Towards a rigourous formulation of nuclear EDF via the FRG method

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ESNT Workshop
Nuclear density functionnal method : Going beyond the minefield

21.11.2023

Outline

Introduction to FRG

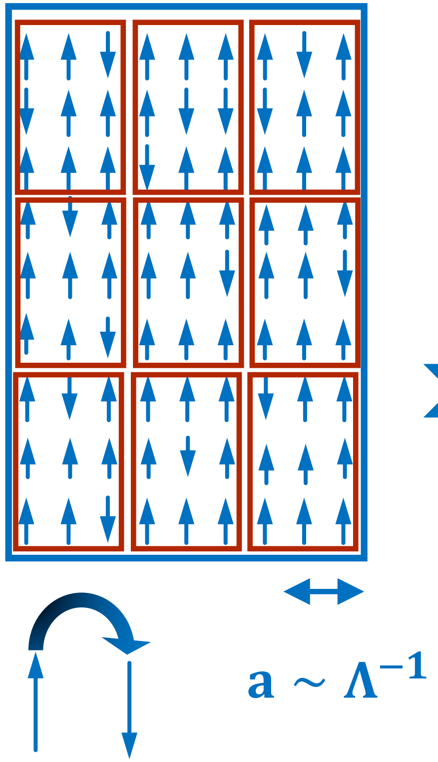
FRG to build EDF

Conclusion

Outline

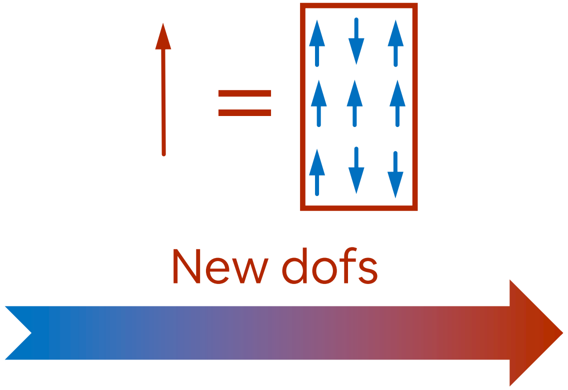
Introduction to FRG

RG : introductive example



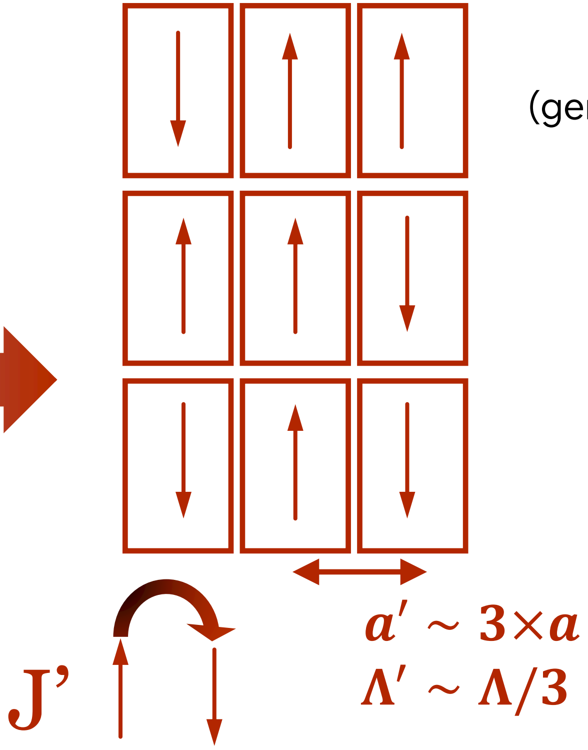
$$H = -\sum_{ij} J\sigma_i\sigma_j$$

$$\langle m \rangle = \sum_{\text{config}} m[\{\sigma\}]e^{-\beta H[\{\sigma\}]}$$



New dofs

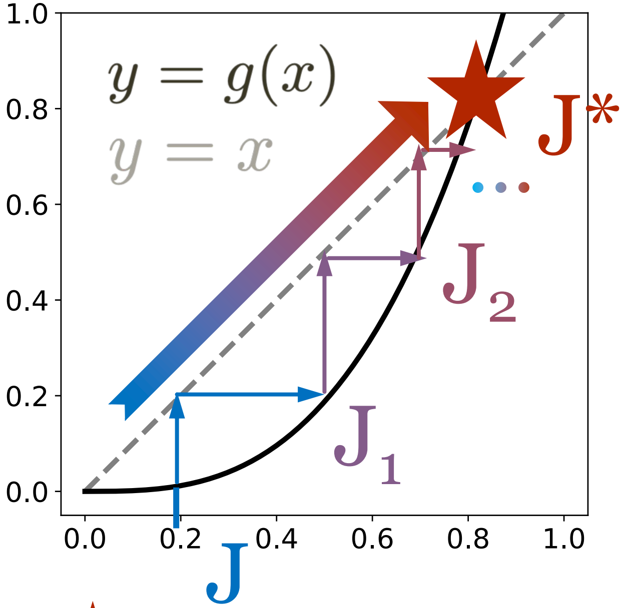
1 RG step
= « unzooming »



$$H' = -\sum_{ij} J'\sigma'_i\sigma'_j$$

$$\langle m \rangle = \sum_{\text{config}} m[\{\sigma'\}]e^{-\beta H'[\{\sigma'\}]}$$

Tricky part :
Find g such that $J' = g(J)$
(generation of non-local couplings, higher order terms,...)

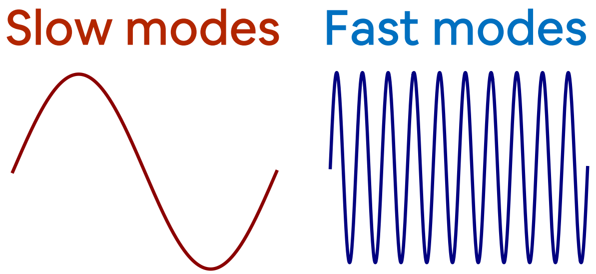


« Fixed point »
Whatever **J** ends in **J***

Can easily compute physical observables with H*

Limitations of Wilsonian FRG

Wilsonian FRG



Λ momentum cutoff
Λ^{-1} spatial resolution

$$Z = \int \mathcal{D}\phi e^{-S[\phi]} = \int \mathcal{D}\phi_{p\leq\Lambda} \mathcal{D}\phi_{p>\Lambda} e^{-S[\phi]} = \int \mathcal{D}\phi_{p\leq\Lambda} e^{-S_\Lambda[\phi_{p\leq\Lambda}]}$$

Integrating out fast modes

$$= e^{-S_\Lambda}$$

Only depends on slow (long range) modes !

<u>**In practice :**</u> hard/impossible to compute S_Λ

(if not <u>perturbative</u>)

$$S_\Lambda \sim \log \int \mathcal{D}\phi_{p>\Lambda} e^{-S[\phi]}$$

Functional Renormalisation Group

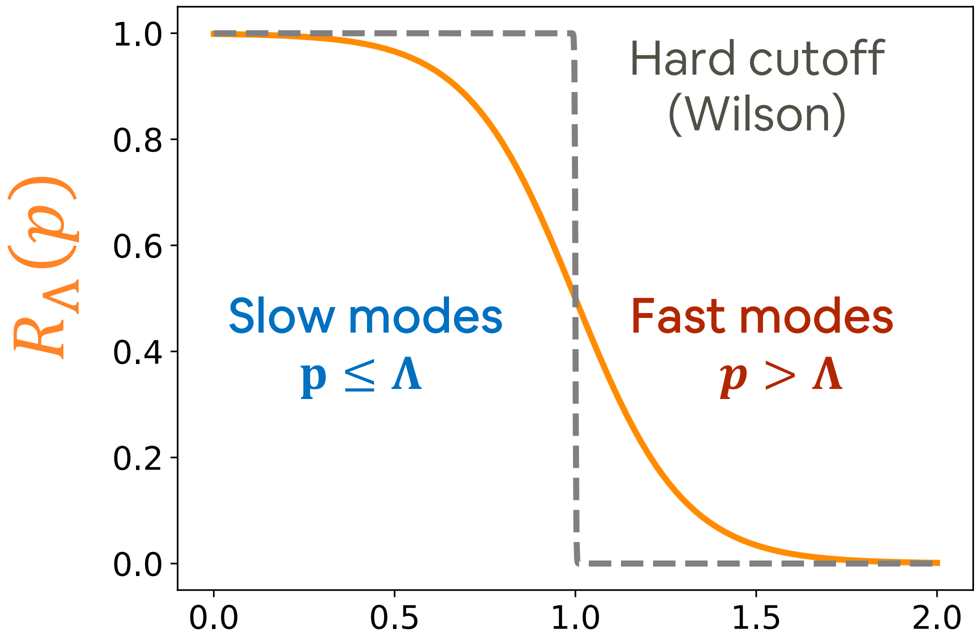
«Functional RG» = Wilson RG +…

Λ momentum cutoff
Λ^{-1} spatial resolution

1. Soften cutoff R_Λ (regulator)
2. Action $S_\Lambda \rightarrow$ Effective Action Γ_Λ (~Legendre transform)
3. Differential equation for Γ_Λ

Wetterich equation (1993)

$$\Lambda \frac{\partial \Gamma_\Lambda}{\partial \Lambda} = \frac{1}{2} \mathrm{Tr} \left[\Lambda \frac{\partial R_\Lambda}{\partial \Lambda} \left(R_\Lambda + \Gamma_\Lambda^{(1,1)} \right)^{-1} \right]$$



« **Exact** » RG : no approximation

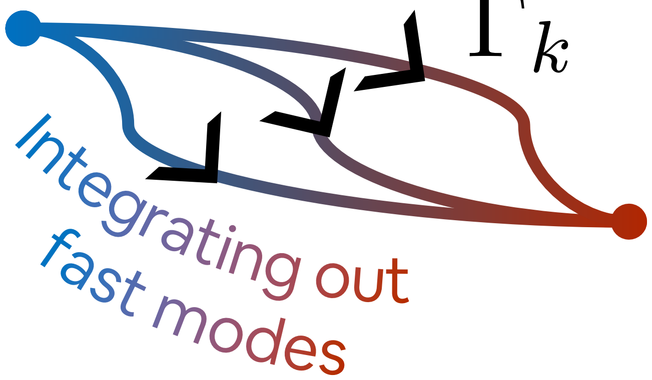
In particular, **non-perturbative**

Impossible to solve exactly BUT

reliable expansions exist

Wetterich equation

$\Gamma_{k=\Lambda} = S$

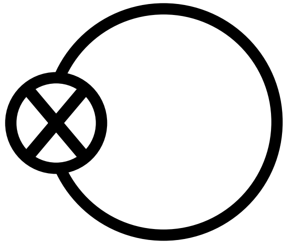


Γ_k

$\Gamma_{k=0} = \Gamma$

Fixed point
Γ^*

Different Regulator
=
Different paths

$$k\frac{\partial \Gamma_k}{\partial k} =$$ 

Simple looking
1 Loop structure

$\otimes \; k\frac{\partial R_k}{\partial k}$

$$\langle \Phi_1 \ldots \Phi_n \rangle_{1PI} = \frac{\delta \Gamma}{\delta^n \Phi_1 \ldots \Phi_n}$$

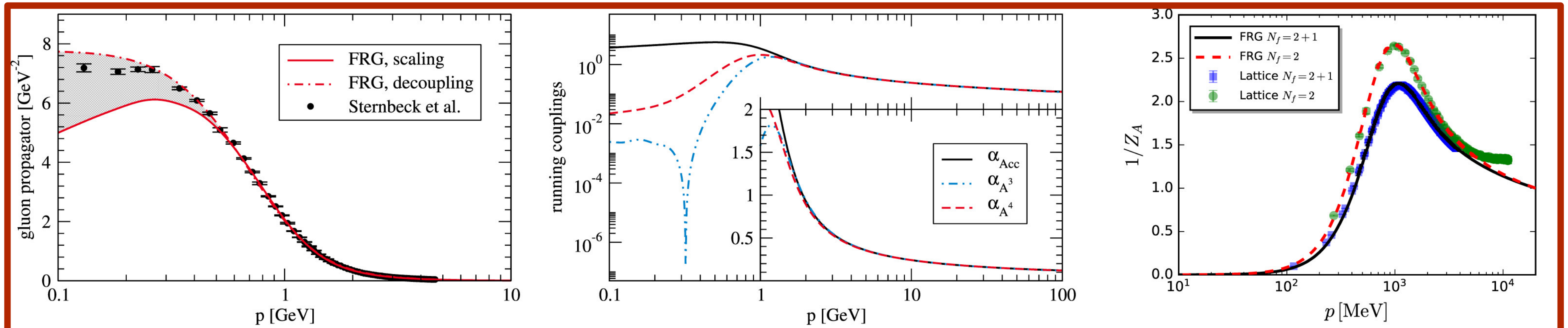
<u>**Derivative expansion**</u> : expand Γ_k in powers of $\nabla\phi$

$$\Gamma_k \sim \int \mathrm{d}^4x \, U_k(\phi) + c_0 Z_k (\nabla\phi)^2 + c_1 Y_k (\nabla\phi)^4 + \ldots$$

First order : Local Potential Approximation <u>**(LPA)**</u>

$Z_k = 1, Y_k = 0$

Applications of FRG



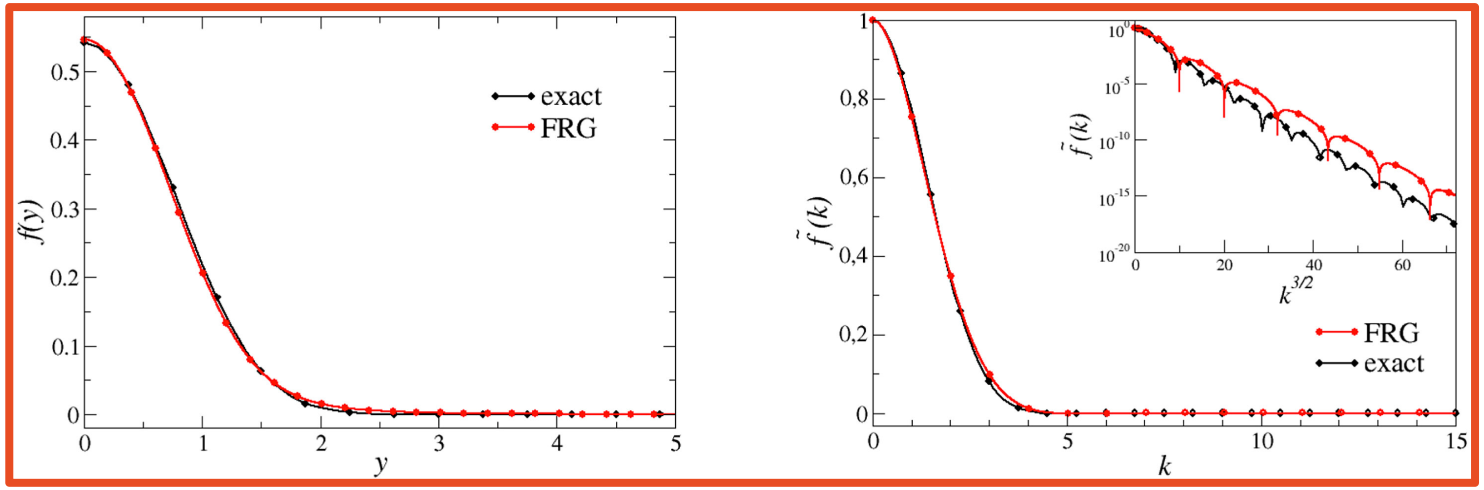
(a) FRG Yang-Mills Gluon propagator $G_A(p^2)$, [721], in comparison to lattice data, [722]. The shaded area for small momenta potentially signals different infrared solutions of Landau gauge Yang-Mills theory, for more details see [721].

(b) Effective running couplings, [721], as obtained from different Yang-Mills vertices: three-gluon vertex: α_{A^3}, four-gluon vertex: α_{A^4}, ghost-gluon vertex: $\alpha_{A\bar{c}c}$. At large momenta the couplings agree due to universality.

(c) 2– and 2 + 1-flavor gluon dressing functions $1/Z_A(p^2) = p^2\, G_A(p^2)$ with the gluon propagator $G_A(p^2)$. FRG: 2-flavor [723], 2 + 1-flavor [724]. Lattice: 2-flavor [725], 2 + 1-flavor [726, 727].

QCD :
Gluon propagator

Taken from FRG review :
Dupuis *et al.* ,
Phys. Rep. **910**, 2021



Turbulence :
Correlation functions for KPZ equation

Outline

Introduction to FRG

FRG to build EDF

FRG to build nuclear EDF

Global Strategy

Free space
N-N Lagrangian

FRG flow

Effective (dressed)
N-N Lagrangian
in medium

Present work

$$\mathcal{L} = \mathcal{L}_{\mathrm{Bonn}}$$

First order : LPA

FRG flow

?

pheno. covariant Lagrangian
for convenience

Benchmark
Properties of nuclear matter

Peter Ring *et al* 2023
J. Phys.: Conf. Ser. **2453** 012031

Some beyond HF correlation with
Dirac-Brueckner approach

Flow equation

Local Potential Approximation :

$$\Gamma_k = \int \mathrm{d}^4x \; \left[U_k(\text{mesons}) + \mathcal{L}_m + \mathcal{L}_N + \mathcal{L}_{\text{int}} - \mu\bar{\psi}\psi\right]$$

❌ For now : fix coupling constants (can be « easily » included)

Freezing heaviest mesons

Heaviest cannot fluctuate : mean-field level

$$\rho \simeq \rho_{\text{MF}}, \omega \simeq \omega_{\text{MF}}, \delta \simeq \delta_{\text{MF}}$$

$$U_k(\text{mesons}) = U_k(\pi, \sigma)$$

Machleidt 1987

meson	Mass (MeV)	J^π	coupling
π	138	0^-	pseudoscalar
σ	550	0^+	scalar
ρ	769	1^-	vector
ω	783	1^-	vector
δ	983	0^+	scalar

Taylor expansion

$$U_k = \frac{1}{2}M_{\sigma,k}^2\sigma^2 + \frac{g_{2,k}}{3}\sigma^3 + \frac{g_{3,k}}{4}\sigma^4 \frac{1}{2} + M_{\pi,k}^2\pi^2$$

$M_{\sigma,\Lambda}, M_{\pi,\Lambda}$

$g_{2,\Lambda} = g_{3,\Lambda} = 0$

FRG flow →

$M_{\sigma,0}, M_{\pi,0}$

$g_{2,0}, g_{3,0}$

FRG for Bonn Lagrangian

Bare N-N Lagrangian :

$$\mathcal{L}_{\mathrm{Bonn,int}} = \sum_{\mathrm{mesons}} g_m \bar{\psi} D_m \psi$$

Lesson from empirical EDF :

$$\mathcal{L}_{\mathrm{Bonn,int}} \sim \mathcal{L}_{\mathrm{NL3,int}}$$

Same analytical form for interaction

Main structural difference

$$\mathcal{L}_{\mathrm{Bonn}} \not\supset g_2 \frac{\sigma^3}{3} + g_3 \frac{\sigma^4}{4} \subset \mathcal{L}_{\mathrm{NL3}}$$

Machleidt 1987

meson	Mass (MeV)	J^π	coupling
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Generated by FRG flow ?

SNM : Parameters flow

$\Lambda \sim M_\rho \sim 750$ MeV

Symmetric nuclear matter

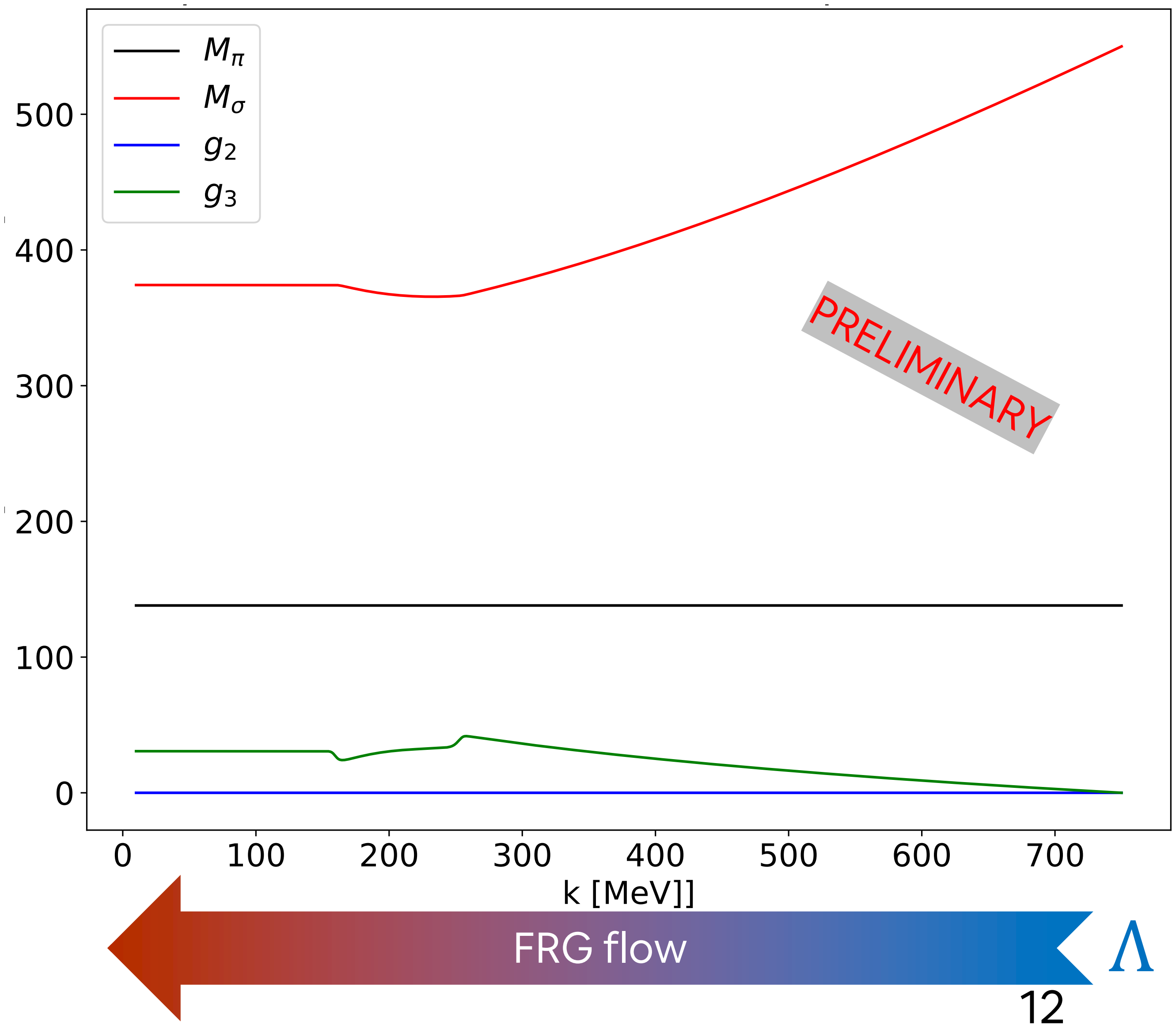
$\rho \simeq \rho_{\mathrm{MF}} = 0$ $\delta \simeq \delta_{\mathrm{MF}} = 0$

Wetterich equation

$$\frac{\mathrm{d}C_{i,k}}{\mathrm{d}k} = f_i(k, \omega_{\mathrm{MF},k}, \mu, \{C_{i,k}\})$$

$$C_{i,k} = M_{\sigma,k}; M_{\pi,k}; g_{2,k}; g_{3,k}$$

M_σ decrease
M_π constant
g_3 non zero
$g_2 = 0$ due to symmetry



Nuclear matter properties

Wetterich equation

$$\frac{\mathrm{d}U_k}{\mathrm{d}k} \equiv \frac{\mathrm{d}C_{i,k}}{\mathrm{d}k} = f_i(k, \omega_{\mathrm{MF},k}, \mu, \{C_{i,k}\})$$

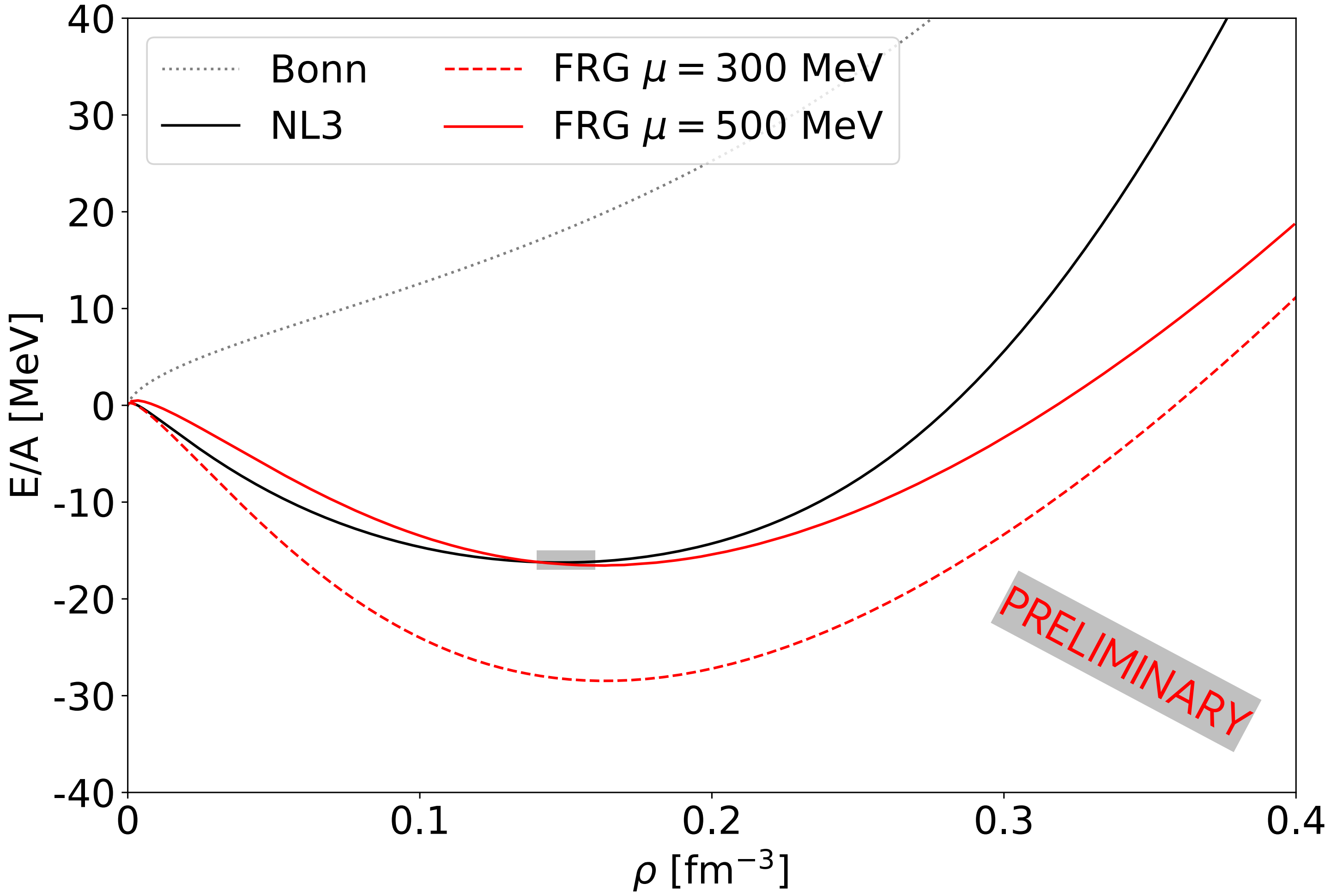
Solve for different μ

Compute E/A(μ)

Drews, PRC 91 035802 (2015)

$$\frac{E}{A}(T=0) = U_{k=0,\mathrm{MF}} + \mu n$$

In principle, with FRG language.
For now, plug into NL3 RMF for a **given μ**



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Conclusion & perspectives

Preliminary results

FRG seems to be promising
to build EDF Lagrangians

Qualitative saturation of nuclear matter

Perspectives

Include flow of Yukawa couplings ⟷ Cutoff dependance ?

Treat Asymetric Nuclear Matter

Extension to finite nuclei

« Beyond the minefield» ...

Thank you for your attention !

Henri Cartier-Bresson
Le Mur de Berlin

Jean-Paul Ebran, Louis Heitz

Back-up slides

Lagrangian in vacuum = UV

Phenomenological lagrangian : **Bonn**

$$
\begin{aligned}
\mathcal{L}_{\text{Bonn}} =& \bar{\psi}\Big[i\partial\!\!\!/ - M - g_\sigma \sigma - g_\delta \vec{\delta}\cdot\vec{\tau} \\
& - \frac{f_\eta}{m_\eta}\gamma^5 \partial\!\!\!/ \eta - \frac{f_\pi}{m_\pi}\gamma^5 \partial\!\!\!/ \vec{\pi}\cdot\vec{\tau} \\
& - g_\omega \omega\!\!\!/ - \frac{f_\omega}{4M}\sigma^{\mu\nu}\Omega_{\mu\nu} - g_\rho \vec{\rho\!\!\!/}\cdot\vec{\tau} - \frac{f_\rho}{4M}\sigma^{\mu\nu}\vec{R}_{\mu\nu}\cdot\vec{\tau}\Big]\psi \\
& + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) + \frac{1}{2}\left(\partial_\mu\vec{\delta}\cdot\partial^\mu\vec{\delta} - m_\delta^2\vec{\delta}^2\right) \\
& + \frac{1}{2}\left(\partial_\mu\eta\partial^\mu\eta - m_\eta^2\eta^2\right) + \frac{1}{2}\left(\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - m_\pi^2\vec{\pi}^2\right) \\
& + \frac{1}{2}\left(\frac{1}{2}\Omega^{\mu\nu}\Omega_{\mu\nu} + m_\omega^2\omega^\mu\omega_\mu\right) + \frac{1}{2}\left(\frac{1}{2}\vec{R}^{\mu\nu}\cdot\vec{R}_{\mu\nu} + m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu\right)
\end{aligned}
$$

Constants fitted to
reproduce NN shifts

Lagrangian in finite nuclei = IR

Structure = same as vacuum Lagrangian (≠ coupling constants)
+ NL σ terms

$$\mathcal{L} = \bar{\psi}\left(\gamma(i\partial - g_\omega\omega - g_\rho\vec{\rho}\vec{\tau} - eA) - m - g_\sigma\sigma\right)\psi + \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega^2 - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^{\,2} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4,$$

Constants fitted to nuclear data (charge radii, binding energy)

Easy to use : Hartree approximation (= slater determinants) OK for finite nuclei.

FRG strategy

First : work at LPA level, T ≠0, homogeneous system

Ansatz for the effective action

$$\Gamma_k = \int \mathrm{d}^4x \left\{ \bar\psi \left(\gamma^E_\mu \partial^E_\mu + g_\sigma \sigma + g_\pi i \gamma_5 \boldsymbol{\pi} \cdot \boldsymbol{\tau} + \gamma^0 \left(\boldsymbol{\mu} - g_\omega \omega_0 - g_\rho \rho^3_0 \tau^3 \right) \right) \psi \right.$$
$$\frac{1}{2} \partial^E_\mu \sigma \partial^\mu_E \sigma + \frac{1}{2} \partial^E_\mu \boldsymbol{\pi} \cdot \partial^\mu_E \boldsymbol{\pi}$$
$$\left. + U_k(T, \mu_p, \mu_n, \boldsymbol{\pi}, \sigma, \omega_0, \rho^3_0) \right\}$$

ω, ρ have M ~ Mnuc, taken at **mean field** level.
π, σ: can **fluctuate**.

Guided by phenomenological IR ansatz :
Taylor expand U in σ, π

$$U_{k,\sigma} = \sum_{n \geq 2} \frac{v_{k,n}}{n!} \sigma^n \qquad U_{k,\pi} = \sum_{n \geq 1} \frac{w_{k,n}}{n!} (\boldsymbol{\pi} \cdot \boldsymbol{\pi})^n$$

Flow equation

$$\frac{\partial U_k}{\partial k} = \frac{k^4}{12\pi^2}\left\{\frac{1+2n_B(E_\sigma)}{E_\sigma} + \frac{2\left(1+2n_B(E'_\pi)\right)}{E'_\pi} + \frac{1+2n_B(E''_\pi)}{E''_\pi} - 4\left[\sum_{i=n,p}\frac{1-\sum_{x=\pm1} n_F(E_{\sigma\pi}+x\mu_{i,\mathrm{eff}})}{E_{\sigma\pi}}\right]\right\} \quad (2.64)$$

où $E_\sigma = k^2 + U''_{k,\sigma}$, $E'_\pi = k^2 + U'_{k,\pi}$ et $E''_\pi = k^2 + U'_{k,\pi} + U''_{k,\pi}\pi^2$,

$$\mu_{p,k,\mathrm{eff}} = \mu_p - g_\omega\omega_{0,k} - g_\rho\rho_{0,k}, \quad \mu_{n,k,\mathrm{eff}} = \mu_p - g_\omega\omega_{0,k} + g_\rho\rho_{0,k}, \quad (2.65)$$

$$E^2_{\sigma\pi} \equiv k^2 + g^2_\sigma\sigma^2 + g^2_\pi\pi^2$$

Mean-Field equations for ρ, ω

$$\frac{\partial}{\partial \omega}\left[U_k(T, \mu_p, \mu - n, \sigma, \pi, \omega, \rho_{0,k}]\right|_{\omega=\omega_{0,k}} = 0, \quad = \frac{\partial}{\partial \rho}\left[U_k(T, \mu_p, \mu - n, \sigma, \pi, \omega_{0,k}, \rho]\right|_{\rho=\rho_{0,k}} = 0$$

Use Wetterich Equation to get rid of U_k

$$g_\omega \omega_{0,k} = \frac{G_\omega}{3\pi^2} \int_k^\Lambda \mathrm{d}p \frac{p^4}{E_{\sigma\pi}} \sum_{x=\pm 1} \partial_\mu \left[n_F(E_{\sigma\pi} - x\mu)|_{\mu=\mu_{p,k,\mathrm{eff}}} + n_F(E_{\sigma\pi} - x\mu)|_{\mu=\mu_{n,k,\mathrm{eff}}} \right],$$

$$g_\rho \rho_{0,k} = \frac{G_\rho}{3\pi^2} \int_k^\Lambda \mathrm{d}p \frac{p^4}{E_{\sigma\pi}} \sum_{x=\pm 1} \partial_\mu \left[n_F(E_{\sigma\pi} - x\mu)|_{\mu=\mu_{p,k,\mathrm{eff}}} - n_F(E_{\sigma\pi} - x\mu)|_{\mu=\mu_{n,k,\mathrm{eff}}} \right],$$