# Nuclear dynamics with quantum computers

## (and some more on the deuteron & measurement problem)

Alessandro Roggero

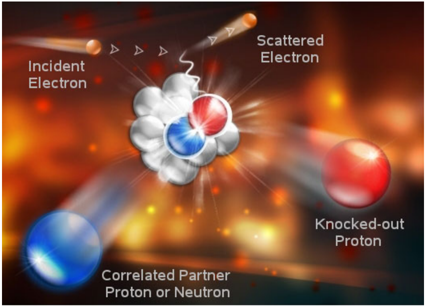



figure credit: JLAB collab.

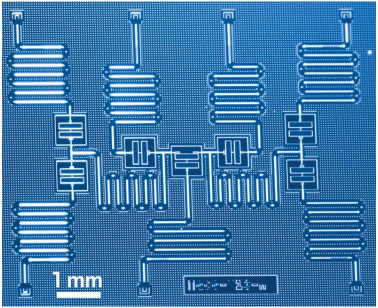


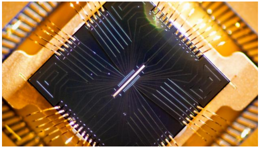

figure credit: IBM

Quantum computing and
scientific research

Saclay – 14 June, 2019

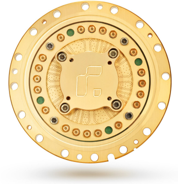
# What is a Quantum computer?

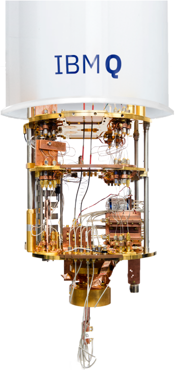
JQI@Univ. of MD



Google

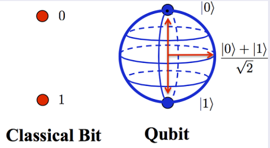

Righetti






- Microsoft?

## Bits vs Qubits




- N bits: an integer number $< 2^N$
- N qubits: a vector $|\psi\rangle$ in $2^N$-dim Hilbert-space

$\Longrightarrow$ exponentially more information available

# Quantum Simulations with qubits

> *"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical."*
>
> — R.Feynman (1982)

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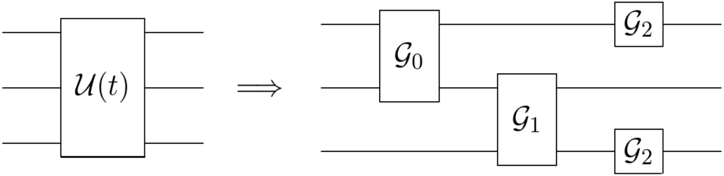
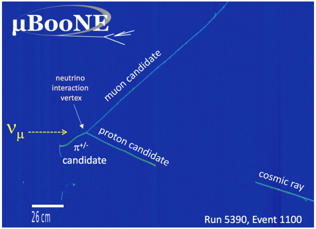
- in 1996 S.Lloyd shows the conjecture is correct for local interactions

- choose a finite basis to discretize system $\longrightarrow dim(\mathcal{H}) = \Omega \propto e^A$
- physical states can be mapped in states of $\sim log_2(\Omega)$ qubits

$$|\Psi(t)\rangle = U(t)\,|\Psi(0)\rangle$$



# Exclusive cross sections in neutrino oscillation experiments



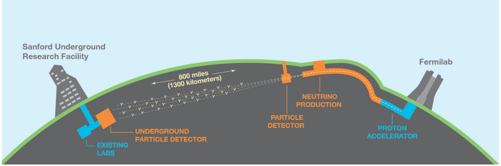

**Goals for $\nu$ oscillation exp.**

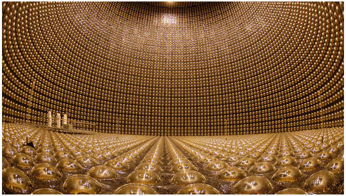
- neutrino masses
- accurate mixing angles
- CP violating phase

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - sin^2(2\theta) sin^2 \left( \frac{\Delta m^2 L}{4 E_\nu} \right)$$

- need to use measured reaction products to constrain $E_\nu$ of the event

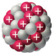
DUNE, MiniBooNE, T2K, Miner$\nu$a, NO$\nu$A,. . .



# Idealized algorithm for exclusive processes at fixed q

- prepare the target ground state



Roggero & Carlson (2018)

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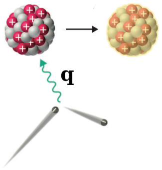


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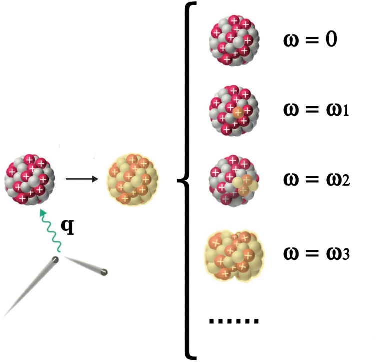


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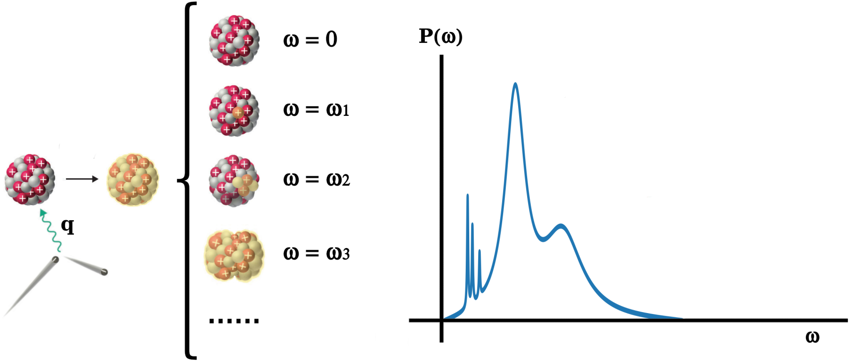


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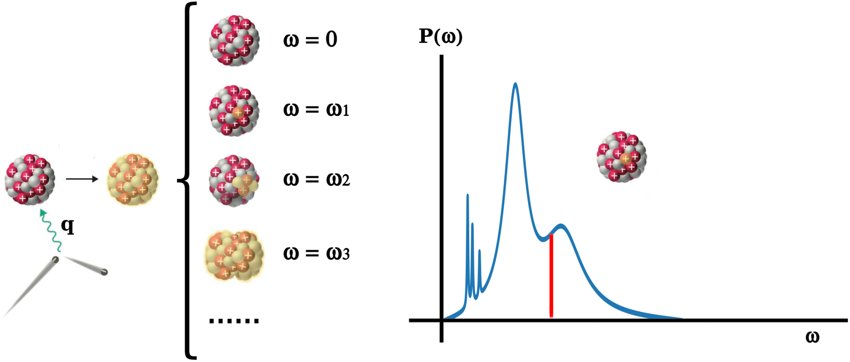


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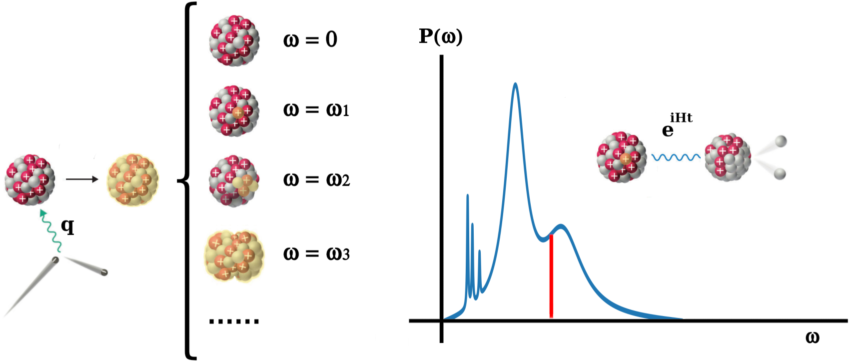


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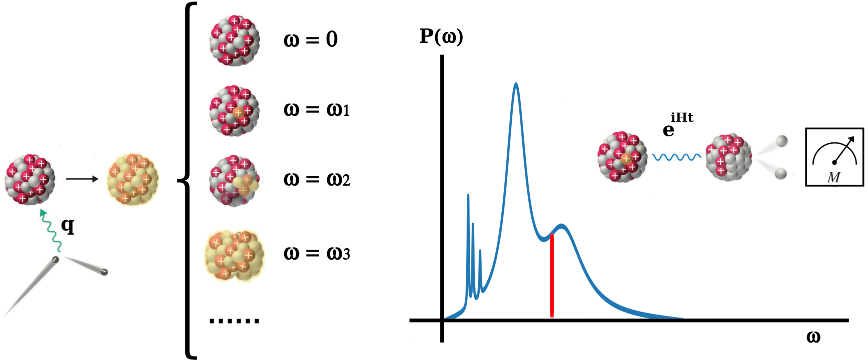


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# Quantum algorithm for exclusive processes at fixed q

- prepare the target ground state on a finite qubit basis
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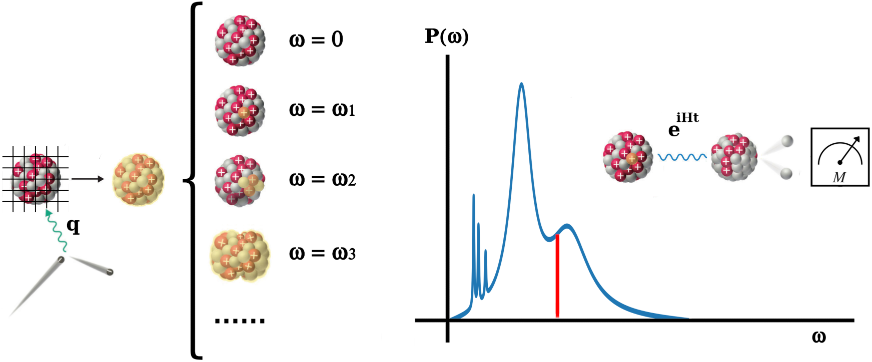


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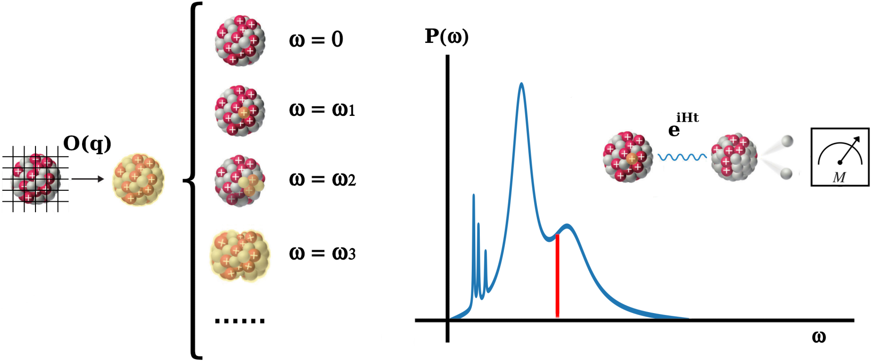


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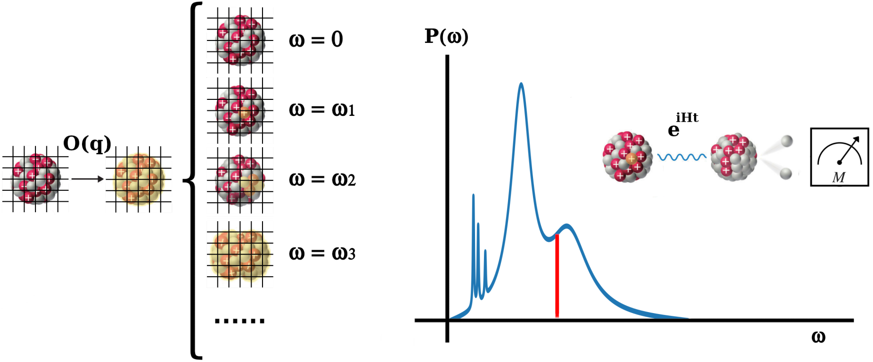


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# Quantum algorithm for exclusive processes at fixed q

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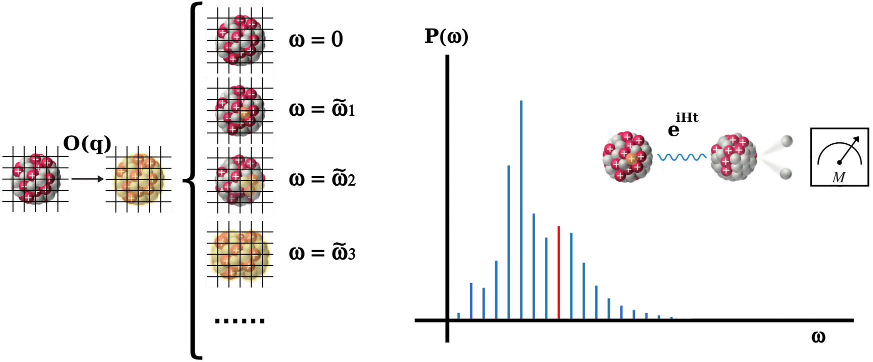


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# Quantum Phase Estimation

Kitaev (1996), Brassard et al. (2002), Svore et. al (2013), Weibe & Granade (2016),. . .

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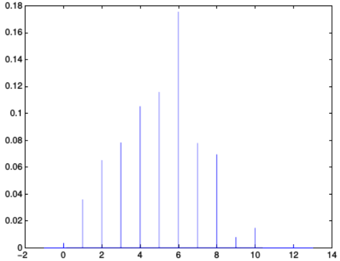

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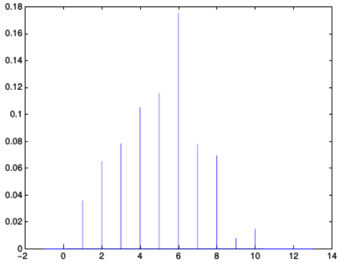


BONUS: final state after measurement is $|\psi_{fin}\rangle \approx \sum_k \delta(\lambda_k - \lambda_n)c_k|\xi_k\rangle$

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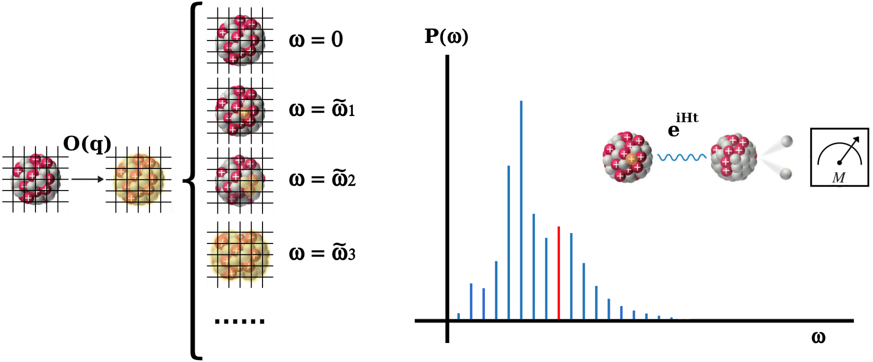


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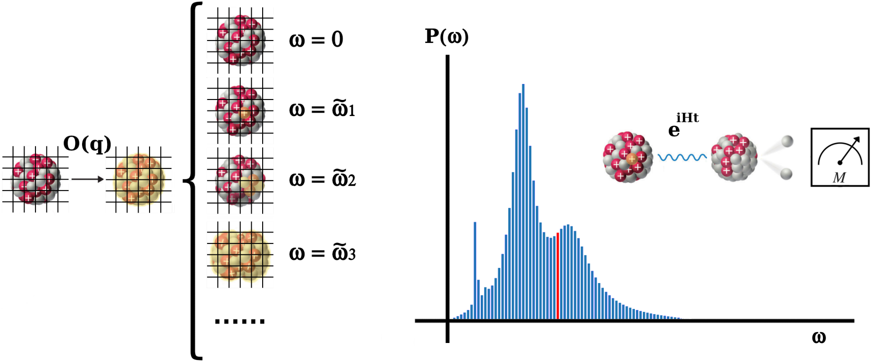


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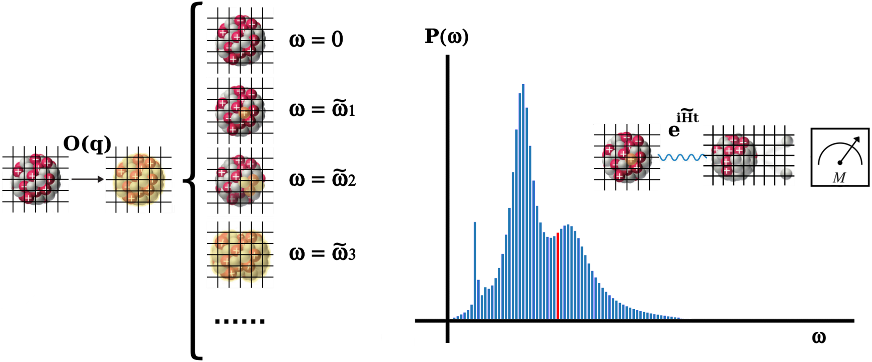


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# How practical is all this?

- pionless EFT on a $10^3$ lattice of size $20$ fm [$a = 2.0$ fm]
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- want $R(q, \omega)$ with $20$ MeV energy resolution

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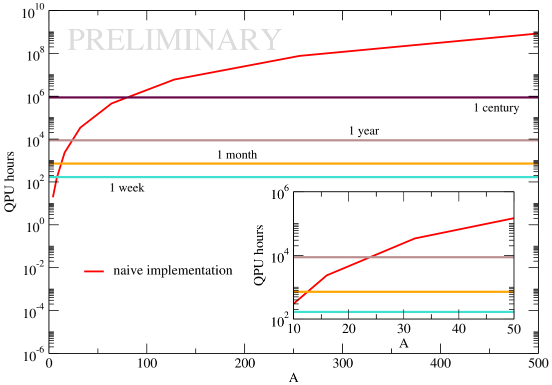

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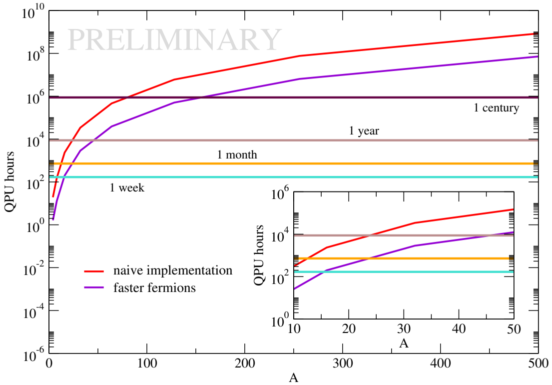

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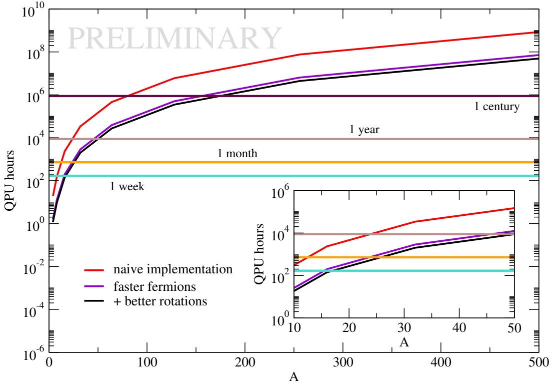

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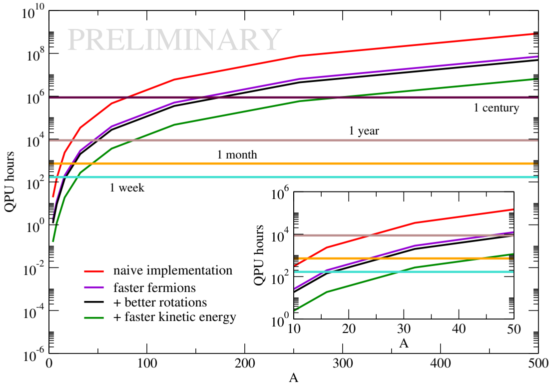

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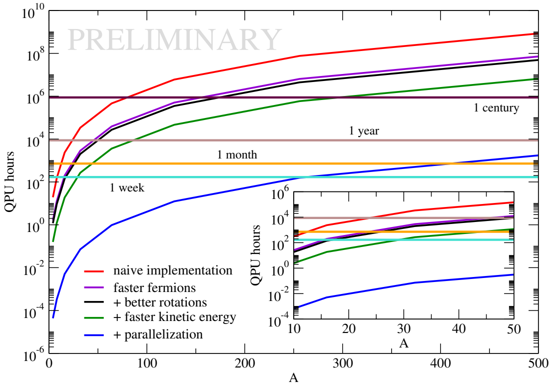

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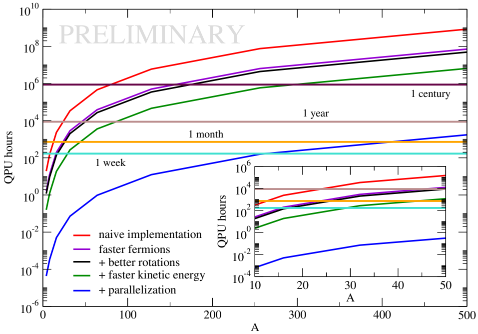
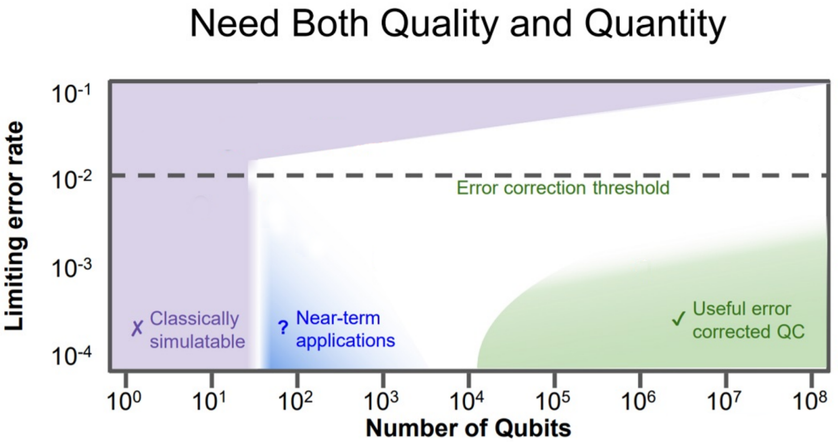



| coherence time for $^{40}$Ar | |
|---|---|
| naive | $\approx 9$ years |
| optimized | $\approx 3$ minutes |

- algorithm efficiency is critical
- there is still a long way to go
- find new algorithms and/or approximations for near term

# Where are we right now?

figure adapted from Google AI



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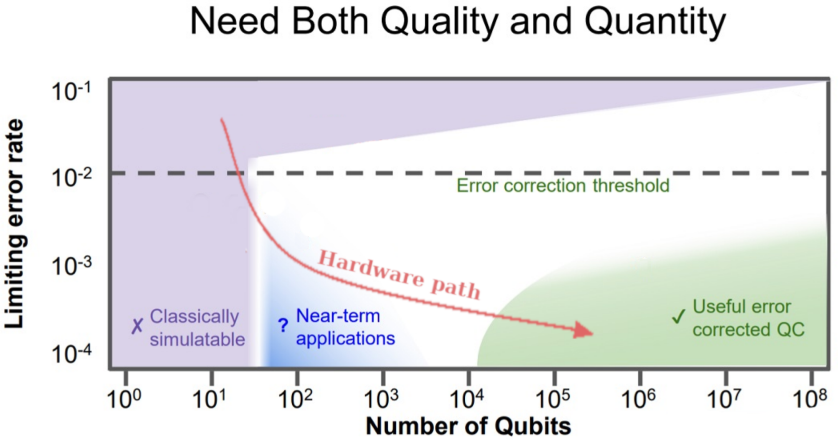

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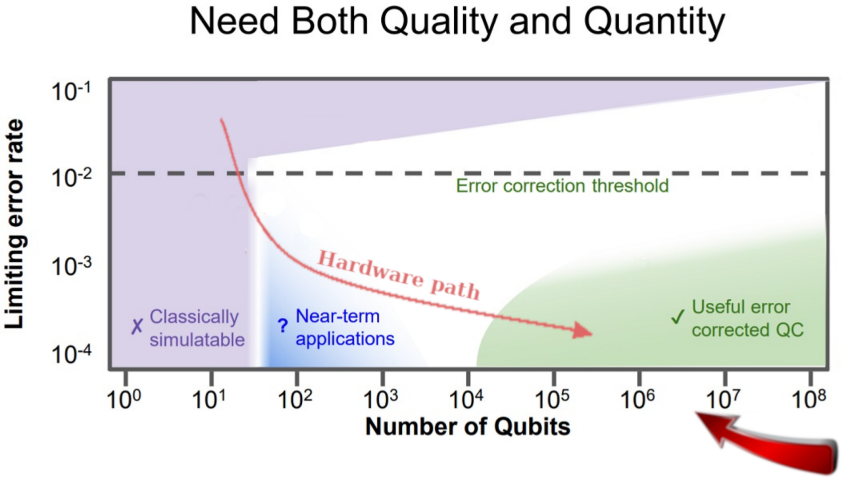

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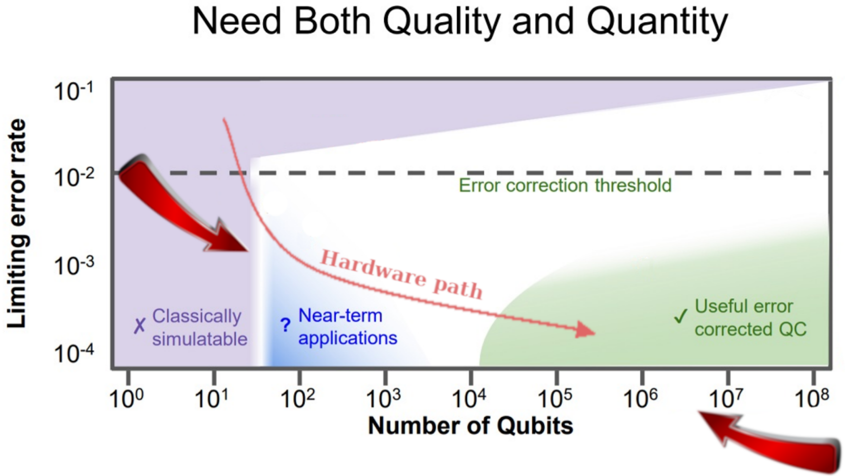
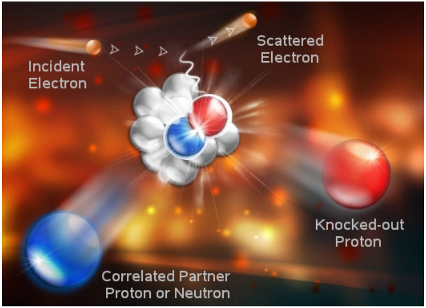



**FNAL - INT - LANL effort**

A.R. (INT), J. Carlson & R. Gupta (LANL), G. Perdue, A. Li & A. Macridin (FNAL)

# Part II: What can we do already?

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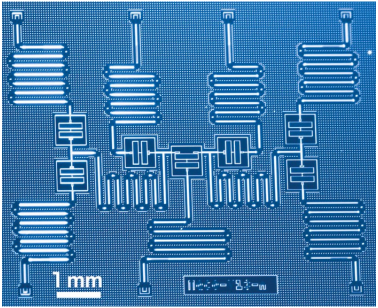


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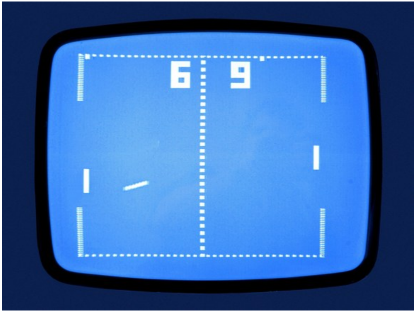
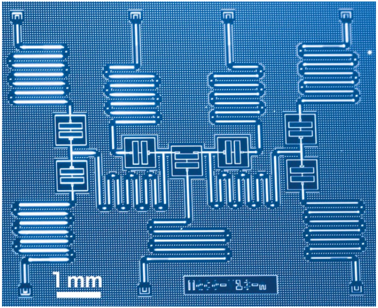



credit: Atari Inc.

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# Pong for a nuclear theorist: the deuteron

- first calculation with $\pi$-less EFT: Dumitrescu et al. (2018)

$$H = K + V_{12}^{s} + V_{12}^{\pi}$$

- $\pi$-exchange introduces S-D mixing $\Rightarrow Q \neq 0$ in the gs.

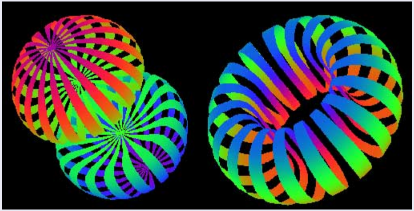

Figure from Forest et al. (1995)

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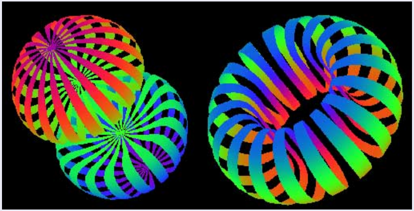

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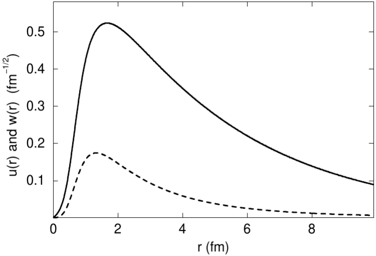


$$H_d \approx \begin{pmatrix} 5 & -35 \\ -35 & 170 \end{pmatrix}$$

completely mapped in just one qubit

$$|gs\rangle = cos(\theta)\,|0\rangle + sin(\theta)\,|1\rangle$$

Roggero & Baroni arXiv:1905.08383

# How hard could this be?

1. first map deuteron Hamiltonian in Pauli basis

$$H_d = \begin{pmatrix} 5 & -35 \\ -35 & 170 \end{pmatrix} = 87.5 \times \mathbb{1} - 35 \times X - 82.5 \times Z$$

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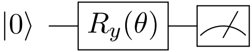
3. prepare the gs with the appropriate rotation and measure polarization

$$|gs\rangle = exp\left(i\theta Y\right)|0\rangle \longrightarrow$$



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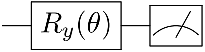


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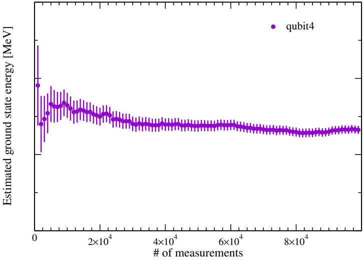


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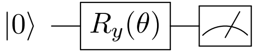


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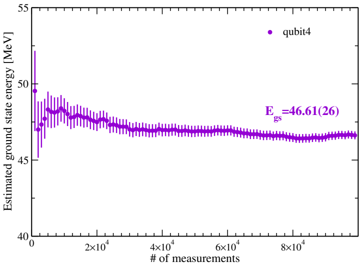


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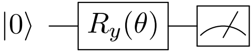


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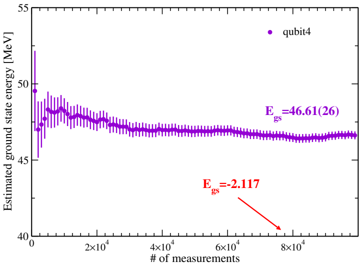


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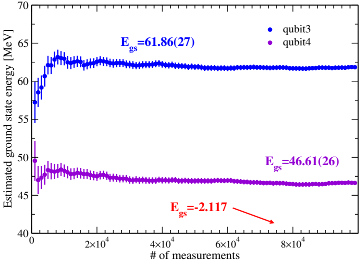


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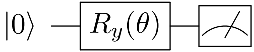


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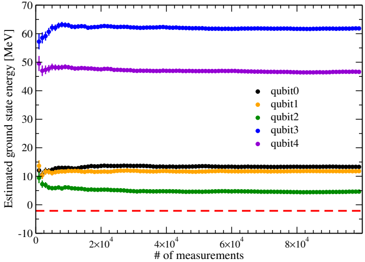


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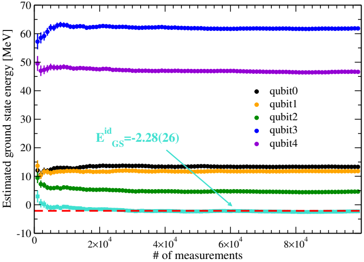


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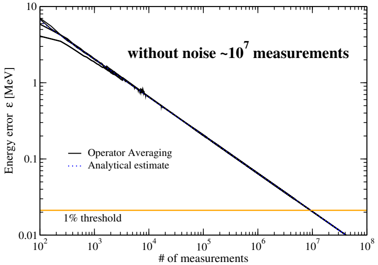


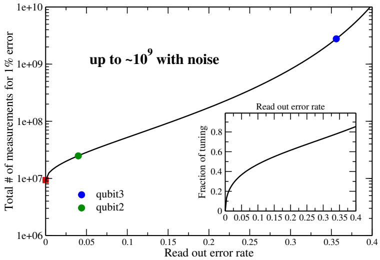


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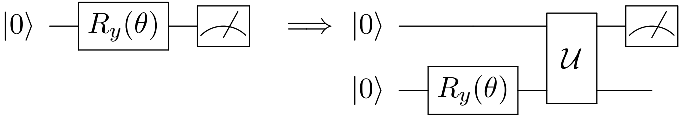
# A (non computer sciency) possible way out

Consider instead a (slightly) more complex circuit



The 2-qubit unitary $\mathcal{U}$ can be engineered so that

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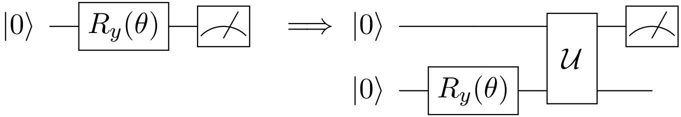


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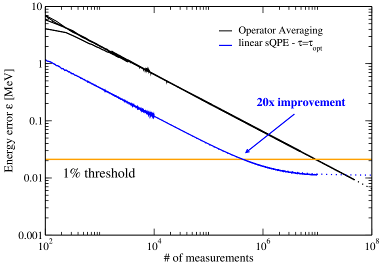


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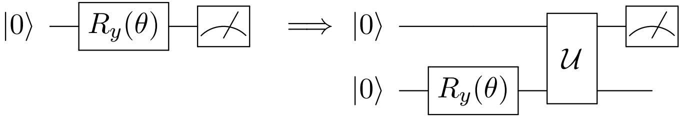


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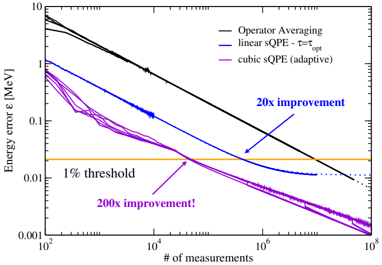


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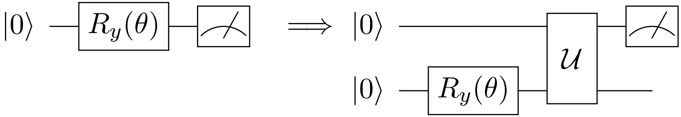


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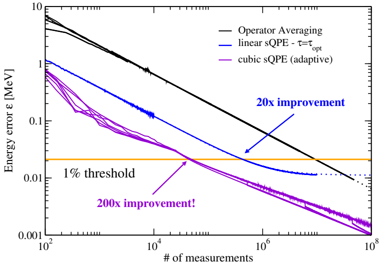


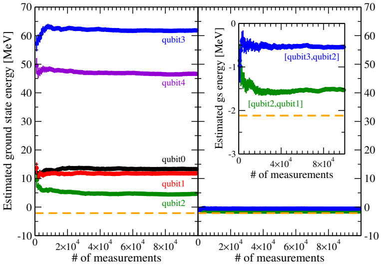


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# Summary

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## Collaborators:

- Joe Carlson (LANL)
- Alessandro Baroni (USC→LANL)