Description of electromagnetic moments in the EDF framework

Jacek Dobaczewski, University of York & University of Warsaw

In collaboration with B.C.T. Backes, M. Bender, J. Bonnard, G. Danneaux, R.F. Garcia Ruiz, R.P. de Groote, M. Kortelainen, D. Muir, A. Nagpal, W. Nazarewicz, A. Sánchez-Fernández, P.L. Sassarini, W. Satuła, X. Sun, J. Wood, H. Wibowo

Nuclear energy density functional method: going beyond the minefield,
Saclay, France, 20-24 November 2023

The bottom line

The nuclear electromagnetic moments of odd nuclei are all about:

1. **Polarization**
2. **Self-consistency**
3. **Symmetry restoration**

About half of the nuclides in nature are odd

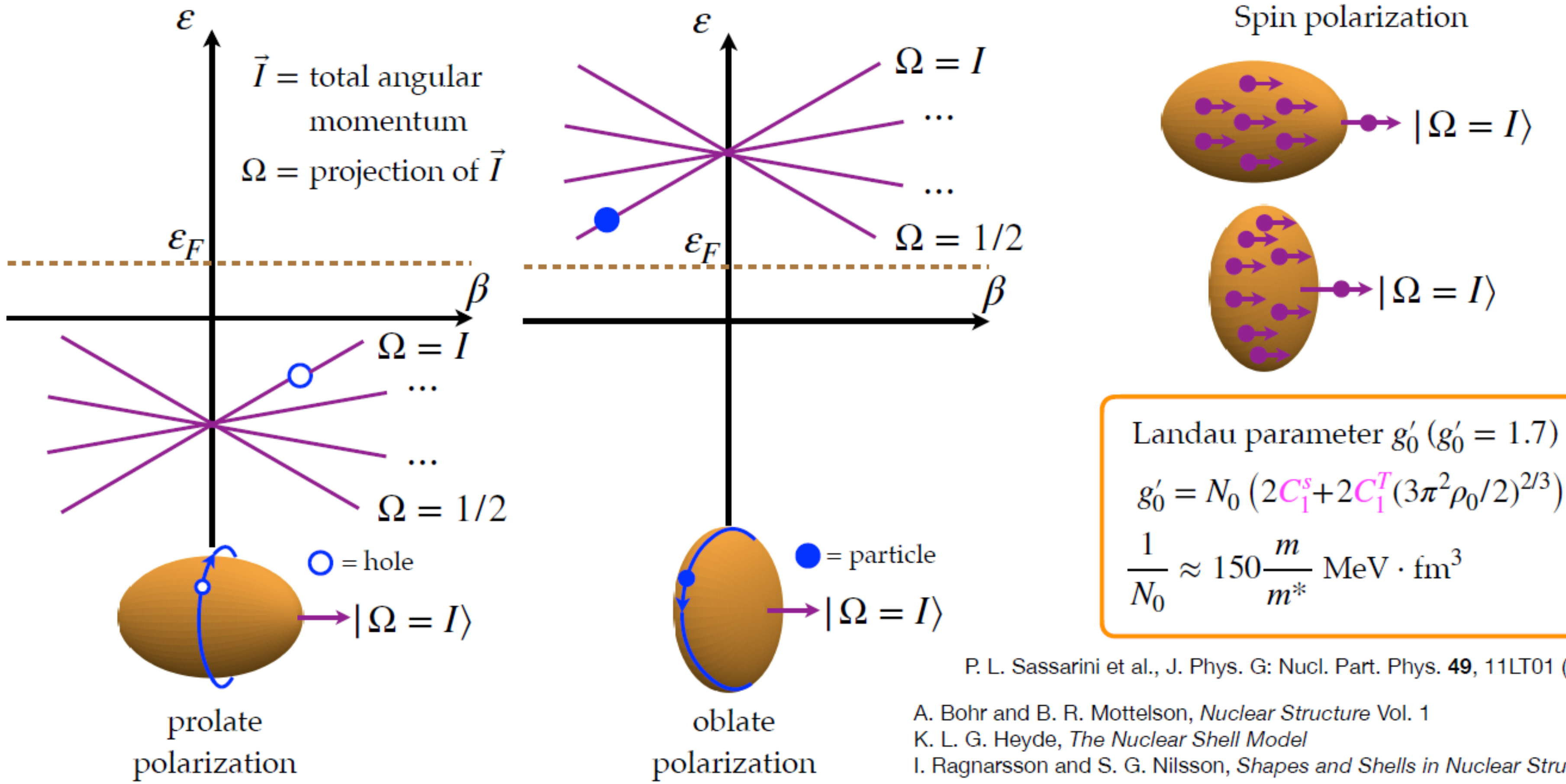
Not on the minefield

Nuclear electromagnetic moments are one-body observables.

Singularities of the Wick theorem cancel exactly:

$$\langle\Psi|O|\Psi'\rangle = \frac{\langle\Psi|O|\Psi'\rangle}{\langle\Psi|\Psi'\rangle} \times \langle\Psi|\Psi'\rangle$$

Shape and spin polarization



Landau parameter g_0' ($g_0' = 1.7$)

$$g_0' = N_0\left(2C_1^s + 2C_1^T(3\pi^2\rho_0/2)^{2/3}\right)$$

$$\frac{1}{N_0} \approx 150\frac{m}{m^*}\ \mathrm{MeV}\cdot\mathrm{fm}^3$$

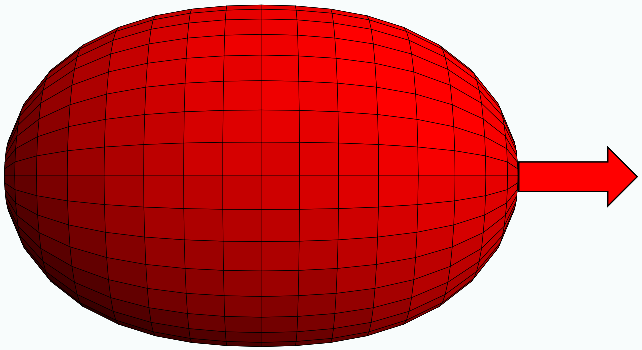
P. L. Sassarini et al., J. Phys. G: Nucl. Part. Phys. **49**, 11LT01 (2022)

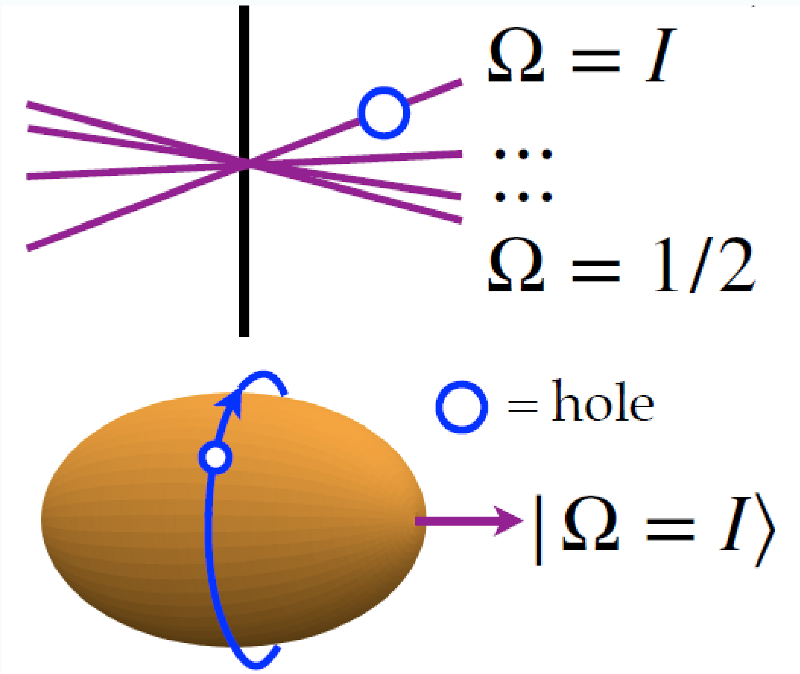
A. Bohr and B. R. Mottelson, *Nuclear Structure* Vol. 1
K. L. G. Heyde, *The Nuclear Shell Model*
I. Ragnarsson and S. G. Nilsson, *Shapes and Shells in Nuclear Structure*

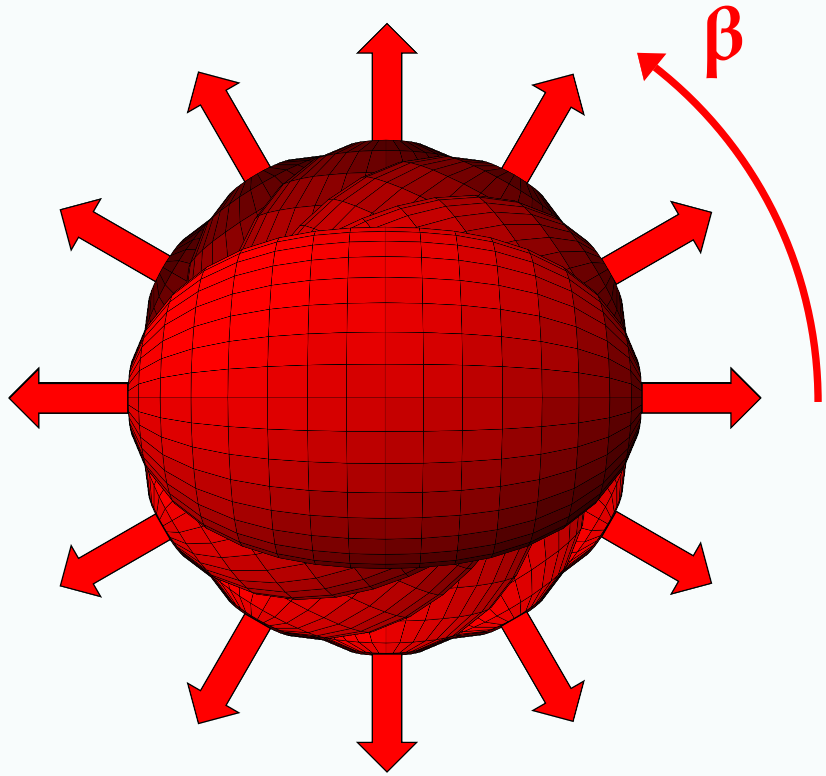
Picture: courtesy H. Wibowo

Time-odd spin alignment & symmetry restoration

"Intrinsic"
Symmetry broken







"Laboratory"
Symmetry restored

$$|IM\rangle = \mathcal{N}_I \int_{\beta=0}^{\pi} \mathrm{d}\beta \, d^I_{M\Omega}(\beta) |\Omega, \beta\rangle$$

J. A. Sheikh et al., J. Phys. G48, 123001 (2021)

Nuclear quadrupole & dipole moments

Spectroscopic electric quadrupole Q and magnetic dipole μ moments are :

$$Q = \sqrt{\frac{16\pi}{5}}\langle II\,|\,\hat{Q}_{20}\,|\,II\rangle \quad \text{and} \quad \mu = \sqrt{\frac{4\pi}{3}}\langle II\,|\,\hat{M}_{10}\,|\,II\rangle\,.$$

P. Ring and P. Schuck, *The Nuclear Many-Body Problem*

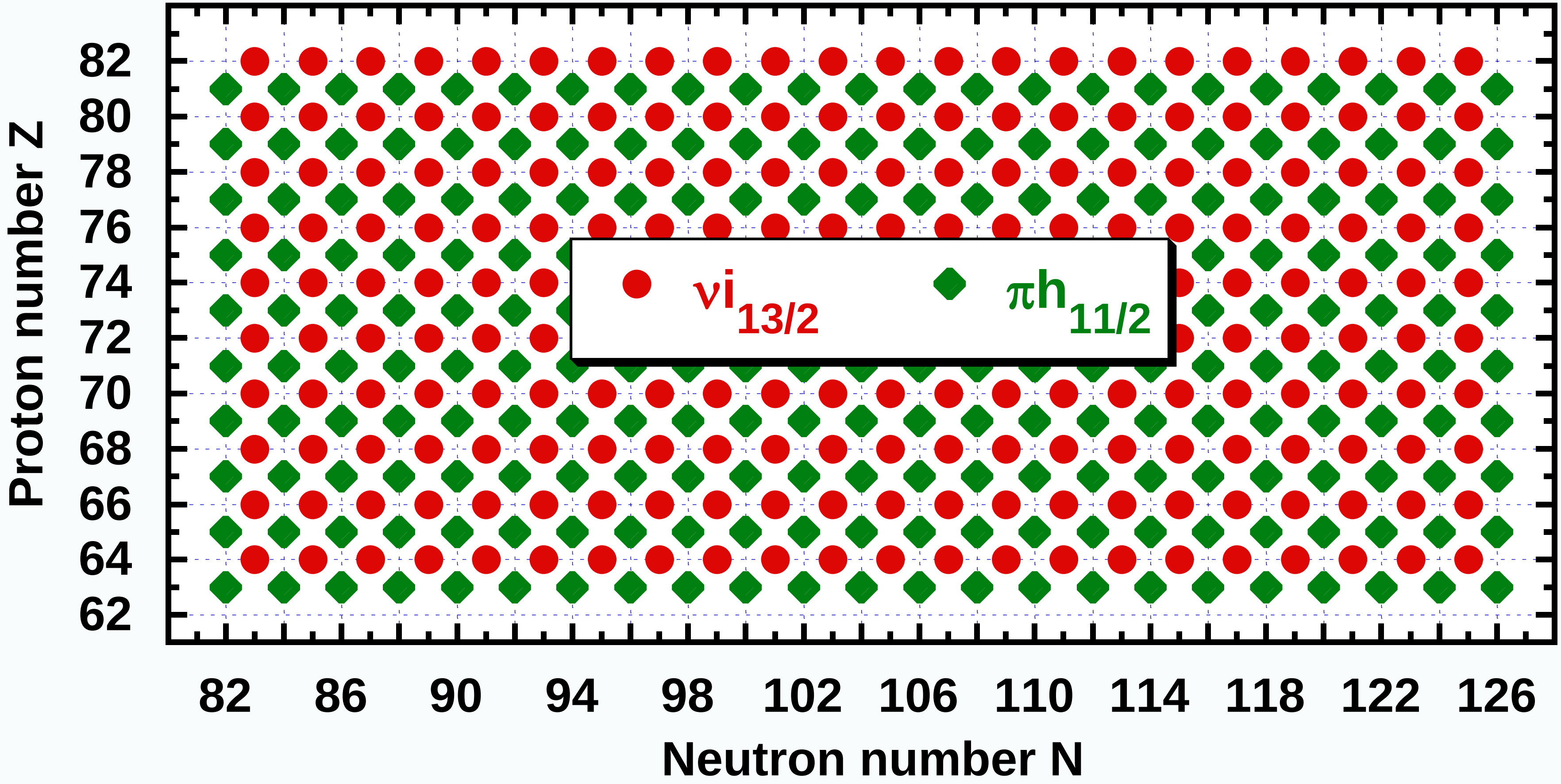
$$\hat{Q}_{20} = \sqrt{\frac{5}{16\pi}}e\sum_{i=1}^{A}\left(\tfrac{1}{2}-t_3^{(i)}\right)\{3z_i^2 - r_i^2\};\quad \hat{M}_{10} = \sqrt{\frac{3}{4\pi}}\mu_N\sum_{i=1}^{A}\left\{g_s^{(i)}s_{zi}+g_\ell^{(i)}\ell_{zi}\right\};$$

$$g_s^{(i)} = g_p(g_n) = 5.59(-3.83)$$
$$g_\ell^{(i)} = 1(0)$$

Intrinsic moments = moments of the symmetry-broken state
Spectroscopic moments = moments of the symmetry-restored state

Spectroscopic moments = moments measured experimentally

The first systematic nuclear-DFT analysis of the electromagnetic moments in heavy deformed open-shell odd nuclei



Blocked quasiparticles were tagged by the neutron $i_{13/2}$ (Ω=+13/2) or proton $h_{11/2}$ (Ω=+11/2) single-particle orbitals

J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

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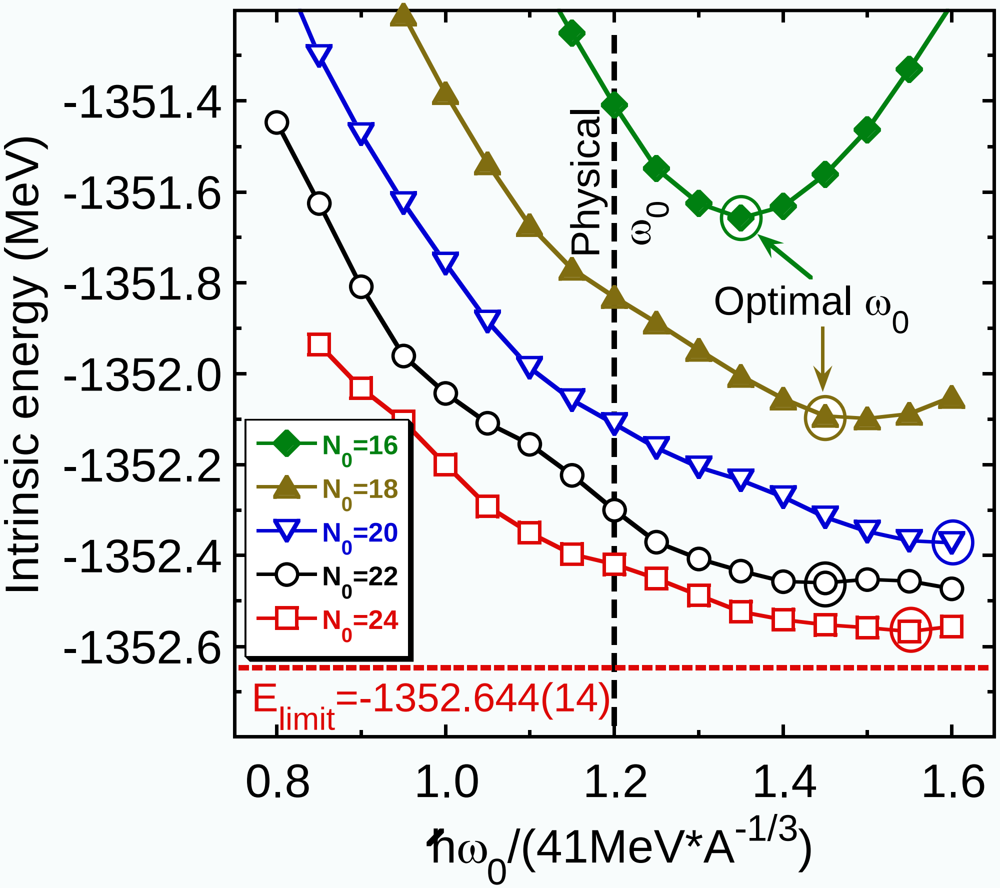
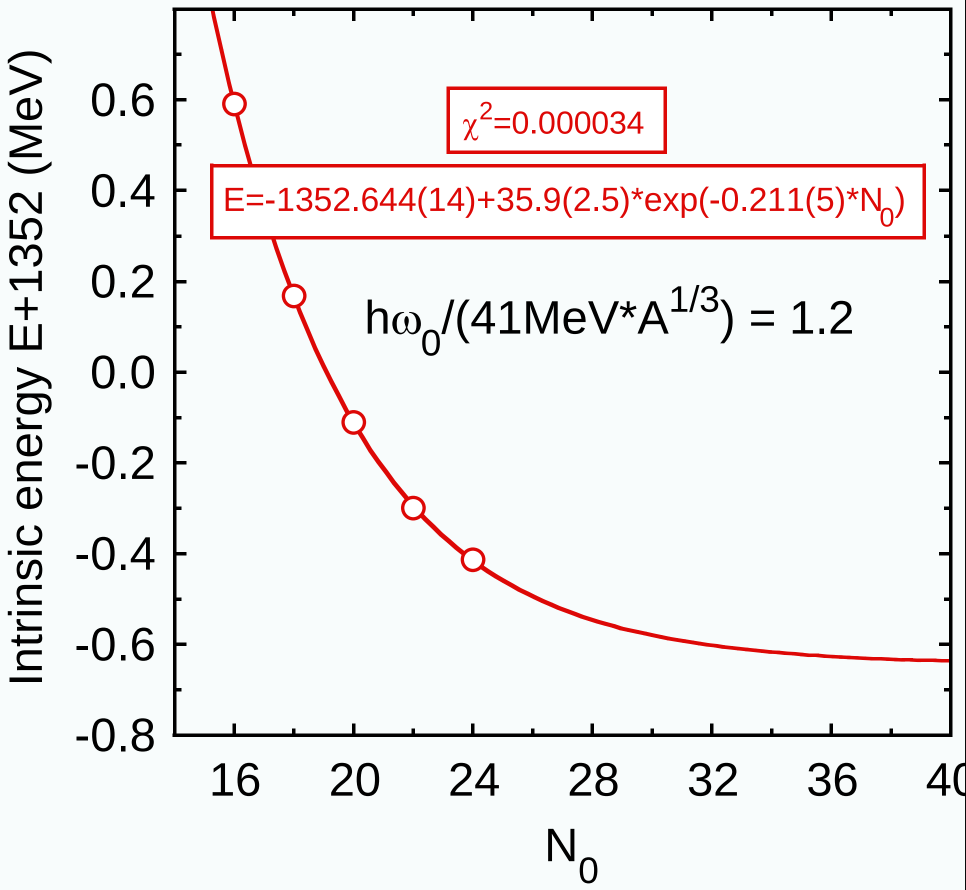
Convergence of the total HFB intrinsic energy

^{167}Ho 11/2$^-$, UNEDF1, g'_0=1.7

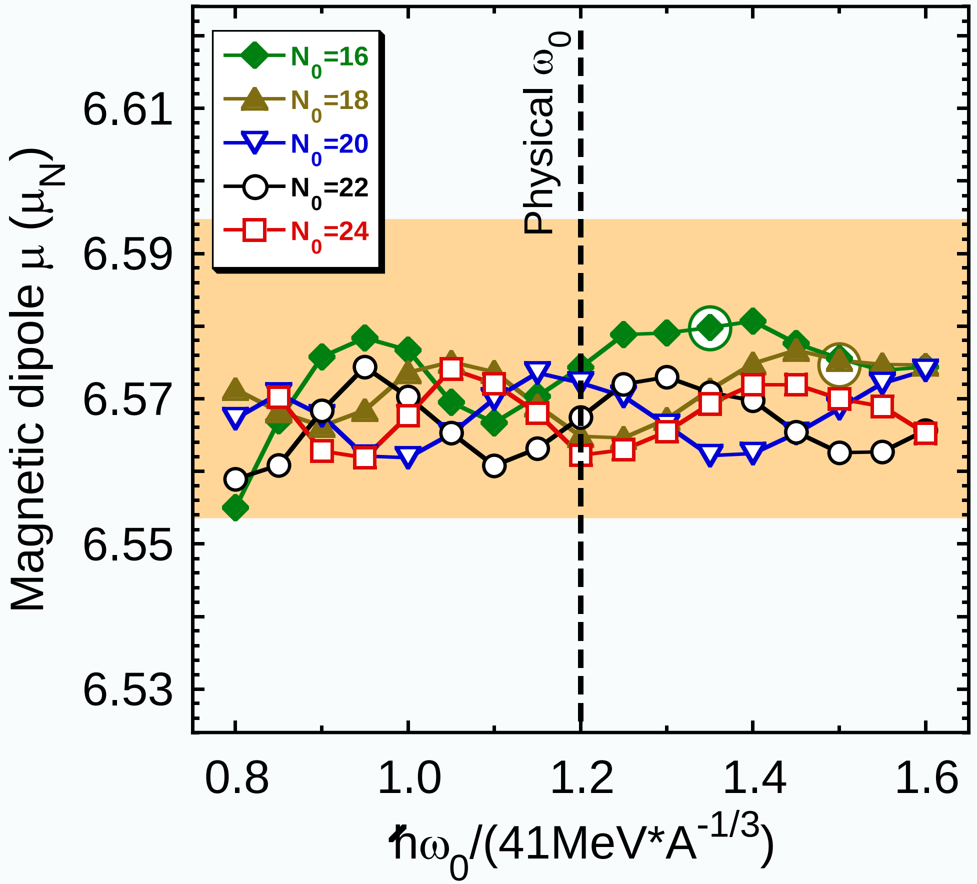


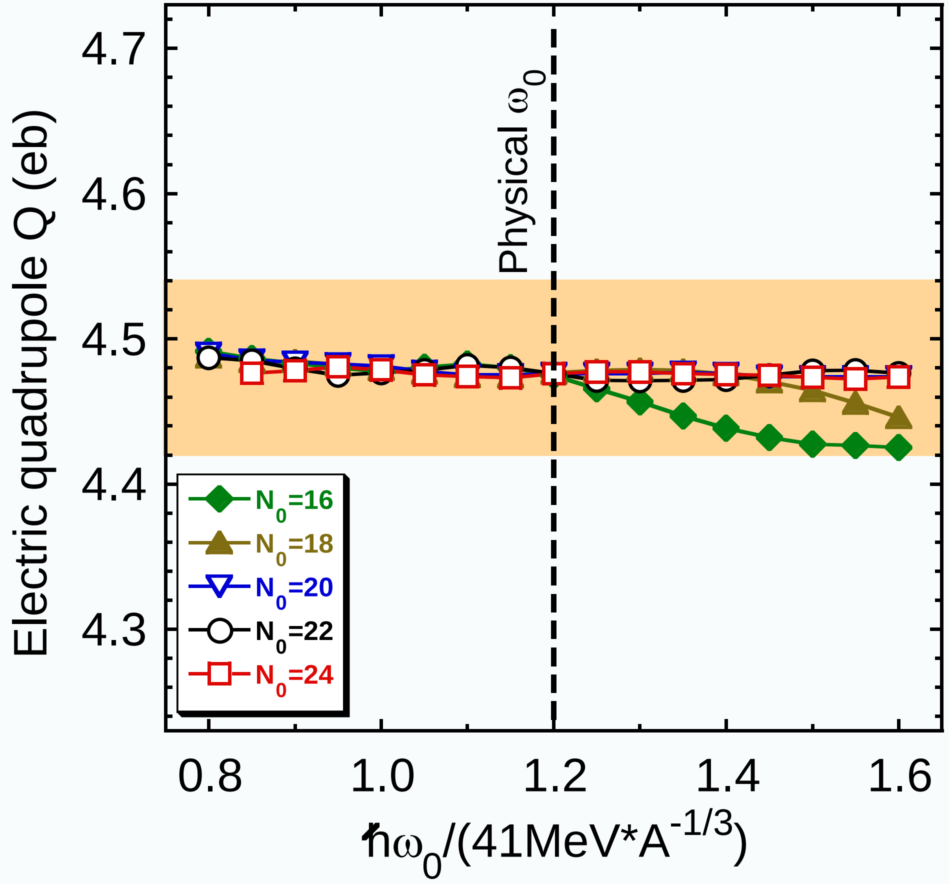


E_{exp}=-1357.77346

Convergence of the spectroscopic moments

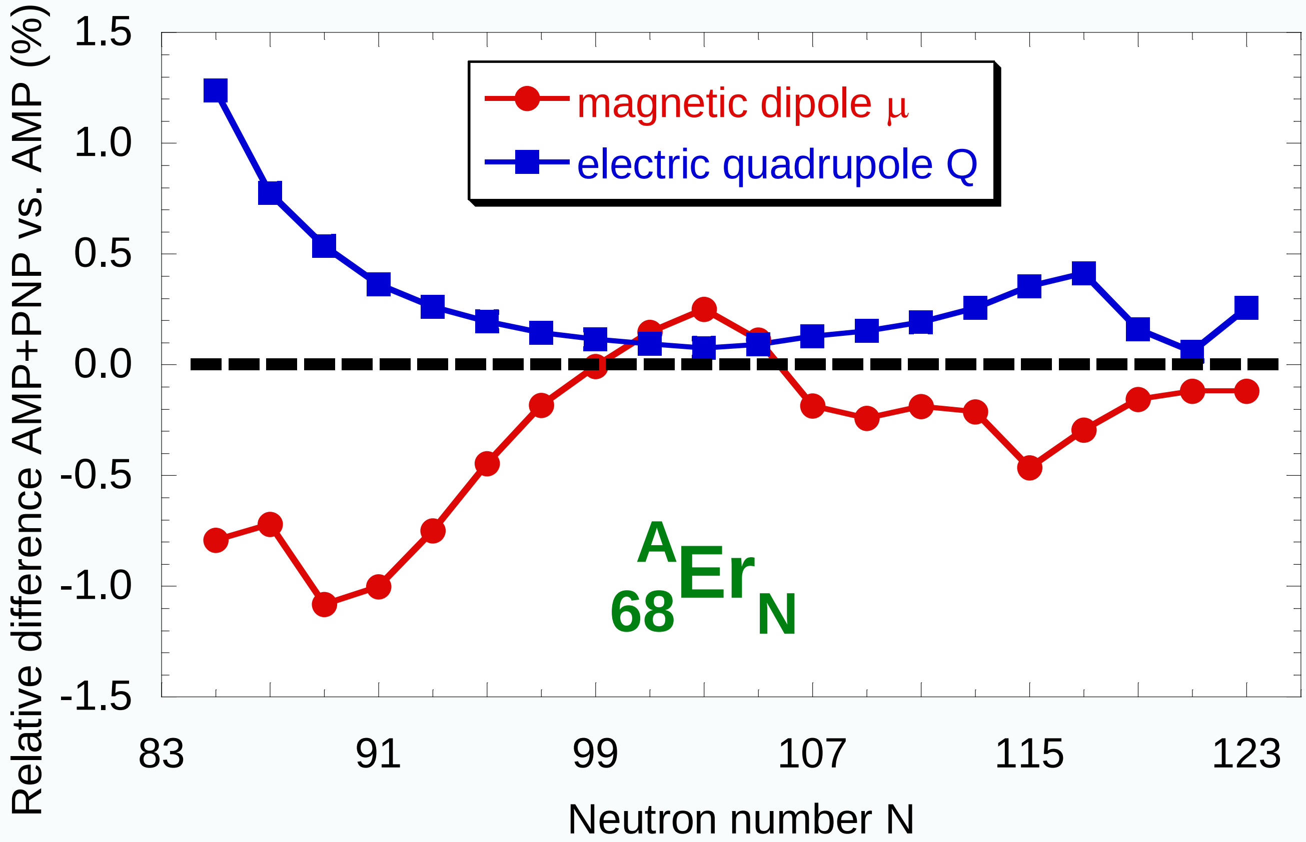
^{167}Ho 11/2$^-$, UNEDF1, g'_0=1.7



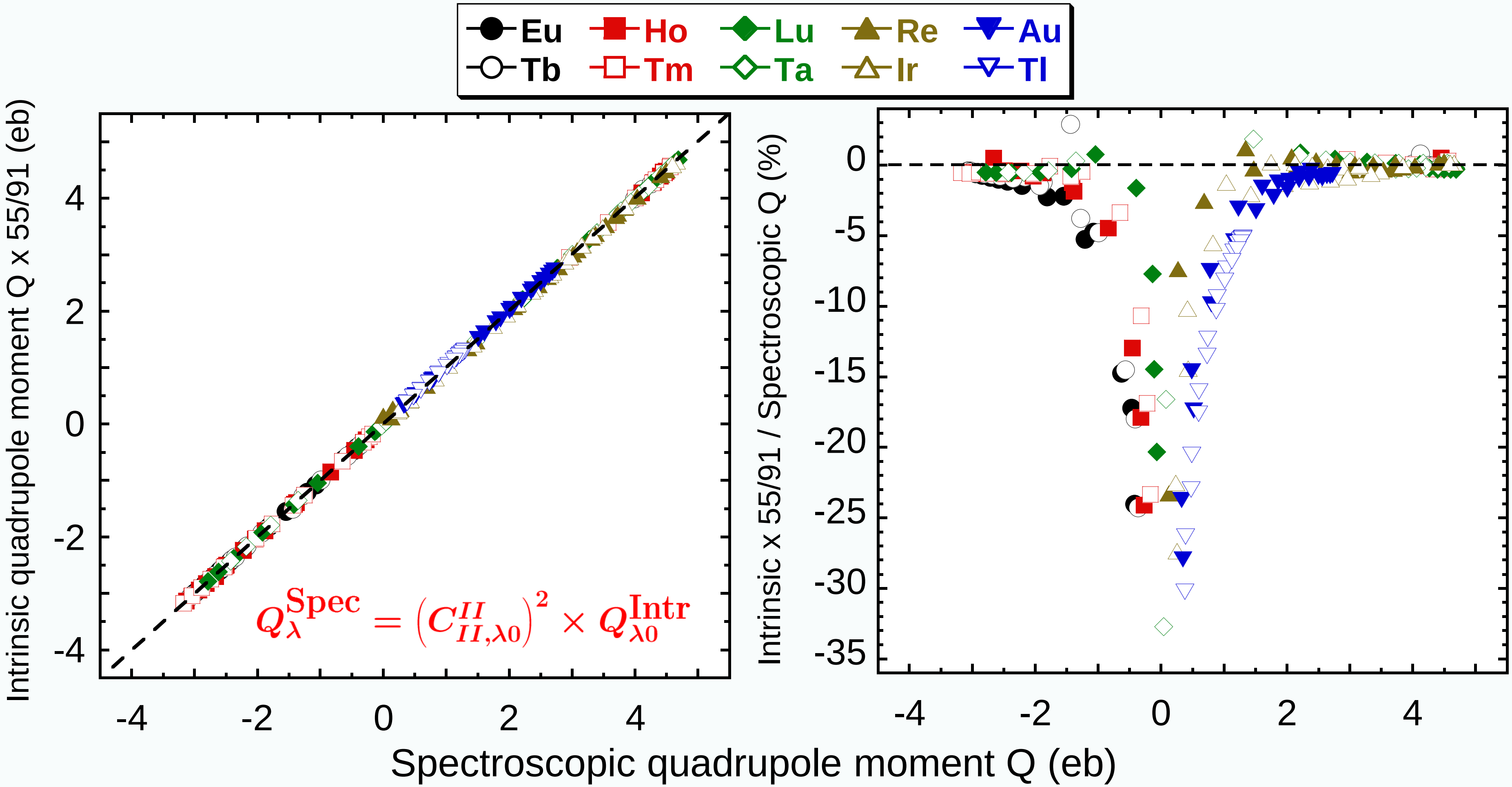


Insignificant impact of the PNP

UNEDF1, g'_0=1.7



Heavy deformed π11/2⁻ odd-Z nuclei

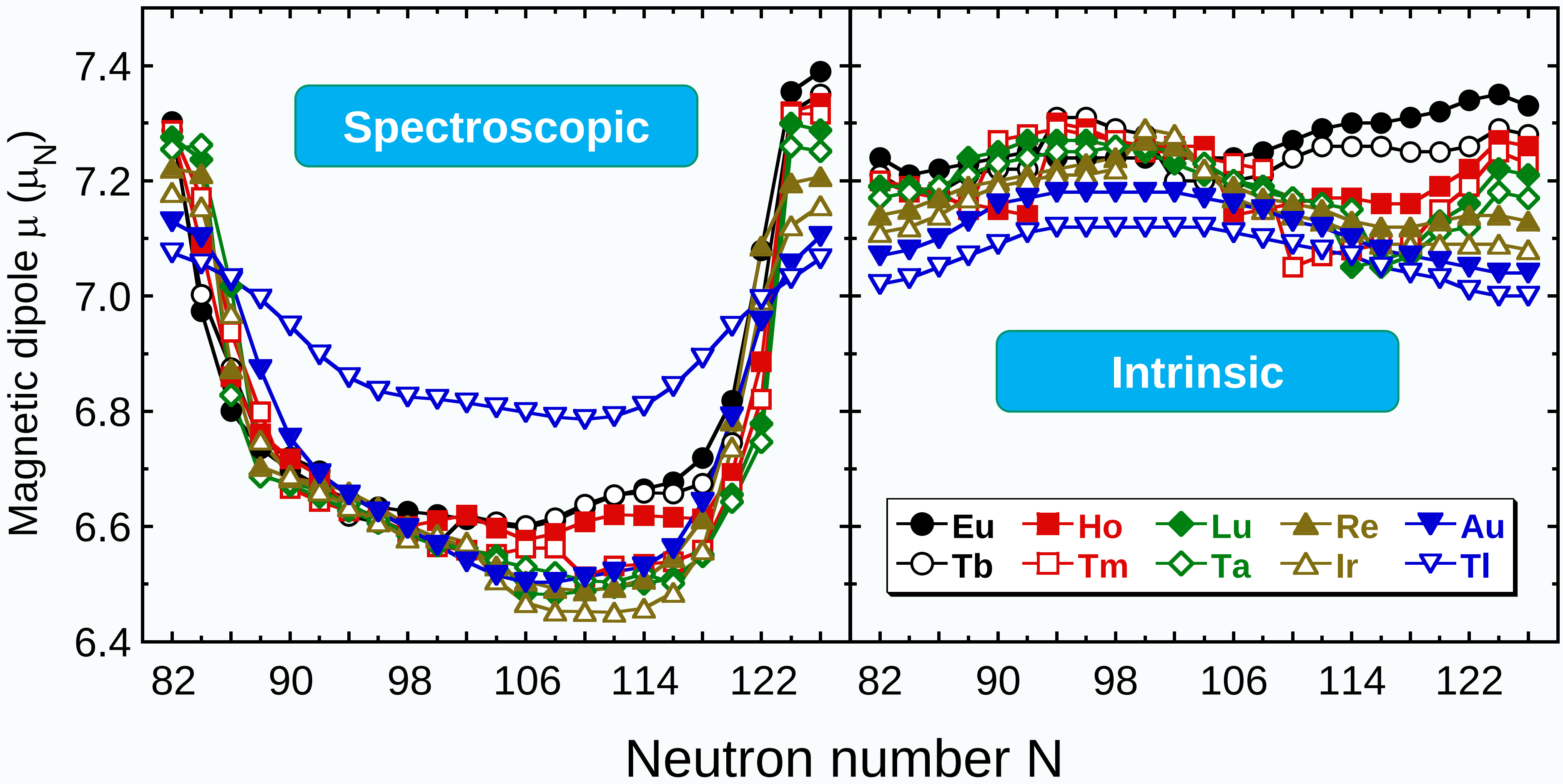


J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

Conclusion:

Spectroscopic electric quadrupole moments can be inferred from the intrinsic ones at ~5% precision only at |Q|>1b)

Heavy deformed π11/2⁻ odd-Z nuclei



J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

Conclusion:

Spectroscopic magnetic dipole moments cannot be inferred from the intrinsic ones

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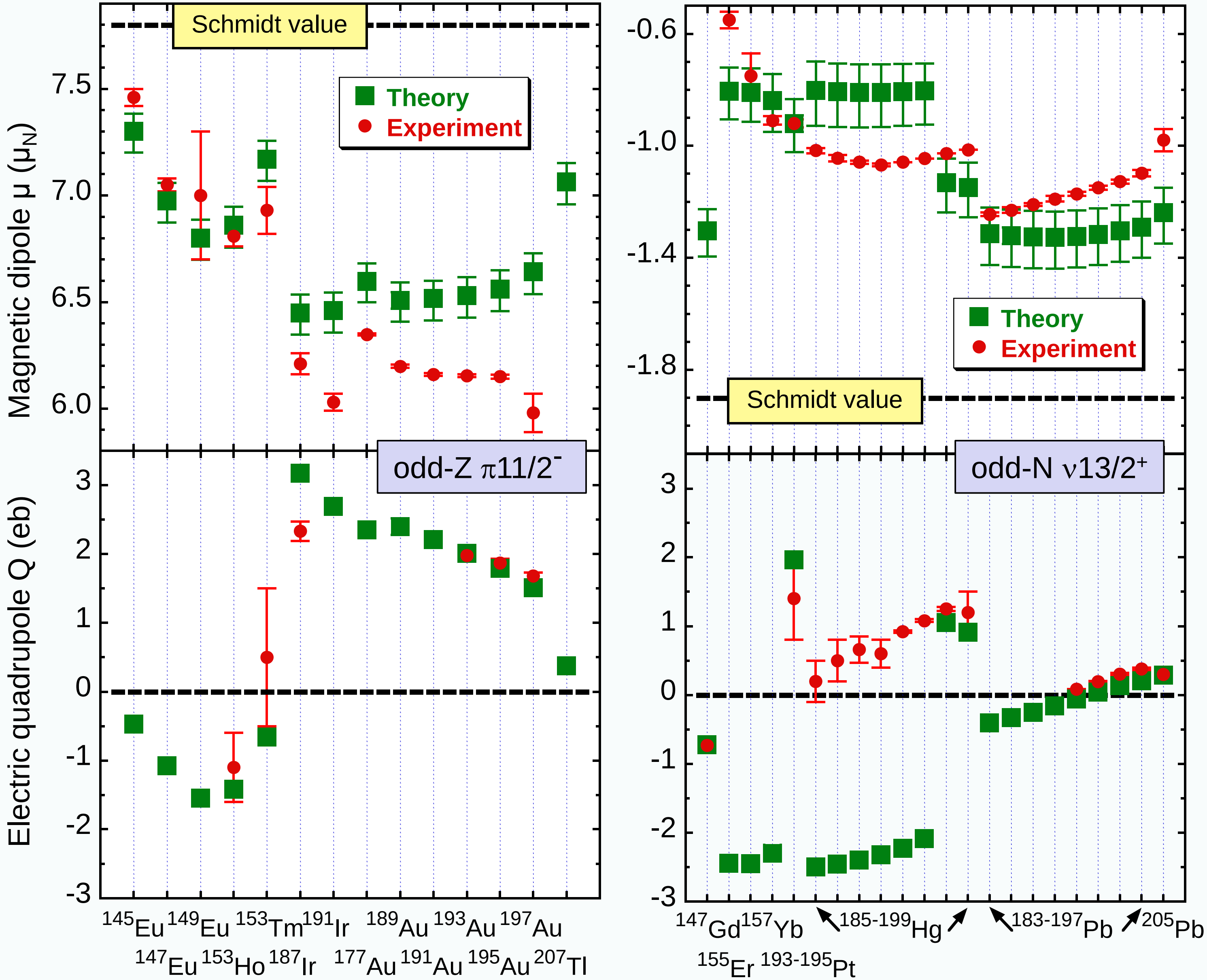
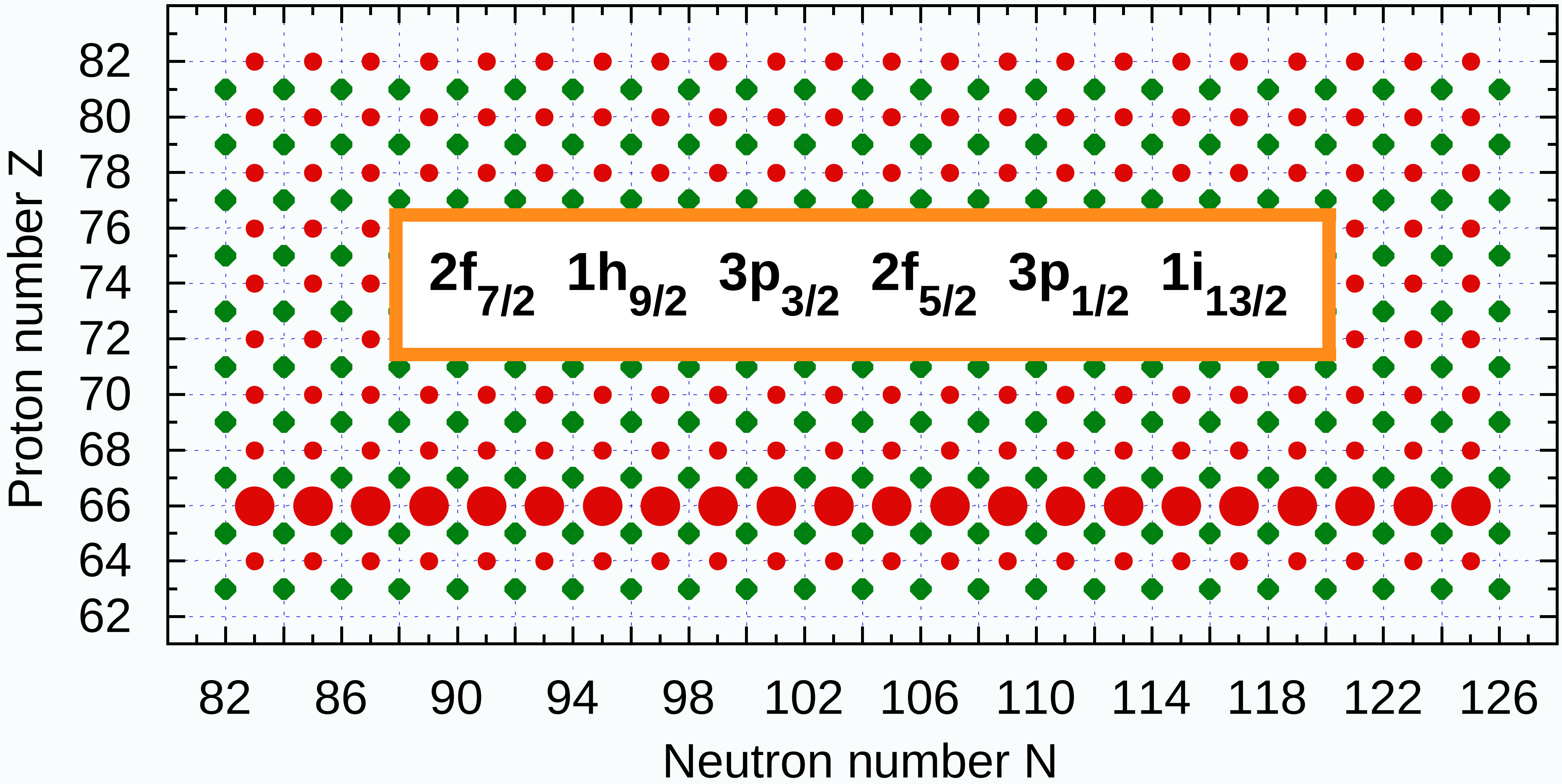
Spectroscopic moments: theory vs. experiment



J. Bonnard *et al.*, Phys. Lett. B 843 (2023) 138014

The first systematic nuclear-DFT analysis of the electromagnetic moments in excited quasiparticle states



J. Dobaczewski *et al.*, to be published

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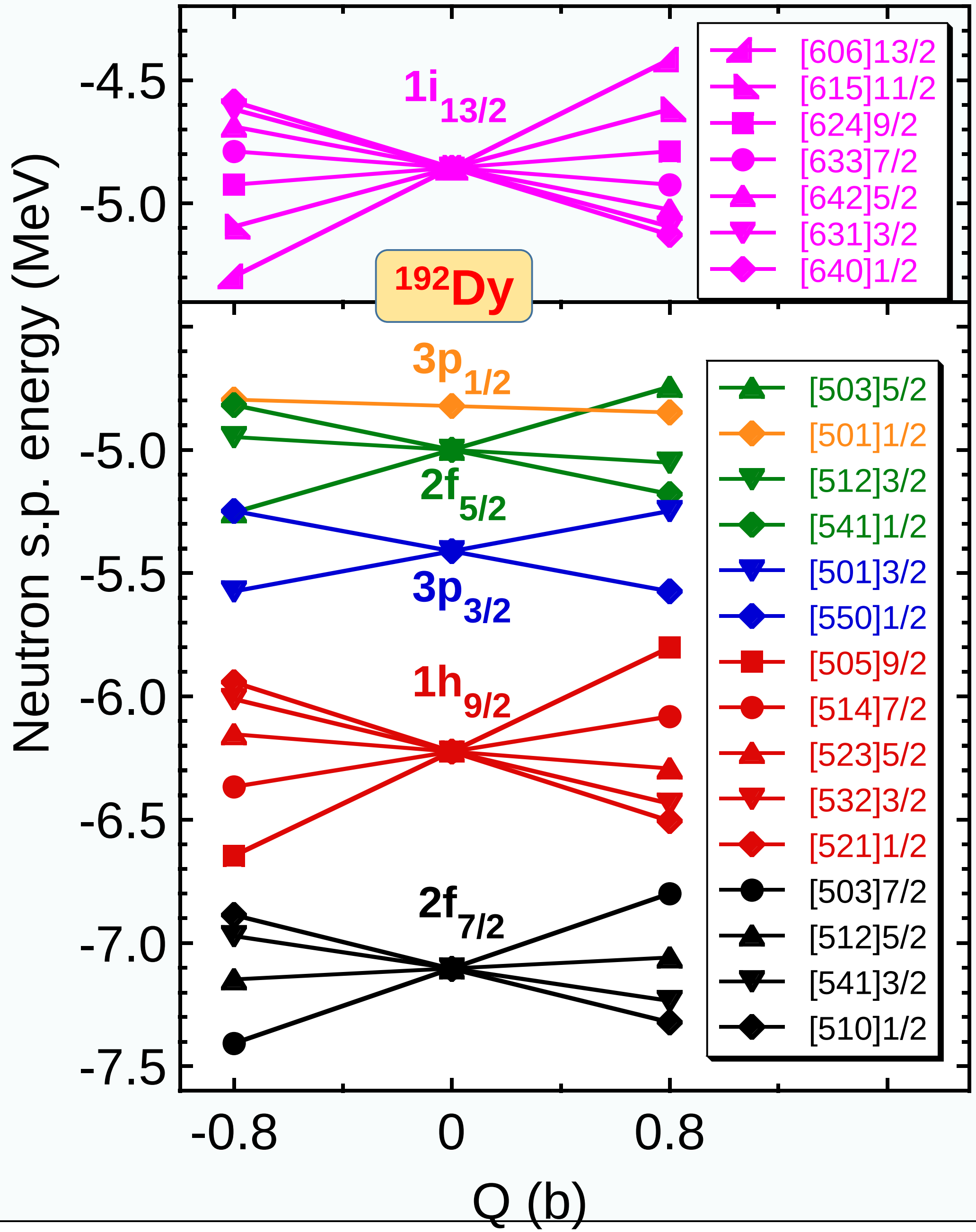
How to calculate odd nuclei in nuclear DFT?



without pairing

A even, $p > A$, $h \leq A$

$$|\Psi\rangle^{\text{even}}_{\text{HF}} = a^+_A \ldots a^+_2 a^+_1 |0\rangle$$

$$|\Psi\rangle^{\text{odd}}_{\text{HF}} = \begin{cases} a^+_p |\Psi\rangle^{\text{even}}_{\text{HF}} \\ a_h |\Psi\rangle^{\text{even}}_{\text{HF}} \end{cases}$$

with pairing

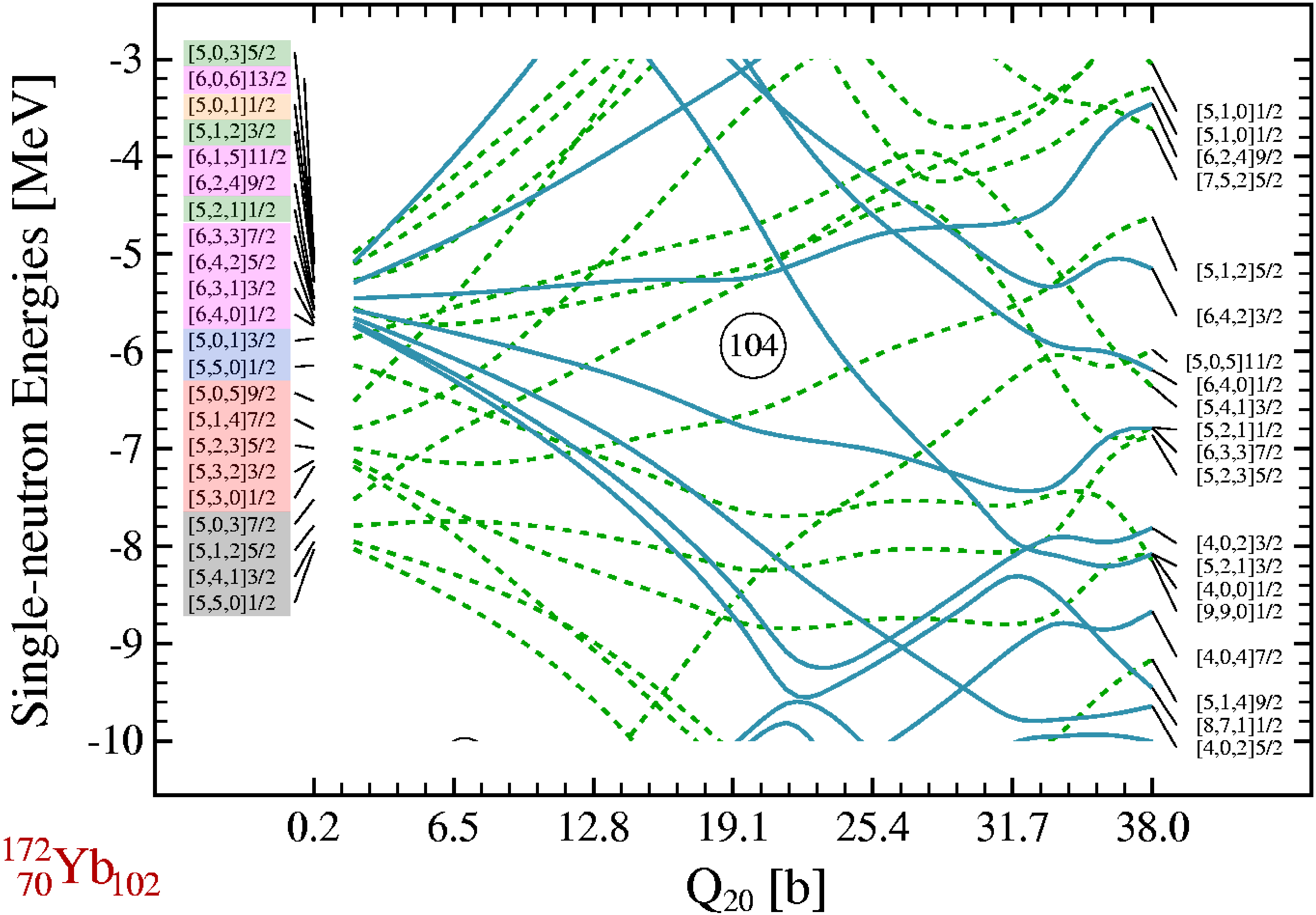
$$|\Psi\rangle^{\text{even}}_{\text{HFB}} = \prod_{\mu>0} \left(u_\mu + v_\mu a^+_{\bar{\mu}} a^+_\mu \right) |0\rangle$$

$$|\Psi\rangle^{\text{odd}}_{\text{HFB}} = \beta^+_\nu |\Psi\rangle^{\text{even}}_{\text{HFB}}$$

$$= a^+_\nu \prod_{\nu\neq\mu>0} \left(u_\mu + v_\mu a^+_{\bar{\mu}} a^+_\mu \right) |0\rangle$$

tagging quasiparticle states

$$\max_\mu \left\{ \langle \varphi_\nu | \phi^{\text{upper}}_\mu \rangle, \langle \varphi_\nu | \phi^{\text{lower}}_\mu \rangle \right\}$$



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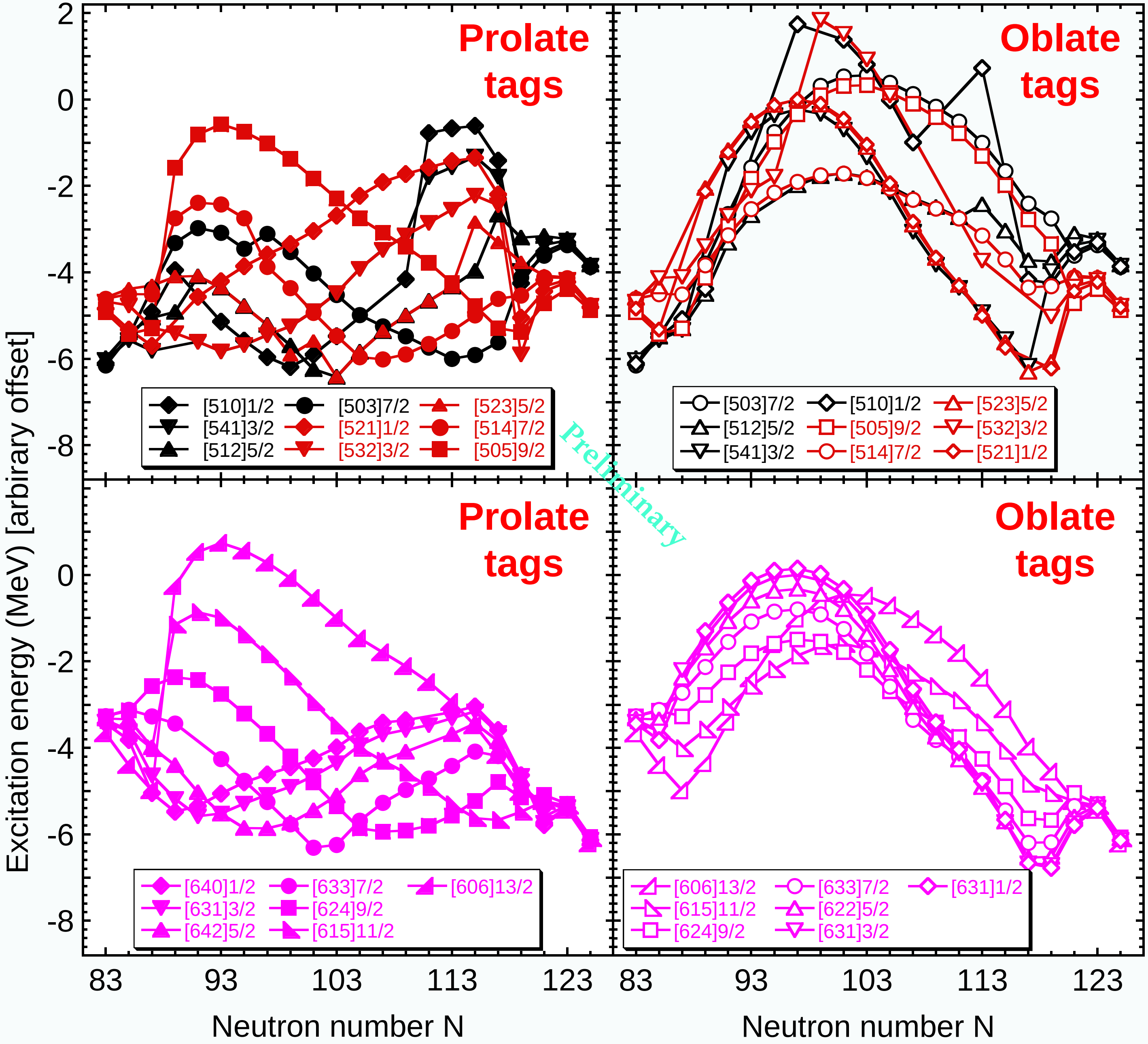
Excitation energies of odd dysprosium isotopes



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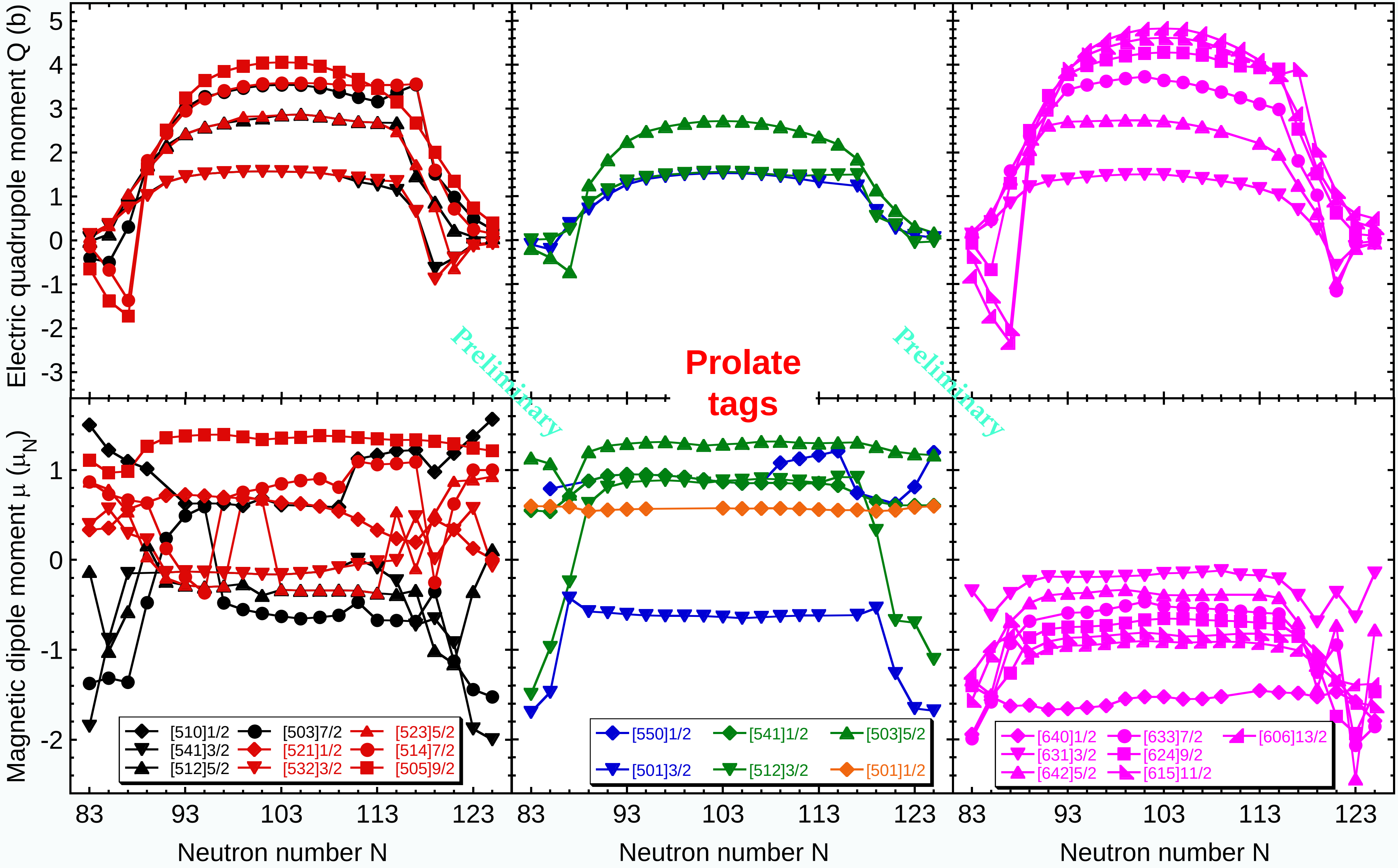
Electromagnetic moments of odd dysprosium isotopes



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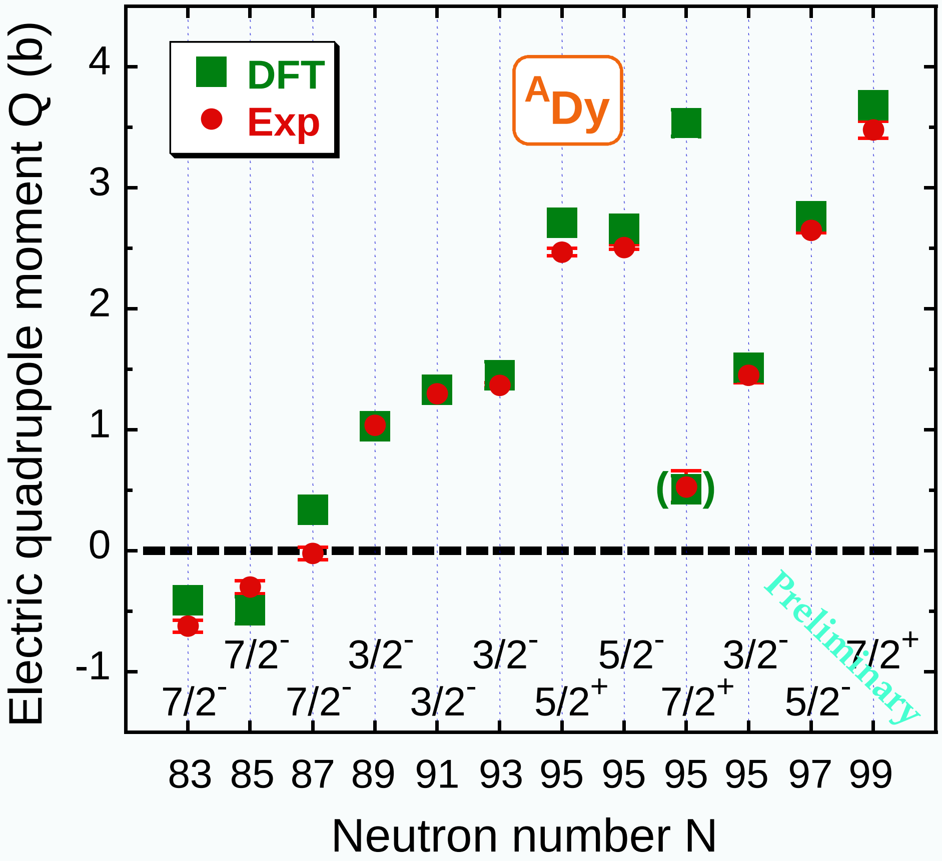
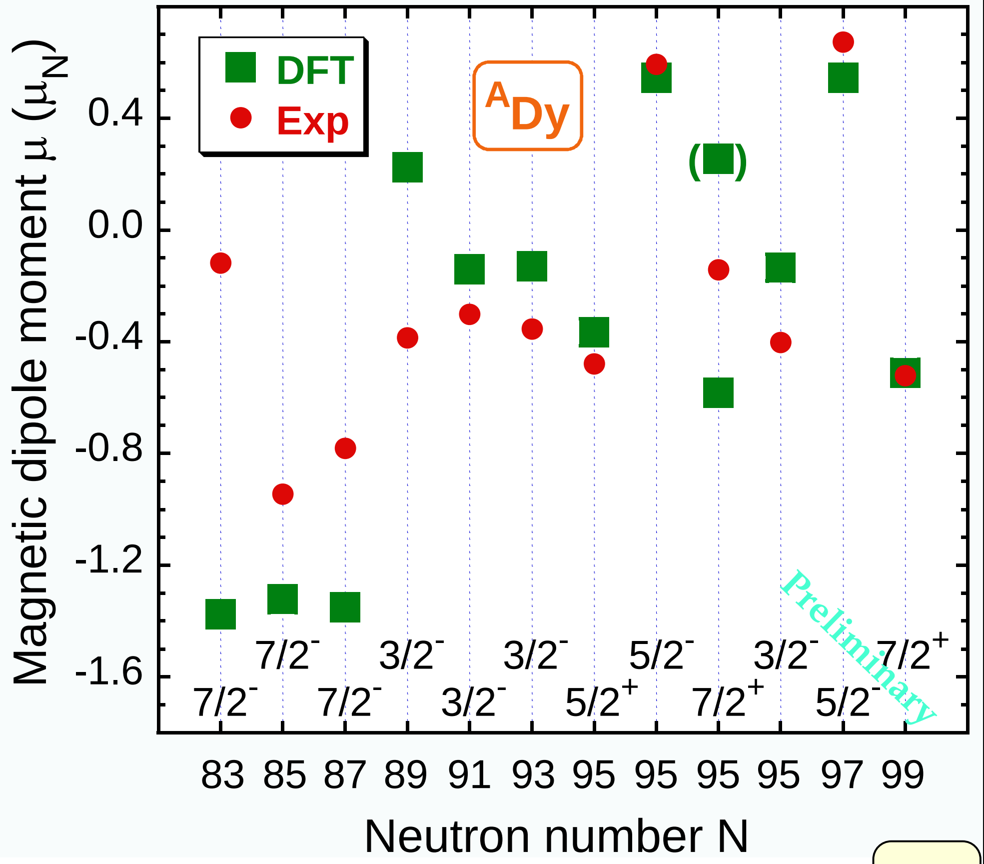
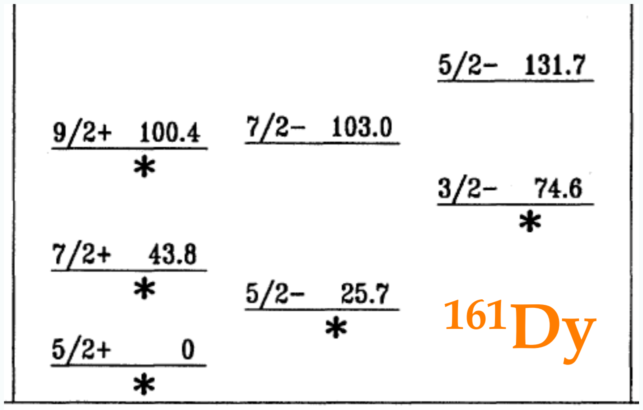
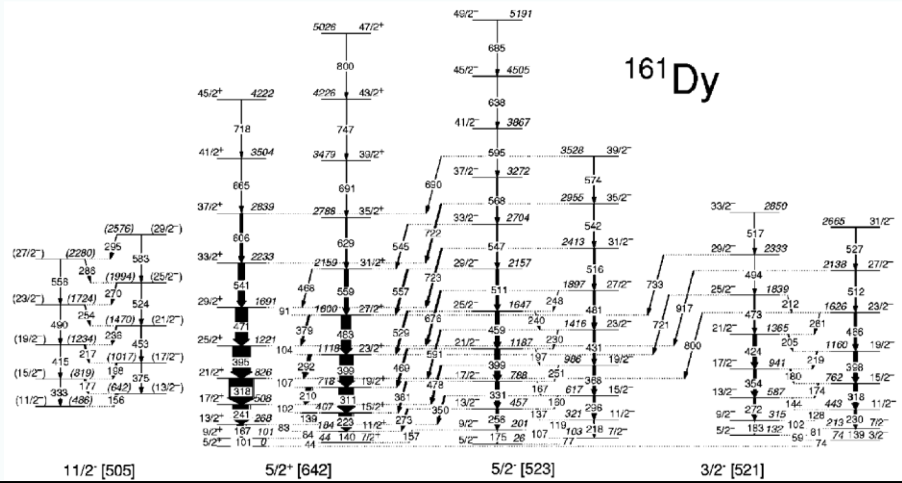
Electromagnetic moments of odd dysprosium isotopes







S. J. Margraf *et al.*, Phys. Rev. C52, 2429 (1995)



A. Jungclaus *et al.*, Phys. Rev. C67, 034302 (2003)

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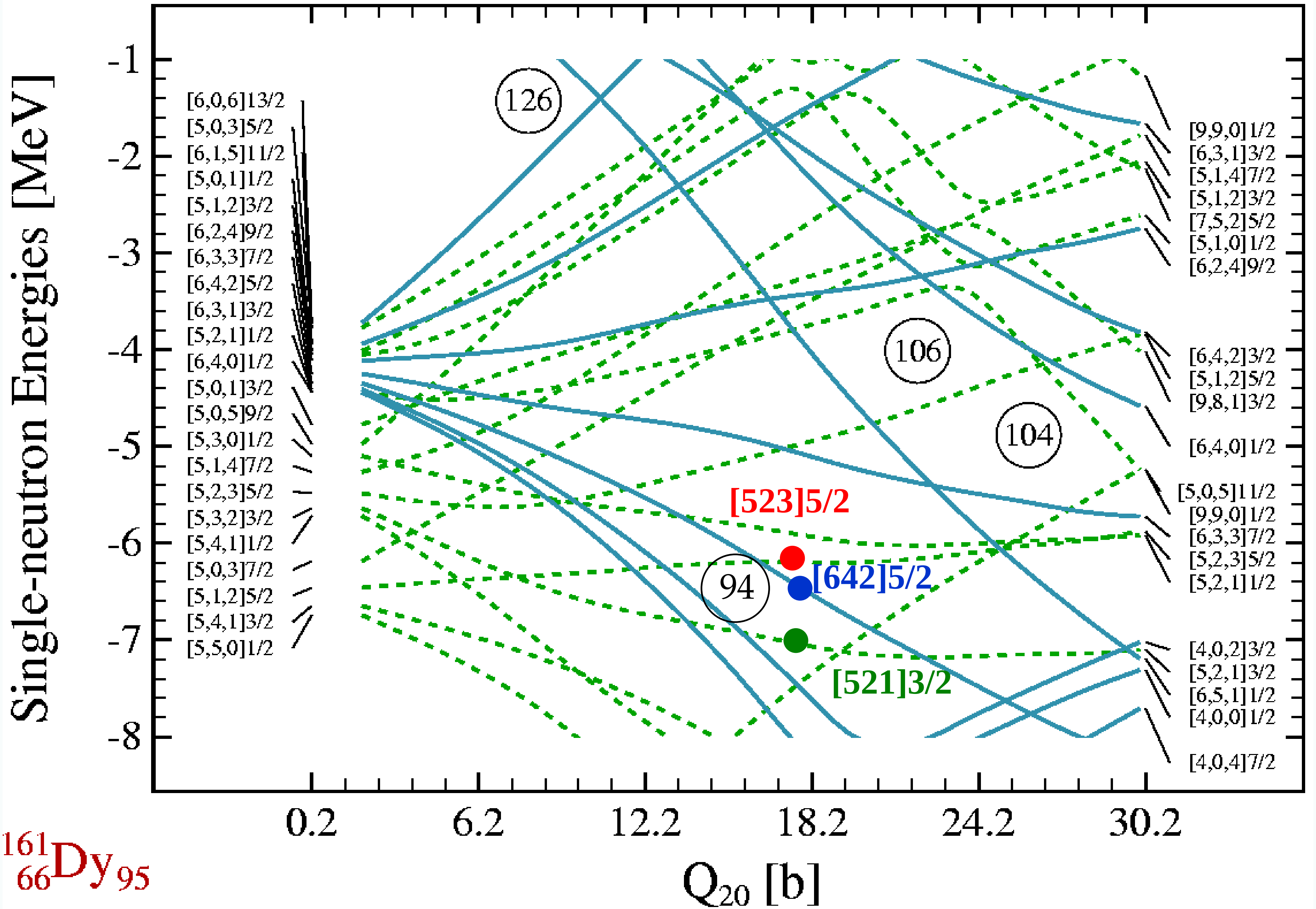

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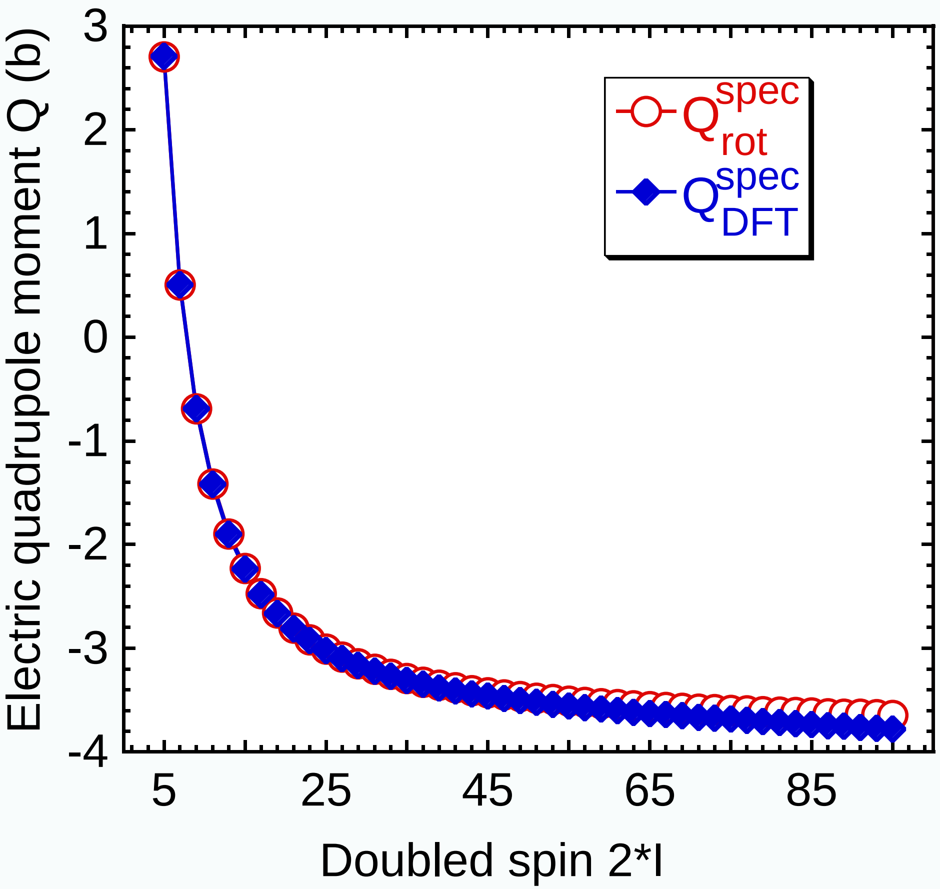
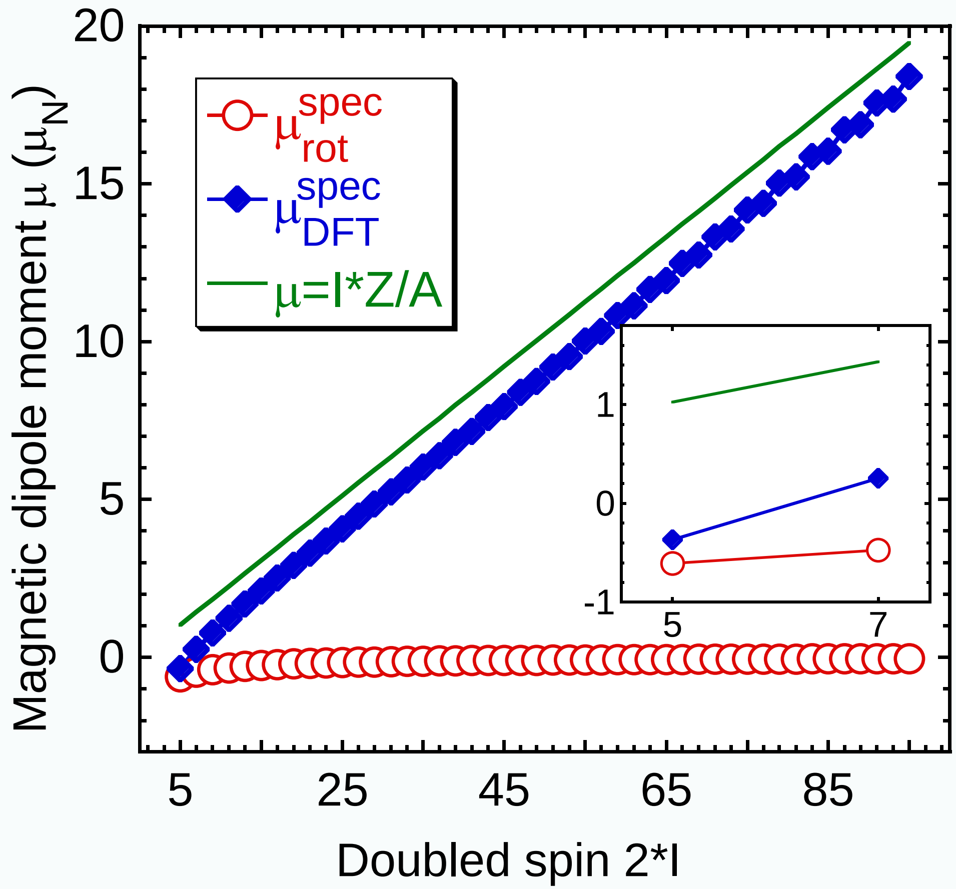
Electromagnetic moments – the rigid-rotor approximation

^{161}Dy 5/2$^+$ UNEDF1, g′$_0$=1.7

$$Q^{spec}_{rot} = Q^{intr}_{20} \times C^{II}_{II,20} \times C^{IK}_{IK,20} \qquad \mu^{spec}_{rot} = \mu^{intr}_{z} \times C^{II}_{II,10} \times C^{IK}_{IK,10}$$





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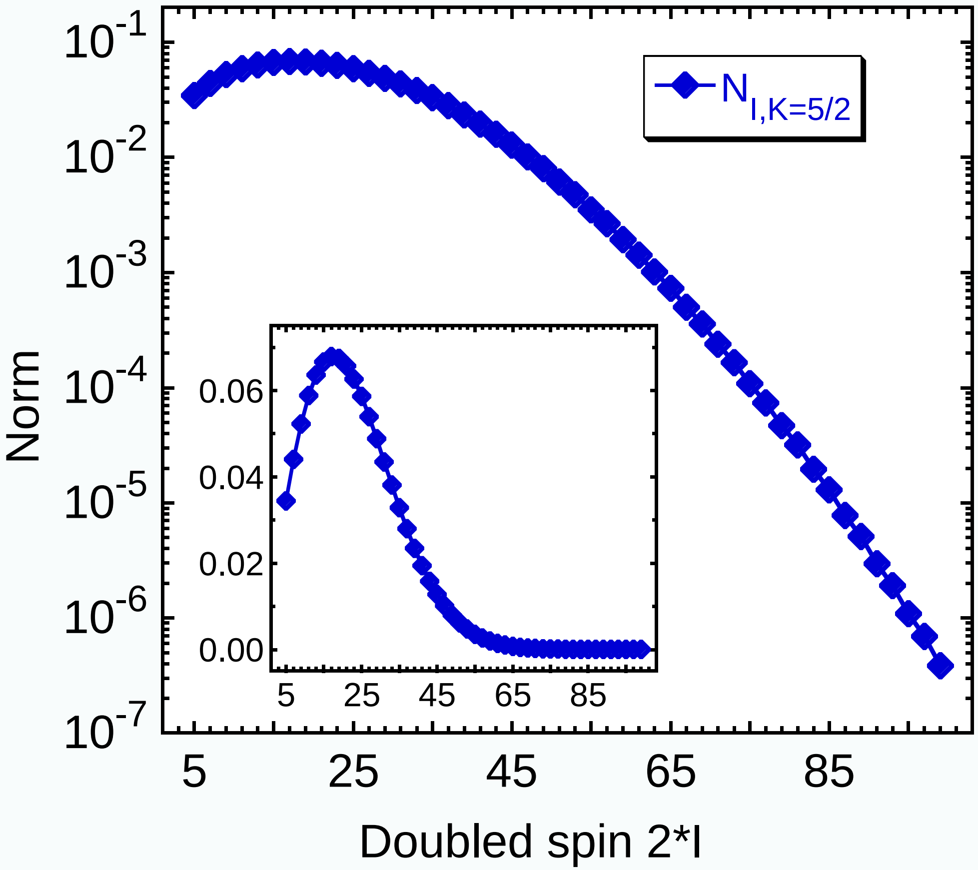
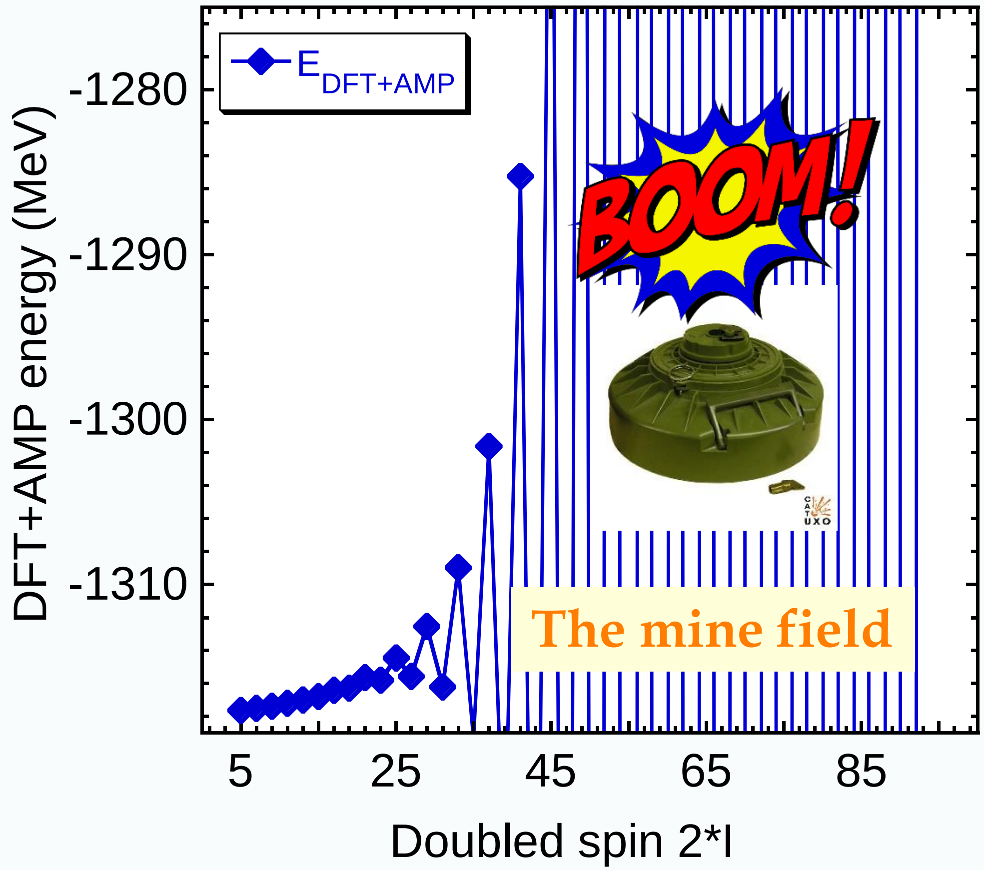
Angular-momentum projection (AMP)

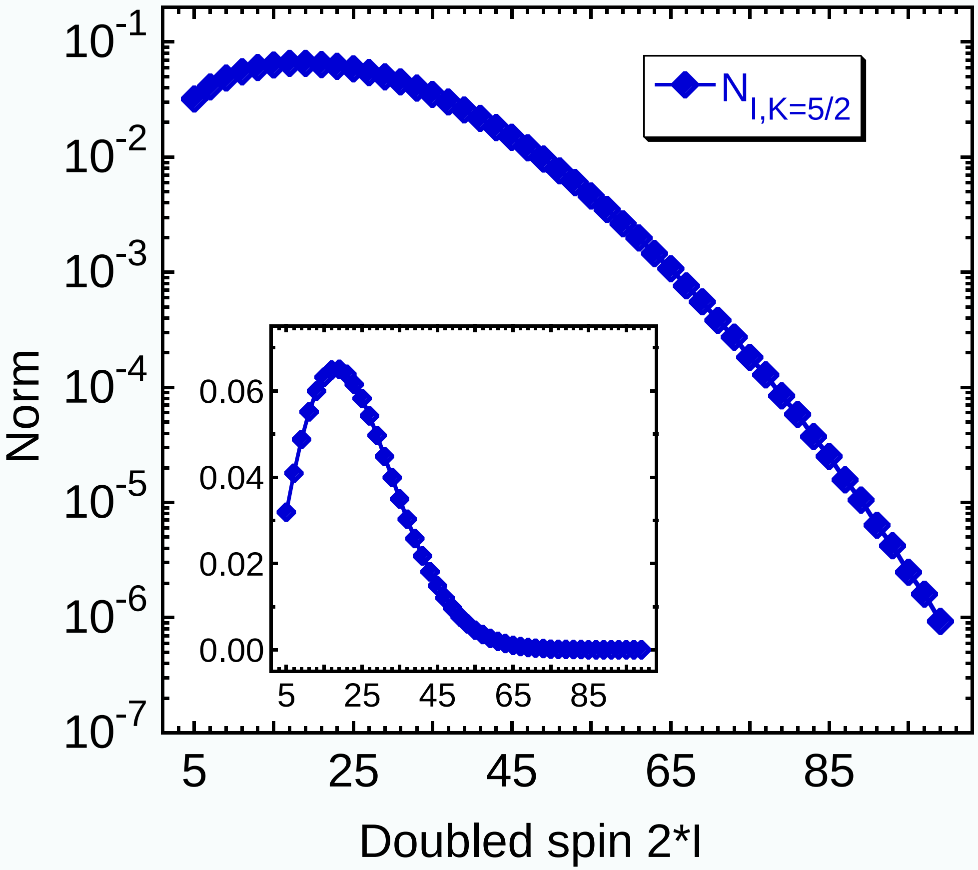
^{161}Dy 5/2^{+} UNEDF1, g'_0=1.7

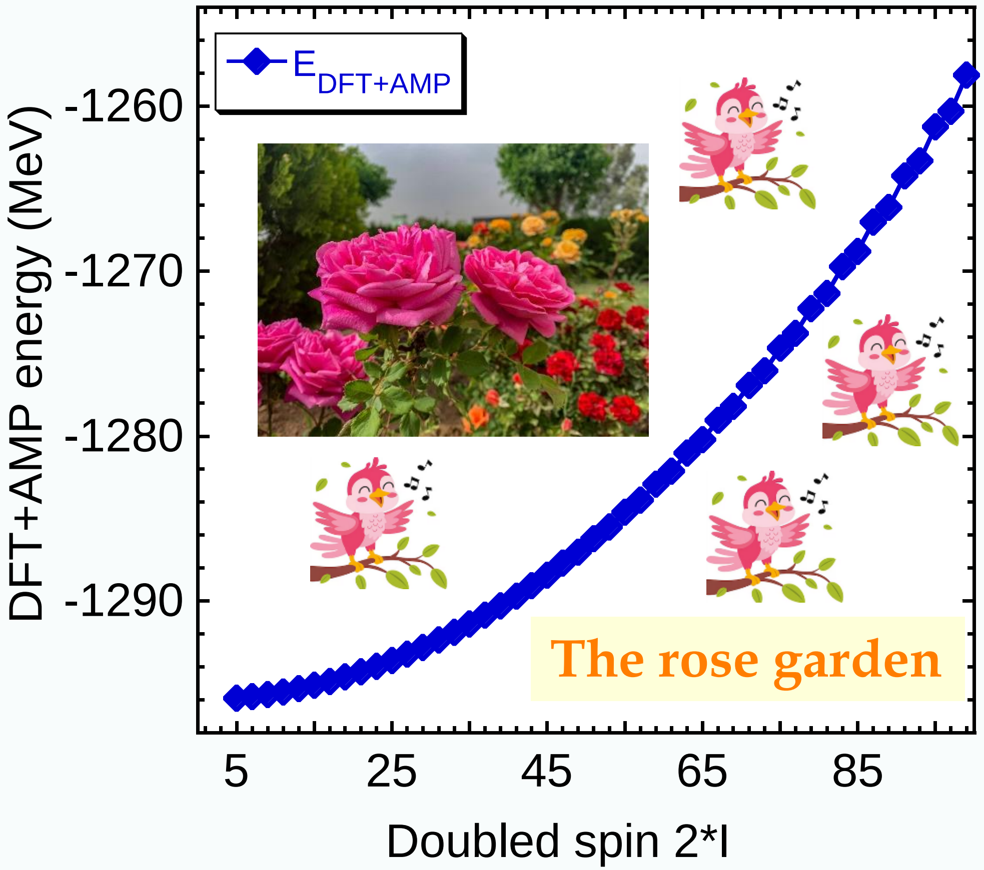




Angular-momentum projection (AMP)

^{161}Dy 5/2^{+} SV$_T$ native TO, exact COU

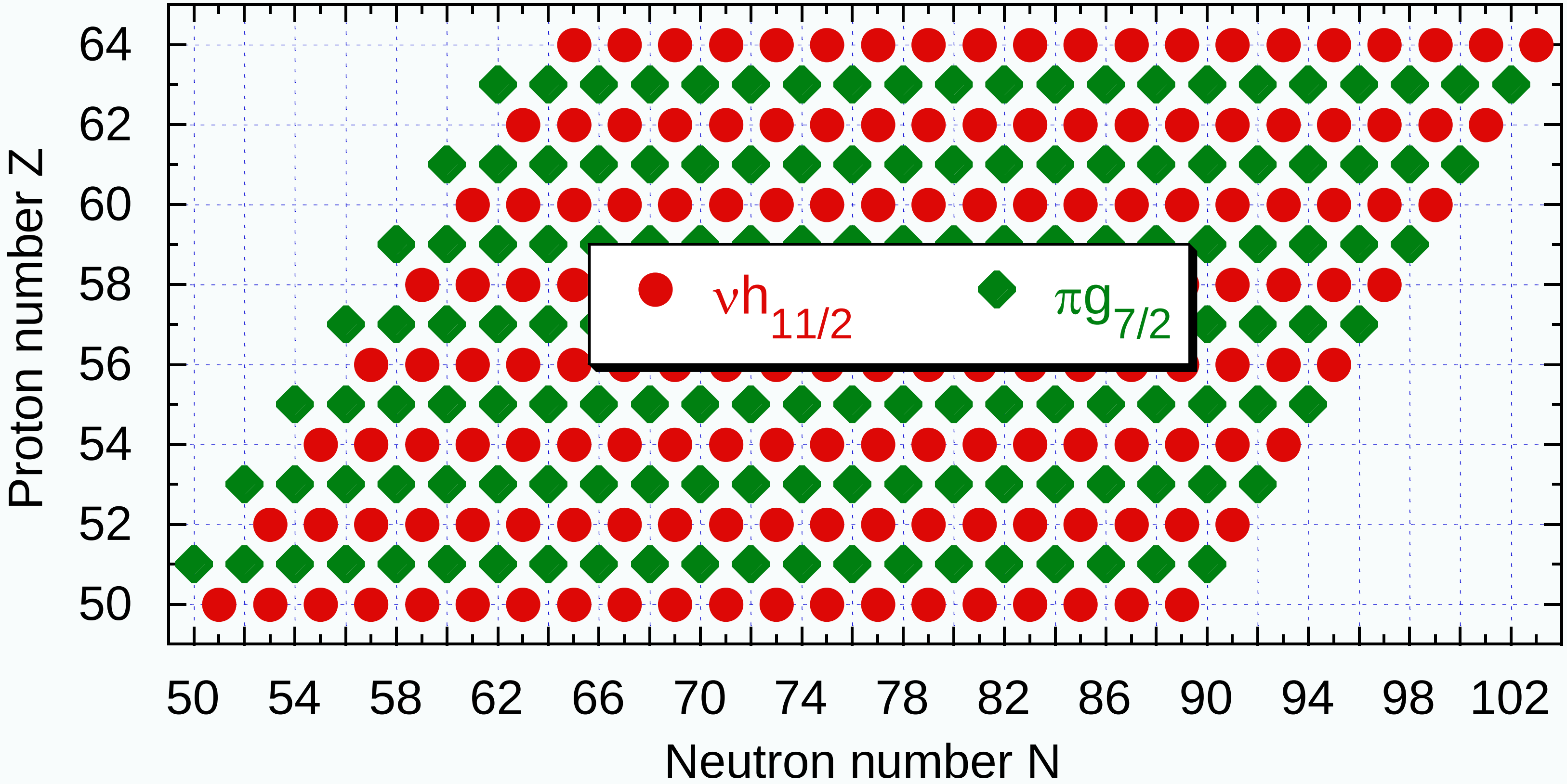




What else on the menu today

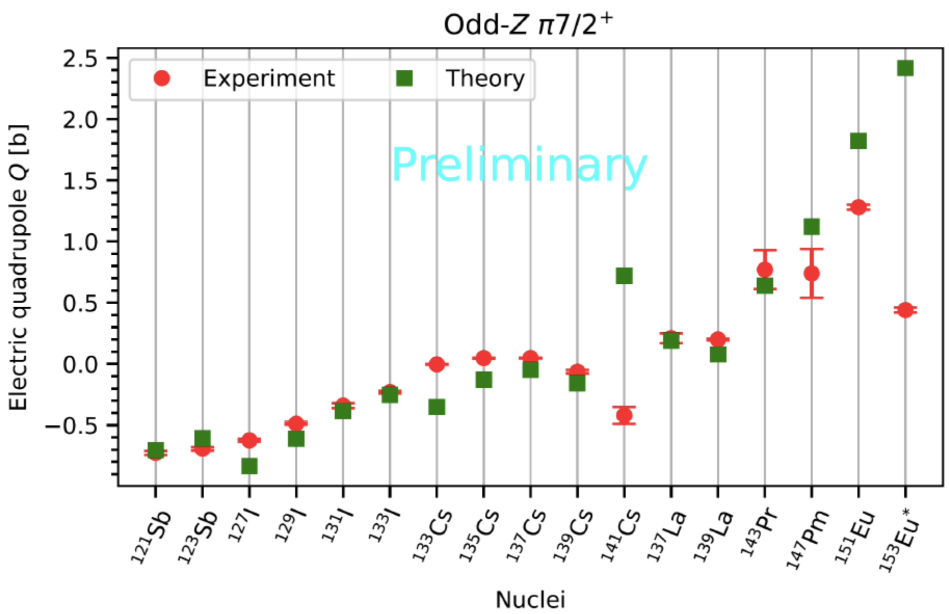
1. Sn-Gd
2. In
3. Ag
4. Sn
5. Sb

Nuclear-DFT analysis of electromagnetic moments between the Sn and Gd isotopes



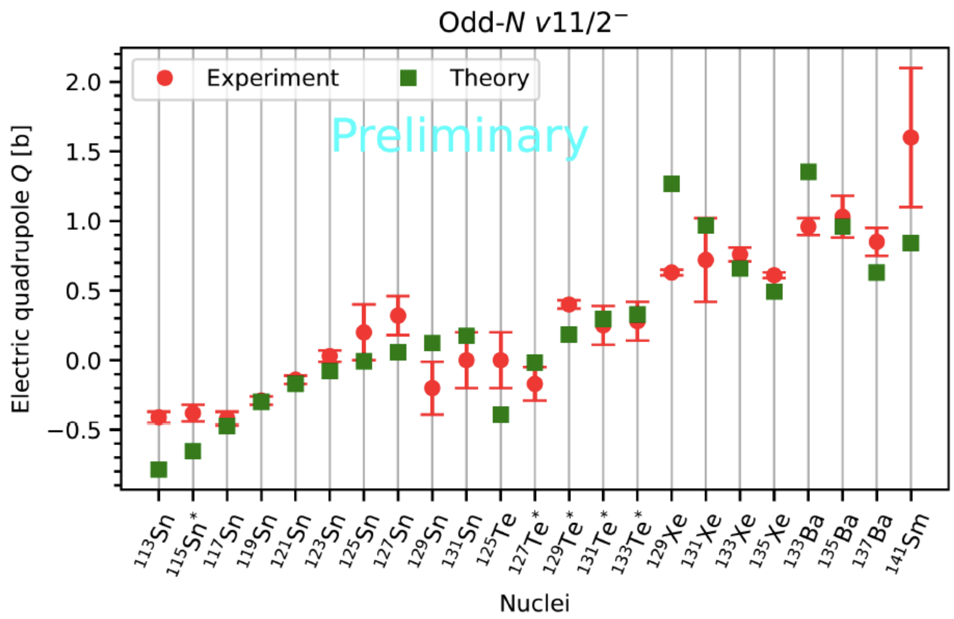
H. Wibowo *et al.*, to be published

Quadrupole moments: theory vs. experiment



N. J. Stone, *Table of nuclear magnetic dipole and electric quadrupole moments* (2014), INDC, report INDC(NDS)-0658

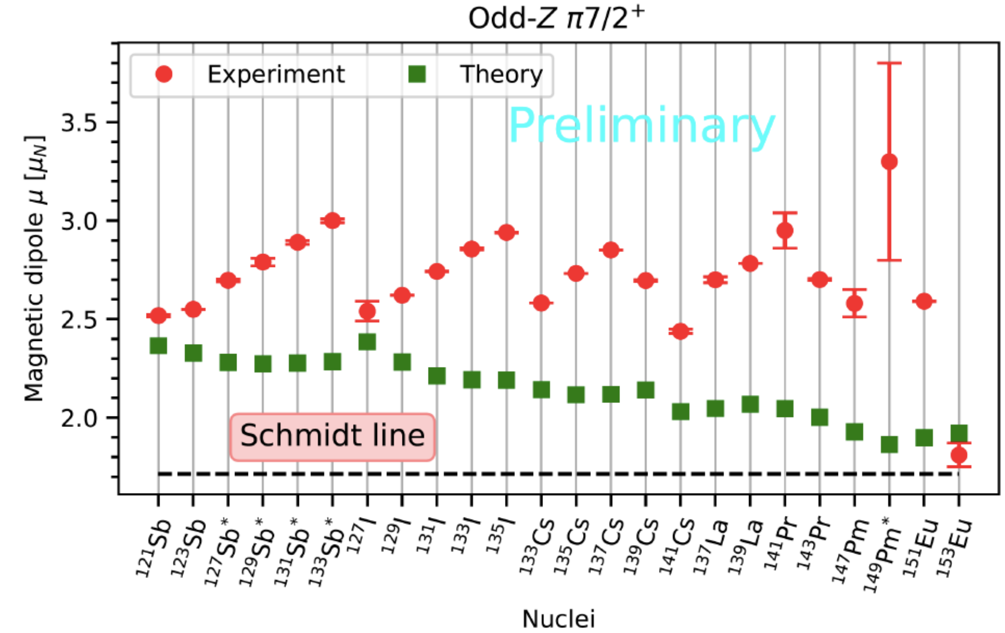
N. J. Stone, *Table of nuclear electric quadrupole moments, ADNDT 111-112*, 1 (2016)



Picture: courtesy H. Wibowo

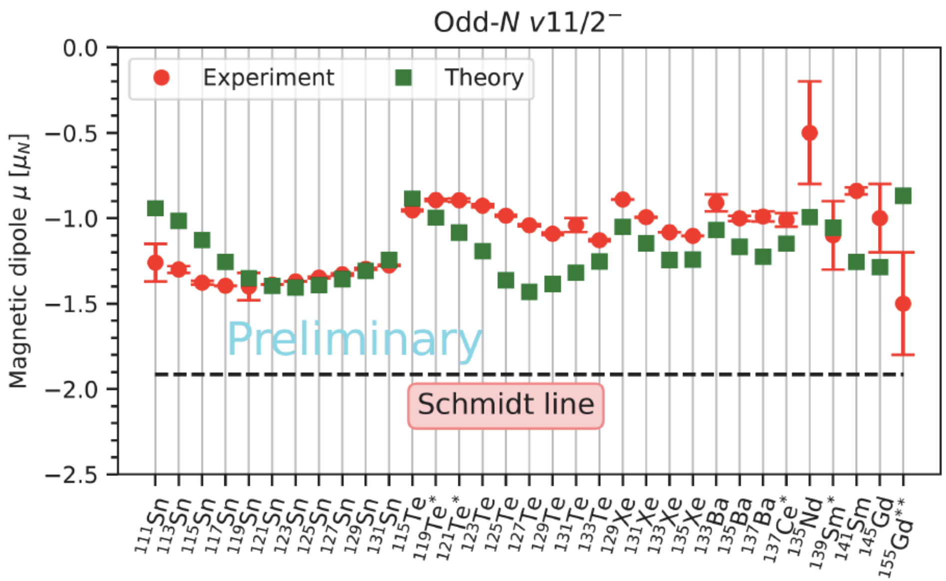
H. Wibowo *et al.*, to be published

Magnetic dipole moments: theory vs. experiment



N. J. Stone, *Table of nuclear magnetic dipole and electric quadrupole moments* (2014), INDC, report INDC(NDS)-0658

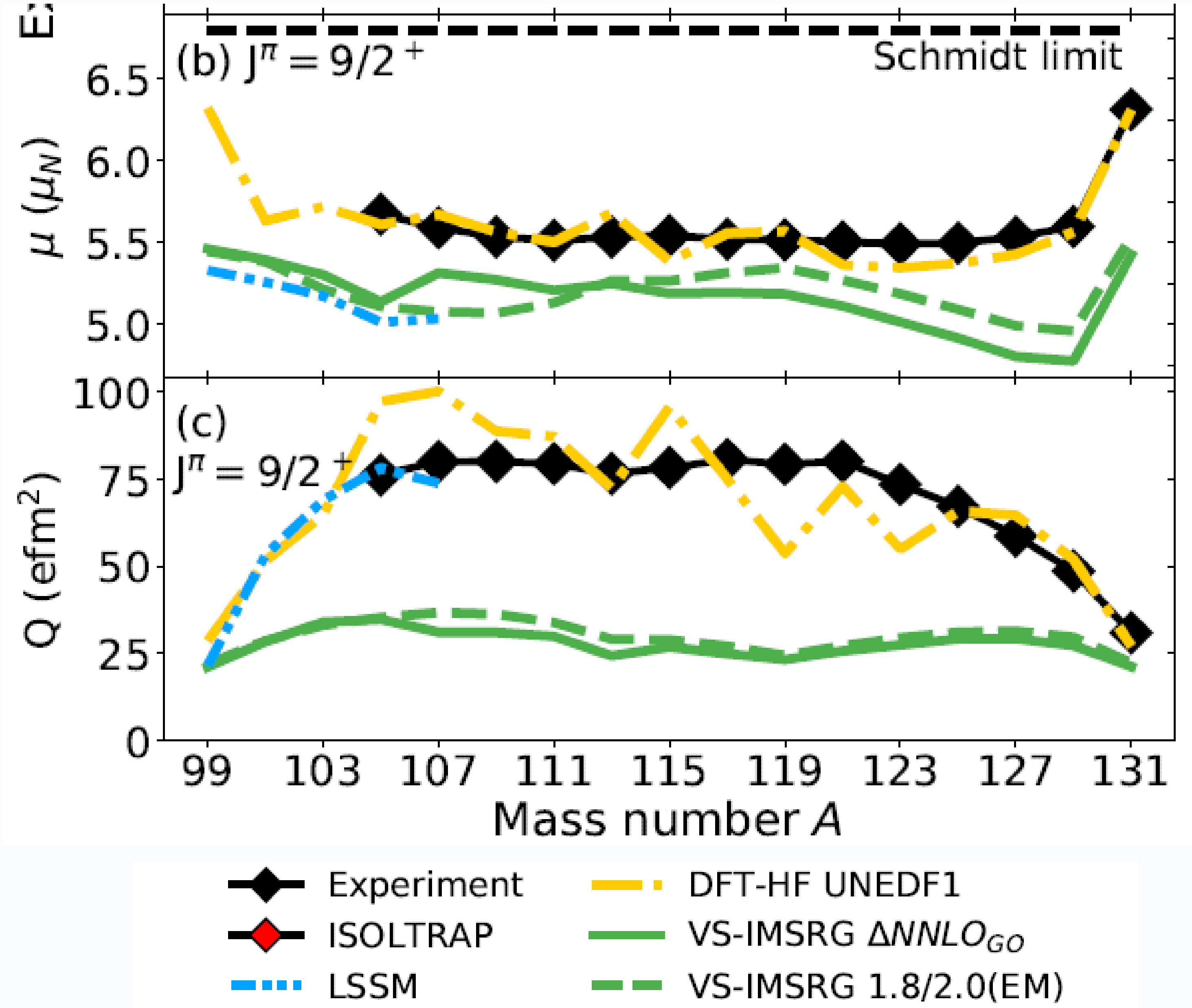
Schmidt lines represent the value of magnetic dipole moment of an odd-mass nucleus which is completely determined by the ℓ and j values of the unpaired nucleon (single-particle model).



Picture: courtesy H. Wibowo

H. Wibowo *et al.*, to be published

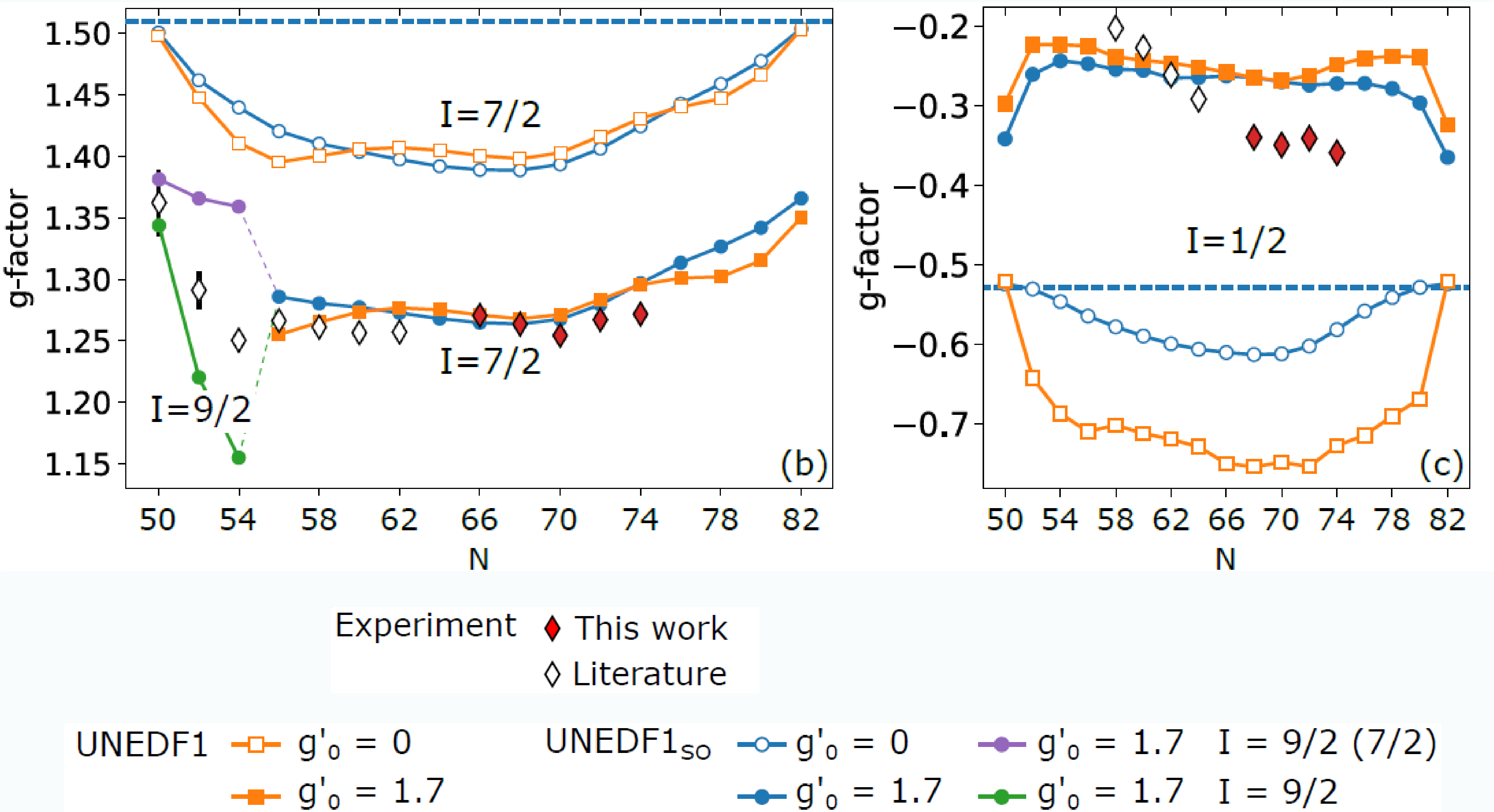
Moments of the 9/2 states in In



A.R. Vernon *et al.*, Nature 607, 260 (2022)

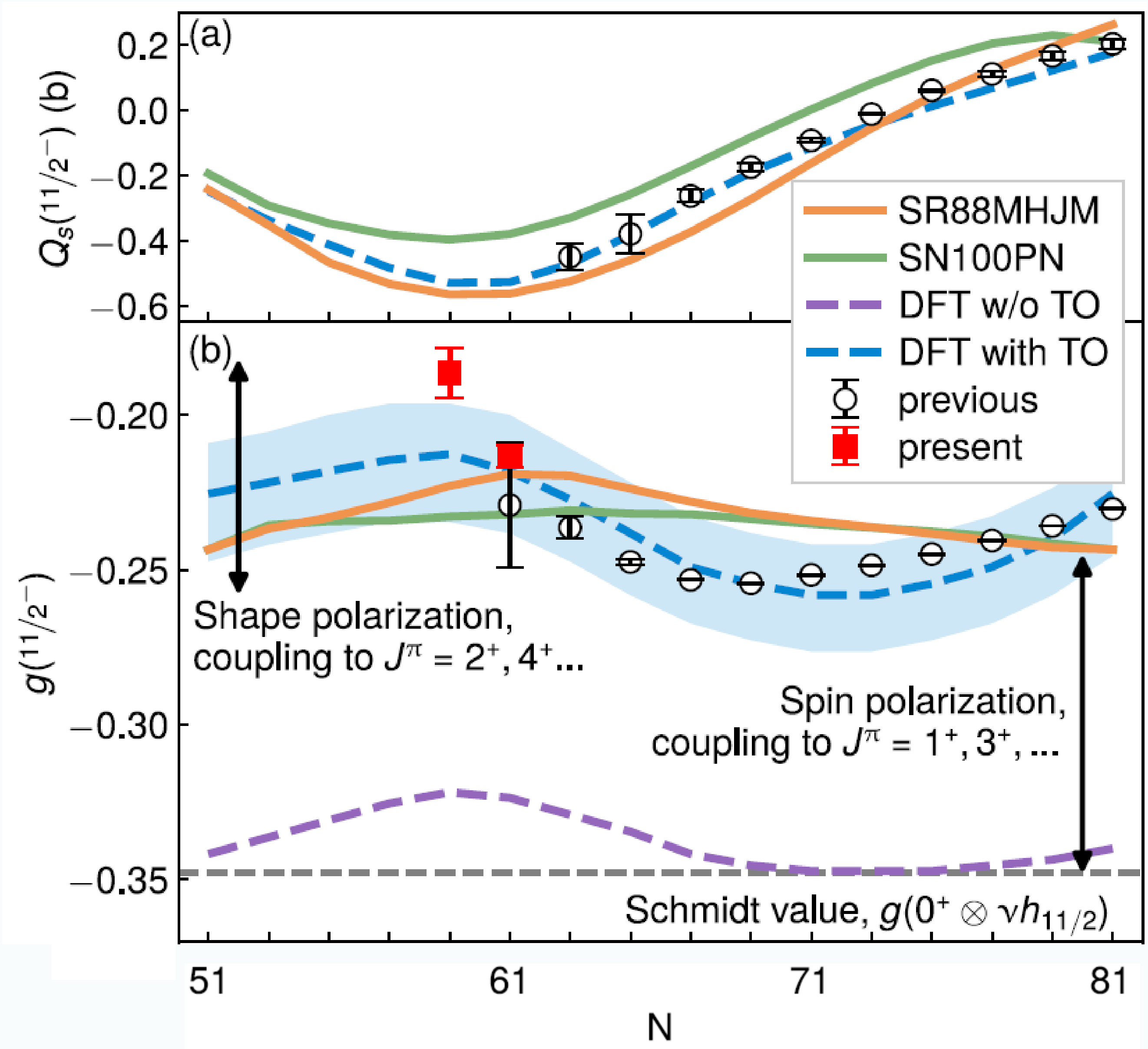
L. Nies *et al.*, Phys. Rev. Lett. 131, 022502 (2023)

Moments of the 1/2, 7/2 & 9/2 states in Ag



R. P. de Groote *et al.*, submitted to Phys. Lett. B

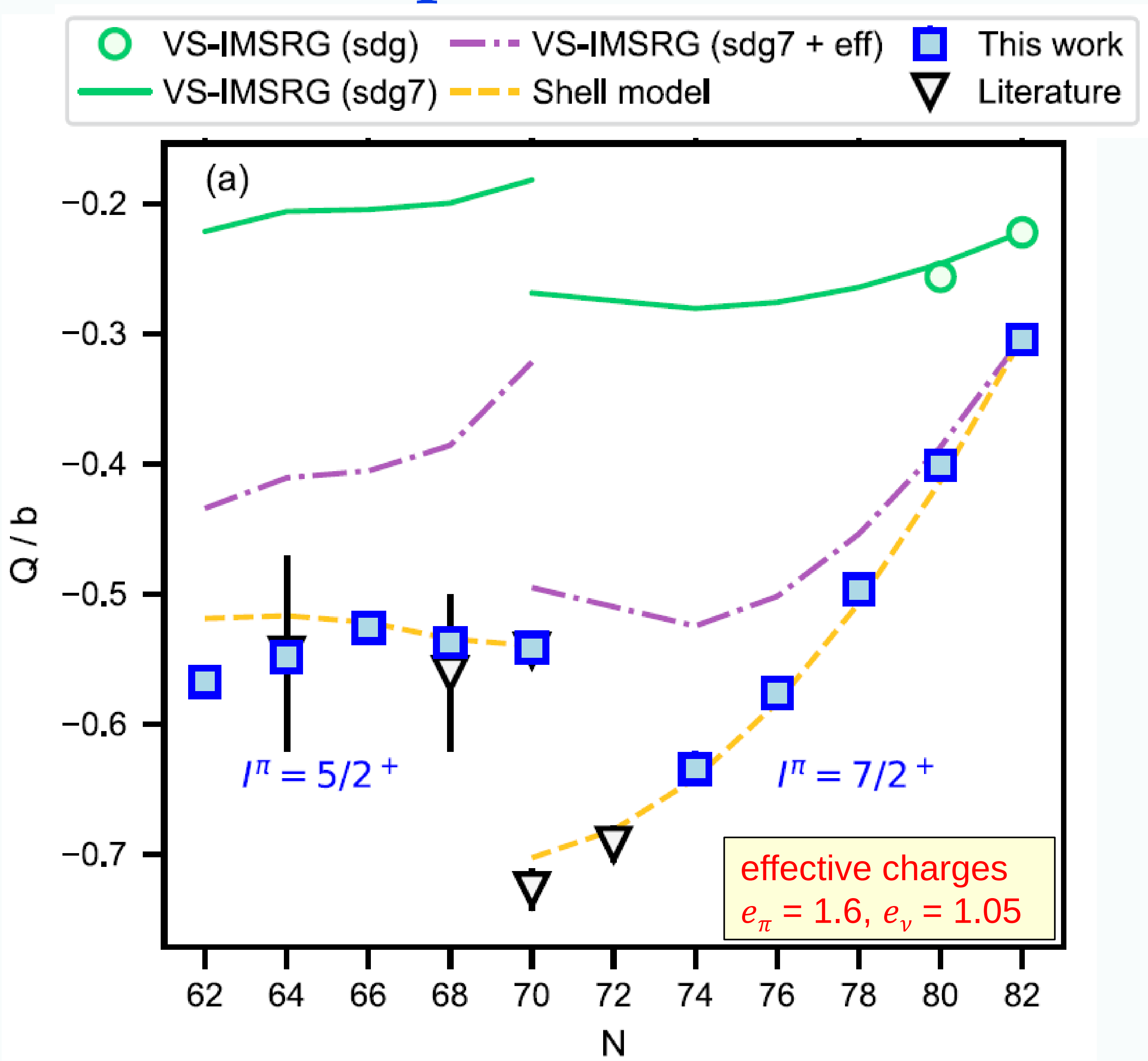
Moments of the $\nu h_{11/2}$ isomers in Sn



g = μ/I

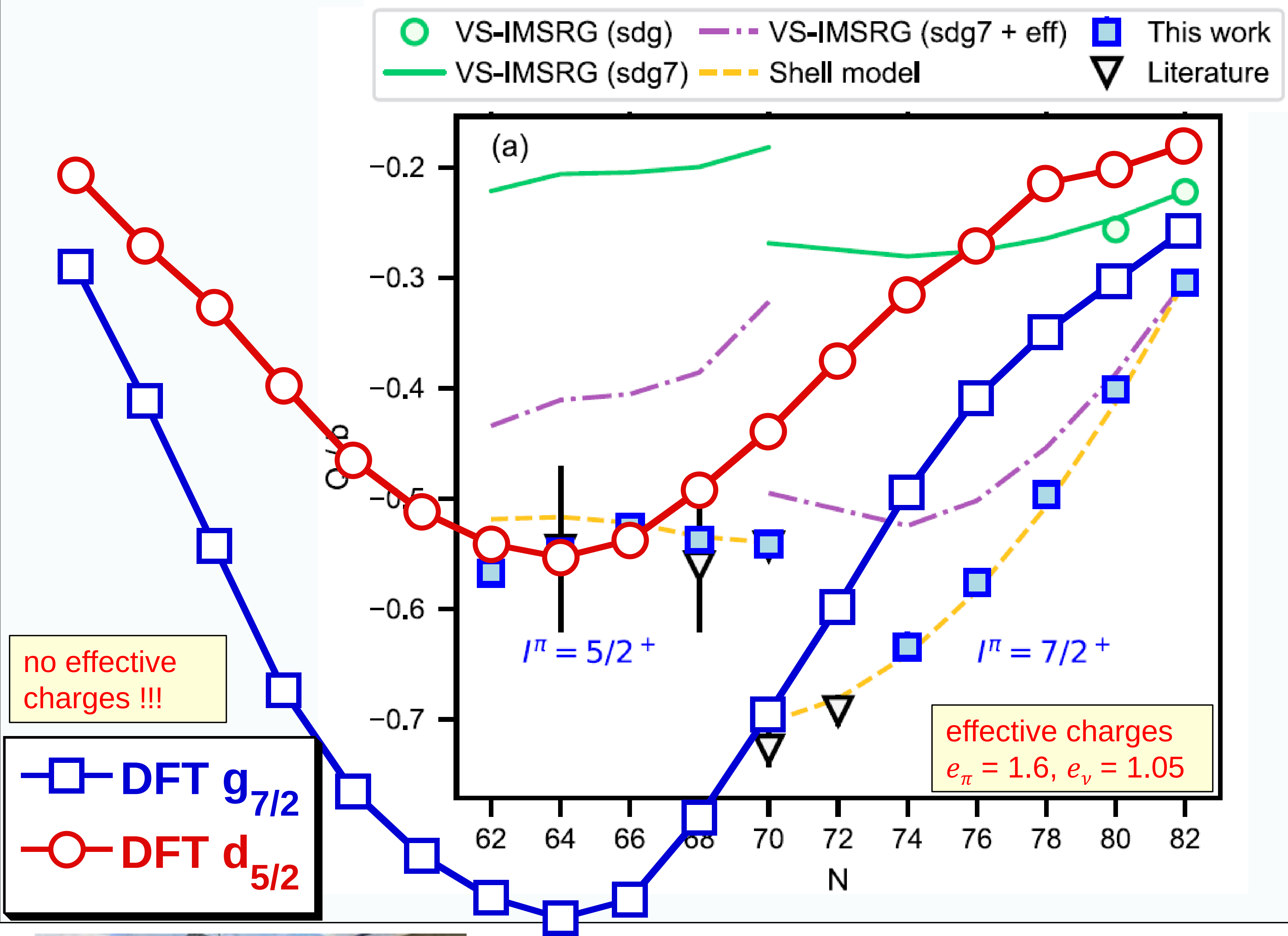
T.J. Gray *et al.*, Phys. Lett. B 847 (2023) 138268

Quadrupole moments in Sb



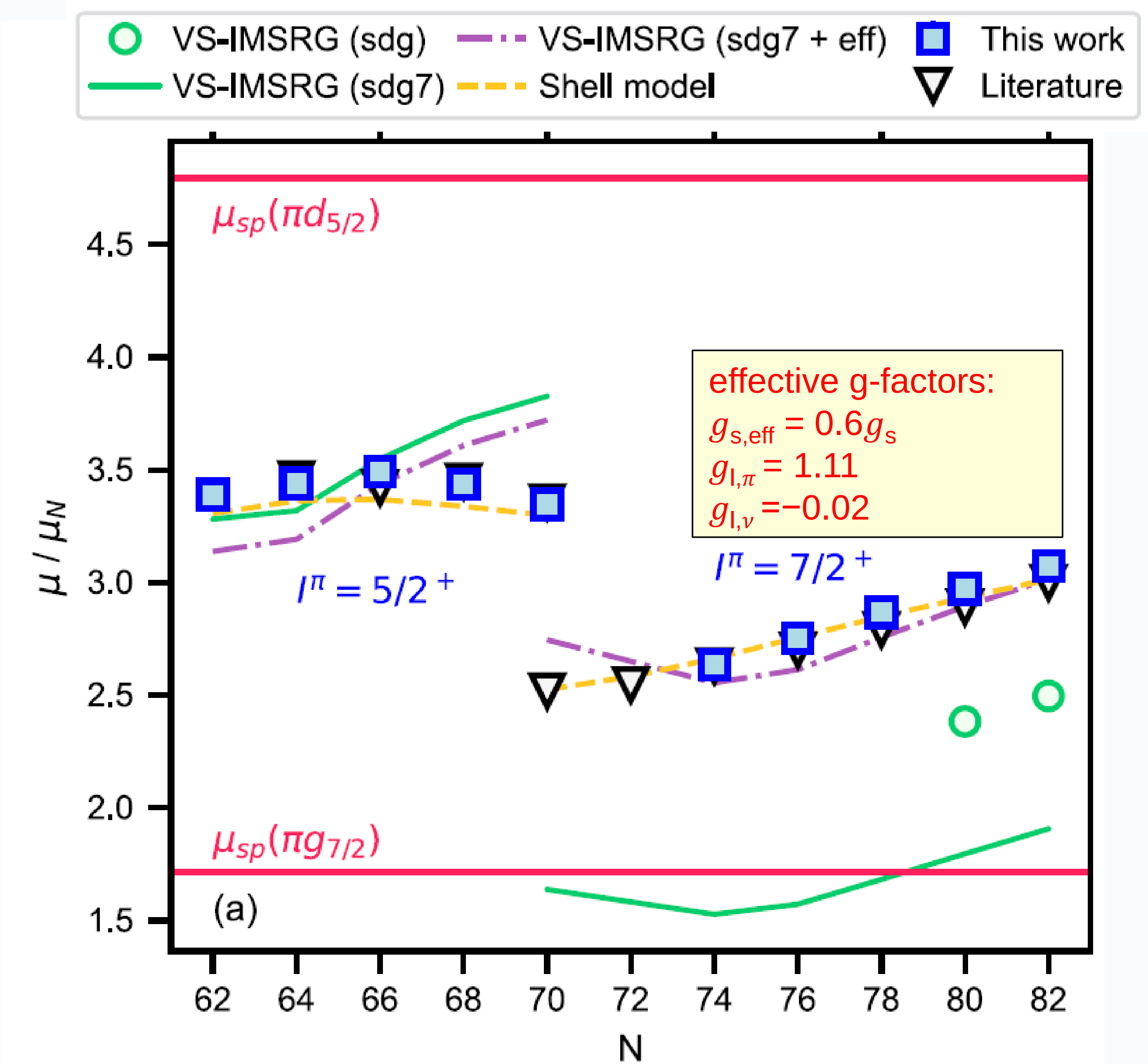
S. Lechner *et al.*, Phys. Lett. B 847 (2023) 138278

Quadrupole moments in Sb



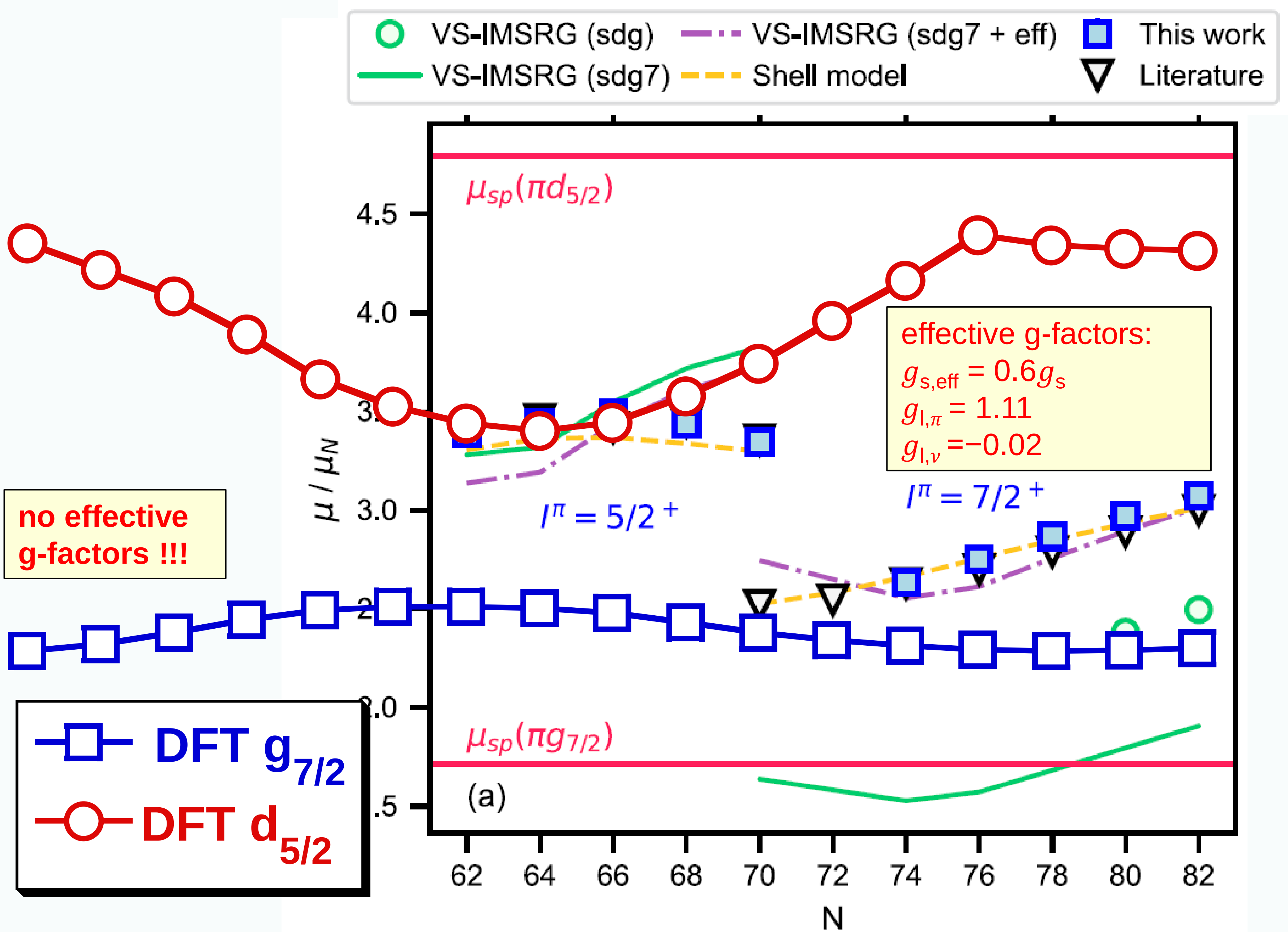
S. Lechner *et al.*, Phys. Lett. B 847 (2023) 138278

Magnetic dipole moments in Sb



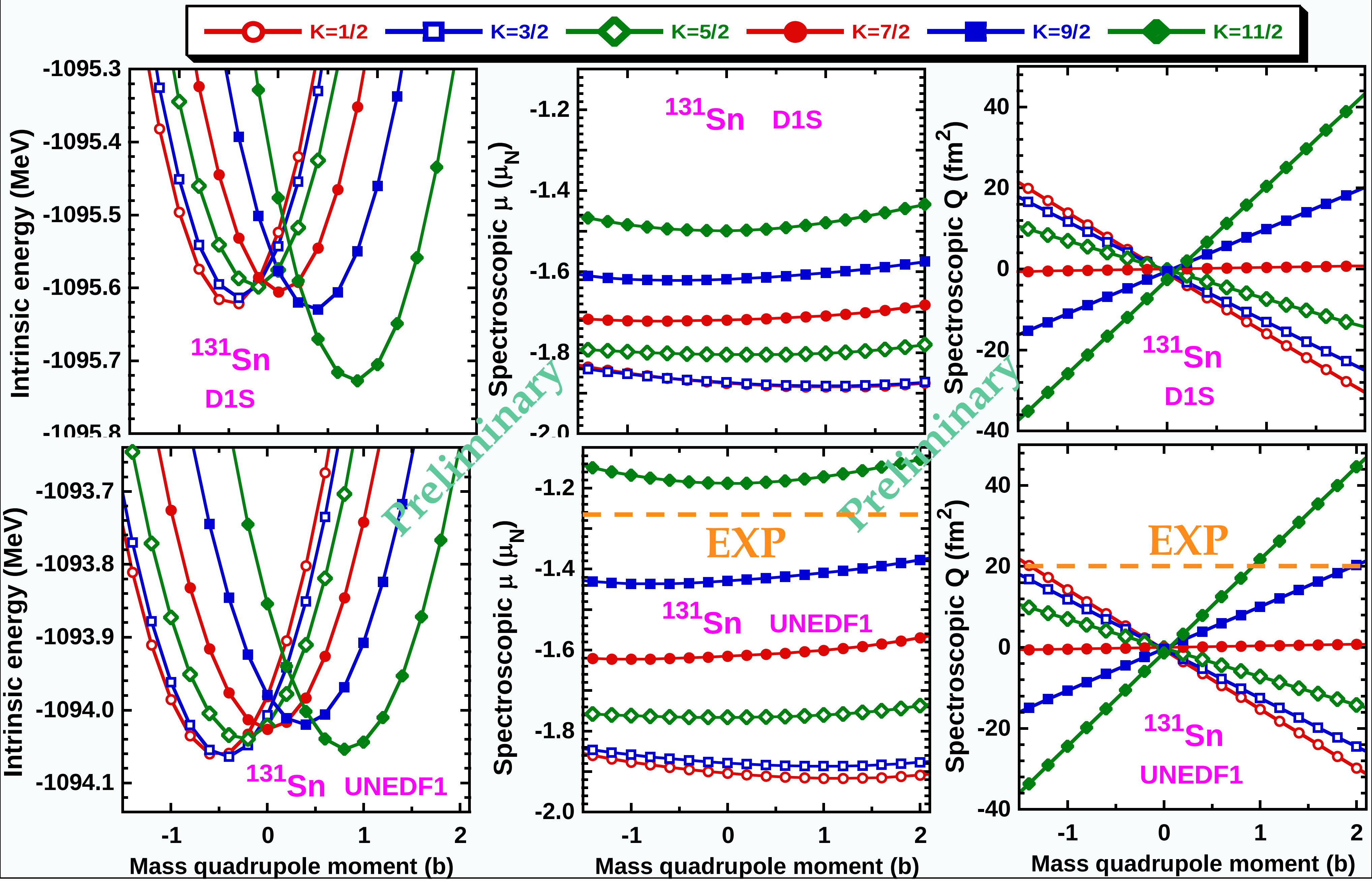
S. Lechner *et al.*, Phys. Lett. B 847 (2023) 138278

Magnetic dipole moments in Sb



S. Lechner *et al.*, Phys. Lett. B 847 (2023) 138278

K-mixing



K=1/2
K=3/2
K=5/2
K=7/2
K=9/2
K=11/2
Intrinsic energy (MeV)
-1095.3
-1095.4
-1095.5
-1095.6
-1095.7
-1095.8
131Sn
D1S
Spectroscopic μ (μN)
-1.2
-1.4
-1.6
-1.8
-2.0
131Sn D1S
Spectroscopic Q (fm2)
40
20
0
-20
-40
131Sn
D1S
Intrinsic energy (MeV)
-1093.7
-1093.8
-1093.9
-1094.0
-1094.1
131Sn UNEDF1
EXP
131Sn UNEDF1
EXP
131Sn
UNEDF1
-1
0
1
2
Mass quadrupole moment (b)
Preliminary
Preliminary

Energies of the K-mixed states

```
*****************************************************************************
*                                                                           *
*                  RESULTS OF THE MULTI-REFERENCE CALCULATION               *
*                                                                           *
*****************************************************************************
*  SPIN  N   EIG_OVERLAP   EIG_ENERGY  |                                    *
*  ----  -   -----------   ----------  |                                    *
*  11/2  1   5.979982E+00 -1092.439526 |                                    *
*  11/2  2   1.575148E-02 -1083.008302 |                                    *
*  11/2  3   2.225877E-03 -1080.289292 |                                    *
*  11/2  4   1.132094E-03 -1078.423819 |                                    *
*  11/2  5   6.412910E-04 -1069.869511 |                                    *
*  11/2  6   2.674966E-04 -1067.359363 |                                    *
*****************************************************************************

E_intrinsic(11/2) = -1092.055162
E_projected(11/2) = -1092.310241
```

Preliminary

What’s next to consider	
Segré chart of electromagnetic moments	
Electromagnetic moments of odd-odd nuclei	★
More advanced functionals	★
Octupole deformation	
Triaxiality	★
Configuration interaction	
K-mixing	★★
Quadrupole/octupole collectivity	
Two-body meson-exchange currents	★★

Thank you