

Towards a rigorous formulation of nuclear EDF via the FRG method

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ESNT Workshop

Nuclear density functional method : Going beyond the minefield

21.11.2023

Outline



Introduction to FRG

FRG to build EDF

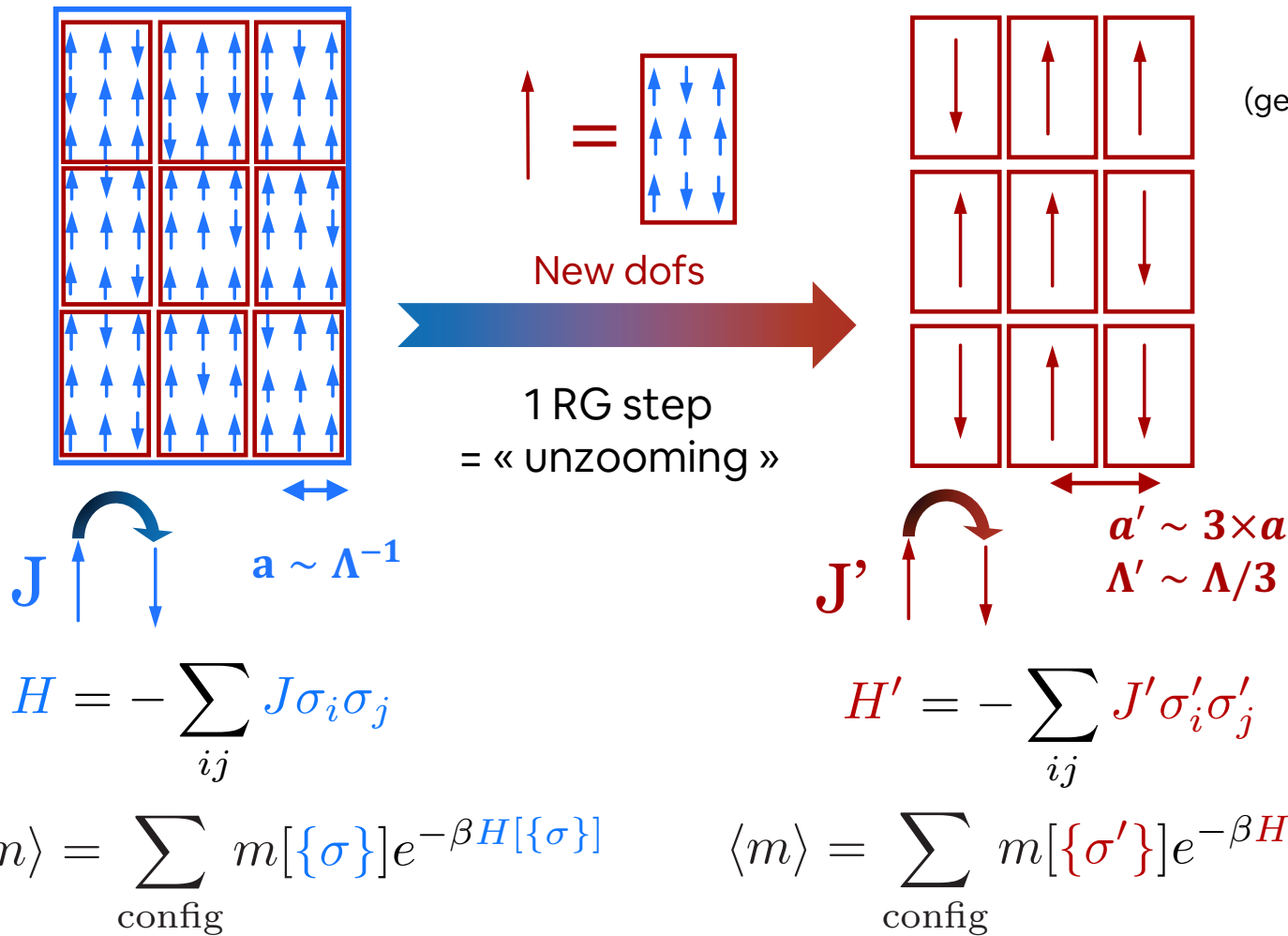
Conclusion

Outline

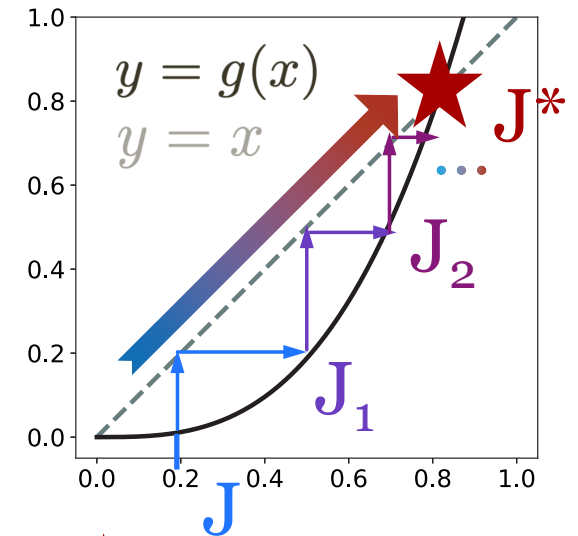


Introduction to FRG

RG : introductive example



Tricky part : Find g such that $J' = g(J)$
(generation of non-local couplings, higher order terms,...)

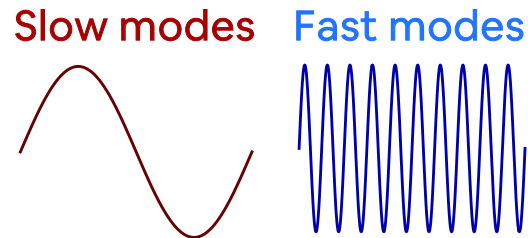


★ « Fixed point »
Whatever J ends in J^*

Can easily compute
physical observables
with H^*

Limitations of Wilsonian FRG

Wilsonian FRG



Λ momentum cutoff
 Λ^{-1} spatial resolution

$$Z = \int \mathcal{D}\phi e^{-S[\phi]} = \int \boxed{\mathcal{D}\phi_{p \leq \Lambda}} \boxed{\mathcal{D}\phi_{p > \Lambda}} e^{-S[\phi]} = \int \mathcal{D}\phi_{p \leq \Lambda} e^{-S_\Lambda[\phi_{p \leq \Lambda}]}$$

Integrating out fast modes

$$= e^{-S_\Lambda}$$

Only depends on slow
(long range) modes !

In practice : hard/impossible to compute S_Λ
 (if not perturbative)

$$S_\Lambda \sim \log \int \mathcal{D}\phi_{p > \Lambda} e^{-S[\phi]}$$

Functional Renormalisation Group

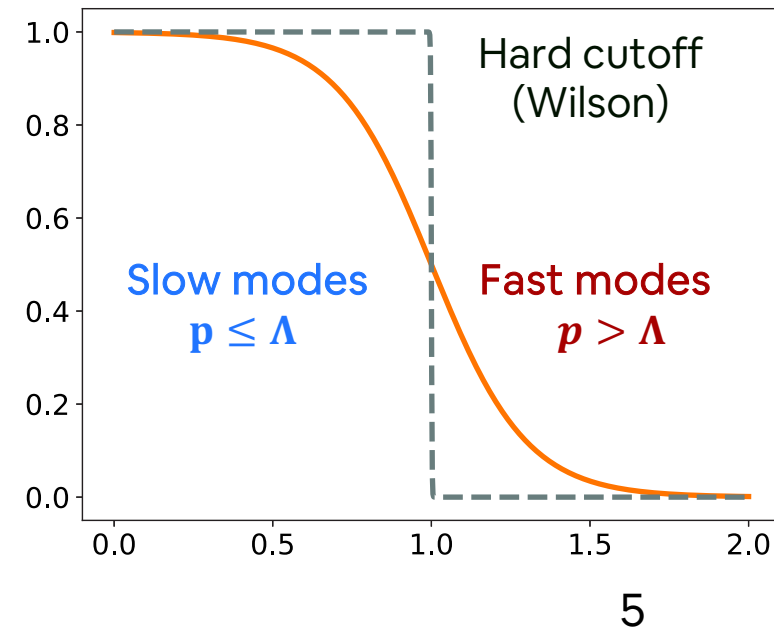
«Functional RG» = Wilson RG + ... Λ momentum cutoff
 Λ^{-1} spatial resolution

1. Soften cutoff R_Λ (regulator)
2. Action $S_\Lambda \rightarrow$ Effective Action Γ_Λ (~Legendre transform)
3. Differential equation for Γ_Λ

Wetterich
equation
(1993)

$$\Lambda \frac{\partial \Gamma_\Lambda}{\partial \Lambda} = \frac{1}{2} \text{Tr} \left[\Lambda \frac{\partial R_\Lambda}{\partial \Lambda} \left(R_\Lambda + \Gamma_\Lambda^{(1,1)} \right)^{-1} \right]$$

$R_\Lambda(p)$



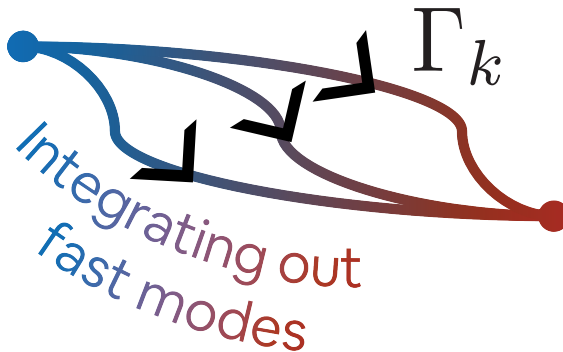
« Exact » RG : no approximation

In particular, non-perturbative

Impossible to solve exactly BUT
 reliable expansions exist

Wetterich equation

$$\Gamma_{k=\Lambda} = S$$



Fixed point
 Γ^*

Different Regulator
=
Different paths

$$\Gamma_{k=0} = \Gamma$$

$$k \frac{\partial \Gamma_k}{\partial k} = \text{Diagram: a circle with a cross inside}$$

Simple looking
1 Loop structure



$$k \frac{\partial R_k}{\partial k}$$

$$\langle \Phi_1 \dots \Phi_n \rangle_{1\text{PI}} = \frac{\delta \Gamma}{\delta^n \Phi_1 \dots \Phi_n}$$

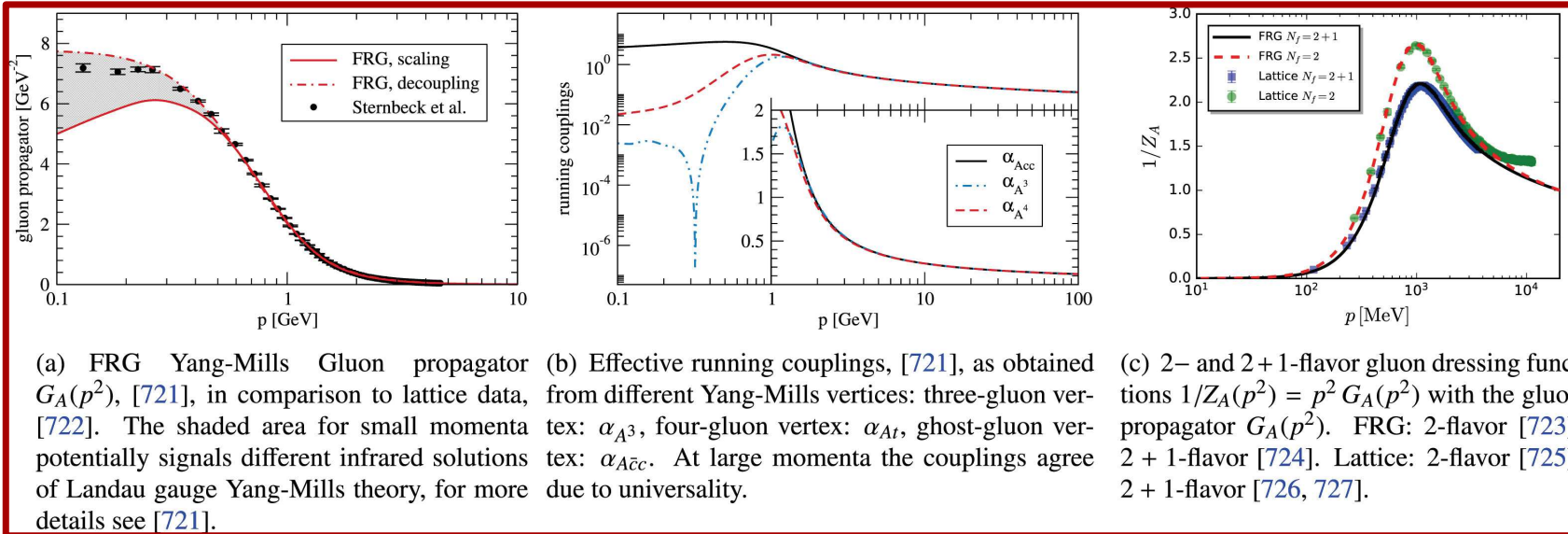
Derivative expansion : expand Γ_k in powers of $\nabla\phi$

$$\Gamma_k \sim \int d^4x U_k(\phi) + c_0 \cancel{Z_k} (\nabla\phi)^2 + c_1 \cancel{Y_k} (\nabla\phi)^4 + \dots$$

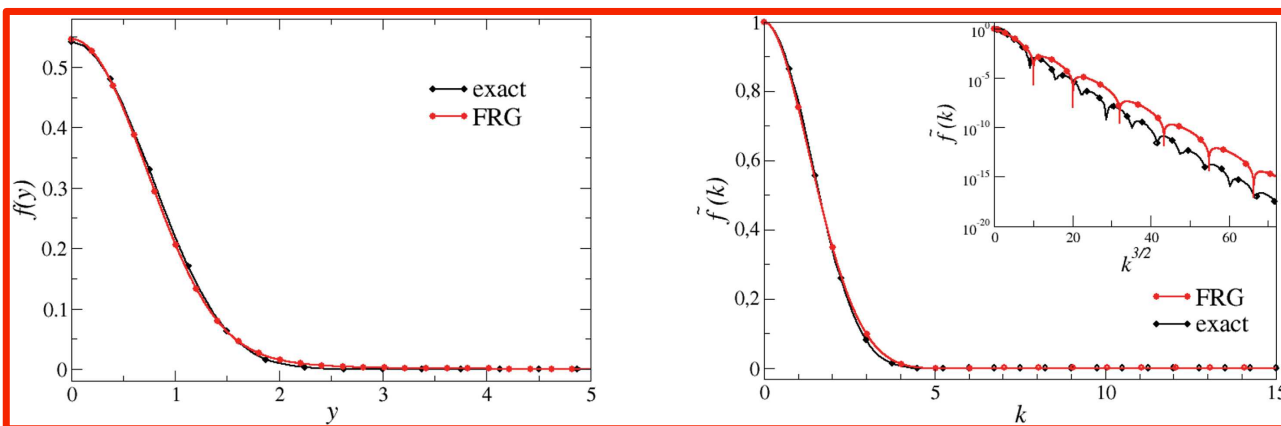
First order : **Local Potential Approximation (LPA)**

$$Z_k = 1, Y_k = 0$$

Applications of FRG



QCD:
Gluon
propagator



Turbulence:
Correlation
functions for
KPZ equation

Taken from FRG
review :
Dupuis *et al.*,
Phys. Rep. 910, 2021

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Introduction to FRG

FRG to build EDF

FRG to build nuclear EDF

Global Strategy

Free space
N-N Lagrangian



FRG flow

Effective (dressed)
N-N Lagrangian
in medium

Present work

$\mathcal{L} = \mathcal{L}_{\text{Bonn}}$

pheno. covariant Lagrangian
for convenience



First order : LPA

FRG flow

?

Benchmark
Properties of nuclear matter

Peter Ring *et al* 2023
J. Phys.: Conf. Ser. 2453 012031

Some beyond HF correlation with
Dirac-Brueckner approach

Flow equation

Local Potential Approximation :

$$\Gamma_k = \int d^4x \left[U_k(\text{mesons}) + \mathcal{L}_m + \mathcal{L}_N + \mathcal{L}_{\text{int}} - \mu \bar{\psi} \psi \right]$$



For now : fix coupling constants
(can be « easily » included)

Machleidt
1987

Freezing heaviest mesons

Heaviest cannot fluctuate :
mean-field level

$$\rho \simeq \rho_{\text{MF}}, \omega \simeq \omega_{\text{MF}}, \delta \simeq \delta_{\text{MF}}$$

$$U_k(\text{mesons}) = U_k(\pi, \sigma)$$

meson	Mass (MeV)	J^π	coupling
π	138	0^-	pseudoscalar
σ	550	0^+	scalar
ρ	769	1^-	vector
ω	783	1^-	vector
δ	983	0^+	scalar

Taylor expansion

$$U_k = \frac{1}{2} M_{\sigma,k}^2 \sigma^2 + \frac{g_{2,k}}{3} \sigma^3 + \frac{g_{3,k}}{4} \sigma^4 \frac{1}{2} + M_{\pi,k}^2 \pi^2$$

$$M_{\sigma,\Lambda}, M_{\pi,\Lambda}$$

$$g_{2,\Lambda} = g_{3,\Lambda} = 0$$

FRG flow

$$M_{\sigma,0}, M_{\pi,0}$$

$$g_{2,0}, g_{3,0}$$

FRG for Bonn Lagrangian

Bare N-N Lagrangian :

$$\mathcal{L}_{\text{Bonn,int}} = \sum_{\text{mesons}} g_m \bar{\psi} D_m \psi$$

Lesson from empirical EDF :

$$\mathcal{L}_{\text{Bonn,int}} \sim \mathcal{L}_{\text{NL3,int}}$$

Same analytical form for interaction

Main structural difference

$$\mathcal{L}_{\text{Bonn}} \not\supset g_2 \frac{\sigma^3}{3} + g_3 \frac{\sigma^4}{4} \subset \mathcal{L}_{\text{NL3}}$$

Machleidt
1987

meson	Mass (MeV)	J^π	coupling
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Generated by FRG flow ?

SNM : Parameters flow

$$\Lambda \sim M_\rho \sim 750 \text{ MeV}$$

Symmetric nuclear matter

$$\rho \simeq \rho_{\text{MF}} = 0 \quad \delta \simeq \delta_{\text{MF}} = 0$$

Wetterich equation

$$\frac{dC_{i,k}}{dk} = f_i(k, \omega_{\text{MF},k}, \mu, \{C_{i,k}\})$$

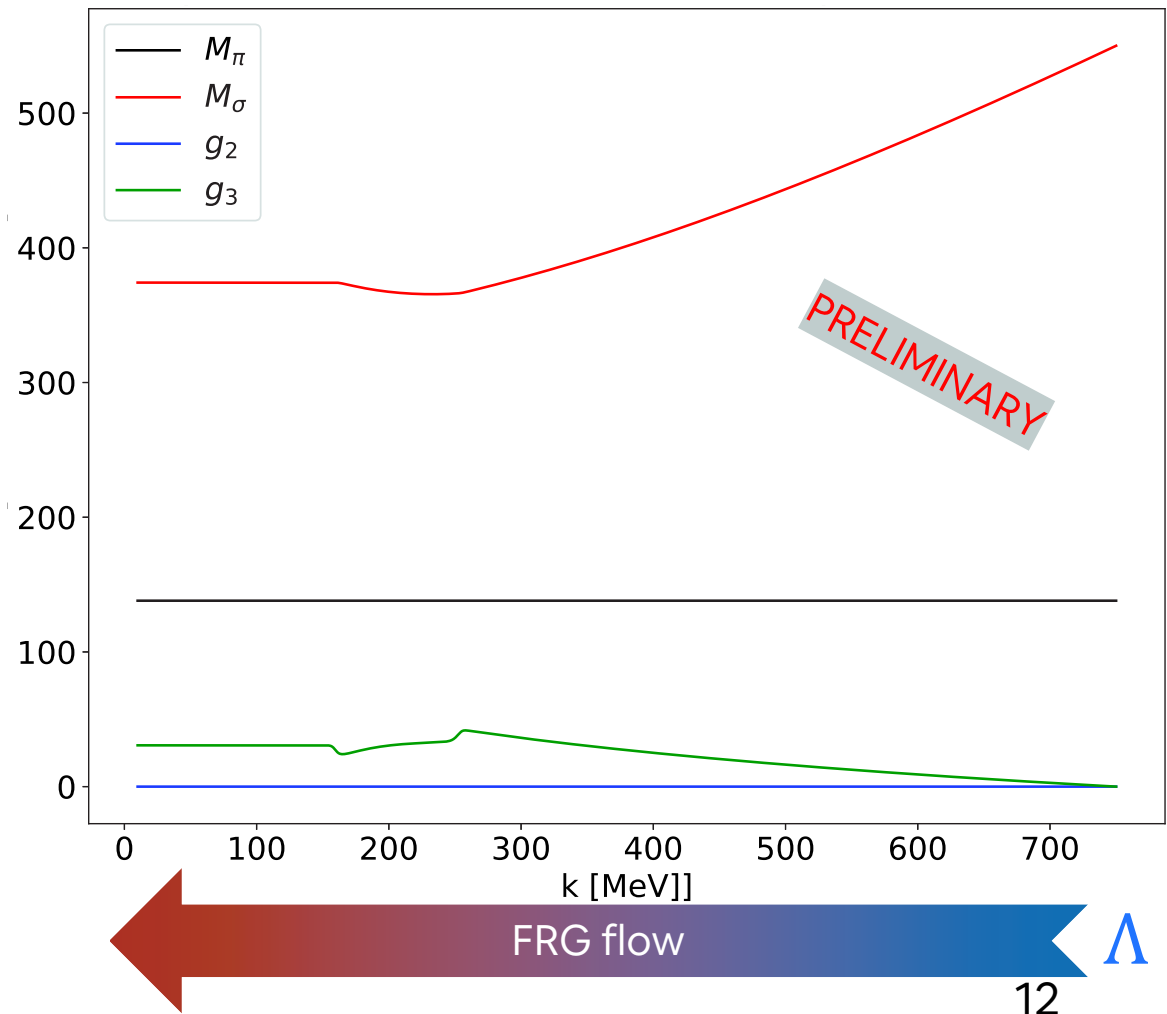
$$C_{i,k} = M_{\sigma,k}; M_{\pi,k}; g_{2,k}; g_{3,k}$$

M_σ decrease

M_π constant

g_3 non zero

$g_2 = 0$ due to symmetry



Nuclear matter properties

Wetterich equation

$$\frac{dU_k}{dk} \equiv \frac{dC_{i,k}}{dk} = f_i(k, \omega_{\text{MF},k}, \mu, \{C_{i,k}\})$$

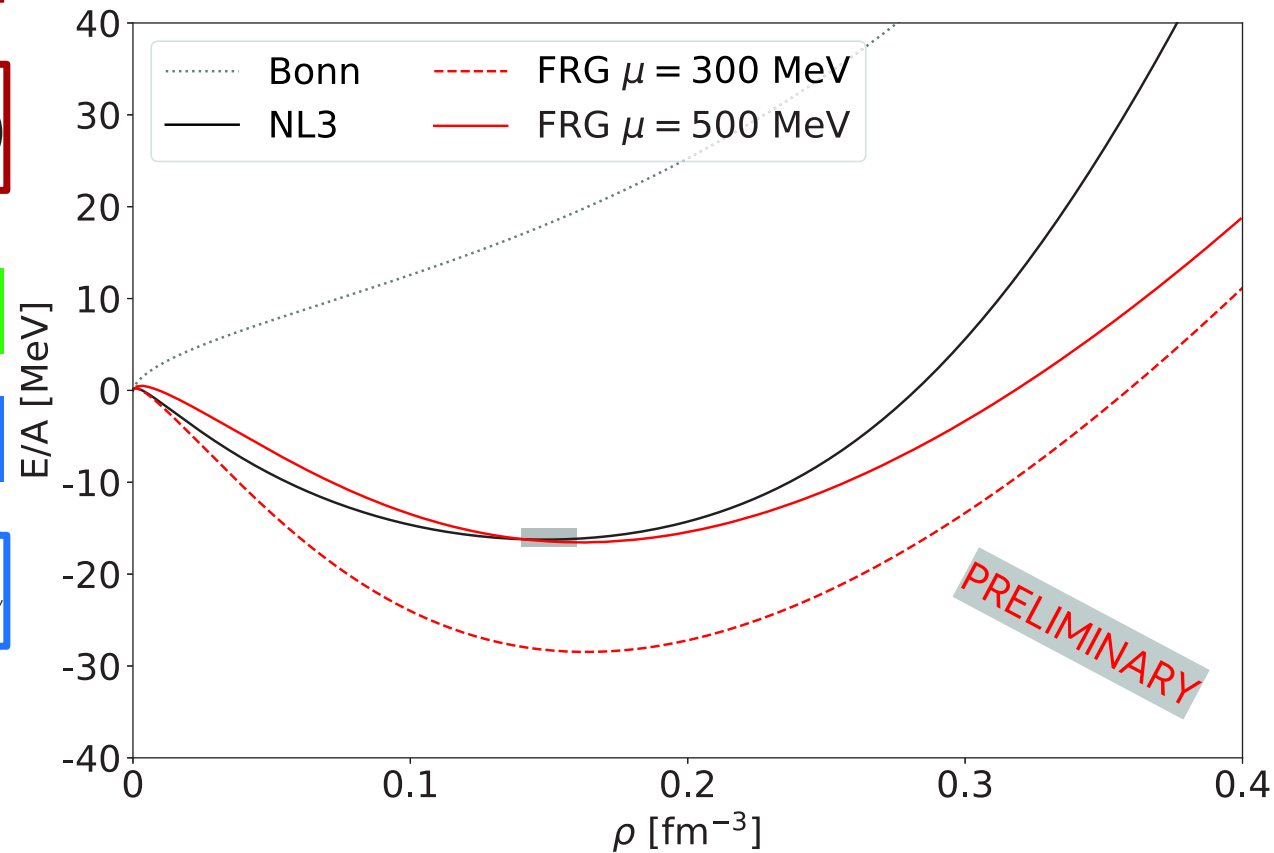
Solve for different μ

Compute $E/A(\mu)$

Drews,
PRC 91 035802
(2015)

$$\frac{E}{A}(T=0) = U_{k=0,\text{MF}} + \mu n$$

In principle, with FRG language.
For now, plug into NL3 RMF for a given μ



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Conclusion & perspectives

Preliminary results

FRG seems to be promising
to build EDF Lagrangians

Qualitative saturation of nuclear matter

Perspectives

Include flow of
Yukawa couplings



Cutoff dependance ?

Treat Asymmetric Nuclear Matter

Extension to finite nuclei

Back-up slides

Lagrangian in vacuum = UV

Phenomenological lagrangian : Bonn

$$\begin{aligned}
 \mathcal{L}_{\text{Bonn}} = & \bar{\psi} \left[i \not{\partial} - M - g_{\sigma} \sigma - g_{\delta} \vec{\delta} \cdot \vec{\tau} \right. \\
 & - \frac{f_{\eta}}{m_{\eta}} \gamma^5 \not{\partial} \eta - \frac{f_{\pi}}{m_{\pi}} \gamma^5 \not{\partial} \vec{\pi} \cdot \vec{\tau} \\
 & - g_{\omega} \not{\omega} - \frac{f_{\omega}}{4M} \sigma^{\mu\nu} \Omega_{\mu\nu} - g_{\rho} \vec{\rho} \cdot \vec{\tau} - \frac{f_{\rho}}{4M} \sigma^{\mu\nu} \vec{R}_{\mu\nu} \cdot \vec{\tau} \left. \right] \psi \\
 & + \frac{1}{2} (\partial_{\mu} \sigma \partial^{\mu} \sigma - m_{\sigma}^2 \sigma^2) + \frac{1}{2} (\partial_{\mu} \vec{\delta} \cdot \partial^{\mu} \vec{\delta} - m_{\delta}^2 \vec{\delta}^2) \\
 & + \frac{1}{2} (\partial_{\mu} \eta \partial^{\mu} \eta - m_{\eta}^2 \eta^2) + \frac{1}{2} (\partial_{\mu} \vec{\pi} \cdot \partial^{\mu} \vec{\pi} - m_{\pi}^2 \vec{\pi}^2) \\
 & + \frac{1}{2} \left(\frac{1}{2} \Omega^{\mu\nu} \Omega_{\mu\nu} + m_{\omega}^2 \omega^{\mu} \omega_{\mu} \right) + \frac{1}{2} \left(\frac{1}{2} \vec{R}^{\mu\nu} \cdot \vec{R}_{\mu\nu} + m_{\rho}^2 \vec{\rho}^{\mu} \cdot \vec{\rho}_{\mu} \right)
 \end{aligned}$$

Constants fitted to
reproduce NN shifts

Lagrangian in finite nuclei = IR

Structure = same as vacuum Lagrangian ($\neq$ coupling constants)
+ NL σ terms

$$\begin{aligned}\mathcal{L} = & \bar{\psi}(\gamma(i\partial - g_{\omega}\omega - g_{\rho}\vec{\rho}\vec{\tau} - eA) - m - g_{\sigma}\sigma)\psi \\ & + \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_{\sigma}^2\sigma^2 - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega^2 \\ & - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_{\rho}^2\vec{\rho}^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\end{aligned}$$

Constants fitted to
nuclear data (charge radii,
binding energy)

Easy to use : Hartree
approximation (= slater
determinants) OK for
finite nuclei.

$$U(\sigma) = \frac{1}{2}m_{\sigma}^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4,$$

FRG strategy

First : work at LPA level, $T \neq 0$, homogeneous system

Ansatz for the effective action

$$\Gamma_k = \int d^4x \left\{ \bar{\psi} \left(\gamma_\mu^E \partial_\mu^E + g_\sigma \sigma + g_\pi i \gamma_5 \boldsymbol{\pi} \cdot \boldsymbol{\tau} + \gamma^0 \left(\mu - g_\omega \omega_0 - g_\rho \rho_0^3 \tau^3 \right) \right) \psi \right. \\ \left. + \frac{1}{2} \partial_\mu^E \sigma \partial_E^\mu \sigma + \frac{1}{2} \partial_\mu^E \boldsymbol{\pi} \cdot \partial_E^\mu \boldsymbol{\pi} \right. \\ \left. + U_k(T, \mu_p, \mu_n, \boldsymbol{\pi}, \sigma, \omega_0, \rho_0^3) \right\}$$

ω, ρ have $M \sim M_{\text{nuc}}$, taken at **mean field** level.
 π, σ : can **fluctuate**.

Guided by phenomenological IR ansatz :
 Taylor expand U in σ, π

$$U_{k,\sigma} = \sum_{n \geq 2} \frac{v_{k,n}}{n!} \sigma^n \quad U_{k,\pi} = \sum_{n \geq 1} \frac{w_{k,n}}{n!} (\boldsymbol{\pi} \cdot \boldsymbol{\pi})^n$$

Flow equation

$$\frac{\partial U_k}{\partial k} = \frac{k^4}{12\pi^2} \left\{ \frac{1 + 2n_B(E_\sigma)}{E_\sigma} + \frac{2(1 + 2n_B(E'_\pi))}{E'_\pi} + \frac{1 + 2n_B(E''_\pi)}{E''_\pi} - 4 \left[\sum_{i=n,p} \frac{1 - \sum_{x=\pm 1} n_F(E_{\sigma\pi} + x\mu_{i,\text{eff}})}{E_{\sigma\pi}} \right] \right\} \quad (2.64)$$

$$\text{où } E_\sigma = k^2 + U''_{k,\sigma}, \quad E'_\pi = k^2 + U'_{k,\pi} \text{ et } E''_\pi = k^2 + U'_{k,\pi} + U''_{k,\pi}\pi^2,$$

$$\mu_{p,k,\text{eff}} = \mu_p - g_\omega\omega_{0,k} - g_\rho\rho_{0,k}, \quad \mu_{n,k,\text{eff}} = \mu_p - g_\omega\omega_{0,k} + g_\rho\rho_{0,k}, \quad (2.65)$$

$$E_{\sigma\pi}^2 \equiv k^2 + g_\sigma^2\sigma^2 + g_\pi^2\pi^2$$

Mean-Field equations for ρ, ω

$$\left. \frac{\partial}{\partial \omega} [U_k(T, \mu_p, \mu - n, \sigma, \pi, \omega, \rho_{0,k})] \right|_{\omega=\omega_{0,k}} = 0, \quad \left. \frac{\partial}{\partial \rho} [U_k(T, \mu_p, \mu - n, \sigma, \pi, \omega_{0,k}, \rho)] \right|_{\rho=\rho_{0,k}} = 0$$

Use Wetterich Equation to get rid of U_k

$$g_\omega \omega_{0,k} = \frac{G_\omega}{3\pi^2} \int_k^\Lambda dp \frac{p^4}{E_{\sigma\pi}} \sum_{x=\pm 1} \partial_\mu \left[n_F(E_{\sigma\pi} - x\mu) |_{\mu=\mu_{p,k,\text{eff}}} + n_F(E_{\sigma\pi} - x\mu) |_{\mu=\mu_{n,k,\text{eff}}} \right],$$

$$g_\rho \rho_{0,k} = \frac{G_\rho}{3\pi^2} \int_k^\Lambda dp \frac{p^4}{E_{\sigma\pi}} \sum_{x=\pm 1} \partial_\mu \left[n_F(E_{\sigma\pi} - x\mu) |_{\mu=\mu_{p,k,\text{eff}}} - n_F(E_{\sigma\pi} - x\mu) |_{\mu=\mu_{n,k,\text{eff}}} \right],$$