Nuclear structure calculations as input for the simulation of relativistic heavy-ion collisions

Benjamin Bally

ESNT workshop - Saclay - 24/11/2023

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  - Provide Woods-Saxon (WS) parameters fitted to our calculations

    Bally *et al.*, PRL 128, 082301 (2022)

    Bally *et al.*, EPJA 59, 58 (2023)

    Ryssens *et al.*, PRL 130, 212302 (2023)

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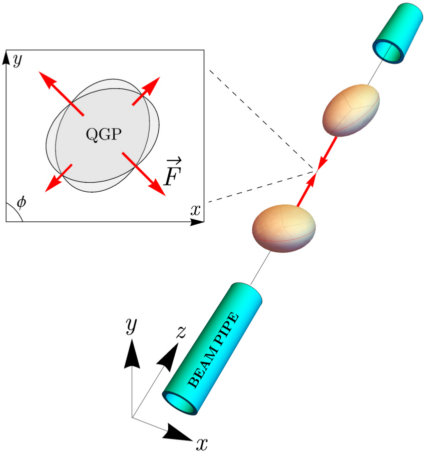
- Species collided: ^{16}O, ^{96}Zr, ^{96}Ru, ^{129}Xe, ^{197}Au, ^{208}Pb, ^{238}U
  $\Rightarrow$ Only EDF can access them all

- Attempt in the other direction: neutron skin of ^{208}Pb

  Giacalone *et al.*, PRL 131, 202302 (2023)

Heavy-ion collisions phenomenology



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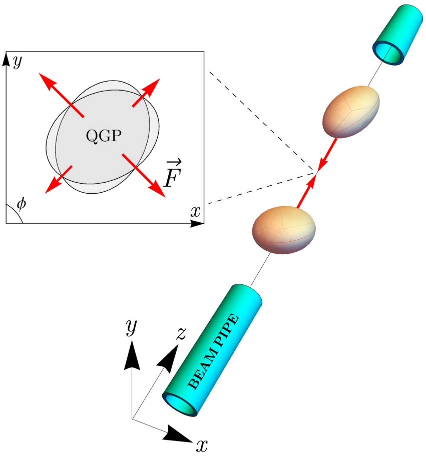

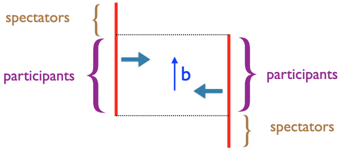

Ollitrault, EPJA 59, 236 (2023)

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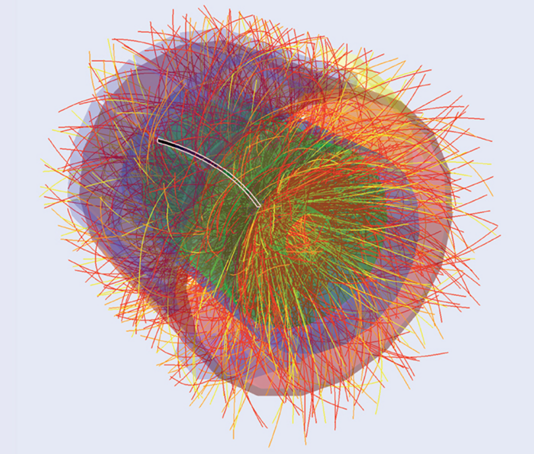

credit: CERN

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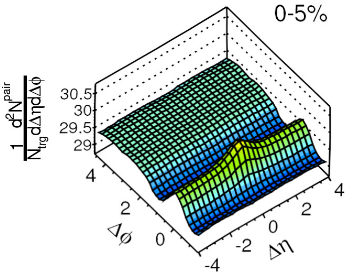
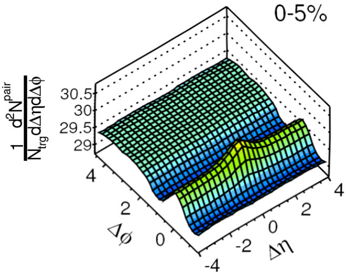


credit: CMS



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- Probability distribution of particle emission

$$P(\phi,\eta) = P(\phi) = \frac{1}{2\pi}\sum_{n=-\infty}^{+\infty} V_n e^{-in\phi}$$

$$V_2 \equiv \text{elliptic flow}$$

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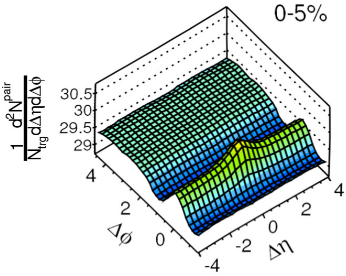

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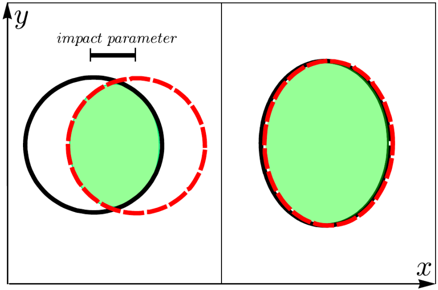
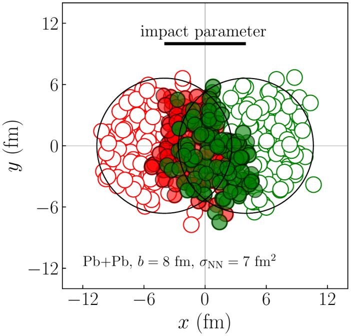
- Average of the pair distribution

$$\left\langle \frac{dN_{\text{pair}}}{d\eta_1 d\eta_2 d\Delta\phi} \right\rangle = \frac{1}{2\pi}\left(1 + 2\sum_{n=1}^{+\infty} \langle |V_n|^2 \rangle \cos(n\Delta\phi)\right)$$

- Anisotropy of particle emission linked to asymmetry of initial condition



Courtesy of V. Somà



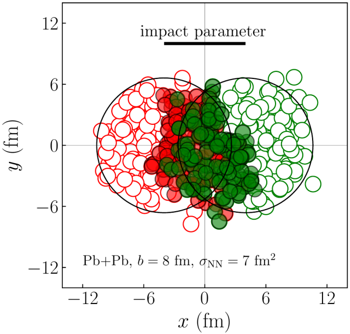
Courtesy of G. Giacalone

- Usually, nucleons sampled using Woods-Saxon density

$$\rho^{WS}(r,\theta,\varphi) = \frac{\rho_0}{1 + \exp\{[r - R(\theta,\varphi)]/a\}}$$

$$R(\theta,\varphi) = R_0 \left\{ 1 + \sum_l \sum_{m=-l}^{l} \beta_{lm}^{WS} Y_{lm}(\theta,\varphi) \right\}$$

Modeling of the nuclei: Monte-Carlo Glauber



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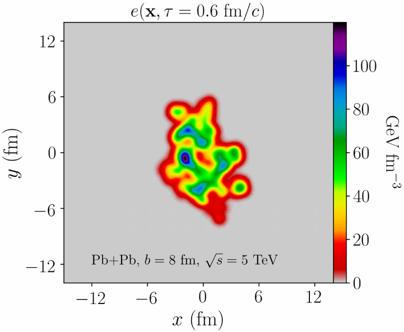
- Better: use directly the densities (e.g. uncorrelated one-body)

Energy deposition: TRENTo model



Courtesy of G. Giacalone

- Nucleons translated into energy density using TRENTo model

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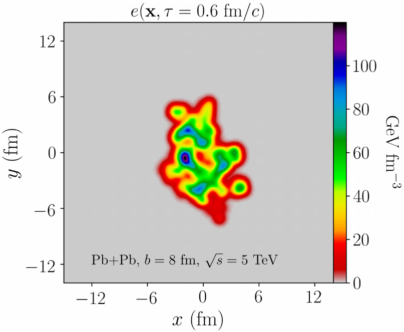
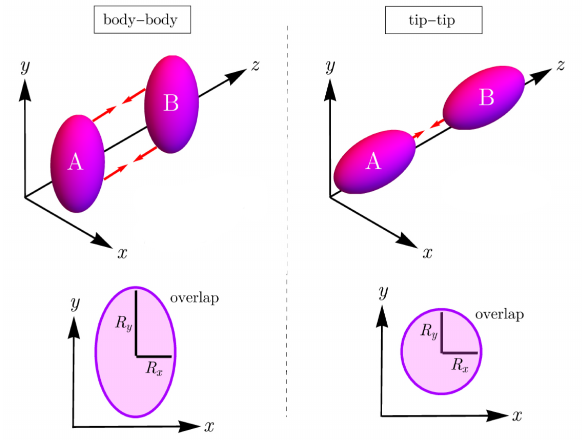


Courtesy of G. Giacalone

- Nucleons translated into energy density using TRENTo model
- Then, you should run relativistic hydrodynamic simulation

How to discern orientations in central collisions?



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Relation between the size of the system and $\langle p_t \rangle$

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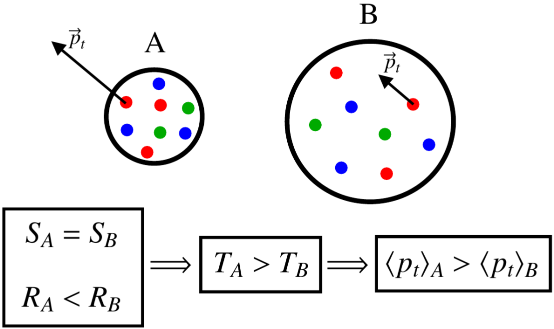

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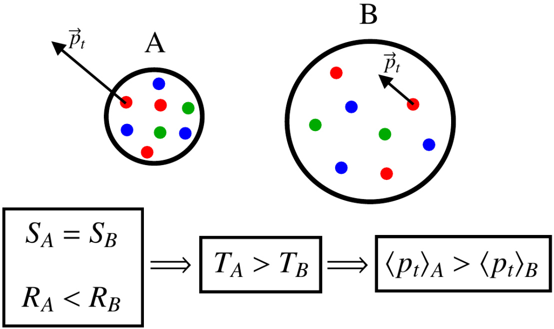

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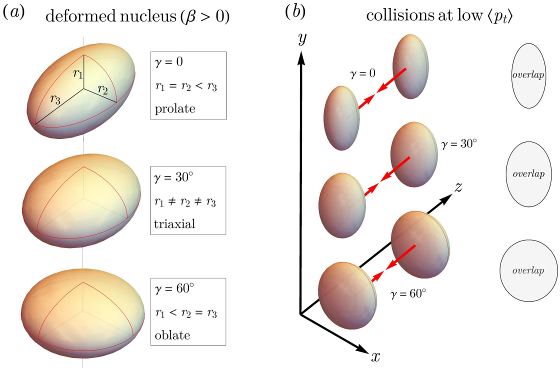
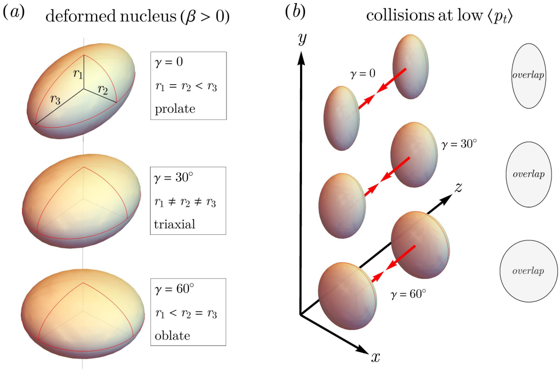
- $\langle p_t \rangle$ is the average transverse momentum of particles
- Looking at low $\langle p_t \rangle$ $\Rightarrow$ looking at larger nuclear overlaps

Orientation at low $\langle p_t \rangle$





- Pearson correlation coefficient

$$\rho(v_2^2, \langle p_t \rangle) = \frac{\left\langle \delta v_2^2 \delta \langle p_t \rangle \right\rangle}{\sqrt{\left\langle \left(\delta v_2^2\right)^2 \right\rangle \left\langle \left(\delta \langle p_t \rangle\right)^2 \right\rangle}}$$

where $\delta o = o - \langle o \rangle$

Calculations for ^{129}Xe and ^{208}Pb

- State-of-the-art multi-reference energy density functional calculations
- Variational calculations

$$\delta \frac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle} = 0 \quad \text{with} \quad |\Psi\rangle = \sum_{(\beta_v,\gamma_v)K} f_{(\beta_v,\gamma_v)K} P^J_{MK} P^N P^Z |\Phi(\beta_v,\gamma_v)\rangle$$

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- Explore triaxial deformations (β_v, γ_v) through constrained minimizations

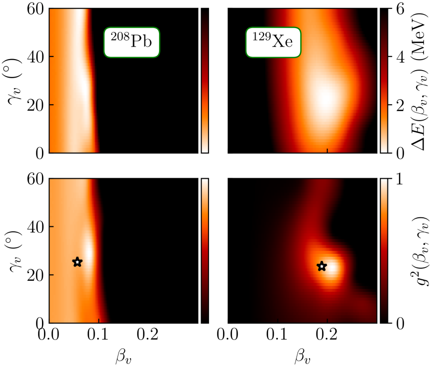
- Calculations performed on a 3d Cartesian mesh

- Skyrme-type SLyMR1 parametrization

  Sadoudi *et al.*, Phys. Rev. C 88, 064326 (2013)

  Jodon, PhD thesis, Université Lyon 1 (2014)

Modeling of the nuclei: inputs from nuclear theory



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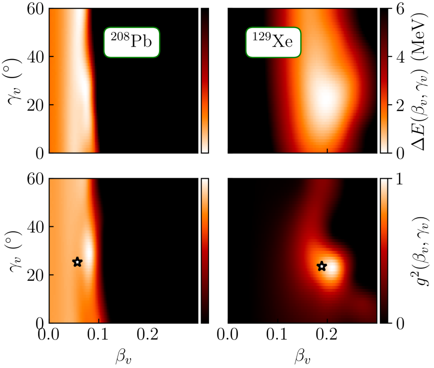

- Strategy

$$\begin{aligned}\bar{\beta}_v &= \textstyle\sum_{(\beta_v,\gamma_v)} \beta_v \, g^2(\beta_v,\gamma_v) \\ \bar{\gamma}_v &= \textstyle\sum_{(\beta_v,\gamma_v)} \gamma_v \, g^2(\beta_v,\gamma_v)\end{aligned} \rightarrow \text{ build } |\Phi(\bar{\beta}_v,\bar{\gamma}_v)\rangle \rightarrow \text{ fit WS parameters}$$

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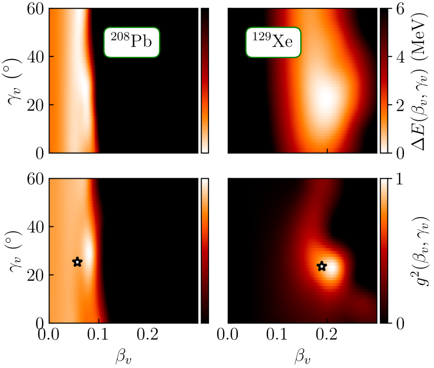

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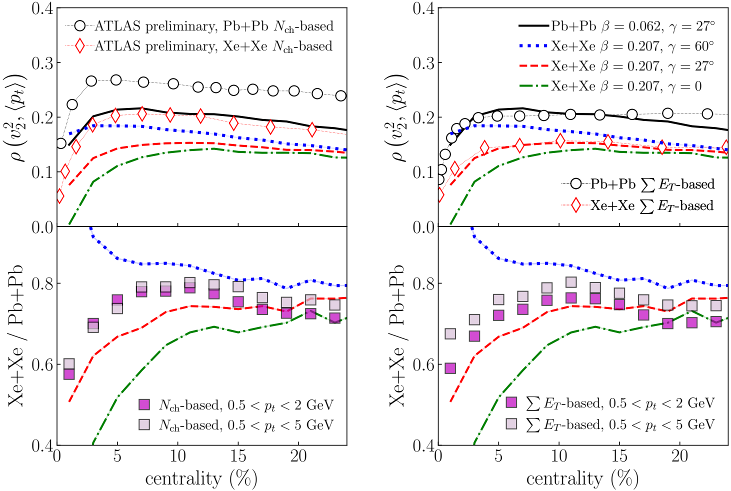
- Results for ^{208}Pb: $a = 0.537$ fm, $R_0 = 6.647$ fm, $\beta = 0.062$, $\gamma = 27.04°$
- Results for ^{129}Xe: $a = 0.492$ fm, $R_0 = 5.601$ fm, $\beta = 0.207$, $\gamma = 26.93°$

Analysis of LHC data



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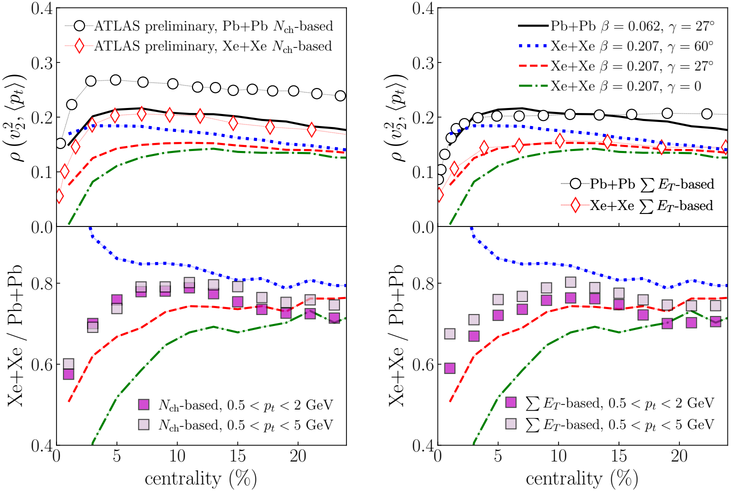
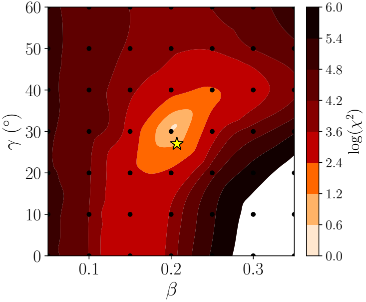
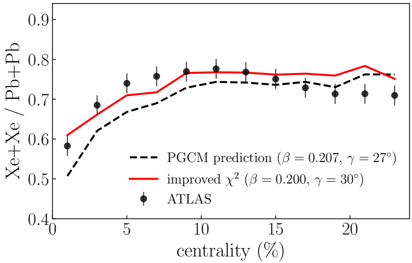


- Only triaxiality explains LHC results!

Best fit of LHC data

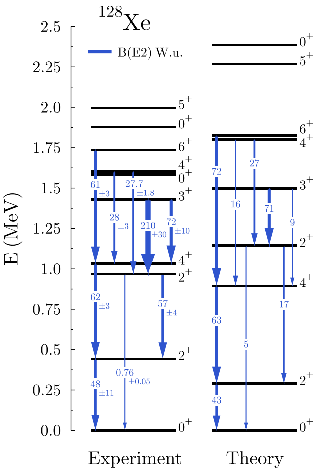




- Simulation of 2×10^7 collisions at 50 deformations (β, γ)
- Best fit obtained for: $(\beta \approx 0.20, \gamma \approx 30^\circ)$

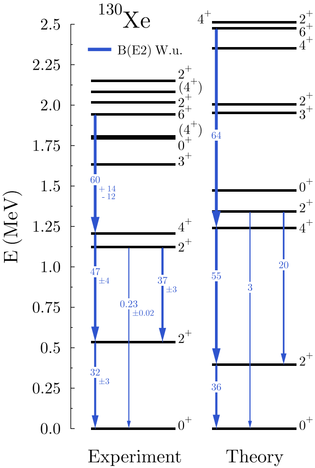
Comparison for ^{128}Xe

Quantity	Theory	Experiment
β_r	0.19	0.20(2)
γ_d	19°	27°
β_c	0.22	
γ_c	21°	
β_k	0.20	≈ 0.21
$\Delta\beta_k$	0.02	
γ_k	39°	≈ 20°?
$\Delta\gamma_k$	21°	



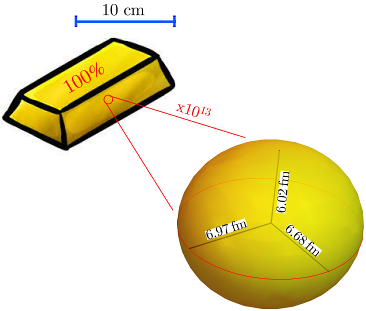
Bally *et al.*, EPJA 59, 58 (2023)

Quantity	Theory	Experiment
β_r	0.18	0.17(1)
γ_d	21°	28°
β_c	0.19	
γ_c	23°	
β_k	0.16	0.17(2)
$\Delta\beta_k$	0.02	
γ_k	28°	23(5)°
$\Delta\gamma_k$	12°	



Bally *et al.*, EPJA 59, 58 (2023)

- Extensive program ^{197}Au+^{197}Au at RHIC
- Similar strategy
  - SLyMR1
  - PGCM with triaxial 1qp
  - Determine "average" deformation
- Give WS parameters for ρ_p^{WS}, ρ_n^{WS}, ρ_a^{WS}



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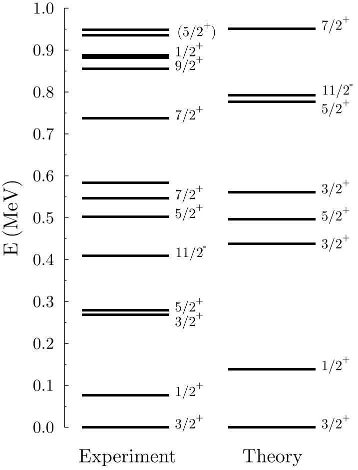
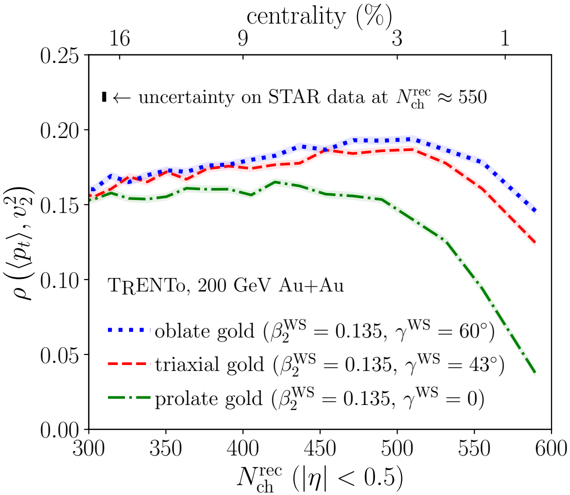
Calculations for ^{197}Au: structure

Quantity	Experiment	Theory
$E(3/2^+_1)$	-1559.384	-1556.044
$r_{\rm rms}(3/2^+_1)$	5.4371(38)	5.389
$\mu(1/2^+_1)$	+0.416(3)	+0.01
$\mu(3/2^+_1)$	+0.1452(2)	-0.38
$\mu(5/2^+_1)$	+0.74(6)	+0.15
$\mu(5/2^+_2)$	+3.0(5)	+0.14
$\mu(7/2^+_1)$	+0.84(7)	+0.51
$\mu(9/2^+_1)$	+1.5(5)	+0.81
$\mu(11/2^-_1)$	(+)5.96(9)	+6.87
$Q_s(3/2^+_1)$	+0.547(16)	+0.65
$Q_s(11/2^-_1)$	+1.68(5)	+2.05

Table: E (MeV), $r_{\rm rms}$ (fm), μ (μ_N), Q_s (eb).



Calculations for ^{197}Au: $\rho(v_2^2, \langle p_t \rangle)$



Calculations for ^{197}Au: neutron skin

- Experiment ("tomography") with ultrarelativistic nuclei
  STAR collab., Science Adv. 9, eabq390 (2023)

  $\Delta r_{np}[\mathrm{STAR}] = 0.17 \pm 0.03\ (\mathrm{stat.}) \pm 0.08\ (\mathrm{syst.})\ \mathrm{fm}$

- Theoretical results

  $\Delta r_{np}[\mathrm{MREDF}] = 0.17\ \mathrm{fm}$

  $\Delta r_{np}[\mathrm{WS\ fit}] = 0.19\ \mathrm{fm}$

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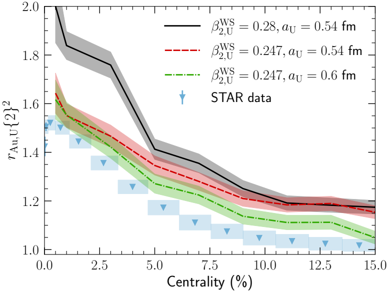
  $\Delta r_{np}[\mathrm{WS\ fit}] = 0.19\ \mathrm{fm}$

- Is this accidental?
  (they use spherical matter WS)

- $\beta_{lm}^{WS} \neq \beta_{lm} \propto \langle Q_{lm} \rangle$
- Consistent treatment → no discrepancy
- Importance of hexadecapole deformations



Ryssens *et al.*, PRL 130, 212302 (2023)

^{16}O and ^{20}Ne with chiral interactions

- Model space
  - $e_{\text{max}} = 6$
  - $e_{3\text{max}} = 18$
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    Hüther *et al.*, PLB 808, 135651 (2019)
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  - real general Bogoliubov quasi-particle states
  - VAPNP minimization
  - Explore explicitly $\beta_{20}, \beta_{22}, \beta_{30}, \beta_{32}$
  - Projection on Z, N, J, M_J, π

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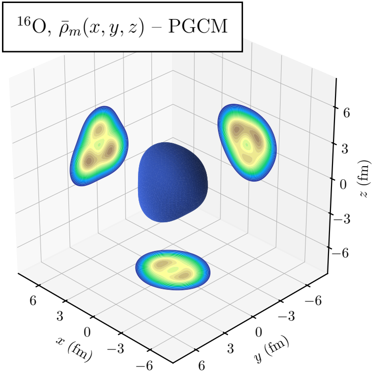
- Cost: $\approx$ 17.5M CPUh

Density at "average" deformation for ^{16}O



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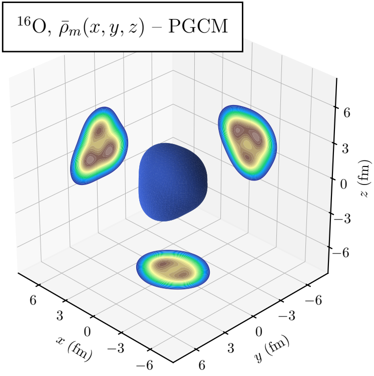
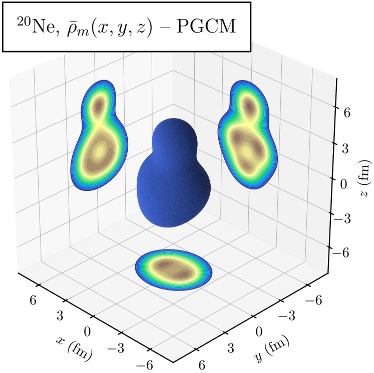
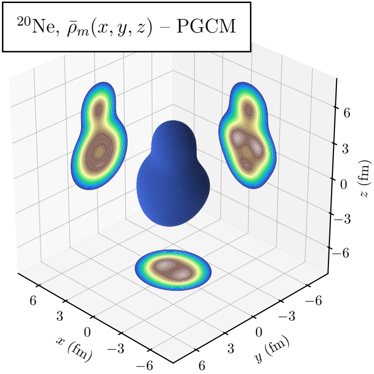


EM1.8/2.0

Density at "average" deformation for ^{20}Ne

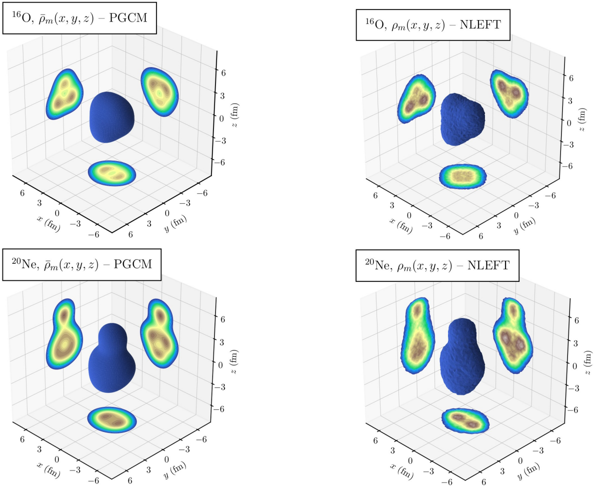


Hüther N3LO



EM1.8/2.0

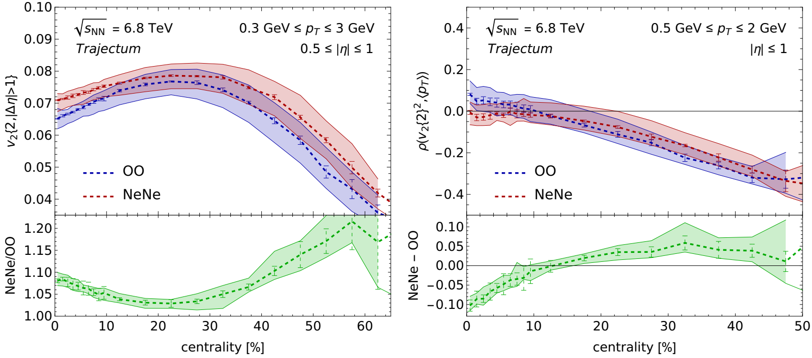
Average deformation of PGCM vs NLEFT



- Nuclear Lattice Effective Field Theory (NLEFT) with pin-hole algorithm

  Lee, Front. Phys. 8, 174 (2020)
- Agreement: effective one-body ("PGCM") and A-body (NLEFT)

Preliminary results



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- In the future, important for $e + A$ at EIC?
  Mäntysaari *et al.*, PRL 131, 062301 (2023)

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