# Functional Renormalization Group of the Nuclear Energy Density Functional Method

Jean-Paul EBRAN
CEA, DAM, DIF

Louis HEITZ
DPhN
IJCLab

*Nuclear energy density functional method : going beyond the minefield*

*ESNT, 20-24 November 2023*

# Outline

## 1 Context

*Where does the EDF method stand within the landscape of nuclear structure theories ?*

## 2 Lessons from empirical EDFs

*1st lesson : Effective (pseudo-)Hamiltonians with simple forms do the job*

*2nd lesson : Static correlations can be optimally grasped via SSBs + bosonic fluctuations of order parameters*

## 3 Towards a rigorous formulation of nuclear EDFs

*WFT, DFT & EA perspectives*

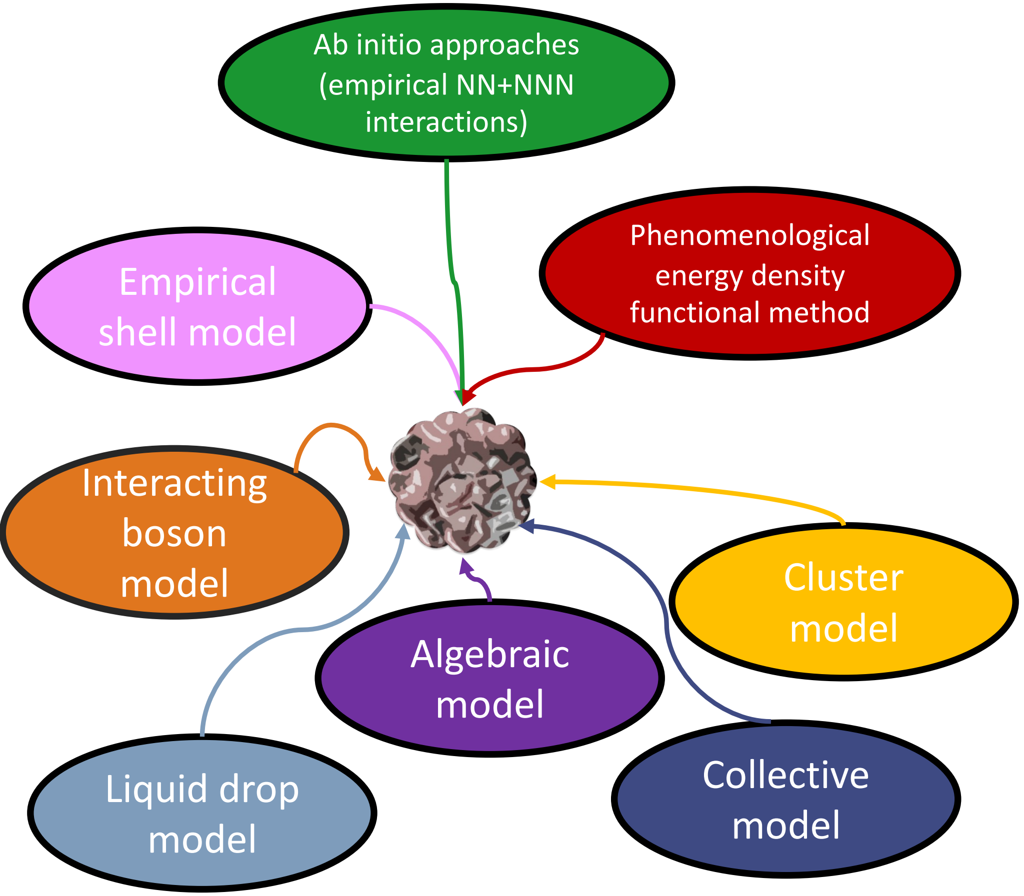
*FRG*

*Application to symmetric nuclear matter*

# ❶ Context : Strategies

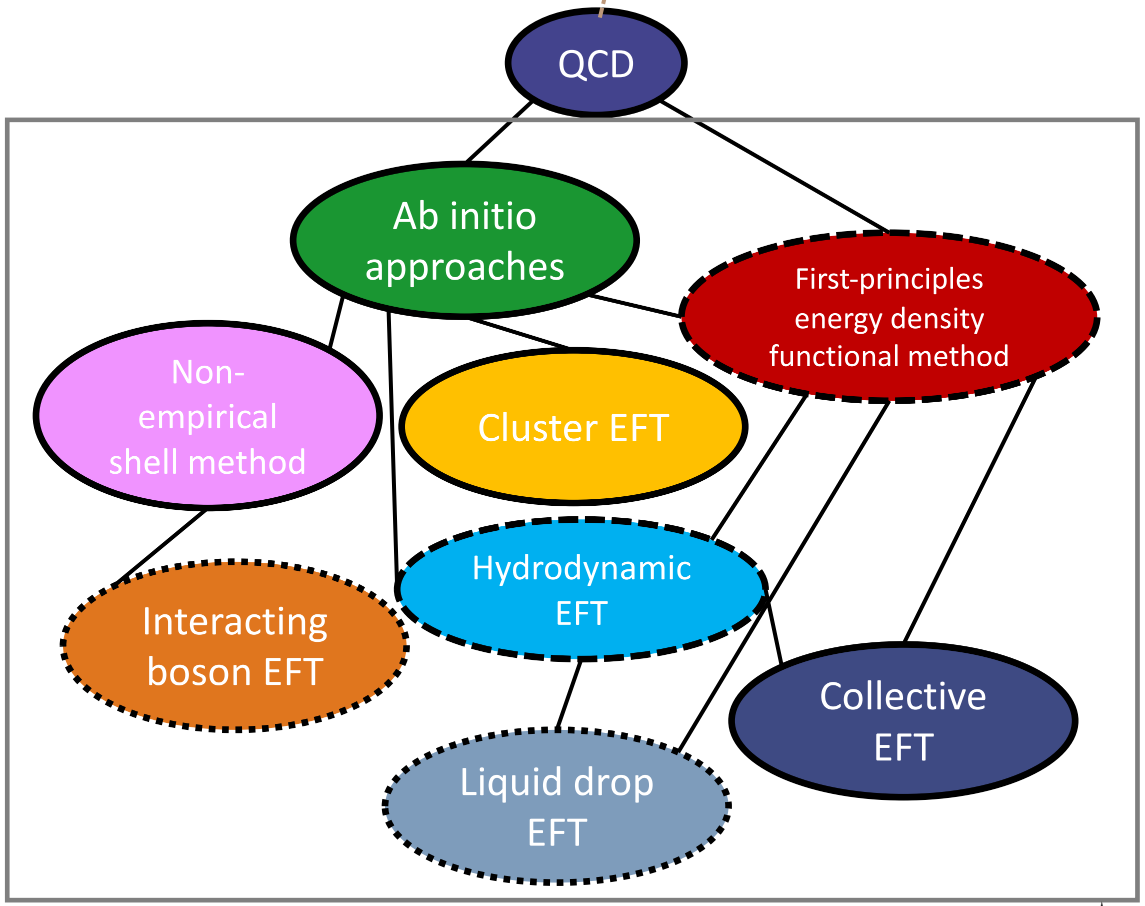
◉ Achieve a { *accurate* / *predictive* / *computationally affordable* } description ?

## Era of models

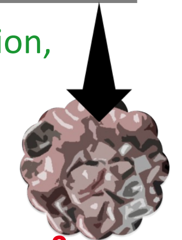



- ☑ Gives insight about relevant scales/dofs
- ☑ Ready to be used
- ☒ Lack of control
  ⇒ double counting issues, error compensation, no error assessment

## Era of effective (field) theories




- ☑ Full control ⇒ systematically improvable, no error compensation, no double counting, possibility of error estimation, …
- ☑ ☒ Force you to step back and rethink



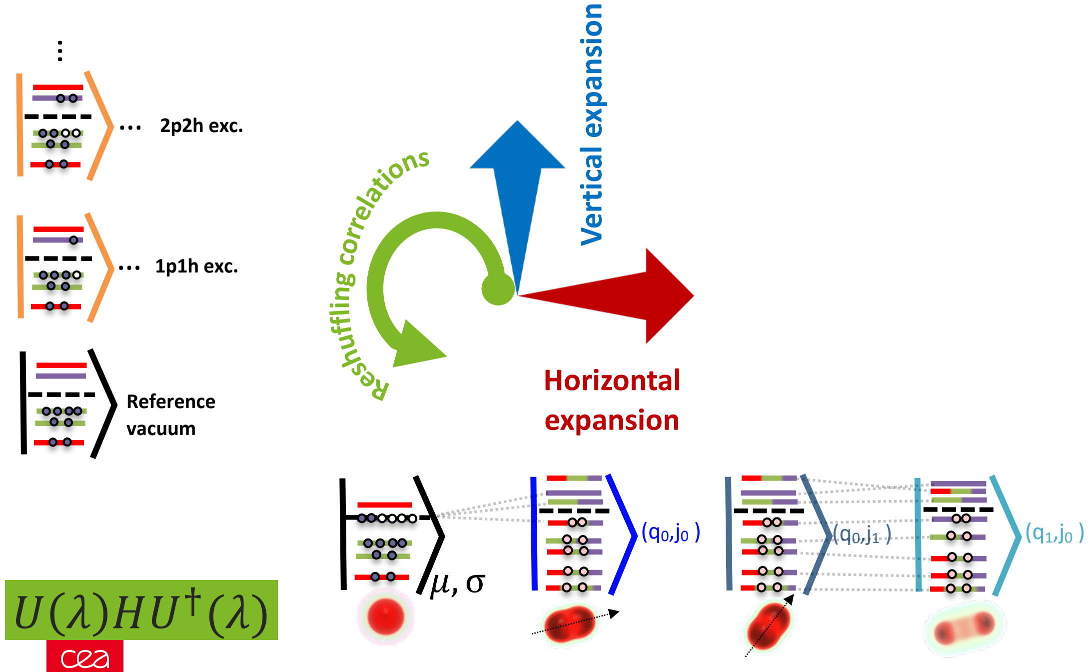
# 1 Context : Nuclear structure from a microscopic viewpoint

1) Nucleus: *A* interacting, structure-less nucleons
2) Structure & dynamic encoded in Hamiltonian, Functional, …
3) Solve *A*-nucleon Schrödinger/Dirac equation to desired accuracy

$$H(\ldots, \ldots, \ldots)|\Psi_{\mu,\sigma}\rangle = E_{\mu\tilde{\sigma}}|\Psi_{\mu,\sigma}\rangle$$

Strongly correlated WF

### *Rationale for grasping nucleon correlations*




$U(\lambda)HU^{\dagger}(\lambda)$

## Ab initio

◉ **Systematically improvable free-space Hamiltonian in $\chi$EFT**

◉ **Solving Schrödinger equation**

◈ Pre-processing H

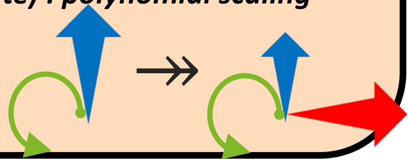
◈ Refined many-body schemes with controlled uncertainties

⇢ *CI (full space diag.) : exponential scaling*

⇢ *Hybrids (valence space diag.) : mixed scaling*

⇢ ***Expansion methods (partition, expand and truncate) : polynomial scaling***

⊗ *How to challenge ab initio frontiers*



## EDF

◉ **Effective pseudo-Hamiltonian**

*Free-space interactions* ↠ *Effective in-medium interactions*

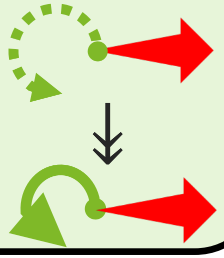
$|\Psi_{\mu,\sigma}\rangle$ *Complicated WF* ↠ $|\Theta_{\mu\sigma}\rangle$ Simplified auxiliary WF

◉ **Various levels of realization**

⇢ *Hartree-Fock-Bogoliubov (HFB)*

⇢ *Projected Generator Coordinate Method (PGCM)*

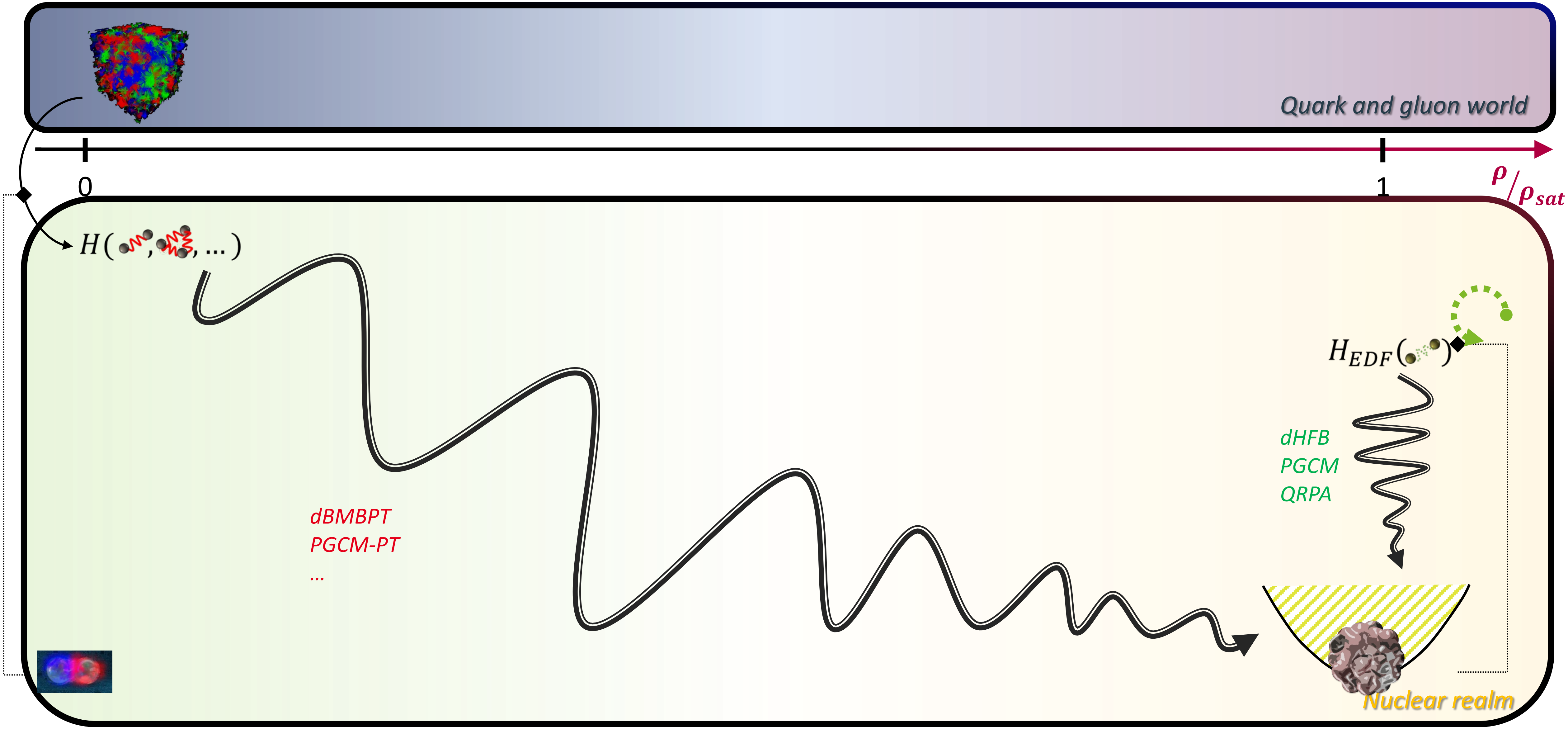
⇢ *Quasiparticule Random Phase Approximation (QRPA)*



⊗ *How to improve current EDFs*

⊗ *How to turn EDF in EFT ?*

# 1 Context : Nuclear structure from a microscopic viewpoint



# Outline

## ❶ Context

*Where does the EDF method stand within the landscape of nuclear structure theories ?*

## ❷ Lessons from empirical EDFs

*1st lesson : Effective (pseudo-)Hamiltonians with simple forms do the job*

*2nd lesson : Static correlations can be optimally grasped via SSBs + bosonic fluctuations of order parameters*

## ❸ Towards a rigorous formulation of nuclear EDFs

*WFT, DFT & EA perspectives*

*FRG*

*Application to symmetric nuclear matter*

# 2 Lessons from empirical EDFs : main idea

◉ **Hamiltonian $H$ acting in $\mathcal{H}_A$** and **Schrödinger equation**

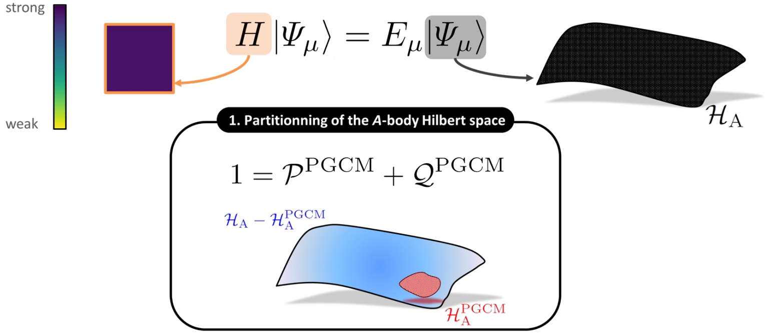
$$H = T + V + W + \cdots$$
$$= \frac{1}{(1!)^2}\sum_{\substack{a_1\\b_1}} t^{a_1}_{b_1} A^{a_1}_{b_1} + \frac{1}{(2!)^2}\sum_{\substack{a_1a_2\\b_1b_2}} v^{a_1a_2}_{b_1b_2} A^{a_1a_2}_{b_1b_2} + \frac{1}{(3!)^2}\sum_{\substack{a_1a_2a_3\\b_1b_2b_3}} w^{a_1a_2a_3}_{b_1b_2b_3} A^{a_1a_2a_3}_{b_1b_2b_3} + \cdots$$

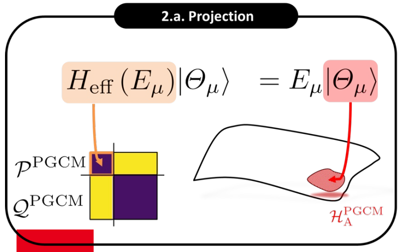
$$A^{a_1\dots a_k}_{b_1\dots b_k} \equiv c^\dagger_{a_1}\dots c^\dagger_{a_k} c_{b_k}\dots c_{b_1}$$

$$H|\Psi_\mu\rangle = E_\mu|\Psi_\mu\rangle$$

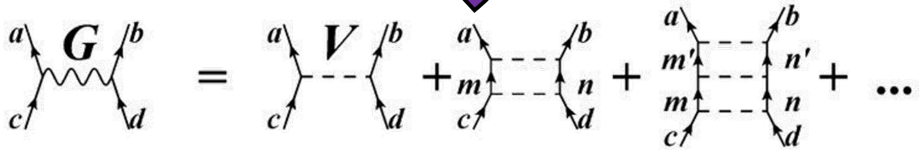
◉ EDF method postulates the existence of **$H_{EDF}$ acting in $\mathcal{H}_A^{\text{PGCM}}$** yielding the same low-energy observables than with $H$

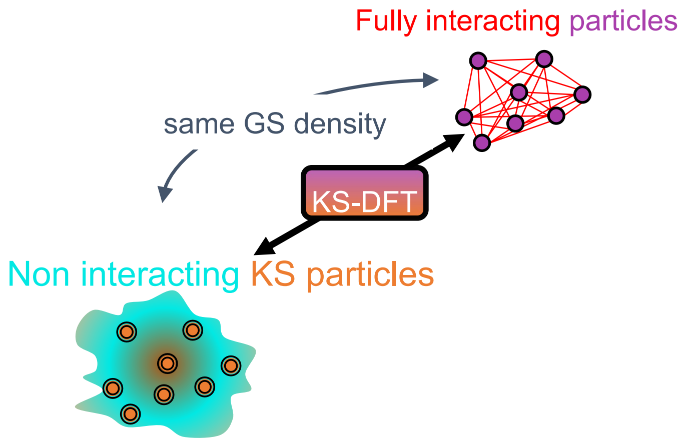
$$E_\mu = \frac{\langle\Psi_\mu|H|\Psi_\mu\rangle}{\langle\Psi_\mu|\Psi_\mu\rangle} = \frac{\langle\Theta_\mu|H_{EDF}|\Theta_\mu\rangle}{\langle\Theta_\mu|\Theta_\mu\rangle}, \quad |\Theta_\mu\rangle \in \mathcal{H}_A^{\text{PGCM}}$$




**Projection** **Resummation** **Mapping**




**⇒ Resulting objects are real monsters !!**

# ❷ Lessons from empirical EDFs : Lesson 1

◉ Empirical effective interactions with simple forms do the job !!

**Galilean EDF** | **Lorentzian EDF**

## Explicit density-dependence

### Gogny D1 vertex

$$V_{12} = \sum_{i=1,2} (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau)\, e^{-\frac{(\vec r_1 - \vec r_2)^2}{\mu_i^2}}$$
$$+ t_0 (1 + x_0 P_\sigma)\, \delta(\vec r_1 - \vec r_2)\, \rho^\alpha\left(\frac{\vec r_1 - \vec r_2}{2}\right)$$
$$+ \mathrm{i} W_{LS}\, \overleftarrow{\nabla}_{12}\, \delta(\vec r_1 - \vec r_2) \times \overrightarrow{\nabla}_{12} \cdot (\vec\sigma_1 + \vec\sigma_2)$$

### DDME Lagrangians

$$\mathcal{L}_{NN} = \overline{\psi}\left(i\gamma^\mu \partial_\mu - M - \sum_b g_b(\rho)\, \phi_b \mathcal{O}_b\right)\psi$$

$$g_i(\rho_v) = g_i(\rho_{sat}) f_i(\xi)\ ,\ i = \sigma,\ \omega,$$
$$f_i(\xi) = a_i \frac{1 + b_i (\xi + d_i)^2}{1 + c_i (\xi + d_i)^2},$$
$$g_\rho(\rho_v) = g_\rho(0) e^{-a_\rho \xi},$$
$$f_\pi(\rho_v) = f_\pi(0) e^{-a_\pi \xi},$$

## Non explicit density-dependence

### Bennaceur et al semi-regularized vertex

$$\hat V(x_1, x_2; x_3, x_4) = \delta(\mathbf{r}_1 - \mathbf{r}_3)\delta(\mathbf{r}_2 - \mathbf{r}_4) g_a(r_{12}) \hat O_j^{(n)}(\mathbf{k}_{12}, \mathbf{k}_{34})$$
$$\times \left\{ W_\nu^{(n)} \hat 1_\sigma \hat 1_\tau + B_\nu^{(n)} \hat{\mathbb{P}}_\sigma \hat 1_\tau - H_\nu^{(n)} \hat 1_\sigma \hat{\mathbb{P}}_\tau - M_\nu^{(n)} \hat{\mathbb{P}}_\sigma \hat{\mathbb{P}}_\tau \right\}$$

$$\hat V = W_3 \left(\hat V_1 + \hat V_2\right)$$

$$\hat V_1 = \hat 1_{\mathbf{r}} \hat 1_q \hat 1_\sigma g_{a_3}(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_2 - \mathbf{r}_3)\,,$$
$$\hat V_2 = \hat 1_{\mathbf{r}} \hat 1_q \hat{\mathbb{P}}^\sigma_{23} g_{a_3}(\mathbf{r}_1 - \mathbf{r}_2)\delta(\mathbf{r}_2 - \mathbf{r}_3)\,.$$

### NL Lagrangians

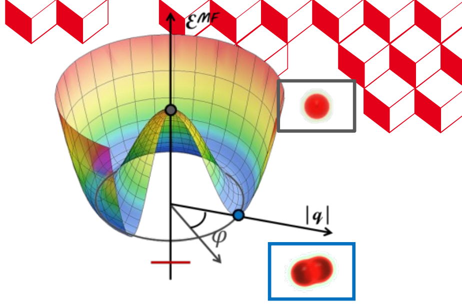
$$\mathcal{L}_{NN} = \overline{\psi}\left(i\gamma^\mu \partial_\mu - M - \sum_b g_b \phi_b \mathcal{O}_b\right)\psi - U[\sigma]$$

$$U[\sigma] = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{g_2}{3}\sigma^3 + \frac{g_3}{4}\sigma^4$$

$$\mathcal{L}^\sigma_{NN} = \left[g_\sigma \overline{\psi}\sigma\psi\right](x), \qquad \mathcal{L}^\pi_{NN} = \left[\frac{f_\pi(\rho_v)}{m_\pi}\overline{\psi}\gamma^5\gamma^\mu \partial_\mu \vec\pi \star \vec\tau\psi\right](x)$$
$$\mathcal{L}^\omega_{NN} = \left[g_\omega \overline{\psi}\gamma^\mu \omega_\mu \psi\right](x),$$
$$\mathcal{L}^\rho_{NN} = \left[g_\rho \overline{\psi}\gamma^\mu \vec\rho_\mu \star \vec\tau\psi\right](x), \qquad \mathcal{L}^{\omega+\rho;T}_{NN} = \left[\overline{\psi}\sigma^{\mu\nu}\left(-\frac{g^T_\omega}{2M}\Omega_{\mu\nu} - \frac{g^T_\rho}{2M}\vec{\mathcal{R}}_{\mu\nu} \star \vec\tau\right)\psi\right](x)$$

--> Simple form ⇔ Fermi-liquid fixed point to be grasped via RG techniques ?

# 2 Lessons from empirical EDFs : Lesson 2

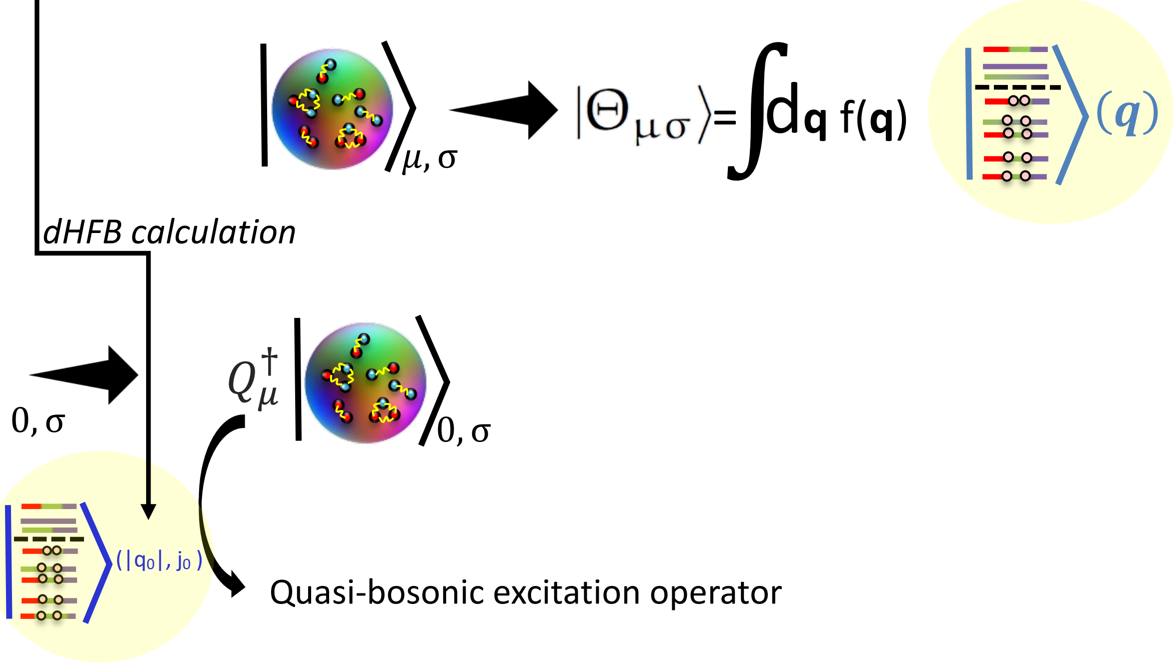



## ◉ Lesson n°2 : GS + low-lying collective excited states via horizontal expansion

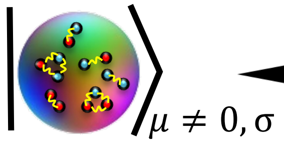
◈ dHFB treatment


◈ Post-HFB treatment : PGCM

--> Symmetry-conserving (non orthogonal) mixture of symmetry-breaking HFB vacua




◈ Post-HFB : QRPA




--> Excitations = coherent mixture of 2-qp excitations

--> Harmonic limit of the GCM

--> Static correlations : fluctuations of bosonic order parameters

⇒ (Partially-)bosonizing the theory ?

# Outline

## ❶ Context

*Where does the EDF method stand within the landscape of nuclear structure theories ?*

## ❷ Lessons from empirical EDFs

*1st lesson : Effective (pseudo-)Hamiltonians with simple forms do the job*

*2nd lesson : Static correlations can be optimally grasped via SSBs + bosonic fluctuations of order parameters*

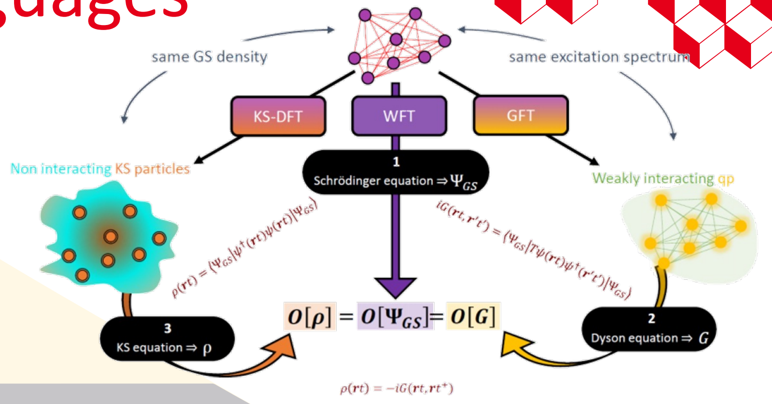
## ❸ Towards a rigorous formulation of nuclear EDFs

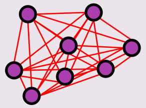
*WFT, DFT & EA perspectives*

*FRG*

*Application to symmetric nuclear matter*

# ❸ Towards a rigorous formulation of nuclear EDFS : Languages




| | **Wave Function theories** | **Functional theories** |
|---|---|---|
| |  |  |
| **Based on** | wave function $\|\Psi\rangle$ | reduced quantity $\varrho$ |
| **Observables** | $O[\|\Psi\rangle] = \langle\Psi\|O\|\Psi\rangle$ | $F[\varrho]$ |

| $\boldsymbol{\varrho}$ | $G(\boldsymbol{r},\boldsymbol{r}';t-t')$ | $\gamma(\boldsymbol{r},\boldsymbol{r}') = G(\boldsymbol{r},\boldsymbol{r}';t-t^{+})$ | $\rho(\boldsymbol{r}) = \gamma(\boldsymbol{r},\boldsymbol{r})$ |
|---|---|---|---|
| **Functional** | $\Phi_{LW}[G]$ or $\Sigma = \frac{\delta\Phi_{LW}}{\delta G}$ | $E_{xc}[\gamma]$ | $E_{xc}[\rho]$ or $v_{xc} = \frac{\delta E_{xc}}{\delta\rho}$ |
| **Approx.** | "easy" | difficult | very difficult |
| **Computationally** | heavy | moderate | light |

1) Nucleus: *A* interacting, structure-less nucleons

2) Structure & dynamic encoded in Hamiltonian, Functional, …

3) Solve master equation to desired accuracy

$$H(\ldots)|\Psi_{\mu,\sigma}\rangle = E_{\mu\tilde{\sigma}}|\Psi_{\mu,\sigma}\rangle$$

$$G^{-1}(x,x') = G_0^{-1}(x,x') - \Sigma(x,x')$$

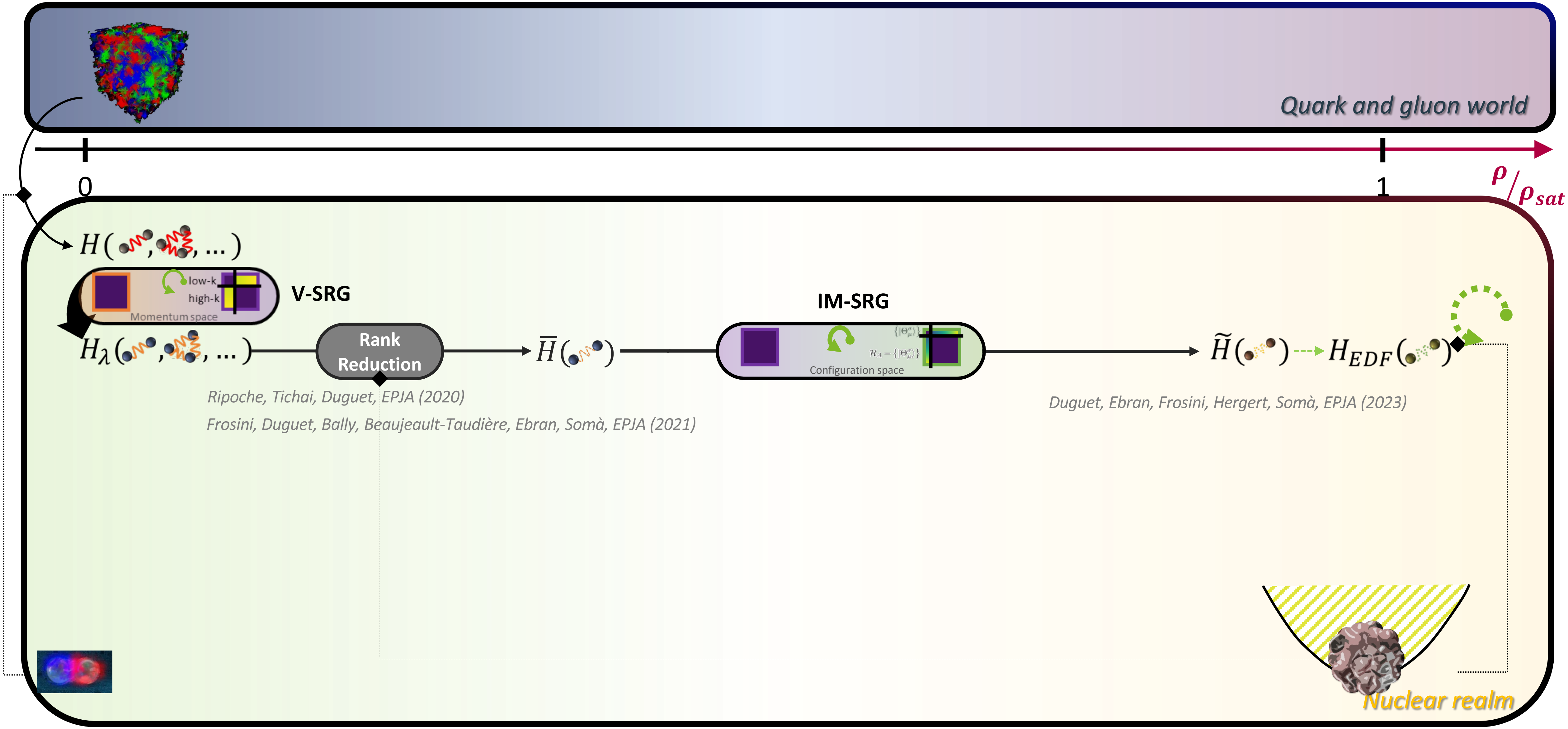
$$h(\mathbf{r})f_\alpha(x) + \int dx'\Sigma(x,x';\varepsilon_\alpha)f_\alpha(x') = \varepsilon_\alpha f_\alpha(x)$$

$$E_{gs} = \min_{\gamma\in N-rep} E[\gamma]$$

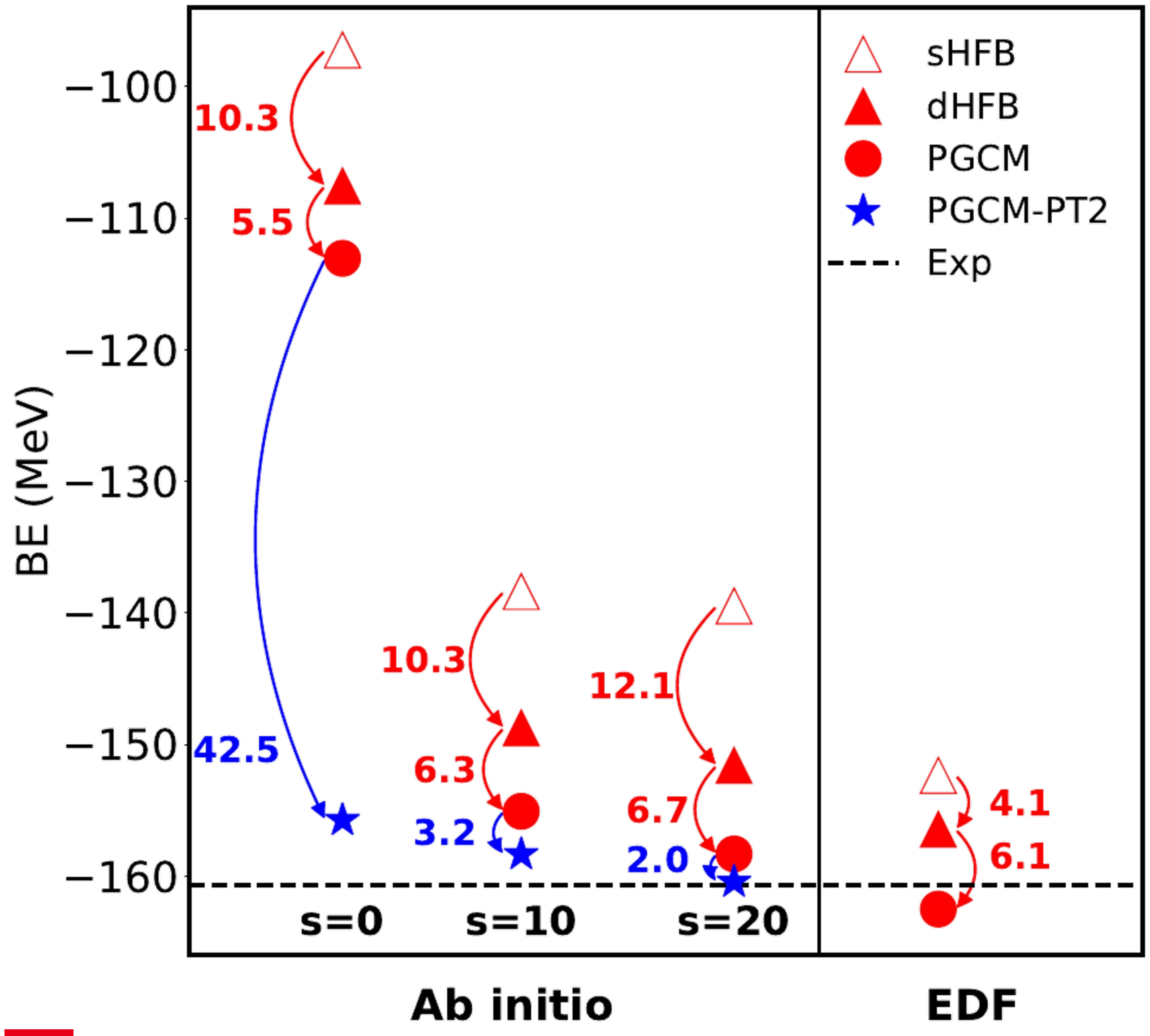
$$\left\{-\frac{\nabla^2}{2m} + v_{KS}(\mathbf{r})\right\}\phi_k(\mathbf{r}) = \varepsilon_k\phi_k(\mathbf{r})$$

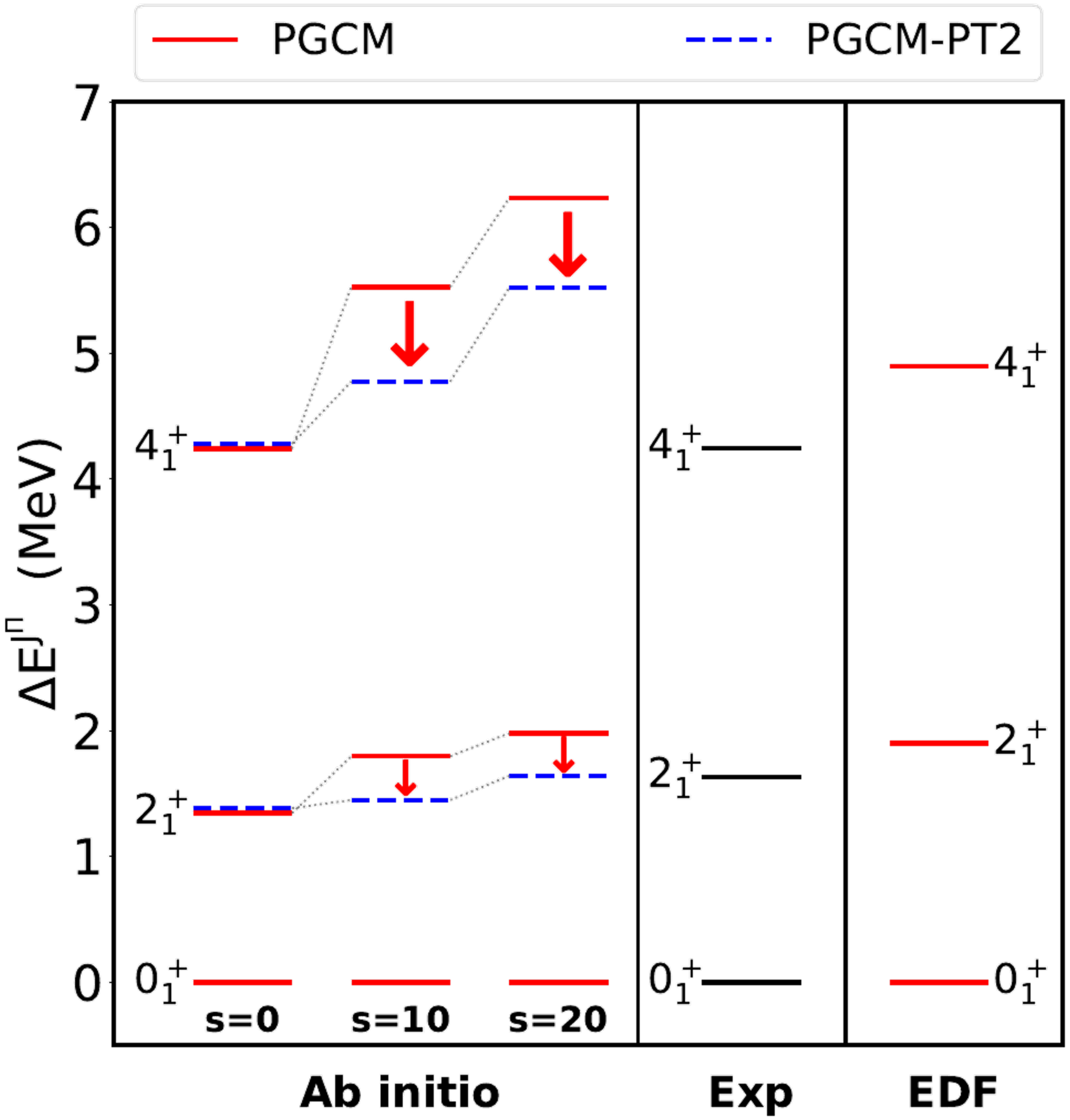
cea

# 3 Towards a rigorous formulation of nuclear EDFS : WFT perspective



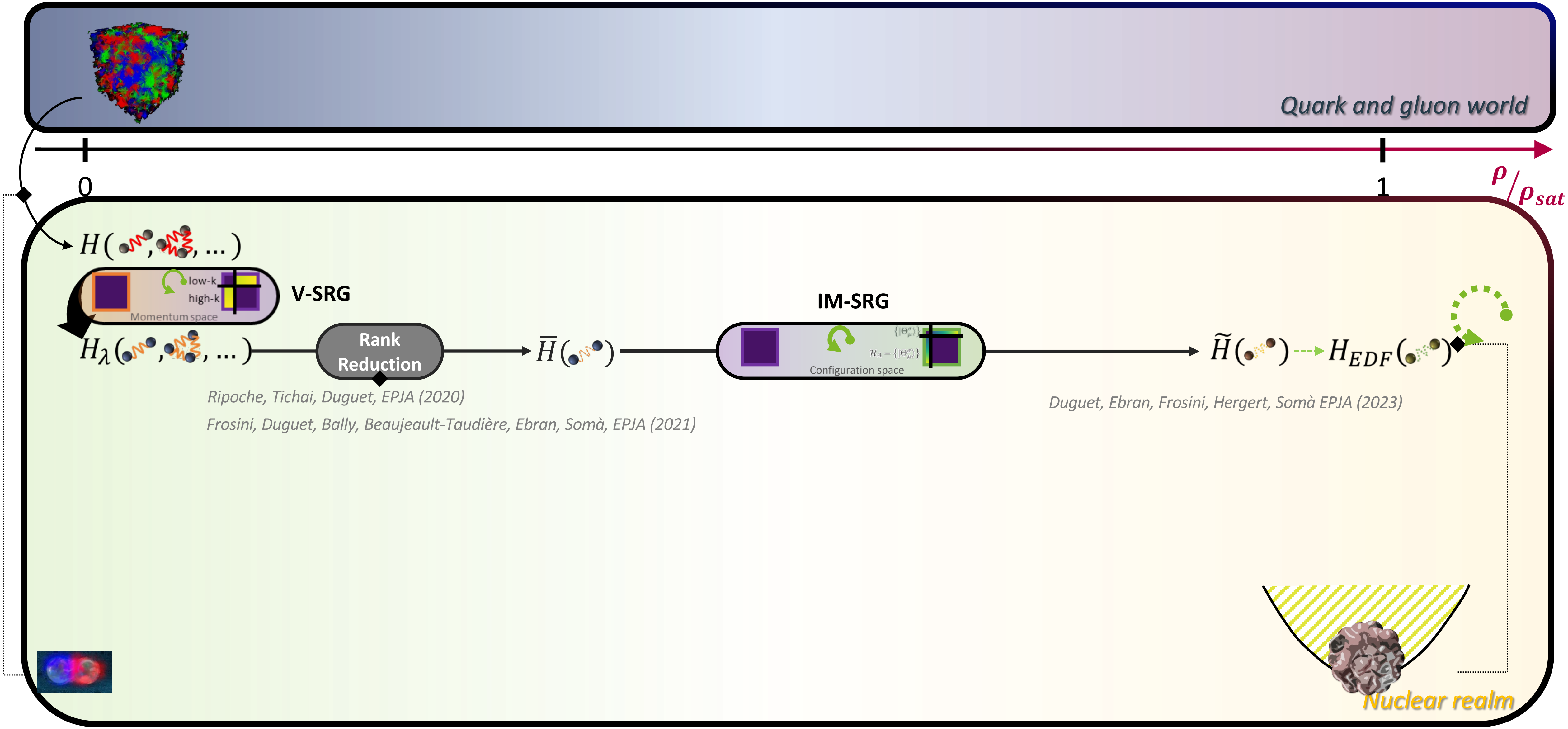
# ❸ Towards a rigorous formulation of nuclear EDFS : WFT perspective




Duguet, Ebran, Frosini, Hergert, Somà, EPJA (2023)

# 3 Towards a rigorous formulation of nuclear EDFS : WFT perspective



◉ All the information needed to compute GS properties is encoded in a simple variable: the constituent density

$$H = T + v + V,$$

i) GS density uniquely defines the system.
In particular, there exists an energy (HK) functional yielding the exact GS energy when evaluated at the exact GS density :

$$\begin{aligned} E^{HK}_{v}[\rho(\mathbf{r})] &= \langle \Psi[\rho] | H[v[\rho], N[\rho]] | \Psi[\rho] \rangle \\ &= \langle \Psi[\rho] | T[N[\rho]] + V[N[\rho]] | \Psi[\rho] \rangle \\ &+ \int d^3 r \rho(\mathbf{r}) v([\rho], \mathbf{r}) \\ &\equiv F^{HK}[\rho(\mathbf{r})] + \int d^3 r \rho(\mathbf{r}) v([\rho], \mathbf{r}) \end{aligned}$$

ii) The exact GS density can be obtained via a variational principle :

$$E_{gs} = \min_{\rho \in \mathfrak{V}_N} E^{HK}_{v}[\rho] \qquad \left.\begin{matrix} \rho_1 \in \mathfrak{V}_N \\ \rho_2 \in \mathfrak{V}_N \end{matrix}\right\} \overset{?}{\Rightarrow} \delta\rho_1 + (1-\delta)\rho_2 \in \mathfrak{V}_N$$

*(Non convex) set of densities originating from a GS WF of some N-particle system subject to a given external potential*

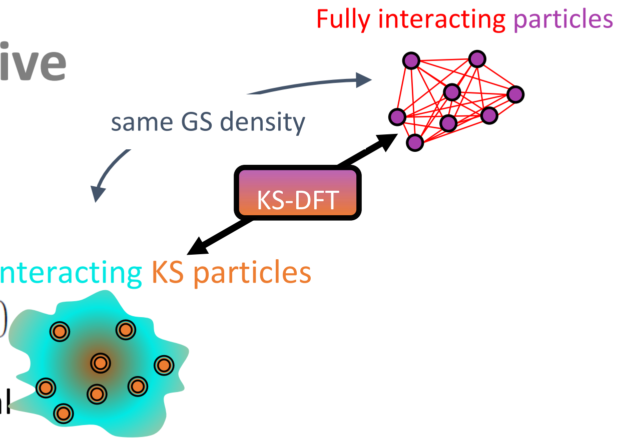
$$E_{gs} = \min_{\rho \in \mathfrak{N}_N} \min_{\Psi \to \rho} \langle \Psi | H | \Psi \rangle \qquad \mathfrak{V}_N \subset \mathfrak{N}_N$$

*(Convex) set of densities originating from arbitrary WFs with finite kinetic energy, satisfying Pauli principle and* $\int d^3 r \rho(\mathbf{r}) = N$

$$= \min_{\rho \in \mathfrak{N}_N} \left\{ \left[ \min_{\Psi \to \rho} \langle \Psi | T + V | \Psi \rangle \right] + \int d^3 r \rho(\mathbf{r}) v([\rho], \mathbf{r}) \right\}$$

$$\equiv \min_{\rho \in \mathfrak{N}_N} \left\{ F^{L}[\rho] + \int d^3 r \rho(\mathbf{r}) v([\rho], \mathbf{r}) \right\}.$$

# ❸ Towards a rigorous formulation of nuclear EDFS : DFT perspective




◇ Let $S[\Theta]$ be an arbitrary functional of some N-particle WF $|\Theta\rangle$ whose form is less complex than the exact WF $|\Psi\rangle$ .

◇ Let $F^S[\rho]$ be a functional of the density : $F^S[\rho] = \min_{\Theta\to\rho} S[\Theta]$ $\quad \rho(\mathbf{r}) = \langle\Theta|\hat{\rho}(\mathbf{r})|\Theta\rangle \quad \hat{\rho}(\mathbf{r}) = \sum_i^N \delta(\hat{\mathbf{r}} - \mathbf{r}_i)$

◇ Let us call $R^S[\rho] \equiv F^L[\rho] - F^S[\rho]$ the difference (remainder) between the Levy functional and the previous functional

◇ The GS energy reads: $E_{gs} = \min_{\rho\in\mathfrak{N}_N}\left\{F^L[\rho] + \int d^3r\rho(\mathbf{r})v([\rho],\mathbf{r})\right\} = \min_{\rho\in\mathfrak{N}_N}\left\{F^S[\rho] + R^S[\rho] + \int d^3r\rho(\mathbf{r})v(\mathbf{r})\right\} = \min_{\rho\in\mathfrak{N}_N}\left\{\left[\min_{\Theta\to\rho} S[\Theta]\right] + R^S[\rho] + \int d^3r\rho(\mathbf{r})v(\mathbf{r})\right\}$

**KS choice** $|\Theta\rangle = |\Phi\rangle$

$\rho\in\mathfrak{V}_N \cap \mathfrak{V}_N^0$

$$\rho(\mathbf{r}) = \sum_i |\phi_i(\mathbf{r})|^2$$

$$E_{gs} = \min_{\rho\in\mathfrak{N}_N}\left\{\left[\min_{\Phi\to\rho} S[\Phi]\right] + R^S[\rho] + \int d^3r\rho(\mathbf{r})v(\mathbf{r})\right\}$$
$$= \min_{\{\phi_i\}\to N}\left\{S[\{\phi_i\}] + R^S[\rho[\{\phi_i\}]] + \int d^3r\rho([\{\phi_i\}];\mathbf{r})v(\mathbf{r})\right\}$$

$$\frac{\delta S[\{\phi_i\}]}{\delta\phi_k^\dagger(\mathbf{r})} + \left\{\frac{\delta R^S[\rho]}{\delta\rho(\mathbf{r})} + v(\mathbf{r})\right\}\phi_k(\mathbf{r}) = \varepsilon_k\phi_k(\mathbf{r})$$

### Standard KS

$$S[\Phi] = \langle\Phi|T|\Phi\rangle = E_k[\{\phi_i\}]$$

$$R^S[\rho] = E_d[\rho] + E_x[\rho] + E_c[\rho]$$

$$E_d[\rho] \equiv \frac{1}{2}\int d^3r d^3r' V(\mathbf{r},\mathbf{r}')\rho(\mathbf{r})\rho(\mathbf{r}'),$$

$$E_x[\rho] \equiv -\frac{1}{2}\sum_{ij}\int d^3r d^3r' V(\mathbf{r},\mathbf{r}')\phi_i^*(\mathbf{r})\phi_j^*(\mathbf{r}')\phi_i(\mathbf{r}')\phi_j(\mathbf{r})$$

$$E_c[\rho] \equiv F^L[\rho] - E_k[\rho] - E_d[\rho] - E_x[\rho].$$

$$\left\{-\frac{\nabla^2}{2m} + v_{KS}(\mathbf{r})\right\}\phi_k(\mathbf{r}) = \varepsilon_k\phi_k(\mathbf{r})$$

$$v_{KS}(\mathbf{r}) \equiv v(\mathbf{r}) + \left(\int d^3r' V(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}')\right) + \frac{\delta(E_x[\rho] + E_c[\rho])}{\delta\rho(\mathbf{r})}$$

### Generalized KS

$$S[\Phi] = \langle\Phi|T + V|\Phi\rangle$$
$$= E_k[\{\phi_i\}] + E_d[\{\phi_i\}] + E_x[\{\phi_i\}]$$

$$R^S[\rho] = E_c[\rho]$$

$$\left\{-\frac{\nabla^2}{2m} + v(\mathbf{r}) + \left(\int d^3r' V(\mathbf{r},\mathbf{r}')\rho(\mathbf{r}')\right) + \frac{\delta E_c[\rho]}{\delta\rho(\mathbf{r})}\right\}\phi_k(\mathbf{r})$$
$$-\left[\int d^3r' V(\mathbf{r},\mathbf{r}')\left(\sum_j \phi_j^*(\mathbf{r})\phi_j(\mathbf{r}')\right)\phi_k(\mathbf{r}')\right] = \varepsilon_k\phi_k(\mathbf{r}).$$

# ❸ Towards a rigorous formulation of nuclear EDFS : DFT perspective

⟐ Let $S[\Theta]$ be an arbitrary functional of some N-particle WF $|\Theta\rangle$ whose form is less complex than the exact WF $|\Psi\rangle$ .

⟐ Let $F^S[\rho]$ be a functional of the density : $F^S[\rho] = \min_{\Theta\to\rho} S[\Theta]$ $\quad \rho(\mathbf{r}) = \langle\Theta|\hat{\rho}(\mathbf{r})|\Theta\rangle \quad \hat{\rho}(\mathbf{r}) = \sum_i^N \delta(\hat{\mathbf{r}} - \mathbf{r}_i)$

⟐ Let us call $R^S[\rho] \equiv F^L[\rho] - F^S[\rho]$ the difference (remainder) between the Levy functional and the previous functional

⟐ The GS energy reads: $E_{gs} = \min_{\rho\in\mathfrak{N}_N}\left\{F^L[\rho] + \int d^3r\rho(\mathbf{r})v([\rho],\mathbf{r})\right\} = \min_{\rho\in\mathfrak{N}_N}\left\{F^S[\rho] + R^S[\rho] + \int d^3r\rho(\mathbf{r})v(\mathbf{r})\right\} = \min_{\rho\in\mathfrak{N}_N}\left\{\left[\min_{\Theta\to\rho} S[\Theta]\right] + R^S[\rho] + \int d^3r\rho(\mathbf{r})v(\mathbf{r})\right\}$

**KS choice** $|\Theta\rangle = |\Phi\rangle$

$\rho\in\mathfrak{V}_N \cap \mathfrak{V}_N^0$

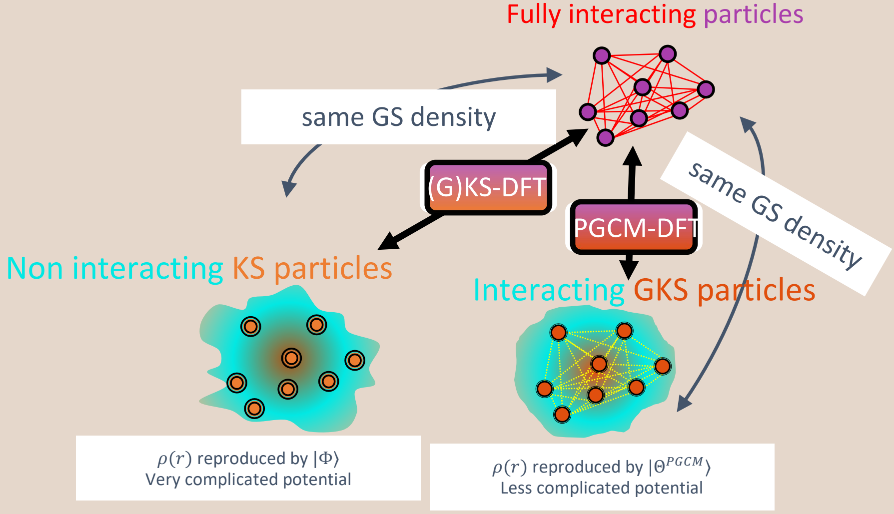
$\rho(\mathbf{r}) = \sum_i |\phi_i(\mathbf{r})|^2$

$$E_{gs} = \min_{\rho\in\mathfrak{N}_N}\left\{\left[\min_{\Phi\to\rho} S[\Phi]\right] + R^S[\rho] + \int d^3r\rho(\mathbf{r})v(\mathbf{r})\right\}$$

$$= \min_{\{\phi_i\}\to N}\left\{S[\{\phi_i\}] + R^S[\rho[\{\phi_i\}]] + \int d^3r\rho(\{\phi_i\};\mathbf{r})v(\mathbf{r})\right\}$$

$$\frac{\delta S[\{\phi_i\}]}{\delta\phi_k^\dagger(\mathbf{r})} + \left\{\frac{\delta R^S[\rho]}{\delta\rho(\mathbf{r})} + v(\mathbf{r})\right\}\phi_k(\mathbf{r}) = \varepsilon_k\phi_k(\mathbf{r})$$

**PGCM choice** $|\Theta\rangle = |\Theta^{PHFB/PGCM}\rangle$



# ❸ Towards a rigorous formulation of nuclear EDFS : DFT perspective

◇ Let $S[\Theta]$ be an arbitrary functional of some N-particle WF $|\,\Theta\rangle$ whose form is less complex than the exact WF $|\,\Psi\rangle$ .

◇ Let $F^S[\rho]$ be a functional of the density : $F^S[\rho] = \min_{\Theta\to\rho} S[\Theta]$ $\quad \rho(\mathbf{r}) = \langle\Theta|\hat\rho(\mathbf{r})|\Theta\rangle \quad \hat\rho(\mathbf{r}) = \sum_i^N \delta(\hat{\mathbf{r}} - \mathbf{r}_i)$

◇ Let us call $R^S[\rho] \equiv F^L[\rho] - F^S[\rho]$ the difference (remainder) between the Levy functional and the previous functional

◇ The GS energy reads: $E_{gs} = \min_{\rho\in\mathfrak{N}_N}\left\{F^L[\rho] + \int d^3r\rho(\mathbf{r})v([\rho],\mathbf{r})\right\} = \min_{\rho\in\mathfrak{N}_N}\left\{F^S[\rho] + R^S[\rho] + \int d^3r\rho(\mathbf{r})v(\mathbf{r})\right\} = \min_{\rho\in\mathfrak{N}_N}\left\{\left[\min_{\Theta\to\rho} S[\Theta]\right] + R^S[\rho] + \int d^3r\rho(\mathbf{r})v(\mathbf{r})\right\}$

**KS choice** $|\Theta\rangle = |\Phi\rangle$

$\rho\in\mathfrak{V}_N \cap \mathfrak{V}_N^0$

$\rho(\mathbf{r}) = \sum_i |\phi_i(\mathbf{r})|^2$

$$E_{gs} = \min_{\rho\in\mathfrak{N}_N}\left\{\left[\min_{\Phi\to\rho} S[\Phi]\right] + R^S[\rho] + \int d^3r\rho(\mathbf{r})v(\mathbf{r})\right\}$$
$$= \min_{\{\phi_i\}\to N}\left\{S[\{\phi_i\}] + R^S[\rho[\{\phi_i\}]] + \int d^3r\rho([\{\phi_i\}];\mathbf{r})v(\mathbf{r})\right\}$$

$$\frac{\delta S[\{\phi_i\}]}{\delta\phi_k^\dagger(\mathbf{r})} + \left\{\frac{\delta R^S[\rho]}{\delta\rho(\mathbf{r})} + v(\mathbf{r})\right\}\phi_k(\mathbf{r}) = \varepsilon_k\phi_k(\mathbf{r})$$

**PGCM choice** $|\,\Theta\rangle = |\,\Theta^{PHFB/PGCM}\rangle$

$$|\Theta_\mu^{JM}\rangle = \sum_q f_{\mu q}^J \frac{P^J_{M0N_0Z_0}|\Phi_q\rangle}{\sqrt{\langle\Phi_q|P^J_{M0N_0Z_0}|\Phi_q\rangle}} \equiv \sum_q f_{\mu q}^J |JMq\rangle .$$

$$\rho^{JM\mu}(\mathbf{r}) = \langle\Theta_\mu^{JM}|\hat\rho(\mathbf{r})|\Theta_\mu^{JM}\rangle = \sum_{qq'} f_{\mu q}^{J*}\langle JMq|\hat\rho(\mathbf{r})|JMq'\rangle f_{\mu q'}^J$$

cea

# ❸ Towards a rigorous formulation of nuclear EDFS : PI perspective

◉ Wightman/Schwinger reconstruction theorem : sequence of (tempered) n-point correlation functions completely determines Hilbert space and algebra of fields (up to unitary equivalence) of a given quantum system.

--> Canonical formulation : $G^{(n)}_{I_1,I_2,\cdots,I_n} \equiv \langle vac|T\hat{\varphi}_{I_1}\hat{\varphi}_{I_2}\cdots\hat{\varphi}_{I_n}|vac\rangle$

--> Path integral formulation : $S_J[\widetilde{\varphi}] = S[\widetilde{\varphi}] - J_I\widetilde{\varphi}_I$

$$J_I\widetilde{\varphi}_I \equiv J_{x,a}\widetilde{\varphi}_{x,a} \equiv \sum_{m_s,c,a}\int_0^\beta d\tau \int_{\mathbb{R}^{D-1}} d^{D-1}r\, J_{m_s,c,a}(\tau,\boldsymbol{r})\widetilde{\varphi}_{m_s,-c,a}(\tau,\boldsymbol{r})$$

$$Z[J] = \mathcal{N}\int_{\mathcal{C}} \mathcal{D}\widetilde{\varphi}\; e^{-\frac{1}{\hbar}S_J[\widetilde{\varphi}]} \equiv e^{\frac{1}{\hbar}W[J]}$$

$$G^{(n)}_{I_1,I_2,\cdots,I_n} \equiv \langle\widetilde{\varphi}_{I_1}\widetilde{\varphi}_{I_2}\cdots\widetilde{\varphi}_{I_n}\rangle_{\text{vac}} = \frac{\int\mathcal{D}\widetilde{\varphi}\;\widetilde{\varphi}_{I_1}\widetilde{\varphi}_{I_2}\cdots\widetilde{\varphi}_{I_n}e^{-\frac{1}{\hbar}S[\widetilde{\varphi}]}}{\int\mathcal{D}\widetilde{\varphi}\;e^{-\frac{1}{\hbar}S[\widetilde{\varphi}]}} = \frac{\hbar^n}{Z[0]}\left.\frac{\delta^n Z[J]}{\delta J_{I_1}\delta J_{I_2}\cdots\delta J_{I_n}}\right|_{J=0}.$$

$$G^{(n),c}_{I_1,I_2,\cdots,I_n} = \hbar^{n-1}\left.\frac{\delta^n W[J]}{\delta J_{I_1}\delta J_{I_2}\cdots\delta J_{I_n}}\right|_{J=0}$$

# 3 Towards a rigorous formulation of nuclear EDFS : PI perspective

◉ Compact way of representing the partition function : Effective action

Classical action

$$S_{JKL^{(3)}\dots L^{(m)}}[\widetilde{\varphi}] \equiv S[\widetilde{\varphi}] - J_I\widetilde{\varphi}_I - \frac{1}{2}K_{IJ}\widetilde{\varphi}_I\widetilde{\varphi}_J - \frac{1}{3!}L^{(3)}_{IJK}\widetilde{\varphi}_I\widetilde{\varphi}_J\widetilde{\varphi}_K - \cdots - \frac{1}{m!}L^{(m)}_{I_1\dots I_m}\widetilde{\varphi}_{I_1}\cdots\widetilde{\varphi}_{I_m},$$

Partition function

$$Z\left[J,K,L^{(3)},\cdots\right] \equiv e^{\frac{1}{\hbar}W\left[J,K,L^{(3)},\cdots\right]} = \mathcal{N}\int_{\mathcal{C}}\mathcal{D}\widetilde{\varphi}\; e^{-\frac{1}{\hbar}S_{JKL^{(3)}\dots}[\widetilde{\varphi}]}.$$

Quantum action

$$\Gamma[\phi,G,V,\cdots] = -W\left[J,K,L^{(3)},\cdots\right] + J_I\phi_I + \frac{1}{2}K_{IJ}(\phi_I\phi_J + \hbar G_{IJ}) + \frac{1}{6}L^{(3)}_{IJK}(\phi_I\phi_J\phi_K + \hbar G_{IJ}\phi_K + \hbar G_{IK}\phi_J + \hbar G_{JK}\phi_I + \hbar^2 V_{IJK}) + \cdots,$$

$$\frac{\delta W\left[J,K,L^{(3)},\cdots\right]}{\delta J_I} = \phi_I,$$

$$\frac{\delta W\left[J,K,L^{(3)},\cdots\right]}{\delta K_{IJ}} = \frac{1}{2}\left[\phi_I\phi_J + \hbar G_{IJ}\right],$$

$$\frac{\delta W\left[J,K,L^{(3)},\cdots\right]}{\delta L^{(3)}_{IJK}} = \frac{1}{6}\left[\phi_I\phi_J\phi_K + \hbar G_{IJ}\phi_K + \hbar G_{IK}\phi_J + \hbar G_{JK}\phi_I + \hbar^2 V_{IJK}\right],$$

$$\vdots$$

◉ Gap equations

$$\left.\frac{\delta\Gamma[\phi,G,V,\cdots]}{\delta\phi_I}\right|_{\phi_{\text{gs}},G_{\text{gs}},V_{\text{gs}},\cdots} = 0,$$

$$\left.\frac{\delta\Gamma[\phi,G,V,\cdots]}{\delta G_{IJ}}\right|_{\phi_{\text{gs}},G_{\text{gs}},V_{\text{gs}},\cdots} = 0,$$

$$\vdots$$

Quantum effective action already contains all correlation functions at tree-level :
= low-energy action with all quantum fluctuations integrated out

$$E_{\text{gs}} = \lim_{\beta\to\infty}\left(-\frac{1}{\beta}\ln(Z[J=0,\cdots])\right) = \lim_{\beta\to\infty}\left(-\frac{1}{\hbar\beta}W[J=0,\cdots]\right) = \lim_{\beta\to\infty}\left(\frac{1}{\hbar\beta}\Gamma^{(n\text{PI})}[\phi=\overline{\phi},\cdots]\right).$$

◉ When minimizing field is homogeneous : $\phi(x) \equiv \phi$= cst

$$\Gamma[\phi] = \beta V U(\phi)$$
$$Z(T,\mu) = e^{-\beta V\, U_{gc}(T,\mu)}$$
$$U_{gc}(T,\mu) = U\big(\phi_{gs}(T,\mu),T,\mu\big)$$

$$p = -U_{\text{gc}}(T,\mu), \qquad n = -\frac{\partial U_{\text{gc}}(T,\mu)}{\partial\mu},$$

$$s = -\frac{\partial U_{\text{gc}}(T,\mu)}{\partial T}, \qquad \epsilon = -p + \mu n + Ts.$$

# 3 Towards a rigorous formulation of nuclear EDFS : Computing the effective action

◉ Classical action

$$S[\psi^\dagger,\psi] = \int d^4x\ \psi^\dagger(x)\left[\partial_t - \frac{\nabla_x^2}{2m}\right]\psi(x) + \frac{1}{2}\int d^3x\, d^3y\, dt\, \psi^\dagger(\boldsymbol{x},t)\psi^\dagger(\boldsymbol{y},t)V(\boldsymbol{x},\boldsymbol{y})\psi(\boldsymbol{y},t)\psi(\boldsymbol{x},t) + \cdots$$

◉ Generator of correlations functions

$$Z[\eta^\dagger,\eta] = e^{W[\eta^\dagger,\eta]} = \int \mathcal{D}\psi^\dagger\mathcal{D}\psi e^{-S[\psi^\dagger,\psi]+\int \eta^\dagger\psi+\psi^\dagger\eta}$$

◉ nP(P)I Effective Action

Legendre transformation

--> Couple source(s) to local field

$$Z[\eta^\dagger,\eta] = e^{W[\eta^\dagger,\eta]} = \int \mathcal{D}\psi^\dagger\mathcal{D}\psi e^{-S[\psi^\dagger,\psi]+\int \eta^\dagger\psi+\psi^\dagger\eta} \longrightarrow \text{1PI EA}\quad \Gamma[\phi^\dagger,\phi]$$

--> Couple source(s) to bi-local composite field

$$Z[K] = e^{W[K]} = \int \mathcal{D}\psi^\dagger\mathcal{D}\psi e^{-S[\psi^\dagger,\psi]+\frac{1}{2}\int K_{xy}\psi_x^\dagger\psi_y} \longrightarrow \text{2PI EA}\quad \Gamma[G]$$

--> Couple source(s) to local composite field

$$Z[J] = e^{W[J]} = \int \mathcal{D}\psi^\dagger\mathcal{D}\psi e^{-S[\psi^\dagger,\psi]+\frac{1}{2}\int J_x\psi_x^\dagger\psi_x} \longrightarrow \text{2PPI EA}\quad \Gamma[\rho] = \sup\left\{-W[J] + \frac{1}{2}\int J_x\rho_x\right\}$$

$$\rho(x) = 2\frac{\delta W}{\delta J(x)}$$

$$E_{gs} = \lim_{\beta\to\infty}\left(\frac{1}{\beta}\Gamma^{(nPI)}[\phi=\bar{\phi},\cdots]\right)$$

◉ Gap equation

$$\frac{\delta\Gamma}{\delta\rho(x)} = \frac{1}{2}J(x)$$

2PPI EA is extremal at the exact, physical density

◉ Inversion method

$$\begin{cases} \Gamma[\rho] = \sum_i \lambda^i\Gamma_i[\rho] \\ J = \sum_i \lambda^i J_i \\ W[J] = \sum_i \lambda^i W_i[J] \end{cases}$$

$$\sum_i \lambda^i\Gamma_i[\rho] = -\sum_i \lambda^i W_i\left[\sum_i \lambda^i J_i\right] + \frac{1}{2}\int\sum_i \lambda^i J_i(x)\rho(x)$$

$$\Gamma_i[\rho] = -W_i[J_0] + \frac{1}{2}\delta_{i,0}\int J_0(x)\rho(x) + \sum_{k=1}^{i-1}\int\frac{\delta W_{i-k}[J_0]}{\delta J_0(x)}J_k(x) + \sum_{m=2}^{i}\frac{1}{m!}\sum_{k_1,\ldots,k_m\geq 1}^{k_1+\cdots+k_m\leq i}\int\frac{\delta^m W_{i-(k_1+\cdots+k_1)}[J_0]}{\delta J_0(x_1)\ldots\delta J_0(x_m)}J_{k_1}(x_1)\ldots J_{k_m}(x_m$$

$$J_0 \to \Gamma_1 \to J_1 \to \Gamma_2 \to \cdots$$

$$\frac{1}{2}J_0(x) = -\frac{\delta\Gamma_{int}}{\delta\rho(x)}$$

At 0th order : $\Gamma_0[\rho] = -W_0[\rho] + \frac{1}{2}\int J_0\rho \quad\Rightarrow\quad \rho(x) = 2\frac{\delta W}{\delta J(x)} = 2\frac{\delta W_0}{\delta J_0(x)}$

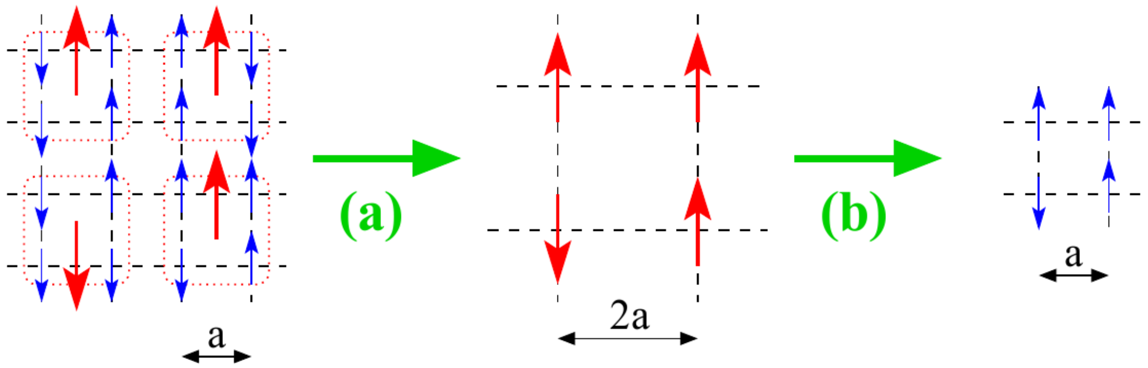
$$\frac{\delta\Gamma_0}{\delta\rho} = \frac{1}{2}J_0 = -\int\frac{\delta W_0}{\delta J_0}\frac{\delta J_0}{\delta\rho} + \frac{1}{2}\int\frac{\delta J_0}{\delta\rho}\rho + \frac{1}{2}J_0$$

**$J_0(x)$ = potential reproducing the exact density $\rho(x)$ in the non-interacting ($\lambda = 0$) system !!!**

*Furnstahl, Turning the nuclear energy density functional method into a proper effective field theory: reflections. Eur. Phys. J. A* **56**, *85 (2020)*
*Fraboulet, Ebran, Addressing energy density functionals in the language of path-integrals I: comparative study of diagrammatic techniques applied to the (0+0)-D O(N)-symmetric φ4-theory. Eur. Phys. J. A* **59**, *91 (2023)*

# ❸ Towards a rigorous formulation of nuclear EDFS : Computing the effective action

## ◉ Renormalization group transformation : Wilson-Kadanoff procedure




$\mathcal{H}_k[\{S_i^k\}]$

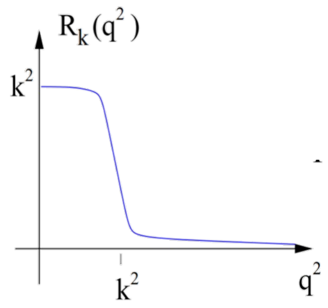
$\mathcal{H}_0[\{S_i^0\}]$ Decimation Renormalization $\mathcal{H}_1[\{S_i^1\}]$

## ◉ Renormalization group transformation : FRG

--> Central object of FRG : scale-dependent (or average) effective action $\Gamma_k$ interpolating between the S and $\Gamma$

$$\Gamma_{k=\Lambda}[\varphi_\Lambda] = S[\psi] \qquad k = \Lambda$$
$$\downarrow$$
$$\Gamma_k[\varphi_k] \qquad k$$
$$\downarrow$$
$$\Gamma_{k=0}[\varphi_0] = \Gamma[\varphi] \qquad k = 0$$

Incorporation of fluctuations of modes with $q > k$



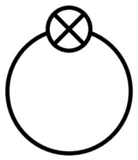

--> Mass term $S + \Delta S_k$

$$\Delta S_k = \frac{1}{2}\int \psi^\dagger(q) R_k(q) \psi(-q)$$

--> Exact RG (or Wetterich) equation

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2}\,\mathrm{tr}\left\{\partial_k R_k \left(\Gamma_k^{(2)}[\varphi] + R_k\right)^{-1}\right\}$$
$$\partial_k \Gamma_k\left[\overline{\psi}, \psi\right] = -\,\mathrm{tr}\left\{\partial_k R_k \left(\Gamma_k^{(1,1)}\left[\overline{\psi}, \psi\right] + R_k\right)^{-1}\right\}$$

=

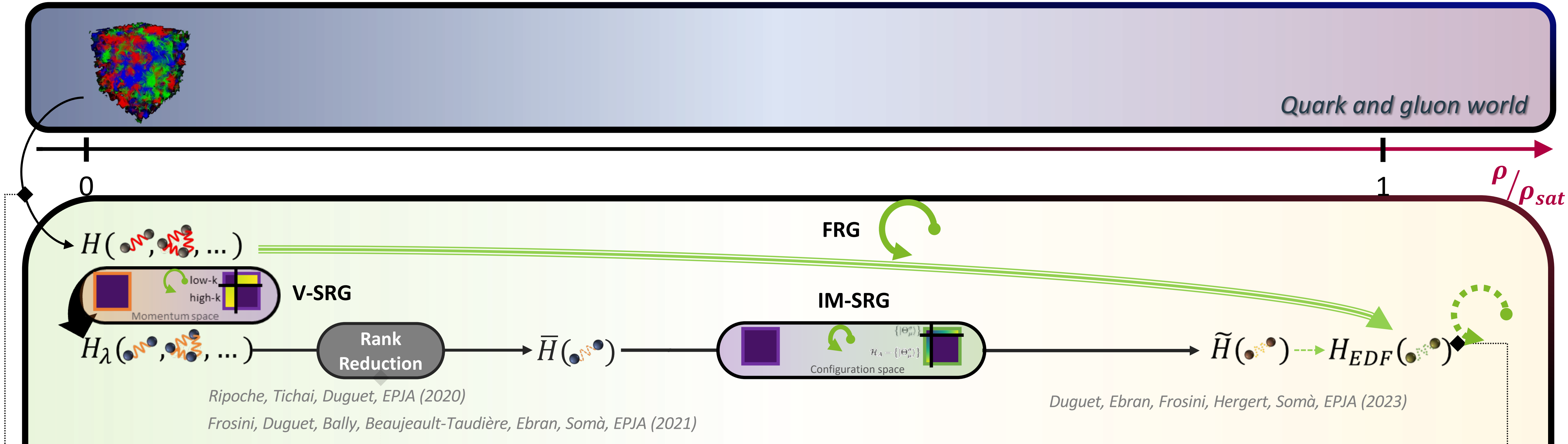


$$\Gamma_k^{(a)}[\varphi] = \frac{\delta^a}{\delta \varphi^a}\Gamma_k[\varphi]$$

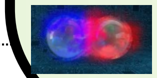
$$\Gamma_k^{(a,b)}\left[\psi, \overline{\psi}\right] = \frac{\overrightarrow{\delta^a}}{\delta \overline{\psi}^a}\Gamma_k\left[\psi, \overline{\psi}\right]\frac{\overleftarrow{\delta^b}}{\delta \psi^b}$$

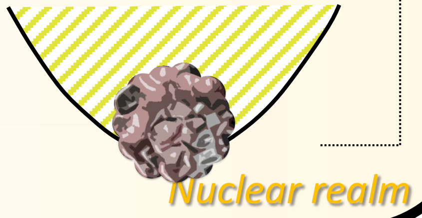
*Fraboulet, Ebran, Addressing energy density functionals in the language of path-integrals II: comparative study of functional renormalization group techniques applied to the (0+0)-D O(N)-symmetric φ4-theory. Accepted in EPJA*

# 3 Towards a rigorous formulation of nuclear EDFS : FRG perspective

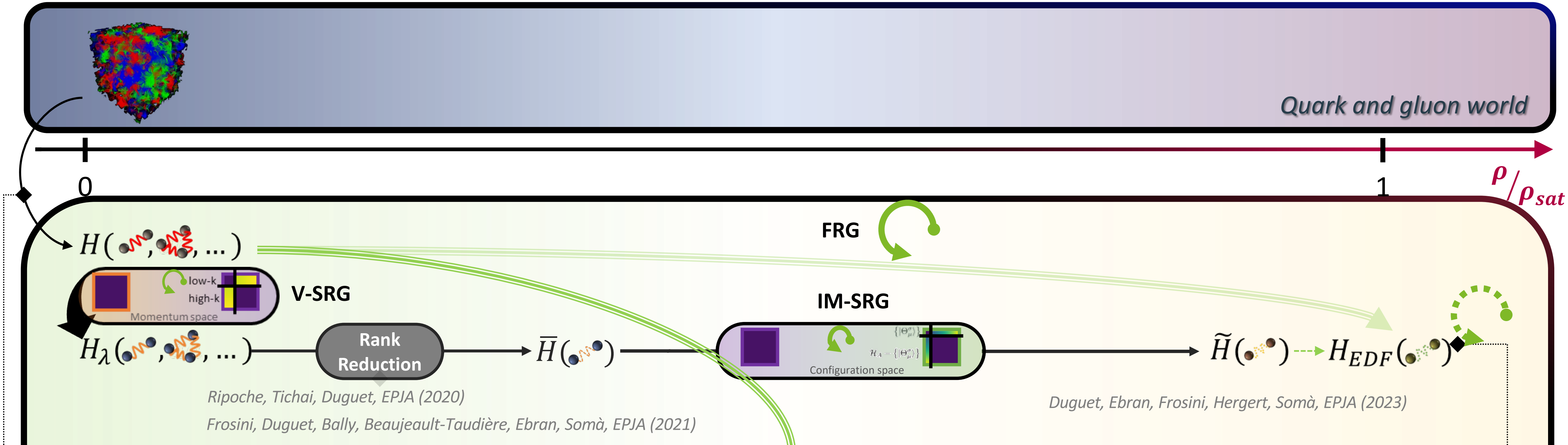



**Option 1** : Start from a Hamiltonian/Lagrangian for nucleons in free space
Compute the effective action for nuclear matter and check saturation properties
Apply to finite nuclei with the usual techniques



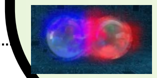


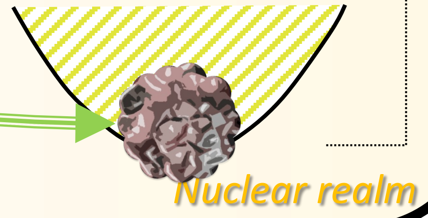
# ❸ Towards a rigorous formulation of nuclear EDFS : FRG perspective



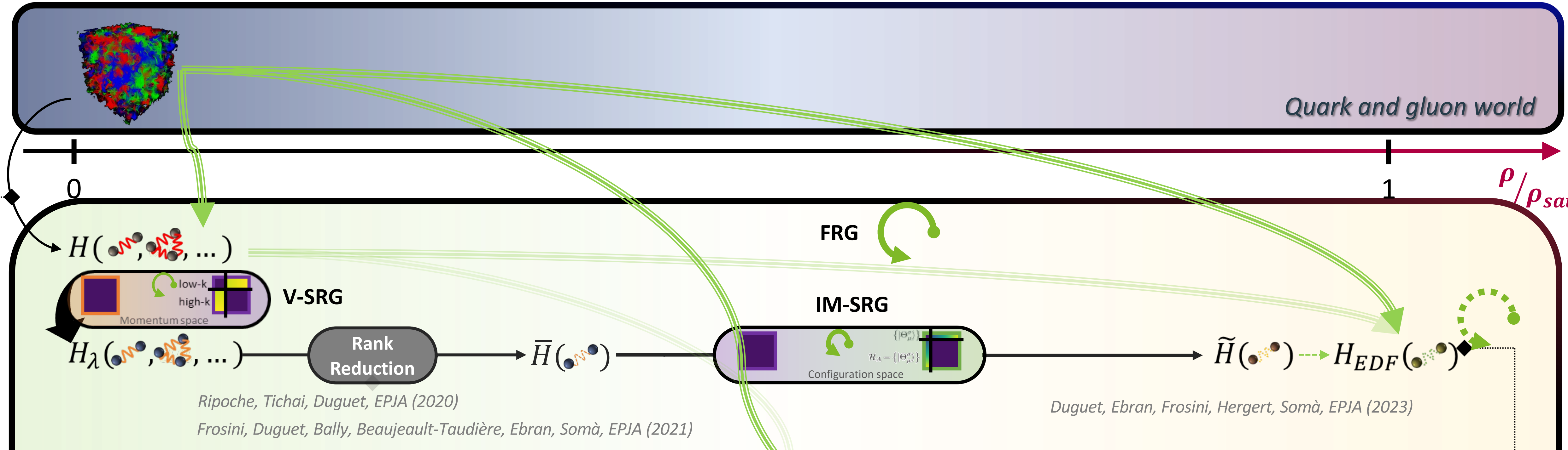

**Option 1** : Start from a Hamiltonian/Lagrangian for nucleons in free space
Compute the effective action for nuclear matter and check saturation properties
Apply to finite nuclei with the usual techniques

**Option 2** : Start from a Hamiltonian/Lagrangian for nucleons in free space
Compute the effective action for finite nuclei by accounting for all types of fluctuation
Compute all the desired correlation functions





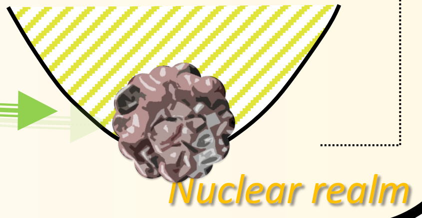
# 3 Towards a rigorous formulation of nuclear EDFS : FRG perspective




**Option 1** : Start from a Hamiltonian/Lagrangian for nucleons in free space
Compute the effective action for nuclear matter and check saturation properties
Apply to finite nuclei with the usual techniques

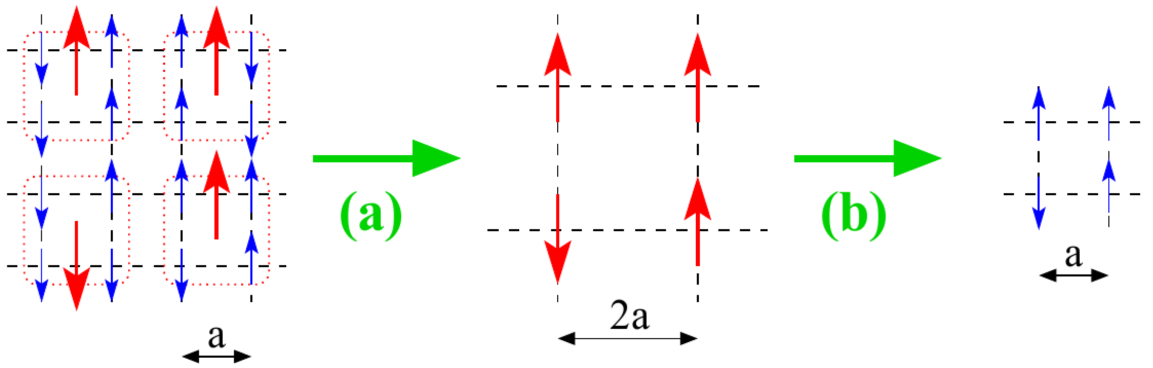
**Option 2** : Start from a Hamiltonian/Lagrangian for nucleons in free space
Compute the effective action for finite nuclei by accounting for all types of fluctuation
Compute all the desired correlation functions

**Option 3** : Start from QCD
Compute the effective action for nucleons in free space/ infinite nuclear matter / finite nuclei



# 3 Towards a rigorous formulation of nuclear EDFS : FRG perspective

◉ Renormalization group transformation : Wilson-Kadanoff procedure




$\mathcal{H}_k[\{S_i^k\}]$

$\mathcal{H}_0[\{S_i^0\}]$ Decimation Renormalization $\mathcal{H}_1[\{S_i^1\}]$

◉ Renormalization group transformation : FRG

--> Central object of FRG : scale-dependent (or average) effective action $\Gamma_k$ interpolating between the S and $\Gamma$

$$\Gamma_{k=\Lambda}[\varphi_\Lambda] = S[\psi] \quad k = \Lambda$$

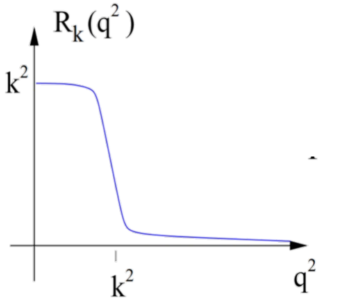
$$\downarrow$$

$$\Gamma_k[\varphi_k] \quad k$$

$$\downarrow$$

$$\Gamma_{k=0}[\varphi_0] = \Gamma[\varphi] \quad k = 0$$

Incorporation of fluctuations of modes with $q > k$

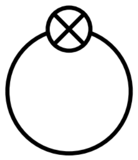



--> Mass term $S + \Delta S_k$ $\qquad \Delta S_k = \frac{1}{2}\int \psi^\dagger(q) R_k(q)\psi(-q)$

--> Exact RG (or Wetterich) equation

$$\partial_k \Gamma_k[\varphi] = \frac{1}{2}\,\mathrm{tr}\left\{\partial_k R_k \left(\Gamma_k^{(2)}[\varphi] + R_k\right)^{-1}\right\}$$

$$\partial_k \Gamma_k\left[\overline{\psi}, \psi\right] = -\,\mathrm{tr}\left\{\partial_k R_k \left(\Gamma_k^{(1,1)}\left[\overline{\psi}, \psi\right] + R_k\right)^{-1}\right\} =$$



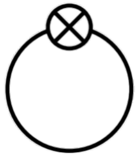
$$\Gamma_k^{(a)}[\varphi] = \frac{\delta^a}{\delta \varphi^a}\Gamma_k[\varphi]$$

$$\Gamma_k^{(a,b)}\left[\psi, \overline{\psi}\right] = \frac{\overrightarrow{\delta^a}}{\delta \overline{\psi}^a}\Gamma_k\left[\psi, \overline{\psi}\right]\frac{\overleftarrow{\delta^b}}{\delta \psi^b}$$

# 3 Towards a rigorous formulation of nuclear EDFS : FRG perspective

◉ Truncation of the effective action

$$\partial_k \Gamma_k\left[\overline{\psi},\psi\right] = -\,\mathrm{tr}\left\{\partial_k R_k \left(\Gamma_k^{(1,1)}\left[\overline{\psi},\psi\right] + R_k\right)^{-1}\right\} =$$



$$\Gamma_k[\Phi] = \int_x \sum_i g_{i,k} \mathcal{O}_i[\Phi]$$

⇢ Derivative expansion :

- Expand in spatial derivative
- Keep all $\Gamma_k^{(n)}$ vertex functions
- Keep the full field dependence

LPA: $\Gamma_k = \int_x \left\{U_k[\varphi] + \frac{1}{2}(\partial_\mu\varphi)^2\right\}$

DE2: $\Gamma_k = \int_x \left\{U_k[\varphi] + \frac{1}{2}Z_k[\varphi](\partial_\mu\varphi)^2\right\}$

DE4 (Canet etal. '03): $\Gamma_k = \int_x \left\{U_k[\varphi] + \frac{1}{2}Z_k[\varphi](\partial_\mu\varphi)^2 + \frac{1}{2}W_{a;k}[\varphi](\partial_\mu\partial_\nu\varphi)^2 + \frac{1}{2}W_{b;k}[\varphi]\phi\partial^2\phi(\partial_\mu\varphi)^2 + \frac{1}{2}W_{c;k}[\varphi](\partial_\mu\varphi)^4\right\}$

- Further expansion in fields $U_k[\varphi] = \sum_i^n U_{i,k}(\varphi - \varphi_0)^i$

# ❸ Towards a rigorous formulation of nuclear EDFS : FRG perspective

--> Start with a phenomenological ab initio Lagrangian

--> Put it in medium : extra $\mu\psi^\dagger\psi$ factor in the Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\text{Bonn}} =& \bar{\psi}\Big[i\partial\!\!\!/ - M - g_\sigma \sigma - g_\delta \vec{\delta}\cdot\vec{\tau} \\
& - \frac{f_\eta}{m_\eta}\gamma^5 \partial\!\!\!/ \eta - \frac{f_\pi}{m_\pi}\gamma^5 \partial\!\!\!/ \vec{\pi}\cdot\vec{\tau} \\
& - g_\omega \omega\!\!\!/ - \frac{f_\omega}{4M}\sigma^{\mu\nu}\Omega_{\mu\nu} - g_\rho \vec{\rho\!\!\!/}\cdot\vec{\tau} - \frac{f_\rho}{4M}\sigma^{\mu\nu}\vec{R}_{\mu\nu}\cdot\vec{\tau}\Big]\psi \\
& + \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2\right) + \frac{1}{2}\left(\partial_\mu \vec{\delta}\cdot\partial^\mu \vec{\delta} - m_\delta^2 \vec{\delta}^2\right) \\
& + \frac{1}{2}\left(\partial_\mu \eta \partial^\mu \eta - m_\eta^2 \eta^2\right) + \frac{1}{2}\left(\partial_\mu \vec{\pi}\cdot\partial^\mu \vec{\pi} - m_\pi^2 \vec{\pi}^2\right) \\
& + \frac{1}{2}\left(\frac{1}{2}\Omega^{\mu\nu}\Omega_{\mu\nu} + m_\omega^2 \omega^\mu \omega_\mu\right) + \frac{1}{2}\left(\frac{1}{2}\vec{R}^{\mu\nu}\cdot\vec{R}_{\mu\nu} + m_\rho^2 \vec{\rho}^\mu \cdot\vec{\rho}_\mu\right)
\end{aligned}
$$

--> Ansatz for the average effective action

$$
\Gamma_k = \int d^4x \left\{\bar{\psi}\left[\gamma_E^\mu \partial_\mu^E + M + g_{\sigma;k}\sigma - \gamma_E^0\left(i g_{\omega;k}\omega_0^E + \mu\right)\right]\psi + \frac{1}{2}\partial_\mu^E \sigma \partial_E^\mu \sigma + \mathcal{U}_k(\mu,\sigma,\omega_0^E)\right\}
$$

--> Plug in Wetterich equation ⇒ Flow equation for effective potential + beta functions for Yukawa couplings

--> Obtain a NL-like Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\text{NL}} =& \bar{\psi}\Big[i\partial\!\!\!/ - M - g_\sigma \sigma - \frac{f_\pi}{m_\pi}\gamma^5 \partial\!\!\!/ \vec{\pi}\cdot\vec{\tau} - g_\omega \omega\!\!\!/ - g_\rho \vec{\rho\!\!\!/}\cdot\vec{\tau}\Big]\psi \\
& + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - U[\sigma] \\
& + \frac{1}{2}\left(\frac{1}{2}\Omega^{\mu\nu}\Omega_{\mu\nu} + m_\omega^2 \omega^\mu \omega_\mu\right) + \frac{1}{2}\left(\frac{1}{2}\vec{R}^{\mu\nu}\cdot\vec{R}_{\mu\nu} + m_\rho^2 \vec{\rho}^\mu \cdot\vec{\rho}_\mu\right)
\end{aligned}
$$

$$
U[\sigma] = \frac{1}{2}m_\sigma^2 \sigma^2 + \frac{g_2}{3}\sigma^3 + \frac{g_3}{4}\sigma^4
$$

--> Check properties of nuclear matter