$\bar{N} - {}^2H$ calculations with Faddeev-Merkuriev equations

R. Lazauskas (IPHC, Strasbourg)

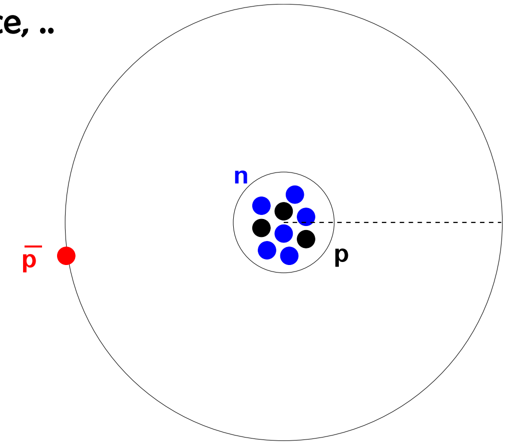
J. Carbonell (IJClab, Orsay)

Goals

The goal of PUMA project is to measure nuclear neuron skins from the $\bar{p}$A annihilation data. We have to answer:

- If the exp. data lead to unambiguous conclusion?
- Can we interpret the data?
- If yes, how and how well?

Accuracy of the solutions, quality of the input, model dependence, ..

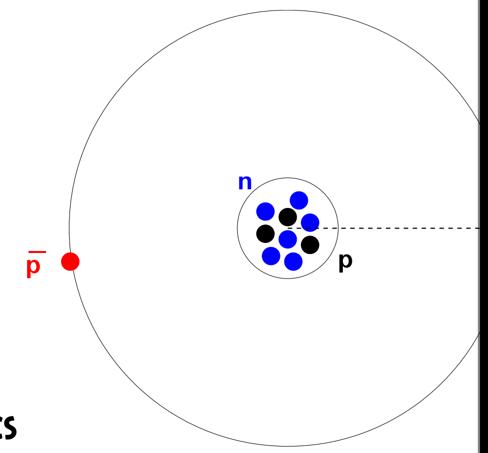


Our aim is to provide the «best» solutions for the accessible systems and use this knowledge to build «antiproton-nucleus» potentials for the rich-neutron systems of experimental realm

Introduction

Provide the « best possible » solution for the NR Schrödinger eq.

$$\widehat{H}|\Psi\rangle = E|\Psi\rangle; \;\; \widehat{H} = \widehat{H_0} + V$$



The problem is extremely complex:

- ~~Relativity~~ and *annihilation dynamics*
- Complexity of the $\bar{p}$N interaction and $\bar{p}$A dynamics
- Presence and coupling between the very different physical scales: atomic (Coulomb), nuclear ($\bar{p}$A), subatomic (annihilation) !!

$\bar{N}N$ interaction

THE FACTS

There are two main sources of experimental info: scattering and protonium

SCATTERING

from $\bar{p}p$ one can measure three contributions to the total cross section

$$\sigma_t = \sigma_e + \sigma_a + \sigma_{ce}$$

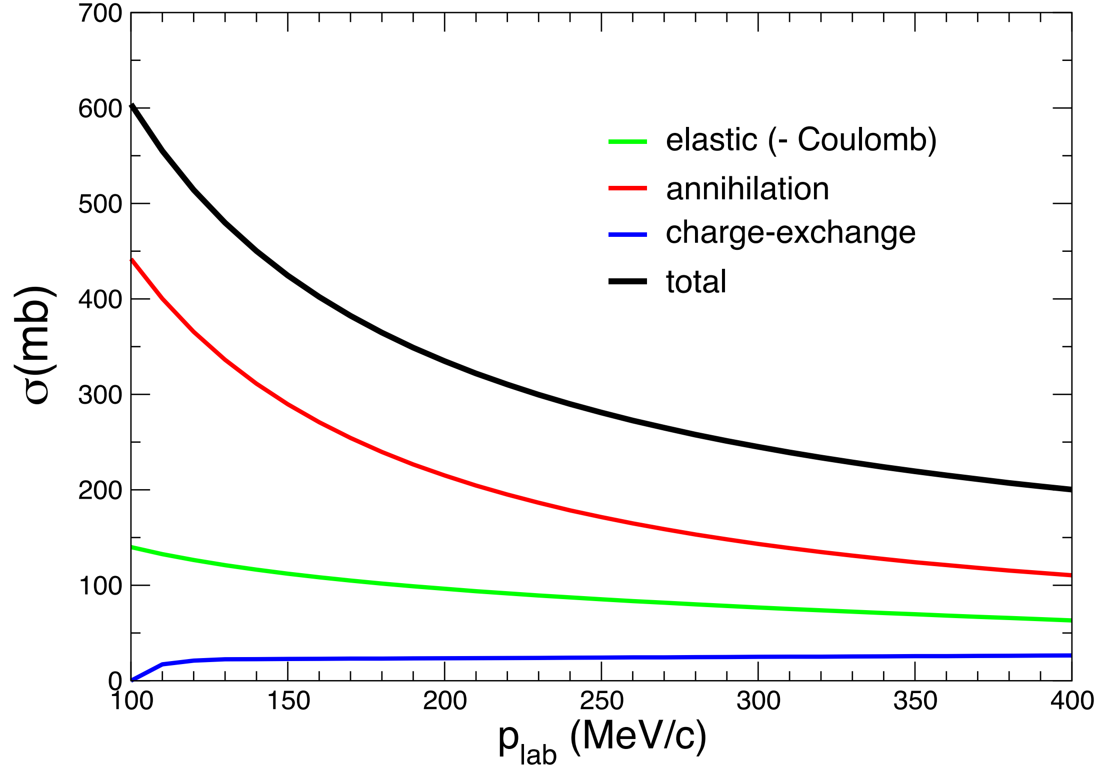
- σ_e elastic
- σ_a annihilation: everything produced beyond $\bar{p}p$ ($\bar{n}n$) channels
- σ_{ce} charge-exchange

$$\bar{p} + p \rightarrow \bar{n} + n$$

from $\bar{n}p$ one gain some interesting low energy results on σ_e and σ_a. One is able to isolate the isospin T=1 component of the interaction and study it by avoiding complications brought by Coulomb interaction. Difficult measurement for it uses the ce to produce the secondary antineutron beam.

$\bar{N}N$ interaction: general properties

THE FACTS



- At low energy (p_L<400 MeV/c) dominated by annihilation $\sigma_a/\sigma_e \simeq 2$
- Partial wave cross sections close to unitary limit $\sigma_a^{(L)}=(2L+1)\pi/k^2_{cm}$
- Cannot be reduced to a black sphere model (for which $\sigma_a=\sigma_e$) :

the strong force of nuclear origin plays a crucial role

$\bar{N}N$ interaction

THE FACTS

There are two main sources of experimental info: scattering and protonium

PROTONIUM

In absence of strong interaction $\bar{p}p$ would form an H-like

$$E_c = -\frac{1}{4}\frac{m_p \alpha^2}{n^2} = -\frac{12.5\ keV}{n^2}$$

with Bohr radius a_0 = 57 fm (a 1000x reduced H-atom)

Strong interaction shifts and broadens the pure Coulomb levels

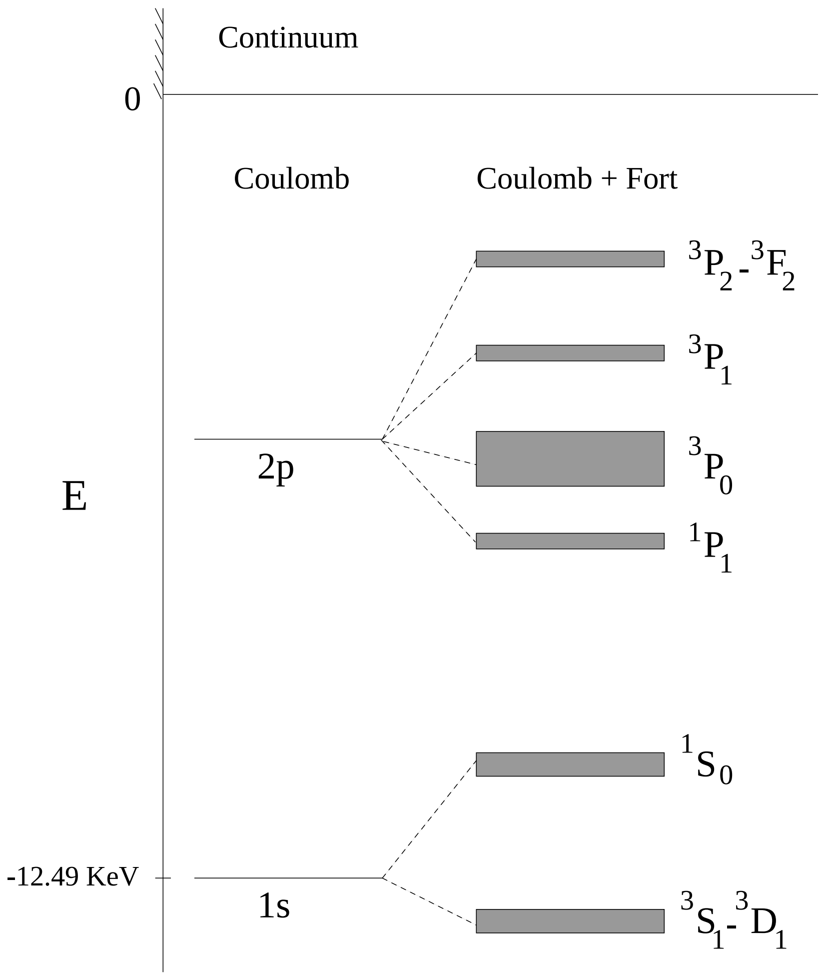
- Difference $\Delta E = \Delta E_R + i\Gamma/2$ is meeasured for low lying states (1s,2p)
- This difference is related to the scattering length $a_{\bar{p}p} = f_{\bar{p}p}$(E=0)

A priviledged open door to $\bar{N}N$ forces at low energy (controlled initial state)

Many other $\bar{\mathrm{p}}\mathrm{A}$ atoms have been measured. It is however very difficult to extract useful information to construct $\bar{\mathrm{p}}\mathrm{N}$ models;

$\bar{N}N$ interaction

Coulomb levels are shifted up/down w.r.t. QED, depending on the state:
Energy shifts $\Delta E = \Delta E_R$ and lifetime $i\Gamma/2$ **(energy spread) are measured**

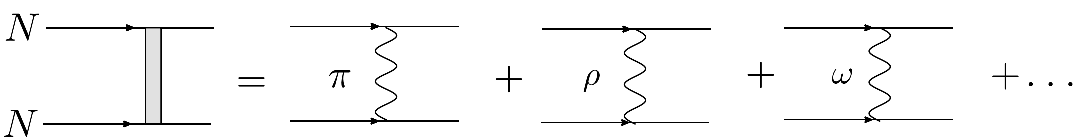


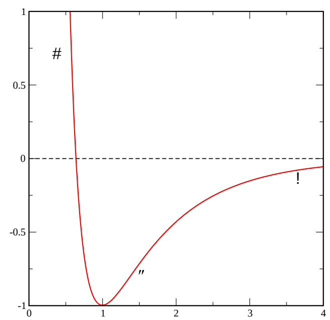
$\bar{N}N$ interaction

THE THEORY

The « traditional » meson exchange approach in Nuclear Physics

$$V_{NN} = +V_\pi + V_\eta + V_\rho + V_\omega + V_{\sigma_0} + V_{\sigma_1}$$





Thouhg outfashioned – it is still remains the most employed model (most of existing calculations are performed based on these models).

$V_{N\bar{N}}$ (real part, T-symmetry) follows from V_{NN} by a G-parity transfomation of the meson-N vertex, providing multiplicative factor:

$$G = C(-)^T$$

$$V_{N\bar{N}} = -V_\pi + V_\eta + V_\rho - V_\omega + V_{\sigma_0} - V_{\sigma_1}$$

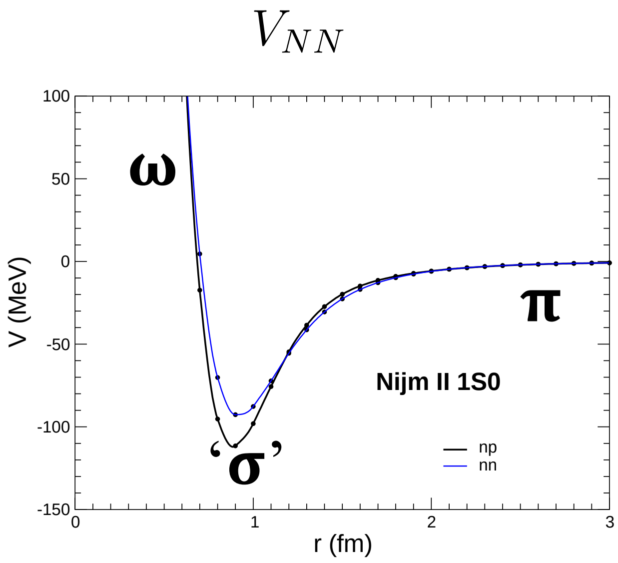
meson	I	J	P	C	G
π	1	0	−	+	−
η	0	0	−	+	+
σ_0	0	0	+	+	+
ρ	1	1	−	−	+
ω	0	1	−	−	−

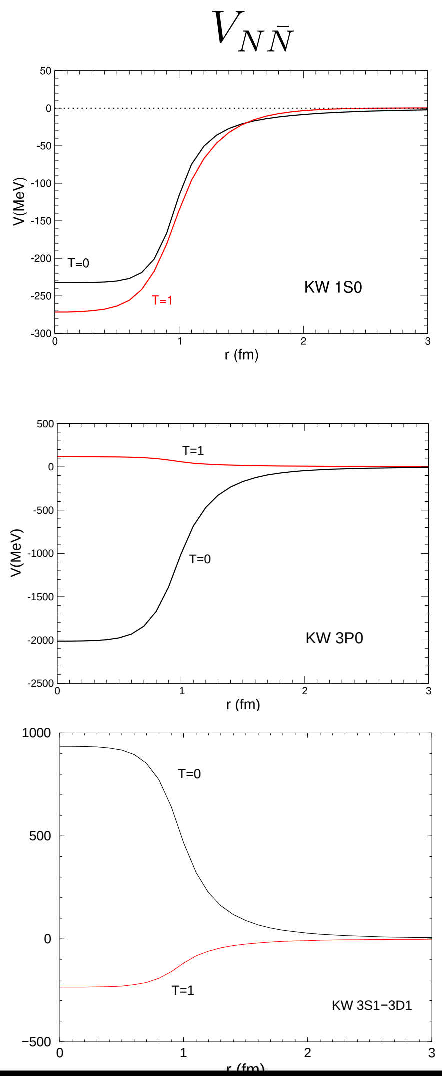
Consequences are dramatic: V_{NN} repulsive core – due to ω - change its sign and becomes strongly attractive (in most of the S-T channels) and the tensor force becomes huge There should exist a rich « quasi-bound » and resonant states ....that have never been directly observed during LEAR time (specificallly built to this aim !!!) despite some intriguing « evidences » ...just before it closes.

$\bar{N}N$ interaction

THE THEORY

PW examples of G-parity transform for a meson exchange V_{NN}





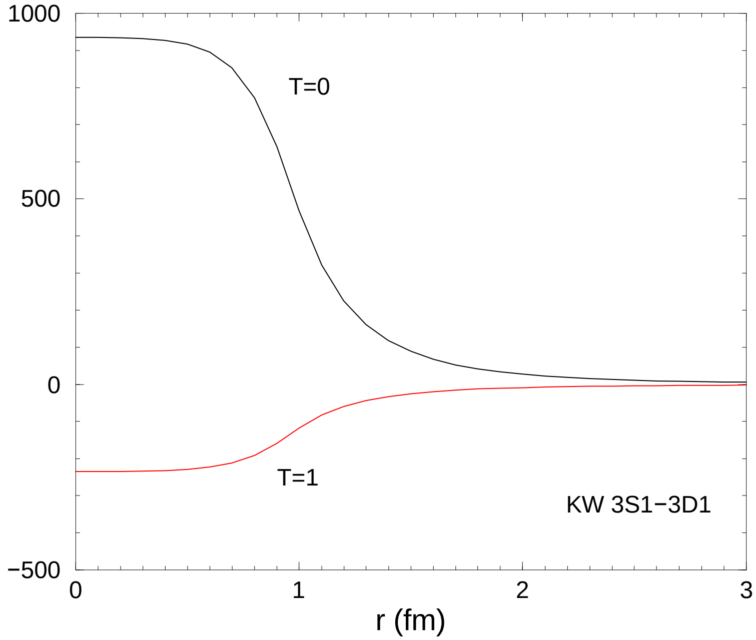
$\bar{N}N$ interaction

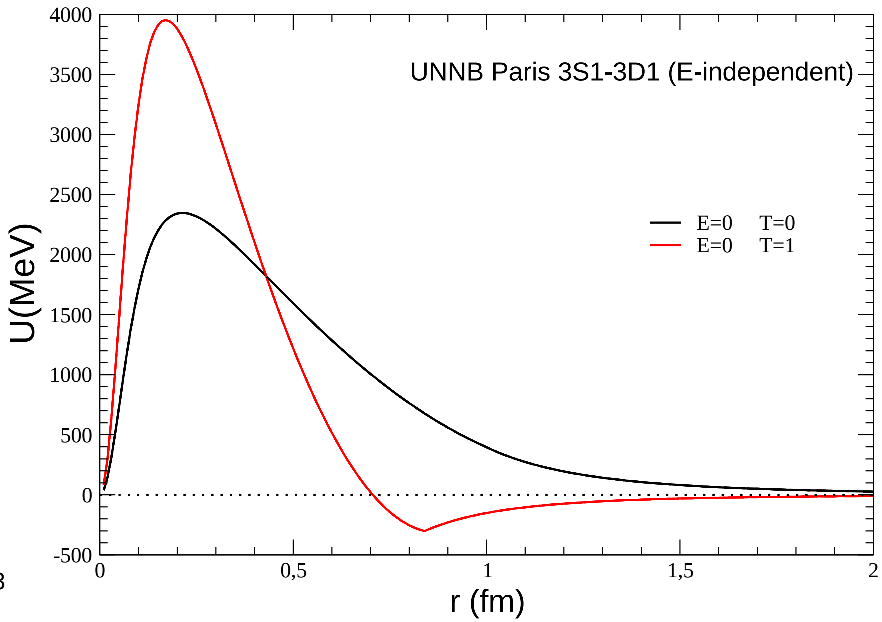
THE THEORY

The « traditional » approach of Nuclear Physics based on meson exchange

Kohno-Weise (KW) versus **Paris 2009** potential

M. Kohno, M. Weise, Nucl. Phys. A454 (1986) 429
B. El-Bennich et al., Phys. Rev. C79 (2009) 054001





$\bar{N}N$ interaction

THE THEORY

EFT approach: at first glance EFT philosophy seems to contradict $V_{\bar{N}N}$ physics (Q>M), still some models based on EFT have been succesfully developped in the recent years:

X. W. Kang, J. Haidenbauer and U.-G. Meißner, JHEP 1402 (2014) 113 (N2LO)
L.Y.. Dai, J. Haidenbauer, Ulf-G. Meißner, JHEP 2017 (2017) 78 (N3LO)

These potentials are built in p-space and are strongly non-local what makes difficult direct comparison.

In EFT, one retains only π (at most!) and so the G-parity rule does not apply here in its full glory. The other terms are regularized contact terms whose constants have been fited to $N\bar{N}$ phase shifts.

Big advantage: possibility of the systematic error estimation

$\bar{N}N$ interaction: annihilation dynamics

THE THEORY

$V_{\bar{N}N}$ constructed in this way does not account the annihilation part:. There are two phenomenological ways to incorporate it: optical and/or coupled channel models

Optical models:

Add to V_{NN} a complex potential $V \rightarrow V_{NN} + W_R - iW_i$

Which allows us to compute the « annihilation density »

In this description, $N\bar{N}$ particles disapear from the flux, go nowhere and never return: $SS^+<1$ (not unitary approach)

The form of W is « guessed » and its parameters detemined by phenomenology.

Quite successful despite its bare simplicity (probably thanks to the poor data)

- Annihilation dynamics is the same for all (T,S,L,J) states !
- Bad analytic properties (mainly in resonances)
- Depressed wave function due to absence of « re-annihilation »

THE THEORY

$V_{\bar{N}N}$ constructed in this way does not account the annihilation part:. There are two phenomenological ways to incorporate it: optical and/or coupled channel models

Coupled channel models:

$\bar{N}N$ channels are coupled with meson production ones $\bar{N}N \leftrightarrow \bar{X}_i X_i$.

- One is not able to account for all possible channels (there are too many), should introduce « effective » one!
- Nicer analytic properties (resonances)
- «Re-annihilation» offers quite different dynamics
- Too many unknowns

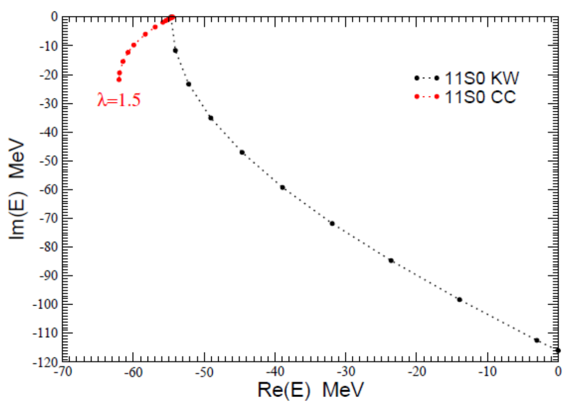


Fig. 4 Complex energy trajectory of a $^{11}S_0$ state as a function of the annihilation strength in optical a unitary coupled channel models

E. Ydrefors, J.C : Eur. Phys. J. **A. 57** (2021) 303

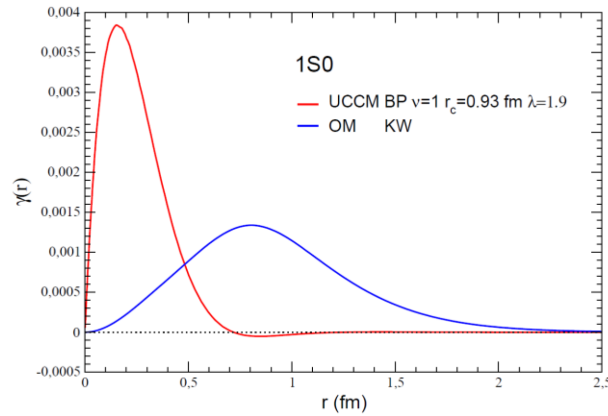


Fig. 14 Protonium annihilation density for the $^1S_0$ state described with the UCCM (in red) and with OM (in blue). Both models reproduce the same experimental complex level shift ΔE value of Table 3.

Calculations in practice

$V_{\bar{N}N}$ For small A=2-4 systems it is useful to consider N and $\bar{N}$ as different particles and use isospin only in separating n from p (or $\bar{n}$ from $\bar{p}$).

$$|\psi\rangle = \begin{pmatrix} \psi(\bar{p}p) \\ \psi(\bar{n}n) \end{pmatrix}; \qquad V_{LL'}^{SJ^{\pi}} = \begin{pmatrix} V_{\bar{p}p} & V_{\bar{p}p\to\bar{n}n} \\ V_{\bar{n}n\to\bar{p}p} & V_{\bar{n}n} \end{pmatrix}$$

For a given J^{π} state one has to solve 2- or 4- coupled channel problem in complex number arithmetics.

When annihilation is mimicked by adding an explicit meson-ameson channel, the solution for a given J^{π} becomes 4-, 8-channel problem in real number arithmethics:

$$|\psi\rangle = \begin{pmatrix} \psi(\bar{p}p) \\ \psi(\bar{n}n) \\ \psi(\overline{x_p}x_p) \\ \psi(\overline{x_n}x_n) \end{pmatrix}$$

Aomic-level energy shifts are tiny, requiring 10^{-10} accuracy, calculations require particular care!!

Protonium

Essentially $\bar{p}p$ or in H-atom like Coulomb orbit with:

$$E_{\bar{p}p}(n) \approx \frac{m_p}{2m_e} E_H(n) = -\frac{12.49\ keV}{n^2}$$

$$a_0 \approx \frac{2m_e}{m_p} a_B = 57.64\ fm; \qquad \langle r \rangle_{nL} = \frac{3n^2 - L(L+1)}{2} a_0$$

- Protonium is produced in $\bar{p}H$ collisions in highly excited states (n,L~34).
- Stark L-mixing and radiative cascade with γ-emission.
- Annihilation happens in lower (S,P,D,..) states with keV γ-detected in coincidence
- Strong (annihilation) forces slightly modifies Coulombic orbits. typically (for g.s.)

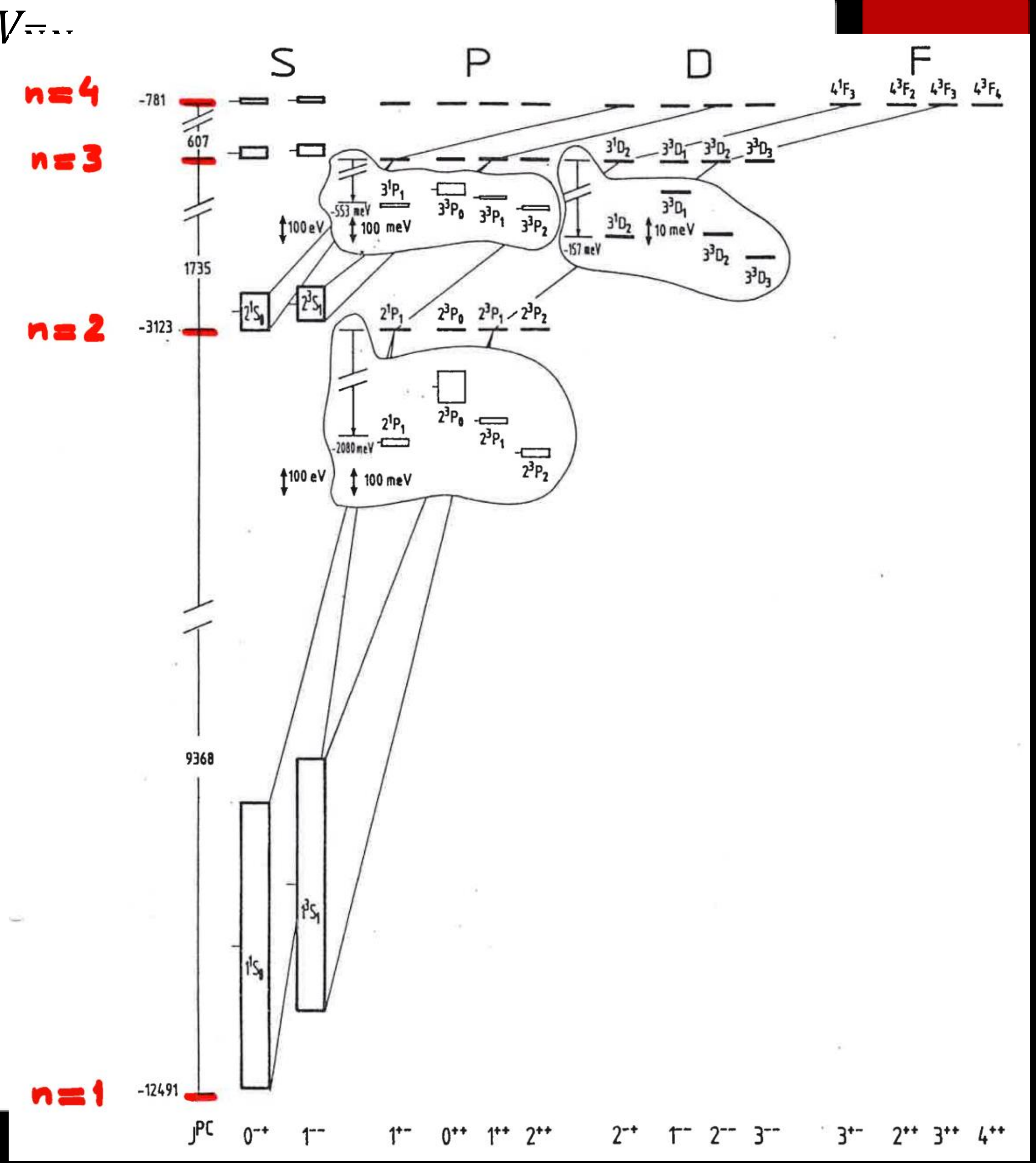
  S-wave: $\Delta E \sim 1\ keV \quad (\frac{\Delta E}{E} \sim 10^{-1})$

  P-wave: $\Delta E \sim 1\ meV \quad (\frac{\Delta E}{E} \sim 10^{-6})$

  D-wave: $\Delta E \sim 1\ \mu eV \quad \left(\frac{\Delta E}{E} \sim 10^{-9}\right)$
- For heavier nuclei, scales with Z as: $\sim Z \frac{Z+N}{Z+N+1}$
- For $Z = 10;\ \ a_0 \approx 2.8\ fm$: the first Coulomb orbit is inside the nucleus

Protonium: level shifts/spreads

$\Delta E = E - E_c = \Delta E_R - i\frac{\Gamma}{2}$



Protonium: level shifts/spreads

Level shifts and widths for lower states by potential models:

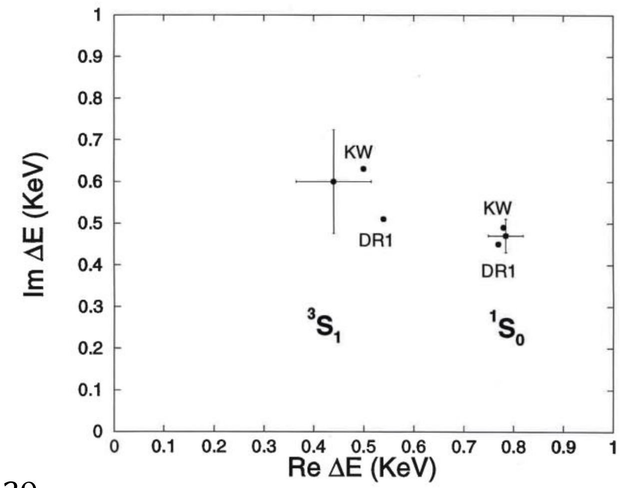
	1S0		3SD1		1P1		3P0		3P1		3PF2	
	keV		keV		meV		meV		meV		meV	
	DER	-EI	DER	-EI	DER	-EI						
DR1	0.54	0.51	0.77	0.45	-26	13	-74	57	36	10	-4.8	15
DR2	0.58	0.52	0.82	0.46	-24	14	-62	40	36	9	-5.9	16
KW	0.50	0.63	0.78	0.49	-29	13	-69	48	29	11	-8.5	18
Paris 09	0.78	0.52	0.69	0.39	-29	13	-67	60	64	45	+7.2	13
EFT	0.44	0.59	0.77	0.58	/		-8	188	/		/	

Quite good agreement within the models!! And an acceptable comparison with data

S-waves (eV) (*)

	Exp [34]	KW	DR1	DR2
$\Delta E_{^1S_0}$	440±075	500	540	580
$\Gamma_{^1S_0}$	1200±250	1260	1020	1040
$\Delta E_{^3S_1}$	785±035	780	770	820
$\Gamma_{^3S_1}$	940±080	980	900	920

*J. Carbonell, G. Ihle, J.M. Richard, Z. Phys. **A 334** (1989) 329



Protonium: level shifts/spreads

Level shifts and widths for lower states by potential models:

	1S0		3SD1		1P1		3P0		3P1		3PF2	
	keV		keV		meV		meV		meV		meV	
	DER	-EI	DER	-EI	DER	-EI						
DR1	0.54	0.51	0.77	0.45	-26	13	-74	57	36	10	-4.8	15
DR2	0.58	0.52	0.82	0.46	-24	14	-62	40	36	9	-5.9	16
KW	0.50	0.63	0.78	0.49	-29	13	-69	48	29	11	-8.5	18
Paris 09	0.78	0.52	0.69	0.39	-29	13	-67	60	64	45	+7.2	13
EFT	0.44	0.59	0.77	0.58	/		-8	188	/		/	

Quite good agreement within the models!! And an acceptable comparison with data

P-waves (eV) (*)

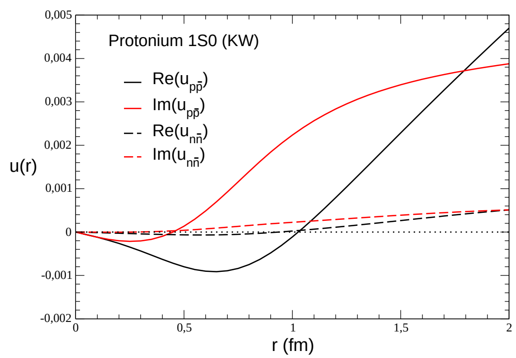
	KW		DR1		DR2		Exp	
State	ΔE_R	$\frac{\Gamma}{2}$	ΔE_R	$\frac{\Gamma}{2}$	ΔE_R	$\frac{\Gamma}{2}$	ΔE_R	$\frac{\Gamma}{2}$
$^1P_1$	-29.	13.	-26.	13.	-24.	14.		
$^3P_0$	-69	48	-74	57	-62	40	-139±28	60±13
$^3P_1$	+29.	11.	+36.	10.	+36.	8.8		
$^3PF_2$	-8.5	18.	-4.8	15.	-5.9	16.		

*J. Carbonell, G. Ihle, J.M. Richard, Z. Phys. **A 334** (1989) 329

General remarks

Trueman relation allows to express $\bar{p}A$ level shifts in terms of $\bar{p}A$ scattering lengths

- Simple and practical. Energy shifts of excited orbits are interrelated
- This relation supposes $r_{strong} \sim a_{\bar{p}A} \ll a_{Coulomb}$: might be easily broken for heay nuclei but perfectly holds for protonium.
- The $\bar{n}n$ component is present together with $\bar{p}p$ one in wave function. But its effect on the energy is «usually » small:



	ΔE	ΔE
	$p\bar{p}$	$p\bar{p} - n\bar{n}$
1S0 keV	0.52- 0.61 i	0.50 - 0.63 i
3S1	0.77- 0.46 i	0.75 - 0.44 i
3P0 meV	- 64 - 34 i	- 69 - 48 i

Quite good agreement within the models!! And an acceptable comparison with data:

	a_R	a_I	a_R	a_I	a_R	a_I	a_R	a_I
$^1P_1$	-1.19	-0.53	-1.07	-0.52	-0.99	-0.58		
$^3P_0$	-2.81	-1.99	-3.01	-2.31	-2.53	-1.62	-5.68±1.14	-2.45±0.53 i
$^3P_1$	+1.22	-0.47	+1.46	-0.42	+1.48	-0.36		
$^3PF_2$	-0.36	-0.75	-0.20	-0.63	-0.25	-0.67		

General remarks

Trueman relation allows to express $\bar{p}A$ level shifts in terms of $\bar{p}A$ scattering lengths

- Simple and practical. Energy shifts of excited orbits are interrelated
- This relation supposes $r_{strong} \sim a_{\bar{p}A} \ll a_{Coulomb}$: might be easily broken for heay nuclei but perfectly holds for protonium.
- The $\bar{n}n$ component is present together with $\bar{p}p$ one in wave function. But its effect on the energy is usually small.
- Non-perturbative! Despite the fact $\Delta E \ll E_{Coulomb}$ the strength of the annihilation potential strngly modifies the Coulomb wave funcion in the overlap region with nucleus ($r_{strong} \sim a_{\bar{p}A}$ domain).

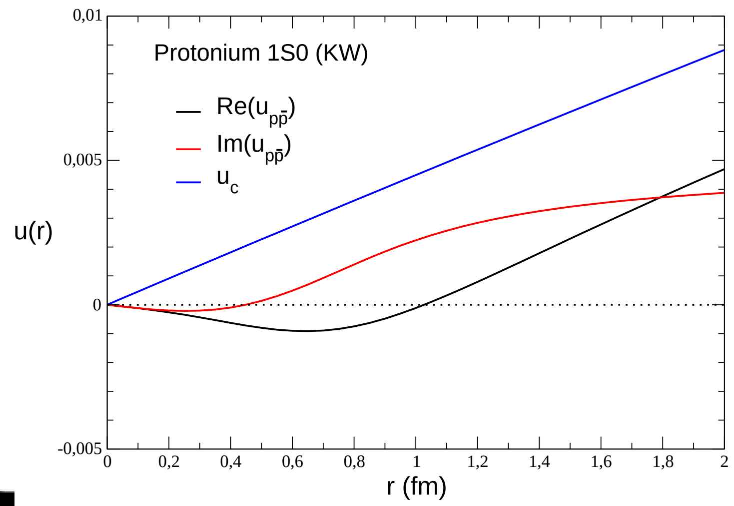
	Exact	Perturbative
$^1S_0$ (eV) n=1	524-602i	-3030-3150i
n=2	65.1-77.8i	-379.-394i
n=3	19.3-23.3i	-112-117i
$^1P_1$ (meV) n=2	-28.1-13.0i	-34.5-7.3i
n=3	-9.9-4.6i	-12.1-2.6i
1D2 (neV) n=3	-378.-9.9i	-363-6.1i

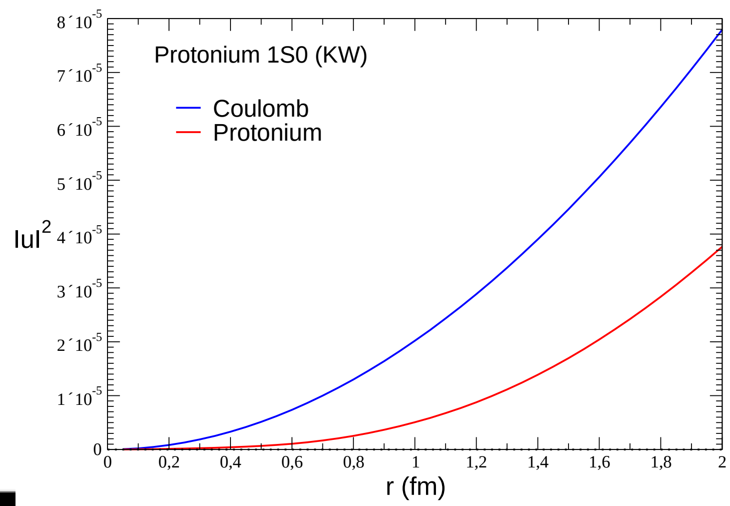
Perturbative result is given by the overlap with the pure Coulomb wf.: $\Delta E = \langle \Psi_C | V_{\bar{p}N}^{st} | \Psi_C \rangle$

General *remarks*

Trueman relation allows to express $\bar{p}A$ level shifts in terms of $\bar{p}A$ scattering lengths

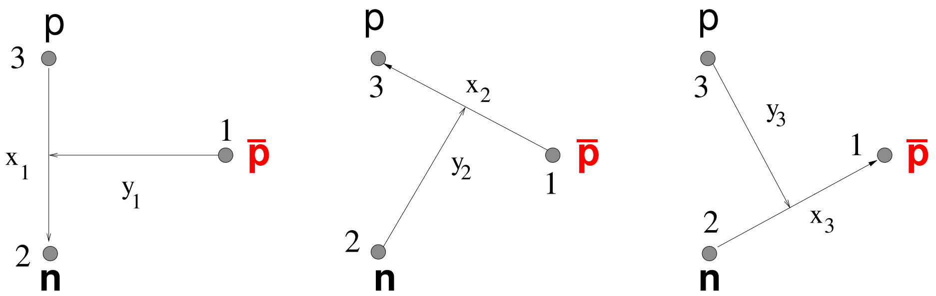
- Simple and practical. Energy shifts of excited orbits are interrelated
- This relation supposes $r_{strong} \sim a_{\bar{p}A} \ll a_{Coulomb}$: might be easily broken for heavy nuclei but perfectly holds for protonium.
- The $\bar{n}n$ component is present together with $\bar{p}p$ one in wave function. But its effect on the energy is usually small.
- **Non-perturbative!** Despite the fact $\Delta E \ll E_{Coulomb}$ the strength of the annihilation potential strongly modifies the Coulomb wave funcion in the overlap region with nucleus ($r_{strong} \sim a_{\bar{p}A}$ domain).





3-body problem: $^2H - \bar{p}$

Faddeev eq's:



$$\begin{cases} \psi_1 = V_1 G_0 \Psi \\ \psi_2 = V_2 G_0 \Psi \\ \psi_3 = V_3 G_0 \Psi \end{cases} \quad \text{or} \quad (E - H_0 - V_i)\psi_i = V_i \left(\psi_j + \psi_k\right); \quad (ijk) = (123)$$

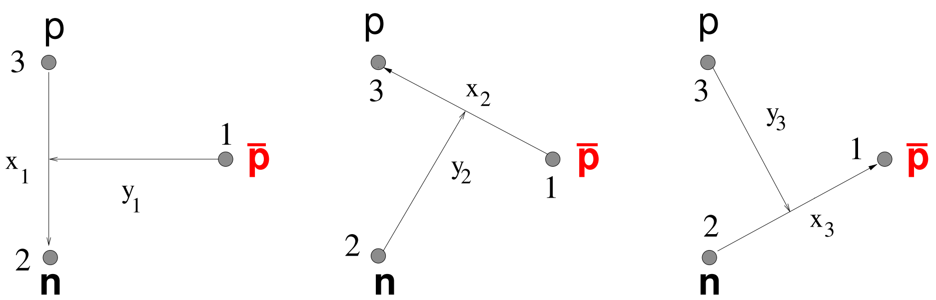
- One gets Schrödinger equation by summing these three eqs with $\Psi = \psi_1 + \psi_2 + \psi_3$
- If particle 1 goes away V_2 & $V_3 \rightarrow 0$ and thus ψ_2 & $\psi_3 \rightarrow 0$; therefore $\psi_1 \rightarrow \Psi$.

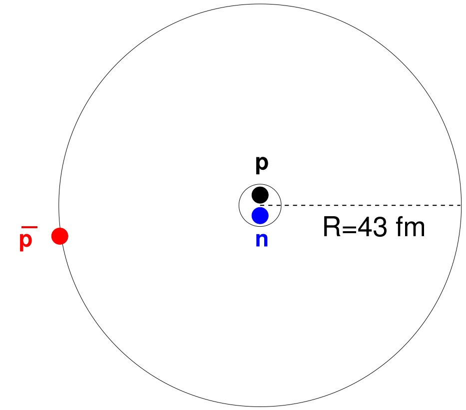
Adapted for scattering problems, since allows to separate assymptotes of 2+1 particle channels

- Should be modified, when longue range interaction is present

3-body problem: $^2H - \bar{p}$

Faddeev-Merkuriev eq's:





$$V_i = V_i^{short} + V_i^{long}$$

$$\psi_i = \left(H_0 + \sum_{j \neq i}^{3} V_j^{long} \right)^{-1} V_i^{short} \Psi; \quad (ijk) = (123)$$

- One gets Schrödinger equation by summing these three eq.,
- FM components separte different 2+1 particle channels equally for long-range interactions

$3 - body\ problem: {}^2H - \bar{p}$

Solution based on Faddeev-Merkuriev eq's:

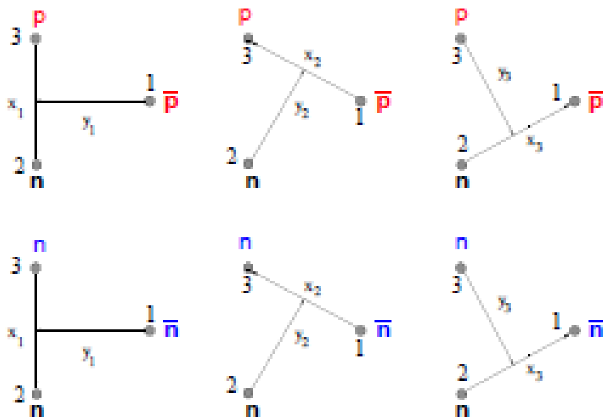
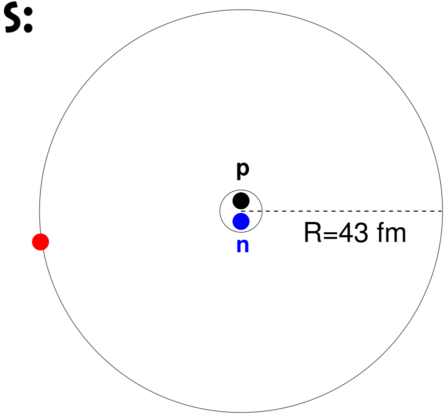


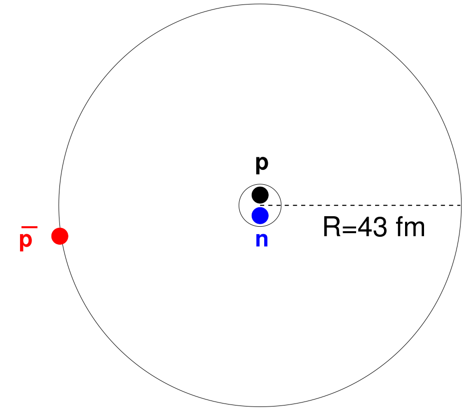
Figure 1: Faddeev components and Jacobi coordinates for the solution of the p̄d problem



$$\hat{\Psi}_{Nn} \equiv \begin{pmatrix} \Psi_{pn,\bar{p}} \\ \Psi_{nn,\bar{n}} \end{pmatrix} (\vec{x}_1, \vec{y}_1) \qquad \hat{\Psi}_{\bar{N}N} \equiv \begin{pmatrix} \Psi_{\bar{p}p,n} \\ \Psi_{\bar{n}n,n} \end{pmatrix} (\vec{x}_2, \vec{y}_2) \qquad \hat{\Psi}_{n\bar{N}} \equiv \begin{pmatrix} \Psi_{n\bar{p},p} \\ \Psi_{n\bar{n},n} \end{pmatrix} (\vec{x}_3, \vec{y}_3)$$

3-body problem: $^2H - \bar{p}$

Pure Coulomb problem:



MT13: pure S-wave (no tensor, no spin-orbit)

ϵ^c_{1s} = -16.6260 keV i.o. 16.666 from Ry
ϵ^c_{2p} = - 4.16655 keV i.o. 4.1666 from Ry/4

AV18: full realistic interaction J=1/2+,-

ϵ^c_{1s} = -16.6123 keV i.o. 16.666 from Ry
ϵ^c_{2p} = - 4.16635 keV i.o. 4.1666 from Ry/4
ϵ^c_{2p} = - 4.16655 keV i.o. 4.1666 from Ry/4

Corrections to Rydberg are quite small, however for realistic interaction there exist visible hyperfine splitting!

$3 - \text{body problem:}\ {}^2H - \bar{p}$

Full problem

	MT13	AV18	INOY	I-N3LO	$-\epsilon_n^{(0)}$ (keV)
S-waves		ΔE (keV)			
${}^2S_{1/2}$, n=1	2.251-1.0045i	2.147-1.0440i	2.214-0.99433i	2.209-1.0509i	16.6662
${}^2S_{1/2}$, n=2	0.294-0.1406i	0.279-0.1454i	0.289-0.13892i	0.288-0.1468i	4.16655
${}^2S_{1/2}$, n=3	0.088-0.0433i	0.084-0.0446i	0.087-0.04271i	0.086-0.0451i	1.85180
P-waves		ΔE (meV)			
${}^2P_{1/2}$, n=2	49.1-258.0i	-55.3-239.2i	-56.2-241.1i	-58.5-244.0i	4.16655
${}^4P_{1/2}$, n=2	24.4-194.8i	200.2-186.4i	200.2-188.2i	200.3-186.1i	4.16655
${}^2P_{1/2}$, n=3	16.1-90.6i	-14.0-83.94i	-14.2-84.57i	-15.0-85.61i	1.85180
${}^4P_{1/2}$, n=3	8.62-68.4i	59.4-65.51i	59.0-66.14i	58.4-65.36i	1.85180

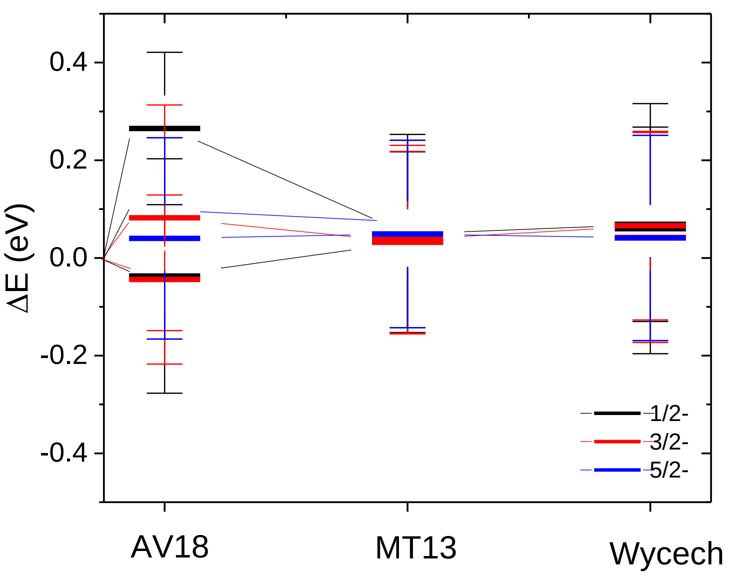
Table 1: Complex p̄d energy shifts ΔE_n obtained for different NN interactions and the KW $\bar{N}N$ model.

- Quite good agreement between the realistic interaction NN model predictions
- MT13 lacking tensor force (ignoring presence of deuterons quadrupole moment) falls short for P-states

$3 - \text{body problem: } {}^2\text{H} - \bar{\text{p}}$

Full problem

		MT13/ KW	AV18/ KW	Wycech[1]	Exp[2,3]
L=0	$\Delta E\ (eV)$	2297	2194	2170	1050+/-250
	$\Gamma\ (eV)$	1982	2129	1250	1100+/-750
L=1	$\Delta E\ (meV)$	26.6	22.5	52	243+/26
	$\Gamma\ (meV)$	428	414	422	489+/-30



Comparison with pionner work (separable app

[1]S. Wycech et al, Phys. Lett B152 (1985) 308

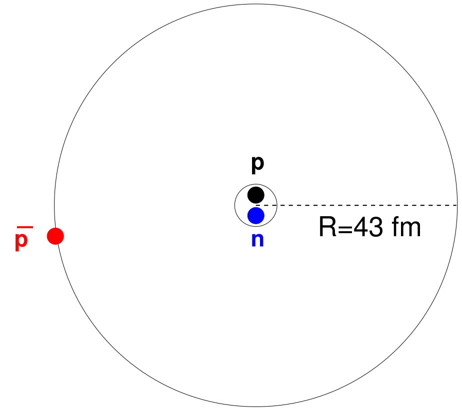
Experiment:

[2]D. Gotta et al., Nucl. Phys. A660 (1999) 283

[3]M. Augsburger et al., Phys. Lett. B461 (1999) 417

$3-\text{body problem: } {}^2H-\bar{p}$

Full problem



	I-N3LO +KW		I-N3LO +Jülich	
	$\bar{p}p$	$\bar{p}p + \bar{n}n$	$\bar{p}p$	$\bar{p}p + \bar{n}n$
${}^2S_{1/2}$, n=1 (keV)	2.179-1.024i	2.209-1.050 i	2.028-0.928i	2.108-1.085i
${}^2S_{1/2}$, n=2 (eV)	284-143i	288-147 i	264-128i	274- 151i
${}^2S_{1/2}$, n=3 (eV)	85.3-43.9i	86.4-45.1 i	79.1-39.3	82.0-46.3i
${}^4S_{3/2}$, n=1 (keV)	2.206-0.970i	2.306-1.045i	2.027-0.916i	2.321-1.216i
${}^4S_{3/2}$, n=2 (eV)	288-136i	302-147i	264-127i	302- 171i
${}^4S_{3/2}$, n=3 (eV)	86.6-41.7i	90.7-45.2i	79.1-38.8	90.7-52.6i
${}^2P_{1/2}$, n=2 (meV)	-61.6-210i	-58.5-244 i	-105-194i	18.7-329i
${}^4P_{1/2}$, n=2 (meV)	214-158i	200-186 i	200-124i	171-194i
${}^2P_{1/2}$, n=3 (meV)	-16.3-73.8i	-15.0-85.6 i	-31.9-68.3i	13.2-120i
${}^4P_{1/2}$, n=3 (meV)	63.5-55.5i	58.4-65.4 i	59.1-43.5i	47.0-63.7i
${}^2P_{3/2}$, n=2 (meV)	-60.3-201i	-76.2-226i	-81.2-144i	-108-207i
${}^4P_{3/2}$, n=2 (meV)	43.6-180i	35.0-191i	55.0-137i	40.4-160i
${}^2P_{3/2}$, n=3 (meV)	-17.3-68.6i	-21.4-79.5i	-23.3-50.6i	-32.7-72.7i
${}^4P_{3/2}$, n=3 (meV)	13.8-63.2i	10.7-67.0i	17.8-48.3i	12.7-56.3i
${}^4P_{5/2}$, n=2 (meV)	57.6-185i	34.7-208i	7.1-132i	-21.6-205i
${}^4P_{5/2}$, n=3 (meV)	18.7-64.8i	10.7-72.9i	1.1-46.2i	-9.1-72.1i

Table 2: Complex level shifts (18) of atomic $\bar{p}d$ states calculated with the same I-N3LO NN interaction (for deuteron) and two different $\bar{N}N$ models: KW [15] and Julich [17].

3 − body problem: $^2H - \bar{p}$

Full problem

	MT13 +KW	AV18 +KW	INOY +KW	I-N3LO +KW	I-N3LO +Jülich	Ref. [30]	Exp.
L=0 ΔE(eV)	2297	2194	2268	2274	2250	2170	1050±250 [24, 25, 26]
L=0 Γ (eV)	1982	2129	1971	2095	2344	1250	1100±750 [24, 25, 26] 2270±260 [25]
L=1 △E (meV)	26.6	22.5	20.7	18.2	-1.1	52	243±26 [25]
L=1 Γ (meV)	428	414	420	420	416	422	489±30 [25]

Table 4: Spin-averaged level shifts (ΔE_R) and widths (Γ) compared to LEAR experimental results

- Coupling $\bar{p}p \leftrightarrow \bar{n}n$ has strong contribution for Jülich χEFT compared to meson exchange potentials
- There is significant $\bar{N}N$ interaction model dependence

Trueman relation: $^2H - \bar{p}$

		MT13 +KW		
	a_0 (fm)	ΔE_1 (keV)	ΔE_2 (keV)	ΔE (keV)
$^2S_{1/2}$ n=1	1.596-0.8569i	2.463-1.322i	2.259-1.014i	2.251-1.004i
$^4S_{3/2}$ n=1	1.647-0.8419i	2.541-1.299i	2.316-0.987i	2.321-0.984i
	a_1 (fm^3)	ΔE_1 (meV)	ΔE_2 (meV)	ΔE (meV)
$^4P_{5/2}$ n=2	0.450-2.68i	34.8-207i	34.8-207i	26.2-215i
		AV18 +KW		
	a_0 (fm)	ΔE_1 (keV)	ΔE_2 (keV)	ΔE (keV)
$^2S_{1/2}$ n=1	1.505-0.8779i	2.323-1.355i	2.155-1.057i	2.147-1.044i
$^4S_{3/2}$ n=1	1.59-0.8771i	2.541-1.354i	2.257-1.039i	2.218-1.075i
	a_1 (fm^3)	ΔE_1 (meV)	ΔE_2 (meV)	ΔE (meV)
$^4P_{5/2}$ n=2	0.469-2.57i	36.4-199i	36.4-199i	39.9-204i

Table 5: Atomic level shifts, calculated from p̄d scattering lengths (a_0 and a_1) employing Trueman relations at first order (ΔE_1) and second order (ΔE_2) are compared with the values obtained from direct binding energy calculations (ΔE).

- Trueman relation works well for spin uncoupled states.
- Should be worked out for spin-coupled ones!

Results: $^{2}\mathrm{H}-\bar{\mathrm{p}}$

Total ^{2}H-$\bar{p}$ Annihilation density:

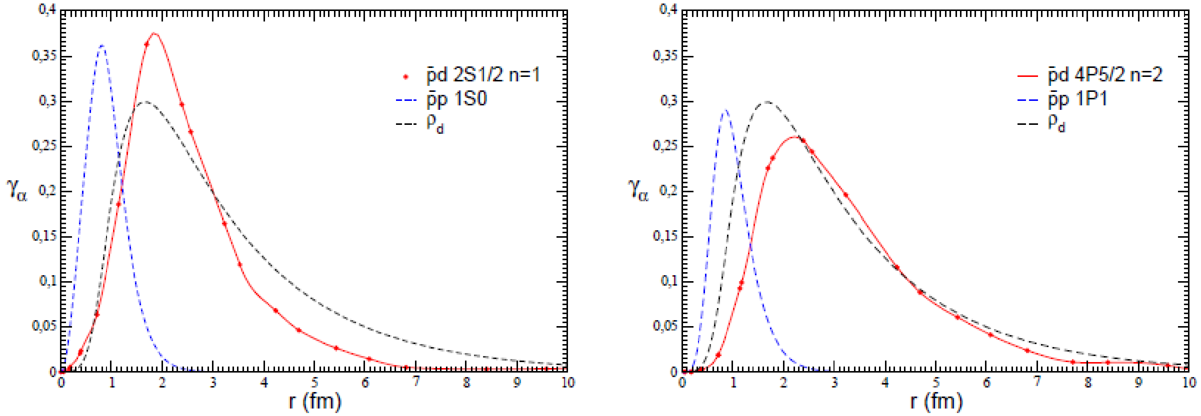


Figure 3: p̄d annihilation densities γ_a for the 2S1/2 (left panel) and 4P5/2 (right panel) states calculated with the MT13+KW model. They are compared with the p̄p $^1S_0$ and $^1P_1$ γ_a's in protonium and with corresponding deuteron matter density ρ_d.

- Annihilation is peripheral for P-wave, however it is not a case for S-wave.