Theory of Quasi-Free Reactions

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TECHNISCHE UNIVERSITÄT DARMSTADT

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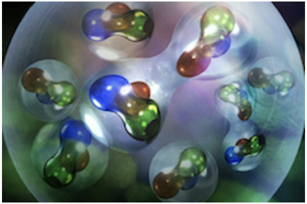

Outline

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- General Reaction Theory
- Cross Sections
- T-Matrix Elements
- Plane-Wave Impulse Approximation for Quasi-Free Reactions
- Deuteron Knockout Reactions
- Conclusions

General Reaction Theory

Preliminary Remarks

- reaction theory
  - lecture with reminder of basic facts
  - non-relativistic description
  - no explicit treatment of antisymmetrisation
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  - specific features

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  - spin and isospin variables mostly suppressed
- quasi-free reactions
  - focus on (p,pd) reactions
  - specific features
- application
  - recent publication:
    'Importance of deuteron breakup in the deuteron knockout reaction'
    Y. Chazono, K. Yoshida, K. Ogata
    Phys. Rev. C 106 (2022) 064613

Quantal Description of Reactions

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    $\Rightarrow$ time evolution of wave packet
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    $\Rightarrow$ fixed energy, two formulations:
    - partial differential equations
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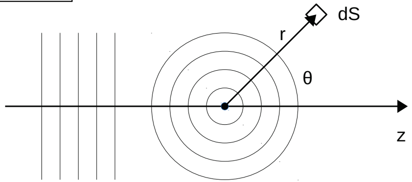

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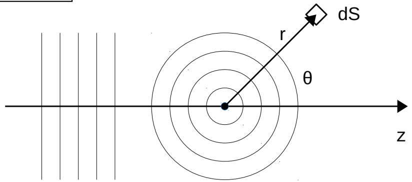

- boundary conditions
  'plane wave + outgoing (incoming) spherical waves'
- asymptotic states: different channels
  - partitions, quantum numbers, energy, momentum, ...
  - relevant quantities: cross sections (from comparison of fluxes)

Operator Formalism

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- example: two-body scattering with interaction V
  - explicit form of scattering wave function

$$\Psi_i^{(\pm)}(\vec{r}) = \Phi_i(\vec{r}) + \int d^3r\, G_0^{(\pm)}(\vec{r},\vec{r}^{\,\prime})V(\vec{r}^{\,\prime})\Psi_i^{(\pm)}(\vec{r}^{\,\prime}) \qquad \Phi_i(\vec{r}) = \exp(i\vec{k}_i \cdot \vec{r})$$

with Green's function $G_0^{(\pm)}(\vec{r},\vec{r}^{\,\prime}) = -\frac{2\mu}{\hbar^2}\frac{\exp(\pm ik_i|\vec{r}-\vec{r}^{\,\prime}|)}{4\pi|\vec{r}-\vec{r}^{\,\prime}|}$

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  - scattering amplitude $f_{fi} = -\frac{\mu}{2\pi\hbar^2}\langle\Phi_f|\hat{V}|\Psi_i^{(+)}\rangle$
    $\Rightarrow$ differential cross section $\frac{d\sigma}{d\Omega} = \frac{v_f}{v_i}|f_{fi}|^2$

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  - define T matrix $\boxed{T_{fi} = \langle\Phi_f|\hat{V}|\Psi_i^{(+)}\rangle = \langle\Phi_f|\hat{T}|\Phi_i\rangle = \langle\Psi_f^{(-)}|\hat{V}|\Phi_i\rangle}$
    with transition operator $\hat{T}$ from $\hat{T} = \hat{V} + \hat{V}\mathcal{G}\hat{T}$ with $\mathcal{G} = \left(E - \hat{H}_0 + \epsilon\right)^{-1}$

$\Rightarrow$ all information in T_{fi}

Cross Sections

Coordinates, Momenta

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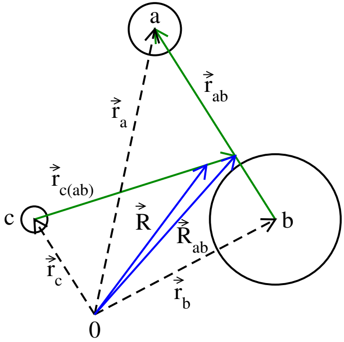
- three-body system with coordinates $\vec{r}_i$ and momenta p_i $(i = a, b, c)$
  - Jacobi coordinates (one of 3 possibilities)

$$\vec{r}_{ab} = \vec{r}_a - \vec{r}_b \qquad \vec{R}_{ab} = \frac{m_a \vec{r}_a + m_b \vec{r}_b}{M_{ab}}$$

$$\vec{r}_{c(ab)} = \vec{r}_c - \vec{R}_{ab} \qquad \vec{R} = \frac{m_c \vec{r}_c + M_{ab} \vec{R}_{ab}}{M_{abc}}$$



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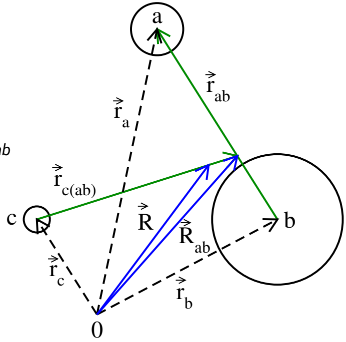
$$\vec{p}_{ab} = \mu_{ab} \left(\frac{\vec{p}_a}{m_a} - \frac{\vec{p}_b}{m_b} \right) \qquad \vec{P}_{ab} = \vec{p}_a + \vec{p}_b$$

$$\vec{p}_{c(ab)} = \mu_{c(ab)} \left(\frac{\vec{p}_c}{m_c} - \frac{\vec{P}_{ab}}{M_{ab}} \right) \qquad \vec{P} = \vec{p}_c + \vec{P}_{ab}$$



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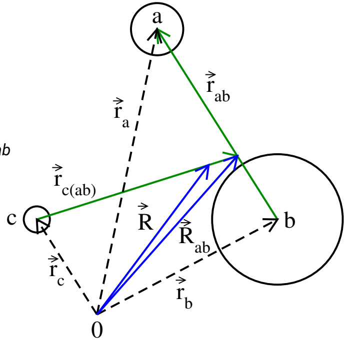
  - total and reduced masses

$$M_{ab} = m_a + m_b \qquad M_{abc} = m_c + M_{ab}$$

$$\mu_{ab} = \frac{m_a m_b}{M_{ab}} \qquad \mu_{c(ab)} = \frac{m_c M_{ab}}{M_{abc}}$$

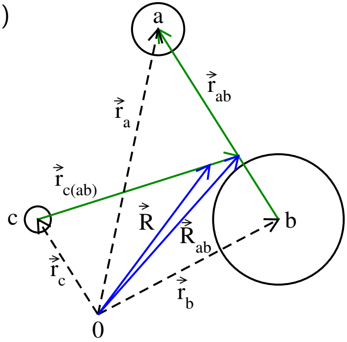


Energies and Transformations

- three-body system with coordinates $\vec{r}_i$ and momenta p_i $(i = a, b, c)$
  - kinetic energies

$$E_i = \vec{p}_i^{\,2}/(2m_i) \qquad E_{ab} = \vec{p}_{ab}^{\,2}/(2\mu_{ab})$$

$$E_{c(ab)} = \vec{p}_{c(ab)}^{\,2}/(2\mu_{c(ab)}) \qquad E = \vec{P}^2/(2M_{abc})$$



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    $E_{c(ab)} = \vec{p}_{c(ab)}^{\,2}/(2\mu_{c(ab)}) \qquad E = \vec{P}^2/(2M_{abc})$
  - useful identities

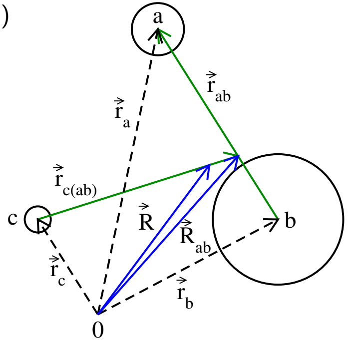
    $\vec{p}_a \cdot \vec{r}_a + \vec{p}_b \cdot \vec{r}_b + \vec{p}_c \cdot \vec{r}_c$

    $= \vec{p}_{ab} \cdot \vec{r}_{ab} + \vec{p}_{c(ab)} \cdot \vec{r}_{c(ab)} + \vec{P} \cdot \vec{R}$

    $E_a + E_b + E_c$

    $= E_{ab} + E_{c(ab)} + E$

    $\Rightarrow$ separation of cm motion possible



Reactions with Two Particles in Final State
a + b → c + d

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- general form of differential cross section

$$d\sigma^{(2)} = \frac{2\pi}{\hbar} \frac{\mu_{ab}}{p_{ab}} \frac{1}{(2J_a+1)(2J_b+1)} \sum_{M_a,M_b} \sum_{M_c,M_d}$$

$$\int \frac{d^3p_c}{(2\pi\hbar)^3} \frac{d^3p_d}{(2\pi\hbar)^3} \left| T_{fi}^{(2)} \right|^2 \delta(Q_2 + E^{(i)} - E^{(f)}) \, (2\pi\hbar)^3 \, \delta(\vec{P}^{(i)} - \vec{P}^{(f)})$$

with Q value $Q_2 = m_a + m_b - m_c - m_d$

and T-matrix element $T_{fi}^{(2)} = (2\pi\hbar)^3 \, \delta(\vec{P}^{(i)} - \vec{P}^{(f)}) \langle \Phi_f | \hat{T}^{(2)} | \Phi_i \rangle$

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- energy integration (with $d^3p_{cd} = \mu_{cd}p_{cd}dE_{cd}d\Omega_{cd}$)

$$\boxed{\frac{d^2\sigma^{(2)}}{d\Omega_{cd}} = \frac{\mu_{ab}\mu_{cd}}{(2\pi)^2\hbar^4}\frac{p_{cd}}{p_{ab}}\frac{1}{(2J_a+1)(2J_b+1)}\sum_{M_a,M_b}\sum_{M_c,M_d}\left|T_{fi}^{(2)}\right|^2}$$

with dependence on 2 angles in final state

Reactions with Three Particles in Final State I
a + b → c + d + e

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$$d\sigma^{(3)} = \frac{2\pi}{\hbar} \frac{\mu_{ab}}{p_{ab}} \frac{1}{(2J_a+1)(2J_b+1)} \sum_{M_a,M_b} \sum_{M_c,M_d,M_e}$$

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- different choices for further integration, e.g.,
  - change to Jacobi momenta (see below)

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- different choices for further integration, e.g.,
  - change to Jacobi momenta (see below)
  - introduce variables $x = \frac{1}{\sqrt{3}}(E_d - E_e)$ $\quad y = \frac{1}{3}(2E_c - E_d - E_c)$

$\Rightarrow E_c = \frac{1}{3}E + y \qquad E_d = \frac{1}{3}E - \frac{1}{2}y + \frac{\sqrt{3}}{2}x \qquad E_e = \frac{1}{3}E - \frac{1}{2}y - \frac{\sqrt{3}}{2}x$

$\Rightarrow$ all allowed states lie in 2-dim. hyperplane (x,y)
in first octant of (E_c, E_d, E_e) space for given $E = E_c + E_d + E_e$

$\Rightarrow$ Dalitz plot

Reactions with Three Particles in Final State II
a + b → c + d + e

- change to Jacobi momenta, momentum integration

$$d\sigma^{(3)} = \frac{2\pi}{\hbar} \frac{\mu_{ab}}{p_{ab}} \frac{1}{(2J_a+1)(2J_b+1)} \sum_{M_a,M_b} \sum_{M_c,M_d,M_e} \int \frac{d^3p_{cd}}{(2\pi\hbar)^3} \frac{d^3p_{e(cd)}}{(2\pi\hbar)^3} \left| T_{fi}^{(3)} \right|^2 \delta(Q_3 + E^{(i)} - E^{(f)})$$

Reactions with Three Particles in Final State II
$a + b \to c + d + e$

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- change to Jacobi momenta, momentum integration

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- energy integration (with $d^3p_{cd} = \mu_{cd} p_{cd} dE_{cd} d\Omega_{cd}$ and $d^3p_{e(cd)} = \mu_{e(cd)} p_{e(cd)} dE_{e(cd)} d\Omega_{e(cd)}$)

$$\frac{d^5\sigma^{(3)}}{dE_{cd}d\Omega_{cd}d\Omega_{e(cd)}} = \frac{\mu_{ab}\mu_{cd}\mu_{e(cd)}}{(2\pi)^5\hbar^7} \frac{p_{cd}p_{e(cd)}}{p_{ab}} \frac{1}{(2J_a+1)(2J_b+1)} \sum_{M_a,M_b} \sum_{M_c,M_d,M_e} \left| T_{fi}^{(3)} \right|^2$$

$\Rightarrow$ differential cross section depends on 5 $(= 3 \cdot 3 - 3 - 1)$ variables in final state

T-Matrix Elements

Transformation of T-Matrix Elements

- impulse approximation (see below)

Transformation of T-Matrix Elements

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- impulse approximation (see below)
- reformulation with distorted waves
  - introduce optical potentials $U_{i,f}$ with known solutions $\chi_{i,f}^{(\pm)}$ of Schrödinger equations $(\hat{H}_0^{(i,)} + \hat{U}_{i,f})\chi_{i,f}^{(\pm)} = E_{i,f}\chi_{i,f}^{(\pm)}$

    ($\chi_{i,f}^{(\pm)}$ sometimes approximated by eikonal wave functions in high-energy scattering)

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  - apply Gell-Mann–Goldberger relation/two-potential formula

    $$T_{fi} = \langle \phi_f \Phi_0^{(f)} | \hat{U}_i | \phi_i \chi_i^{(\pm)} \rangle + \langle \phi_f \chi_f^{(-)} | \hat{V}_f - \hat{U}_f | \Psi_i^{(+)} \rangle$$

    (ϕ_i, ϕ_f: internal wave functions of particles, $\Phi_0^{(i,f)}$: plane waves)

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    (ϕ_i, ϕ_f: internal wave functions of particles, $\Phi_0^{(i,f)}$: plane waves)
  - for rearrangement reactions ($i \neq f \Rightarrow \langle \phi_f \Phi_0^{(f)} | \hat{U}_i | \phi_i \chi_i^{(\pm)} \rangle = 0$)
    - 'post' form: $T_{fi} = \langle \phi_f \chi_f^{(-)} | \hat{V}_f - \hat{U}_f | \Psi_i^{(+)} \rangle$
    - 'prior' form: $T_{fi} = \langle \Psi_f^{(-)} | \hat{V}_i - \hat{U}_i | \phi_i \chi_i^{(+)} \rangle$

    still exact but problem to find full solutions $\Psi_{i,f}^{(\pm)}$ remains!

Approximations of T-Matrix Elements

- distorted-wave Born approximation (DWBA):
  replace full solutions $\Psi_{i,f}^{(\pm)}$ with distorted waves $\chi_{i,f}^{(\pm)}$
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- use coupled-channel (CC) approximation for full solution
  with boundary condition

$$\Psi_i^{(+)} \to \delta_{c0} \phi_i^{(c)} \exp(i\vec{p}_i^{(c)} \cdot \vec{r}_i/\hbar) + \sum_c f_i^{(c)} \phi_i^{(c)} \frac{\exp(ip_i^{(c)} r_i/\hbar)}{r_i} \quad \text{for } r_i \to \infty$$

  with different internal channel wave functions $\phi_i^{(c)}$ in initial state i
  - particular realisation:
    continuum-discretized coupled channel (CDCC) approximation

Plane-Wave Impulse Approximation for Quasi-Free Reactions

Quasi-Free (p,pd) Reactions

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- detection of deuteron-like $n+p$ correlation in nuclei by proton scattering
  - consider reaction $\boxed{p+T \to p+d+S}$
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- establish relation between T-matrices
  - $\boxed{T_{fi}^{(3)} = \langle \Phi_{pdS}(\vec{p}_{pd}, \vec{p}_{S(pd)}) | \hat{T}^{(3)} | \Phi_{pT}(\vec{p}_{pT}) \rangle}$

    with $\Phi_{pT}(\vec{p}_{pT}) = \phi_p \phi_T \exp[i(\vec{p}_{pT} \cdot \vec{r}_{pT})/\hbar]$
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    with $\Phi_{pd}(p_{pd}^{(i/f)}) = \phi_p \phi_d \exp[i(\vec{p}_{pd}^{(i/f)} \cdot \vec{r}_{pT})/\hbar]$

Transformation of T-Matrix I

- assumption: target wave-function ϕ_T contains deuteron-like cluster correlation ϕ_d
  - consider overlap function with momentum space wave function χ_{dS}

$$\varphi_T^{dS}(\vec{r}_{dS}) = \langle \phi_d \phi_S | \phi_T \rangle = \int \frac{d^3 Q_{dS}}{(2\pi\hbar)^3} \chi_{dS}(\vec{Q}_{dS}) \exp(-i\vec{Q}_{dS} \cdot \vec{r}_{dS}/\hbar)$$

(norm of $\varphi_T^{dS} \to$ 'deuteron spectroscopic factor')

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  - change of Jacobi variables in initial state of T-matrix element $T_{fi}^{(3)}$ from $\vec{r}_{pT}$ and $\vec{r}_{dS}$ to $\vec{r}_{pd}$ and $\vec{r}_{S(pd)}$

Transformation of T-Matrix II

- apply plane-wave impulse approximation (PWIA)
  - neglect interaction of residual nucleus S with p and d
    $\Rightarrow$ approximation of transition operator $\hat{T}^{(3)} \approx \hat{T}^{(2)}$ in $T_{fi}^{(3)}$

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    - momentum transfer to residual (= spectator) nucleus S

      $$\boxed{\vec{Q} = \vec{p}_{S(pd)} + \frac{m_S}{M_{dS}}\vec{p}_{pT} = \vec{p}_{S(pd)} - \frac{m_S}{m_T}\vec{p}_{Tp}}$$

      (argument of deuteron wave function in momentum space)
    - initial-state momentum in T-matrix element

      $$\boxed{\vec{q}_{pd}^{\,(i)} = \vec{p}_{pT} + \frac{m_p}{M_{pd}}\vec{p}_{S(pd)}}$$

Relation of Cross Sections

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with

- kinematic factor $K = \frac{2J_S+1}{(2\pi\hbar)^3} \frac{\mu_{pT}\mu_{p(dS)}\mu_{dS}}{\mu_{pd}^2} \frac{q_{pd}^{(i)}}{p_{pd}^{(f)}} \frac{p_{p(dS)}p_{dS}}{p_{pT}}$
- momentum distribution $W(\vec{Q}) = |\chi_{dS}(\vec{Q})|^2$
- half-off-energy-shell cross section $\frac{d^2\sigma^{(2)HOES}}{d\Omega_{pd}}$ of reaction $d + p \to d + p$

$\left(\text{in general} \quad \frac{[\vec{q}_{pd}^{(i)}]^2}{2\mu_{pd}} + Q_2 \neq \frac{[p_{pd}^{(f)}]^2}{2\mu_{pd}}\right)$

Kinematics of Quasi-Free (p,pd) Reactions

- quasi-free scattering condition $\boxed{\vec{Q} = \vec{p}_{S(pd)} - \frac{m_S}{m_T}\vec{p}_{Tp} = 0}$
  - no momentum transfer to residual = spectator nucleus *S*
  - correlation of final-state momenta (angles and energies)

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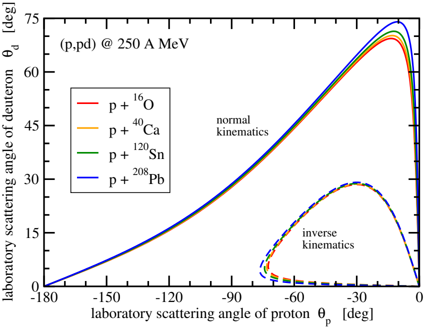

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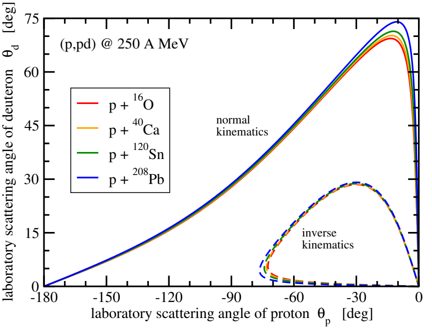

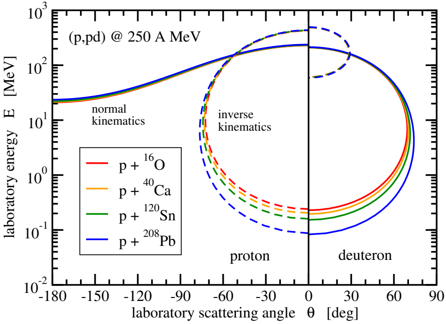

Deuteron Knockout Reaction

Proton-Induced Deuteron Knockout Reaction T(p,pd)S

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- fragility of deuteron
  $\Rightarrow$ final state can be reached via different processes, e.g.,
  - $p_1 + d \to p_1 + d$ elastic scattering (quasi-free process)
  - $p_1 + d \to p_1 + (p_2 + n) \to p_1 + d$ breakup and reformation
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  transition between deuteron bound and breakup states
  $\Rightarrow$ action of final-state interaction (FSI)
- new reaction model presented in recent publication:
  'Importance of deuteron breakup in the deuteron knockout reaction'
  (Y. Chazono, K. Yoshida, K. Ogata. Phys. Rev. C 106 (2022) 064613)
  - application for 250 MeV protons on targets ^{16}O, ^{40}Ca, ^{56}Ni in normal kinematics
  - general kinematic conditions, not only quasi-free scattering

Reaction Model I

- combination of different methods
  - distorted-wave impulse approximation (DWIA):
    replace plane waves in PWIA with distorted waves
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    $\Rightarrow$ takes proton-target interaction explicitly into account

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- distorted waves $\chi_{pT}^{(+)}, \chi_{pS}^{(-)}$ for $p + T$ and $p + S$ scattering
- wave function for $d + S$ scattering $\Psi_{dS}^{(-)}$ in CDCC approach (only S waves)
- internal deuteron ground-state wave function ϕ_d
- deuteron wave function in target φ_T^{dS}
- $\mathcal{A}$ antisymmetrization operator

Reaction Model II

- interactions
  - pn interaction in deuteron system

    one-range Gaussian form $v_{pn}(r) = v_0 \exp\left(-r^2/r_0^2\right)$ to reproduce binding energy and radius (T. Ohmura et al., Prog. Theor. Phys. 43 (1970) 347)

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  - T operator in impulse approximation: $\hat{T} = \hat{t}_{pp} + \hat{t}_{pn}$
    effective pp and pn potentials for pd interaction (M. A. Franey and W. G. Love, PRC 31 (1985) 488)

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    Dirac phenomenology, EDAD1 parameter set (S. Hama et al., PRC 41 (1990) 297; ...)
- mismatch of arguments in distorted waves
  - asymptotic momentum approximation (AMA)
    $\chi^{(\pm)}(\vec{p}, \vec{r} + \Delta\vec{r}) \approx \chi^{(\pm)}(\vec{p}, \vec{r}) \; \exp(\pm i\vec{p} \cdot \Delta\vec{r}/\hbar)$

Elastic p+d Scattering and d(p,p)pn Breakup Reaction

- differential cross sections for proton energies:

  120, 135, 155, 170, 190, 250 MeV

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  (K. Ermisch et al., PRC 71 (2005) 064004

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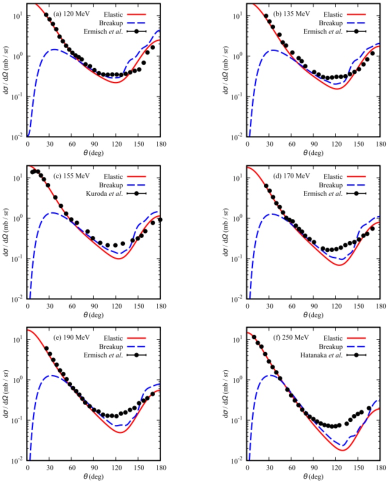

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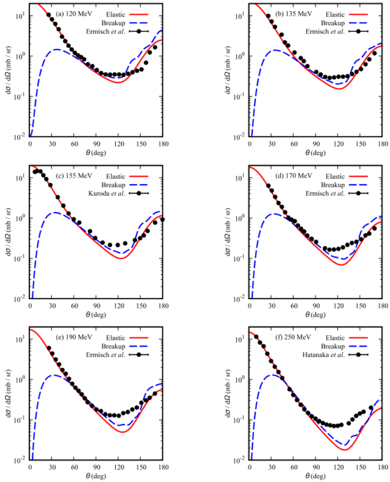
  K. Hatanaka et al. PRC 66 (2002) 044002)
- correction factor $C \approx 0.69$ for cross sections
  - needed to reproduce forward-angle elastic p+d scattering
  - also used in calculation of (p,pd) reactions



Differential Cross Sections for (p,pd) Reactions

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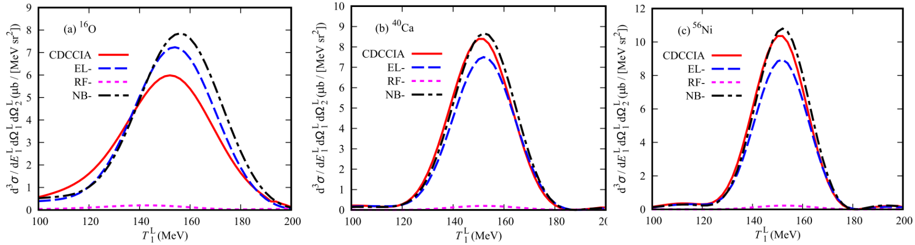
- 250 MeV protons, targets: ^{16}O, ^{40}Ca, ^{56}Ni, dependence on energy of outgoing proton T_1^L

Differential Cross Sections for (p,pd) Reactions

- 250 MeV protons, targets: ^{16}O, ^{40}Ca, ^{56}Ni, dependence on energy of outgoing proton T_1^L
  - full CDCCIA calculation (red)
  - EL: without reformation path in CDCCIA (blue)
  - RF: only deuteron reformation path in CDCCIA (pink)
  - NB: no breakup of deuteron (black)



Conclusions

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- general reaction theory: transparent formulation for
  - cross sections
  - T-matrix elements
- plane-wave impulse approximation (PWIA)
  - factorization of differential cross section:
    kinematic factor $\times$ momentum distribution $\times$ HOES two-body cross section
- quasi-free reactions
  - specific kinematic correlations of particles in final state
  - suppression of other reaction mechanism
- fragility of deuteron in (p,pd) reactions
  - possible effects of deuteron breakup and reformation
- reactions with three particles
  - full Faddeev approach required $\Rightarrow$ future

Thank You for Your Attention!