Overview of nuclear deformation and shape coexistence around ^{96}Zr and ^{96}Ru

- overall quadrupole deformation and shape coexistence
- triaxiality
- octupole collectivity

Nuclear shapes

- general description of a shape:

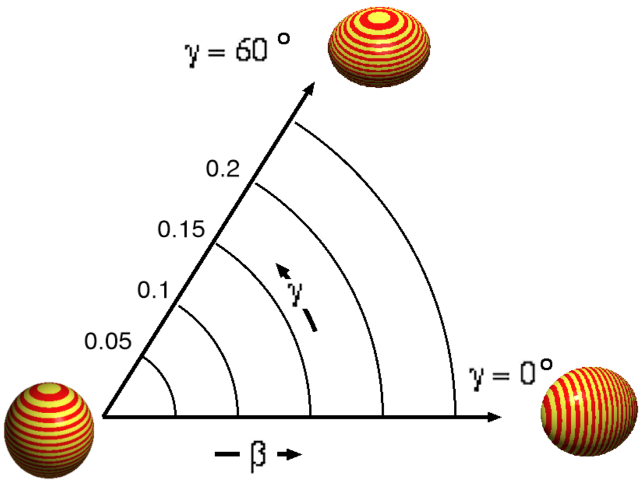
  $$R(\theta, \phi) = R_0 \left[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} a_{\lambda,\mu} Y_{\lambda\mu}(\theta, \phi)\right]$$

- important nuclear shapes:
  - $a_{2,\mu}$ quadrupole deformation (triaxial ellipsoid)
  - $a_{3,\mu}$ octupole deformation (pear shape)

- in the principal axes frame $a_{2,1} = a_{2,-1} = 0$ and only two parameters are enough to describe all possible quadrupole shapes:

$$a_{2,0} = \beta \cos\gamma$$

$$a_{2,2} = a_{2,-2} = \frac{\beta \sin\gamma}{\sqrt{2}}$$

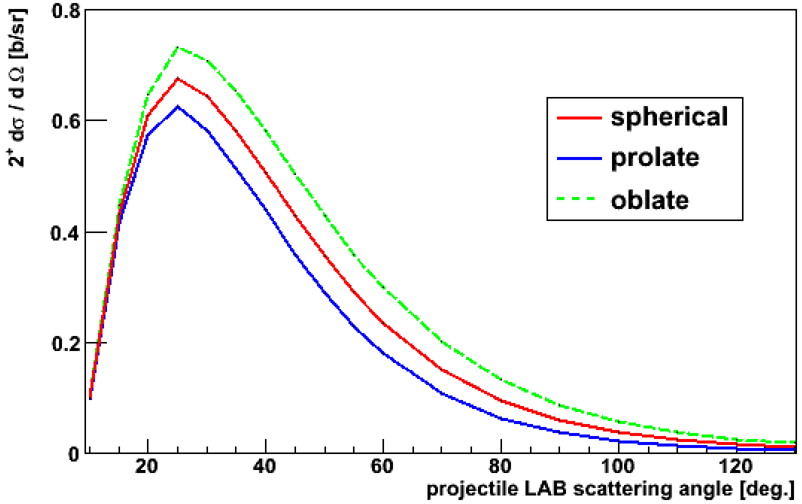


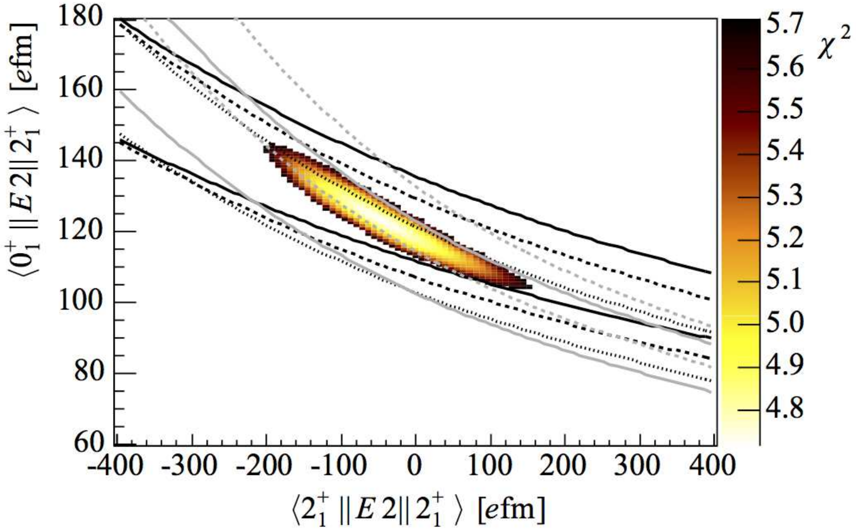
How can we measure shapes of nuclei?

- level energies
  - energy of the first 2^+ state: the simplest measure of collectivity
- transition probabilities: $B(E2; 0^+ \rightarrow 2^+) = ((3/4\pi)eZR_0^2)^2\ {\beta_2}^2$
- quadrupole moments: measure of the charge distribution in a given state (always zero for spin 0 and 1/2, even if there is non-zero intrinsic deformation)
  - laser spectroscopy for long-lived states
  - reorientation effect in Coulomb excitation for short-lived states: influence of the quadrupole moment of an excited state on its excitation cross section
- deformation lengths from inelastic scattering: need for accurate potentials to describe the nuclear interaction between collision partners
- complete sets of E2 matrix elements:
  possibility to determine quadrupole invariants and level mixing
- monopole transition strengths: enhancements observed for shape coexistence with strong mixing

Measuring quadrupole moments of excited states

- differential cross section measurements: possible at $\sim 10^4$ pps (statistics of at least 1000 counts needed)





^{202}Rn, ISOLDE

L. Gaffney *et al.* PRC 91, 064313 (2015)

M. Zielińska *et al.* EPJA 52, 99 (2016)

- independent lifetime measurements increase precision of quadrupole moments determined in this way

Quadrupole sum rules

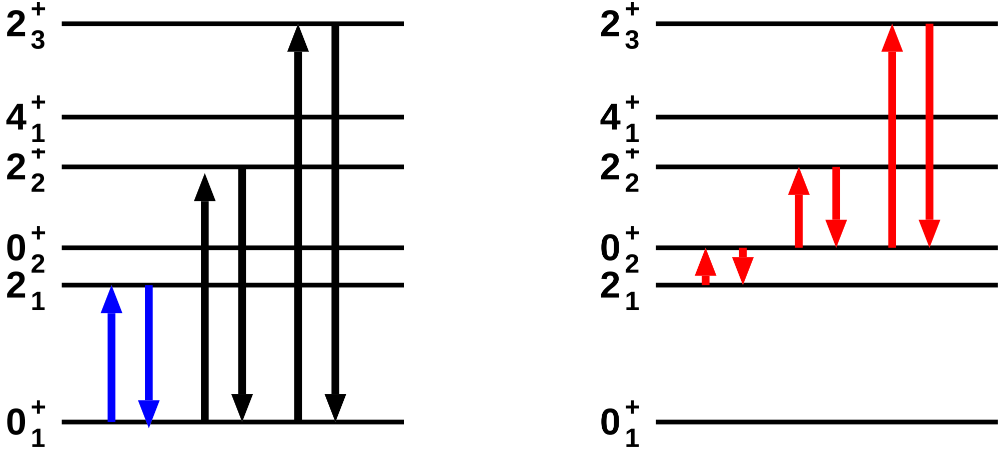
D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683
K. Kumar, PRL 28 (1972) 249

- electromagnetic multipole operators are spherical tensors – products of such operators coupled to angular momentum 0 are rotationally invariant

- in the intrinsic frame of the nucleus, the E2 operator may be expressed using two parameters Q and δ related to charge distribution:

$$\begin{aligned} E(2,0) &= Q\cos\delta \\ E(2,2)=E(2,-2) &= \frac{Q}{\sqrt{2}}\sin\delta \\ E(2,1)=E(2,-1) &= 0 \end{aligned}$$

$$\frac{\langle Q^2\rangle}{\sqrt{5}} = \langle i|\,[E2\times E2]^0\,|i\rangle = \frac{1}{\sqrt{(2I_i+1)}}\sum_t \langle i\|E2\|t\rangle\langle t\|E2\|i\rangle \begin{Bmatrix} 2 & 2 & 0 \\ I_i & I_i & I_t \end{Bmatrix}$$



$\langle Q^2\rangle$: measure of the overall deformation;
for the ground state – extension of $B(E2;\,0^+\rightarrow 2^+) = ((3/4\pi)eZR_0^2)^2\ \beta_2{}^2$

Contributions to $\langle Q^2\rangle$ in ^{100}Mo: K. Wrzosek-Lipska *et al.*, PRC 86 (2012) 064305

$\langle Q^2 \rangle$ for ^{96}Zr and ^{96}Ru ground states

- Extensive lifetime measurements for low-spin states in ^{96}Zr and ^{96}Ru:
- ^{96}Zr: (n,n'γ) + (e,e') for 2_2^+; ^{96}Ru: (p,p'γ), (^{3}He,2nγ)

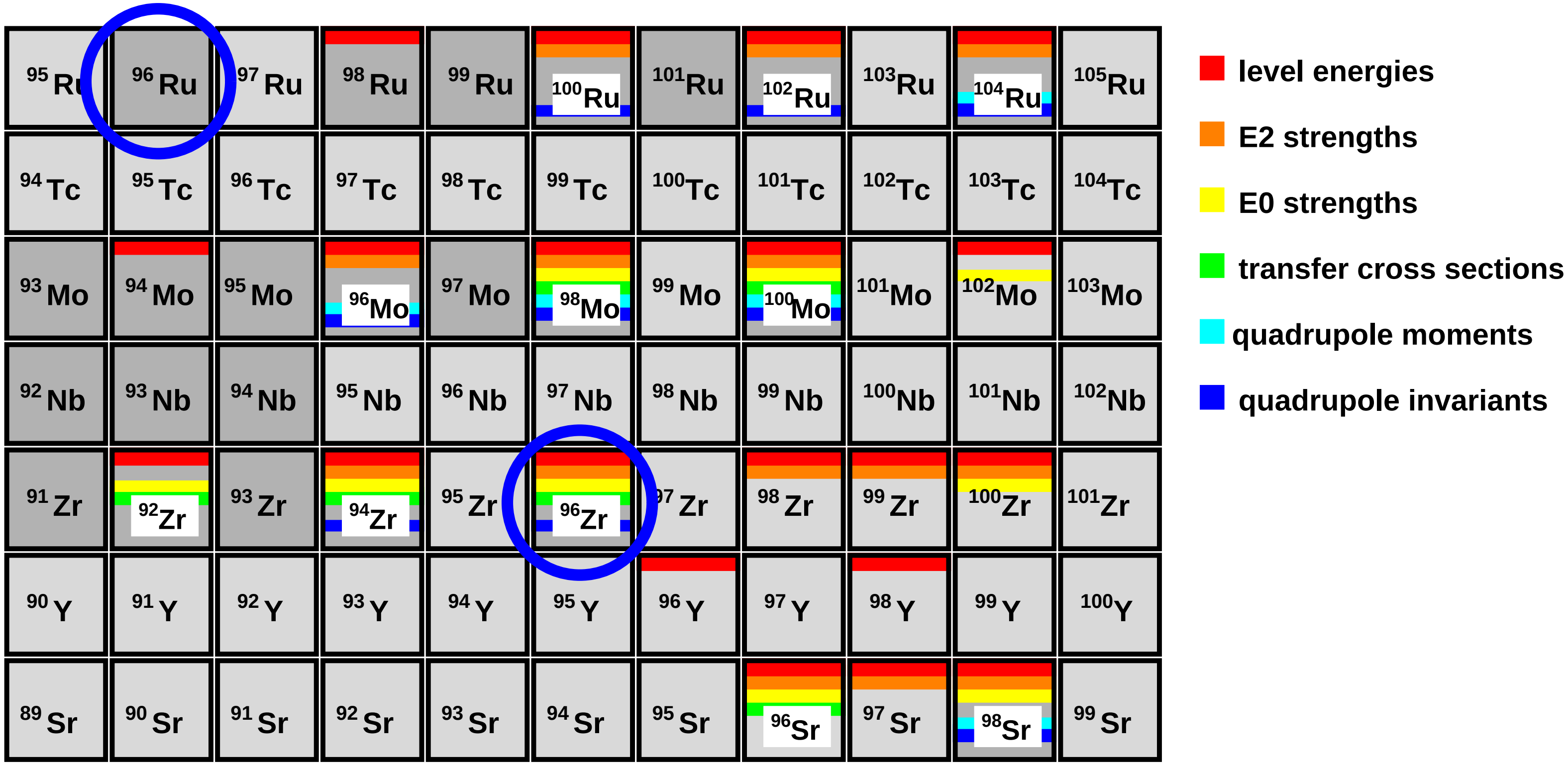
- ^{96}Zr:
  - B(E2; $2_1^+ \rightarrow 0_1^+$) = 2.3(3) W.u. $\rightarrow \langle\, 2_1^+ \,\|\, \mathrm{E2} \,\|\, 0_1^+ \rangle$ = 0.173(11) eb
  - B(E2; $2_2^+ \rightarrow 0_1^+$) = 0.26(8) W.u. $\rightarrow \langle\, 2_2^+ \,\|\, \mathrm{E2} \,\|\, 0_1^+ \rangle$ = 0.058(9) eb
  - $\langle Q^2 \rangle$ = 0.033(5)e^{2}b^2, β=0.06(1)

  ^{96}Ru:
  - B(E2; $2_1^+ \rightarrow 0_1^+$) = 18.4(4) W.u. $\rightarrow \langle\, 2_1^+ \,\|\, \mathrm{E2} \,\|\, 0_1^+ \rangle$ = 0.490(5) eb
  - B(E2; $2_2^+ \rightarrow 0_1^+$) = 0.16(4) W.u. $\rightarrow \langle\, 2_2^+ \,\|\, \mathrm{E2} \,\|\, 0_1^+ \rangle$ = 0.050(6) eb
  - $\langle Q^2 \rangle$ = 0.243(6)e^{2}b^2, β=0.155(4)

- $\langle Q^2 \rangle = q_0^2 \langle \beta_2^2 \rangle$; $q_0 = \frac{3}{4\pi} ZeR_0^2$ and $R_0 = 1.2A^{1/3}$ fm
- includes both dynamic and static deformation and assumes that mass and charge distributions are the same
- errors in ENSDF for ^{96}Ru: wrong B(E2; $2_2^+ \rightarrow 0_1^+$)=35 W.u, 2_4^+ lifetime 0.15 fs, 15 fs (it is 0.15 ps)

Shape coexistence: experimental information for A $\approx$ 100

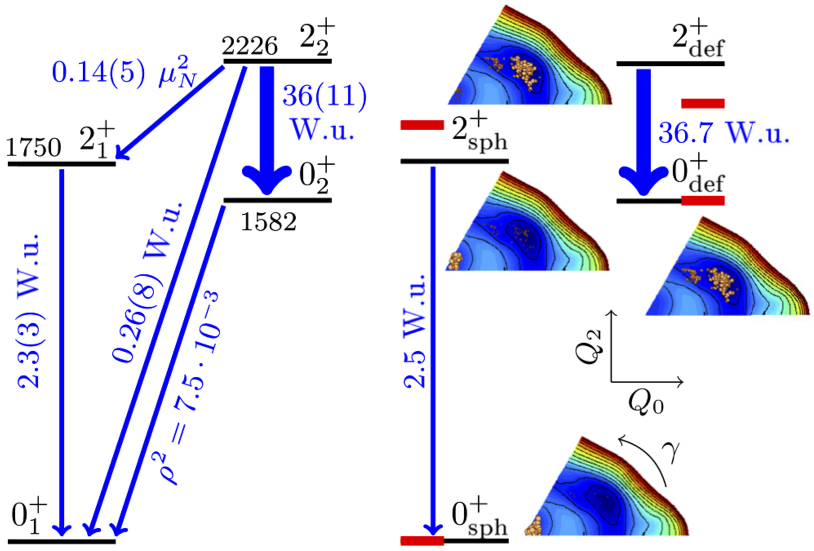
- dramatic increase of ground-state deformation at N=60
- multitude of coexisting shapes predicted by theory



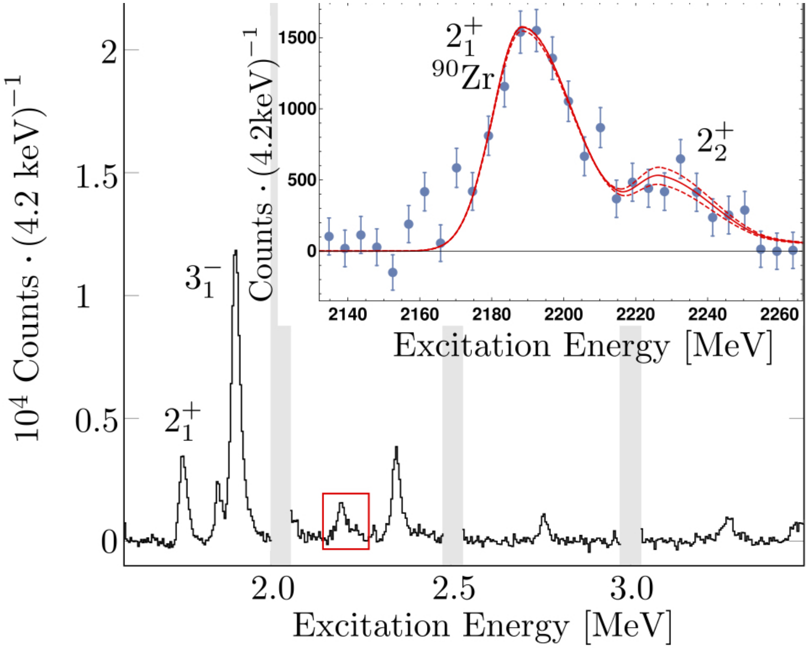
^{95}Ru	^{96}Ru	^{97}Ru	^{98}Ru	^{99}Ru	^{100}Ru	^{101}Ru	^{102}Ru	^{103}Ru	^{104}Ru	^{105}Ru
^{94}Tc	^{95}Tc	^{96}Tc	^{97}Tc	^{98}Tc	^{99}Tc	^{100}Tc	^{101}Tc	^{102}Tc	^{103}Tc	^{104}Tc
^{93}Mo	^{94}Mo	^{95}Mo	^{96}Mo	^{97}Mo	^{98}Mo	^{99}Mo	^{100}Mo	^{101}Mo	^{102}Mo	^{103}Mo
^{92}Nb	^{93}Nb	^{94}Nb	^{95}Nb	^{96}Nb	^{97}Nb	^{98}Nb	^{99}Nb	^{100}Nb	^{101}Nb	^{102}Nb
^{91}Zr	^{92}Zr	^{93}Zr	^{94}Zr	^{95}Zr	^{96}Zr	^{97}Zr	^{98}Zr	^{99}Zr	^{100}Zr	^{101}Zr
^{90}Y	^{91}Y	^{92}Y	^{93}Y	^{94}Y	^{95}Y	^{96}Y	^{97}Y	^{98}Y	^{99}Y	^{100}Y
^{89}Sr	^{90}Sr	^{91}Sr	^{92}Sr	^{93}Sr	^{94}Sr	^{95}Sr	^{96}Sr	^{97}Sr	^{98}Sr	^{99}Sr

P. Garrett, MZ, E. Clément, Prog. Part, Nucl. Phys. 124, 123931 (2022)

Shape coexistence in ^{96}Zr – experimental information



S. Kremer et al, Phys. Rev. Lett. 117 (2017) 172503

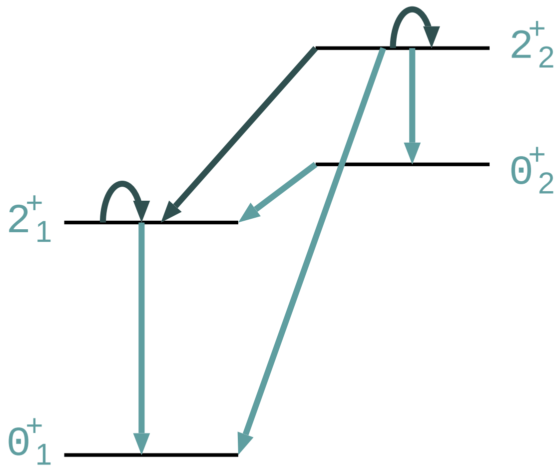


- B(E2; $2_2^+ \rightarrow 0_1^+$) measured using electron scattering, combined with known branching and mixing ratios:
  $\rightarrow$transition strengths from the 2_2^+ state
- B(E2; $2_1^+ \rightarrow 0_1^+$) = 2.3(3) Wu vs B(E2; $2_2^+ \rightarrow 0_2^+$) = 36(11) Wu: nearly spherical and a well-deformed structure ($\beta \approx$ 0.24)
- very low mixing of coexisting structures: cos$^2\theta_0$=99.8%, cos$^2\theta_2$=97.5%,

Two-state mixing model

- we assume that physical states are linear combinations of pure spherical and deformed configurations:

$$| I_1^+ \rangle = +\cos\theta_I \times | I_d^+ \rangle + \sin\theta_I \times | I_s^+ \rangle$$
$$| I_2^+ \rangle = -\sin\theta_I \times | I_d^+ \rangle + \cos\theta_I \times | I_s^+ \rangle$$



with transitions between the pure spherical and deformed states forbidden:

$$\langle 2_d^+ \| E2 \| 0_s^+ \rangle = \langle 2_d^+ \| E2 \| 2_s^+ \rangle = \langle 2_s^+ \| E2 \| 0_d^+ \rangle = 0$$

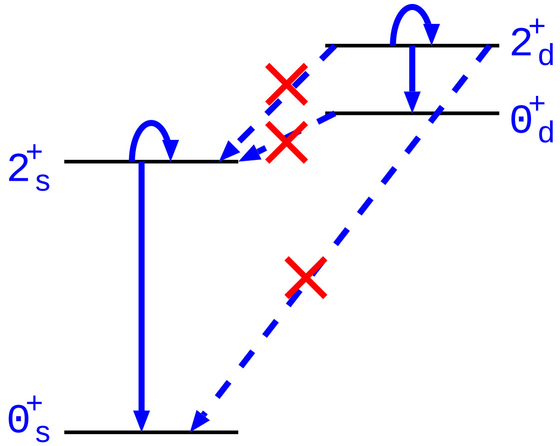
- the measured matrix elements can be expressed in terms of the "pure" matrix elements and the mixing angles:

$$\langle 2_1^+ \| E2 \| 0_1^+ \rangle =$$
$$\sin\theta_0 \sin\theta_2 \ \langle 2_s^+ \| E2 \| 0_s^+ \rangle + \cos\theta_0 \cos\theta_2 \ \langle 2_d^+ \| E2 \| 0_d^+ \rangle$$

$$\langle 2_1^+ \| E2 \| 0_2^+ \rangle =$$
$$\cos\theta_0 \sin\theta_2 \ \langle 2_s^+ \| E2 \| 0_s^+ \rangle - \sin\theta_0 \cos\theta_2 \ \langle 2_d^+ \| E2 \| 0_d^+ \rangle$$

$$\langle 2_2^+ \| E2 \| 0_1^+ \rangle =$$
$$\sin\theta_0 \cos\theta_2 \ \langle 2_s^+ \| E2 \| 0_s^+ \rangle - \cos\theta_0 \sin\theta_2 \ \langle 2_d^+ \| E2 \| 0_d^+ \rangle$$

$$\langle 2_2^+ \| E2 \| 0_2^+ \rangle =$$
$$\cos\theta_0 \cos\theta_2 \ \langle 2_s^+ \| E2 \| 0_s^+ \rangle + \sin\theta_0 \sin\theta_2 \ \langle 2_d^+ \| E2 \| 0_d^+ \rangle$$



E0 strengths, shape coexistence and mixing

- E0 transitions are sensitive to the changes in the nuclear charge-squared radii

- their strengths depends on the mixing of configurations that have different mean-square charge radii:

$$\rho^2(E0) = \frac{Z^2}{R^4} \cos^2\theta_0 \, \sin^2\theta_0 \, \left(\langle r^2\rangle_A - \langle r^2\rangle_B\right)^2$$

$$= \left(\frac{3Z}{4\pi}\right)^2 \cos^2(\theta_0)\sin^2(\theta_0) \cdot \left[\left(\beta_1^2 - \beta_2^2\right) + \frac{5\sqrt{5}}{21\sqrt{\pi}}\left(\beta_1^3 \cos\gamma_1 - \beta_2^3 \cos\gamma_2\right)\right]^2$$

J.L. Wood *et al.*, NPA 651, 323 (1999)

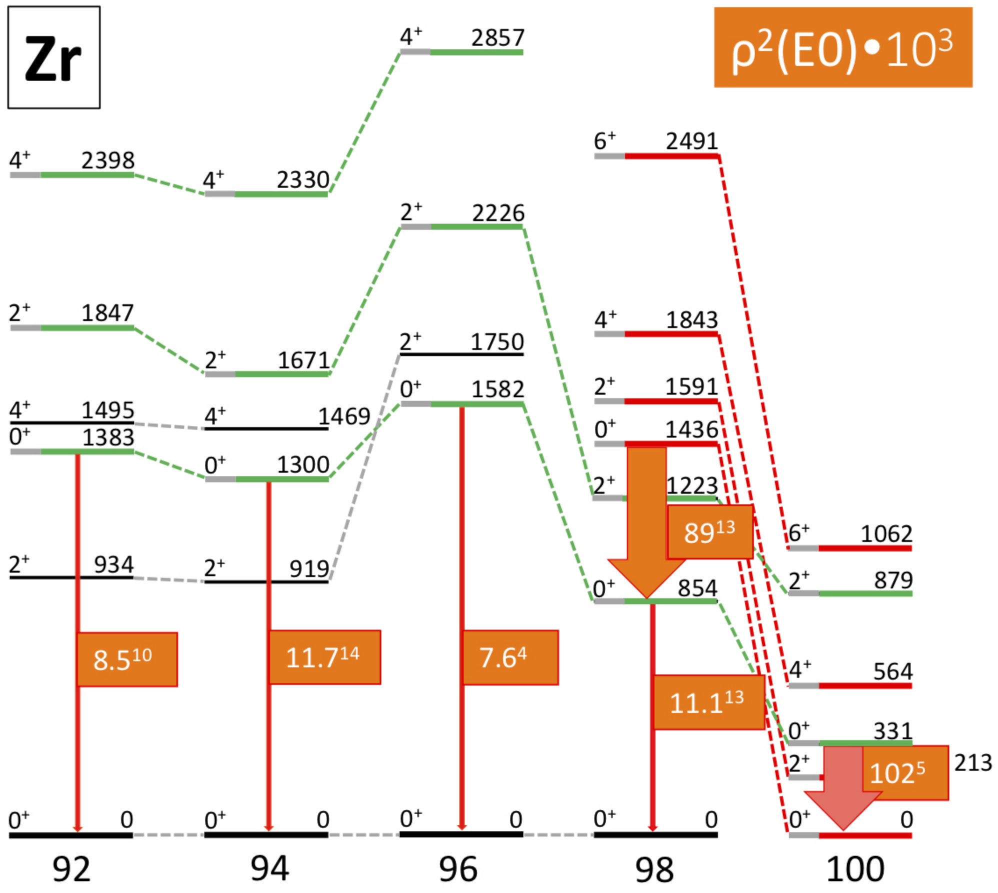
Example of ^{42}Ca: K. Hadyńska-Klęk *et al.*, PRC 97 (2018) 024326 (Coulomb excitation), J.L. Wood *et al.*, NPA 651, 323 (1999) (E0)

	from E2 matrix elements [KHK]	from $\rho^2(E0)$ [JLW] + sum rules results [KHK]
$\cos^2(\theta_0)$	0.88(4)	0.84(4)
$\cos^2(\theta_2)$	0.39(8)	-

- good agreement of the $\cos^2(\theta_0)$ values obtained with the two methods
- $\cos^2(\theta_2)<0.5$: two-state mixing model cannot be applied to 2^+ states in ^{42}Ca

E0 strengths in Zr and Ru isotopes

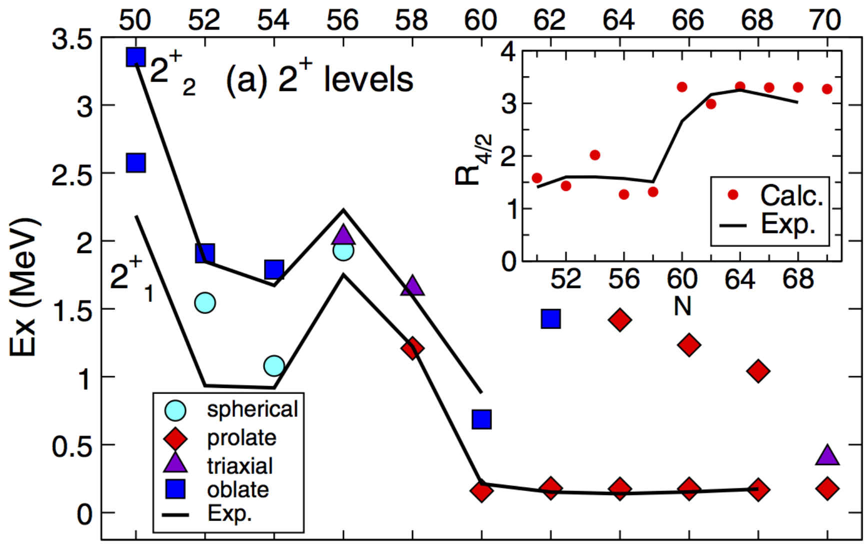
T. Kibedi *et al.*, Prog. Part. Nucl. Phys. 120 (2021)



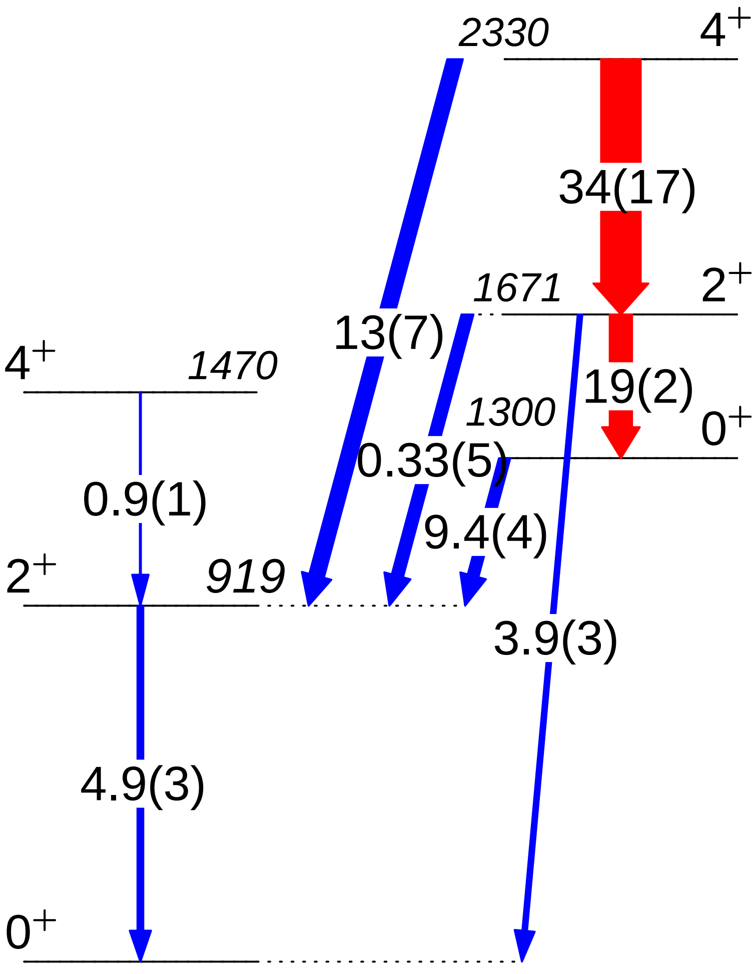
- ^{100}Ru: 11(2) 10^{-3} between 0_2^+ and 0_2^+, no data for lighter Ru isotopes

Shape coexistence in ^{94}Zr

A. Chakraborty et al, PRL 110, 022504 (2013)



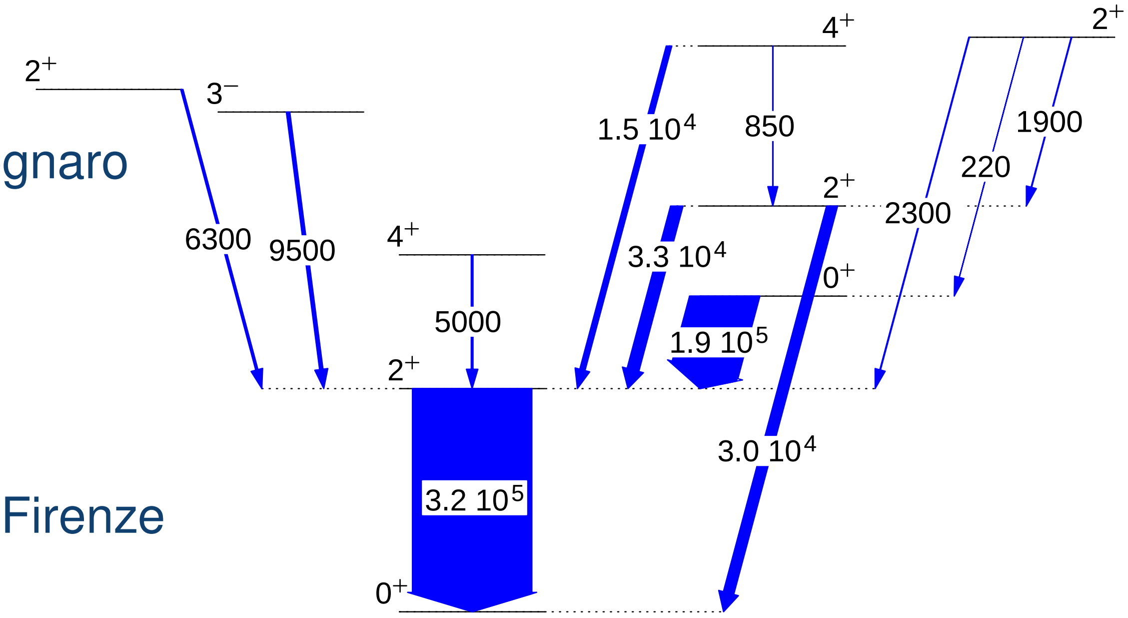
T. Togashi et al, PRL 117, 172502 (2016)

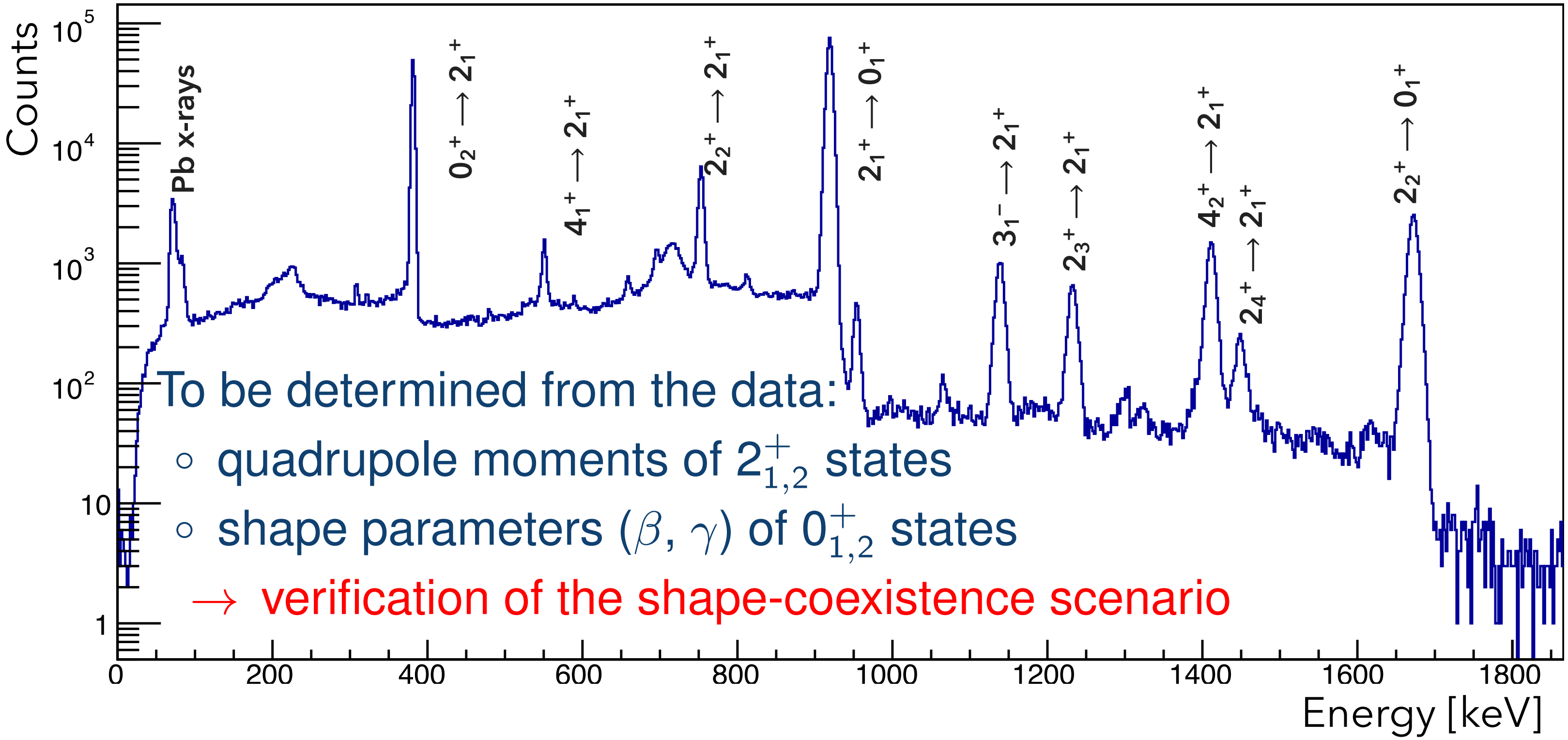


- observation of a strong $2^+_2 \rightarrow 0^+_2$ transition (19 W.u.) – deformed band built on 0^+_2
- shell model calculations suggest an oblate shape

Coulomb excitation of ^{94}Zr

- experiment performed at LNL Legnaro (March 2018)
- GALILEO + SPIDER
- ^{94}Zr beam on ^{208}Pb target
- analysis: Naomi Marchini, INFN Firenze





To be determined from the data:

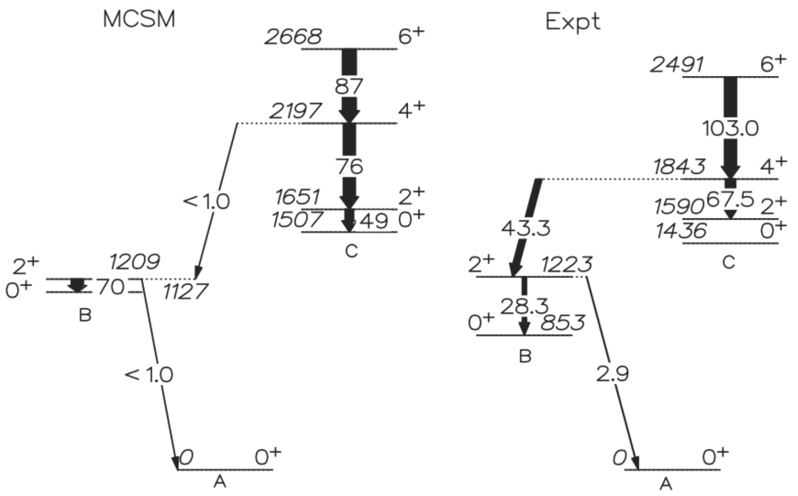
- ○ quadrupole moments of $2^+_{1,2}$ states
- ○ shape parameters (β, γ) of $0^+_{1,2}$ states

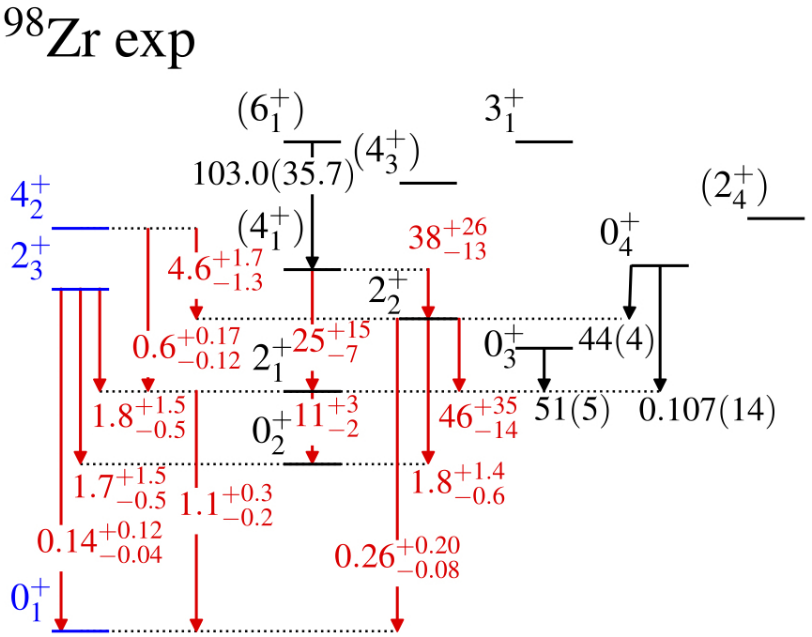
→ verification of the shape-coexistence scenario

Lifetime measurements in ^{98}Zr

- Lifetimes measured in ^{9}Be induced fission of ^{238}U, and ^{96}Zr+^{18}O 2p transfer

P. Singh et al., PRL 121, 192501 (2018) V. Karayonchev et al., PRC 102, 064314 (2020)



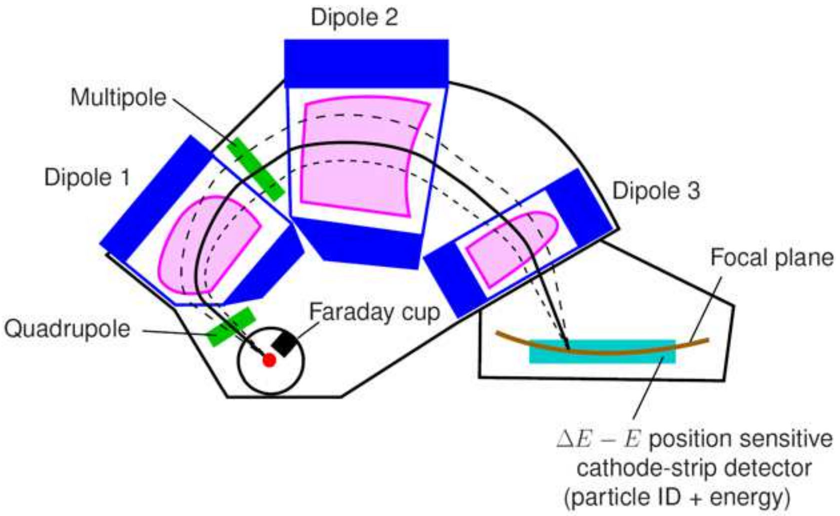


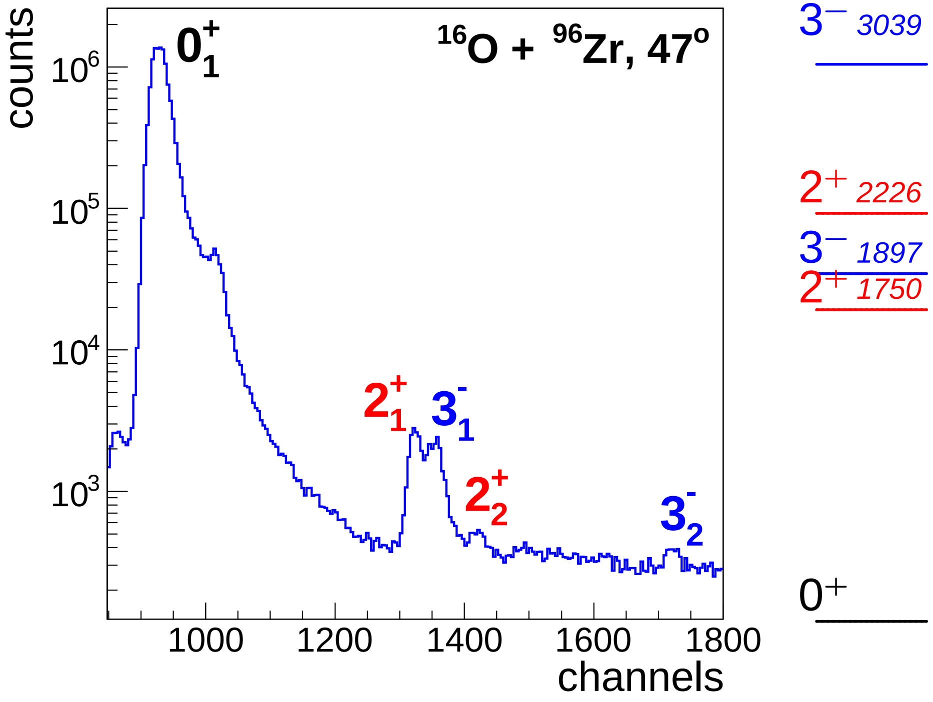
- substantial differences in measured lifetimes and interpretations
- $2_2^+ \to 0_3^+$ is expected to be either enhanced in-band transition, or a forbidden three- to two-phonon transition
- combination of 2_2^+ lifetime and branching ratio points to an unphysical value of 500 W.u.
- β-decay data from TRIUMF (under analysis) expected to resolve this issue

Coulomb excitation with the Q3D spectrometer

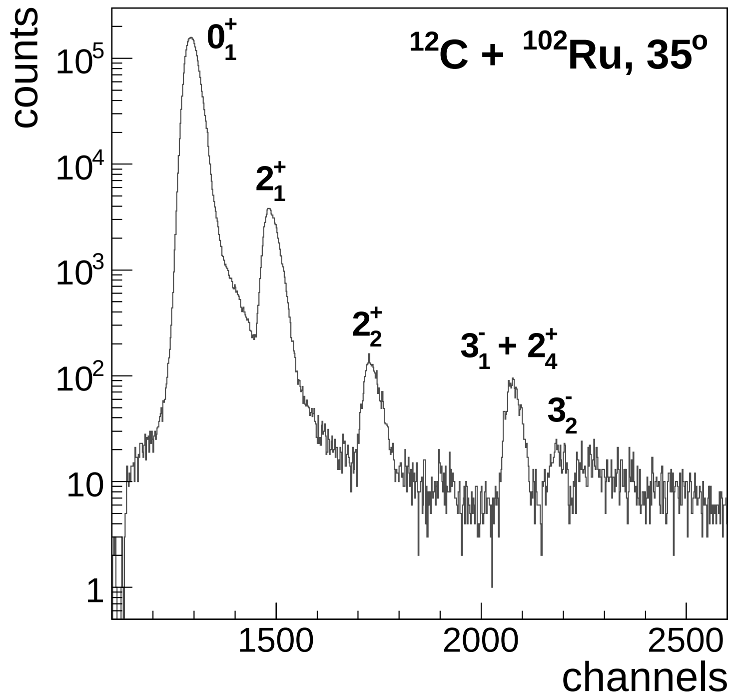
- Coulomb-excitation measurements with magnetic spectrometers common in 1970s, but completely abandoned in favour of γ-ray spectroscopy
- still a very attractive option, especially to populate higher-lying low-spin states: very high beam intensities (100 pnA) can compensate for low cross sections
- campaigns with ^{12}C, ^{16}O beams: direct measurement of 2^+ and 3^- population $\rightarrow$ precise B(E2; $2^+_i \rightarrow 0^+_1$) and B(E3; $3^-_i \rightarrow 0^+_1$) values

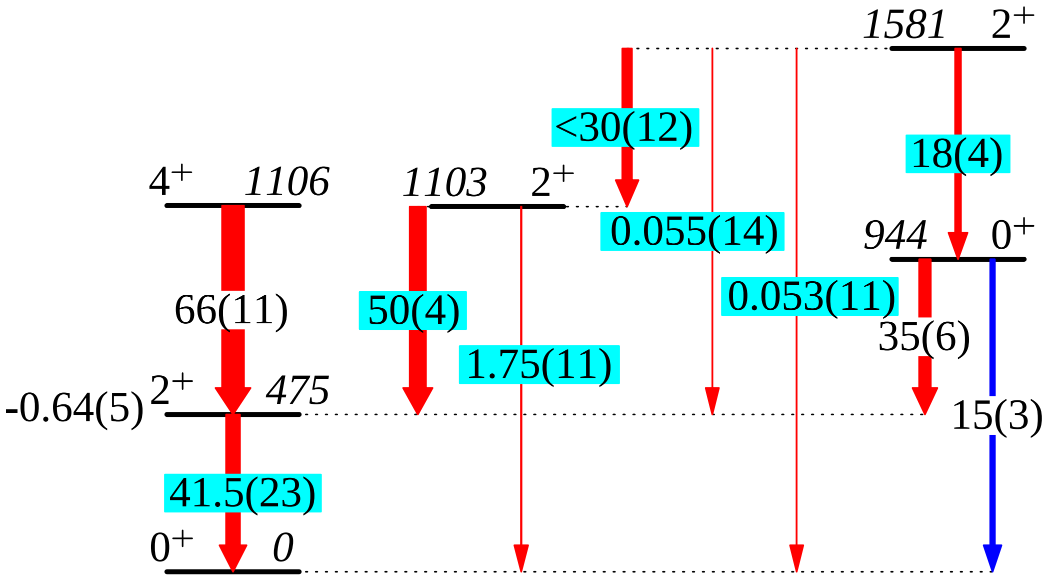
Q3D magnetic spectrometer, MLL





Results: shape coexistence in ^{102}Ru

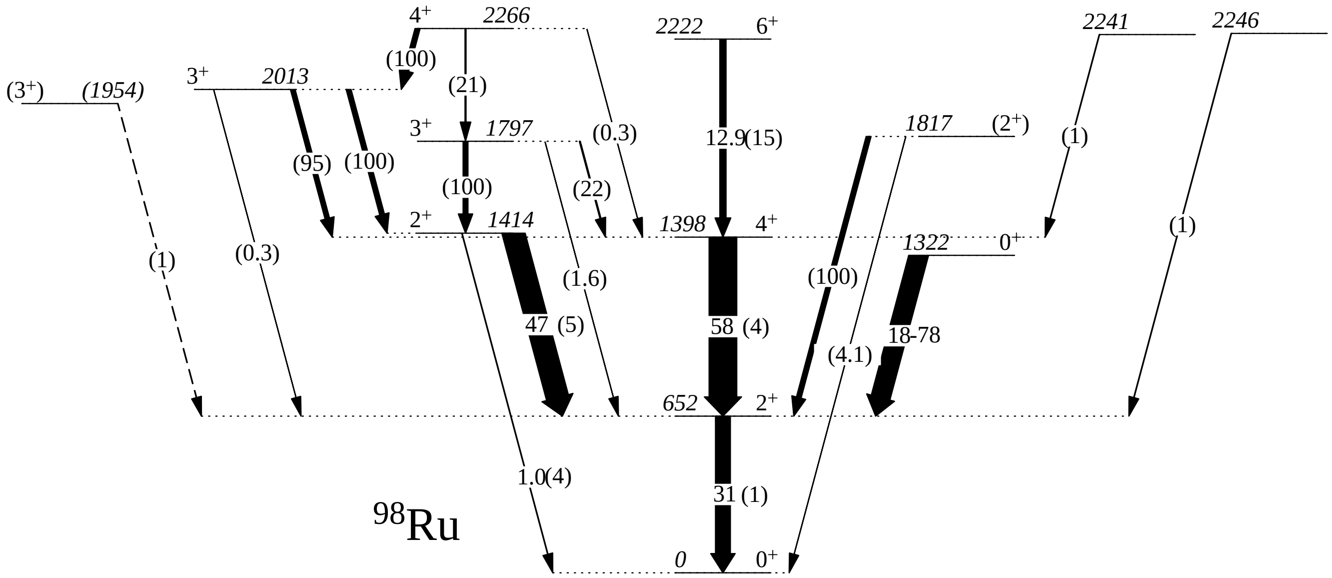




P. Garrett, MZ et al, submitted to Phys. Rev. C

- first measurement of the B(E2; $2^+_3 \rightarrow 0^+_1$) value
- combined with known branching ratios yields B(E2) values in the two bands differing by a factor of 2
- coexistence of two structures with different overall deformation ($\beta \approx 0.24$ and $\beta \approx 0.18$)

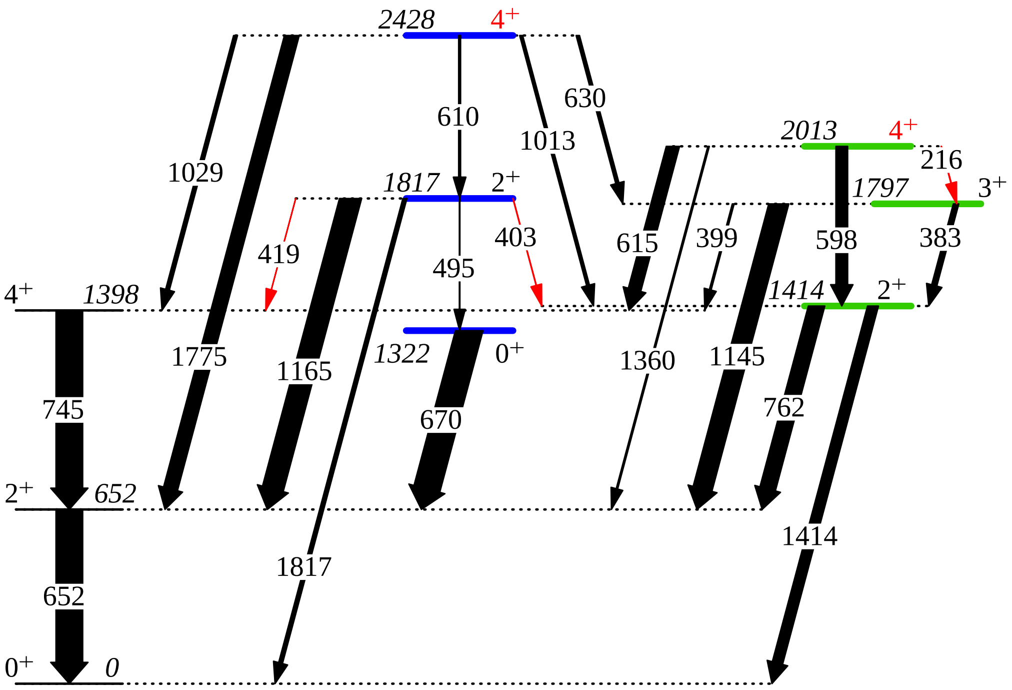
^{98}Ru level scheme a few years ago

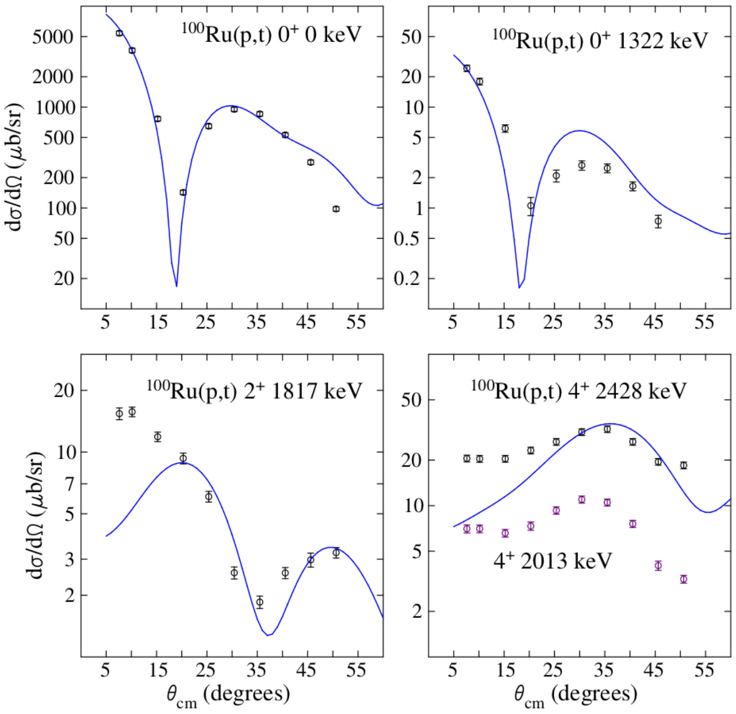


- highly unlikely that there are three closely-lying 3^+ states
- level scheme incomplete with missing decays and spin assignments

Reevaluation of ^{98}Ru level scheme

P. Garrett et al., PLB 809, 135762 (2020)





- combined β-decay study (iTHEMBA Labs) and (p,t) transfer (MLL)
- resulting level scheme suggestive of shape coexistence and triaxiality

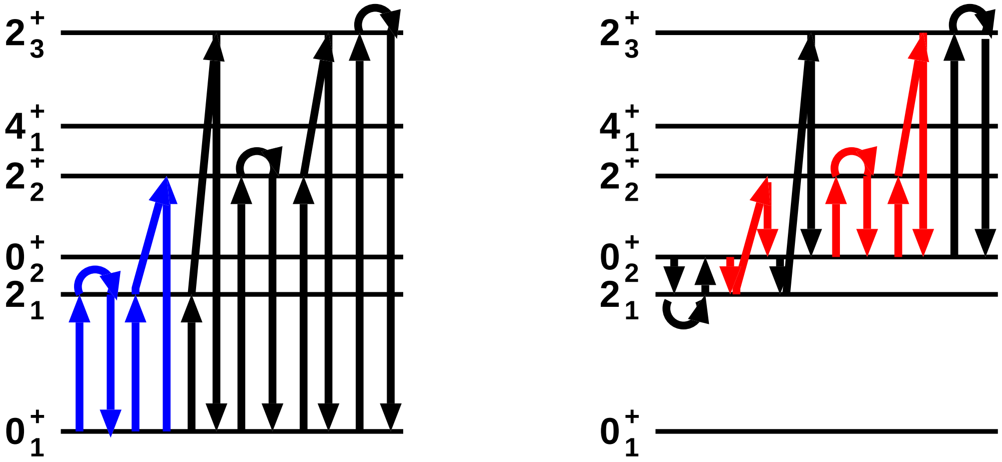
Quadrupole sum rules: triaxiality

D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683

K. Kumar, PRL 28 (1972) 249

$$\sqrt{\frac{2}{35}}\langle Q^3 \cos 3\delta\rangle = \langle i|\{[E2\times E2]^2\times E2\}^0|i\rangle$$

$$= \frac{1}{(2I_i+1)}\sum_{t,u}\langle i||E2||u\rangle\langle u||E2||t\rangle\langle t||E2||i\rangle\begin{Bmatrix} 2 & 2 & 2 \\ I_i & I_t & I_u \end{Bmatrix}$$



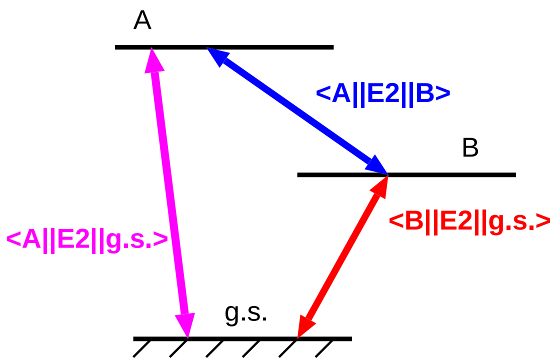
$\langle\cos 3\delta\rangle$: measure of triaxiality

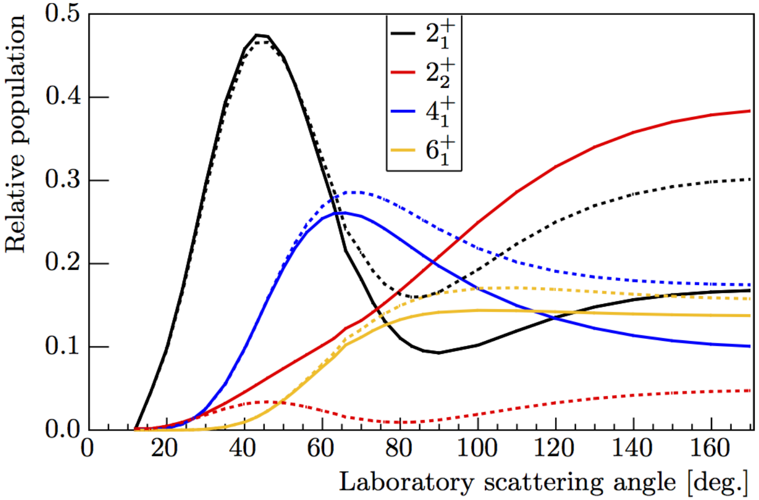
- relative signs of E2 matrix elements are needed: can we get them experimentally?

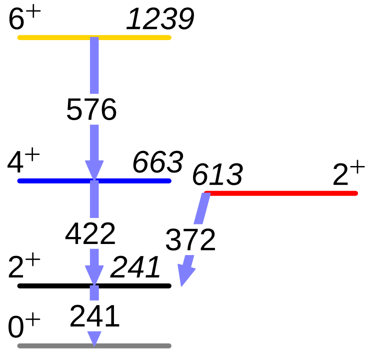
Contributions to $\langle Q^3\cos 3\delta\rangle$ in ^{100}Mo: K. Wrzosek-Lipska *et al.*, PRC 86 (2012) 064305

Relative signs of E2 matrix elements

- Coulomb-excitation cross section are sensitive to relative signs of MEs: result of interference between single-step and multi-step amplitudes
- excitation amplitude of state A: $a_A \sim \langle A\|E2\|g.s.\rangle + \langle B\|E2\|g.s.\rangle\langle A\|E2\|B\rangle$
- excitation probability ($\sim a_A^2$) contains interference terms $\langle A\|E2\|g.s.\rangle\langle B\|E2\|g.s.\rangle\langle A\|E2\|B\rangle$







^{110}Ru

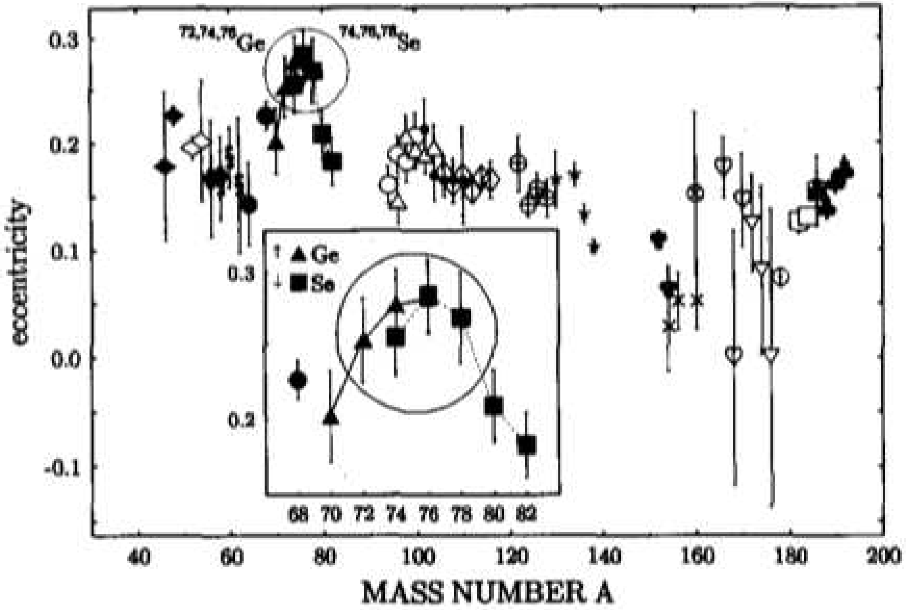
- negative $\langle 2_1^+\|E2\|2_2^+\rangle$ (solid lines): much higher population of 2_2^+ at high CM angles
- sign of a product of matrix elements is an observable

Quadrupole sum rules: triaxiality

A. Andrejtscheff *et al*, Phys. Lett. B 329 (1994) 1

For the ground state, two terms dominate the sum:

$$\langle \cos 3\delta \rangle \approx -\sqrt{\frac{7}{10}}\langle Q^2_{0^+_1}\rangle^{-3/2}\Big(\left|\langle 0^+_1 \| E2 \| 2^+_1 \rangle\right|^2 \langle 2^+_1 \| E2 \| 2^+_1 \rangle + 2\langle 0^+_1 \| E2 \| 2^+_1 \rangle \langle 2^+_1 \| E2 \| 2^+_2 \rangle \langle 2^+_2 \| E2 \| 0^+_1 \rangle \Big)$$



still, sign of the $\langle 0^+_1 \| E2 \| 2^+_1 \rangle \langle 2^+_1 \| E2 \| 2^+_2 \rangle \langle 2^+_2 \| M(E2) \| 0^+_1 \rangle$ product is necessary

Do we know all states that should enter the sum?

- especially for the (E2 x E2 x E2), where terms can cancel out – can we say that terms involving higher lying levels (the 2^+_4 state etc) do not significantly influence the rotational invariant?
  - if such state were coupled to the state in question via a large E2 matrix element, it would be populated in the experiment
  - comparison with GBH calculations for ^{100}Mo: Q^2, $Q^3\cos(3\delta)$ calculated directly from probability density distributions and from theoretical values of matrix elements, limited to the same three intermediate states

⇒ difference below 3% for both 0^+ states

	GBH		exp
$0^+_1 : \overline{\beta}$	0.20	0.20	0.22 ±0.01
$0^+_1 : \overline{\gamma}$	27°	27°	29°±3°
$0^+_2 : \overline{\beta}$	0.24	0.24	0.25 ±0.01
$0^+_2 : \overline{\gamma}$	18°	17°	10°±3°

K. Wrzosek-Lipska, PRC 86 (2012) 064305

PHYSICAL REVIEW C **86**, 064305 (2012)

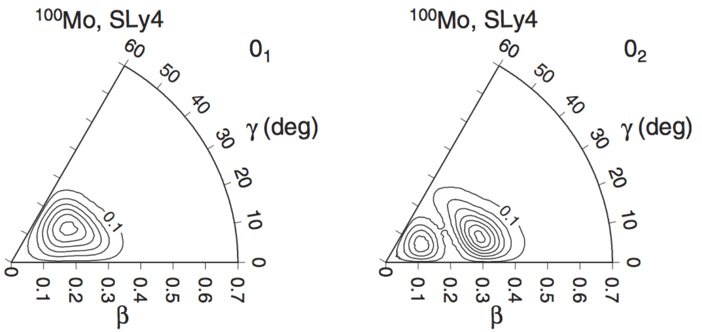
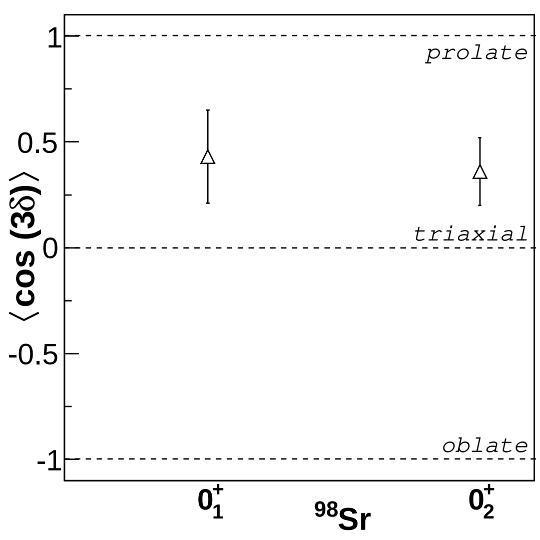
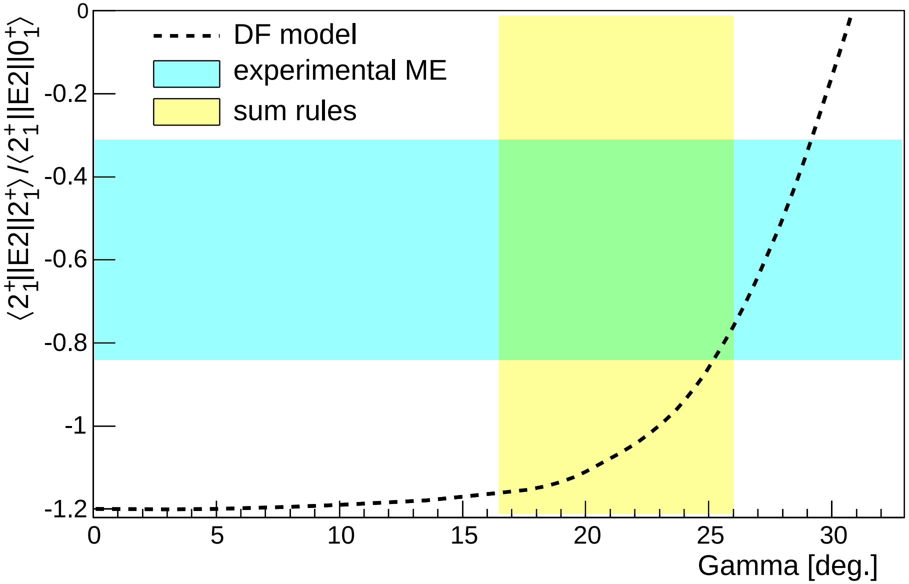


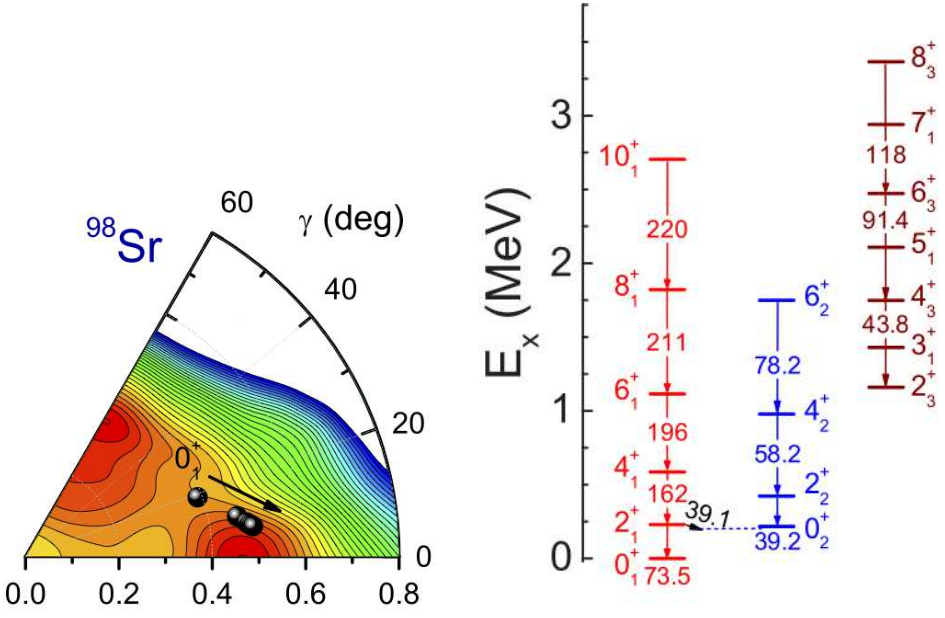
FIG. 15. Probability density [Eq. (26)] for the 0^+_1 and 0^+_2 states for the Skyrme SLy4 interaction. The contour interval is 0.3.

Triaxiality in ^{98}Sr



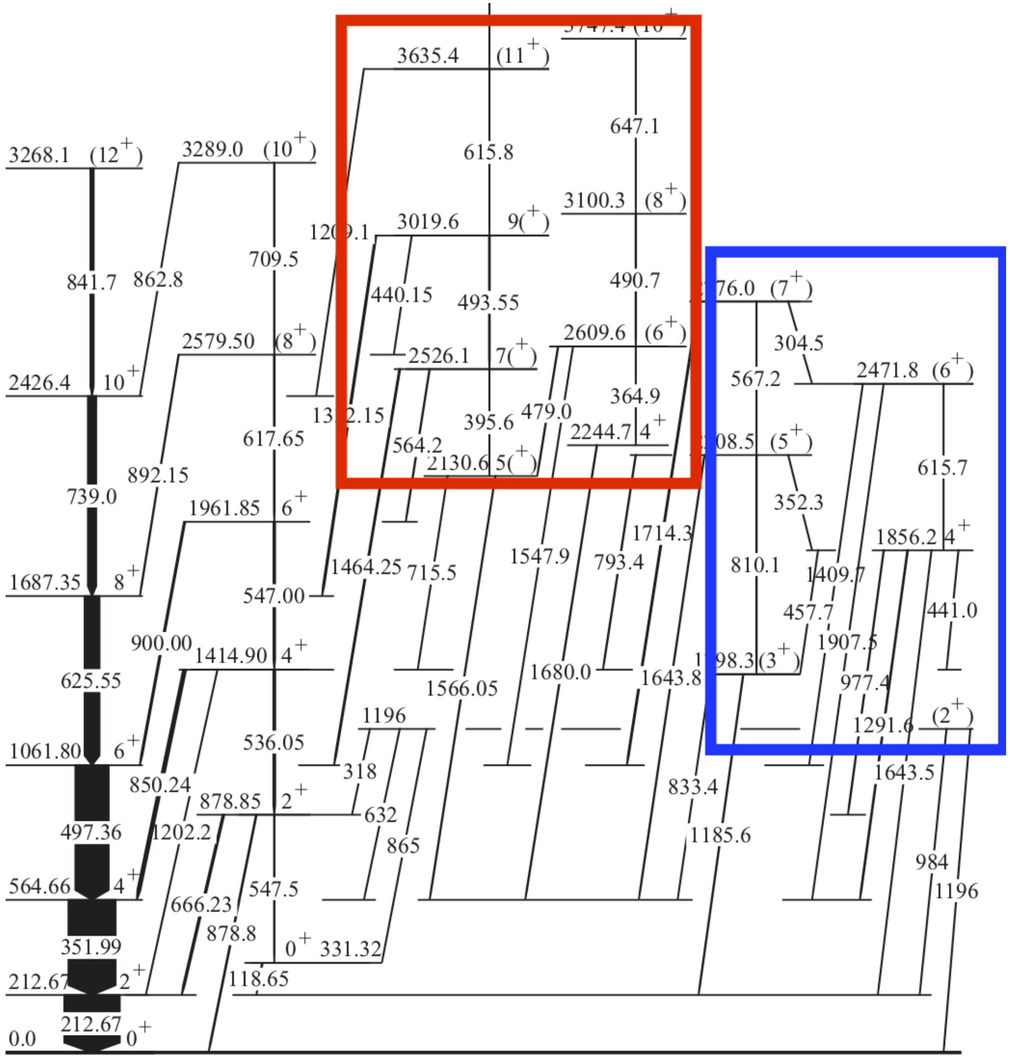


- gamma ≈ 25° would explain the reduction of $Q_s(2_1^+)$ in ^{98}Sr
- but where is the gamma band?



J. Xiang *et al.*, PRC 93, 054324 (2016), 5DCH with PC-PK1 interaction

Gamma and ‘triaxial’ structures in ^{100}Zr



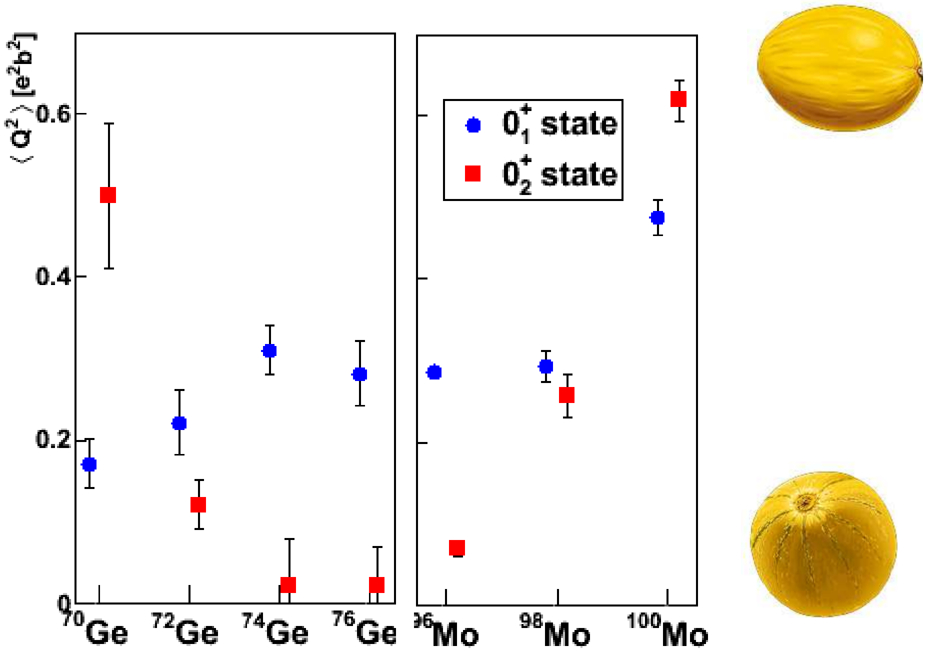
W. Urban et al, PRC 100, 014319 (2019)

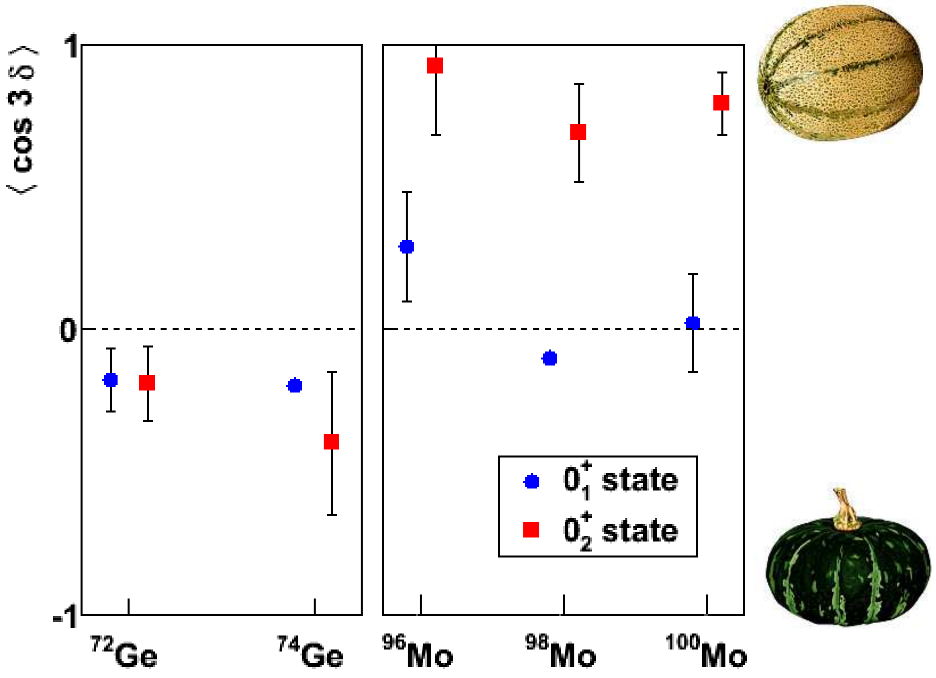
- “gamma” band proposed (related to the softness in the γ degree of freedom) and “triaxial” band (related to a rotation of an non-axial shape, like in the Davydov-Filippov model)
- transition to low-spin states missing, or even candidates missing

Shape evolution of $^{96-100}$Mo

MZ *et al.*, Nucl. Phys. A 712 (2002) 3

K. Wrzosek-Lipska *et al.*, PRC 86 (2012) 064305

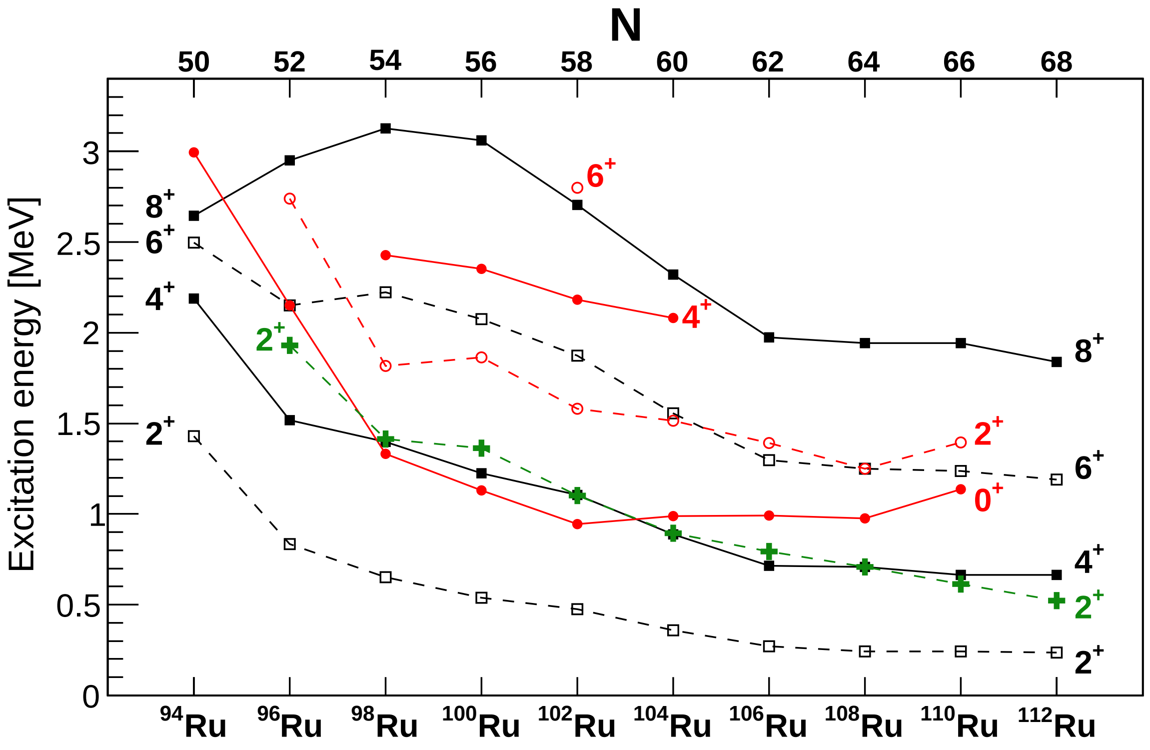




- ^{72,74,76}Ge, ^{96}Mo: coexistence of the deformed ground state with a spherical 0_2^+
- ground states of the Mo isotopes triaxial, deformation of 0_2^+ increasing with N
- shape coexistence in ^{98}Mo manifested in a different triaxiality of 0_1^+ and 0_2^+

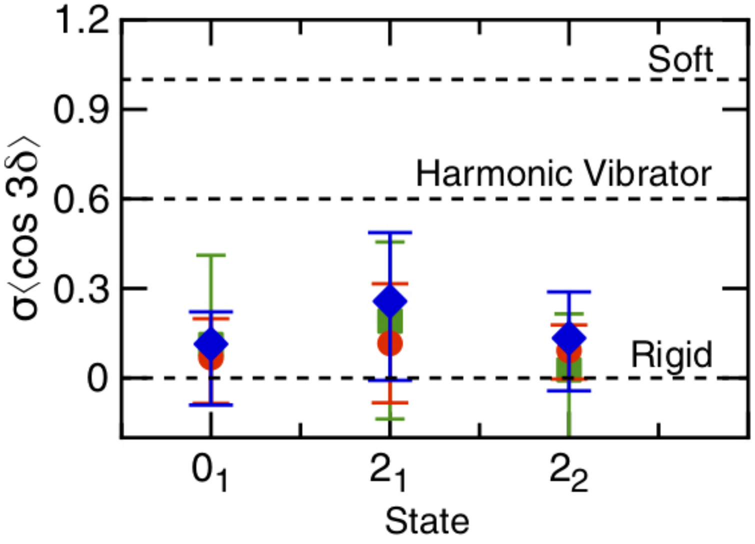
Energy systematics in Ru isotopes

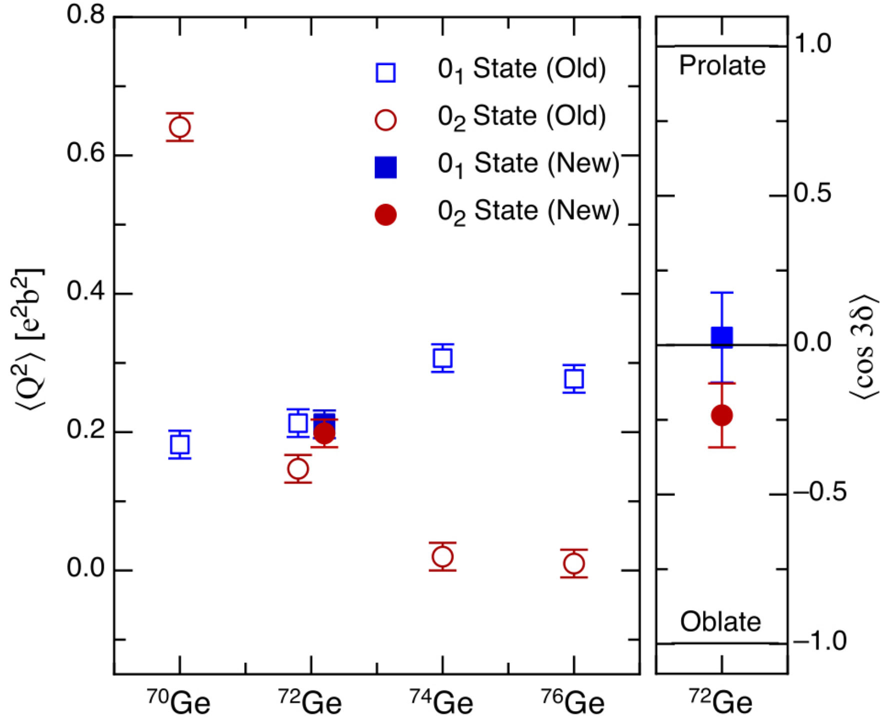
- transition from potentially γ-rigid ^{110,112}Ru (D. Doherty et al, PLB 776, 334 (2017)) to γ-soft nuclei
- parabolic intrusion of potentially shape-coexisting shapes
- experimental data on shape coexistence less detailed than in the Zr, Mo isotopic chains



Higher-order quadrupole invariants – example of ^{72,76}Ge

A.D. Ayangeakaa *et al.*,
PRL 123, 102501 (2019)
PLB 754, 254 (2016)





- ^{76}Ge: unique example of determination of softness in γ from experimental data

- ^{72}Ge: much higher number of transitions observed in a new measurement → slight change of the deduced invariants due to extra states entering the sum

Experimental information on octupole collectivity in even-even nuclei

- energy of the first 3^- state (first hint)

- $B(E3; 3_1^- \rightarrow 0_1^+)$ value; $B(E3; I_i \rightarrow I_f) = \frac{7}{16\pi}(I_f 0 3 0 | I_i 0)^2 Q_3^2$
  $Q_3 = \frac{3}{\sqrt{7\pi}} Z\, e\, R_0^3 \beta_3$

- negative-parity states decay predominantly by fast E1 transitions; large B(E1) values usually correlate with octupole collectivity, but the inverse is not true

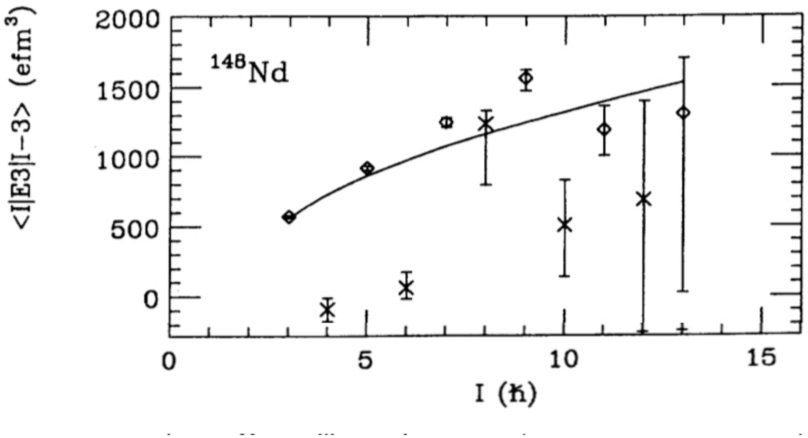
- lifetime of a negative-parity state is a very poor indicator of octupole collectivity

- direct E3 decay is rarely observed

- Coulomb excitation and inelastic scattering are the methods of choice to determine E3 strength

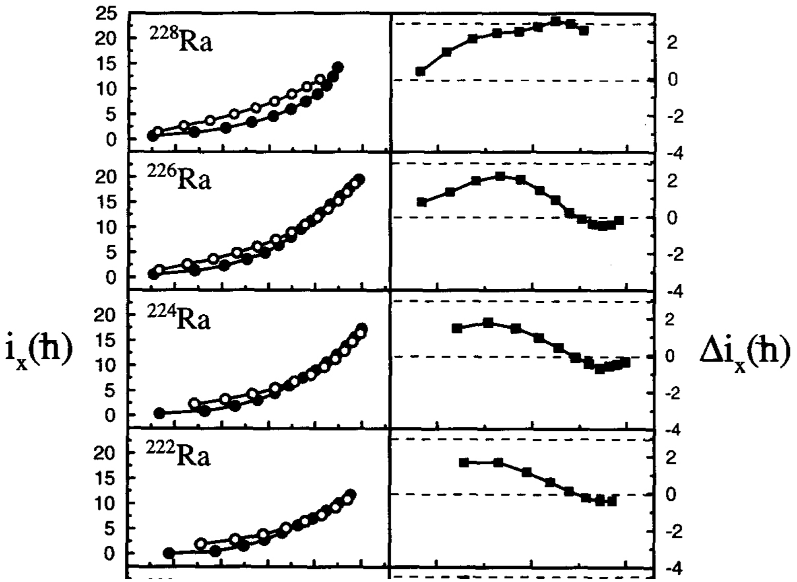
Rigid octupole deformation versus octupole vibration

- apart from actinides, E3 collectivity is usually attributed to surface vibrations
- rigid octupole deformation can be claimed on the basis of B(E3) values between the ground-state band and the negative-parity band, or identical rotational alignments in these bands (→ interleaving of positive and negative-parity states)



R. Ibbotson et al, PRL 71, 27 (1993)

J.F.C. Cocks et al. / Nuclear Physics A 645 (1999) 61–91

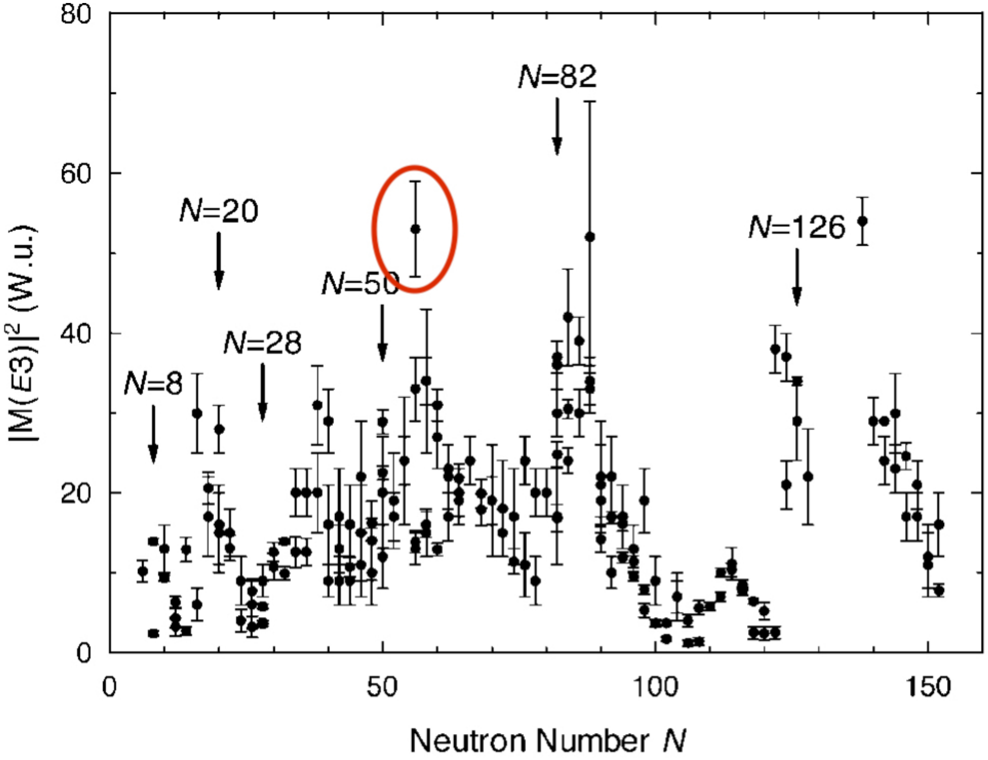


More info: P. A. Butler and W. Nazarewicz Rev. Mod. Phys. 68, 349 (1996); P. Butler, Proc. R. Soc. A 476, 202 (2020)

Octupole collectivity in Zr isotopes: anomalous value for ^{96}Zr

- evaluated B(E3; $3_1^- \rightarrow 0_1^+$) strength for ^{96}Zr strikingly high (57(6) W.u.), comparable with those known for nuclei with rigid pear shapes

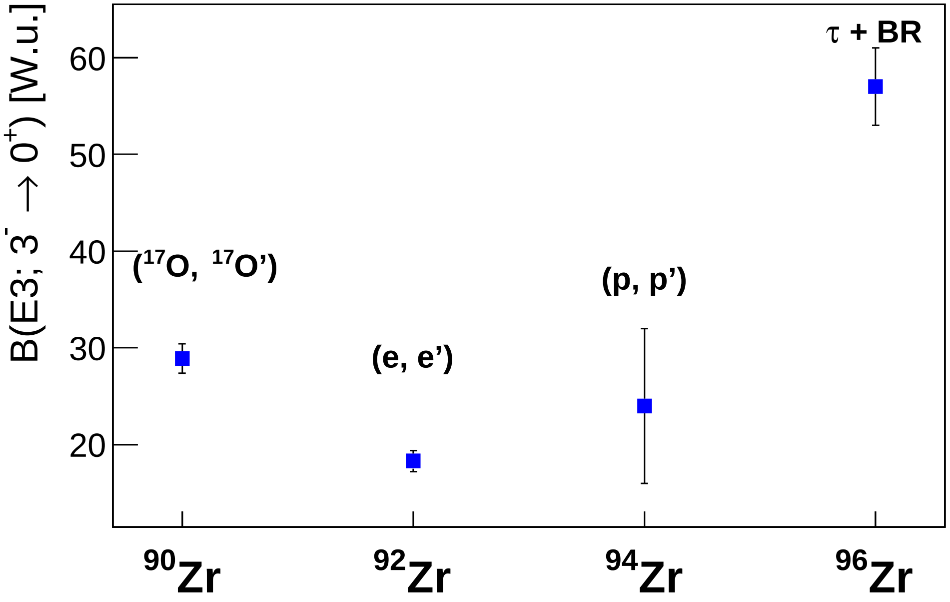
- long-standing challenge for theory

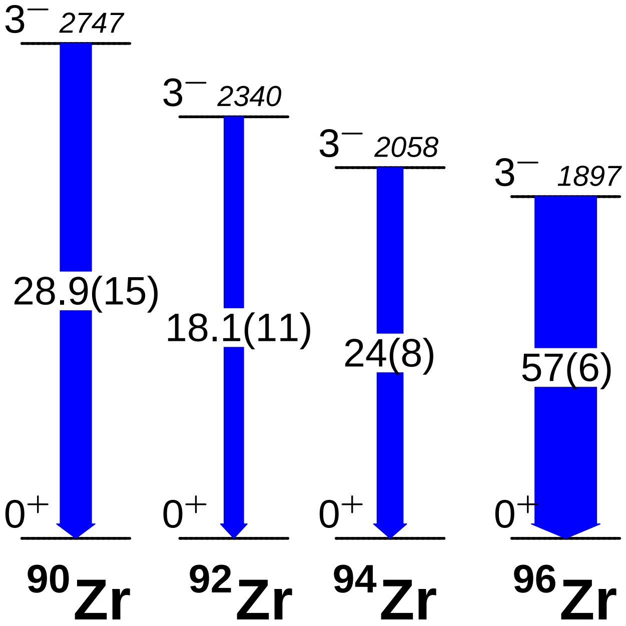


T. Kibédi and R.H. Spear, At. Data Nucl. Data Tables 80, 35 (2002)

Octupole collectivity in Zr isotopes

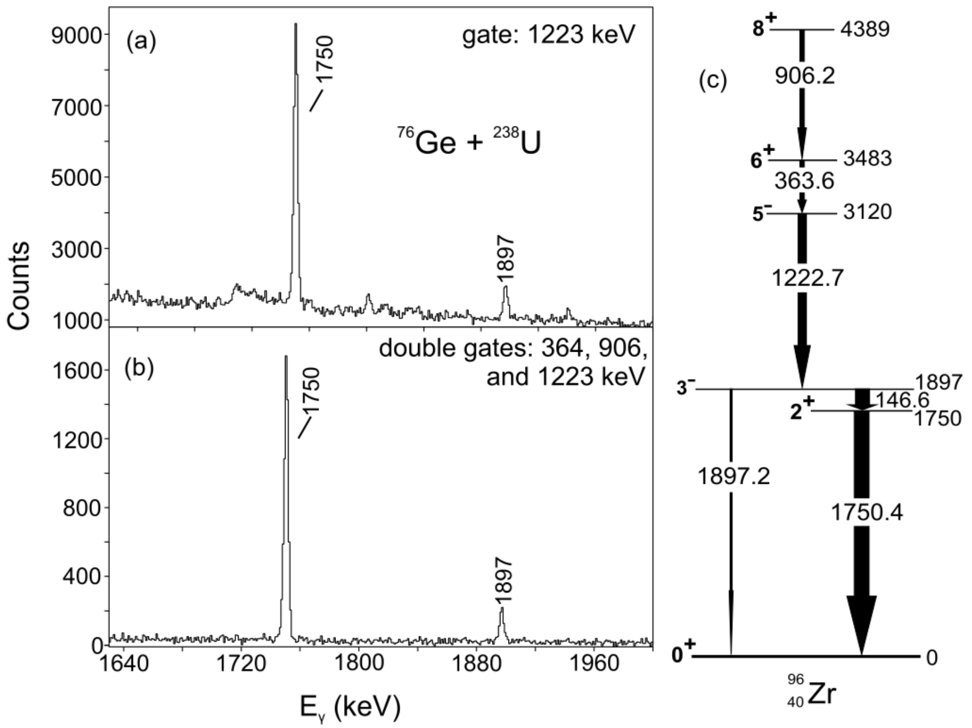
- evaluated B(E3; $3_1^- \rightarrow 0_1^+$) strength for ^{96}Zr strikingly high (57(6) W.u.)
- each of evaluated B(E3; $3_1^- \rightarrow 0_1^+$) values for ^{90,92,94,96}Zr results from a different experimental approach
- observed trend of B(E3; $3_1^- \rightarrow 0_1^+$) values in Zr isotopes inconsistent with 3_1^- energies and hard to explain





Revision of the E3 strength in ^{96}Zr

- determination of E3 strength in ^{96}Zr using gamma-ray spectroscopy requires two measurements:
  - lifetime ($\approx$ 70ps – plunger measurements)
  - branching ratio E3/E1
- if the 147 keV / 1897 keV intensity ratio is directly measured, the efficiency must be known precisely
  - walk effect, conversion at 147 keV

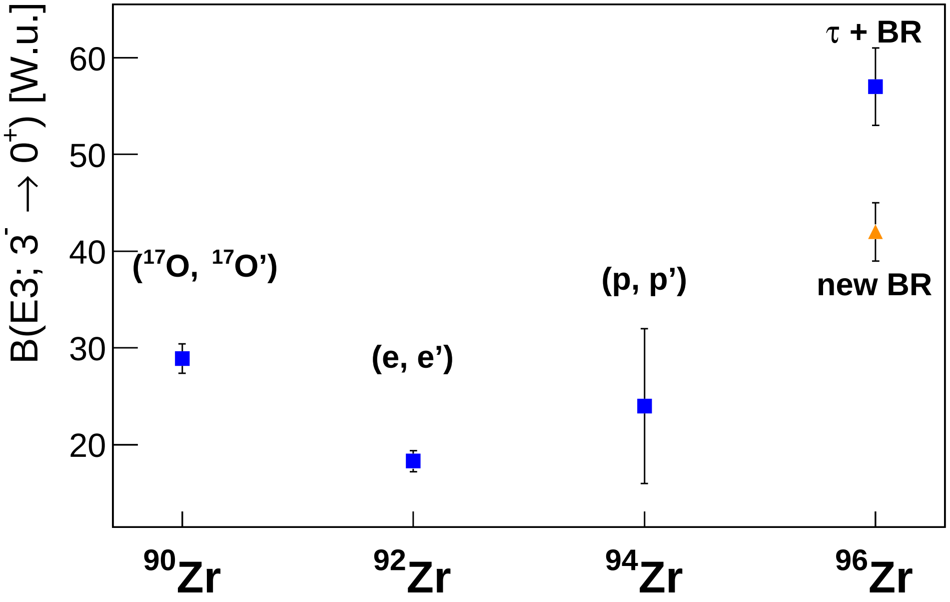


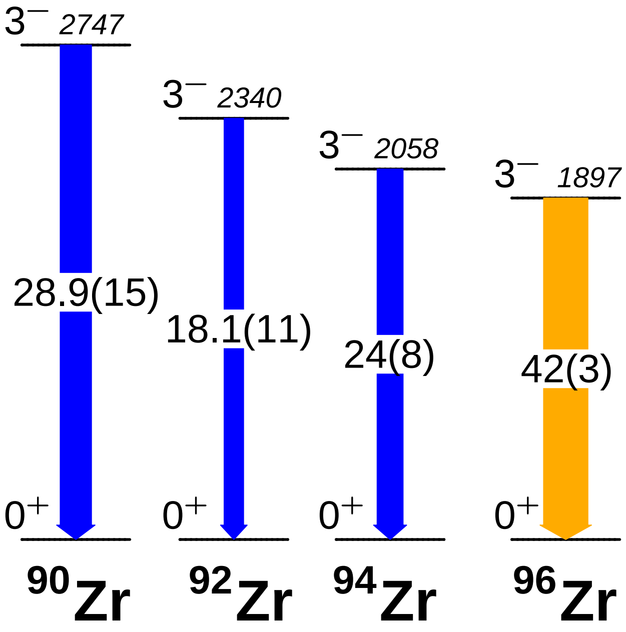
- new measurement – gating from above and comparison of 1750 keV and 1897 keV intensities

Ł. Iskra et al, Phys. Lett. B 788 (2019) 396

Octupole collectivity in Zr isotopes: new BR measurement for ^{96}Zr

- new measurement of E1/E3 branching ratio in ^{96}Zr (Ł. Iskra et al, Phys. Lett. B 788 (2019) 396) points to lower octupole collectivity, but the overall trend remains puzzling

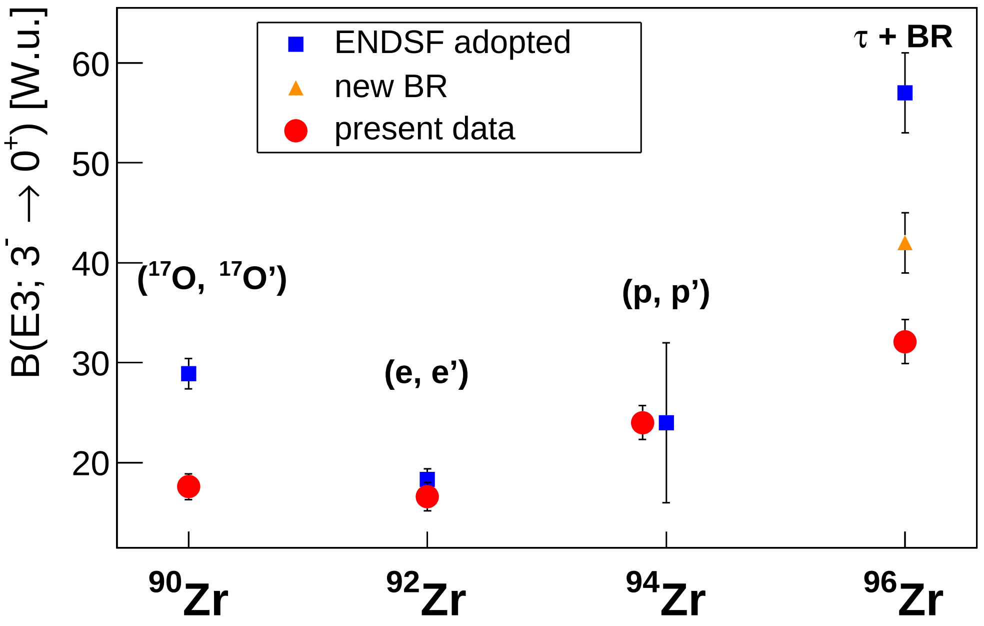


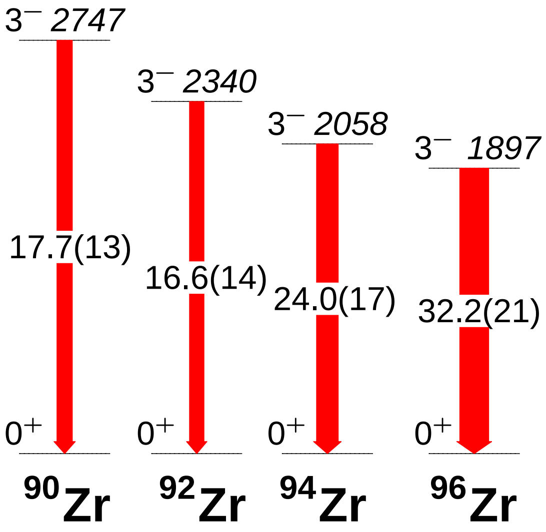


→ new systematic study of quadrupole and octupole collectivity in stable Zr isotopes at MLL

Results: octupole collectivity in Zr isotopes

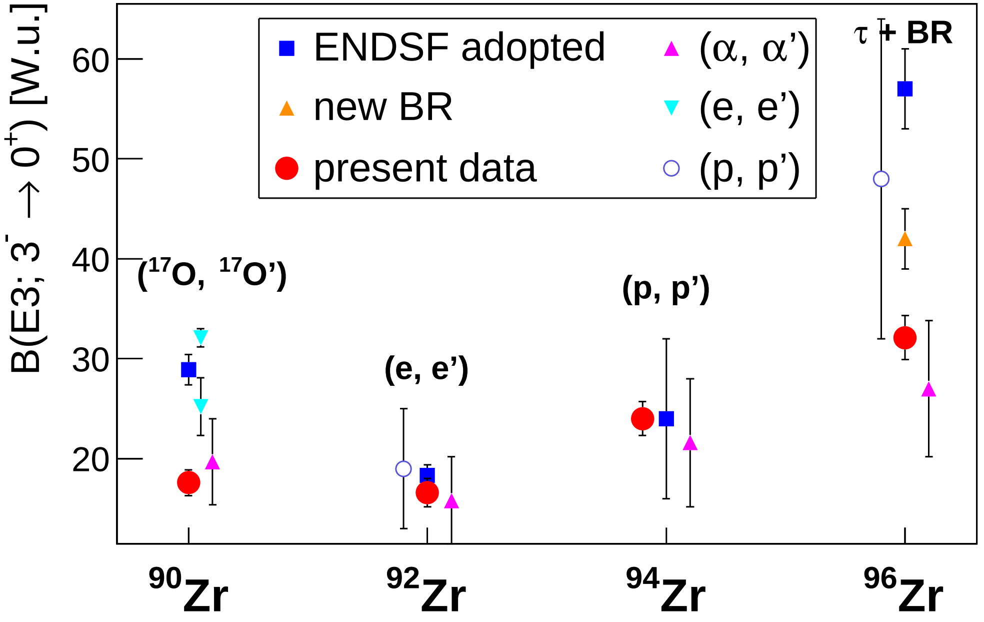
- overall trend of B(E3; $3^-_1 \rightarrow 0^+_1$) values in Zr more consistent with evolution of 3^-_1 energies than that of evaluated values

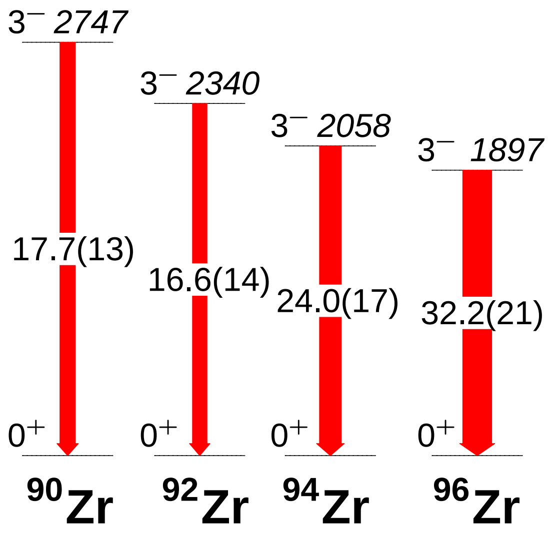




Results: octupole collectivity in Zr isotopes

- overall trend of B(E3; $3_1^- \rightarrow 0_1^+$) values in Zr more consistent with evolution of 3_1^- energies than that of evaluated values

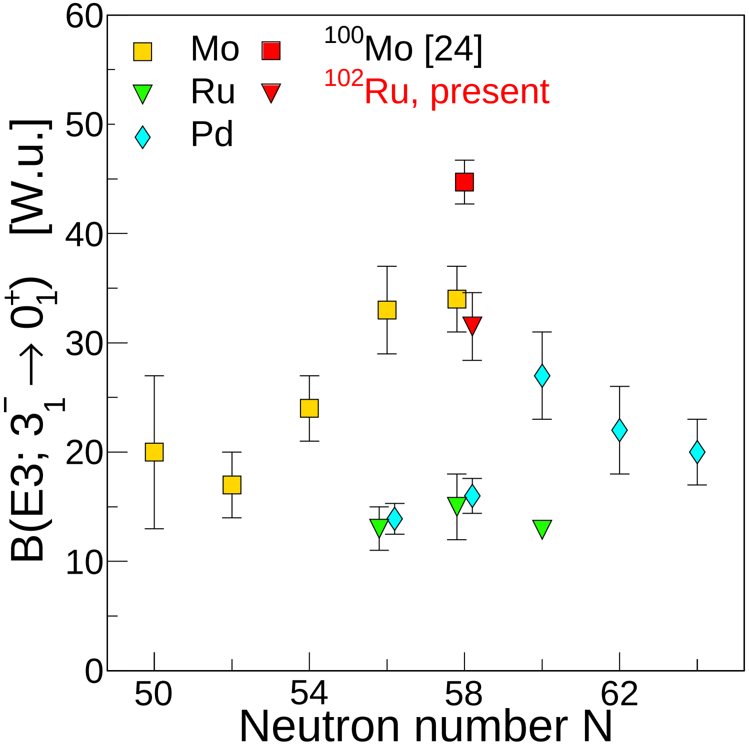


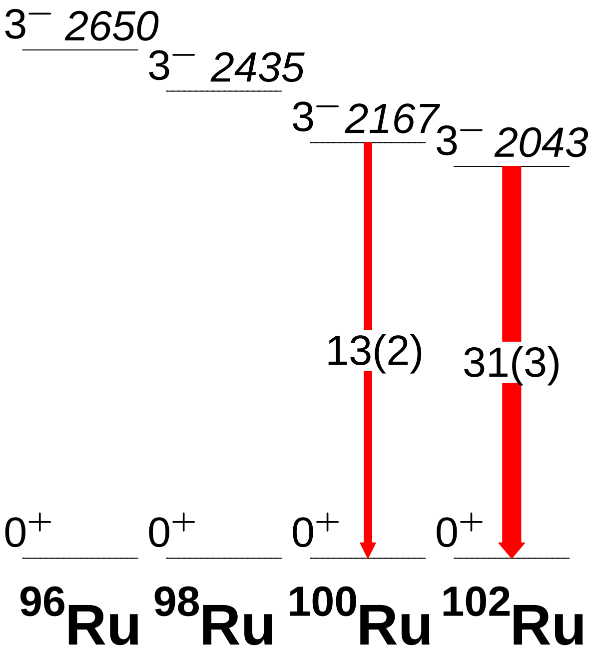


- similarities with results of (α, α') (D. Rychel et al, Z. Phys. A 326, 455 (1987) – the only other systematic study of β_3 in Zr) but considerably higher precision

Octupole collectivity in Ru isotopes

- no B(E3) values for Ru isotopes lighter than ^{100}Ru
- smooth evolution of 3^- energies
- conflicting B(E3) results in Ru and Mo nuclei



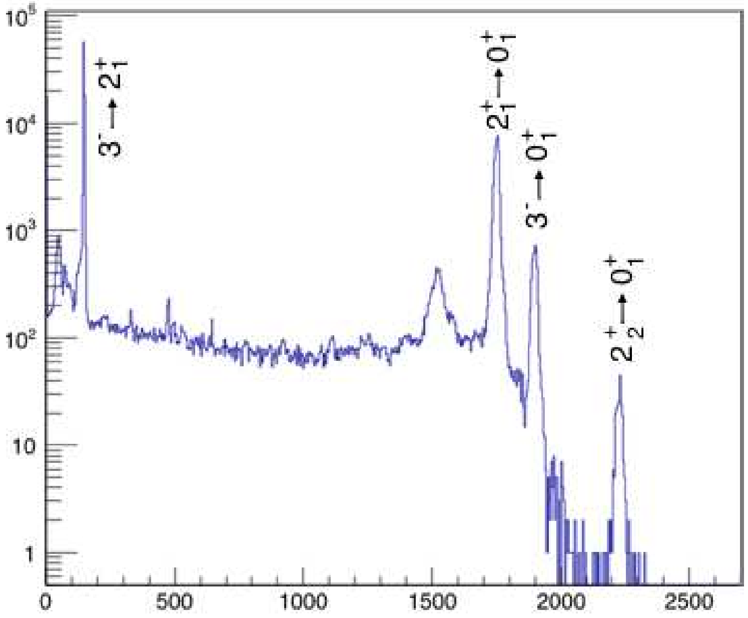


P. Garrett, MZ et al, submitted to Phys. Rev. C

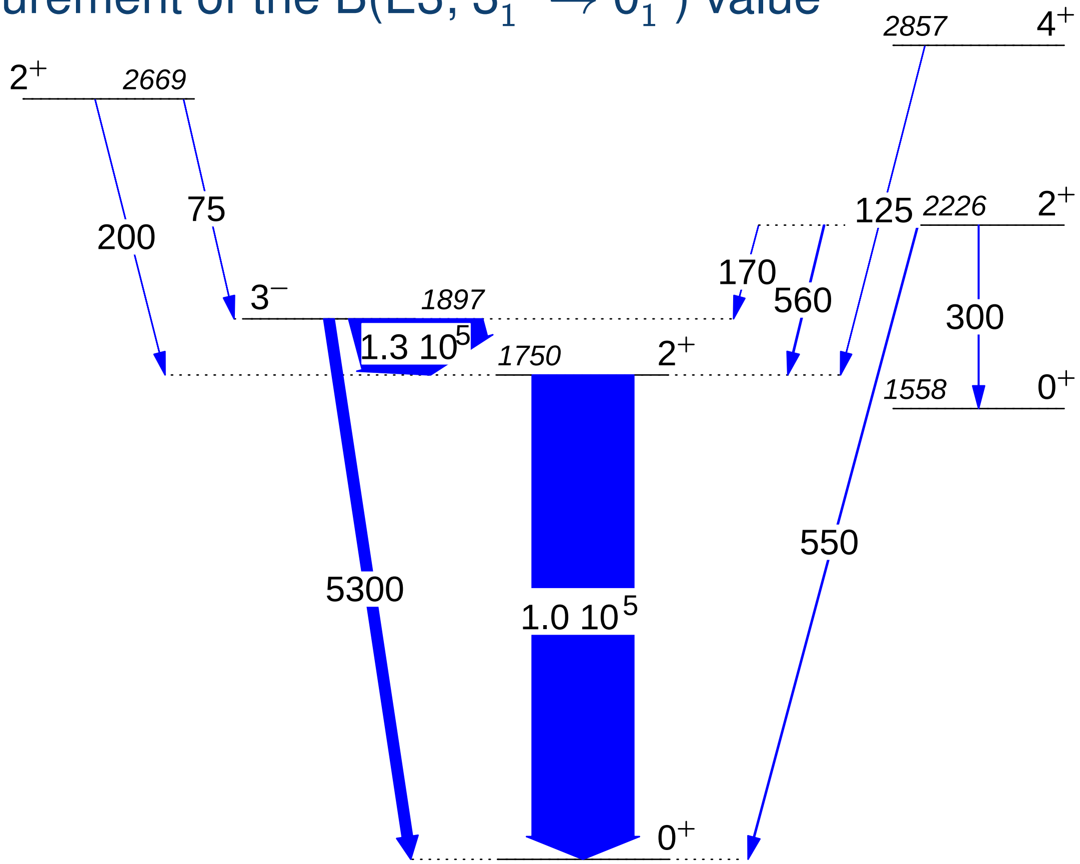
Outlook: Coulomb excitation of ^{96}Zr

D.T. Doherty (Surrey), N. Marchini (Florence), M. Zielińska (Saclay) et al

- AGATA + SPIDER, ^{58}Ni beam on ^{96}ZrO$_2$ target
- goals of the measurement:
  - to measure quadrupole moments of $2^+_{1,2}$ states
  - to provide firm exidence for shape coexistence in ^{96}Zr
  - to provide independent measurement of the B(E3; $3^-_1 \rightarrow 0^+_1$) value



simulation by A. Goasduff



Quadrupole sum rules

D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683

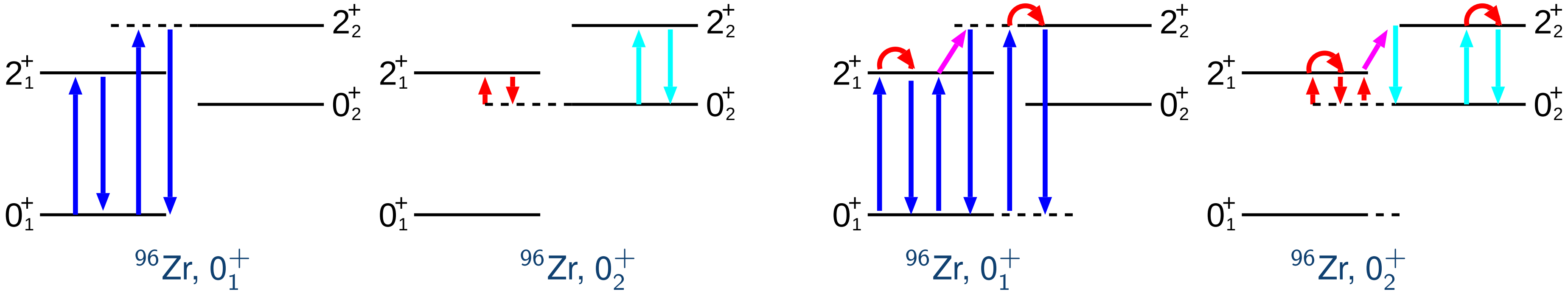
K. Kumar, Phys. Rev. Lett. 28 (1972) 249

$$\frac{\langle Q^2\rangle}{\sqrt{5}} = \langle i|\,[\mathrm{E2}\times \mathrm{E2}]^0\,|i\rangle = \frac{1}{\sqrt{(2I_i+1)}}\sum_t \langle i\|\mathrm{E2}\|t\rangle\langle t\|\mathrm{E2}\|i\rangle \begin{Bmatrix} 2 & 2 & 0\\ I_i & I_i & I_t\end{Bmatrix}$$

$$\sqrt{\frac{2}{35}}\langle \mathrm{Q}^3\cos 3\delta\rangle = \langle i|\{[\mathrm{E2}\times \mathrm{E2}]^2\times \mathrm{E2}\}^0|i\rangle = \frac{1}{(2I_i+1)}\sum_{t,u}\langle i\|\mathrm{E2}\|u\rangle\langle u\|\mathrm{E2}\|t\rangle\langle t\|\mathrm{E2}\|i\rangle \begin{Bmatrix} 2 & 2 & 2\\ I_i & I_t & I_u\end{Bmatrix}$$

matrix elements needed to determine $\langle Q^2\rangle$

matrix elements needed to determine $\langle Q^3\cos(3\delta)\rangle$



blue arrows: matrix elements determined from MLL data;

light blue: matrix elements determined from MLL data combined with branching ratio measured at TRIUMF;

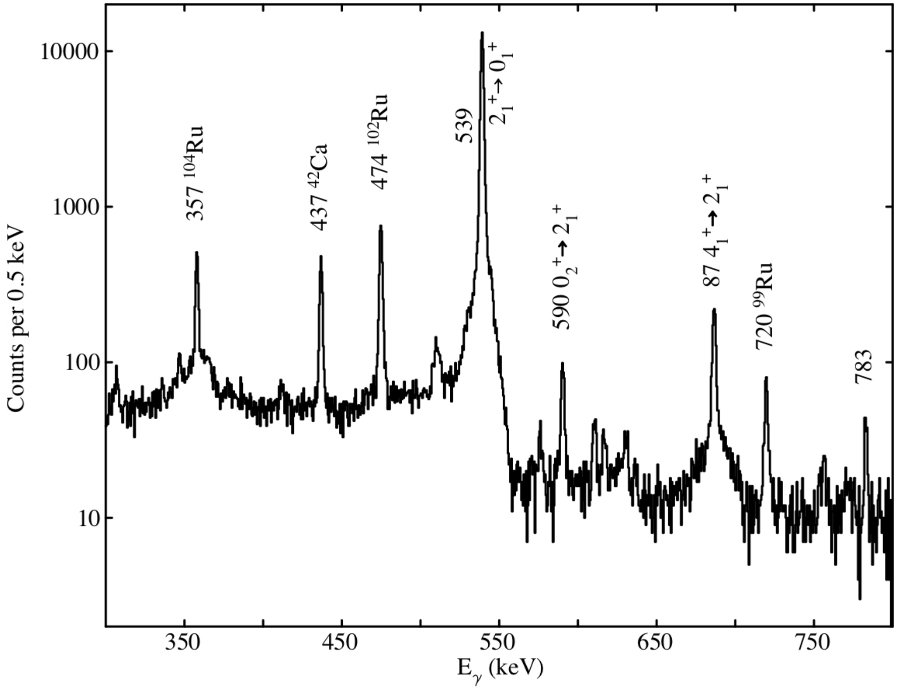
red: matrix elements determined from the future LNL study;

magenta: matrix elements determined from the future LNL study and mixing ratio measured at TRIUMF

(decay of the (2^+_3) state is known only to $2^+_{1,2}$ and 3^-_1, so the contribution of this state to the invariants is neglected)

Coulomb excitation of ^{100}Ru

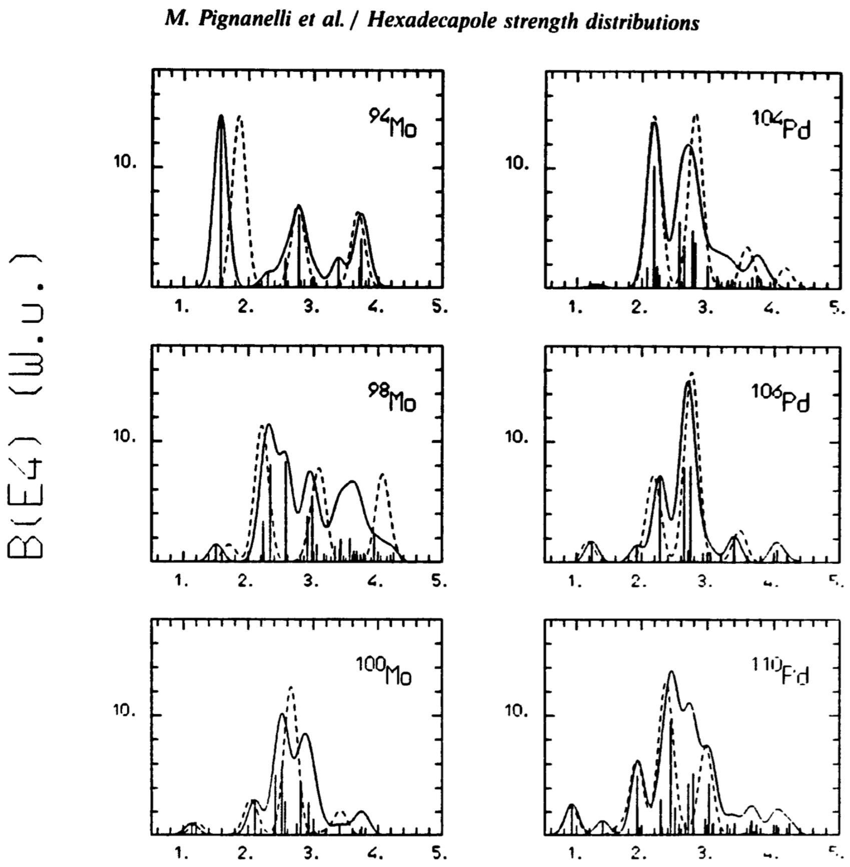
- low-energy Coulomb excitation of ^{100}Ru with a ^{32}S beam performed at HIL Warsaw in April 2022 (PI P. Garrett, K. Wrzosek-Lipska, MZ)
- in order to better constrain the properties of the 2_2^+ state, data will be completed by a second measurement with a ^{14}N beam
- additional lines in the spectrum due to target oxidation
- decay of the 3_1^- state at the observation limit



Outlook: challenges for future Coulomb-excitation studies

- abundance: 5.54 % ^{96}Ru, 2.80 % ^{96}Zr
- difficult to get material with high enrichment (even more since the war has started); to my knowledge, no suppliers offer ^{96,98}Ru
- difficult to produce Ru and Zr targets (material often available in oxide form, Ru targets produced by electrodeposition proven very fragile)
- high excitation energies in ^{96}Zr and ^{96}Ru with respect to other isotopes make it more difficult to populate levels of interest

Hexadecapole strength in A$\approx$ 100 nuclei



M. Pignanelli et al, NPA 540, 27 (1992)