Opportunities for QGP studies from Bayesian analyses with several ions

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Deciphering nuclear phenomenology across energy scales
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FAPESP
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Outline

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Bayesian inference

- Parameter estimation: Given a model, what parameter values are compatible with experiment, and with what precision can we determine them?
- Can use Bayesian inference — useful for systematic treatment of uncertainty
- Experimental data (D) and model parameters (p) associated with probability distributions
- Bayes' theorem relates conditional probabilities.

$$\Pr(p\,\&D) = \Pr(p) \times \Pr(D|p) = \Pr(D) \times \Pr(p|D)$$
$$\text{prior} \times \text{likelihood} = \text{evidence} \times \text{posterior}$$

- We want $\Pr(p|D) = \dfrac{\Pr(p)\Pr(D|p)}{\Pr(D)}$
- Obtain likelihood $\Pr(D|p)$ from comparison with data

$$\Pr(D|p) \propto e^{-\chi^2/2}$$
$$\text{with } \chi^2 = (D - \text{Model}(p))^T\,\Sigma^{-1}\,(D - \text{Model}(p))$$
$$\text{and } \Sigma = \text{uncertainty covariance (exp. and theor.)}$$

- Also need prior $\Pr(p)$

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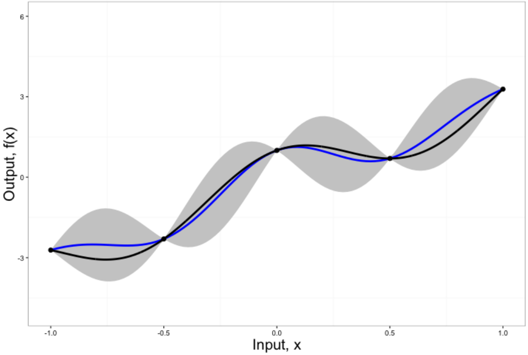
Model emulation

- Probing posterior requires many samples of model
- An emulator can serve as fast proxy
- More than a few parameters $\Longrightarrow$ large multidimensional space $\Longrightarrow$ need emulator
- Gaussian process emulators have been successful
    - Uses Bayesian statistics to represent outputs as a Gaussian process on parameter space
    - Requires fairly smooth dependence on parameters
- Other techniques and recent developments can help further — PCA, transfer learning, multi-fidelity emulation, etc.



Outline

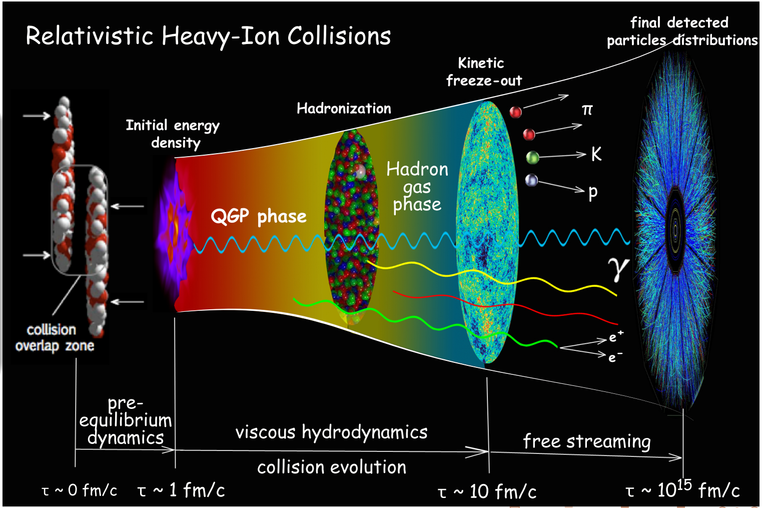
Recent multi-ion analyses

- Various multi-system analyses have been done, e.g.:
  - Duke (pPb & PbPb) — PRC 101 (2020) 2, 024911
  - Trajectum (pPb & PbPb) — PRL 126 (2021) 20, 202301; PRC 103 (2021) 5, 054909
  - JETSCAPE (PbPb & AuAu) — PRL. 126 (2021) 24, 242301; PRC 103 (2021) 5, 054904;
- Let's review the models and parameterizations used:

Time line of heavy-ion collision

Collision model

- Incoming nuclei
- Initial scattering
- Hydrodynamization
- Relativistic Fluid
    - Quark-Gluon Plasma
    - Hadrons
- Hadronic scattering



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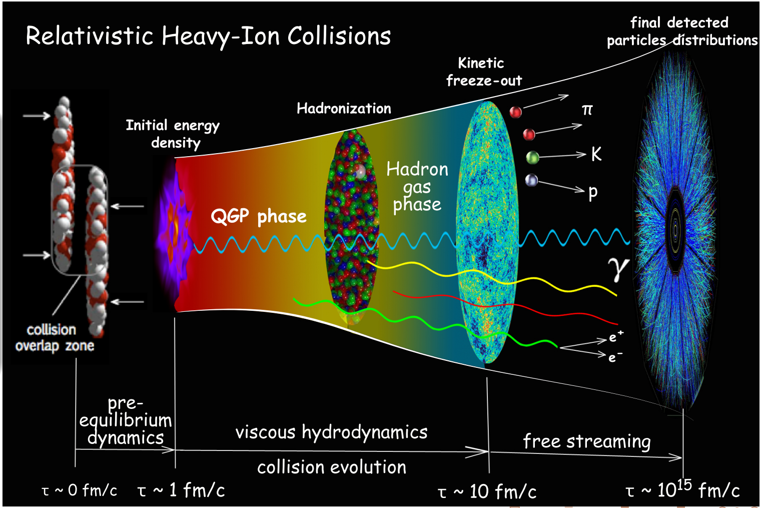
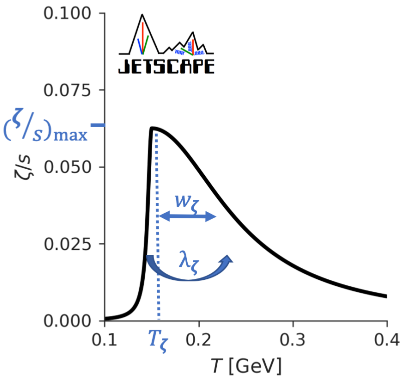
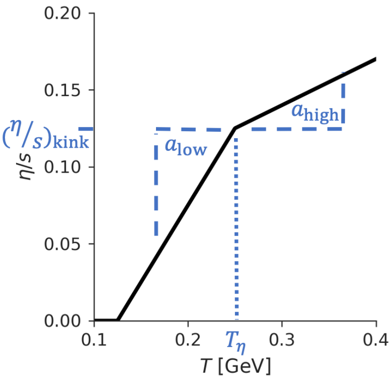


HYDRODYNAMICS

- Main workhorse: 2nd order relativistic viscous hydrodynamics
- Equation of state from Lattice $\epsilon(p)$
- Unknown quantities: transport coefficients
- Shear $\frac{\eta}{s}(T)$ and bulk viscosity $\frac{\zeta}{s}(T)$
- 2nd order transport coefficients τ_π (JETSCAPE, Trajectum), τ_Π, $\tau_{\pi\pi}$ (Trajectum)





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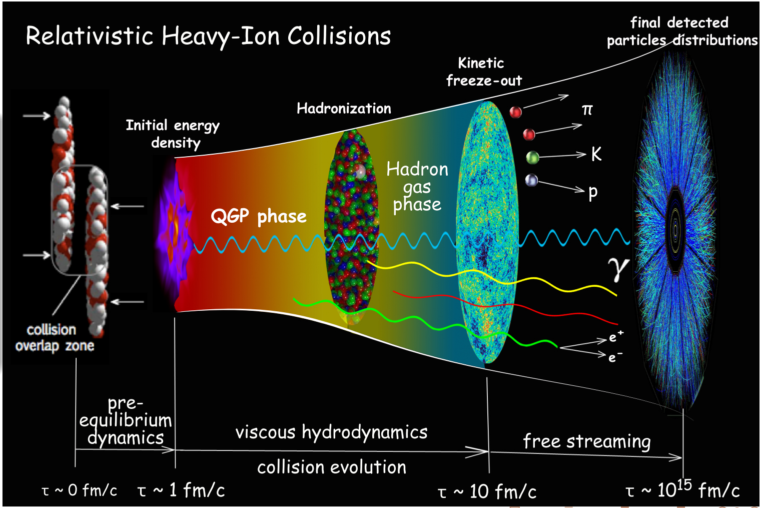

Initial conditions for hydrodynamics

- Nucleus
  - Nucleon positions sampled from Woods-Saxon
  - Reject nuclei with nucleons closer than d_{min}
- Trento
  - Boost invariant
  - Participant nucleons determined by b-dependent cross section with width parameter w
  - Energy density at time $\tau = 0^+$ proportional to generalized mean of nuclear thickness functions multiplied by a random fluctuation γ of variance σ_k^2.

$$\tau\epsilon(\mathbf{x}) = N T_R\left(\mathbf{x}_\perp; p\right) = N\left(\frac{T_A^p\left(\mathbf{x}_\perp\right) + T_B^p\left(\mathbf{x}_\perp\right)}{2}\right)^{1/p}$$

$$T_A(\mathbf{x}_\perp) = \sum_{i\in A} \gamma_i \rho(\mathbf{x}_\perp - \mathbf{x}_{i,\perp})$$

  - Nucleon substructure (Duke, Trajectum): Nucleon consists of n_c constituents of width ν
- Free steaming
  - Energy spreads out isotropically with transverse velocity $v = 1$ (Duke, JETSCAPE) or $v \leq 1$ (Trajectum) for time τ_{fs}, which can depend on energy via exponent α (JETSCAPE)
  - Full energy-momentum tensor at τ_{fs} used as initial condition for hydro

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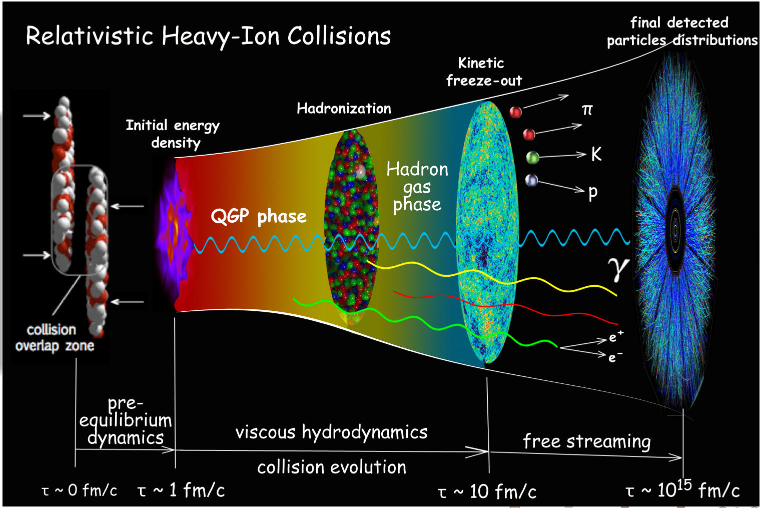


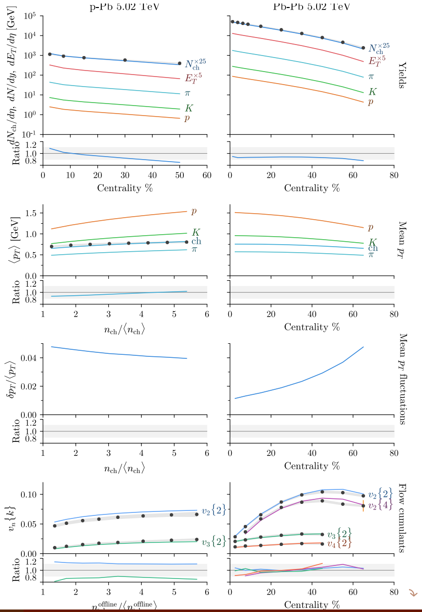
Hadronic afterburner

- Switch from fluid to particles (hadrons) at T_{sw}
- Equilibrium distribution function given by kinetic theory, but viscous corrections non-universal
- Estimate uncertainty via 3 models
  - Grad (JETSCAPE)
  - Chapman-Enskog (JETSCAPE)
  - Pratt-Torrieri-Bernhard (Duke, Trajectum & JETSCAPE)
- Collisions and decays via SMASH (JETSCAPE & Trajectum) or UrQMD (Duke)

Outline

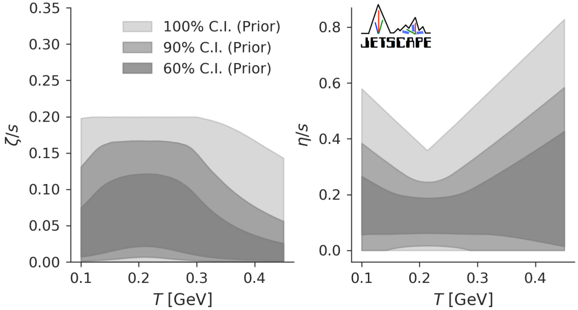
SYSTEMS AND OBSERVABLES

- Systems
  - Duke (pPb & PbPb)
  - Trajectum (pPb & PbPb)
  - JETSCAPE (PbPb & AuAu)
- Observables
  - Charged hadrons
    - Multiplicity $dN_{ch}/d\eta$
    - Transverse energy $dE_T/d\eta$ (Trajectum, JETSCAPE)
    - p_T fluctuations $\delta p_T/\langle p_T\rangle$ (Trajectum, JETSCAPE)
    - Integrated anisotropic flow (Duke, JETSCAPE) $v_2\{2\}$, $v_3\{2\}$, $v_4\{2\}$
    - $\langle p_T\rangle$ (Duke)
  - Identified hadrons (pion, kaon , proton)
    - Yield dN/dy (Trejectum, JETSCAPE)
    - $\langle p_T\rangle$ (Trajectum, JETSCAPE)
    - Differential anisotropic flow (Trajectum) $v_2\{2\}(p_T)$, $v_3\{2\}(p_T)$
    - p_T spectra (Trajectum)



BAYES

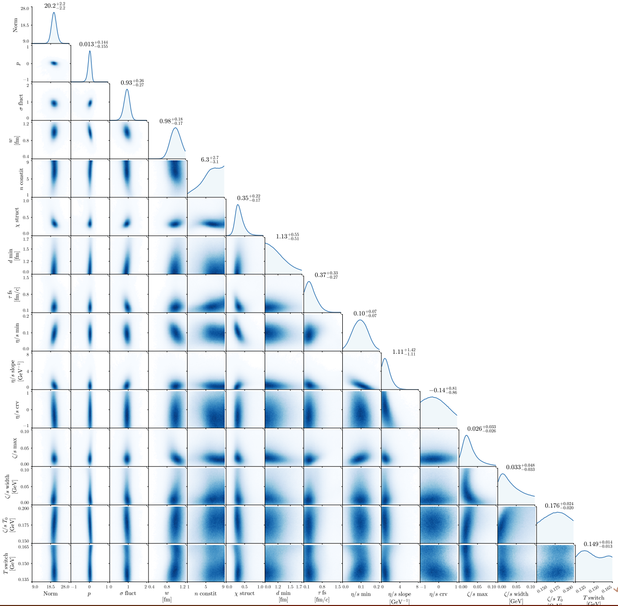
- Prior $\Pr(p)$: each parameter given uniform prior within pre-defined range
- Compare model output to data to obtain likelihood
- $\Pr(p|D) \propto \Pr(p)\,\Pr(D|p)$



Parameter	Symbol	Prior	Parameter	Symbol	Prior
Norm. Pb-Pb 2.76 TeV	N[2.76 TeV]	[10, 20]	temperature of (η/s) kink	T_η	[0.13, 0.3] GeV
Norm. Au-Au 200 GeV	N[0.2 TeV]	[3, 10]	(η/s) at kink	$(\eta/s)_{\mathrm{kink}}$	[0.01, 0.2]
generalized mean	p	[–0.7, 0.7]	low temp. slope of (η/s)	a_{low}	[–2, 1] GeV^{-1}
nucleon width	w	[0.5, 1.5] fm	high temp. slope of (η/s)	a_{high}	[–1, 2] GeV^{-1}
min. dist. btw. nucleons	d^3_{min}	[0, 1.7^3] fm^3	shear relaxation time factor	b_π	[2, 8]
multiplicity fluctuation	σ_k	[0.3, 2.0]	maximum of (ζ/s)	$(\zeta/s)_{\mathrm{max}}$	[0.01, 0.25]
free-streaming time scale	τ_R	[0.3, 2.0] fm/c	temperature of (ζ/s) peak	T_ζ	[0.12, 0.3] GeV
free-streaming energy dep.	α	[–0.3, 0.3]	width of (ζ/s) peak	w_ζ	[0.025, 0.15] GeV
particlization temperature	T_{sw}	[0.135, 0.165] GeV	asymmetry of (ζ/s) peak	λ_ζ	[–0.8, 0.8]

Duke

- Multiply prior by likelihood to obtain posterior — multidimensional probability density
- Visualize by marginalizing over all but 1 or 2 parameters:



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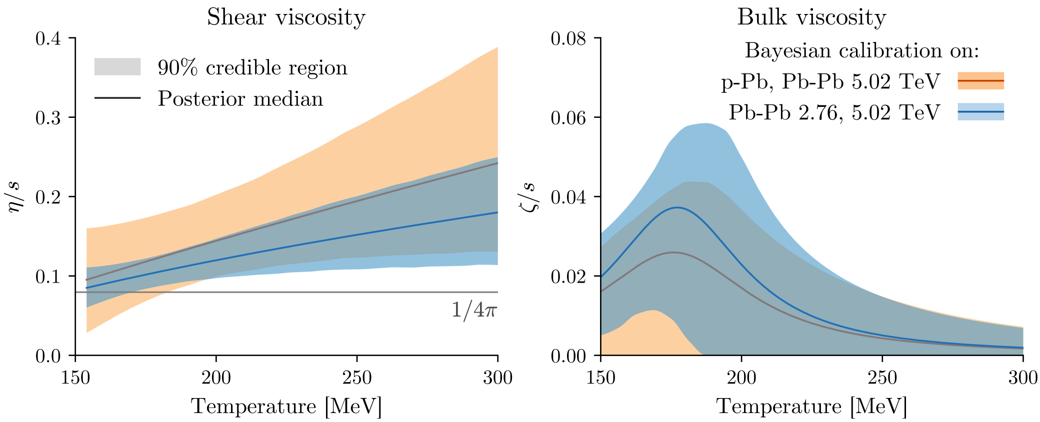


- Note: not a simple comparison of adding an additional ion

Trajectum

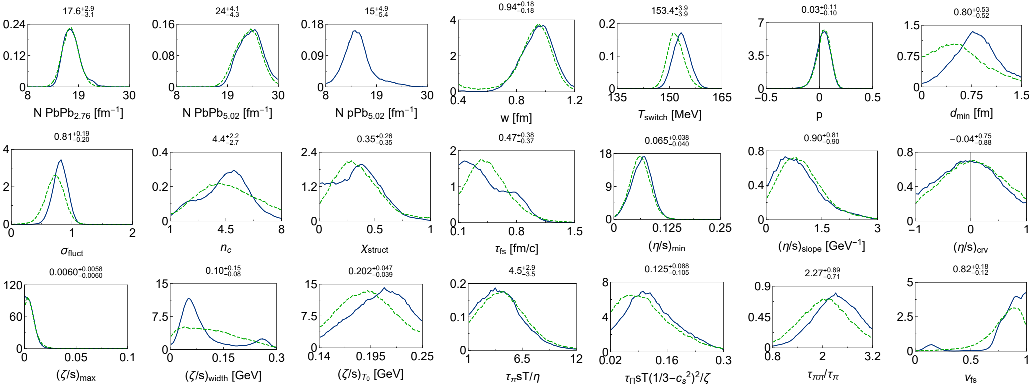
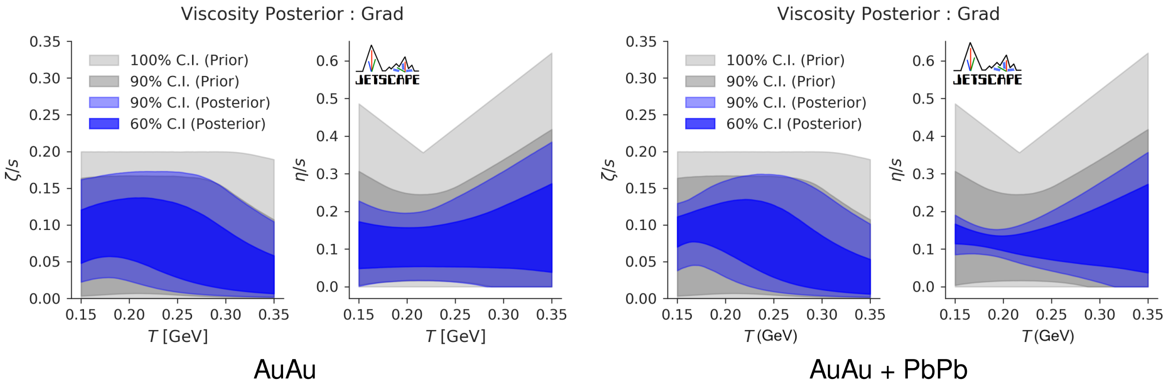


FIG. 1. Posterior distributions for all model parameters fitted to PbPb and pPb (solid) or PbPb only (dashed, not applicable to pPb norm) data. Values indicate the expectation values with the 90% highest posterior density credible interval.

- QGP properties: better constraint on bulk viscosity by adding pPb to PbPb data?

JETSCAPE



- Combining ions changes posterior — but mostly due to collision energy?

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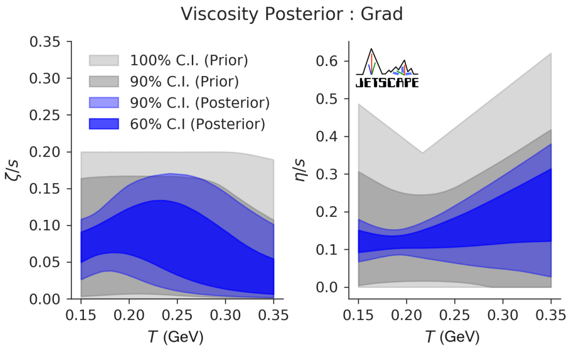
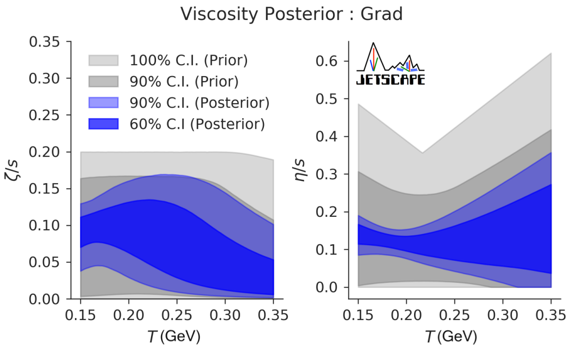


PbPb



AuAu + PbPb

- Combining ions changes posterior — but mostly due to collision energy?

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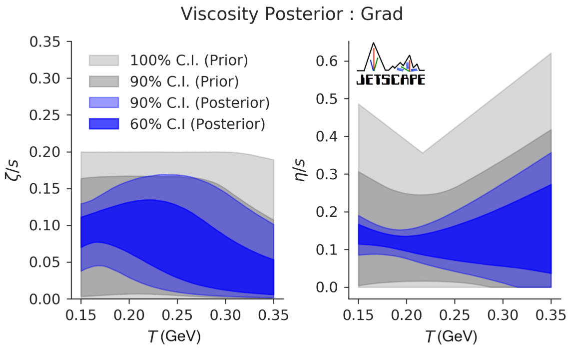

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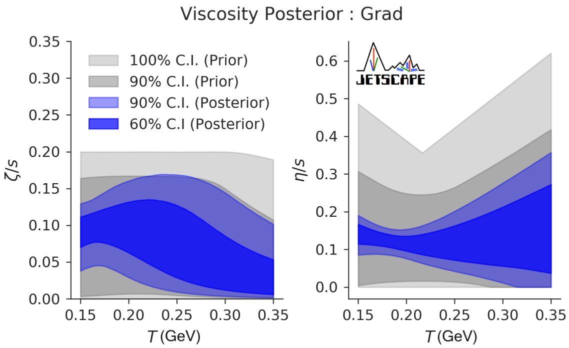

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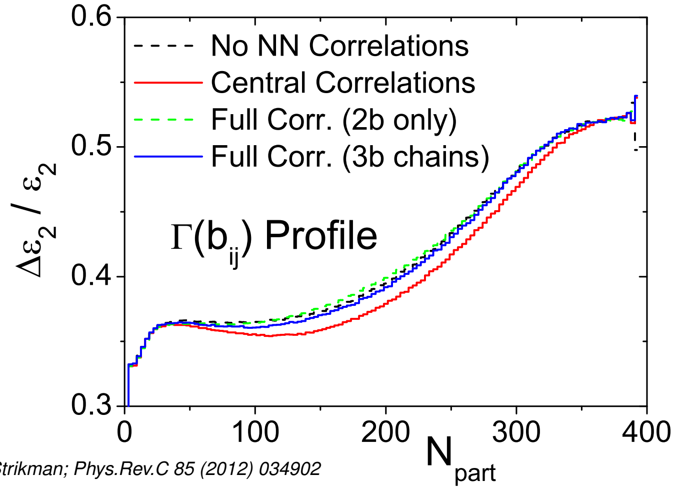
Questions

1. What can we learn about hot QGP by colliding multiple ions?
   - Most obvious: scan system **size**
     - Viscous effects $\implies$ gradients $\implies$ size
     - Deviation from hydrodynamic behavior as size decreases
     - Centrality also scans size. What do we gain from colliding smaller ions?
   - Secondary effects: better knowledge of initial state $\implies$ better measurement of QGP properties
   - Other?
2. What benefit do we get by adding ions to, e.g., p-A + A-A analysis?
3. We already have various ions
   - LHC: p-Pb, Xe-Xe, Pb-Pb, (O-O?)
   - RHIC: p-Au, d-Au, ^{3}He, Au-Au, Ru-Ru, Zr-Zr, U-U, Cu-Cu, O-O...

   what benefit do we get by adding more?
   - Fill in more sizes to better probe onset of viscous effects and/or breakdown of hydrodynamics?
   - Sensitivity of sub-nuclear scales a can be slowly turned on
   - Do we need better quantitative estimates of the benefit of adding specific ions?

Extra Slides

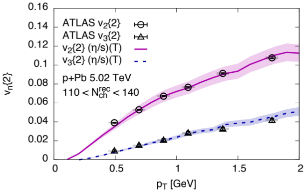
Short-range correlations and d_{min}



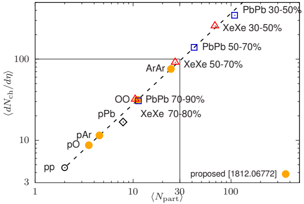
Alvioli, Holopainen, Eskola, Strikman; Phys.Rev.C 85 (2012) 034902

- Ignoring correlations can be better than including only radial repulsion

Heavy-Ion collisions



Mäntysaari, Schenke, Shen, Tribedy, PLB 772 (2017) 681-686



Mazeliauskas talk at Initial State 2021

Effects of non-trivial proton geometry essential for description of flown p+A collision

Not clear if Hydrodynamics quantitatively accurate for p+A

Heavy-Ion phenomenology can not properly distinguish geometry (e_2) and magnitude of response ($v_2 \sim k_{22}\, e_2$)

Crucial test in upcoming $O_{16}+O_{16}$ where medium properties are similar to high mult. p+A but geometry is (well?) constrained from nuclear structure

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S. Schlichting, talk yesterday