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# Quantum advantages

of analog quantum simulators probing strongly correlated systems and of near-term quantum computers

Jens Eisert, Freie Universität Berlin  
CEA/Saclay workshop, June 2019



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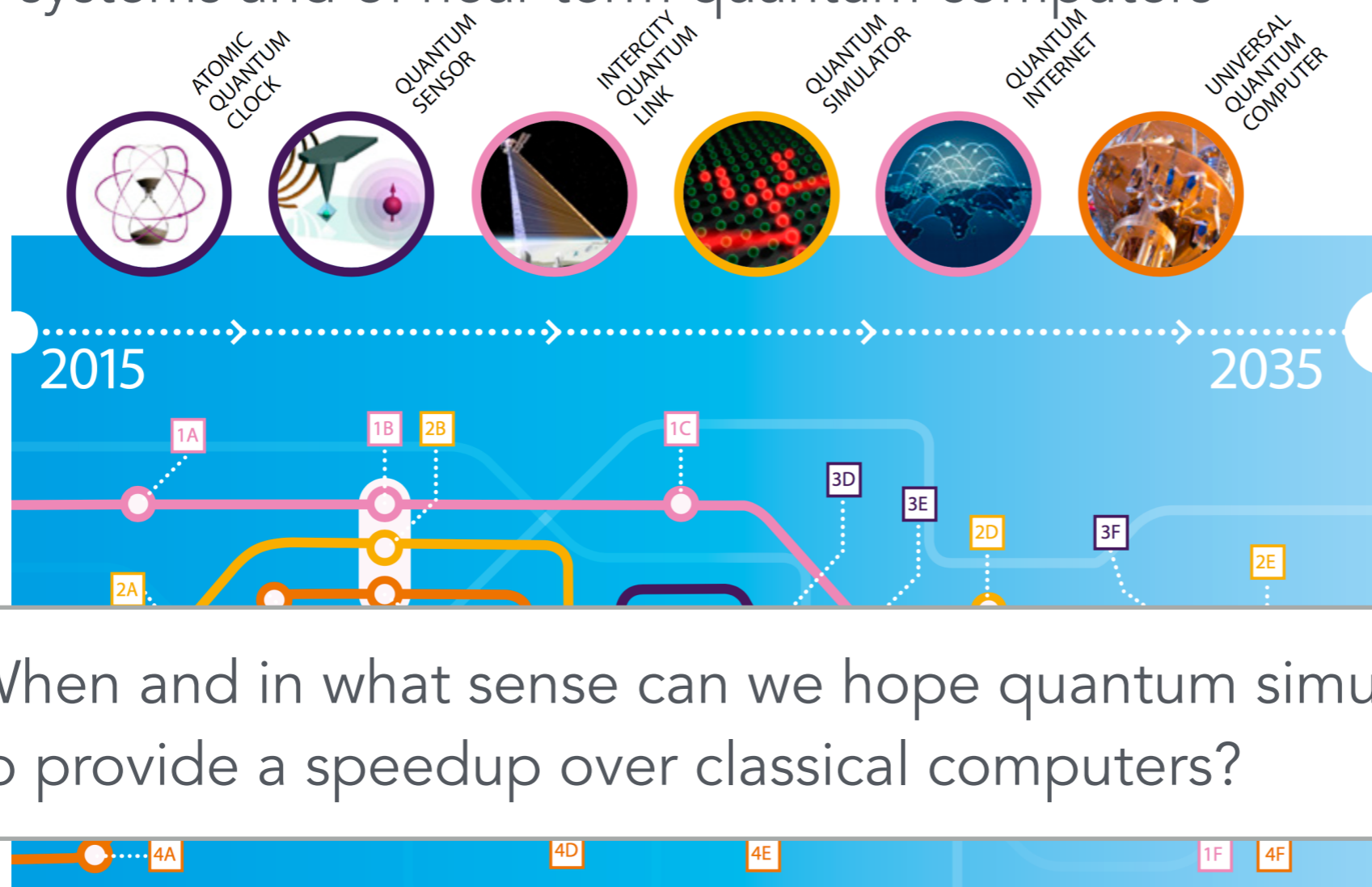
# Quantum advantages

of analog quantum simulators probing strongly correlated systems and of near-term quantum computers

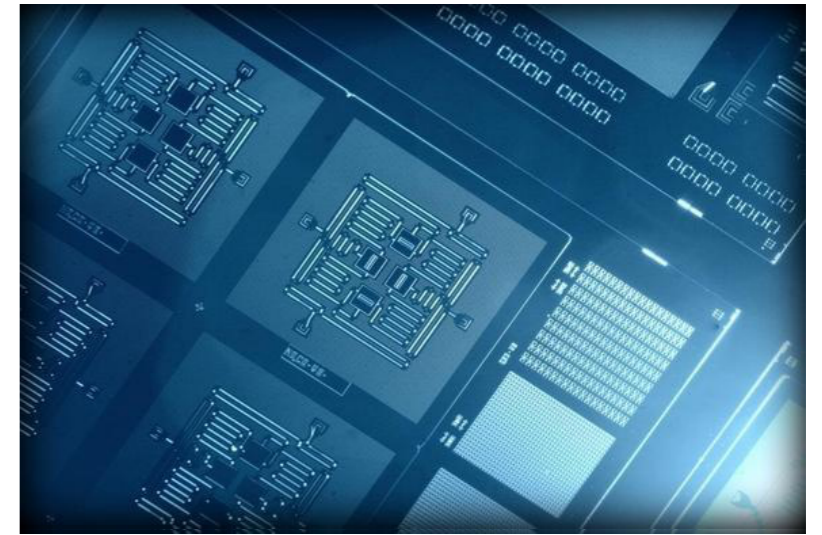
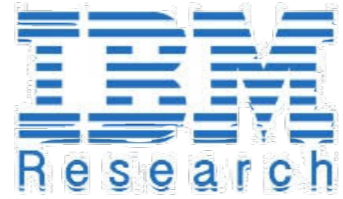
- What is an analog quantum simulator? What are relevant problems?

# Quantum advantages

of analog quantum simulators probing strongly correlated systems and of near-term quantum computers



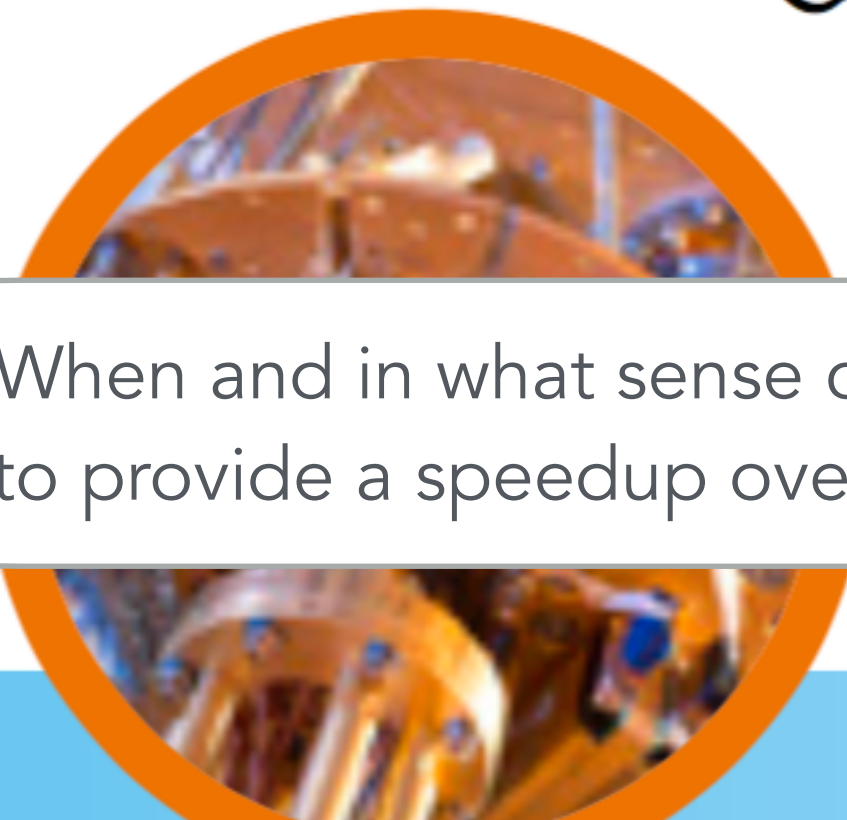
- When and in what sense can we hope quantum simulators to provide a speedup over classical computers?



- 50-128 qubit quantum devices
- Noisy intermediate scale quantum computers

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COMPUTER

- 
- A circular inset image showing a close-up of quantum hardware, likely a superconducting qubit chip, with various components and wiring.
- When and in what sense can we hope quantum simulators to provide a speedup over classical computers

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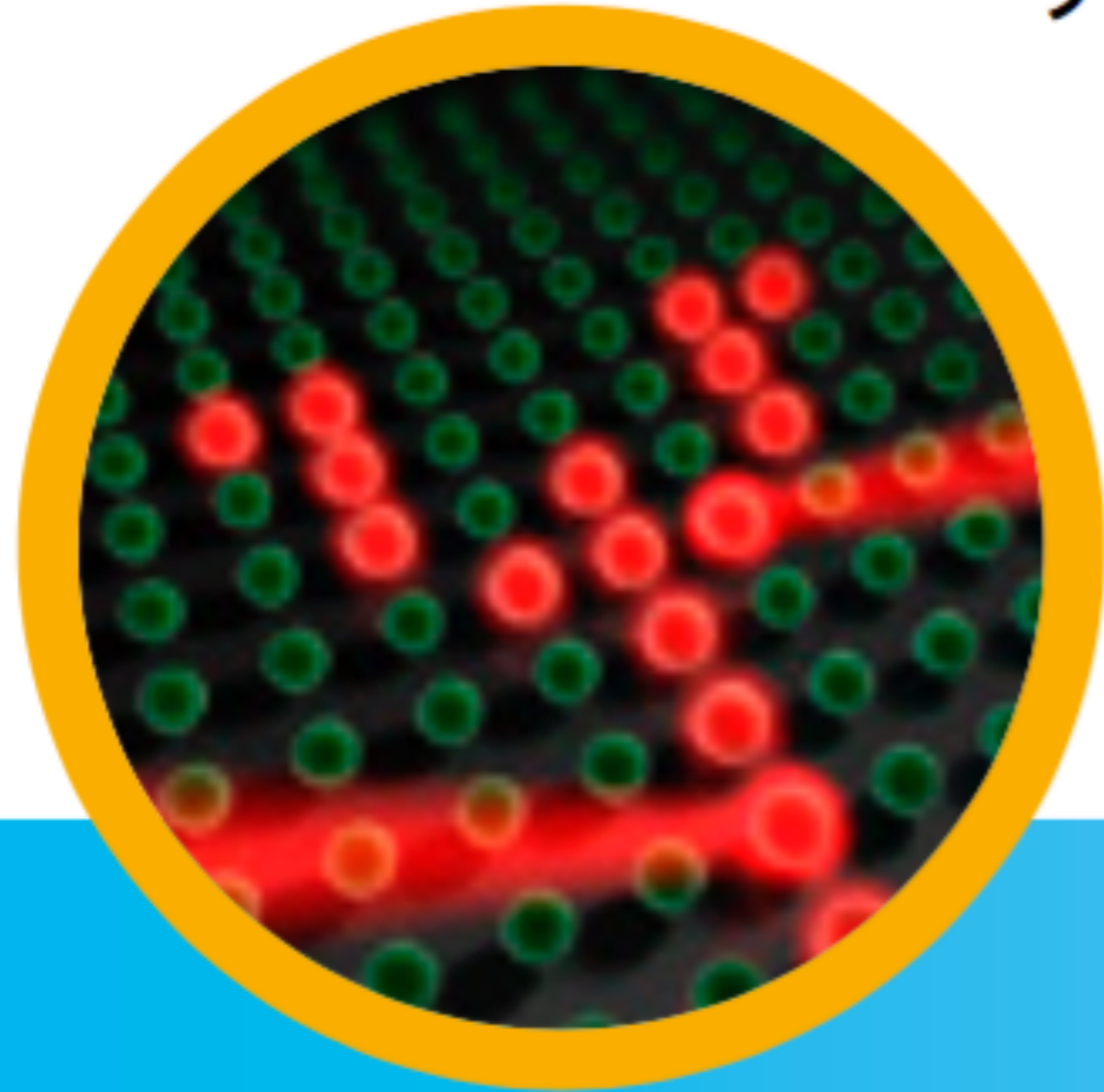
- Analog(ue) quantum simulators
  - Address interesting physics problems
  - Not BQP-complete, what is computational power?
  - Error correction/fault tolerance unavailable
  - Robustness?



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ERGENCY  
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- When can it be claimed that a system has been successfully simulated?
- Testable advantage?

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# Analog quantum simulators



# Analog quantum simulators



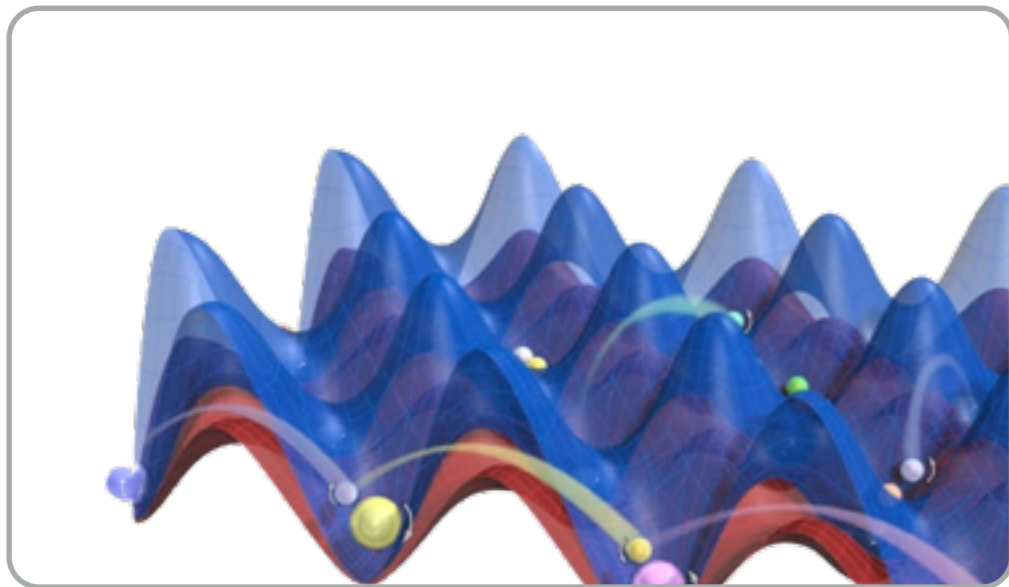
- “Analog”, rather than discrete
- Probing questions in physics (including nuclear physics)

- System size  $n$
- Local Hamiltonians with some levels of control
- Noise levels
- Classes of preparations and measurements

# Analog quantum simulators



- Cold atoms in optical lattices most advanced



- Global control over  $n \sim 10^5$  sites (1D-3D)
- Bosons and fermions
- Some tuneability
- Time-of-flight and in-situ measurements

Bloch, Dalibard, Nascimbene, Nature Physics 8, 267 (2012)

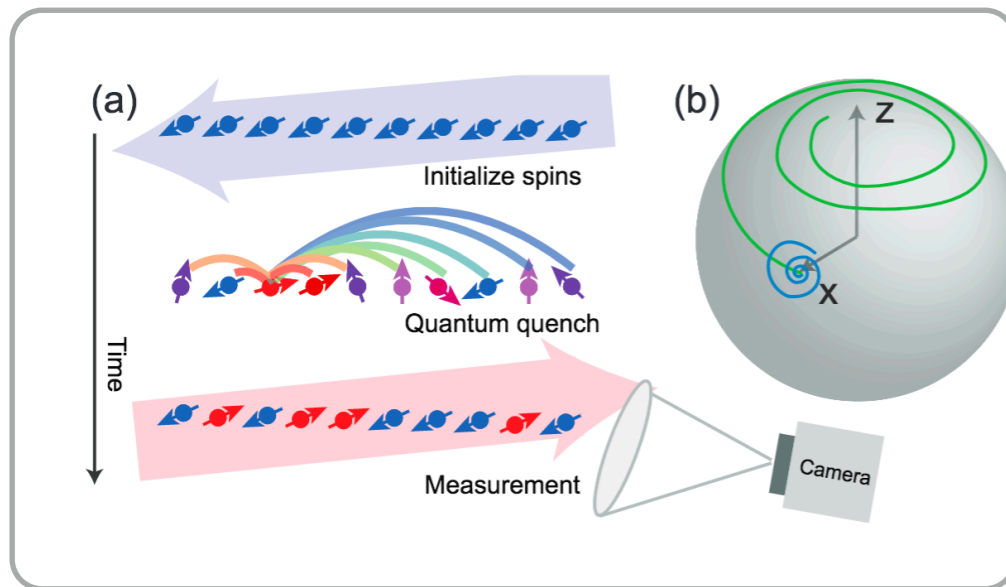
Parsons, Mazurenko, Chiu, Ji, Greif, Greiner, Science, 353, 1253 (2016)

- Towards programmable potentials

# Analog quantum simulators



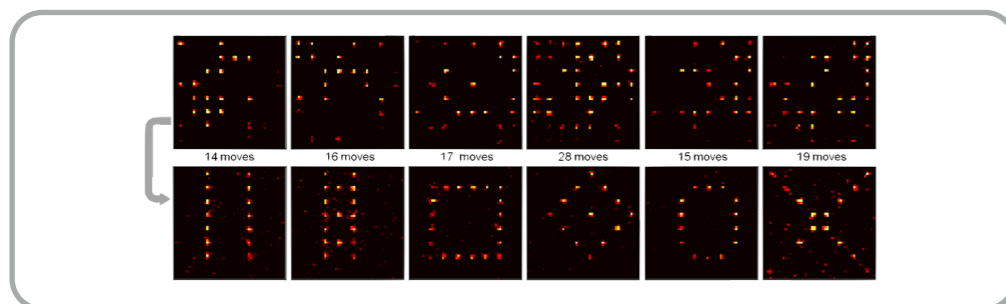
## • Trapped ions



- $n \leq 53$
- Universal control
- Some global gates easier than others
- Tomographically complete measurements

Zhang, Pagano, Hess, Kyprianidis, Becker, Kaplan, Gorshkov, Gong, Monroe 551, 601 (2017)  
Blatt, Roos, Nature Phys 8, 277 (2012)

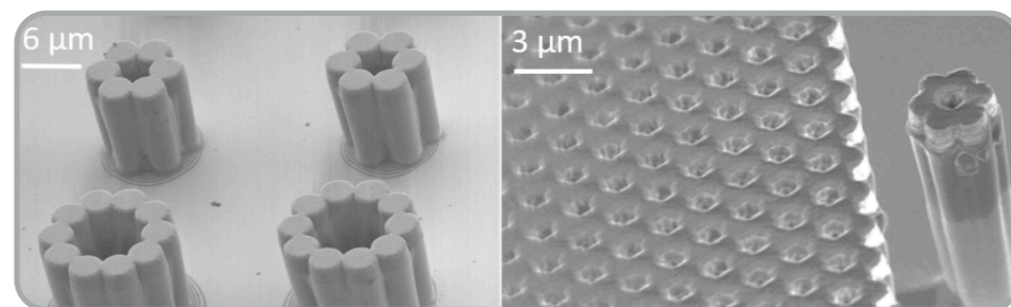
## • Optical microtraps



Labuhn, Barredo, Ravets, Léséleuc, Macrì, Lahaye, Browaeys, Nature 534, 667 (2016)

- $n \sim 50 \times 50$ , long-ranged Ising

## • Polaritonic/photonic architectures



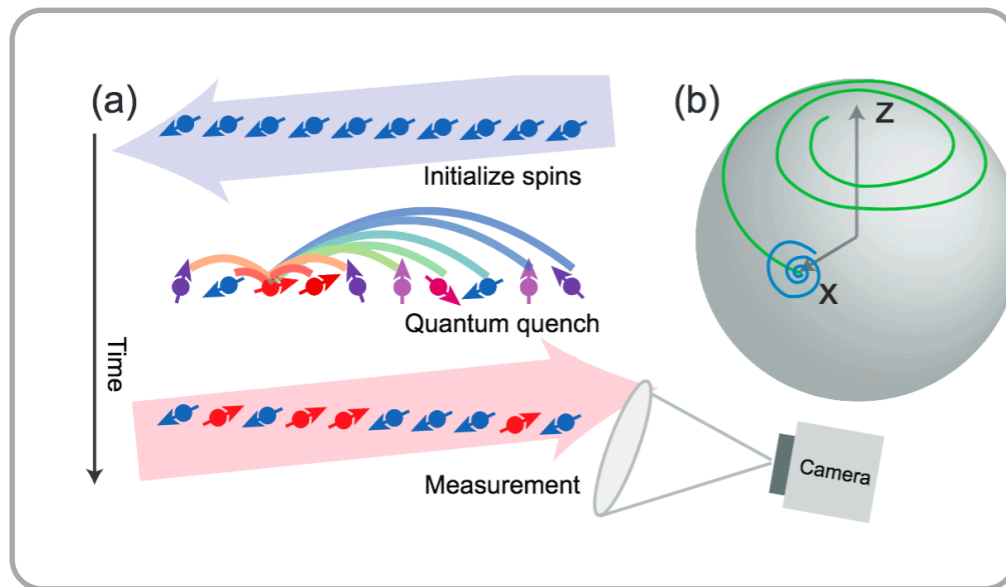
Wertz, Ferrier, Solnyshkov, Johne, Sanvitto, Lemaître, Sagnes, Grousseau, Kavokin, Senellart, Malpuech, Bloch, Nature Phys 6, 860 (2010)

- Large, but intrinsically open and noisy

# Analog quantum simulators



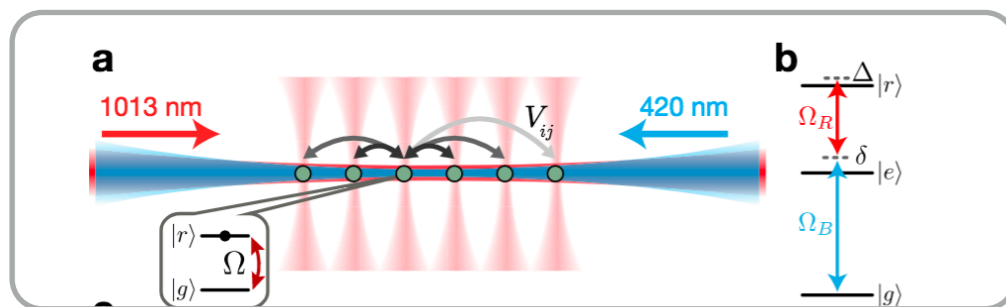
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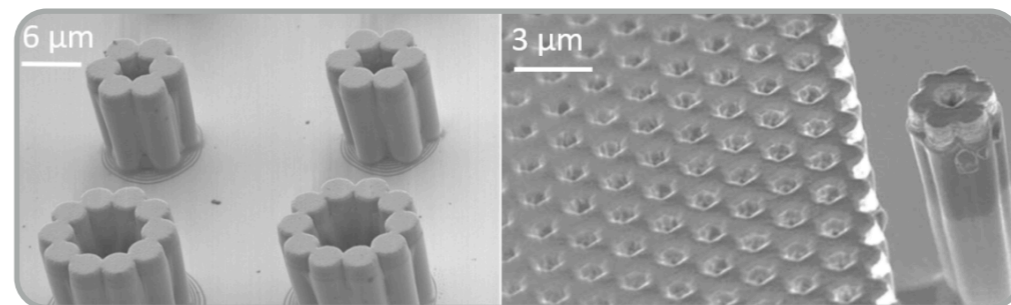
## • Cold atoms in Rydberg states



Bernien, Schwartz, Keesling, Levine, Omran, Pichler, Choi, Zibrov, Endres, Greiner, Vuletic, Lukin, Nature 551, 579 (2017)

## • Programmable

## • Polaritonic/photonic architectures



Wertz, Ferrier, Solnyshkov, Johne, Sanvitto, Lemaître, Sagnes, Grousson, Kavokin, Senellart, Malpuech, Bloch, Nature Phys 6, 860 (2010)

## • Large, but intrinsically open and noisy



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What can they probe?

# What can they probe?

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- **Time-dependent** problems ("quenches")

$$\rho(t) = e^{-itH} \rho e^{itH}$$

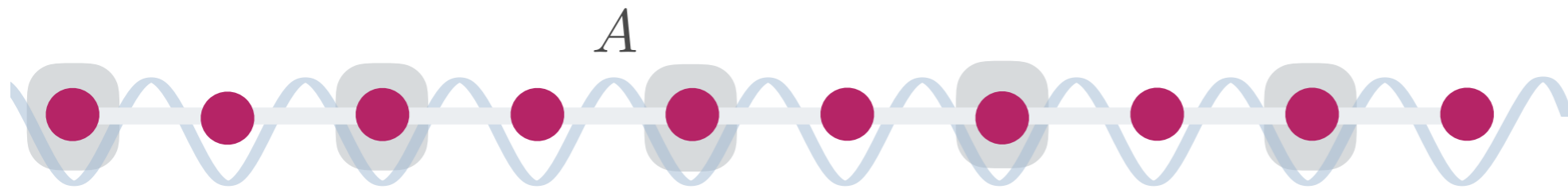
- E.g. probe **equilibration and thermalisation**

Eisert, Friesdorf, Gogolin, Nature Phys 11, 124 (2015)

- **Dynamical phase transitions**

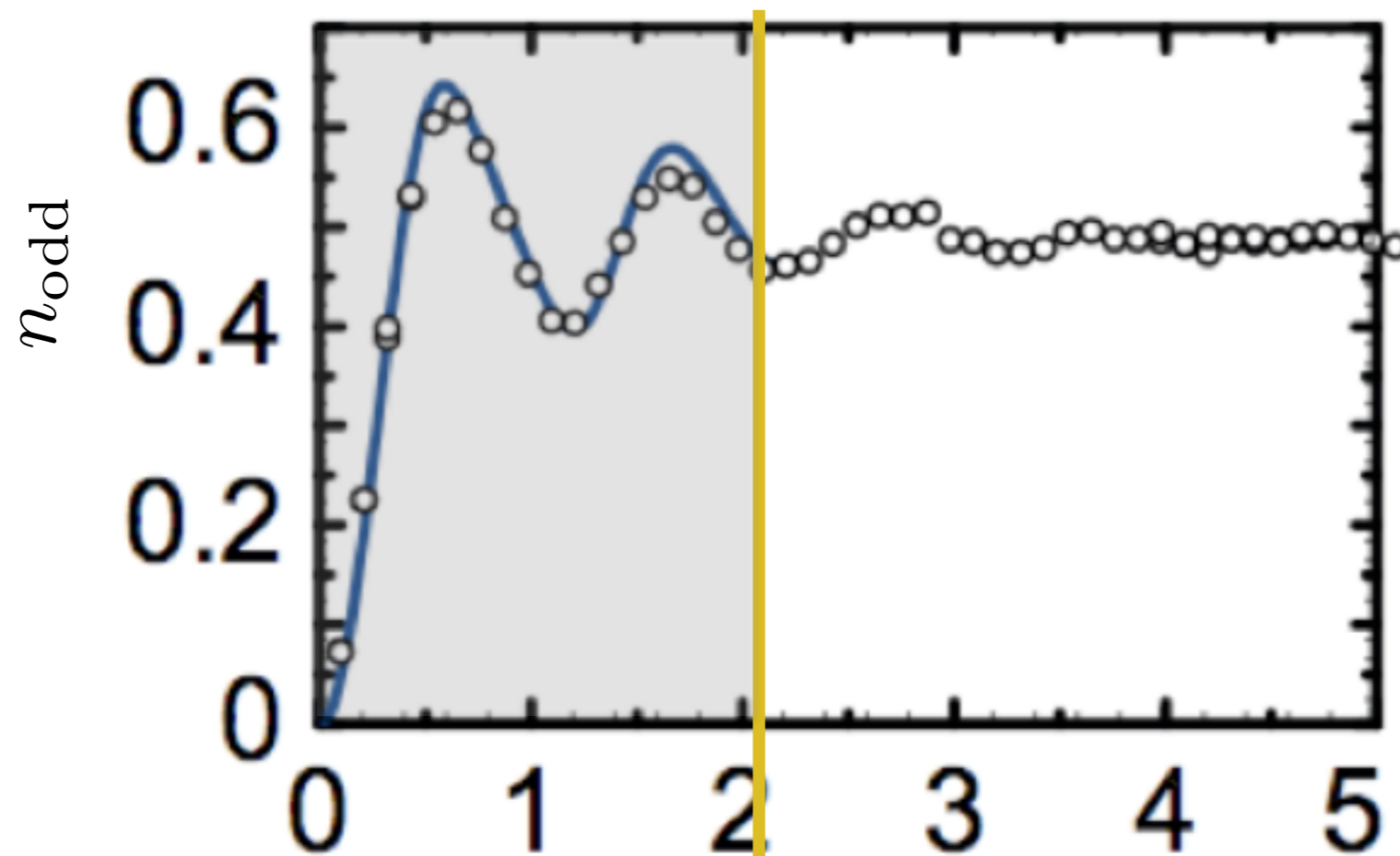
Zhang, Pagano, Hess, Kyprianidis, Becker, Kaplan, Gorshkov, Gong, Monroe, Nature 551, 601 (2017)

# What can they probe?



- **Time-dependent** problems ("quenches")

- Imbalance as function of time for  $|\psi(0)\rangle = |0, 1, \dots, 0, 1\rangle$  under Bose-Hubbard Hamiltonian (MPQ)



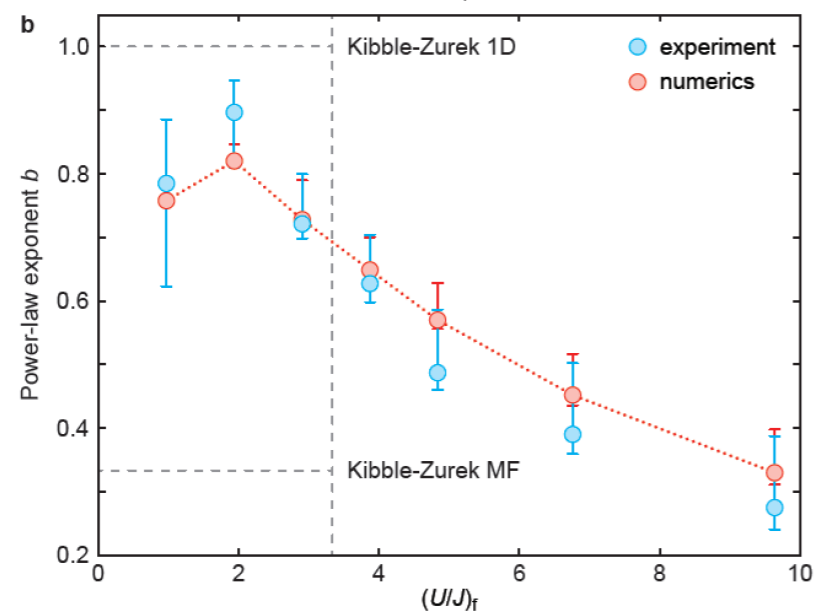
Best available classical tensor network simulation, bond dimension 5000

# What can they probe?



- **Slow parameter variations** (reminiscent of adiabatic quantum algorithms)

- E.g., Kibble-Zurek dynamics (1D-2D)



- Probing scaling laws of correlations

Braun, Friesdorf, Hodgman, Schreiber, Ronzheimer, Riera, del Rey, Bloch, Eisert, Schneider, Proc Natl Acad Sci 112, 3641 (2015)

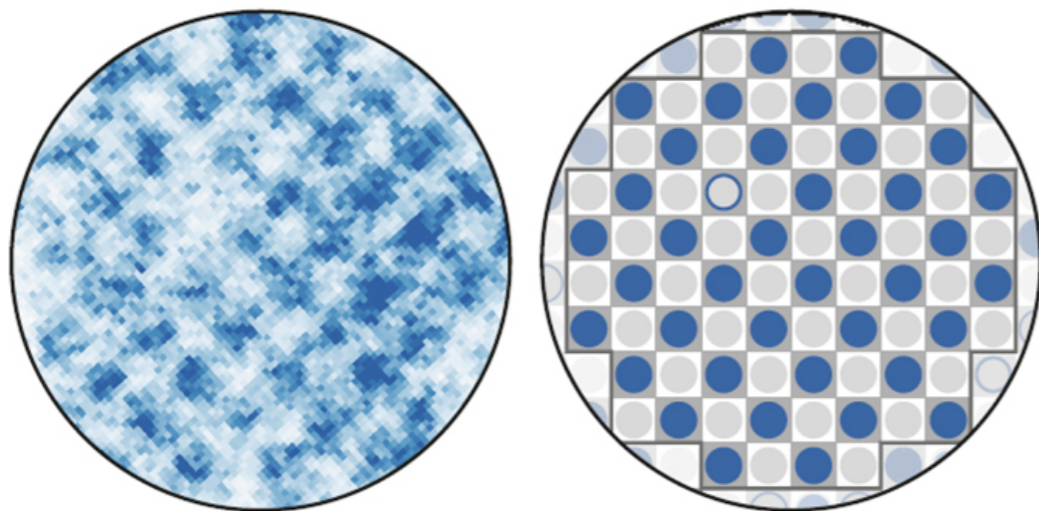


# What can they probe?



## • Ground state and static problems

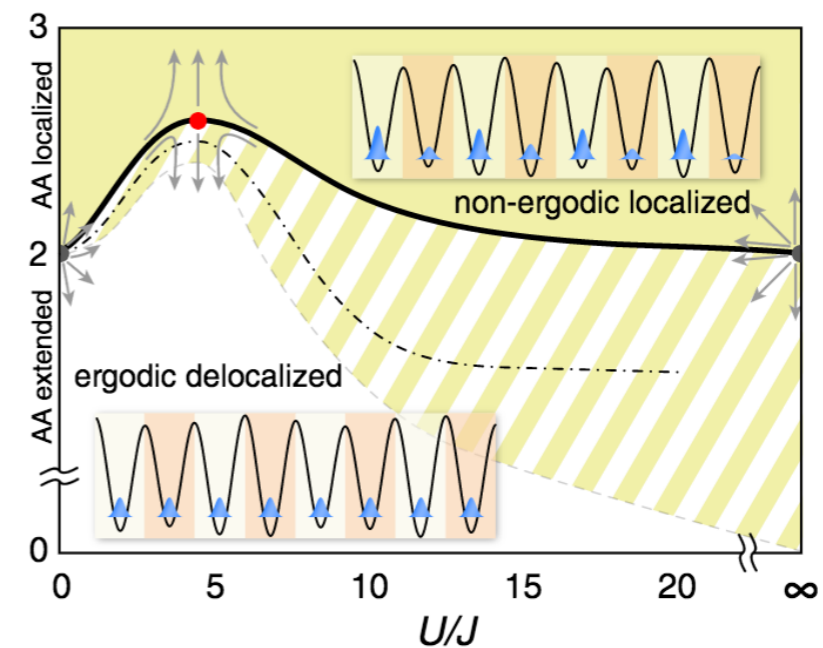
- Hubbard model, probing high- $T_c$  superconductivity



- Cooled to create a magnetic state with long-range order

Mazurenko, Chiu, Ji, Parsons, Kanász-Nagy, Schmidt, Grusdt, Demler, Greif, Greiner, Nature 545, 462 (2017)  
Esslinger, Ann Rev Cond Mat Phys 1, 129 2010

- Many-body localization (1D-2D)



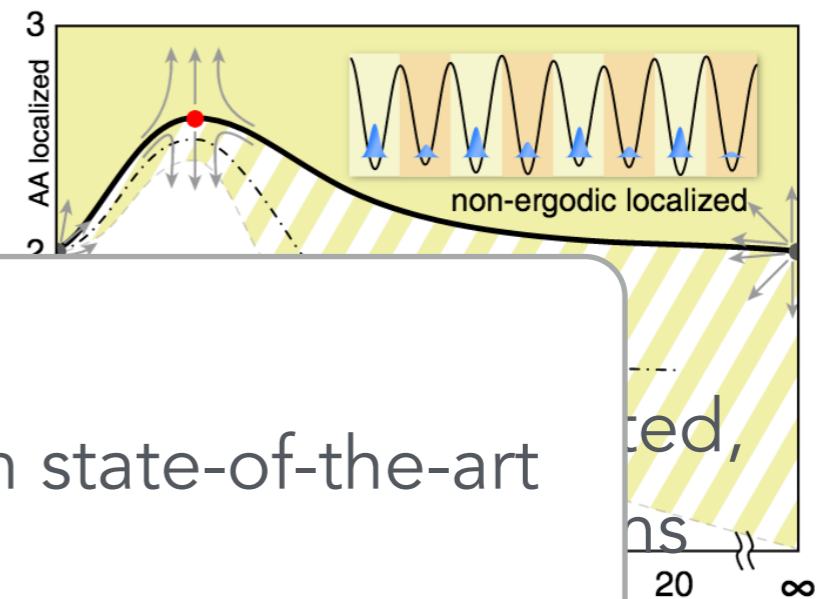
- Debated in 2D

Schreiber, Hodgman, Bordia, Lüschen, Fischer, Vosk, Altman, Schneider, Bloch, Science 349, 842 (2015)

# What can they probe?



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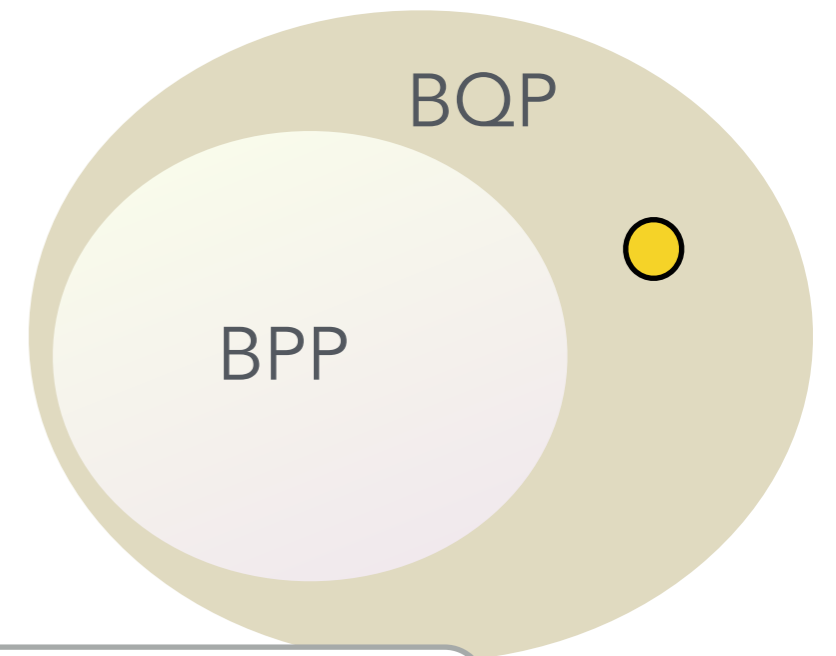


- **Quantum simulators**

Existing quantum simulators outperform state-of-the-art algorithms on classical supercomputers

- Debated in 2D

- Cleverer simulation method?



- **Intermediate problems**

To be safe against "lack of imagination", we must prove the hardness of the task in a complexity-theoretic sense

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Super-polynomial quantum advantages?

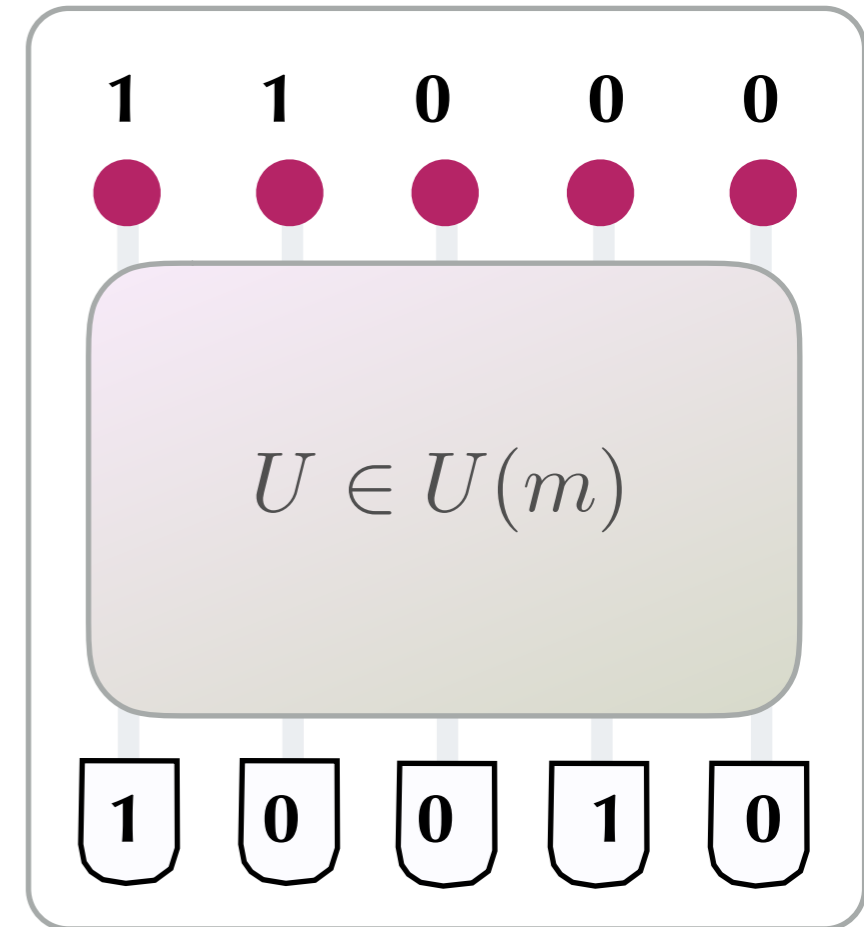


# Complexity-theoretic quantum advantages

- **Aim:** Find **some** problem with strong evidence for quantum advantage

- **Boson sampling**

Aaronson, Arkhipov, Th Comp 9, 143 (2013)

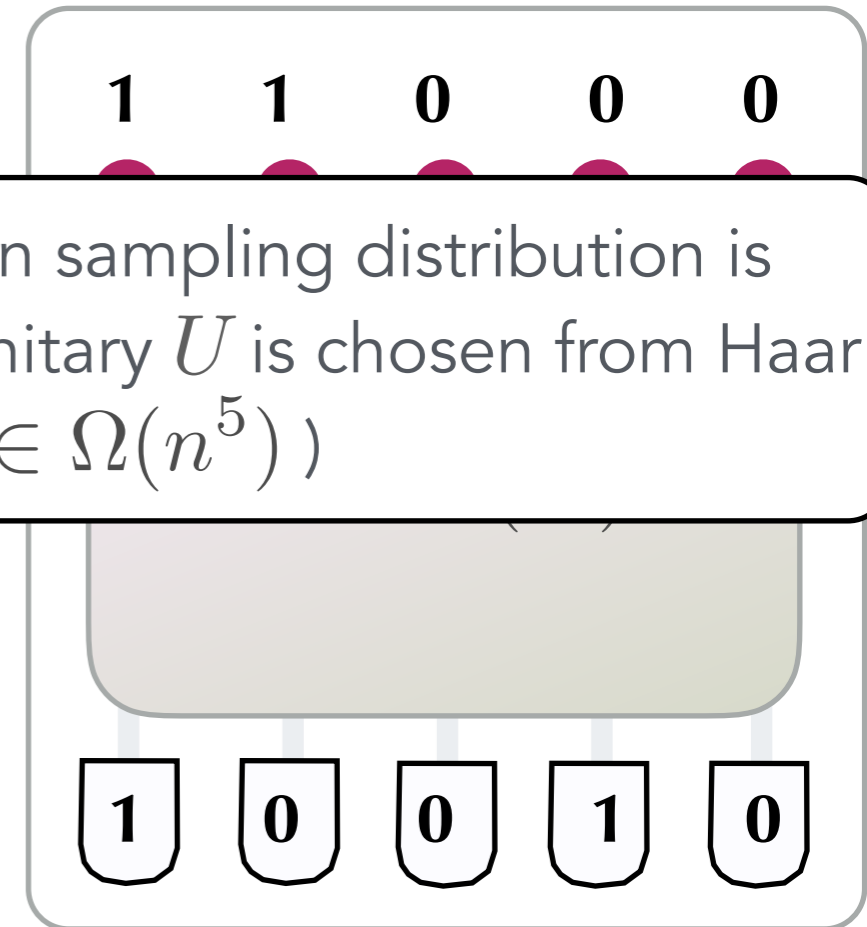


# Complexity-theoretic quantum advantages

- **Aim:** Find **some** problem with strong evidence for quantum advantage
- **Boson sampling**

Aaronson, Arkhipov, Th Comp 9, 143 (2013)

Sampling from a distribution close in  $l_1$  norm to boson sampling distribution is "computationally hard" with high probability if the unitary  $U$  is chosen from Haar measure and  $m$  increases sufficiently fast with  $n$  ( $m \in \Omega(n^5)$ )



# Complexity-theoretic quantum advantages

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- **Aim:** Find **some** problem with strong evidence for quantum advantage

- Verification and testing? Black-box verification seems out of question

# Hamiltonian quantum simulation architectures

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- **Aim:** Find **some** problem with strong evidence for quantum advantage

- **Challenging prescription:** Is it possible to scale it up to provably hard regimes, in an architecture close to a quantum simulation?

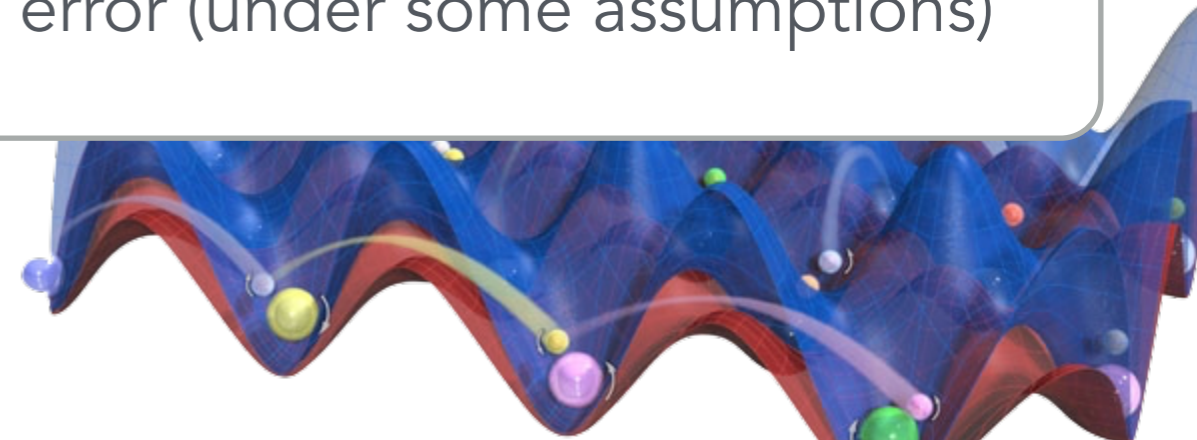
# Hamiltonian quantum simulation architectures

- **Aim:** Find some problem with strong evidence for quantum advantage

Combine benefits of both worlds



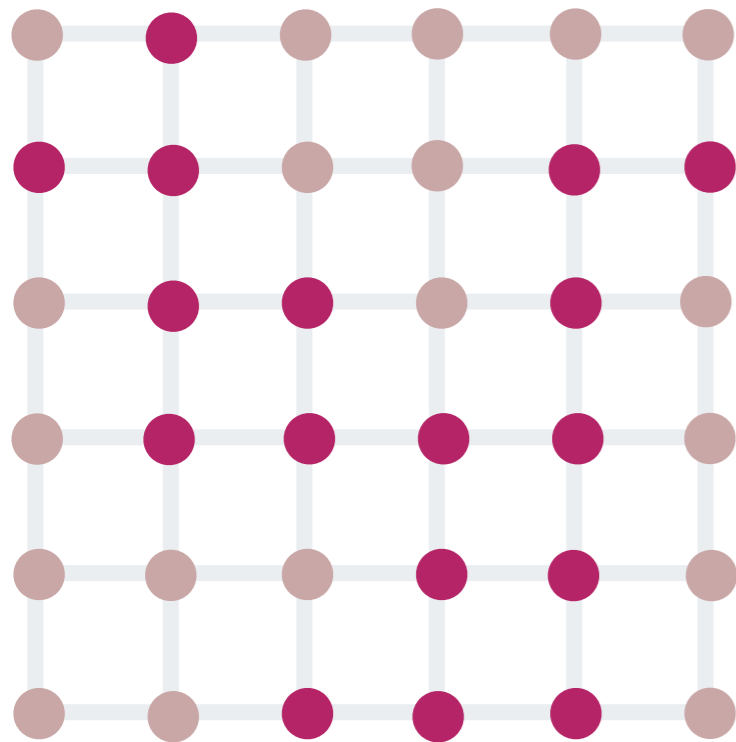
- **Hamiltonian quench architecture**
- **Low periodicity** of the interaction Hamiltonian (NN or NNN)
- **Hardness proofs** with  $l_1$ -norm error (under some assumptions)



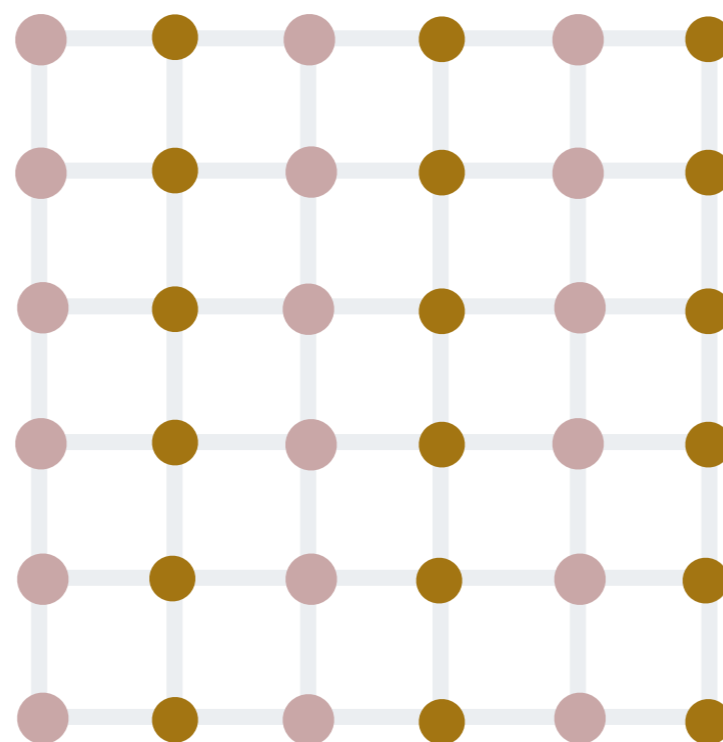
# Hamiltonian quantum simulation architectures

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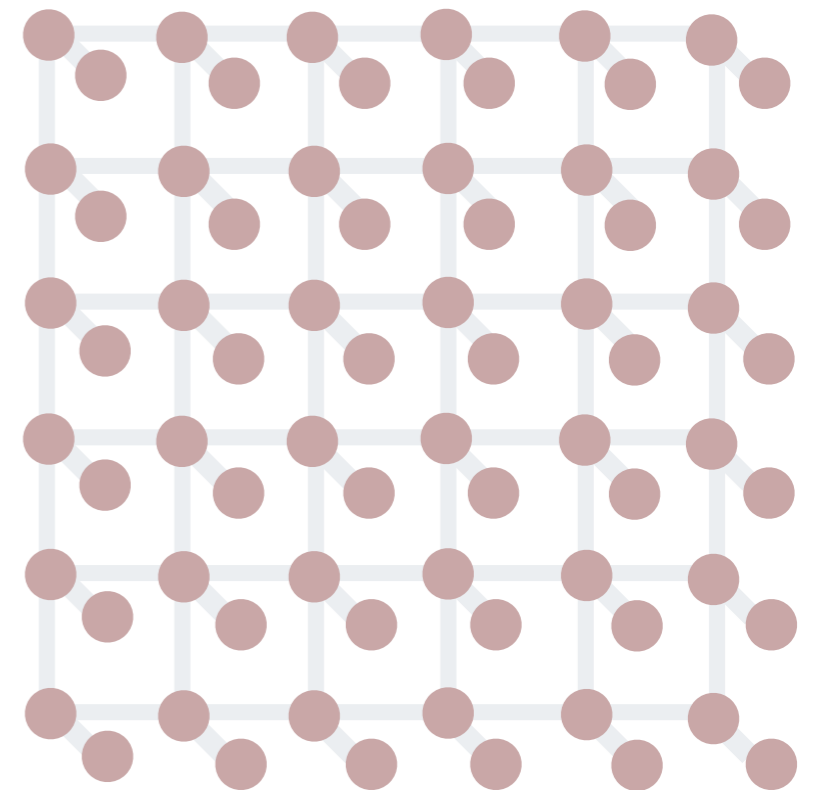
Combine benefits of both worlds



Random



Quasi-periodic



Translationally invariant

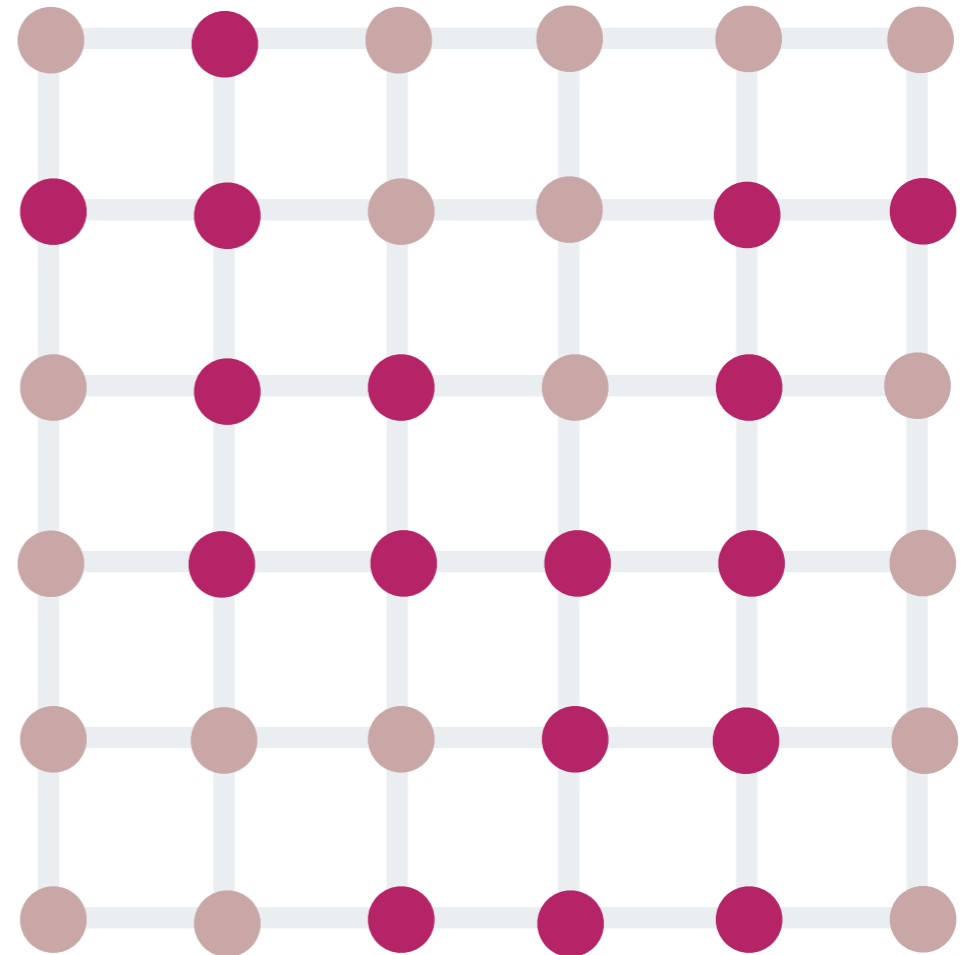


# Simple Ising models

- Prepare  $N$  qubits in  $n \times m$  square lattice in product

$$|\psi_\beta\rangle = \bigotimes_{i,j=1}^{n,m} (|0\rangle + e^{i\beta_{i,j}} |1\rangle)$$

with  $\beta_{i,j} \in \{0, \pi/4\}$ ,  $\{\bullet, \bullet\}$  i.i.d. randomly



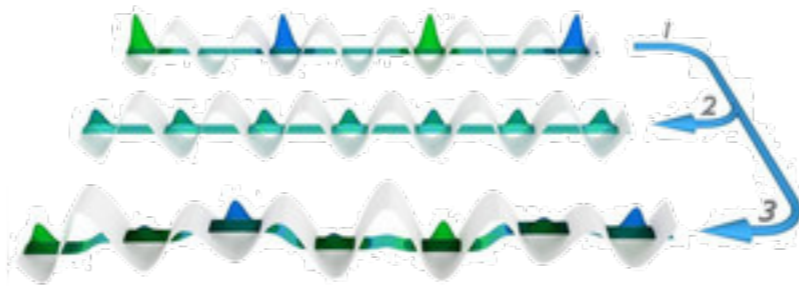
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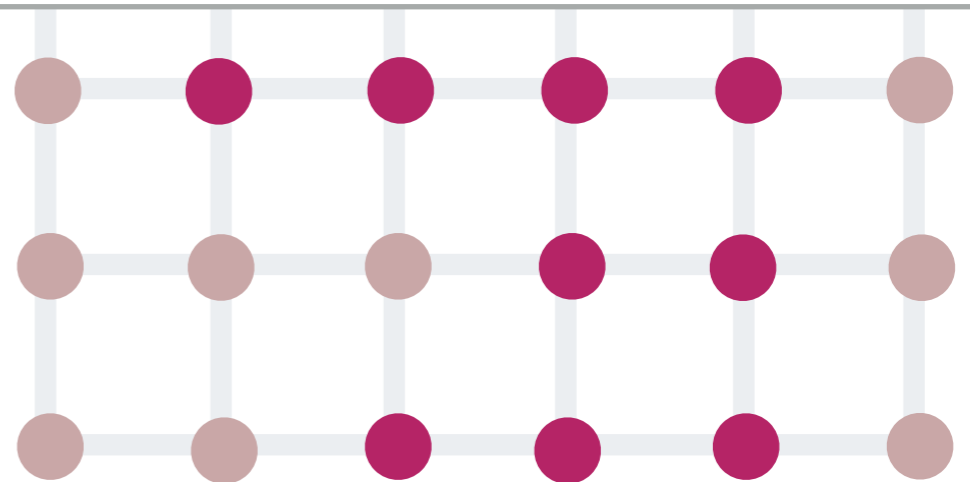
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- Reminiscent of disordered optical lattices



Schreiber, Hodgman, Bordia, Lüschen, Fischer, Vosk, Altman, Schneider, Bloch, Science 349, 842 (2015)



# Simple Ising models

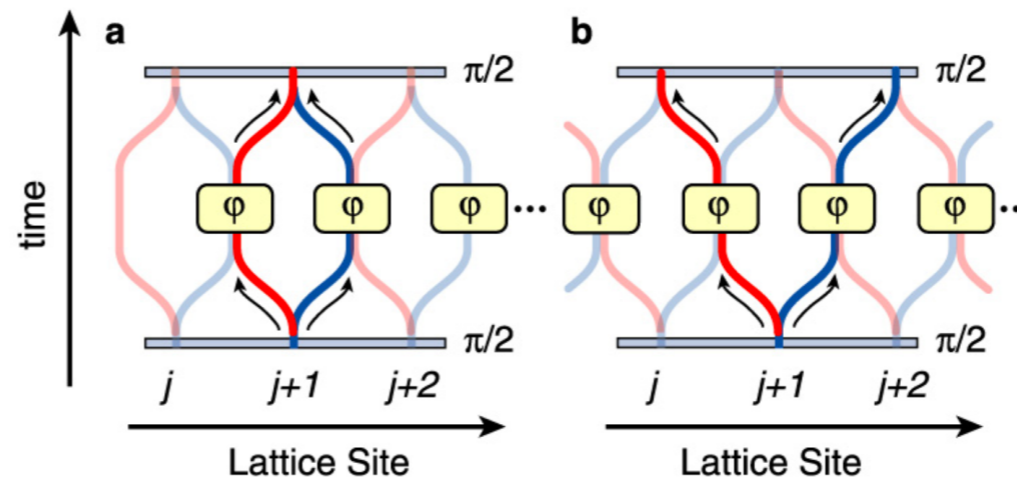
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- Quench to  $H = \sum_{(i,j) \in E} Z_i Z_j + \frac{\pi}{4} \sum_{i \in V} Z_i$  and evolve under  $U = e^{iH}$

- Controlled coherent collisions long realized



Mandel, Greiner, Widera, Rom, Hänsch, Bloch, Nature, 425, 937 (2003)

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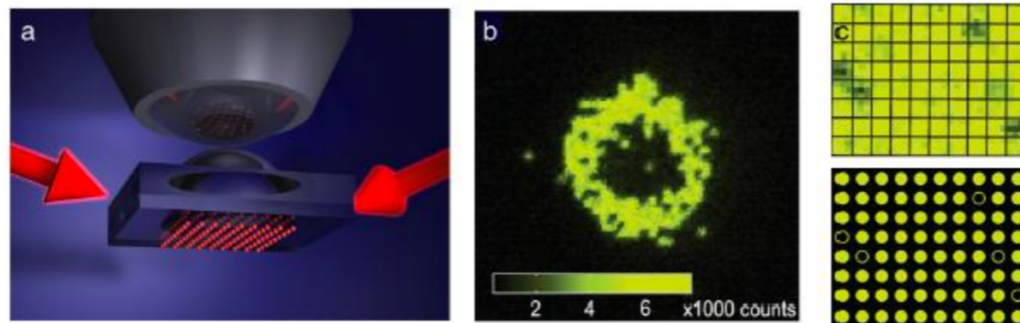
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- Measure all qubits in  $X$ -basis

- Single-site addressing possible (within limits)



Bakr, Gillen, Peng, Foelling, Greiner, Nature 462, 74–77 (2009)

Weitenberg, Endres, Sherson, Cheneau, Schauß, Fukuhara, Bloch, Kuhr, Nature 471, 319 (2011)

# Simple Ising models

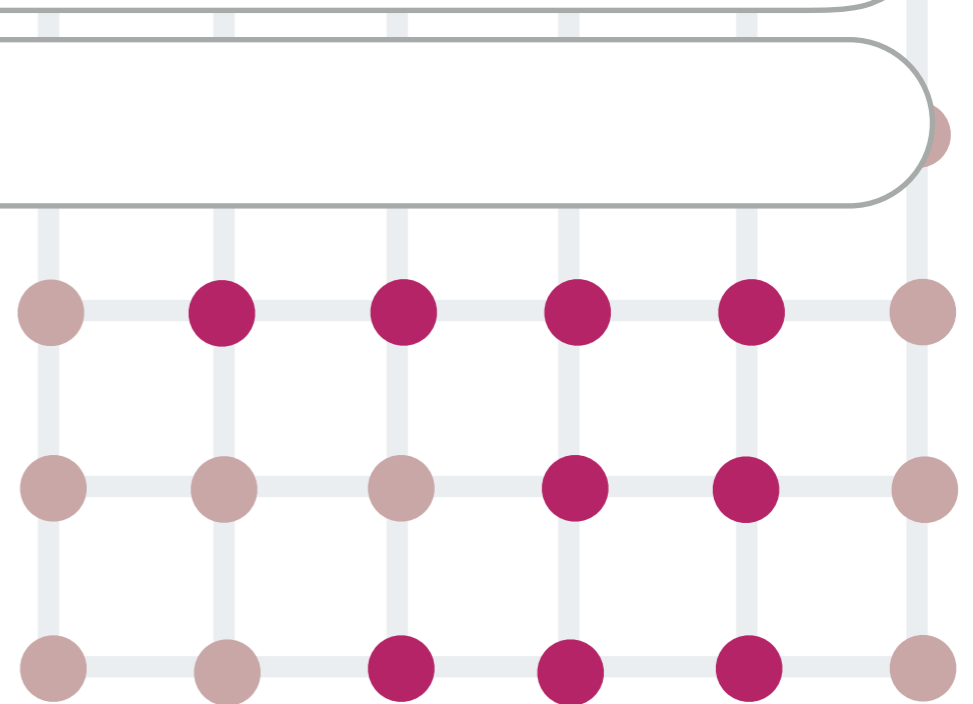
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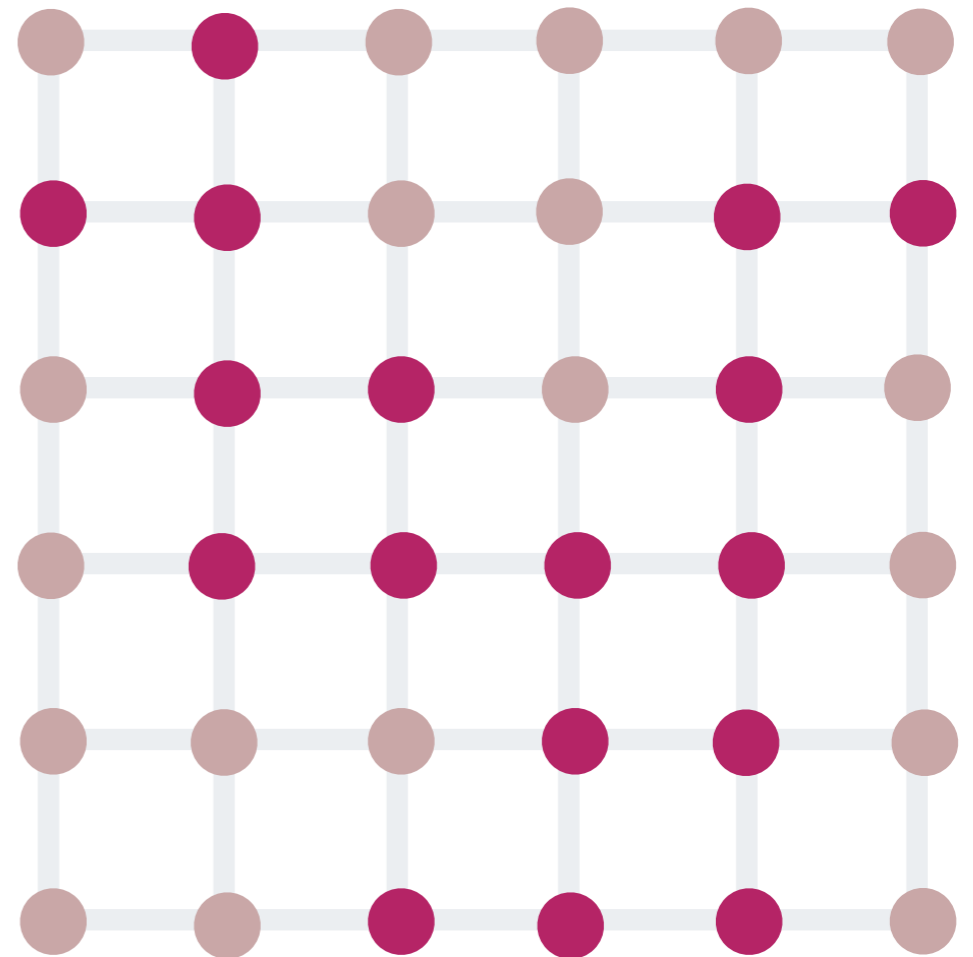
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- **Theorem** (Hardness of classical sampling):

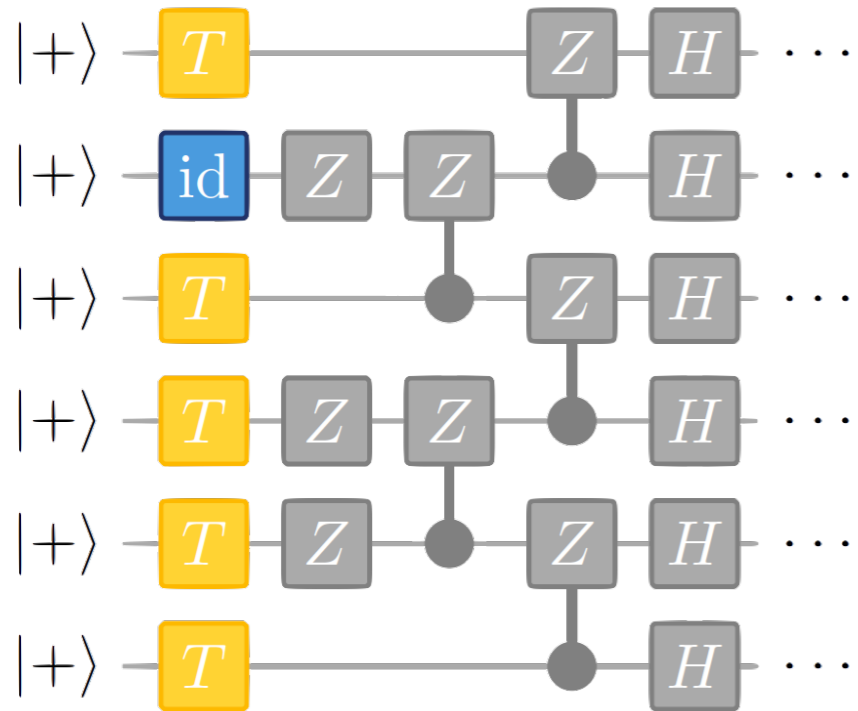
Assuming three highly plausible complexity-theoretic conjectures are true a classical computer cannot efficiently sample from the outcome distribution of our scheme up to constant error in  $l_1$  distance





- Relate quench architecture to post-selected measurement-based quantum computing

- Universal quantum circuit for postBQP



- It is #P-hard to approximate the outcome distribution

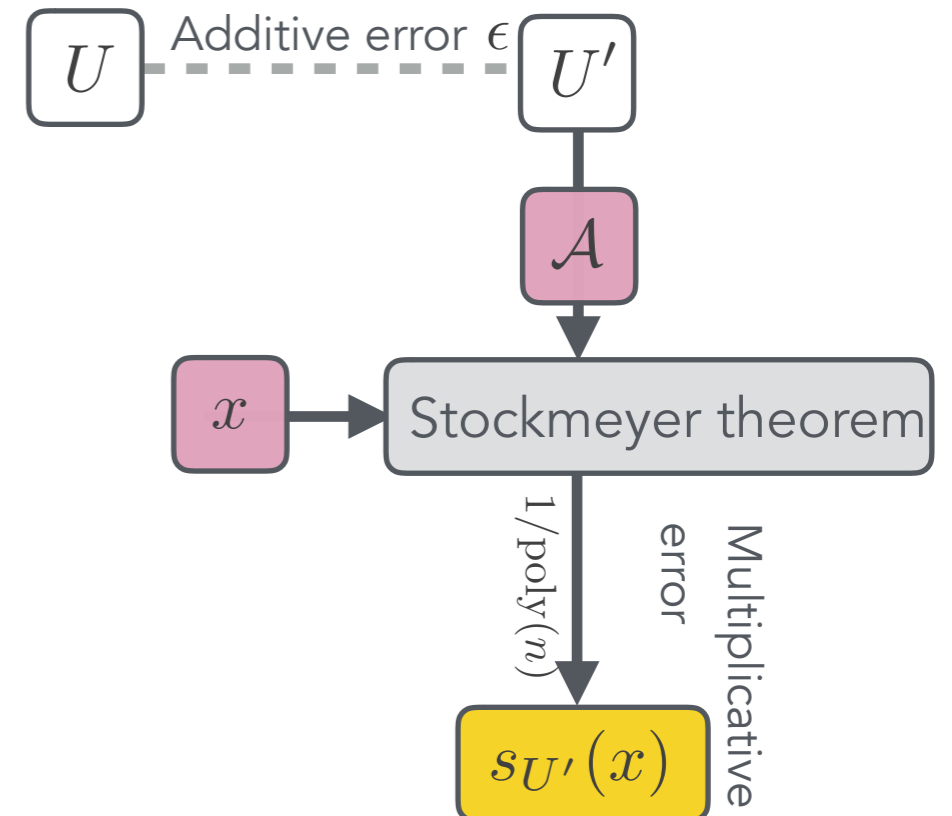
- Polynomial hierarchy (similar  $P \neq NP$ )
- Average-case complexity
- Anti-concentration

Bouland, Fefferman, Nirkhe, Vazirani, arXiv:1803.04402

Hangleiter, Bermejo-Vega, Schwarz, Eisert, Quantum 2, 65 (2018)

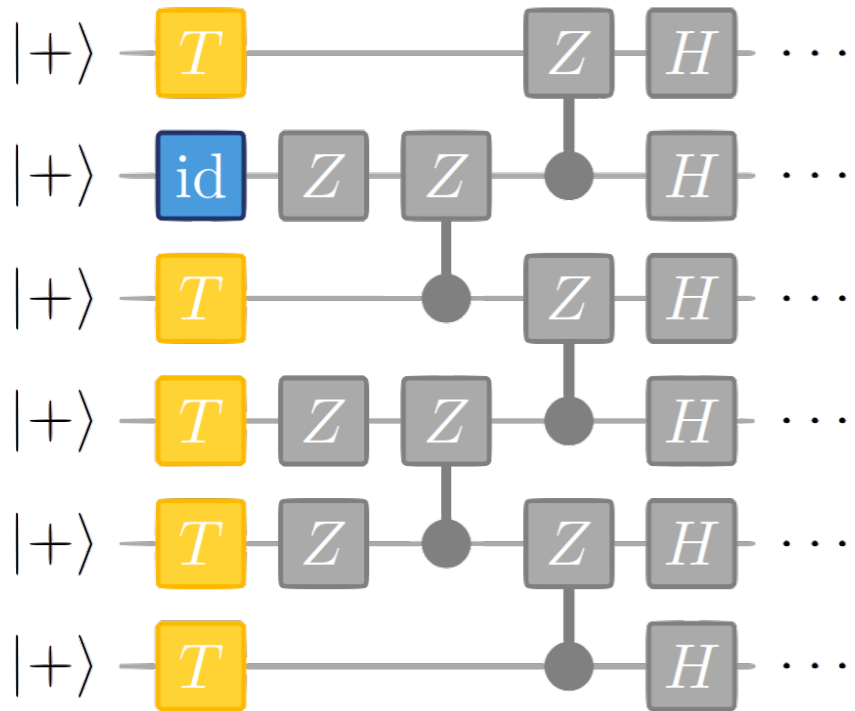
Mann, Bremner, arXiv:1711.00686

- Relate hardness of computing probabilities to hardness of sampling with additive errors



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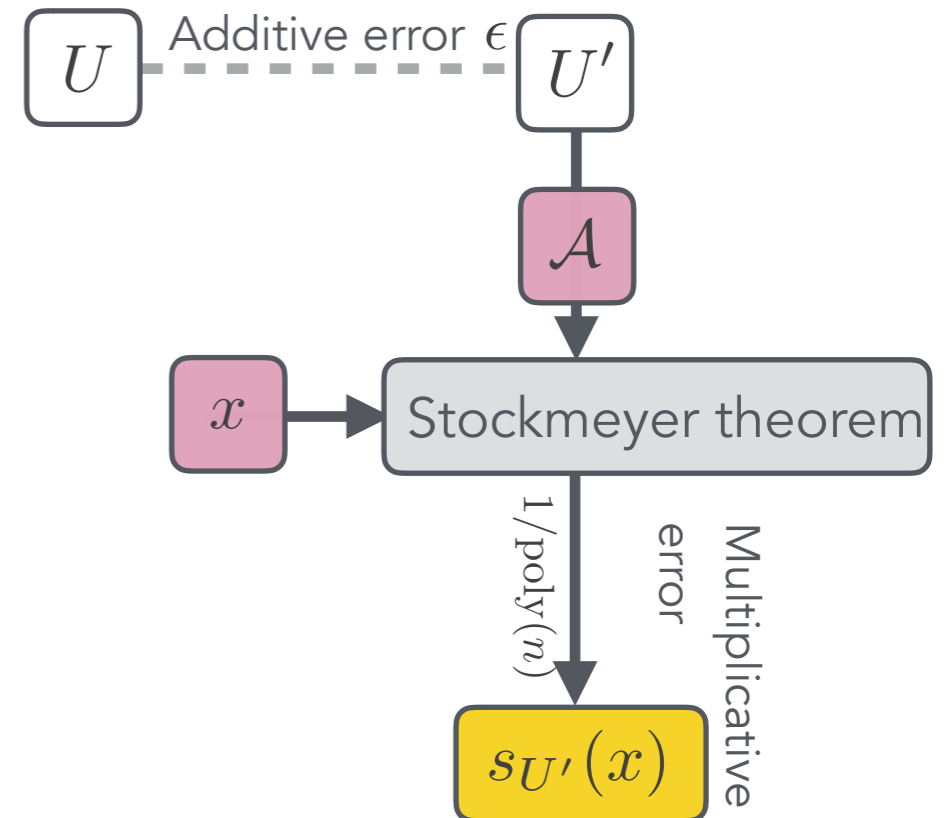
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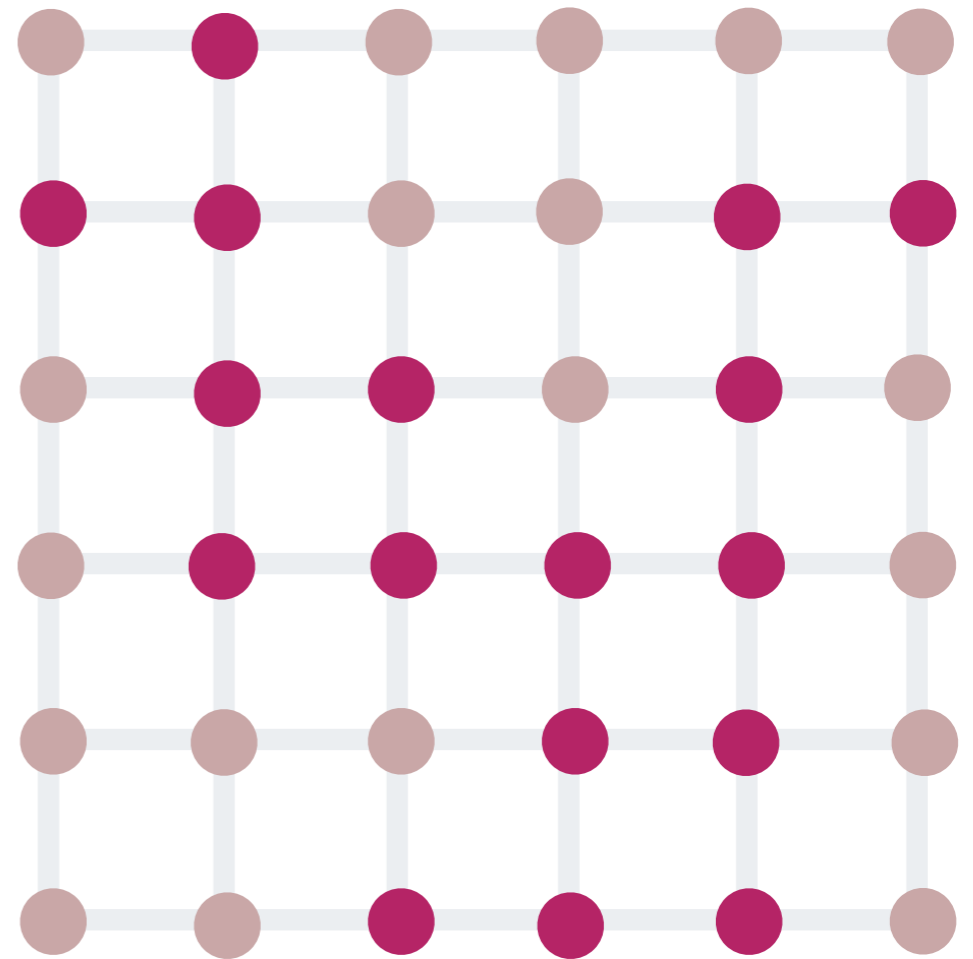
theoretic conjectures are true a  
 ple from the outcome distribution  
 instance

- Relate hardness of computing probabilities to hardness of sampling with additive errors



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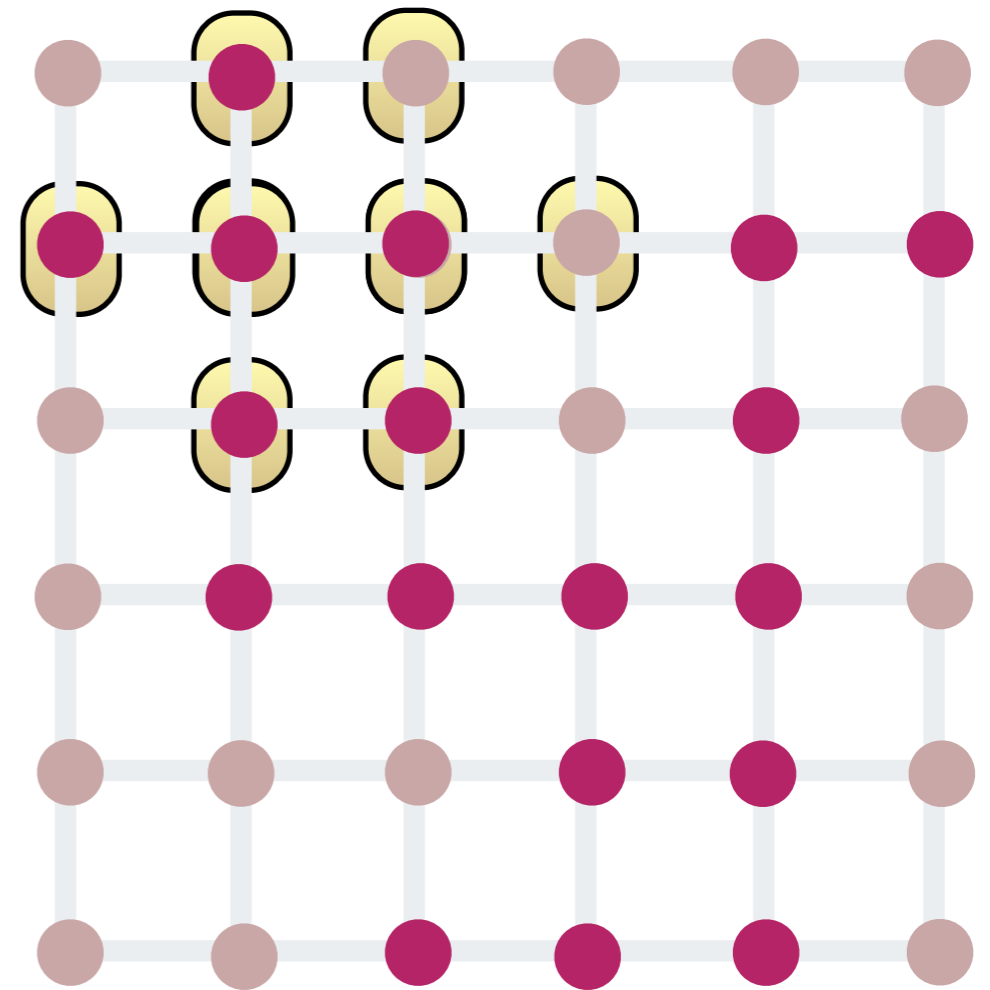


- This quantum simulation is intractable for classical computers



# Verifiable quantum devices showing a quantum advantage

- One can with  $\theta(N)$  many measurements detect closeness in  $l_1$ -norm!
- Ground state of fictitious frustration-free Hamiltonian
- Much simpler than fault tolerance



# Verifiable quantum devices showing a quantum advantage

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# Verifiable quantum devices showing a quantum advantage

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- **Common prejudice:** In order to be able to verify a quantum simulation, one needs to be able to efficiently simulate it

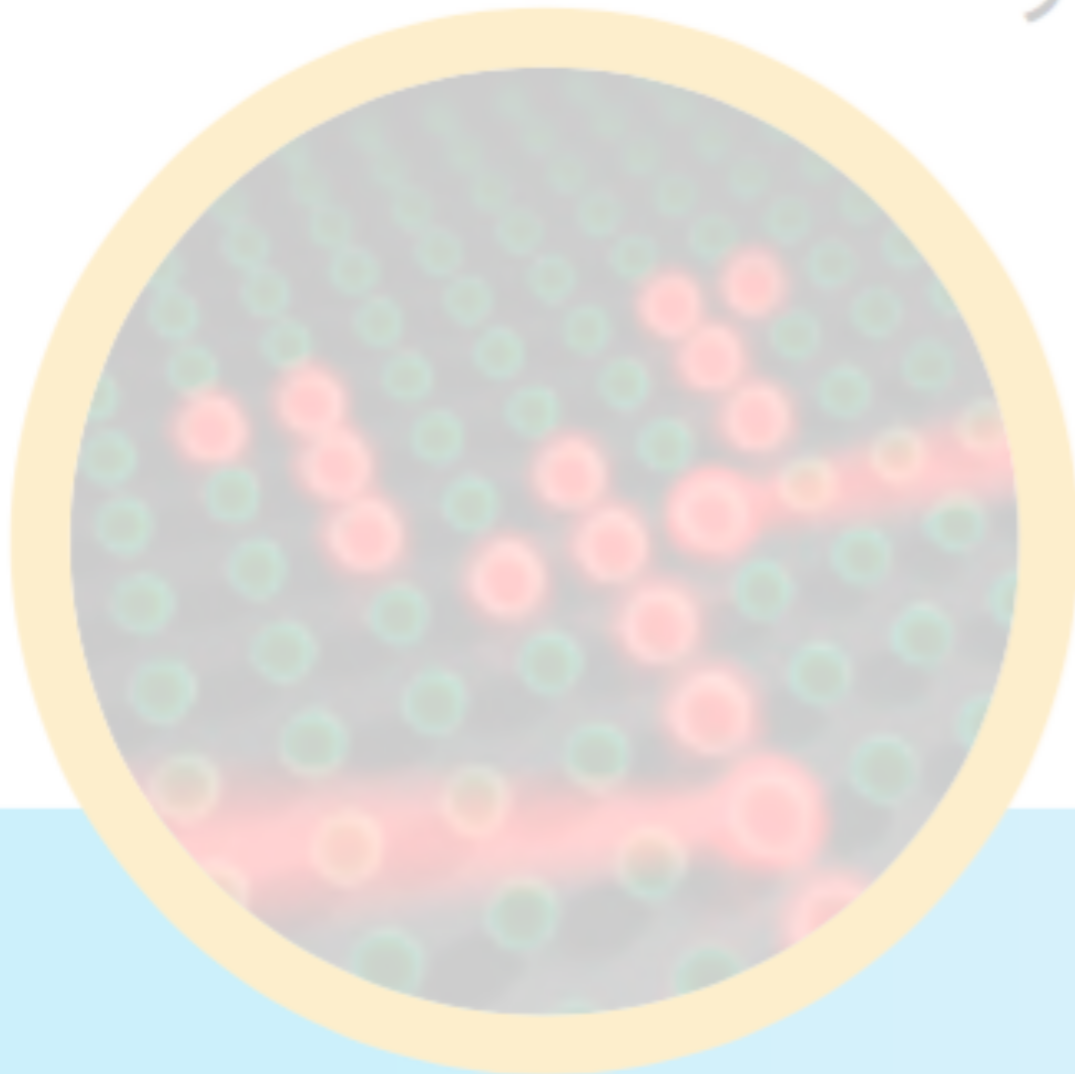
## Summary, outlook and open questions

- Analog quantum simulators already outperform good classical algorithms

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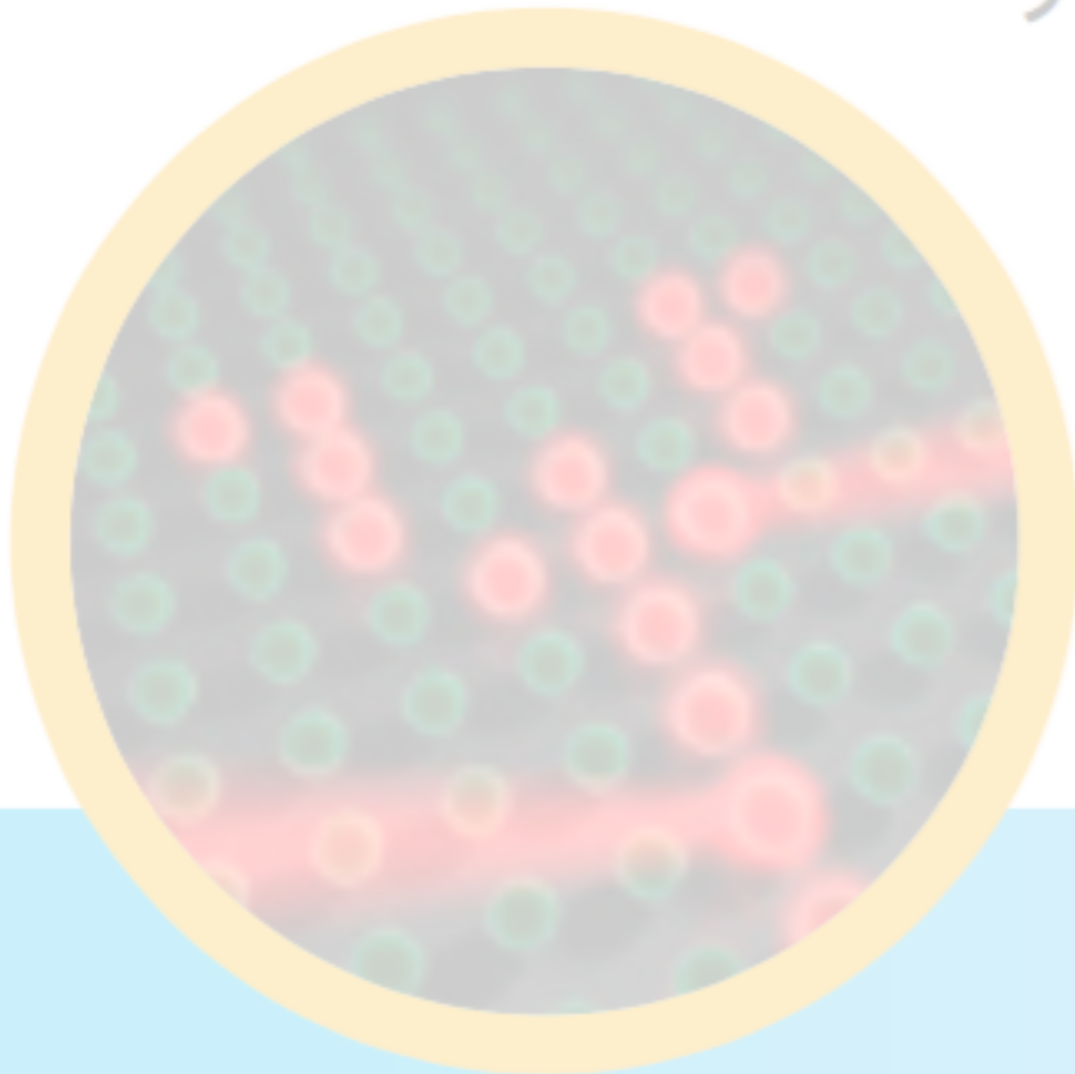
## Summary, outlook and open questions

- Analog quantum simulators already outperform good classical algorithms
- Hope for **feasible** quantum simulators with **superpolynomial speedup**

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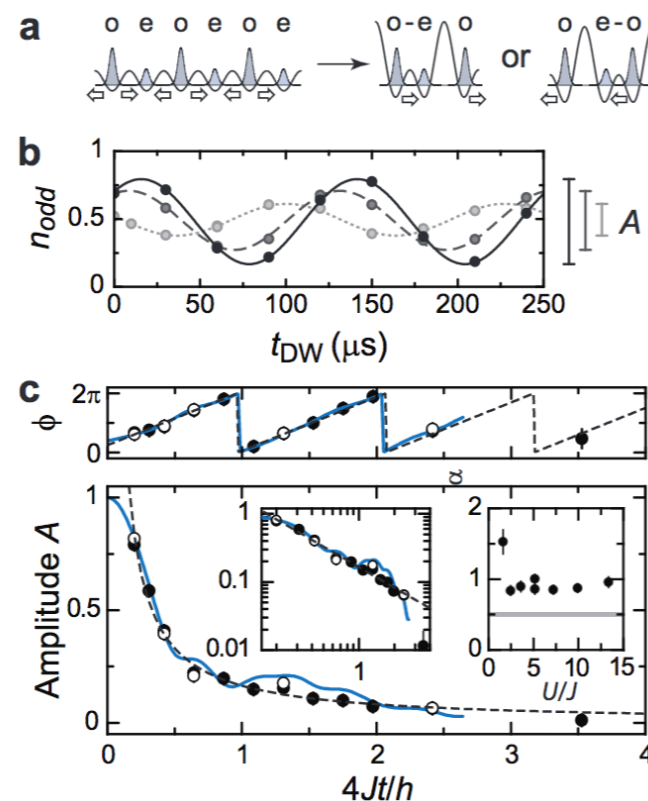
QUANTUM  
INTEGRATION



## Summary, outlook and open questions

- Analog quantum simulators already outperform good classical algorithms
- Hope for **feasible** quantum simulators with superpolynomial speedup
- Not fault tolerant, but can be certified: Bell test for quantum computing  
- even if simulators exhibit quantum computational speedup

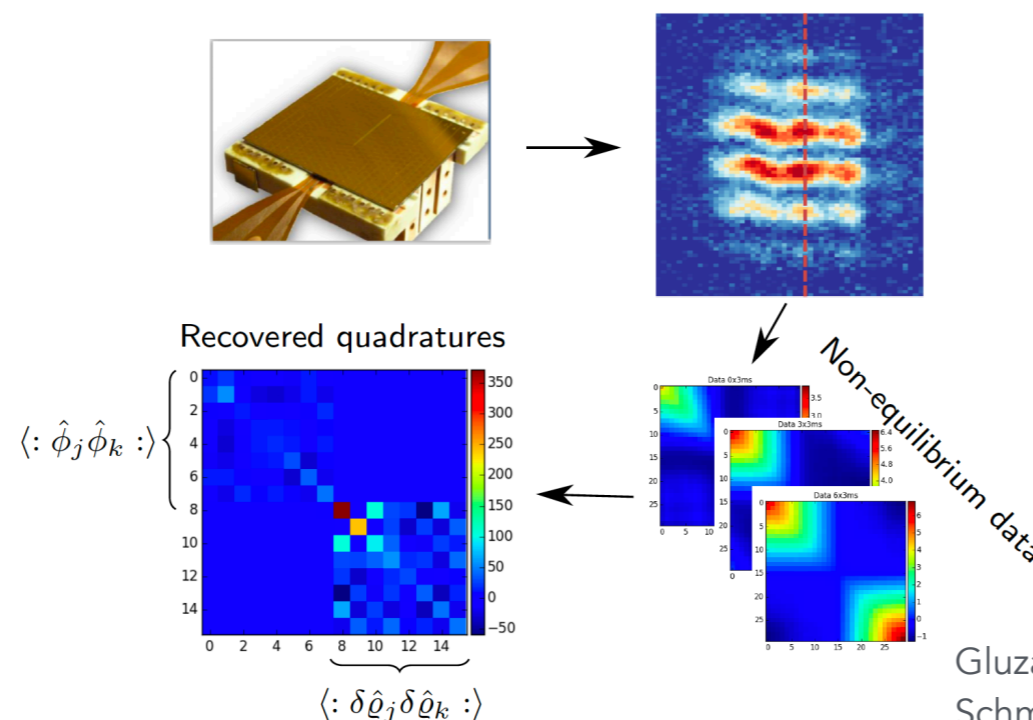
- Closer to physically more interesting schemes?
- More structured problems, optimization?



# Summary, outlook and open questions

- Analog quantum simulators already outperform good classical algorithms
- Hope for **feasible** quantum simulators with **superpolynomial speedup**
- Not fault tolerant, but can be **certified**: Bell test for quantum computing - even if simulators exhibit quantum computational speedup

- Closer to nuclear physics questions?
- More structured problems, optimization?
- Robustness of quantum simulators? Readout?



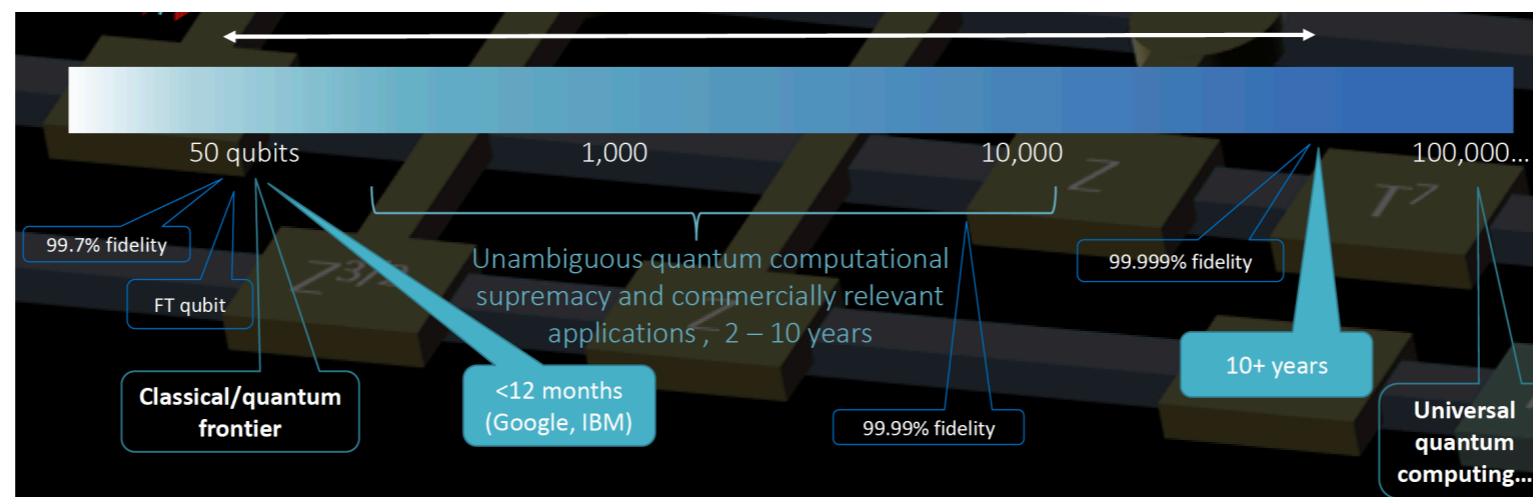
Gluza, Schweigler, Krumnow, Rauer, Schmiedmayer, Eisert, in preparation



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- Space time trade offs?



(Mick Bremner)



## Summary, outlook and open questions

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# Thanks for your attention!