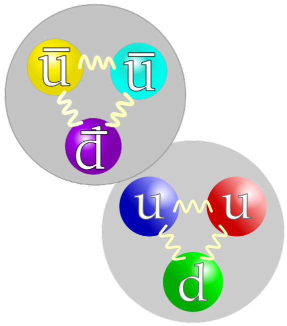
$\hbar = c = 1$



The antinucleon-nucleon interaction

Rob G. E. Timmermans

Workshop on "*Nuclear physics with antiprotons: A theory endeavor*"

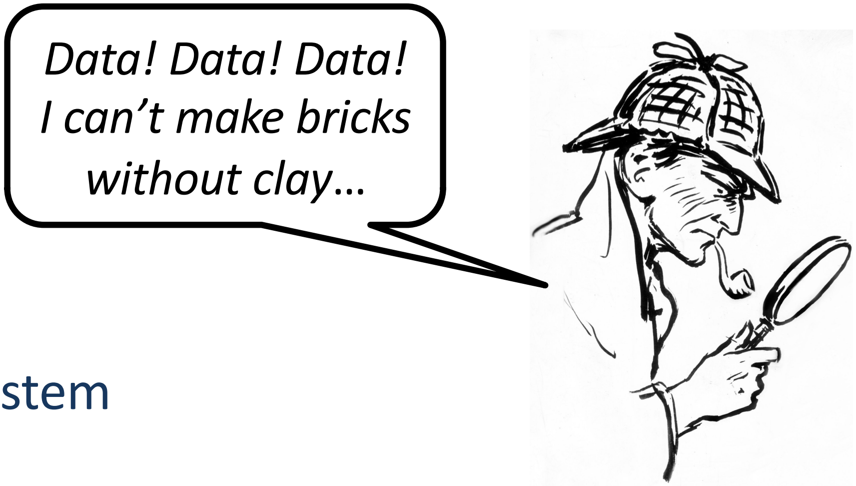
ESNT, November 15, 2021

university of groningen
faculty of science and engineering
van swinderen institute for particle physics and gravity
NWO

Game plan

- ✓ The antinucleon-nucleon interaction 101
  - Phenomenology, modeling
  - The existing database from LEAR (and pre-LEAR)
- ✓ Partial-wave analysis (PWA, a.k.a. PSA)
  - Why? How?
  - "Spin physics"
- ✓ PWA as a "tool"
  - What did we learn?
  - What is needed?
- ✓ Conclusion
- ✓ Throughout: Comparison to the *NN* system

RGET, Rijken, de Swart:
PRL '91, PLB '91, PRD '92, PRC '94, '95
Rentmeester, RGET, de Swart: PRL '99, PRC '01, '02, '03
Zhou, RGET: PRC '12, '13



Modeling the antinucleon-nucleon interaction I

- ✓ Black-disk model: $\sigma_{\text{el}} = \sigma_{\text{ann}} = \sigma_{\text{total}}/2 = \pi R^2$, radius R varies ≈ 1-2 fm

- ✓ Boundary-condition model (> 1967)
  - Parametrize boundary condition
  - Include long-range interaction V_L for $r > b$
  - Example: modified black disk: $P = -ipb$, $V_L = 0$

$$P = b\left(\frac{\mathrm{d}\psi}{\mathrm{d}r}\psi^{-1}\right)_{r=b}$$

- ✓ Optical-potential model (> 1968)
  - Pick your favorite *NN* model, apply charge conjugation (or *G* parity)
  - Add an optical potential
    - U, W constants
    - Pick a form for $f(r)$, *e.g*. Woods-Saxon, with a range parameter b
    - Fit U, W, and b qualitatively to some cross section
    - Make strong claims about baryonia, *etc*.

$$V(r) = (U - iW)f(r)$$

Modeling the antinucleon-nucleon interaction II

- ✓ Paris optical-potential model (*cf*. Loiseau's talk):
  - *G*-parity transformed Paris *NN* model
  - Spin- and isospin-dependent optical potential
  - Fit the available data (# parameters = ?)

- ✓ Nijmegen coupled-channels model
  - CC84: P. H. Timmers *et al*., PRD **29**, 1928 (1984), *NN* OBE model D
  - CC93: RGET, PhD thesis, *NN* soft-core model

$$V = \begin{pmatrix} V_{\overline{N}N} & V_{\mathrm{A}} \\ \tilde{V}_{\mathrm{A}} & 0 \end{pmatrix}$$

  - Effective two-particle annihilation channels
    - *e.g.* $2M_1$ = 1700 MeV, $2M_2$ = 420 MeV

$$V_A^{(i,I)}(r) = V(i,I)\frac{1}{1+\exp(m_a r)}$$

  - Fit the available data, # parameters = 14-20

Modeling the antinucleon-nucleon interaction III

- ✓ *NN* and *NN*bar are related by charge conjugation
  - Or, assuming isospin symmetry, *G* parity

- ✓ One-boson exchange picture:

$$V(pp) = V_\pi + V_\varrho + V_\omega + V_\varepsilon + \ldots$$

  - Vector mesons have negative *C* parity

$$V(\overline{p}p) = V_\pi - V_\varrho - V_\omega + V_\varepsilon + \ldots$$

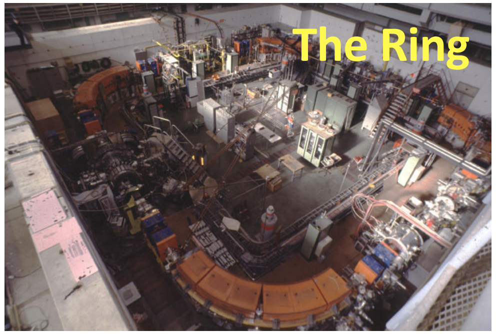
  - Charge-exchange reaction

$$V(\overline{p}p \to \overline{n}n) = 2(V_\pi - V_\varrho + \ldots)$$

- ✓ In *NN*: Coherent spin-orbit forces, relatively weak central & tensor forces

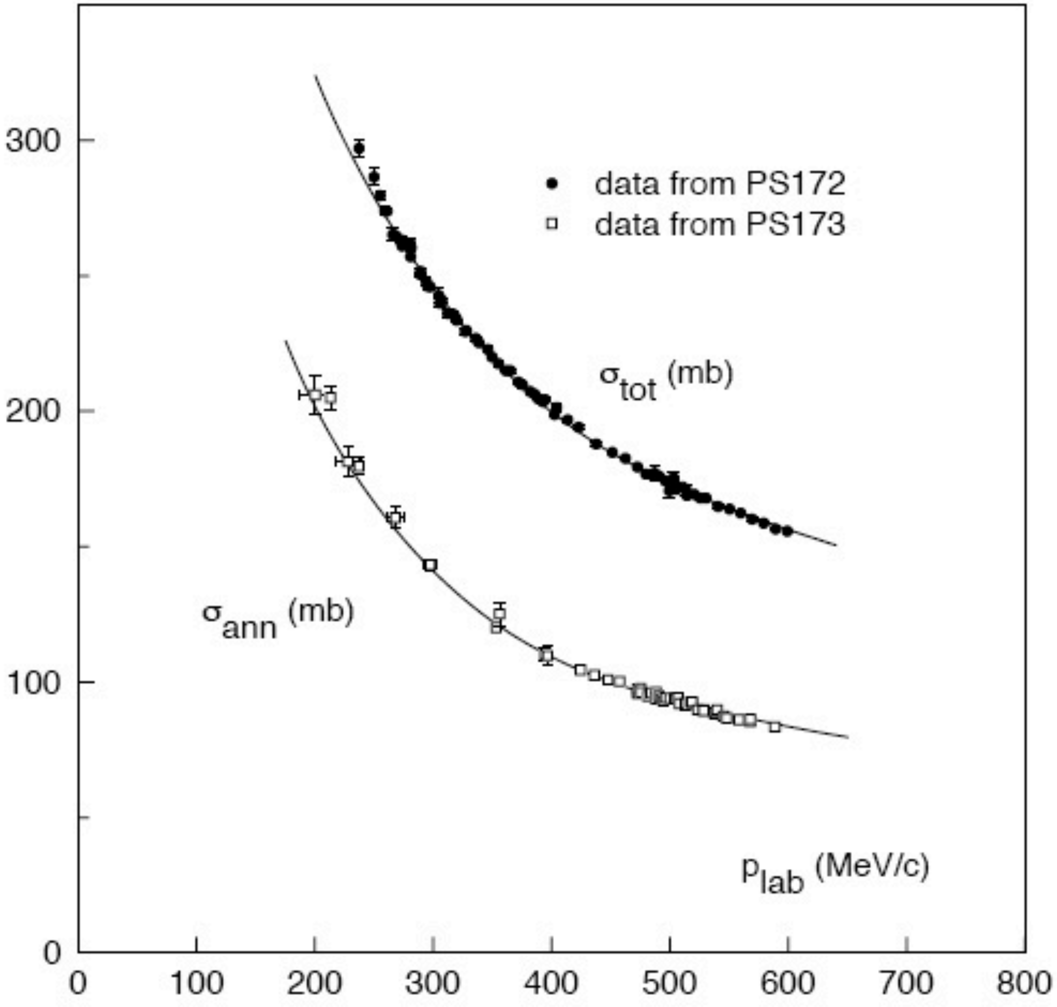
- ✓ In *NN*bar: Strong central attraction, strong coherent tensor force
  - Similar things hold in chiral EFT (LECs c_1, c_3, c_4, …)

Experiments @ LEAR



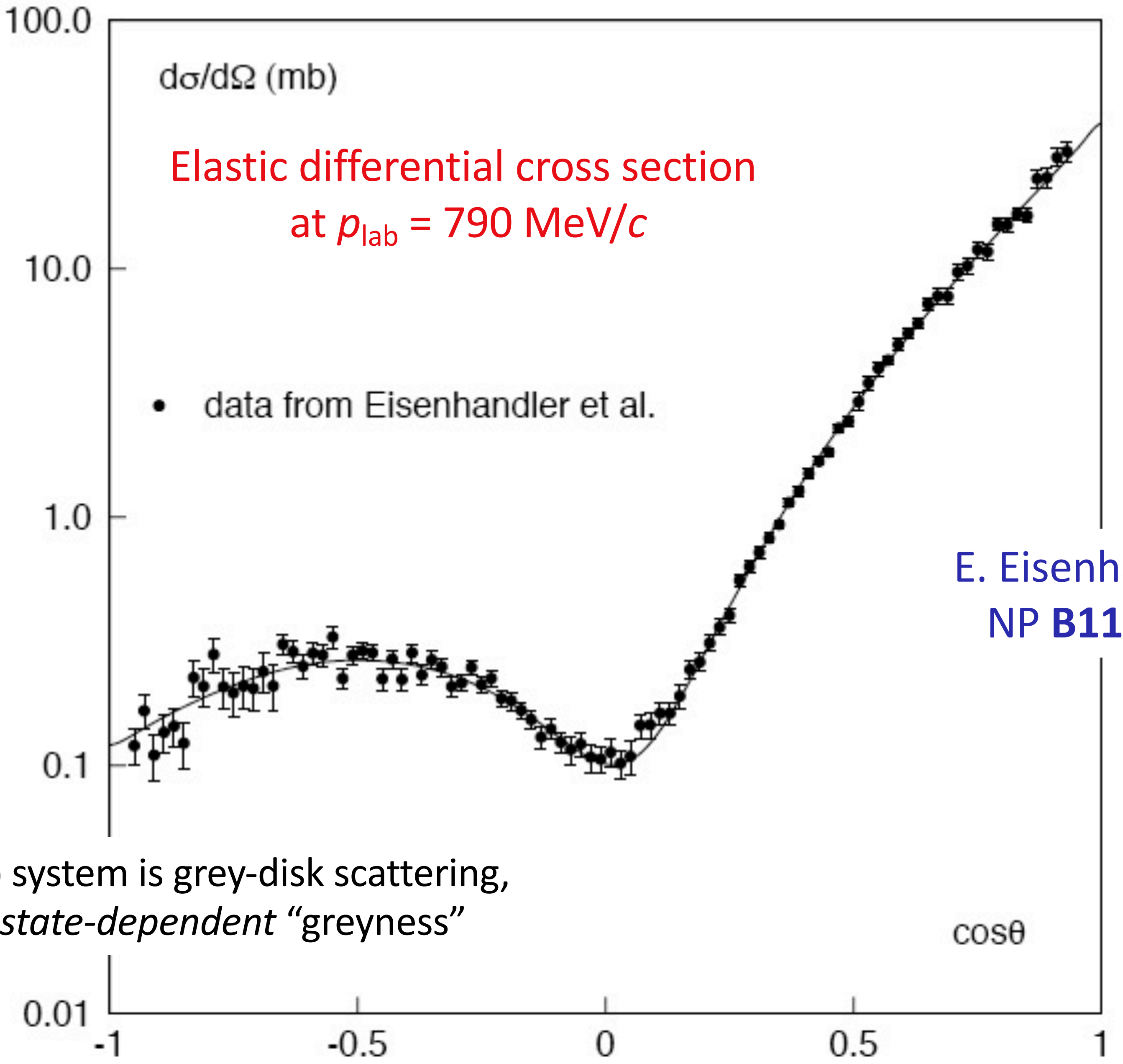
Group	Reaction	Observable	p_{lab} (MeV/c)
PS172	$\bar{p}+p \to \bar{p}+p$	σ_{tot} , $d\sigma/d\Omega$, A_y , D_{yy}	200-1500
PS173	$\bar{p}+p \to \bar{p}+p$	σ_{ann} , $d\sigma/d\Omega$	180-600
	$\bar{p}+p \to \bar{n}+n$	σ_{cex} , $d\sigma/d\Omega$	180-600
PS185	$\bar{p}+p \to \bar{Y}+Y$	$d\sigma/d\Omega$, P_y , C_{ij} , D_{nn} , K_{nn}	1435-1900
PS198	$\bar{p}+p \to \bar{p}+p$	$d\sigma/d\Omega$, A_y , D_{yy}	440-700
PS199	$\bar{p}+p \to \bar{n}+n$	$d\sigma/d\Omega$, A_y	600-1300
PS206	$\bar{p}+p \to \bar{n}+n$	$d\sigma/d\Omega$	693

Total and annihilation cross section



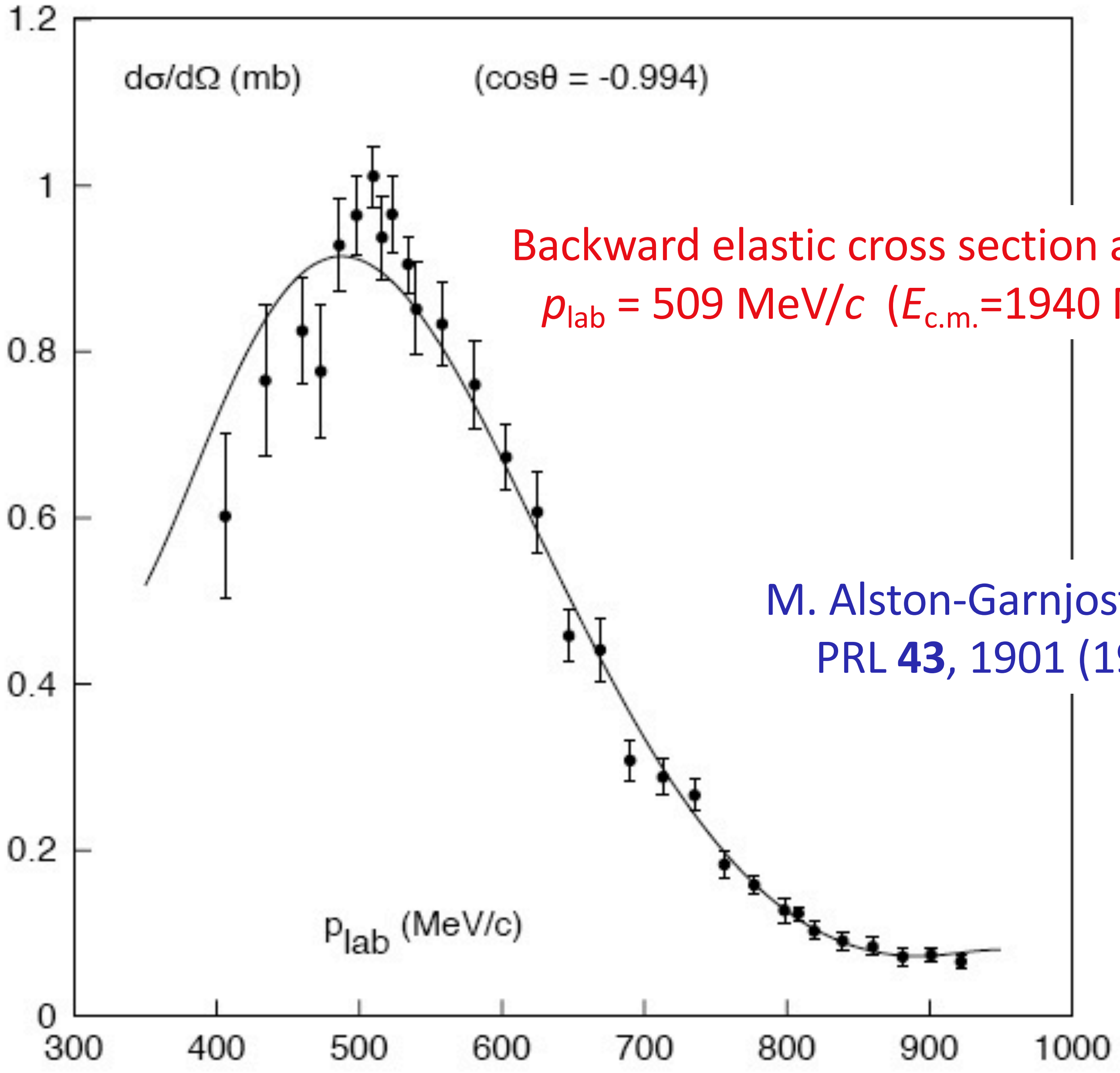
- ✓ Annihilation: Complex boundary condition at radius *b* = 1.25 fm

Elastic differential cross section at $p_{\mathrm{lab}} = 790$ MeV/c



E. Eisenhandler *et al.*, NP **B113**, 1 (1976)

The $\bar{p}p$ system is grey-disk scattering, with a *state-dependent* “greyness”



Backward elastic cross section around p_{lab} = 509 MeV/c ($E_{\text{c.m.}}$=1940 MeV)

M. Alston-Garnjost *et al.*, PRL **43**, 1901 (1979)

The database of the *NN* PWA below T_{lab} = 350 MeV

✓ Proton-proton

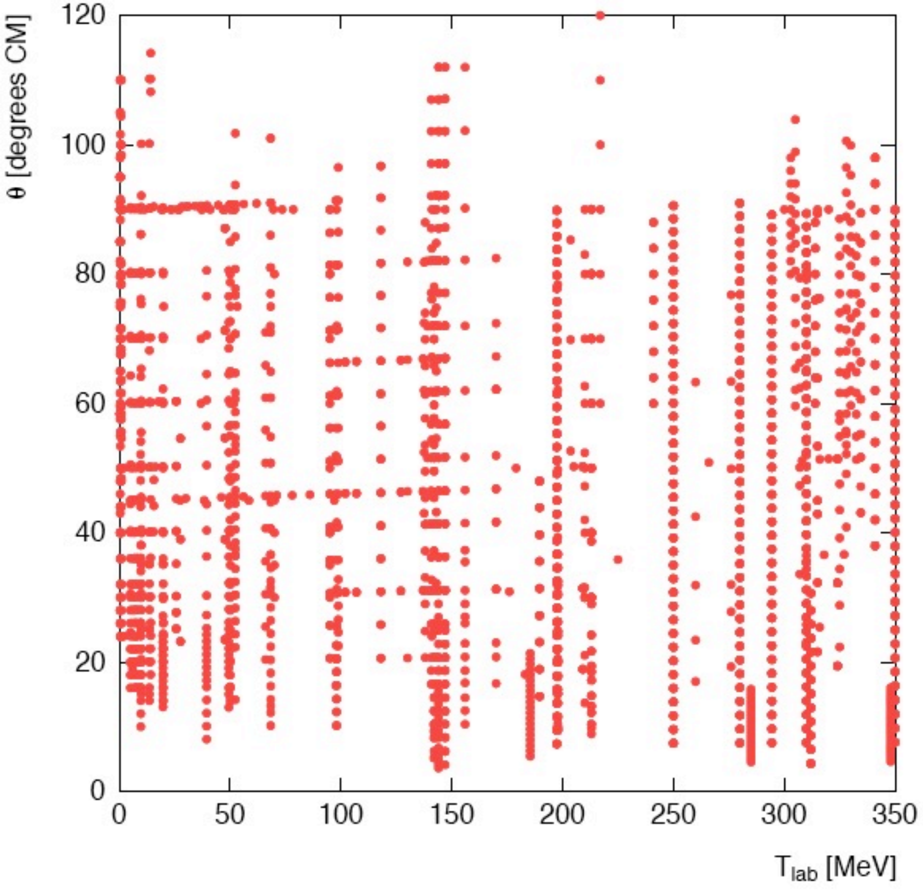
Type	# data
σ_{tot} , $\Delta\sigma_L$, $\Delta\sigma_T$	—
$d\sigma/d\Omega$	947
A_y	816
A_{ii} , C_{nn}	876
D, D_t	114
R , R' , A , A'	237
Rest	36
All	*3026*

Neutron-proton

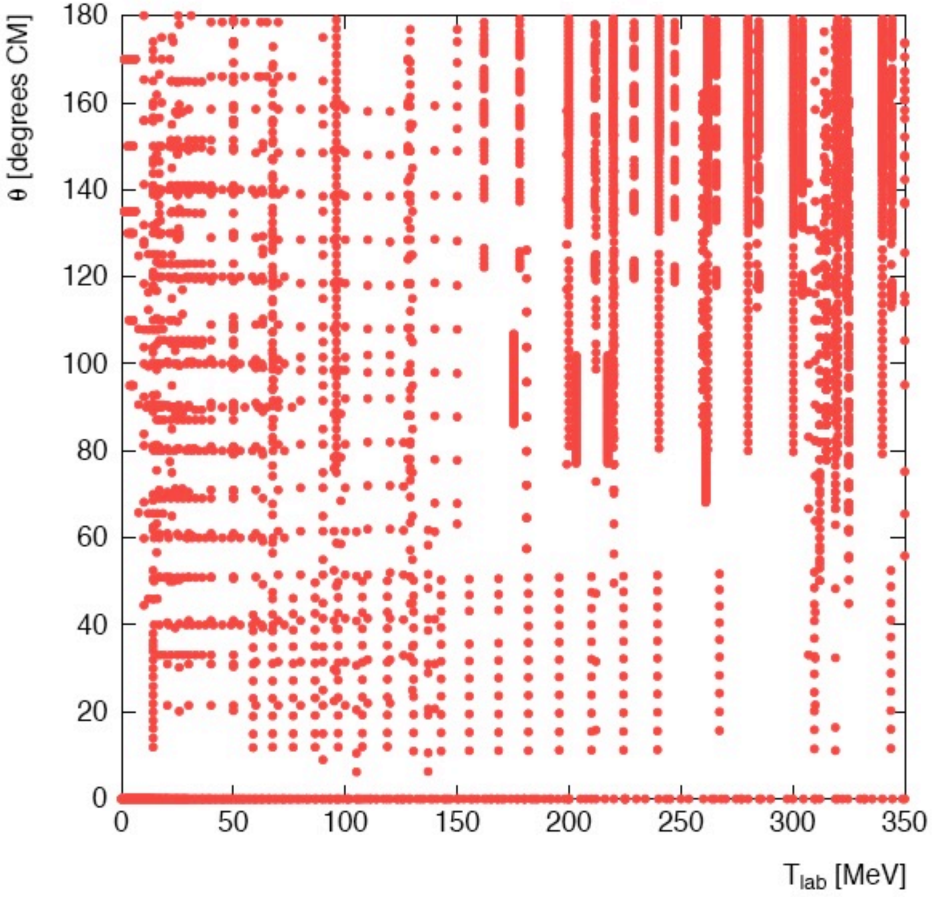
Type	# data
σ_{tot} , $\Delta\sigma_L$, $\Delta\sigma_T$	275
$d\sigma/d\Omega$	1475
A_y	1213
A_{yy} , A_{zz}	327
D_t	122
R_t , R_t' , A_t , A_t'	162
Rest	78
All	*3652*

✓ High quality (∼15% rejected) Reasonable quality (∼25% rejected)

“Abundance plots” of the *NN* scattering data



✓ Proton-proton



Neutron-proton

The database of the *NN*bar PWA below p_{lab} = 925 MeV/*c*

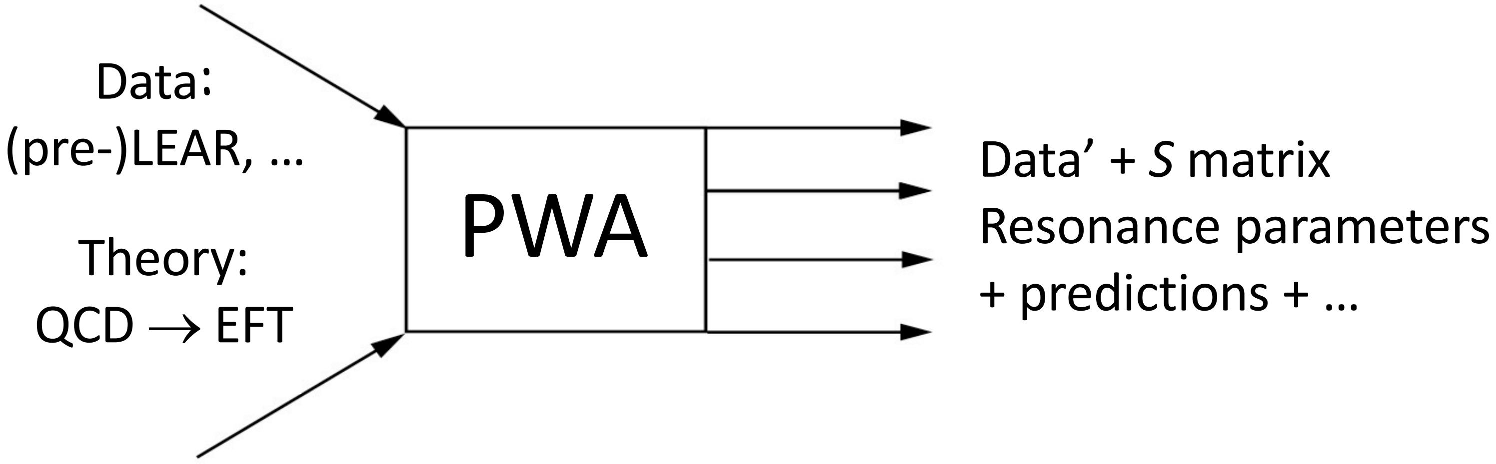
✓

	Elastic		Charge-exchange	
Type	LEAR	rest	LEAR	rest
$\sigma_{\mathrm{tot}}, \sigma_{\mathrm{ann}}$	124	–	–	63
$d\sigma/d\Omega$	281	2507	91	154
A_y	200	29	89	–
D_{yy}	5	–	9	–
Total	*610*	*2536*	*189*	*217*

- Reasonable quality ~17% rejected
- PWA has χ^2/N_{data} = 1.085 with 30 parameters
- Only ~25% of the data comes from LEAR

Partial-wave analysis (PWA) as bridge

- ✓ PWA is *impossible* without theory input



Data:
(pre-)LEAR, …

Theory:
QCD → EFT

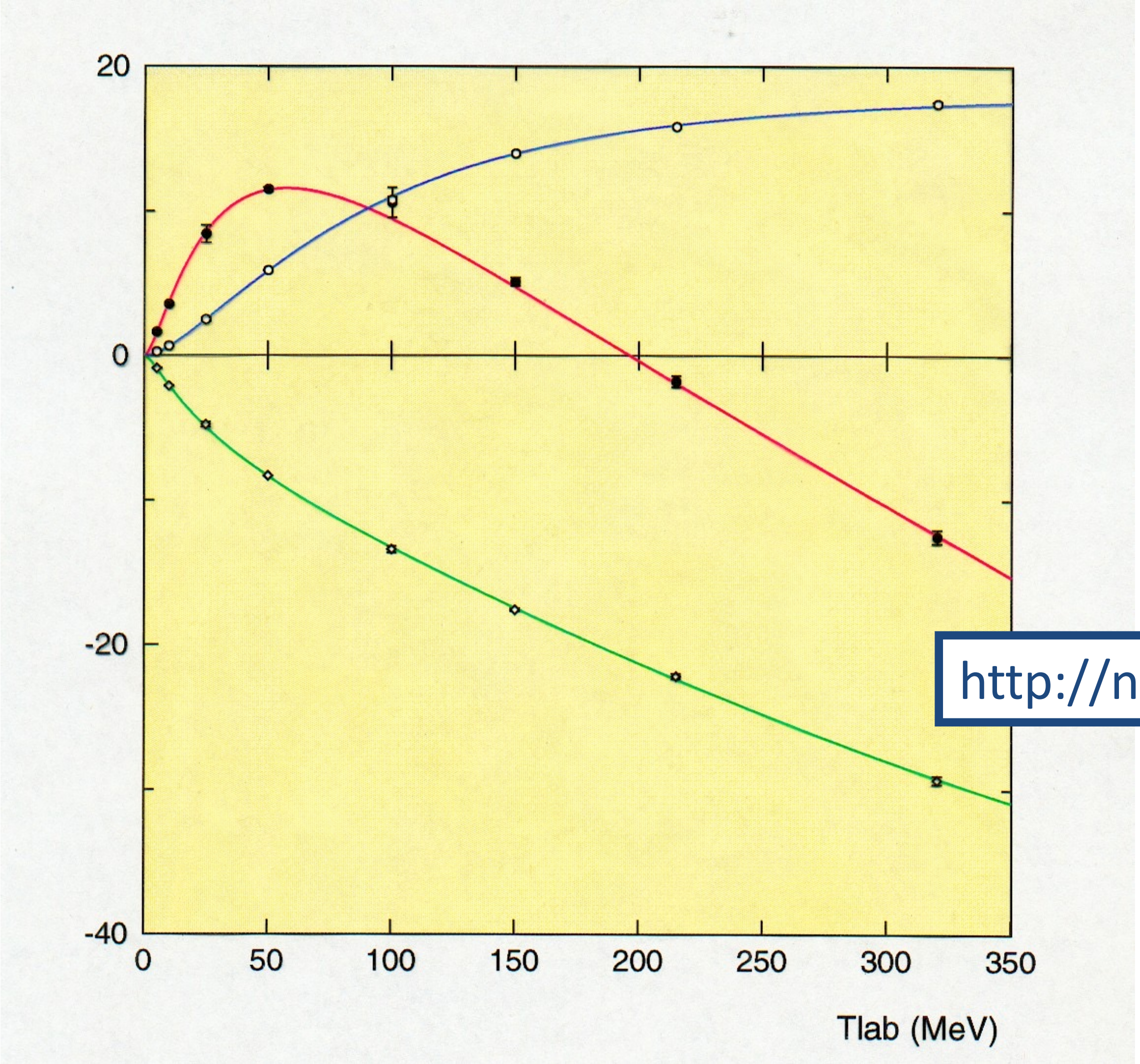
PWA

Data’ + *S* matrix
Resonance parameters
+ predictions + …

- ✓ Input into the PWA
  - Complete database
  - Theory (model independent, *e.g.* Coulomb, EFT)

- ✓ Output of the PWA
  - Database, correlations (χ^2-hypersurface)
  - Multichannel *S* matrix (phase shifts, …)
  - Physics (coupling constants, resonance parameters, ...)

The *pp* ${}^3P_{0,1,2}$ phase shifts after PWA93



http://nn-online.org

- ✓ After PWA93, all phase shifts are accurately known!

Amplitude analysis vs. PWA

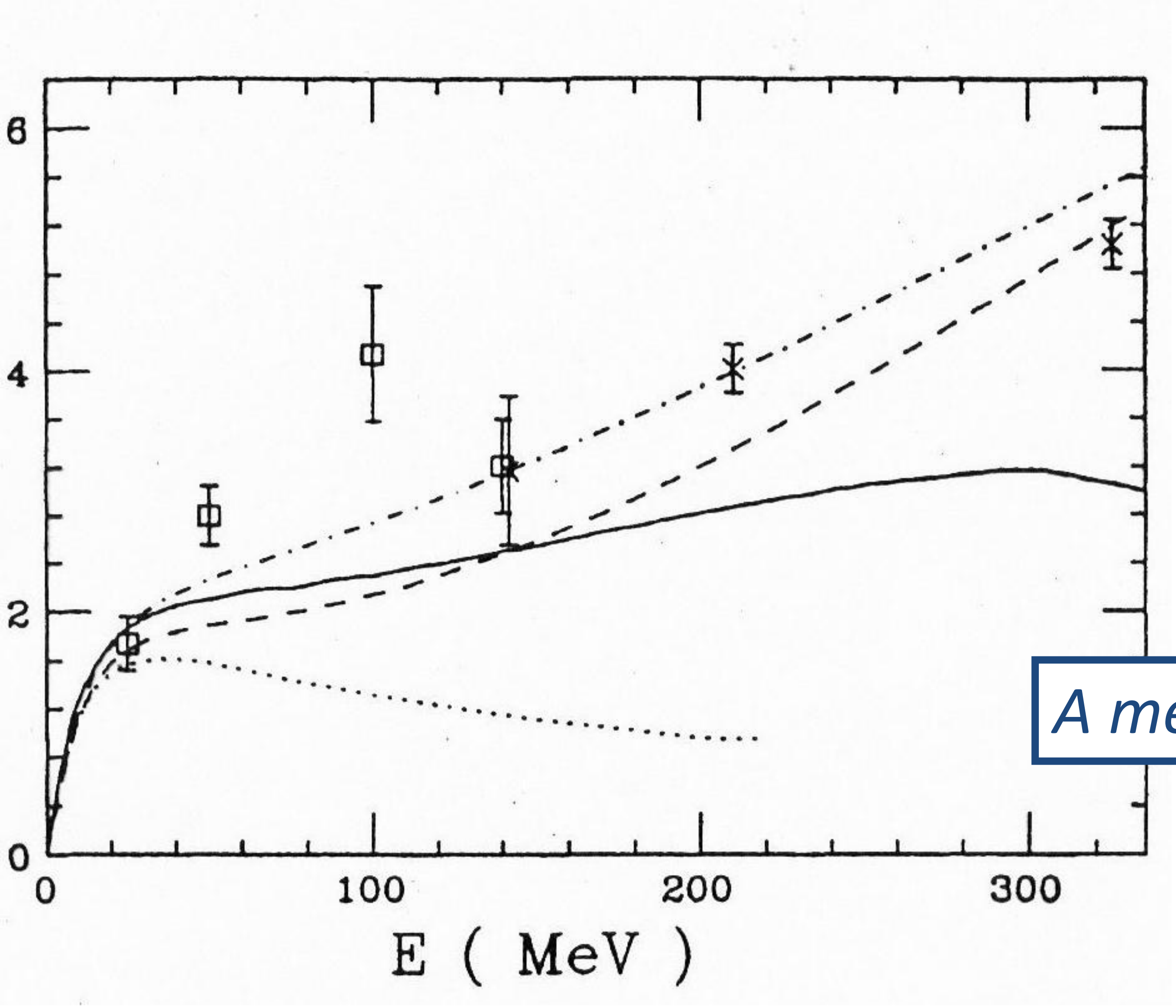
- ✓ Commonly used (but wrong) reasoning, based on “amplitude analysis”:
  - Determine f^2 or $\varepsilon_1 \rightarrow$ OPE $\rightarrow S_{12}$ (tensor force)
  - Best experiments: A_{yy}, $\Delta\sigma_L$, $\Delta\sigma_T$, D_{nn}

- ✓ In a single energy amplitude analysis, one must determine five complex amplitudes $a(\theta)$, $b(\theta)$, $c(\theta)$, $d(\theta)$, $e(\theta)$, for every angle at a fixed energy. Therefore one needs to perform, *for each energy*, 9 experiments, *at all angles*! This was never done...

- ✓ In a single-energy phase-shift analysis, one needs for each value of the total angular momentum J:
  - 2.5 parameters (on average) for *pp;* 5 parameters for *np* (2 for J=0)
  - 20 parameters for *pp*bar (5 for J=0)

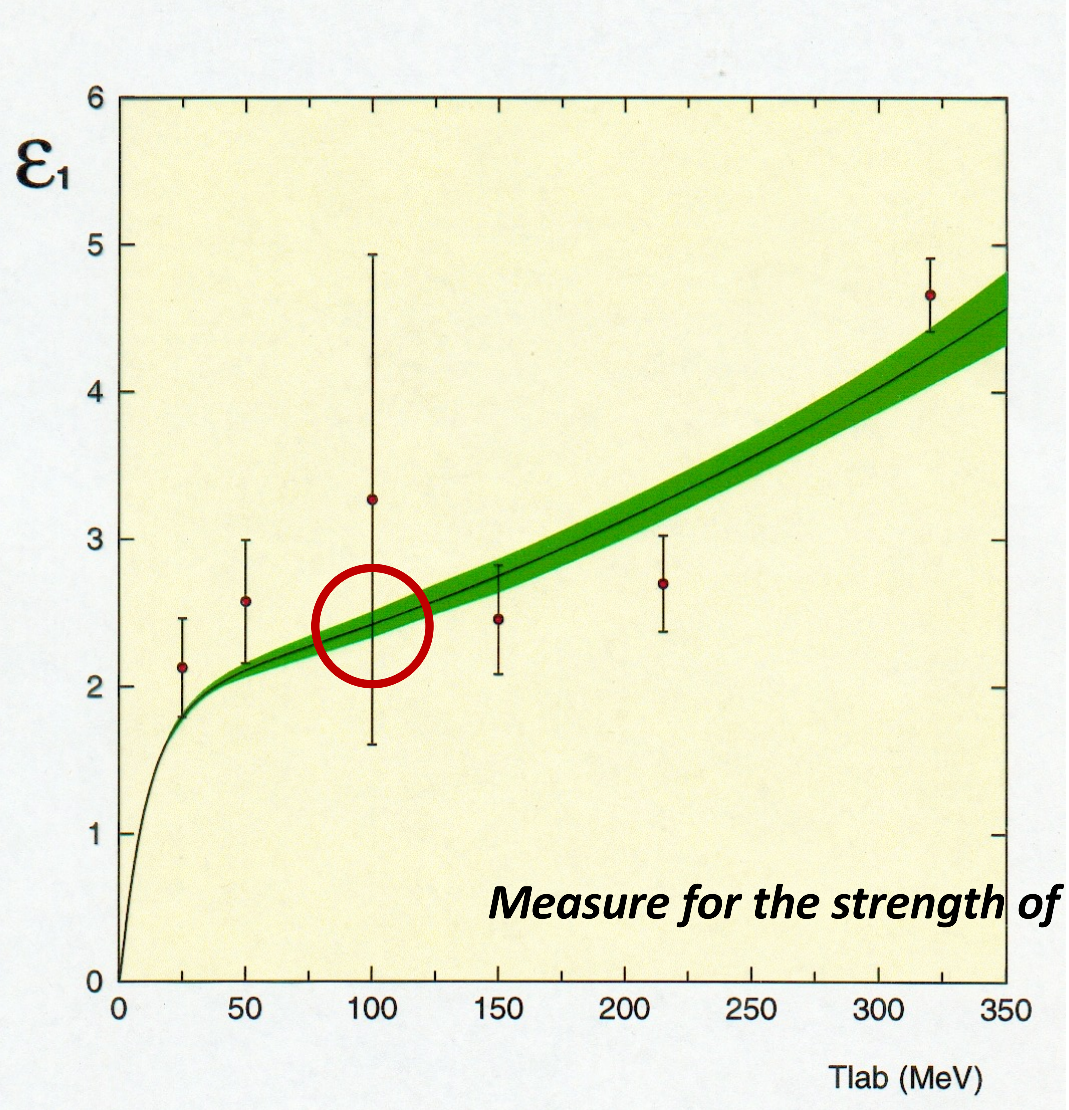
- ✓ However, to perform an energy-dependent PWA, one needs much less:
  - 21 parameters for all *pp* scattering data below 350 MeV
  - 30 parameters for all *pp*bar scattering data below 925 MeV/*c*

The $^3S_1$-$^3D_1$ mixing parameter in *np* scattering



A mess!

The $^3S_1$-$^3D_1$ mixing parameter *after* PWA93



Measure for the strength of the tensor force

- ✓ Note: multi-energy (with error!) versus single-energy

Spin physics (the darling of every PAC world-wide…)

- ✓ Proton-proton PWA below 350 MeV, # data = 1787
  - $d\sigma/d\Omega$, A_y : # = 1381
  - “spin data” : # = 40

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ $\chi^2_{min}/N_{data} = 1.00$

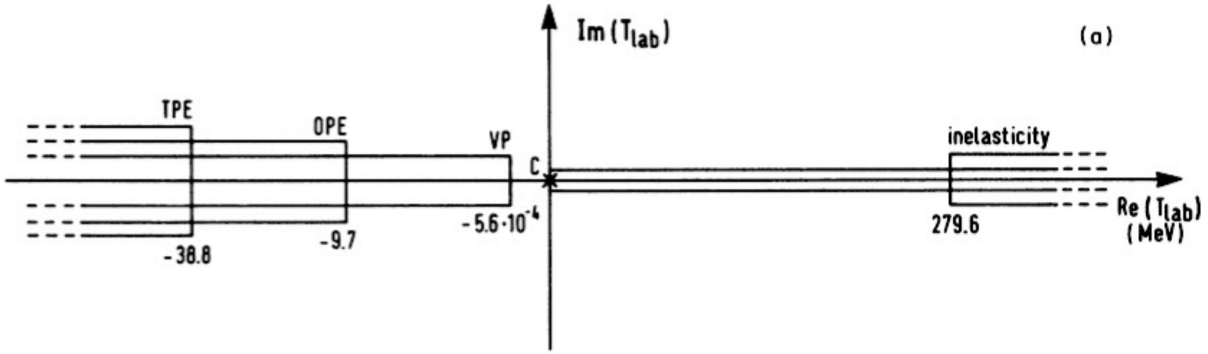
- ✓ Now do a PWA of only the 1381 “non-spin” data:
  - $\chi^2_{min} = 1404$, or $\chi^2_{min}/N_{data} = 1.02$

- ✓ Use this PWA of the “non-spin” data to *predict* the “spin data”:
  - for $N_{data} = 1787$ we get $\chi^2/N_{data} = 1.23$!

- ✓ How many “spin data” are actually *useful* ?
  - 406 “spin data” with $\chi^2 = 800$
  - 64 points (8 sets) have $\chi^2 = 477$; 342 points have $\chi^2 = 321$!

Analyticity of the *S*-matrix

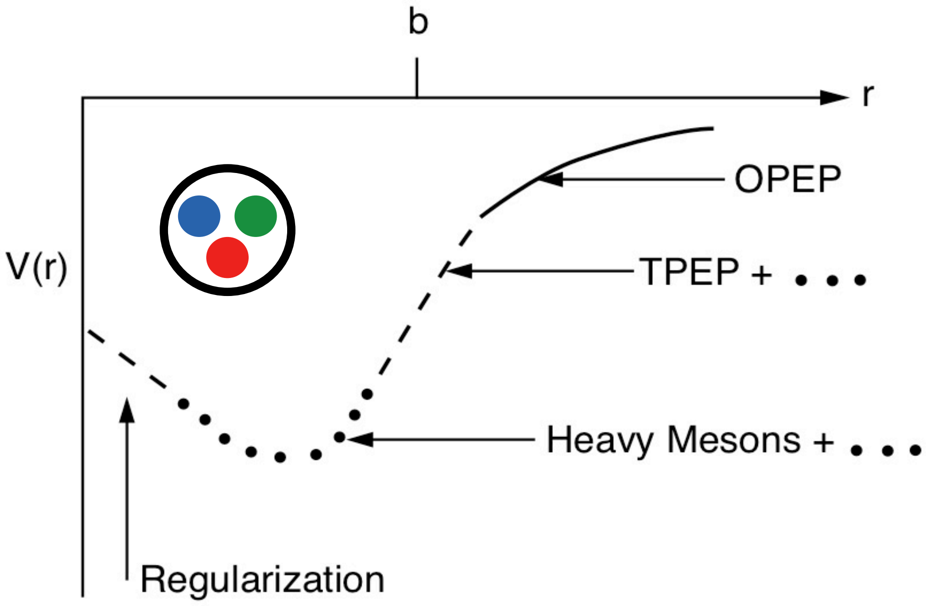
- ✓ Need good description of the energy dependence of the amplitudes
  - Forces are “left-hand cuts”
  - Rapid energy dependence ← nearby cuts ← long-range interactions
  - Slow energy dependence ← far-away cuts ← short-range interactions
- ✓ Strategy:
  - Calculate long-range interaction V_L from field theory
  - Treat short-range interaction V_S completely general



Coulomb	$1/r$
Rel.corr.+2γ	$1/r^2$
Magn.mom.	$1/r^3$
Vac.pol.	$\exp(-2m_e r)/r^{3/2}$
OPE	$\exp(-m_\pi r)/r$
TPE	$\exp(-2m_\pi r)/r^{5/2}$

PWA: Implementation

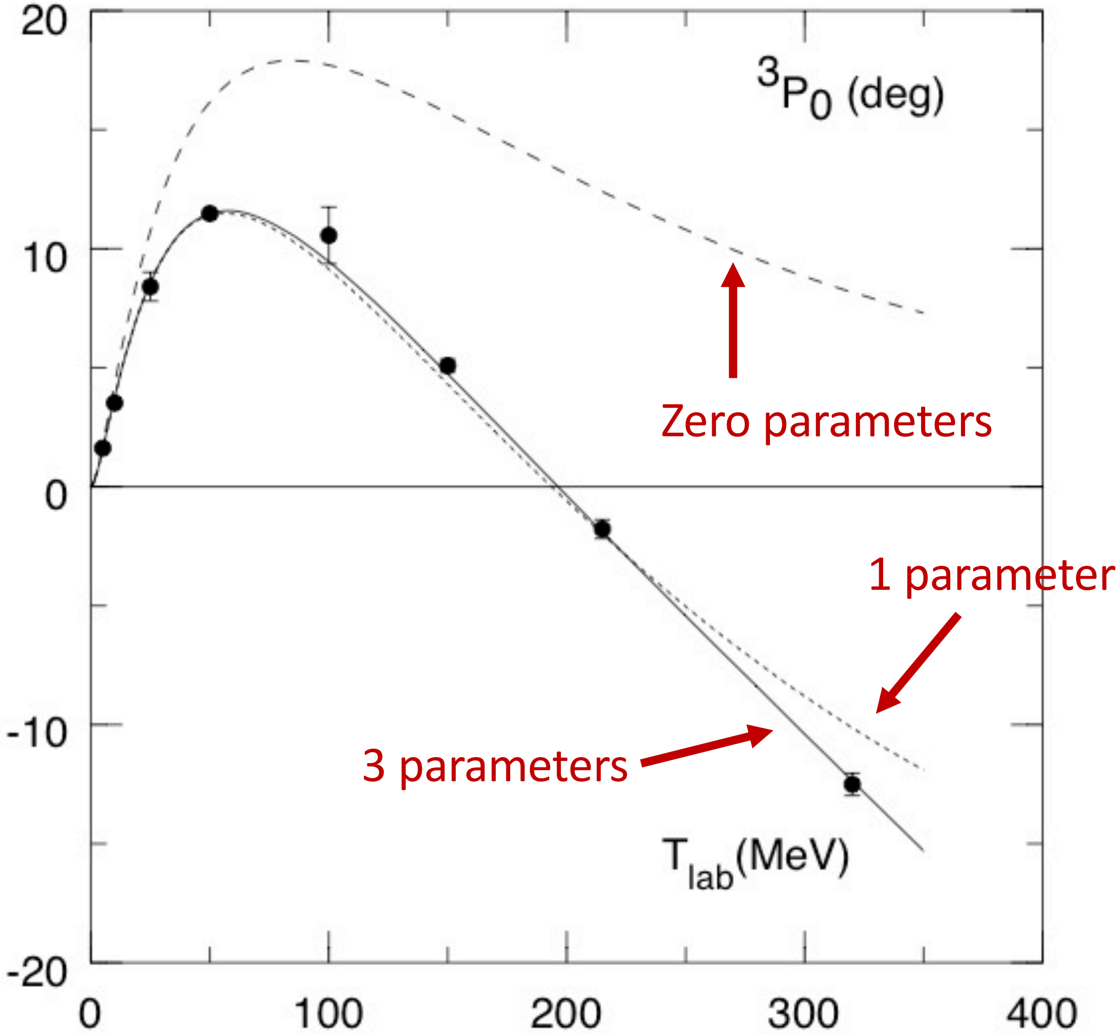
- ✓ Do not parametrize $\delta_L(E)$ directly
- ✓ Instead, boundary condition at $r = b$
  - Jaffe & Low, PRD 19, 2105 (1979)
  - BC = $b[d\psi/dr]\psi^{-1}|_{r=b}$
  - Analytic function of energy
  - $V_S = C_0 + C_2p^2 + C_4p^4 + \ldots$
- ✓ Explicit long-range interaction
  - $V_L(r) = V_{EM}(r) + V_{OPE}(r) + V_{\chi TPE}(r)$
  - $V_{EM} = V_C + V_{2\gamma} + V_{VP} + V_{MM}$
- ✓ Inelasticity $\eta_L(E)$: complex BC



<u># BC parameters in *pp* PWA:</u>

$^1S_0$: 4 $^3P_0$: 3 $^3P_1$: 2
$^1D_2$: 2 $^3P_2$: 3 ε_2: 2 $^3F_2$: 1 $^3F_3$: 3
$^1G_4$: 1 $^3F_4$: 2 ε_4: - $^3H_4$: - $^3H_5$: -
$^1I_6$: - etc. Total: #=21

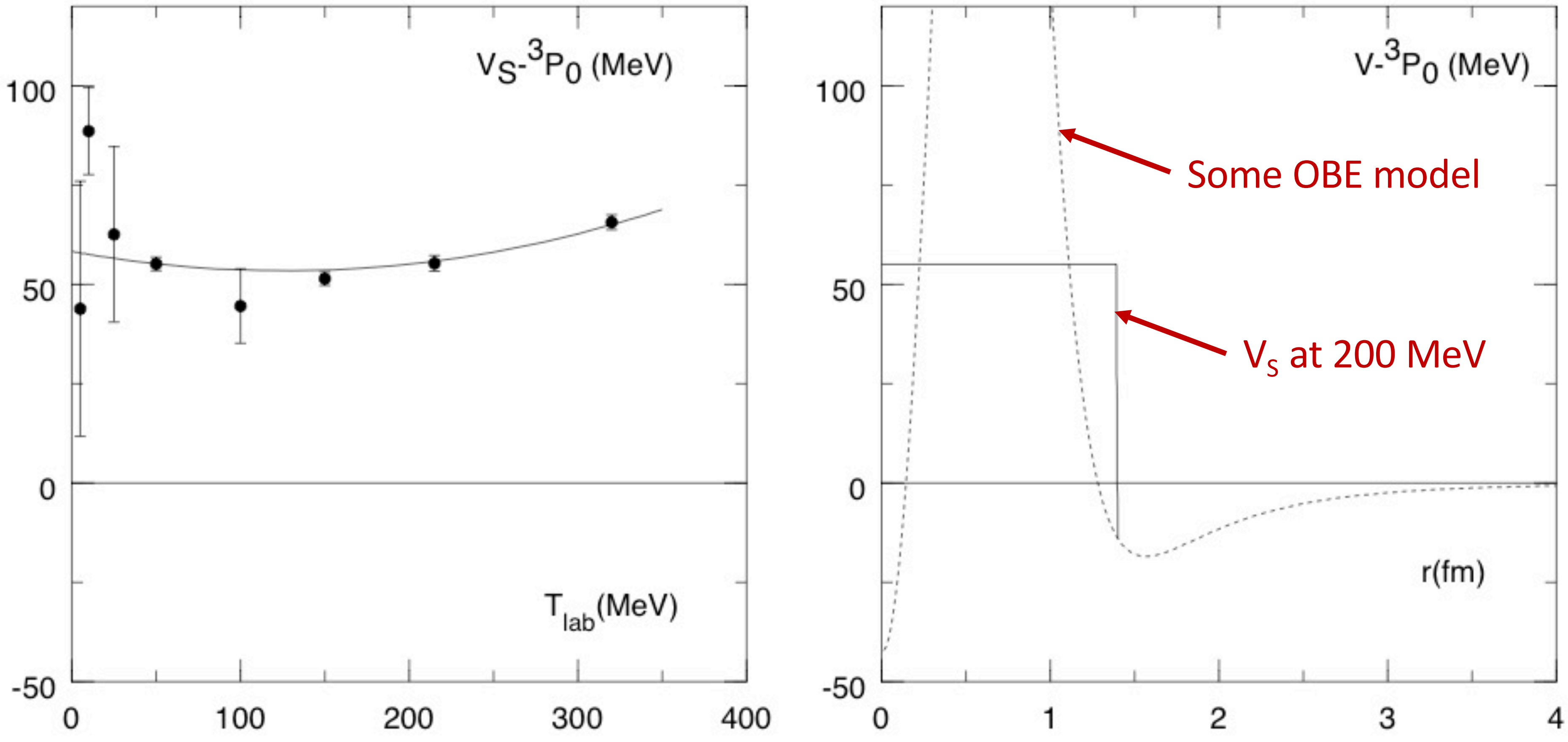
The $^3P_0$ phase shift in the *pp* PWA



- ✓ “The details of the short-range interactions do not matter”

The short-range interaction in the $^3P_0$ wave

- ✓ Energy dependence of the short-range interaction for $^3P_0$:
- ✓ Radial dependence of the potential for the $^3P_0$ wave:



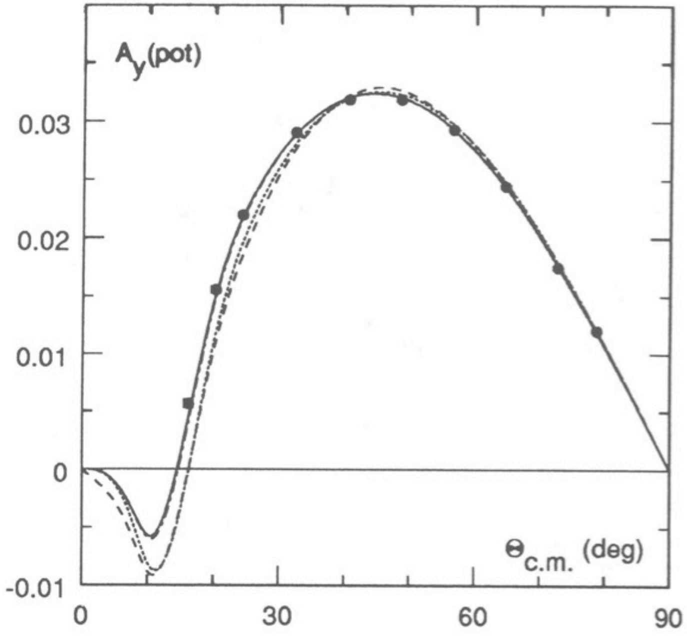
Some OBE model

V_S at 200 MeV

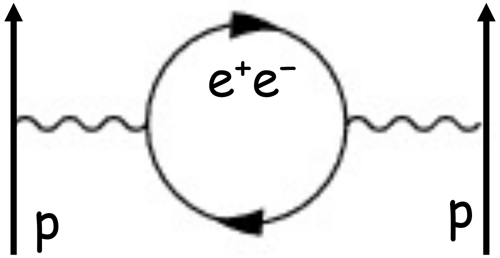
Long-range electromagnetic effects

- ✓ Coulomb interaction

- ✓ Magnetic-moment interaction
  - $V_{MM} = -\alpha[\mu_p{}^2 S_{12} + (6+8\kappa_p) L\cdot S]/4m^2r^3$
  - Charge & magn. mom. → spin-orbit
  - Magn. mom. & magn. mom. → tensor
  - In *pp* PWA: $\Delta\chi^2_{min} \approx +400$, so 20 s.d.

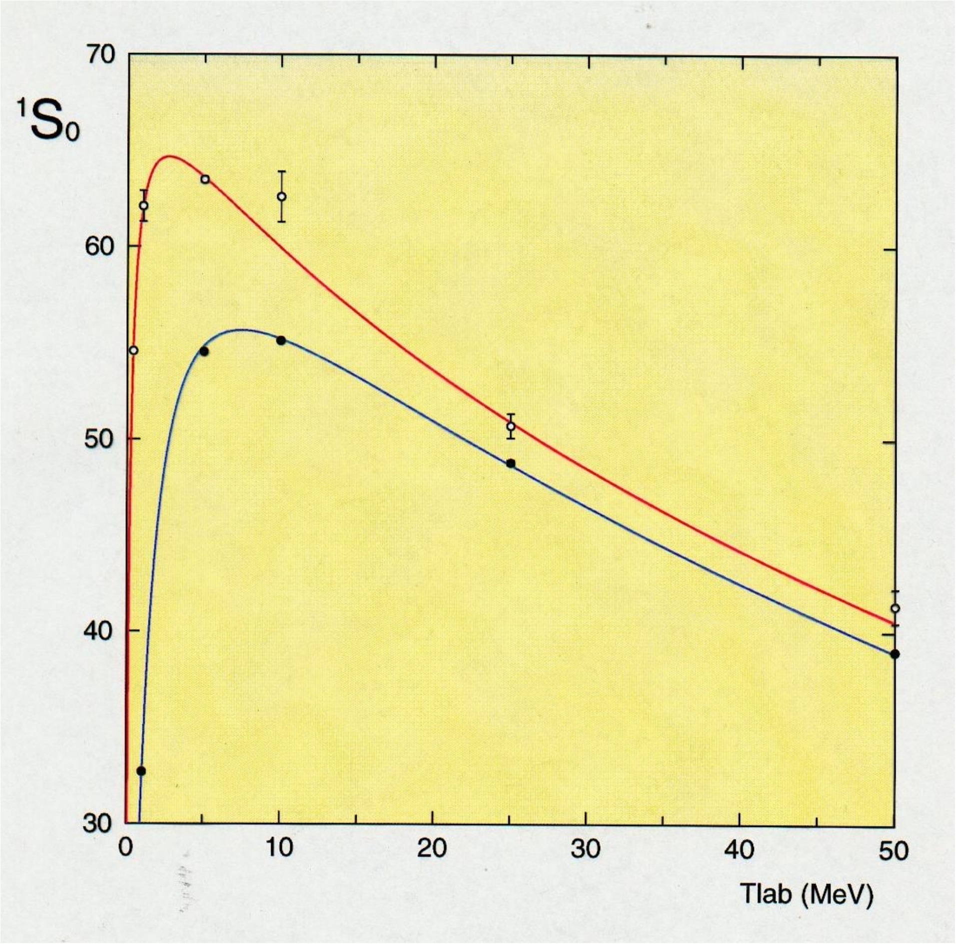


- ✓ Vacuum-polarization interaction
  - $V_{VP} = \alpha' I(r)/r \approx \alpha\alpha' \exp(-2m_e r)/r^{3/2}$
  - Enhances Coulomb force V_C
  - Long-range: $1/2m_e \approx 200$ fm
  - Relevant in proton-proton $^1S_0$ wave
  - In *pp* PWA: $\Delta\chi^2_{min} \approx +215$, so 15 s.d.



Charge-independence breaking, *pp* vs. *np*

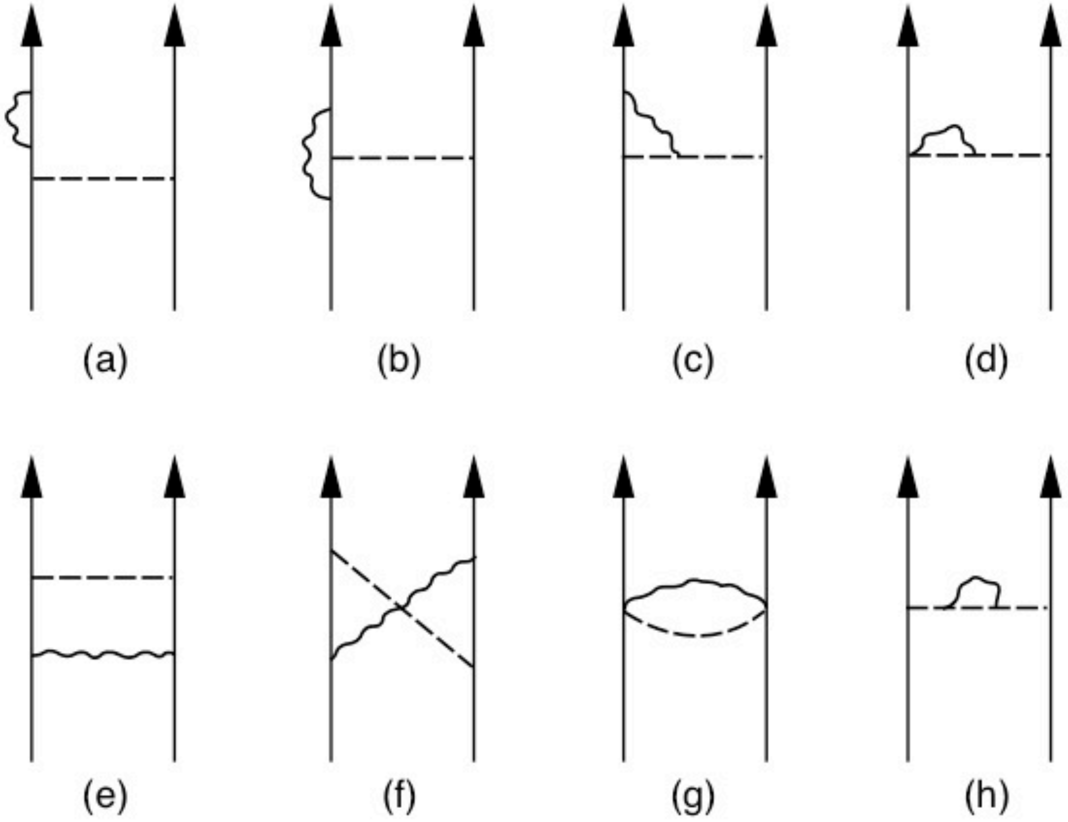
- ✓ Isospin violation is due to EM effects + the up-down quark-mass difference
  - Our long-range interaction (strong + EM) contains the relevant isospin violation predicted by χPT



- ✓ Study CIB for all low waves
  - *pp* and *np* $^1S_0$ wave
  - Also *e.g.* the ^3P waves

Pion-photon exchange*

- ✓ Studied in *np* PWA 0-500 MeV
- ✓ $V_{\gamma\pi}$ from χPT (19 diagrams)
- ✓ $N_{\text{data}} \approx 4100$, $\Delta\chi^2_{\text{min}} \approx -0.8$
  - No effect of $f^2_{NN\pi}$



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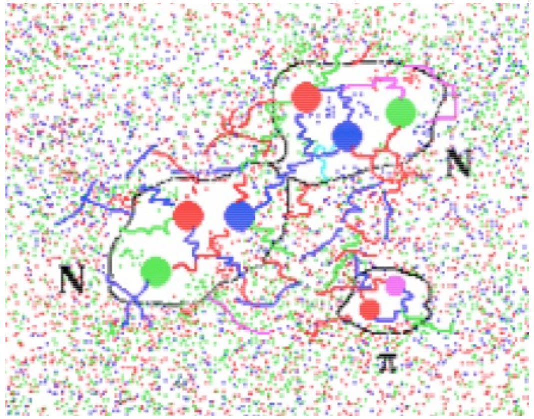
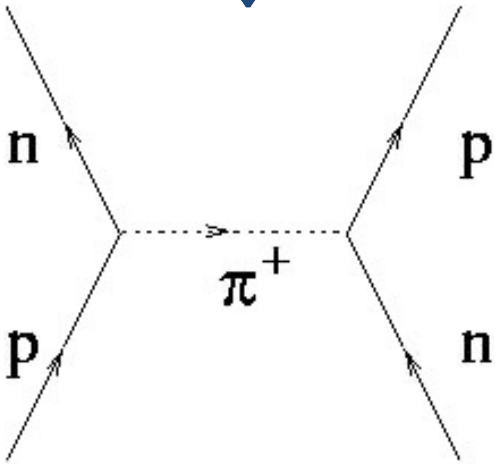
* M.C.M. Rentmeester *et al.*, PRL **80**, 4386 (1998).

One-pion exchange: The glue of nuclei





- ✓ Strong interaction: $V_{\text{nuc}}(\text{r}) = V_{\text{OPE}} + V_{\chi\text{TPE}}$
  - One-pion exchange ($x = m_\pi r$):
  - $V_{\text{OPE}} = f_{NN\pi}{}^2\, m_\pi[\sigma_1 \cdot \sigma_2 + \xi(x) S_{12}]\exp(-x)/3x$
  - Charge-dependent:
    - $m_{\pi 0}$ vs. $m_{\pi\pm}$
    - $f_{\text{pp}\pi 0}$ vs. $f_{\text{nn}\pi 0}$ vs. $f_{\text{np}\pi\pm}$
- ✓ Best value: $\boxed{f^2{}_{NN\pi} = 0.0750(9)^*}$
- ✓ Goldberger-Treiman relation: $f_{NN\pi} / m_\pi = g_A / F_\pi$
  - *i.e.* the “discrepancy” is only 1-2%

---

* Or $g^2{}_{NN\pi}/4\pi \approx 13.6$

“Seeing” one-pion exchange

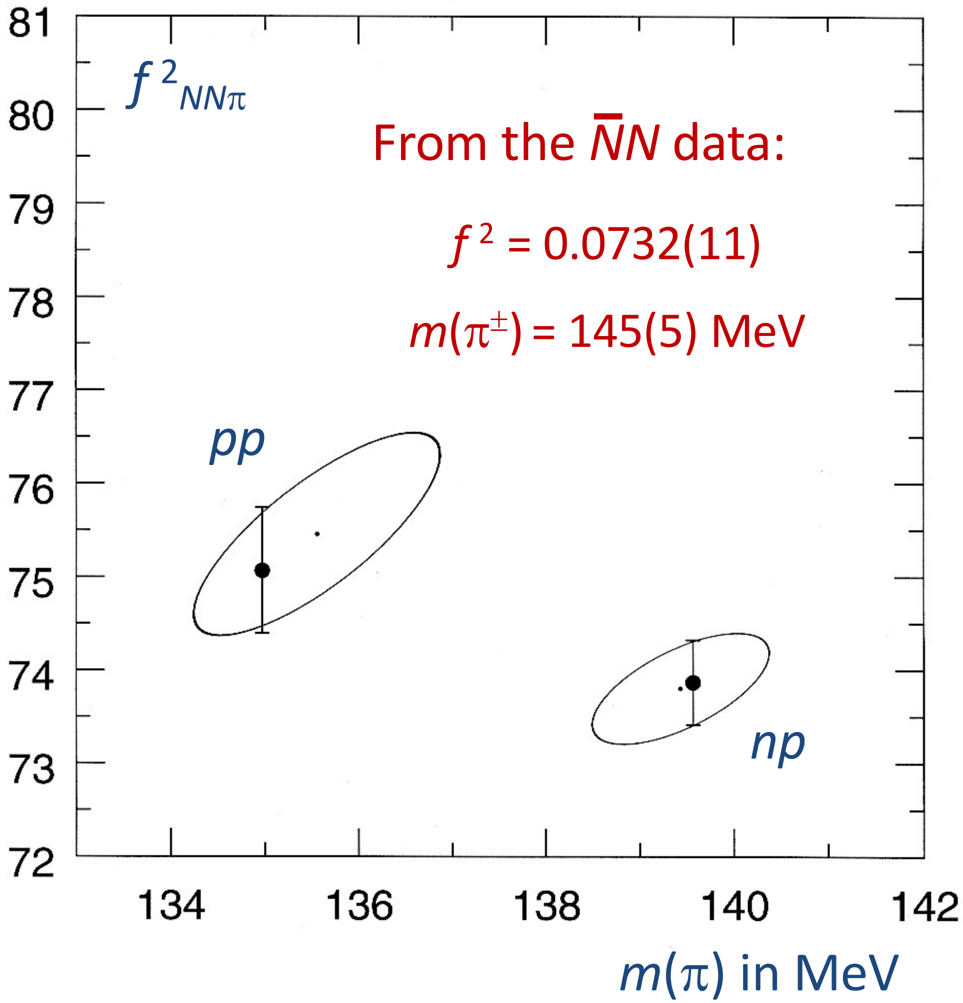
- ✓ The *$NN\pi$* coupling constant is determined *at the pion pole* from the long-range OPE

- ✓ We can also *fit* the pion masses
  - From the *pp* and the *np* data

$m(\pi^0) = 135.6(10)$ MeV

$m(\pi^\pm) = 139.6(13)$ MeV

- ✓ No evidence for isospin violation:
  - $f(pp\pi^0) \approx f(nn\pi^0) \approx f(np\pi^\pm)$

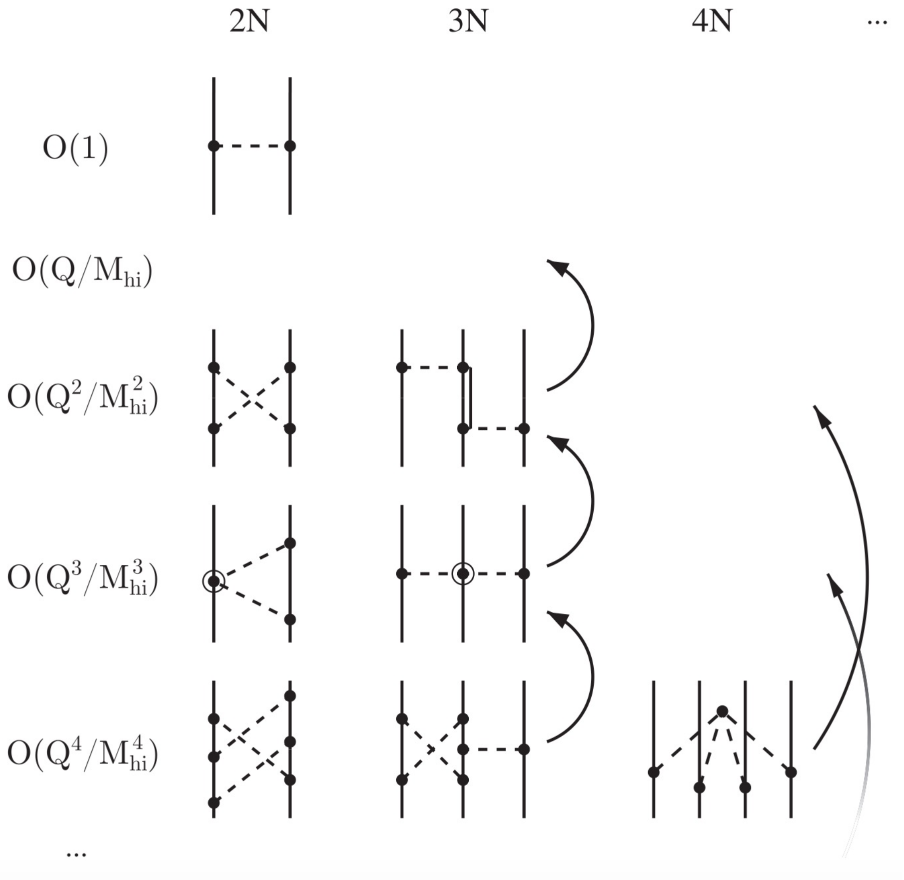


From the $\bar{N}N$ data:

$f^2 = 0.0732(11)$

$m(\pi^\pm) = 145(5)$ MeV

How to connect nuclear physics to QCD



Lattice QCD ⇒
low-energy constants of χPT

⇓

NN & few-nucleon systems
⇒ χPT: *controlled* expansion

⇓

Nuclear structure ⇒
via many-body EFT?

✓ Chiral effective field theory * → Chiral two-pion exchange interaction!

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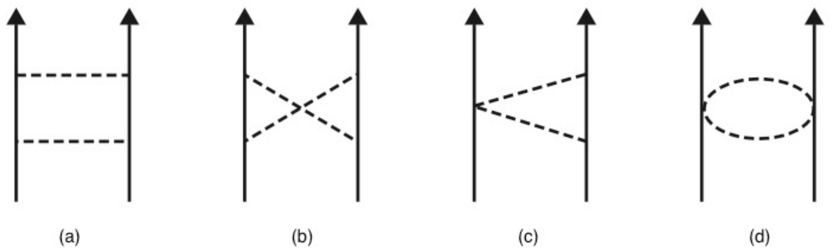
* Hammer, König, van Kolck, RMP **92**, 025004 (2020).

“Seeing” two-pion exchange

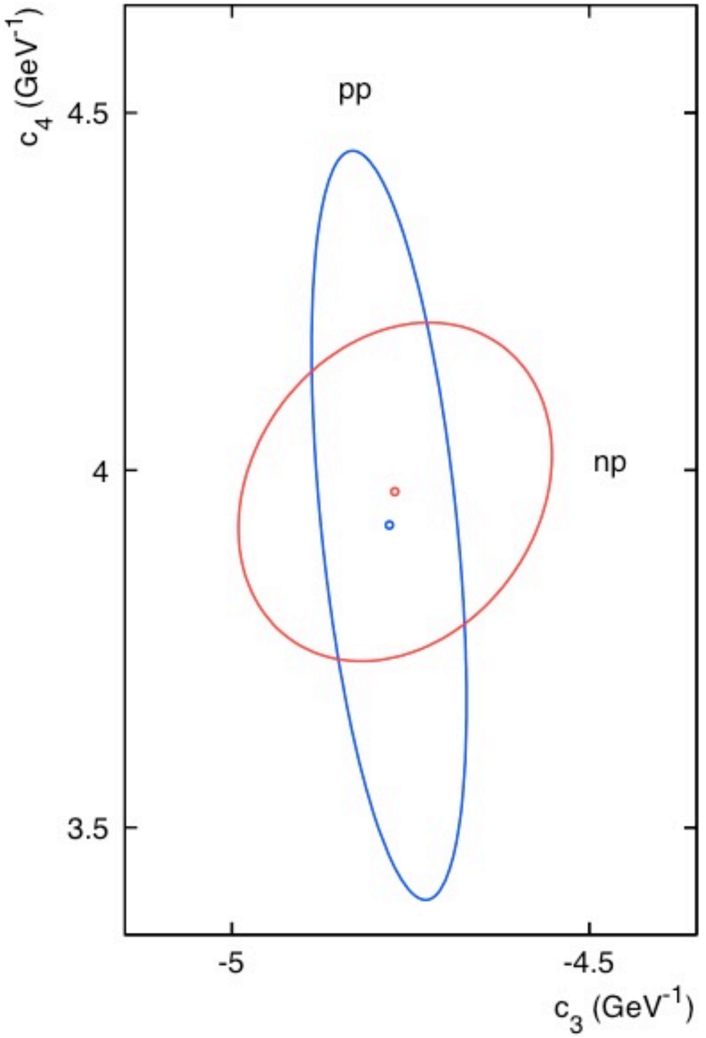


- ✓ With long-range OPE+χTPE:
  - An excellent χ^2/data $\simeq 1$ is possible!
  - No need for the “sigma-meson”

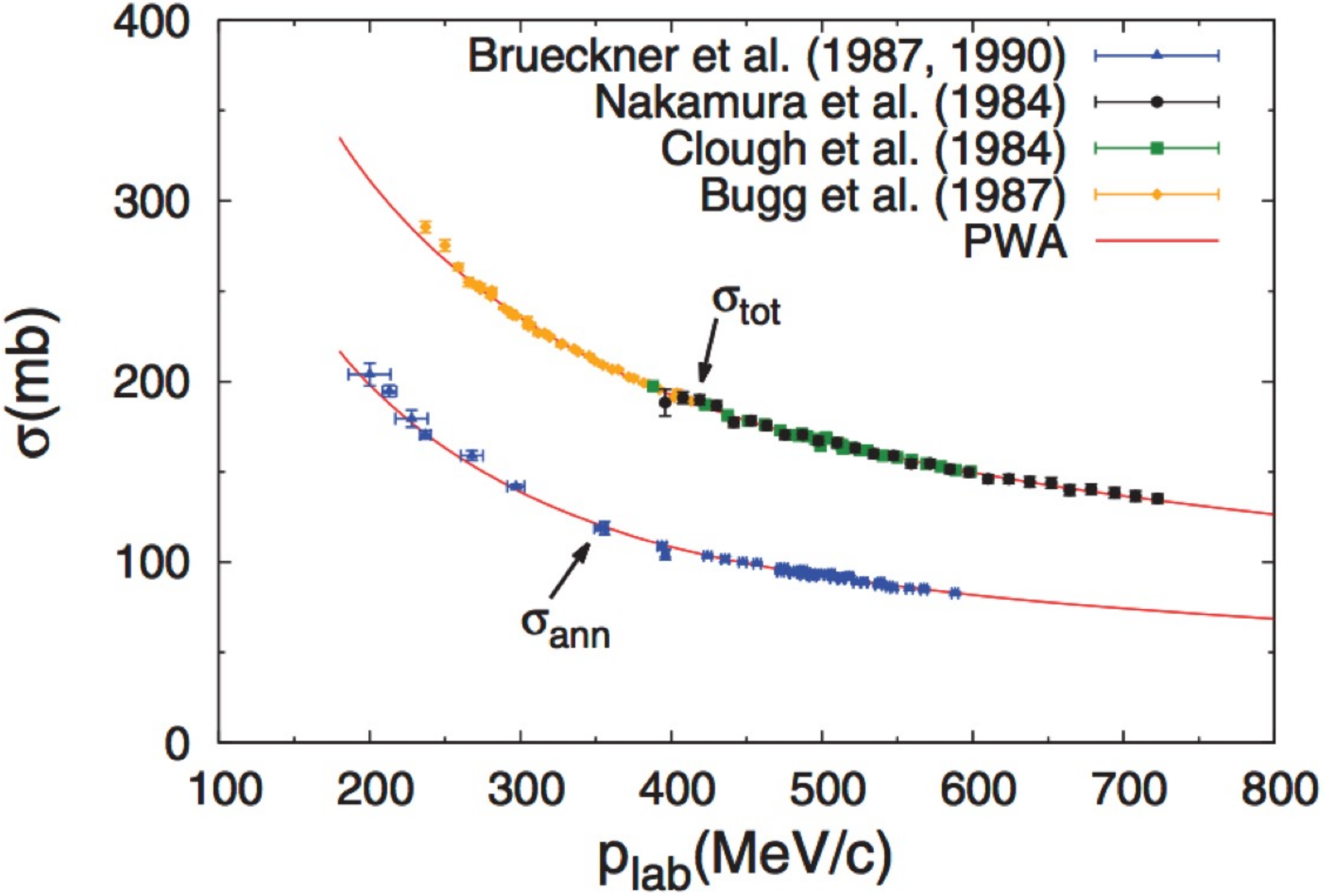
- ✓ $NN\pi\pi$ coupling constants:
  - $c_3 = -4.78(10)$/GeV
  - $c_4 = +3.96(22)$/GeV

- ✓ Pion mass from long-range χTPE:
  - $m_\pi = 128(9)$ MeV

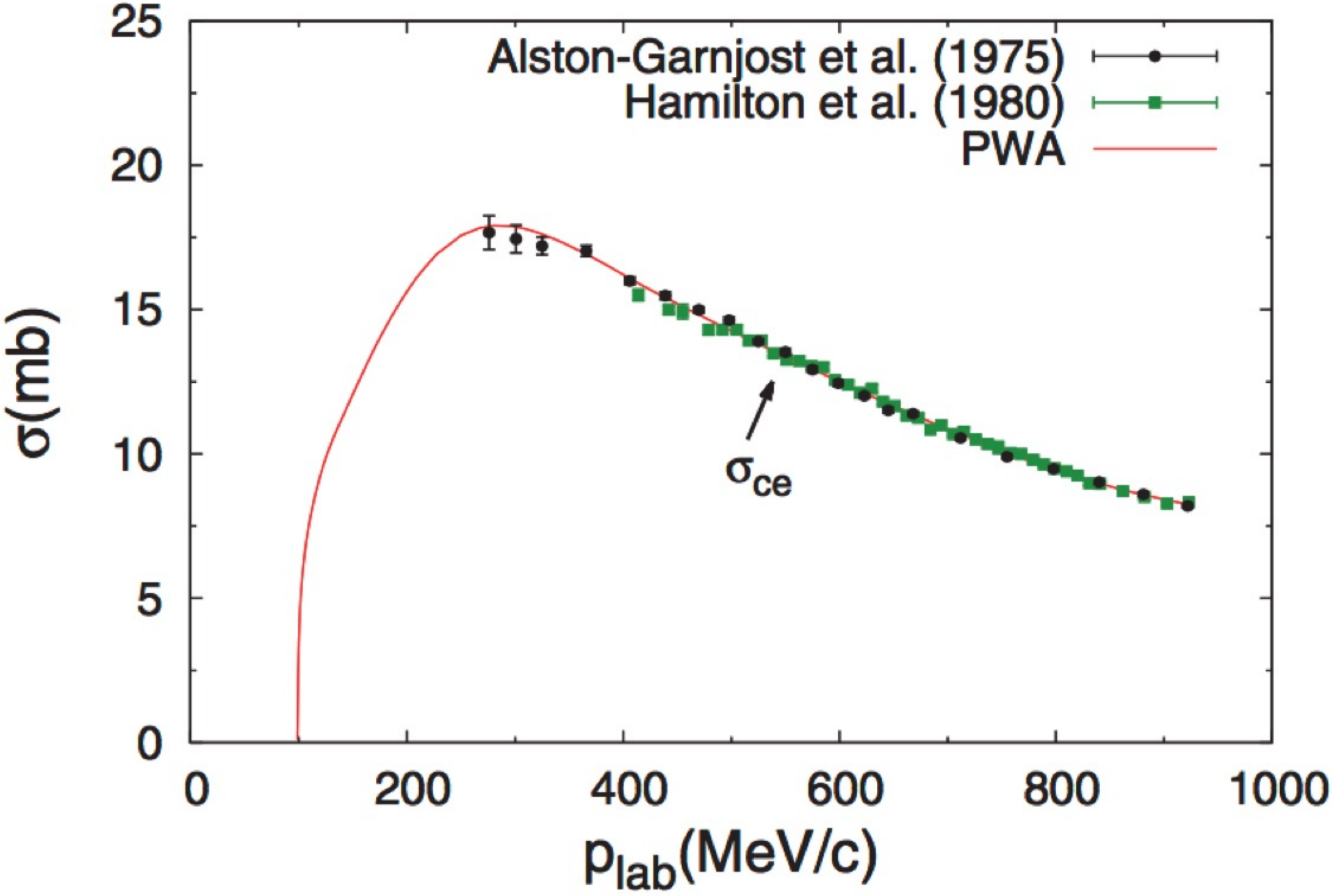
OPE + (charge-conj.) χTPE provides an excellent long-range $\overline{NN}$ interaction.



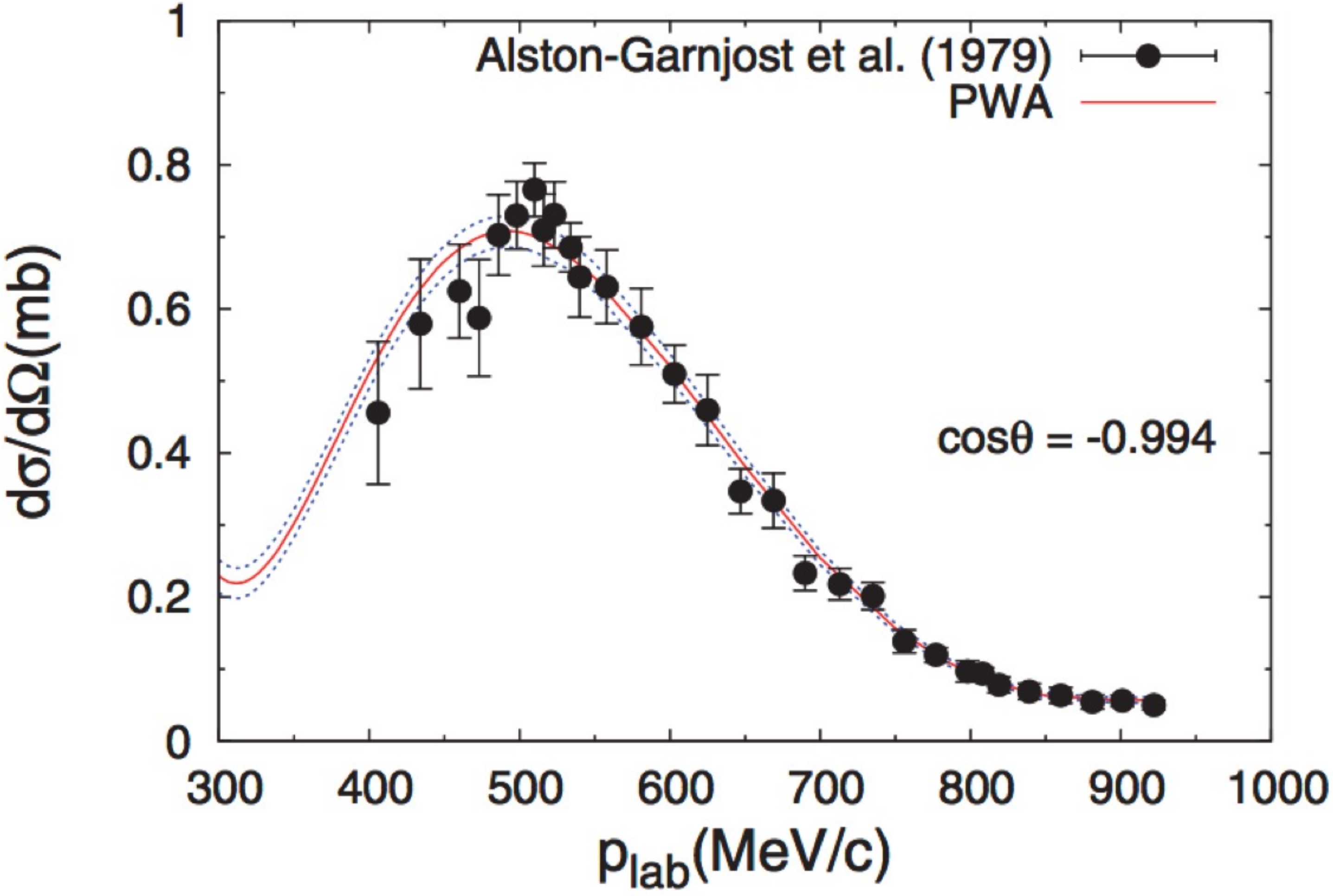
Total and annihilation cross section



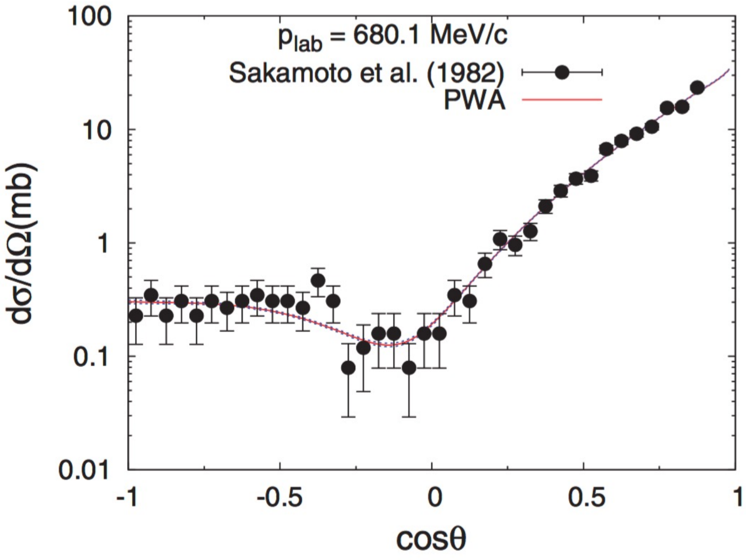
Total charge-exchange cross section

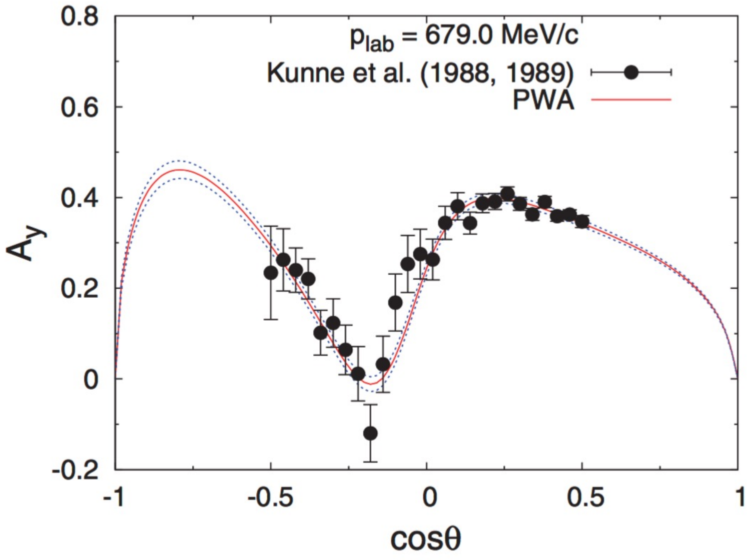


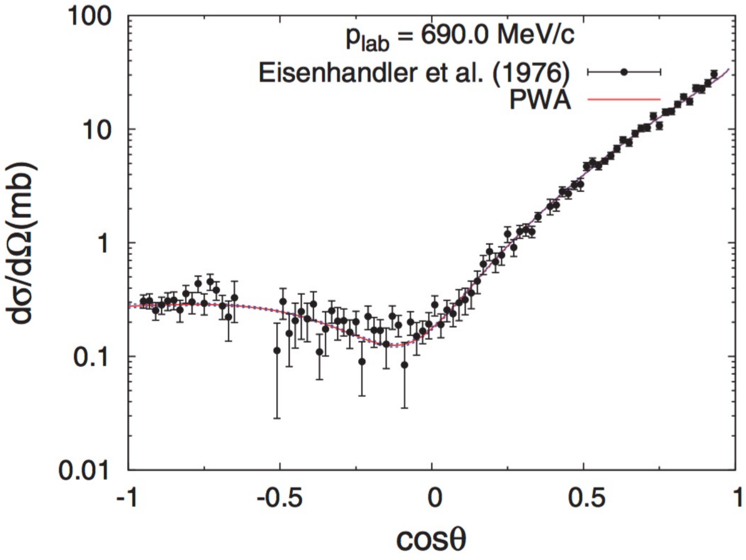
Backward elastic differential cross sections

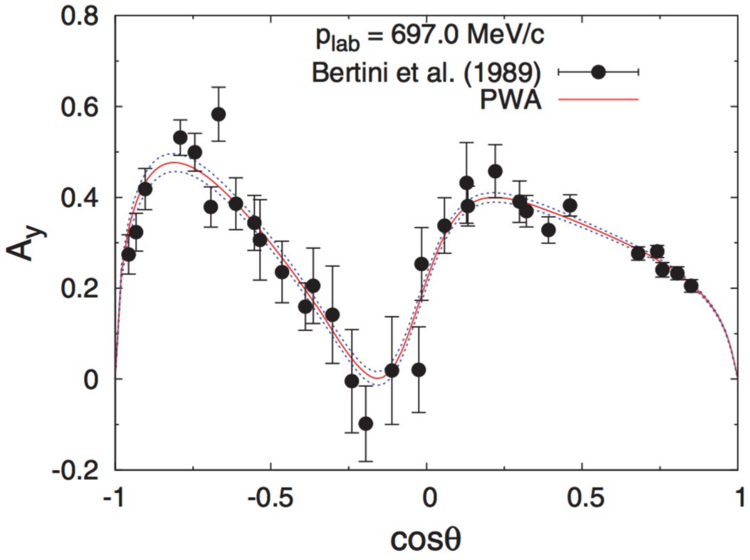


Elastic differential cross section and polarization

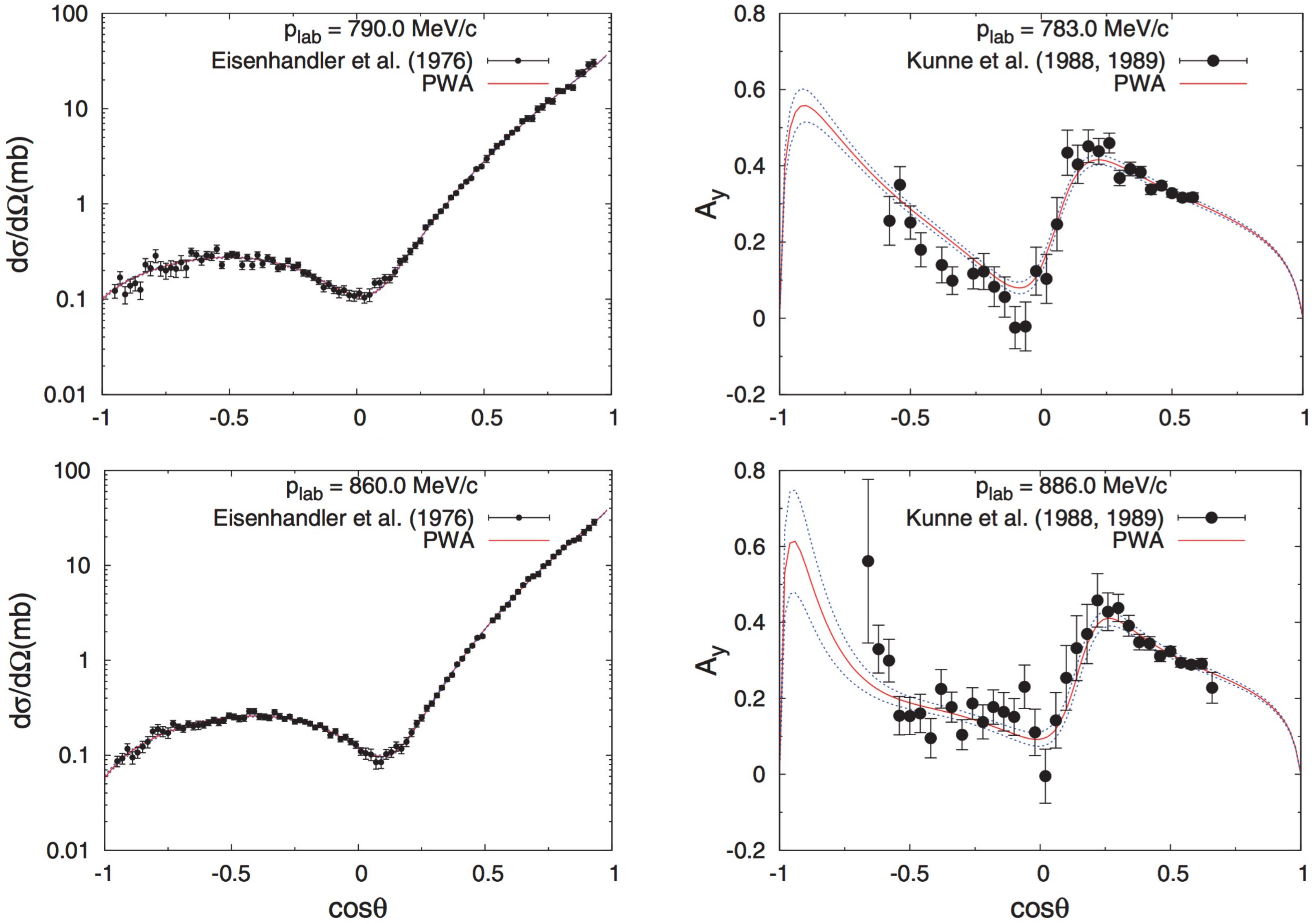




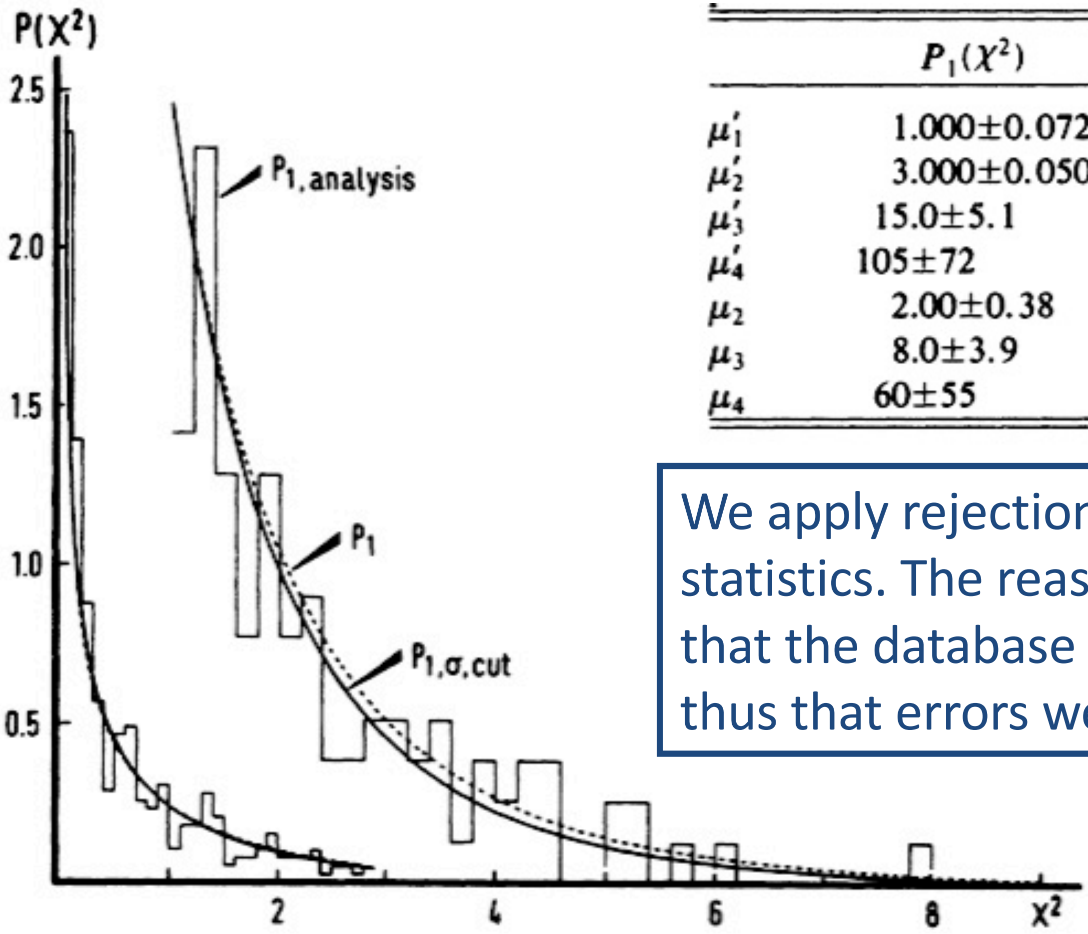


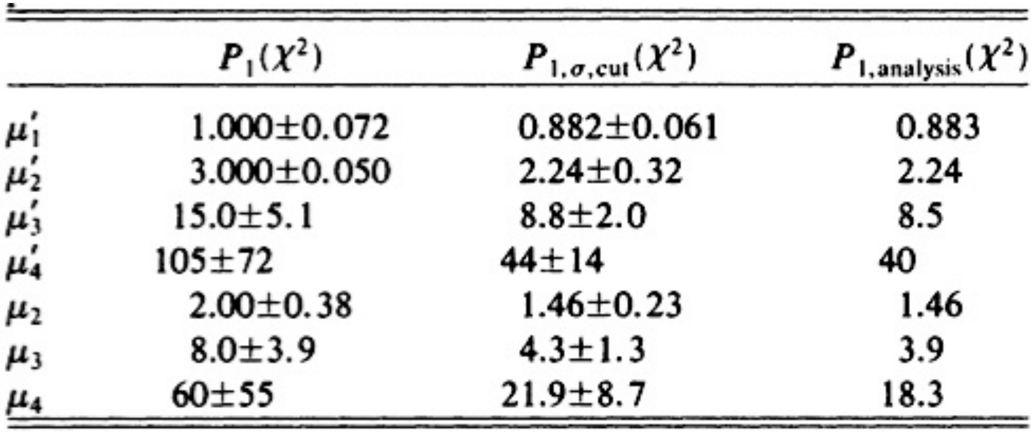


Elastic differential cross section and polarization



Lies, damned lies, and statistics!



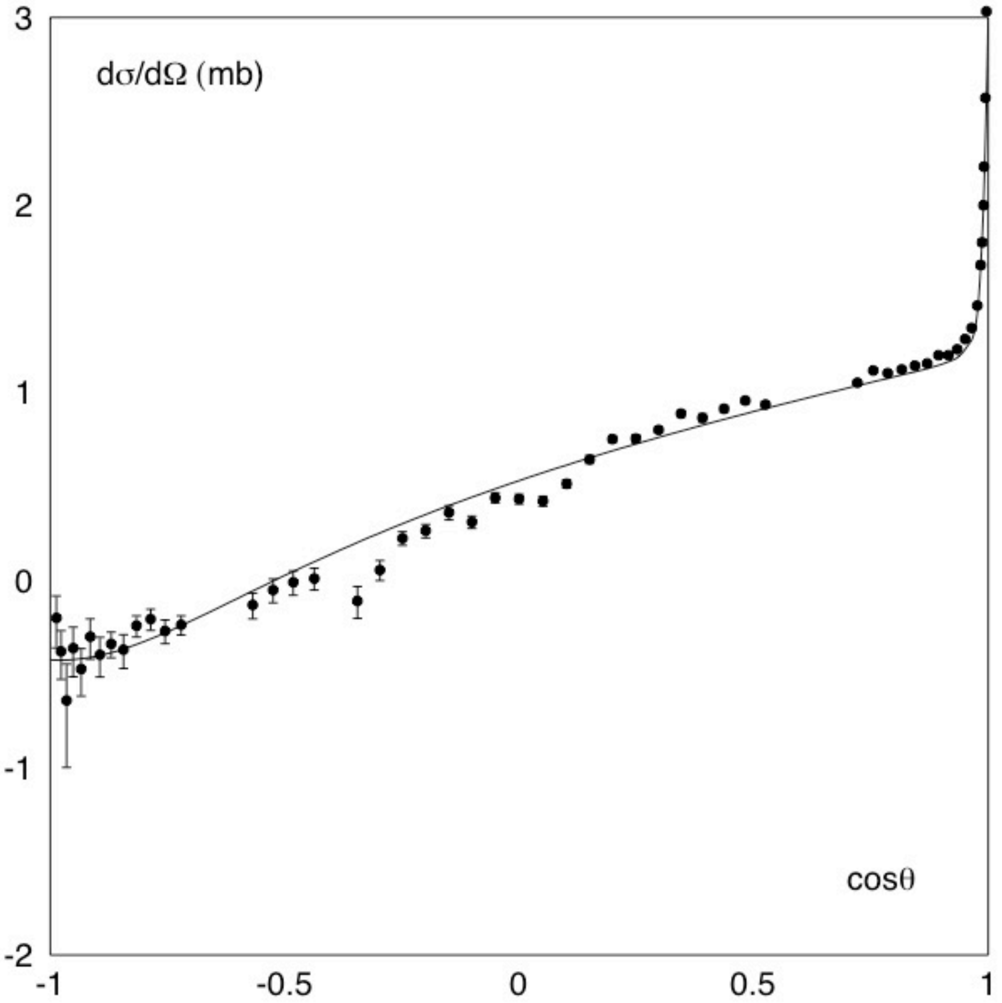


	$P_1(\chi^2)$	$P_{1,\sigma,\mathrm{cut}}(\chi^2)$	$P_{1,\mathrm{analysis}}(\chi^2)$
μ_1'	1.000 ± 0.072	0.882 ± 0.061	0.883
μ_2'	3.000 ± 0.050	2.24 ± 0.32	2.24
μ_3'	15.0 ± 5.1	8.8 ± 2.0	8.5
μ_4'	105 ± 72	44 ± 14	40
μ_2	2.00 ± 0.38	1.46 ± 0.23	1.46
μ_3	8.0 ± 3.9	4.3 ± 1.3	3.9
μ_4	60 ± 55	21.9 ± 8.7	18.3

We apply rejection criteria based on *standard* statistics. The reason is very simple: to make sure that the database is a *statistical ensemble* and thus that errors we quote are really statistical!

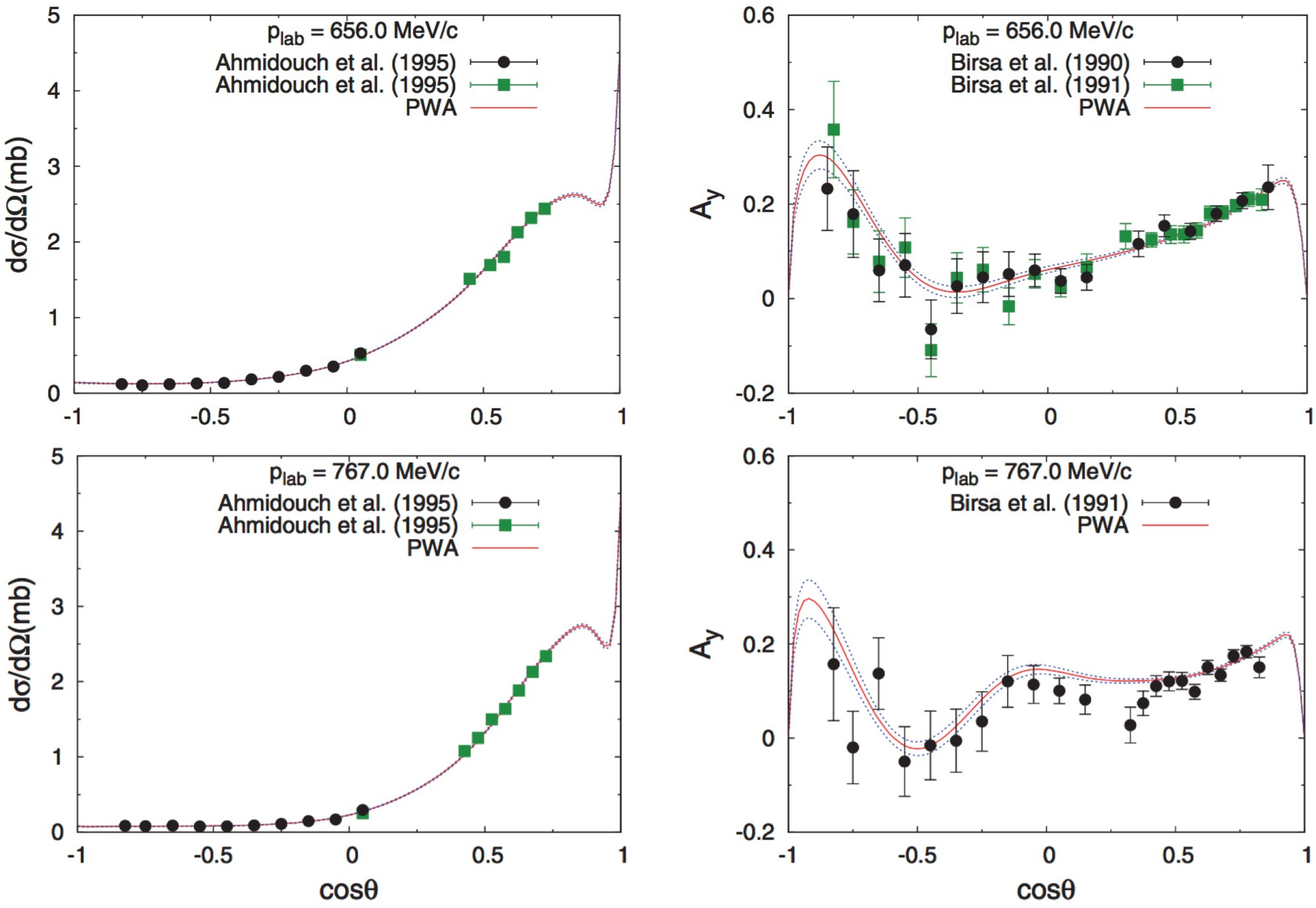
- ✓ *We do not determine if experiments are right or wrong, but we do decide whether they are statistically acceptable, yes or no*

Problematic data sets from LEAR

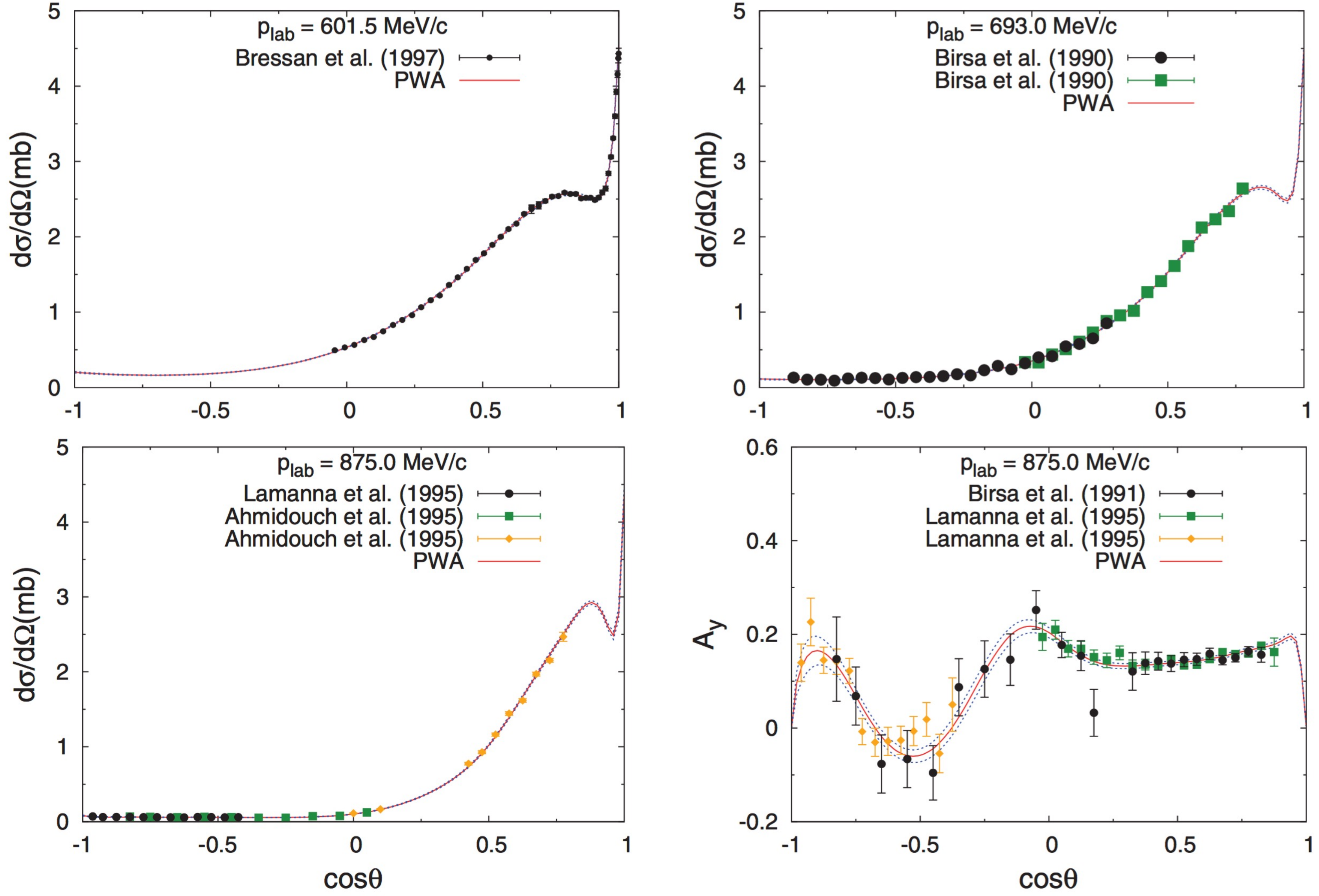


Group	χ^2	# data
PS172	504	84
PS173	505	173
PS198	1743	84
KEK	3096	173

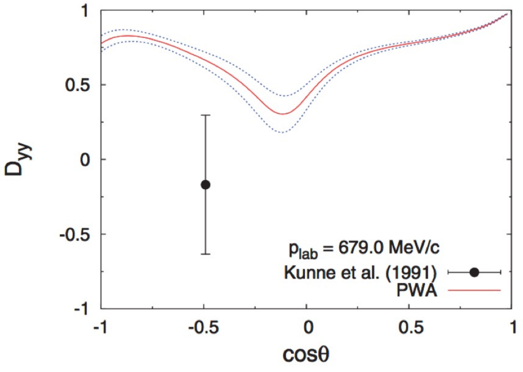
Charge-exchange cross section and analyzing power

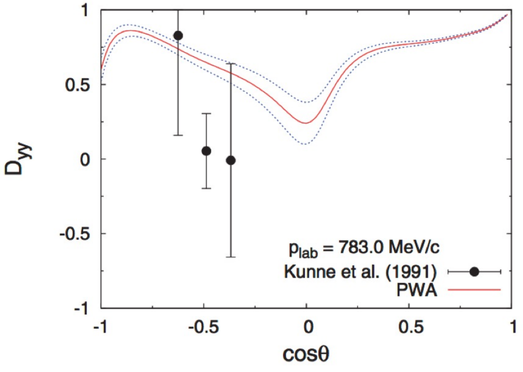


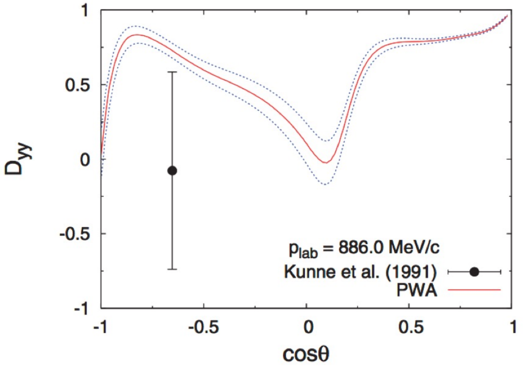
Charge-exchange cross section and analyzing power

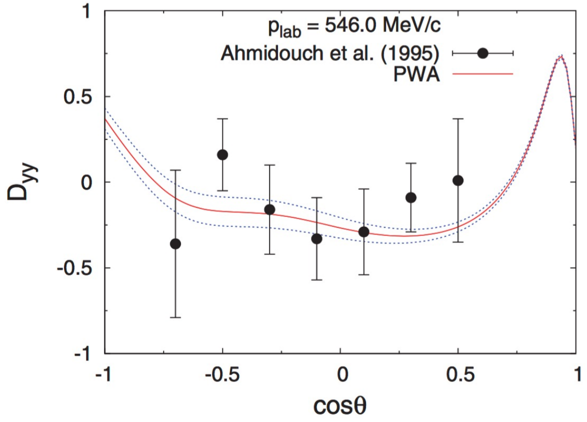


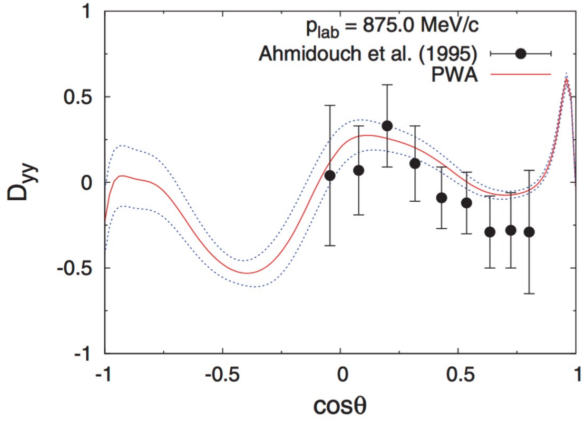
Depolarization & spin transfer

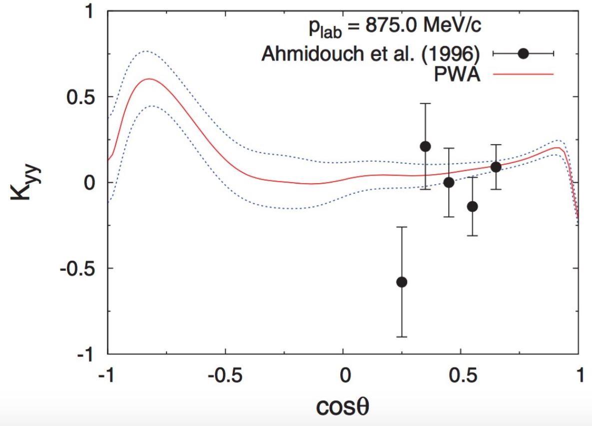




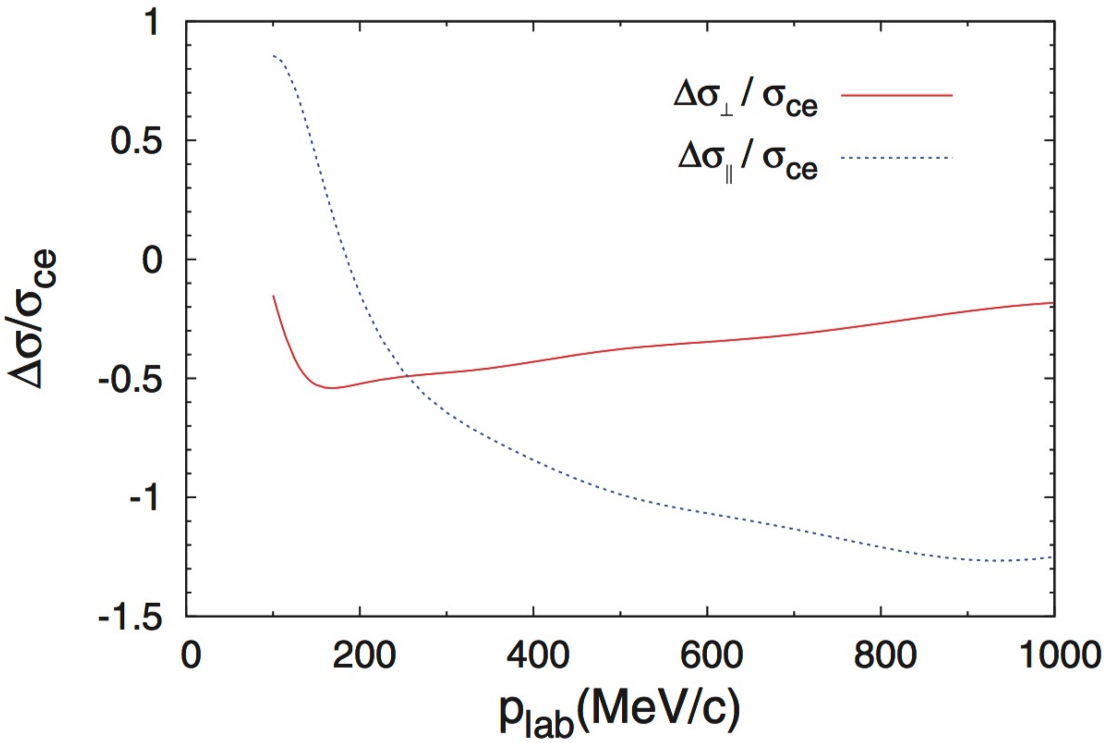








Spin-dependent total cross sections

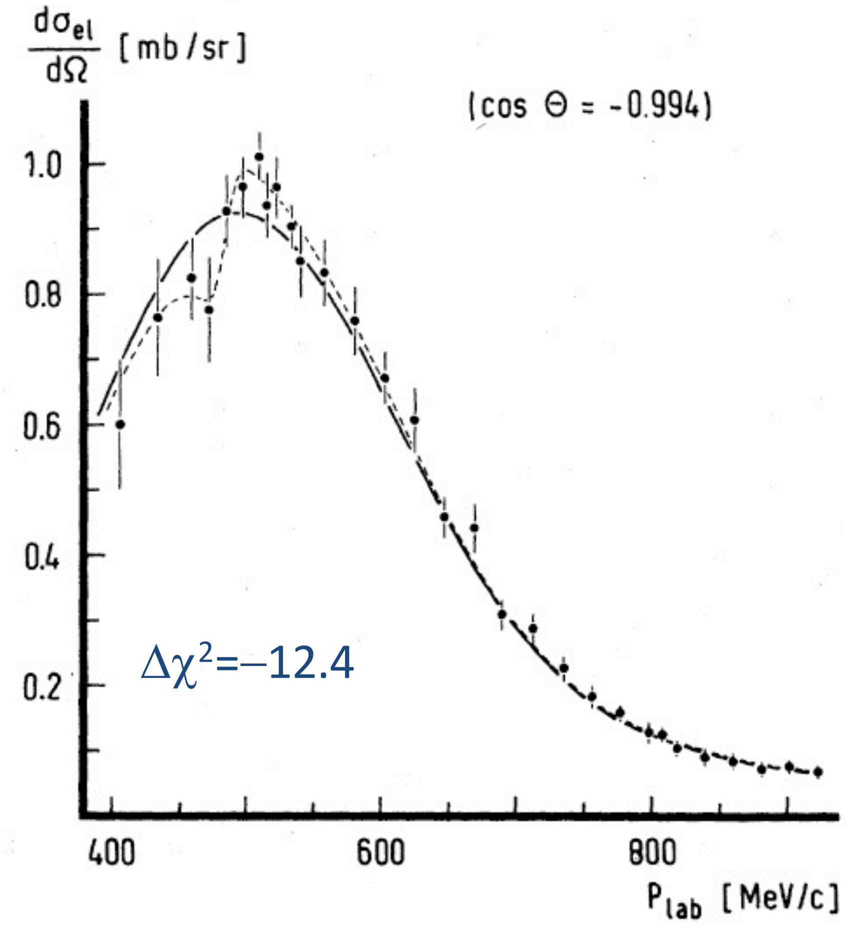


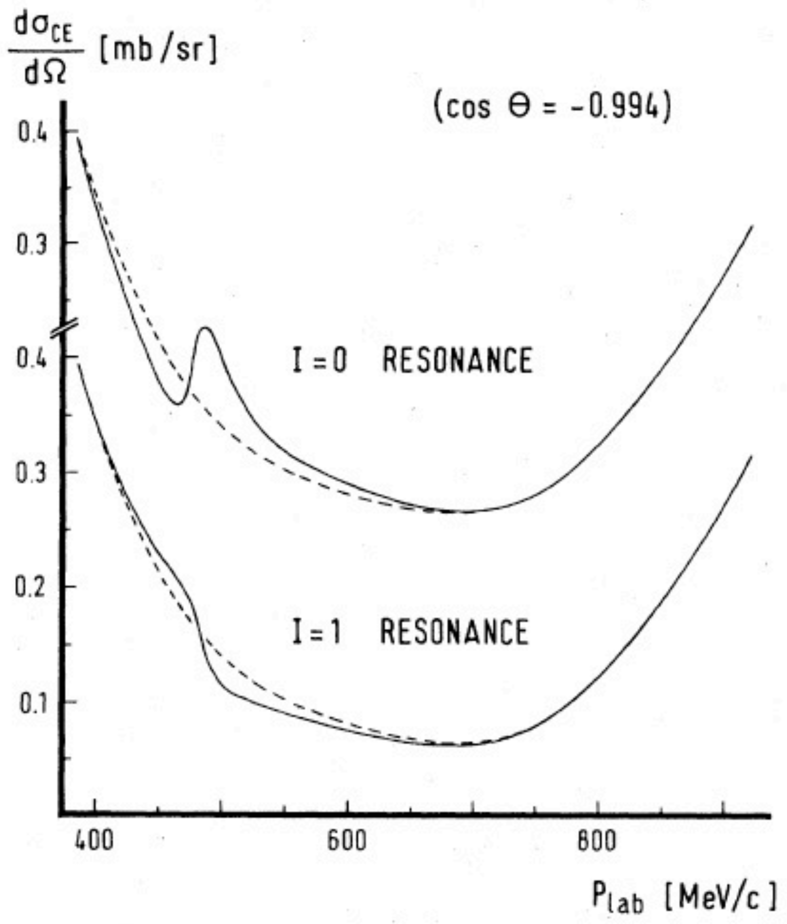
$$\Delta\sigma_{\perp} = \sigma_{\uparrow\downarrow} - \sigma_{\uparrow\uparrow},$$

$$\Delta\sigma_{\parallel} = \sigma_{\rightleftarrows} - \sigma_{\rightrightarrows}.$$

p_{lab} (MeV/c)	σ_{tot} (mb)	$\Delta\sigma_{\mathrm{T}}$(mb)	$\Delta\sigma_{\mathrm{L}}$(mb)	$\Delta\sigma_{\mathrm{T}}/\sigma_{\mathrm{tot}}$ (%)	$\Delta\sigma_{\mathrm{L}}/\sigma_{\mathrm{tot}}$ (%)
200	314.8	−91.0	−19.4	−28.9	−6.2
400	194.0	−45.6	−51.8	−23.5	−26.7
600	151.8	−31.5	−58.6	−20.8	−38.6
800	128.5	−25.8	−54.1	−20.1	−42.1

Hunting resonances





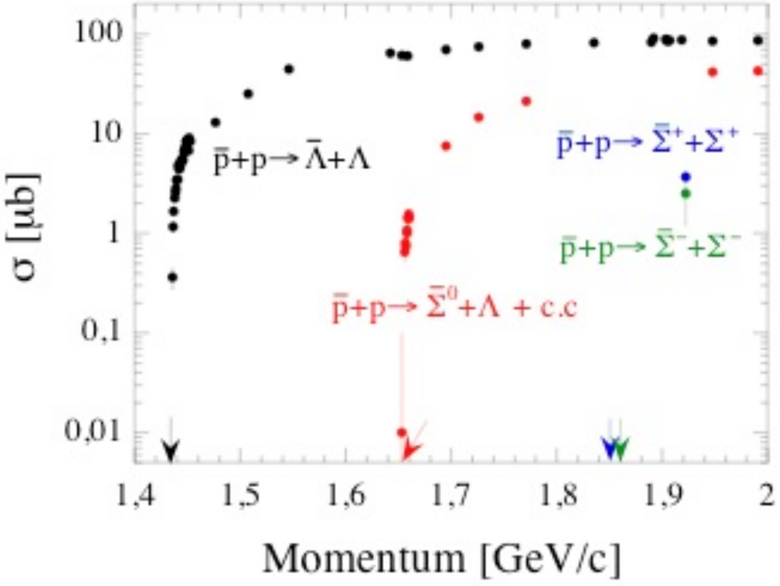
- ✓ With a resonance in the $^{11}D_2$ wave, with E_R=1934(3) MeV, Γ=6(4) MeV

P. H. Timmers *et al.*, PRD **31**, 99 (1985).

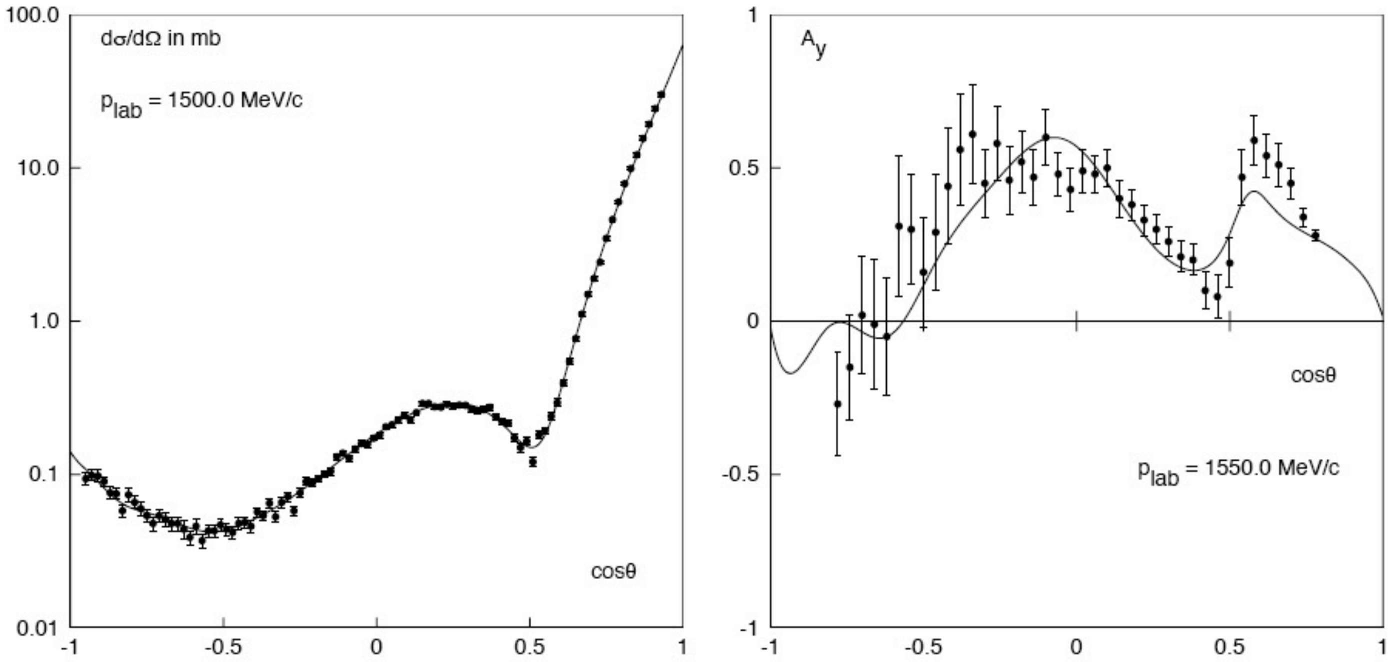
PS185 @ LEAR: Strangeness exchange

- ✓ Coupled-channels PWA:
  - Long-range interaction: Charge-conjugated Nijmegen *YN* model
  - Short-range interaction: Complex boundary condition

	p_{lab} MeV/c	ΔQ	ΔS
$\bar{p}p$	0	0	0
$\bar{n}n$	100	1	0
$\bar{\Lambda}\Lambda$	1435	1	1
$\bar{\Lambda}\Sigma^0, \bar{\Sigma}^0\Lambda$	1653	1	1
$\bar{\Sigma}^+\Sigma^+$	1853	0	1
$\bar{\Sigma}^0\Sigma^0$	1871	1	1
$\bar{\Sigma}^-\Sigma^-$	1899	2	1
$\bar{\Xi}^0\Xi^0$	2582	1	2
$\bar{\Xi}^-\Xi^-$	2620	2	2



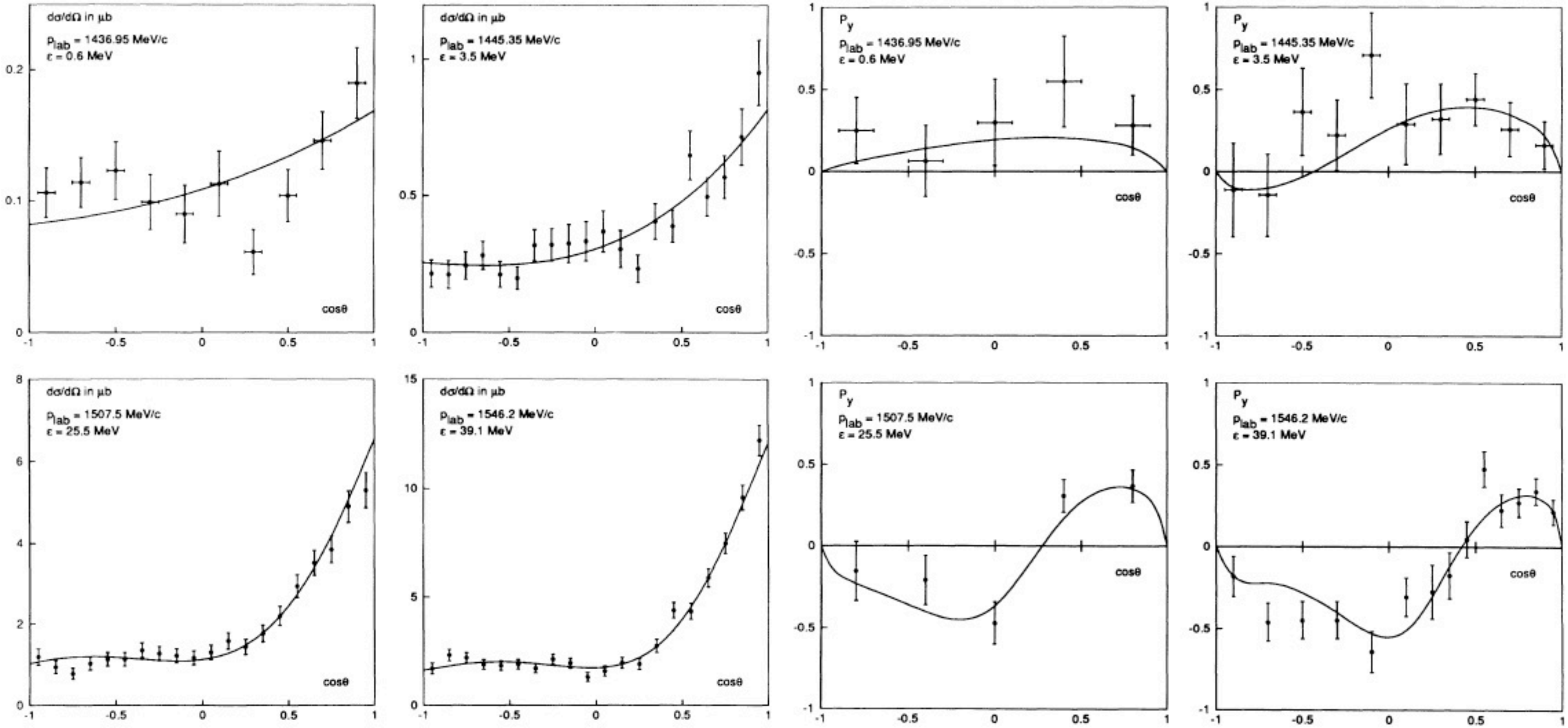
Initial- and final-state interaction



- ✓ Initial-state *p*pbar interaction
  - – 2/3 annihilation, black/grey disk scattering
  - – Strong spin and isospin dependence

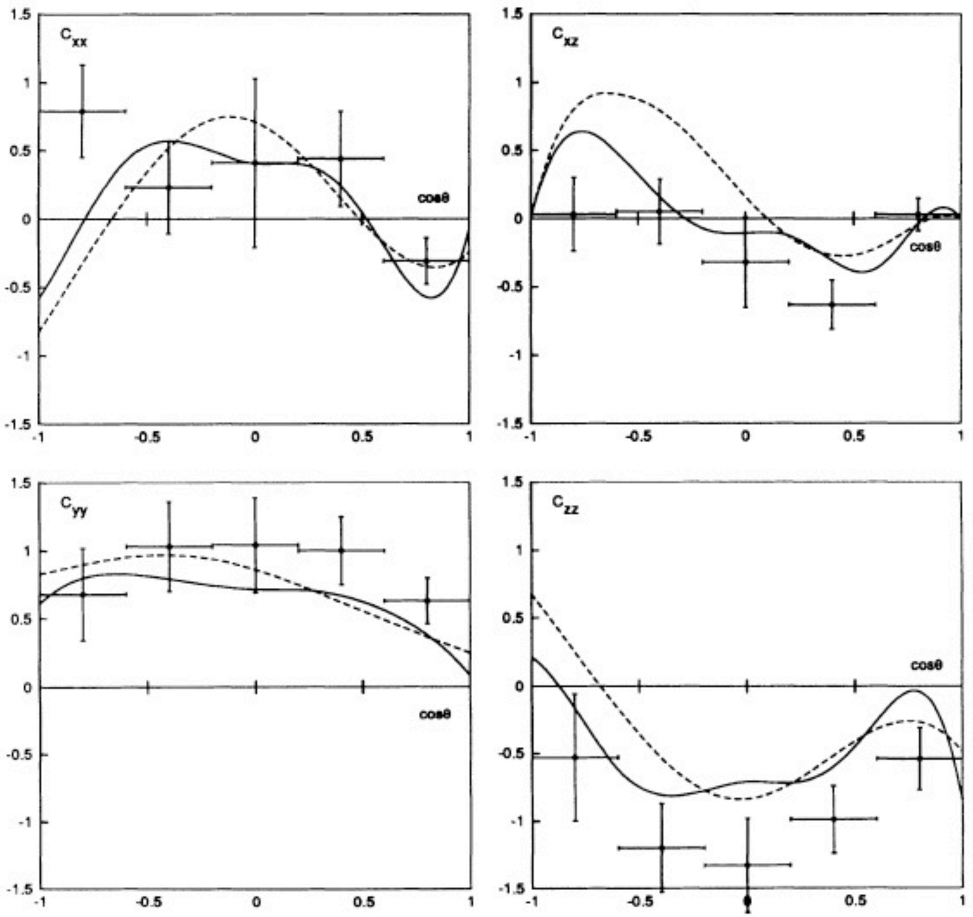
- ✓ Final-state *ΛΛ*bar interaction
  - – Not much known, related by flavor-*SU*(3) symmetry

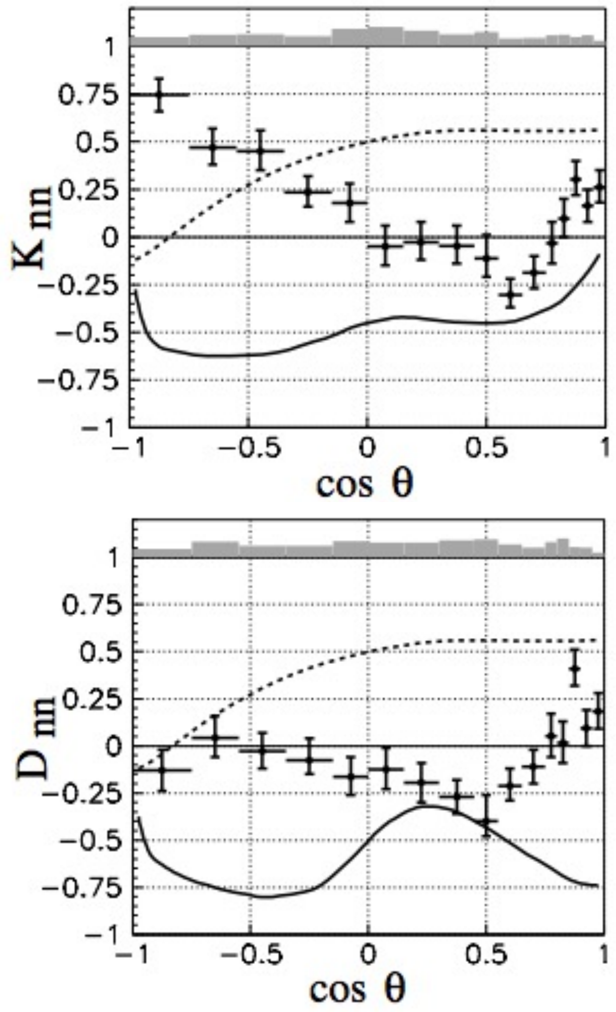
Cross sections & polarizations



- ✓ Differential cross sections: strongly forward peaked, flat backward angles
- ✓ Large polarizations, positive forward, negative backward angles

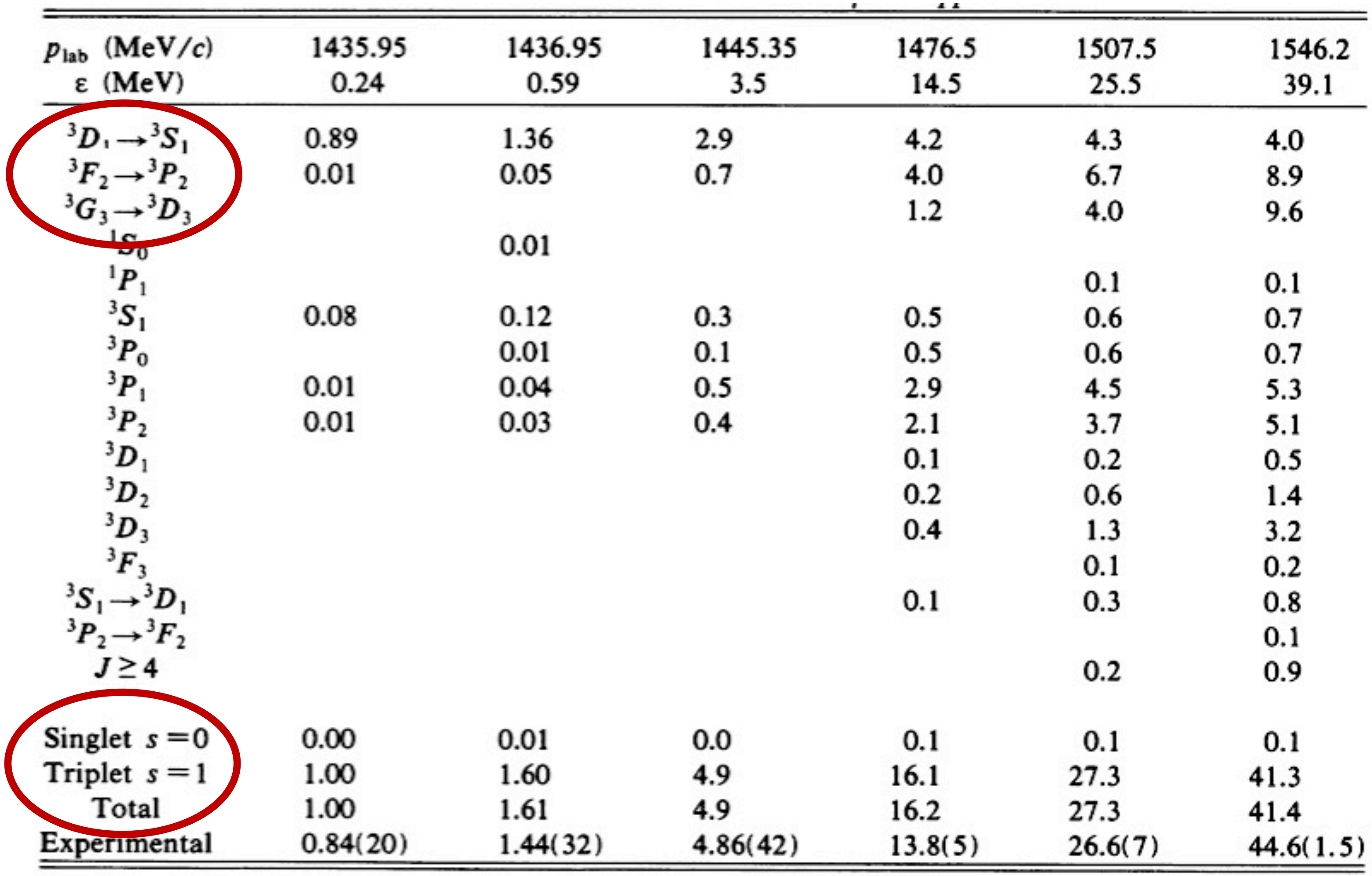
Spin physics!





- ✓ Large spin correlations, problematic for models
- ✓ With polarized target @ 1.64 GeV/*c*: depolarization D_{nn} and transfer K_{nn}

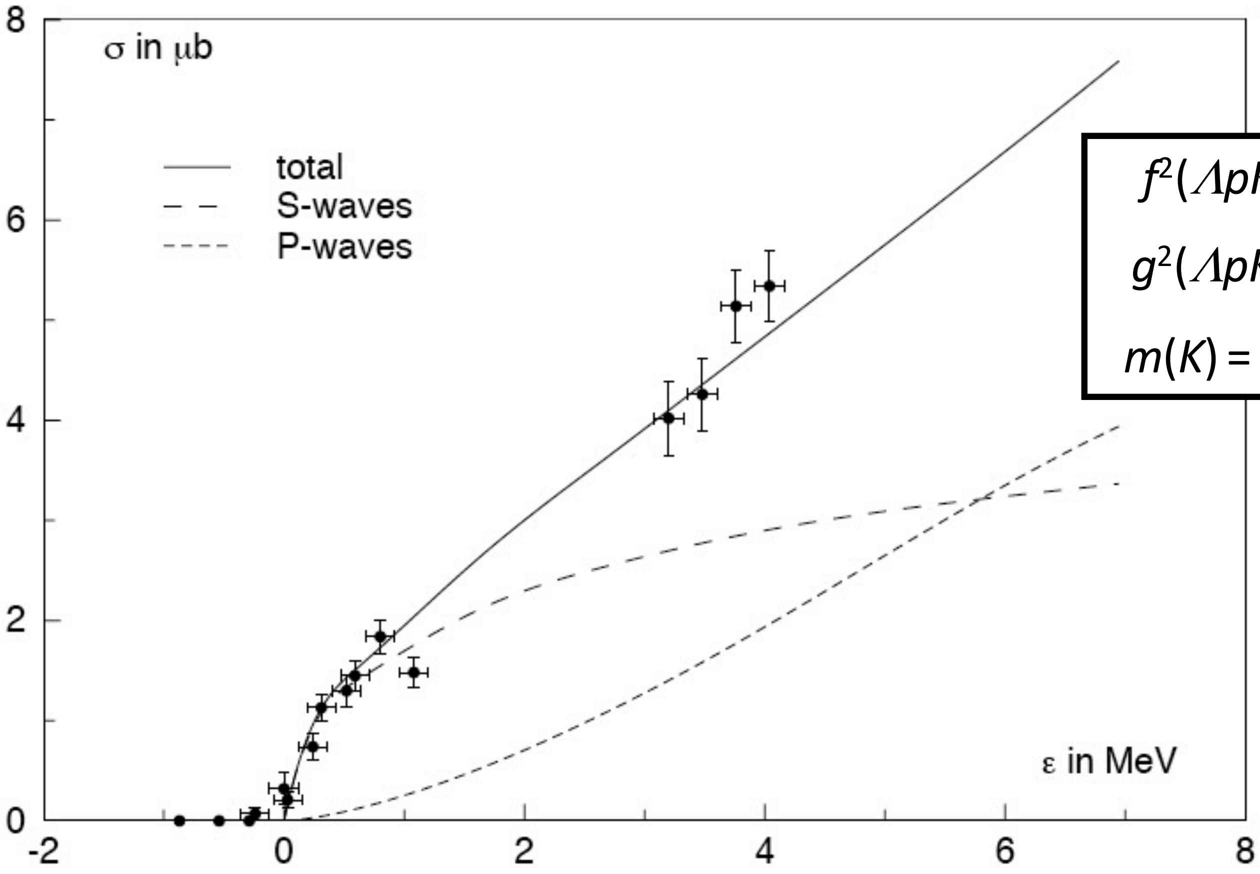
Partial-wave cross sections



p_{lab} (MeV/c)	1435.95	1436.95	1445.35	1476.5	1507.5	1546.2
ε (MeV)	0.24	0.59	3.5	14.5	25.5	39.1
$^3D_1 \to {}^3S_1$	0.89	1.36	2.9	4.2	4.3	4.0
$^3F_2 \to {}^3P_2$	0.01	0.05	0.7	4.0	6.7	8.9
$^3G_3 \to {}^3D_3$				1.2	4.0	9.6
$^1S_0$		0.01				
$^1P_1$					0.1	0.1
$^3S_1$	0.08	0.12	0.3	0.5	0.6	0.7
$^3P_0$		0.01	0.1	0.5	0.6	0.7
$^3P_1$	0.01	0.04	0.5	2.9	4.5	5.3
$^3P_2$	0.01	0.03	0.4	2.1	3.7	5.1
$^3D_1$				0.1	0.2	0.5
$^3D_2$				0.2	0.6	1.4
$^3D_3$				0.4	1.3	3.2
$^3F_3$					0.1	0.2
$^3S_1 \to {}^3D_1$				0.1	0.3	0.8
$^3P_2 \to {}^3F_2$						0.1
$J \geq 4$					0.2	0.9
Singlet $s=0$	0.00	0.01	0.0	0.1	0.1	0.1
Triplet $s=1$	1.00	1.60	4.9	16.1	27.3	41.3
Total	1.00	1.61	4.9	16.2	27.3	41.4
Experimental	0.84(20)	1.44(32)	4.86(42)	13.8(5)	26.6(7)	44.6(1.5)

- ✓ Naturally explained by coherent tensor force from *K* and *K** exchange

“Seeing” one-kaon exchange



$f^2(\Lambda pK) = 0.071(7)$

$g^2(\Lambda pK) = 15.4(1.5)$

$m(K) = 480(60)$ MeV

- ✓ Test of Goldberger-Treiman relation for $SU(3)\times SU(3)$ Goldstone bosons
  - Chiral symmetry for $NN\pi$, $\Lambda\Sigma\pi$, $\Sigma\Sigma\pi$, ΛNK, ΣNK coupling constants
  - Should be pursued *e.g.* for Ξ, Ω

Conclusion

- ✓ To make progress:

Data! Data! Data!
I can't make bricks
without clay…