Probing alpha-clustering in N=Z nuclei by elastic alpha scattering
(work in progress)

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Outline

1. **Motivation;**
2. **Partial wave analysis based on optical potential models;**
3. **Folding model potential;**
4. **Preliminary results for α - ^{40}Ca elastic scattering;**
5. **Outlook and open questions.**

Motivation

- **Experimental data** [1] reveal at low energies an **anomalous large angle scattering** (ALAS) - strong enhancement of the cross section at backward angles - in the elastic alpha scattering on light N=Z nuclei.

  [1] G. Gaul *et al.*, Nucl. Phys. A137 (1969) 177.

- These **data cannot be reasonably explained** in the standard approach of alpha scattering based on an optical potential model.

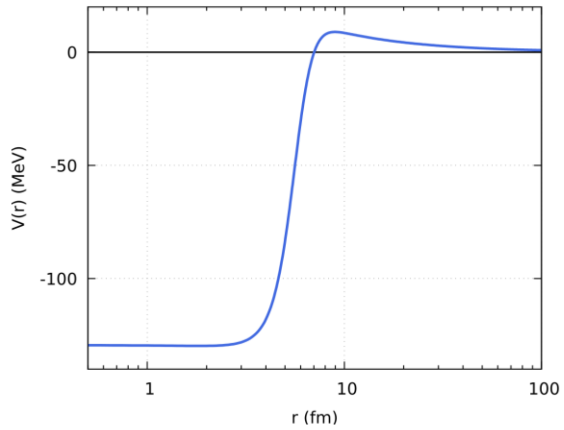
- Several studies [2] indicate that ALAS is **related to the alpha-like correlations** in N=Z nuclei.

  [2] N. C. Schmeing, Nucl. Phys. A142 (1970) 449.

Thus, these intriguing observations represent a promising premise for further exploration.

Partial wave analysis employing global optical potential models

Partial wave expansion of the scattering amplitude



- Solve numerically the radial Schrödinger equation:

$$\left[-\frac{\hbar^2}{2\mu_r}\nabla^2 + V(r)\right]\psi_{\boldsymbol{k}}(\boldsymbol{r}) = E_{\boldsymbol{k}}\psi_{\boldsymbol{k}}(\boldsymbol{r})$$

for an alpha particle in an effective potential of the form:

$$V(r,E) = V_{\mathrm{C}}(r) + V_{\mathrm{R}}(r,E) + i\left[W_{\mathrm{V}}(r,E) + W_{\mathrm{S}}(r,E)\right]$$

- Expand the wavefunction as a distorted wave:

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) = \frac{4\pi}{kr}\sum_{\ell=0}^{\infty} \mathrm{i}^{\ell} e^{\mathrm{i}\boxed{\delta_\ell}} P_\ell(kr) \sum_{m=-\ell}^{\ell} Y_{\ell m}(\hat{\boldsymbol{r}}) Y^*_{\ell m}(\hat{\boldsymbol{k}})$$

phase shifts

- Luckily, the scattering amplitude depends only on the phase shifts:

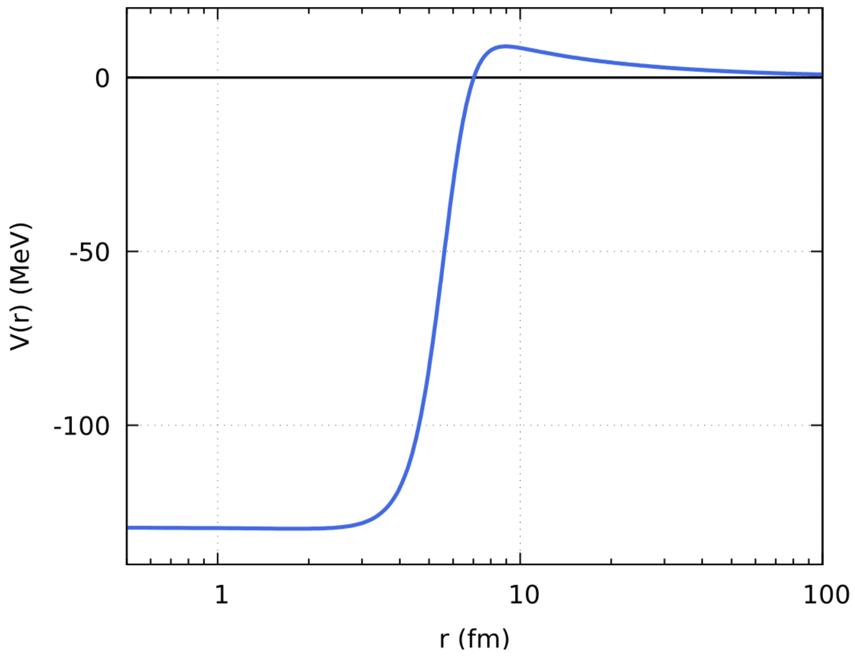
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\hat{\Omega}} = |f(\theta)|^2 \qquad f(\theta) = \frac{1}{2\mathrm{i}k}\sum_{\ell=0}^{\infty} \mathrm{i}^{\ell}(2\ell+1)\left(\mathrm{e}^{\mathrm{i}2\delta_\ell} - 1\right) P_\ell(\cos\theta)$$

DWBA angular distributions for α elastic scattering on nuclei

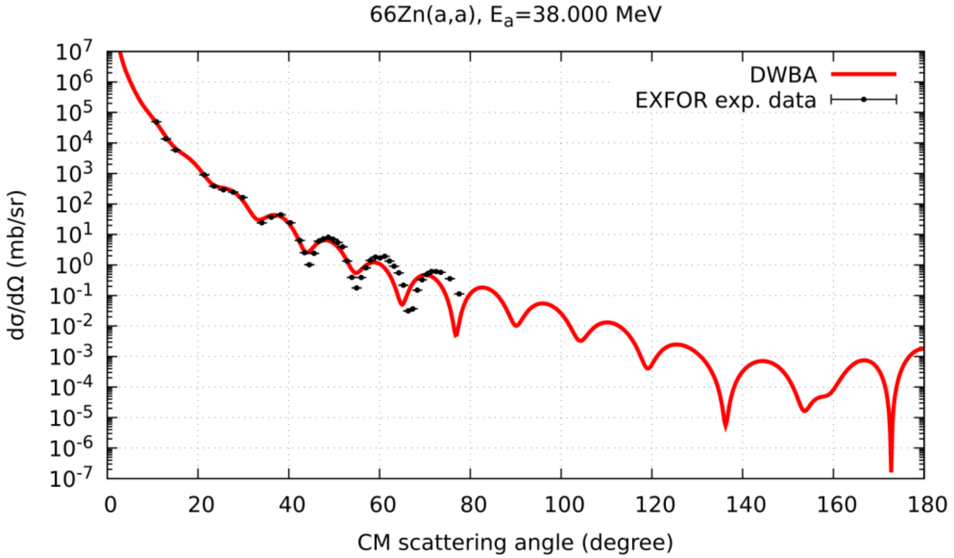
Custom DWBA code → **Solve numerically radial Schrödinger eq.** employing the **RADIAL Fortran subroutine package** [3] → **Phase shifts** → **Differential cross section** (dxs)

$$\frac{d\sigma}{d\hat{\Omega}} = |f(\theta)|^2$$

$$f(\theta) = \frac{1}{2ik}\sum_{\ell=0}^{\infty} i^{\ell}(2\ell+1)\left(e^{i2\delta_\ell}-1\right)P_\ell(\cos\theta)$$



DWBA = distorted wave Born approximation



(Global) OPM:

X.-W. Su and Y.-L. Han [4].

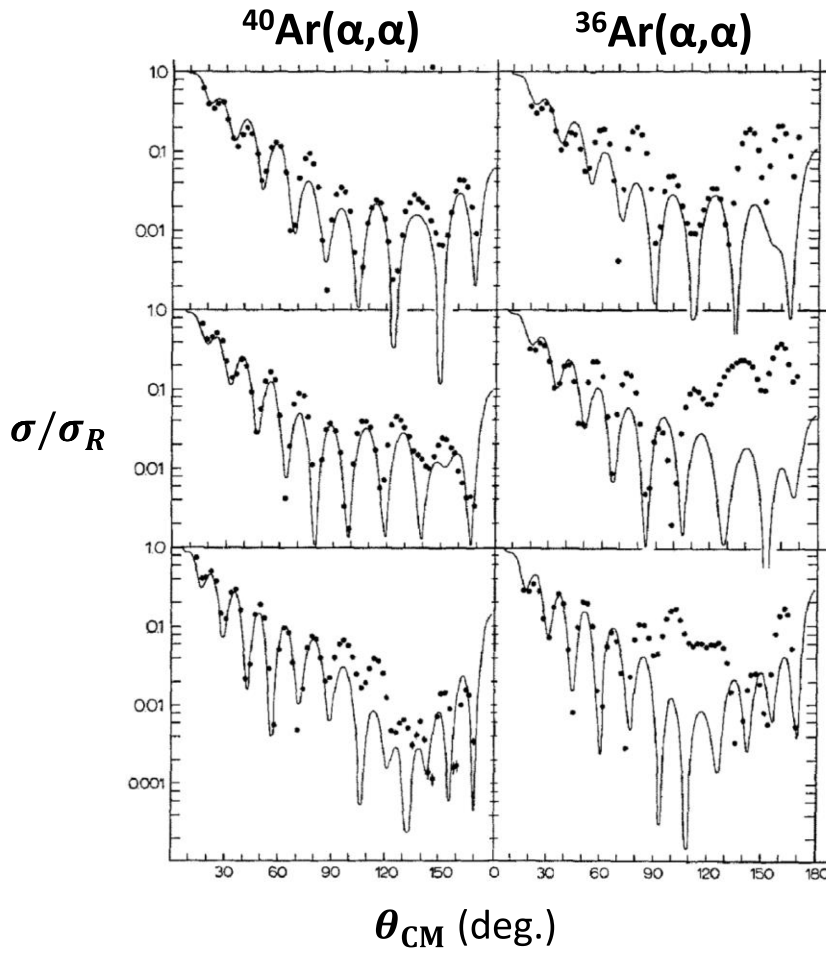
[3] F. Salvat, J. M. Fernandez-Varea, Comp. Phys. Comm. 240, 165-177 (2019).
[4] X.-W. Su, and Y.-L. Han, Int. J. Mod. Phys. E, vol. 24, no. 12 (2015).

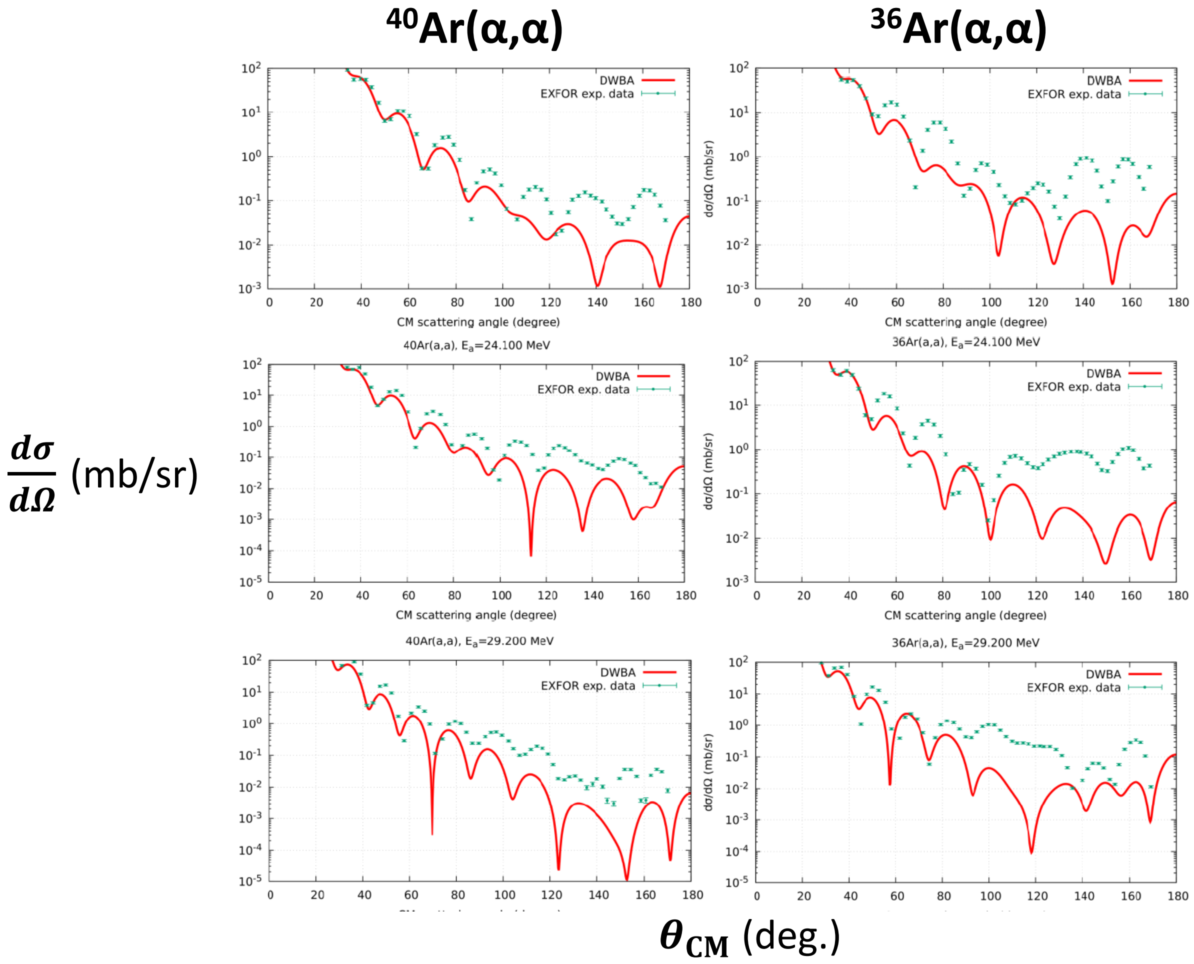
Experimental data vs. DWBA angular distributions

$E_\alpha = 22.1$ MeV

$E_\alpha = 24.1$ MeV

$E_\alpha = 29.2$ MeV





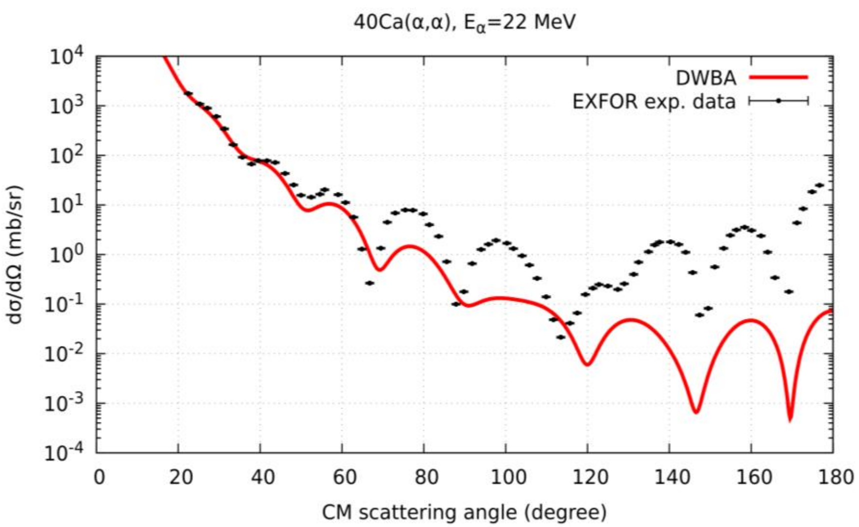
- **Anomalous large angle scattering** is **more accentuated for N=Z nuclei** as highlighted in Ref. [1] (shown on the left).
- Running the custom DWBA code using the OPM of Ref. [4], the **effect is also visible** (plots on the right).
- **The OPM does not reproduce well the backward scattering region!**

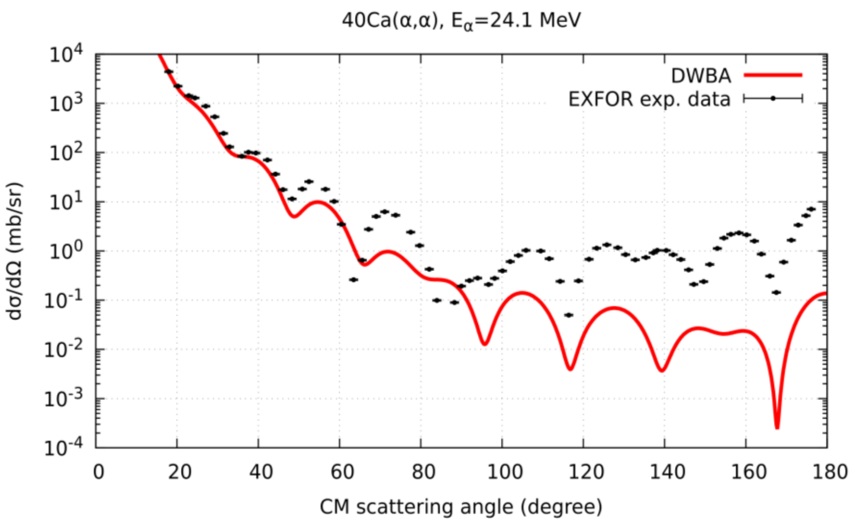
[1] G. Gaul et al., Nucl. Phys. A137 (1969) 177.
[4] X.-W. Su, and Y.-L. Han, Int. J. Mod. Phys. E, vol. 24, no. 12 (2015).

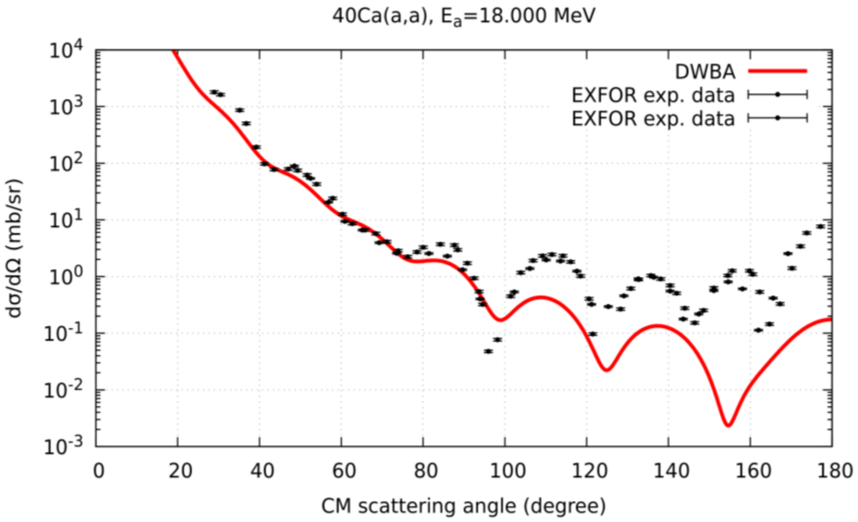
Beyond OPM: Folding model potential

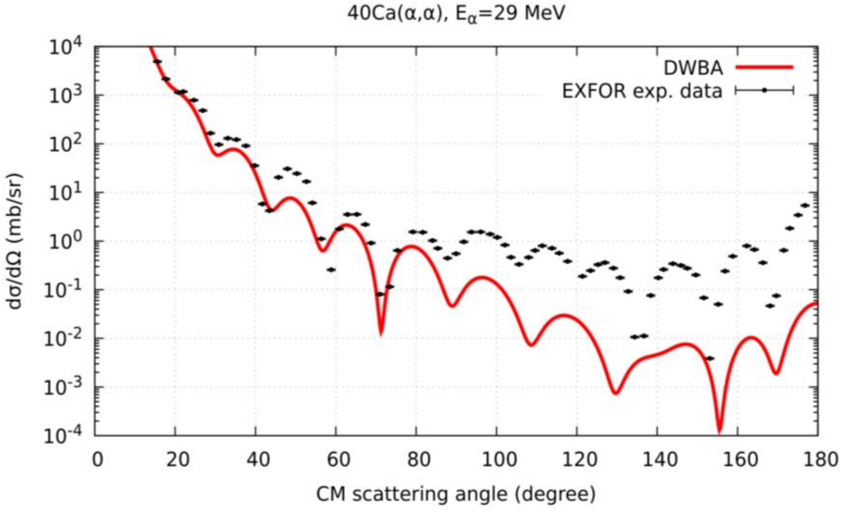
Study case: α – ^{40}Ca elastic scattering

Experimental data vs. DWBA predictions







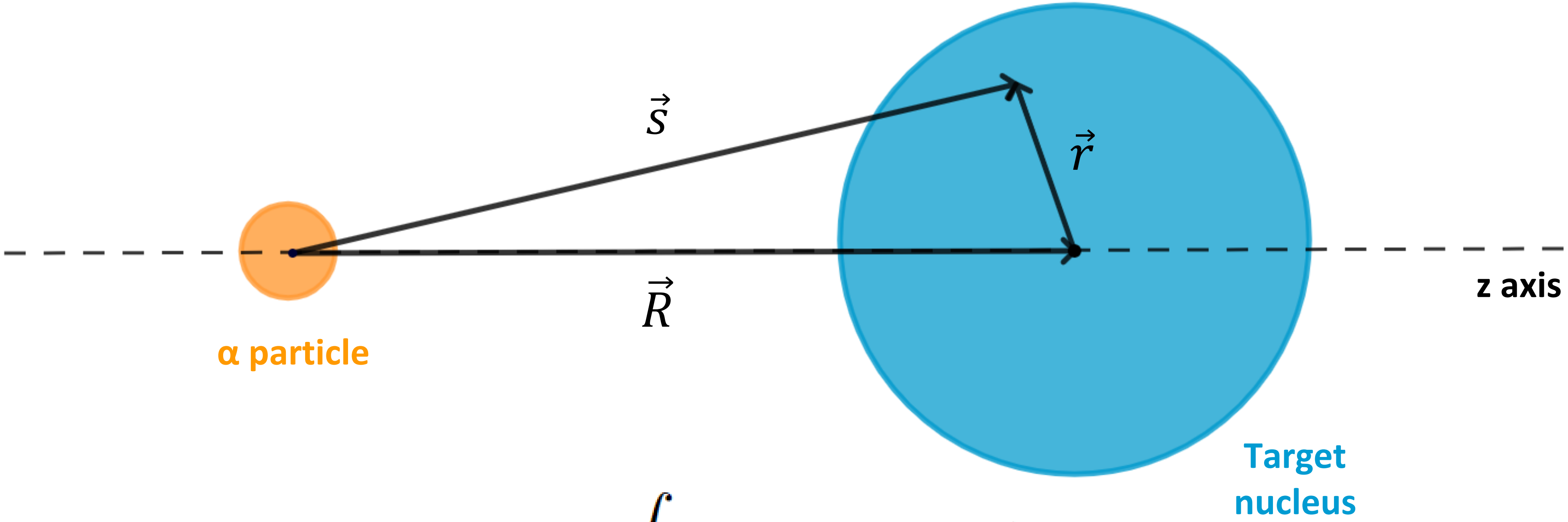


At low energies of the α particles on ^{40}Ca, the experimental data show the anomalous large angle scattering.

Based on OPM, the feature is not resolved.

Folding model potential analysis.

Single folding potential



$$V_{\alpha A}(R) = \int \underbrace{\rho(r)}_{\text{Target nuclear density}} \underbrace{v_{\alpha n}(s)}_{\alpha\text{ - nucleon potential}} d^3r$$

[5] G. R. Satchler, and W. G. Love, Phys. Rep. 55 (1979) 183-254

1st ingredient: α – nucleon potential

$$V_{\alpha A}(R) = \int \rho(r)\, v_{\alpha n}(s)\, d^3 r$$

Density independent α – nucleon potential:

$$\boxed{v_{\alpha n}(s) = -V_{\mathrm{F}}\, \exp\left(-\frac{s^2}{\alpha^2}\right) - i\, W_{\mathrm{F}}\, \exp\left(-\frac{s^2}{\alpha^2}\right)}$$

[6] F. E. Bertrand *et al.*, Phys. Rev. C22 (1980) 1832

$$\left.\begin{array}{l} V_{\mathrm{F}} = 37.4\ \mathrm{MeV},\ \text{and}\ W_{\mathrm{F}} = 21.5\ \mathrm{MeV}. \\ \alpha = 1.94\ \mathrm{fm}. \end{array}\right\}$$

Parameters fitted <u>only</u> for 141.7 MeV α particles on ^{40}Ca!

2^{nd} ingredient: target nuclear density

$$V_{\alpha A}(R) = \int \rho(r) v_{\alpha n}(s)\, d^3 r$$

^{40}Ca nuclear density based on 3 parameter Fermi (3pF) model:

$$\boxed{\rho(r) = \rho_0 \frac{1 + \frac{w r^2}{c^2}}{1 + \exp\left(\frac{r-c}{z}\right)}}$$

The constants are: $w = -0.161$, $c = 3.766$ fm, and $z = 0.586$ fm.

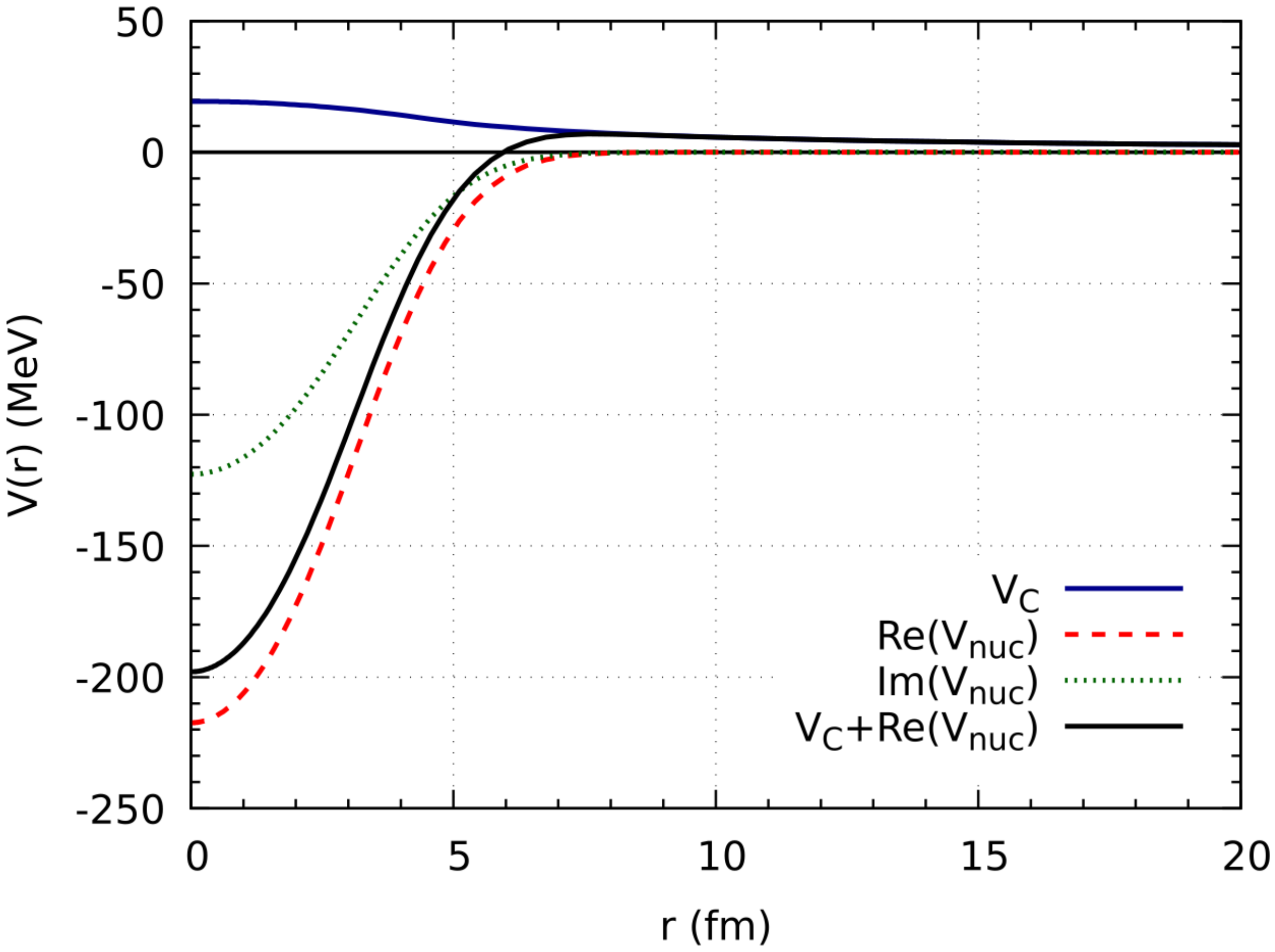
The normalization ρ_0 was determined such that,

$$\int \rho(r)\, d^3 r = A$$

[7] H. de Vries *et al*., Atom. Data Nucl. Data Tabl. 36 (1987) 495-536

α – ^{40}Ca interaction potential

PRELIMINARY RESULTS!



Obtained by
numerical integration

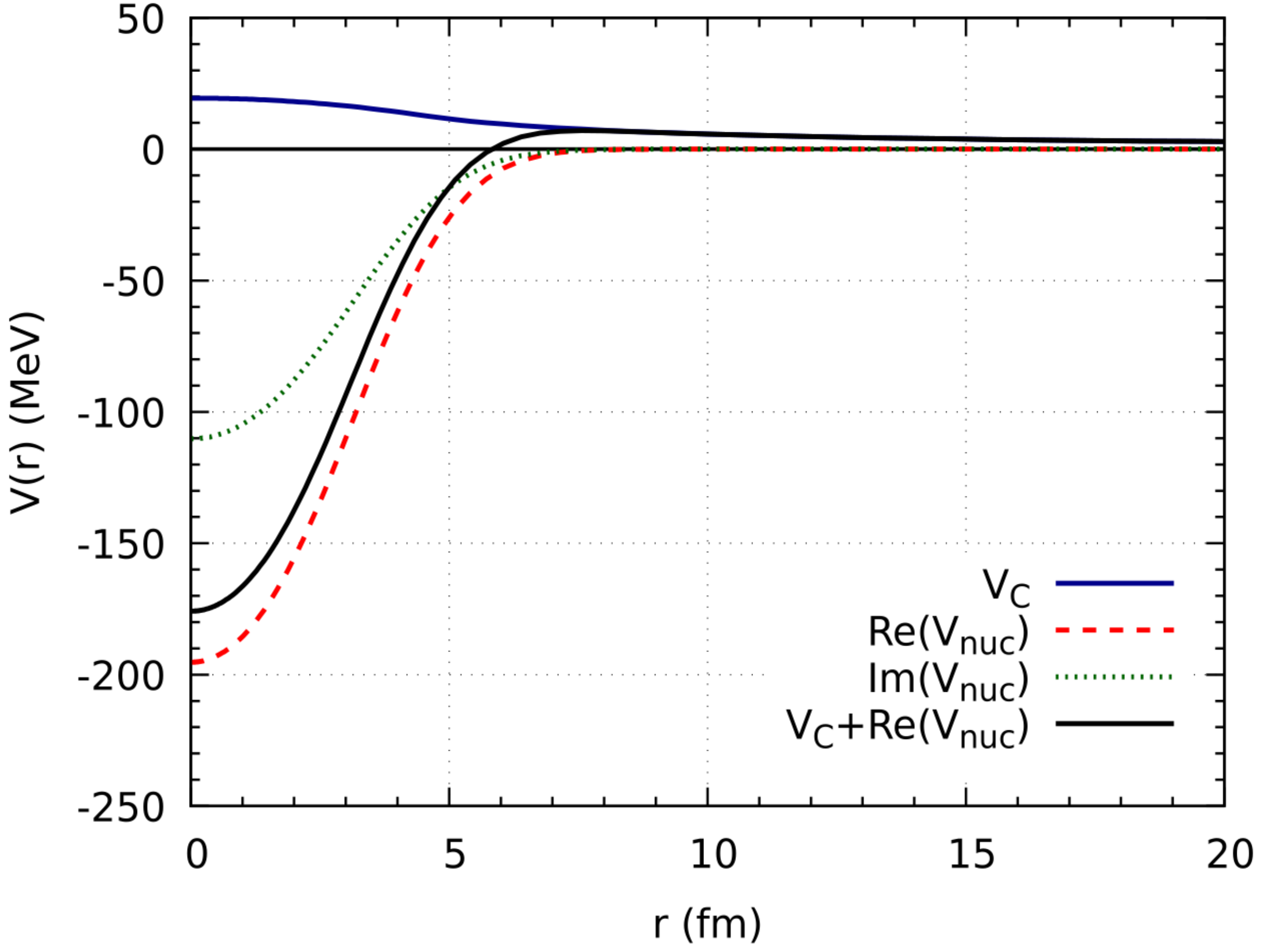
$$V(R) = V_{\mathrm{C}}(R) + V_{\alpha A}(R)$$

$$V_{\mathrm{C}}(R) = \begin{cases} \dfrac{zZe^2}{2R_{\mathrm{C}}}\left(3 - \dfrac{r^2}{R_C^2}\right) & R < R_{\mathrm{C}}, \\ \dfrac{zZe^2}{r} & R \geq R_{\mathrm{C}}. \end{cases}$$

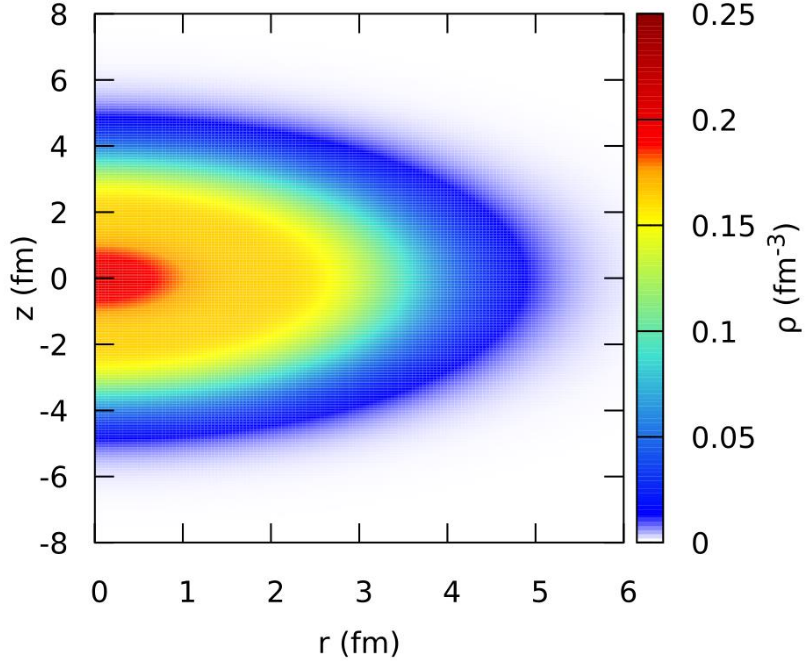
$$R_{\mathrm{C}} = r_{\mathrm{C}} A^{1/3} \qquad r_{\mathrm{C}} = 1.3 \text{ fm}$$

α – ^{40}Ca interaction potential – RMF density

PRELIMINARY RESULTS!



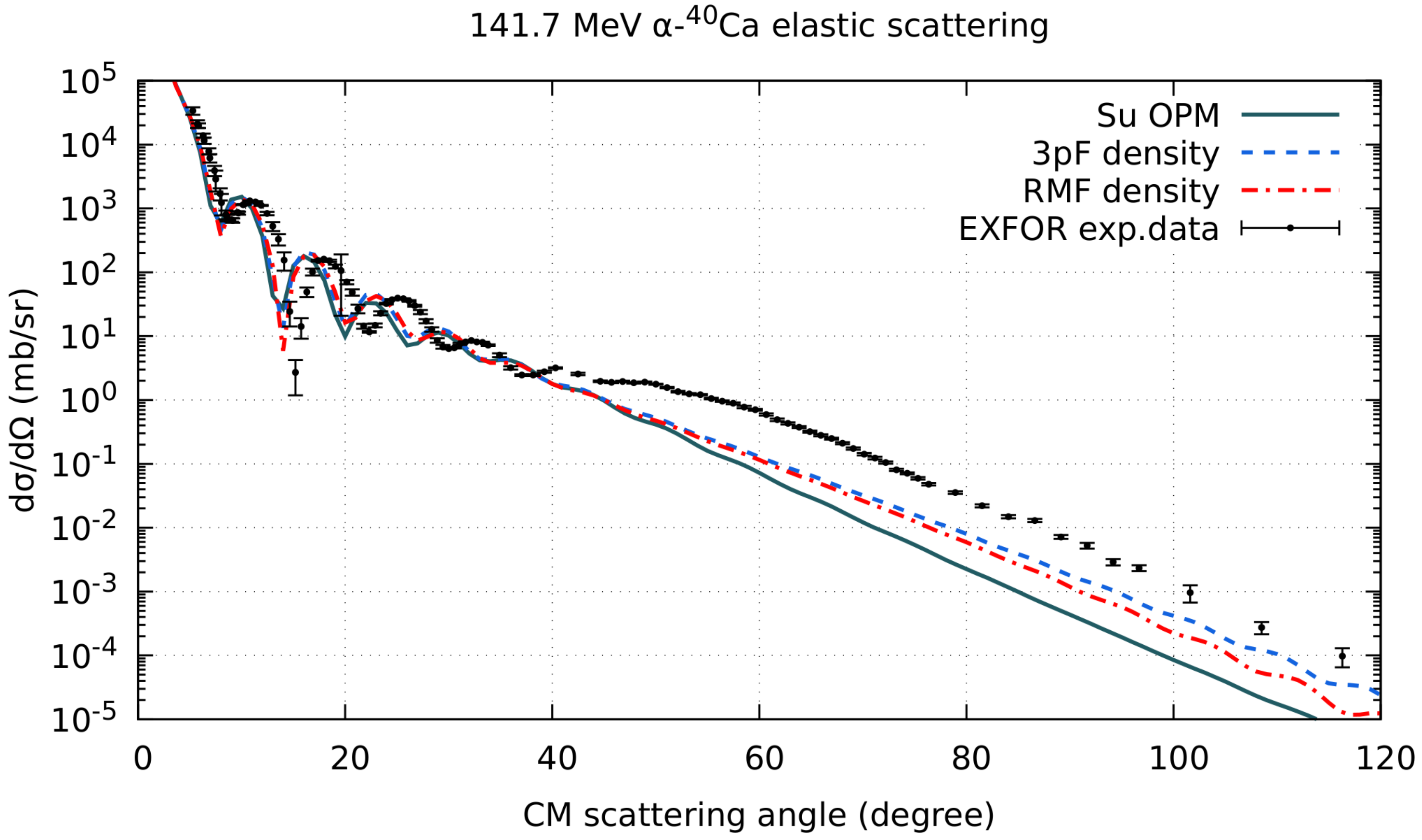
^{40}Ca nuclear density calculated in **RMF** approach (provided by Luis Heitz, PhD student at University Paris-Saclay).



Shallower depths of both real and imaginary parts of the folding potential are obtained.

DWBA angular distribution for 141.7 MeV α on ^{40}Ca

PRELIMINARY RESULTS!



First step:
using the folding model potential we reproduce a differential cross section comparable to the one calculated in the OPM framework.

29 MeV α on ^{40}Ca

PRELIMINARY RESULTS!

Interaction potential: $V(R) = V_{\mathrm{C}}(R) + V_{\alpha A}(R) + iW(R)$

Real part: folding potential $V_{\alpha A}(R) = \int \rho(r)\, v_{\alpha n}(s)\, d^3 r$ $\quad v_{\alpha n}(s) = -V_{\mathrm{F}} \exp\left(-\frac{s^2}{\alpha^2}\right)$

where, $V_{\mathrm{F}} = 37\ \mathrm{MeV}$, and $\alpha = 2\ \mathrm{fm}$.

[8] A.M. Bernstein, and W.A. Seidler, Phys. Lett. B34 (1971) 569-571.

Imaginary part: Wood-Saxon terms $W(R) = -W_{\mathrm{V}} f_{\mathrm{V}}^2(R) + 4 a_{\mathrm{D}} W_{\mathrm{D}} \frac{\mathrm{d} f_{\mathrm{D}}^2(R)}{\mathrm{d}r}$

$$f_i(R) = \frac{W_i}{1 + \exp\left(\frac{R - R_i}{a_i}\right)}$$

where, $i = \mathrm{V}, \mathrm{D}$.

W_{V} (MeV)	26.75
r_{V} (fm)	1.056
a_{V} (fm)	0.251
W_{D} (MeV)	0.95
r_{D} (fm)	2.174
a_{D} (fm)	0.538

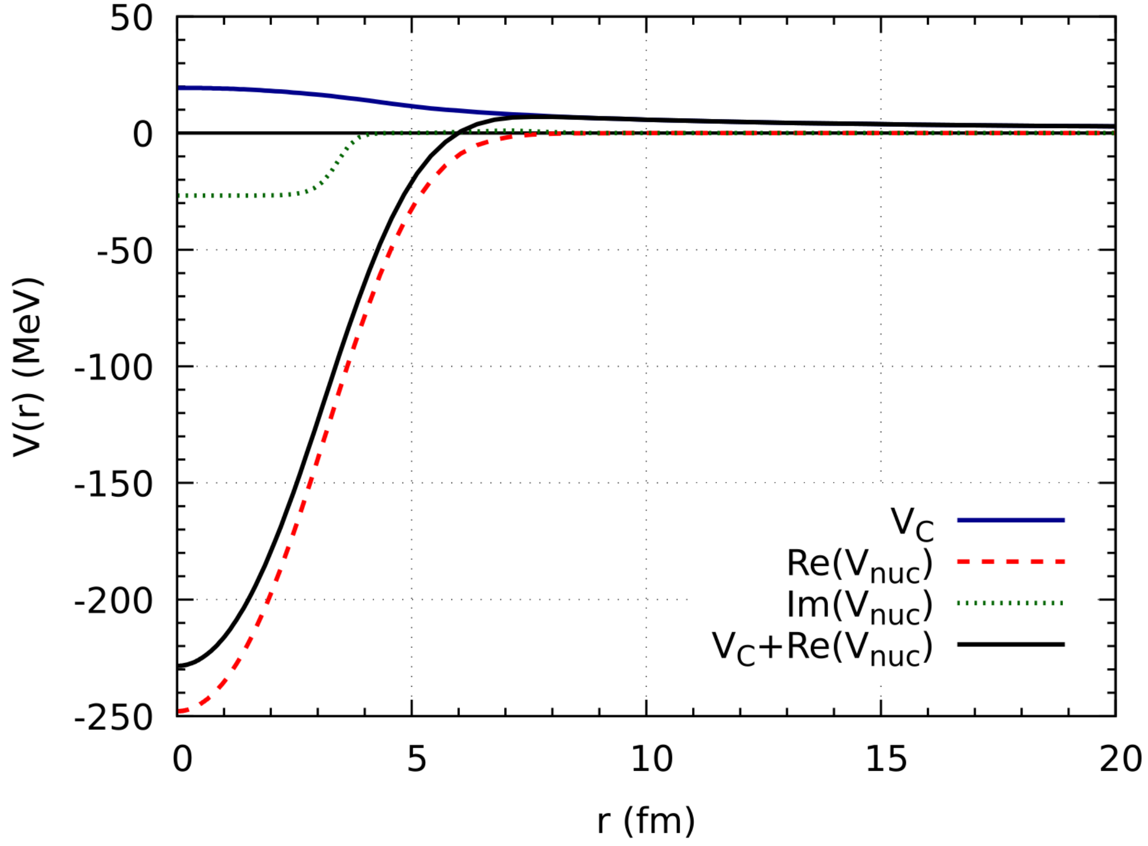
Parameters fitted only for 29 MeV α particles on ^{40}Ca!

[9] A.M. Kobos *et al.*, Nucl. Phys. A425 (1984) 205-232.

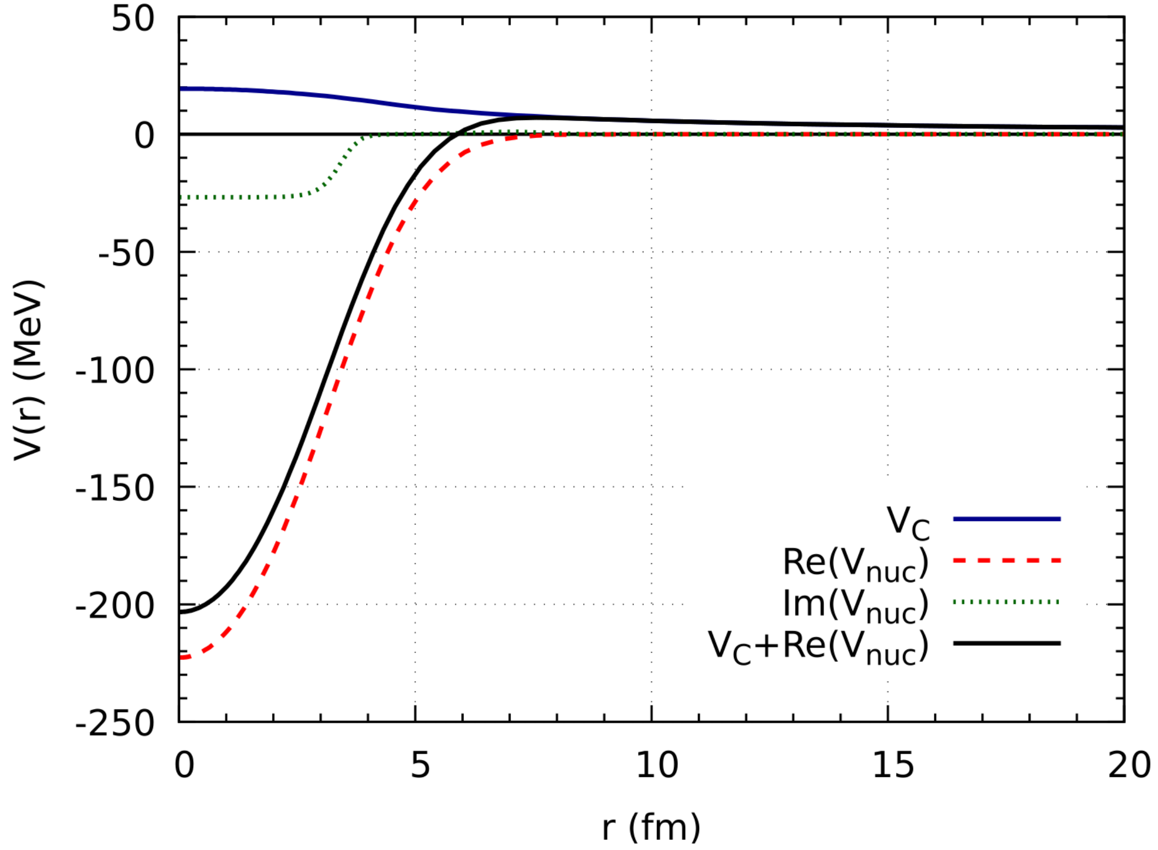
29 MeV α on ^{40}Ca – interaction potential

PRELIMINARY RESULTS!

3pF density



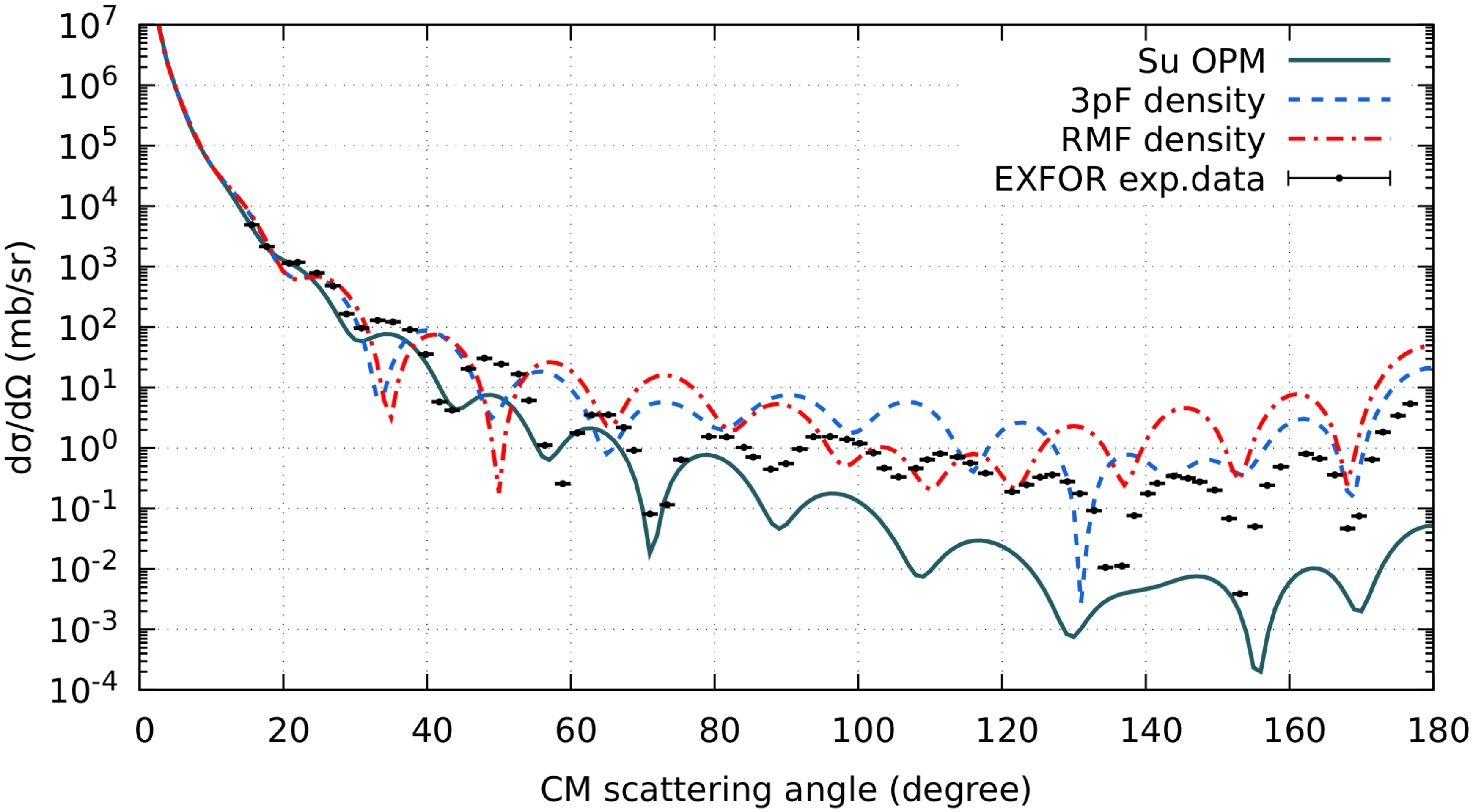
RMF density



- **Imaginary part**: both the **shape** and the **depth** are different when using Wood-Saxon forms;
- **Real part**: different **depths** when using **different density parametrizations**.

29 MeV α on ^{40}Ca – angular distribution

PRELIMINARY RESULTS!



- Using the **folding model potential** for the **real part** and **Wood-Saxon** forms for the **imaginary part** of the potential we start to **enhance the backward scattering**.
- The **shape** of the differential cross section is **sensitive** to different **density approximations**.

Next steps and open questions

- Use the **density** generated by **RMF+QCM calculations** including the effect of pn pairing, expected to **increase the clusterisation** and, eventually, the differential cross section in the backscattering region.

Is there a more suitable **α - nucleon interaction**?

- including an **energy dependence** such that we can probe more energies of the α particles;

- valid for **various target nuclei**.

Maybe a **double folding** procedure could give more accurate interaction potentials?

Should **only the real part of the potential be folded**, while for the **imaginary part** to use **Wood-Saxon terms** fitted on available experimental data?

Thank you for your attention!

References

1. G. Gaul *et al.*, Effects of α-particle correlations in elastic α-scattering, Nucl. Phys. A137 (1969) 177.
2. N. C. Schmeing, An explanation of large backward scattering of α-particles on ^{40}Ca, Nucl. Phys. A142 (1970) 449
3. F. Salvat, J. M. Fernandez-Varea, RADIAL: A Fortran subroutine package for the solution of the radial Schrödinger and Dirac wave equations, Comp. Phys. Comm. 240 (2019) 165-177.
4. X.-W. Su, and Y.-L. Han, Global optical model potential for alpha projectile, Int. J. of Mod. Phys. E, vol. 24, no. 12 (2015).
5. G. R. Satchler, and W. G. Love, Folding model potentials from realistic interactions for heavy-ion scattering, Phys. Rep. 55 (1979) 183-254.
6. F. E. Bertrand *et al.*, Giant multipole resonances from inelastic scattering of 152-MeV alpha particles, Phys. Rev. C22 (1980) 1832.
7. H. de Vries *et al.*, Nuclear charge-density-distribution parameters from elastic electron scattering , Atom. Data Nucl. Data Tabl. 36 (1987) 495-536.
8. A.M. Bernstein, and W.A. Seidler, An alpha-particle optical potential from the nuclear density distribution, Phys. Lett. B34 (1971) 569-571.
9. A.M. Kobos *et al.*, Folding-model analysis of elastic and inelastic α -particle scattering using a density-dependent force, Nucl. Phys. A425 (1984) 205-232.