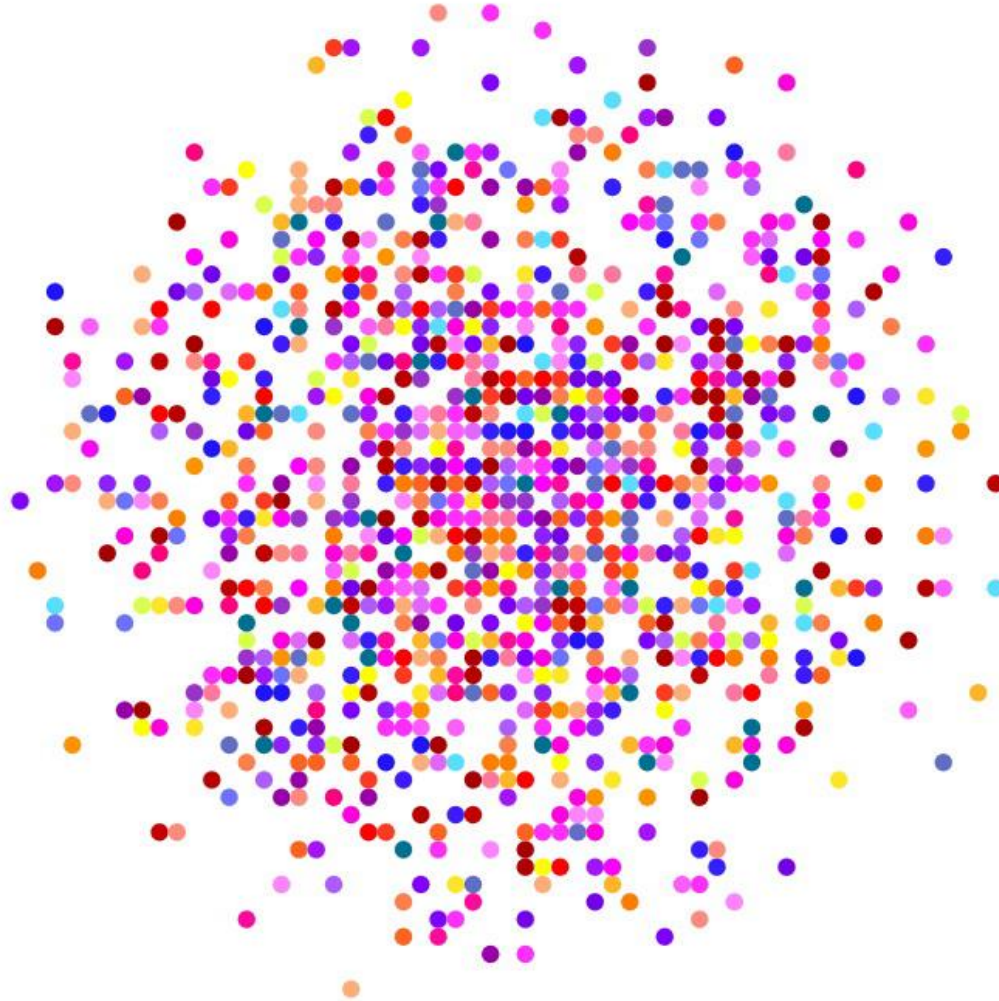


# Energy-density-functional results for magic nuclei and the extrapolation to the neutron matter equation of state



Nuclear Physics A 627 (1997) 710–746

# A Skyrme parametrization from subnuclear to neutron star densities

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$$\mathcal{H} = \mathcal{K} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\text{eff}} + \mathcal{H}_{\text{fin}} + \mathcal{H}_{\text{so}} + \mathcal{H}_{\text{sg}} + \mathcal{H}_{\text{Coul}},$$

$$\mathcal{H}_0 = \frac{1}{4} t_0 [(2 + x_0) \rho^2 - (2x_0 + 1) (\rho_p^2 + \rho_n^2)],$$

$$\mathcal{H}_3 = \frac{1}{24} t_3 \rho^\sigma [(2 + x_3) \rho^2 - (2x_3 + 1) (\rho_p^2 + \rho_n^2)],$$

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & \frac{1}{8} [t_1 (2 + x_1) + t_2 (2 + x_2)] \tau \rho \\ & + \frac{1}{8} [t_2 (2x_2 + 1) - t_1 (2x_1 + 1)] (\tau_p \rho_p + \tau_n \rho_n), \end{aligned}$$

$$\begin{aligned} \mathcal{H}_{\text{fin}} = & \frac{1}{32} [3t_1 (2 + x_1) - t_2 (2 + x_2)] (\nabla \rho)^2 \\ & - \frac{1}{32} [3t_1 (2x_1 + 1) + t_2 (2x_2 + 1)] [(\nabla \rho_p)^2 + (\nabla \rho_n)^2], \end{aligned}$$

$$\mathcal{H}_{\text{so}} = \frac{1}{2} W_0 [\mathbf{J} \cdot \nabla \rho + \mathbf{J}_p \cdot \nabla \rho_p + \mathbf{J}_n \cdot \nabla \rho_n],$$

$$\mathcal{H}_{\text{sg}} = -\frac{1}{16} (t_1 x_1 + t_2 x_2) \mathbf{J}^2 + \frac{1}{16} (t_1 - t_2) [\mathbf{J}_p^2 + \mathbf{J}_n^2].$$

### 3.2. Asymmetric infinite nuclear matter

As function of  $Y_p = Z/A$  (or  $I = (N - Z)/A$ ), we can use the density functional given by Eqs. (2.5), (2.6) to write the energy per particle of an asymmetric infinite nuclear matter:

$$\begin{aligned} \frac{E}{A}(Y_p \text{ or } I, \rho) = & \frac{3}{5} \frac{\hbar^2}{2m} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{2/3} F_{5/3} + \frac{1}{8} t_0 \rho [2(x_0 + 2) - (2x_0 + 1) F_2] \\ & + \frac{1}{48} t_3 \rho^{\sigma+1} [2(x_3 + 2) - (2x_3 + 1) F_2] \\ & + \frac{3}{40} \left( \frac{3\pi^2}{2} \right)^{2/3} \rho^{5/3} \left\{ [t_1(x_1 + 2) + t_2(x_2 + 2)] F_{5/3} \right. \\ & \left. + \frac{1}{2} [t_2(2x_2 + 1) - t_1(2x_1 + 1)] F_{8/3} \right\}, \end{aligned} \quad (3.18)$$

with the following definition for the asymmetry factors:

$$F_m(Y_p) = 2^{m-1} [Y_p^m + (1 - Y_p)^m], \quad F_m(I) = \frac{1}{2} [(1 + I)^m + (1 - I)^m].$$

All of the curves obtained by Skyrme EDF are given by the analytical expression

$$F(\rho) = a\rho + b\rho^\gamma + c\rho^{2/3} + d\rho^{5/3} \quad (1)$$

nuclear-matter EOS

$$F_m = (E/A) \quad (2)$$

neutron EOS

$$F_n = (E/N) \quad (3)$$

symmetry energy

$$S = F_{sym} = F_n - F_m \quad (4)$$

$\rho$  from s-wave

$\rho^\gamma$  from density dependence  $\gamma=(1+\sigma)$

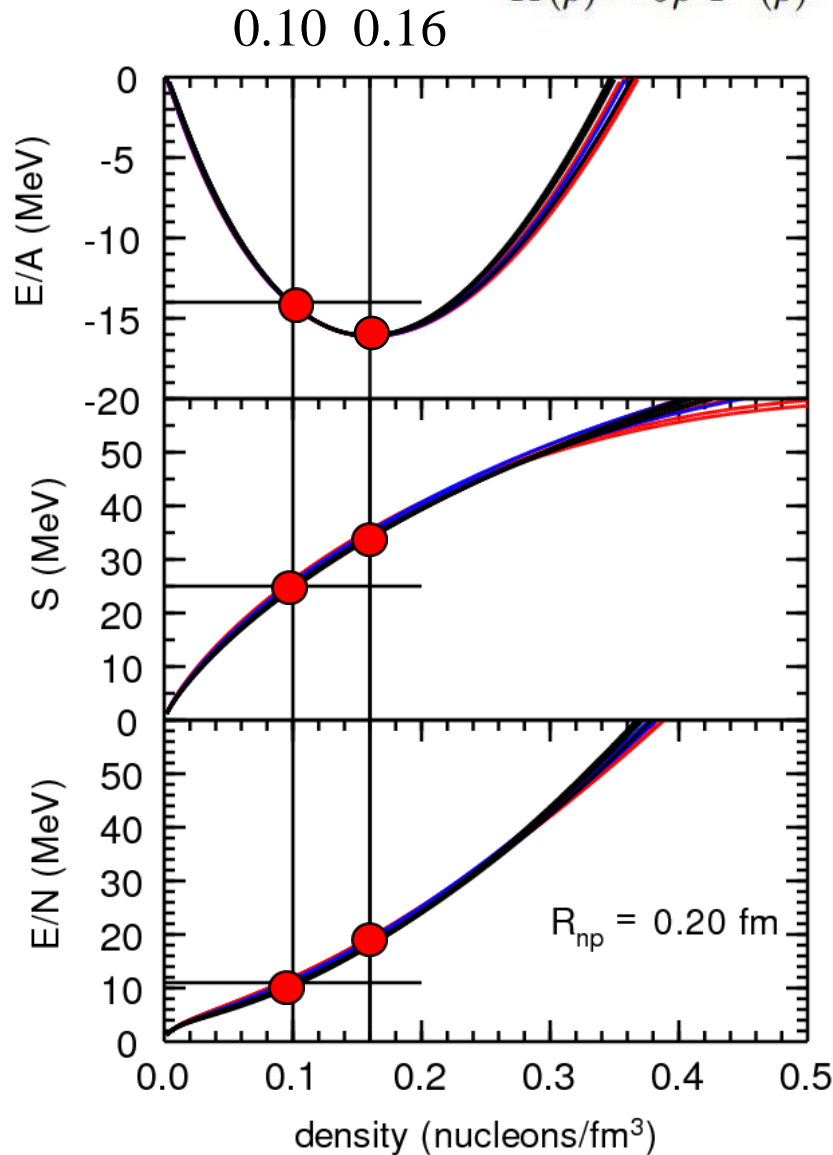
$\rho^{2/3}$  from kinetic energy

$\rho^{5/3}$  from p-wave (effective mass)

$$J(\rho) = F(\rho)$$

$$L(\rho) = 3\rho F'(\rho)$$

$$K(\rho) = 9\rho^2 F''(\rho)$$



Typical values (MeV) at  
0.10 0.16 nucleons/ $\text{fm}^3$

$J_m$	-14	-16
$L_m$		0
$K_m$		240
$J_s$	25	34 = $S_v$
$L_s$		68 = $L$
$K_s$		-100
$J_n$	11	18
$L_n$		68
$K_n$		140

In terms of

$$\alpha = \frac{(\rho_n - \rho_p)}{(\rho_n + \rho_p)}$$

A more general EOS can be expanded in the form

$$\begin{aligned} \mathcal{E}(\rho, \alpha) &= \mathcal{E}(\rho, \alpha = 0) + S_2(\rho) \alpha^2 + S_4(\rho) \alpha^4 \\ &= \mathcal{E}(\rho, \alpha = 0) + \frac{1}{2} \left[ \frac{\partial^2 \mathcal{E}(\rho, \alpha)}{\partial \alpha^2} \right]_{\alpha=0} \alpha^2 + \frac{1}{24} \left[ \frac{\partial^4 \mathcal{E}(\rho, \alpha)}{\partial \alpha^2} \right]_{\alpha=0} \alpha^4 \end{aligned}$$

and for the symmetry energy

$$S(\rho) = S_2(\rho) + S_4(\rho)$$

The  $S_4$  derived from nuclear properties has a very large uncertainty, e.g. an  $(N - Z)^4$  term in the liquid drop model has a very large uncertainty.

With Skyrme we assume that this comes from a specific functional form.

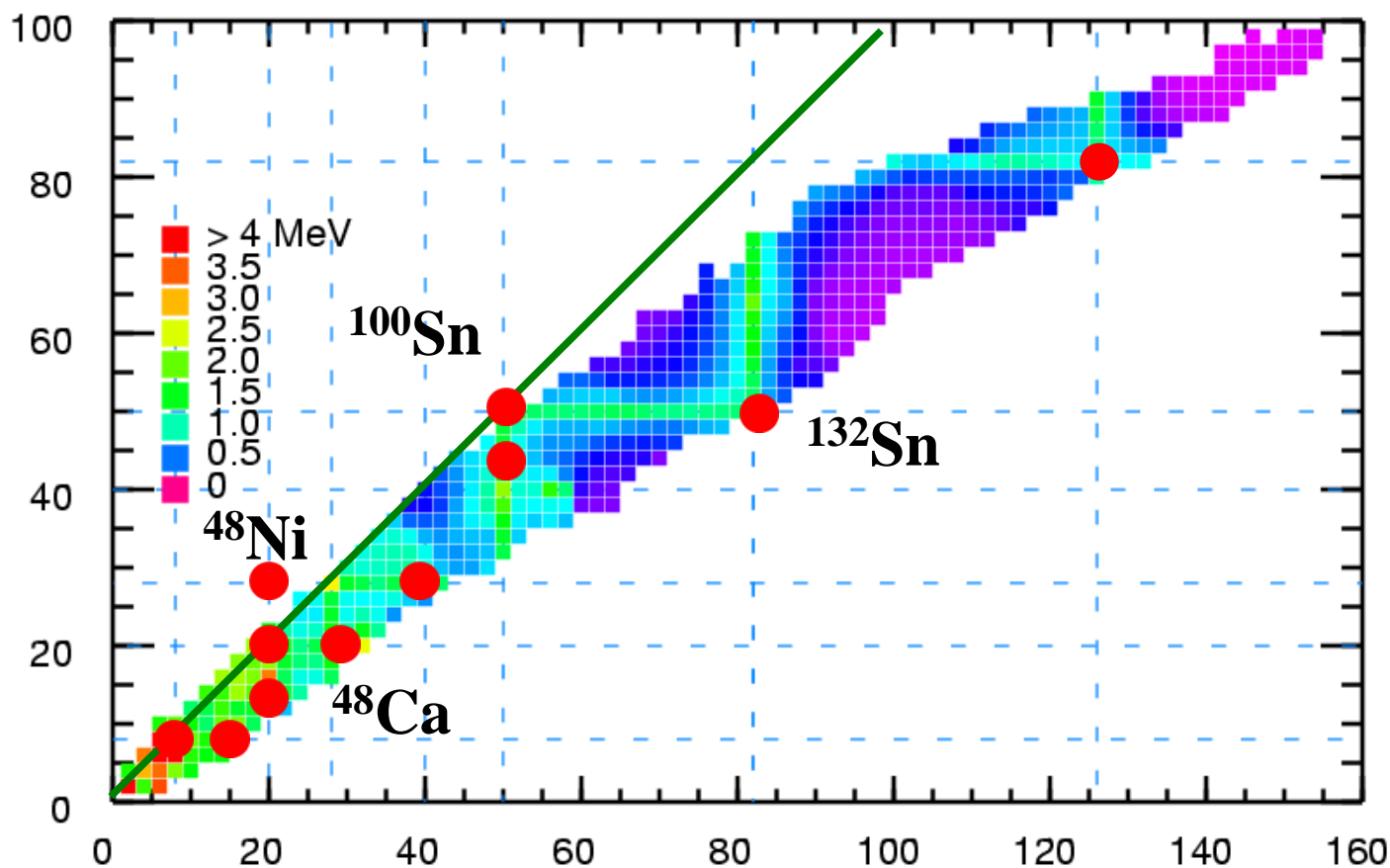
## New Skyrme interaction for normal and exotic nuclei

B. Alex Brown

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East Lansing, Michigan 48824-1321*

*and Department of Physics, University of Stellenbosch, Stellenbosch 7600, South Africa*

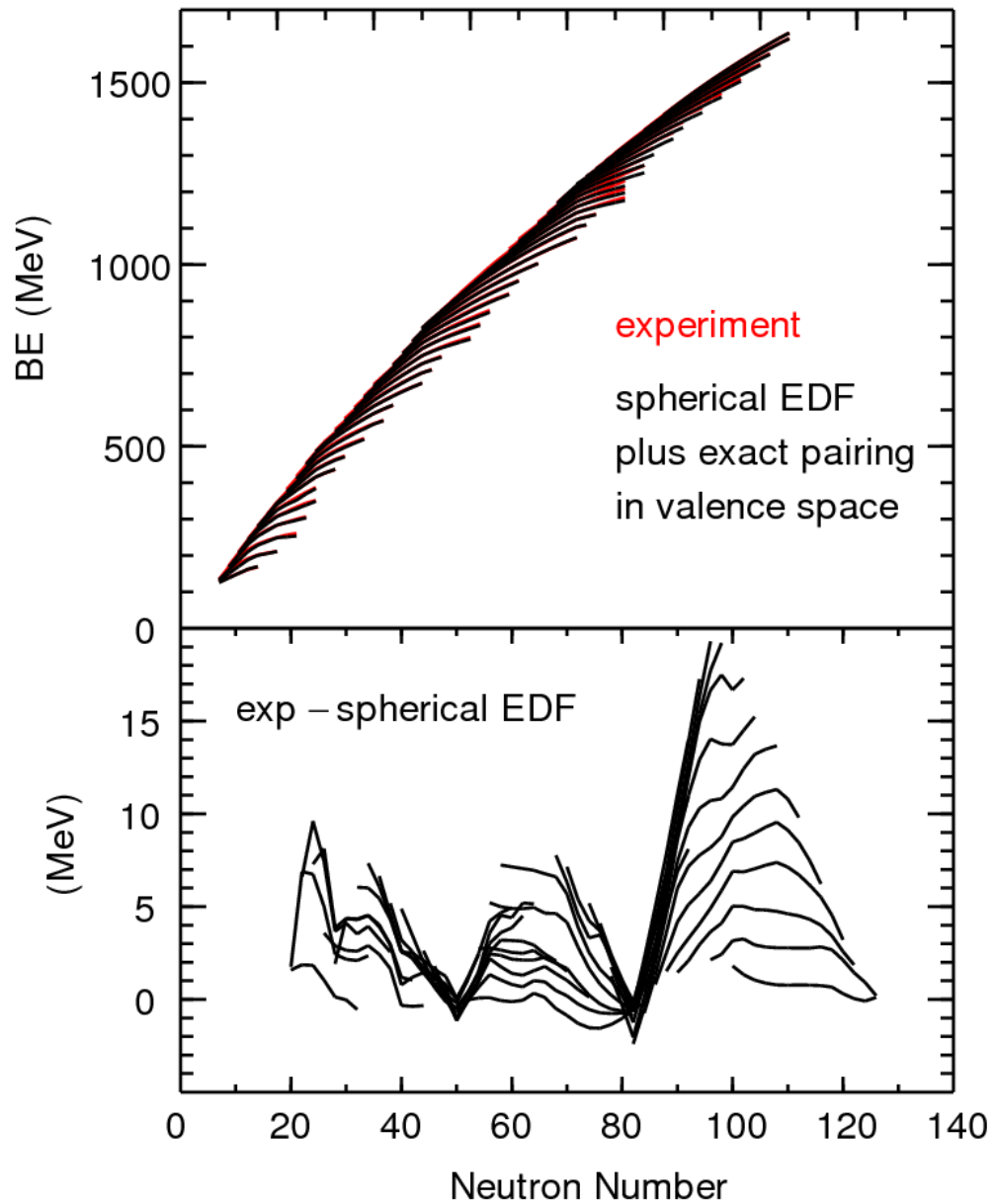
(Received 5 May 1997)

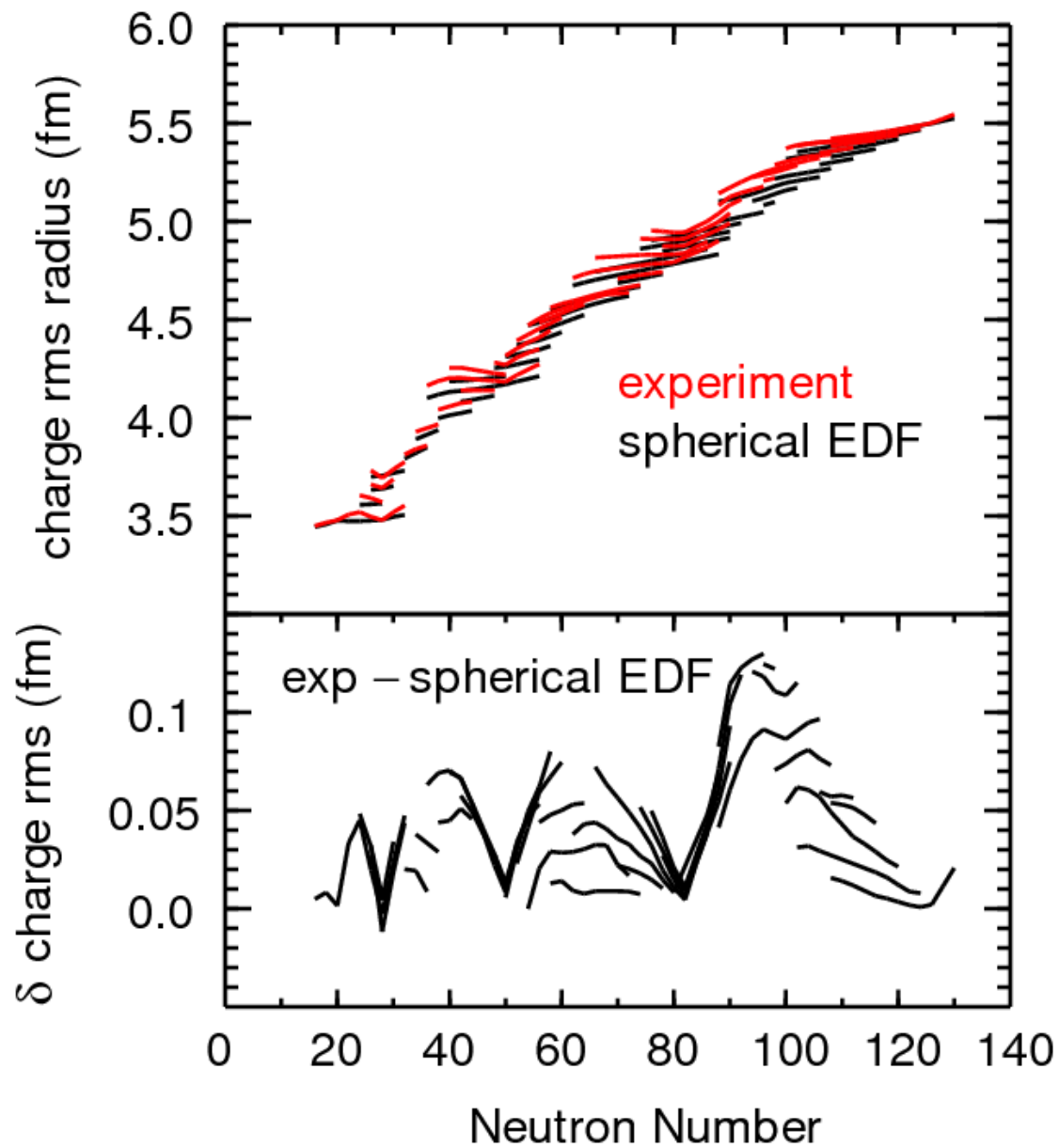


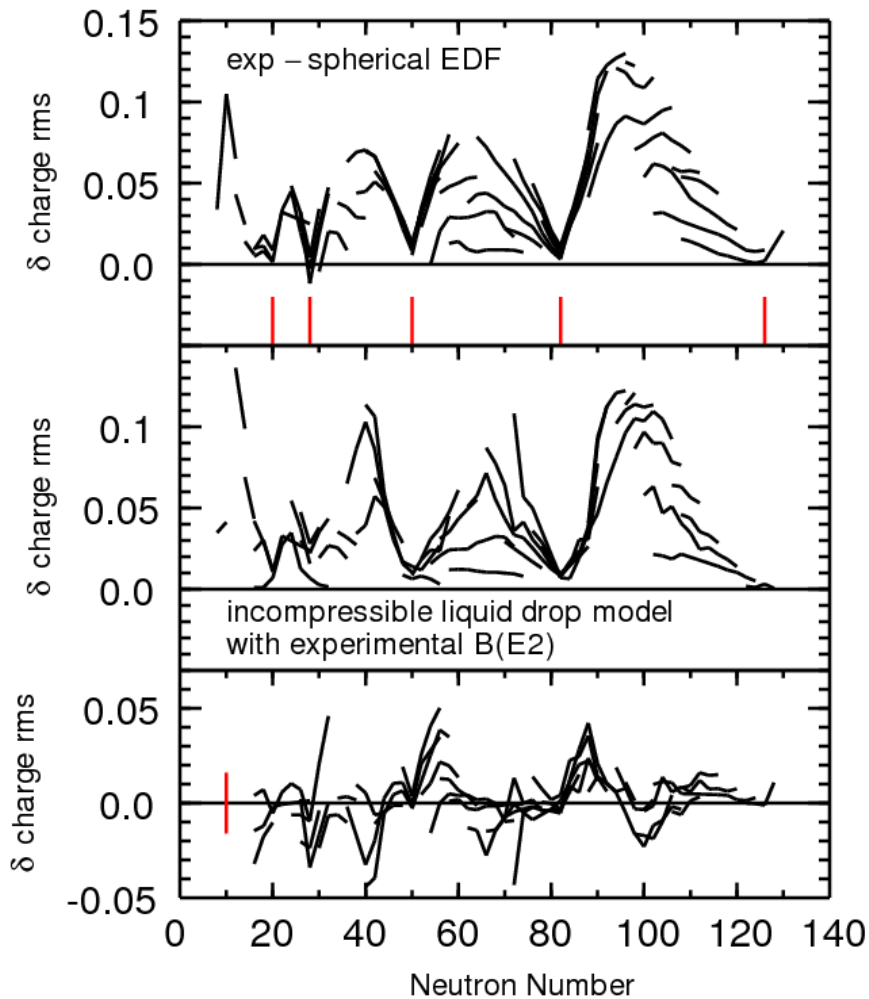


# Nuclear data for the **Skx** the parameters

- Properties of “closed-shell” nuclei  
 $^{16}\text{O}$ ,  $^{24}\text{O}$ ,  $^{34}\text{Si}$ ,  $^{40}\text{Ca}$ ,  $^{48}\text{Ca}$ ,  $^{48}\text{Ni}$ ,  $^{68}\text{Ni}$ ,  $^{88}\text{Sr}$ ,  $^{100}\text{Sn}$ ,  $^{132}\text{Sn}$  and  $^{208}\text{Pb}$
- Binding energies,  $[Z,N]$   $[Z+1,N]$   $[Z-1,N]$   $[Z,N+1]$   $[Z,N-1]$   
  
fitted to within about 1 MeV
- rms charge radii  
  
fitted to within about 0.02 fm
- Some assumptions about the neutron equation of state







$$\delta \langle r^2 \rangle = \frac{B(E2 \uparrow)}{(5/4\pi) \langle r_0^2 \rangle (eZ)^2}.$$

Except for the increase just after the magic numbers 28, 50, 82 and 126 the rms charge radii are understood to the level of about 0.02 fm

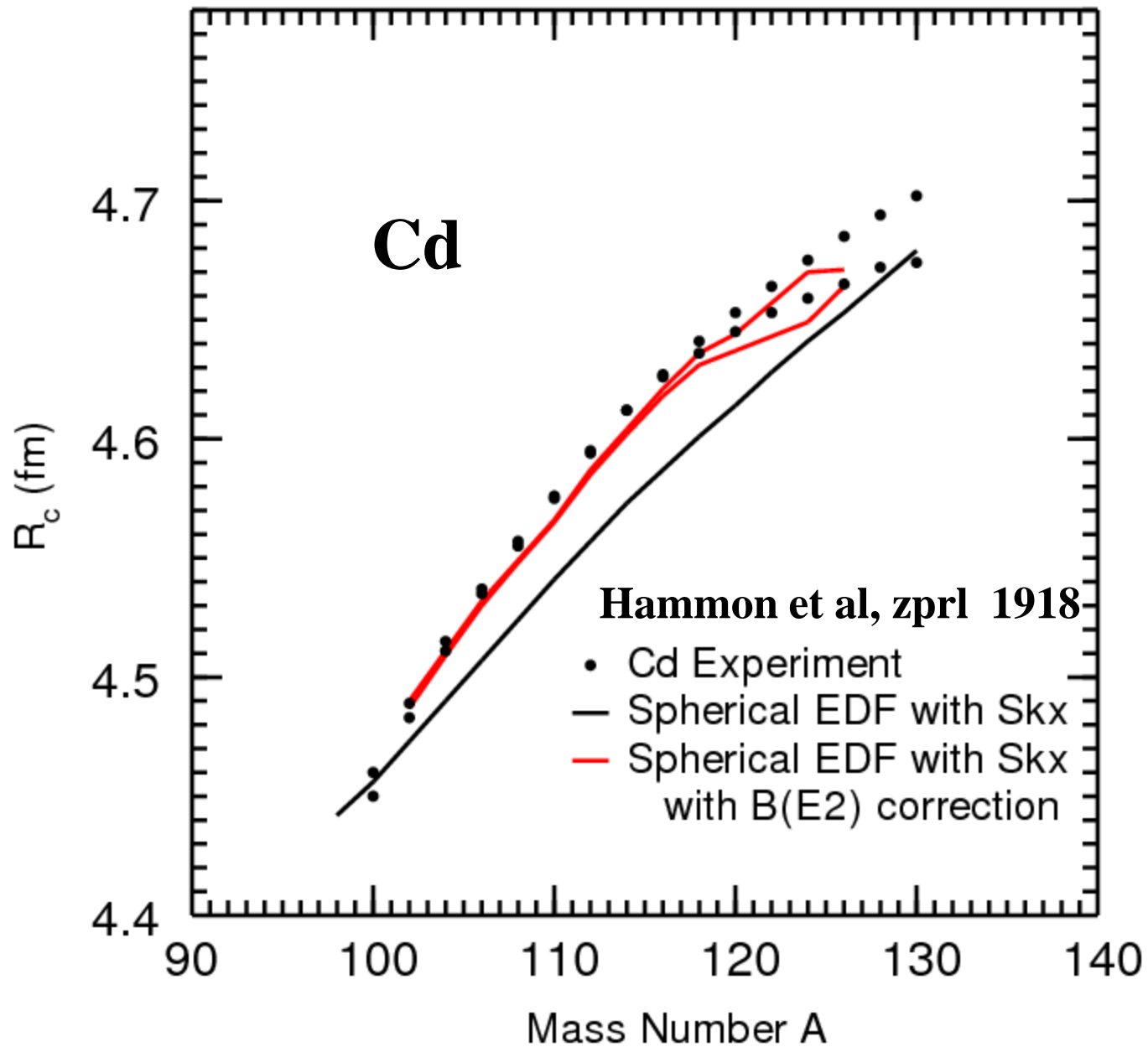
The “proton charge radius puzzle” (in fm)

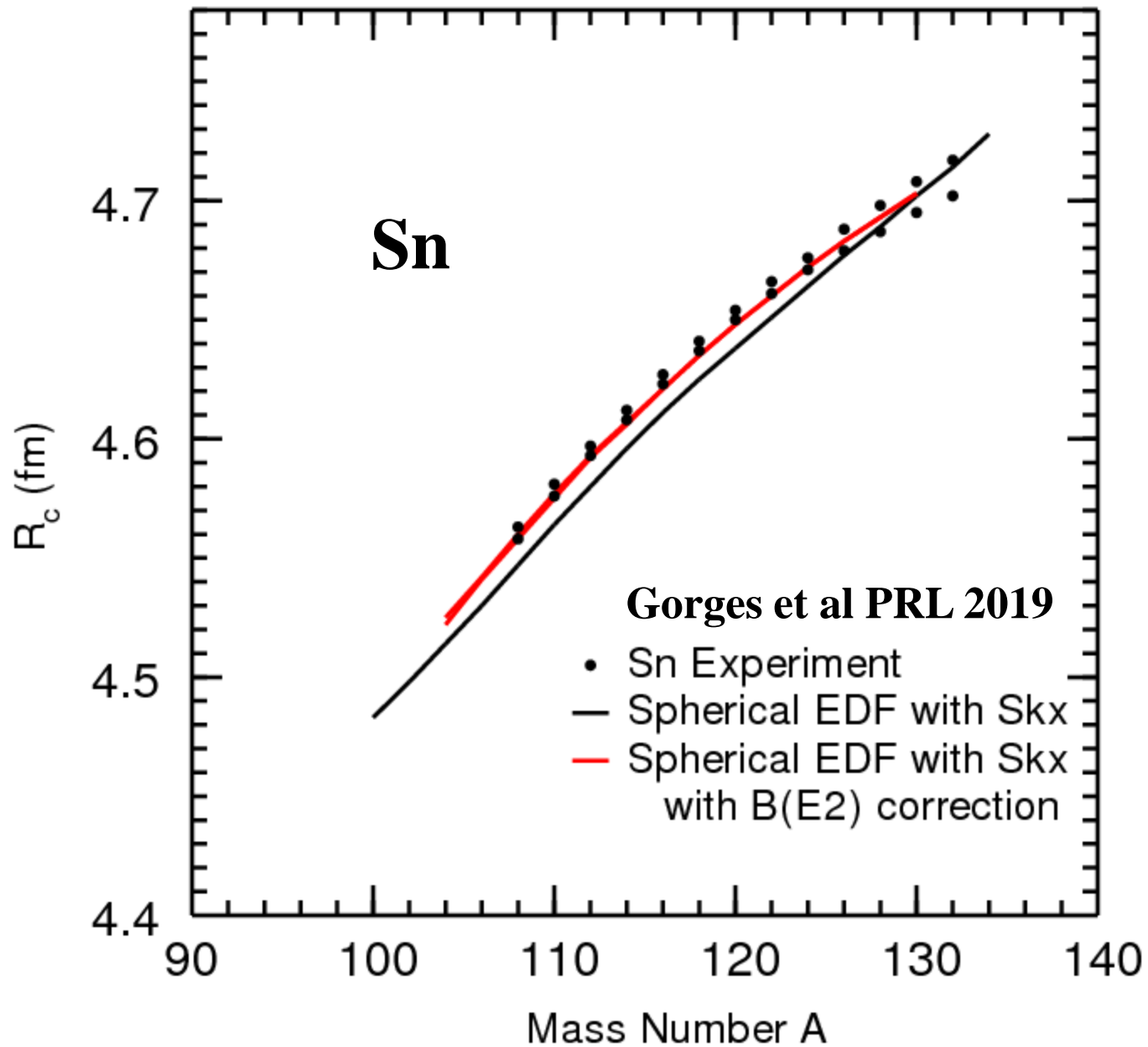
0.879(11) from electron scattering

0.833(10) from atomic Lamb shift (Berginov et al., Science 365, 1007 (2019)).

0.8409(4) from muon Lamb shift

results differ 0.04 fm ( $7 \sigma$ )





# Structure of even-even nuclei using a mapped collective Hamiltonian and the D1S Gogny interaction

J.-P. Delaroche,<sup>1,\*</sup> M. Girod,<sup>1</sup> J. Libert,<sup>2</sup> H. Goutte,<sup>1</sup> S. Hilaire,<sup>1</sup> S. Péru,<sup>1</sup> N. Pillet,<sup>1</sup> and G. F. Bertsch<sup>3,\*</sup>

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(Received 22 September 2009; published 13 January 2010)

## 5DCH 5 dimensional collective Hamiltonian

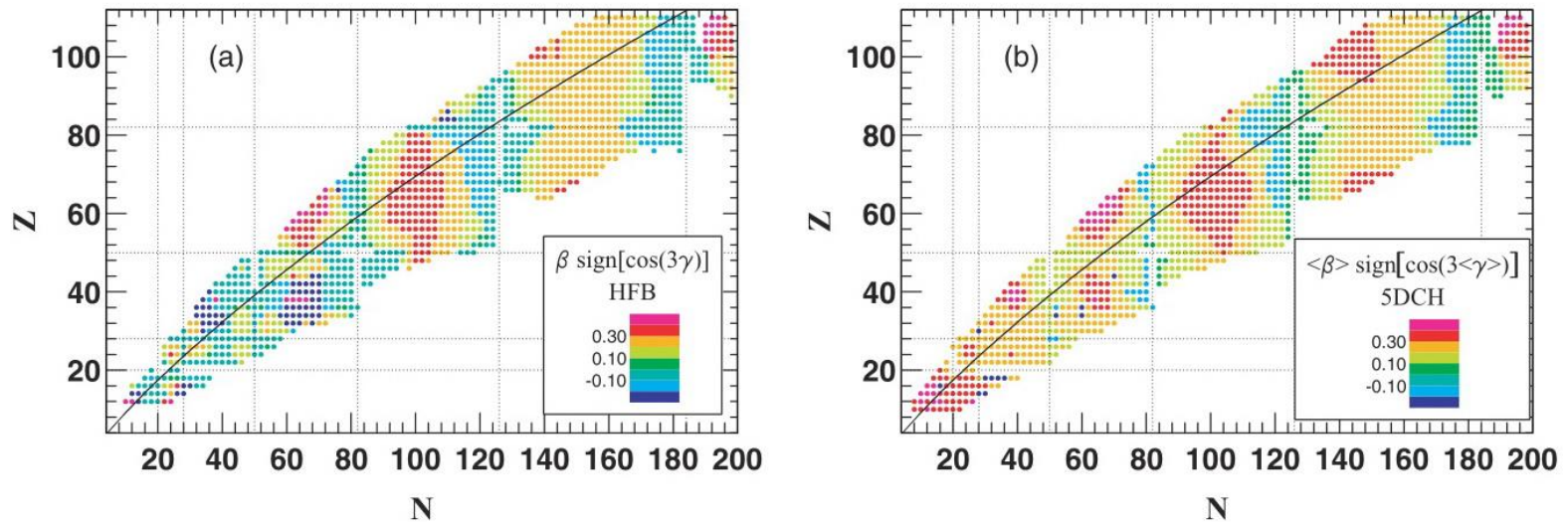
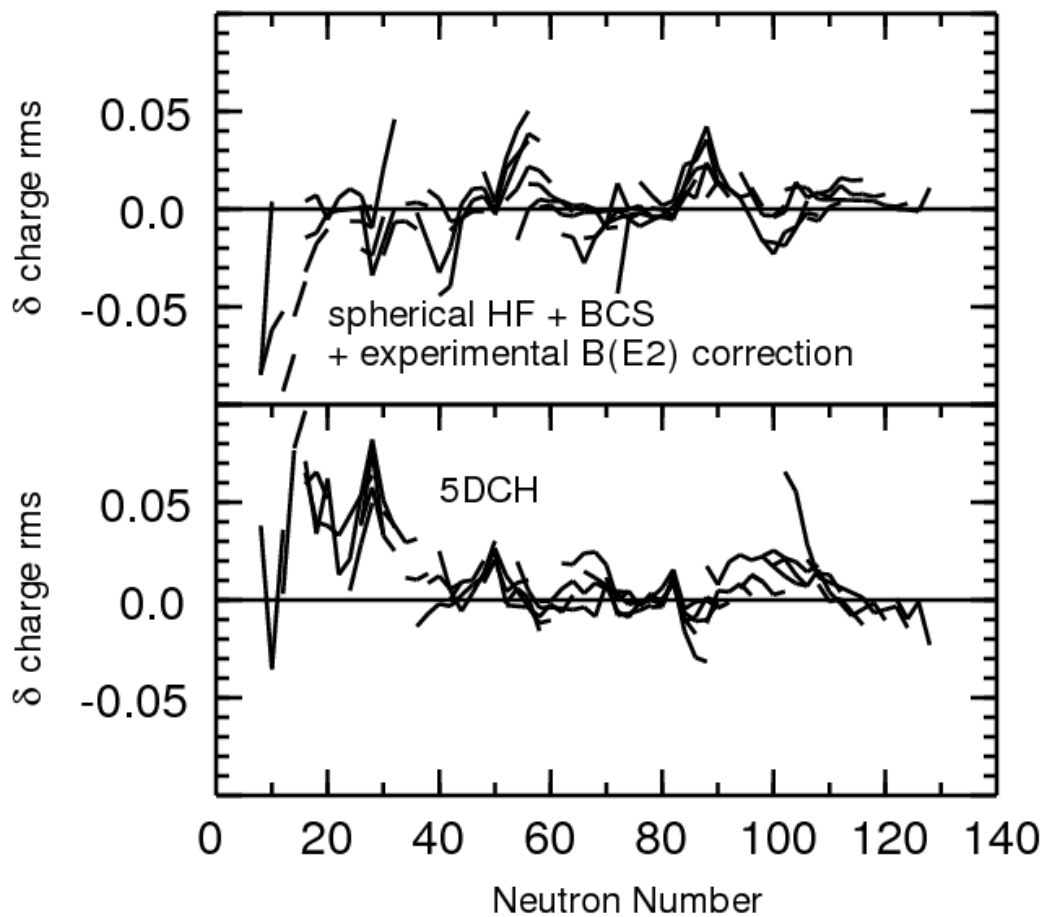


FIG. 3. (Color online) Chart of nuclides showing ground state deformations. (a) HFB minimum; (b) expectation value in the 5DCH ground state. The black curve shows the  $\beta$ -stability line.





Question – how can we extrapolate the properties of nuclei to neutron matter and neutron stars?

Path A - measure neutron skin or mirror charge radii differences

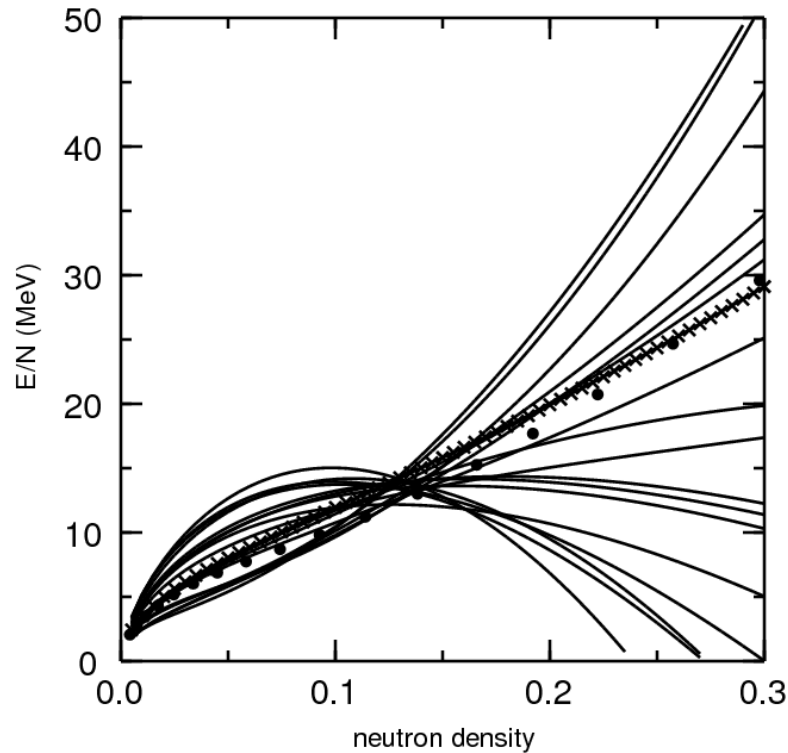
Path B - use low-density ab-initio theory neutron matter

Path C - use maximum mass of neutron stars

Remember the basic assumption that all of these are linked by the Skyrme EDF functional form

**Neutron Radii in Nuclei and the Neutron Equation of State**

B. Alex Brown

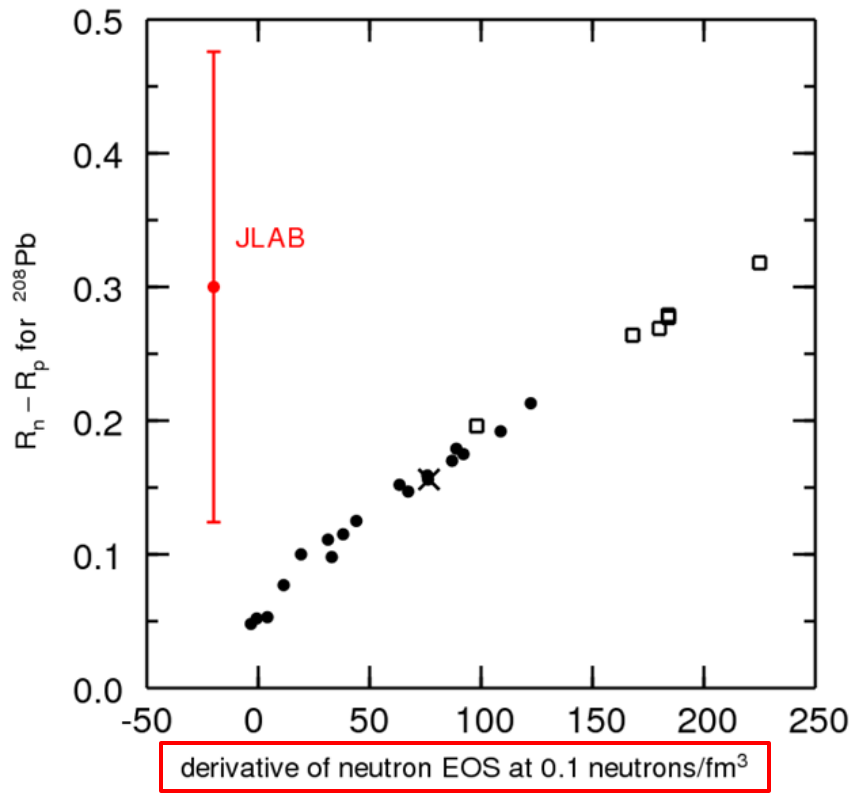


● Friedman-Pandharipanda

x Skx

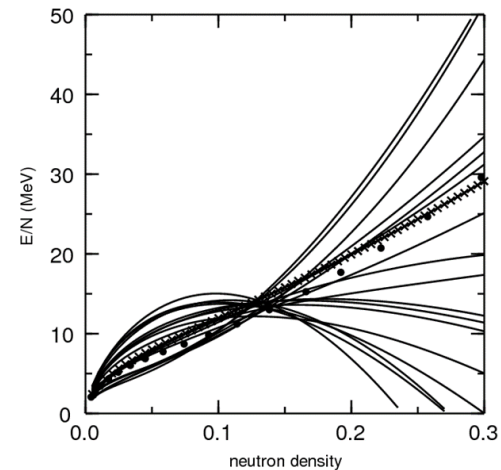
lines – other Skyrme type  
functions that fit nuclear  
properties

# Question - what determines the size of the neutron skin in heavy nuclei like $^{208}\text{Pb}$ ?



(circles) Skyrme-type EDF

(square) relativistic mean field (RMF)



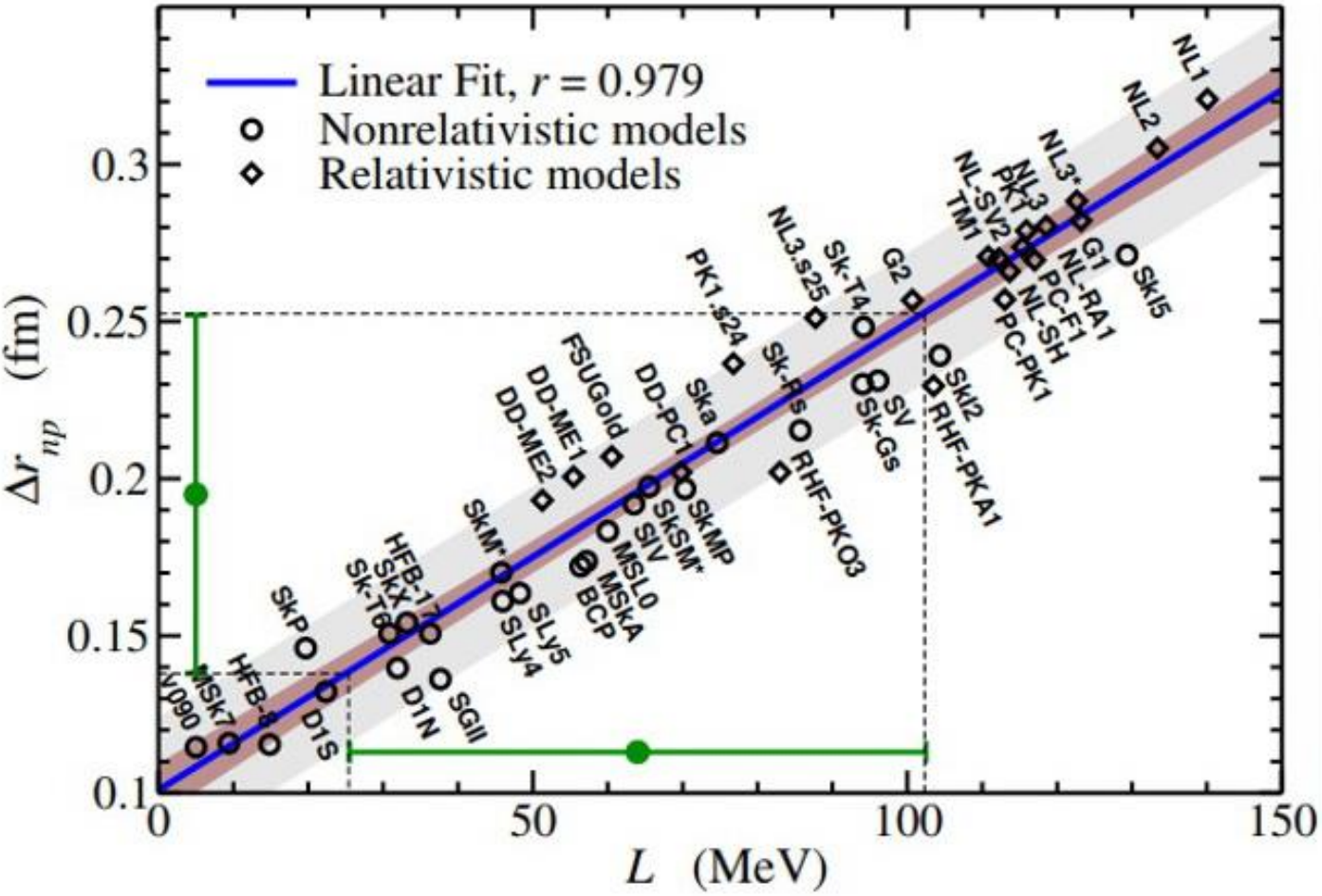
PHYSICAL REVIEW C, VOLUME 64, 027302

## Neutron radii and the neutron equation of state in relativistic models

S. Typel and B. Alex Brown

Path A

One of the most complete versions from 2011 -  
 Note: it is the skin vs L (at 0.16 nucleons/fm<sup>3</sup>)



Roca-Maza et al., PRL106,252501(2011)



**Constraints on the Skyrme Equations of State from Properties of Doubly Magic Nuclei**

B. Alex Brown

**Started with 12 Skyrme-type EDF - most from CSkP of Dutra et al. PRC 85, 035201 (2012).**

PHYSICAL REVIEW C 85, 035201 (2012)

**Skyrme interaction and nuclear matter constraints**M. Dutra,<sup>\*</sup> O. Lourenço,<sup>\*</sup> J. S. Sá Martins, and A. Delfino*Instituto de Física–Universidade Federal Fluminense, Avenida Litorânea s/n, 24210-150 Boa Viagem, Niterói RJ, Brazil*

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(Received 16 November 2011; revised manuscript received 27 January 2012; published 5 March 2012)*

Path A

name		$\sigma$	$K_0$ (MeV)	$m_0^*/m$
KDE0v1	s3	1/6	217	0.81
NRAPR	s6	0.14	221	0.73
Ska25	s7	0.25	220	0.98
Ska35	s8	0.35	238	0.99
SKRA	s9	0.14	213	0.80
SkT1	s10	1/3	238	0.97
SkT2	s11	1/3	238	0.96
SkT3	s12	1/3	236	0.97
SQMC750	s15	1/6	223	0.75
SV-sym32	s16	0.30	232	0.91
SLy4	s17	1/6	222	0.76
SkM*	s18	1/6	219	0.79

incompressibility      effective mass      for nuclear matter

$K_m = 215 - 240$        $m^*/m = 0.70$  to  $1.00$       these are reasonable ranges

**Constraints on the Skyrme Equations of State from Properties of Doubly Magic Nuclei**

B. Alex Brown

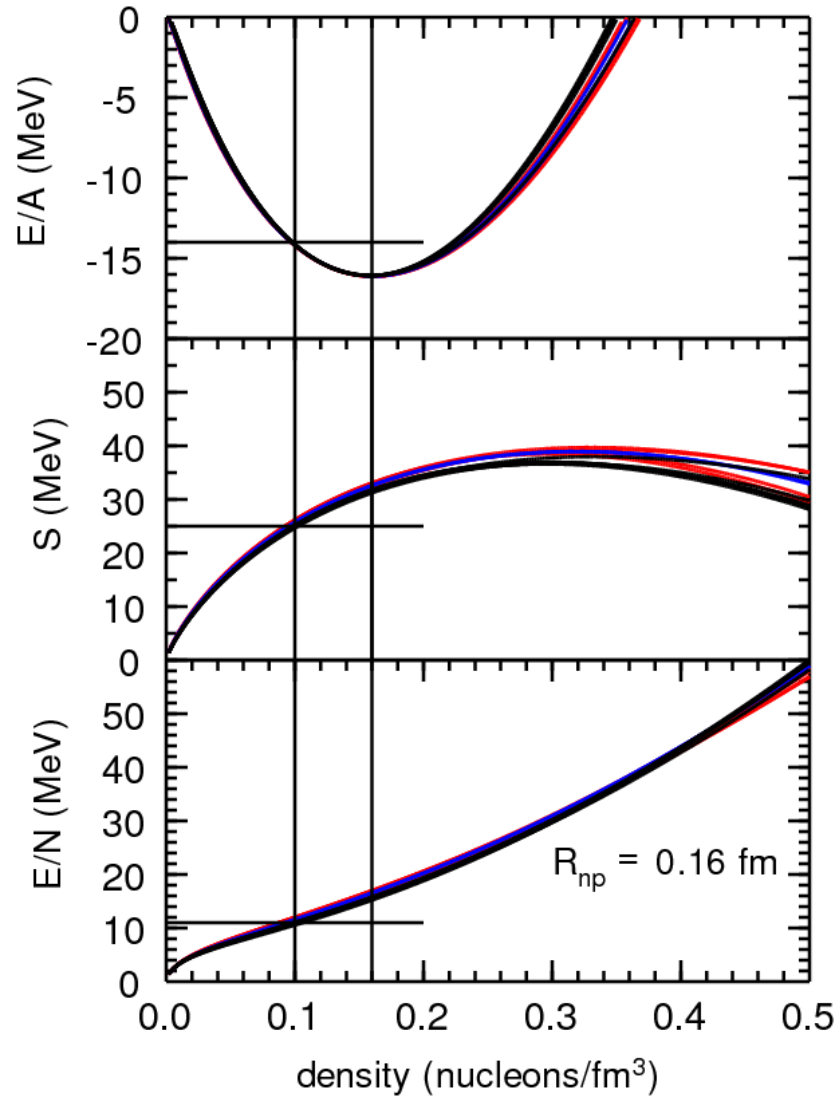
Some combinations of parameters were refit to my SkX data set

But I also add a constraint to have fixed values for the neutron skin of  $^{208}\text{Pb}$

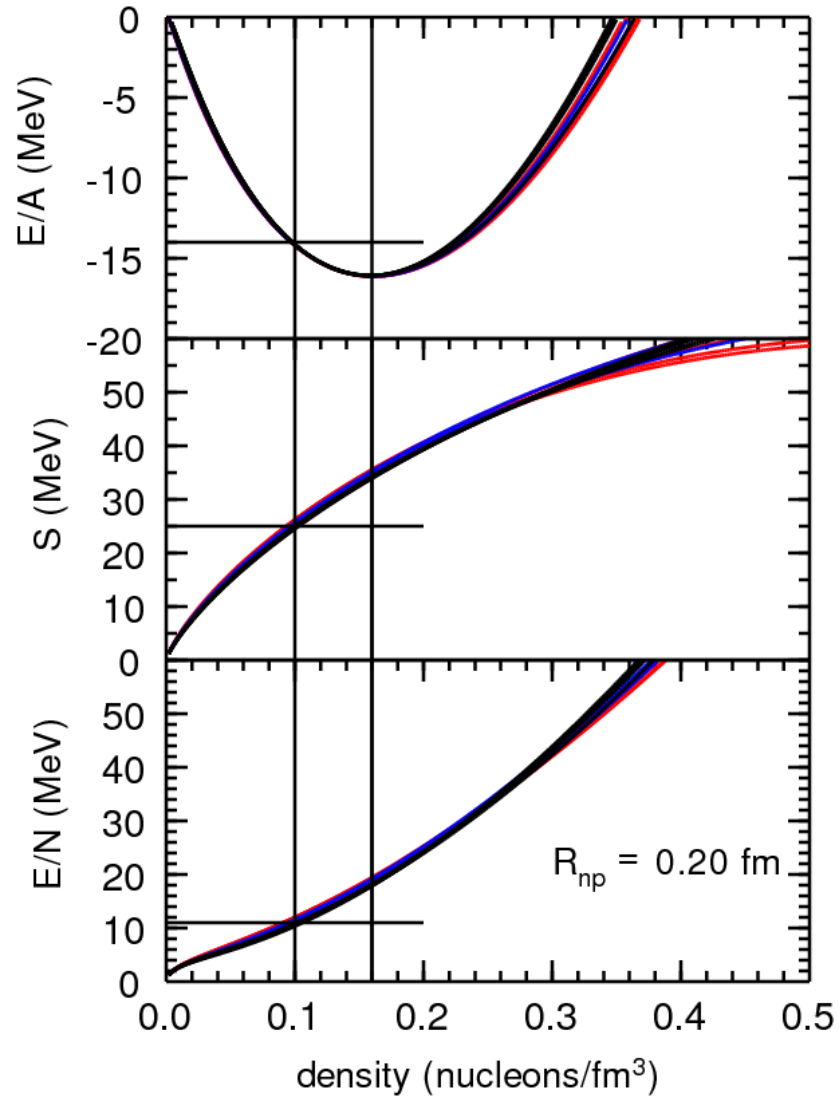
0.16, 0.20, 0.24 fm



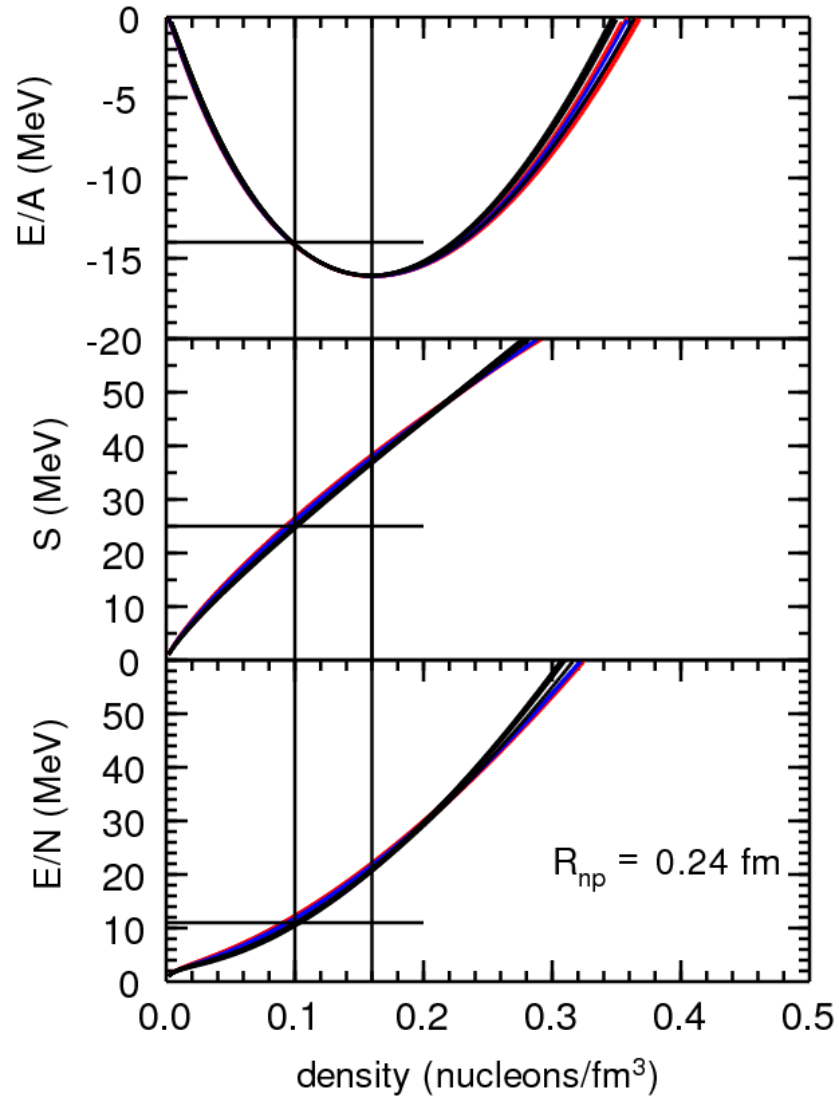
Path A



# Path A



# Path A



Considering the entire CSkP set, the neutron EOS is best determined at

$$\rho_{on} = 0.10$$

with a value of

$$[E/N](\rho_{on}) = 11.3(8) \text{ MeV} \quad (17)$$

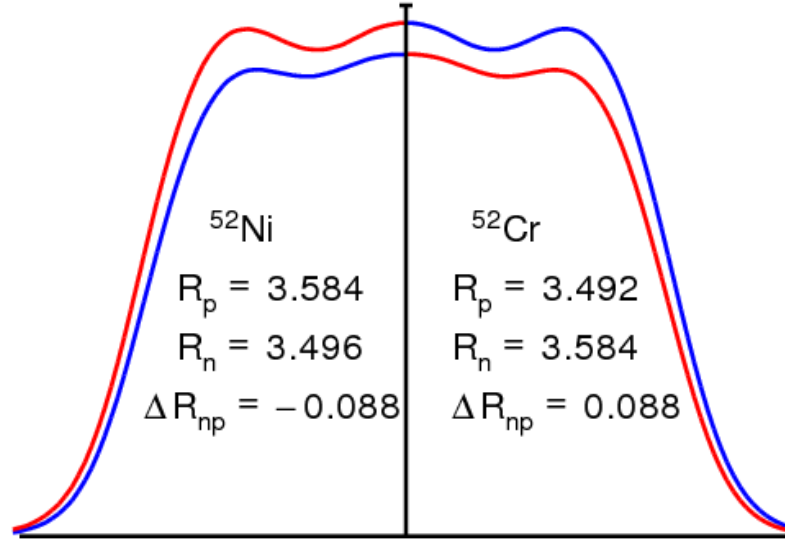
The symmetry energy at this point is

$$\begin{aligned} S(\rho_{on}) &= [E/N](\rho_{on}) - [E/A](\rho_{on}) \\ &= 11.3(8) + 14.1(1) = 25.4(8) \text{ MeV} \end{aligned} \quad (18)$$

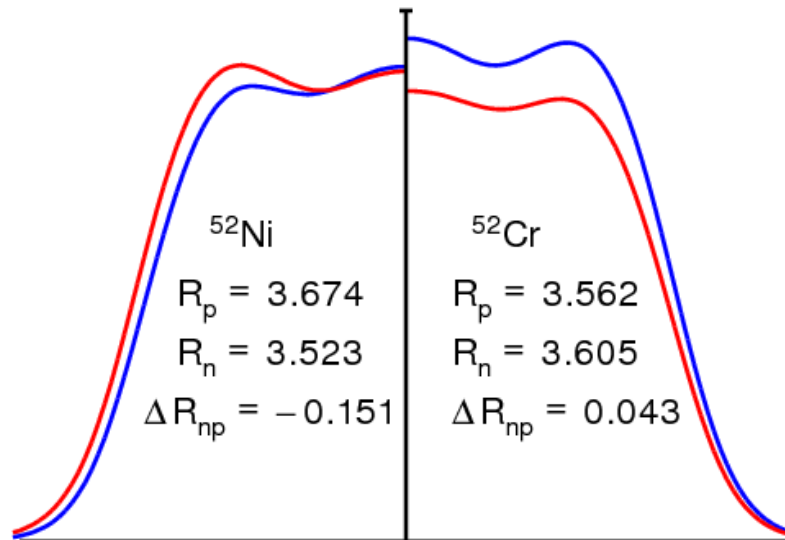
Mirror Charge Radii and the Neutron Equation of State

				48 28,Ni	49 21	50 22	51 23	52 24	53 25	54 26	55 27	56 28,Ni
							50	51	52	53	54	55 27,Co
			45 26,Fe	46 26,Fe	47	48	49	50	51	52	53	54 26,Fe
			44		46	47	48	49	50	51	52	53 25,Mn
		42	43	44 24,Cr	45	46	47	48	49	50	51	52 24,Cr
			42	43 23,V	44	45	46	47	48	49	50	51 23,V
		39	40	41 22,Ti	42	43	44	45	46	47	48	49 22,Ti
			39	40 21,Sc	41	42	43	44	45	46	47	48 21,Sc
36 16	37 17	38 18	39 19	40 20,Ca	41 21	42 22	43 23	44 24	45 25	46 26	47 27	48 20,Ca
35	36	37	38 19,K	39	40	41	42	43	44	45	46	47 19,K
34	35	36 18,Ar	37	38	39	40	41	42	43	44	45	46 18,Ar
33	34	35	36	37 17,Cl	38	39	40	41	42	43	44	45 17,Cl
32	33	34	35	36 16,S	37	38	39	40	41	42	43	44 16,S

Path A

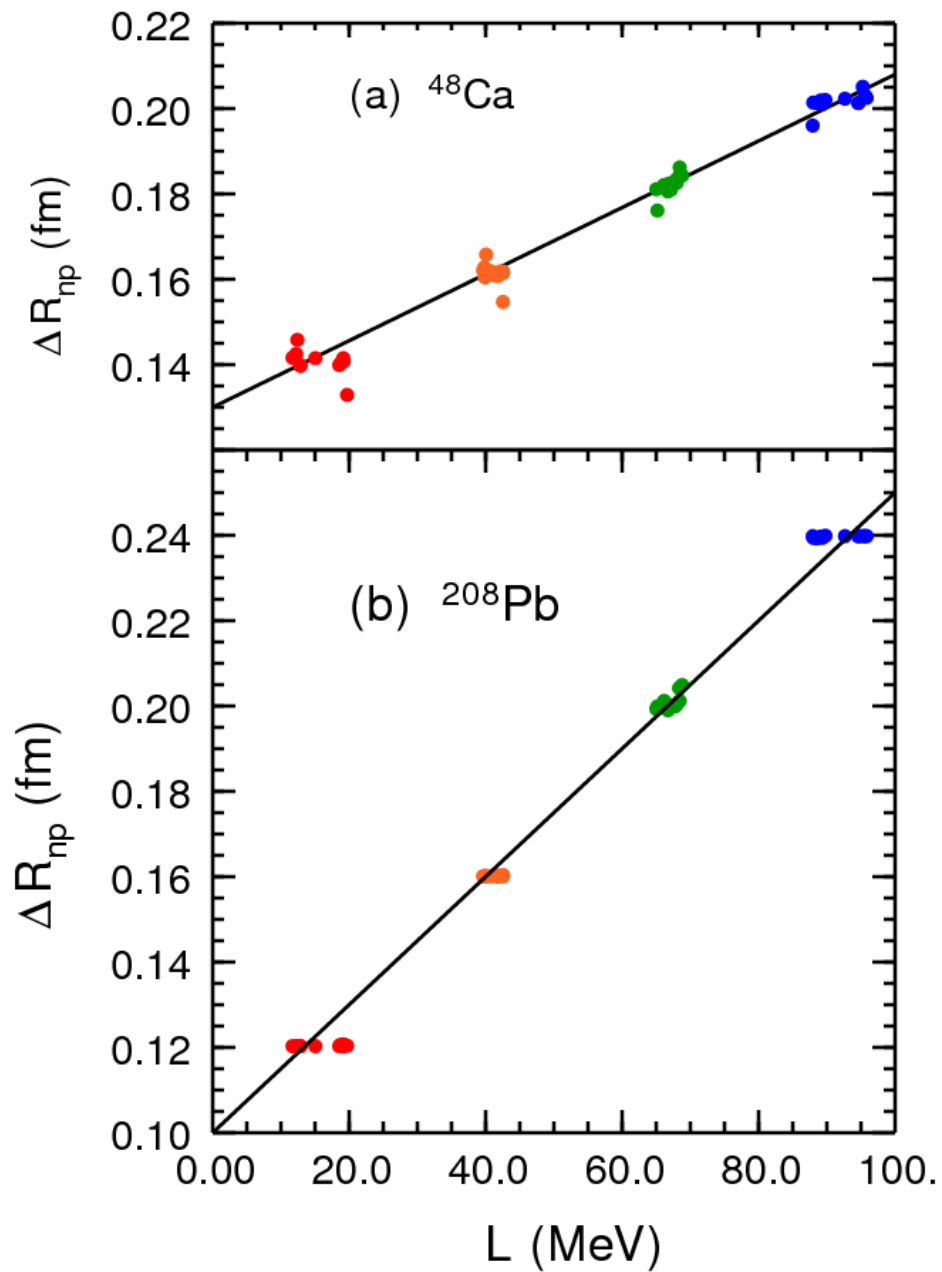


No Coulomb

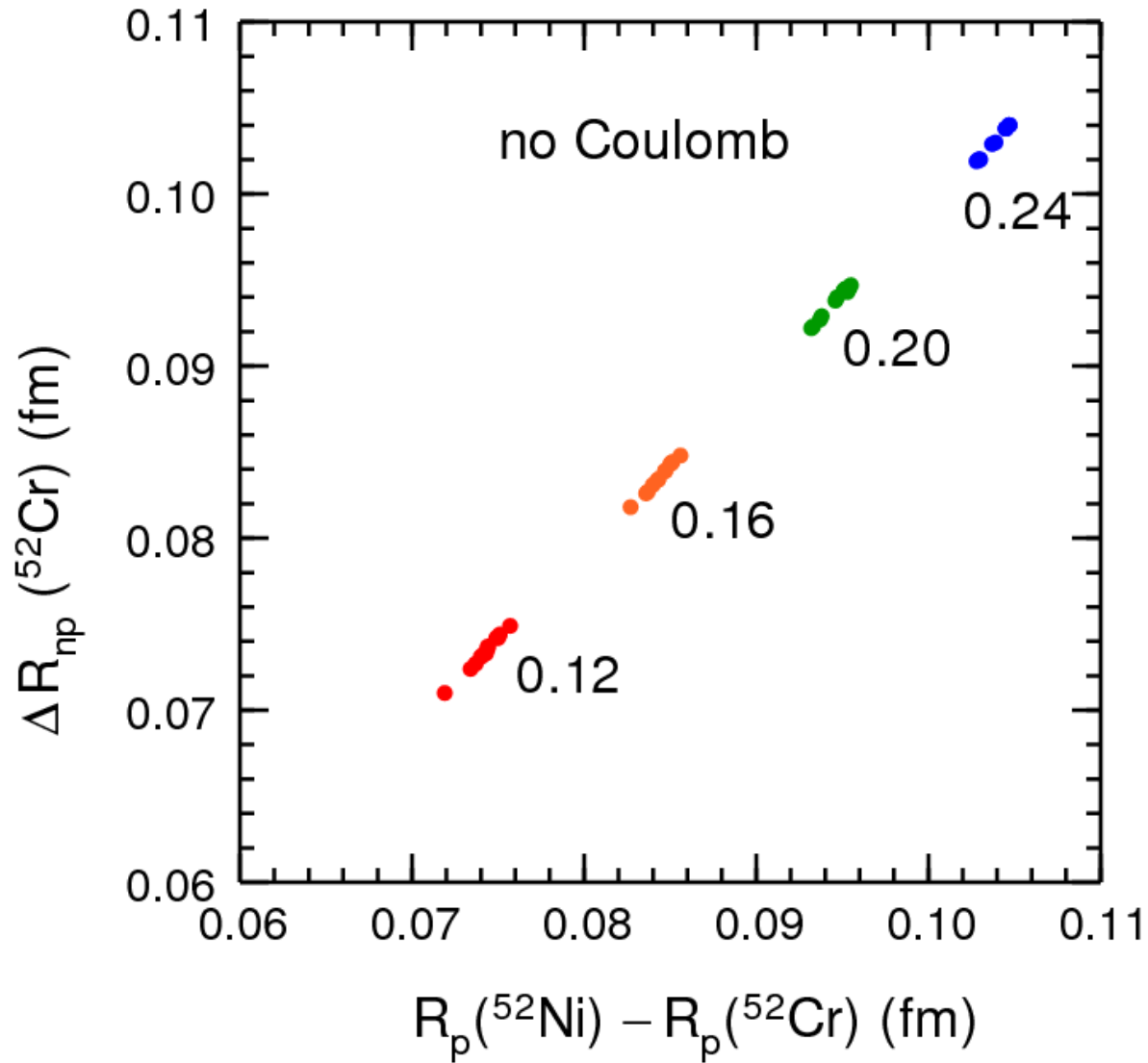


With Coulomb

Path A

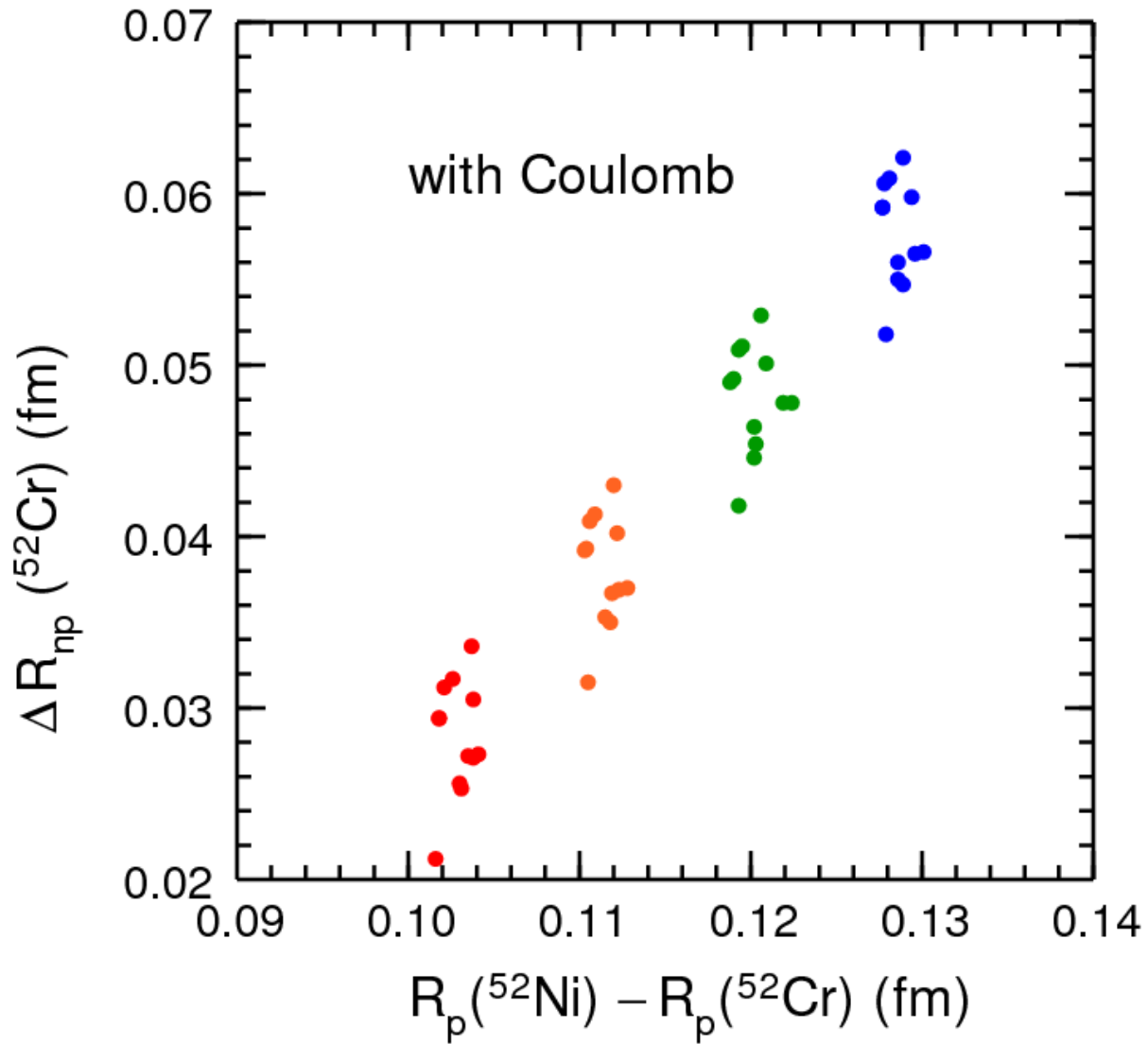


Path A

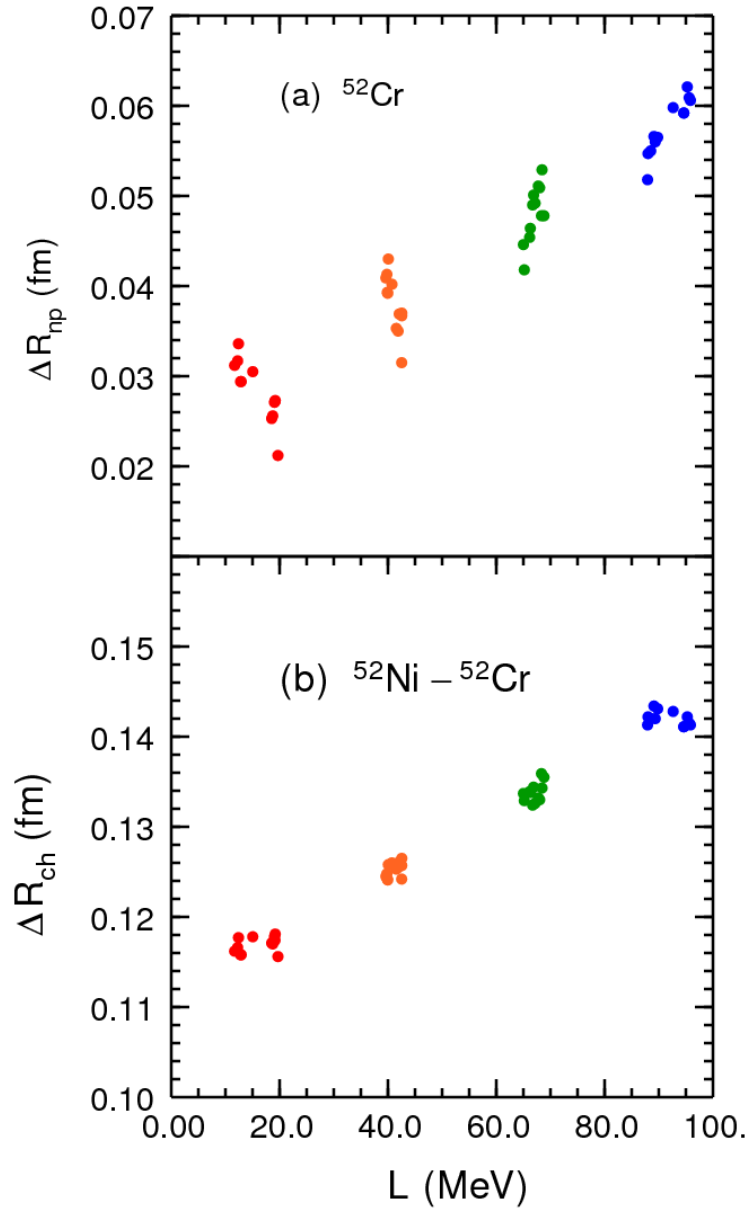




Path A



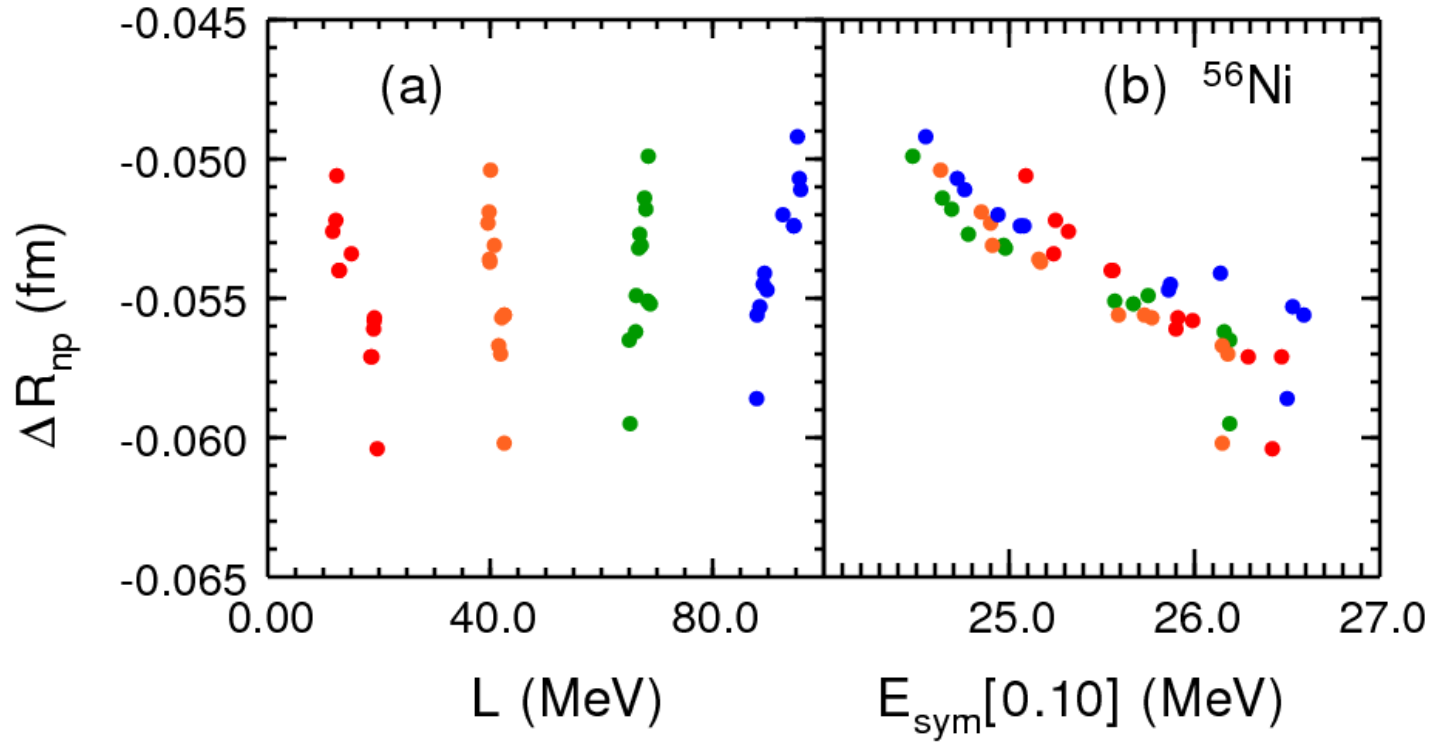
Path A



With Coulomb

With Coulomb

Path A



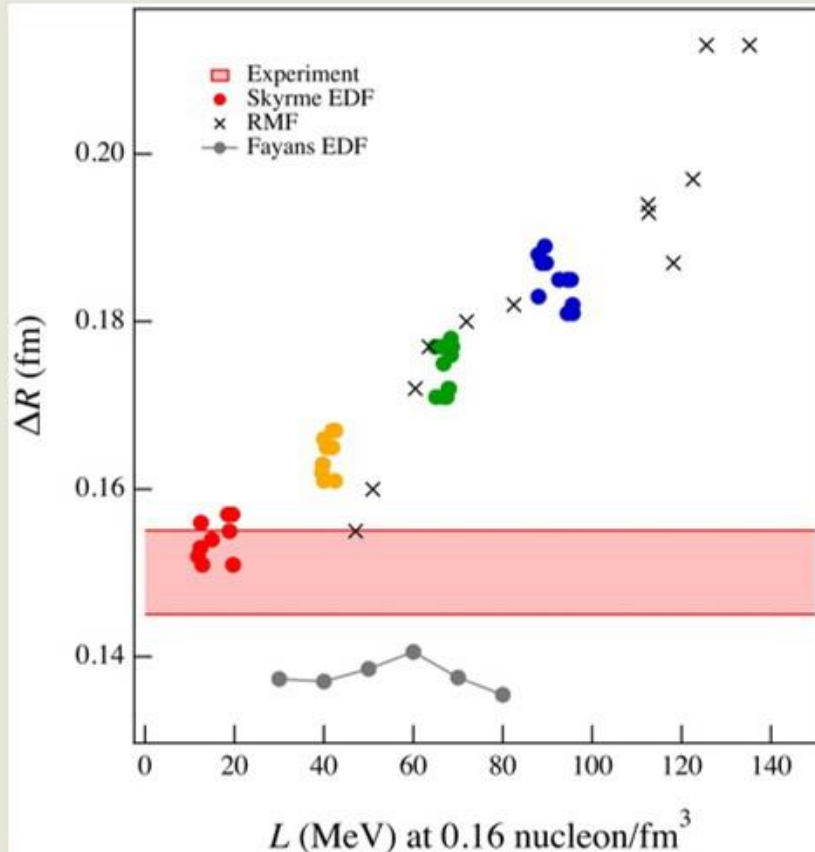
# Path A

				48 28,Ni	49 21	50 22	51 23	52 24	53 25	54 26	55 27	56 28,Ni
							50	51	52	53	54	55 27,Co
			45 26,Fe	46 26,Fe	47	48	49	50	51	52	53	54 26,Fe
			44		46	47	48	49	50	51	52	53 25,Mn
		42	43	44 24,Cr	45	46	47	48	49	50	51	52 24,Cr
			42	43 23,V	44	45	46	47	48	49	50	51 23,V
	39	40	41	42 22,Ti	43	44	45	46	47	48	49	50 22,Ti
		39	40	41 21,Sc	42	43	44	45	46	47	48	49 21,Sc
36 16	37 17	38 18	39 19	40 20,Ca	41 21	42 22	43 23	44 24	45 25	46 26	47 27	48 20,Ca
35	36	37	38	39 19,K	40	41	42	43	44	45	46	47 19,K
34	35	36	37	38 18,Ar	39	40	41	42	43	44	45	46 18,Ar
33	34	35	36	37 17,Cl	38	39	40	41	42	43	44	45 17,Cl
32	33	34	35	36 16,S	37	38	39	40	41	42	43	44 16,S



# Difference of Mirror Charge Radii $^{36}\text{Ca}$ - $^{36}\text{S}$ and Neutron Equation of State

K. Minamisono<sup>1,2</sup>, A. J. Miller<sup>1,2</sup>, A. Klose<sup>3</sup>, B. A. Brown<sup>1,2</sup>, D. Garand<sup>1</sup>, C. Kujawa<sup>3</sup>, J. D. Lantis<sup>1,4</sup>, Y. Liu<sup>5</sup>, B. Maaß<sup>6</sup>, P. F. Mantica<sup>4,7</sup>, W. Nazarewicz<sup>2,7</sup>, W. Nörtershäuser<sup>6</sup>, M. R. Pearson<sup>8</sup>, S. Pineda<sup>3</sup>, P.-G. Reinhard<sup>9</sup>, D. M. Rossi<sup>6</sup>, F. Sommer<sup>6</sup>, C. Sumithrarachchi<sup>1</sup>, A. Teigelhöfer<sup>8</sup> and J. Watkins<sup>1</sup>



Experiment: Present mirror charge radii difference between  $^{36}\text{Ca}$  and  $^{36}\text{S}$ . The band includes all systematic errors in quadratic sum.

Skyrme EDF: from ref. [4]

RMF: present relativistic mean-field calculations

Fayans EDF: present calculations

- Skyrme and RMF calculations show systematic trend of positive slope.
- Crossing with experiment suggests small value of  $L$ .
- However, Fayans has almost flat dependence on  $L$  and shows no systematics.
- It has been shown that the effect of coupling to the proton continuum on charge radius is critical for weakly bound nucleus such as  $^{36}\text{Ca}$ .
- Only Fayans includes the continuum effect.

## Path A

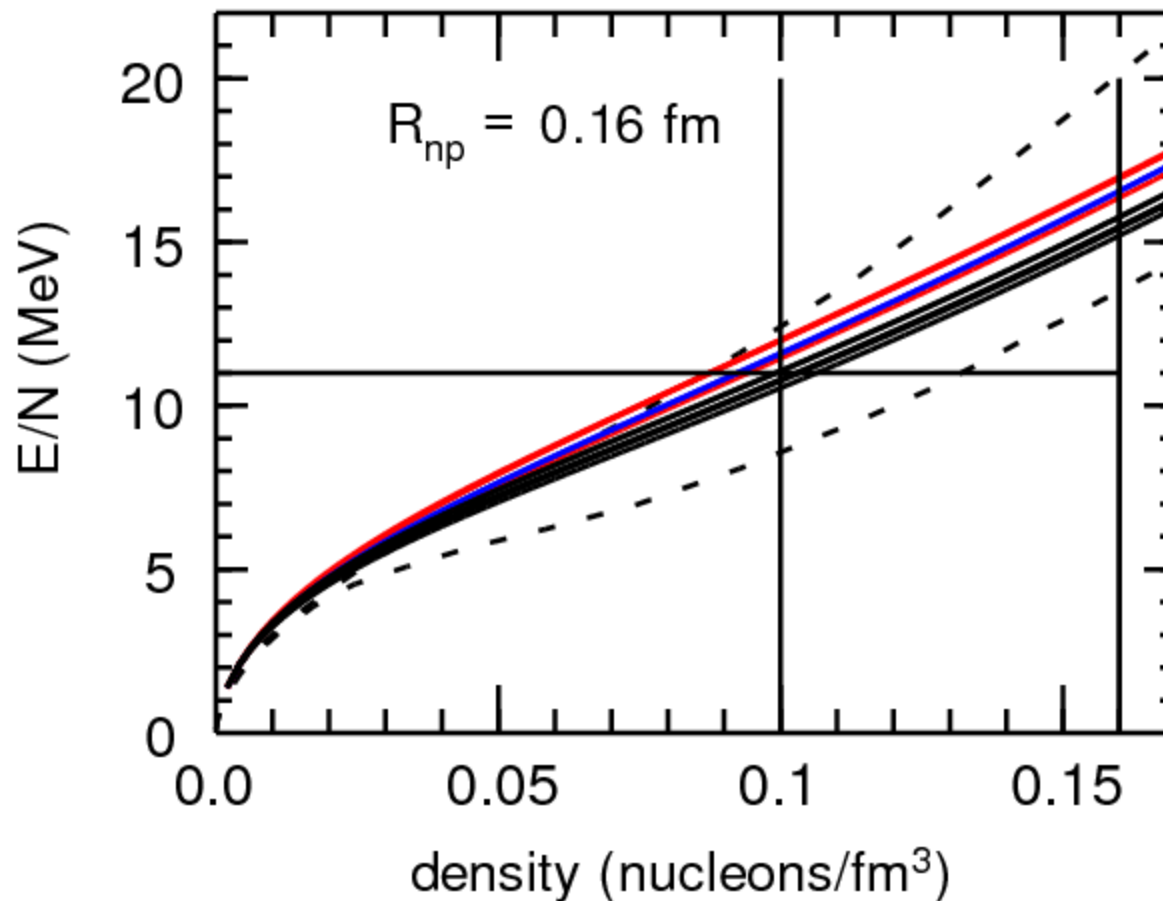
0.05 fm is a very small radius scale – we need to consider

- 1) Proton size problem (0.04 fm)
- 2) Spin-orbit relativistic correction to the charge radius (0.02 fm)
- 3) Mirror asymmetry in the deformation,  $B(E2)$ , correction (0.02 fm)
- 4) Near the proton continuum for proton-rich nuclei (?)

## Path B

Upper and lower values of the error band of calculations based on the Entem and Machleidt  $N^3\text{LO}$  NN potential with a cutoff at 500 MeV that include  $N^2\text{LO}$  NNN forces (the dashed lines).

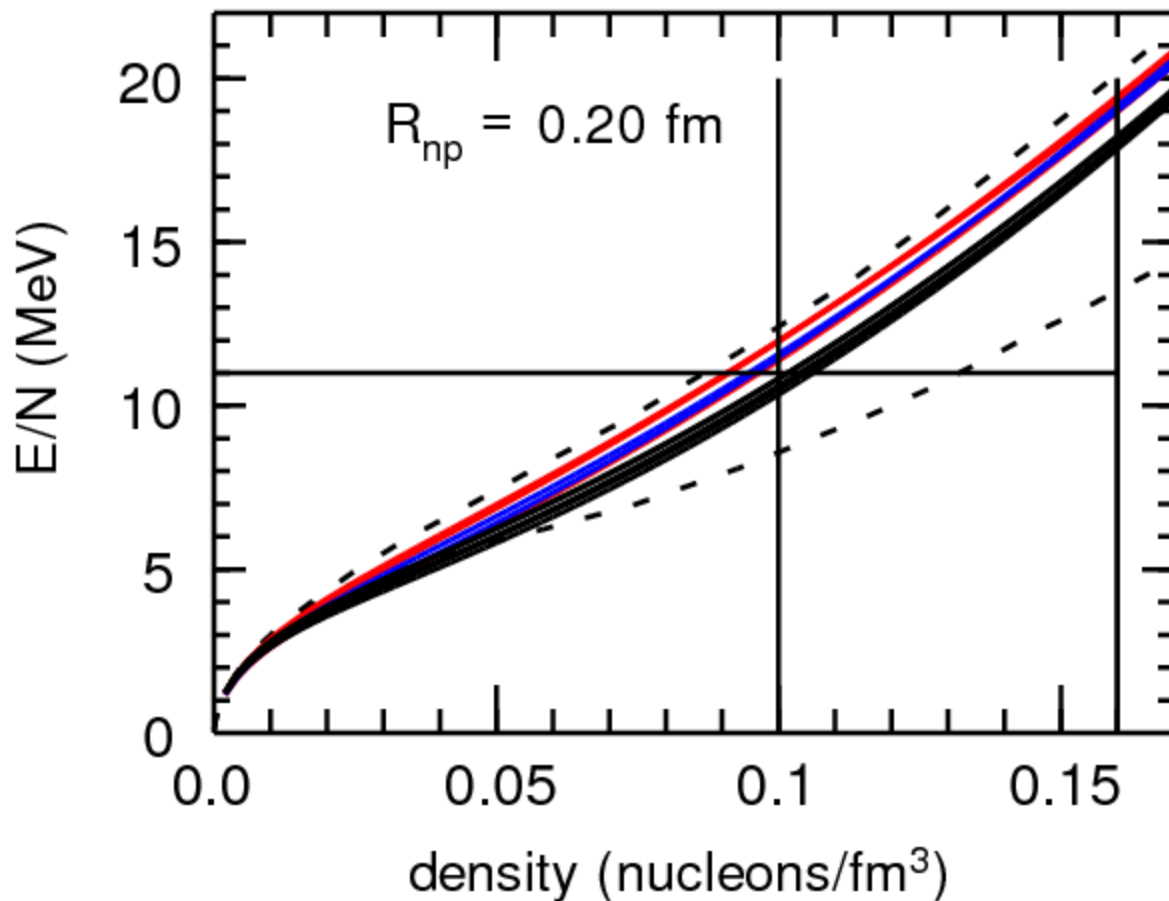
K. Hebeler and A. Schwenk, Phys. Rev. C **82**, 014314 (2010).



## Path B

Upper and lower values of the error band of calculations based on the Entem and Machleidt  $N^3\text{LO}$  NN potential with a cutoff at 500 MeV that include  $N^2\text{LO}$  NNN forces (the dashed lines).

K. Hebeler and A. Schwenk, Phys. Rev. C **82**, 014314 (2010).

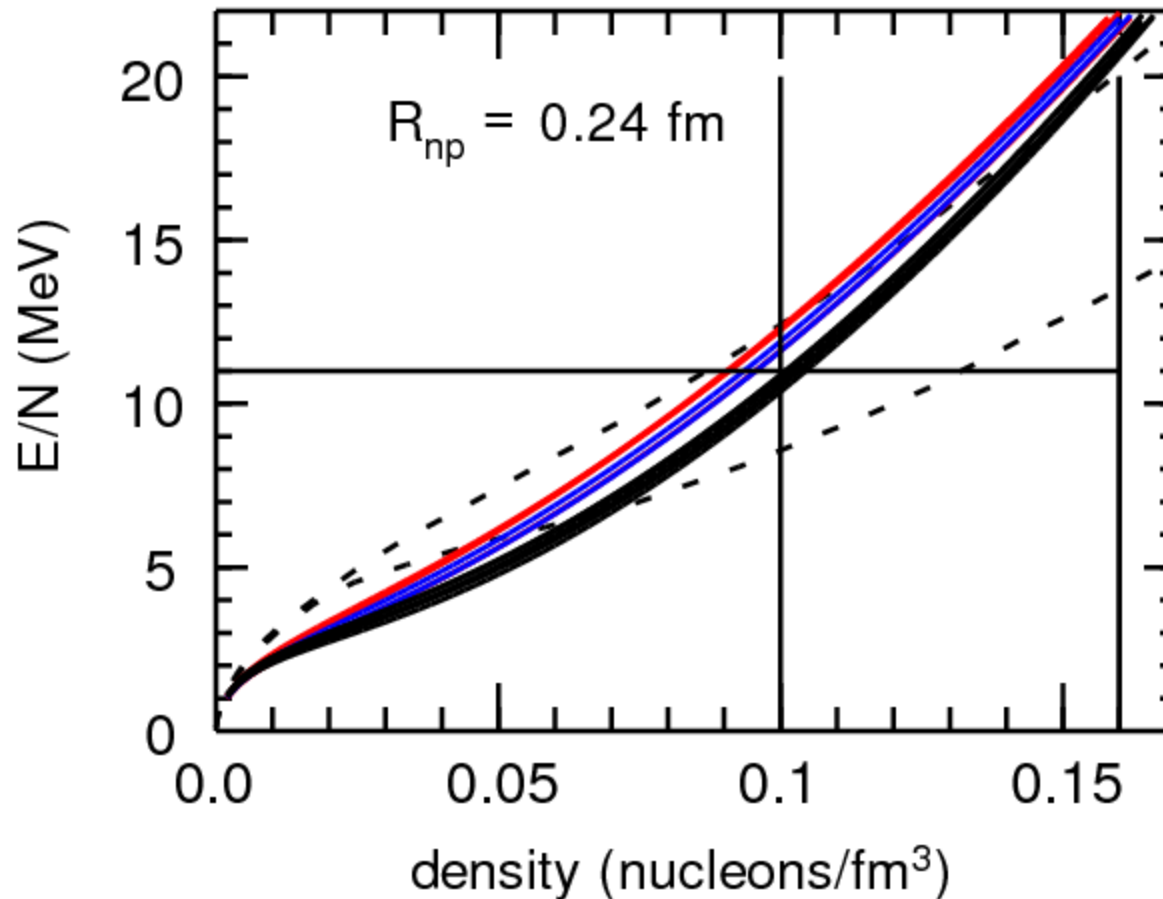




## Path B

Upper and lower values of the error band of calculations based on the Entem and Machleidt  $N^3\text{LO}$  NN potential with a cutoff at 500 MeV that include  $N^2\text{LO}$  NNN forces (the dashed lines).

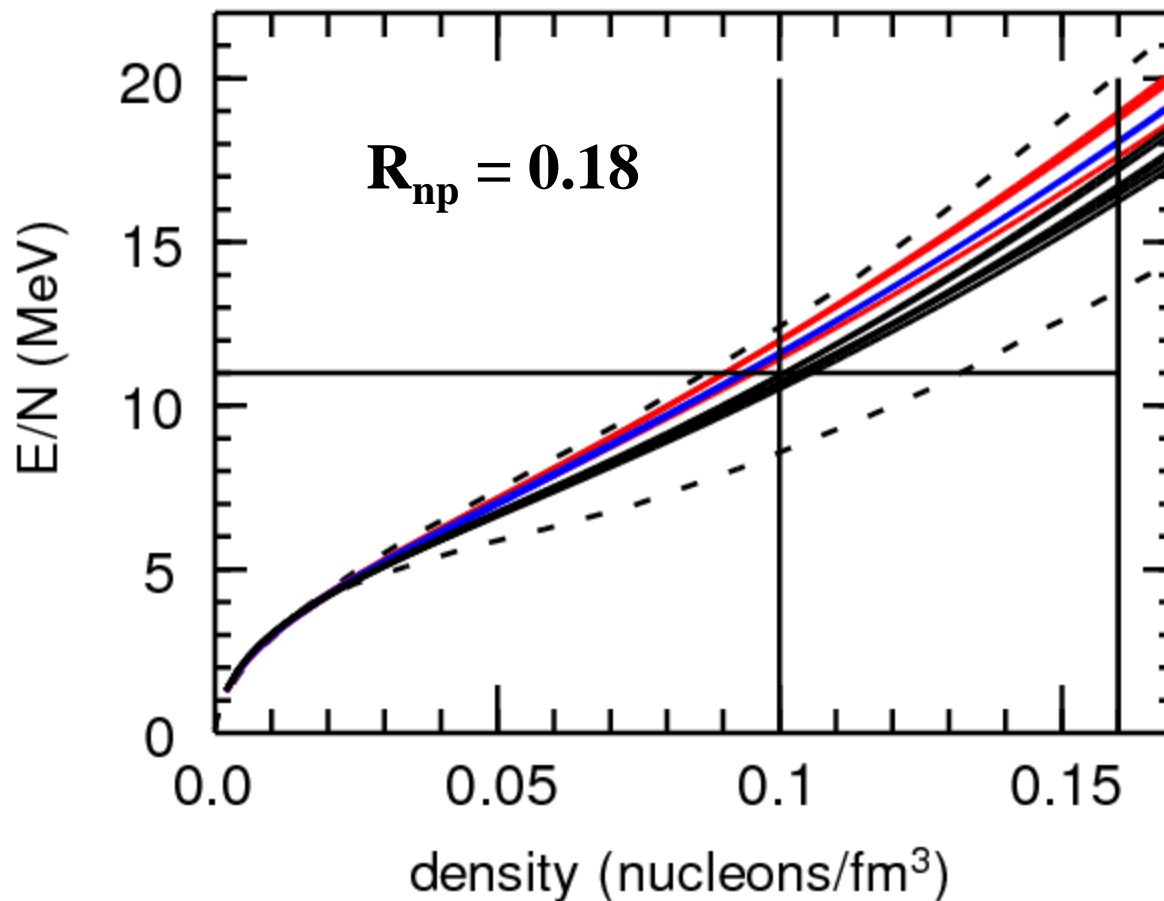
K. Hebeler and A. Schwenk, Phys. Rev. C **82**, 014314 (2010).



## Path B

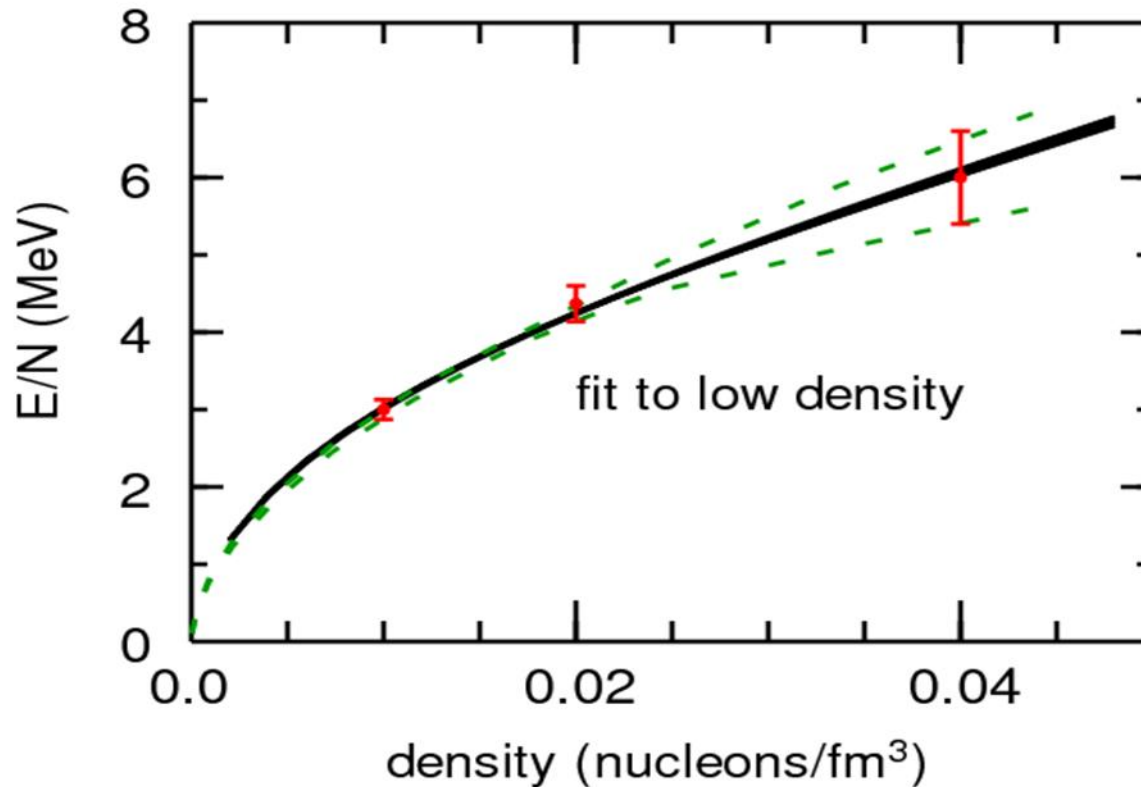
Upper and lower values of the error band of calculations based on the Entem and Machleidt  $N^3LO$  NN potential with a cutoff at 500 MeV that include  $N^2LO$  NNN forces (the dashed lines).

K. Hebeler and A. Schwenk, Phys. Rev. C **82**, 014314 (2010).



PHYSICAL REVIEW C **89**, 011307(R) (2014)

## Constraints on Skyrme equations of state from properties of doubly magic nuclei and *ab initio* calculations of low-density neutron matter

B. Alex Brown<sup>1</sup> and A. Schwenk<sup>2,3</sup>

## Path B

The low-density and nuclear fit was not sensitive to the neutron effective mass, and we assumed that  $[m_n^*/m]$  (at 0.16 nucleons/fm<sup>3</sup>) = 0.85

$J$ (MeV)	$L$ (MeV)	$K_s$ (MeV)	$R_{np}$ (fm) <sup>208</sup> Pb	$R_{np}$ (fm) <sup>48</sup> Ca
34.9	61	-130	0.192	0.172
35.1	61	-142	0.193	0.178
32.5	51	-138	0.176	0.170
32.8	54	-144	0.180	0.172
33.7	55	-139	0.181	0.172
33.3	56	-140	0.183	0.172
33.5	58	-135	0.186	0.174
32.7	53	-144	0.179	0.172
34.8	59	-148	0.190	0.176
32.3	51	-148	0.176	0.174
34.1	56	-145	0.184	0.174
34.2	58	-139	0.187	0.175
33.8(13)	56(5)	-138(8)	0.184(9)	0.174(4)

## Path B

The result is that we predict !

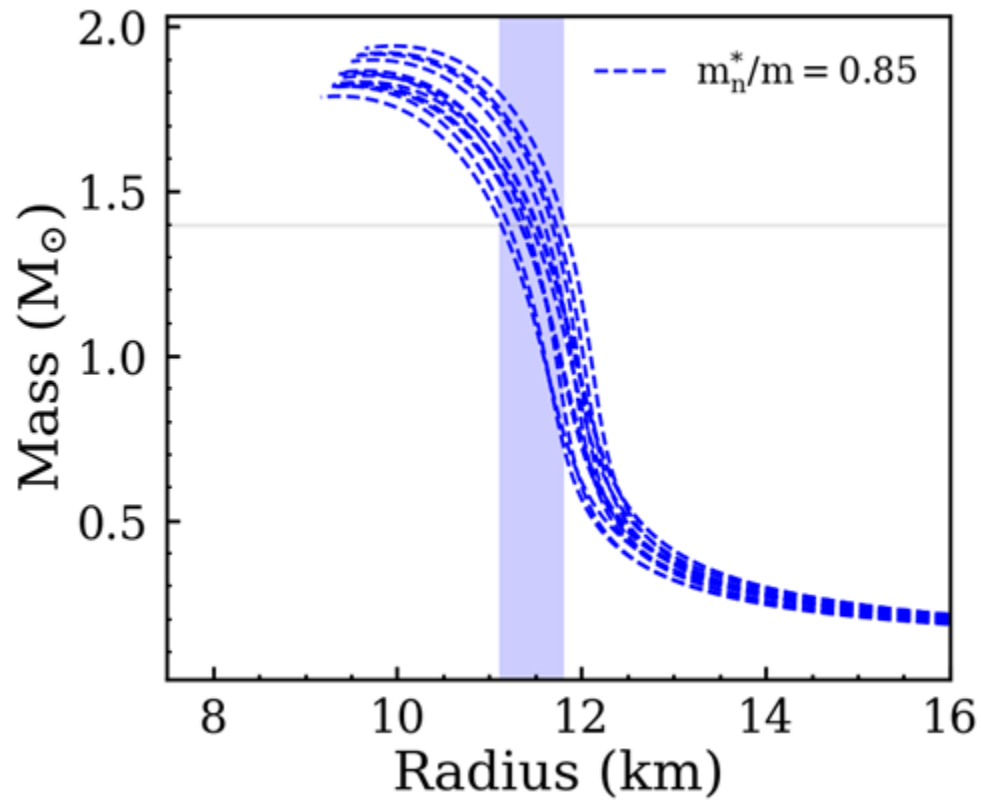
$$L_v = 56(5) \text{ MeV}$$

$$R_{np}(^{208}\text{Pb}) = 0.184(9) \text{ fm}$$

$$R_{np}(^{48}\text{Ca}) = 0.173(4) \text{ fm}$$

What if these turn out to be wrong? – then

- 1) The ab initio low-density neutron EOS is wrong
- 2) The Skyrme functional form is wrong

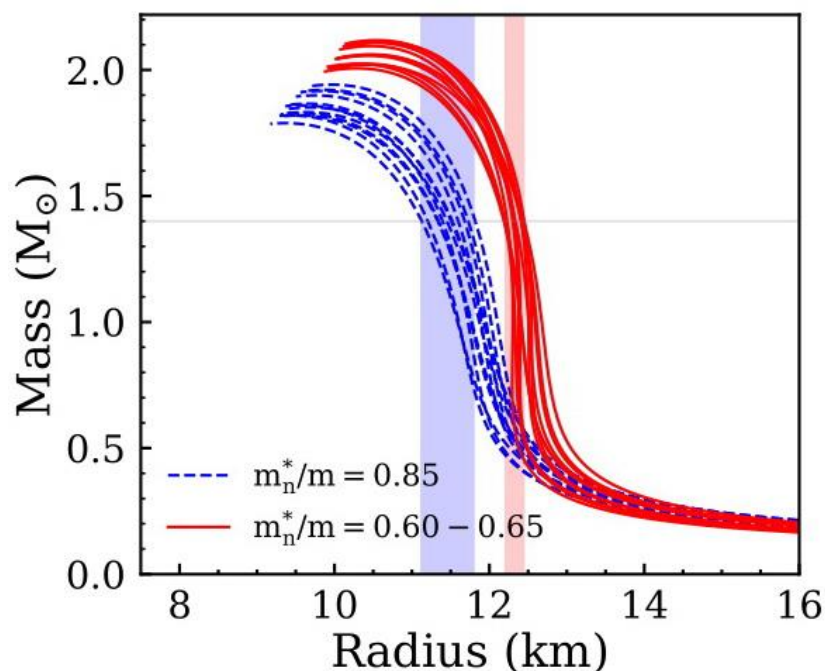


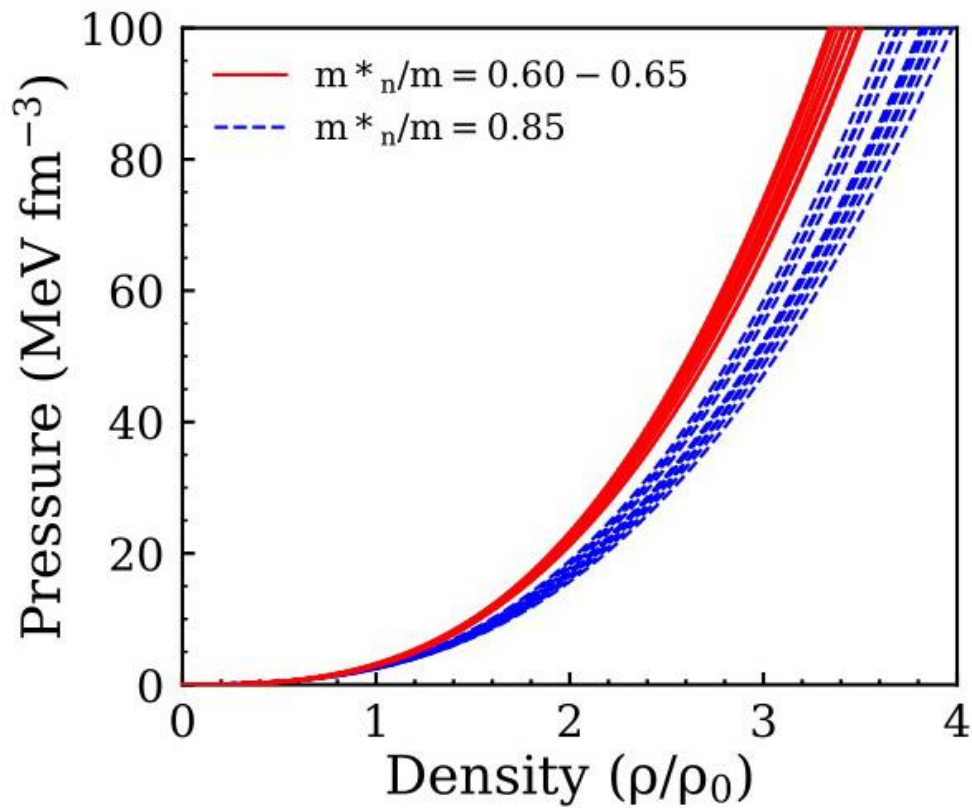
# Constraints on Skyrme Equations of State from Doubly Magic Nuclei, Ab-Initio Calculations of Low-Density Neutron Matter, and Neutron Stars

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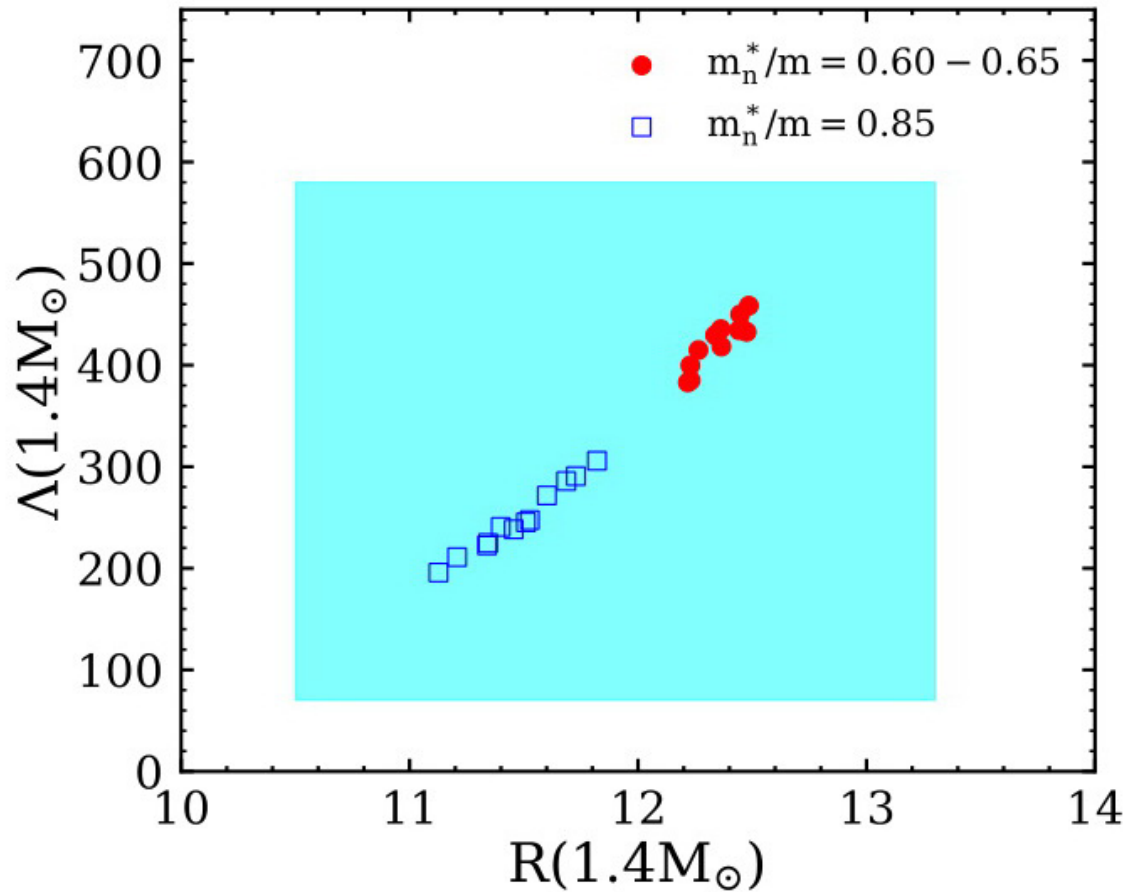
<sup>2</sup>*Department of Physics, Manhattan College, Riverdale, NY 10471, USA*







Path C



Blue area is the region allowed by the LIGO event GW170817

# Path C

Max neutron star mass requires  $[m_n^*/m]$  (at 0.16 nucleons/fm<sup>3</sup>) = 0.60-0.65

$J$ (MeV)	$L$ (MeV)	$K_s$ (MeV)	$R_{np}$ (fm) <sup>208</sup> Pb	$R_{np}$ (fm) <sup>48</sup> Ca
34.9	61	-130	0.192	0.172
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## Path C

Max neutron star mass requires  $[m_n^*/m]$  (at 0.16 nucleons/fm<sup>3</sup>) = 0.60-0.65

This is determined from nuclear matter high 3-5 times normal nuclear matter

Maybe for high density Skyrme EDF it is just mocking up new physics

But at least Skyrme EDF is capable of provided a smooth parameterization for

- 1) ab-initio low-density neutron matter
- 2) BE and radii of nuclei
- 3) Max mass of neutron stars