Symmetry breaking and restoration for trapped electrons, ions, and neutral atoms

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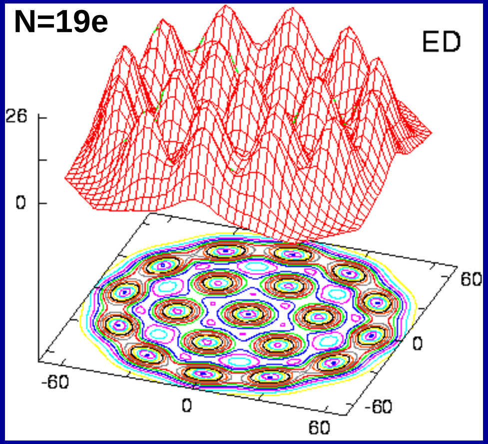
R_W=5, B=0



Rep. Prog. Phys. 70, 2067 (2007);

PRA 96, 043610 (2017)

Symmetry breaking and symmetry preserving schemes : how to efficiently grasp collective correlations in mesoscopic many-body systems?
CEA-Saclay, May 13-17, 2019

Supported by the U.S. AFOSR (FA9550-15-1-0519)

CONTROL PARAMETERS FOR SYMMETRY BREAKING

IN SINGLE QD'S: WIGNER CRYSTALLIZATION

- Essential Parameter at B=0: (parabolic confinement)

$$R_W = (e^2/\kappa l_0)/\hbar\omega_0 \sim 1/(\hbar^3\omega_0)^{1/2}$$

e-e Coulomb repulsion kinetic energy

$$l_0 = (\hbar / m^* \omega_0)^{1/2} \quad \}$$ Spatial Extent of 1s s.p. state

κ : dielectric const. (12.9)

m^* : e effective mass ($0.067\ m_e$) GaAS

$\hbar\omega_0$ (5 - 1 meV) => R_W (1.48 - 3.31)

- In a magnetic field, essential parameter is B itself

IN QDM'S: DISSOCIATION (Electron puddles, Mott transition)

Essential parameters: Separation (d)
Potential barrier (V_b)
Magnetic field (B)

$$R_\delta = gm/(2\pi\hbar^2)$$

Neutral bosons

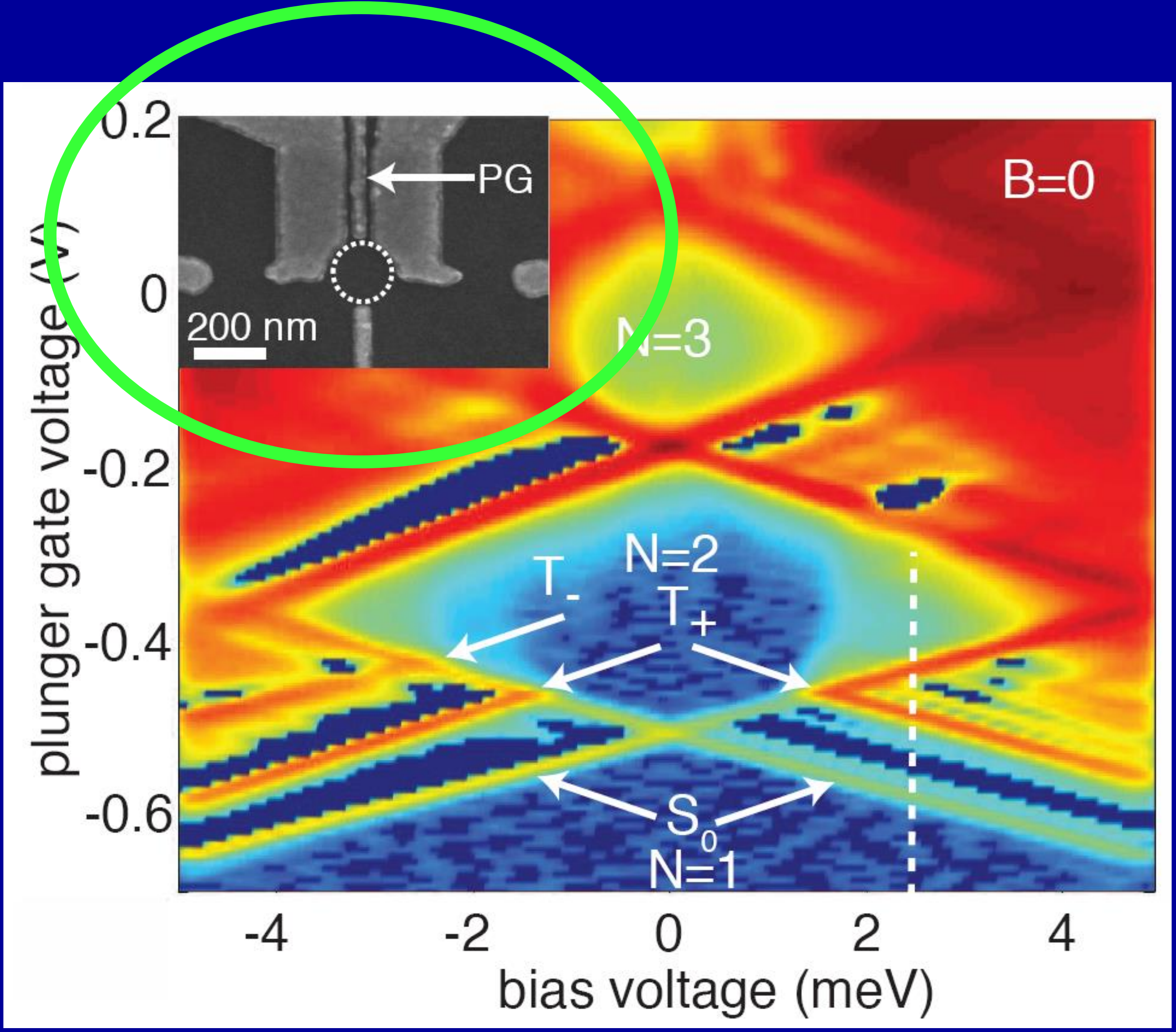
Towards controlling symmetry breaking and symmetry restoration in both space and time in manmade nanosystems (SMALL IS DIFFERENT)

Unprecedented experimental control of few-body systems of trapped ultracold ions and neutral atoms, and of few-electron assemblies in quantum dots

Time-evolution phenomena in quantum mechanical finite systems which is not captured by mean-field approaches

Different direction from that in the book: "Basic Notions of Condensed Matter Physics", by P.W. Anderson, 1984 **(MORE IS DIFFERENT)**

ETH (Zurich) single QD



Ellenberger, Ensslin, Yannouleas, Landman et al.,
Phys. Rev. Lett. **96,** 126806 (2006)

Control and measurement of three-qubit entangled states

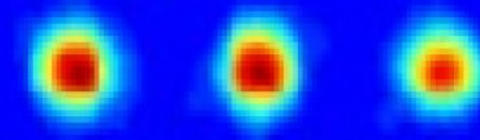
C. F. Roos[1], Mark Riebe[1], H. Häffner[1], W. Hänsel[1], J. Benhelm[1], G. P. T. Lancaster[1], C. Becher[1], F. Schmidt-Kaler[1] & R. Blatt[1,2]

[1]Institut für Experimentalphysik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria
[2]Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften

- Basics of ion trap quantum computers
- Entangling operations (Bell states, CNOT)
- Generation of W- and GHZ-states
- Selective read-out of a quantum register
- Entanglement transformation by condional operations

Entangled states with three ions

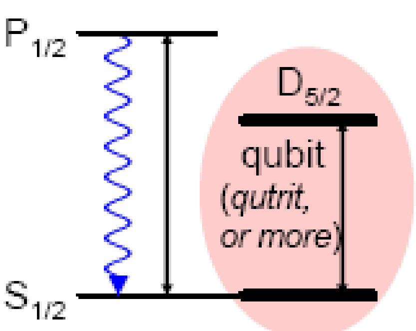
GHZ-states: $|SSS + DDD\rangle$



W-states: $|SDD + DSD + DDS\rangle$

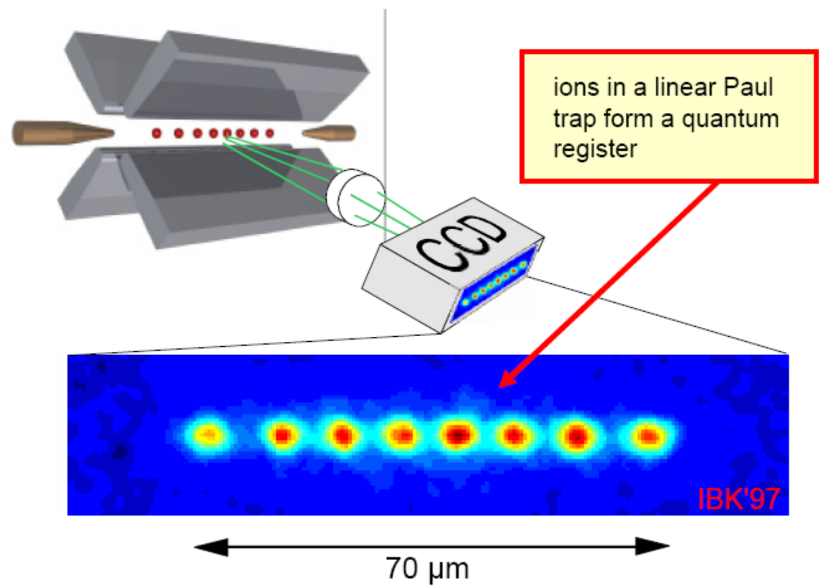
H. Haeffner et al., Nature 438, 643 (2005)

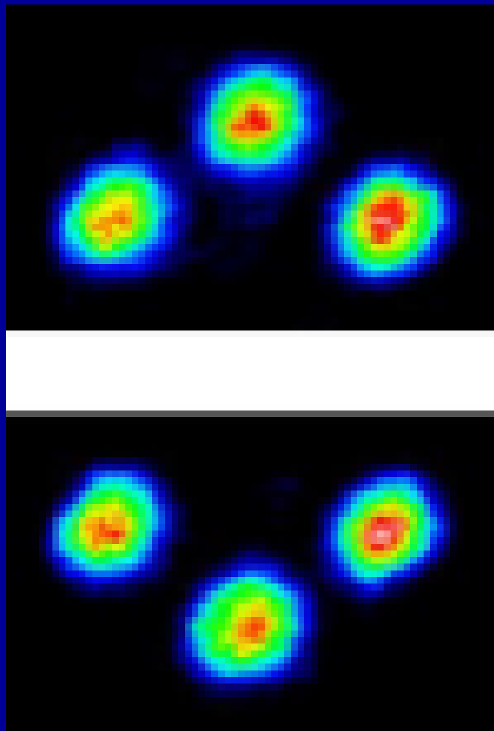
Qubits with trapped ions

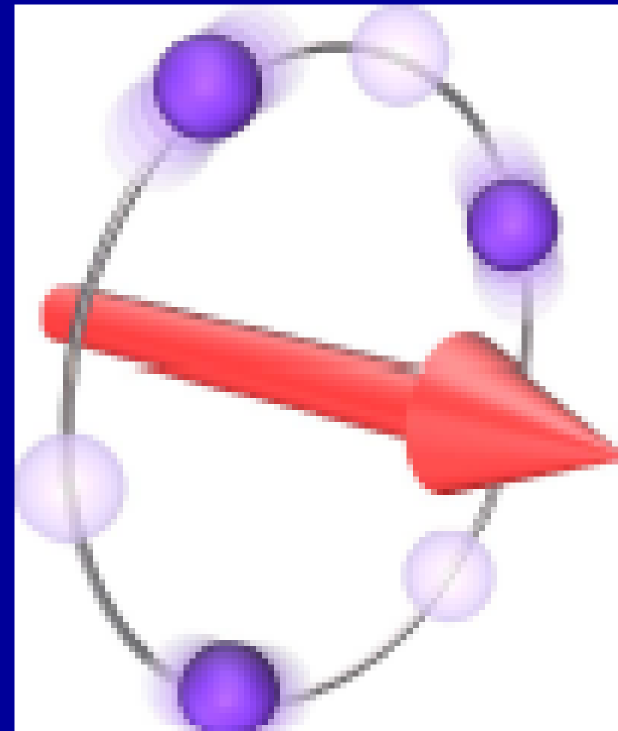


S – D transitions in alkaline earths:
Ca^+, Sr^+, Ba^+, Ra^+, (Yb^+, Hg^+) etc.

The system: String of $^{40}Ca^+$ ions in a linear Paul trap



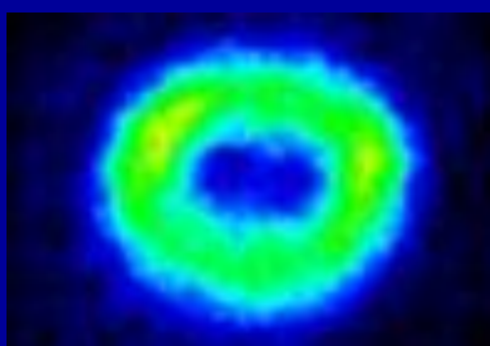


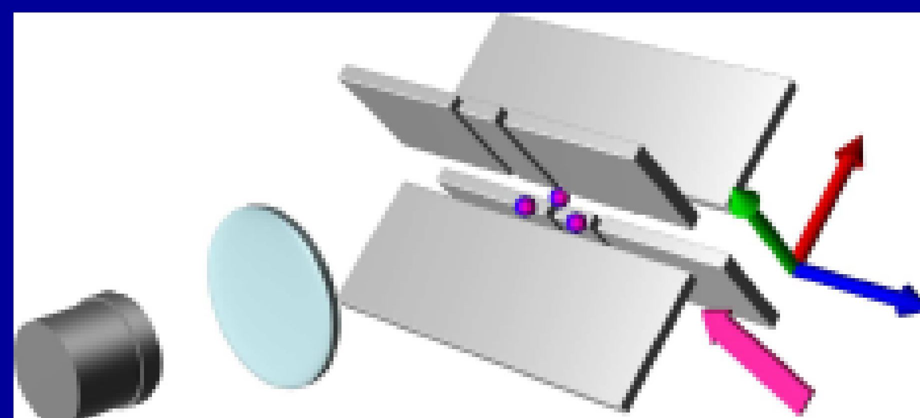


$^{40}Ca^{+}$

Fermion

B





Noguchi, A. et al., Nat. Commun. 5:3868 (2014).

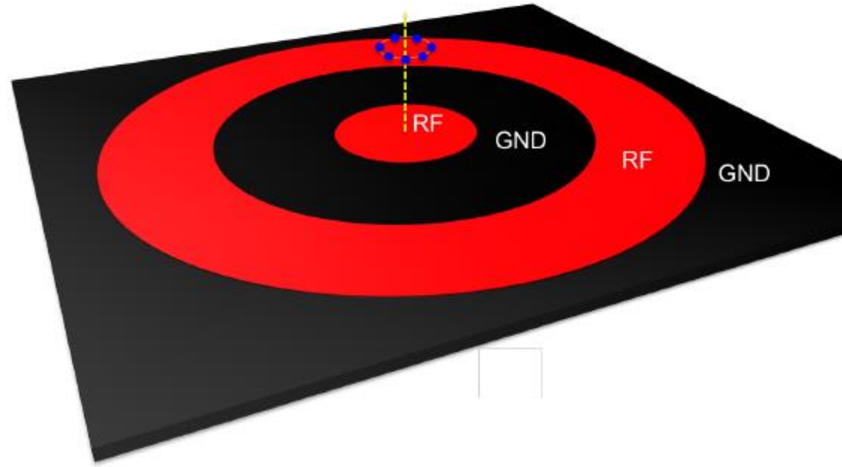
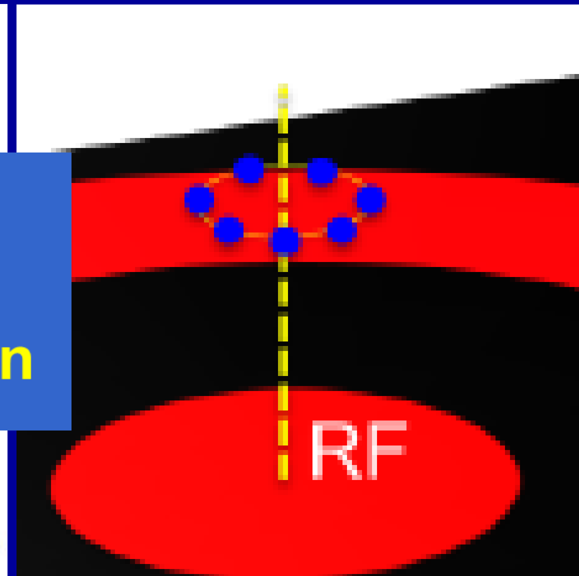


Figure 4.7: Concept for symmetric ring trap. RF electrodes are shown in red, ground in black. Ions trapped above the surface shown in blue.

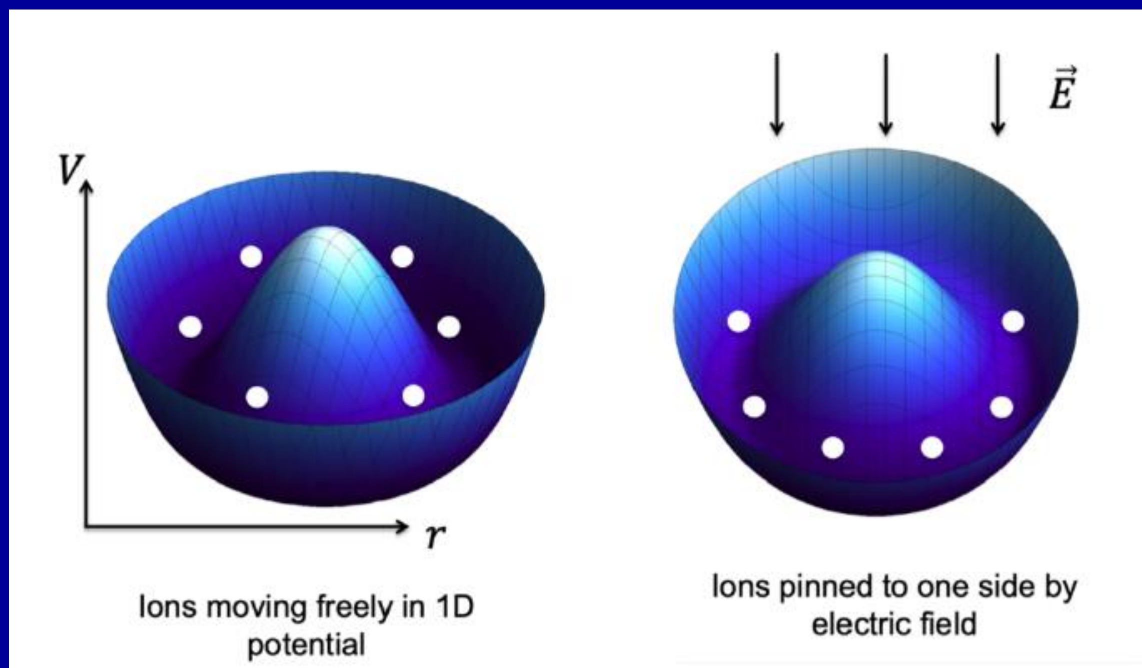
$^{40}Ca^{+}$

Fermion



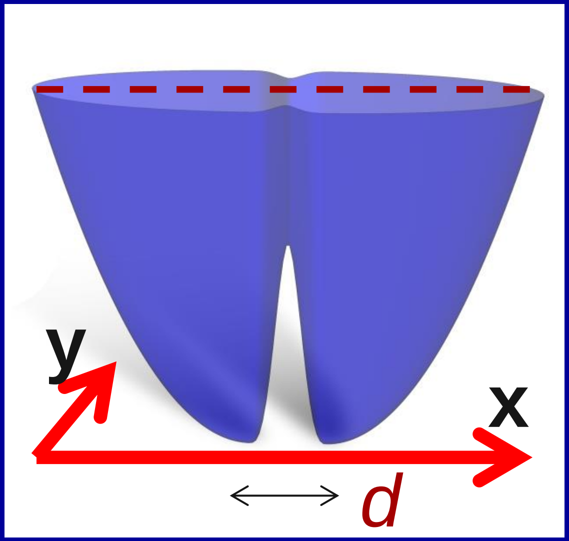
Erik Urban, Ph.D. Thesis, Univ. of California, Berkeley, 2019

Urban et al.,
Coherent Control of the Rotational Degree of Freedom of a Two-Ion Coulomb Crystal,
arXiv:1903.05763



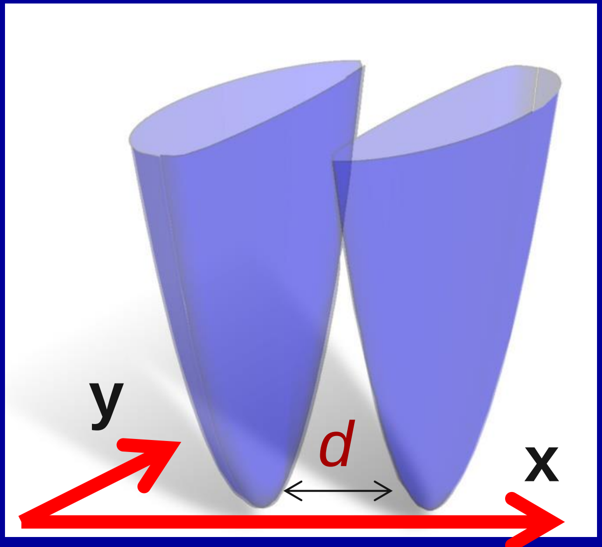
2 ^{6}Li ATOMS IN A DOUBLE OPTICAL TRAP

Linear arrangement (LI)

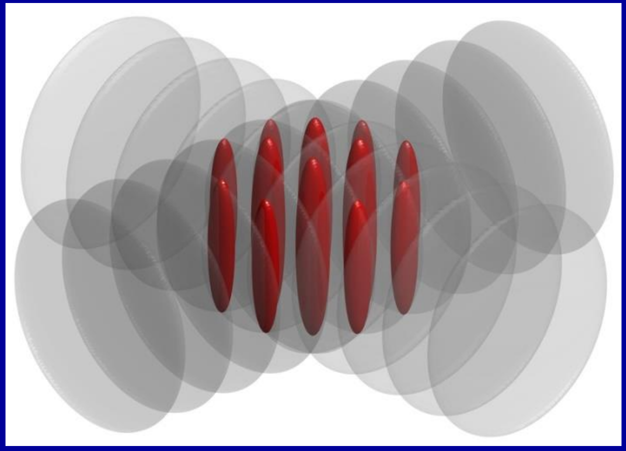


Strictly 1D

Parallel arrangement (PA)



2D aspects



Experimental depictions of 1D optical traps

LOWERING OF ENERGY AND SYMMETRY BREAKING (SB) IN SELF-CONSISTENT MEAN-FIELD APPROACHES

Restricted vs unrestricted and fermions vs bosons

Attraction

Restricted approach (RHF)

Same orbital and
symmetry breaking (SB):

Nilsson potential
(fermions, nucleons)

Gross-Pitaevskii (GP)
(attractive bosons,
Lump of ultracold neutral
atoms)

GP for repulsive bosons:
SB raises the energy !

Repulsion

Unrestricted approach (UHF)

Different orbitals and
symmetry breaking (SB):

Hydrogen molecule, dissociation
(electrons, chemistry)

Wigner molecules
(electrons, quantum dots)

Space-time crystals on rings
(fermion or bosons, ultracold ions)

UNRESTRICTED HF FOR REPELLING FERMIONS

(Self-consistent Pople-Nesbet Eqs.)

Different orbitals for different spins

Two coupled equations/ Spin-up coupled to spin-down

Self-consistent solution → orbital localization

[example from chemistry:

dissociation of Hydrogen molecule (next slide)]

UNRESTRICTED HF FOR REPELLING BOSONS?

A different orbital for each particle

Self-consistent set of Eqs. is not practical

Reason: orbitals for bosons are not orthogonal/

spin-orbitals for fermions are orthogonal

In case of crystals, employ ansatz = permanent of orbitals approximated as displaced Gaussians

The Hydrogen molecule

Szabo and Ostlund, Modern Quantum Chemistry

3.8.7 The Dissociation Problem and Its Unrestricted Solution

At very large bond lengths, however, one is really trying to describe two individual hydrogen atoms. A proper description will have one electron on one H atom and the other electron on the other H atom, i.e., the two electrons will have quite different spatial distributions. They should not have identical

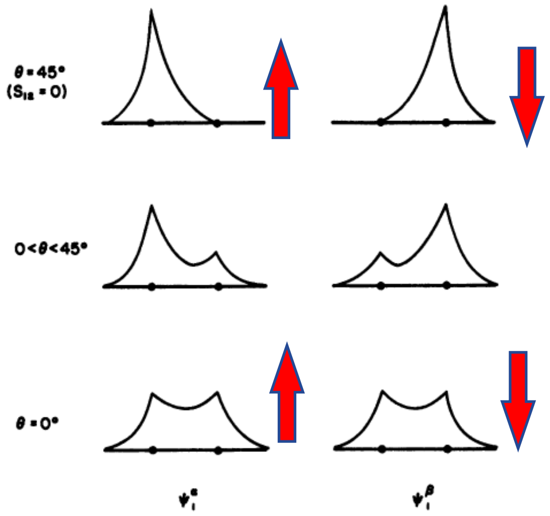
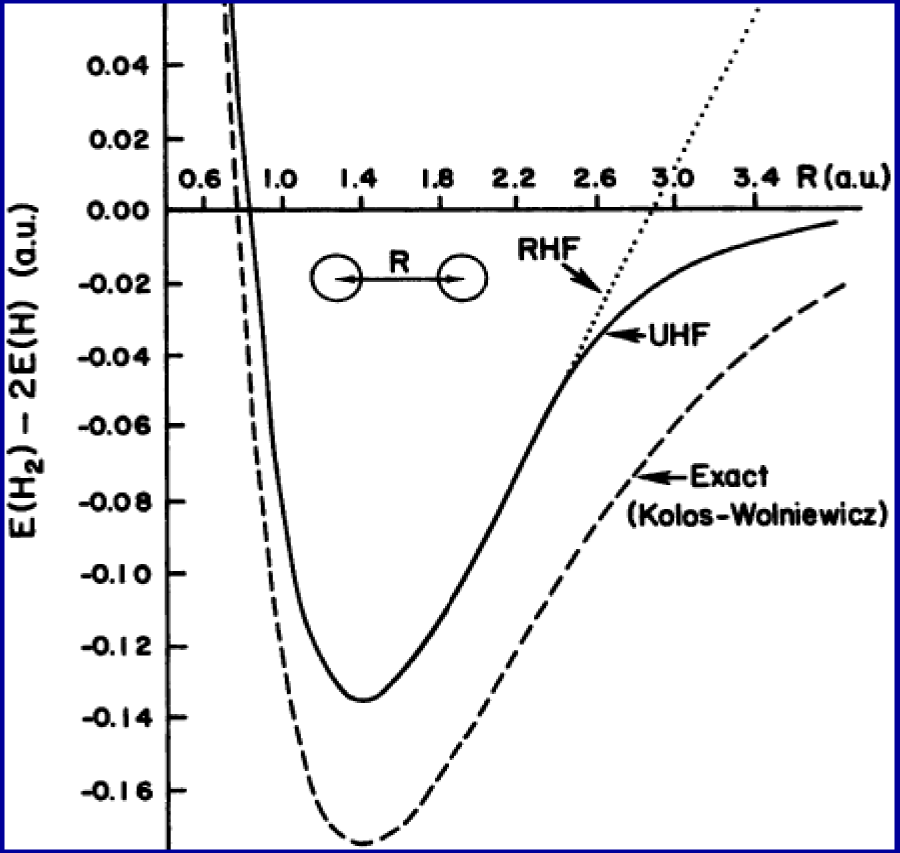
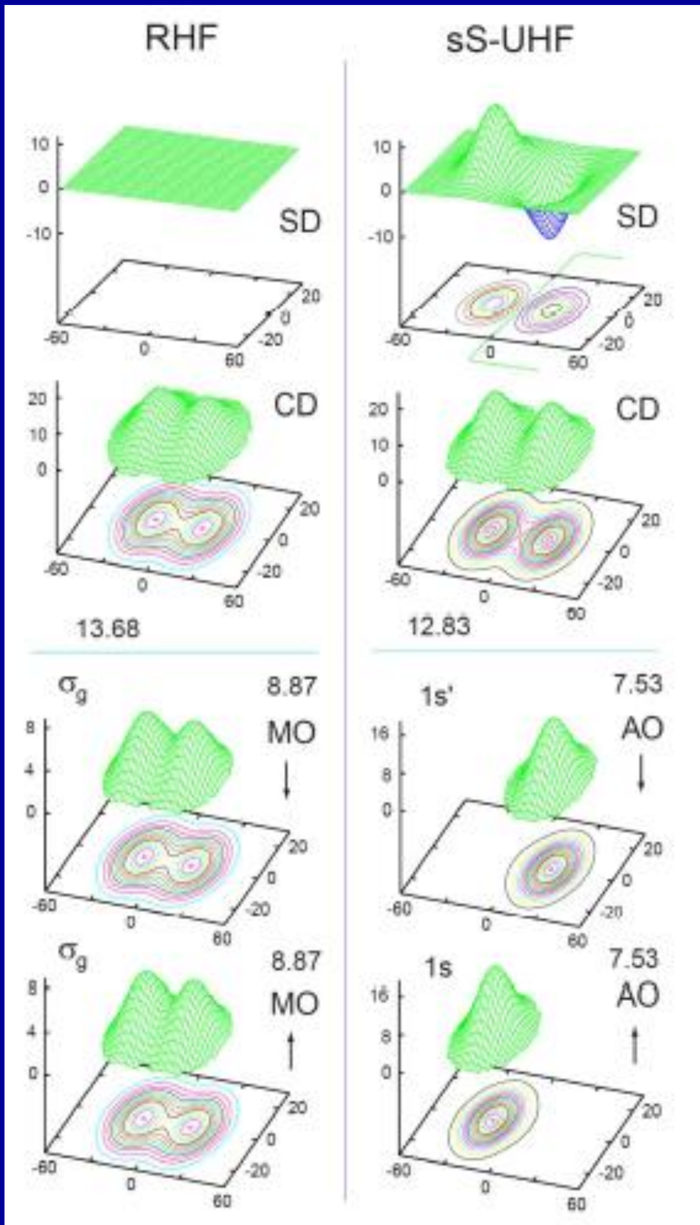


Figure 3.16 Qualitative behavior of unrestricted molecular orbitals ψ_1^α ar
function of θ.



Next step: Heitler-London

H_2-QDM DOUBLE-WELL



$$\sqrt{2}\Psi_{\mathrm{UHF}}(1,2) = \begin{vmatrix} u(\mathbf{r}_1)\alpha(1) & v(\mathbf{r}_1)\beta(1) \\ u(\mathbf{r}_2)\alpha(2) & v(\mathbf{r}_2)\beta(2) \end{vmatrix} \equiv |u(1)\bar{v}(2)\rangle,$$

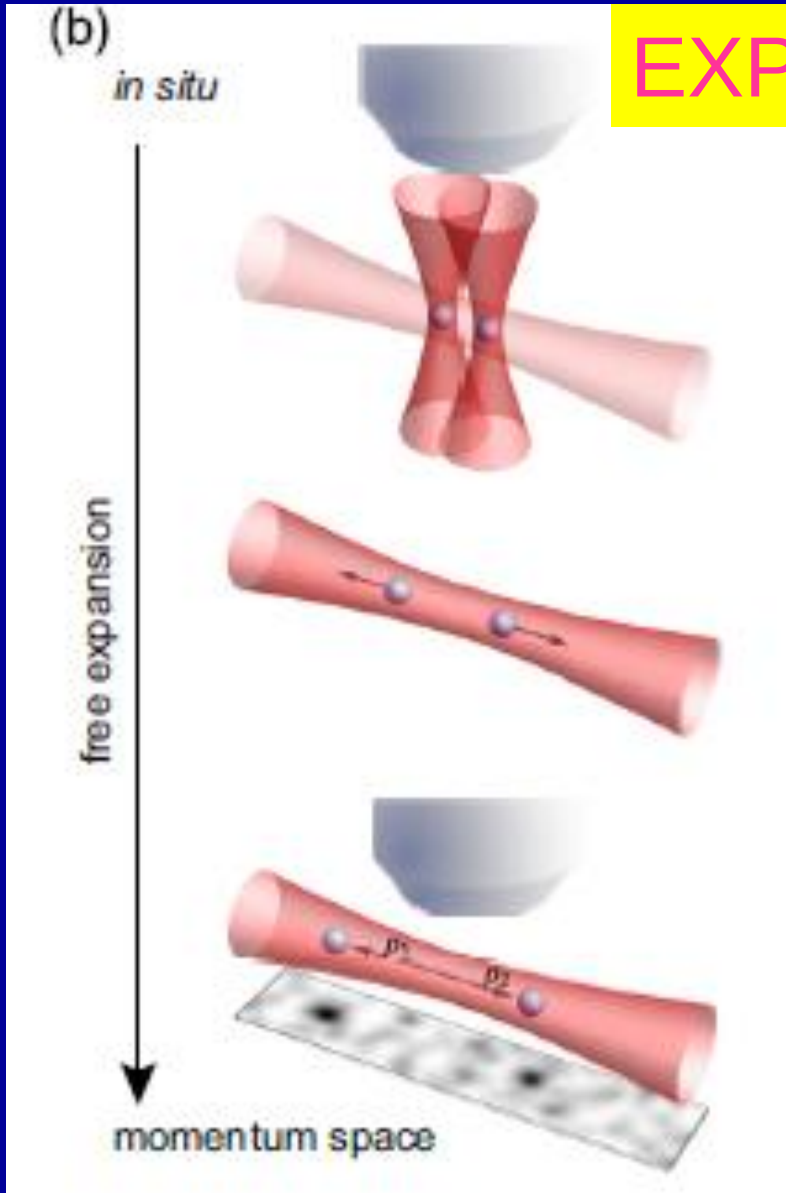
$$\mathcal{P}^{s,t}_{\mathrm{spin}} = 1 \mp \varpi_{12}$$

$$\sqrt{2}\mathcal{P}^{s}_{\mathrm{spin}}\Psi_{\mathrm{UHF}}(1,2) =$$
$$(u(\mathbf{r}_1)v(\mathbf{r}_2) + u(\mathbf{r}_2)v(\mathbf{r}_1))\chi(0,0)$$

$$\chi(s=0, S_z=0) = (\alpha(1)\beta(2) - \alpha(2)\beta(1))/\sqrt{2}$$

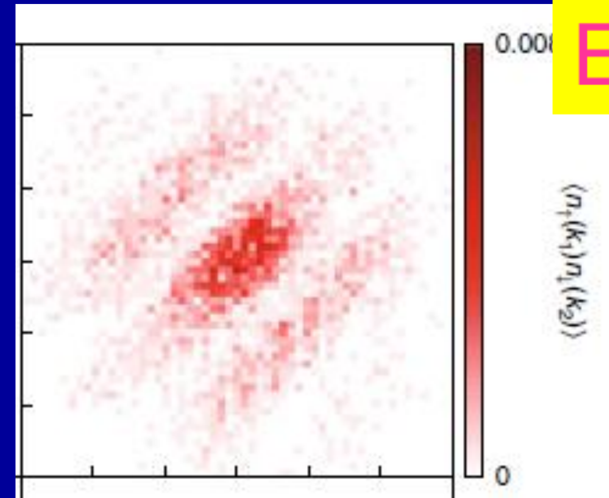
Heitler-London/ EPR-Bohm-Bell

Energy gain vs ENTANGLEMENT

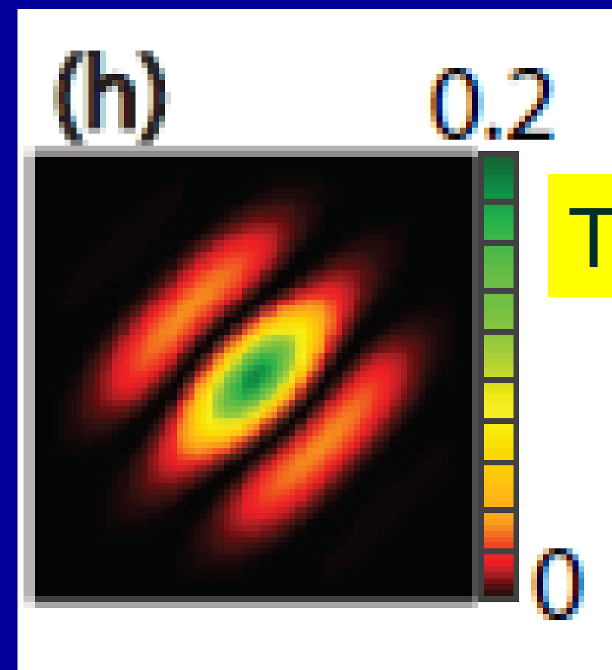


EXP

^{6}Li
Fermion



EXP



THE

Andrea Bergschneider et al. (Heidelberg), Nature Physics, 22 April 2019

Brandt, Yannouleas, Landman, PRA **97**, 053601 (2018)

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9. Vogelstein, B. *et al. Science* **339**, 1546–1558 (2013).
10. Craigie, R. & Bushman, F. D. *Cold Spring Harb. Perspect. Med.* **2**, a006890 (2012).
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12. Reyes, A. *et al. Nature* **466**, 334–338 (2010).
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QUANTUM PHYSICS

Getting the measure of entanglement

A property called entanglement entropy helps to describe the quantum states of interacting particles, and it has at last been measured. The findings open the door to a deeper understanding of quantum systems. SEE ARTICLE P.77

STEVEN ROLSTON

famously bothered by the idea that measuring

To understand what many-particle entanglement means, let's start by considering a non-entangled system. If I create a system that has N particles, each in an identical state independent of their $N-1$ neighbours, then its many-body description is simple, and measuring one particle or partitioning the sample has little impact on the overall system. Not that such states are uninteresting — this is a good description of a state of matter called a Bose–Einstein condensate, for example. Similarly, if each particle is in its own different state, with no relationship to its neighbours, then measurement or partitioning has no global effect.

But if the particles are entangled with one

Measuring entanglement entropy in a quantum many-body system

Rajibul Islam[1], Ruichao Ma[1], Philipp M. Preiss[1], M. Eric Tai[1], Alexander Lukin[1], Matthew Rispoli[1] & Markus Greiner[1]

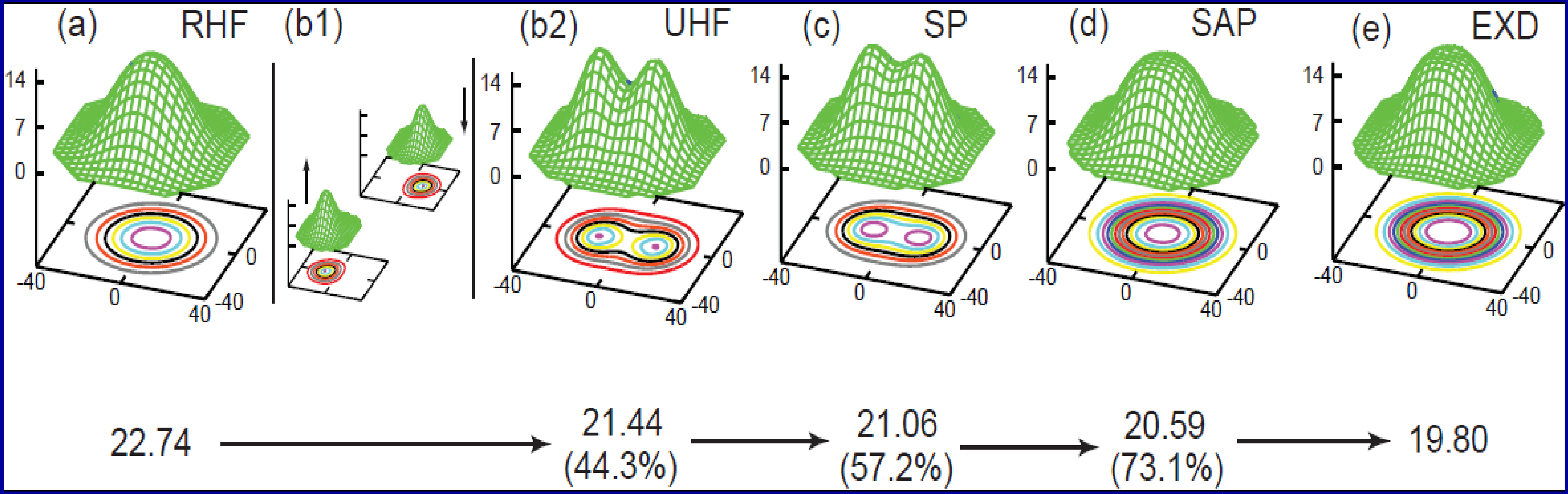
3 DECEMBER 2015 | VOL 528 | NATURE | 77

Even weirder, it's possible for two particles to become "entangled," meaning they will retain a sort of causal relationship with each other, no matter their distance in time and space. If you measure one particle and it spins clockwise, for example, then its entangled companion would instantly collapse into a counterclockwise spin, even if it's on the other side of the universe. That either means that one communicated with the other in an instant, or the state of each particle only popped into existence once one was measured.

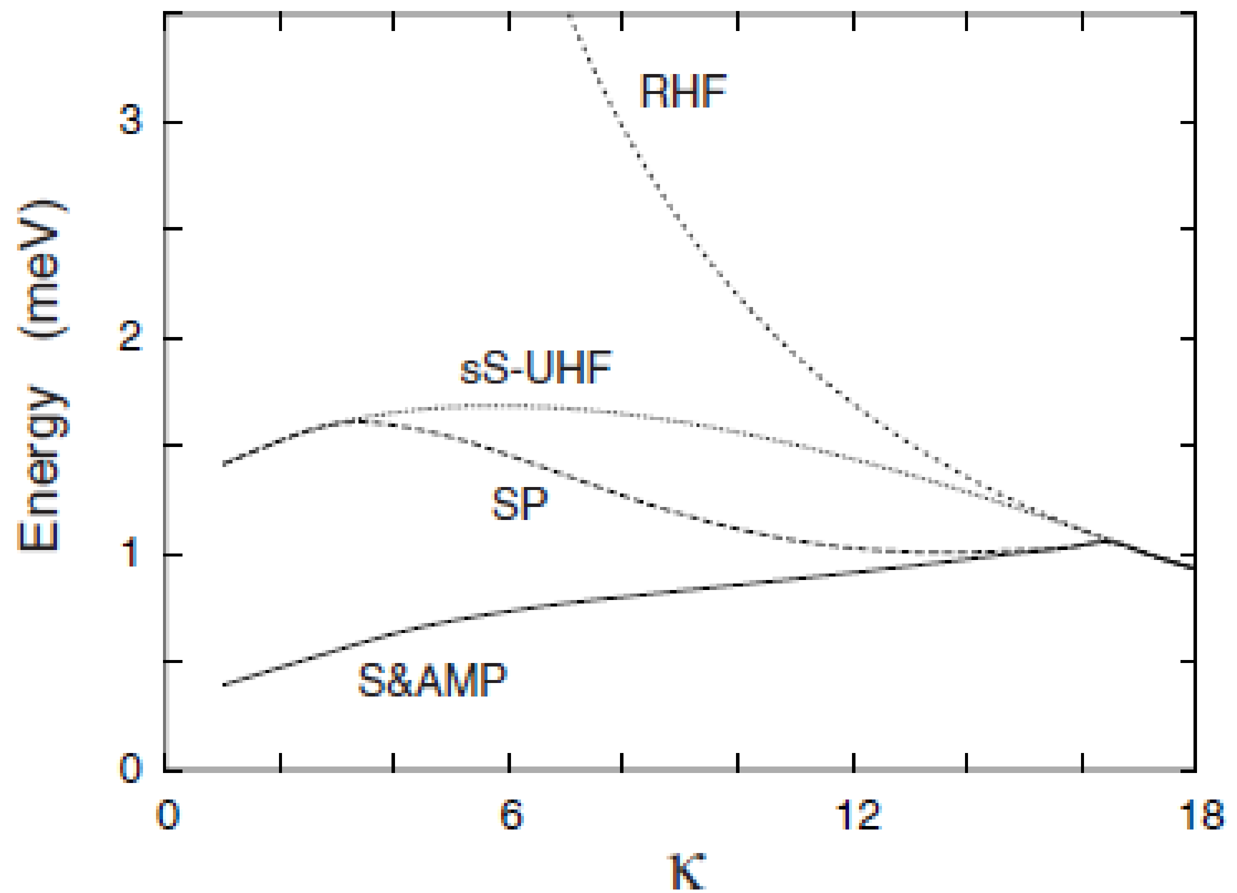
We know what you're thinking. For one thing, this whole idea is ridiculous: Things are what they are regardless of whether you're looking at them. For another, nothing can go faster than light, so how can two particles communicate across the universe in an instant? Einstein thought the same thing, derisively calling the idea "spooky action at a distance." Those in Einstein's camp are in favor of a concept called "local realism." "Locality" says that no signal can travel faster than light, and "realism" says that particles have definite states even before you measure them.

Self-consistent HF → Exact $R_W = 2.40$ $B = 0$

N=2e



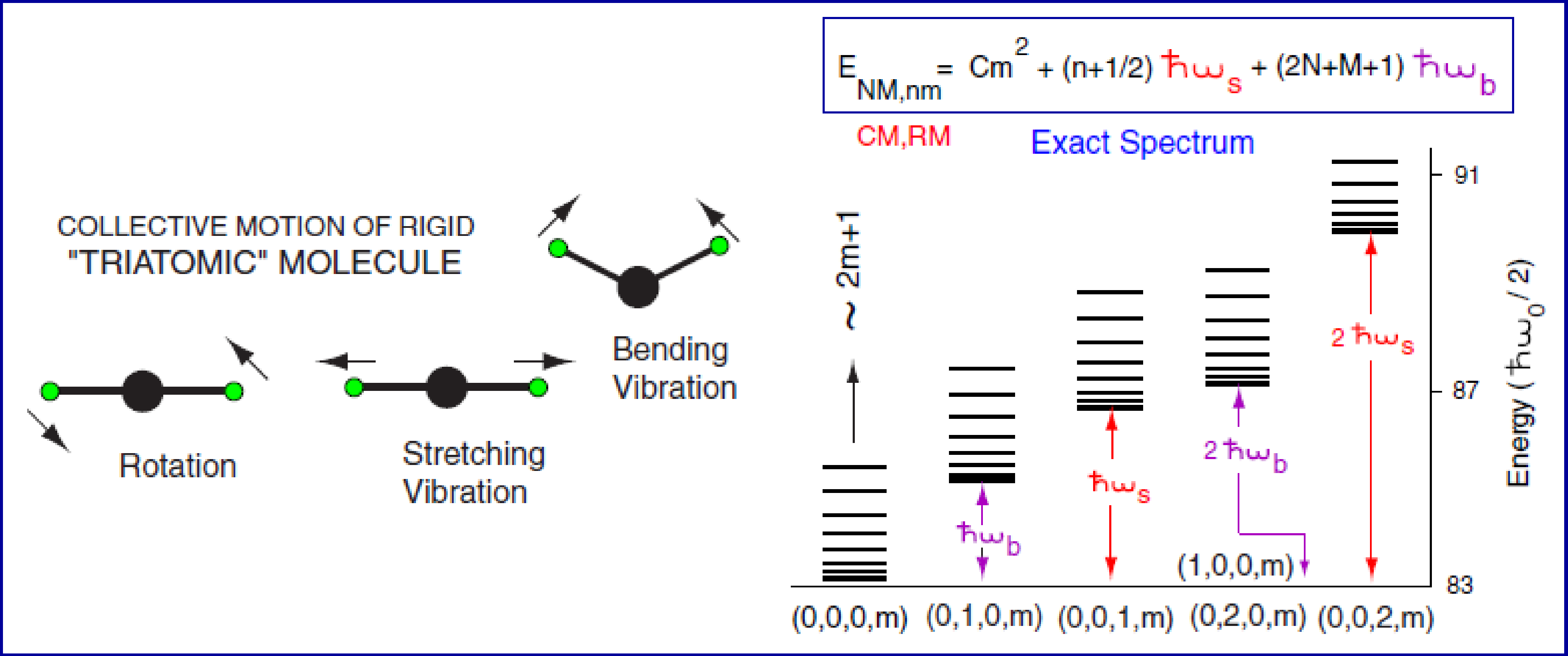
COMBINE PROJECTIONS/
SPIN/ ANGULAR MOMENTUM



RHF
sS-UHF
SP
S&AMP
Energy (meV)
K
0
1
2
3
0
6
12
18

N=2e QD, R_W=200; EXACT

RIGID ROTOR



Y&L, PRL **85**, 1726 (2000)

PROJECTION/ ANGULAR MOMENTUM

$$\mathcal{P}_L = \frac{1}{2\pi}\int_0^{2\pi} e^{i\gamma(L-\hat{L})}d\gamma$$

$$\Phi_L^{\mathrm{PROJ}} = \frac{1}{2\pi}\int_0^{2\pi} d\gamma \Psi^{\mathrm{SB}}(\gamma)e^{i\gamma L}$$

$$E^{\mathrm{PROJ}}(L) = \int_0^{2\pi} h(\gamma)e^{i\gamma L}d\gamma \Big/ \int_0^{2\pi} n(\gamma)e^{i\gamma L}d\gamma,$$

where

$$h(\gamma) = \langle \Psi^{\mathrm{SB}}(0)|\mathcal{H}|\Psi^{\mathrm{SB}}(\gamma)\rangle,$$

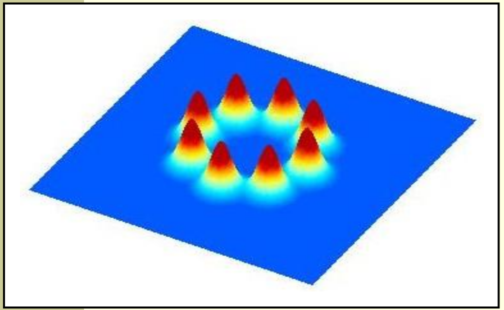
and the norm overlap

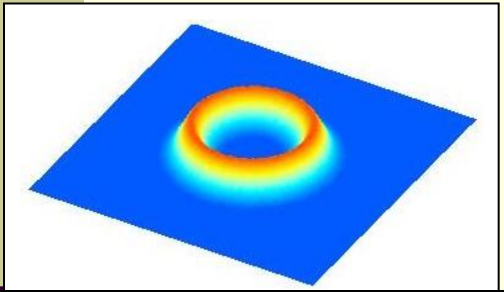
$$n(\gamma) = \langle \Psi^{\mathrm{SB}}(0)|\Psi^{\mathrm{SB}}(\gamma)\rangle$$

Bosons in the ring trap.

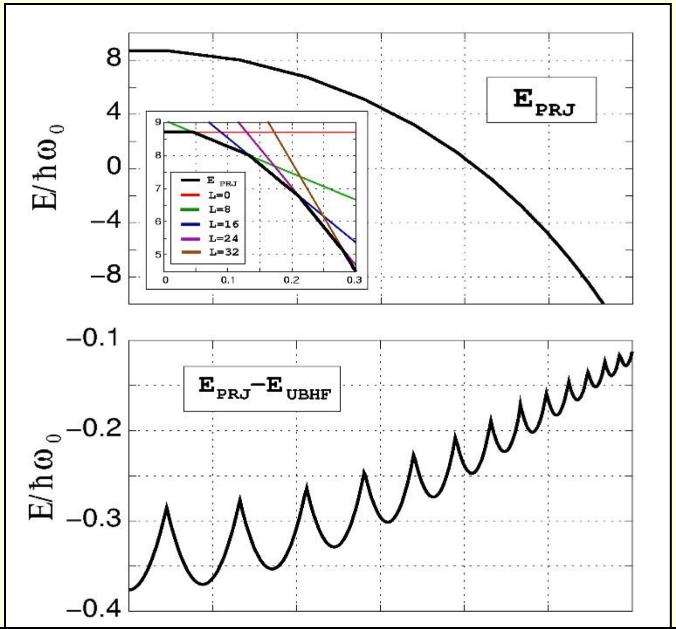
Energy, angular momentum and probability densities.

Probability densities

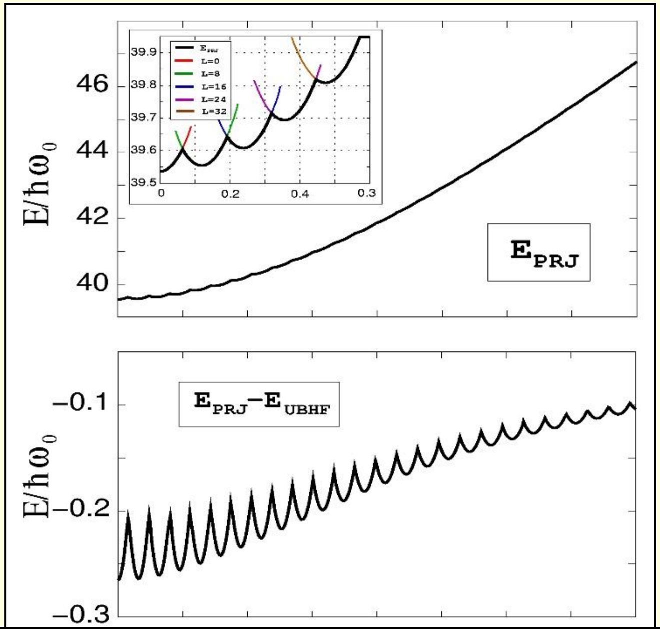


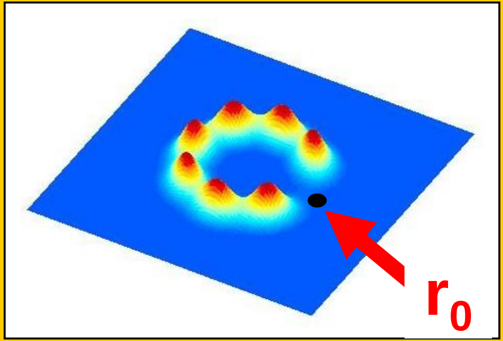


Rotating Frame



Magnetic Field





The hidden crystalline structure in the projected function can be revealed through the use of conditional probability density (CPD).

$$\rho(\mathbf{r}|\mathbf{r_0}) = \langle\Phi|\sum_{i\neq j}\delta(\mathbf{r}_i-\mathbf{r})\delta(\mathbf{r}_j-\mathbf{r}_0)|\Phi\rangle/\langle\Phi|\Phi\rangle$$

A HIERARCHY OF APPROXIMATIONS

TOTAL ENERGY

Restricted Hartree-Fock (RHF)

All spin and space symmetries are preserved

Double occupancy / e-densities: circularly symmetric

Single Slater determinant (central mean field)

Correlations

Unrestricted Hartree-Fock (UHF)

Total-spin and space symmetries (rotational or parity) are broken / Different orbitals for different spins

Solutions with lower symmetry (point-group symmetry)

Lower symmetry explicit in electron densities

Single Slater determinant (non-central mean field)

Implementation of UHF: Pople-Nesbet Eqs.

2D harmonic-oscillator basis set

Two coupled matrix Eqs. (for up and down spins)

Non-linear equations

Bifurcations

EMERGENT

PHENOMENA

Restoration of symmetry via projection techniques

Superposition of UHF Slater det.'s (beyond mean field)

e-densities: circularly symmetric

Good total spin and angular momenta

Lower symmetry is INTRINSIC (or HIDDEN)

Detection of broken symmetry:

CPDs and rovibrational excitations of quantum dots

CPDs and dissociation of quantum dot molecules

Restoration of linearity of many-body equatons

TIME EVOLUTION

ENTANGLEMENT

HAMILTONIAN FOR CLEAN 2D QD'S AND QDM'S

$$\mathcal{H} = \sum_{i=1}^{N_e} H(i) + \sum_{i=1}^{N_e} \sum_{j>i}^{N_e} \boxed{g\,\delta(\mathbf{r}_i - \mathbf{r}_j)}$$

$$H(i) = H_0(i) + H_B(i)$$

$$\frac{\vec{p}_i^{\,2}}{2m^*} + V(x_i, y_i)$$

External confinement

Parabolic, single QD

Two-center oscillator with V_b control } QDM

$$\boxed{\Omega}$$

$$[(\vec{p}_i - e\vec{A}_i/c)^2 - \vec{p}_i^{\,2}]/2m^* + g^* \mu_B \vec{B}\cdot\vec{S}_i/\hbar$$

$$\vec{A}_i = B(-y_i, x_i, 0)/2$$

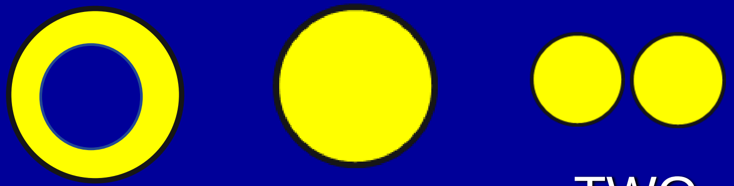
Zeeman

$\mathcal{H}$ can be generalized to Multi-component syste

Neutral Bosonic systems

External Confinement

$$\frac{(r_i - R)^2}{2l_0^2/(\hbar\omega_0)}$$



2D RING TRAP

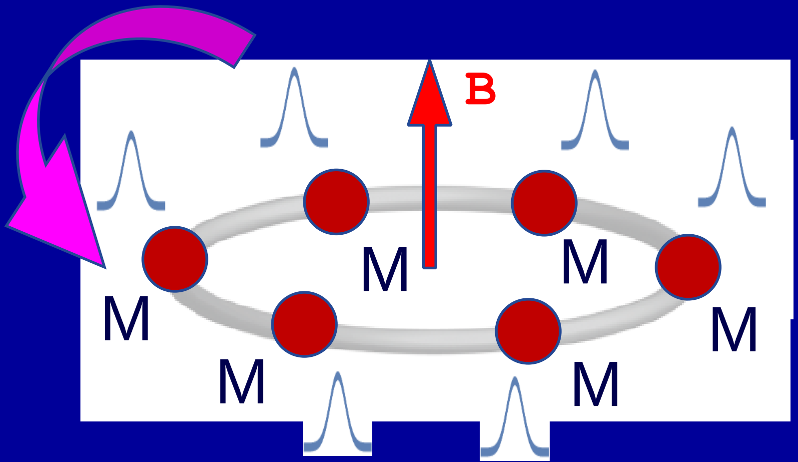
2D TRAP

TWO WELLS

1 hertz [Hz] = 4.13566553853599E-15 electron-volt [eV]

“ROTATING” QUANTUM-MECHANICAL SP DENSITY SHOULD EXHIBIT PERIODICITY IN BOTH SPACE AND TIME

BREAKING OF TIME TRANSLATIONAL SYMMETRY



T. Li et al., PRL 109, 163001 (2012)

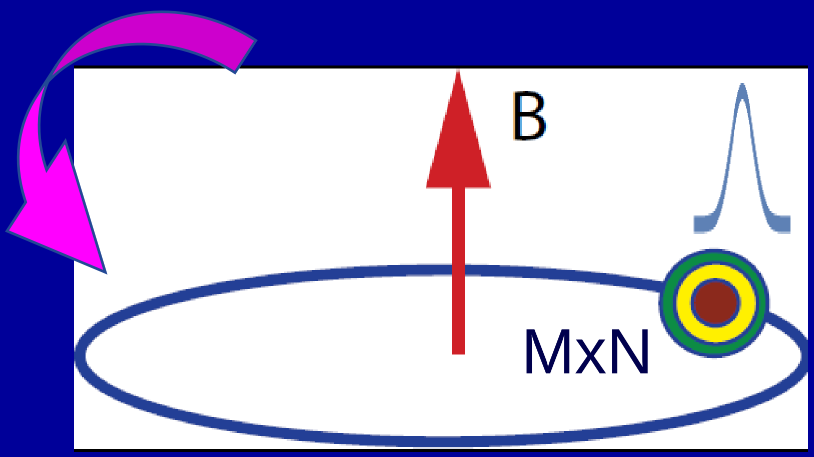
Ion crystal

Ultracold ions/ Coulomb repulsion

Both fermions ($^{24}Mg^{+}$) and

Bosons ($^{9}Be^{+}$)

A different orbital for each particle



F. Wilczek, PRL 109, 160401 (2012)

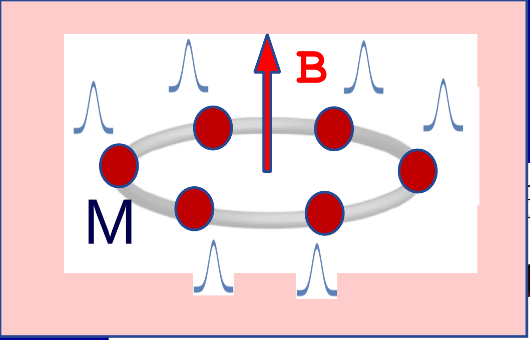
Lump/ Bose-Einstein soliton

Ultracold neutral atoms

Attractive contact interaction

Bosons (^{87}Rb or ^{85}Rb)

The same orbital for all particles



e each particle localized at position $\mathbf{R}_j$ as a ussian function

$$u(\mathbf{r},\mathbf{R}_j) = \frac{1}{\sqrt{\pi}\lambda} \exp\left(-\frac{(\mathbf{r}-\mathbf{R}_j)^2}{2\lambda^2} - i\varphi(\mathbf{r},\mathbf{R}_j;B) \right), \quad (3)$$

with $\lambda = \sqrt{\hbar/(M\Omega)}$; $\Omega = \sqrt{\omega_0^2 + \omega_c^2/4}$ where $\omega_c = \eta B/M$ is the cyclotron frequency. The phase in Eq. (3) is due to the gauge invariance of magnetic translations [57, 58]) and is given by $\varphi(\mathbf{r},\mathbf{R}_j;B) = (xY_j - yX_j)/(2l_B^2)$, with $l_B = \sqrt{\hbar/(\eta B)}$ being the magnetic length. For

$$\varphi(\mathbf{r},\mathbf{R}_j;B) = (xY_j - yX_j)/(2l_B^2)$$

Construct determinant/ permanent Ψ^{SB}

(MF symmetry-breaking ansatz)

Rotational spectrum: quantum rigid rotor

$R_W = 1000$

$R_\delta = 50$

$$E^{\mathrm{PROJ}}(L) \approx V_{\mathrm{int}} + C_R(L - N\Phi/\Phi_0)^2$$

V_{int} Band head/ where interaction and Correlations show up

Φ Magnetic flux

$$C_R \approx C_R^{\mathrm{cl}} = \hbar^2/[2\mathcal{I}(R_{\mathrm{eq}})]$$

$$\mathcal{I}(R_{\mathrm{eq}}) = NMR_{\mathrm{eq}}^2$$

L => magic (fermions spin polarized)

Repelling, polygon crystal:

$$L_m = kN; \quad k = 0, \pm1, \pm2, \pm3, \ldots$$

Fermions, N odd; bosons

$$L_m = (k + \frac{1}{2})N; \quad k = 0, \pm1, \pm2, \pm3, \ldots$$

Fermions, N even

$$L_m = 0, \pm1, \pm2, \ldots$$

Attractive bosons, lump

Floppy rotor (high B)

Analytic approximation to REM (quantum)

$$E^{\mathrm{REM}}_{\mathrm{app},L}(N) = \hbar(\Omega - \omega_c/2)L + \sum_{q=1}^{r} \frac{C_{V,q}}{L_q^{1/2}} + \sum_{q=1}^{r-1}\sum_{s>q}^{r} V_C\left(\lambda\sqrt{\frac{L_q}{n_q}}, \lambda\sqrt{\frac{L_s}{n_s}}\right)$$

Total *L*

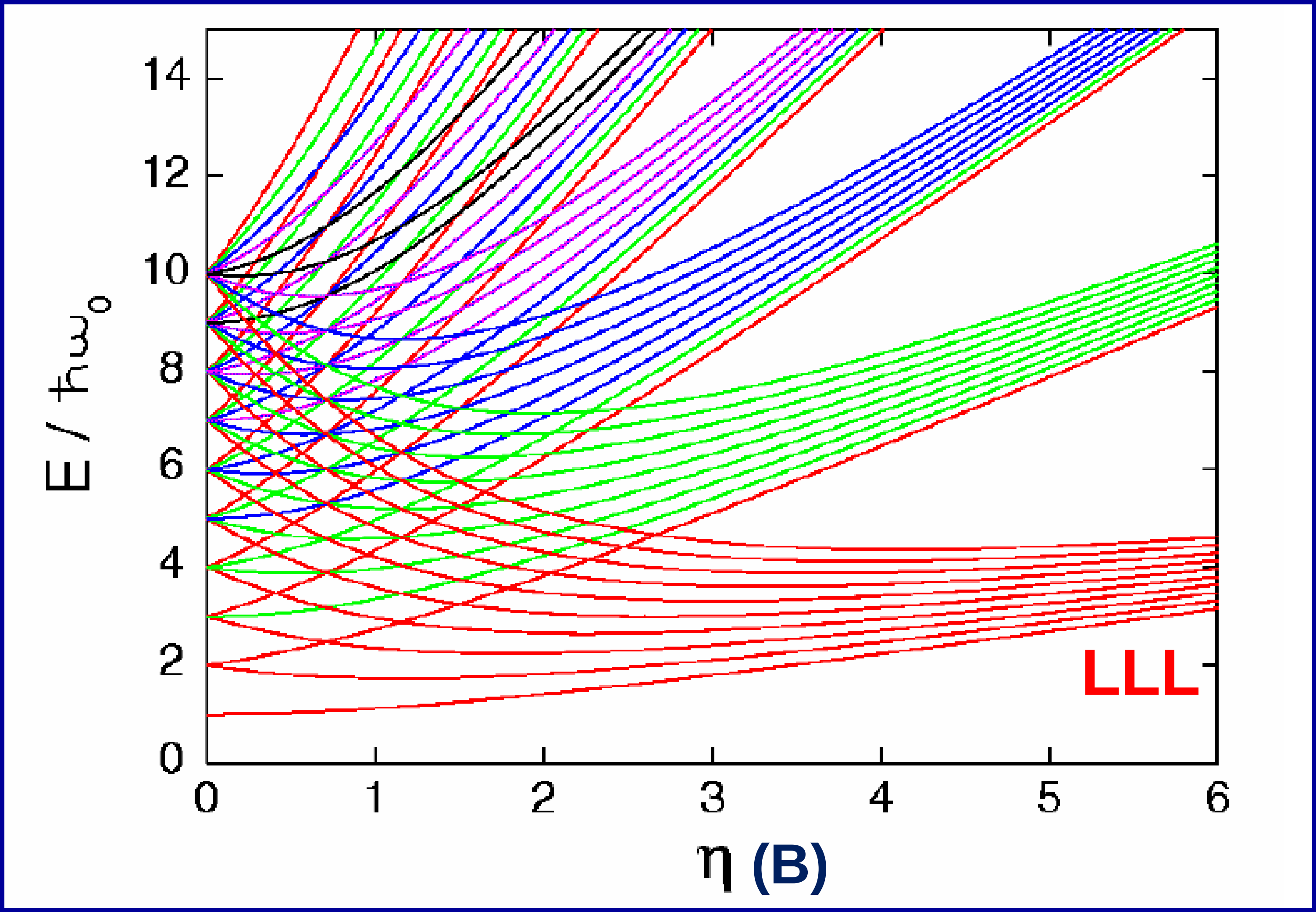
Partial angular momenta (corresponding to rings)

$$V_C(a_q, a_s) = n_q n_s {}_2F_1[3/4, 1/4; 1; 4a_q^2 a_s^2 (a_q^2 + a_s^2)^{-2}] e^2 (a_q^2 + a_s^2)^{-1/2}/\kappa$$

RIGID / Classical

$$\mathcal{J}_{\mathrm{cl}} = \sum_{i=1}^{N} m^* |Z_i|^2$$

$$E^{\mathrm{RCL}}_L = \hbar^2 L^2/(2\mathcal{J}_{\mathrm{cl}}) + 0.5\sum_{i=1}^{N} m^* \omega_0^2 |Z_i|^2 + \sum_{i=1}^{N}\sum_{j>i}^{N} e^2/(\kappa|Z_i - Z_j|)$$

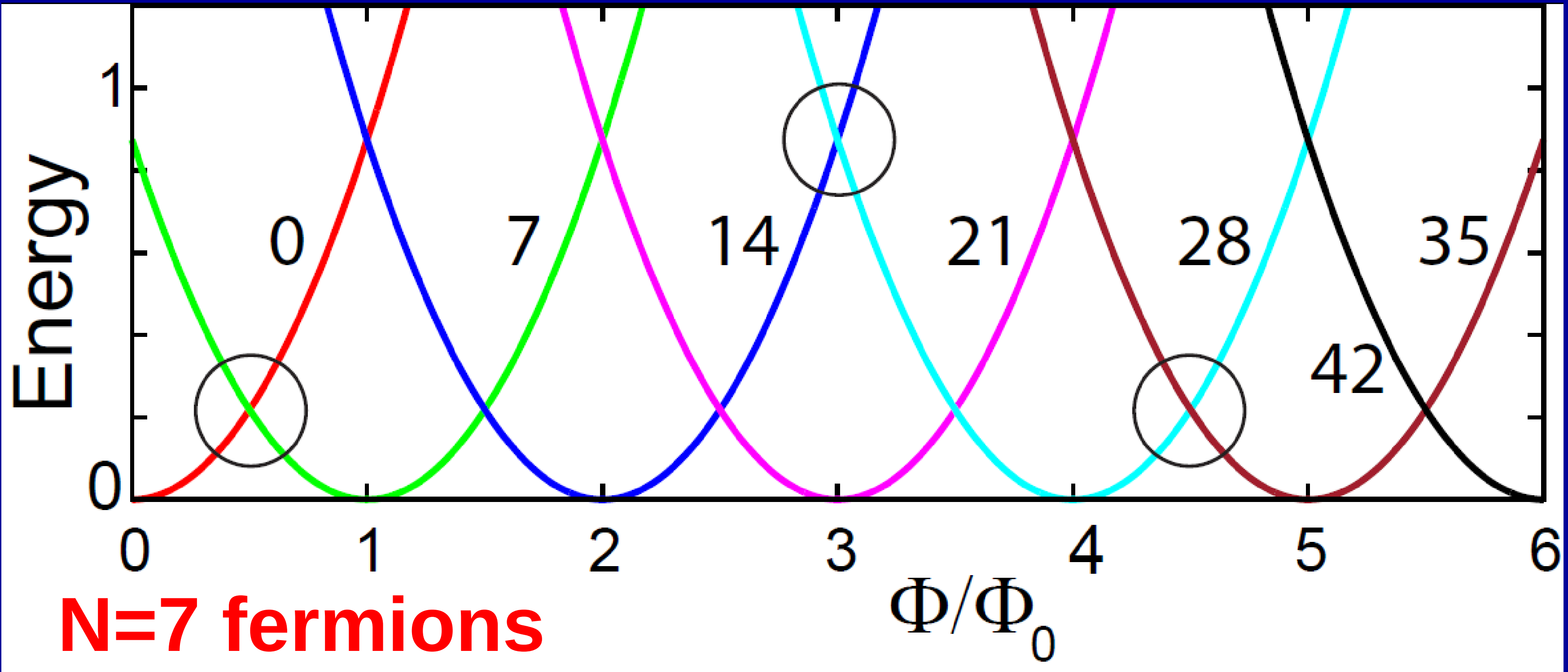


Effect of breaking time reversal symmetry
2D oscillator under a magnetic field
Darwin-Fock spectrum

Rotational spectrum: quantum rigid rotor

$$E^{\mathrm{PROJ}}(L) \approx V_{\mathrm{int}} + C_R(L - N\Phi/\Phi_0)^2$$

SECOND TERM/ AHARONOV-BOHM TYPE SPECTRUM



N=7 fermions

GOA/ GCM

Norm overlap/ Time-reversal

where in such a way that the quotient $h(\alpha)/n(\alpha)$ is a rather smooth
ction. This is a general property of many-body wave functions (which
discussed in great detail in Sec. 10.7). Following the arguments of
ion 10.7.4, we obtain*

$$n(\alpha)\simeq\exp\left(i\langle\hat{J}\rangle\cdot\alpha-\tfrac{1}{2}\langle\Delta\hat{J}^2\rangle\alpha^2\right), \tag{11.79}$$

$$\Delta\hat{J}=\hat{J}-\langle\hat{J}\rangle,$$

nly difference now being that we also allow for time odd components
e wave functions, which does not give a pure Gaussian, but an
ional phase in Eq. (11.79) ($\langle\hat{J}\rangle\neq0$). The idea of Kamlah is now to
he operator

$$\hat{I}:=-\langle\hat{J}\rangle+\frac{1}{i}\frac{\partial}{\partial\alpha},$$

$\langle\hat{J}\rangle \rightarrow \Phi/\Phi_0$

KAMLAH expansion

projected energy now has the form:

$$E^{I}_{\text{proj}}=\langle H\rangle-\frac{\langle\Delta\hat{J}^2\rangle}{2\mathcal{J}_Y}+\frac{\langle\hat{J}\rangle}{\mathcal{J}_{\text{sc}}}(I-\langle\hat{J}\rangle)+\frac{1}{2\mathcal{J}_Y}(I-\langle\hat{J}\rangle)^2. \tag{11.91}$$

t us first study the method of *variation before projection*, which was
nally proposed by Peierls and Yoccoz [PY 57, Yo 57]. Here the wave
tion $|\Phi\rangle$ is obtained from a minimization of H without constraint. It
erefore, time-reversal invariant and has vanishing expectation values
and $\langle H\hat{J}\rangle$. The spectrum then has the form

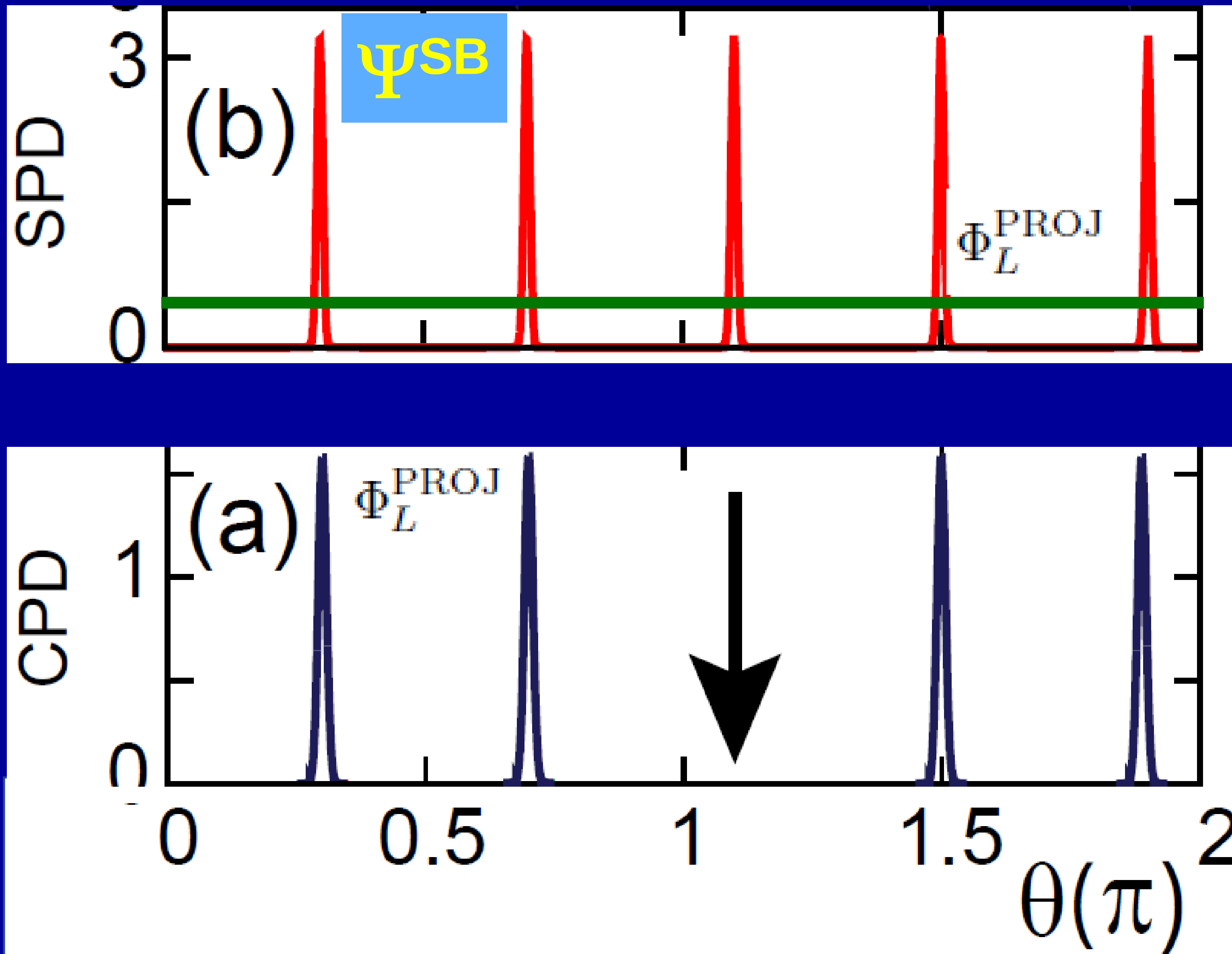
$$E^{I}_{\text{proj}}=\left\langle H-\frac{\hat{J}^2}{2\mathcal{J}_Y}\right\rangle+\frac{I^2}{2\mathcal{J}_Y}. \tag{11.92}$$

he spectrum of a one-dimensional rotor with the Yoccoz value $\mathcal{J}_Y$ for
oment of inertia (see Sec. 11.4.5).† The band head is obtained from

Ring & Schuck, Ch. 11

Structure of many-body wave functions on ring

N=5 ultracold ions (fermions)



Wave packets/ Time evolution

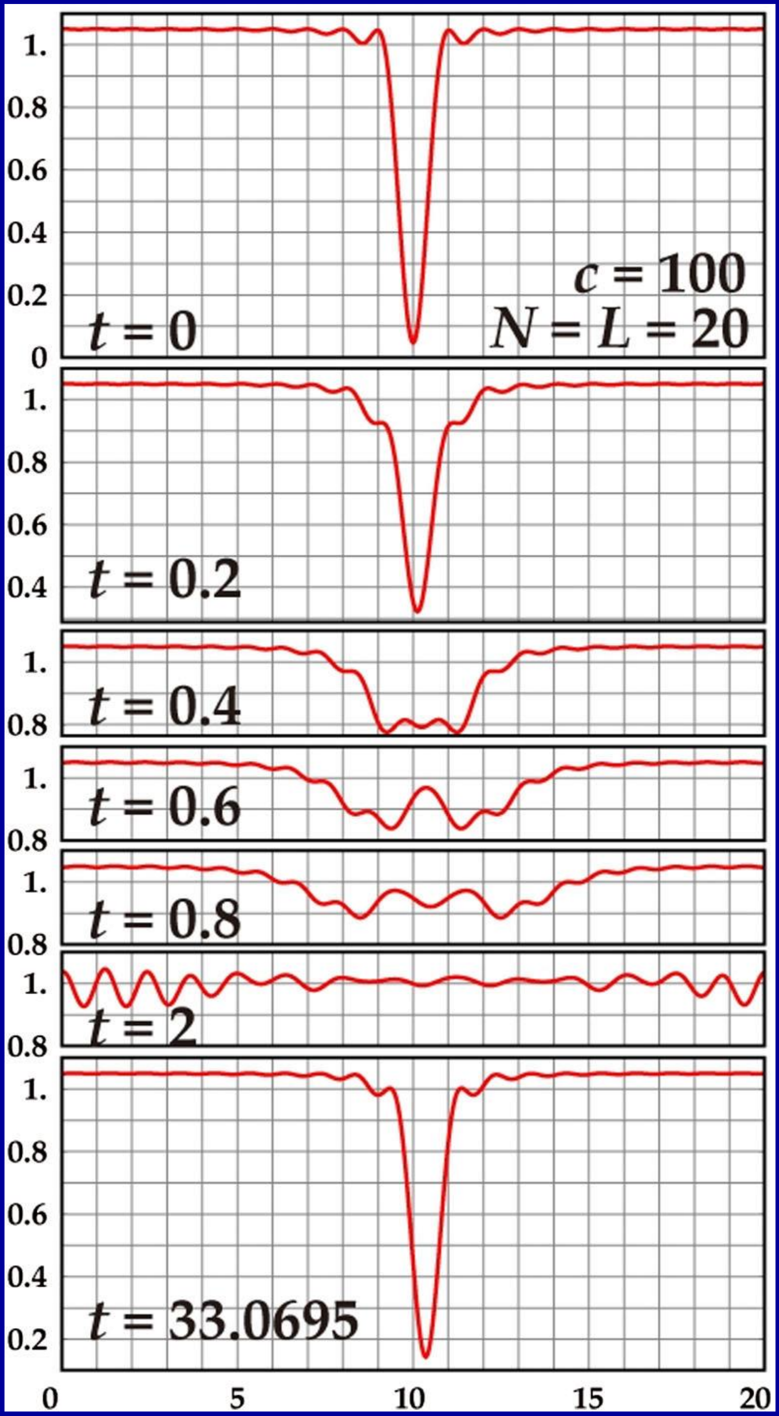
$$\Psi^{\mathrm{SB}} = \sum_L \mathcal{C}_L \Phi_L^{\mathrm{PROJ}}$$

Invert the projection

$$\mathcal{C}_L = \frac{1}{2\pi} \int_0^{2\pi} d\gamma e^{-i\gamma L} n(\gamma)$$

Many frequencies, terms e^{-iE_L}

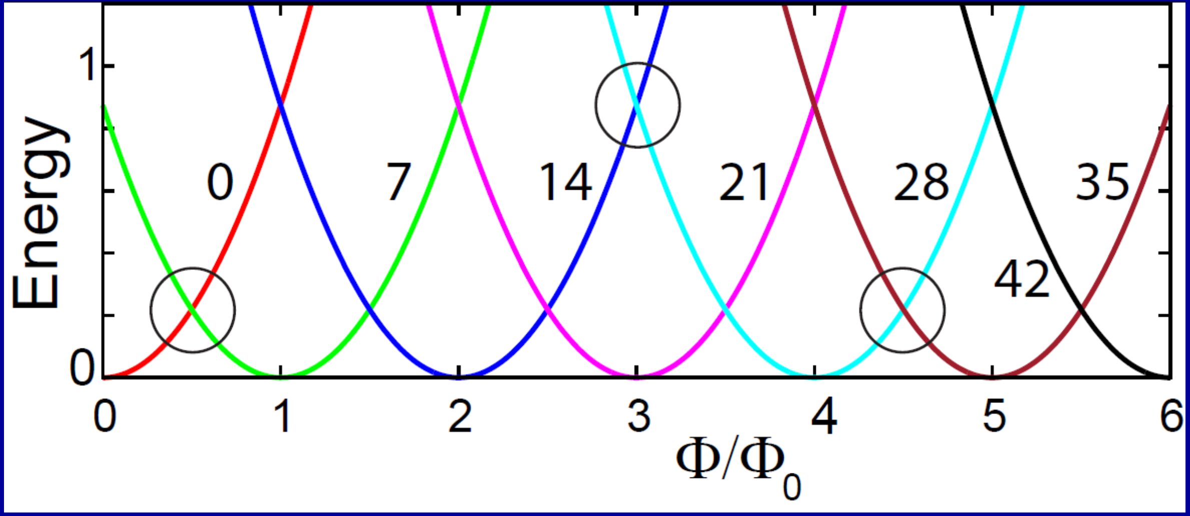
Diffusion and Revival



J. Sato et al.,
PRL **108**, 110401 (2012)

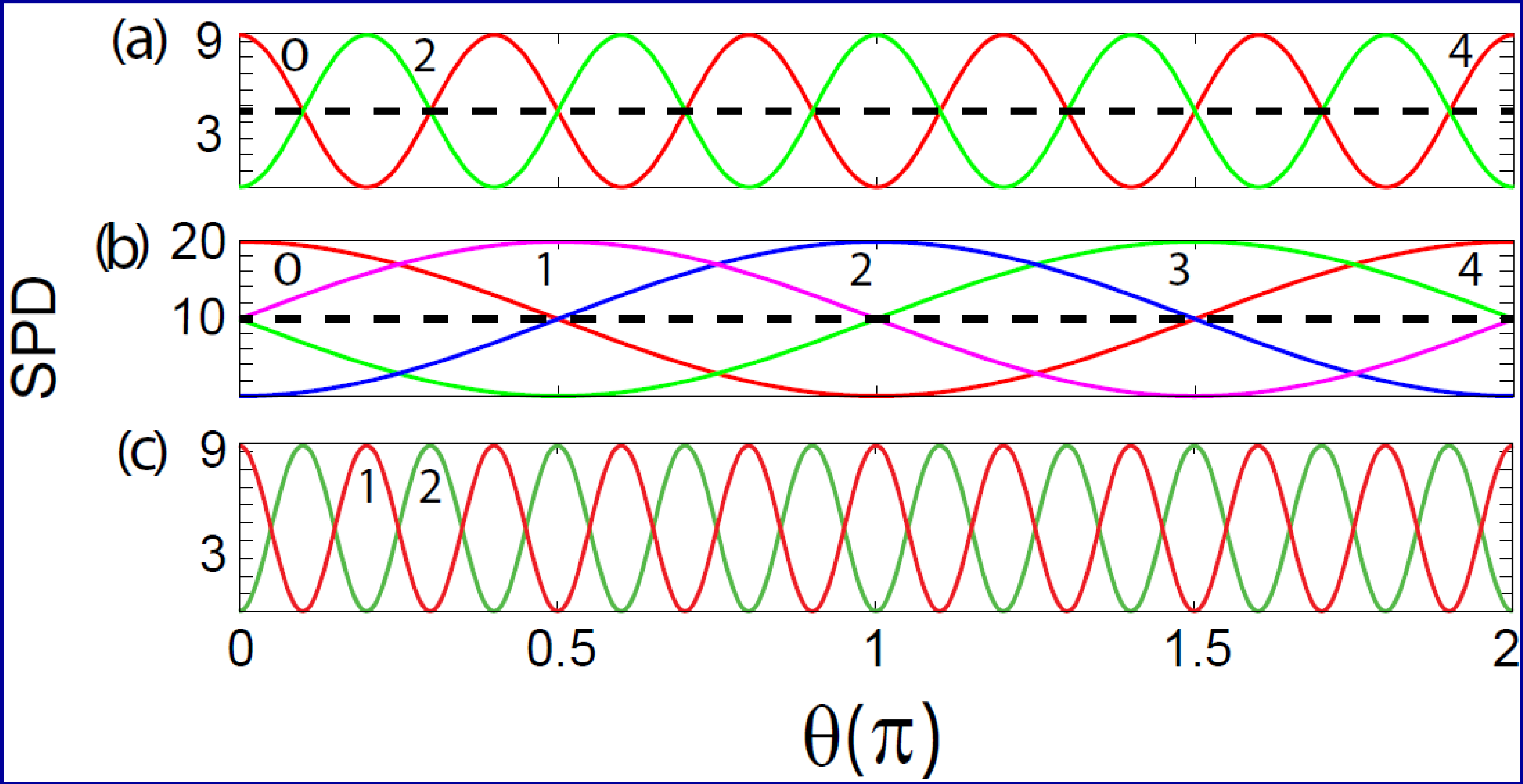
PINNED WIGNER MOLECULE (PWM)/ TWO-PROJECTED-STATE SUPERPOSITION

$$\Phi^{\mathrm{PWM}}(L_1, L_2; t=0) = \alpha \Phi^{\mathrm{PROJ}}_{L_1} + \beta e^{i\phi(t=0)} \Phi^{\mathrm{PROJ}}_{L_2}$$



PWM: TIME EVOLUTION OF SPD (snapshots)

$$\tau = 2\pi\hbar / |E_1 - E_2|$$



BOTH SPACE AND TIME TRANSLATIONAL SYMMETRY ARE BROKEN

CONCLUSIONS:

SYMMETRY RESTORATION IS A NATURAL METHOD TO BE USED IN ADDRESSING THE PHYSICS IN MESOSCOPIC SYSTEMS OTHER THAN NUCLEI

Physics

VIEWPOINT

How to Create a Time Crystal

A detailed theoretical recipe for making time crystals has been unveiled and swiftly implemented by two groups using vastly different experimental systems.

by Phil Richerme*

The story of time crystals—whose lowest-energy configurations are periodic in time rather than space—epitomizes the creative ideas, controversy, and vigorous discussion that lie at the core of the scientific process. Originally theorized by Frank Wilczek in 2012 [1] (see 15 October 2012 Viewpoint), time crystals were met with widespread attention, but also a healthy dose of skepticism [2]. This ignited a debate in the literature, culminating in a proof that time crystals cannot exist in thermal equilibrium, as originally imagined by Wilczek [3]. But the

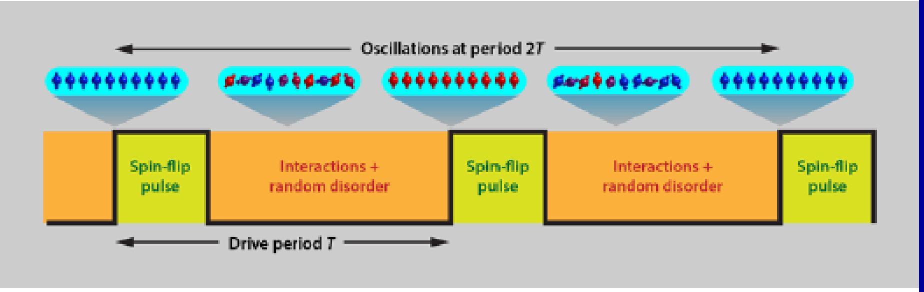


Figure 1: Yao *et al.* [7] have developed a blueprint for creating a time crystal and a method for detecting it, which has been followed by two experimental groups [8, 9]. Quantum spins are subjected to imperfect spin-flip driving pulses and then allowed to interact with each other in the presence of strong random disorder in the local

From Wikipedia

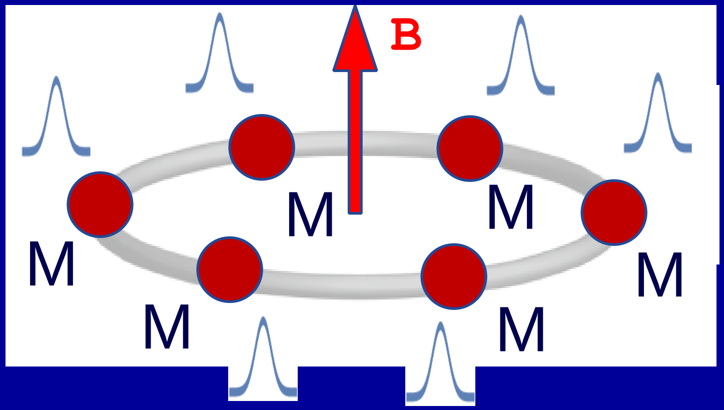
"A time crystal or space-time crystal is a structure that repeats periodically in time, as well in space. Normal three-dimensional crystals have a repeating pattern in space, but remain unchanged with respect to time; time crystals repeat themselves in time as well, leading the crystal to change from moment to moment.
The idea of a time crystal was first described by Nobel laureate and MIT professor Frank Wilczek in 2012"

Time reflection and time translational symmetries are broken
Quantum Space Time Crystal (symmetry breaking in all four dimensions: space and time)

Time evolution phenomena in quantum mechanical finite systems

Unprecedented experimental control of few-body systems of trapped ultracold ions and neutral atoms

THE CONCEPT/ INTUITION/ TWO PROPOSALS/ STARTING POINT BASED ON SYMMETRY BREAKING



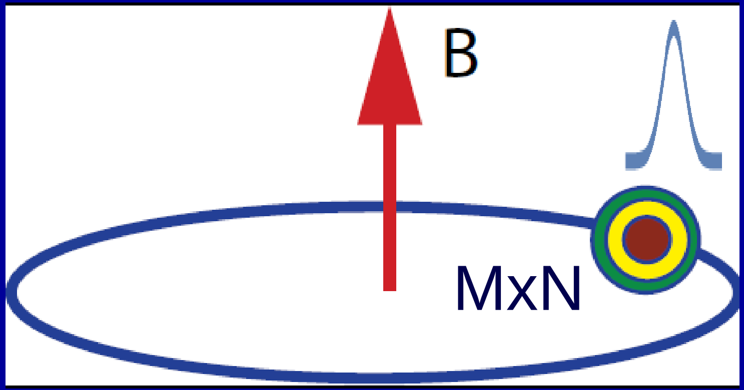
T. Li et al., PRL 109, 163001 (2012)

Ion crystal

Ultracold ions/ Coulomb repulsion

Both fermions ($^{24}Mg^{+}$) and

Bosons ($^{9}Be^{+}$)



F. Wilczek, PRL 109, 160401 (2012)

Lump/ Bose-Einstein soliton

Ultracold neutral atoms

Attractive contact interaction

Bosons (^{87}Rb or ^{85}Rb)

<u>RESOLUTION OF SYMMETRY DILEMMA:</u>

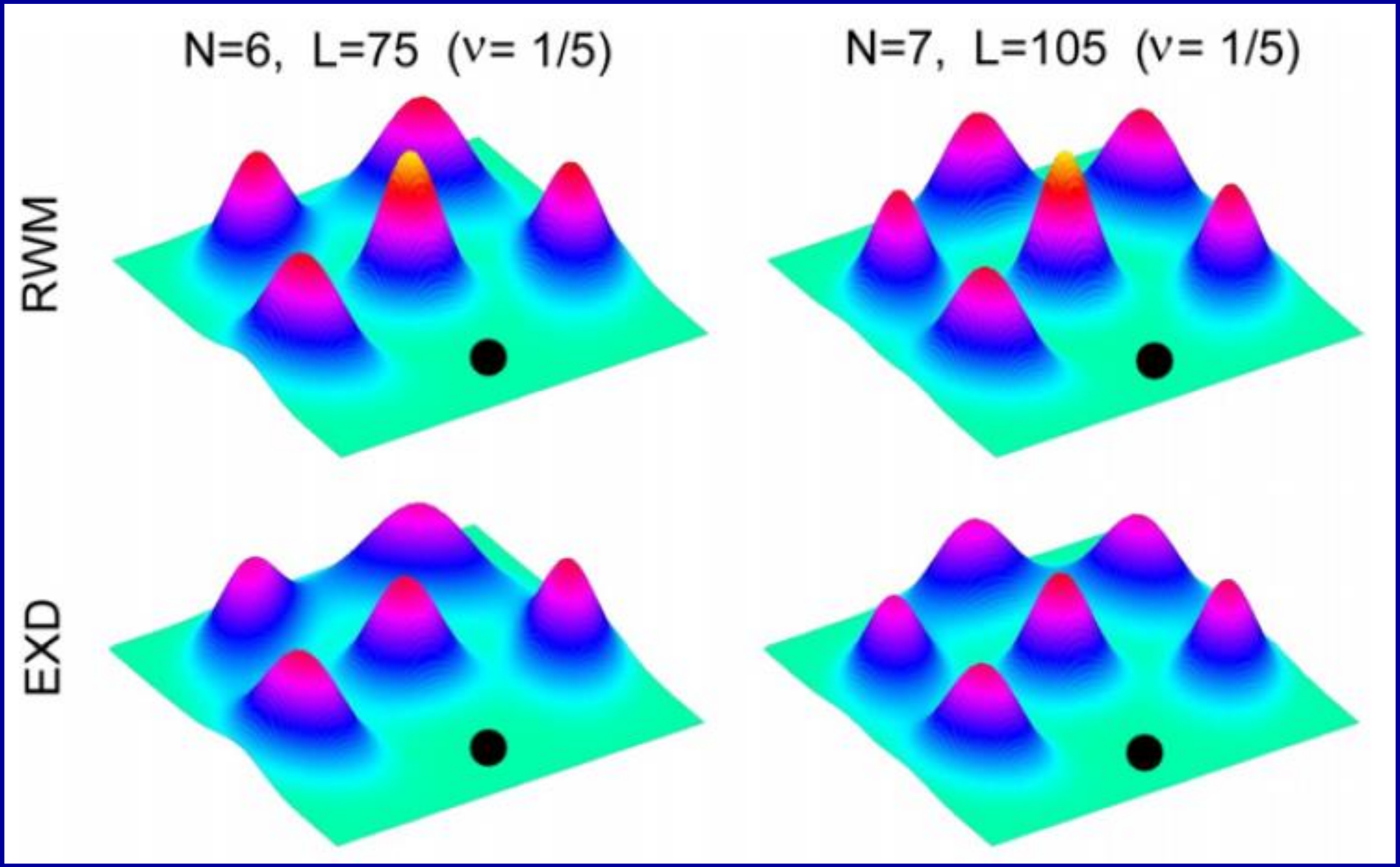
<u>RESTORATION OF BROKEN SYMMETRY</u>

<u>BEYOND MEAN FIELD (Projection)!</u>

- Per-Olov Löwdin
  (Chemistry - Spin)
- R.E. Peierls and J. Yoccoz
  (Nuclear Physics – *L, rotations*)

<u>Ch. 11 in the book by P. Ring and P. Schuck</u>

CPDs for QDs, fully spin polarized



Lowest Landau level B → Infinity