The Paris nucleon-antinucleon potential and Baryonia

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Nuclear physics with antiprotons: a theory endeavor
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1) Introduction. ⇒ Some (incomplete list) publications 2019 → 1991:

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2) Paris $N\bar{N}$ 2009 model and results

- ⋆ A) Brief reminder
- ⋆ B) Scattering observables
- ⋆ C) Antiprotonic hydrogen level shifts
- ⋆ D) Bound states and resonance
- ⋆ E) The $N\bar{N}$ optical potential

3) In search of Baryonia

4) Revisiting the Paris potential

5) Concluding remarks and outlook

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i) broad states due to fast annihilation and heavy backgrounds in experiments,
ii) exclusion principle not operative and large number of partial waves.

B) Specific $N\bar{N}$ states can be reached in formation experiments:
i) near threshold enhancements in the $p\bar{p}$, invariant mass spectrum of heavy meson decays such as $J/\psi \to \gamma p\bar{p}$ but no structure in $J/\psi \to \pi^0 p\bar{p}$ decays,
ii) explanation with updated $N\bar{N}$ Paris potential, ⋆ [Paris99] B. El-Bennich, M. Lacombe, B. Loiseau and S. Wycech, *Paris $N\bar{N}$ potential constrained by recent antiprotonic-atom data and $\bar{n}p$ total cross sections*, Phys. Rev. C **79**, 054001 (2009).

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$V_{N\bar{N}}(\mathbf{r}, T_{Lab}) = U_{N\bar{N}}(\mathbf{r}, T_{Lab}) + i\, W_{N\bar{N}}(\mathbf{r}, T_{Lab})$; kinetic energy $T_{Lab} = 2E_{CM}$

⋆ The real potential $U_{N\bar{N}}(\mathbf{r}, T_{Lab})) =$
$U_0(r, T_{Lab})\,\Omega_0 + U_1(r, T_{Lab})\,\Omega_1 + U_{LS}(r)\Omega_{LS} + U_T(r)\Omega_T + U_{SO2}(r)\Omega_{SO2}$

⋆ The five non-relativistic invariants:
$\Omega_0 = (1 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)/4$, $\Omega_1 = (3 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)/4$, $\Omega_{LS} = \mathbf{L} \cdot \mathbf{S}$,
$\Omega_T = 3(\boldsymbol{\sigma}_1 \cdot \mathbf{r}\, \boldsymbol{\sigma}_2 \cdot \mathbf{r})/r^2 - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$, $\Omega_{SO2} = (\boldsymbol{\sigma}_1 \cdot \mathbf{L}\, \boldsymbol{\sigma}_2 \cdot \mathbf{L} + \boldsymbol{\sigma}_2 \cdot \mathbf{L}\, \boldsymbol{\sigma}_1 \cdot \mathbf{L})/2$.

⋆ The linear nonlocality in the central singlet, U_0 and central triplet, U_1:
$U(r, T_{Lab}) = U^a(r) + T_{Lab}\, U^b(r)$

⇒ The real potential, for $r \geq r_c$ ($r_c \leq 1$ fm), is the *G*-parity transform of the theoretical Paris *NN* potential, M. Lacombe, B. Loiseau, J-M. Richard, R. Vinh Mau, J. Côté, P. Pirès, and R. de Tourreil, *Parametrization of the Paris NN potential*, Phys. Rev. C**21**, 861 (1980), based on a theoretical dispersion-relation treatment of the correlated and uncorrelated two-pion exchange, W.N. Cottingham, M. Lacombe, B. Loiseau, J.-M. Richard, and R. Vinh Mau, *Nucleon-Nucleon interaction from pion-nucleon phase shift analysis* Phys. Rev. D**8**, 800 (1973).

→ For the one pion exchange $g_\pi^2/4\pi = 14.43$; for the three pion exchange it uses the ω meson, $m_\omega = 782.7$ MeV, $g_\omega^2/4\pi = 11.75$ and the A_1 meson, $m_{A_1} = 1100$ MeV, $g_{A_1}^2 = 10.4$.

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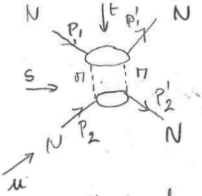
$\boxed{\text{S-Matrix}}$

Analyticity - Unitarity - crossing symmetry



Mandelstam invariant
$$\begin{cases} s = (p_1+p_2)^2 \\ t = (p_1-p_1')^2 \\ u = (p_2-p_1')^2 \end{cases}$$

$$S = 1 + iT$$

Unitarity: $SS^+ = 1 \Leftrightarrow (1+iT)(1-iT) = 1$

$$\Rightarrow TT^+ = i(T-T^+) \Leftrightarrow \mathrm{Im}\,T = -\frac{1}{2}TT^+$$

$$\sum_a |a\rangle\langle a| = 1 \rightarrow \sum_{2\pi} |2\pi\rangle\langle 2\pi| \simeq 1$$

Long range

$$\langle N\bar{N}|\mathrm{Im}\,T|N\bar{N}\rangle \simeq -\frac{1}{2}\sum_{2\pi} \langle N\bar{N}|T|2\pi\rangle\langle 2\pi|T^+|2\pi\rangle$$

From πN scattering

Crossing

$$\mathrm{Im}\, T_{N\bar{N}}(s,t) \simeq \frac{-1}{2} \int d\Omega_{2\pi} \langle N\bar{N}|T|2\pi\rangle\langle 2\pi|T^{+}|2\pi\rangle$$

1) Correlated S, P waves $N\bar{N} \to 2\pi$ analyticity

2) uncorrelated D waves + ... $\pi N \to \pi N$ scattering PSA

Analyticity (Cauchy formula)

$$\boxed{\mathrm{Re}\, T_{N\bar{N}}(s,t) = \frac{1}{4\pi^2} \int_{4m^2}^{\infty} \frac{ds'}{s'-s} \int_{4\mu^2}^{\infty} \frac{dt'}{t'-t}\, \mathrm{Im}\, T_{N\bar{N}}(s',t')}$$

Mandelstam representation

1 A) Optical potential: ii) phenomenological short range $r \leqslant r_c$ ($r_c \leqslant 1$ fm) real part

→ For $r \leqslant r_c$, the empirical central potentials $U_0^a(r)$, $U_1^a(r)$:
$U(r) = a_3 r^3 + a_2 r^2 + a_1 r + a_0$

→ For, $U_{0,1}^b(r)$, $U_{LS}(r)$, $U_T(r)$ and $U_{S02}(r)$: $U(r) = b_2 r^2 + b_1 r + b_0$

⋆ Parameters a_i ($i = 0$ to 3), b_i ($i = 0$ to 2) determined:
(i) by matching to the theoretical potential at $r = r_0 = r_c$ and $r = r_1 = r_0 + \Delta r$ with $\Delta r = 0.15$ fm,
(ii) choosing a phenomenological height at $r_2 = 0.587$ fm and, only for $U_{0,1}^a(r)$, at $r = 0.188$ fm

⋆ For all isospin-0 potentials $r_c = 1$ fm

⋆ For all isospin-1 potentials $r_c = 0.84$ fm except for $U_0^a(r)$ where $r_c = 1$ fm.

⇒ While solving the Schödinger equation, the tensor potential $U_T(r)$ at small r regularized by multiplying it by $F(r) = \dfrac{(pr)^2}{1 + (pr)^2}$ with $p = 10$ fm^{-1}.

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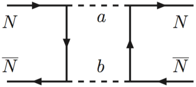

→ This Feynman diagram calculated by B. Moussallam [PhD Thesis, Universite Paris VI, (1980), unpublished)] can be approximated by a short range radial function:

$W_{N\bar{N}}(\mathbf{r}, T_{Lab}) =$

$$\left[g_C\left(1 + f_C T_{Lab}\right) + g_{SS}\left(1 + f_{SS} T_{Lab}\right)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + g_T \Omega_T + \frac{f_{LS}}{4m^2}\Omega_{LS}\frac{1}{r}\frac{d}{dr}\right]\frac{K_0(2mr)}{r}$$

→ Modified Bessel function $K_0(2mr)$ is the Fourier transform of a dispersion integral from the calculation of the $N\bar{N}$ annihilation box diagram into two mesons with a nucleon-antinucleon intermediate state in the crossed t-channel

$K_0(2mr) = \frac{1}{2}\int_{4m^2}^{\infty} dt' \frac{e^{-\sqrt{t'}r}}{\sqrt{t'(t'-4m^2)}}$, here $m = 940$ MeV

→ To avoid the singular behavior at $r = 0$:

⋆ Imaginary central and spin-spin potentials multiplied by $G(r) = (1 - e^{-2mr})^4$

⋆ Imaginary tensor and spin-orbit potentials multiplied by $H(r) = (1 - e^{-2mr})^7$

1 A) Imaginary part of the optical potential $W_{N\bar{N}}(\mathbf{r}, T_{Lab})$ [see Appendix (Paris94)]

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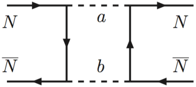

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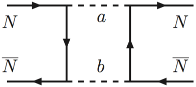

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Table: All quantities determined by the fit to experimental observables.

	Isospin $T = 0$		Isospin $T = 1$	
	This work	Paris 99	This work	Paris 99
$U_0^a(r_3)$	8692.49	8594.41	-5300.64	-1917.54
$U_0^a(r_2)$	-378.44	-489.08	-664.40	-1716.76
$U_0^b(r_2)$	0.5857	1.307	-0.327	-0.132
$U_1^a(r_3)$	-6508.93	-5286.67	5001.23	3121.01
$U_1^a(r_2)$	-1041.26	-810.89	-1115.79	-1135.07
$U_1^b(r_2)$	-1.306	-1.741	-1.676	-1.931
$U_{LS}(r_2)$	917.12	788.30	-436.31	-423.71
$U_T(r_2)$	481.68	397.14	216.46	128.14
$U_{SO2}(r_2)$	105.43	75.03	203.28	172.48
g_c	153.57	124.86	153.82	78.40
$f_c(\mathrm{MeV}^{-1})$	0.0153	0.0190	0.0121	0.0335
g_{SS}	-15.56	-3.83	45.49	19.94
$f_{SS}(\mathrm{MeV}^{-1})$	0.0076	-0.0373	0.0135	0.0412
g_{LS}	0.010	35.369	0.026	12.027
g_T	0.023	2.057	0.027	5.073

→ Important parameters: (i) 6 values central singlet & triplet, tensor, $L \cdot S$ terms at $r_2 = 0.587$ fm, (ii) 4 couplings singlet & triplet central terms imaginary part. Fine tuning adjusting 5 remaining parameters.

Table: All quantities determined by the fit to experimental observables.

	Isospin $T=0$		Isospin $T=1$	
	This work	Paris 99	This work	Paris 99
$U_0^a(r_3)$	8692.49	8594.41	-5300.64	-1917.54
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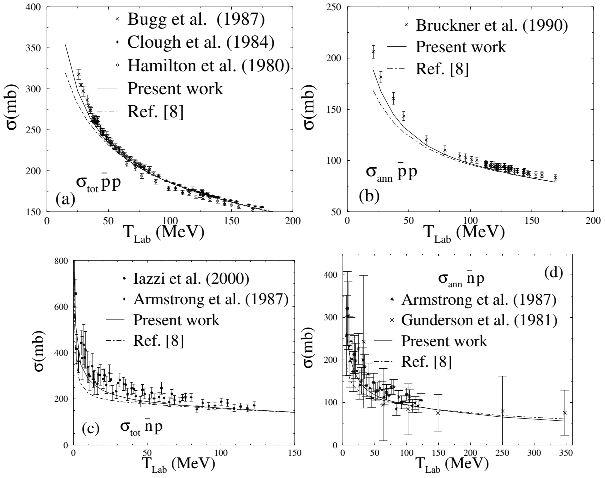
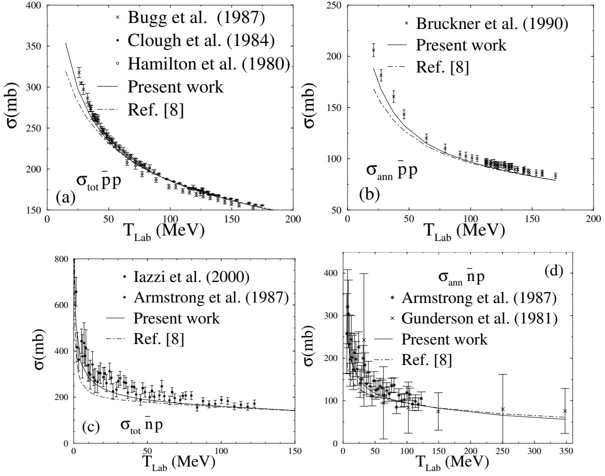


- ★ Scattering amplitude: solving the Schödinger equation.
- → Real & imaginary short range parameters fitting data: 915 in1982, 3800 in 1994, 4259 in 1999.
- ★ In 2009, 4259 (1999) + 64 data total $\bar{n}p$ cross sections [Iazzi et al. (2000)] + 10 level shifts & widths antiprotonic hydrogen [Augsburger et al. & Gotta et al. both (1999)]
- ⇒ χ^2/data = 4.52 [4.59 for Paris99]



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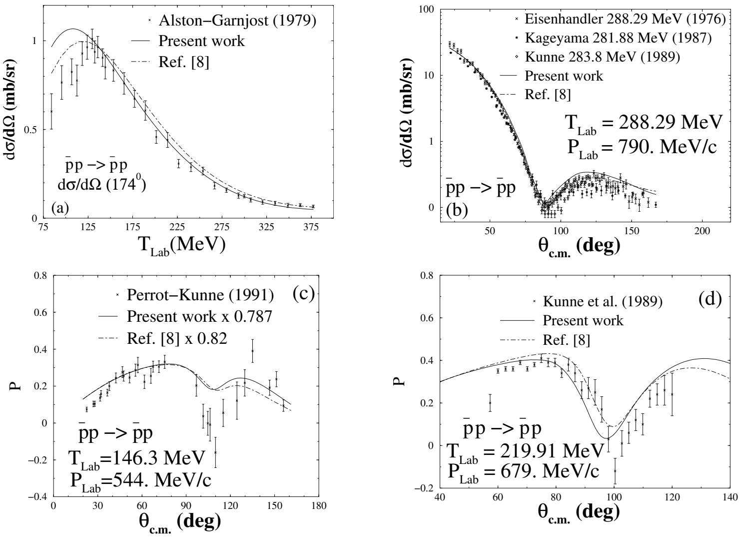
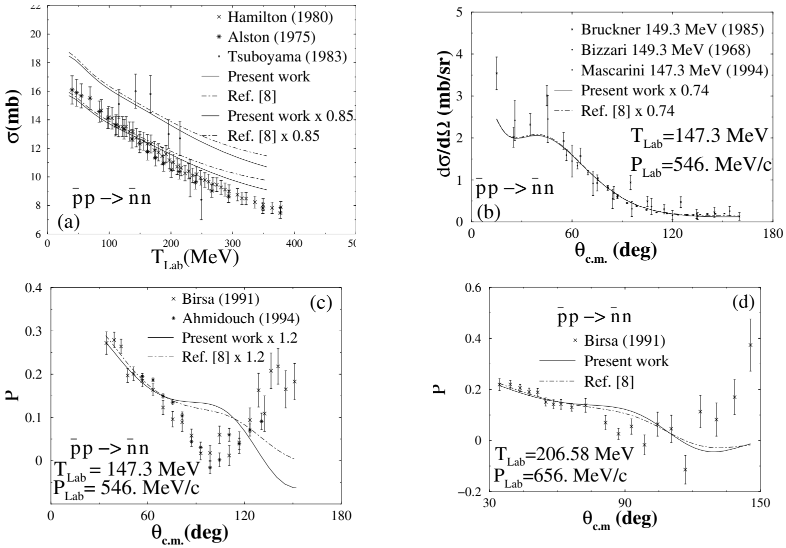

Alston−Garnjost (1979)
Present work
Ref. [8]
p̄p −> p̄p
dσ/dΩ (174⁰)
(a)
dσ/dΩ (mb/sr)
T_Lab(MeV)
Eisenhandler 288.29 MeV (1976)
Kageyama 281.88 MeV (1987)
Kunne 283.8 MeV (1989)
Present work
Ref. [8]
T_Lab = 288.29 MeV
P_Lab = 790. MeV/c
p̄p −> p̄p
(b)
dσ/dΩ (mb/sr)
θ_c.m. (deg)
Perrot−Kunne (1991)
Present work x 0.787
Ref. [8] x 0.82
(c)
p̄p −> p̄p
T_Lab=146.3 MeV
P_Lab=544. MeV/c
P
θ_c.m. (deg)
Kunne et al. (1989)
Present work
Ref. [8]
(d)
p̄p −> p̄p
T_Lab = 219.91 MeV
P_Lab = 679. MeV/c
P
θ_c.m. (deg)

1 B) $\bar{p}p \to \bar{n}n$ σ_{tot} & $d\sigma/d\Omega$ & polarization P. Present work: [Paris09], Ref.[8]: [Paris99]



⋆ One has [Trueman 1961)]: $\Delta E_S - i\frac{\Gamma_S}{2} = \frac{2\pi}{\mu_{p\bar{p}}} \left|\psi^{\text{coul}}(0)\right|^2 a_c^S \left(1 - 3.154\, \frac{a_c^S}{B}\right)$

→ a_c^S: coulomb corrected S-wave $p\bar{p}$ complex scattering length, $\mu_{p\bar{p}} = M_P/2$: $p\bar{p}$ reduced mass, $M_P = 938.27$ MeV: proton mass, Bohr radius: $B = 1/(\alpha\mu_{p\bar{p}}) = 57.6399$ fm, $\alpha = 1/137.036$: fine structure constant, $\left|\psi^{\text{coul}}(0)\right|^2 = 1/(\pi B^3)$.

If $a_c^S \simeq 1$ fm $\Rightarrow a_c^S/B$ few per cent correction, term negligible in higher angular momentum.

For P wave ($n = 2$) we use [Lambert (1970)] $\Delta E_P - i\frac{\Gamma_P}{2} = \frac{3}{16\mu_{p\bar{p}}B^5}\, a_c^P$.

→ Calculate Coulomb corrected scattering lengths following [J. Carbonell, J.M. Richard and S. Wycech, On the relation between protonium level shifts and nucleon-antinucleon scattering amplitudes, Zeit. Phys. A 343, 325 (1992)].

⋆ $N\bar{N}$ Phase shifts: solving $p\bar{p}$ Schrödinger equation in configuration space with the Paris $N\bar{N}$ optical potential plus the Coulomb potential.

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⋆ $N\bar{N}$ Phase shifts: solving $p\bar{p}$ Schrödinger equation in configuration space with the Paris $N\bar{N}$ optical potential plus the Coulomb potential.

Table: ΔE_L, Γ_L [keV] for S waves, [meV] for P waves. Standard notation $^{2S+1}L_J$.

	$\Delta E_L - i\Gamma_L/2$ $\{a_c^L\ (\mathrm{fm}^{2L+1})\}$		
State	Experimental	Present work	Paris 99
$^1S_0$	0.440(75)-i0.60(12) [Augsburger99]	0.778-i0.519	0.755-i0.243
	{0.492(92)-i0.732(146)}	{ 0.920-i0.666}	{0.911-i0.312}
$^3S_1$	0.785(35)-i0.47(4) [Augsburger99]	0.693-i0.393	0.654-i0.323
	{0.933(45)-i0.604(51)}	{0.823-i0.498}	{0.778-i407}
S-world	0.712(20)-i0.527(33) [Augsburger99]	0.714-i0.425	0.680-i0.303
	{0.835(25)-i0.669(42)}	{0.847-i0.540}	{0.812-i0.384}
$^3P_0$	-139(30)-i60(12) [Gotta99]	-67.0-i60	-68.0-i66.8
	{-5.68(1.23)-i2.45(49)}	{-2.74-i2.460}	{-2.78-i2.730}
Sum-P	15(25)-i15.2(1.5) [Gotta99]	6.10-i21.7	4.40-i10.9
	{0.613(1.02)-i0.621(60)}	{0.250-i0.886}	{0.180-i0.445 }
$^1P_1$	$a(p\bar{p}) = [a(\mathrm{T{=}0}) + a(\mathrm{T{=}1})]/2$	-29.4-i13.2	-29.6-i13.7
		{-1.20-i0.539}	{-1.21-i0.561}
$^3P_1$	$a(S\text{-world}) = [a(\text{singlet}) + 3a(\text{triplet})]/4$	63.8-i44.8	59.7-i12.6
		{2.61-i1.83}	{2.44-i0.516}
$^3P_2$	$a(\text{Sum-}P) = [3a(^1P_1) + 3a(^3P_1) + 5a(^3P_2)]/11$	7.22-i12.9	-8.44-i8.12
		{-0.295-i0.528}	{-0.345-i0.332}

Real central singlet and triplet potentials relatively strong medium + short range attractive parts.
If imaginary potentials set to zero $\Rightarrow$ several bound or resonant states.
Many disappear if the necessary annihilation is introduced.

- ⋆ Search for S-matrix poles in the complex energy plane as in M. Lacombe, B. Loiseau, B. Moussallam, R. Vinh Mau, *Nucleon-antinucleon resonance spectrum in a potential model*, Phys. Rev. C **29**, 1800 (1984):

Table: Binding energy in MeV of the close to threshold quasibound states

${}^{2T+1\ 2S+1}L_J$	Present work	Paris 99
${}^{11}S_0$	-4.8-i26	
${}^{33}P_1$	-4.5-i9.0	-17-i6.5

- ⋆ Search for other P-wave poles:

Table: Close to threshold resonances. In parenthesis resonance Paris 99. ${}^{13}P_0$, ${}^{13}P_1$: identical positions in Paris 99 . No ${}^{33}P_0$ resonance in Paris 99.

${}^{2T+1\ 2S+1}L_J$	${}^{11}P_1$	${}^{13}P_0$	${}^{13}P_1$	${}^{33}P_0$
Mass (MeV)	1877 (1872)	1876	1872	1871
Width (MeV)	26 (12)	10	20	21

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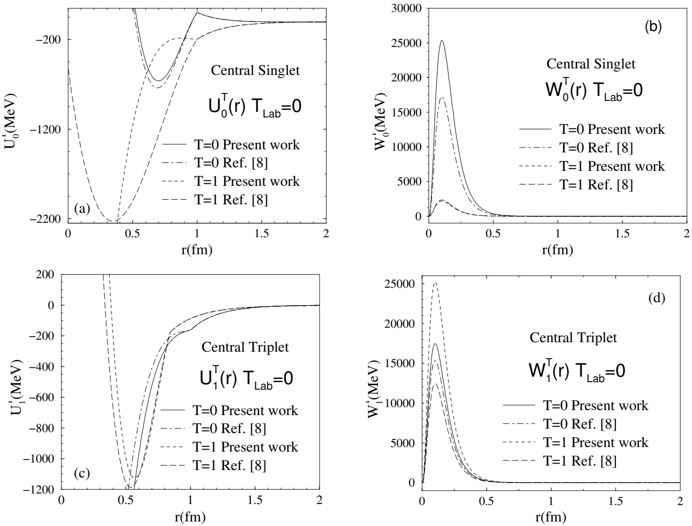
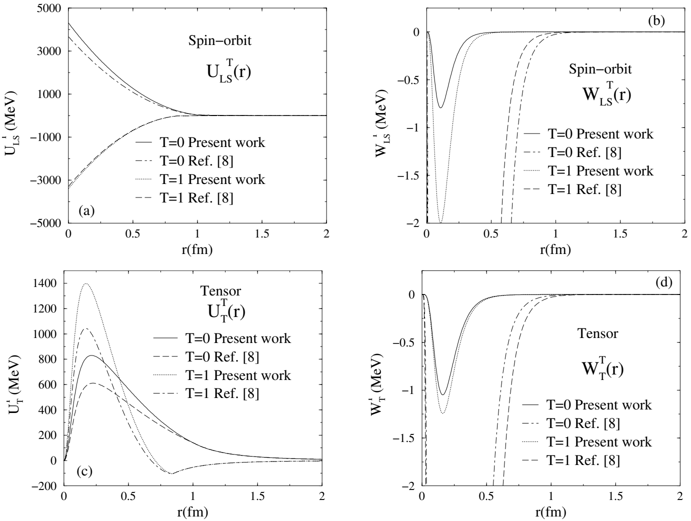
1 E) Central real + imaginary potentials (see p. 4 to 7). Present work: Paris09, Ref. [8]: Paris99.

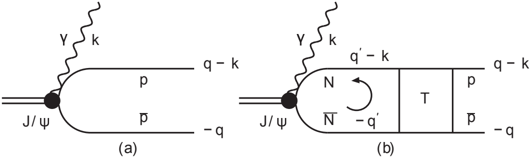


1 E) Spin-orbit, tensor real + imaginary potentials (p. 4 to 7). Present work: Paris09, Ref. [8]: Paris99.

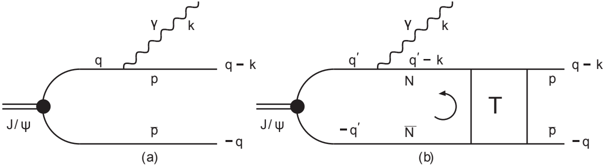


Two processes (DWBA) to describe the BES Collaboration data on γ (or ω).

a) Direct Emission (DE):



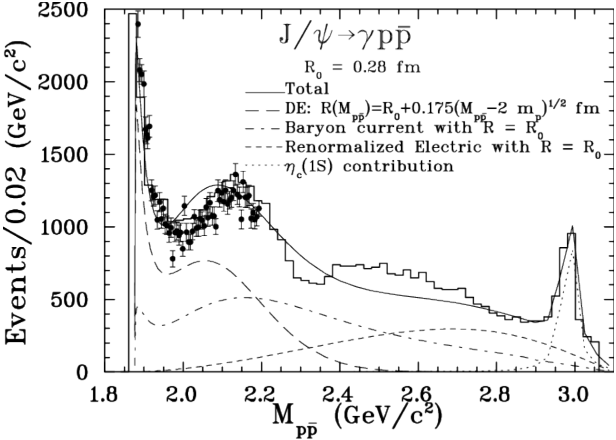
b) Emission from Baryonic Current (BC):



=●: J/ψ source, phenomenological Gaussian function with radius $R(M_{p\bar{p}}) = R_0 + \beta\sqrt{M_{p\bar{p}} - 2m} \to R_0 = 0.28$ fm, $\beta = 0.175$ fm$^{3/2}$ represent the data fairly well.

(a) Born term, (b) T: Final State Interaction (FSI) with *S*-wave half-off shell function from Paris $N\bar{N}$ potential [B. El-Bennich, M. Lacombe, B. Loiseau, S. Wycech, PRC **79**, 054001 (2009)]

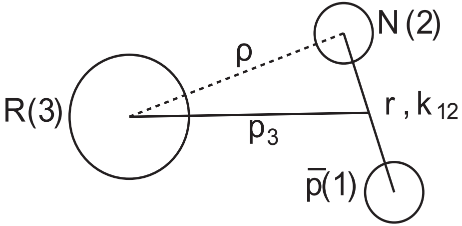
Peak related to strong nucleon-antinucleon attraction essentially in the $N\bar{N}$ $^{11}S_0$ state



Baryonic current peak: strongly suppressed due to interference of intermediate $p\bar{p}$, $n\bar{n}$ channels.

$\to$ 7 free parameters,
i) normalization source function fixed by $J/\psi \to p\bar{p}$ decay rate,
ii) magnetic and electric amplitudes calculated independently for DE and BC emission modes,
iii) emission rates added and the normalizations of the DE and BC rates fixed to reproduce the experimental ratio $\mathcal{R} = \Gamma(p\bar{p}\gamma)/\Gamma(p\bar{p})$ and the invariant mass distribution,
iv) Electric contribution (P-wave) is renormalized,
v) the $\eta_c(2983)$ formation is fitted by a relativistic Breit-Wigner.

Level shifts and width for ^{2}H($2P$), ^{3}He($2P$, $3D$), ^{4}He($2P$, $3D$) expressed in terms of $\bar{p}N$ sub-threshold scattering lengths and volumes.



Quasi-three-body system, 1: antiproton, 2: nucleon, 3: residual system.
Jacobi coordinates, momentum: $\boldsymbol{p}_3$, $\boldsymbol{k}_{12}$, space: $\boldsymbol{\rho}$, $\boldsymbol{r}$.

Outline for the S-wave interaction:

$$V_{\bar{p}N}(E_{cm}, S) = \frac{2\pi}{\mu} \widetilde{T}_0(r, E_{cm})$$

$$\widetilde{T}_0(r, E) = \frac{\mu_{N\bar{N}}}{2\pi} V_{N\bar{N}}(r, E) \frac{\Psi(r, E, k'(E))}{\psi_o(r, k'(E))}$$

$\rightarrow$ $E < 0$, $V_{N\bar{N}}(r, E)$: Paris model, $k'(E) = \sqrt{2\mu_{N\bar{N}}E}$, regular free wave: $\psi_o(r, k) = \sin(rk)/(rk)$, $\Psi(r, E, k'(E))$ solution of the Lippman-Schwinger equation: $\Psi = \psi_o + G^+ V \Psi$.

$\Rightarrow$ If $\bar{p}$ bound into an atomic orbital, energy shifts of upper levels (levels of small atomic-nucleus overlap) generated by perturbation and at leading order: $\Delta E_{nL} - i\Gamma_{nL}/2 = \Sigma_j \langle \psi_L \varphi | V_{\bar{p}N_j}(E, S) | \varphi \psi_L \rangle$, Σ_j over all nucleons of the nucleus. $\varphi(\rho)$: wave function of the struck nucleon; $\psi_L(\beta\rho)$: Coulomb-atomic-wave functions of given angular momentum L with $\beta = \frac{M_R}{M_R+M_N}$.

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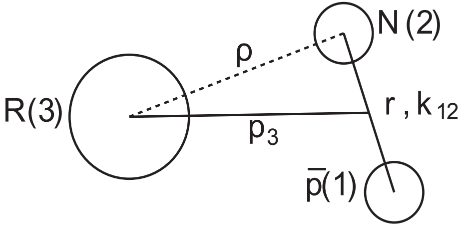

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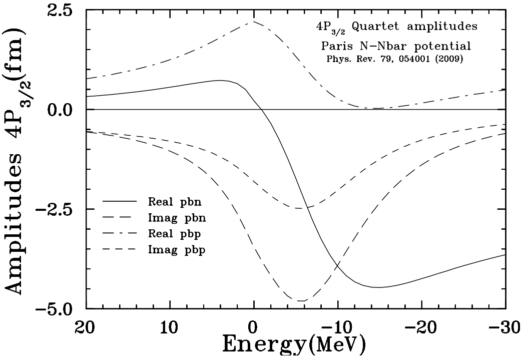
$\varphi(\boldsymbol{\rho})$: wave function of the struck nucleon; $\psi_L(\beta\boldsymbol{\rho})$: Coulomb-atomic-wave functions of given angular momentum L with $\beta = \frac{M_R}{M_R+M_N}$.

order	Shift	Width
S wave	100{113}	210{145}
P wave	-9{58}	365{206}
Sum	91{171}	575{341}
Data [1, 2]	243±26	489±30

[1] D. Gotta, Prog.Part.Nuc.Phys. **52**, 133 (2004).
[2] D. Gotta *et al.*, NPA **660**, 283 (1999).

Results for 2P-deuterium level corrections in meV calculated with the spin averaged amplitudes of the Paris 2009 potential. Numbers curly brackets are results with the Paris 1999 potential.



Subthreshold amplitudes generating the $4P_{3/2}$ hyperfine structure in deuterium. With Paris 09 solution it is strongly dominated by the resonant $a(^{33}P_1)$ at -4.8 MeV. Relevant $\bar{p}N$ c.m. energies fall in the region -7.6 ± 1 MeV. Downward shift of the resonance position by a -4 MeV or more will strongly reduce the attraction calculated in this component. In this way the hyperfine structure splitting is practically nullified.

As seen (p. 14, 15) some Paris potential terms discontinuity in their first derivative at r_c.

$\rightarrow$ Artifact short range regularization, no physical justification.

$\Rightarrow$ Possible difficulties numerical treatment of loosely bound antiprotonic atomic states.

- ⋆ Expand the isopin 0 & 1, U_S^a,U_S^b,U_{LS},$U_{S_{12}}$,U_Q($\equiv U_{SO2}$), below r_c in terms of four cubic splines $S_i(x)$ defined in the interval $[0, r_c]$:

$$U(r < r_c) = \sum_{i=0}^{3} c_i(S_i(r) = c_0 S_0(r) + c_1 S_1(r) + c_2 S_2(r) + c_3 S_3(r)$$

$$c_0 = U(0) \quad c_1 = U'(0) \quad c_2 = U(r_c) \quad c_3 = U'(r_c)$$

- For $U_0^{T,a}$, $U_0^{T,b}$, $U_1^{T,a}$ with a cusp, defining $U_2 \equiv U(r_2)$, $U_3 \equiv U(r_3)$

$$c_0 S_0(r_2) + c_1 S_1(r_2) = U_2 - c_2 S_2(r_2) - c_3 S_3(r_2)$$
$$c_0 S_0(r_3) + c_1 S_1(r_3) = U_3 - c_2 S_2(r_3) - c_3 S_3(r_3)$$
$\Rightarrow$ c_0 and c_1.

- $U_1^{T,b}$, U_{LS}^T, $U_{S_{12}}^T$, U_Q^T, only one adjustable parameter $U_2 \equiv U(r_2) \rightarrow$ some additional constrain, for $U_1^{T,b}$, U_{LS}^T, U_Q^T impose zero derivative at $r = 0 \rightarrow c_1 = 0$ and c_0:

$$c_0 S_0(r_2) = U_2 - c_2 S_2(r_2) - c_3 S_3(r_2)$$

As seen (p. 14, 15) some Paris potential terms discontinuity in their first derivative at r_c.

$\rightarrow$ Artifact short range regularization, no physical justification.

$\Rightarrow$ Possible difficulties numerical treatment of loosely bound antiprotonic atomic states.

- $\star$ Expand the isopin 0 & 1, U_S^a,U_S^b,U_{LS},$U_{S_{12}}$,$U_Q(\equiv U_{SO2})$, below r_c in terms of four cubic splines $S_i(x)$ defined in the interval $[0, r_c]$:

  $$U(r < r_c) = \sum_{i=0}^{3} c_i(S_i(r) = c_0 S_0(r) + c_1 S_1(r) + c_2 S_2(r) + c_3 S_3(r)$$

  $$c_0 = U(0) \quad c_1 = U'(0) \quad c_2 = U(r_c) \quad c_3 = U'(r_c)$$

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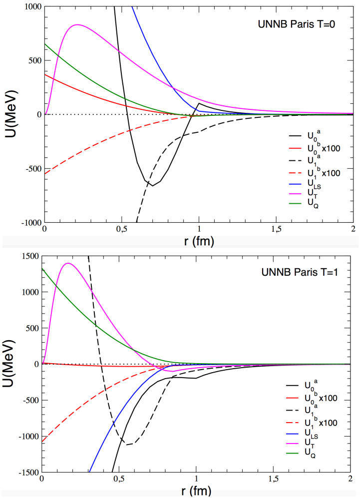
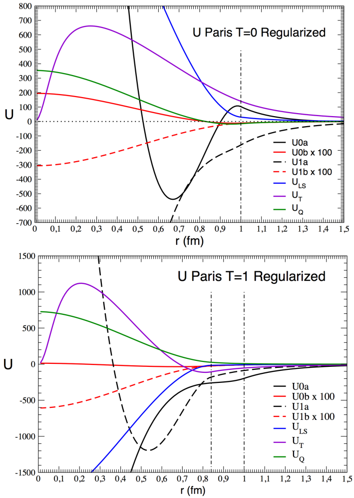
4) Smooth Paris 2009 potentials

Original Isospin Paris 2009 potentials



Smooth Paris 2009 potentials



Next step: fit to the scattering observables + constraints from light antiprotonic nuclei data: $\bar{p}H$, $\bar{p}^2H$, $\bar{p}^{3(4)}He$

5) Concluding remarks and outlook

- 1)Introduction: importance to have a good nucleon-antinucucleon interaction
- 2) In the Paris 2009 brief reminder:

→ fit to scattering observables - to scattering lengths from antiprotonic level shifts should be improved

→ pole positions: - quasi bound states ${}^{11}S_0$, ${}^{33}P_1$ or resonances ${}^{11}P_1$, ${}^{13}P_0$, ${}^{13}P_1$, ${}^{33}P_0$ should be readjusted to obtain better agreement with light antiprotonic-atom data

→ discontinuity between theoretical long range and phenomenological short range potentials should be cured

- 3) Baryonia could show up:

→ in formation experiment such as in the peak at $\bar{p}p$ threshold in $J/\psi \to \gamma p\bar{p}$ decays

→ in subthreshold analysis of antiprotonic $\bar{p}H$, $\bar{p}He$ atoms data. Better agreement with those should be looked at.

- 4) Revisiting the Paris nucleon-antinucleon potential:

→ the junction between long and short range has been smoothed using cubic spline functions

→ adjustement of the phenomenological short part - to reproduce at best the scattering data and - to displace the poles close to the real axis to agree better with the subthreshold data from antiprotonic atoms, to be done.

⇒ Resulting revisited Paris potential could be a good tool for PUMA project.

MERCI POUR VOTRE ATTENTION

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MERCI POUR VOTRE ATTENTION

BACKUP MATERIAL

Summary: BES Collaboration data on $J/\psi \to \gamma(\omega)p\bar{p}$ is described by two processes.

1) Direct emission process before formation of final baryons.
   $\star$ FSI for $\gamma \Rightarrow$ 2 resonant states:
   a) a very sharp peak close to threshold due to a baryonium - broad 52 MeV wide quasi-bound state at 4.8 MeV below threshold in $^{11}S_0$ wave of Paris potential,
   b) a resonant state at 2170 MeV - shape resonance in the same partial wave.
   $\star$ For (ω) Born contribution describes full ω spectrum at large $M_{p\bar{p}}$ and $M_{\omega p}$.
   $\star$ For γ (or ω) weak energy dependence for the source radius is necessary.
2) Emission from baryonic current. Occurs after J/ψ decay into an $N\overline{N}$ pair.
   $\star$ For γ not sufficient to reproduce final resonant states $\Rightarrow$ need DE model.
   $\star$ For $(\omega, \pi, \phi$, not shown here) the Born term is the dominant mode.
   $\star$ But the ω mass distribution $M_{p\bar{p}}$ needs a strong reduction in the lower mass region: obtained by introducing a FSI involving a $N^*(3/2)$ or $\overline{N}^*(3/2)$ resonance created by an ω-p (ω-$\bar{p}$) interaction via an ω exchange between $\bar{p}$-p (p-$\bar{p}$) pairs.

Outlook

- J/ψ and $\psi(2S)$ different internal structure: DE$\Rightarrow$ no peak in $\psi(2S)$ formation.
- Paris potential fits data $M_{p\bar{p}} \lesssim 2.1$ GeV but produces reasonable results beyond.
- Present approach could be applied with other interaction like the χEFTN^{3}LO [Ling-Yun Dai, J. Haidenbauer, U.-G. Meissner, JHEP **07**, 078 (2017)]
- Present work $\Rightarrow$ related $\bar{p}p \to J/\psi +$ *meson* reaction on nuclei sooner or later at FAIR (Facility for Antiproton and Ion Research), at GSI, Darmstadt, Germany.

	2P shift	2P width	3D width
S wave	6.68{9.22}	17.5{11.5}	0.69{0.49}
P wave	-6.36{1.44}	15.0{26.9}	1.46{2.08}
Sum	0.31{10.71}	32.5{38.4}	2.15{2.57}
Data [3]	17±4	25±9	2.14 ±0.18

[3] M. Schneider *et al.*, Zeit. Phys. A **338**, 217 (1991).

Leading order calculations in eV for $2P$ and in meV for $3D$ (widths only) level corrections in ^{3}He obtained with the spin averaged amplitudes of the Paris 2009 potential. Numbers in curly brackets are obtained with the Paris 1999 potential.

	2P shift	2P width
S wave	6.66{7.83}	12.3{7.70}
P wave	-5.20{4.75}	19.1{22.13}
Sum	1.46{12.59}	31.4{29.8}
Data [3]	17±4	25±9

As in the above Table but only for $2P$ level including higher order corrections. Now the contribution of the P wave interaction depends also on S wave interaction as a result of multiple scattering summation method.

	2P shift	2P width	3D width
S wave	9.72{17.6}	26.0{19.8}	0.66{0.50}
P wave	-9.01{-10.4}	14.9{14.8}	0.91{0.91}
Sum	0.708{7.2}	40.9{34.6}	1.57{1.41}
Data [3]	18±2	45±5	2.36 ±0.10

[3] M. Schneider *et al.*, Zeit. Phys. A **338**, 217 (1991).

Leading order calculations in eV for $2P$ and in meV for $3D$ (widths only) level corrections in ^{4}He obtained with the spin averaged amplitudes of the Paris 2009 potential. Numbers in curly brackets are obtained with the Paris 1999 potential.

	2P shift	2P width
S wave	8.94{12.3}	14.7{10.4}
P wave	-8.71{-10.9}	19.0{18.6}
Sum	0.23{1.4}	33.7{29.0}
Data [3]	18±2	45±5

As in the above Table but only for $2P$ level including higher order corrections. Now the contribution of the P wave interaction depends also on S wave interaction as a result of multiple scattering summation method.

The Ratios of neutron over proton capture rates from atomic states: $R_{n/p} = N(\bar{p}n)/N(\bar{p}p)$.

atom	$N(\bar{p}n)/N(\bar{p}p)$
^{96}Zr [4]	2.6±0.3
^{124}Sn [4]	5.0±0.6
^{106}Cd [4]	0.5±0.1
^{112}Sn [4]	0.79±0.14

[4] P. Lubiński, J. Jastrzębski, A. Trzcińska, W. Kurcewicz, F. J. Hartmann, W. Schmid, T. vonEgidy, R. Smolańczuk, S. Wycech, Composition of the nuclear periphery from antiproton absorption, PRC **57**, 2962 (1997).

The ratios of $N(\bar{p}n)$ and $N(\bar{p}p)$ capture rates from atomic states. The second column gives the experimental numbers obtained in radiochemical experiments [4]. Two normal cases ^{96}Zr and ^{124}Sn: neutron haloes.
Anomalous results for ^{106}Cd and ^{112}Sn, partly due to a sizable differences ($\sim$ 3 MeV) in p and n separation energies valence nucleons. Additional explanation: fairly narrow N-$\bar{N}$ quasibound state boosting $\bar{p}$-p absorptions over $\bar{p}$-n ones.

Atom	Experiment	Paris 09	Paris 99
$\bar{p}\,^2$H [5]	0.81±0.03	$1.09\ \{0.55\rangle$	$0.84\ \{0.61\}$
$\bar{p}\,^2$H [6]	0.749±0.018	$1.09\ \{0.55\rangle$	$0.84\ \{0.61\}$
$\bar{p}\,^3$He [7]	0.70±0.14	.65	1.00
$\bar{p}\,^4$He [7]	0.48±0.03	.48	0.59

[5] R. Bizzari *et al.*, Nuovo Cim. **22A**, 225 (1974)
[6] T. E. Kalogeropoulos and G.S Tsanakos, PRD **22**, 2585 (1980).
[7] F. Balestra *et al.*, Nucl. Phys. **A474**, 651 (1987).

$R_{n/p}$ ratios. Second column: experimental results from $\bar{p}$ stopped in bubble chambers. Third and fourth columns Paris potential calculation It is assumed that capture occurs from nP atomic levels. Results for captures in deuterons from nS states: curly brackets.

Antiprotonic atomic levels characterized with very small nuclear-atom overlap are a powerful method to study $\bar{p}N$ amplitudes below the threshold, down to some -40 MeV.

- Paris 2009: S-wave $\bar{p}N$ amplitudes dominated by a broad $^{11}S_0$ quasibound state, E =-4.8 MeV, $\Gamma = 50$ MeV, $\rightarrow$ strong repulsion levels in the light atoms.
- P-wave interactions are attractive and balance with S wave $\rightarrow$uncertain position of the $^{33}P_1$-quasibound state predicted by Paris 2009 and 1999.
- Repulsion from the $^{11}S_0$ wave not strong enough: $\rightarrow$ new phenomenon below -40 MeV - In Paris 2009: I=1 quasibound S-wave state at -80 MeV - Data requires a shift to -60 MeV. ⋆ Medium and heavy atoms - higher nuclear densities and level shifts far from Born approximations - require attraction.
- Consistency with the ^{2}H , ^{3}He atomic levels + understanding of the $R_{n/p}$ anomalies require the $^{33}P_1$ quasibound state (-17 MeV in Paris 99; -4.5 MeV in Paris 09) to be located in the [-11, -9] MeV region.
- In antiprotonic deuterium measurement of the $4P_{3/2}$ fine structure would be valuable and would fix the energy of $^{33}P_1$ quasibound state.

⇒ Paris 2009 versus Paris 1999: level shifts Paris 99 better (strongly bound $^{33}P_1$); Paris 09 fits on additional $\bar{n}p$ and capture rates $R_{n/p}$ better $\rightarrow$ advantage PUMA project at CERN; Paris 09 better for the BES Collaboration enhancement results.

⋆ **Outlook** Paris 09 starting point for successful description of atomic, bubble chamber, radiochemical data if P-wave baryonium position shifted down by few MeV + deeply bound S state pushed up by some 20 MeV.
⇒ Update of this potential model, work is in progress.