

# Multidimensional PES in spontaneous fission

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ESNT, Saclay

# Motivation

Multidimensional PES  
in spontaneous fission

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2D PES

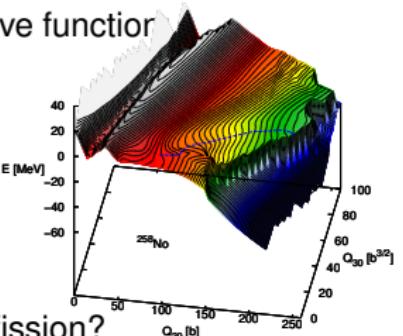
Fission paths  
Discontinuities

3D PES

Fission channels  
Neck constraint

Summary &  
Conclusions

- Potential Energy Surface (PES)** -  
basic ingredient in description of fission observables
  - **Spontaneous fission half-lives** → shape of fission barrier
  - **Fission dynamics** → topography of PES
- In **self-consistent** models PES is defined as a function of **constraints** (not deformation parameters) ⇒ The self-consistent solutions are not always univocal
  - Dependence on the starting wave function  
2 minima with different value of the non-constraint collective coordinate
- Which (and how many) collective degrees of freedom** in the deformation space allows to obtain an accurate description of fission?
- Can one obtain a **continuous PES at scission**?



# 2D PES: Fission paths of $^{258}\text{No}$

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2D PES

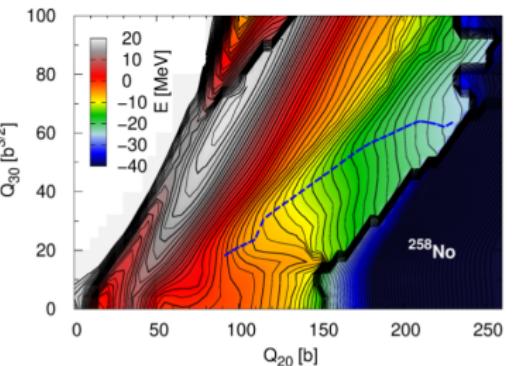
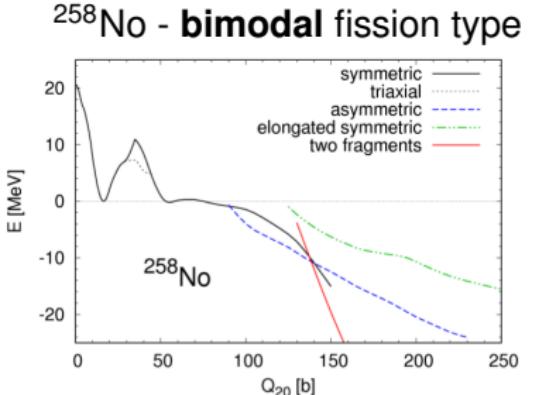
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Considered degrees of freedom:

- **Elongation:**  $\hat{Q}_{20} = \frac{1}{2} \sqrt{\frac{16\pi}{5}} \sum_{i=1}^A r_i^2 Y_{20}(\theta)$
- **Mass asymmetry:**  $\hat{Q}_{30} = \sqrt{\frac{4\pi}{7}} \sum_{i=1}^A r_i^3 Y_{30}(\theta)$



# 2D PES of $^{258}\text{No}$ : Discontinuities

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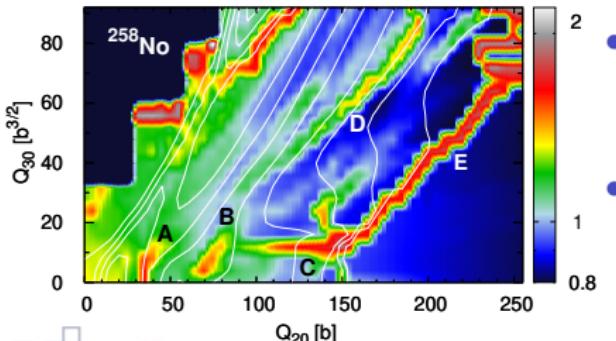
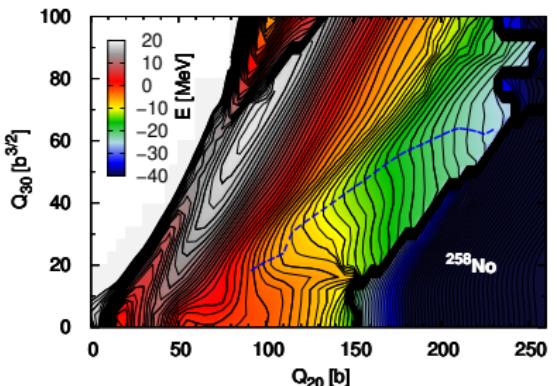
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## Density distance<sup>1</sup>:

$$D_{\rho\rho'} = \int |\rho(\vec{r}) - \rho'(\vec{r})| d\vec{r}$$

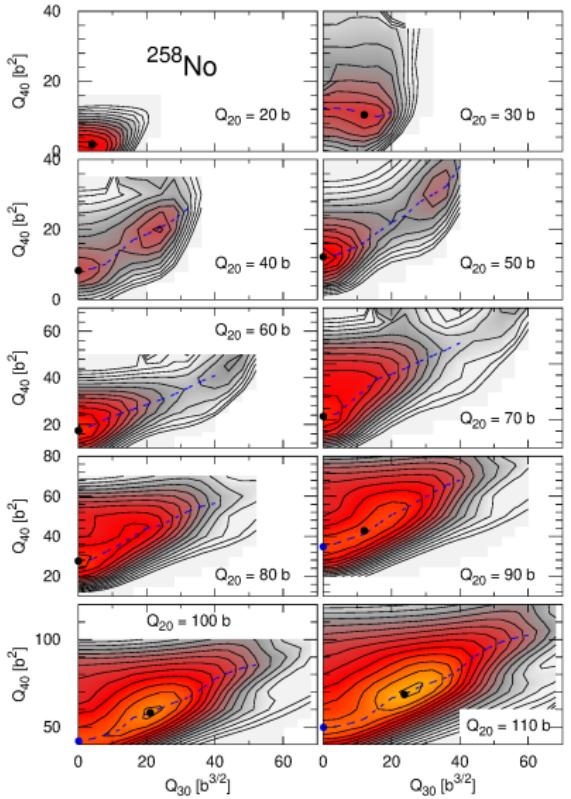
- **A:** first barrier
- **B:** the second minimum region
- **C:** between symmetric and asymmetric valley
- **D:** bordering asymmetric valley at large octupole deformation
- **E:** scission line



1. N. Dubray, D. Regnier, CPC **183** 2035 (2012).

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# 3D PES of $^{258}\text{No}$ : ( $Q_{20}$ , $Q_{30}$ , $Q_{40}$ )



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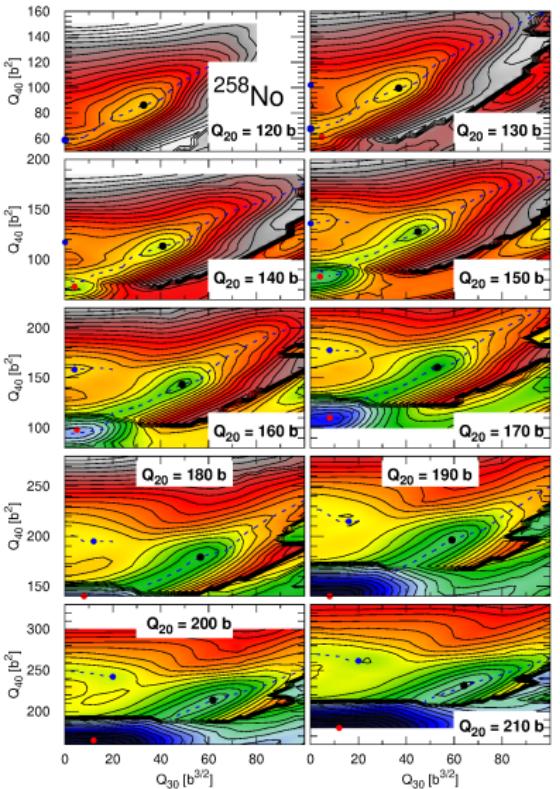
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## Hexadecapole moment:

$$\hat{Q}_{40} = \sqrt{\frac{4\pi}{9}} \sum_{i=1}^A r_i^4 Y_{40}$$

Visible 1-2 minima in the region  
below the second fission barrier

# 3D PES of $^{258}\text{No}$ : ( $Q_{20}$ , $Q_{30}$ , $Q_{40}$ )



$Q_{20} \geq 130 \text{ b}$  - 3 fission valleys spread

- asymmetric mode
- symmetric mode
- symmetric elongated

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# 2D PES: Fission channels of $^{258}\text{No}$

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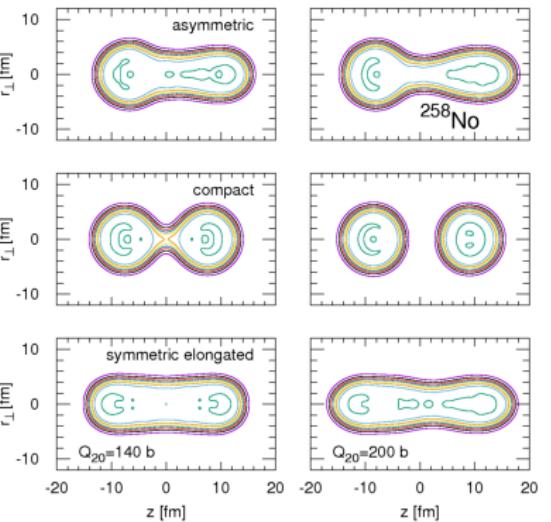
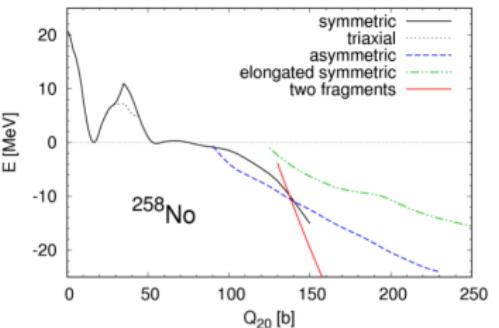
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- multiplication of the surfaces on the 2D PES  $\Rightarrow$  misinterpretation of the results from different valleys
- possible accidental change of the configuration



# Nuclear shape evolution on the way to fission

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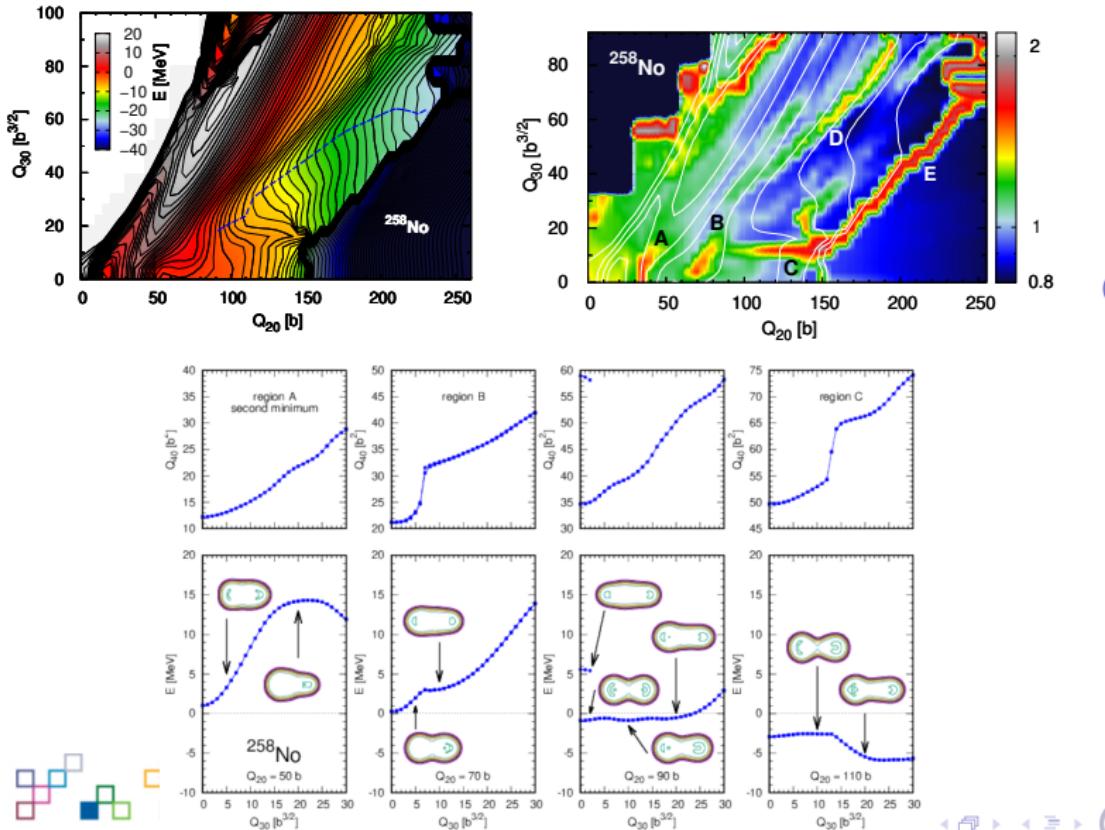
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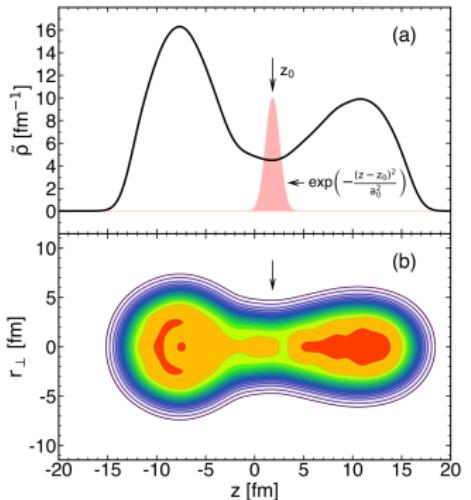
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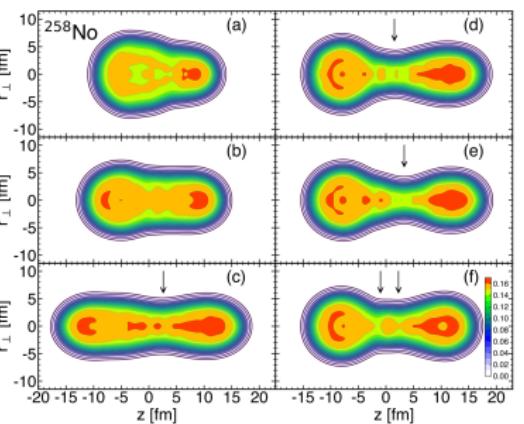


# Neck constraint



$$\hat{Q}_N = \exp \left[ -\frac{(z-z_0)^2}{a_0^2} \right]$$

$z_0$  - positioned at the local minimum of the linear density



Possible problems:

- No reduction of the linear density
- Neck as thick as fragment
- Symmetric-elongated shape
- Rapid change of  $z_0$  position
- Double minimum of the linear density

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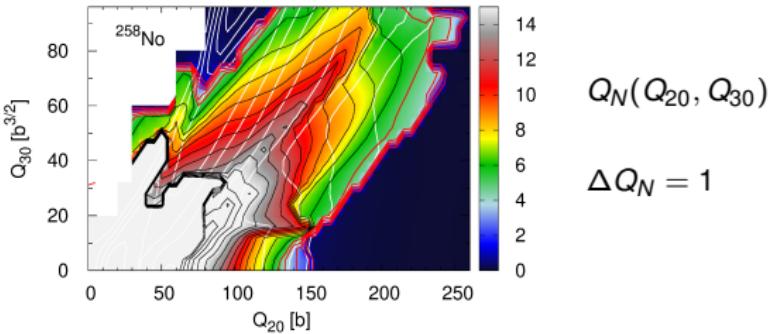
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# Neck thickness

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$$Q_N(Q_{20}, Q_{30})$$

$$\Delta Q_N = 1$$

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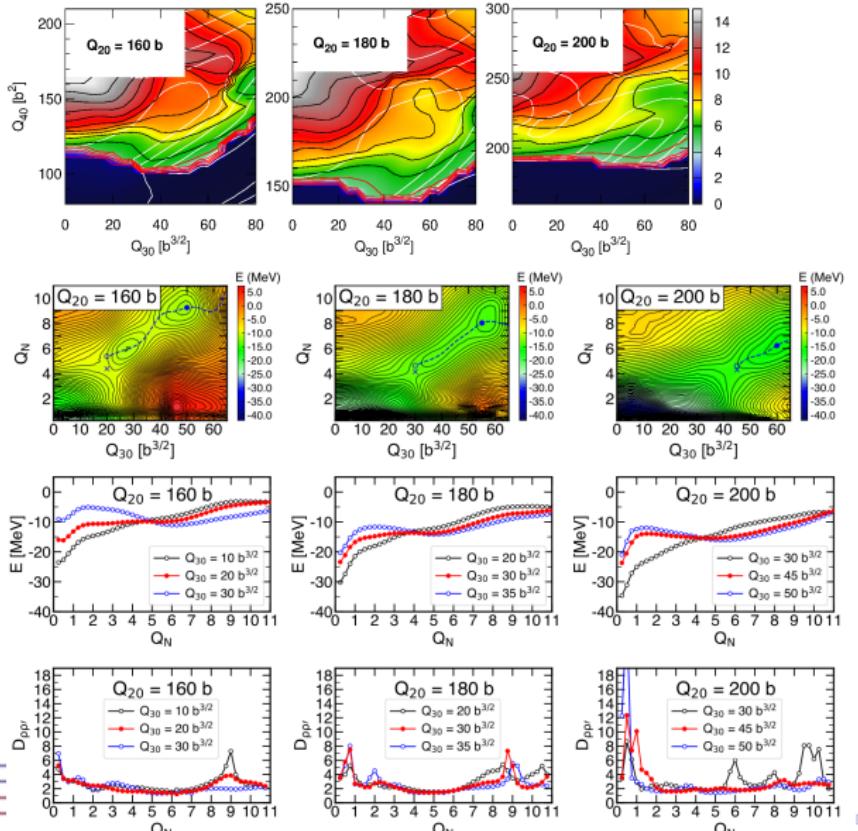
- $Q_N = 4 - 5 \Rightarrow$  Pre-scission shape
- Neck radius (def. as  $r$  of a cylinder with a height  $\sqrt{\pi}a_0$ ):

$$r_{NECK} = \sqrt{Q_N / (\rho_0 \pi^{3/2} a_0)} = 1.059 \sqrt{Q_N} \Rightarrow \text{rms radius of:}$$

- deuteron: 2.76 fm
- $\alpha$  particle: 2.13 fm



# $^{258}\text{No}$ - scission region in asymmetric valley



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# $^{258}\text{No}$ - scission region in symmetric valley

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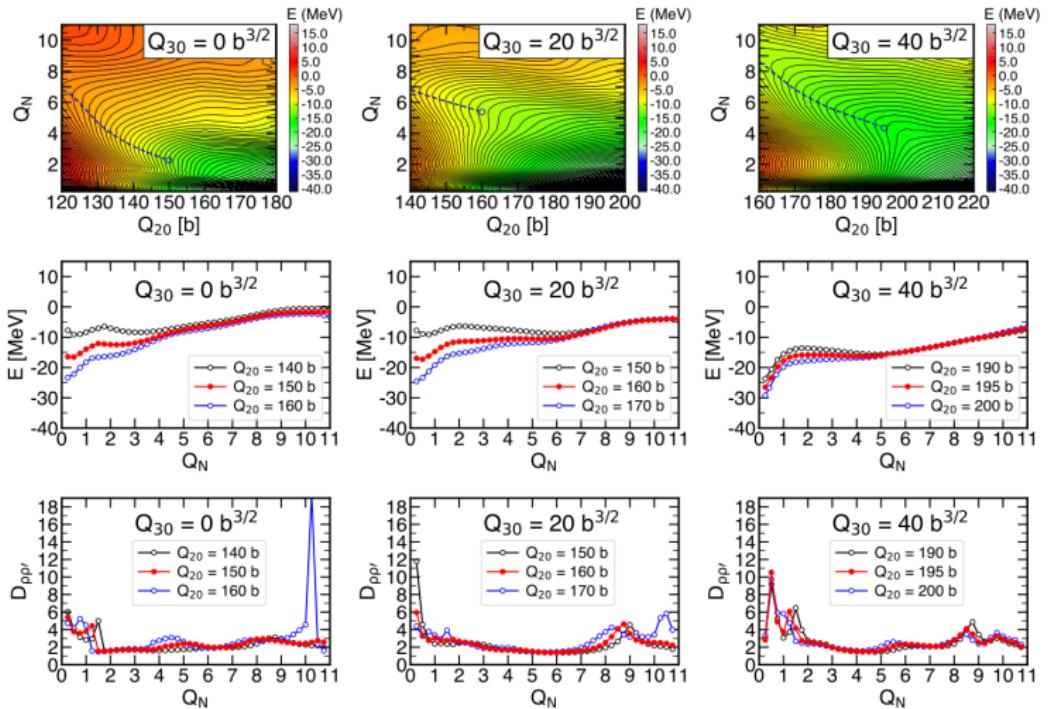
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# Scission point

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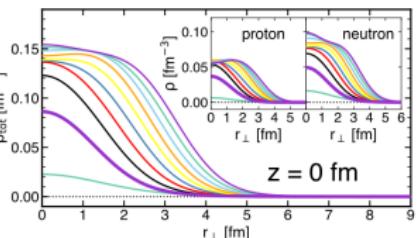
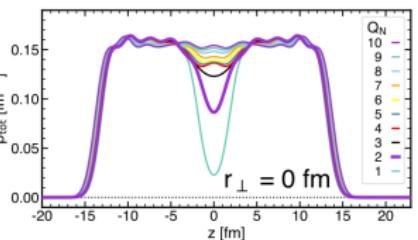
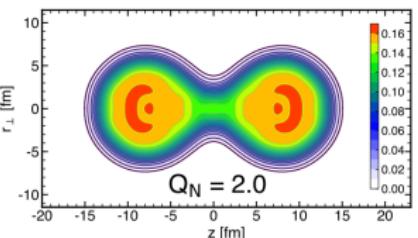
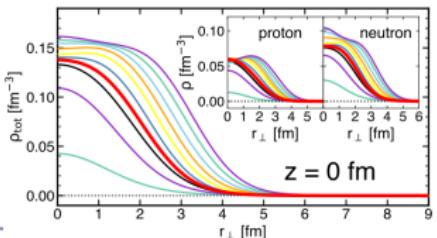
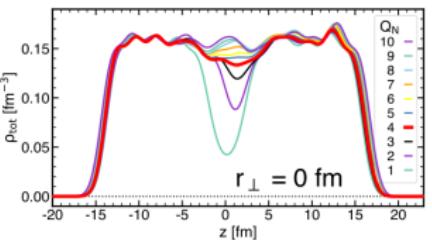
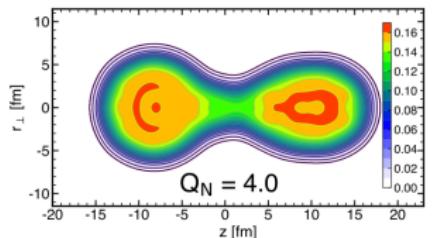
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# Summary & Conclusions

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1. The HFB model with well-reputed Gogny D1S interaction was applied in order to investigate self-consistent PES' in fission.
2. For high  $Q_{20}$  values a complicated topography was found and analyzed.
3. The **competing fission channels** responsible for observed **bimodal** mass yield in fission of  $^{258}\text{No}$  were discussed: asymmetric, symmetric compact and elongated modes.
4. One can obtain a **continuous PES in the scission region** when  $Q_N$  is considered as a collective degree of freedom.
5. There is a **limit** value of the neck parameter after which the system becomes **unstable**: 2 in symmetric mode, 4 in asymmetric.



# Thank you!

