



Bundesministerium  
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TECHNISCHE  
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# EDFs with local chiral interactions: Pionifying Skyrme

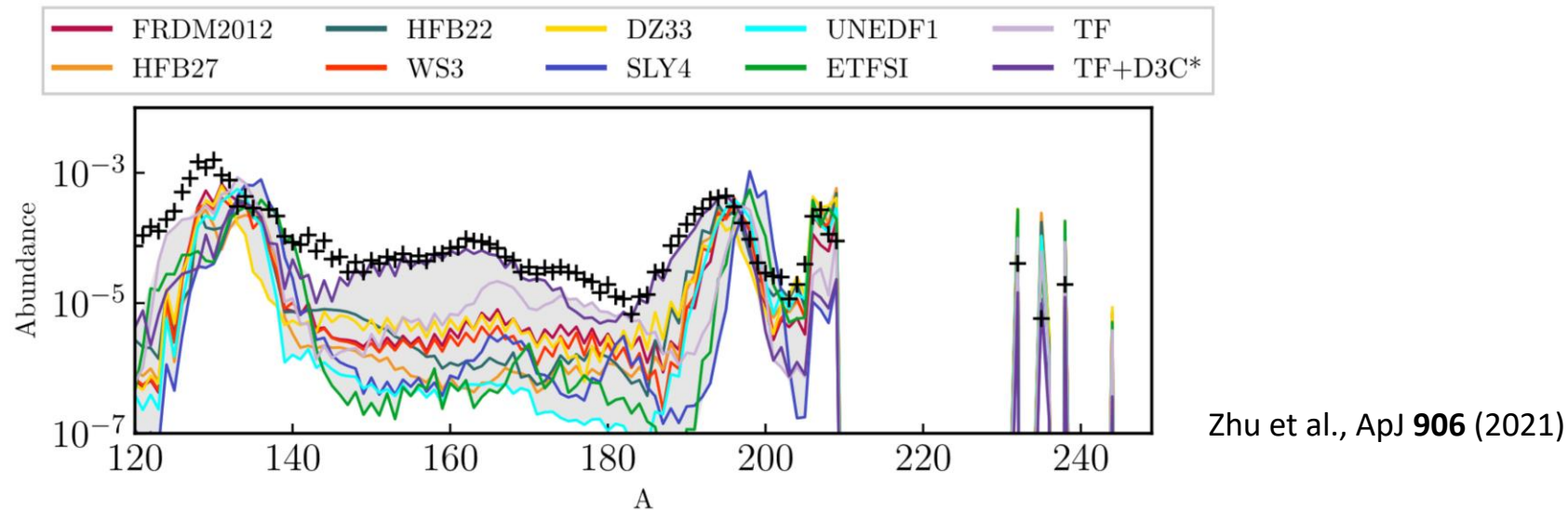
arXiv: 2307.13568

**Lars Zurek**

ESNT EDF workshop, November 22, 2023

# Some EDF issues

- Extrapolation outside fitting region potentially uncontrolled

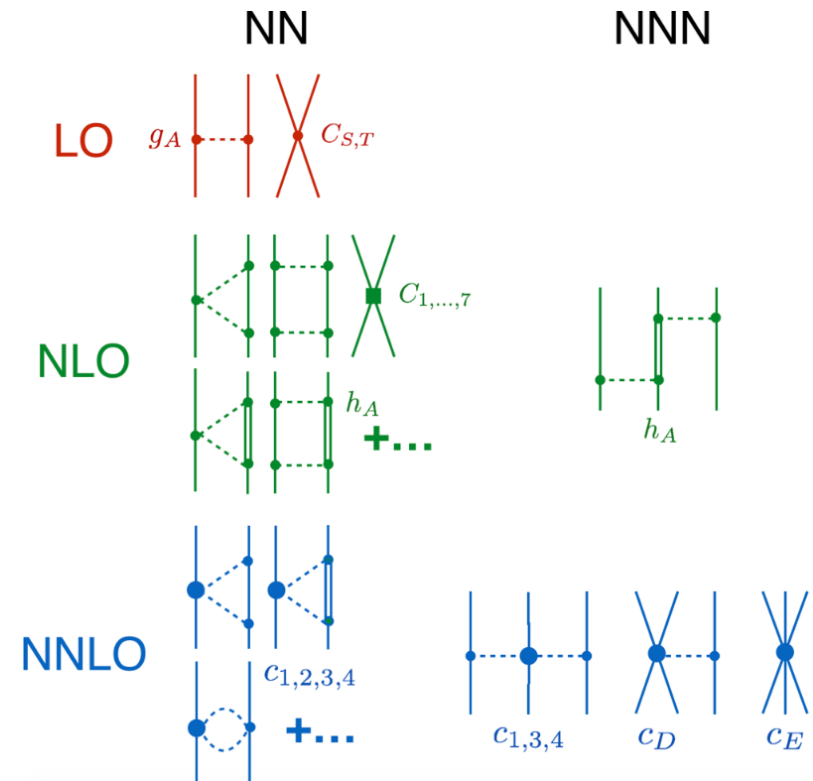


- How to assess uncertainties?
- Have standard EDFs reached their accuracy limit?
- Phenomenological construction of EDFs

McDonnell et al., PRL **114** (2015)

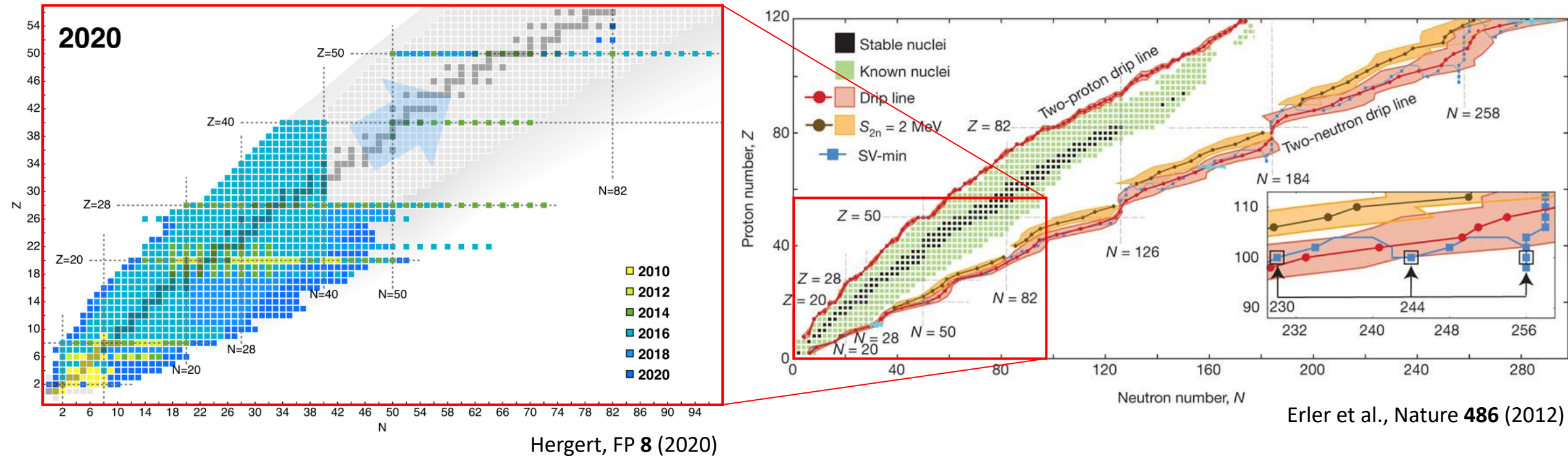
# Ab initio calculations with chiral EFT

- Chiral effective field theory:  
expansion of nuclear interaction in typical momenta over breakdown scale
- Systematically improvable
- Uncertainty estimates “built-in”



# Ab initio calculations with chiral EFT

- Challenging to achieve similar accuracies as EDFs
- Computationally more expensive



# Quest for ab initio energy density functionals

- How to connect energy density functionals and the ab initio approach?

Salvioni et al., JPG **47** (2020)

Furnstahl, EPJA **56** (2020)

Marino et al., PRC **104** (2021)

Duguet et al., EPJA **59** (2023)

- Schematic of ab initio calculations:

- Potential from chiral EFT
- Solve on mean-field level
- Build correlations on top via correlation expansion method

→ Adjust short-range part of potential instead of using correlation expansion

- cf. encapsulate triples from CCSD(T) in CCSD by adjusting 3N contact

Sun et al., PRC **106** (2022)

# Quest for ab initio energy density functionals

- Semi-phenomenological hybrid EDFs
  - Increase resolution: add pions from chiral EFT to Skyrme
  - No additional fit parameters
  - Refit Skyrme parameters
  - Encouraging results, but puzzling systematics

Drut et al., PPNP **64** (2010)  
Gebremariam et al., PRC **82** (2010)

Navarro Pérez et al., PRC **97** (2018)

**Zurek** et al., arXiv:2307.13568 (2023)

→ Revisit EDF construction

# Semi-phenomenological hybrid EDFs

- Employed at HFB level
- Consider only even-even nuclei

$$E = E_H^\chi + E_F^\chi + E_{\text{Skyrme}} + E_{\text{Coulomb}} + E_{\text{pair}} + E_{\text{kin}}$$

$$E_{\text{Skyrme}} = \sum_{t=0,1} \int d\mathbf{R} \left[ (C_t^{\rho\rho} + C_{tD}^{\rho\rho} \rho_0^\gamma) \rho_t^2 + C_t^{\rho\tau} \rho_t \tau_t + C_t^{\rho\Delta\rho} \rho_t \Delta\rho_t \right. \\ \left. + C_t^{\rho\nabla J} \rho_t \nabla \cdot \mathbf{J}_t + C_t^{JJ} J_{t,ab} J_{t,ab} \right]$$

$$E_{\text{pair}} = \frac{1}{4} \sum_{q=n,p} \int d\mathbf{R} V_0^q \left( 1 - \frac{1}{2} \frac{\rho_0}{0.16 \text{ fm}^{-3}} \right) \tilde{\rho}_q^2$$

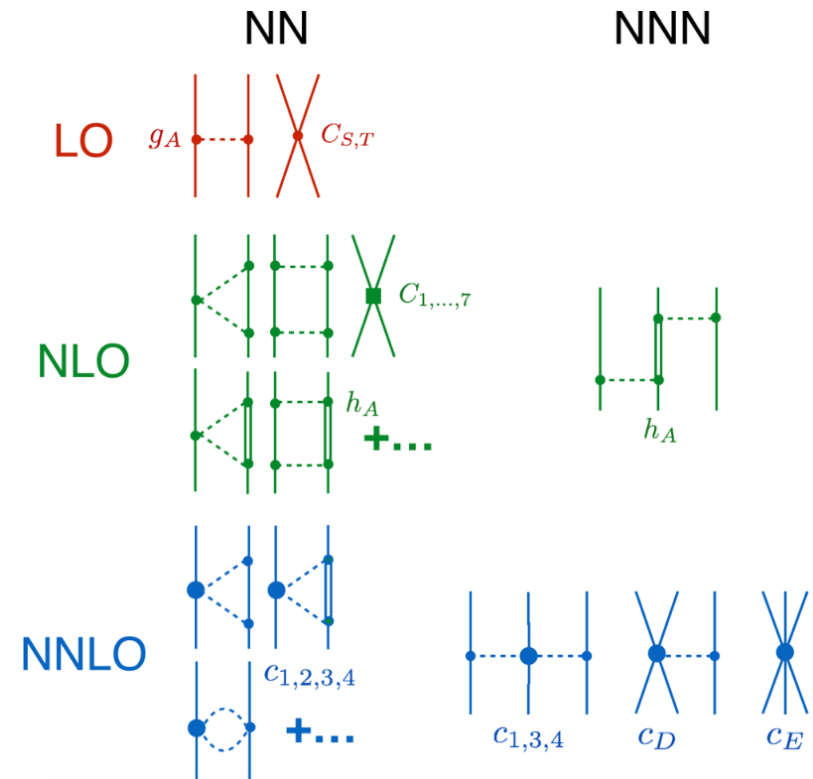
- 14 parameters

# Chiral pion exchanges

- At different chiral orders up to N<sup>2</sup>LO
- With / without  $\Delta$  isobars and 3N forces
- Local coordinate-space regulator

Krebs et al., EPJA **32** (2007)

Epelbaum et al., EPJA **51** (2015)

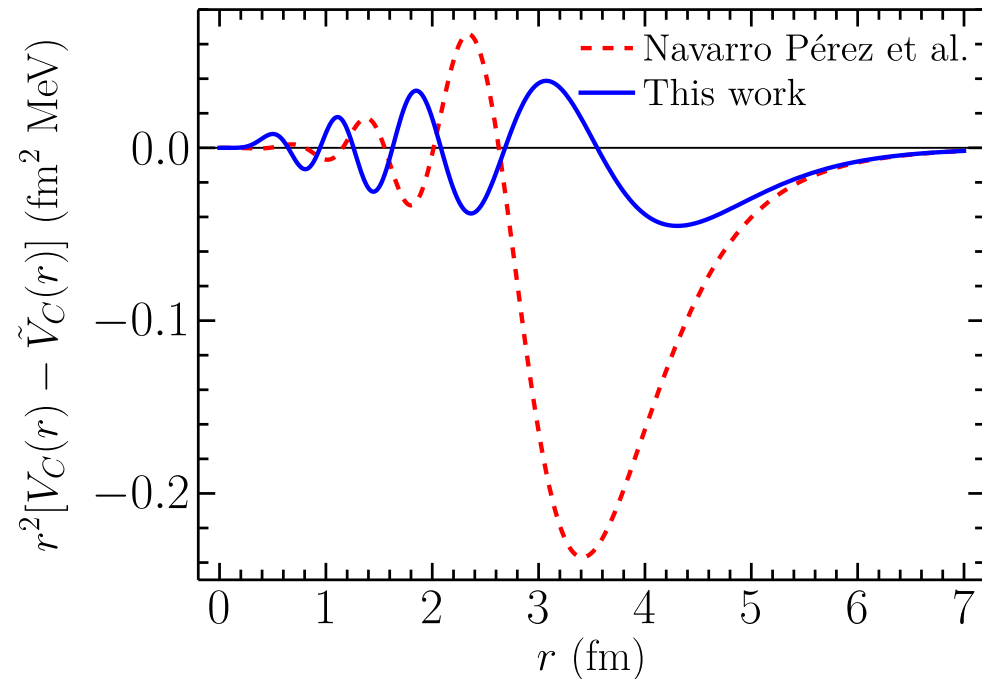


Ekström et al., PRC **97** (2018)



# Chiral pion exchanges

- Hartree term: approximate potentials by sums of 5 Gaussians



- 5 MeV deviation from exact Hartree energy in  $^{208}\text{Pb}$  at N<sup>2</sup>LO (4 GeV)

# Chiral pion exchanges

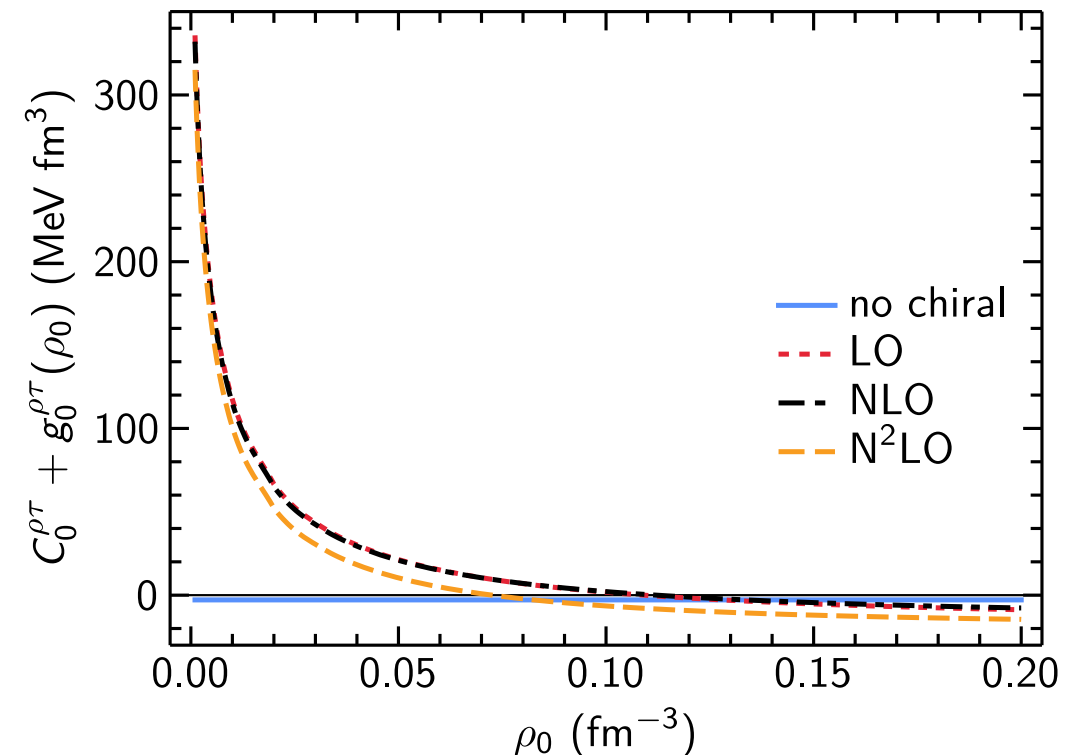
- Fock term in local form
  - Approximate one-body density matrices via density-matrix expansion
  - Extension of Slater approximation

Gebremariam et al., PRC **82** (2010), NPA **851** (2011)

Slater, PR **81** (1951)

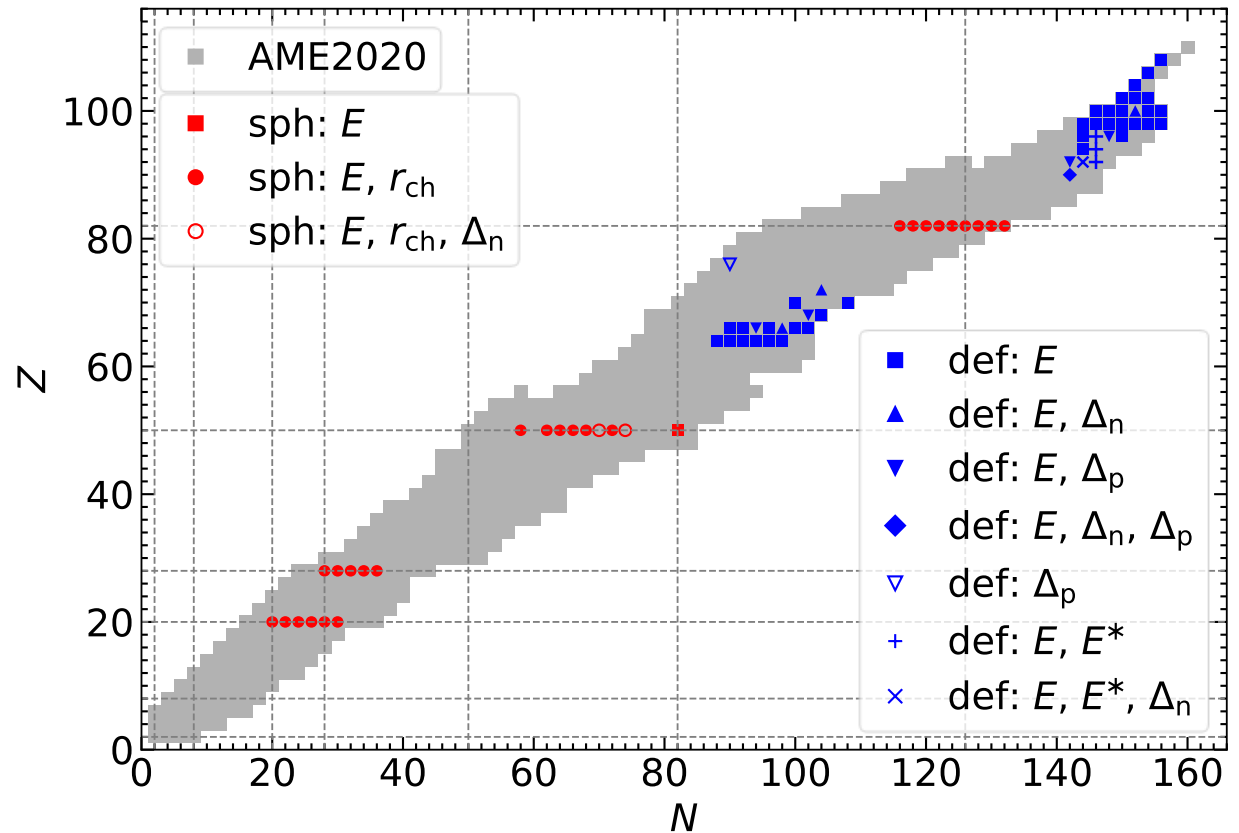
$$\rho(\mathbf{x}_1, \mathbf{x}_2) = \frac{3j_1(k_F(\mathbf{R})r)}{k_F(\mathbf{R})r} \rho(\mathbf{R})$$

- Finite-range potential  $\rightarrow$  EDF with density-dependent contacts



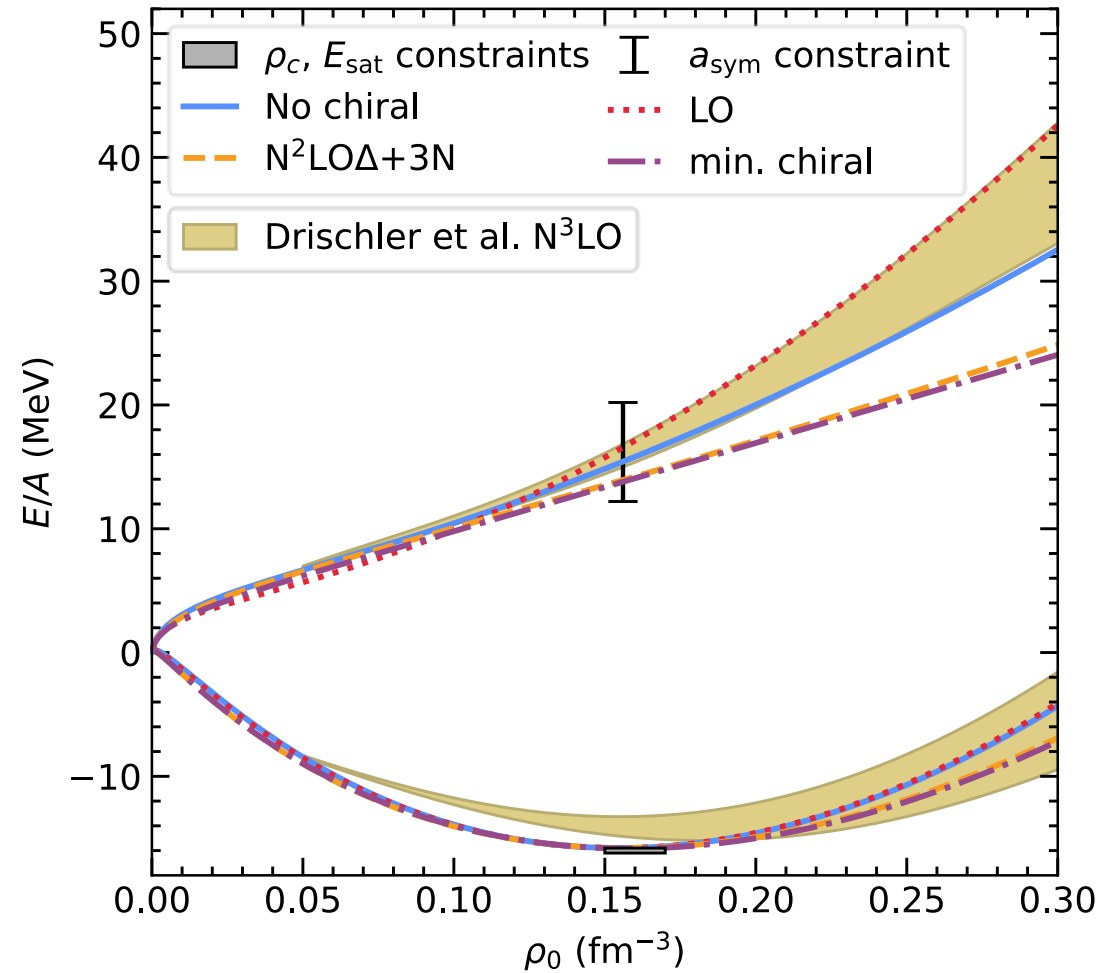
# Skyrme parameter optimization

- Fit to 81 nuclei
  - Binding energies, charge radii, odd-even mass staggerings, fission isomer energies
- Weights for observables from Bayesian posterior estimate for UNEDF1  
Schunck et al., JPG **47** (2020)
- Find constrained minimum using POUNDERS  
Wild et al., JPG **42** (2015)



# Skyrme parameter optimization

- Infinite nuclear matter parameters are constrained to physically plausible region in fit



# GUDE [guːdə]

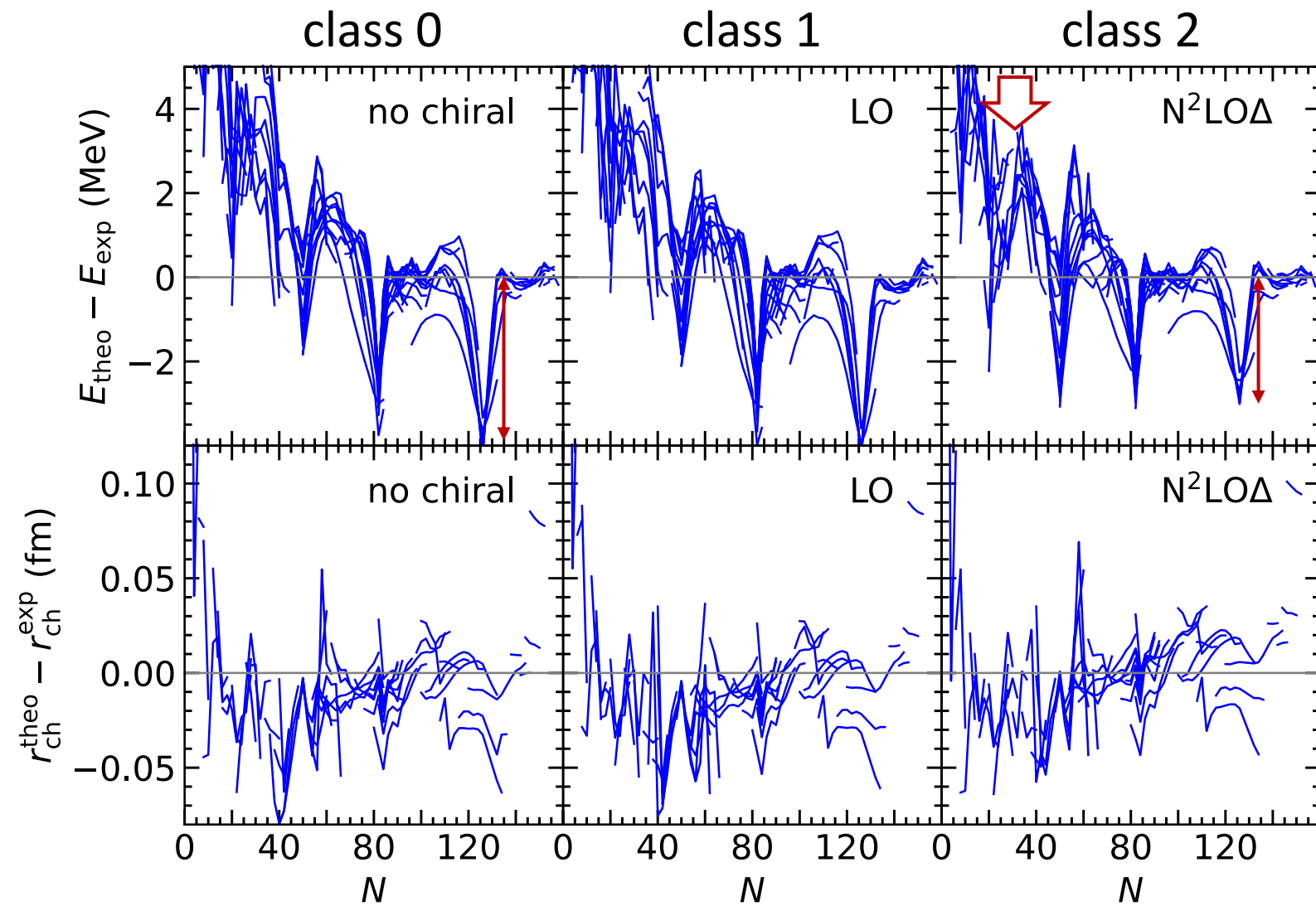
Germany-USA Density-matrix expansion Energy density functionals

- We group EDFs in 3 classes

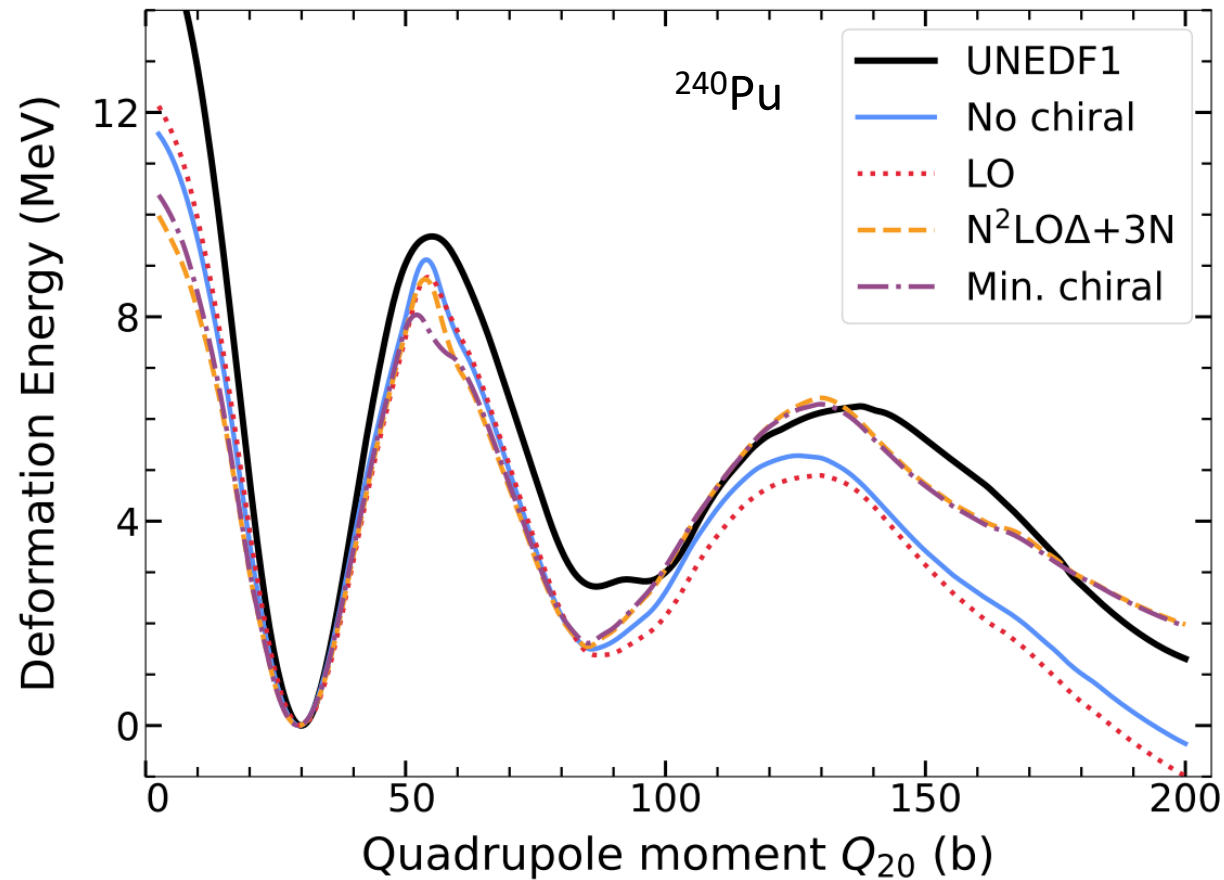
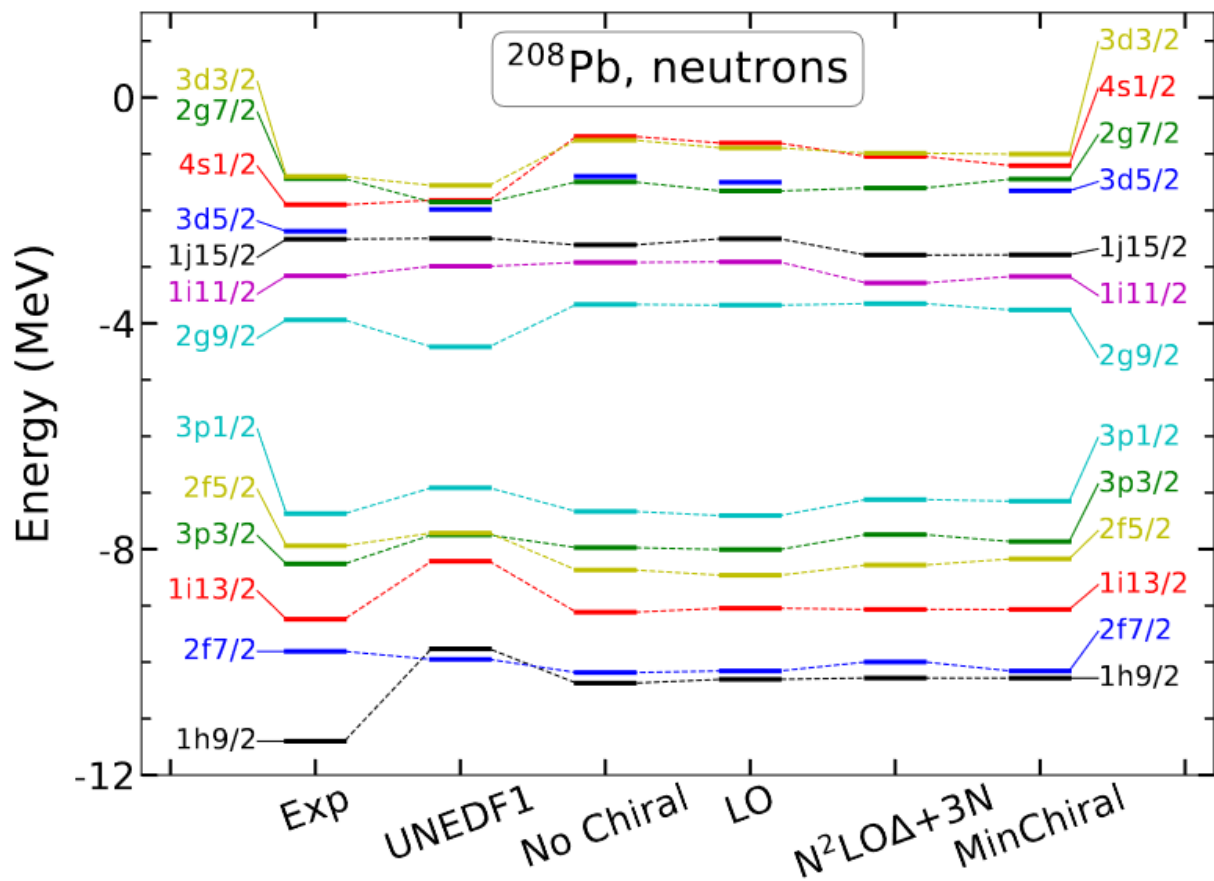
Class	0	1	2
Variants	no chiral	LO, NLO	NLO $\Delta$ (+3N), N <sup>2</sup> LO( $\Delta$ ,+3N)
$\chi^2$	122	145	86 – 91
Binding energy rms deviation (MeV)	2.11	2.09 – 2.13	1.41 – 1.56

- Terms beyond NLO move global minima “closer” towards region allowed by nuclear matter parameter bounds
- Significant improvement beyond NLO

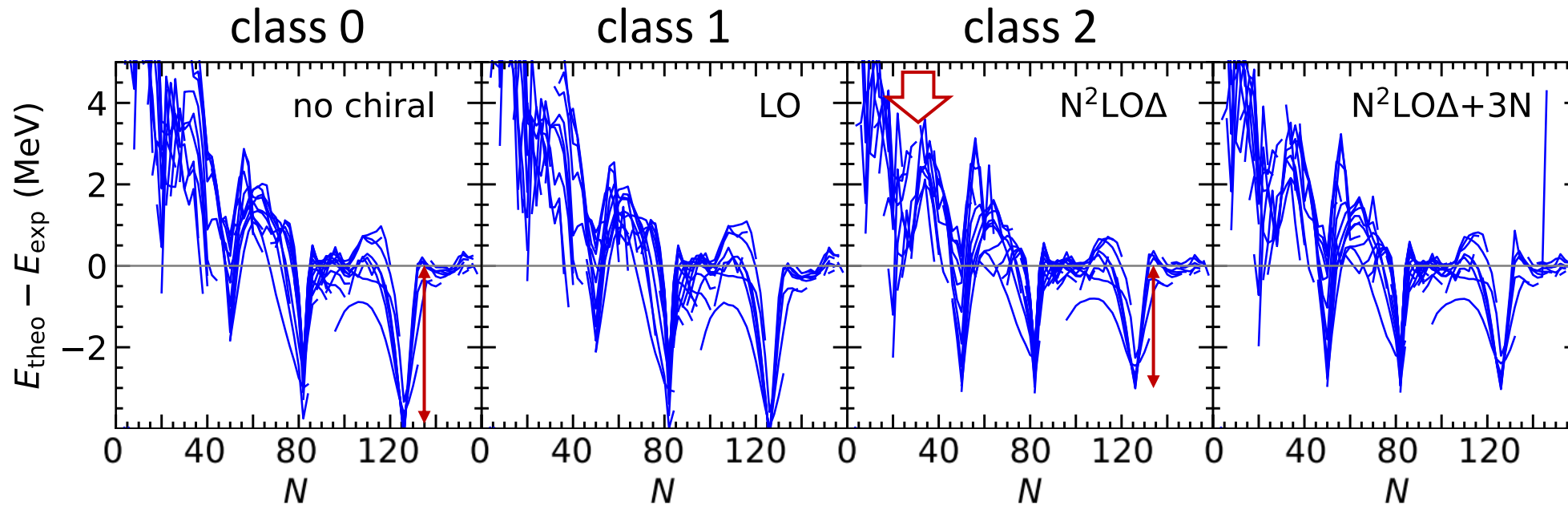
# GUDE results



# GUDE results



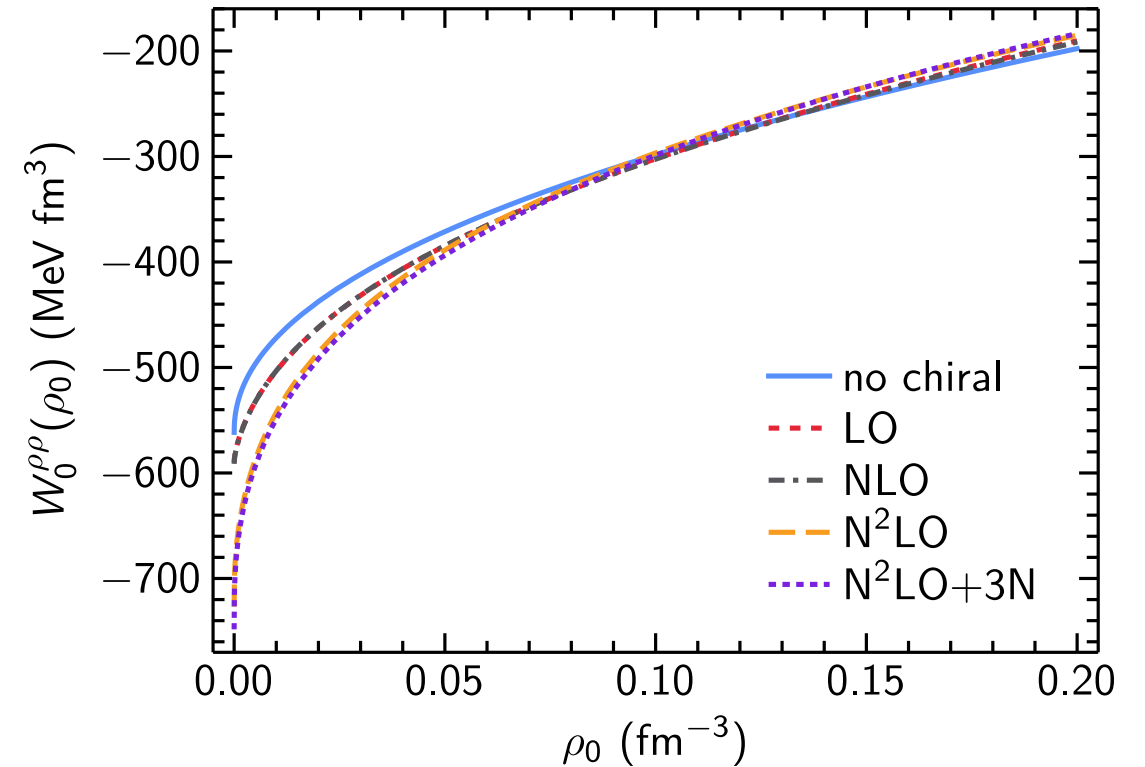
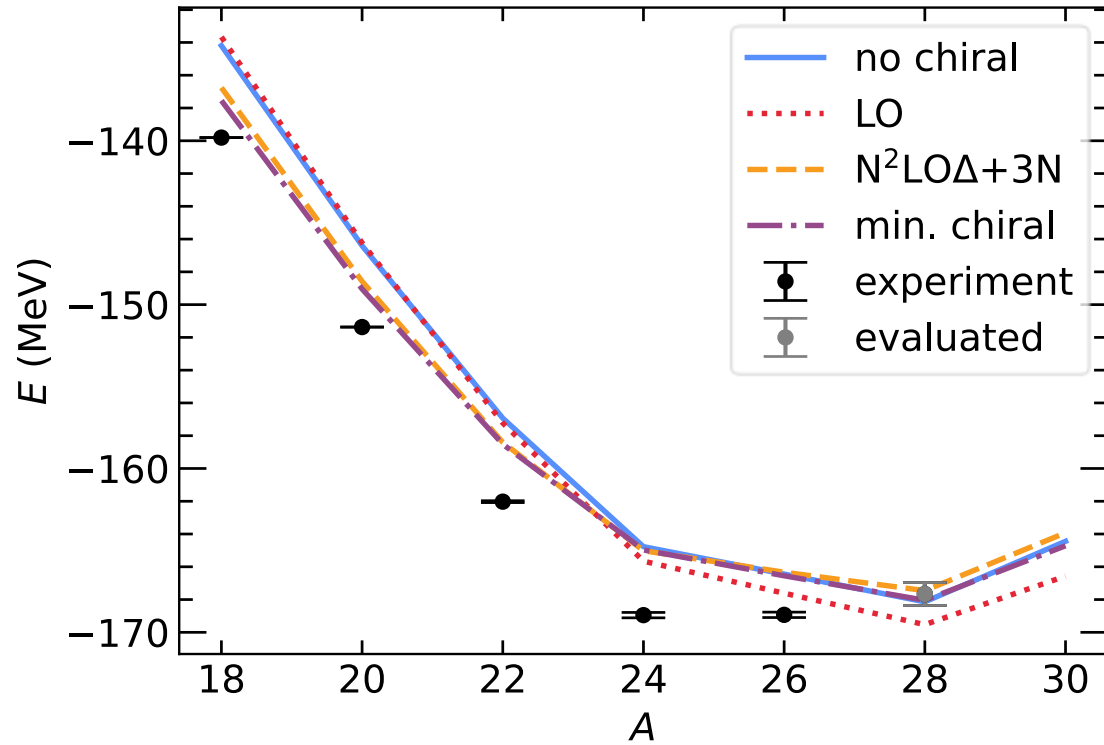
# Analysis of pion contributions



- 3N pion exchanges have no effect



# Analysis of pion contributions

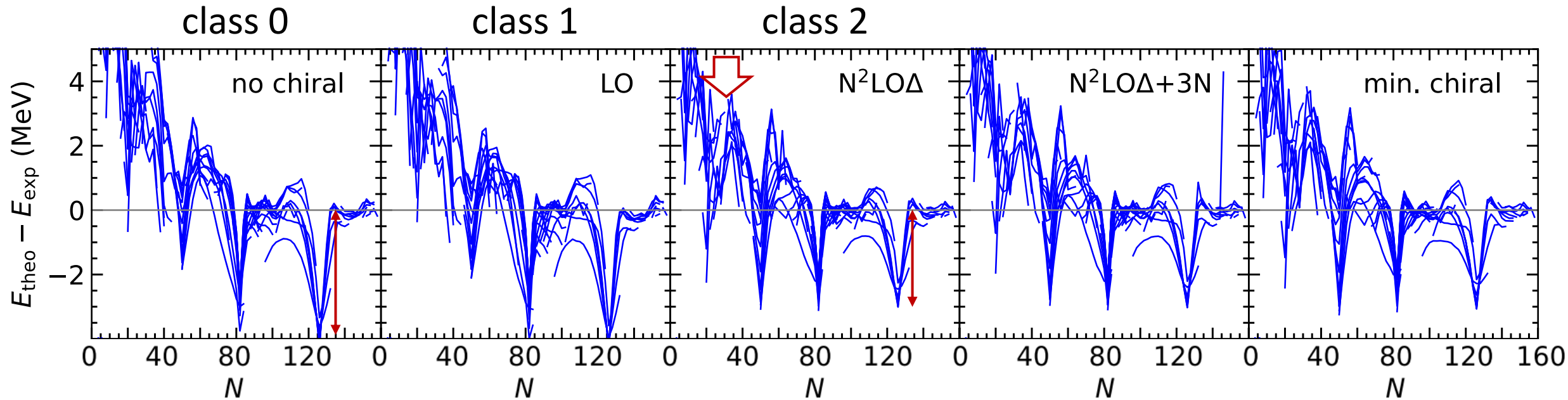


- 3N pion exchanges have no effect
  - Already effectively incorporated via density-dependent Skyrme contact

# Analysis of pion contributions

- 3N pion exchange effect might appear from
  - fitting to neutron-rich nuclei
  - Changing regularization scheme
- Leave out density dependent contact?

# Analysis of pion contributions

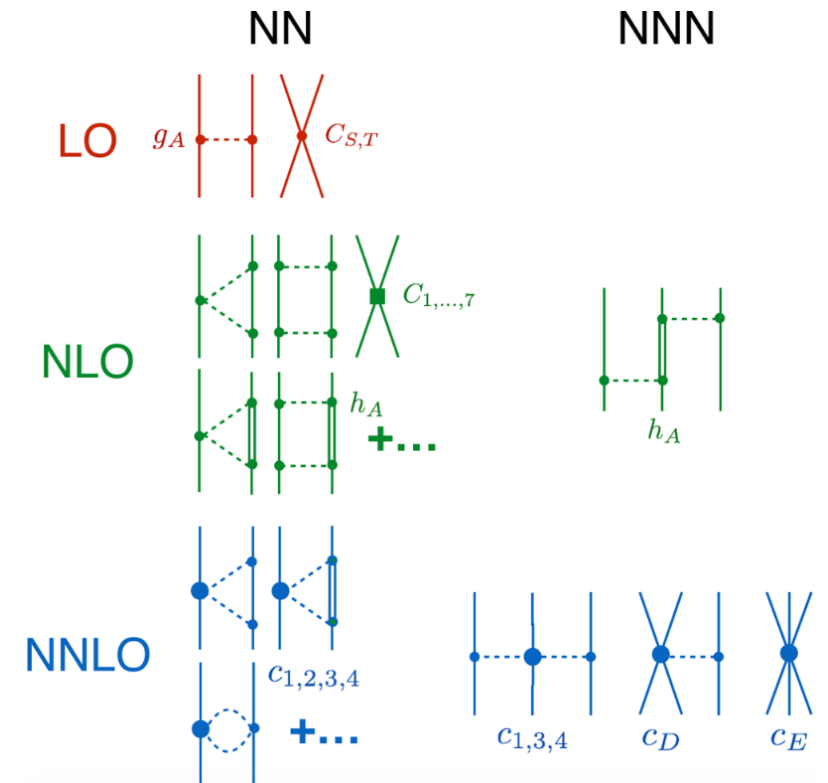


- Min. chiral EDF: minimal set of chiral terms that yield improvement
  - isoscalar  $N^2\text{LO}$  Hartree + LO Fock (in Slater approximation)

# Analysis of pion contributions

- Chiral power counting does not carry over
  - Here: structure of contacts does not change with chiral order

$$\begin{aligned}
 E_{\text{Skyrme}} = \sum_{t=0,1} \int d\mathbf{R} & \left[ (C_{t0}^{\rho\rho} + C_{tD}^{\rho\rho} \rho_0^\gamma) \rho_t^2 + C_t^{\rho\tau} \rho_t \tau_t \right. \\
 & + C_t^{\rho\Delta\rho} \rho_t \Delta\rho_t + C_t^{\rho\nabla J} \rho_t \nabla \cdot \mathbf{J}_t \\
 & \left. + C_t^{JJ} J_{t,ab} J_{t,ab} \right],
 \end{aligned}$$



Ekström et al., PRC 97 (2018)

# Summary

- Towards ab initio EDFs: add parameter-free pion exchanges to Skyrme EDF
  - Significant improvement beyond NLO:  
binding energy rms deviation decreased by 30%
- Pions help with simultaneous description of finite nuclei and infinite matter

# Outlook

- Different regularization scheme
- Changes of fit data set
  - Fit to ab initio pseudodata
- Do pions help other EDF forms?

Thanks for your attention

and to my collaborators

Scott Bogner, Toño Coello Pérez, Dick Furnstahl,  
Rodrigo Navarro Pérez, Nicolas Schunck, and Achim Schwenk

