Introduction to Nuclear Effective Field Theory

Bira van Kolck

Outline

- Why EFT?
- What is EFT?
- QCD + ...
- Chiral EFT
- Pionless EFT
- Halo/Cluster EFT
- Conclusion

Why EFT?

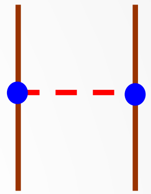
Central goal of nuclear physics: How does nuclear structure emerge from the Standard Model?

R. Machleidt, arxiv:nucl-th/0609050

Table 1. Seven Decades of Struggle: The Theory of Nuclear Forces

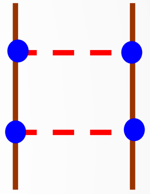
1935	**Yukawa: Meson Theory**
1950's	*The "Pion Theories"* One-Pion Exchange: o.k. Multi-Pion Exchange: disaster
1960's	Many pions ≡ multi-pion resonances: $\sigma, \rho, \omega, \ldots$ The One-Boson-Exchange Model
1970's	Refine meson theory: Sophisticated 2π exchange models (Stony Brook, Paris, Bonn)
1980's	Nuclear physicists discover **QCD** Quark Cluster Models
1990's and beyond	Nuclear physicists discover **EFT** Weinberg, van Kolck **Back to Meson Theory!** *But, with Chiral Symmetry*

Long-range physics
cf. photon exchange



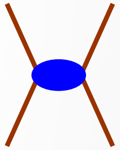
$R \sim \hbar / m_\pi c$

No renormalization!
cf. QED

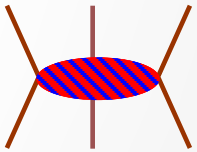


+ ... $\to \infty$

Phenomenological models
for short-range physics;
three-body forces?



$r \ll R$



$= ?$

Fail to account
for pion physics

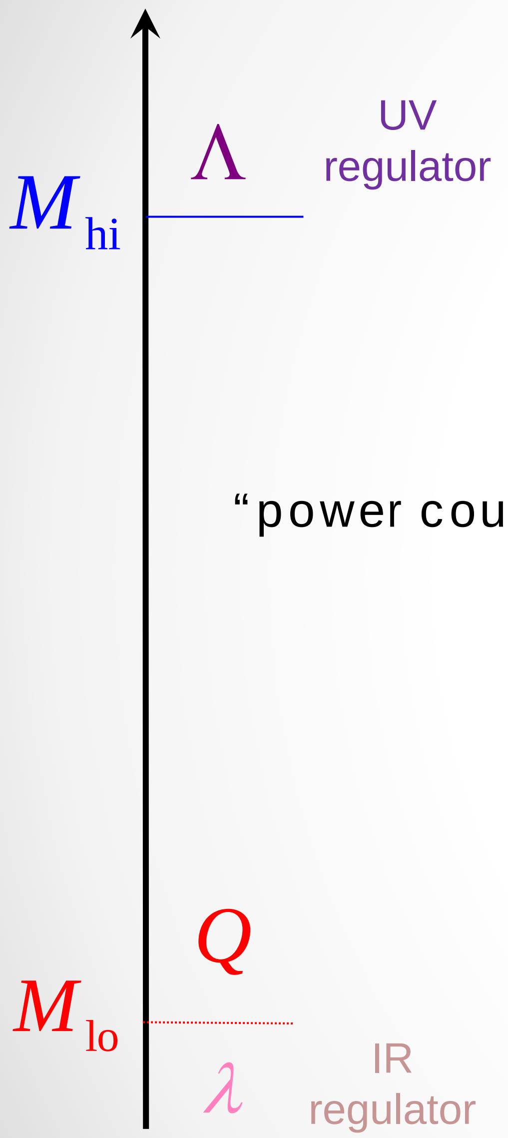
Next: 30 y in 30 min…

And, much more!

What is EFT?

Modern *S*-Matrix Theory!

Euler + Heisenberg '36
Weinberg '67 … '79
…



most general Lagrangian

"power counting" $\nu = \nu(s, d, N, \ldots)$

most general *S* matrix

$$\mathcal{L}_{\rm EFT} = \sum_{\nu=0}^{\infty} \underbrace{\gamma_i^{(\nu)}\left(\frac{M_{\rm lo}}{\Lambda}, \frac{\lambda}{M_{\rm lo}}\right) \frac{M_{\rm lo}^{s}}{M_{\rm hi}^{\nu}}}_{\text{"low-energy constants"/"Wilson coefficients"}} \underbrace{O^{(\nu)}\left(\partial^d \psi^N\right)}_{\text{operators}}$$

$$S^{(\bar{\nu})}(Q \sim M_{\rm lo} \ll M_{\rm hi}) - 1 \propto \sum_{\nu=0}^{\bar{\nu}} \left[\frac{Q}{M_{\rm hi}}\right]^{\nu} \overbrace{F^{(\nu)}\left(\frac{Q}{M_{\rm lo}}, \frac{Q}{\Lambda}, \frac{\lambda}{Q}; \gamma_i^{(\le \nu)}\left(\frac{M_{\rm lo}}{\Lambda}, \frac{\lambda}{M_{\rm lo}}\right)\right)}^{\text{regulator-independent, non-analytic functions, from (finite or infinite number of) loops}}$$

$$\times \left\{1 + \mathcal{O}\left(\frac{Q^{\bar{\nu}+1}}{M_{\rm hi}^{\bar{\nu}+1}}, \underbrace{\frac{Q^{\bar{\nu}+1}}{M_{\rm hi}^{\bar{\nu}}\Lambda}, \frac{\lambda Q^{\bar{\nu}}}{M_{\rm hi}^{\bar{\nu}+1}}}_{\text{RENORMALIZATION}}\right)\right\}$$

N$^{\bar{\nu}}$LO

CONTROLLED UNCERTAINTY

prior

reproduces underlying theory

MODEL INDEPENDENCE

QC(+E)D (LITE)

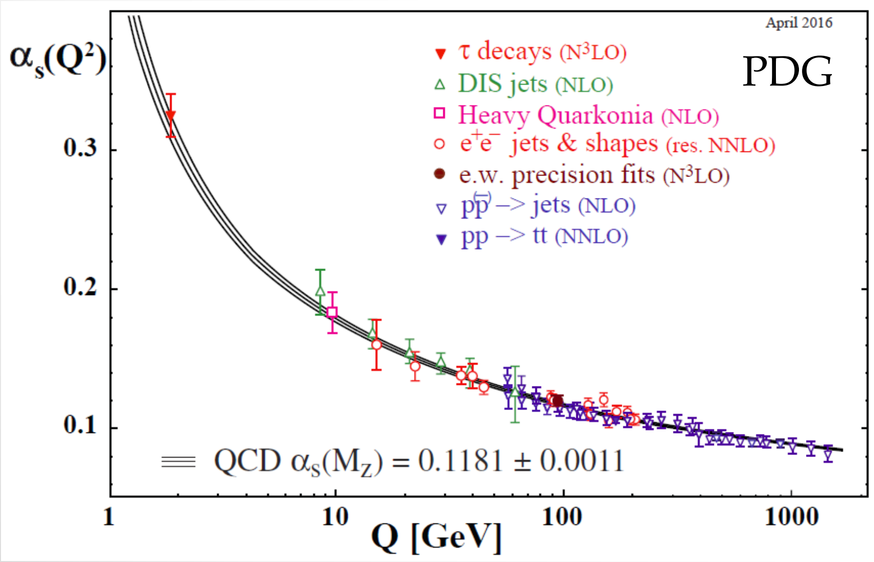
d.o.f.s quarks: $q = \begin{pmatrix} u \\ d \end{pmatrix}$ gluons: G^a_μ (+ photon)

symmetries SO(3,1) global, $SU(3)_c$ $(+U(1)_{em})$ gauge

$Q \ll m_{W,Z}$

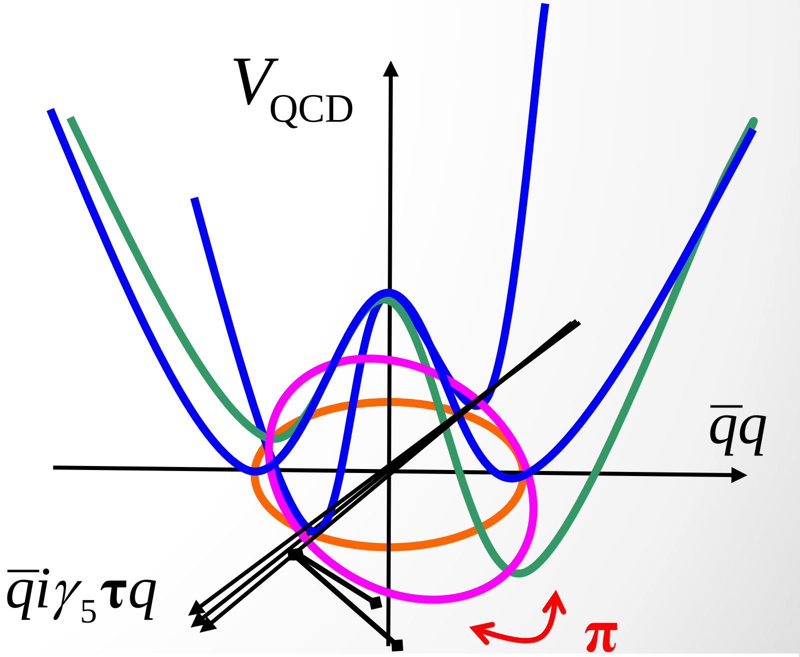
$$\mathcal{L}_{QCD} = \underbrace{\bar{q}\left(i\partial\!\!\!/ + g_s G\!\!\!/\right)q - \frac{1}{2}\mathrm{Tr}\, G^{\mu\nu}G_{\mu\nu}}_{M_{QCD} \sim m_N, m_\rho, 4\pi f_\pi, \ldots \sim 1\ \mathrm{GeV}} + \underbrace{\bar{m}\,\bar{q}\left(1 - \varepsilon\tau_3\right)q}_{m_\pi \sim \sqrt{\bar{m} M_{QCD}} \simeq 140\ \mathrm{MeV}} + \ldots$$

Basic
mass scales



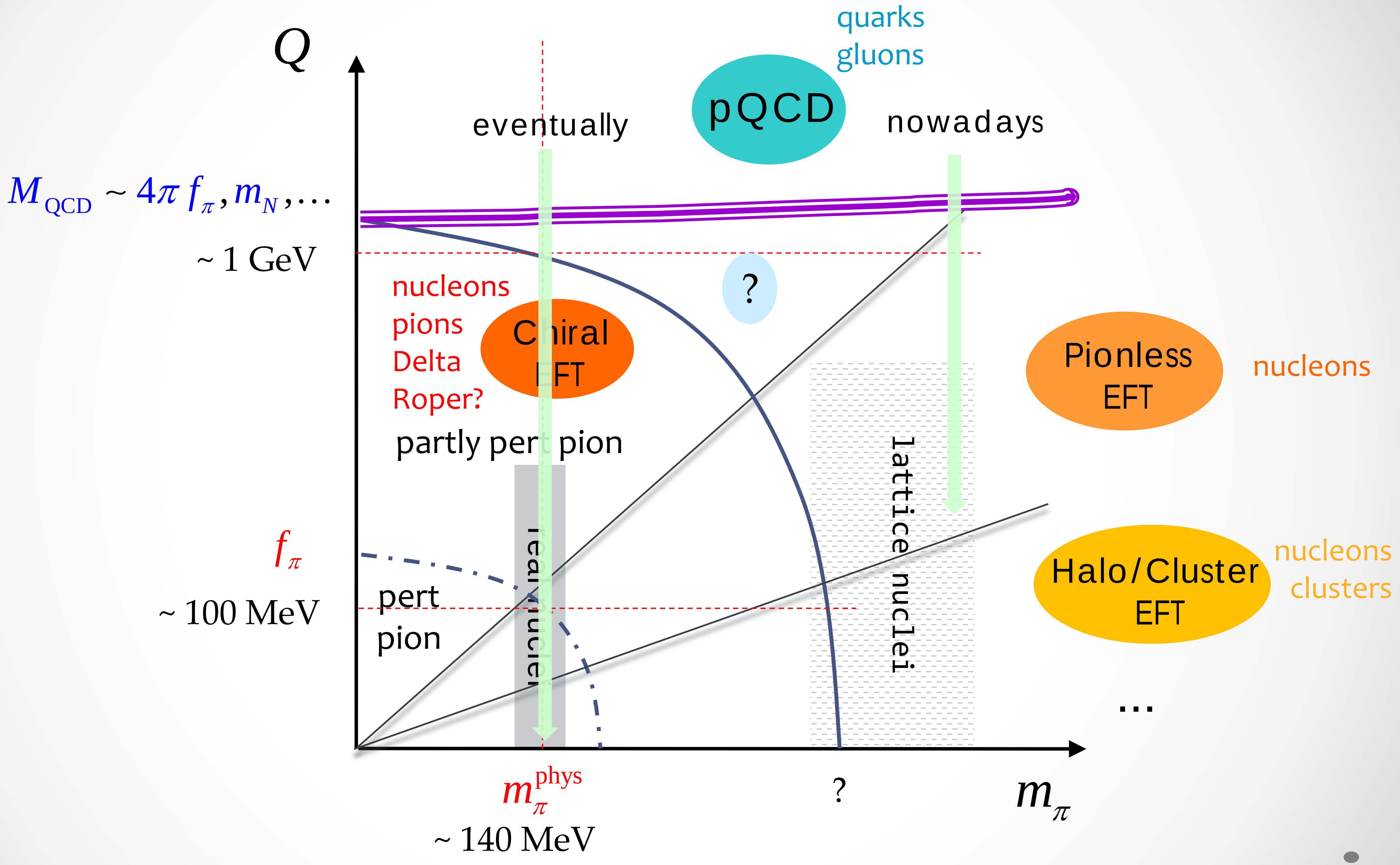
Gross + Wilczek '73
Politzer '73
…

Nambu '64
…



$$f_\pi \sim M_{QCD}/4\pi + \mathcal{O}(\bar{m}) \simeq 100\ \mathrm{MeV}$$

The Nuclear EFT Landscape



Chiral EFT

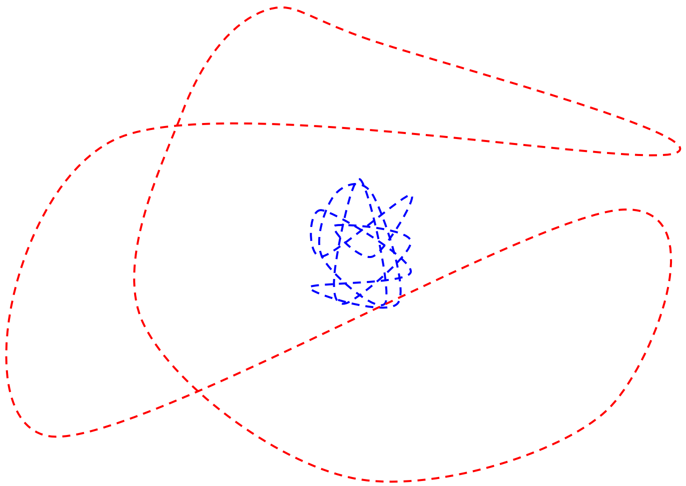
$Q \sim m_\pi \ll M_{\text{QCD}}$

slow-moving nucleon

nonrelativistic expansion

$$\frac{Q}{m_N} \sim \frac{Q}{M_{\text{QCD}}}$$

long-ranged but sparse



$\lambda \sim 1/m_\pi$

chiral symmetry:
pion interactions $\propto Q,\ m_\pi^2$

weak!

$1/M_{\text{QCD}} \approx 0.3$ fm

dense but short-ranged

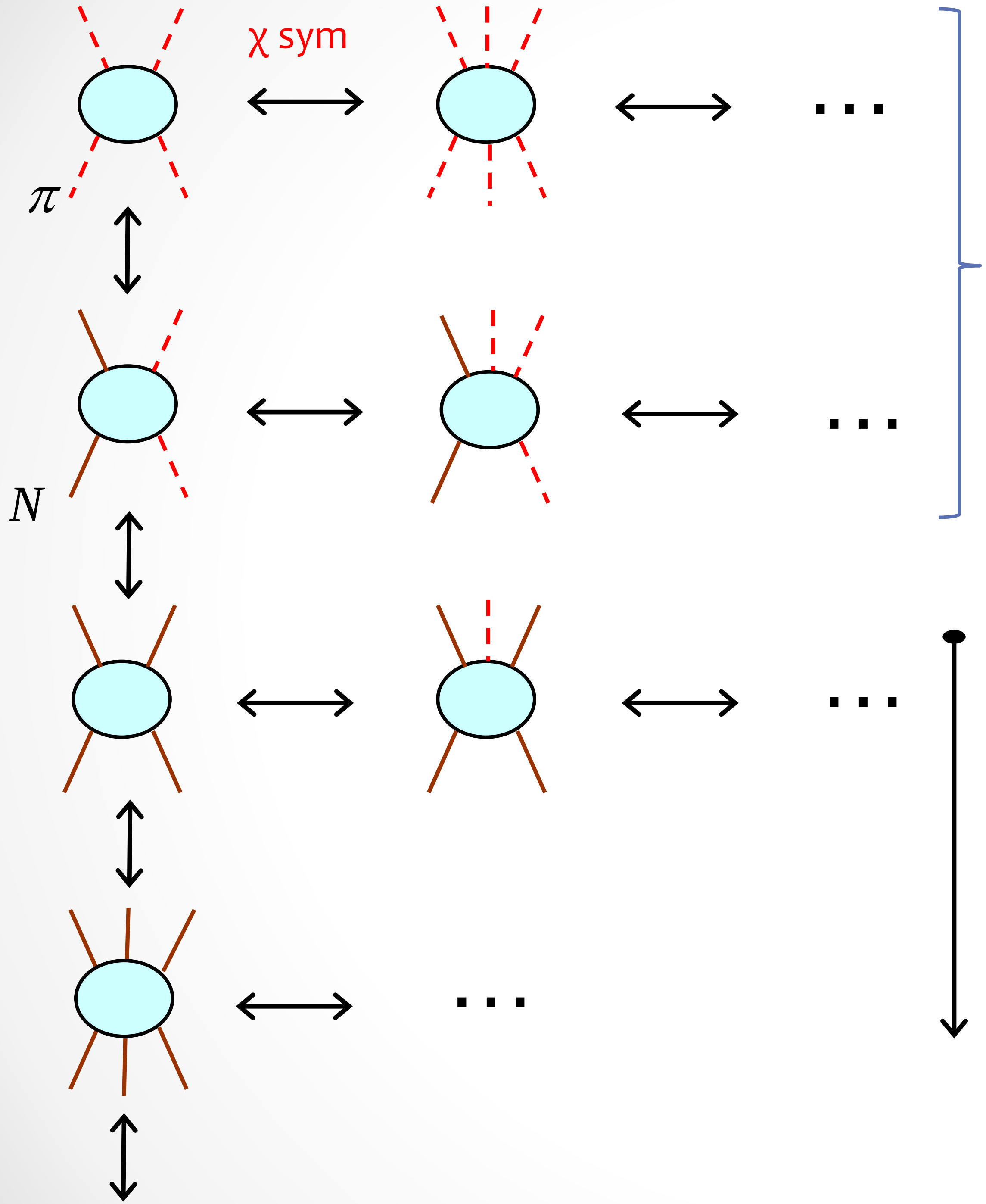
multipole expansion

$$\frac{Q}{m_\rho}, \ldots \sim \frac{Q}{M_{\text{QCD}}}$$

pion loop expansion

$$\frac{Q}{4\pi f_\pi} \sim \frac{Q}{M_{\text{QCD}}}$$

$1/m_\pi \cong 1.4$ fm



Weinberg ’79
Gasser + Leutwyler ’84
…

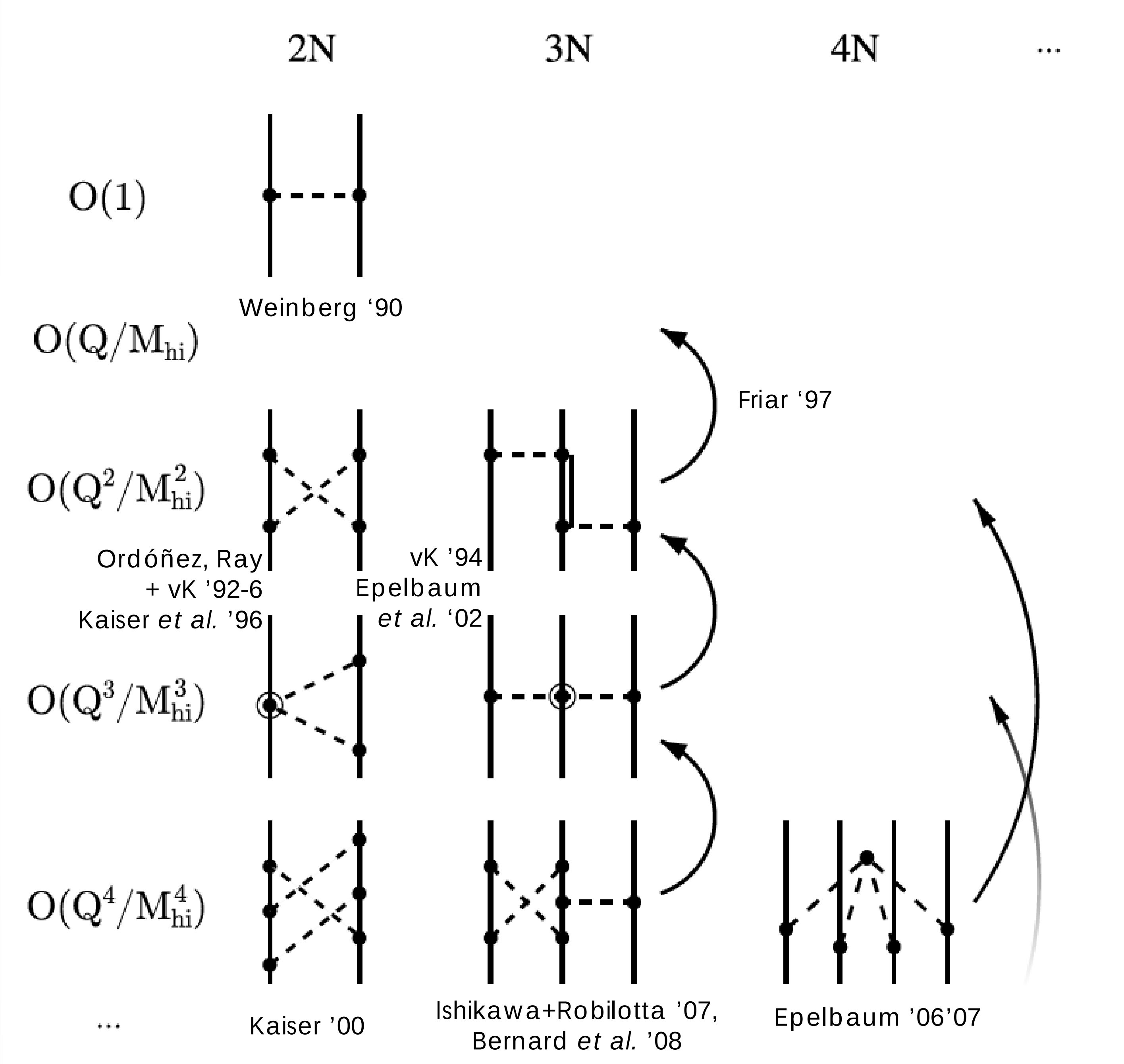
Chiral Perturbation Theory

Gasser, Sainio + Švarc ’87
Bernard, Kaiser + Meißner ‘90
Jenkins + Manohar ’91
…

Non-perturbative at leading order!

Weinberg ’90
Rho ’91
Weinberg ‘91
Ordóñez + vK ’92
Weinberg ‘92
vK ’94
…

Long-range, isospin-symmetric nuclear potential



Hammer, König, vK, *Rev. Mod. Phys.* 92 (2020) 025004

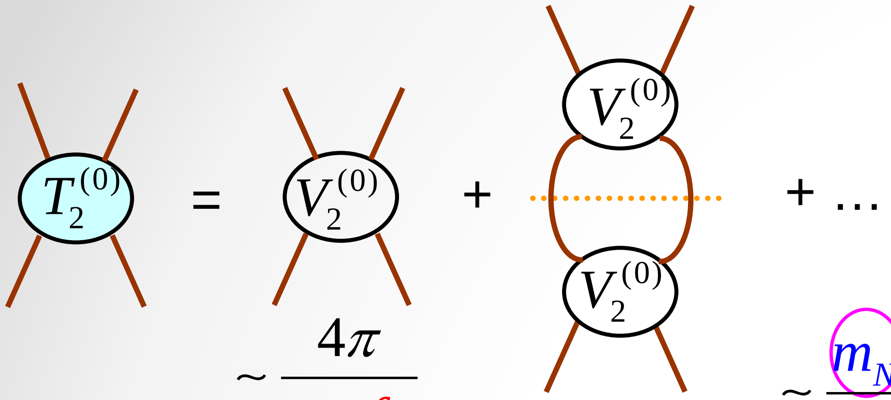
\+

isospin violation

vK '95

vK, Friar + Goldman '96

...



$$\sim \frac{4\pi}{m_N f_\pi} \qquad \sim \frac{m_N Q}{4\pi}$$

Weinberg '90

infrared
enhancement!

$$\sim \frac{4\pi}{m_N f_\pi} \frac{1}{1 - \frac{Q}{f_\pi}}$$

Bedaque
+ vK '02

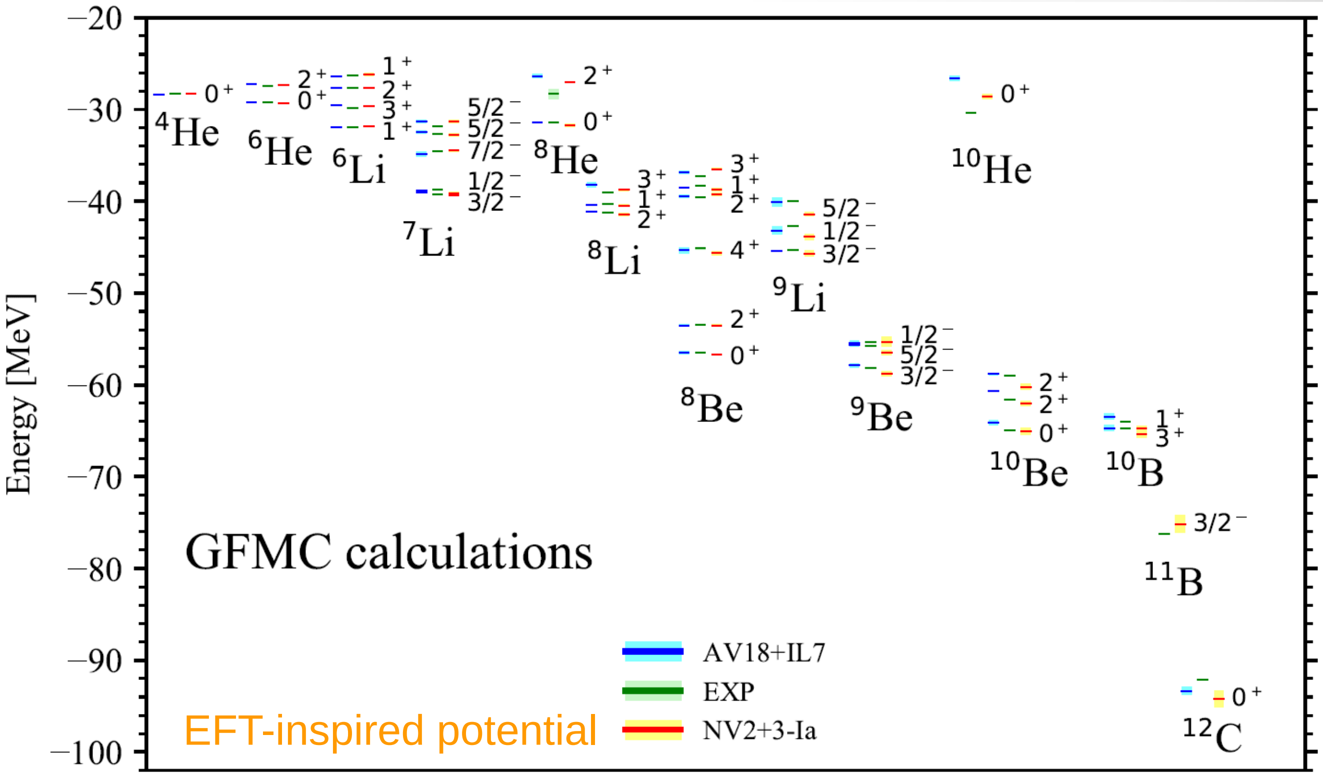
bound state

$$Q \sim f_\pi \ll M_{\rm QCD}$$

$$-E \sim \frac{f_\pi^2}{M_{\rm QCD}} \sim 10 \text{ MeV}$$

Nuclear scale arises naturally
in QCD due to spontaneous
chiral symmetry breaking

Example
(contact interactions suggested
by naïve dimensional analysis)



M. Piarulli *et al.*, *Phys. Rev. Lett.* 120 (2018) 052503

but $E(r) \sim \frac{1}{m_N r^2} - \frac{g_{\pi N}^2}{m_N} \frac{e^{-m_\pi r}}{r^3} S_{12}(\hat{r})$ fall to the center

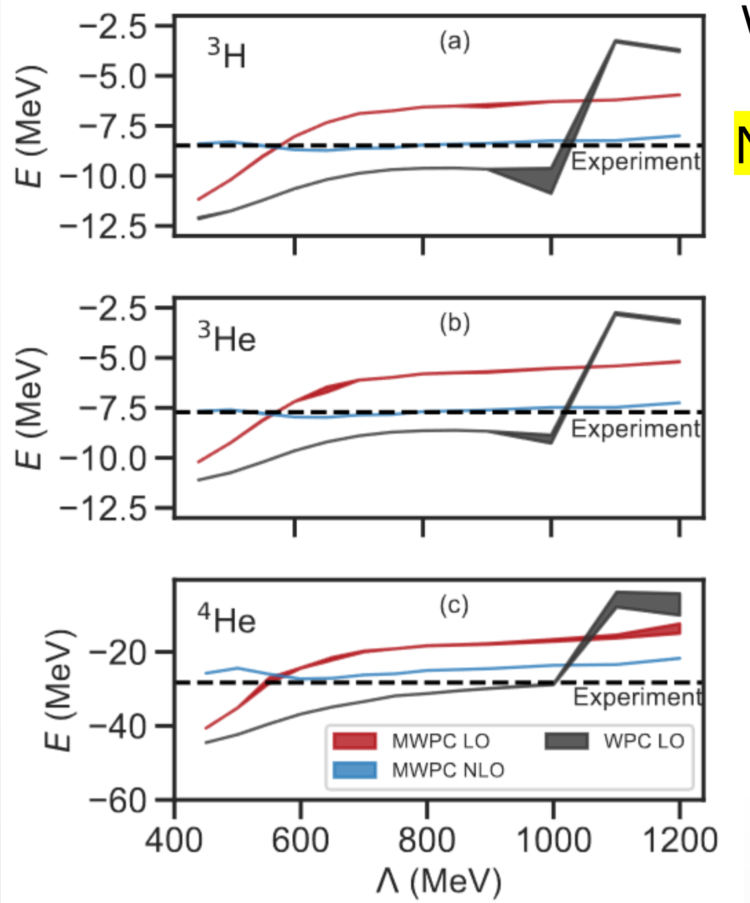
Nogga, Timmermans + vK '05
Pavón Valderrama + Ruiz Arriola '06
…

+ breakdown of naïve pion-mass expansion

Kaplan, Savage + Wise '96
…

- enhanced contact interactions
- pions not iterated in high partial waves
- perturbative treatments of corrections

works well for light nuclei

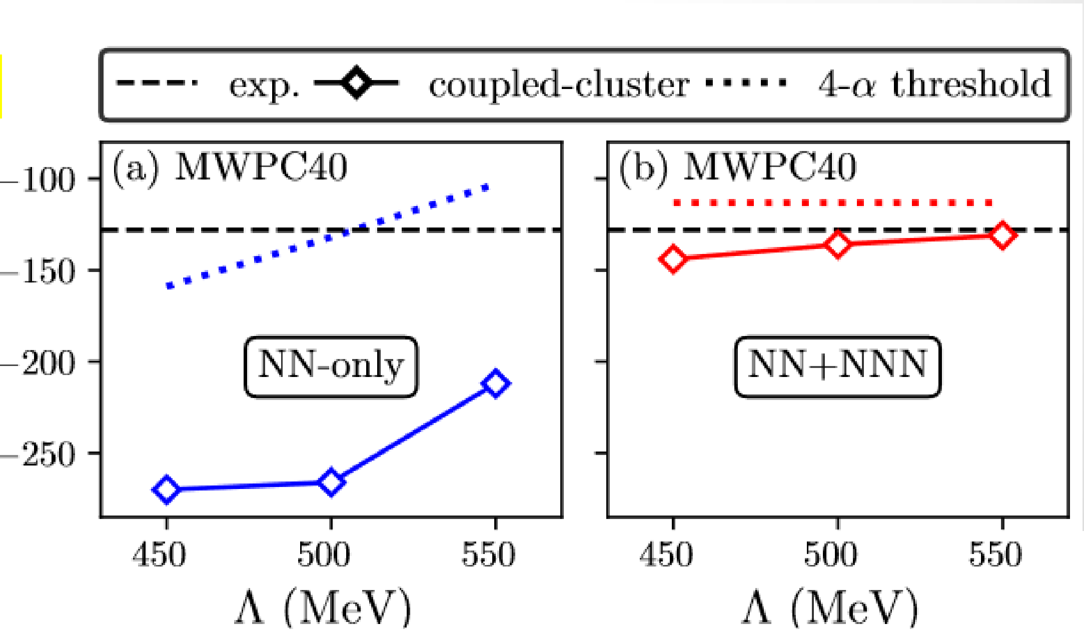


Weinberg LO
LO
NLO

Yang, Ekström, Forssén, Hagen, *Phys. Rev. C* 103 (2021) 054304

+ combinatorial enhancement of few-body forces for larger nuclei?

LO



Yang, Ekström, Forssén, Hagen, Rupak, vK, arXiv: 2109.13303

External probes

→ Electroweak form factors

Rho '91
Park, Min + Rho '93
...

→ Pion elastic scattering

Weinberg '92
Beane *et al.* '98
...

→ Pion photoproduction

Beane, Lee + vK '95
Beane *et al.* '97
...

→ Pion production

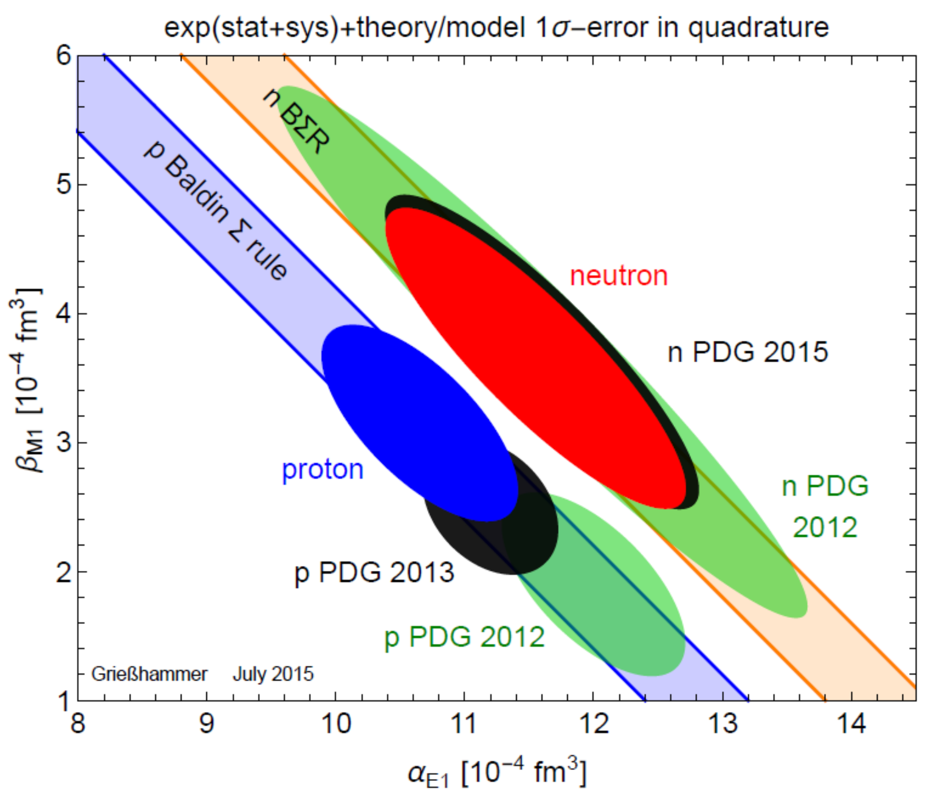
Kubodera *et al.* '96
Cohen, Friar, Miller + vK '96
...

→ Compton scattering

Beane, Malheiro, Phillips + vK '99
Beane, Malheiro, McGovern, Phillips + vK '03
...

→ *etc.*

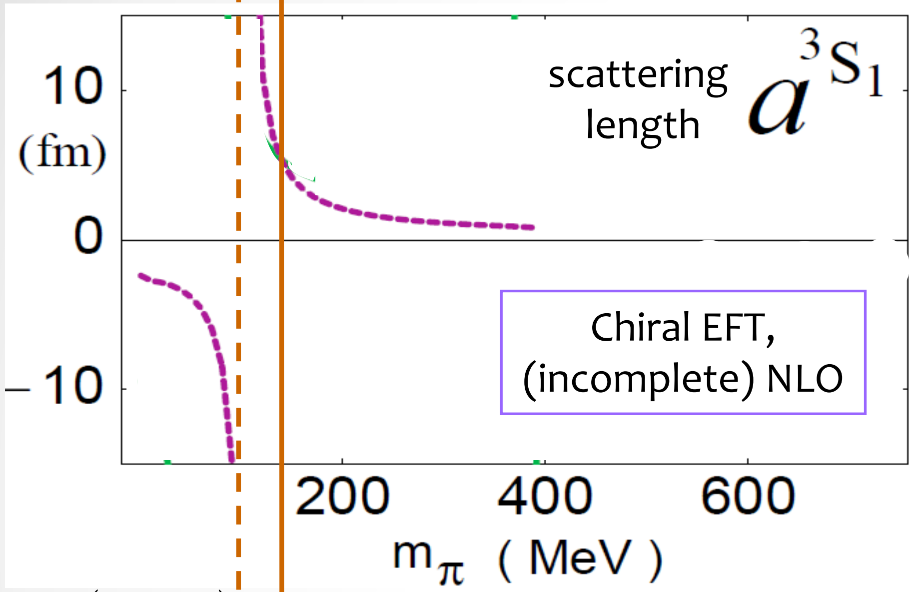
Example
Nucleon polarizabilities



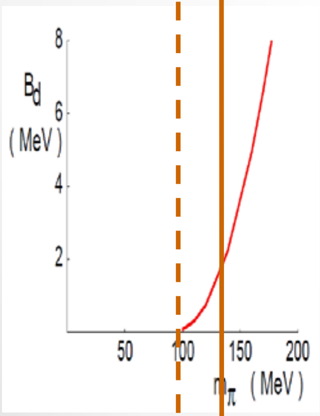
Grießhammer, McGovern, Phillips,
arXiV: 1509.09177

Possible quark-mass dependence

unitarity limit



$m_\pi^*\left(M_{QCD}\right)$ $\quad m_\pi \simeq 140\text{ MeV}$



Beane, Bedaque, Savage, vK, *Nucl. Phys. A* 700 (2002) 377

new scale

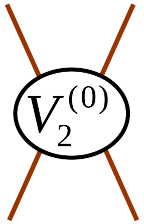
$$a_2^{-1} \sim \frac{m_\pi - m_\pi^*}{m_\pi^*} f_\pi \equiv \aleph$$

Feshbach resonance
in quark masses
~ pion mass!

Pionless EFT

Bedaque + vK '97
vK '97
Kaplan, Savage + Wise '98
...

pion exchange → contact



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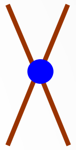
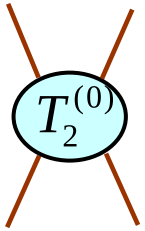


$$\sim \frac{4\pi}{m_N \aleph}$$



$$\sim \frac{4\pi}{m_N \aleph} \frac{1}{1 - \frac{Q}{\aleph}}$$

bound state

$$Q \sim \aleph \sim 30\text{ MeV}$$

$$-E \sim \frac{\aleph^2}{M_{QCD}} \sim 1\text{ MeV}$$

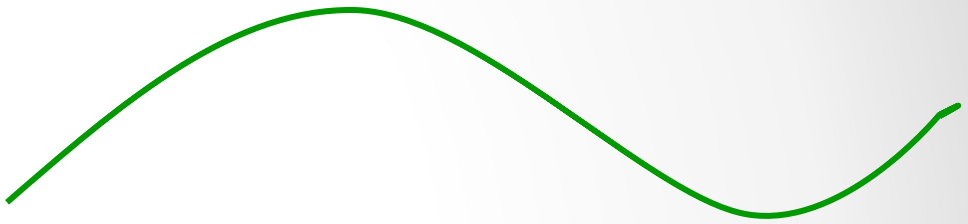
Light scale emerges and
accounts for large size of light nuclei

Pioniess EFT

$Q \sim \sqrt{2m_N B_A / A} \ll m_\pi$

slow-moving nucleon

nonrelativistic expansion $\dfrac{Q}{m_N}$



$\lambda \sim 1 \big/ \sqrt{2m_N B_A / A}$

$1/m_\pi \cong 1.4 \text{ fm}$

e.g.

$1 \big/ \sqrt{2m_N B_3 / 3} \approx 2.8 \, \text{fm}$

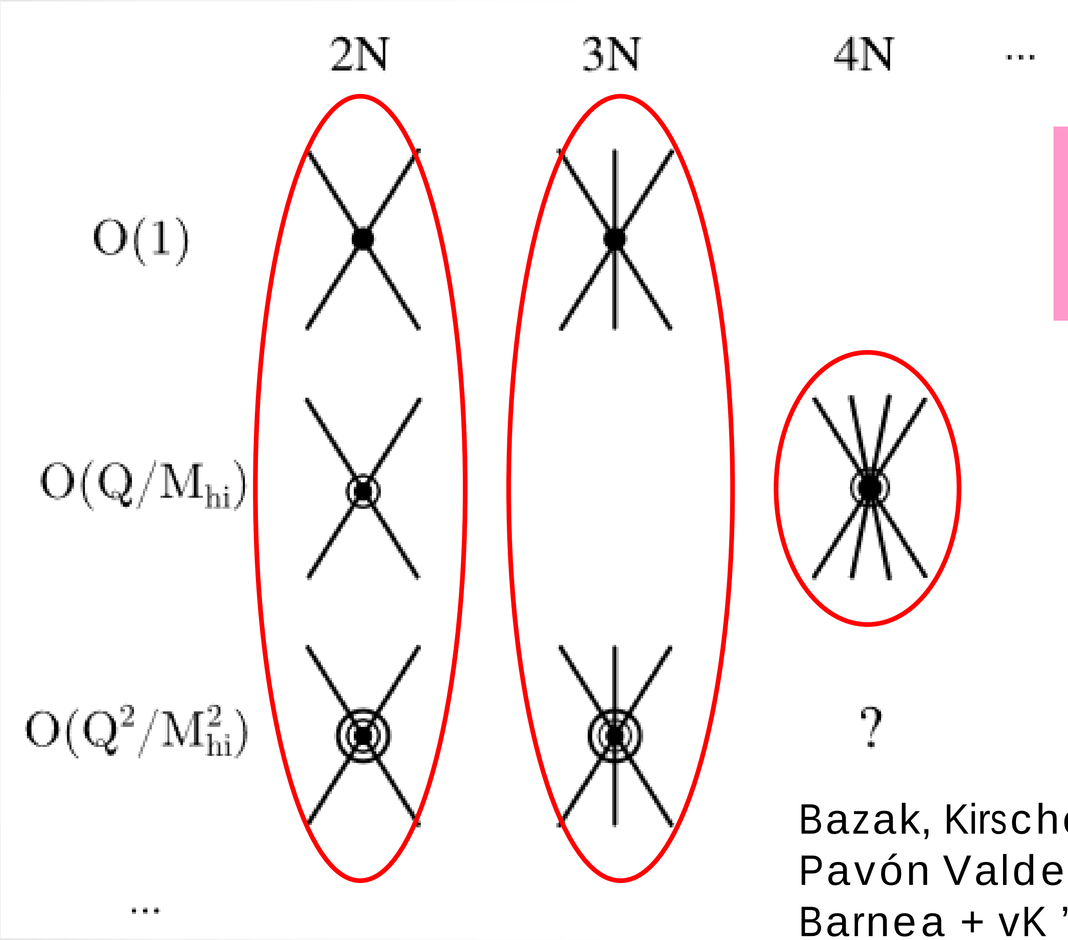
$1 \big/ \sqrt{2m_N B_A / A}$

short-ranged

multipole expansion $\dfrac{Q}{m_\pi}, \ldots$

no role for chiral symmetry

Isospin-symmetric potential



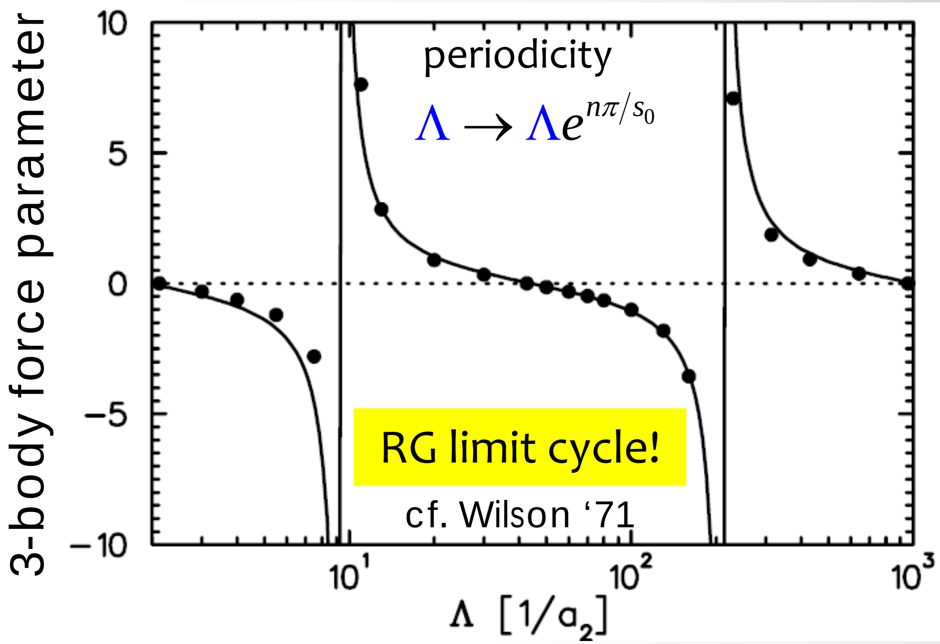
discrete scale invariance!

Bazak, Kirscher, König, Pavón Valderrama, Barnea + vK '19

vK '97'99
Kaplan, Savage + Wise '98

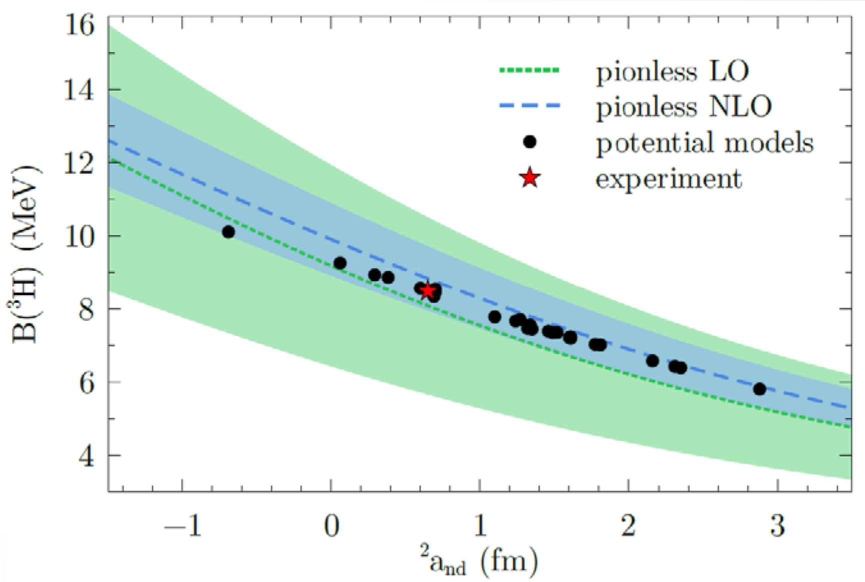
Bedaque, Hammer + vK '99 '00

Hammer, König, vK, *Rev. Mod. Phys.* 92 (2020) 025004



Bedaque, Hammer, vK, *Phys. Rev. Lett.* 82 (1999) 463

position of the cycle: one parameter Λ_*
→ correlations, e.g.



Hammer, König, vK, *Rev. Mod. Phys.* 92 (2020) 025004

External probes

→ Electroweak form factors
Chen, Rupak + Savage ’99
Phillips, Rupak + Savage ’00
…

→ Radiative capture/fusion
Chen, Rupak + Savage ’99
Kong + Ravndal ‘99
…

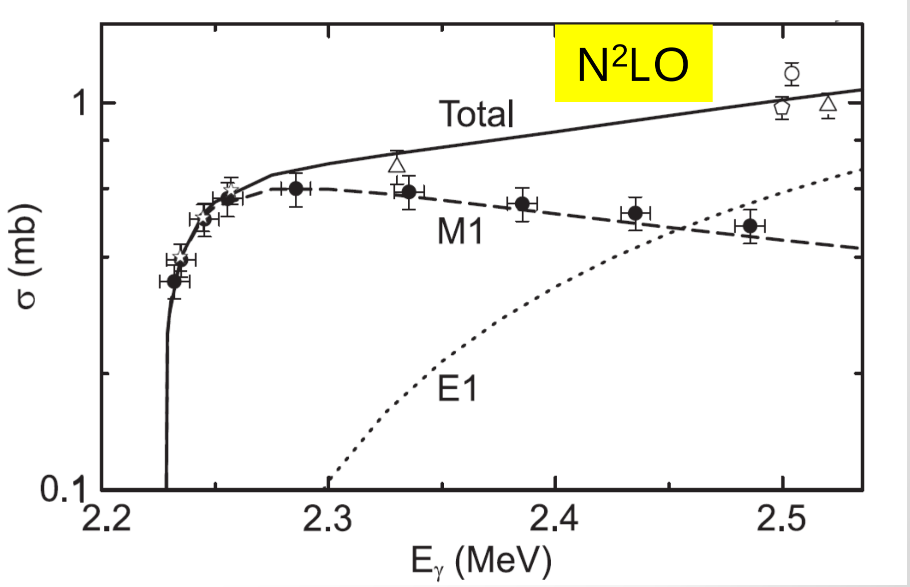
→ Photo/electrodisintegration
Christlmeier + Grießhammer ’08
Ryezawa *et al.* ‘08
…

→ Compton scattering
Grießhammer +Rupak ‘02
…

→ Neutrino scattering
Butler + Chen ’00
…

→ *etc.*

Example
Deuteron electrodisintegration



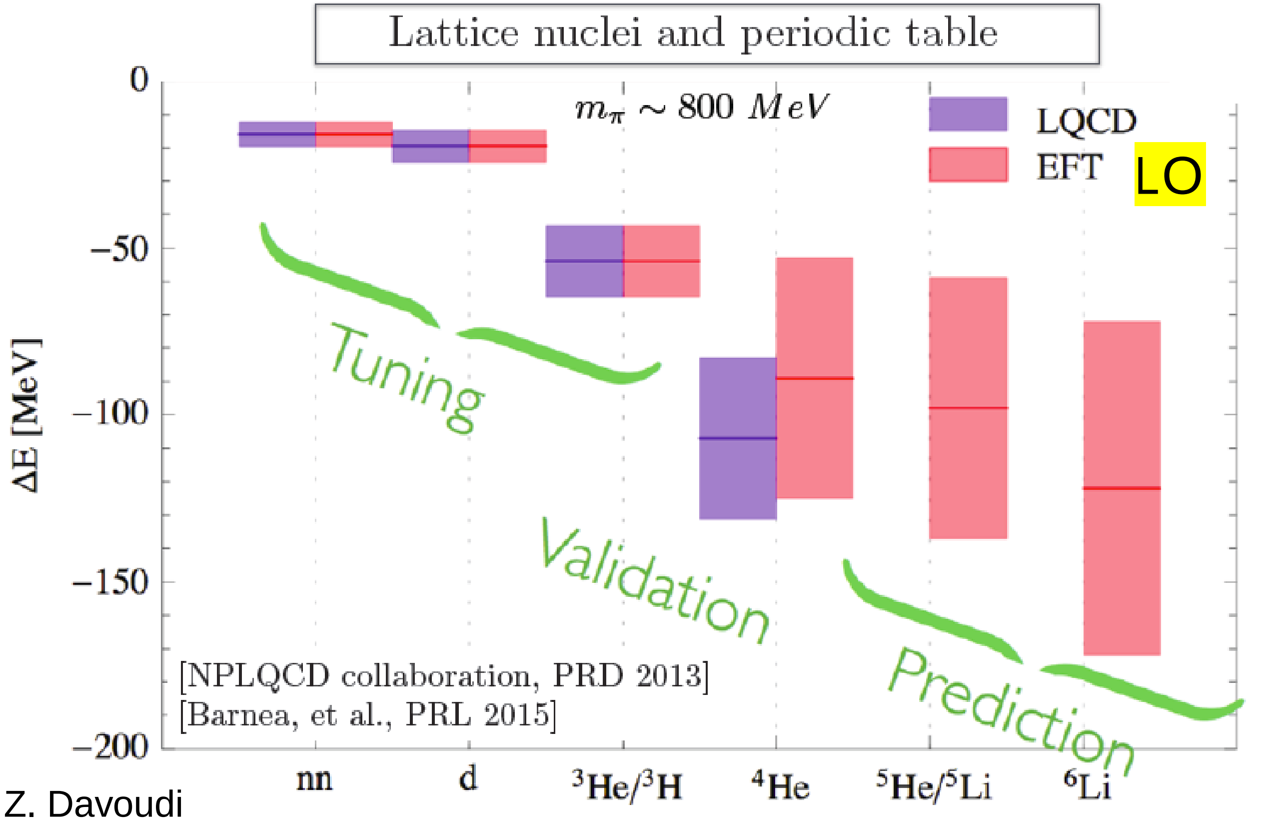
Ryezayeva *et al.*, *Phys. Rev. Lett.* 100 (2008) 172501

Unphysical quark masses

three parameters at LO: $\Lambda_*, \aleph_{^3S_1}, \aleph_{^1S_0}$

Barnea, Contessi, Gazit, Pederiva + vK '15
Kirscher, Barnea, Gazit, Pederiva + vK '15
Contessi, Lovato, Pederiva, Roggero, Kirscher + vK '17
Bansal *et al.* '18

Proof of principle for extending lattice QCD to larger nuclei



Z. Davoudi

Qualitatively similar to physical pion mass, just more bound by a factor ~5

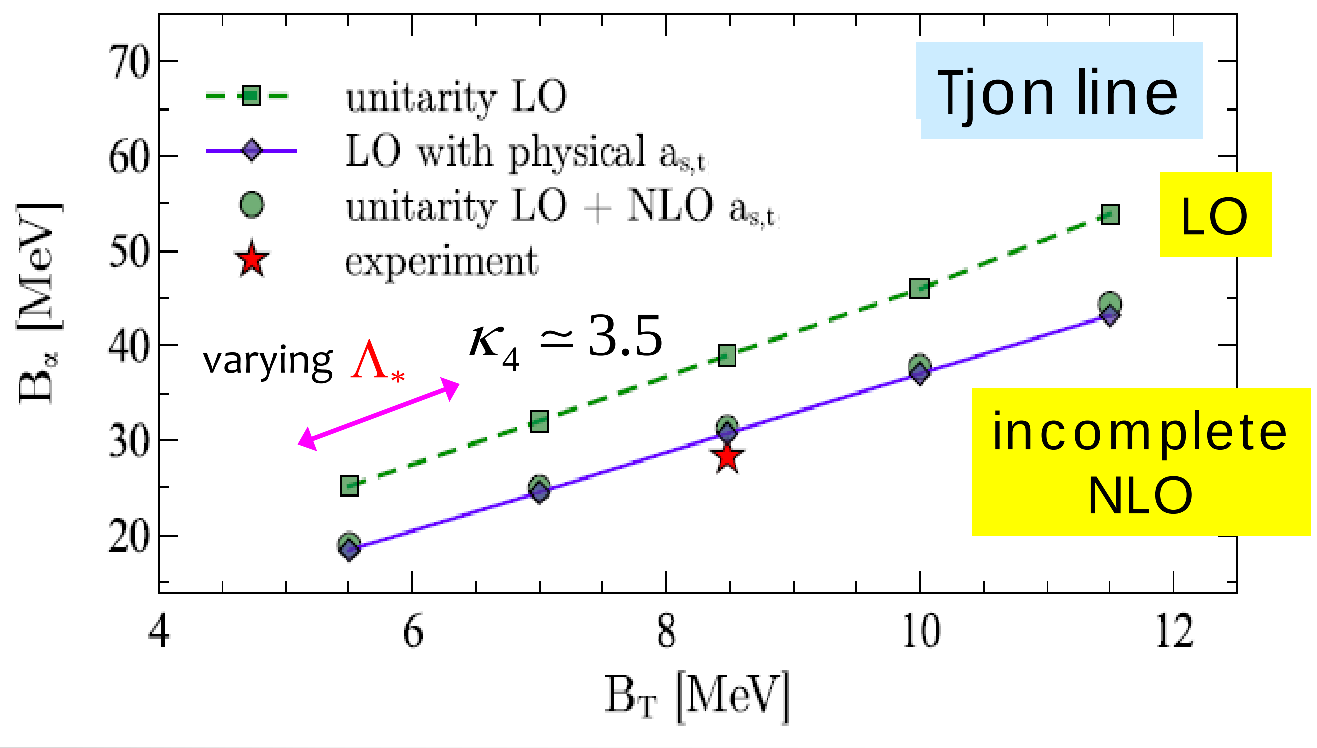
Physical quark masses (near unitarity)

König, Grießhammer, Hammer + vK ’17
König ’20
...

one parameter at LO: Λ_*

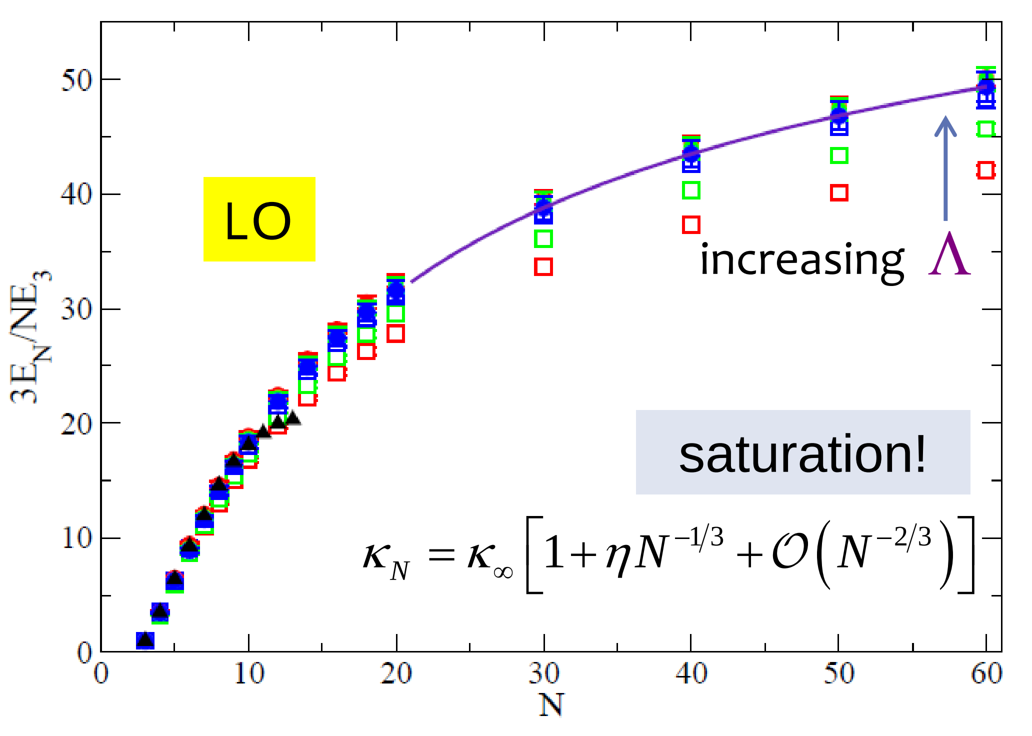
discrete scale invariance
- geometric towers of states (Efimov ’71, Hammer + Platter ’07, ...)
- ground states: $\frac{B_A^{(0)}(\Lambda_*)}{A} = \kappa_A \frac{B_3(\Lambda_*)}{3}$

universal numbers



König, Grießhammer, Hammer, vK, *Phys. Rev. Lett.* 118 (2017) 202501

Bosons at unitarity

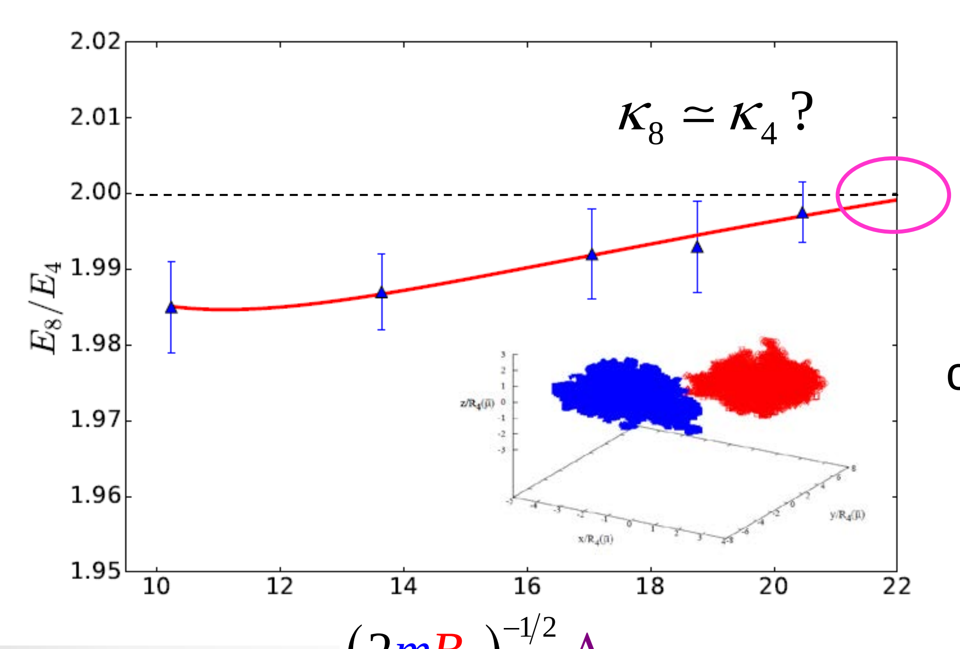


Gandolfi, Carlson, Vitiello, vK, *Phys. Rev. Lett.* 119 (2017) 223002

Bertulani, Hammer + vK ’02
Bedaque, Hammer + vK ’03
...

Could this provide the saturation mechanism for nucleons?

A first step: $A = 8$ at unitarity



$(2mB_4)^{-1/2}\,\Lambda$

consistent with ^{8}Be

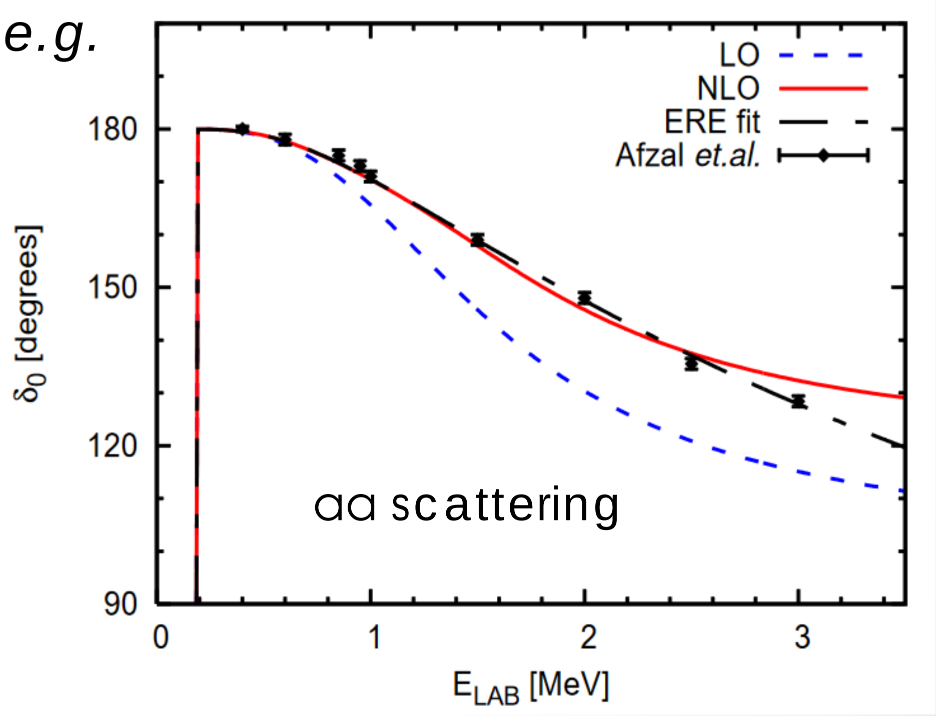
Dawkins, Carlson, vK, Gezerlis, *Phys. Rev. Lett.* 124 (2020) 143402

Clustering a universal property of multi-component unitary fermions?

Halo/Cluster EFT

tight cluster → “elementary” field

e.g.



Hammer, Higa, vK, *Nucl. Phys. A* 809 (2008) 171

useful for very low-energy reactions

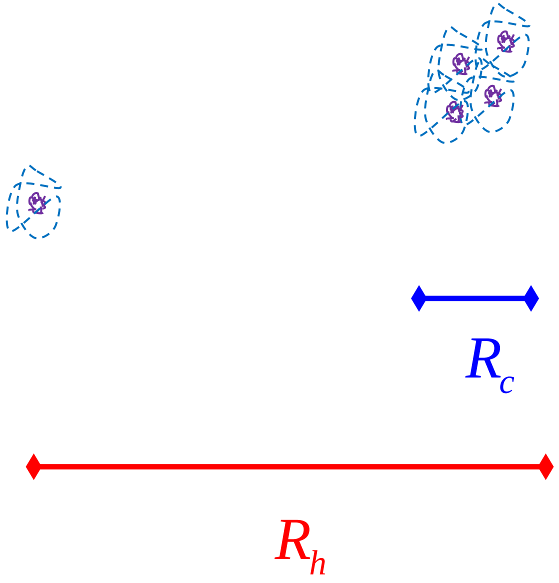
Halo/Cluster EFT

$$Q \sim R_h^{-1} \ll A^{-1/3} m_\pi \sim R_c^{-1}$$

slow-moving cluster

nonrelativistic expansion $\dfrac{Q}{A_c m_N}$

e.g.
$R_{^4\mathrm{He}} \approx 1.5\ \mathrm{fm}$
$R_{^6\mathrm{He}} \approx 2.4\ \mathrm{fm}$



$\lambda \sim R_h$

short-ranged

multipole expansion $Q\,R_c, \ldots$

no role for chiral symmetry

External probes

→ Electromagnetic form factors

Canham + Hammer '08'10
Hammer + Phillips '11
...

→ Radiative capture

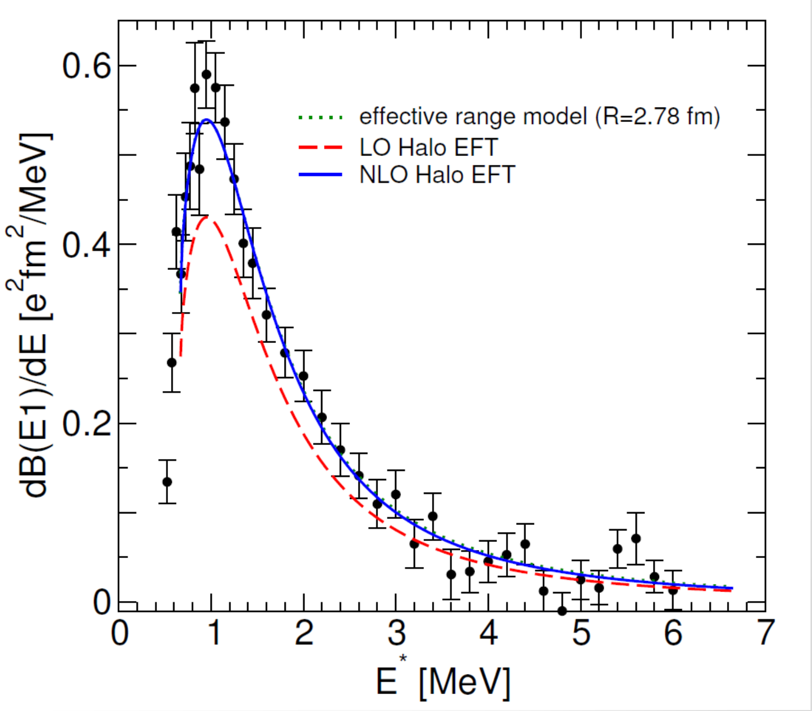
Rupak + Higa '11
Fernando, Higa + Rupak '12
...

→ Electro/photodisintegration

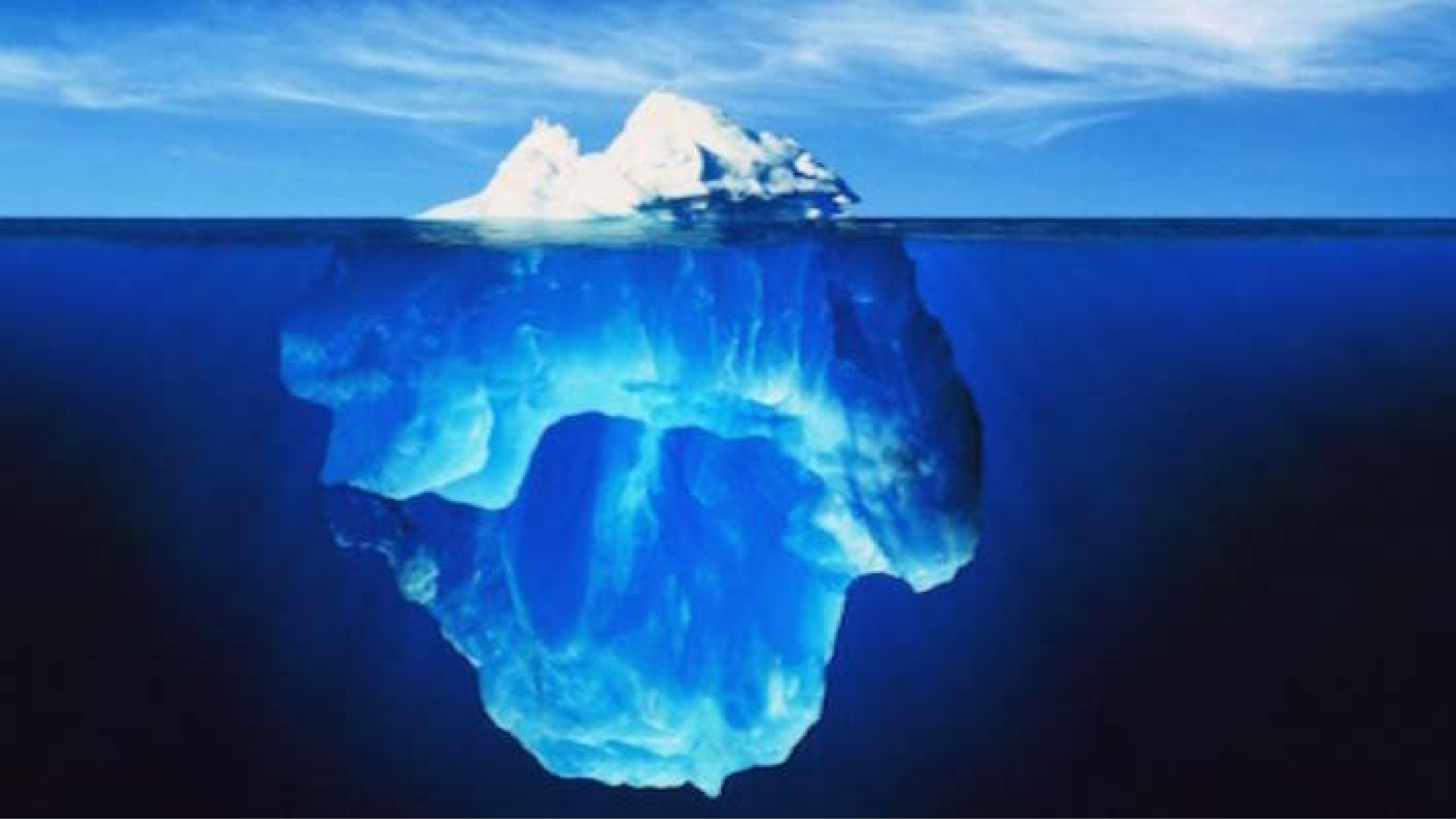
Hammer + Phillips '11
Acharya + Phillips '13
...

→ *etc.*

Example
^{11}Be Coulomb dissociation



Hammer, Phillips, *Nucl. Phys. A* 865 (2011) 17



Nuclear effective field theory: status and perspectives

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(Dated: July 1, 2019)

The nuclear physics landscape has been redesigned as a sequence of effective field theories (EFTs) connected to the Standard Model through symmetries and lattice simulations of Quantum Chromodynamics (QCD). EFTs in this sequence are expansions around different low-energy limits of QCD, each with its own characteristics, scales, and ranges of applicability regarding energy and number of nucleons. We review each of the three main nuclear EFTs—Chiral, Pionless, Halo/Cluster—highlighting their similarities, differences, and connections. In doing so, we survey the structural properties and reactions of nuclei that have been derived from the *ab initio* solution of the few- and many-body problem built upon EFT input.

Conclusion

EFT

a general framework

for theory construction

- ✓ same method across scales
  (but not all scales at once!)
- ✓ model independent
- ✓ controlled expansion

a paradigm

in nuclear physics

- ✓ encodes QCD (more generally, [B]SM)
- ✓ incorporates hadronic physics
- ✓ generates nuclear structure

the frontier:

many bodies & lattice QCD

- ❑ interplay with *ab initio* methods
- ❑ new EFTs