Quasi-deuteron model to effectively embed short range correlations in relativistic mean field approaches

4^{th} International Workshop on Quantitative Challenges in SRCs and the EMC Effect Research

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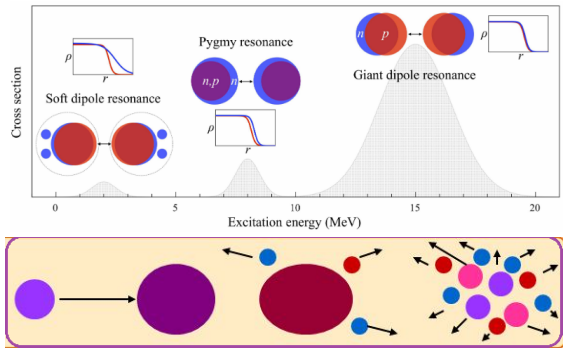
Equation of State: theoretical approaches

- Equation of State (**EoS**) of nuclear matter (**NM**):
  - **Structure** and **reaction** dynamics of finite nuclei
  - Modelization of **compact stellar objects**
- **Theoretical** models (only baryons degrees of freedom)
  - *Ab-initio* approaches based on **realistic** interactions
  - **Phenomenological** models with **effective** interaction
- **Mean-field** approximation $\Rightarrow$ Energy **density functional**
  - Non-relativistic **Skyrme** type interaction
  - **Relativistic** models based on **mesons exchange**
- **Short-range** induced many-body **correlations**
  - Formation of bound states (**clusters**) at



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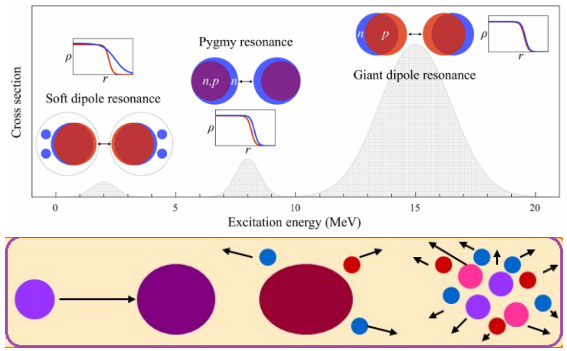

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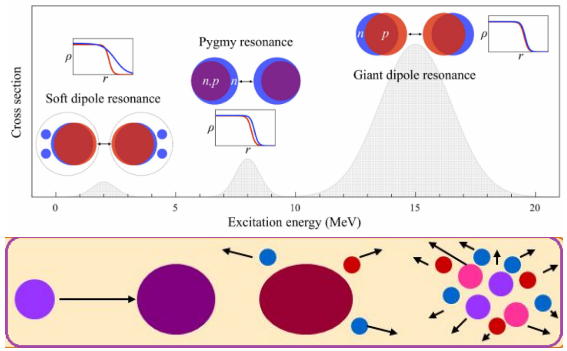

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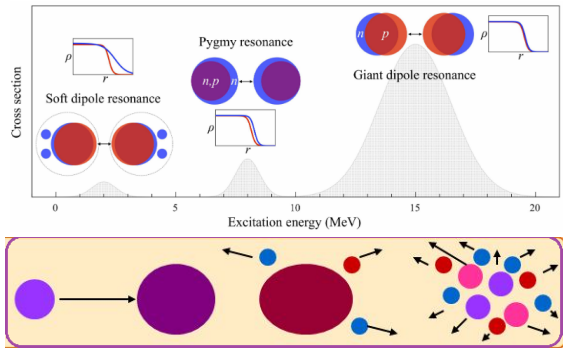

Cluster-based phenomenological models

- **Phenomenological** models with **clusters**
  - **Dilute** matter as a **mixture** of nucleons and nuclei
    $\Rightarrow$ **Nuclear statistical equilibrium** model

    [A. R. Raduta, F. Gulminelli, PRC 82, 065801 (2010)]
- Cluster **dissolution** at saturation n_0 $\Rightarrow$ **Mott effect**
  - Geometrical **excluded-volume** mechanism
  - Microscopic **in-medium effects**
- Generalized relativistic density functional (**GRDF**)
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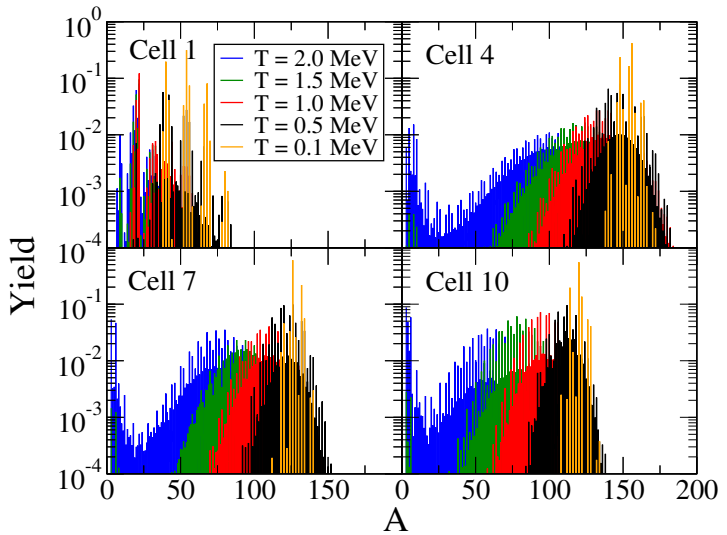

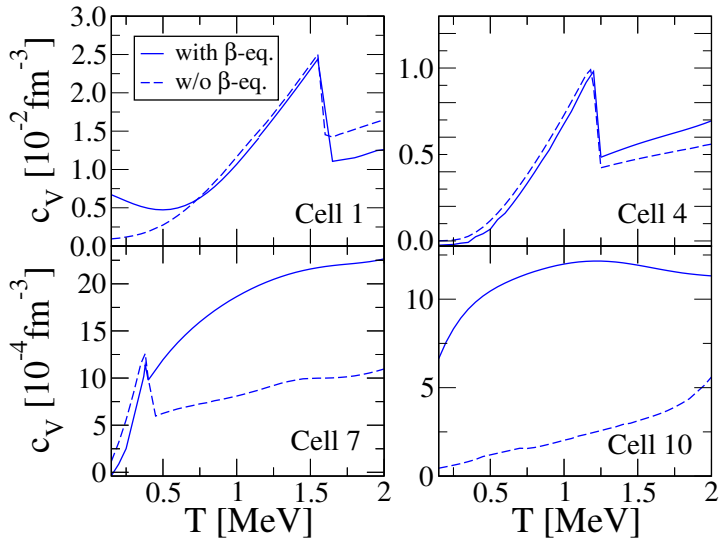

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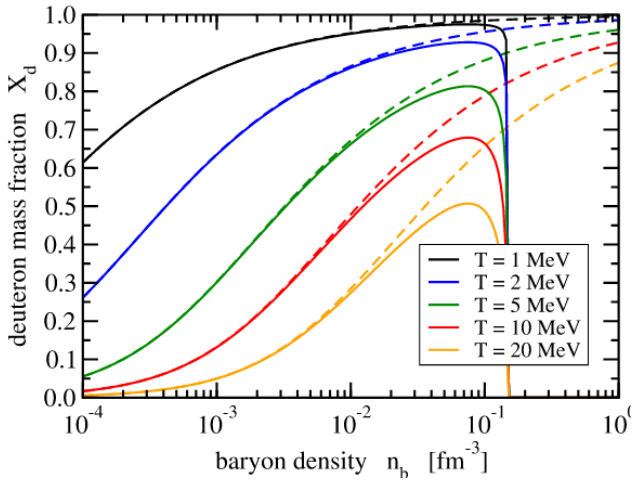

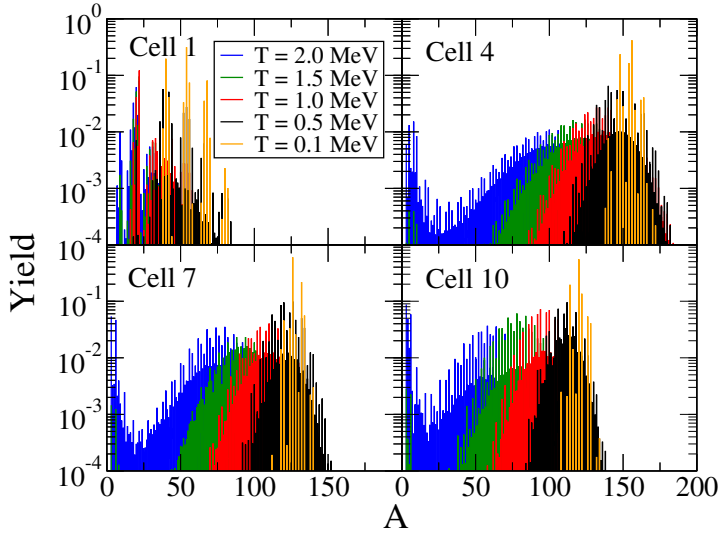

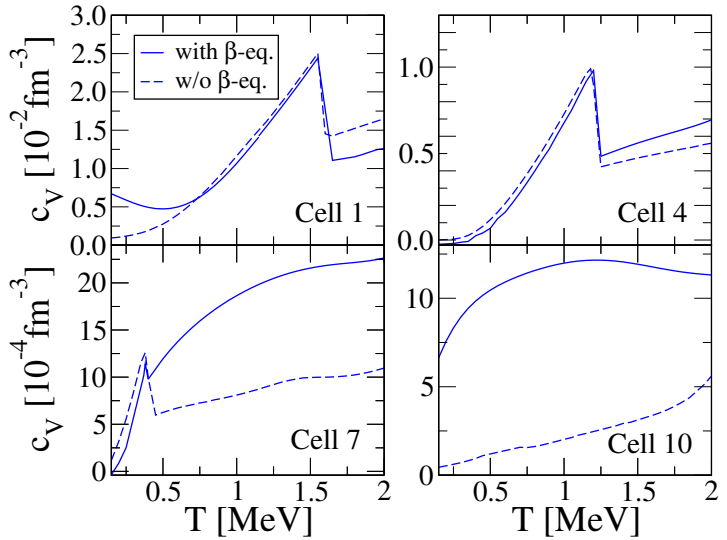

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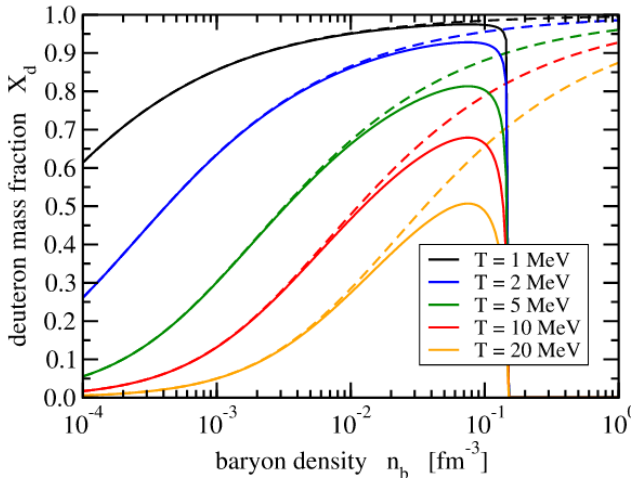

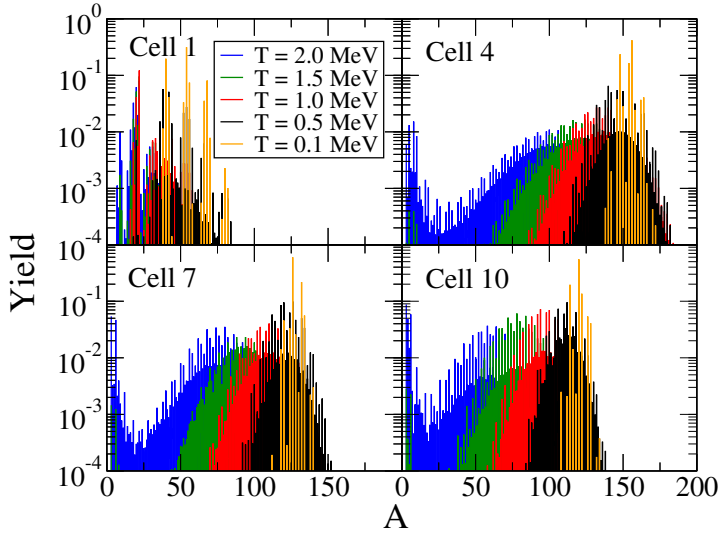

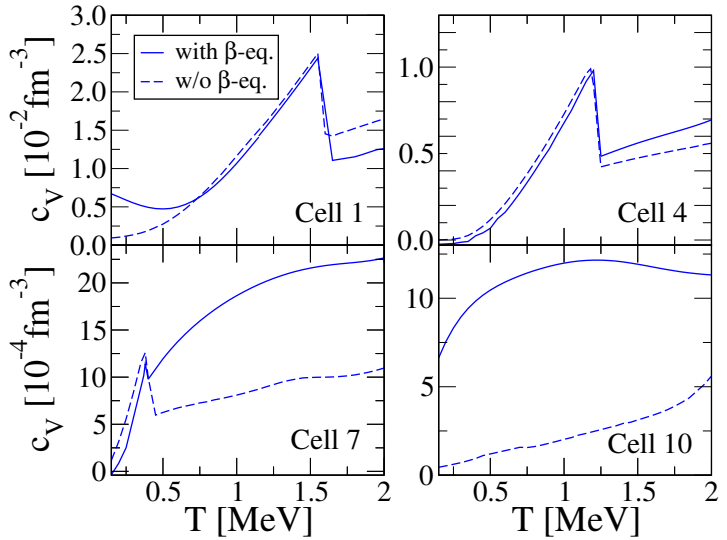

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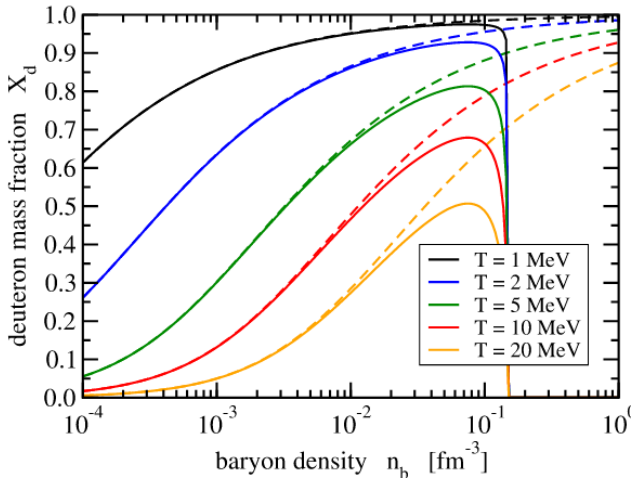

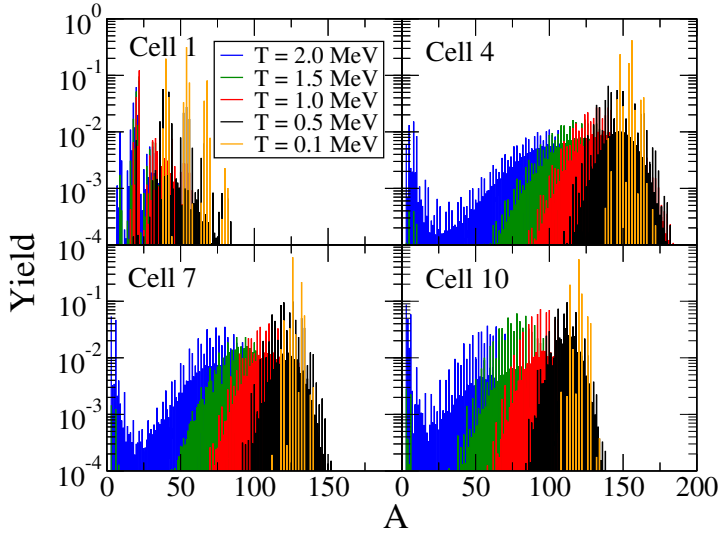

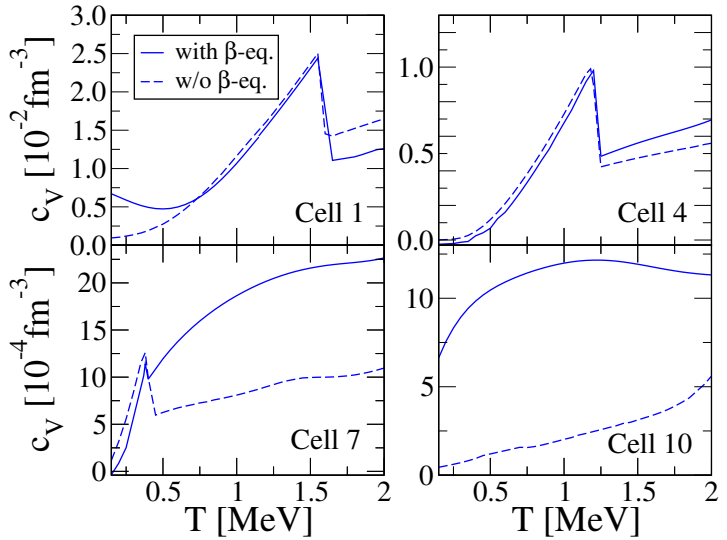

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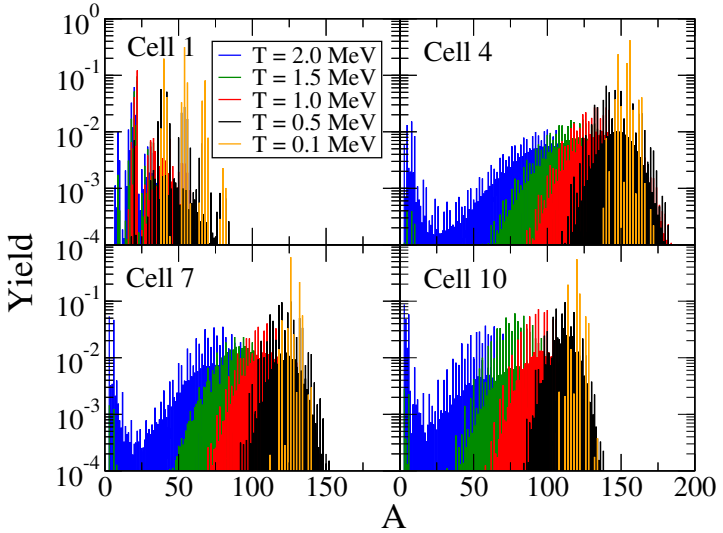

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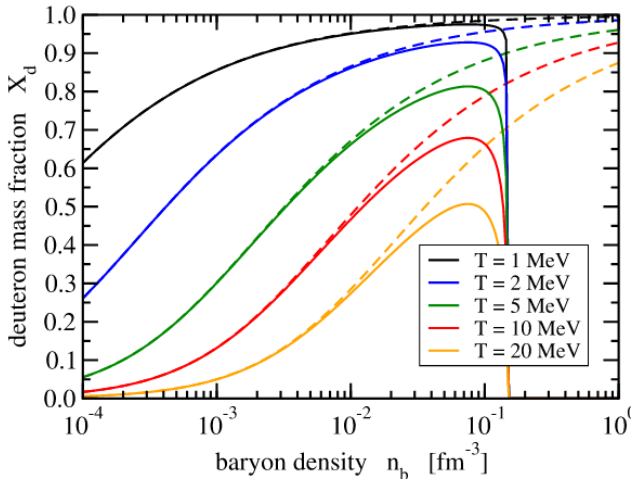

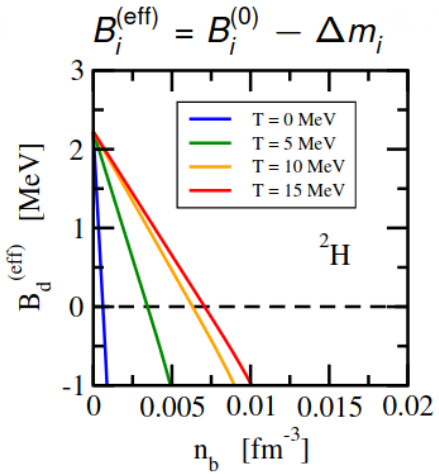

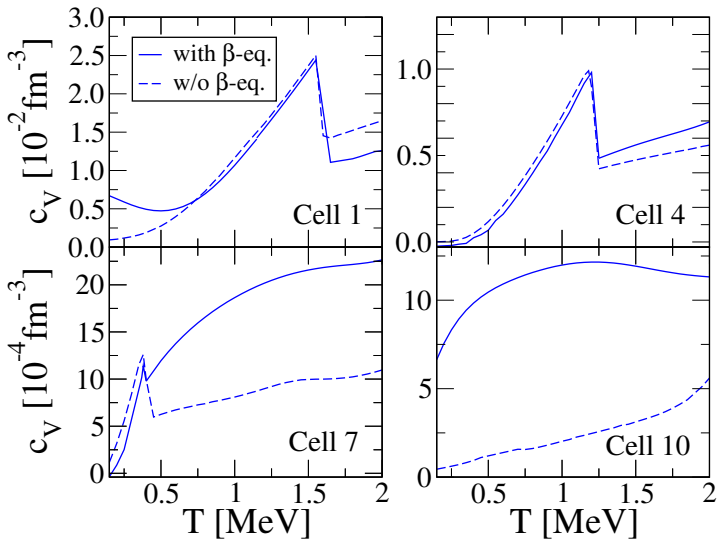

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Mass-shift parameterizations at low density

- **Mass-shift** obtained by solving the **in-medium** many-body **Schrödinger equation**
  - Contributions from **Pauli blocking** or screening of **electronic background**
  - **Parameterization** as function of density (n_b), asymmetry (β), temperature (T)
- Symmetric NM (**SNM**) with clusters at rest $\Rightarrow \Delta m_i^{(\mathrm{low})} \propto n_b$

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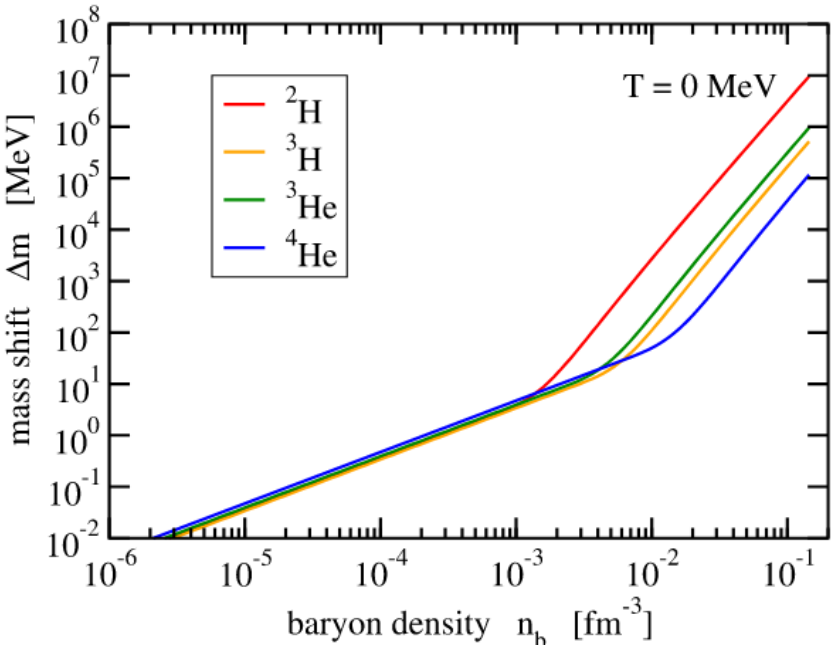

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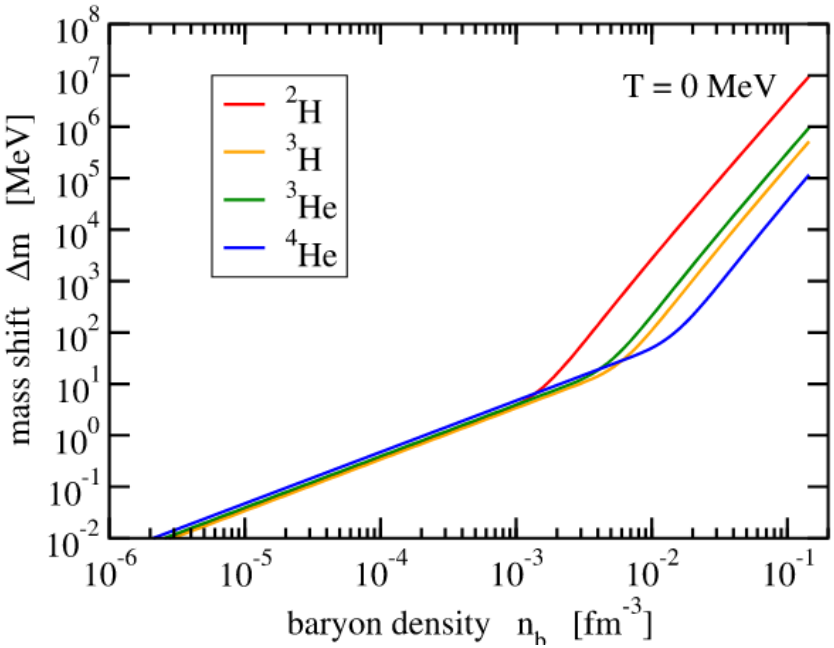
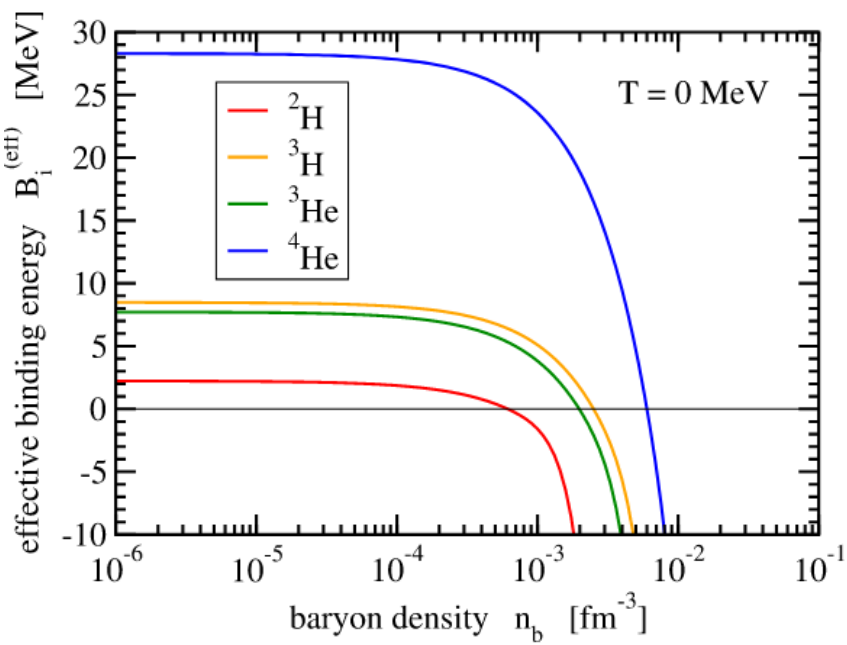
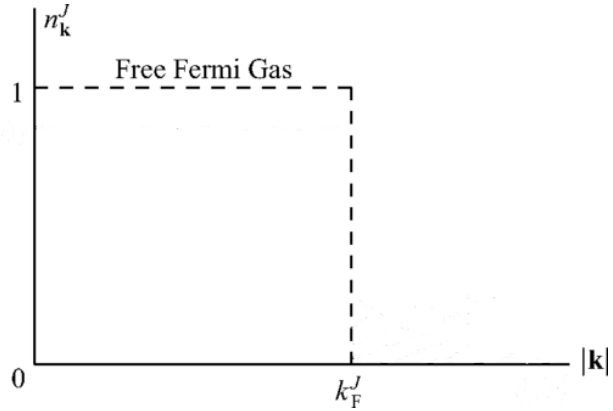




Mean-field framework and short-range correlations

- **Cluster-free** NM above n_0: Free Fermi gas (**FFG**)
  $\Rightarrow$ **step function** in momentum distribution at zero T
- Nucleon knock-out in **inelastic electron scattering**
  [O. Hen et al. (CLAS Coll.), Science 346, 614 (2014)]
  - **Smearing** of Fermi surface in cold nucleonic matter
  - **High momentum tail (HMT)** decreasing with $\sim |k|^{-4}$
- Nucleon-nucleon short-range correlations (**SRCs**)
  - Tensor components or **repulsive** core of nuclear forces
- Same **height** for two species in asymmetric systems
  - Enhancement of HMT of **minority** to the **majority** one
- **Isospin**-dependence of distribution
  - Kinetic **symmetry energy** very different from FFG
  $\Rightarrow$ Importance of incorporating SRCs in **realistic** EoS



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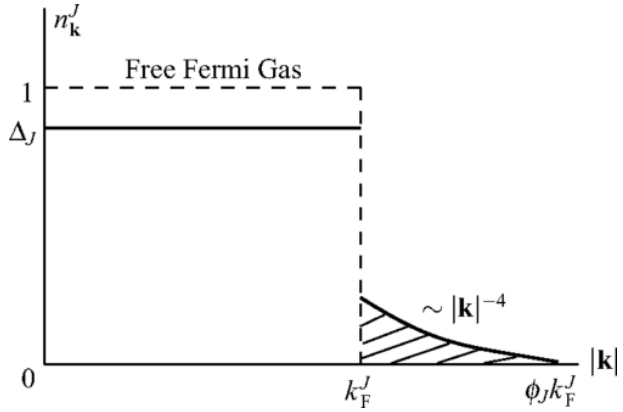

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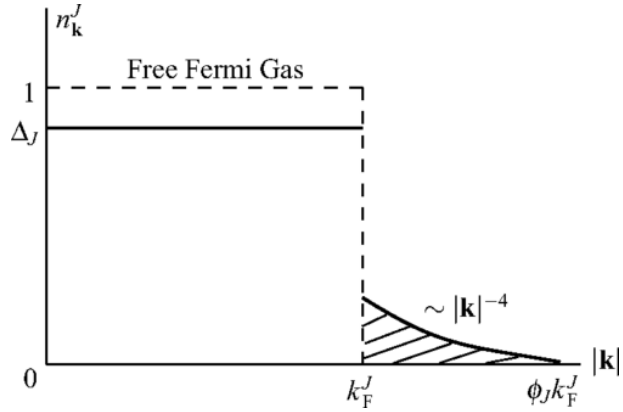

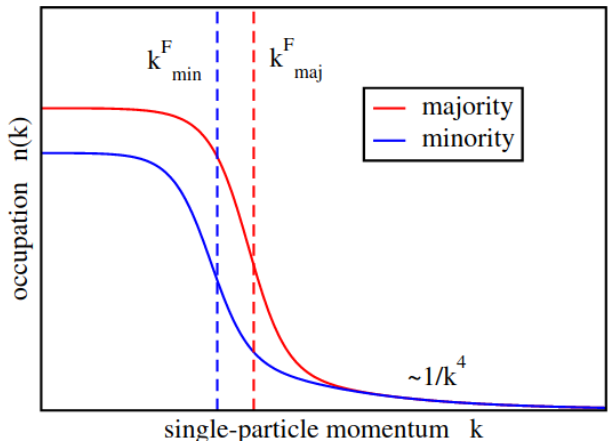

Quasi-deuterons as surrogate for SRCs in GRDF

- Effective resonances (**quasi-clusters**) for treatment of SRCs at **supra-saturation**
  - Embedded in **GRDF** model through **in-medium modifications** of $\Delta m_d^{(\mathrm{high})}$
- **Two-body** correlations in $np\ {}^3S_1$ channel $\Rightarrow$ quasi-deuteron
- $T = 0 \Rightarrow$ **boson condensate** of deuterons under chemical potentials **equilibrium**
- $m^*_{\mathrm{nuc}} \geq 0 \Rightarrow 0 \leq X_d \leq \min\left\{X_d^{(\max)}, 1-|\beta|\right\}$, $X_d^{(\max)} = \frac{m_{\mathrm{nuc}}}{\chi_d C_\sigma n_b} \xrightarrow[n_b \to \infty]{} 0$
- Crucial role of **scaling factor** $\chi_d \equiv \chi$ for **bound nucleon**-meson coupling strenght

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$$\mu_d = \mu_n + \mu_p \Rightarrow \boxed{m_d^* + \Delta m_d^{(\mathrm{high})} + V_d' = \sqrt{k_n^2 + (m_n^*)^2} + V_n' + \sqrt{k_p^2 + (m_p^*)^2} + V_p'}$$

$$m_i^* = m_i - S_i \qquad S_i = \chi_i A_i C_\sigma n_\sigma \qquad V_i = \chi_i A_i \left(C_\omega n_\omega + C_\rho n_\rho\right)$$

$$V_i' = V_i + W_i + W_i^{(r)} \qquad W_i = \frac{1}{2}\left(C_\omega' n_\omega^2 + C_\rho' n_\rho^2 - C_\sigma' n_\sigma^2\right)$$

$$W_i^{(r)} = n_d \frac{\partial \Delta m_d^{(\mathrm{high})}}{\partial n_i} \qquad C_j = \frac{\Gamma_j^2(n_b)}{m_j^2} \qquad C_j' = \frac{dC_j}{dn_b}, \quad j = \sigma, \omega, \rho$$

- $m_{\mathrm{nuc}}^* \geq 0 \Rightarrow 0 \leq X_d \leq \min\left\{X_d^{(\max)}, 1 - |\beta|\right\}$, $X_d^{(\max)} = \dfrac{m_{\mathrm{nuc}}}{\chi_d C_\sigma n_b} \xrightarrow[n_b \to \infty]{} 0$
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Quasi-deuterons as surrogate for SRCs in GRDF

- Effective **resonances** (**quasi-clusters**) for treatment of SRCs at **supra-saturation**
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  - With **scalar** (S_i), **vector** (V_i) and **rearrangement** (W_i, $W_i^{(r)}$) potentials ($i = nuc, d$)

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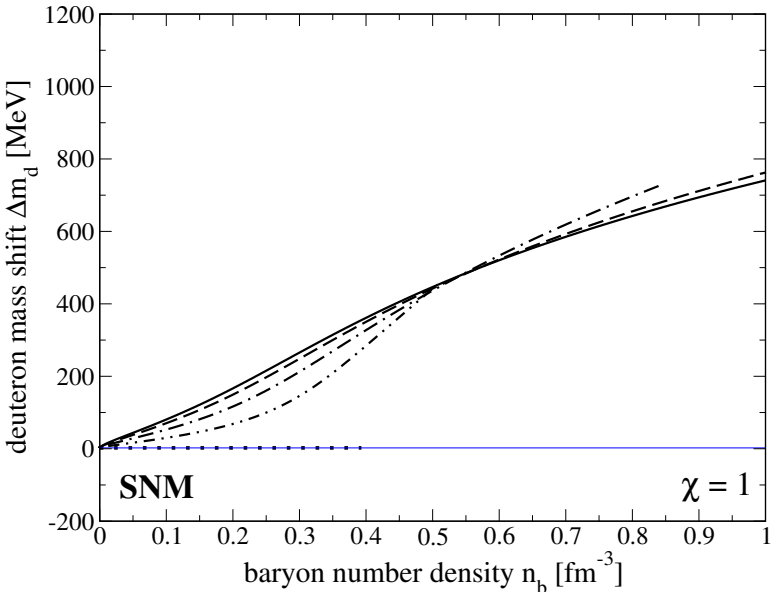

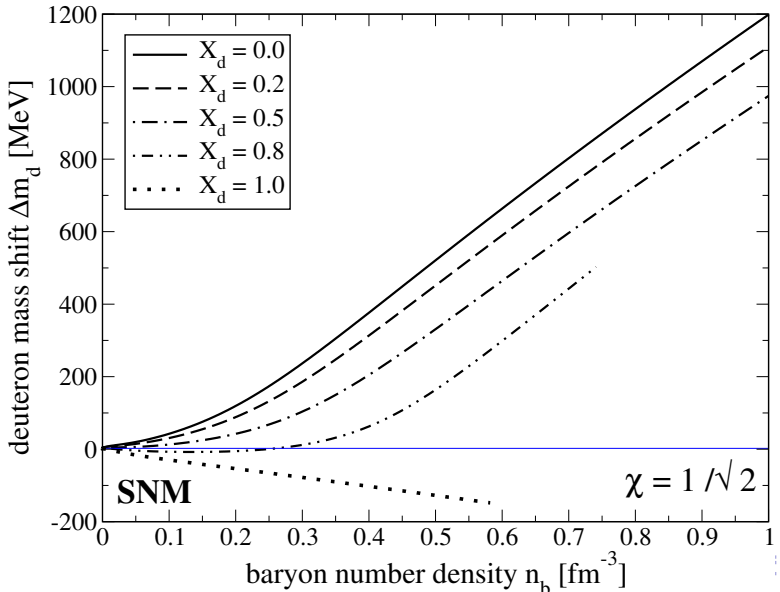

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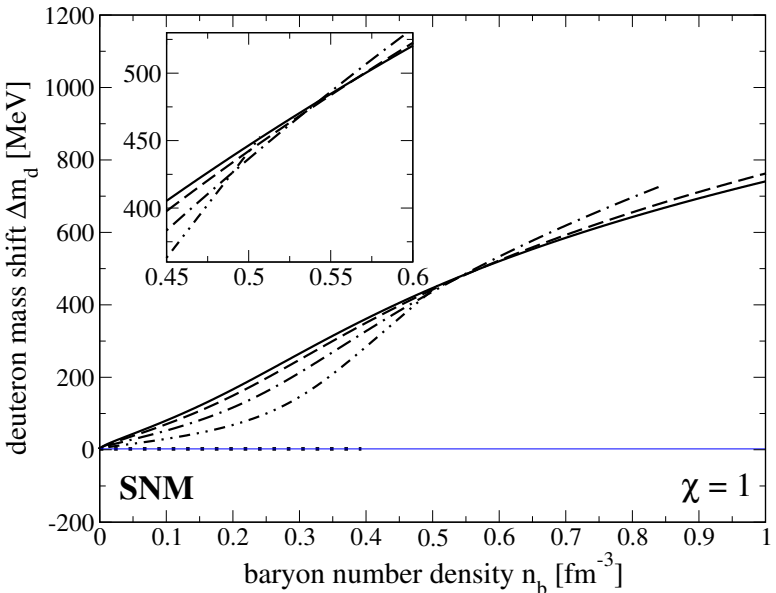

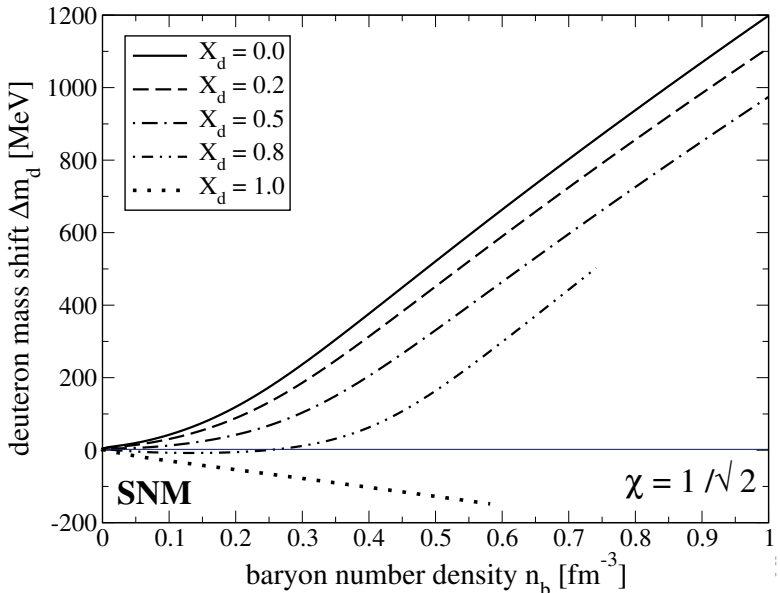

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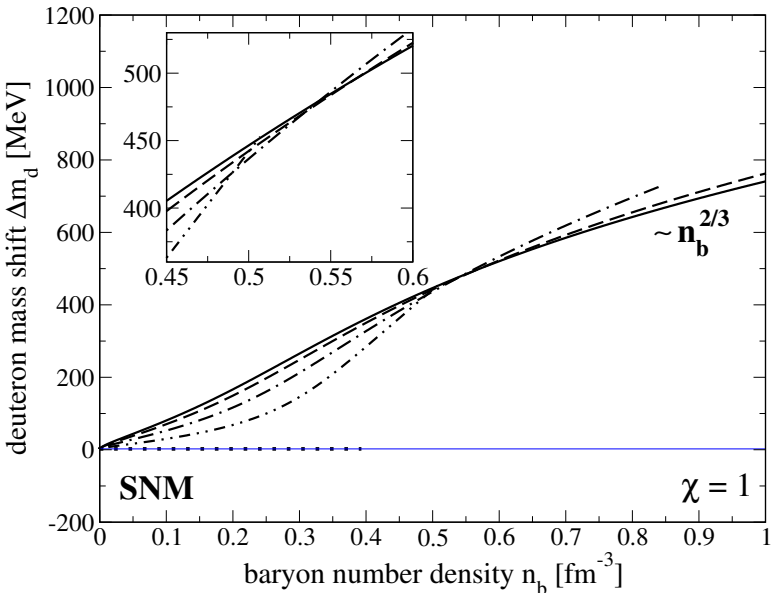

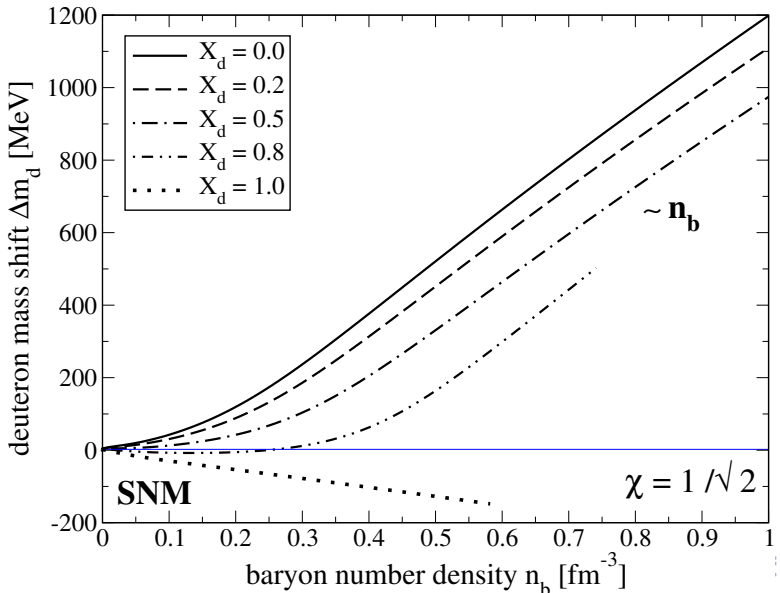

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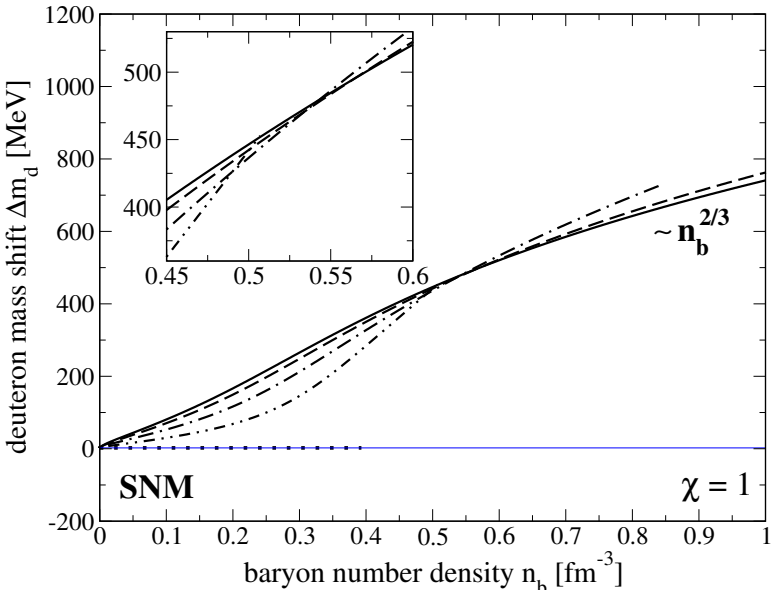

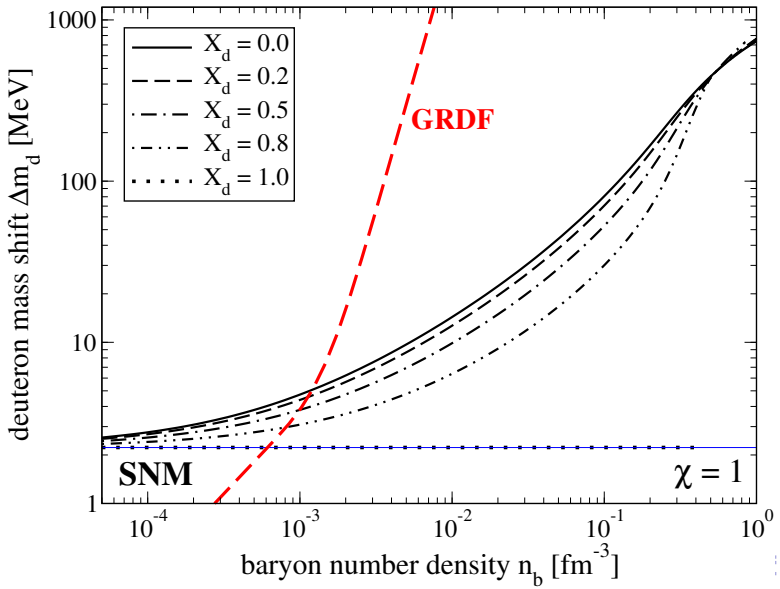

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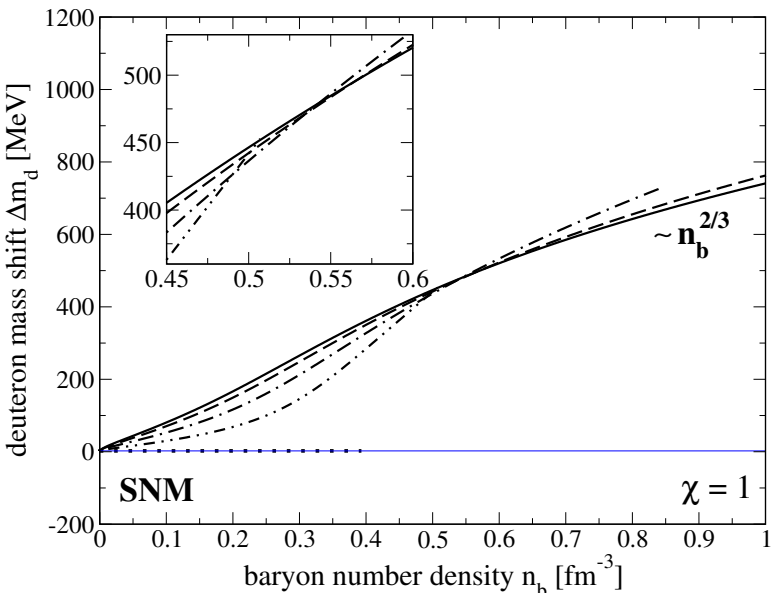
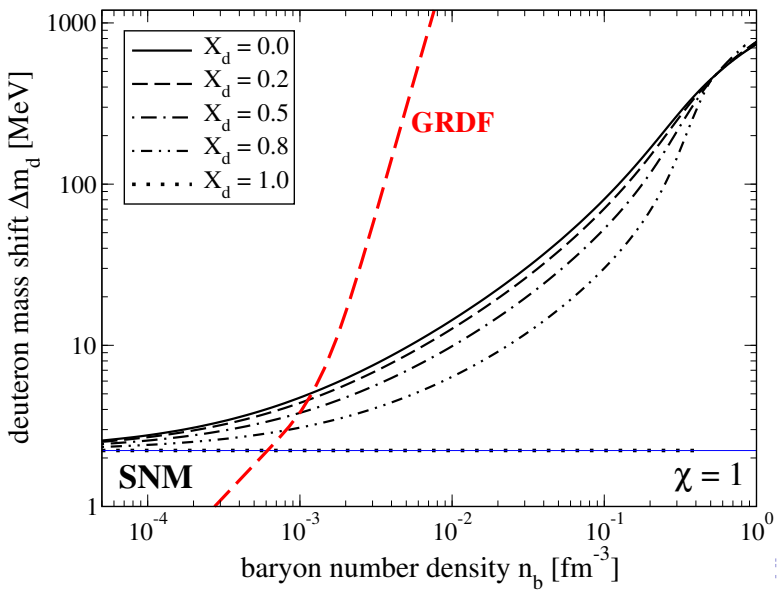
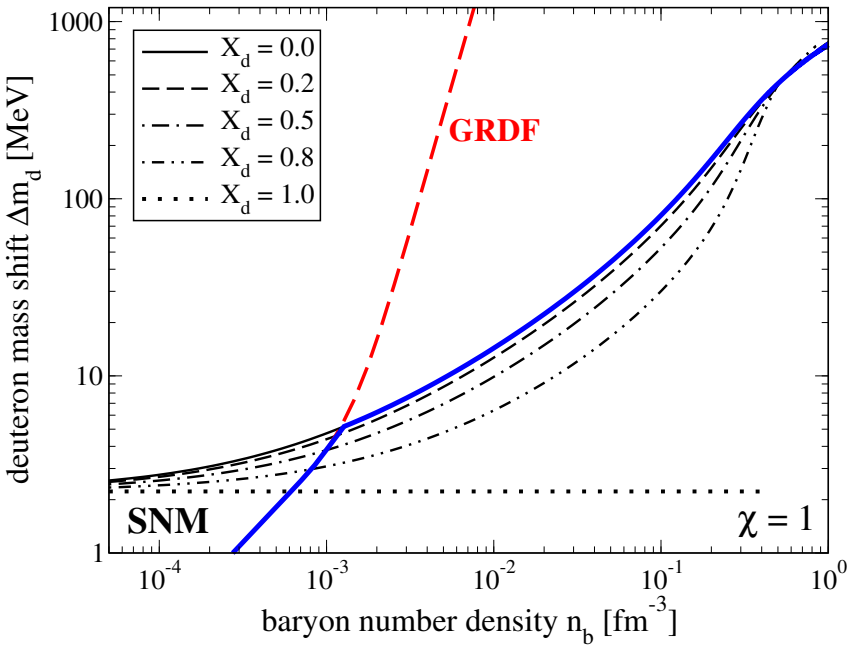




Piecewise interpolation and saturation constraints

- **Piecewise** parameterization: $\Delta m_d(n_b, X_d) = \min\left\{\Delta m_d^{(\mathrm{low})}(n_b), \Delta m_d^{(\mathrm{high})}(n_b, X_d)\right\}$



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	χ	$\Gamma_{\sigma,0}$	$\Gamma_{\omega,0}$	$\Gamma_{\rho,0}$	$\Delta m_{d,0}$ [MeV]	$\varepsilon_{\mathrm{bind}}$ [MeV]
	1	10.580042	13.217226	3.556424	[illegible]	[illegible]
	$1/\sqrt{2}$	10.919963	13.719324	3.400187	[illegible]	[illegible]
DD2	—	10.686681	13.342362	3.626940	[illegible]	[illegible]

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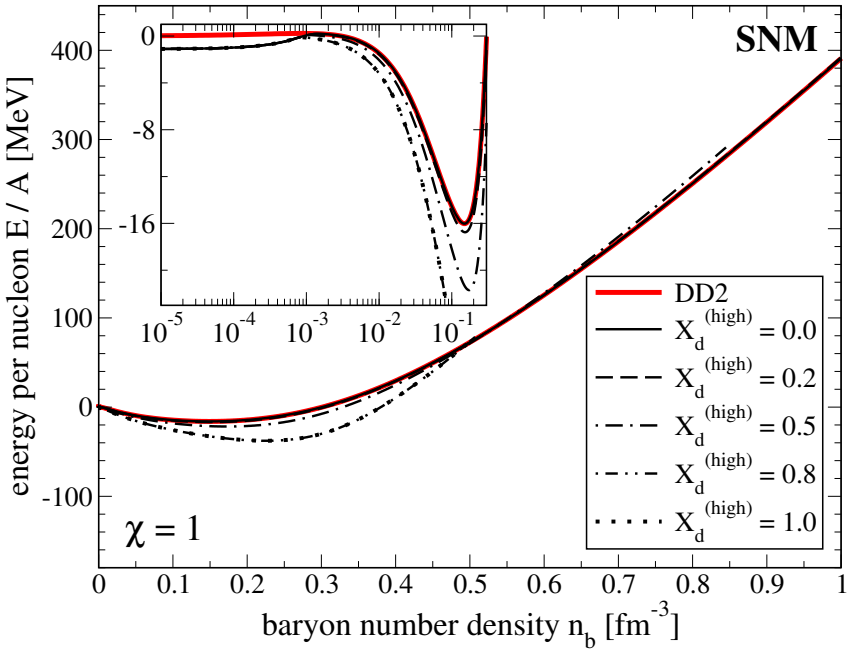

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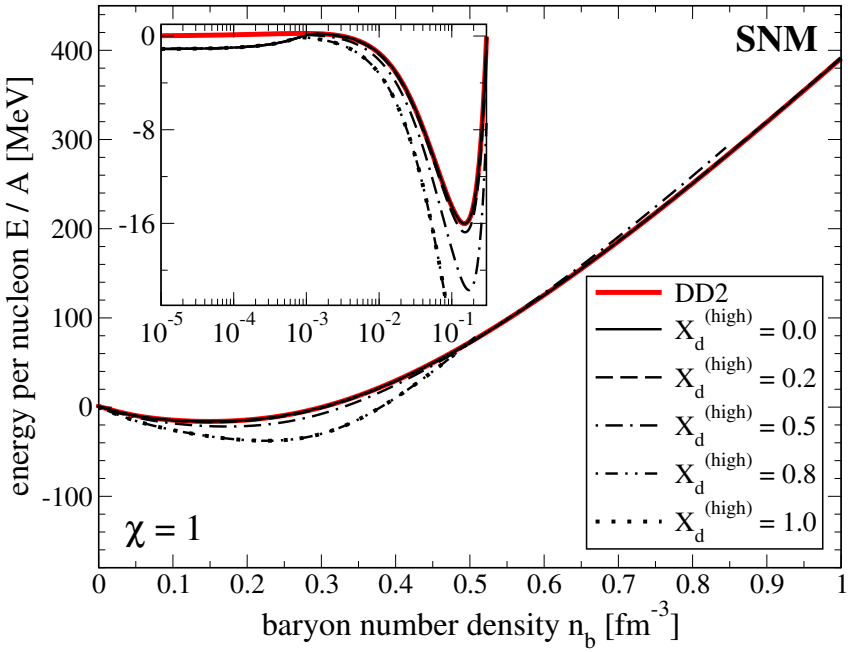

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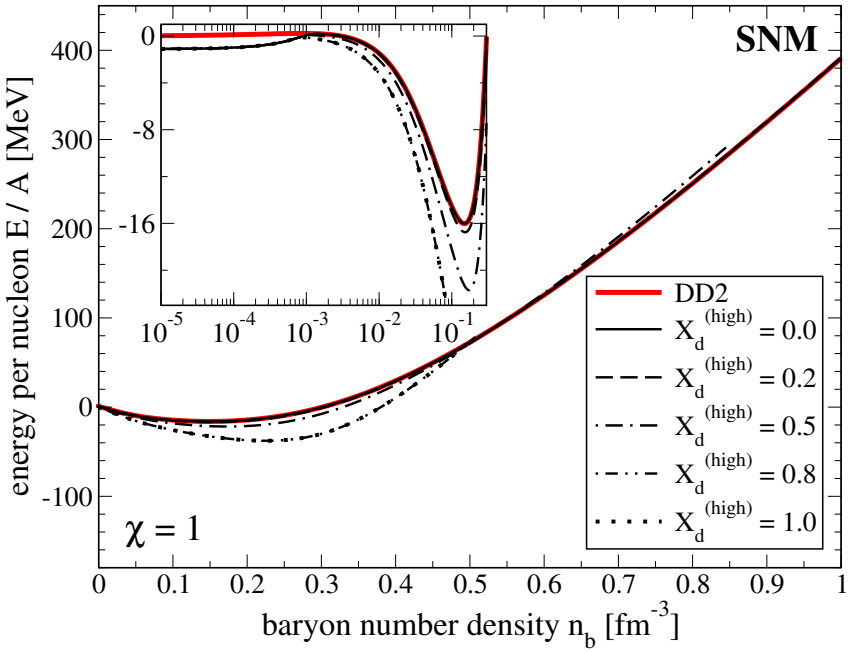

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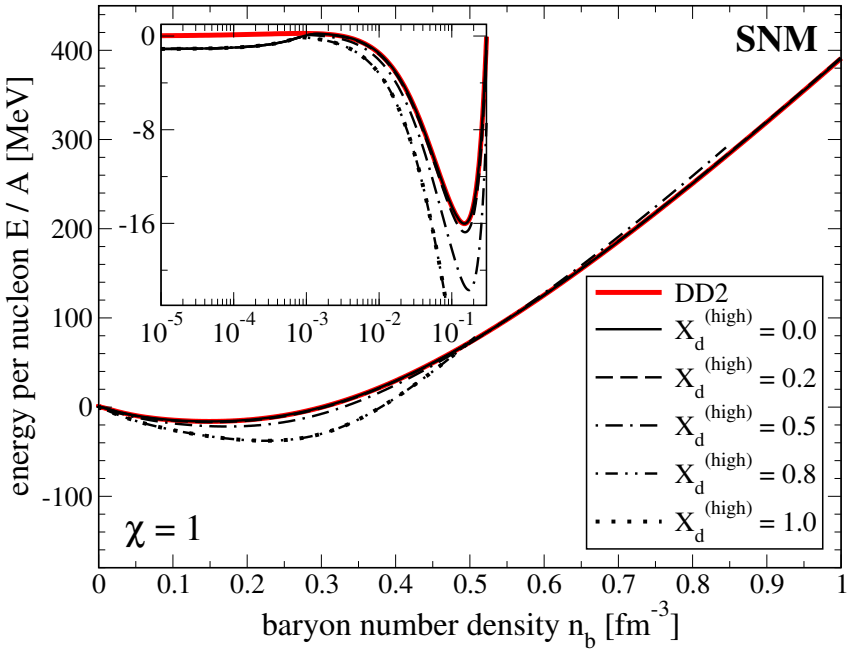

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Deuteron mass-shift parametrization

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$$\Delta m_d(x) = \frac{ax}{1+bx} + cx^{\eta+1}\left[1 - \tanh(x)\right] + fx^{\gamma}\tanh(gx), \qquad x = \frac{n_b}{n_0}$$

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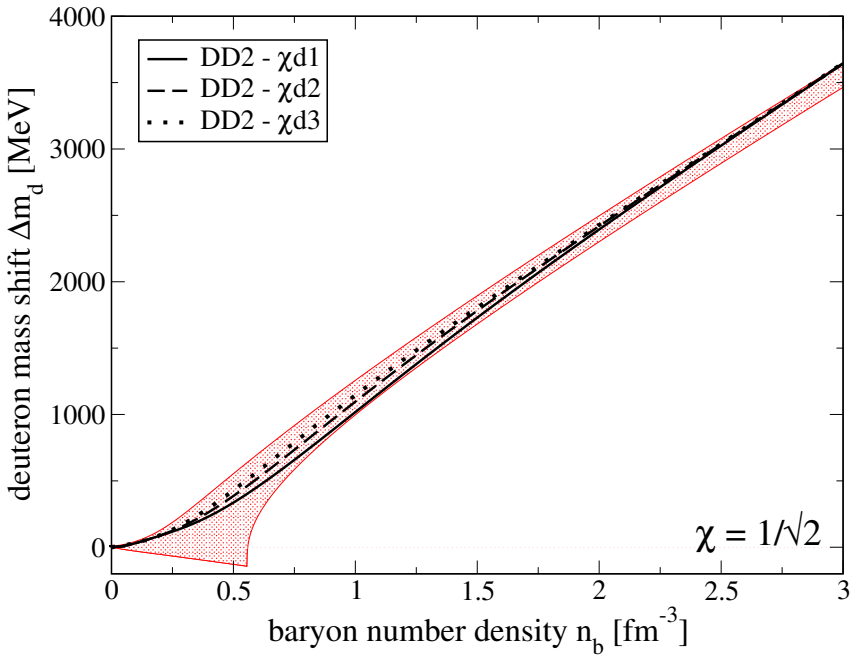

	a	b	c	η	f	g
DD2 - χd1	541.726060	243.472387	99.677247	1.656159	181.113975	0.18
DD2 - χd2	541.726060	243.472387	70.476986	1.230947	181.113975	0.22
DD2 - χd3	541.726060	243.472387	41.777908	0.257252	181.113975	0.26

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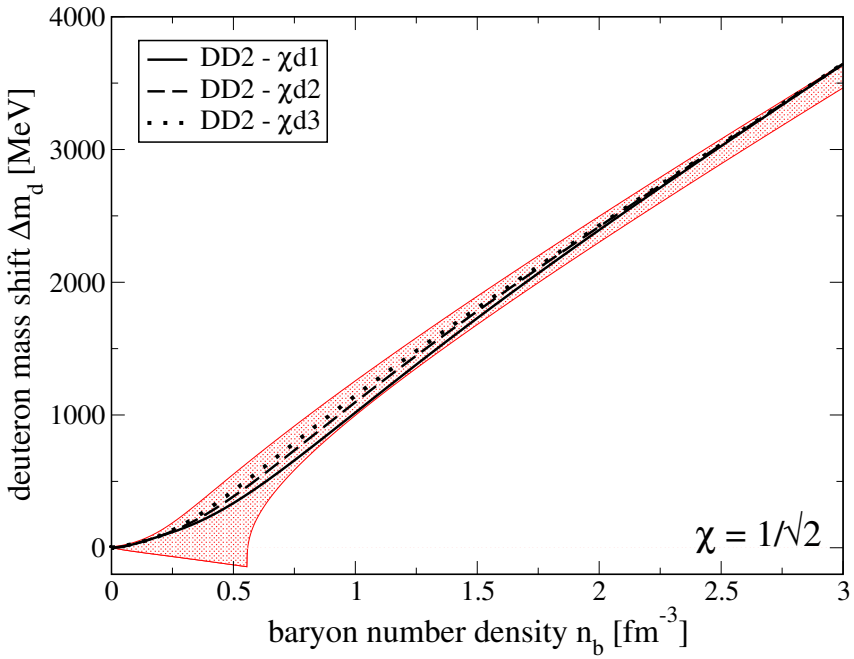

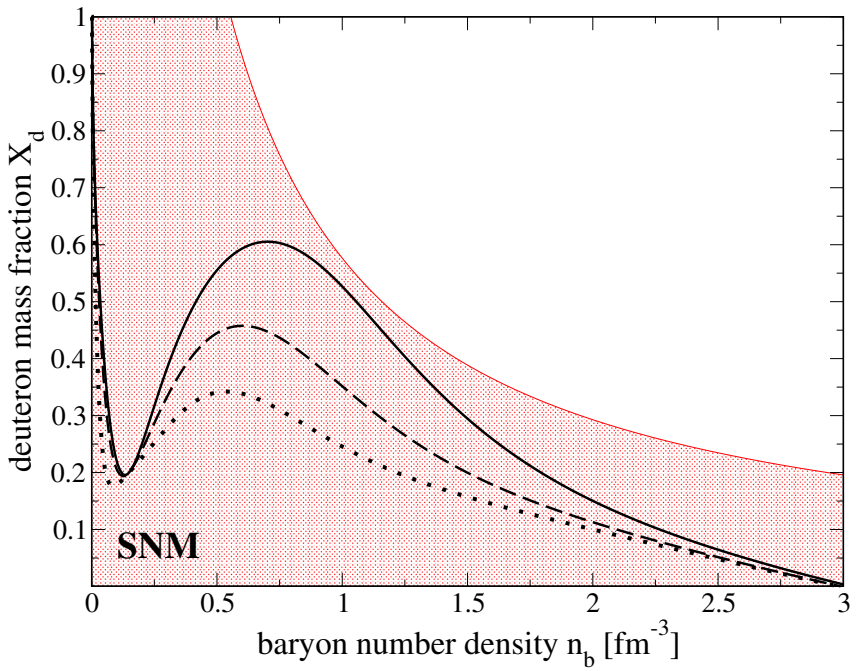

	a	b	c	η	f	g
DD2 - χd1	541.726060	243.472387	99.677247	1.656159	181.113975	0.18
DD2 - χd2	541.726060	243.472387	70.476986	1.230947	181.113975	0.22
DD2 - χd3	541.726060	243.472387	41.777908	0.257252	181.113975	0.26

Deuteron mass-shift parametrization: $\chi = 1/\sqrt{2}$

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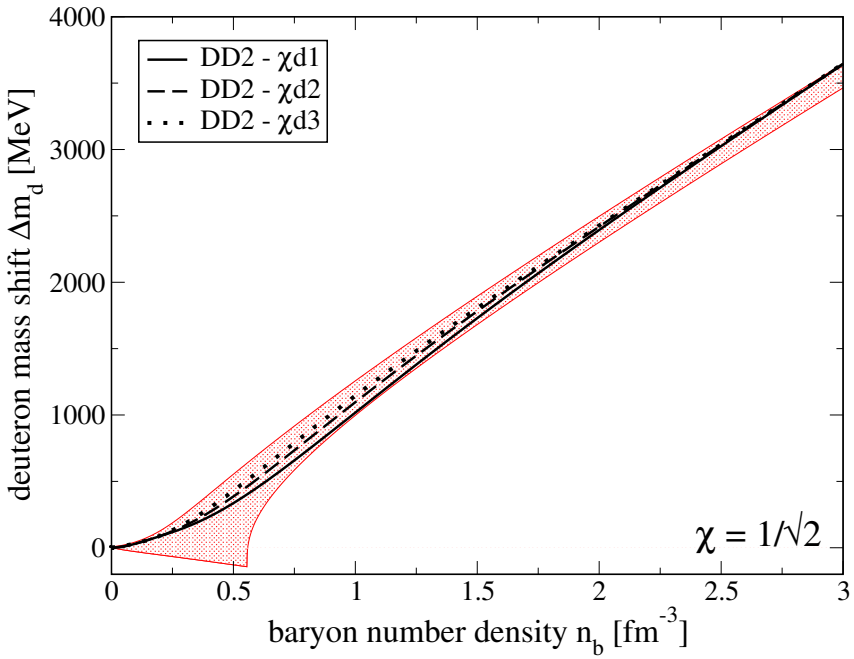

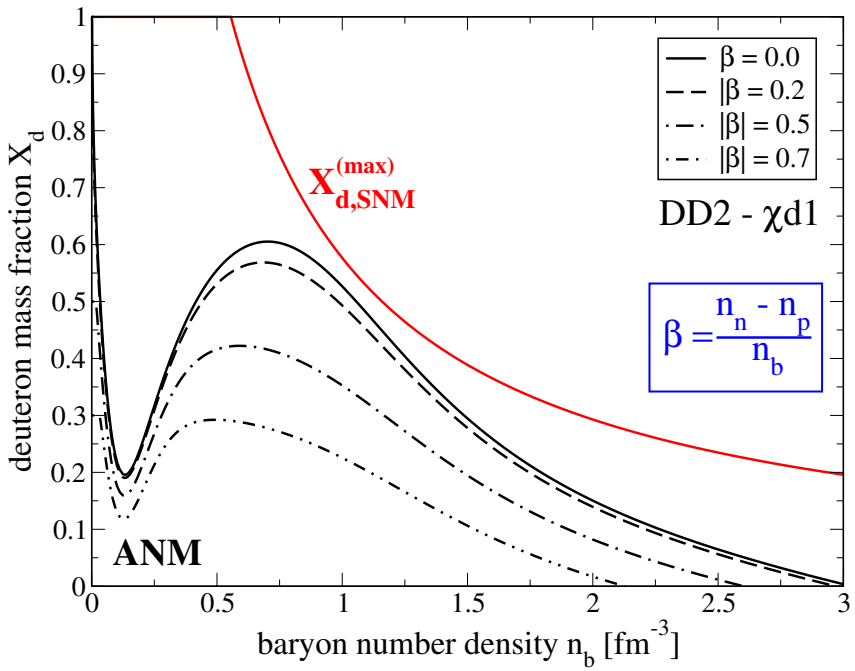

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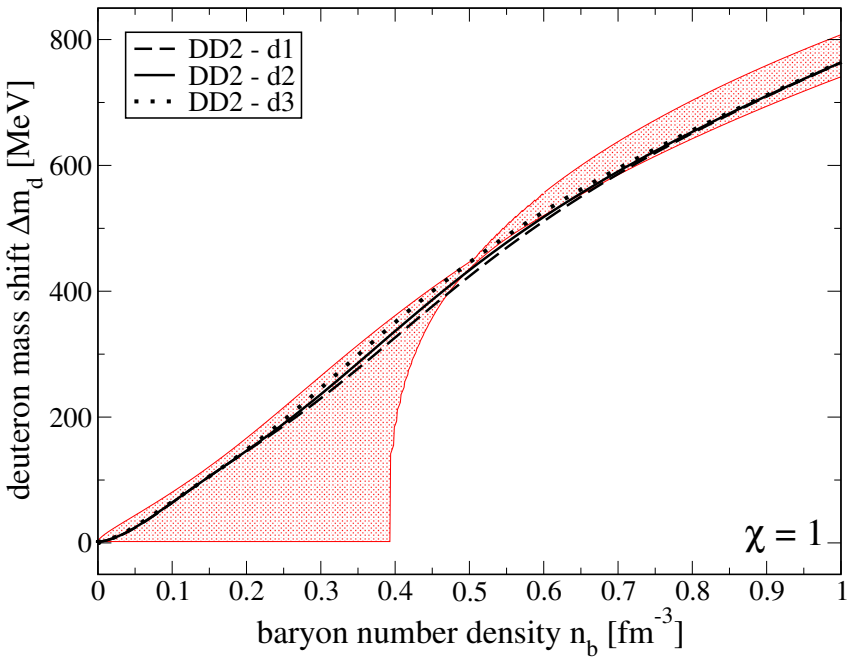
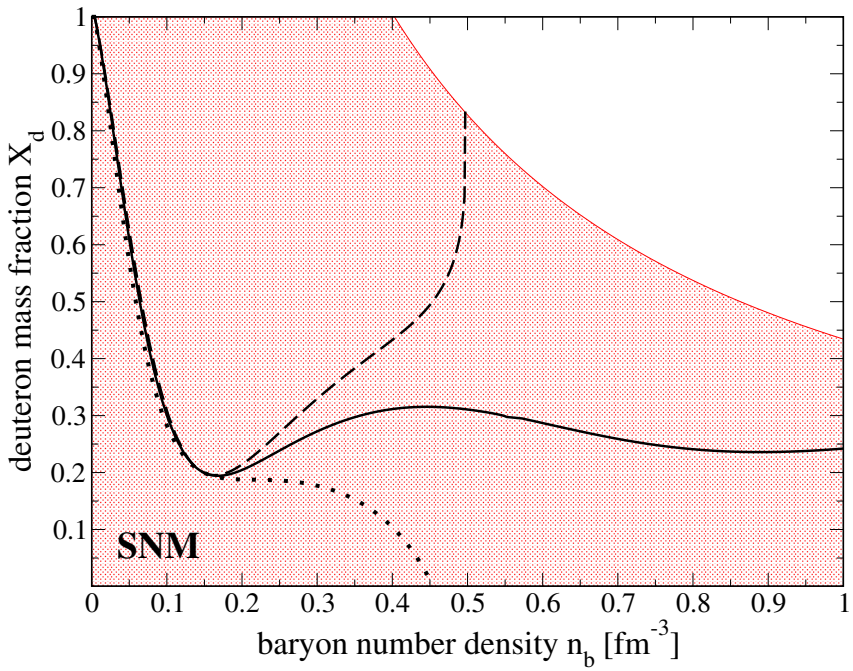
Deuteron mass-shift parametrization: $\chi = 1$

- **Unified** mass-shift **parameterization** ($\gamma = 2/3$) [S. Burrello, S. Typel, EPJA 58, 120 (2022)]

$$\Delta m_d(x) = \frac{ax}{1+bx} + cx^{\eta+1}\left[1 - \tanh(x)\right] + fx^{\gamma}\tanh(gx), \qquad x = \frac{n_b}{n_0}$$



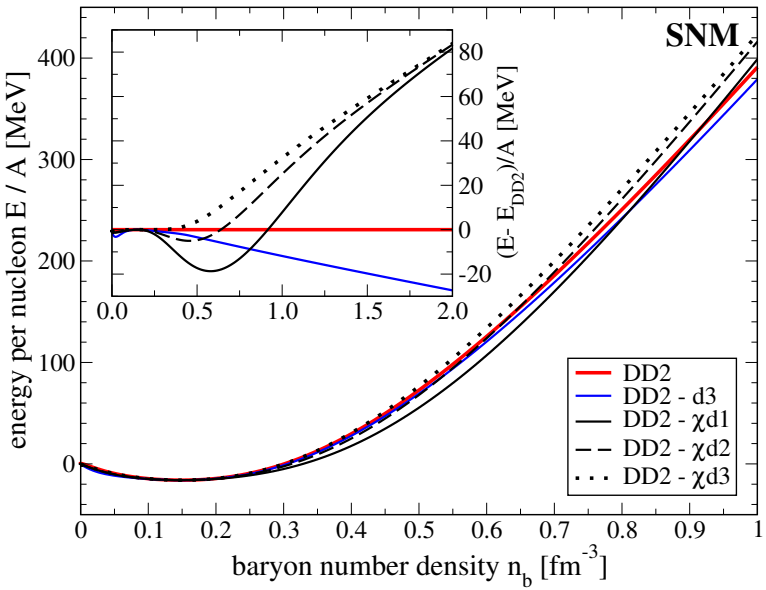


	a	b	c	η	f	g
DD2 - d1	541.726060	243.472387	−83.230901	3.491787	214.368137	0.65
DD2 - d2	541.726060	243.472387	−98.923123	3.200967	214.368137	0.67632
DD2 - d3	541.726060	243.472387	−140.309501	2.715545	214.368137	0.75

SNM: impact on EoS and matter incompressibility

- **Attraction** in presence of **quasi-deuterons** $\Longleftrightarrow$ attraction/repulsion for Γ_i-**refit**

[S. Burrello, S. Typel, EPJA 58, 120 (2022)]



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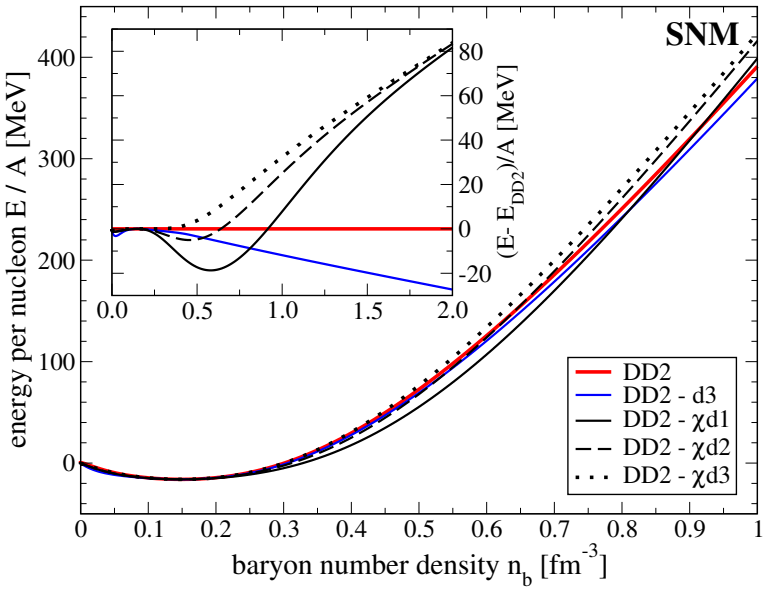

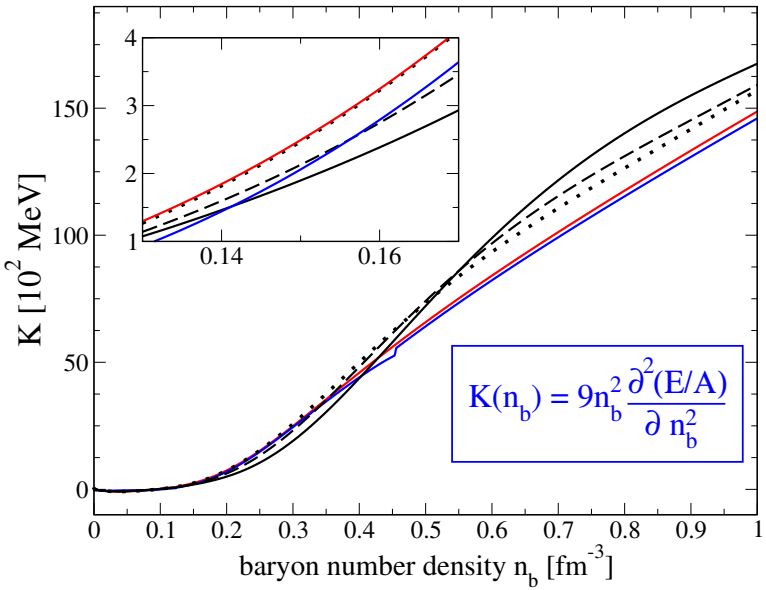

	DD2	DD2-d3	DD2-χd1	DD2-χd2	DD2-χd3
K_0 [MeV]	242.7	199.6	185.3	207.3	240.3

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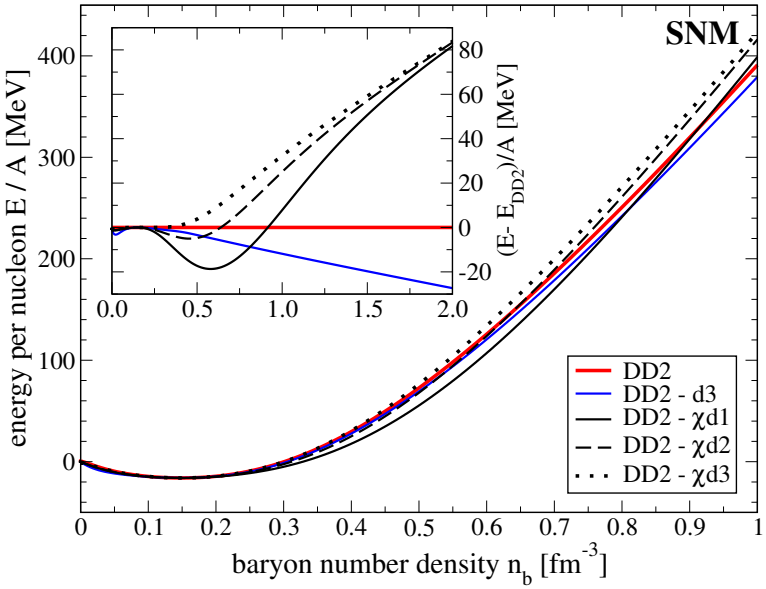
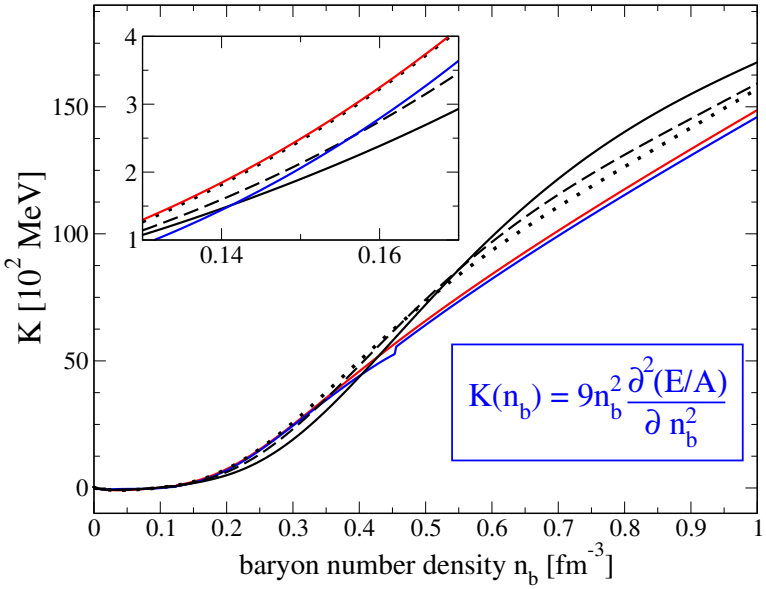
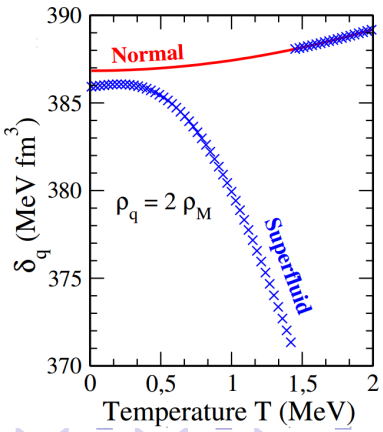




	DD2	DD2-d3	DD2-χd1	DD2-χd2	DD2-χd3
K_0 [MeV]	242.7	199.6	185.3	207.3	240.3

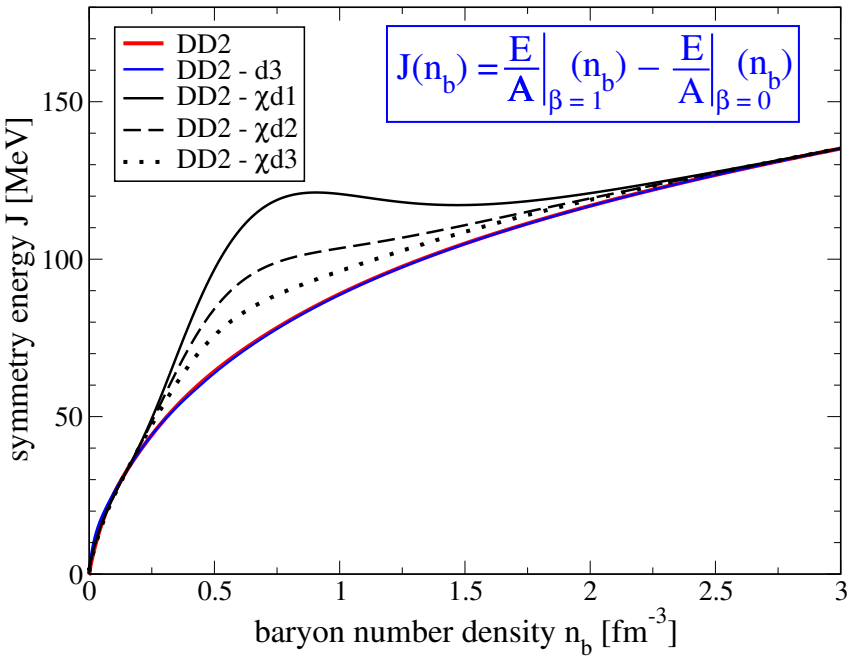
- Discontinuity $\Rightarrow$ 2^{nd} order phase transition?



[S. Burrello, M. Colonna, F. Matera, PRC 94, 012801(R) (2016)]
[S. Burrello, M. Colonna, F. Matera, PRC 89, 057604 (2014)]

Effect on symmetry energy and its slope

[S. Burrello, S. Typel, EPJA 58, 120 (2022)]



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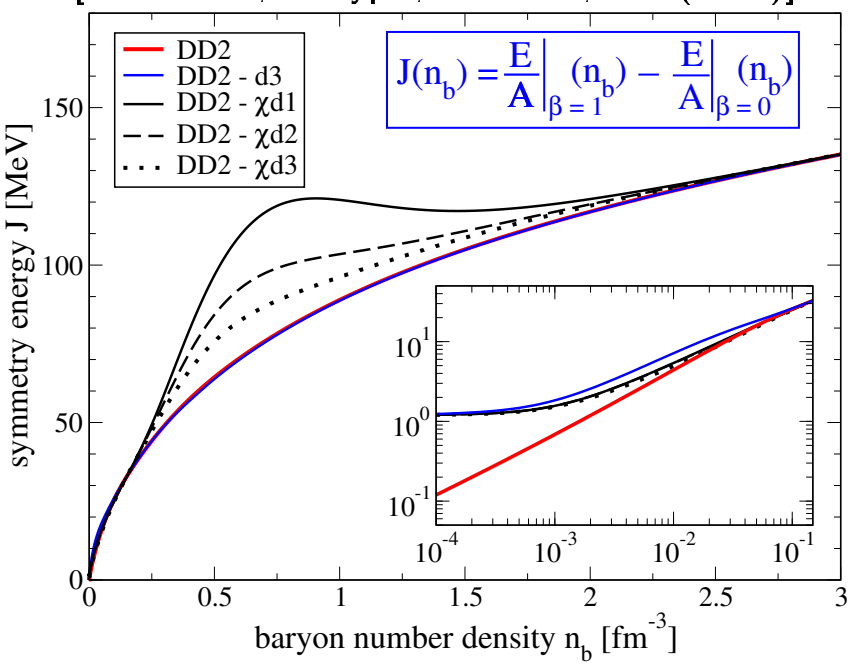

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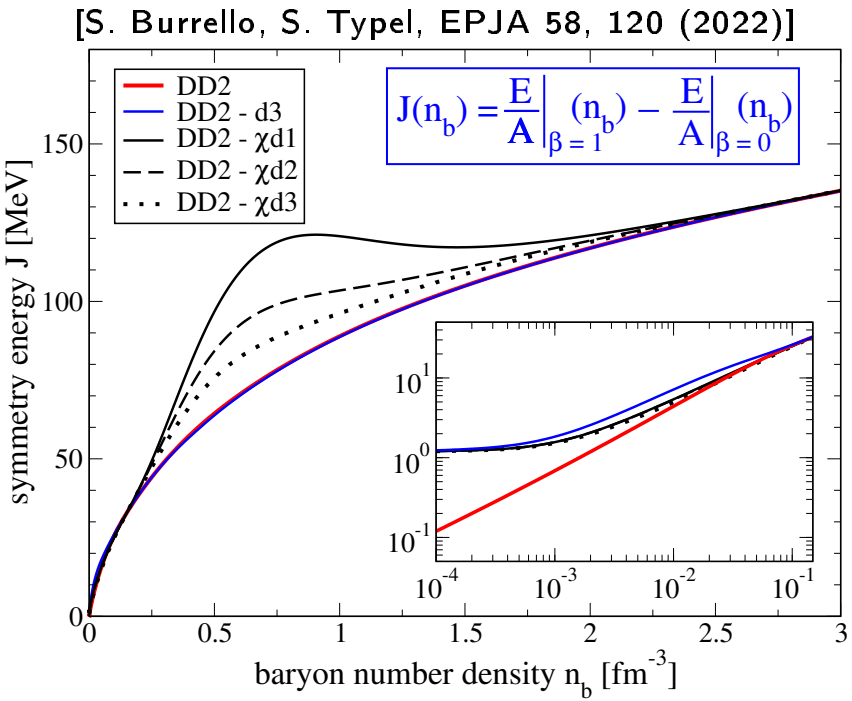

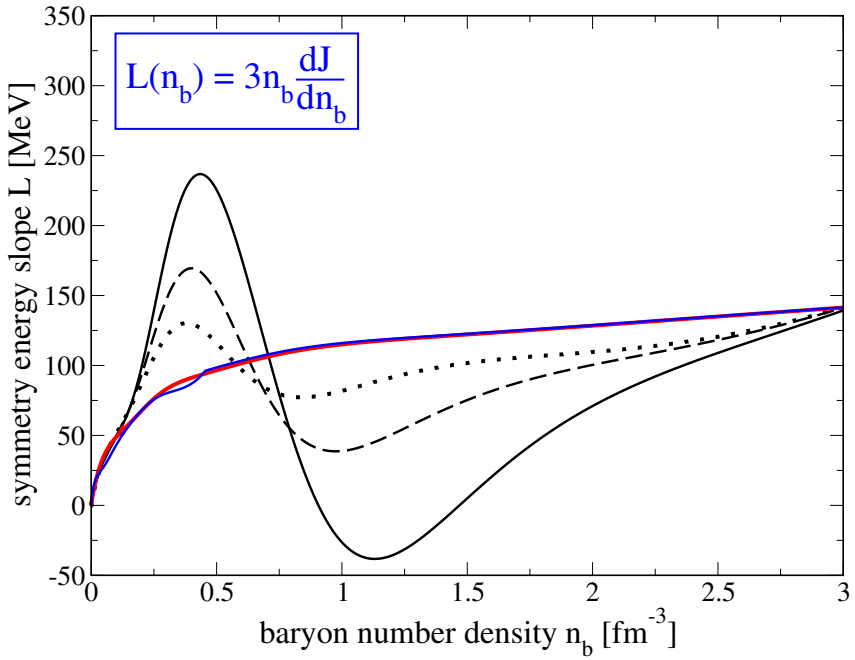

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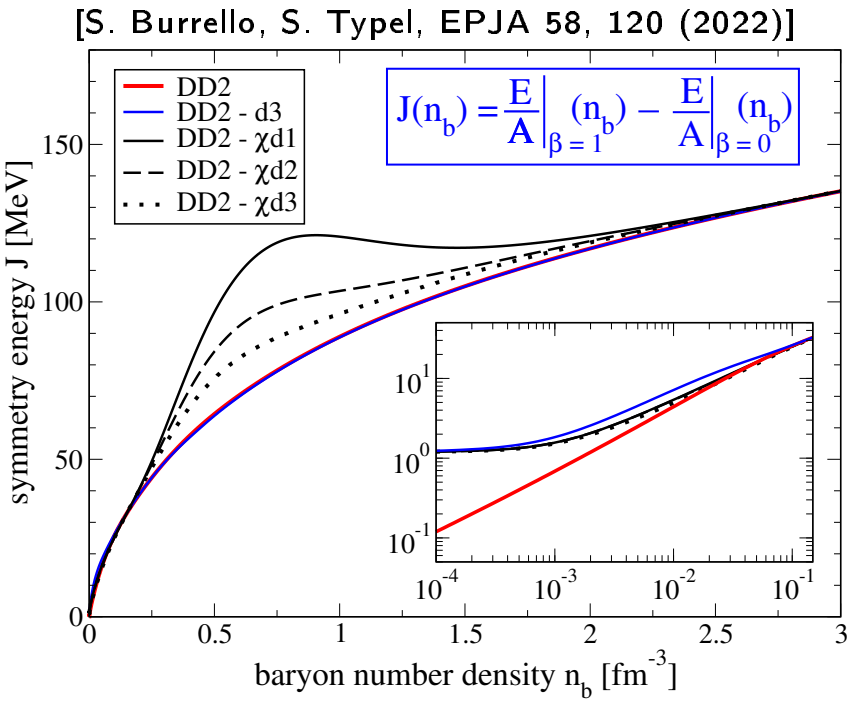

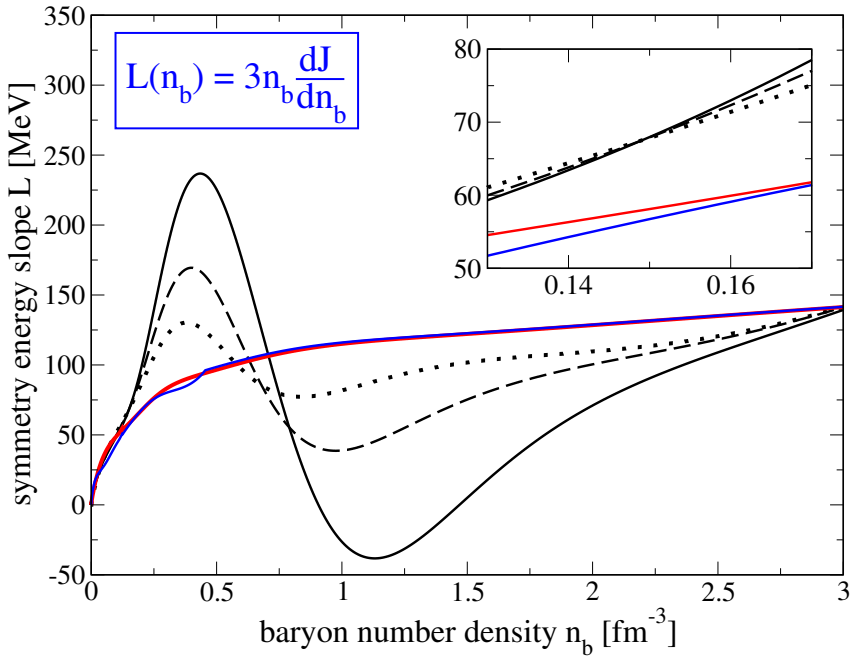

	DD2	DD2-d3	DD2-χd1	DD2-χd2	DD2-χd3
L_0 [MeV]	57.94	56.49	67.50	67.50	67.50

Final remarks and conclusions

Main topic

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THANK YOU FOR YOUR KIND ATTENTION!