Adding a 3rd gaussian to the Gogny EDF : a new mass model suited for astrophysical applications

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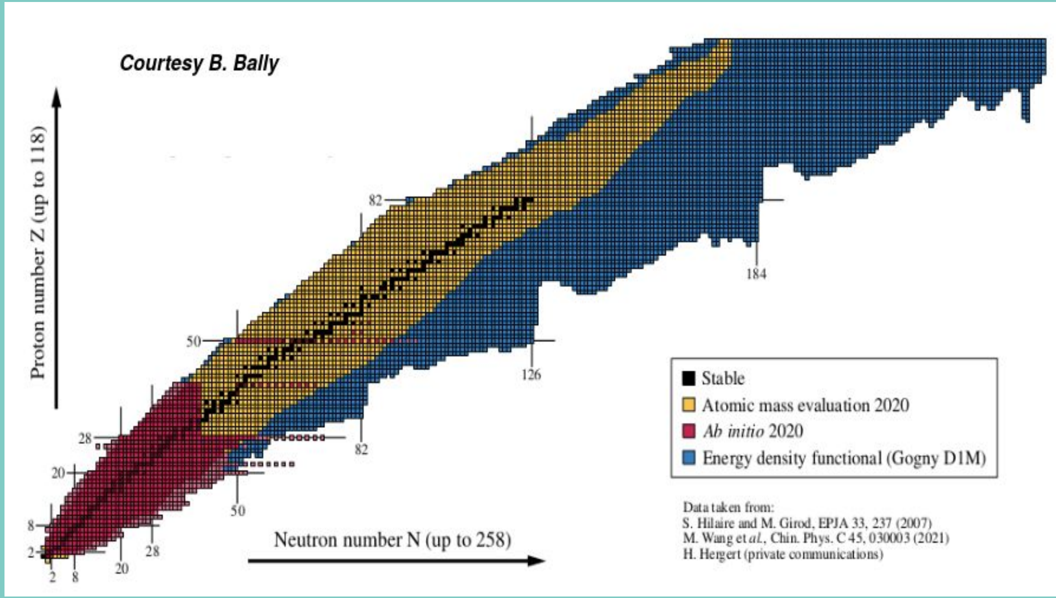
$$V_{12} = \sum_{j=1}^{\not{2}\,3} (W_j + B_j P_\sigma + H_j P_\tau + M_j P_\sigma P_\tau) e^{\frac{-(\vec{r}_1 - \vec{r}_2)^2}{\mu_j^2}}$$
$$+ t_3 (1 + x_0 P_\sigma) \delta(\vec{r}_1 - \vec{r}_2) \rho^\alpha \left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right)$$
$$+ i W_{LS} \overleftarrow{\nabla}_{12} \delta(\vec{r}_1 - \vec{r}_2) \wedge \overrightarrow{\nabla}_{12} (\vec{\sigma}_1 + \vec{\sigma}_2)$$

- How to best describe nuclear properties over the chart of known nuclei ?
- How to get reliable extrapolation over the unknown ?

Nuclear = N-body problem

- ab initio ➤ loads of recent progress the last years, but do not cover yet the region of heavy nuclei

- **Energy density functionals :**
all nuclei treated on an equal footing
reliable for A> few nucleons
extrapolation to drip-lines



- liquid drop ➤ low rms over masses but quite unreliable on extrapolation

We aim at staying as microscopic as possible ! (But with the least parameters...)

- How to best describe nuclear properties over the chart of known nuclei ?
- How to get reliable extrapolation over the unknown ?

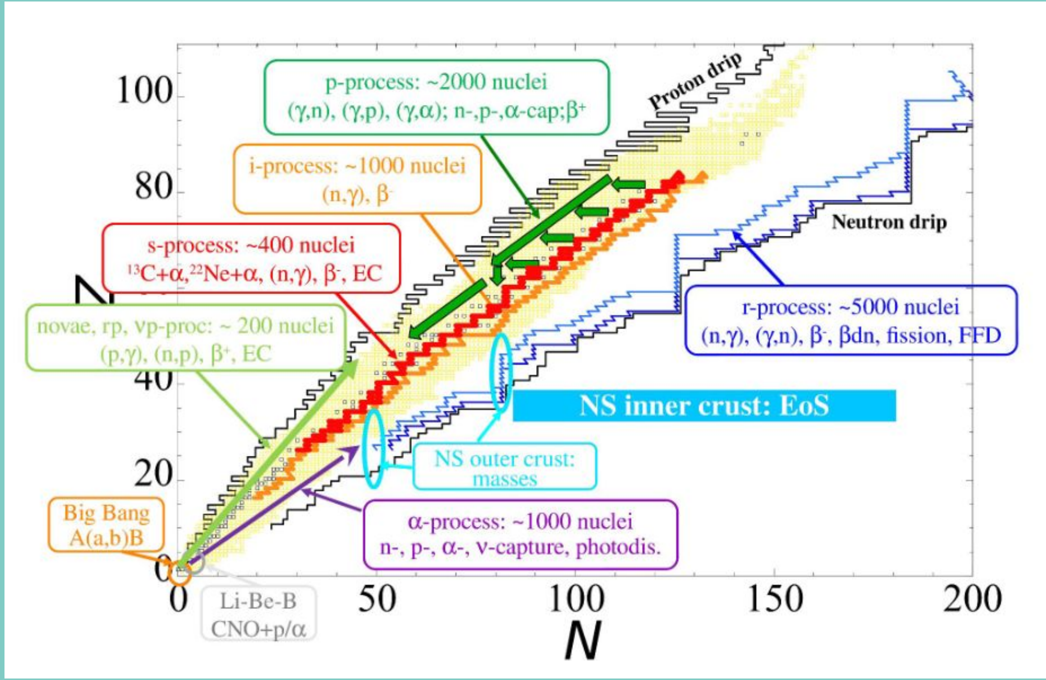
Energy density functionals :
Skyrme, Gogny, relativistic… Are complementary

Nuclear interaction = the “basic” input for astro calculations !

Strong impact on nucleosynthesis, neutron star masses, reaction mechanisms etc

➤ importance to be as precise as possible over what is measured to extrapolate beyond !

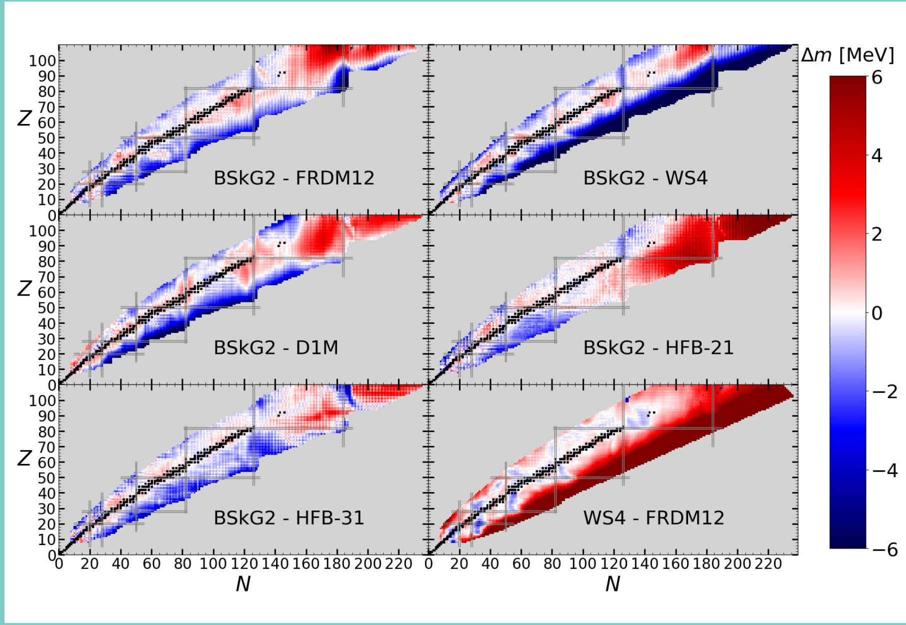
➤ Nuclear **mass** models

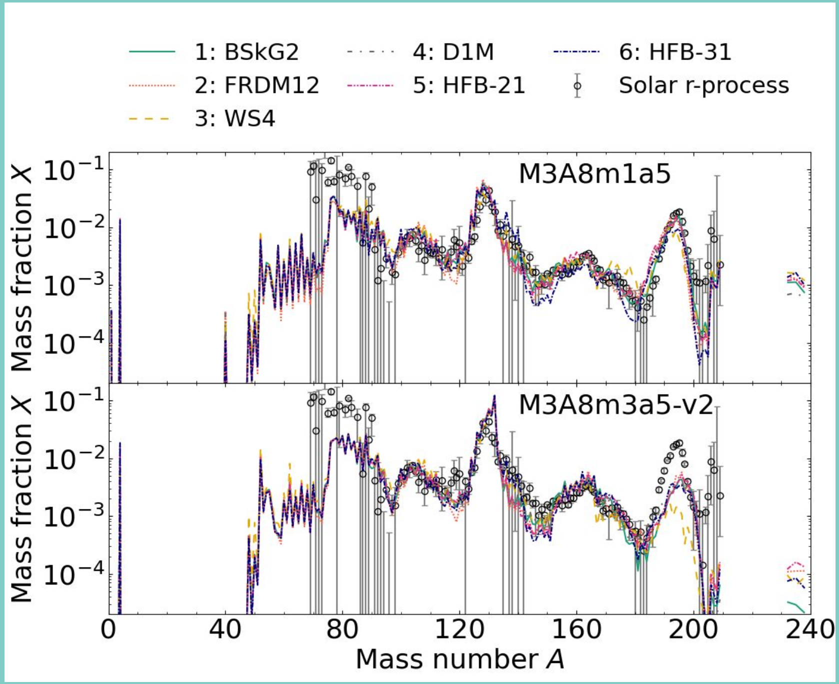


(That’s right, you’ve seen this figure in Wouter’s talk)

- How to best describe nuclear properties over the chart of known nuclei ?
- How to get reliable extrapolation over the unknown ?

Both the change of a model and the choice of a parameter set can have a huge impact on nucleosynthesis ...
Example : courtesy of I. Kullmann





(more in : I Kullmann, S Goriely & al. "Impact of systematic nuclear uncertainties on composition and decay heat of dynamical and disc ejecta in compact binary mergers", MNRAS, 523, 2 (2023))

The Gogny interaction

its free parameters to fit...

$$V_{12} = \sum_{j=1}^{2} (W_j + B_j P_\sigma + H_j P_\tau + M_j P_\sigma P_\tau) e^{\frac{-(\vec{r}_1 - \vec{r}_2)^2}{\mu_j^2}}$$ central terms

$$+ t_3 (1 + x_0 P_\sigma) \delta(\vec{r}_1 - \vec{r}_2) \rho^\alpha \left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right)$$ 3-body emulation

$$+ i W_{LS} \overleftarrow{\nabla}_{12} \delta(\vec{r}_1 - \vec{r}_2) \wedge \vec{\nabla}_{12} (\vec{\sigma}_1 + \vec{\sigma}_2)$$ spin-orbit term

originally : 14 parameters

A finite range for all the terms ? *G. Zietek's talk*
A regularized 3-body term ? *P. Da Costa's talk*

… And why we work on an extension !

—Infinite/Asymmetric Matter requirements — | *— Finite nuclei —*

	effective masses hierarchy m*n>m*p ?	*Pure Neutron Matter Eq. of state*	*Sym Energy*	*rms (MeV)*
D1S(1989) *Berger & al.*	Yes	~**<u>flat</u>** E(0.8)=22 MeV	**<u>collapsing after 0.2 fm^-3</u>**	**~5 (AME03)**
D1N(2007) *Chappert & al.*	**<u>No</u>**	**<u>very soft</u>** E(0.8)=80 MeV	**<u>collapsing after 0.3 fm^-3</u>**	**~5 (AME03)**
D1M(2008) *Goriely & al.*	Yes	~ **<u>quite soft</u>** E(0.8)=115 MeV	~**<u>flat</u>** E(0.8)=30 MeV	0.811 (AME20)
D1M*(2018) *Gonzalez-B. & al.*	**<u>No</u>**	stiff E(0.8)=180 MeV	stiff E(0.8)=110 MeV	>1.34 (AME03)
D2(2017) *N.Pillet & al.*	Yes	stiff E(0.8)=180 MeV	stiff E(0.8)=80 MeV	**~6(AME03)**

<u>IM Objectives :</u>
m*n>m*p (from micro. calc
+IsoVector Giant Res.)
+ non collapsing Esym at high density + EoS PNM not too low (max masses of Neutron Stars not too low)+ …

<u>Finite nuc Objectives :</u>
overall masses rms < 0.8 MeV (and < 0.03 fm on radii) + good pairing properties (ex. Sn)+…

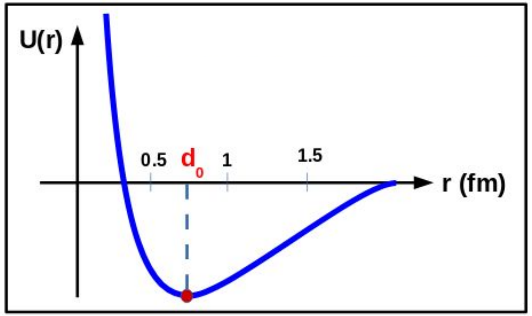
Long story short : fitting is a tedious task !

<u>And a bunch of other less known from standard or extensions : none satisfying all conditions at once !</u>

Recent motivations for an improved interaction

Why a third gaussian ?

- 3 ranges in the central term : short, medium and long range of the nuclear potential



- Ranges values connected to more microscopic potentials (meson exchange) : ratio of the exchange (Fock) over the direct (Hartree) contributions to the nucleon self-energy

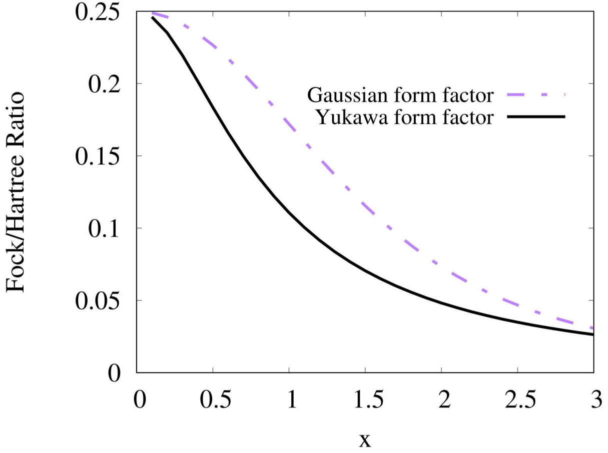
- "Relatively easy" implementation

$$e^{-\mu_Y r}/\mu_Y r \qquad \mu_Y = mc/\hbar$$

clear interpretation of the range for a Yukawa

$$R_Y = \frac{3}{16x_Y^4}\left[2x_Y^2 - 2x_Y \arctan 2x_Y + \operatorname{arctanh}\frac{2x_Y^2}{1+2x_Y^2}\right] \qquad x_Y = k_F/\mu_Y$$

$$R_G = \frac{3}{4x_G^4}\left[-2 + 2\,e^{-x_G^2} + \sqrt{\pi}\,x_G\,\mathrm{Erf}(x_G)\right] \qquad x_G = k_F\ \mu_G$$



$$R_Y(\mu_Y^{-1} = 0.256\ \text{fm}) = 0.214$$
$$R_Y(\mu_Y^{-1} = 0.402\ \text{fm}) = 0.178$$
$$R_Y(\mu_Y^{-1} = 1.407\ \text{fm}) = 0.052$$

rho, sigma and pion mesons from M3Y

$$\mu_1 = 0.475\ \text{fm},\ \ \mu_2 = 0.716\ \text{fm},\ \ \mu_3 = 1.78\ \text{fm}$$

Recent motivations for an improved interaction

Why a third gaussian ?

- The longest-range gaussian parameters linked to the One Pion Exchange Potential and the pion itself such as $(\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2)$
- $\rightarrow$ 4 contributions to the S,T channels are proportional

(W3,B3,H3,M3) →(W3,-2*W3,2*W3,-4*W3)

$(W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau)$

And then ?

A fit : Infinite Matter properties + a few finite nuclei binding energies +...

Allowing ourselves a few percentage of variation on estimated ranges (Gaussians are not Yukawas!)

—-------➤ Fitting and then... D3G3 was born

(L. Batail & al. EPJA 59, 173 (2023), details in PhD manuscript)

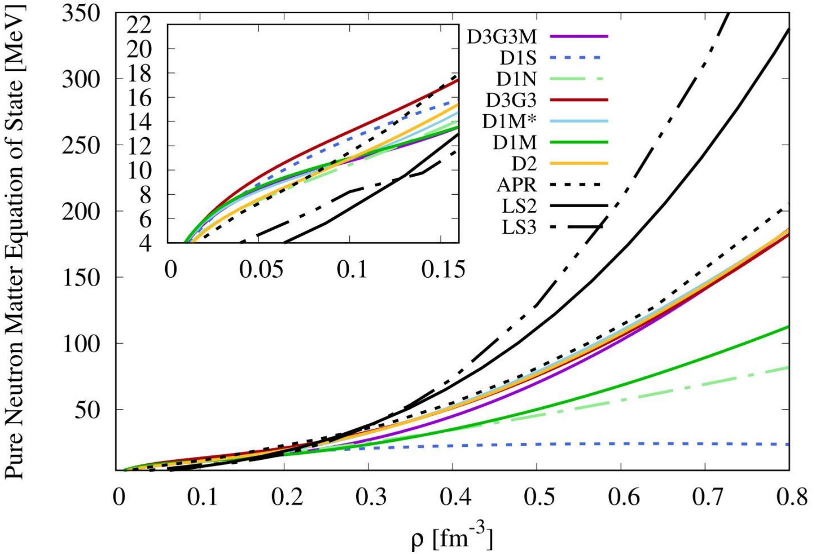
A few key values in homogeneous matter

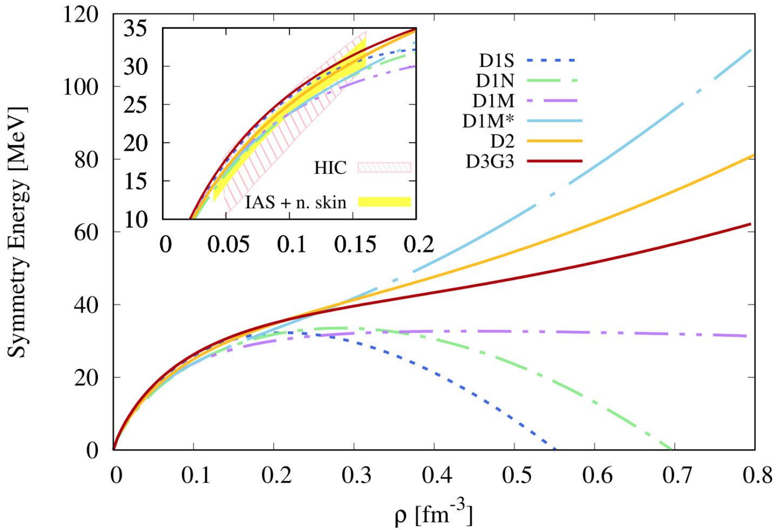
	Kinf (MeV)	**m*/m**	**Esym(rho_0)=J (MeV)**
target val.	210<Kinf<240 ?	0.74<m*/m<0.9?	28<J<32,34?
D1S	203	0.70	32.0
D1M	225	0.746	28.6
D1M*	225	0.746	30.3
D2	209	0.746	31.1
D3G3	227	0.68	32.6

Open question : which target values to refer to ?

- +/- wide ranges from micro. calculations but systematic fitting on masses reduce them naturally
- Some contradictory values (*i.e.* on J and L from neutron skins)
- **Whatever the protocole, competition : improving IM properties degrades finite nuc. and *vice versa***

Pure neutron matter equation of state and symmetry energy





Maximum Neutron Star mass beared with APR : ~2.15 Msun

Maximum Neutron Star mass beared with FP : ~1.8 Msun (D1M)

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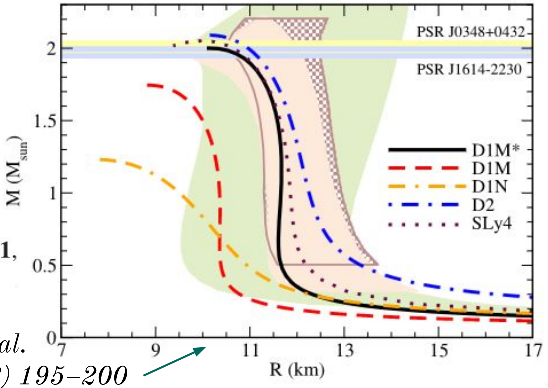
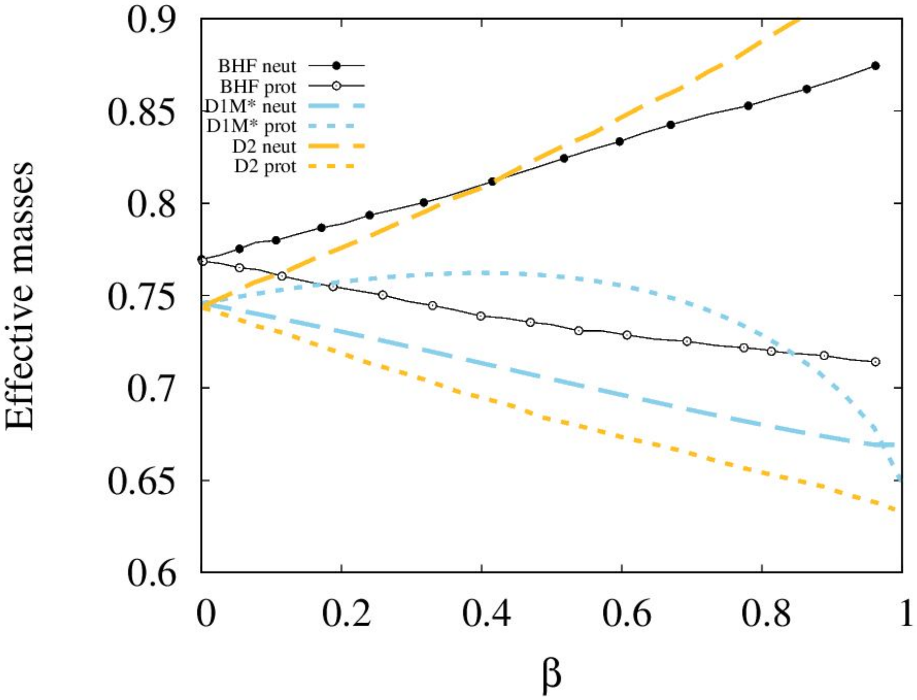
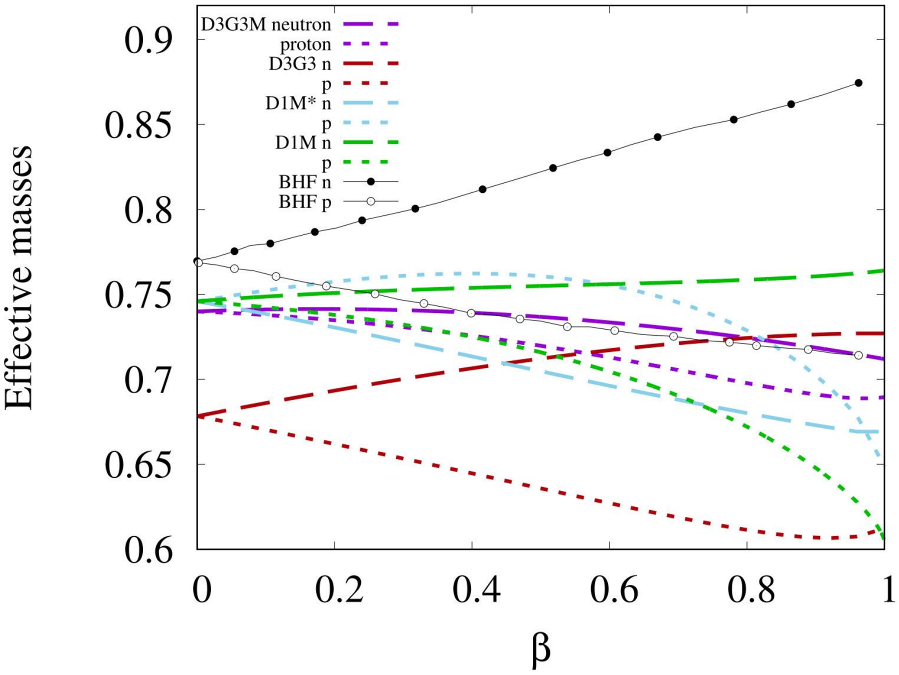


from C. Gonzalez-Boquera et al. Physics Letters B 779 (2018) 195–200

Ideally (future) : a PNM EoS stiffer than APR

Effective masses m*/m (=nucleon masses in nuclear medium)





asymmetry parameter β=ρn-ρp/ρ

IVGDR constrain the proton to neutron hierarchy around saturation density
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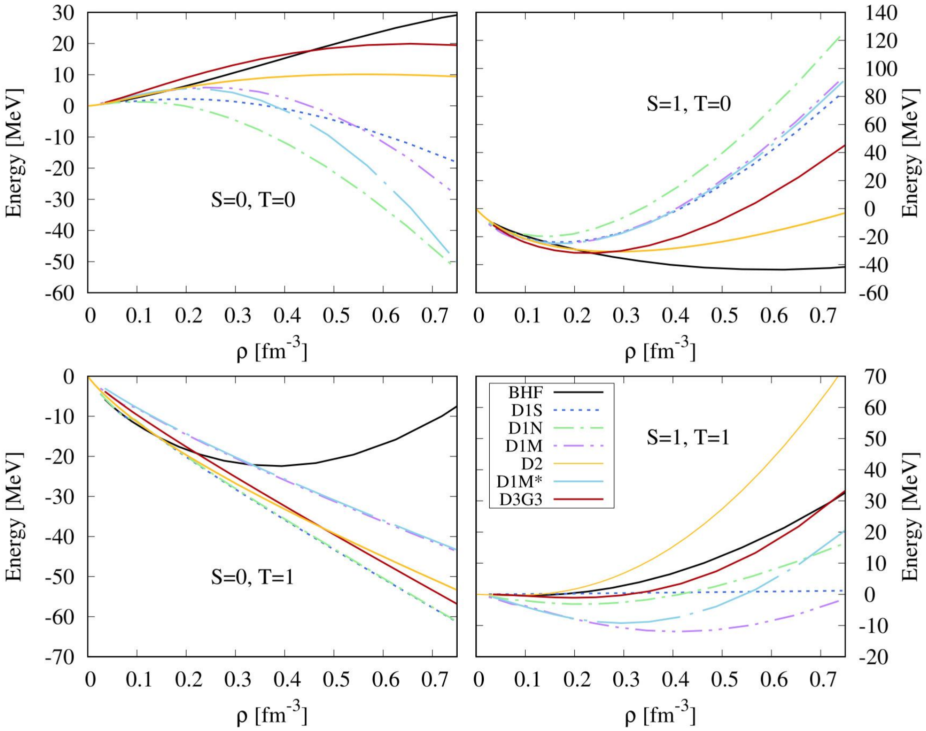

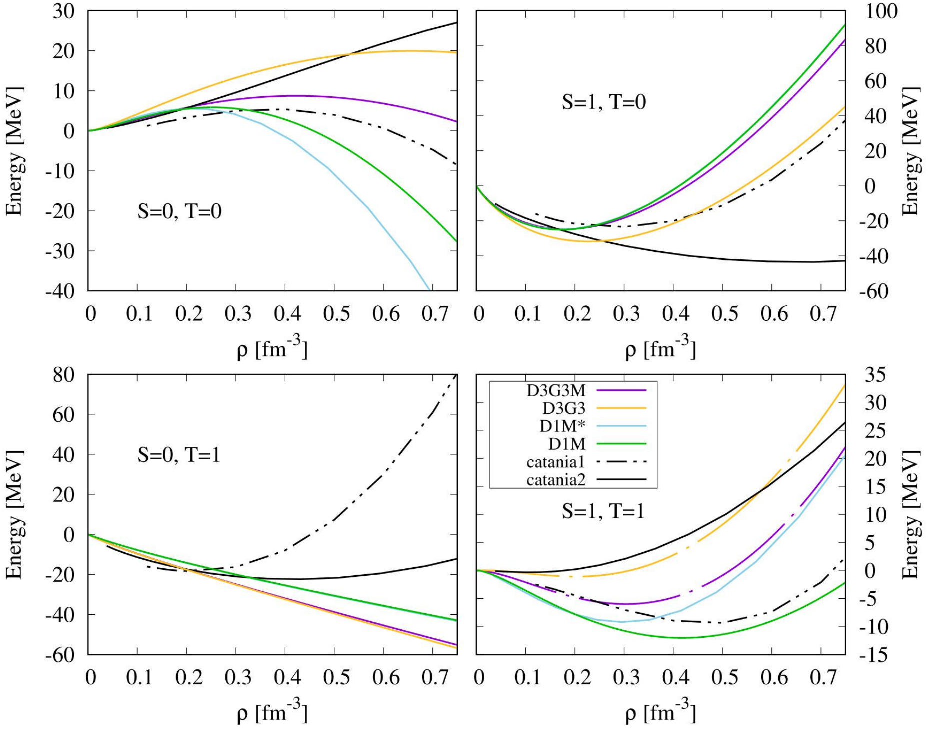

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- Quite some differences between catania 1 et 2 : which microscopic calculations to compare to ? What to expect at high densities ?

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D3G3 in a nutshell

	—Infinite/Asymmetric Matter requirements —			*— Finite nuclei —*
	effective masses hierarchy m*n>m*p ?	*Pure Neutron Matter Eq. of state*	*Sym Energy*	*rms (MeV)*
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D3G3(2023) *L.Batail & al.*	Yes	stiff E(0.8)=180 MeV	stiff E(0.8)=62 MeV	**~6 MeV (2000 nuc)**

Objectives in nuclei :
overall masses rms < 0.8 MeV (and < 0.03 fm on radii) + good pairing properties (ex. Sn)+...

Objectives in nuclear matter :
m*n>m*p (from micro. calc. +IsoVector Giant Res.) + non collapsing Esym at high density + EoS PNM not too low (max masses of Neutron Stars not too low)+ ...

D3G3 has very good nuclear matter properties but rms on masses **way too high** !

-------> Let's go for a more systematic fitting with Brussels protocole

Fitting procedure over Infinite Matter properties and finite nuclei

protocole similar to the D1M one

Inclusion of the 3rd Gaussian in all codes and then :

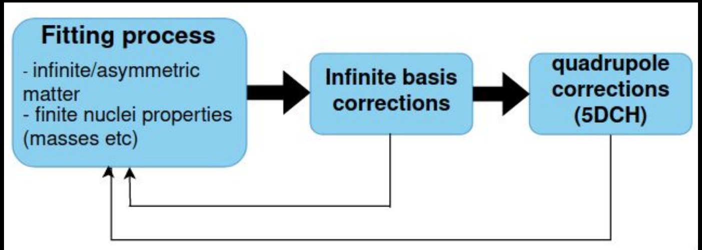
Free parameters and INM properties : **analytical expressions**

- ➢ matrix inversion
- ➢ INM properties = the new parameters fixed or let vary
    - In our case : 9 params inverted, 5 fixed, 2 letting vary and 3 related to another
    - *NB : only W3 out of the 3rd range is let vary (same for D3G3)*

- ➢ 2457 nuc, A>30, including odd ones
- ➢ self-consistent quadrupole corrections (5DCH beyond mean-field) *not at each iter.*
- ➢ infinite basis corrections *not at each iter.*

D3G3M is born !

Numerical tools and other considerations



- HFB code used to determine ground state properties on axially deformed HO basis
- Use of triaxial code outputs as inputs for 5DCH code to get quadrupole corrections

- What about infinite basis correction ?

$$B_{inf} = \Delta E(\frac{1}{3}e^{4-\Delta base} + 1)$$

Results with D3G3M

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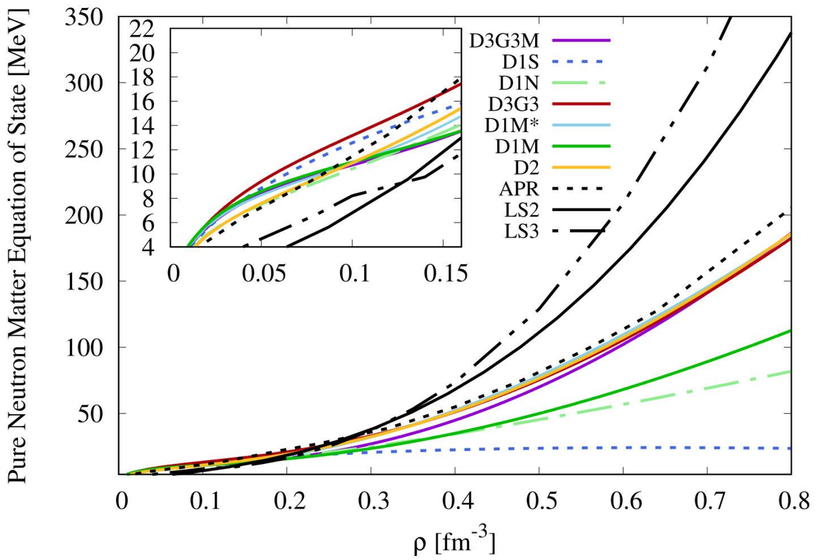

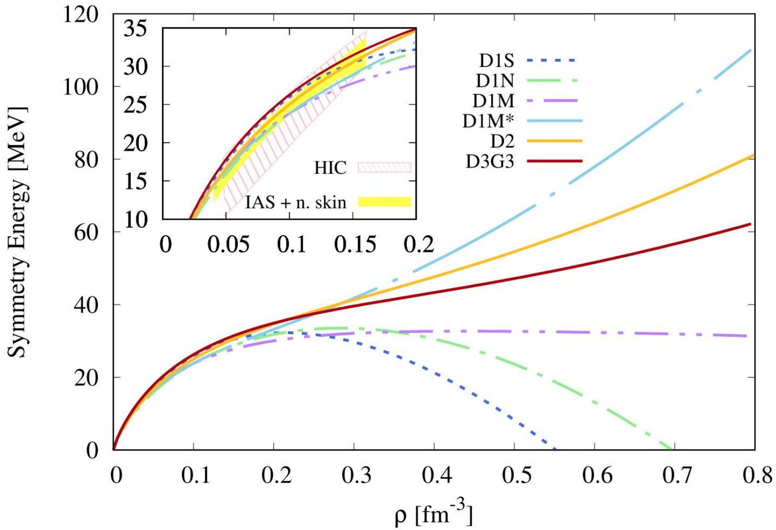

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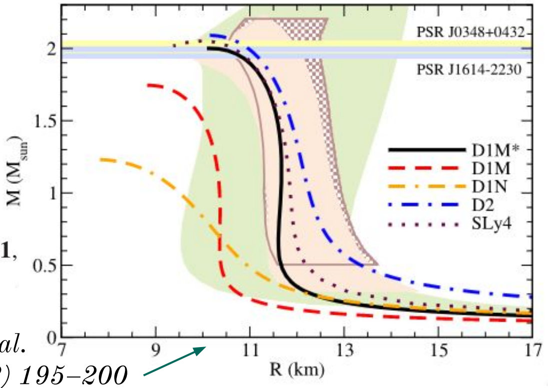

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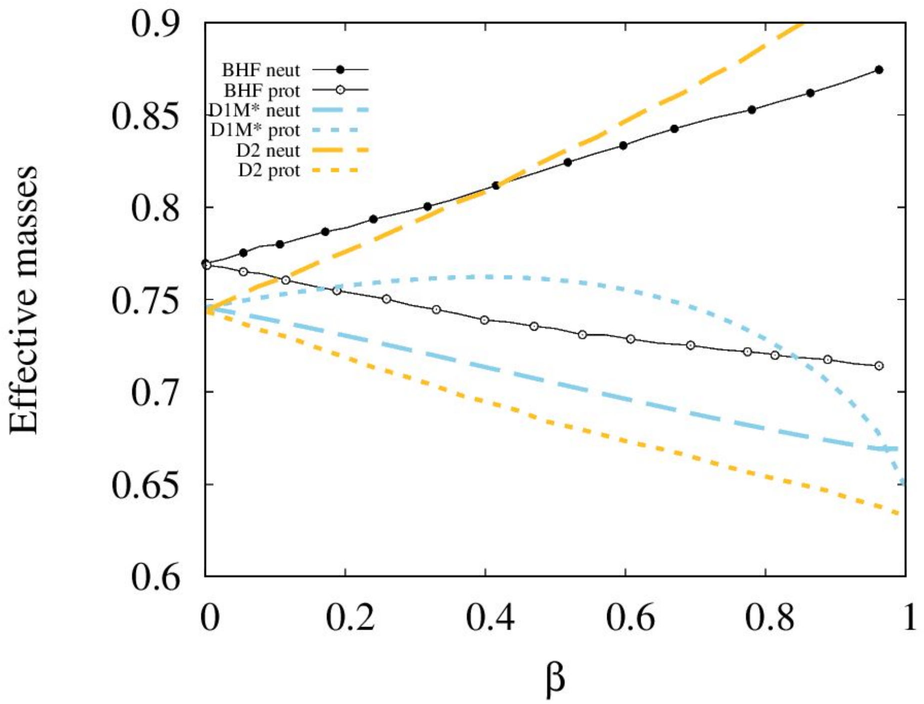

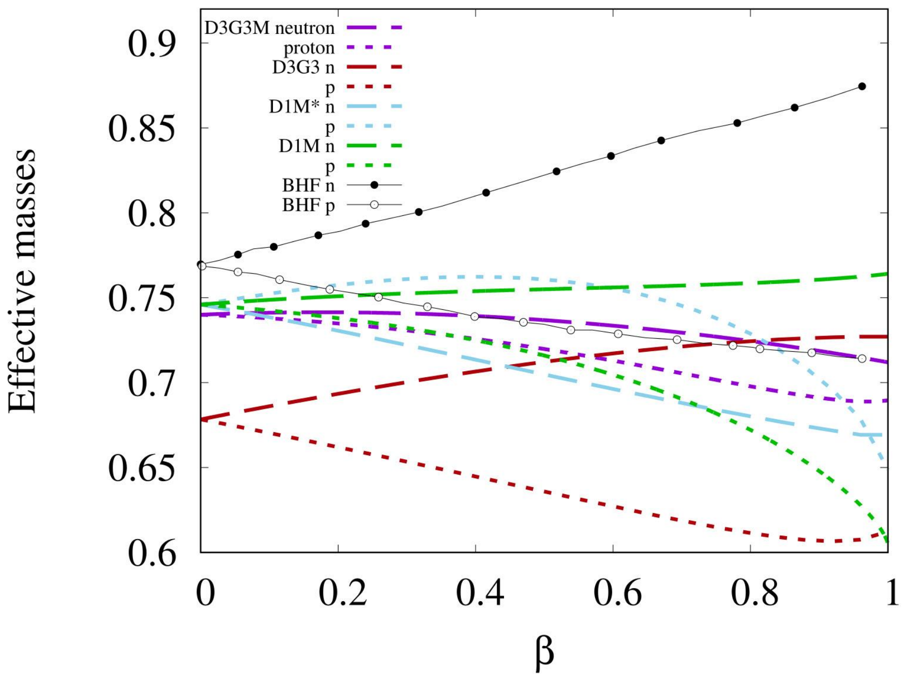

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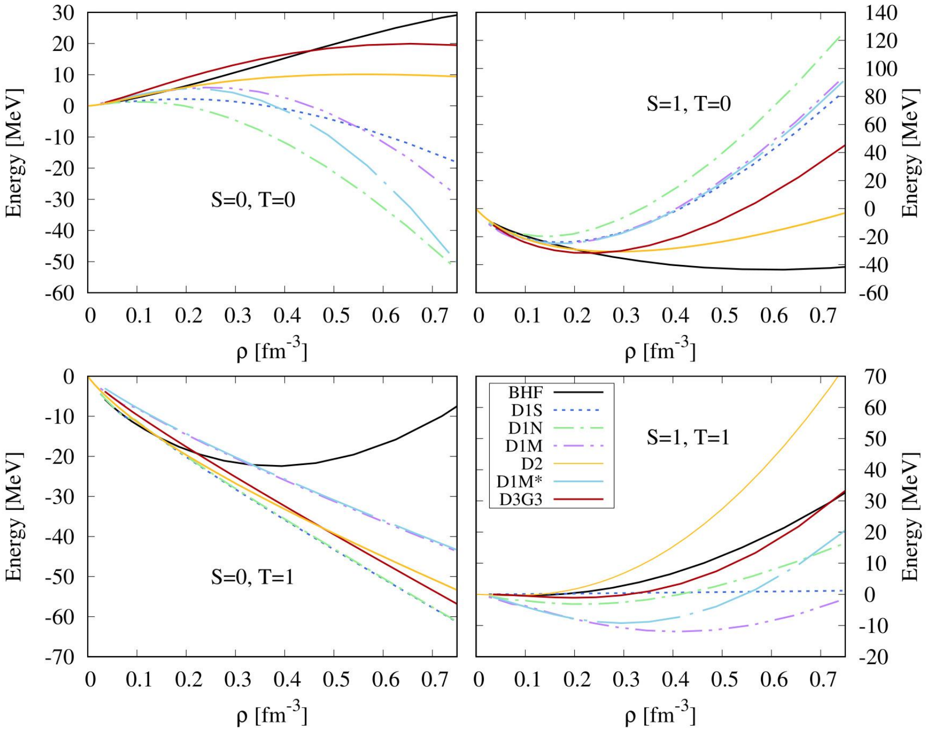

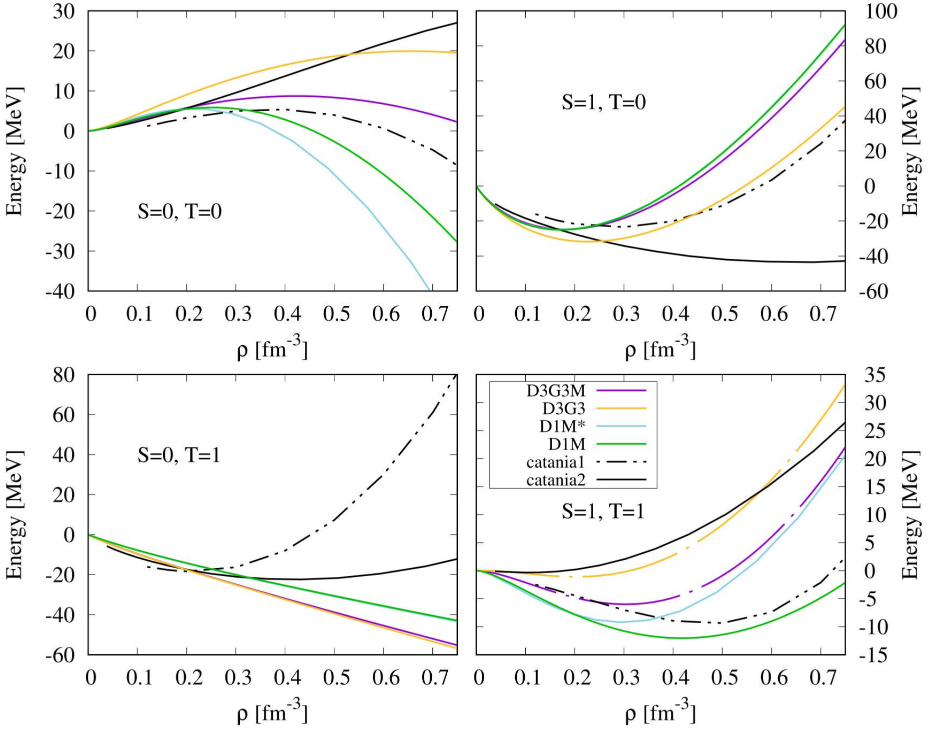

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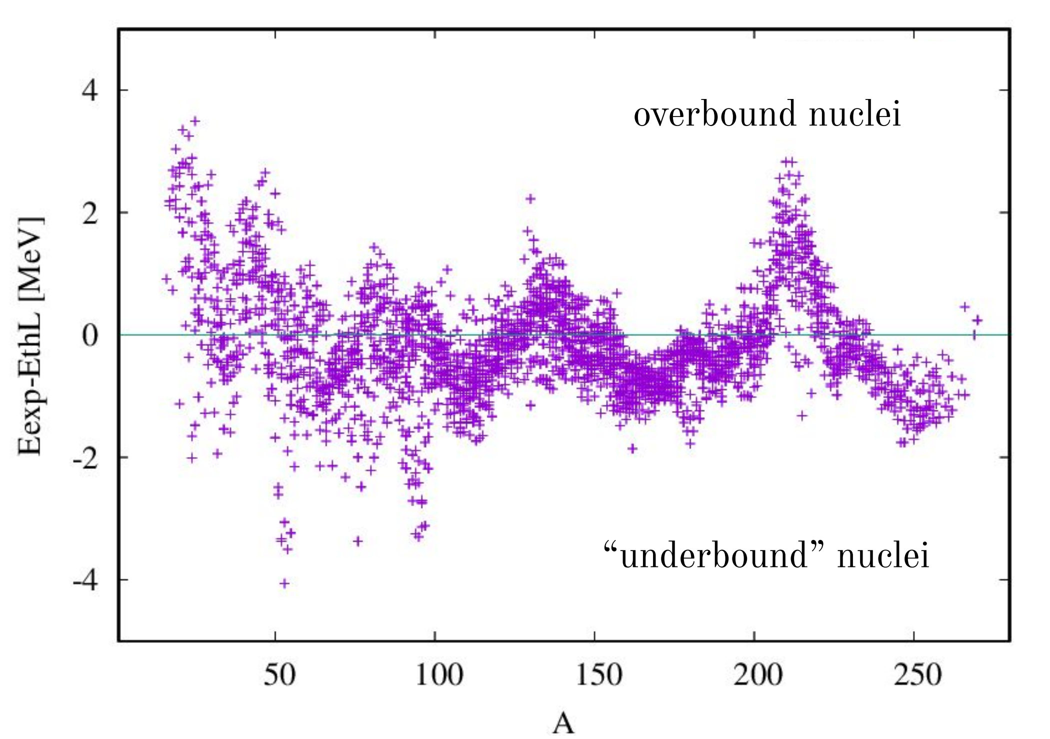
Finite nuclei

similar trends with Skyrme mass models...

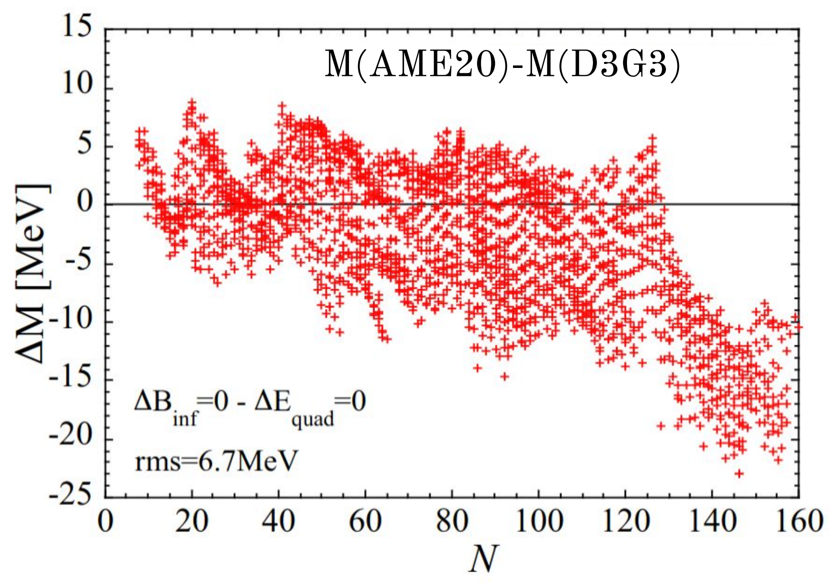
	Masses rms (MeV)	radii rms (MeV)
D3G3M (AME20)	0.875	0.029
D1M (AME20)	0.811	0.031

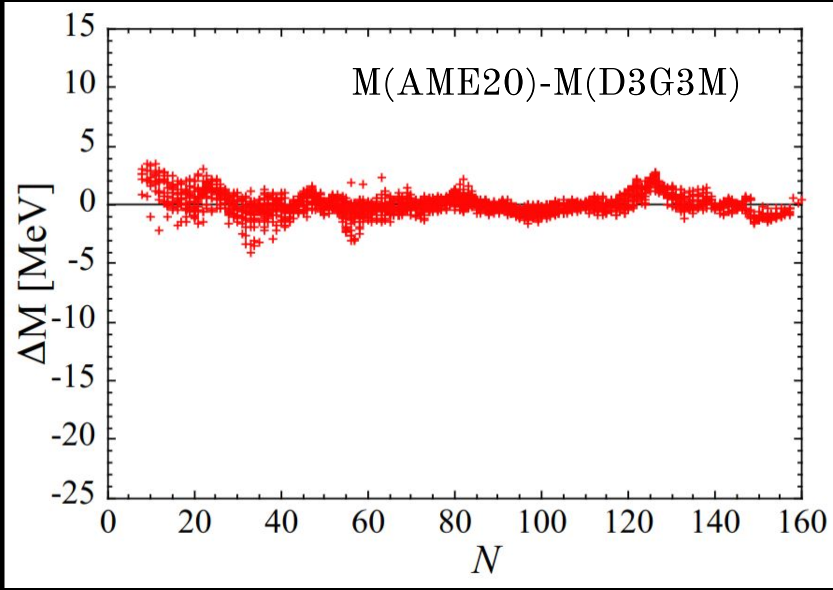


What to improve with light nuc ?

Which direction to take for magic ones ?

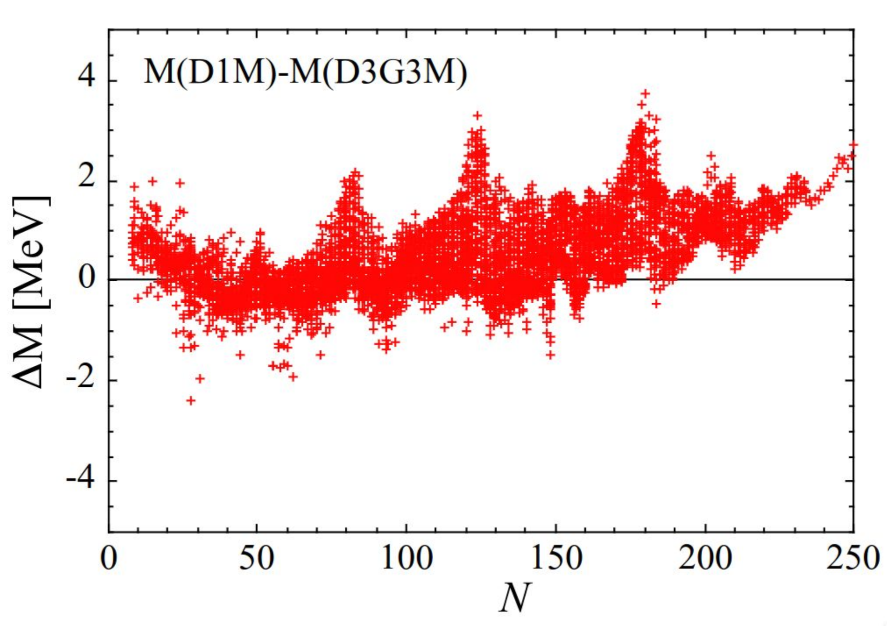
Finite nuclei : fitting on a large scale

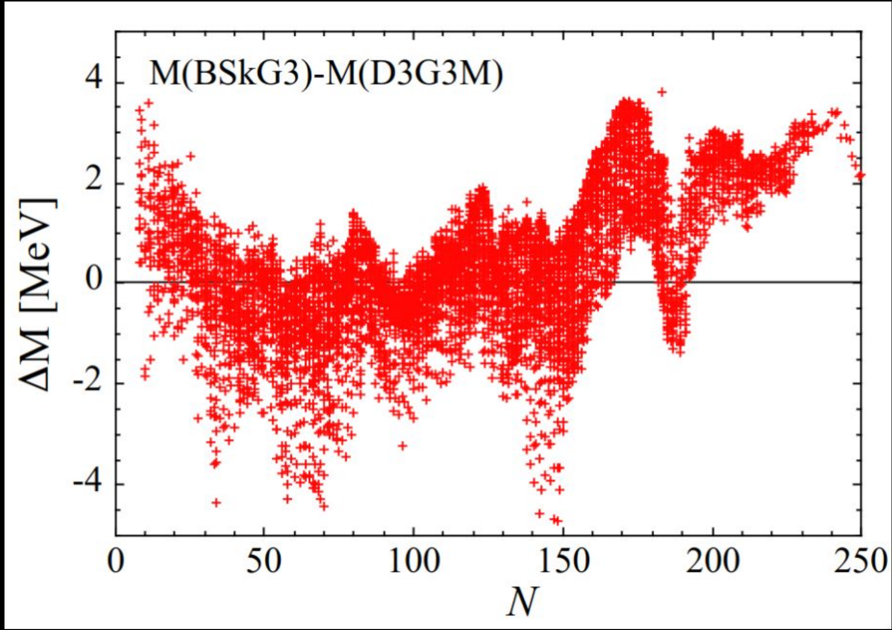




To give you an idea of the improvements from D3G3 to D3G3M : the mass fitting TREMENDOUSLY reduces the dispersion !

What happens for 8000 nuclei ?





5 MeV maximum, including between two Skyrme and Gogny mass models

Take away messages

With 3 gaussians in the central term :

- better overall Infinite Matter properties than in the past
- pretty good mass rms

D3G3M is all together the best Gogny mass model so far

With D3G3M in particular, we still have to check :

- instabilities
- S2N for driplines
- giant resonances
- partial waves etc

& perspectives

- release constraint on the 3rd range parameter ?
- going beyond ?
- fitting over actual ground state spin+parity for odd nuclei (Sophie's statement) ?

S. Goriely
W. Ryssens
G. Grams
...

D.Davesne

J.Navarro

Thank you for your attention and your collaboration !

S. Hilaire
S. Péru
A. Pastore