theor. Exp. Phys. **2017**, 073D05 (10 pages)
DOI: 10.1093/ptep/ptx071

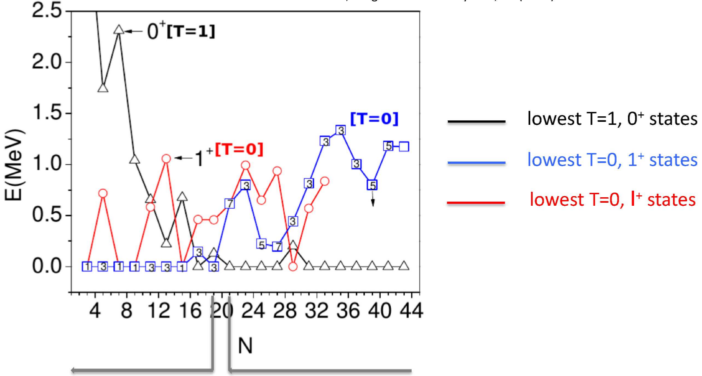
Isovector and isoscalar pairing in odd–odd $N = Z$ nuclei within a quartet approach

D. Negrea[1,*], N. Sandulescu[1,*], and D. Gambacurta[2,*]

[1] *National Institute of Physics and Nuclear Engineering, 76900 Bucharest-Magurele, Romania*
[2] *Extreme Light Infrastructure—Nuclear Physics (ELI-NP), 76900 Bucharest-Magurele, Romania*
[*] E-mail: negrea.daniel@theory.nipne.ro; sandulescu@theory.nipne.ro; danilo.gambacurta@eli-np.ro

Proton-neutron pairing in odd-odd nuclei

S. Frauendorf and A. O. Macchiavelli, Progr. Part. Nucl. Phys. 78, 24 (2014)



A<40
T=0 ground state

A≥40
T=1 ground state

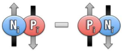
⟹ **Competition between T=1 and T=0 pairing !**

Pairing and quartetting in odd-odd N=Z nuclei

Isovector T=1, J_z=0 pairs

$$P^+_{i,0} = \frac{1}{\sqrt{2}}(\nu^+_i \pi^+_{\bar{i}} + \pi^+_i \nu^+_{\bar{i}}) \qquad P^+_{i,1} = \nu^+_i \nu^+_{\bar{i}} \qquad P^+_{i,-1} = \pi^+_i \pi^+_{\bar{i}}$$

Isoscalar T=0, J_z=0 pairs



$$D^+_{i,0} = \frac{1}{\sqrt{2}}(\nu^+_i \pi^+_{\bar{i}} - \pi^+_i \nu^+_{\bar{i}})$$

Hamiltonian

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \underbrace{\sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^+_{i,t_z} P_{j,t_z}}_{\text{isovector}} + \underbrace{\sum_{i,j} V^{T=0}(i,j) D^+_{i,J_z=0} D_{j,J_z=0}}_{\text{isoscalar}}$$

Isovector-isoscalar quartets

$$\Delta^+_0 = \sum_i y_i D^+_{i,0} \qquad Q^+ = 2\Gamma^+_1 \Gamma^+_{-1} - \Gamma^{+2}_0 + \Delta^{+2}_0 \qquad \Gamma^+_t = \sum_i x_i P^+_{i,t}$$

T=0 state: $|is, QCM\rangle = \tilde{\Delta}^+_0 (Q^+)^{n_q} |0\rangle$

(exact solution for degenerate states)

collective isoscalar odd pair $\tilde{\Delta}^+_0 = \sum_i z_i D^+_{i,0}$

T=1 state: $|iv, QCM\rangle = \tilde{\Gamma}^+_0 (Q^+)^{n_q} |0\rangle$

(exact solution for degenerate states)

collective isovector odd pair $\tilde{\Gamma}^+_0 = \sum_i z_i P^+_{i,0}$

Calculation scheme

Hamiltonian: $\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^{+}_{i,t_z} P_{j,t_z} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,j_z=0} D_{j,j_z=0}$

Pair-quartet condensate:

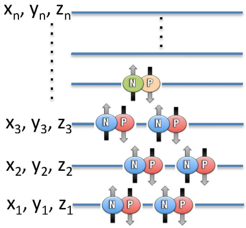
(T=0 state)

$|is, QCM\rangle = \tilde{\Delta}^{+}_0 (2\Gamma^{+}_1 \Gamma^{+}_{-1} - \Gamma^{+2}_0 + \Delta^{+2}_0)^{n_q} |0\rangle$ $\quad \tilde{\Delta}^{+}_0 = \sum_i z_i D^{+}_{i,0}$ $\quad \Gamma^{+}_t = \sum_i x_i P^{+}_{i,t}$

$|iv, QCM\rangle = \tilde{\Gamma}^{+}_0 (2\Gamma^{+}_1 \Gamma^{+}_{-1} - \Gamma^{+2}_0 + \Delta^{+2}_0)^{n_q} |0\rangle$ $\quad \tilde{\Gamma}^{+}_0 = \sum_i z_i P^{+}_{i,0}$ $\quad \Delta^{+}_0 = \sum_i y_i D^{+}_{i,0}$

(T=1 state)

Unknown parameters: mixing amplitudes xi, yi and z_i



Minimization: $\delta_{x,y,z} \langle \Psi | \hat{H} | \Psi \rangle = 0$

Constraint: $\langle \Psi | \Psi \rangle = 1$

The method of recurrence relations

Auxiliary states:

$|n_1 n_2 n_3 n_4 n_5\rangle = (\Gamma^{+}_1)^{n_1} (\Gamma^{+}_{-1})^{n_2} (\Gamma^{+}_0)^{n_3} (\Delta^{+}_0)^{n_4} \tilde{\Delta}^{+}_0 |0\rangle$ (T=0 state)

$|m_1 m_2 m_3 m_4 m_5\rangle = (\Gamma^{+}_1)^{m_1} (\Gamma^{+}_{-1})^{m_2} (\Gamma^{+}_0)^{m_3} (\Delta^{+}_0)^{m_4} \tilde{\Gamma}^{+}_0 |0\rangle$ (T=1 state)

Strength of the pairing force in T=1 and T=0 channels

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^{+}_{i,t_z} P_{j,t_z} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,j_z=0} D_{j,j_z=0}$$

- **s.p. states given by Skyrme-HF calculations for axially deformed m.f.**
- **zero range delta interaction** $V^{T=\{0,1\}}_{pairing}(\vec{r}_1 - \vec{r}_2) = V_0^{T=\{0,1\}} \delta(\vec{r}_1 - \vec{r}_2) \hat{P}_{S=\{0,1\}}$

Strength of the isovector pairing force

$$V_0^{T=1} = 465 \text{ MeV fm}^{-3}$$

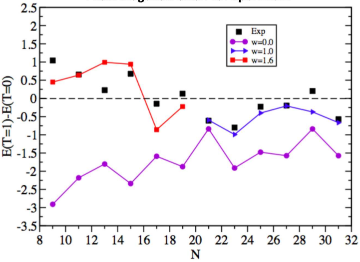
Strength of the isoscalar pairing force

$$V_0^{T=0} = w\, V_0^{T=1}$$

w = ?

A<40: w=1.6 **A≥40: w=1.0**

Exact diagonalization vs Experiment



The structure of the lowest T=0 and T=1 states of odd-odd nuclei

Correlation energies (MeV): $E_{corr} = E_0 - E$

T=0 ground state

		Exact	$\tilde{\Delta}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}$	$\tilde{\Delta}_0^+(Q_{iv}^+)^{n_q}$	$(\Delta_0^+)^{2n_q+1}$	$\tilde{\Delta}_0^+(\Gamma_0^{+2})^{n_q}$
^{30}P	T=0	12.66	12.60 (0.44%)	12.55 (0.86%)	11.96 (5.86%)	11.94 (5.95%)

T=1 ground state

		Exact	$\tilde{\Gamma}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}$	$\tilde{\Gamma}_0^+(Q_{iv}^+)^{n_q}$	$\tilde{\Gamma}_0^+(\Delta_0^{+2})^{n_q}$	$(\Gamma_0^+)^{2n_q+1}$
^{54}Co	T=1	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)

D.N., N. Sandulescu, D. Gambacurta, PTEP 2017, 073D05

Conclusions:

QCM describes well the low-lying states of odd-odd nuclei.

The pn pair condensates (isovector or isoscalar) are less accurate than the quartet condensates.

Isovector and isoscalar pairing correlations coexist in the even-even core.

^{50}Mn	**Exact**	$\tilde{\Gamma}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}$	$\tilde{\Gamma}_0^+(Q_{iv}^+)^{n_q}$	$\tilde{\Gamma}_0^+(\Delta_0^{+2})^{n_q}$	$(\Gamma_0^+)^{2n_q+1}$
$T=1$	12.77	12.76 (0.07%)	12.75 (0.14%)	12.52 (2.02%)	12.62 (1.22%)
$T=0$	12.37	12.36 (0.04%)	12.34 (0.24%)	12.18 (1.61%)	12.19 (1.48%)
	Exact	$\tilde{\Delta}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}$	$\tilde{\Delta}_0^+(Q_{iv}^+)^{n_q}$	$(\Delta_0^+)^{2n_q+1}$	$\tilde{\Delta}_0^+(\Gamma_0^{+2})^{n_q}$

^{54}Co	$T=1$	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)
	$T=0$	15.93	15.92 (0.04%)	15.89 (0.22%)	15.53 (2.56%)	15.66 (1.73%)

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Isovector and isoscalar proton-neutron pairing in $N > Z$ nuclei

D. Negrea,[1] P. Buganu,[1] D. Gambacurta,[2] and N. Sandulescu[1,*]
[1] *National Institute of Physics and Nuclear Engineering, 077125 Măgurele, Romania*
[2] *Extreme Light Infrastructure - Nuclear Physics (ELI-NP), National Institute of Physics and Nuclear Engineering, 077125 Măgurele, Romania*

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Pairing and quartetting in even-even N > Z nuclei

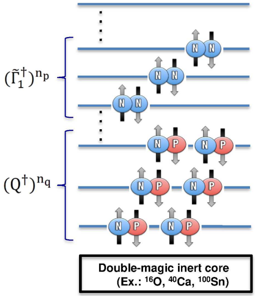
Isovector-isoscalar quartets

$\Gamma_t^+ = \sum_i x_i P_{i,t}^+$ $\quad Q^+ = 2\Gamma_1^+\Gamma_{-1}^+ - \Gamma_0^{+2} + \Delta_0^{+2}$ $\quad \Delta_0^+ = \sum_i y_i D_{i,0}^+$

$$|QCM\rangle = (\tilde{\Gamma}_1^\dagger)^{n_p} (Q^\dagger)^{n_q} |0\rangle$$

Collective nn pairs $\tilde{\Gamma}_1^\dagger = \sum_i z_i P_{i,t=1}^+$

$\tilde{\Gamma}_1^\dagger$ and Γ_1^+ have different structures: $z_i \neq x_i$



Calculation scheme

Hamiltonian: $\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^{+}_{i,t_z} P_{j,t_z} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,j_z=0} D_{j,j_z=0}$

Pair-quartet condensate: $|QCM\rangle = (\tilde{\Gamma}_1^{\dagger})^{n_p} \left(2\Gamma_1^{\dagger}\Gamma_{-1}^{\dagger} - \Gamma_0^{\dagger 2} + \Delta_0^{\dagger 2}\right)^{n_q} |0\rangle$

Unknown parameters: mixing amplitudes xi, yi and z_i

$$\Gamma_t^{+} = \Sigma_i x_i P^{+}_{i,t}$$
$$\Delta_0^{+} = \Sigma_i y_i D^{+}_{i,0}$$
$$\tilde{\Gamma}_1^{\dagger} = \sum_i z_i P^{+}_{i,t=1}$$

Minimization: $\delta_{x,y,z} \langle \Psi | \hat{H} | \Psi \rangle = 0$

Constraint: $\langle \Psi | \Psi \rangle = 1$

The method of recurrence relations

Auxiliary states: $|n_1 n_2 n_3 n_4 n_5\rangle = (\Gamma_1^{\dagger})^{n_1} (\Gamma_{-1}^{\dagger})^{n_2} (\Gamma_0^{\dagger})^{n_3} (\Delta_0^{\dagger})^{n_4} (\tilde{\Gamma}_1^{\dagger})^{n_5} |0\rangle$

Strength of the pairing force in T=1 and T=0 channels

$$\hat{H} = \hat{H}_0 + \hat{H}_p$$

$$\hat{H}_0 = \sum_{i,\tau=\pm 1} \varepsilon_{i,\tau} N_{i,\tau}$$ - s.p. states given by Skyrme-HF calculations for axially deformed m.f.

Pairing interactions:

(I) State-independent pairing force

$$\hat{H}_p = \boxed{V^{(T=1)}} \sum_{\substack{i,j,\\ t=-1,0,1}} P^\dagger_{i,t} P_{j,t} + \boxed{V^{(T=0)}} \sum_{i,j} D^\dagger_{i,0} D_{j,0}$$

Strength of the isovector pairing force

$$V^{(T=1)} = -24/A$$

Strength of the isoscalar pairing force

$$V^{(T=0)} = \boxed{w \cdot} V^{(T=1)}$$

w = ? ⟹
- sd-shell nuclei: w=1.2
- heavier nuclei: w=0.8

(II) Zero range delta interaction

$$\hat{H}_p = \sum_{i,j} V^{(T=1)}_{i,j} \sum_{t=-1,0,1} P^\dagger_{i,t} P_{j,t} + \sum_{i,j} V^{(T=0)}_{i,j} D^\dagger_{i,0} D_{j,0}$$

$$V^{T=\{0,1\}}_{pairing}(\vec{r}_1 - \vec{r}_2) = \boxed{V^{T=\{0,1\}}_0} \delta(\vec{r}_1 - \vec{r}_2) \hat{P}_{S=\{0,1\}}$$

Strength of the isovector pairing force

$$V^{T=1}_0 = 465 \ \ \mathrm{MeV\ fm^{-3}}$$

Strength of the isoscalar pairing force

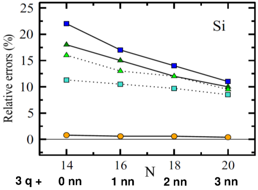
$$V^{T=0}_0 = \boxed{w \cdot} V^{T=1}_0$$

w = ? ⟹
- sd-shell nuclei: w=1.6
- heavier nuclei: w=1.0

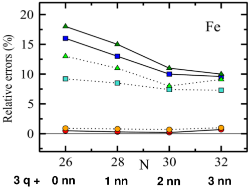
QCM in N>Z nuclei

Isovector and isoscalar pn pairing in N > Z nuclei: results (I)

sd-shell nucleus



pf-shell nucleus



Correlation energies (MeV): $E_{corr} = E_0 - E$

(I) State-independent pairing force **(II) Zero range delta interaction**

$$|C_{iv}\rangle = (\tilde{\Gamma}_1^\dagger)^{n_p}\left(\Gamma_0^{\dagger 2}\right)^{n_q}|0\rangle$$

$$|C_{is}\rangle = (\tilde{\Gamma}_1^\dagger)^{n_p}\left(\Delta_0^{\dagger 2}\right)^{n_q}|0\rangle$$

$$|QCM\rangle = (\tilde{\Gamma}_1^\dagger)^{n_p}(Q^\dagger)^{n_q}|0\rangle$$

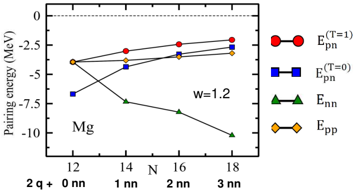
|QCM> describes well the ground state pairing correlations (errors < 1%).

|Civ> and |Cis> (larger errors): not a fast transition to a pure condensate of iv/is pn pairs.

D. N., P. Buganu, D. Gambacurta, and N. Sandulescu, Phys. Rev. C98, 064319 (2018)

Isovector and isoscalar pn pairing in N > Z nuclei: results (II)

sd-shell nucleus



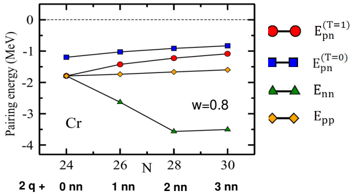
State-independent pairing force

Pairing energies (MeV):

$$E_t^{(T=1)} = V^{(T=1)} \sum_{i,j,t} \langle QCM | P^\dagger_{i,t} P_{j,t} | QCM \rangle$$

$$E_{pn}^{(T=0)} = V^{(T=0)} \sum_{i,j} \langle QCM | D^\dagger_{i,0} D_{j,0} | QCM \rangle$$

pf-shell nucleus



Pn pairing energies are decreasing, but remain significantly large even when 3 extra nn pairs are added.

Isovector and isoscalar pn pairing correlations coexist in both N = Z and N > Z nuclei.

D. N., P. Buganu, D. Gambacurta, and N. Sandulescu, Phys. Rev. C98, 064319 (2018)