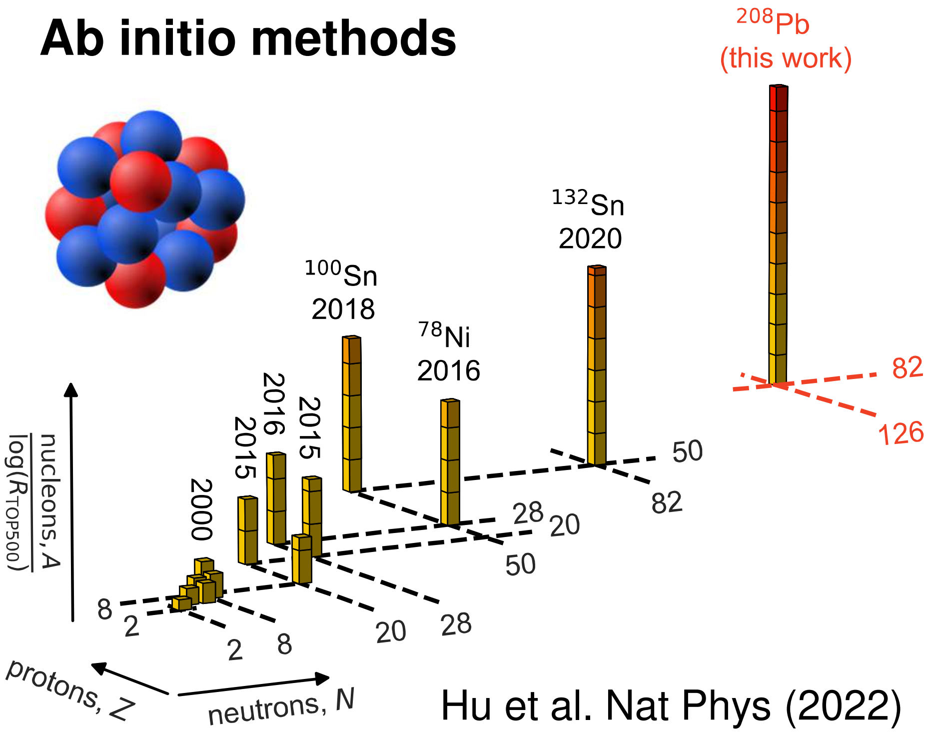
Effective theories of QCD for nuclei at low energy

Ab initio methods



Hu et al. Nat Phys (2022)

Thomas DUGUET

DPhN, CEA-Saclay, France
IKS, KU Leuven, Belgium

Workshop on *Deciphering nuclear phenomenology across energy scales*

September 20-23 2022, ESNT, Saclay, France

Contents

Contents

Elementary facts and « big » questions about nuclei

○ 252 **stable** isotopes, ~3100 synthesized in the lab

○ **How many** bound (w.r.t strong force) nuclei exist; 9000?

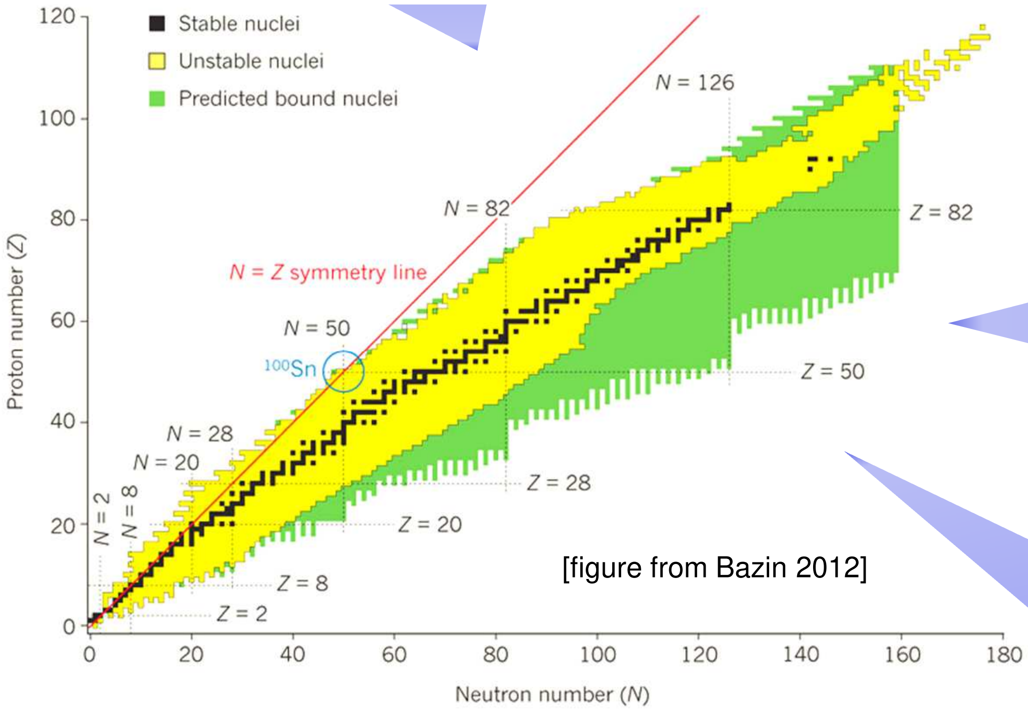
Less than 50% known (>10^{-22}s) → Discovery of ~15 per year in the years 2010s
→ Several 100s from next generation facilities

Oganesson $_{118}$Og added to Mendeleïev table in 2016

○ **Heaviest** synthesized element Z=118

○ **Heaviest possible** element?

○ Enhanced stability near Z=120?126?



[figure from Bazin 2012]

2p decay beyond the proton drip line in ^{45}Fe in 2002

○ Modes of **instability** (α, p, β, γ decays, fission)

○ Are there more exotic/rare decay modes?

Ex: ν-less 2β decay = test of standard model?

Gravitational wave + kilonova from neutron stars merger in 2017

○ Elements **up to Fe** produced in stellar fusion

○ How have heavier elements been produced?

○ Exotic r-process nucleosynthesis ; but where?

Updated in 2019 to Z=9 (22 neutrons) and Z=10 (24 neutrons)

○ Neutron **drip-line** known up to ~~Z=8 (16 neutrons~~)

○ Where is the neutron drip-line beyond Z=10?

Shown to disappear away from stability in 1975/1993

○ Over-stable "magic" nuclei (2, 8, ~~20, 28~~, 50, 82, …)

○ How **other magic numbers** evolve with N-Z?

The atomic nucleus as a 4-components quantum mesoscopic system
An extremely rich and diverse phenomenology

Nucleus: bound (or resonant) state of *Z* protons and *N* neutrons

Several scales at play:

p & n momenta ~ $\mathbf{10^8}$ eV

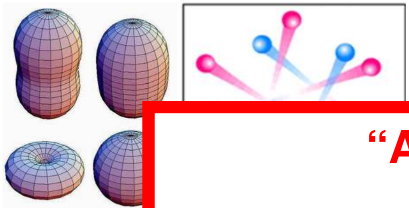
Separation energies ~ $\mathbf{10^7}$ eV

Vibrational excitations~ $\mathbf{10^6}$ eV

$\mathbf{0^4}$ eV

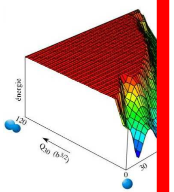
Ground state

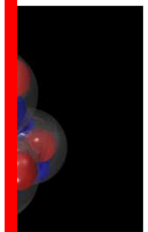
Mass, size, superfluidity, e.m. moments…



Radioa

β, 2β, (





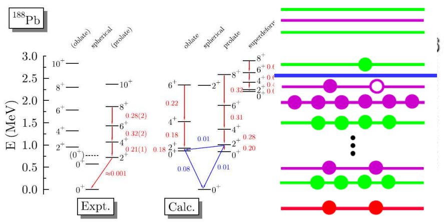
"Ab initio", i.e. Chiral-EFT in A-body sector, long-term endeavor

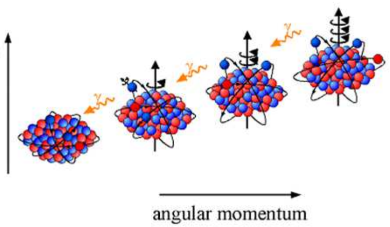
Can nuclear systems be described

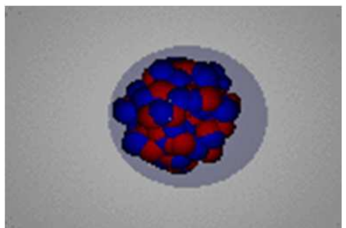
1) Consistently (from a single theoretical rationale?)
2) Systematically (complete phenomenology?)
3) Accurately enough (relevant to experimental uncertainty?)
4) From inter-nucleon interactions (right balance between reductionism/emergence?)
5) Rooted in QCD (sound connection to underlying EFT?)

Spectroscopy

Excitation modes

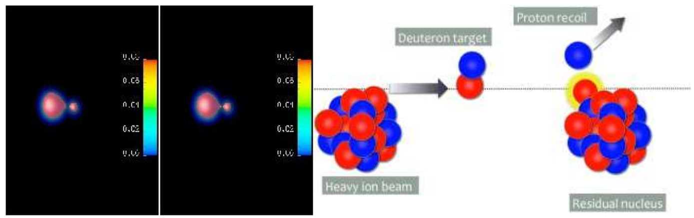






Reaction processes

Fusion, transfer, knockout, …



Contents

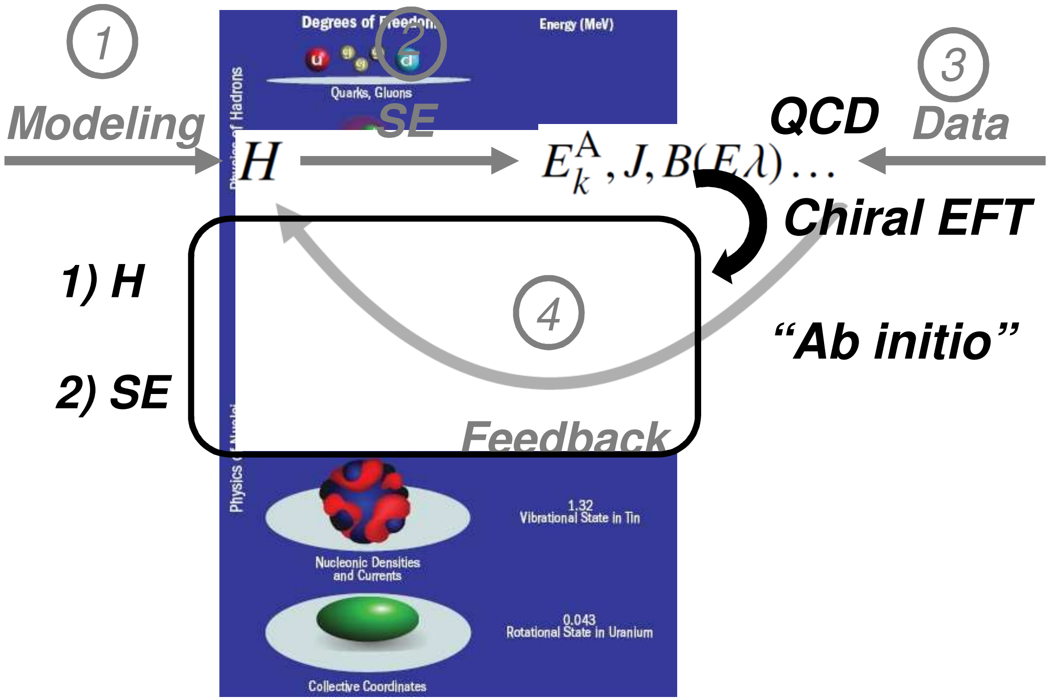
Ab initio (i.e. In medias res) quantum many-body problem

Ab initio nuclear many-body theory = Chiral Effective Field Theory (χEFT) in A-nucleon sector

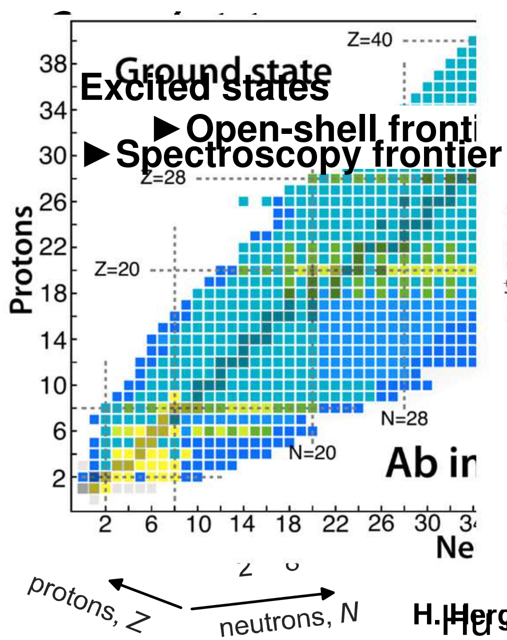
1) *A* structure-less nucleons as degrees of freedom at low energy
2) Interactions mediated by pions and contact operators based on, e.g., Weinberg, power counting
3) **Solve A-body Schrödinger equation to relevant accuracy**

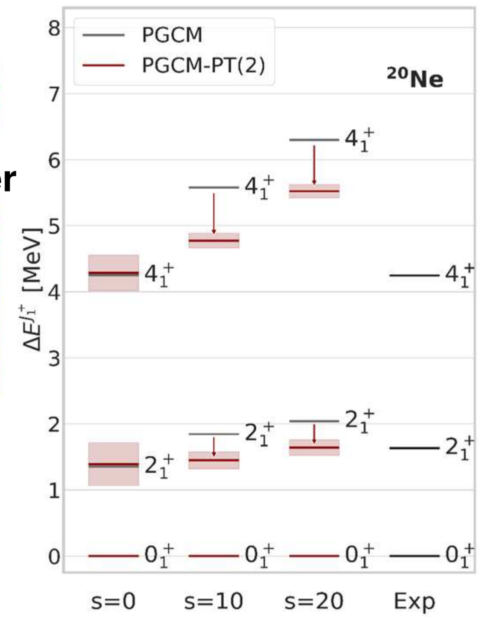
A-body Schrödinger Equation

$$H|\Psi_k^{\mathrm{A}}\rangle = E_k^{\mathrm{A}}|\Psi_k^{\mathrm{A}}\rangle$$



Rapidly evolving field in the last 15 years





Contents

The nuclear Hamiltonian

Build H (and other operators) with χEFT at various orders

- Non-trivial formal task whose difficulty increases with order (e.g. **3N at N²LO, 4N at N³LO**…)
- Fit LECs of mode-2k tensors to experimental data (or lattice QCD) in A = k-body systems

Organization = power counting
Importance of interaction terms

A-body Schrödinger Equation

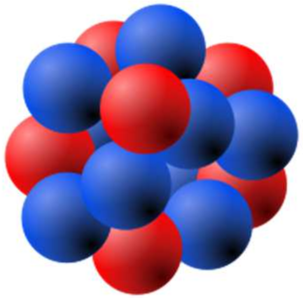
$$H|\Psi_k^{\mathrm{A}}\rangle = E_k^{\mathrm{A}}|\Psi_k^{\mathrm{A}}\rangle$$

Effective description = *A*-body operator in principle

$$H = T + \boxed{V^{2\mathrm{N}} + V^{3\mathrm{N}}} + V^{4\mathrm{N}} + \ldots + V^{A\mathrm{N}}$$

At least 3N necessary = major difficulty to solve SE next

Symmetries of the nuclear Hamiltonian



$$\vec{P} = \sum_{i=1}^{A} \vec{p}_i$$

Total center-of-mass momentum

$$\vec{J} = \vec{L} + \vec{S} = \sum_{i=1}^{A} \vec{l}_i + \sum_{i=1}^{A} \vec{s}_i$$

Total (internal) angular momentum

Nuclear systems are

❶ **Translationally invariant: T(1)**

$[H,P_i] = 0$ ➲ $| \Phi^{P}_{cm} > | \Psi_{\mathbf{im}} >$

❷ **Rotationally invariant: SU(2)**

$[H,J^2] = [H,J_z] = 0$ ➲ $| \Psi^{\mathbf{JM}} >$

❸ **Carry fixed neutrons/protons numbers: U(1)**

$[H,N] = [H,Z] = 0$ ➲ $| \Psi^{\mathbf{JMNZ}} >$

❹ + additional symmetries (time reversal, parity, ~isospin)

Symmetries

❶ Strongly constrain the mathematical form of H

❷ Dictates quantum numbers of its eigenstates

e.g. factorization of cm hard to ensure in practice

Phenomenology of inter-nucleon interactions

$$H \equiv \sum_{i=1}^{A}\frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2}\sum_{i\neq j}^{A} V^{2N}(i,j) + \frac{1}{6}\sum_{i\neq j\neq k}^{A} V^{3N}(i,j,k) + \ldots$$

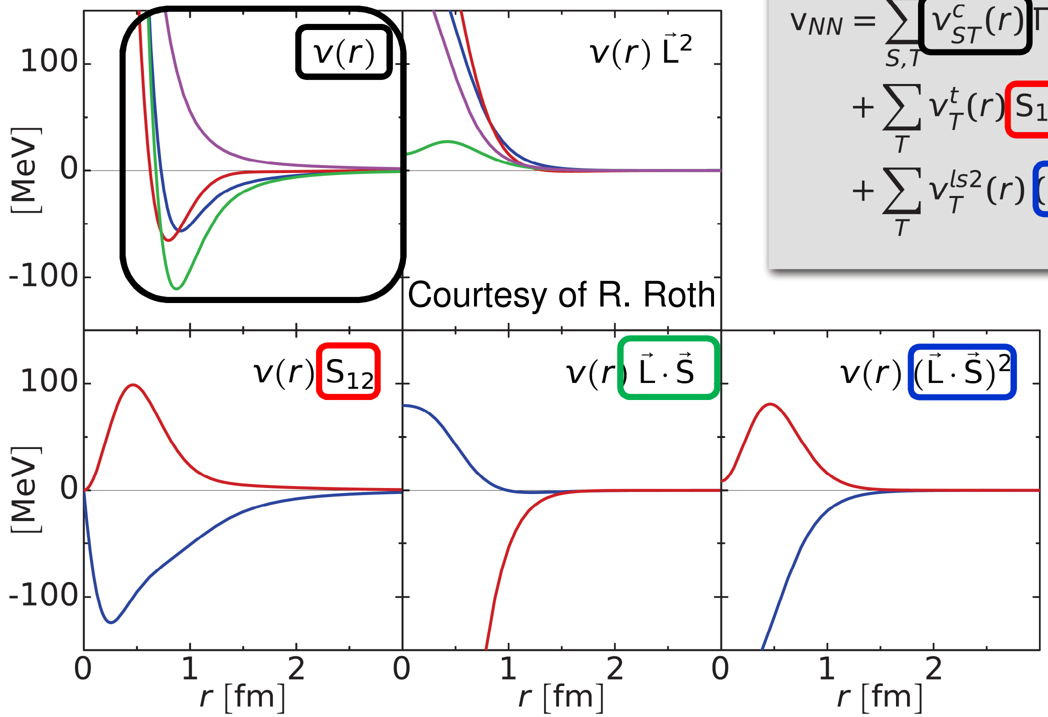
$$= \sum_{\alpha\beta} t_{\alpha\beta}\, a_\alpha^\dagger a_\beta + \left(\frac{1}{2!}\right)^2 \sum_{\alpha\beta\gamma\delta} \bar{v}^{2N}_{\alpha\beta\gamma\delta}\, a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma + \left(\frac{1}{3!}\right)^2 \sum_{\alpha\beta\gamma\delta\zeta\epsilon} \bar{v}^{3N}_{\alpha\beta\gamma\delta\zeta\epsilon}\, a_\alpha^\dagger a_\beta^\dagger a_\gamma^\dagger a_\epsilon a_\zeta a_\delta + \cdots$$

Interactions between effective 4-components point fermions

➲ nucleons = ±½ isospin & ±½ spin projections

1. **Complex operator structure in r⊗σ⊗τ spaces (constrained by symmetries)**

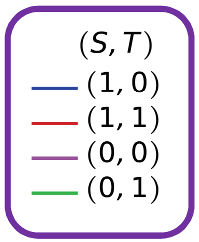
AV18 2N interaction



$$v_{NN} = \sum_{S,T} v^{c}_{ST}(r)\,\Pi_{ST} + \sum_{S,T} v^{l2}_{ST}(r)\,\vec{L}^2\,\Pi_{ST} + \sum_{T} v^{t}_{T}(r)\,S_{12}\,\Pi_{1T} + \sum_{T} v^{ls}_{T}(r)\,(\vec{L}\cdot\vec{S})\,\Pi_{1T} + \sum_{T} v^{ls2}_{T}(r)\,(\vec{L}\cdot\vec{S})^2\,\Pi_{1T} + \ldots$$

✶Central operator
~Lennard-Jones
Short-range repulsion
✶Tensor operator
✶Spin-orbit
✶Spin-orbit²

[Wiringa et al. 1995]



4 two-body spin-isospin channels

⌦AV18 model local but generally nuclear interactions are non-local in space

Phenomenology of inter-nucleon interactions

$$H \equiv \sum_{i=1}^{A} \frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2}\sum_{i\neq j}^{A} V^{2N}(i,j) + \frac{1}{6}\sum_{i\neq j\neq k}^{A} V^{3N}(i,j,k) + \ldots$$

$$= \sum_{\alpha\beta} t_{\alpha\beta} a^\dagger_\alpha a_\beta + \left(\frac{1}{2!}\right)^2 \sum_{\alpha\beta\gamma\delta} \bar{v}^{2N}_{\alpha\beta\gamma\delta} a^\dagger_\alpha a^\dagger_\beta a_\delta a_\gamma + \left(\frac{1}{3!}\right)^2 \sum_{\alpha\beta\gamma\delta\zeta\epsilon} \bar{v}^{3N}_{\alpha\beta\gamma\delta\zeta\epsilon} a^\dagger_\alpha a^\dagger_\beta a^\dagger_\gamma a_\epsilon a_\zeta a_\delta + \cdots$$

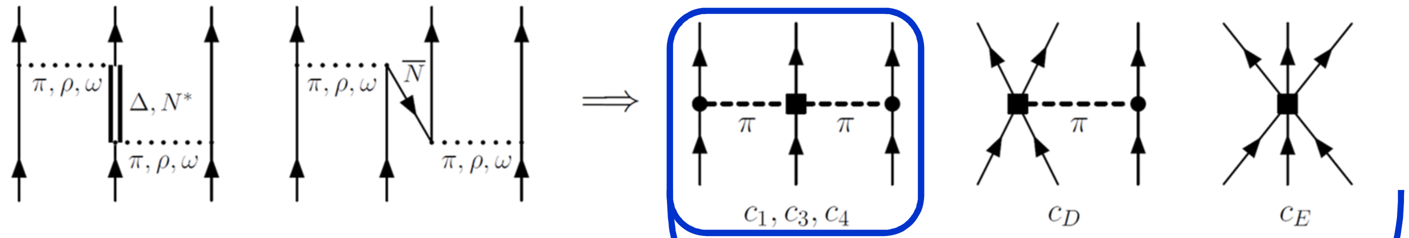
Interactions between effective 4-components point fermions

➲ nucleons = ±½ isospin & ±½ spin projections

2. **Dominant 2-nucleon + sub-leading (but mandatory) 3-nucleon and (minor?) 4-nucleon forces**

« Integrating out » DOFs lead to multi-nucleon forces

Example 3N



2π-exchange diagram

First contributions to 3N interaction in chiral-EFT (N²LO)

Example of analytical expression

Tensor operator

$$\langle \vec{p}^{\,\prime}_1 \vec{p}^{\,\prime}_2 \vec{p}^{\,\prime}_3 | V^{3N}_{2\pi \mathrm{N^2LO}} | \vec{p}_1 \vec{p}_2 \vec{p}_3 \rangle = \frac{g_A^2}{8F_\pi^4} \frac{\boxed{\vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3}}{[q_1^2 + m_\pi^2][q_3^2 + m_\pi^2]} \Big[\vec{\tau}_1 \cdot \vec{\tau}_3 \left(2\boxed{c_3} \vec{q}_1 \cdot \vec{q}_3 - 4\boxed{c_1} m_\pi^2 \right)$$

Spin-orbit-like operator

$$+ \boxed{c_4} \vec{\tau}_1 \times \vec{\tau}_3 \cdot \vec{\tau}_2 \boxed{\vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2} \Big] \delta(\vec{P}^{\prime} - \vec{P})$$

$$+ \text{all permutations of } (1,2,3)$$

ME in plane-wave basis of $\mathcal{H}_3$

$$\vec{q}_i = \vec{p}^{\,\prime}_i - \vec{p}_i \qquad \vec{P} = \sum_{i=1}^{3} \vec{p}_i$$

Low-Energy Constants (LECs)

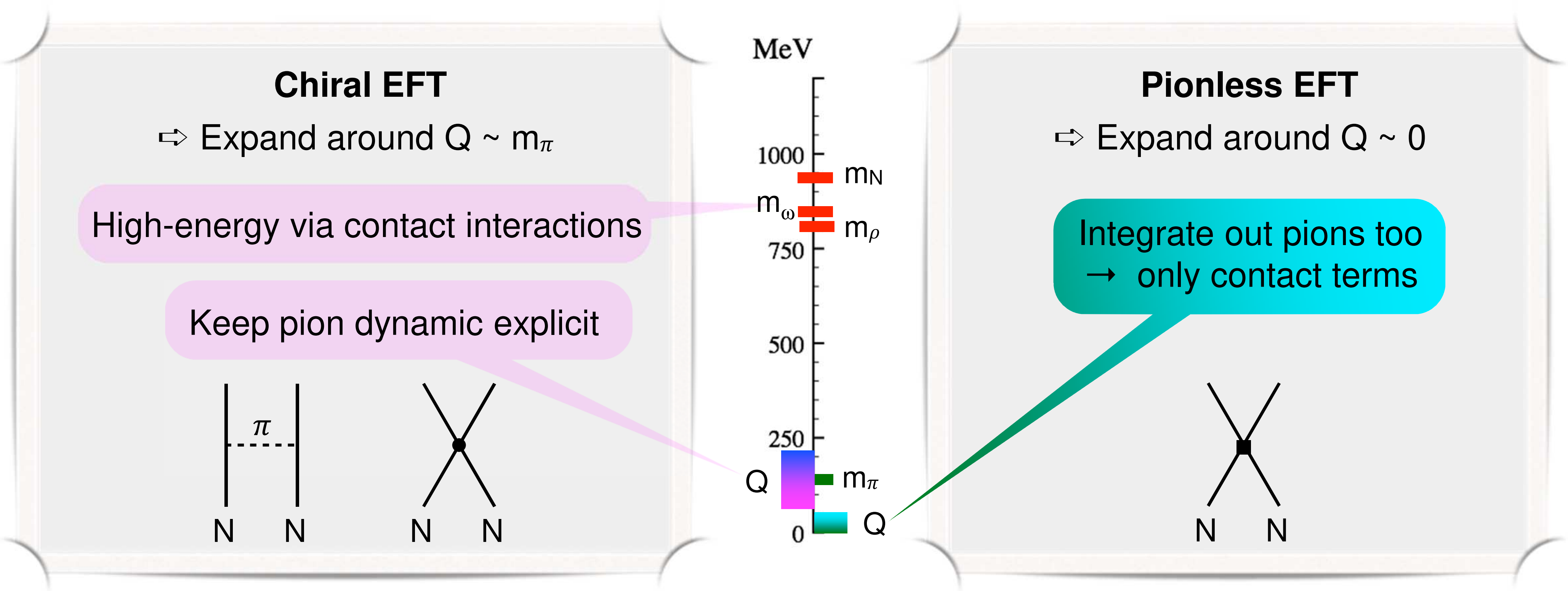
Fixed on π-nucleon scatt. exp.

Modern constructive approach = effective field theory

1. Use **separation of scales to define d.o.f** & expansion parameter [Weinberg, Gasser, Leutwyler, van Kolck, ..]

Typical momentum at play ← Q/Λ → High energy scale (physics beyond not included explicitly)

2. Parametrize physics beyond Λ + write #∞ terms allowed by **(broken) symmetries of underlying QCD**
3. Order by size all possible terms → **systematic expansion ("power counting")** → theoretical error
4. Truncate at a given order and **adjust low-energy constants (LECs) via underlying theory or data**
5. Regularize UV divergences and (hopefully) **achieve order-by-order renormalization of observables**



Chiral effective field theory = Weinberg power counting

1) Interaction diagrams are made out of a) nucleon and pion propagators
b) pion-nucleon and (derivative) k-nucleon contact vertices

Goal of PC: estimate the power ν of the law $(Q/\Lambda_\chi)^\nu$ with which each diagram scales

2) Naive Dimensional Analysis
a) nucleon propagator carries Q^{-1}
b) pion propagator carries Q^{-2}
b) derivative operator carries Q
c) loop integration brings Q^4

$$\frac{1}{m_\omega^2+Q^2} \approx \frac{1}{m_\omega^2}\underbrace{\left(1-\frac{Q^2}{m_\omega^2}+\frac{Q^4}{m_\omega^4}-+\ldots\right)}$$

Fits with PC in powers of $Q/m_\omega \approx Q/\Lambda_\chi$

Connected diagrams

$$\nu = \boxed{2k} - 4 + 2L + \sum_i \Delta_i$$

with $\Delta_i \equiv d_i + \frac{n_i}{2} - 2$

k = k-nucleon sector ➡ **Hierarchy of k-body forces: 3N (4N) starts 2 (4) orders after 2N**

L = # of loops

d_i = # of derivatives/pion masses at vertex i

n_i = # of nucleon fields at vertex i

$\Delta_i \geq 0$

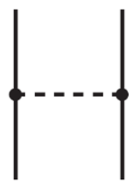
$\nu \geq 0$

Chiral symmetry

Finite # at given order

Weinberg PC for interaction potential ➡ **Insert into dynamical, i.e. A-body Schroedinger, equation to access observables**

3) Examples: diagrams in 2-nucleon sector at Leading Order (LO) with ~Q^0 (ν = 0 from k=2, L=0, Δ_i=0)



$$V_{1\pi}^{(0)}(\boldsymbol{p}',\boldsymbol{p}) = -\frac{g_A^2}{4f_\pi^2}\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,\frac{\boxed{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\;\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}}{q^2+m_\pi^2}$$

Tensor operator

$$V_{\mathrm{ct}}^{(0)}(\boldsymbol{p}',\boldsymbol{p}) = C_S + C_T\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$

Central operator (no q dependence)

1π exchange (1PE)

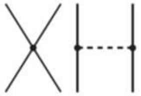
Pure contact term (CT)

4) **Consistent construction of other operators (e.g. coupling to electroweak or WIMP probes)**

Chiral effective field theory = interactions expansion

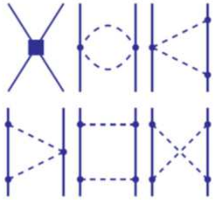
2N Force 3N Force 4N Force

LO $(Q/\Lambda_\chi)^0$



2N tree level CT (key for large a_s)
2N tree-level 1PE (key for deuteron)

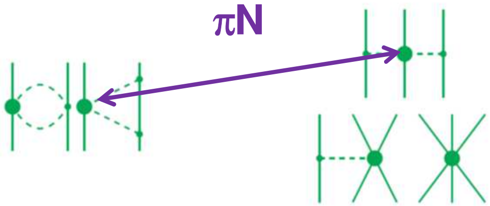
NLO $(Q/\Lambda_\chi)^2$



2N tree level CT (spin-orbit, tensor)
2N one-loop 2PE (weak)

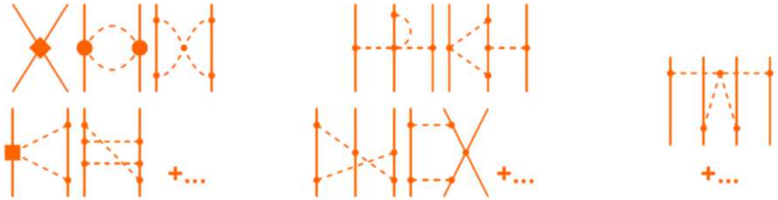
- ▶ **Finite number of terms up to given ν**
- ▶ Proliferation = need fast convergence
- ▶ **3N at N²LO and 4N at N³LO**
- ▶ **Consistency between k-body sectors**
- ▶ **LECs of contacts fitted to few-body data**
  Ex: 16 at NNLO
- ▶ Estimate of error from $(Q/\Lambda_\chi)^{\nu+1}$

NNLO $(Q/\Lambda_\chi)^3$



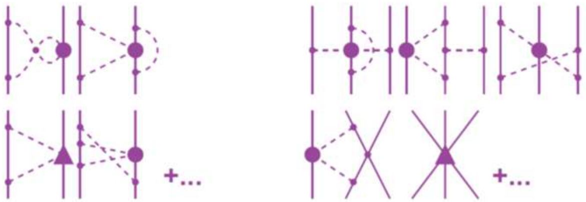
2N one-loop 2PE (brings intermediate range attraction; no new param)
3N tree-level 2PE, 1PE-CT, CT (key for saturation, 3N scattering, drip lines…)

N³LO $(Q/\Lambda_\chi)^4$



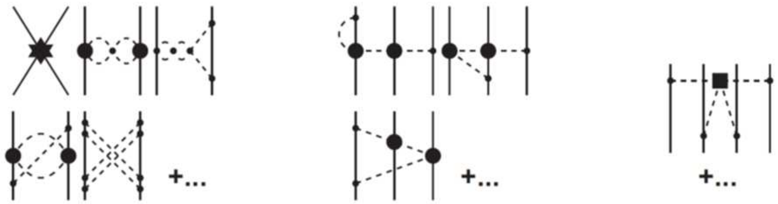
2N tree-level CT, one-loop 2PE, two-loop 2PE, 3PE
3N one-loop 3PE, 2PE-CT… (no new param)
4N… (weak; no new param)

N⁴LO $(Q/\Lambda_\chi)^5$



2N two-loop 2PE, 3PE
3N… (derived but little used; large + solve the n-d A_y puzzle)
4N… (not yet derived)

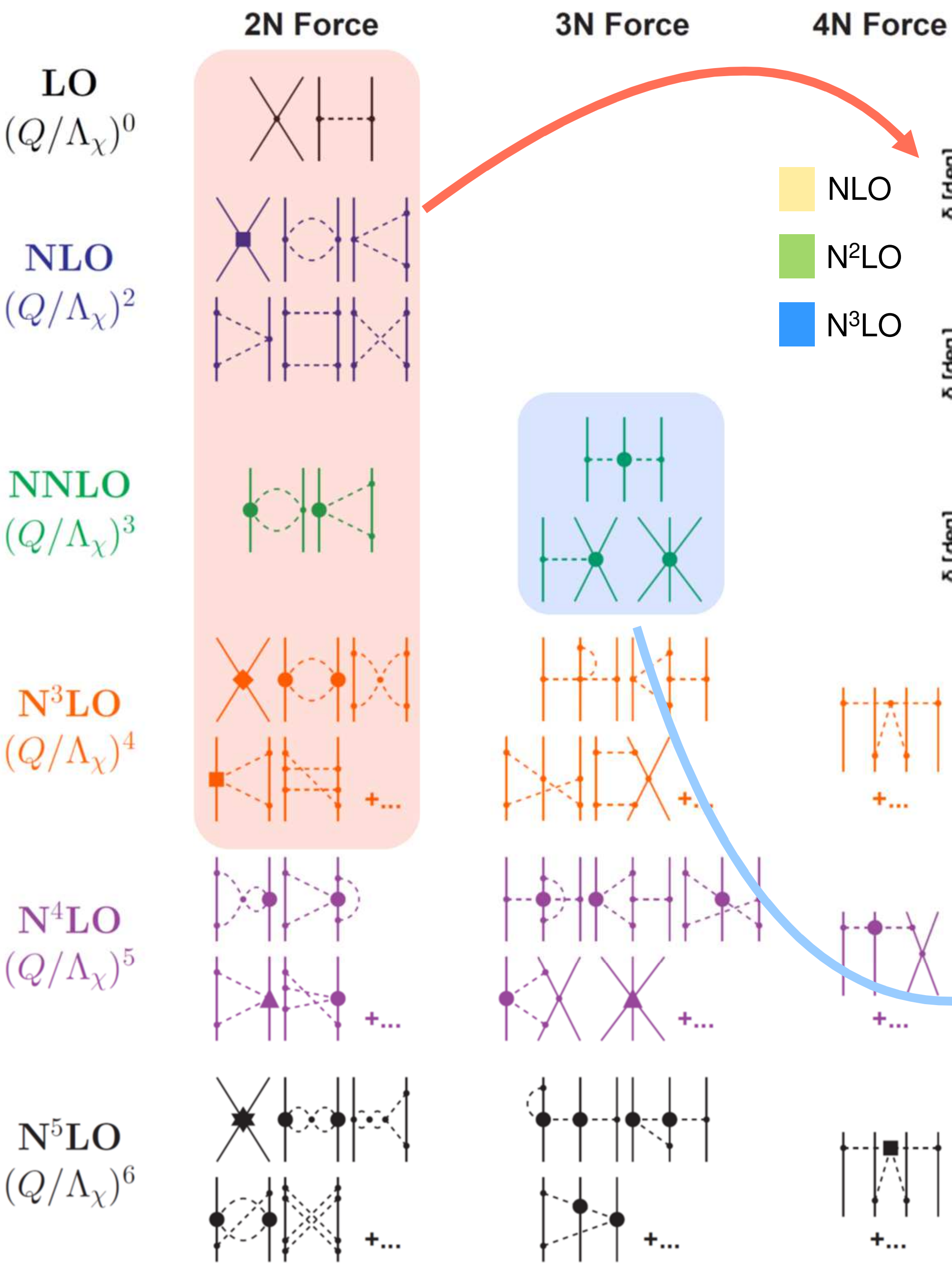
N⁵LO $(Q/\Lambda_\chi)^6$



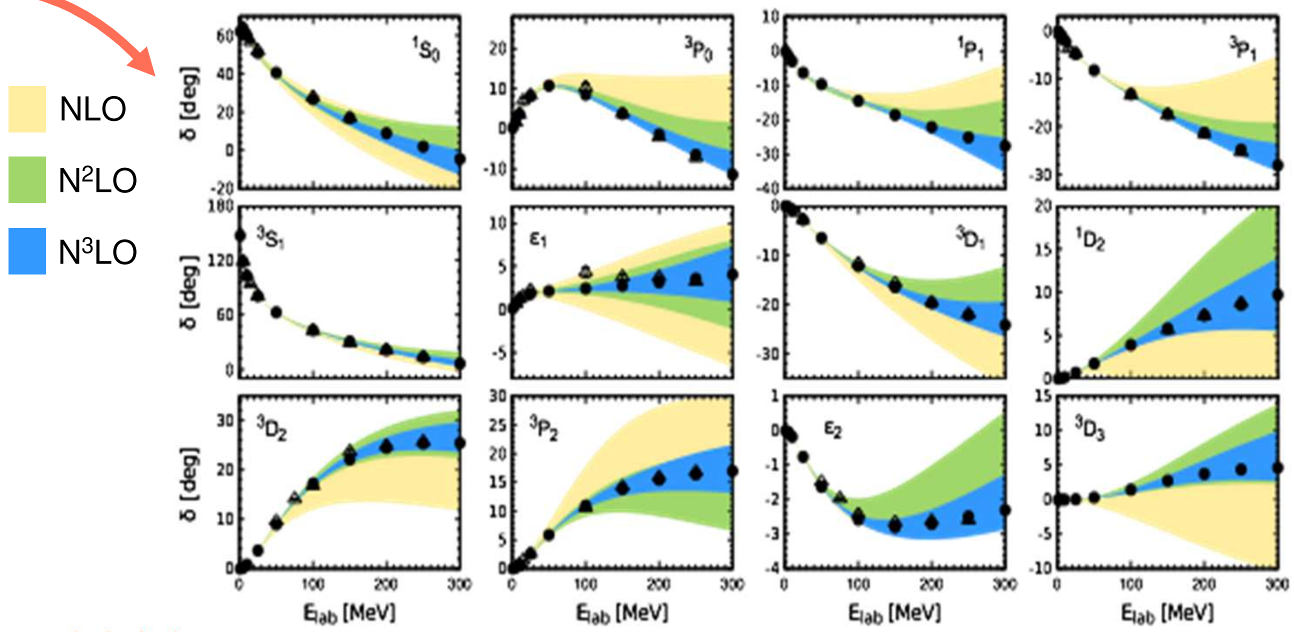
Major challenges

1) Can k-body, k>3, be omitted in A>>3?
2) $N^{3/4}$LO 2N for high precision; 3N? 4N?

Chiral effective field theory = interactions expansion

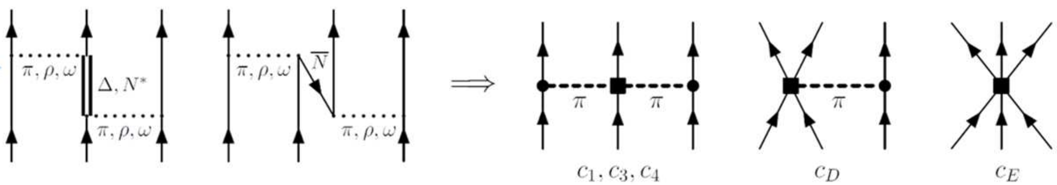


2N phase shifts versus Nijgmegen analysis



[E. Epelbaum, H. Krebs, U. Meißner, arXiv:1412.0142]

« Integrating out » DOFs lead to multi-nucleon forces



Contents

Nuclear many-body problem

◉ Memory load for JT-coupled matrix elements

→ SRG evolved in mass A~50 (e1max = 13 / e2max = 26 / e3max = 16)

2NF ~ 2GB 3NF ~ 25GB

→ SRG evolved in mass A~100 (e1max = 15 / e2max = 30 / e3max = 20)

2NF ~ 7GB 3NF ~ 350GB

◉ Transform to JT-coupled matrix elements for storage (on-the-fly decoupling)

$$\langle pq|V|rs\rangle \longrightarrow \langle \tilde{p}\tilde{q};J;T|V|\tilde{r}\tilde{s};J;T\rangle$$

$$\langle pqr|W|stu\rangle \longrightarrow \langle \tilde{p}\tilde{q}\tilde{r};J_{pq}J;T_{pq}T|W|\tilde{s}\tilde{t}\tilde{u};J_{st}J;T_{st}T\rangle$$

→ typical reduction by factor ~30

More needed to reduce the load to go beyond A~100

Ex: $H_{\mathrm{N^2LO}} = T + V^{2N}_{\mathrm{N^2LO}} + V^{3N}_{\mathrm{N^2LO}} + \emptyset$ because of the truncation of χEFT expansion of the operator

A-body Schrödinger Equation

$$H_{\mathrm{N^2LO}}|\Psi^{\mathrm{JMNZ}}_k\rangle = E^{\mathrm{JNZ}}_k|\Psi^{\mathrm{JMNZ}}_k\rangle$$

◉ HO single-particle basis

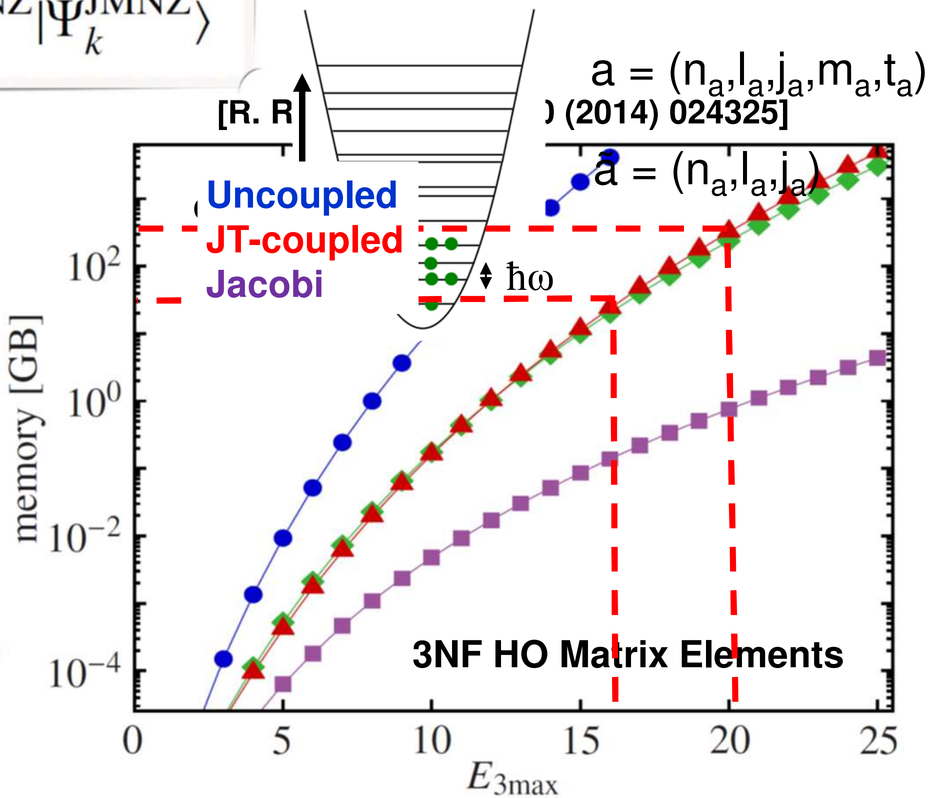
◉ Second-quantized form

$$H \equiv \frac{1}{(1!)^2}\sum_{pq} t_{pq}c^\dagger_p c_q$$

Four-index tensor → $$+\frac{1}{(2!)^2}\sum_{pqrs} \bar{v}_{pqrs}c^\dagger_p c^\dagger_q c_s c_r$$

Six-index tensor → $$+\frac{1}{(3!)^2}\sum_{pqrstu} \bar{w}_{pqrstu}c^\dagger_p c^\dagger_q c^\dagger_r c_u c_t c_s$$

$$+\ldots$$

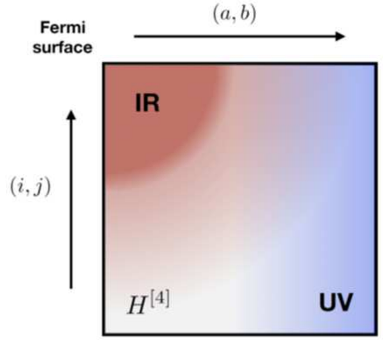


Contents

Pre-processing of short-range correlations

1) Short-r repulsion
2) Bound np/Virtual nn

Two-body ➡ interaction matrix



Many-body ➡ correlations

1) Dynamic corr. in UV
2) Strong static corr. in IR
2a) Pairing in SOS
2b) Collect. Quad. in DOS

1) Taming down the short-range/coupling to UV in the Hamiltonian

Unitary **Similarity Renormalization Group** (SRG) transformation

$$H(s) \equiv U(s) H U^{\dagger}(s) = T + V^{2N}(s) + V^{3N}(s) + \ldots$$

♦ Paramaterize the *change* of the Hamiltonian $\frac{dH(s)}{ds} = \boxed{[\eta(s),} H(s)]$

Anti-hermitian generator $\eta(s) = \frac{dU(s)}{ds} U^{\dagger}(s)$ specifies the transformation

$H \equiv H_{\mathrm{D}} + H_{\mathrm{OD}}$ ➡ $\eta(s) \equiv [H_{\mathrm{D}}, H(s)]$ ➡ $\frac{d}{ds} H(s) = 0$ when $[H_{\mathrm{D}}, H(s)] = 0$

Trivial fixed point H(s) diagonal

♦ To tame short-range choose $H_{\mathrm{D}} \equiv T$ = diagonal in momentum space basis

♦ Do not go all the way to fixed point because $[\eta(s), H(s)]$ ***induces multi-body operators in H(s)***

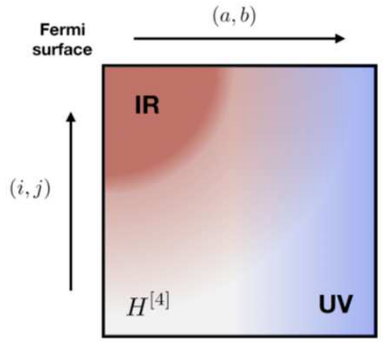
Run until appropriate s_{final} = pre-diagonalization

Pre-processing of short-range correlations

1) **Short-r repulsion**
2) **Bound np/Virtual nn**

Two-body

interaction matrix



Many-body

correlations

1) **Dynamic corr. in UV**
2) **Strong static corr. in IR**
  2a) **Pairing in SOS**
  2b) **Collect. Quad. in DOS**

1) Taming down the short-range/coupling to UV in the Hamiltonian

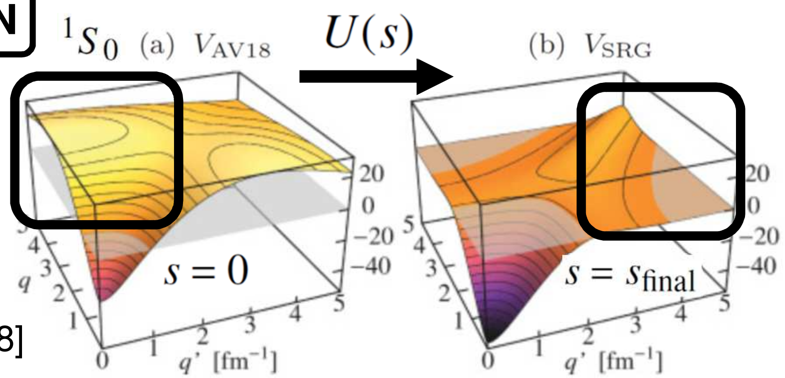
Unitary **Similarity Renormalization Group** (SRG) transformation

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$

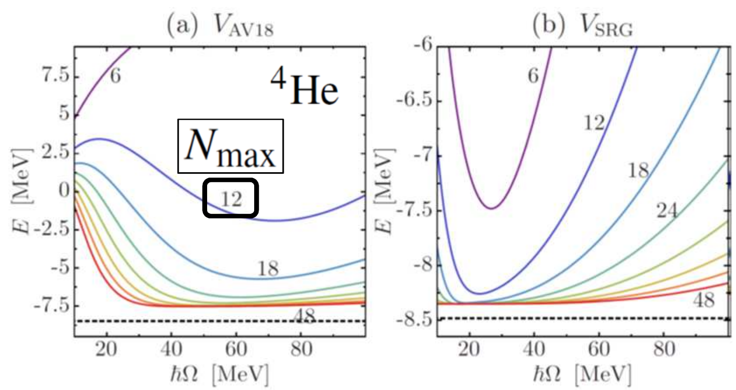
$$\eta(s) \equiv [T, H(s)]$$

[Roth, Reinhardt, Hergert 2008]

2N



Relative momentum basis of $\mathcal{H}_2$



Drastically accelerated convergence
More perturbative behavior in the UV

$$V^{2N}(s)(q,q') \approx V^{2N}(0)(q,q')\, e^{-s(q^2-q'^2)^2}$$

Low-to-high off-diagonal matrix elements suppressed
➲ Similar SRG procedure and achievement for 3N in $\mathcal{H}_3$

Pre-processed nuclear many-body problem

$U(s)$ **SRG** ➡

A-body Schrödinger Equation

$$\boxed{H_{\mathrm{N^2LO}}}|\Psi_k^{\mathrm{JMNZ}}\rangle = E_k^{\mathrm{JNZ}}|\Psi_k^{\mathrm{JMNZ}}\rangle$$

Rather strong coupling to UV

Ex: $H_{\mathrm{N^2LO}} = T + V^{\mathrm{2N}}_{\mathrm{N^2LO}} + V^{\mathrm{3N}}_{\mathrm{N^2LO}}$ **+ $\varnothing$ because of the truncation of chiral-EFT expansion of the operator**

Pre-processed nuclear many-body problem

$$\rho_k^{\mathrm{JMNZ}}(\vec r_1,\vec r_2,\vec r_3,\vec r_4) \equiv \langle \Psi_k^{\mathrm{JMNZ}} | c^\dagger(\vec r_1) c^\dagger(\vec r_2) c(\vec r_3) c(\vec r_4) | \Psi_k^{\mathrm{JMNZ}} \rangle$$

Two-body density matrix

$$= \langle \Psi_k^{\mathrm{JMNZ}}(s) \boxed{| U(s) c^\dagger(\vec r_1) c^\dagger(\vec r_2) c(\vec r_3) c(\vec r_4) U^\dagger(s) |} \Psi_k^{\mathrm{JMNZ}}(s) \rangle$$

$$\neq \langle \Psi_k^{\mathrm{JMNZ}}(s) | c^\dagger(\vec r_1) c^\dagger(\vec r_2) c(\vec r_3) c(\vec r_4) | \Psi_k^{\mathrm{JMNZ}}(s) \rangle$$

Sum of up to A-body operator/density matrices

► discussion AV18 vs χEFT on tuesday

given that $|\Psi_k^{\mathrm{JMNZ}}(s)\rangle \equiv U(s) |\Psi_k^{\mathrm{JMNZ}}\rangle$

« Soft » Hamiltonian
=
reduced coupling to UV

A-body Schrödinger Equation

$$\boxed{H_{\mathrm{N^2LO}}(s)} |\Psi_k^{\mathrm{JMNZ}}(s)\rangle = \boxed{E_k^{\mathrm{JNZ}}} |\Psi_k^{\mathrm{JMNZ}}(s)\rangle$$

A-body observables independent of s

Ex: $H_{\mathrm{N^2LO}}(s) = T + V^{\mathrm{2N}}_{\mathrm{N^2LO}}(s) + V^{\mathrm{3N}}_{\mathrm{N^2LO}}(s) \boxed{+ \cancel{\dots}}$ **to be tractable** $\Rightarrow$ $\frac{d}{ds} E_k^{\mathrm{JNZ}} \neq 0$ **violate unitarity**

Induced k-body forces (k≤ A)

Choose truncation & s

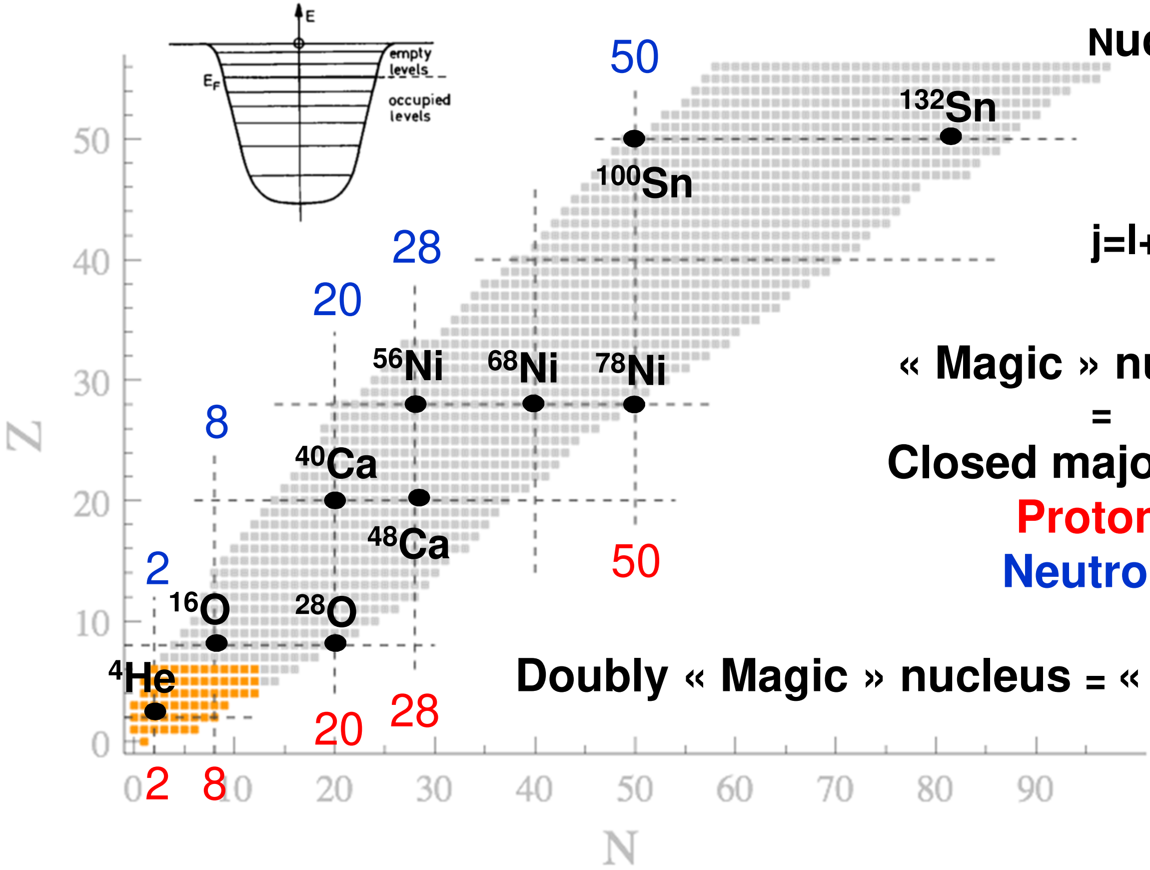
$$\frac{d}{ds} E_k^{\mathrm{JNZ}} \sim 0$$

SRG transformation is a compromised between

- ➲ Reduction of coupling to UV
- ➲ Size of induced k-body interactions that cannot be handled

Contents

Categories of nuclei vs correlations vs expansion method

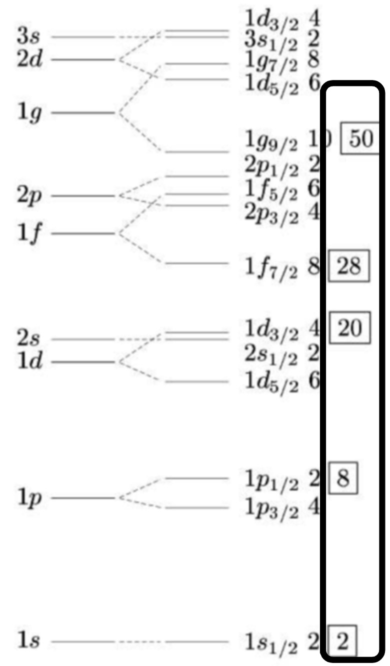


Nuclear shells from SC ([H,J^2]=0) Hartree-Fock

j=l+s → 2j+1 degeneracy

« Magic » numbers
=
Closed major shells
Protons
Neutrons

Doubly « Magic » nucleus = « double noble gas »



1 set for protons
1 set for neutrons

Counting even-even nuclei belonging to each category

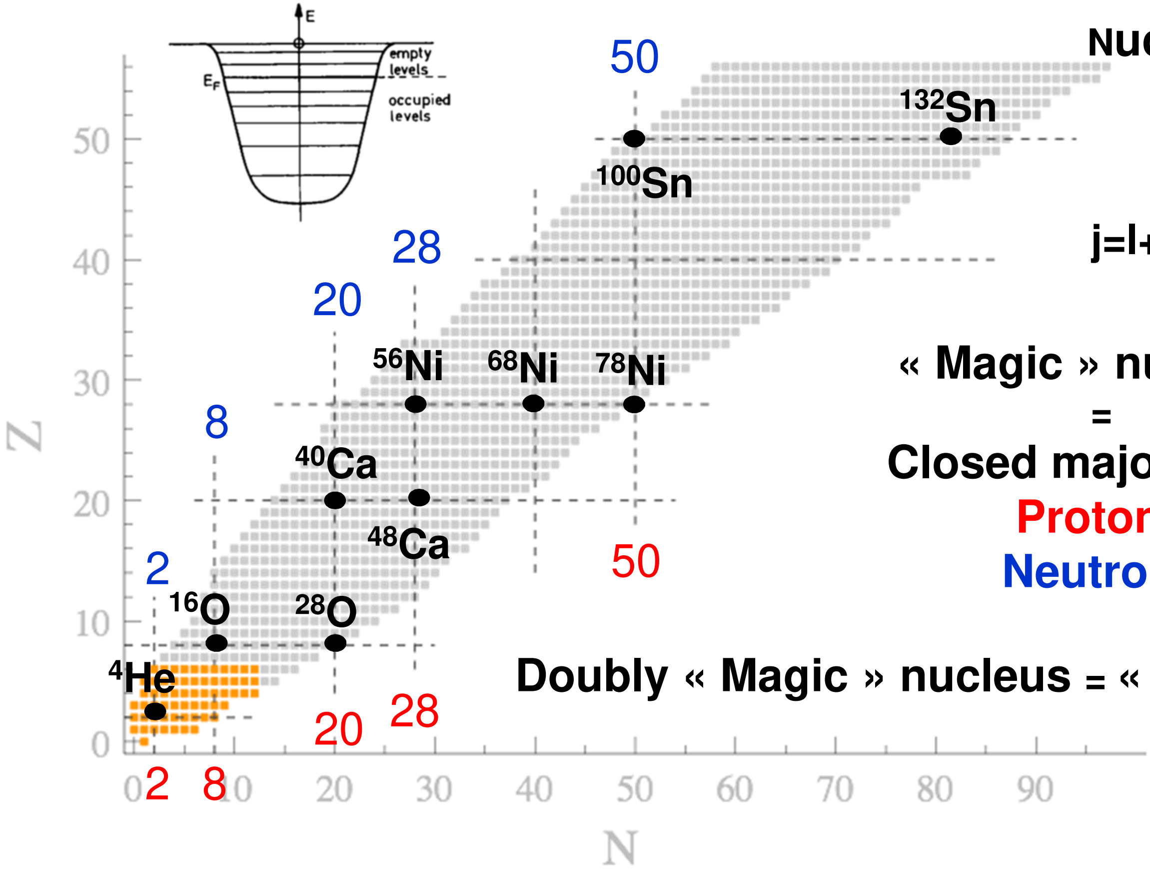
Even-even nuclei	Number estimated	Percentage estimated
Total	2075	100%
Doubly closed-shell	16	0.8 %
Singly open-shell	246	11.9%
Doubly open-shell	1813	87.3 %

99.2%

c » correlations

Table: Based on nuclear shells from a Hartree-Fock calculation of ^{16}O. Courtesy of B. Bally.

Categories of nuclei vs correlations vs expansion method

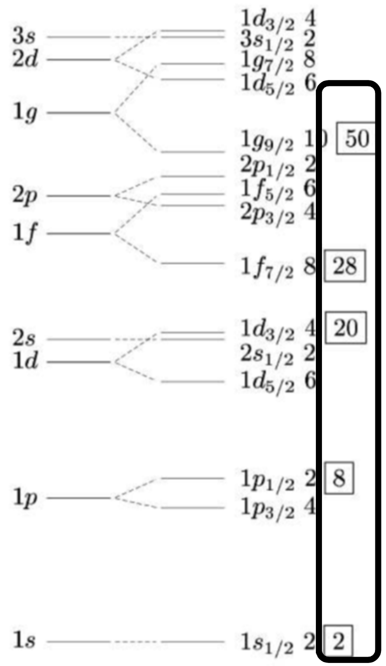


Nuclear shells from SC ($[H,J^2]=0$) Hartree-Fock

j=l+s → 2j+1 degeneracy

« Magic » numbers
=
Closed major shells
Protons
Neutrons

Doubly « Magic » nucleus = « double noble gas »



1 set for protons
1 set for neutrons

More pertinent categorization

Even-even nuclei	Number estimated	Percentage estimated
Total	2075	100%
Doubly closed-(sub)shell	181	$\boxed{8.7\ \%}$
Singly open-(sub)shell	838	40.4%
Doubly open-(sub)shell	1056	50.9 %

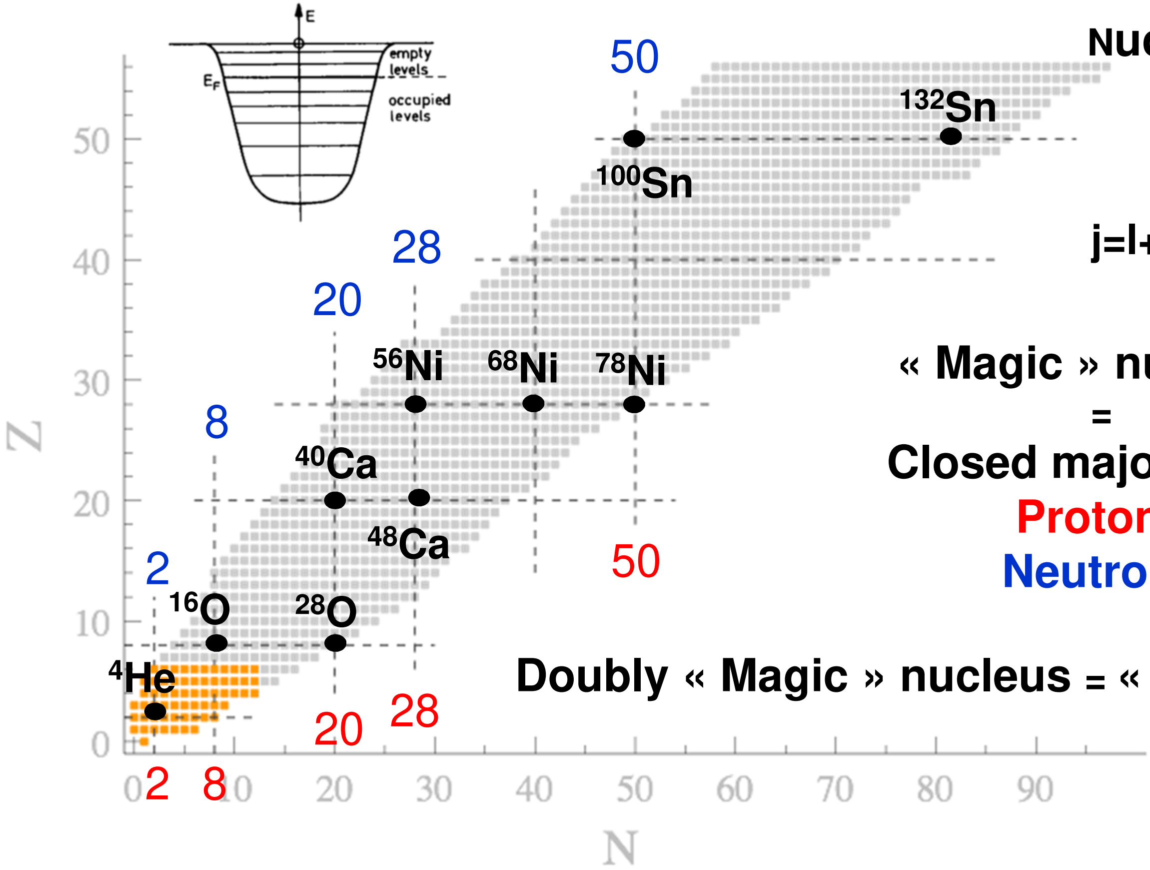
$\left.\begin{matrix}40.4\% \\ 50.9\%\end{matrix}\right\}$ **91.3%**

Nuclei	Number
Odd-even	4050
odd-odd	2014

Via particle-attached methods

Table: Based on nuclear shells from a Hartree-Fock calculation of ^{16}O. Courtesy of B. Bally.

Categories of nuclei vs correlations vs expansion method



Nuclear shells from SC ($[H,J^2]=0$) Hartree-Fock

j=l+s → 2j+1 degeneracy

« Magic » numbers

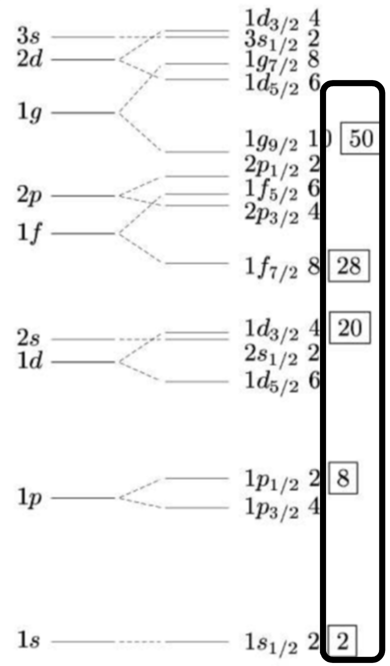
=

Closed major shells

Protons

Neutrons

Doubly « Magic » nucleus = « double noble gas »



1 set for protons
1 set for neutrons

Any « universal » (expansion) method must deal with static and dynamical correlations

- Consistently (no double counting)
- Efficiently (at reasonal polynomial cost)

Evolution of ab initio nuclear chart vs type of method

$$H|\Psi_n^A\rangle = \mathrm{E}_n^A|\Psi_n^A\rangle$$

One-body Hilbert space

$$\mathcal{H}(1)$$

$$\dim \mathcal{H}(1) \equiv n_{\dim}$$

A-body Hilbert space

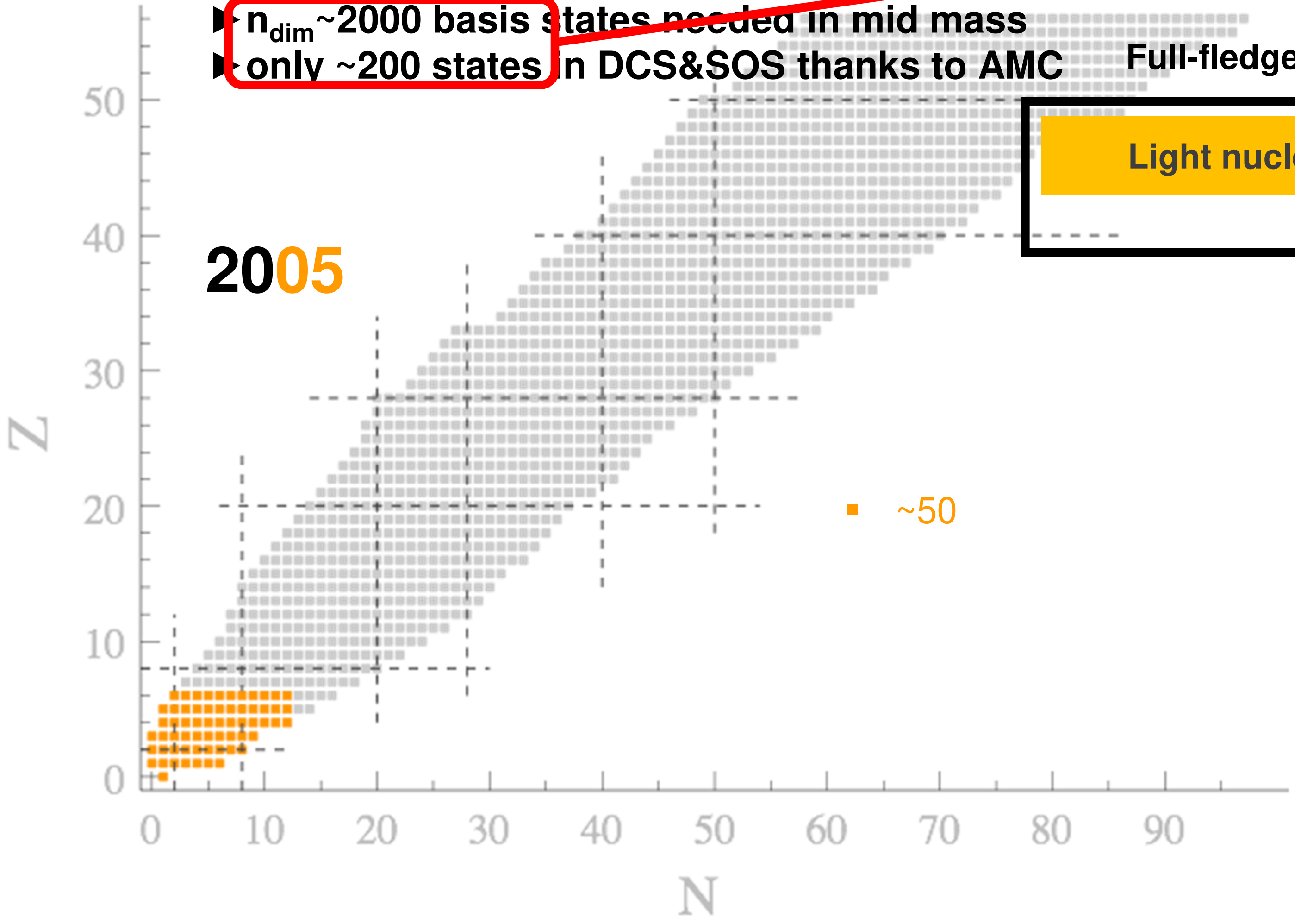
$$\mathcal{H}_{\mathrm{A}} = \mathcal{H}(1) \otimes \ldots \otimes \mathcal{H}(\mathrm{A})$$

$$\boxed{\dim \mathcal{H}(\mathrm{A}) \equiv n_{\dim}^{\mathrm{A}}}$$

« The curse of dimensionality »

Dynamical/static correlations

- **n_{dim}~2000 basis states needed in mid mass**
- **only ~200 states in DCS&SOS thanks to AMC**



Full-fledged method, e.g. diagonalization

Light nuclei	Exponential cost n^A
	GFCM, NCSM

Evolution of ab initio nuclear chart vs type of method

$$H|\Psi_n^A\rangle = \mathrm{E}_n^A|\Psi_n^A\rangle$$

One-body Hilbert space

$$\mathcal{H}(1)$$

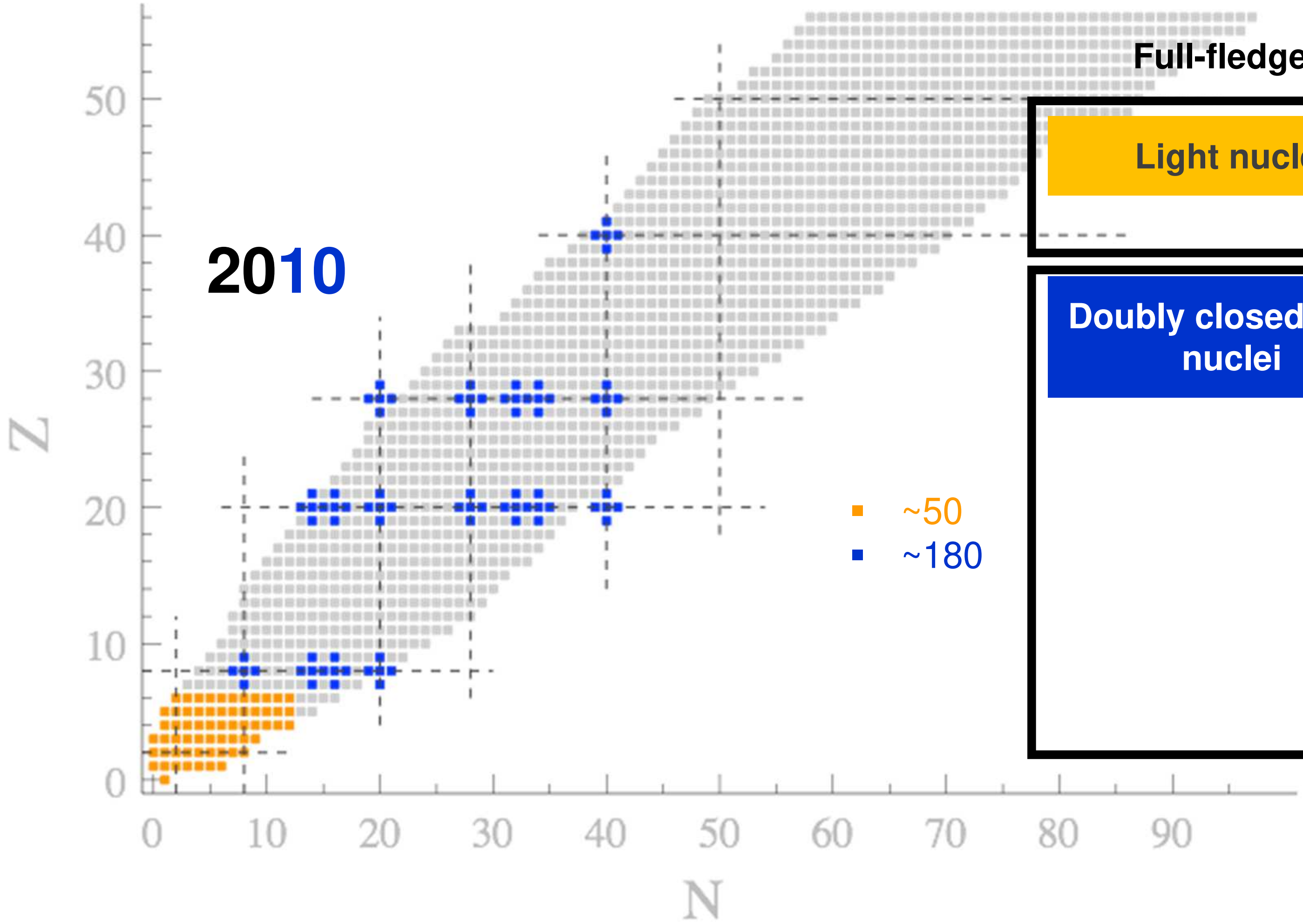
$$\dim\mathcal{H}(1) \equiv n_{\dim}$$

A-body Hilbert space

$$\mathcal{H}_{\mathrm{A}} = \mathcal{H}(1)\otimes\ldots\otimes\mathcal{H}(\mathrm{A})$$

$$\boxed{\dim\mathcal{H}(\mathrm{A}) \equiv n_{\dim}^{\mathrm{A}}}$$

« The curse of dimensionality »



Full-fledged method, e.g. diagonalization

Light nuclei	Exponential cost n^A **GFCM, NCSM**
Doubly closed-shell nuclei	Exponential cost n'^p **CC, SCGF, MRSRG**

Expansion methods

Evolution of ab initio nuclear chart vs type of method

$$H|\Psi_n^A\rangle = \mathrm{E}_n^A|\Psi_n^A\rangle$$

One-body Hilbert space

$$\mathcal{H}(1)$$

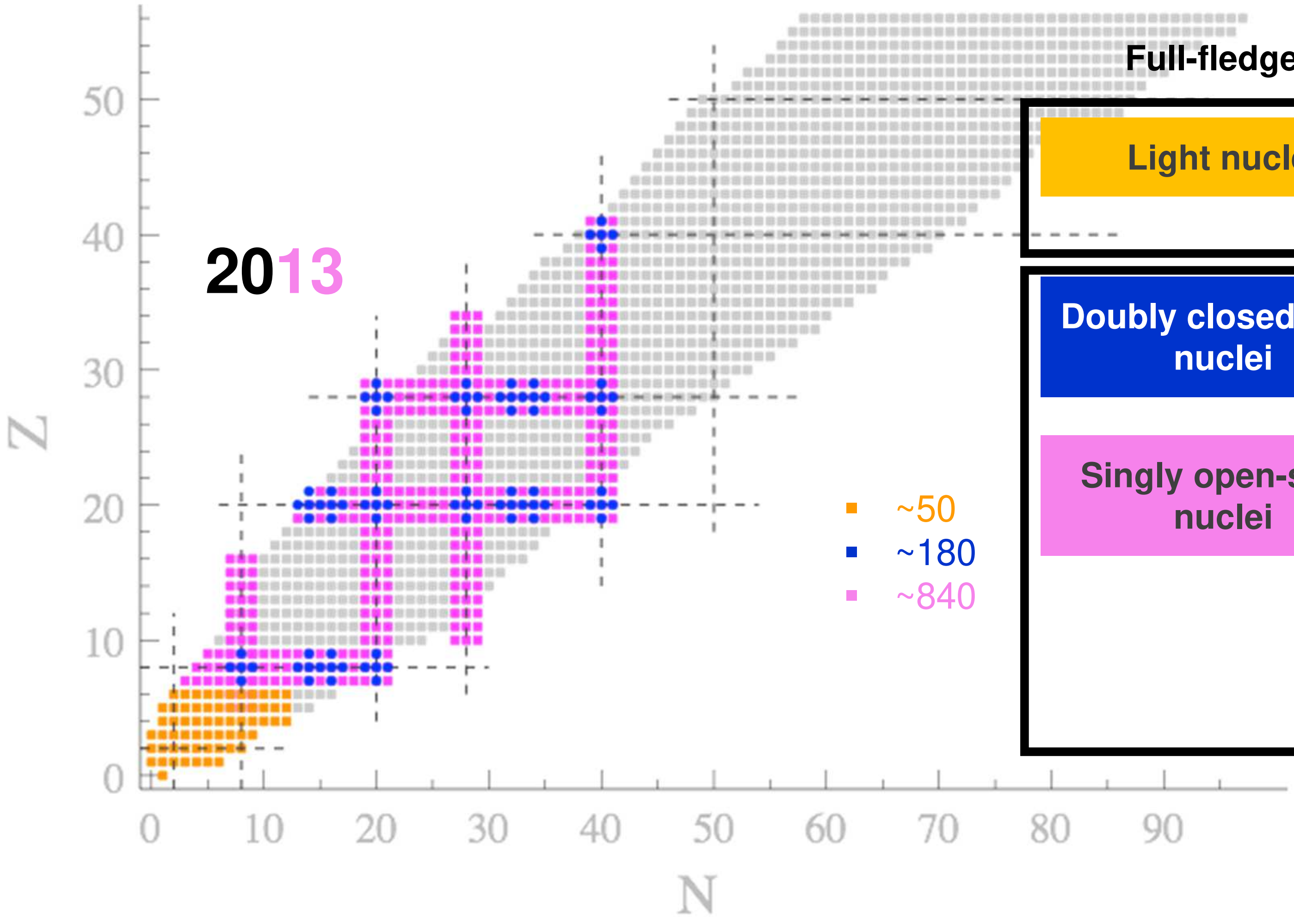
$$\dim \mathcal{H}(1) \equiv n_{\text{dim}}$$

A-body Hilbert space

$$\mathcal{H}_{\mathrm{A}} = \mathcal{H}(1) \otimes \ldots \otimes \mathcal{H}(\mathrm{A})$$

$$\boxed{\dim \mathcal{H}(\mathrm{A}) \equiv n_{\text{dim}}^{\mathrm{A}}}$$

« The curse of dimensionality »



Full-fledged method, e.g. diagonalization

Light nuclei	Exponential cost n^A
	GFCM, NCSM

Doubly closed-shell nuclei	Exponential cost n'^p
	CC, SCGF, MRSRG
Singly open-shell nuclei	Exponential cost n''^p
	sMBPT, sBCC, sGSCGF

Expansion methods

Evolution of ab initio nuclear chart vs type of method

$$H|\Psi_n^A\rangle = \mathrm{E}_n^A|\Psi_n^A\rangle$$

One-body Hilbert space

$$\mathcal{H}(1)$$

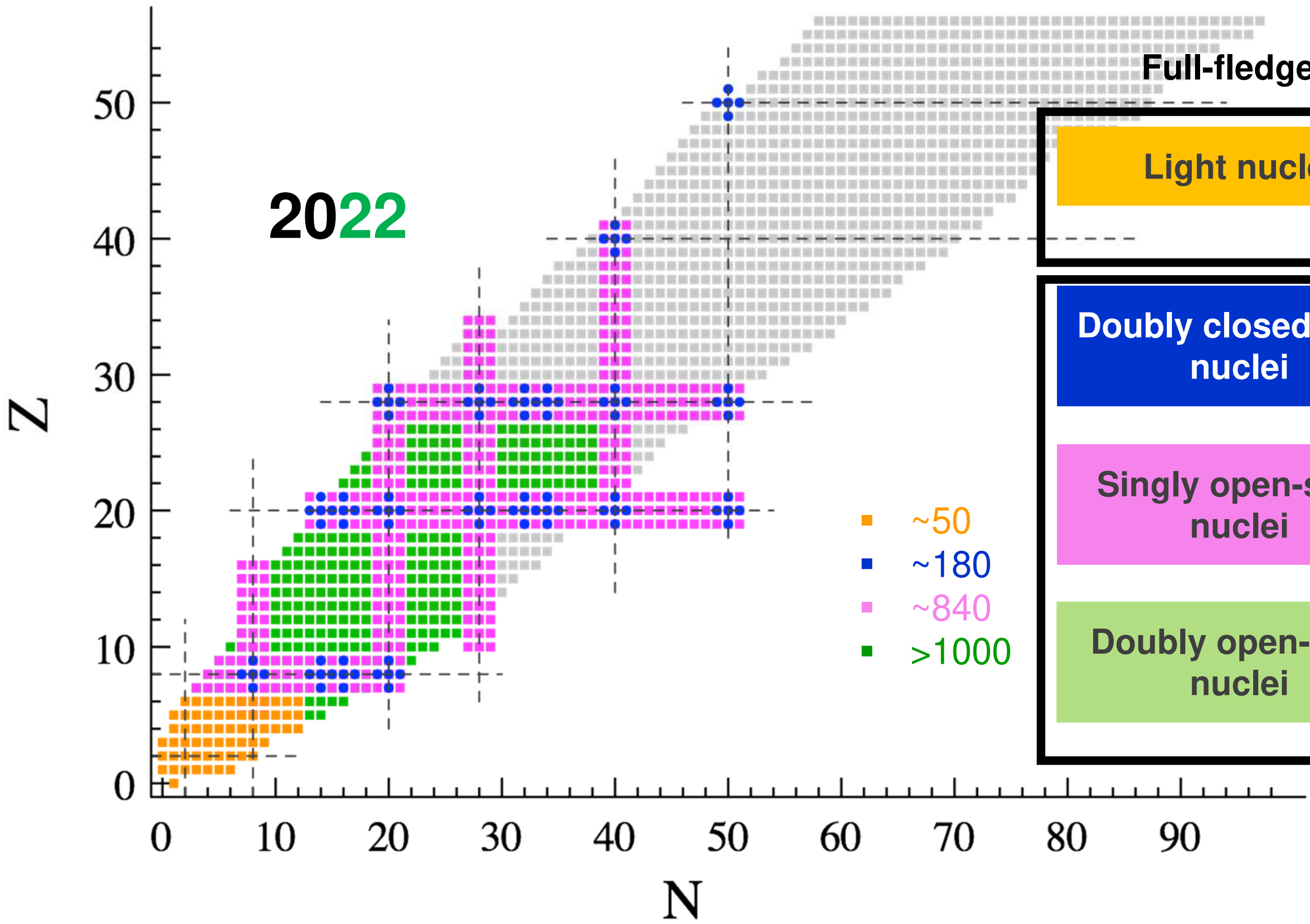
$$\dim \mathcal{H}(1) \equiv n_{\mathrm{dim}}$$

A-body Hilbert space

$$\mathcal{H}_{\mathrm{A}} = \mathcal{H}(1) \otimes \ldots \otimes \mathcal{H}(\mathrm{A})$$

$$\boxed{\dim \mathcal{H}(\mathrm{A}) \equiv n_{\mathrm{dim}}^{\mathrm{A}}}$$

« The curse of dimensionality »



Full-fledged method, e.g. diagonalization

Light nuclei	Exponential cost n^A **GFCM, NCSM**
Doubly closed-shell nuclei	Exponential cost n'^p **CC, SCGF, MRSRG**
Singly open-shell nuclei	Exponential cost n''^p **sMBPT, sBCC, sGSCGF**
Doubly open-shell nuclei	Exponential cost n^p **dBCC, dGSCGF, PGCM-PT**

Expansion methods

Expansion many-body methods: general rationale

H Hamiltonian

Good neutron/proton number N/Z

$[H, R(\theta)] = 0$ Symmetries (U(1), SU(2), ...) Good angular momentum (J^2,J_z)

$H|\Psi\rangle = E|\Psi\rangle$ Secular equation

Solution to static correlations lies here

$H = H_0 + H_1$ Partitioning

« Easy » to solve

$H_0|\Theta^{(0)}\rangle = E^{(0)}|\Theta^{(0)}\rangle$ Unperturbed state

Expansion series

$|\Psi\rangle = \Omega|\Theta^{(0)}\rangle$ Fully correlated state

Key questions

- **Unperturbed state**
  - **-what nature for which systems?**
  - **-what about symmetries?**
  - **-how « simple »/low-cost can it remain?**
- Nature of the expansion
  - -perturbative? non-perturbative?
  - **-formulation depends on unperturbed state?**

Impacts algebraic&numerical complexity there

Closed-shell systems

$$H$$
$$[H, R(\theta)] = 0$$
$$H|\Psi\rangle = E|\Psi\rangle$$

$$H = H_0 + H_1$$

No correlations

$$H_0|\Theta^{(0)}\rangle = E^{(0)}|\Theta^{(0)}\rangle$$

Dynamical correlations

$$|\Psi\rangle = \Omega|\Theta^{(0)}\rangle$$

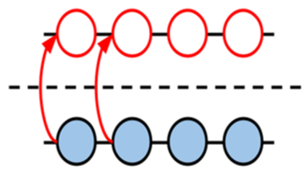
Closed-shell

$$[H_0, R(\theta)] = 0$$

Spherical Hartree Fock

$$|\Theta^{(0)}\rangle \equiv |\Phi\rangle \text{ (sHF)}$$

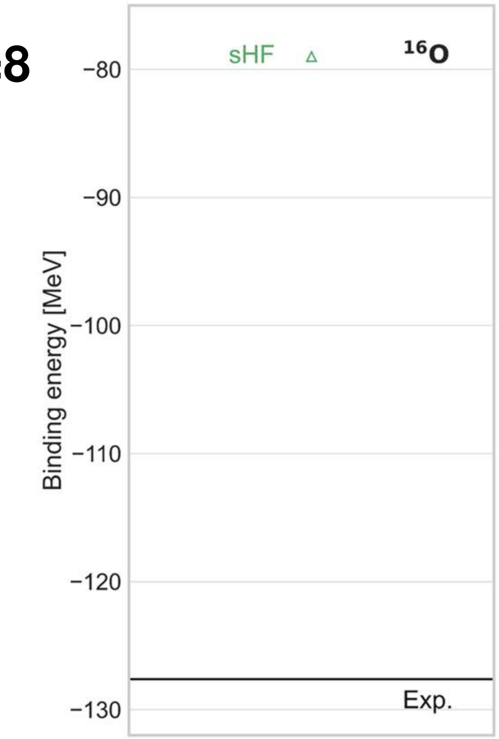
Controlled np-nh expansion

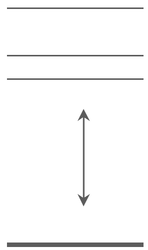


sMBPT, sCC, sDSCGF…

1) Partition
2) Expand

Z=8,N=8





$$|\Phi^{p_1\cdots}_{h_1\cdots}\rangle$$

$$|\Phi\rangle$$

$$\delta E^{p_1\cdots}_{h_1\cdots} > 0$$

Slater determinant reference state and normal ordering

Slater determinant unperturbed state

$$b_\alpha \equiv \sum_l U^*_{l\alpha} c_l \qquad |\Phi^A\rangle \equiv \prod_{i=1}^{A} b_i^\dagger |0\rangle$$

Particle states a,b,c…

Hole states i,j,k…

Ritz variational principle

$$\delta \frac{\langle \Phi^A | H | \Phi^A \rangle}{\langle \Phi^A | \Phi^A \rangle} = 0$$

HF one-body eigenvalue problem

Normal ordering via Wick's theorem with respect to $|\Phi^A\rangle$

$$H \equiv \Lambda^{00}$$
$$+ \frac{1}{1!1!} \sum_{l_1 l_2} \Lambda^{11}_{l_1 l_2} : b^\dagger_{l_1} b_{l_2} :$$
$$+ \frac{1}{2!2!} \sum_{l_1 l_2 l_3 l_4} \Lambda^{22}_{l_1 l_2 l_3 l_4} : b^\dagger_{l_1} b^\dagger_{l_2} b_{l_4} b_{l_3} :$$

$$+ \frac{1}{3!3!} \sum_{l_1 l_2 l_3 l_4 l_5 l_6} \Lambda^{33}_{l_1 l_2 l_3 l_4 l_5 l_6} : b^\dagger_{l_1} b^\dagger_{l_2} b^\dagger_{l_3} b_{l_6} b_{l_5} b_{l_4} :$$

Anti-symmetric fields Λ^{ij} function of

$$t_{pq} \quad \overline{v}_{pqrs} \quad \overline{w}_{pqrstu} \quad U_{pk}$$

➡ **Six-index tensor**
Too expensive to handle

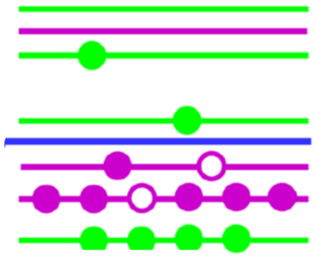
➡ **NO2B approximation**
1-3% error in closed shell
[R. Roth *et al.*, PRL 109 (2012) 052501]

➡ **Effective 2-body operators**
Captures essential of 3-body
Many-body method with 2-body

Spherical coupled cluster expansion method

Slater determinant reference state

$$|\Phi^{\mathrm{A}}\rangle \equiv \prod_{i=1}^{\mathrm{A}} b_i^\dagger |0\rangle$$



Individual excitations

$$|\Phi^{\mu}\rangle \equiv |\Phi^{ab\ldots}_{ij\ldots}\rangle \equiv b_a^\dagger b_{a_n}^\dagger \ldots b_j b_i |\Phi^{\mathrm{A}}\rangle$$

np-nh excitations of the vacuum
Orthonormal basis of $\mathcal{H}_{\mathrm{A}}$

CC ansatz

CC wave operator Ω

$$|\Psi_0^{\mathrm{A}}\rangle \equiv e^{T^{\mathrm{A}}} |\Phi^{\mathrm{A}}\rangle \quad \text{with} \quad \begin{cases} T^{\mathrm{A}} \equiv \sum_{n=1}^{\mathrm{A}} T_n^{\mathrm{A}} \\ T_n^{\mathrm{A}} \equiv \frac{1}{(n!)^2} \sum_{ijk\ldots abc\ldots} \overbrace{T^{abc\ldots}_{ijk\ldots}}^{\text{Cluster amplitudes / Unknowns of the problem}} \underbrace{b_a^\dagger b_b^\dagger b_c^\dagger \ldots b_k b_j b_i}_{\text{Pure excitation operators}} \end{cases}$$

Cluster amplitudes
Unknowns of the problem

$$T_1^{\mathrm{A}}|\Phi\rangle \rightarrow |\Phi_i^a\rangle \quad T_2^{\mathrm{A}}|\Phi\rangle \rightarrow |\Phi_{ij}^{ab}\rangle$$

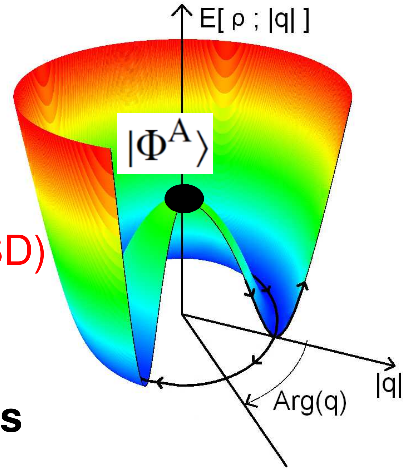
n-tuple connected cluster operator

Pure excitation operators

Energy and amplitude equations

$$H|\Psi_0^{\mathrm{A}}\rangle = E_0^{\mathrm{A}}|\Psi_0^{\mathrm{A}}\rangle \Rightarrow \begin{array}{l} E_0^{\mathrm{A}} = \langle\Phi^{\mathrm{A}}|H\, e^{T^{\mathrm{A}}}|\Phi^{\mathrm{A}}\rangle_C \\ 0 = \langle\Phi^{\mu}|H\, e^{T^{\mathrm{A}}}|\Phi^{\mathrm{A}}\rangle_C \end{array}$$

Truncate, e.g. $T^{A} = T^{A}_1 + T^{A}_2$ (CCSD)
Solve for n=1,2



Connected = terminating exponential
Algebraic expression through Wick's theorem/diagrammatic rules

Ex: for the energy

$$E_0^{\mathrm{A}} = \boxed{\Lambda^{00}} + \boxed{\sum_{ia} \Lambda^{11}_{ia} T_i^a + \frac{1}{2}\sum_{ijab} \Lambda^{22}_{ijab} T_i^a T_j^b + \frac{1}{4}\sum_{ijab} \Lambda^{22}_{ijab} T_{ij}^{ab}}$$

Unperturbed

Dynamical correlations

Open-shell systems - 1

$$H$$
$$[H, R(\theta)] = 0$$
$$H|\Psi\rangle = E|\Psi\rangle$$

What to do?
-can one keep the simplicity of a single-reference method?
-if so, is there a price to play?

Open-shell

$$H = H_0 + H_1$$

No correlations

$$H_0|\Theta^{(0)}\rangle = E^{(0)}|\Theta^{(0)}\rangle$$

Dynamical correlations

$$|\Psi\rangle = \Omega|\Theta^{(0)}\rangle$$

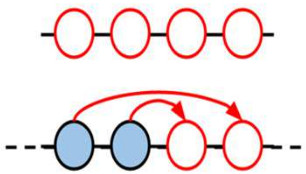
$$[H_0, R(\theta)] = 0$$

Spherical Hartree Fock

$$|\Theta^{(0)}\rangle \equiv |\Phi\rangle \text{ (sHF)}$$

Symmetry-conserving single-reference expansion
►state misses crucial IR static correlations

***Uncontrolled* np-nh expansion**



$|\Phi^{p_1\cdots}_{h_1\cdots}\rangle$

$|\Phi\rangle$

$\delta E^{p_1\cdots}_{h_1\cdots} \sim 0$

***Degenerate* unperturbed Slater determinant**

Ex: $\Delta E^{(2)}_{\text{MBPT}} = -\frac{1}{4}\sum_{ijab} \frac{|h^{(2)}_{ijab}|^2}{e_a + e_b - e_i - e_j} = 0$

Open-shell systems - 2

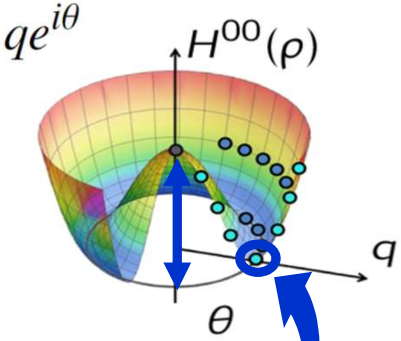
$$H$$
$$[H, R(\theta)] = 0$$
$$H|\Psi\rangle = E|\Psi\rangle$$

order parameter $\varrho \equiv q e^{i\theta}$

$U(1) \rightarrow \cancel{A}$

$SU(2) \rightarrow \cancel{J}^2$

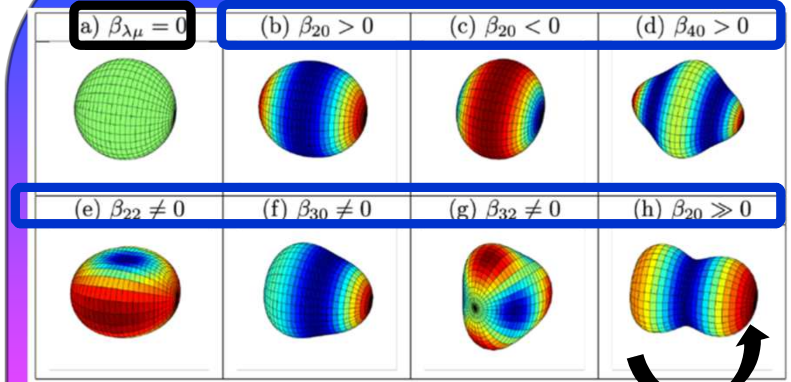
Open-shell



Symmetry breaking minimum

$\equiv$ "q"

Example for SU(2)



$$J^2|\Phi(\beta_{\lambda\mu} = 0)\rangle = J(J+1)|\Phi(0)\rangle$$ (with J=0)

$$J^2|\Phi(\beta_{\lambda\mu} \neq 0)\rangle \neq J(J+1)|\Phi(0)\rangle$$

Rotation Euler angles $(\alpha, \beta, \gamma) \equiv$ "θ"

$$H = H_0 + H_1$$

$$[H_0, R(\theta)] \neq 0$$

$$q \neq 0$$

Static IR correlations

Deformed Hartree Fock Bogoliubov

$$H_0|\Theta^{(0)}\rangle = E^{(0)}|\Theta^{(0)}\rangle \qquad |\Theta^{(0)}\rangle \equiv |\Phi(q)\rangle \text{ (dHFB)}$$

Dynamical correlations

$$|\Psi\rangle = \Omega|\Theta^{(0)}\rangle$$

Bogoliubov transformation

$$\beta_k(q) \equiv \sum_l U^*_{lk}(q)\, c_l + V^{*l}_{\ k}(q)\, c^\dagger_l$$

$$\beta^\dagger_k(q) \equiv \sum_l U^{lk}(q)\, c^\dagger_l + V^k_l(q)\, c_l.$$

Sum over different j

Bogoliubov Vacuum

$$|\Phi(q)\rangle, \,/\; \beta_k(q)|\Phi(q)\rangle = 0 \;\; \forall q$$

Product state as Slater determinant

Symmetry breaking

$$A|\Phi(q)\rangle \neq A|\Phi(q)\rangle$$

$$J^2|\Phi(q)\rangle \neq J(J+1)|\Phi(q)\rangle$$

Elementary excitations

$$\beta^\dagger_{k_1}(q) \cdots \beta^\dagger_{k_n}(q)|\Phi(q)\rangle = |\Phi^{k_1 \cdots k_n}(q)\rangle$$

Generalize np-nh excitations

Open-shell systems - 2

H

$[H, R(\theta)] = 0$

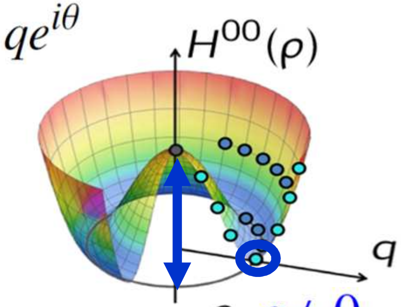
$H|\Psi\rangle = E|\Psi\rangle$

order parameter $\varrho \equiv q e^{i\theta}$

$U(1) \rightarrow \cancel{A}$

$SU(2) \rightarrow \cancel{J^2}$

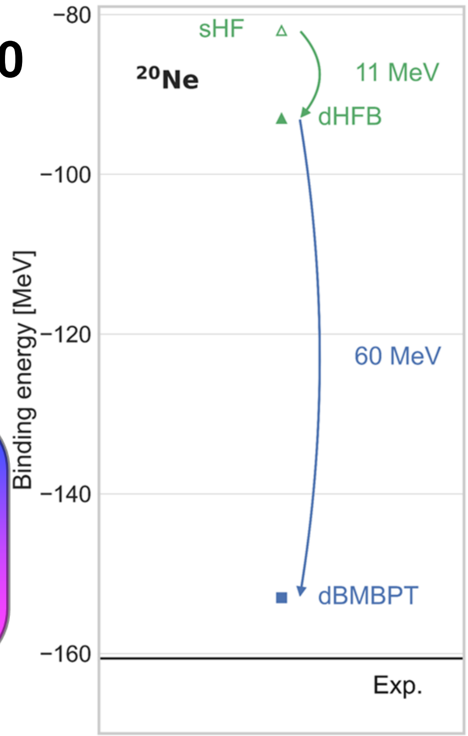
Open-shell



$\theta \quad q \neq 0$

Symmetry breaking minimum

Z=10,N=10



$H = H_0 + H_1$

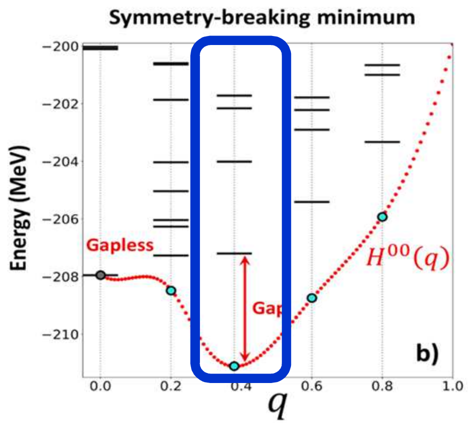
$[H_0, R(\theta)] \neq 0$

Static IR correlations

Deformed Hartree Fock Bogoliubov

$H_0|\Theta^{(0)}\rangle = E^{(0)}|\Theta^{(0)}\rangle$

$|\Theta^{(0)}\rangle \equiv |\Phi(q)\rangle$ (dHFB)



Non-degenerate unperturbed Bogoliubov state

Dynamical correlations

Controlled qp expansion

$|\Psi\rangle = \Omega|\Theta^{(0)}\rangle$

dBMBPT, dBCC, dGSCGF…

Ex: $\Delta E^{(2)}_{\text{BMBPT}} = \frac{1}{4!} \sum_{k_1 k_2 k_3 k_4} \frac{|\Omega^{40}_{k_1 k_2 k_3 k_4}|^2}{E_{k_1} + E_{k_2} + E_{k_3} + E_{k_4}} > 0$

1) **Break - partition**

2) **Expand**

Bogoliubov reference state and normal ordering

Bogoliubov reference state

$$\beta_k = \sum_p U^*_{pk} c_p + V^*_{pk} c^\dagger_p$$

$$\beta^\dagger_k = \sum_p U_{pk} c^\dagger_p + V_{pk} c_p$$

$$|\Phi\rangle \equiv C \prod_k \beta_k |0\rangle$$

$q_{U(1)}$ ≠0 a priori and reduce to SD otherwise

$q_{SU(2)}$ omitted everywhere for simplicity

$$\Omega \equiv H - \lambda A$$

Ritz variational principle

$$\delta \frac{\langle\Phi|\Omega|\Phi\rangle}{\langle\Phi|\Phi\rangle} = 0$$

HFB eigenvalue problem

Normal ordering via Wick's theorem with respect to $|\Phi\rangle$ in quasi-particle basis

$$H \equiv \sum_{n=0}^{3} \sum_{i+j=2n} \frac{1}{i!\,j!} \sum_{l_1 \ldots l_{i+j}} H^{ij}_{l_1 \ldots l_{i+j}} \beta^\dagger_{k_1} \ldots \beta^\dagger_{k_i} \beta_{k_{i+j}} \ldots \beta_{k_{i+1}}$$

$$\equiv H^{00} + [H^{20} + H^{11} + H^{02}] + [H^{40} + H^{31} + H^{22} + H^{13} + H^{04}] \boxed{+ \sum_{i+j=6} H^{ij}}$$

$$\equiv \sum_{n=0}^{2} H^{[2n]} \boxed{+ H^{[6]}}$$

6-qp operators

H^{ij} **matrix elements function of**

$$t_{pq} \quad \overline{v}_{pqrs} \quad \overline{w}_{pqrstu} \quad U_{pk} \quad V_{pk}$$

Similarly for A and $\Omega = H - \lambda A$

➡ **Six-index tensors**
Too expensive to handle

➡ **NO2B approximation**
Simply remove H[6]?

➡ **PNO2B approximation**
[Ripoche, Tichai, Duguet, EPJA 2020]
SC rank-reduction
[Frosini et *al*. EPJA 2021]

Deformed Bogoliubov coupled cluster expansion method

Bogoliubov reference state

$$|\Phi\rangle \equiv C \prod_k \beta_k |0\rangle$$

Quasi-particle excitations

$$|\Phi^{\mu}\rangle \equiv |\Phi^{k_1\dots k_{2n}}\rangle \equiv \beta^{\dagger}_{k_1}\dots\beta^{\dagger}_{k_{2n}}|\Phi\rangle$$

Orthonormal basis of Fock space

Reduces to npnh excit. in closed-shell

Bogoliubov CC ansatz

$$|\Psi_0^{\cancel{(A)}}\rangle \equiv e^{T}|\Phi\rangle \quad \text{with} \quad \begin{cases} T = \sum_{n=1} T_n \\ T_n \equiv \dfrac{1}{(2n)!}\sum_{k_1\dots k_{2n}} \underbrace{T^{2n0}_{k_1\dots k_{2n}}}_{\text{Cluster a, Unknowns of the problem}} \underbrace{\beta^{\dagger}_{k_1}\dots\beta^{\dagger}_{k_{2n}}}_{\text{Pure excitation operators}} \end{cases}$$

Cluster a $T_1|\Phi\rangle \rightarrow |\Phi^{k_1k_2}\rangle$ $T_2|\Phi\rangle \rightarrow |\Phi^{k_1k_2k_3k_4}\rangle$

Unknowns of the problem

n-tuple connected cluster operator

Pure excitation operators

as soon as T is truncated

➲symmetry contamination

➲but « simple » theory with n^p cost

Energy and amplitude equations

$$H|\Psi_0^{A}\rangle = E_0^{A}|\Psi_0^{A}\rangle$$

$$A|\Psi_0^{A}\rangle = \mathrm{A}|\Psi_0^{A}\rangle$$

$\langle\Phi|$, $\langle\Phi^{\mu}|$ ➜

$$E_0^{\cancel{A}} = \langle\Phi|H\,e^{T}|\Phi\rangle_C$$

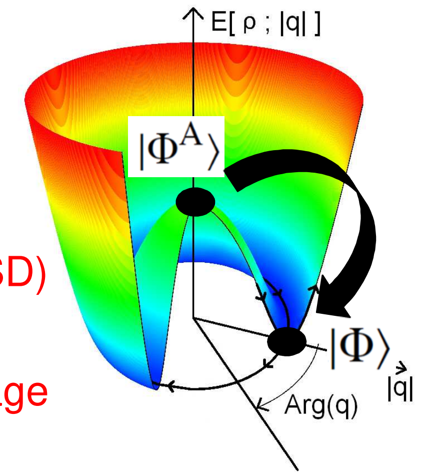
$$0 = \langle\Phi^{\mu}|H\,e^{T}|\Phi\rangle_C$$

$$\cancel{\mathrm{A}} = \langle\Phi|A\,e^{T}|\Phi\rangle_C$$

Truncate, e.g. T= T_1+T_2 (BCCSD)

Solve for n=1,2

Constrained to be true in average



Connected = terminating exponential

Algebraic expression through Wick's theorem/diagrammatic rules

Ex: for the energy

$$E_0^{\cancel{A}} = \boxed{H^{00}} + \frac{1}{2}\sum_{k_1k_2} H^{02}_{k_1k_2}T^{20}_{k_1k_2} + \frac{1}{8}\sum_{k_1k_2k_3k_4} H^{04}_{k_1k_2k_3k_4}T^{20}_{k_1k_2}T^{20}_{k_3k_4} + \frac{1}{4!}\sum_{k_1k_2k_3k_4} H^{04}_{k_1k_2k_3k_4}T^{40}_{k_1k_2k_3k_4}$$

Unperturbed

Open-shell systems - 2

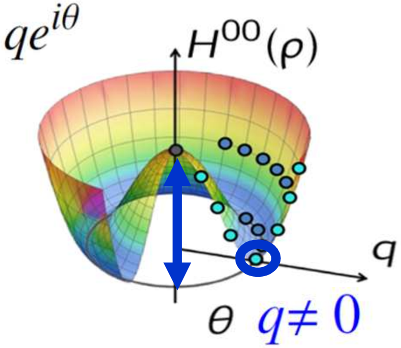
$$H$$
$$[H, R(\theta)] = 0$$
$$H|\Psi\rangle = E|\Psi\rangle$$

order parameter $\varrho \equiv q e^{i\theta}$

$U(1) \rightarrow \cancel{A}$

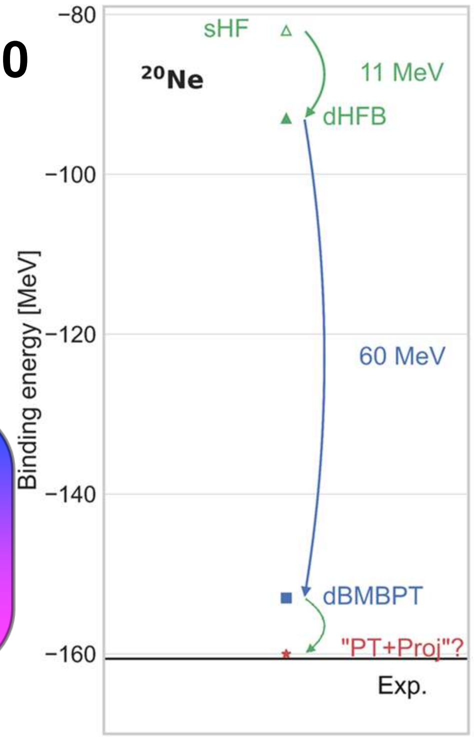
$SU(2) \rightarrow \cancel{J}^2$

Open-shell



Symmetry breaking minimum

Z=10,N=10



$$H = H_0 + H_1$$

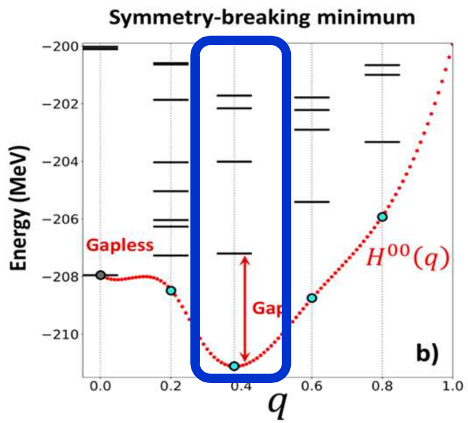
$$[H_0, R(\theta)] \neq 0$$

Static IR correlations

Deformed Hartree Fock Bogoliubov

$$H_0|\Theta^{(0)}\rangle = E^{(0)}|\Theta^{(0)}\rangle$$

$$|\Theta^{(0)}\rangle \equiv |\Phi(q)\rangle \text{ (dHFB)}$$



Non-degenerate unperturbed Bogoliubov state

Dynamical correlations

Controlled qp expansion

$$|\Psi\rangle = \Omega|\Theta^{(0)}\rangle$$

dBMBPT, dBCC, dGSCGF…

1) **Break** - partition
2) **Expand**

Open-shell systems - 3

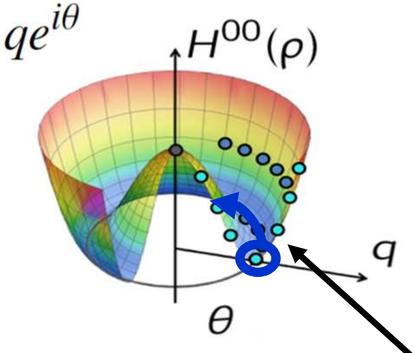
$$H$$
$$[H, R(\theta)] = 0$$
$$H|\Psi\rangle = E|\Psi\rangle$$

order parameter $\varrho \equiv q e^{i\theta}$

$\mathrm{U}(1) \rightarrow \cancel{N}$

$\mathrm{SU}(2) \rightarrow \cancel{J}^2$

Open-shell



Symmetry breaking minimum

$$H = H_0 + H_1$$

$$[H_0, R(\theta)] \neq 0$$

Static IR correlations

Deformed Hartree Fock Bogoliubov

$$H_0|\Theta^{(0)}\rangle = E^{(0)}|\Theta^{(0)}\rangle$$

$$|\Theta^{(0)}\rangle \equiv |\Phi(q)\rangle \text{ (dHFB)}$$

Dynamical correlations

Generalized qp expansion

$$|\Psi\rangle = \Omega|\Theta^{(0)}\rangle$$

PBMBPT, PBCC

1) **Break - partition**
2) **Expand - Project**

Insert into expanded energy:
- ►**keep same partitioning and expansion of Ω**
- ►**P superfluous/impactful if Ω exact/truncated**

Further capture static correlations

Successful novel approach
Natural access to rotational excitations

Particle-number projected BCC formalism

Projection operator on good A

$$P^{\mathrm{A}} \equiv \frac{1}{2\pi}\int_0^{2\pi} d\varphi\, e^{-i\mathrm{A}\varphi}\boxed{R(\varphi)} \equiv e^{iA\varphi}$$

Rotation is gauge space

Projected BCC ansatz

$$|\Psi^{\mathrm{A}}_{\mathrm{PBCC}}\rangle \equiv \boxed{P^{\mathrm{A}}}|\Psi^{\cancel{(\mathrm{A})}}_{\mathrm{BCC}}\rangle$$

Always true!

Rotated state

Thouless operator

$$\langle\Phi(\varphi)| = \langle\Phi|R(\varphi) = \langle\Phi(\varphi)|\Phi\rangle\langle\Phi|e^{\boxed{Z(\varphi)}}$$

Known from (U,V,φ)

with $$Z(\varphi) \equiv \frac{1}{2}\sum_{k_1k_2}\boxed{Z^{02}_{k_1k_2}(\varphi)}\boxed{\beta_{k_2}\beta_{k_1}}$$

Pure de-excitation operator

Similarity transformed operator

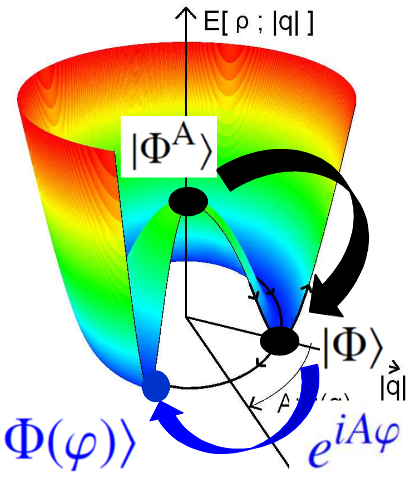
$$O_Z(\varphi) \equiv e^{Z(\varphi)}Oe^{-Z(\varphi)}$$

Projected BCC energy

$$H|\Psi^{\mathrm{A}}\rangle = E^{\mathrm{A}}|\Psi^{\mathrm{A}}\rangle \quad \Rightarrow \quad E^{\mathrm{A}} = \frac{\int_0^{2\pi} d\varphi\, e^{-i\mathrm{A}\varphi}\,\mathcal{H}(\varphi)}{\int_0^{2\pi} d\varphi\, e^{-i\mathrm{A}\varphi}\,\mathcal{N}(\varphi)}$$

Not a pure excitation operator…

with
$$\left|\begin{aligned}\mathcal{N}(\varphi) &\equiv \langle\Phi(\varphi)|e^{U}|\Phi\rangle = \langle\Phi(\varphi)|\Phi\rangle\langle\Phi|e^{\boxed{U_Z(\varphi)}}|\Phi\rangle \\ \mathcal{H}(\varphi) &\equiv \langle\Phi(\varphi)|He^{U}|\Phi\rangle = \langle\Phi(\varphi)|\Phi\rangle\langle\Phi|H_Z(\varphi)e^{U_Z(\varphi)}|\Phi\rangle\end{aligned}\right.$$



[Duguet, Signoracci JPG 2016]
[Qiu *et al.* PRC 2019]

Off-diagonal norm and Hamiltonian kernels

Particle-number projected BCC formalism

Disentangled cluster operators Disantengling the algebra to extract pure excitation terms

$$e^{U_Z(\varphi)}|\Phi\rangle \equiv e^{\boxed{W(\varphi)}}|\Phi\rangle$$

1) Pure excitation operator BUT contains a constant term
2) Allows algebraic expressions of kernels later on following standard steps
3) Explicit relation between W(φ) and U_Z(φ) too complicated (need other approach)

$$W(\varphi) = \sum_{n=0} W_n(\varphi) \equiv \underbrace{W_0(\varphi)}_{\text{Constant}} + \underbrace{\mathcal{T}(\varphi)}_{\text{Standard cluster operator form}} \quad \text{with} \quad W_n(\varphi) \equiv \frac{1}{2n!}\sum_{k_1\ldots k_{2n}} W^{2n0}_{k_1\ldots k_{2n}}(\varphi)\underline{\beta^\dagger_{k_1}\ldots\beta^\dagger_{k_{2n}}}$$

Connected kernels and PBCC energy

But how to determine W(φ)?

$$\mathcal{N}(\varphi) \equiv e^{W_0(\varphi)}\langle\Phi(\varphi)|\Phi\rangle$$

Norm kernel determined by W_0(φ)

$$h(\varphi) \equiv \frac{\mathcal{H}(\varphi)}{\mathcal{N}(\varphi)} = \langle\Phi|H_Z(\varphi)e^{\mathcal{T}(\varphi)}|\Phi\rangle_C$$

Connected part of energy kernel determined by $\mathcal{T}$(φ)
Same algebraic/terminating form as standard BCC kernel!

$$\boxed{E^{\mathrm{A}} = \frac{\int_0^{2\pi} d\varphi\, e^{-i\mathrm{A}\varphi}\, h(\varphi)\mathcal{N}(\varphi)}{\int_0^{2\pi} d\varphi\, e^{-i\mathrm{A}\varphi}\, \mathcal{N}(\varphi)}}$$

1) Reduction to BCC

$\varphi = 0$ in $h(\varphi)$

$$E^{\mathrm{A}} = \langle\Phi|H\, e^{U}|\Phi\rangle_C$$

2) Reduction to PHFB

$$W(\varphi) = 0$$

$$\mathcal{N}^{\mathrm{PHFB}}(\varphi) \equiv \langle\Phi(\varphi)|\Phi\rangle$$

$$h^{\mathrm{PHFB}}(\varphi) = \langle\Phi|H_Z(\varphi)|\Phi\rangle_C$$

[Duguet, Signoracci JPG 2016]
[Qiu *et al.* PRC 2019]

Particle-number projected BCC formalism

Gauge-rotated cluster amplitudes $W_k(\varphi)$

$\frac{d}{d\varphi}$ ➡

$\langle \Phi^\mu |$ ➡

$$e^{Z(\varphi)}\, e^{U} |\Phi\rangle = e^{W(\varphi)} |\Phi\rangle$$

Initial conditions

$$W_0(0) = 0$$

$$W_k(0) = U_k$$

Even when U truncated ➡ **All ranks in $W(\varphi)$ coupled**

⬇

Second truncation on $W_k(\varphi)$

⬇

Integrate coupled ODEs and insert in PBCC energy

Coupled ordinary differential equations

Kernel of particle number operator = generator of U(1)

$$\frac{d}{d\varphi} W_0(\varphi) = \frac{i}{2} \sum_{k_1 k_2} \boxed{A^{02}_{k_1 k_2}(\varphi)} W_{k_1 k_2}(\varphi)$$

$$\frac{d}{d\varphi} W_{k_1 k_2}(\varphi) = i \sum_{k_3 k_4} A^{02}_{k_3 k_4}(\varphi) \Big[\frac{1}{2} W_{k_3 k_4 k_1 k_2}(\varphi) - W_{k_1 k_3}(\varphi) W_{k_2 k_4}(\varphi) \Big],$$

$$\frac{d}{d\varphi} W_{k_1 k_2 k_3 k_4}(\varphi) = i \sum_{k_5 k_6} A^{02}_{k_5 k_6}(\varphi) \Big[\frac{1}{2} W_{k_5 k_6 k_1 k_2 k_3 k_4}(\varphi) + W_{k_1 k_5}(\varphi) W_{k_6 k_2 k_3 k_4}(\varphi) + W_{k_2 k_5}(\varphi) W_{k_1 k_6 k_3 k_4}(\varphi) + W_{k_3 k_5}(\varphi) W_{k_1 k_2 k_6 k_4}(\varphi) + W_{k_4 k_5}(\varphi) W_{k_1 k_2 k_3 k_6}(\varphi) \Big]$$

$$\vdots$$

[Duguet, Signoracci JPG 2016]
[Qiu *et al.* PRC 2019]

Open-shell systems - 3

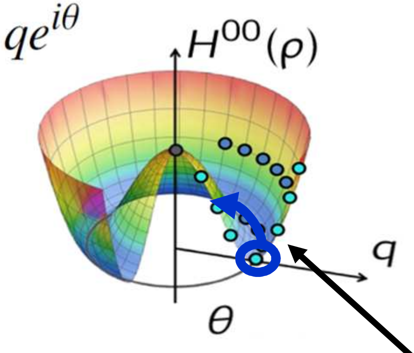
$$H$$
$$[H, R(\theta)] = 0$$
$$H|\Psi\rangle = E|\Psi\rangle$$

order parameter $\varrho \equiv q e^{i\theta}$

$\mathrm{U}(1) \to \cancel{N}$

$\mathrm{SU}(2) \to \cancel{J^2}$

Open-shell



Symmetry breaking minimum

$$H = H_0 + H_1$$

$$[H_0, R(\theta)] \neq 0$$

Static IR correlations

Deformed Hartree Fock Bogoliubov

$$H_0|\Theta^{(0)}\rangle = E^{(0)}|\Theta^{(0)}\rangle$$

$$|\Theta^{(0)}\rangle \equiv |\Phi(q)\rangle \text{ (dHFB)}$$

Dynamical correlations

Generalized qp expansion

$$|\Psi\rangle = \Omega|\Theta^{(0)}\rangle$$

PBMBPT, PBCC

1) **Break - partition**
2) **Expand - Project**

Insert into expanded energy:
- ►**keep same partitioning and expansion of Ω**
- ►**P superfluous/impactful if Ω exact/truncated**

Further capture static correlations

Successful novel app**roach**
Natural access to rotational excitations
Projection however approximated
Handling symmetry prior to partition?

Open-shell systems - 4

H

$[H, R(\theta)] = 0$

$H|\Psi\rangle = E|\Psi\rangle$

Open-shell

$H = H_0 + H_1$

Static IR correlations

$H_0|\Theta^{(0)}\rangle = E^{(0)}|\Theta^{(0)}\rangle$

Dynamical correlations

$|\Psi\rangle = \Omega|\Theta^{(0)}\rangle$

$[H_0, R(\theta)] = 0$

Multi-configuration sHF

$|\Theta^{(0)}\rangle \equiv \sum_{I \in \mathcal{P}} c_I |\Phi^I(0)\rangle$ (MC)

Generalized np-nh expansion

MCPT

$|\Phi^{p_1 \cdots}_{h_1 \cdots}(0)\rangle$

$\mathcal{P}$ $|\Phi(0)\rangle$

Final requirements for unperturbed state

- ❑ Handles **degeneracy**
- ❑ Preserves **symmetry**
- ❑ **Low dimensionality**

Open-shell systems - 4

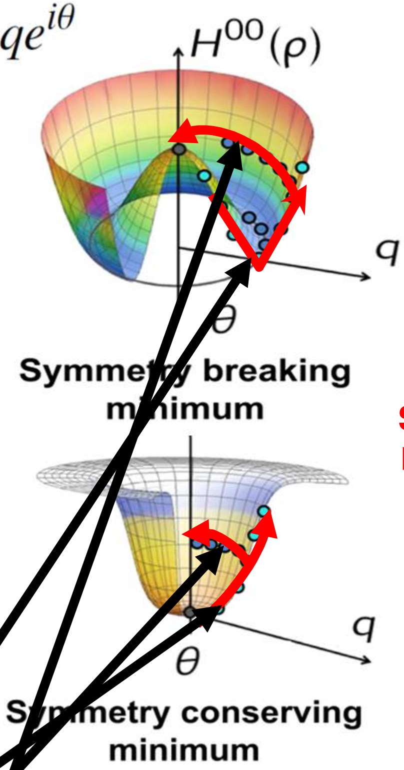
$$H$$
$$[H, R(\theta)] = 0$$
$$H|\Psi\rangle = E|\Psi\rangle$$

order parameter

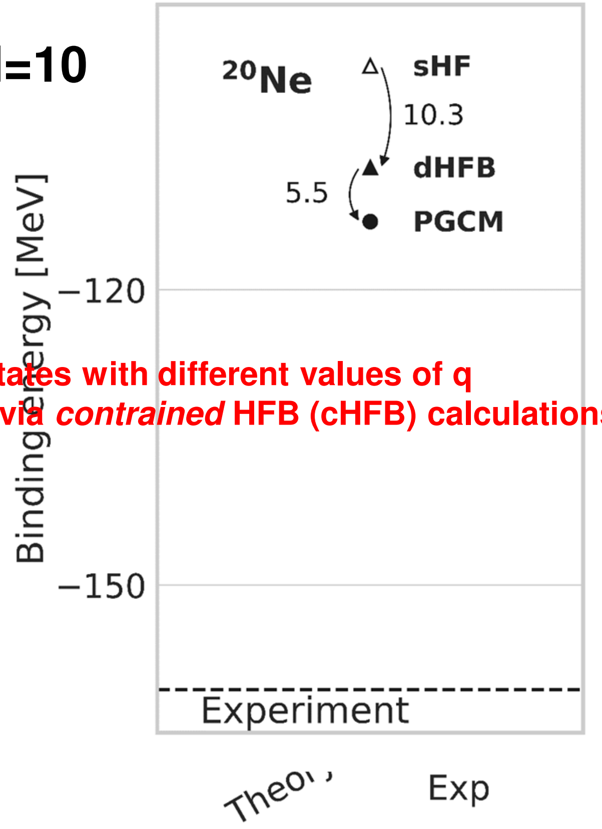
$\varrho \equiv q e^{i\theta}$

$$\mathrm{U}(1) \rightarrow \mathrm{A}$$
$$\mathrm{SU}(2) \rightarrow \mathrm{J}^2$$

Open-shell



Z=10,N=10



Set of HFB states with different values of q
►generated via *contrained* HFB (cHFB) calculations

$$H = H_0 + H_1$$

$$[H_0, R(\theta)] = 0$$

Static IR correlations

PGCM

$$H_0|\Theta^{(0)}\rangle = E^{(0)}|\Theta^{(0)}\rangle$$

$$|\Theta^{(0)}\rangle \equiv \sum_q f(q) P|\Phi(q)\rangle \ \text{(PGCM)}$$

Dynamical correlations

$$|\Psi\rangle = \Omega|\Theta^{(0)}\rangle$$

Projected generator coordinate method

***Low dimensional* mixing of non-orthogonal cHFB states**

- ►Symmetry restored = fluctuation of angle θ of order parameter
- ►Further includes quantum fluctuations of norm q of order parameter

Perform low-dimensional diagonalization in *non-orthogonal basis*

- **►further include static ground-state correlations**
- **►access both vibrational (q) and rotational (θ) excitations**

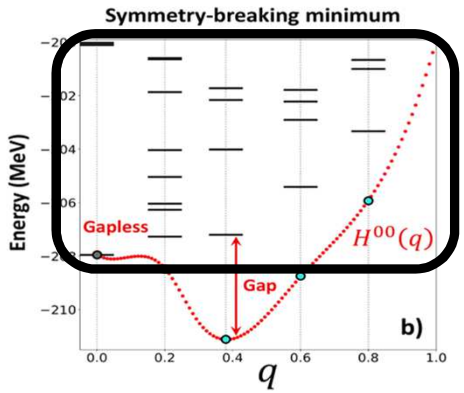
Open-shell systems - 4

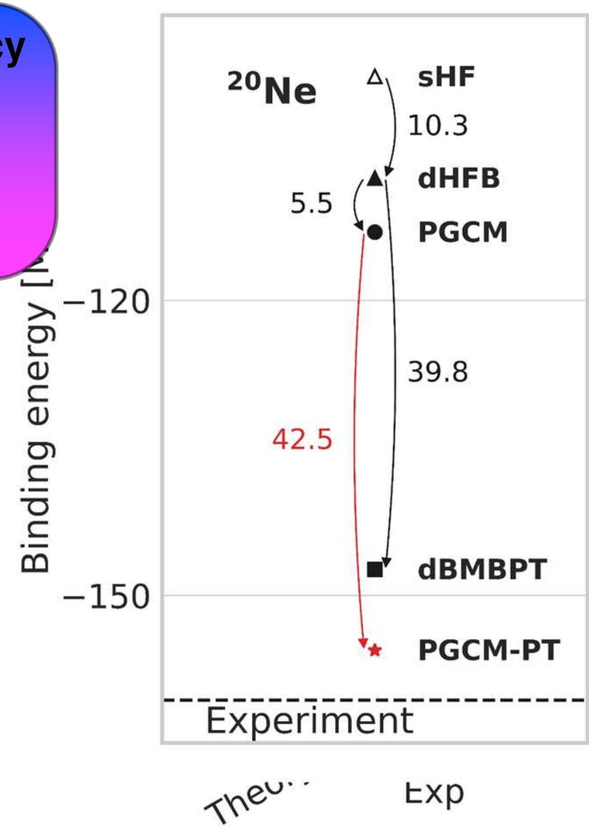
$$H$$
$$[H, R(\theta)] = 0$$
$$H|\Psi\rangle = E|\Psi\rangle$$

$$\mathrm{U(1)} \rightarrow \mathrm{A}$$
$$\mathrm{SU(2)} \rightarrow \mathrm{J}^2$$

Open-shell

Absolute energies: **1-2% accuracy**
-improves over dBMBPT(2)
Spectra: **20% accuracy**
-Negligible PT corrections
-Enrich PGCM state?





$$H = H_0 + H_1$$

Static IR correlations

$$H_0|\Theta^{(0)}\rangle = E^{(0)}|\Theta^{(0)}\rangle$$

Dynamical correlations

$$|\Psi\rangle = \Omega|\Theta^{(0)}\rangle$$

$$[H_0, R(\theta)] = 0$$

PGCM

$$|\Theta^{(0)}\rangle \equiv \sum_q f(q) P |\Phi(q)\rangle \ \text{(PGCM)}$$

Generalized multi vacua qp expansion

PGCM-PT

PGCM Perturbation Theory
Only weak remaining dynamical correlations to include
Multi copies of Fock space = large linear system to solve
► handling redundancies, intruders, large dimension
State-specific expansion for each PGCM state
► access ground-state and spectroscopy

1) **Break - project - partition**
2) **Expand**

[M. Frosini et al., EPJA 58 (2022) 62; 63; 64]

Selected references

Symmetry-breaking single-reference expansion methods

GSCGF [V. Somà, T. Duguet, C. Barbieri, PRC 84 (2011) 064317]
[V. Somà, A. Cipollone, C. Barbieri, P. Navratil, T. Duguet, PRC 89 (2014) 061301]
[C. Barbieri, T. Duguet, V. Somà, PRC 105 (2022) 044330]

BMBPT [A. Tichai, P. Arthuis, T. Duguet, H. Hergert, V. Somà, R. Roth]
[P. Arthuis, T. Duguet, A. Tichai, R.-D. Lasseri, J.-P. Ebran, CPC 240 (2019) 202]

BCC [A. Signoracci, T. Duguet, G. Hagen, G. R. Jansen, PRC 91 (2015) 064320]
[T. M. Henderson, G. E. Scuseria, J. Dukelsky, A. Signoracci, T. Duguet, PRC89 (2014) 054305]
[A. Tichai, P. Demol, T. Duguet, in preparation (2022)]

Symmetry-breaking and restored single-reference expansion methods

PBMBPT [T. Duguet, JPG 42 (2015) 025107]

PBCC [T. Duguet, A. Signoracci, JPG 44 (2016) 015103]
[Y. Qiu, T. M. Henderson, J. Zhao, and G. E. Scuseria, JCP 147 (2017) 064111]
[Y. Qiu, T. M. Henderson, T. Duguet, G. E. Scuseria, PRC 99 (2019) 044301]
[G. Hagen, S. J. Novario, Z. H. Sun, T. Papenbrock, G. R. Jansen, J. G. Lietz, T. Duguet, A. Tichai, PRC 105 (2022) 064311]

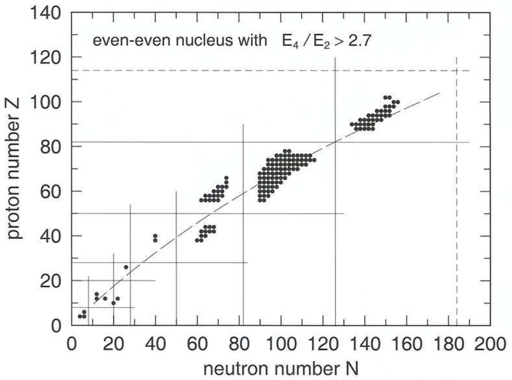
Symmetry-conserving multi-reference expansion method

PGCM-PT [M. Frosini, T. Duguet, J.-P. Ebran, V. Somà, EPJA 58 (2022) 62]
[M. Frosini, T. Duguet, J.-P. Ebran, B. Bally, T. Mongelli, T. R. Rodriguez, R. Roth, V. Somà, EPJA 58 (2022) 63]
[M. Frosini, T. Duguet, J.-P. Ebran, B. Bally, H. Hergert, T. R. Rodriguez, R. Roth, J. M. Yao, V. Somà, EPJA 58 (2022) 64]

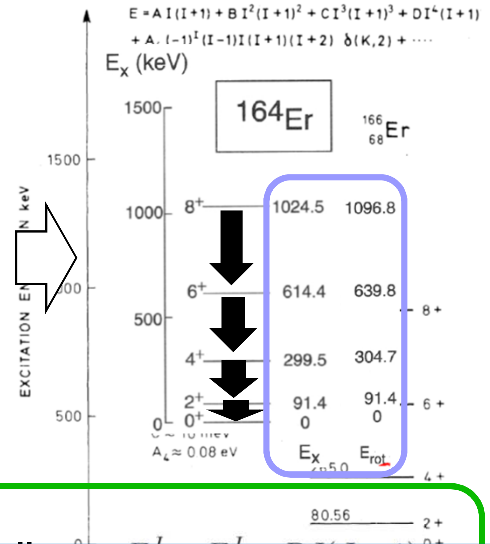
Contents

Rotational properties: from experiment to theory

I. The phenomenology

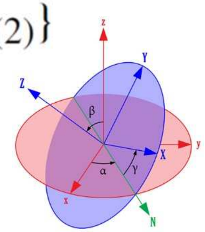


A. Bohr, Nobel Lecture (1975)



Observable patterns
- Set of states
- Strong E2
- Happens fo

II. The symmetry $SU(2) \equiv \{R(\Omega), \Omega \in D_{SU(2)}\}$



Symmetry

$$[H, R(\Omega)] = 0 \Rightarrow \begin{cases} H|\Psi_n^{JM}\rangle = E_n^J|\Psi_n^{JM}\rangle \\ \langle\Psi_n^{JM}|T_\mu^\lambda|\Psi_{n'}^{J'M'}\rangle \equiv (JM\lambda\mu|J'M')\langle Jn||T^\lambda||J'n'\rangle \end{cases}$$

IRREP $\langle\Psi_n^{JM}|R(\Omega)|\Psi_{n'}^{J'M'}\rangle \equiv \delta_{nn'}\delta_{JJ'}D_{MM'}^J(\Omega)$

Observable patterns dictated by SU(2) symmetry
- Energies labeled by J and independent of M
- $\ldots \mu|J'M')$

Ab initio nuclear A-body problem viewpoint

Do rotational properties emerge from basic interactions between the nucleons?
- ►Non-trivial as $B \ll$ energy scale for individual excitations
- ►2N (+3N) are adjusted on 2-body (+3-body) systems

Do perturbations due to coupling to vibrations and individual dof emerge?

III. The mod

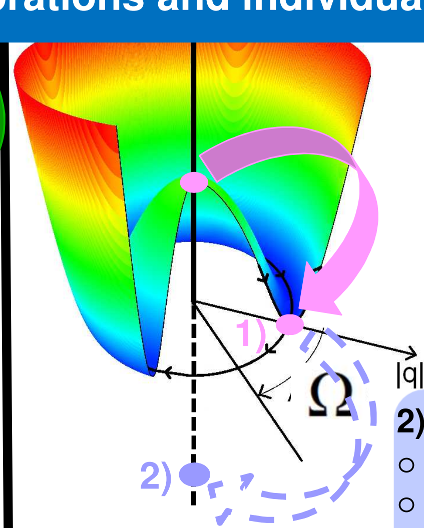
$$|\Psi_{KM}^J\rangle \propto \int d\ldots$$

True eigenstate **Intrinsic state** **Rotational motion**

Features
$$E_n^J = E_0^J + BJ(J+1) \quad \text{with} \quad B \equiv \hbar^2/2I$$
$$Q(J) = [3K^2 - J(J+1)][(J+1)(2J+3)]^{-1} Q_0$$
$$B(E2; J \to J') = 5(16\pi)^{-1}(JK20|J'K)(eQ_0)^2$$

Lessons
- Links a specific subset of states together
- Excellent account of idealized patterns
- Built in separation of rotational degrees of freedom
- Disturbed by coupling of rot. to vib. and ind. dynamics

...etry breaking



...from dof + H
Is not fully realized
Similar for other symmetries of H
***Emerging* lower ~Sp(3,R) symmetry**

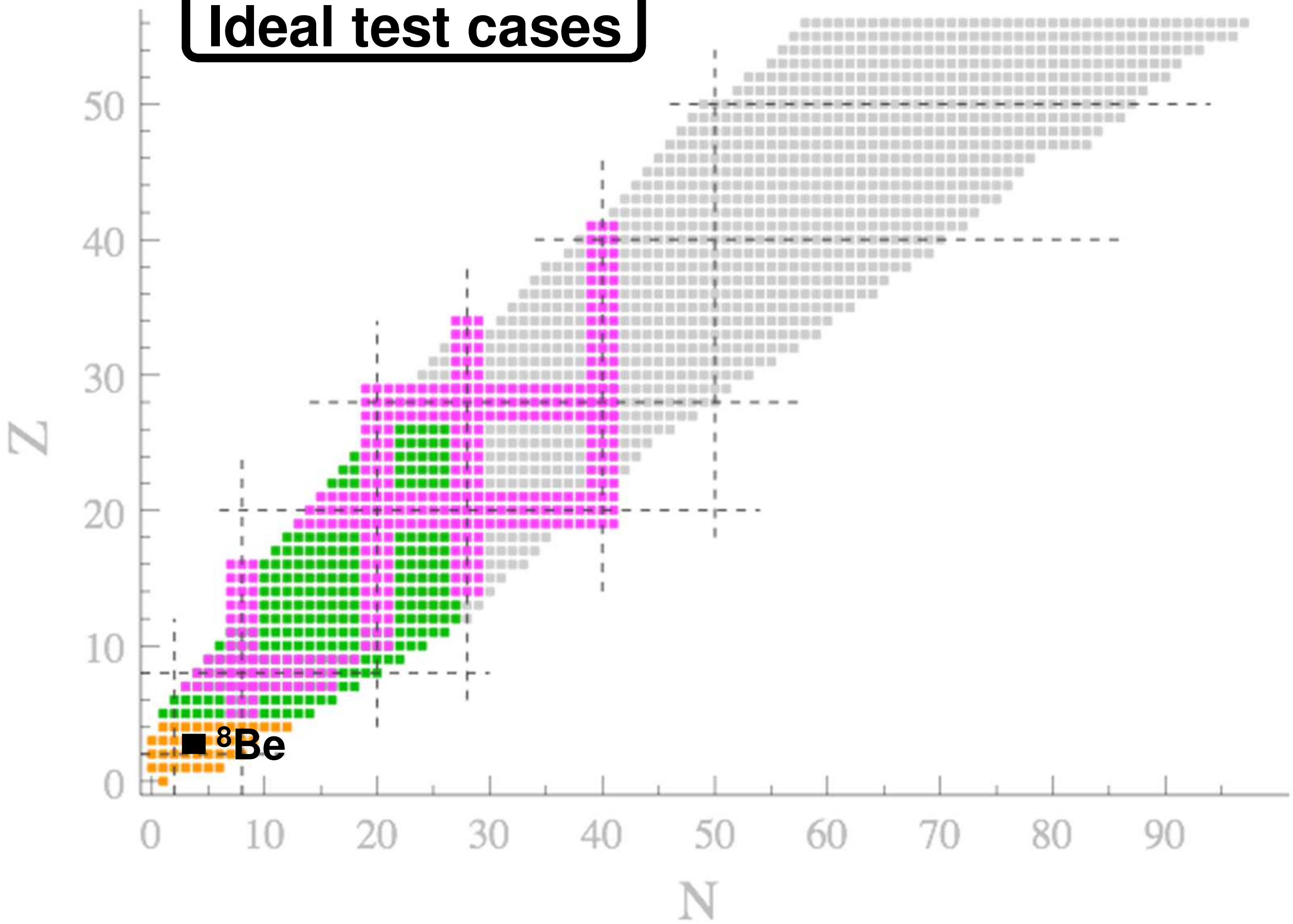
1) Spontaneous breaking of SU(2)
- GS has lower symmetry than H
- GS = wave packet mixing IRREPs
- Goldstone boson = rotations
- Higgs modes = vibrations

2) Finite system = breaking only emergent
- SU(2) symmetry actually satisfied
- Lower symmetry imprints excitations
- Rotational bands and transitions

Ab initio many-body methods: rotational properties

$$H|\Psi_n^{JM}\rangle = E_n^J|\Psi_n^{JM}\rangle$$

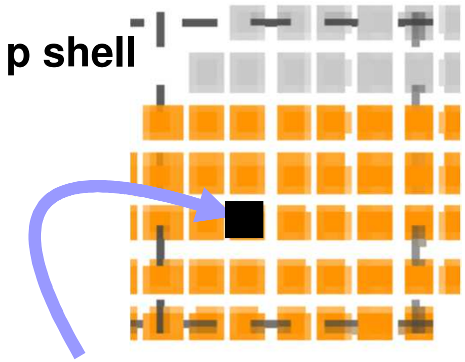
Ideal test cases



1) ^{8}Be [2015]
- No-core shell model
- **Symmetry conserving**

No-core shell model calculation of ^{8}Be isotopes

[M.A. Caprio *et al.*, JMPE 24 (2015) 1541002]

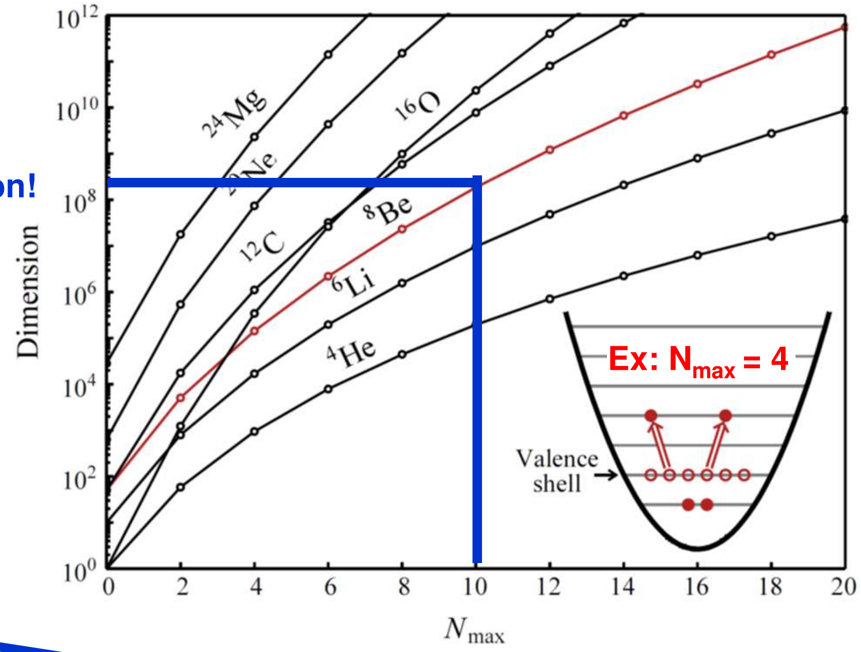


No core shell model calculation of ^{8}Be

^{8}Be at N_{max} =10 ~2.10^8 matrix dimension!

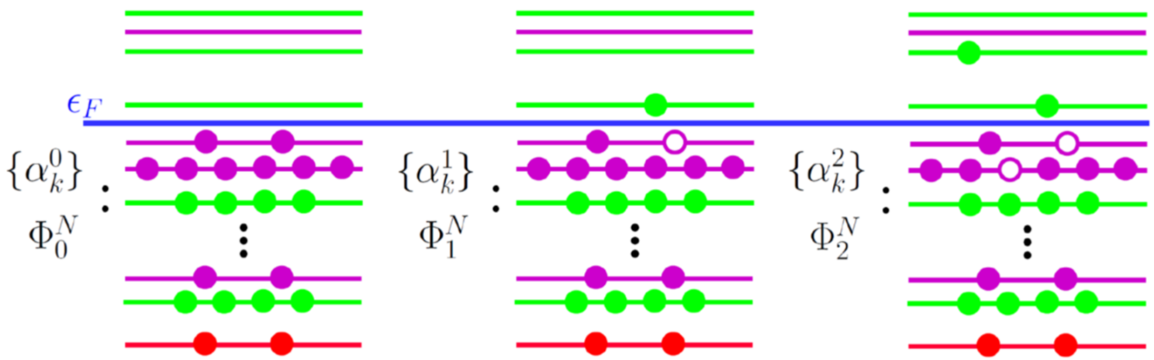
SD basis truncated according to N_{max}

From diagonalization of H in SD basis



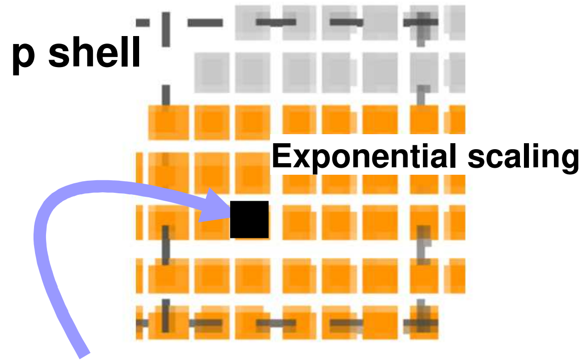
CI expansion over basis of symmetry conserving SD

$$|\Psi_0^N\rangle = |\Phi\rangle + \sum_{ai} C_i^a(0)|\Phi_i^a\rangle + \left(\frac{1}{2!}\right)^2 \sum_{abij} C_{ij}^{ab}(0)|\Phi_{ij}^{ab}\rangle + \left(\frac{1}{3!}\right)^2 \sum_{abcijk} C_{ijk}^{abb}(0)|\Phi_{ijk}^{abc}\rangle + \dots$$



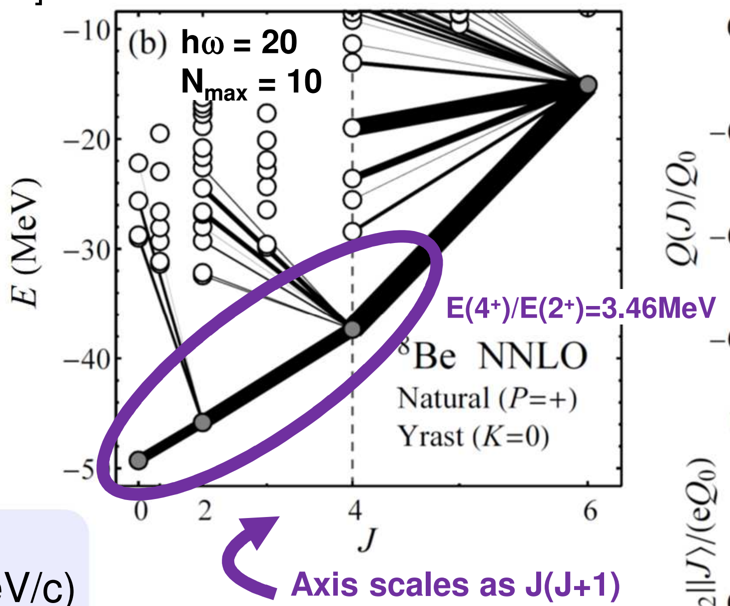
No-core shell model calculation of ^{8}Be isotopes

[M.A. Caprio *et al.*, JMPE 24 (2015) 1541002]

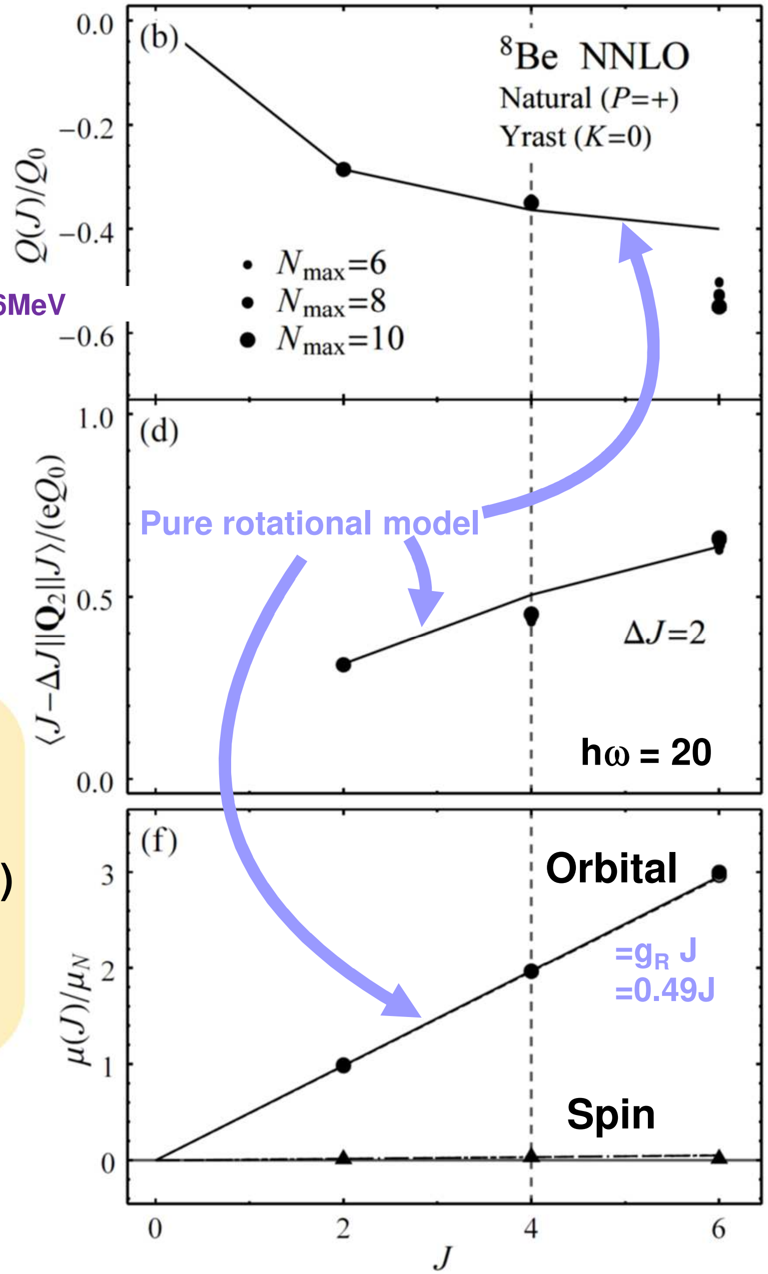


No core shell model calculation of ^{8}Be

Yrast positive parity band



J_{max}=4 in pure p shell



Two-nucleon interactions

- Chiral 2N (N^{2}LO ; Λ_{2NF} = 500 MeV/c)

[Ekstrom *et al.* , PRL 110 (2013) 192502]

Results

Rotational behavior emerge convincingly

- Energies, E2/M1 moments and transitions
- Converged quadrupole strength *ratios* **(absolute moment unsettled)**
- Null spin contribution to μ(J) consistent with α-clustering
- Similar for (un)natural parity/excited bands in ^{7,9}Be (not shown)

Robust against (modest) variation of 2N interaction (not shown)

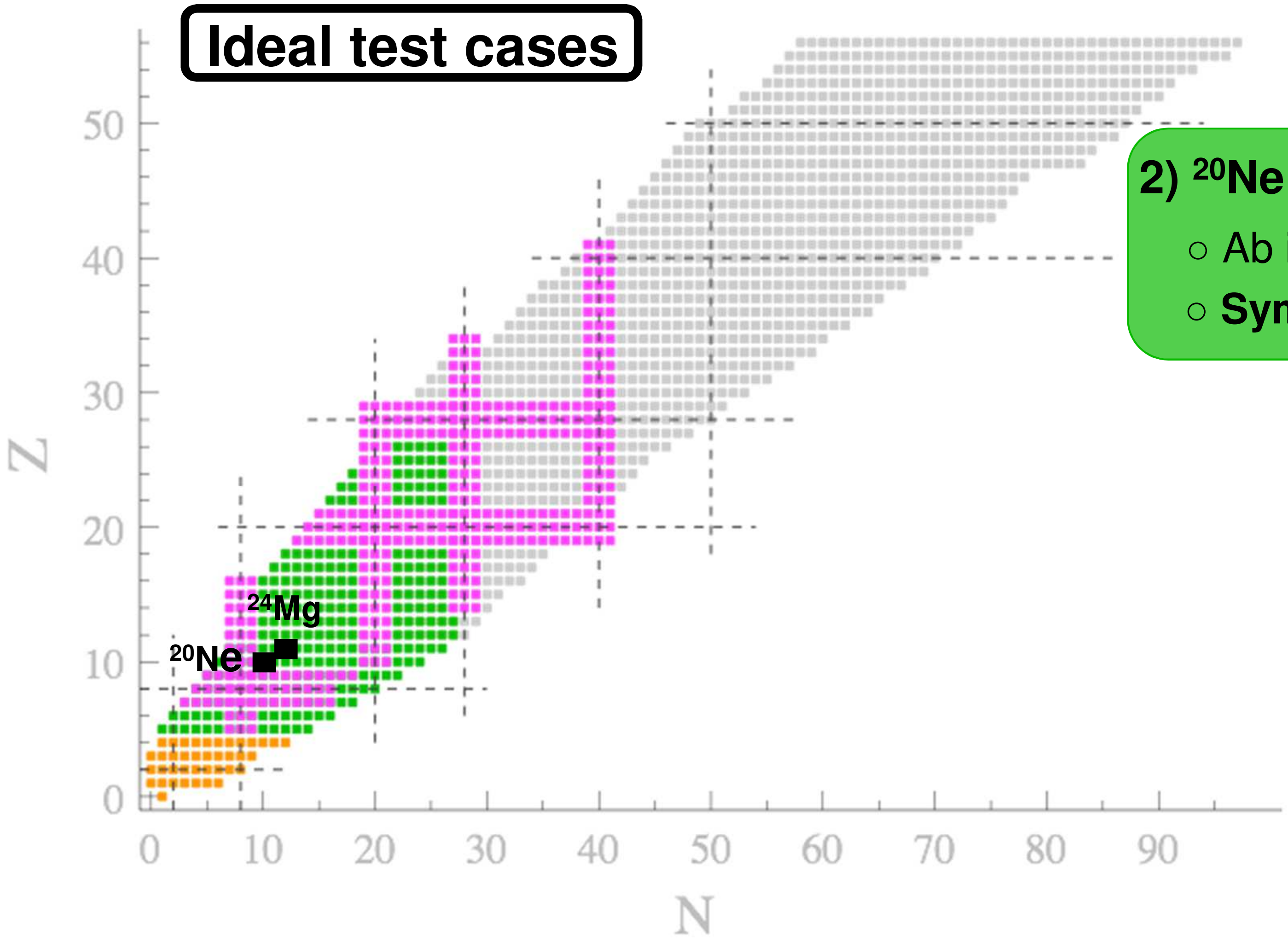
Perpectives

- **Check in non-ideal rotor nuclei**
- **Add 3N interaction + test at various N^{k}LO orders**

Similar for GFMC calculations [R. B. Wiringa *et al.*, PRC62 (2000) 014001]

Ab initio many-body methods: rotational properties

$$H|\Psi_n^{JM}\rangle = E_n^J|\Psi_n^{JM}\rangle$$



1) ^{8}Be [2015]

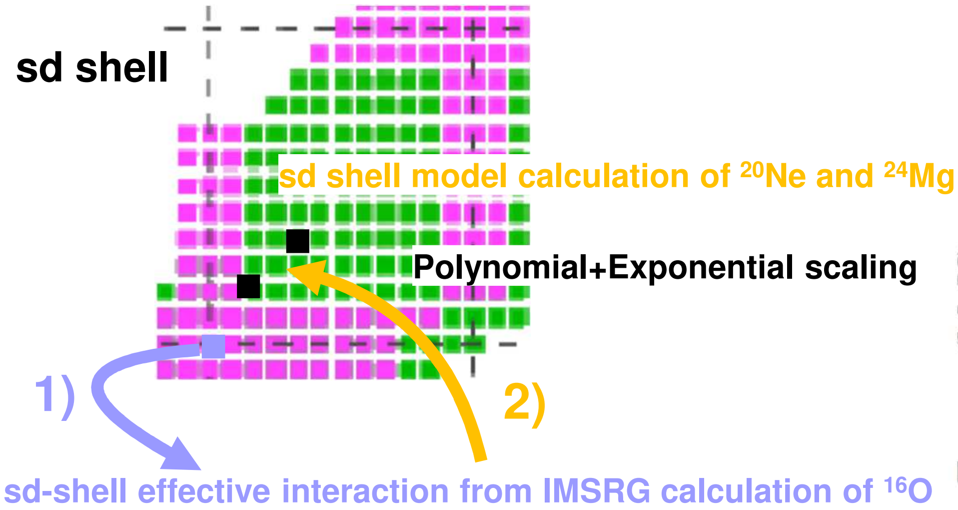
- No-core shell model
- **Symmetry conserving**

2) ^{20}Ne and ^{24}Mg [2016]

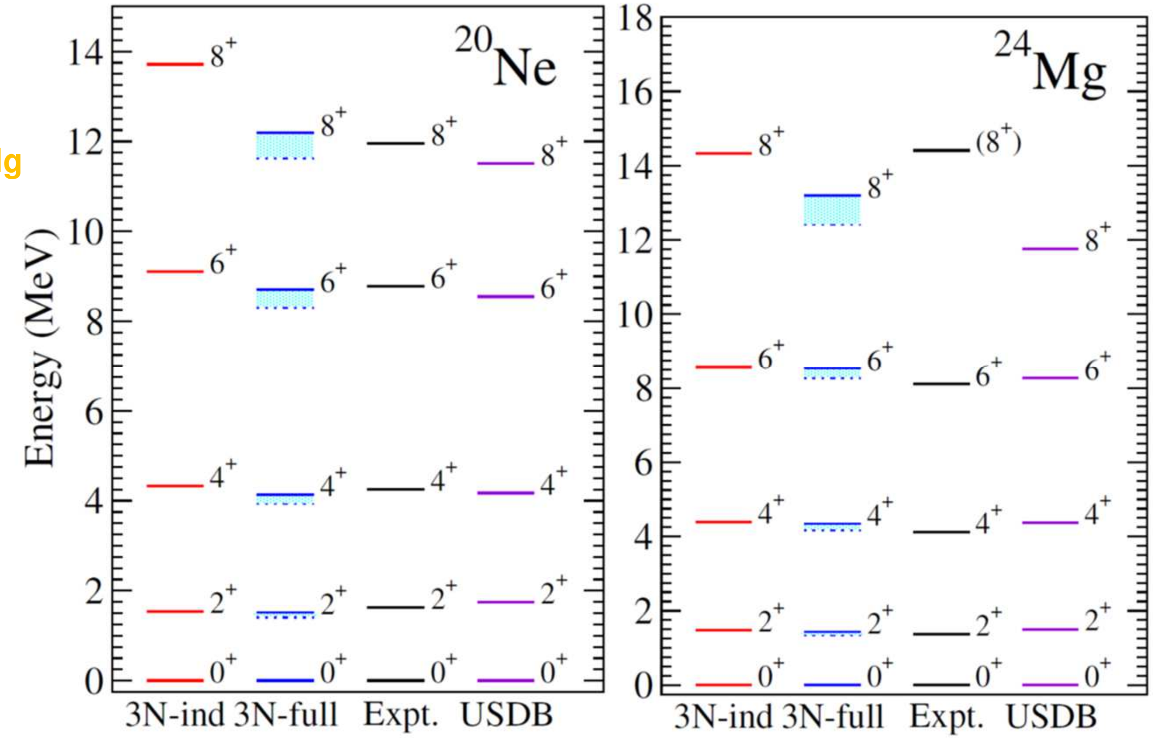
- Ab initio valence-space shell model
- **Symmetry conserving**

Valence-space shell model calculation of ^{20}Ne and ^{24}Mg

[S. R. Stroberg *et al.*, PRC93 (2016) 051301]



Yrast spectroscopy of ideal rotor nuclei



Inter-nucleon interactions

- **Chiral 2N (N^3LO ; Λ_{2NF} = 500 MeV/c)**
  [D.R. Entem, R. Machleidt, PRC 68, 041001 (2003)]
- **Chiral 3N (N^2LO ; Λ_{3NF} = 400 MeV/c)**
  [P. Navratil, FBS 41, 117 (2007)]
- **SRG evolved down to λ = 2.0 fm^{-1}**

Results

- **Rotational bands emerge convincingly**
- **Quantitatively as good as empirical model**
- **Insensitive to 3N interaction at low spins**
  Unlike overall spectroscopy in sd shell

Perpectives

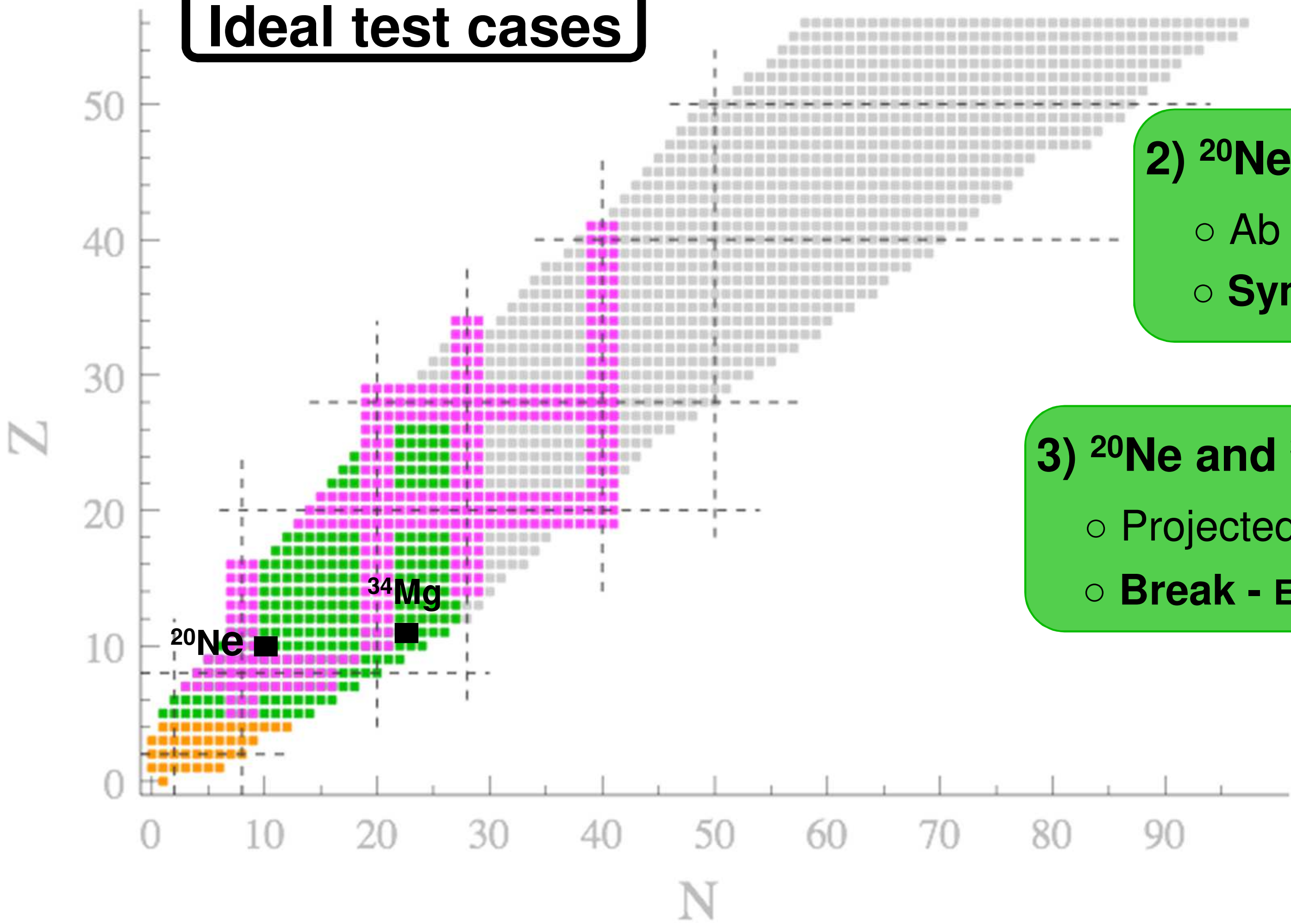
- **Check in non-ideal rotor nuclei**
- **Various EFT orders and uncertainty propagation**
- **Investigate E2/M1 moments and transitions**

Similarly for CC-based valence-space shell model
[G. Hagen *et al.*, Phys. Scr. 91 (2016) 063006]

Ab initio many-body methods: rotational properties

$$H|\Psi_n^{JM}\rangle = E_n^J|\Psi_n^{JM}\rangle$$

Ideal test cases



1) ^{8}Be [2015]

- No-core shell model
- **Symmetry conserving**

2) ^{20}Ne and ^{24}Mg [2016]

- Ab initio valence-space shell model
- **Symmetry conserving**

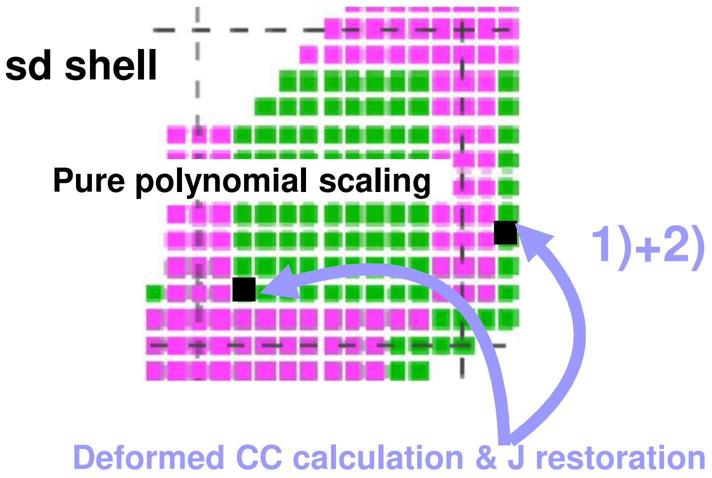
3) ^{20}Ne and ^{34}Mg [2022]

- Projected Coupled cluster theory
- **Break - Expand - Restore**

SU(2) broken &restored CC calculation of ^{20}Ne and ^{34}Mg

[T.Duguet. JPG 42 (2015) 025107]

[G. Hagen et al., PRC 105, 064311 (2022)]

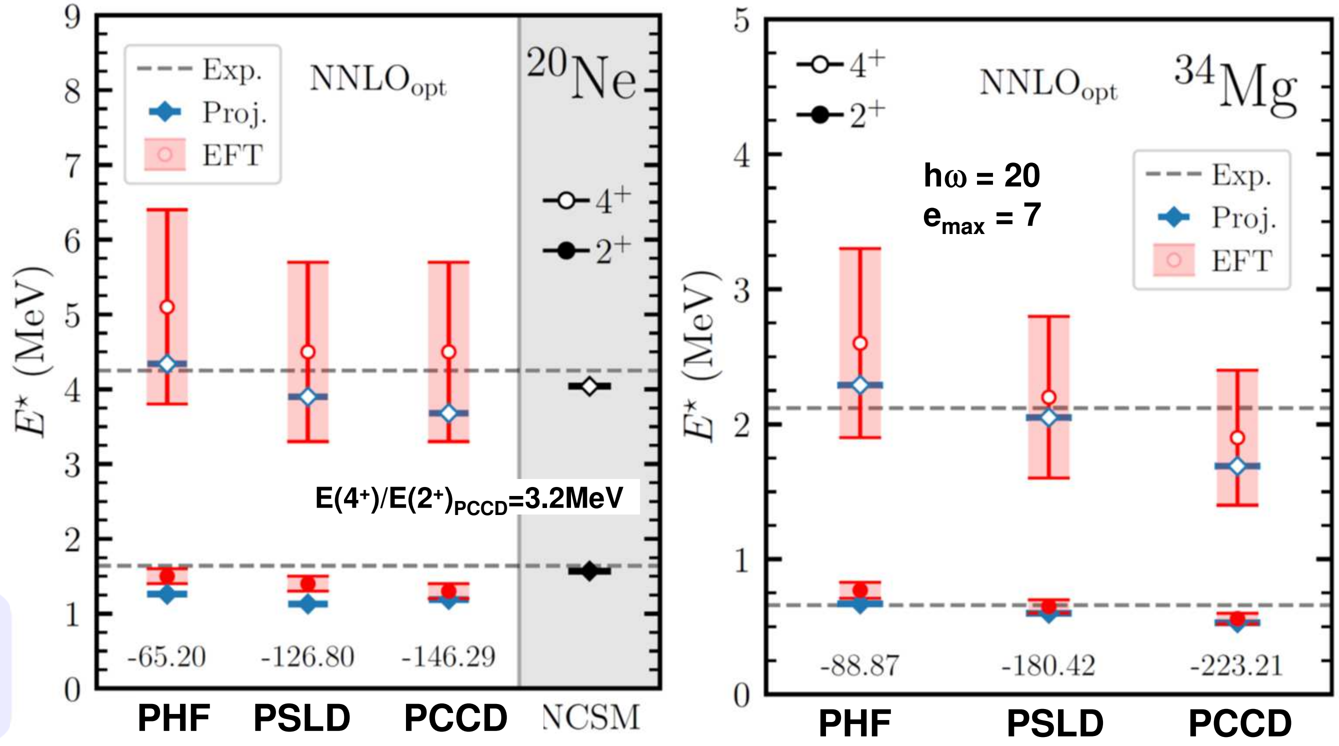


Two-nucleon interactions

➢ **Chiral 2N (N^{2}LO ; Λ_{2NF} = 500 MeV/c)**

[Ekstrom *et al.* , PRL 110 (2013) 192502]

SA-NCSM results in ^{20}Ne from [T. Dytrych *et al.*, PRL124 (2020) 042501]



Perpectives

➢ Inclusion of 3N interaction (done)

➢ Better “bra state” (done)

➢ **Check in non-ideal rotor nuclei**

➢ **EFT orders and uncertainty propagation**

➢ **E2/M1 moments and transitions**

Results

Low-lying rotational states consistently described

►**vs NCSM benchmark and experiment**

►vs LO=RRM (+uncertainty) of EFT for deformed nuclei

[T. Papenbrock, NPA 852, 36 (2011)]

Dynamical correlations

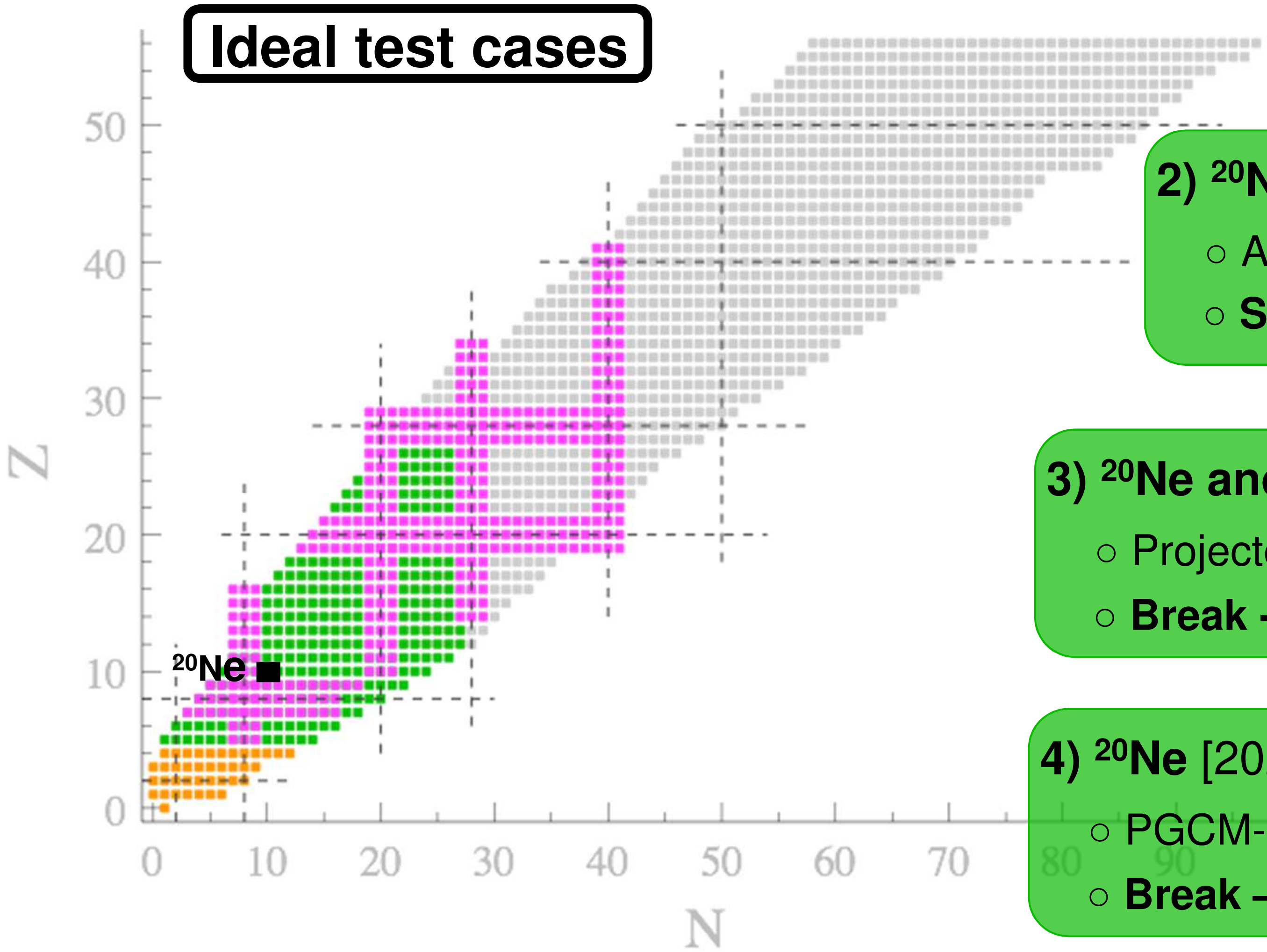
►strongly impact absolute energies

►**only slightly increase moment of inertia in ^{20}Ne**

►**impact ^{34}Mg more significantly**

Ab initio many-body methods: rotational properties

$$H|\Psi_n^{JM}\rangle = E_n^J|\Psi_n^{JM}\rangle$$



1) ^{8}Be [2015]

- No-core shell model
- **Symmetry conserving**

2) ^{20}Ne and ^{24}Mg [2016]

- Ab initio valence-space shell model
- **Symmetry conserving**

3) ^{20}Ne and ^{34}Mg [2022]

- Projected Coupled cluster theory
- **Break - Expand - Restore**

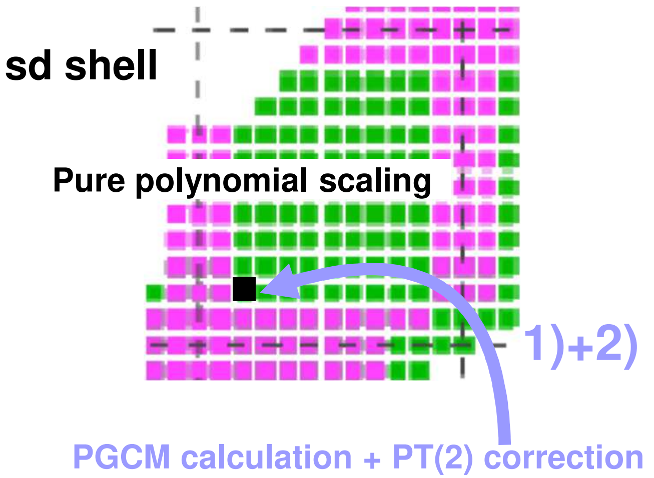
4) ^{20}Ne [2022]

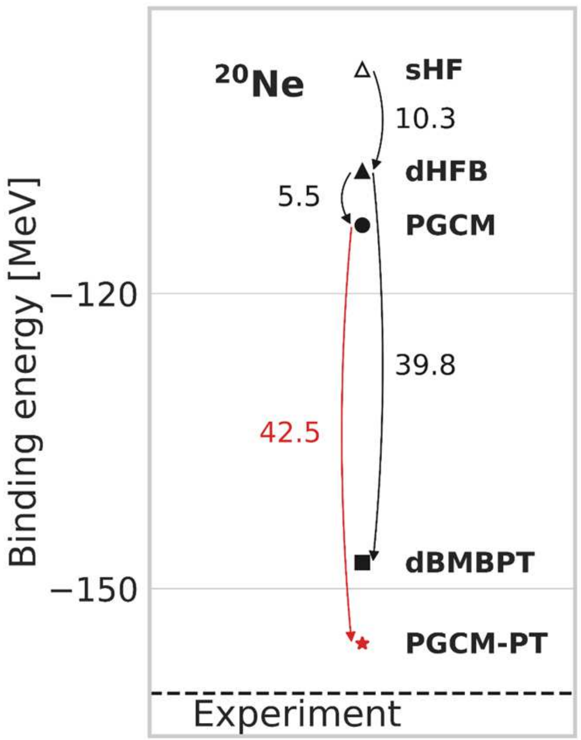
- PGCM-Perturbation theory
- **Break – Restore - Expand**

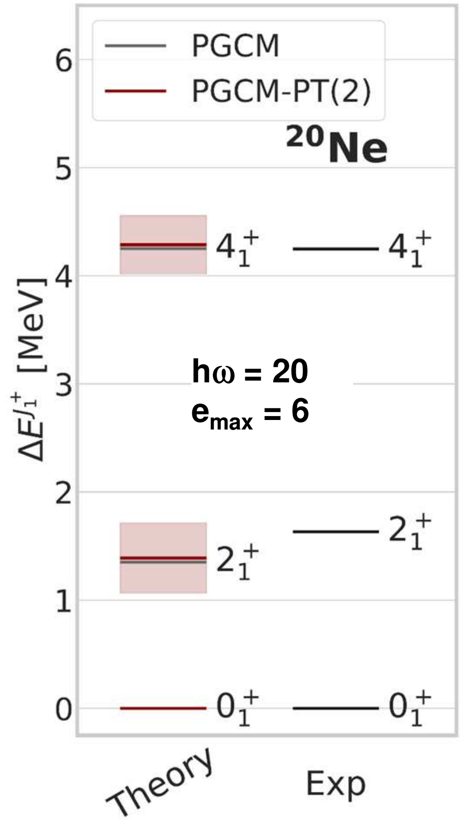
PGCM-PT calculation of ^{20}Ne

[M. Frosini et al., EPJA 58 (2022) 62]

[M. Frosini et al., EPJA 58 (2022) 64]







Inter-nucleon interactions

- **Chiral 2N (N^3LO) λ_{SRG} = 1.8 fm^{-1}**
- **Chiral 3N (N^2LO ; Λ_{3NF} = 2.0 fm^{-1})**

[K. Hebeler et al.. PRC 83, 031301 (2011)]

Perspectives

- Check in non-ideal rotor nuclei
- ***EFT orders and uncertainty propagation***
- **E2/M1 moments and transitions**
- **Systematic comparisons with PCC**
- Study vibrational excitations

Results

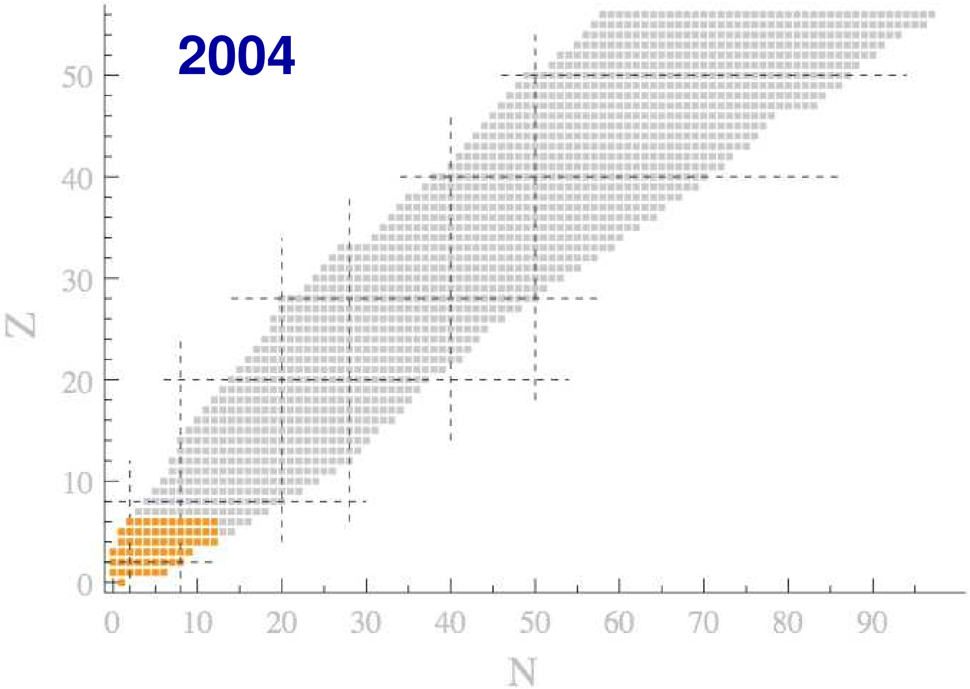
Low-lying rotational states

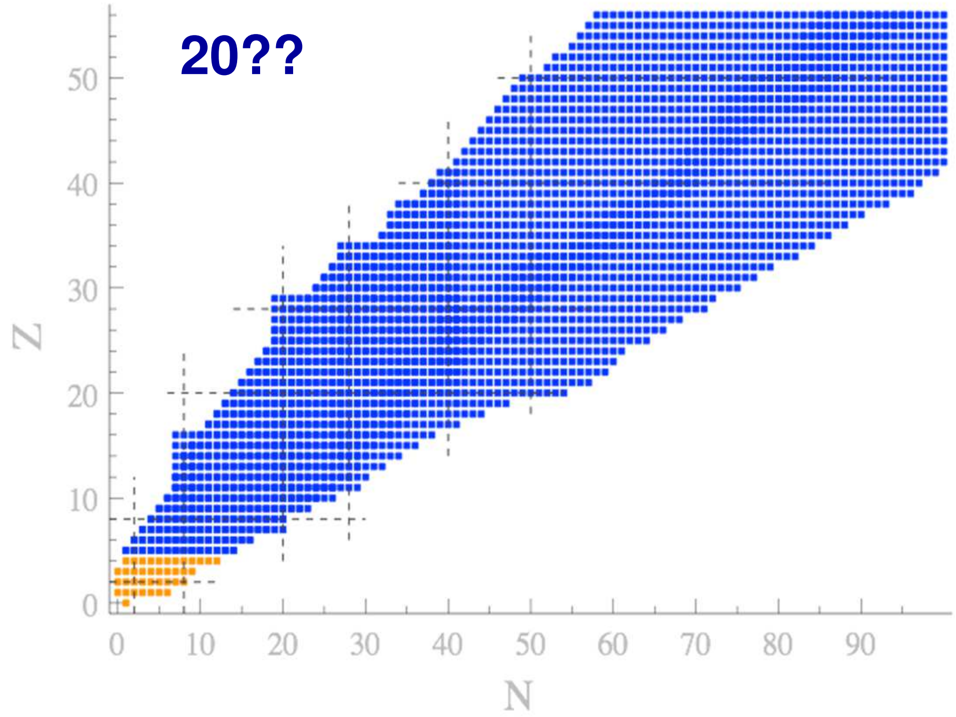
- **►consistently emerge**
- **►reproduce well experimental data**

Dynamical correlations

- ►strongly impact absolute energies
- ►do not impact moment of inertia in ^{20}Ne

Long-term perspectives of ab initio methods





Emergence from nucleons and their interactions?

- ○ Binding, size, limit of existence, collectivity, superfluidity…

Limits of such a description with A/in accuracy?

- ○ Modified ab initio effective theory when A increases?
- ○ More effective but explicitly connected approaches?

Detailed and systematic description of nuclei

Rotational properties

- ❖ **Dominance of prolate over oblate?**
- ❖ **Superdeformation?**
- ❖ **Physics of transitional nuclei?**
- ❖ **Shape coexistence phenomenon?**
- ❖ **New features?**

What specific features of AN interactions probed?

Contents

- ◉ Introduction to low-energy nuclear physics
    - ○ Phenomenology
    - ○ Rationale from the theoretical viewpoint
- ◉ Strong inter-nucleon forces
    - ○ Basic phenomenology and modelling
- ◉ The ab initio nuclear many-body problem
    - ○ Pre-processing short-range correlations
    - ○ Expansion methods handling both « weak/strong » dynamical/static correlations
    - ○ Nuclear deformation from ab initio calculations
- ◉ Conclusions

Conclusions