

Many-body correlations in light nuclei, from pairing to clustering

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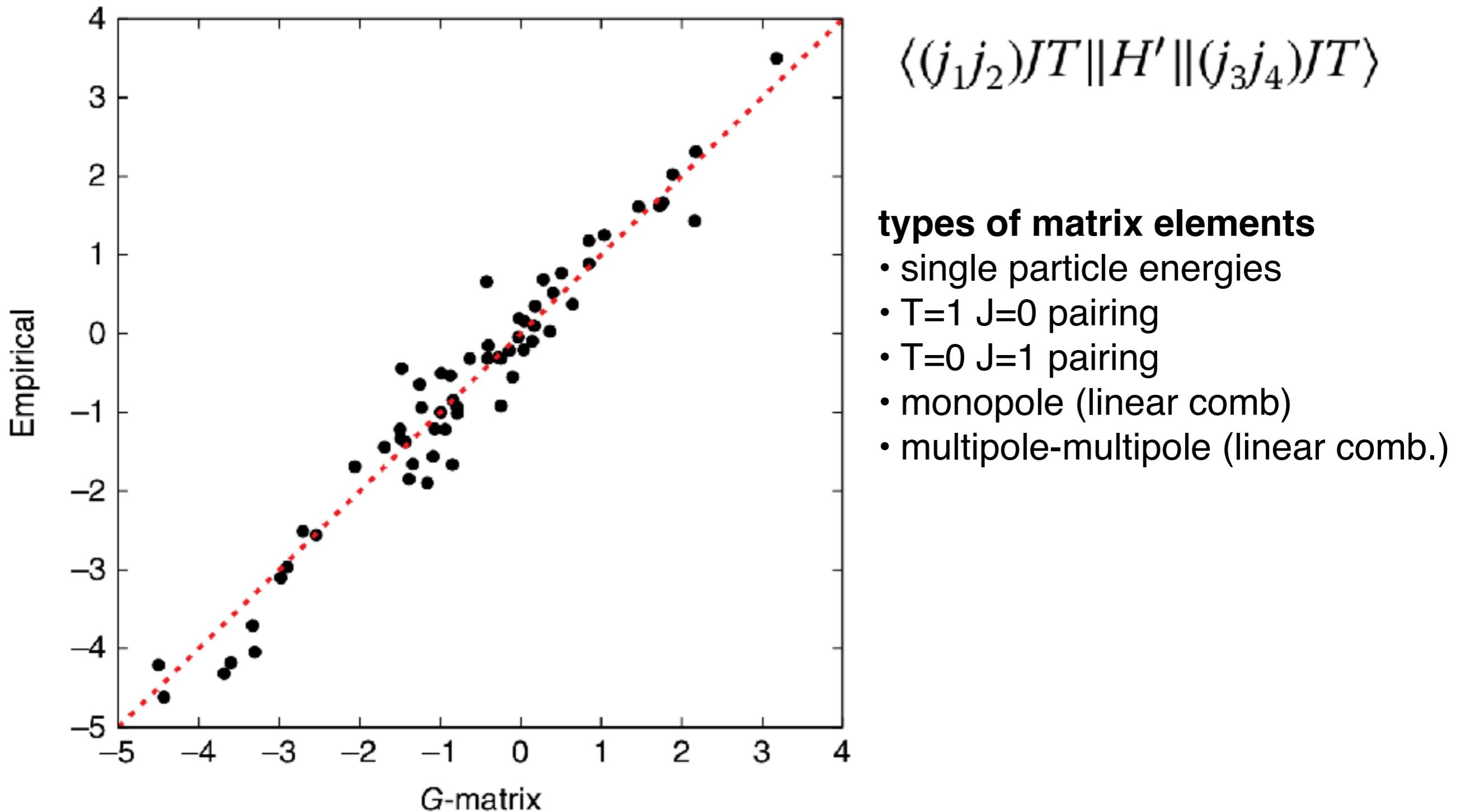
Questions

- How collective features emerge from NN interactions
- What is the nature of collectivities
- What is the role of pairing and clustering in nuclear structure
- What is the interplay between collectivities.
- How collective dynamics influences reactions

Methodology

- Configuration interaction approach.
- Consider sd valence space: 63 matrix elements+ 3 sp. energies
- Excellent agreement with experiment.
- Well connected with fundamental NN-force

sd-shell model matrix elements



Ground state energy and role of pairing

Symmetry+nn+pp pairing

											⁴⁰ Ca	
											³⁸ K ³⁹ K	
											³⁶ Ar ³⁷ Ar ³⁸ Ar	
						³⁴ Cl	³⁵ Cl	³⁶ Cl	³⁷ Cl			
						³² S	³³ S	³⁴ S	³⁵ S	³⁶ S		
					³⁰ P	³¹ P	³² P	³³ P	³⁴ P	³⁵ P		
				²⁸ Si	²⁹ Si	³⁰ Si	³¹ Si	³² Si	³³ Si	³⁴ Si		
			²⁶ Al	²⁷ Al	²⁸ Al	²⁹ Al	³⁰ Al	³¹ Al	³² Al	³³ Al		
		²⁴ Mg	²⁵ Mg	²⁶ Mg	²⁷ Mg	²⁸ Mg	²⁹ Mg	³⁰ Mg	³¹ Mg	³² Mg		
	²² Na	²³ Na	²⁴ Na	²⁵ Na	²⁶ Na	²⁷ Na	²⁸ Na	²⁹ Na	³⁰ Na	³¹ Na		
	²⁰ Ne	²¹ Ne	²² Ne	²³ Ne	²⁴ Ne	²⁵ Ne	²⁶ Ne	²⁷ Ne	²⁸ Ne	²⁹ Ne	³⁰ Ne	
¹⁸ F	¹⁹ F	²⁰ F	²¹ F	²² F	²³ F	²⁴ F	²⁵ F	²⁶ F	²⁷ F	²⁸ F	²⁹ F	
¹⁶ O	¹⁷ O	¹⁸ O	¹⁹ O	²⁰ O	²¹ O	²² O	²³ O	²⁴ O	²⁵ O	²⁶ O	²⁷ O	²⁸ O

Binding Energy Loss (MeV)
0.00000
1.38889
2.77778
4.16667
5.55556
6.94444
8.33333
9.72222
11.11111
12.50000

Isovector pairing

											⁴⁰ Ca
											³⁸ K
											³⁹ K
											³⁶ Ar
											³⁷ Ar
											³⁸ Ar
						³⁴ Cl	³⁵ Cl	³⁶ Cl	³⁷ Cl		
						³² S	³³ S	³⁴ S	³⁵ S	³⁶ S	
						³⁰ P	³¹ P	³² P	³³ P	³⁴ P	³⁵ P
						²⁸ Si	²⁹ Si	³⁰ Si	³¹ Si	³² Si	³³ Si
						²⁶ Al	²⁷ Al	²⁸ Al	²⁹ Al	³⁰ Al	³¹ Al
						²⁴ Mg	²⁵ Mg	²⁶ Mg	²⁷ Mg	²⁸ Mg	²⁹ Mg
						³⁰ Mg	³¹ Mg	³² Mg			
						²² Na	²³ Na	²⁴ Na	²⁵ Na	²⁶ Na	²⁷ Na
						²⁸ Na	²⁹ Na	³⁰ Na	³¹ Na		
						²⁰ Ne	²¹ Ne	²² Ne	²³ Ne	²⁴ Ne	²⁵ Ne
						²⁶ Ne	²⁷ Ne	²⁸ Ne	²⁹ Ne	³⁰ Ne	
						¹⁸ F	¹⁹ F	²⁰ F	²¹ F	²² F	²³ F
						²⁴ F	²⁵ F	²⁶ F	²⁷ F	²⁸ F	²⁹ F
						¹⁶ O	¹⁷ O	¹⁸ O	¹⁹ O	²⁰ O	²¹ O
						²² O	²³ O	²⁴ O	²⁵ O	²⁶ O	²⁷ O
						²⁸ O					

Energy Gain (MeV)
0.00000
1.16667
2.33333
3.50000
4.66667
5.83333
7.00000
8.16667
9.33333
10.50000

Isovector+isoscalar pairing

																⁴⁰ Ca
																³⁸ K ³⁹ K
																³⁶ Ar ³⁷ Ar ³⁸ Ar
																³⁴ Cl ³⁵ Cl ³⁶ Cl ³⁷ Cl
								³² S	³³ S	³⁴ S	³⁵ S	³⁶ S				
						³⁰ P	³¹ P	³² P	³³ P	³⁴ P	³⁵ P					
					²⁸ Si	²⁹ Si	³⁰ Si	³¹ Si	³² Si	³³ Si	³⁴ Si					
				²⁶ Al	²⁷ Al	²⁸ Al	²⁹ Al	³⁰ Al	³¹ Al	³² Al	³³ Al					
		²⁴ Mg	²⁵ Mg	²⁶ Mg	²⁷ Mg	²⁸ Mg	²⁹ Mg	³⁰ Mg	³¹ Mg	³² Mg						
	²² Na	²³ Na	²⁴ Na	²⁵ Na	²⁶ Na	²⁷ Na	²⁸ Na	²⁹ Na	³⁰ Na	³¹ Na						
²⁰ Ne	²¹ Ne	²² Ne	²³ Ne	²⁴ Ne	²⁵ Ne	²⁶ Ne	²⁷ Ne	²⁸ Ne	²⁹ Ne	³⁰ Ne						
¹⁸ F	¹⁹ F	²⁰ F	²¹ F	²² F	²³ F	²⁴ F	²⁵ F	²⁶ F	²⁷ F	²⁸ F	²⁹ F					
¹⁶ O	¹⁷ O	¹⁸ O	¹⁹ O	²⁰ O	²¹ O	²² O	²³ O	²⁴ O	²⁵ O	²⁶ O	²⁷ O	²⁸ O				

Energy Gain (MeV)
0.00000
1.07222
2.14444
3.21667
4.28889
5.36111
6.43333
7.50556
8.57778
9.65000

nn and pp pairing broken symmetry

															⁴⁰ Ca
															³⁸ K
															³⁹ K
															³⁶ Ar
															³⁷ Ar
															³⁸ Ar
															³⁴ Cl
															³⁵ Cl
															³⁶ Cl
															³⁷ Cl
															³² S
															³³ S
															³⁴ S
															³⁵ S
															³⁶ S
															³⁰ P
															³¹ P
															³² P
															³³ P
															³⁴ P
															³⁵ P
															²⁸ Si
															²⁹ Si
															³⁰ Si
															³¹ Si
															³² Si
															³³ Si
															³⁴ Si
															²⁶ Al
															²⁷ Al
															²⁸ Al
															²⁹ Al
															³⁰ Al
															³¹ Al
															³² Al
															³³ Al
															²⁴ Mg
															²⁵ Mg
															²⁶ Mg
															²⁷ Mg
															²⁸ Mg
															²⁹ Mg
															³⁰ Mg
															³¹ Mg
															³² Mg
															²² Na
															²³ Na
															²⁴ Na
															²⁵ Na
															²⁶ Na
															²⁷ Na
															²⁸ Na
															²⁹ Na
															³⁰ Na
															³¹ Na
															²⁰ Ne
															²¹ Ne
															²² Ne
															²³ Ne
															²⁴ Ne
															²⁵ Ne
															²⁶ Ne
															²⁷ Ne
															²⁸ Ne
															²⁹ Ne
															³⁰ Ne
															¹⁸ F
															¹⁹ F
															²⁰ F
															²¹ F
															²² F
															²³ F
															²⁴ F
															²⁵ F
															²⁶ F
															²⁷ F
															²⁸ F
															²⁹ F
															¹⁶ O
															¹⁷ O
															¹⁸ O
															¹⁹ O
															²⁰ O
															²¹ O
															²² O
															²³ O
															²⁴ O
															²⁵ O
															²⁶ O
															²⁷ O
															²⁸ O

Binding Energy Loss (MeV)
0.00000
0.61667
1.23333
1.85000
2.46667
3.08333
3.70000
4.31667
4.93333
5.55000

full, time conjugate variational

															⁴⁰ Ca
															³⁸ K
															³⁹ K
															³⁶ Ar
															³⁷ Ar
															³⁸ Ar
							³⁴ Cl	³⁵ Cl	³⁶ Cl	³⁷ Cl					
							³² S	³³ S	³⁴ S	³⁵ S	³⁶ S				
							³⁰ P	³¹ P	³² P	³³ P	³⁴ P	³⁵ P			
							²⁸ Si	²⁹ Si	³⁰ Si	³¹ Si	³² Si	³³ Si	³⁴ Si		
							²⁶ Al	²⁷ Al	²⁸ Al	²⁹ Al	³⁰ Al	³¹ Al	³² Al	³³ Al	
							²⁴ Mg	²⁵ Mg	²⁶ Mg	²⁷ Mg	²⁸ Mg	²⁹ Mg	³⁰ Mg	³¹ Mg	³² Mg
							²² Na	²³ Na	²⁴ Na	²⁵ Na	²⁶ Na	²⁷ Na	²⁸ Na	²⁹ Na	³⁰ Na
							²⁰ Ne	²¹ Ne	²² Ne	²³ Ne	²⁴ Ne	²⁵ Ne	²⁶ Ne	²⁷ Ne	²⁸ Ne
							¹⁸ F	¹⁹ F	²⁰ F	²¹ F	²² F	²³ F	²⁴ F	²⁵ F	²⁶ F
							¹⁶ O	¹⁷ O	¹⁸ O	¹⁹ O	²⁰ O	²¹ O	²² O	²³ O	²⁴ O
							²⁵ O	²⁶ O	²⁷ O	²⁸ O					

Binding Energy Loss (MeV)
0.00000
0.38333
0.76667
1.15000
1.53333
1.91667
2.30000
2.68333
3.06667
3.45000

Sensitivity of wave functions to two-body matrix elements

Invariant Correlational Entropy sensitivity of wave functions to matrix elements

- Averaged density matrix $|\alpha(\lambda)\rangle = \sum \langle \alpha(\lambda)|k\rangle |k\rangle$

$$\rho_{kk'}^\alpha = \langle k|\alpha\rangle\langle\alpha|k\rangle \quad \overline{\rho_{kk'}^\alpha} = \frac{1}{\delta\lambda} \int_\lambda^{\lambda+\delta\lambda} \rho_{kk'}^\alpha d\lambda'$$

- ICE

$$I^\alpha(\lambda) = -\text{Tr} [\overline{\rho^\alpha} \ln (\overline{\rho^\alpha})]$$

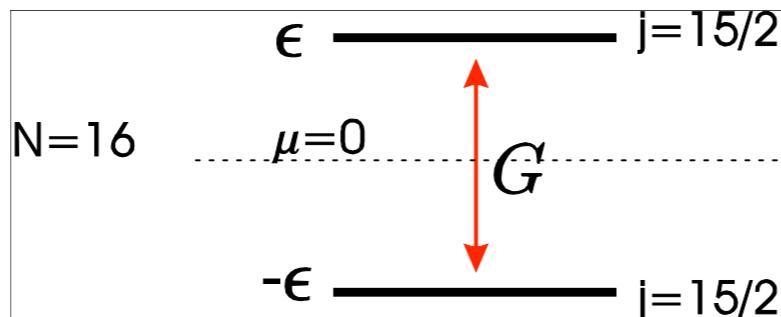
Advantages

- Basis independent
- Explore individual quantum states
- Needs no heat bath
- No equilibration, thermalization and particle number conservation issues.
- Probe sensitivity of states to noise in external parameter(s)
- **Phase transitions -> peaks in ICE**

Pairing phase transitions

- ★ Phase transitions in small systems
- ★ Phase transitions in individual quantum states
- ★ Thermodynamics

Two-level system



BCS solution

$$V_{\text{critical}} = \frac{4\epsilon}{\Omega} = 0.25$$

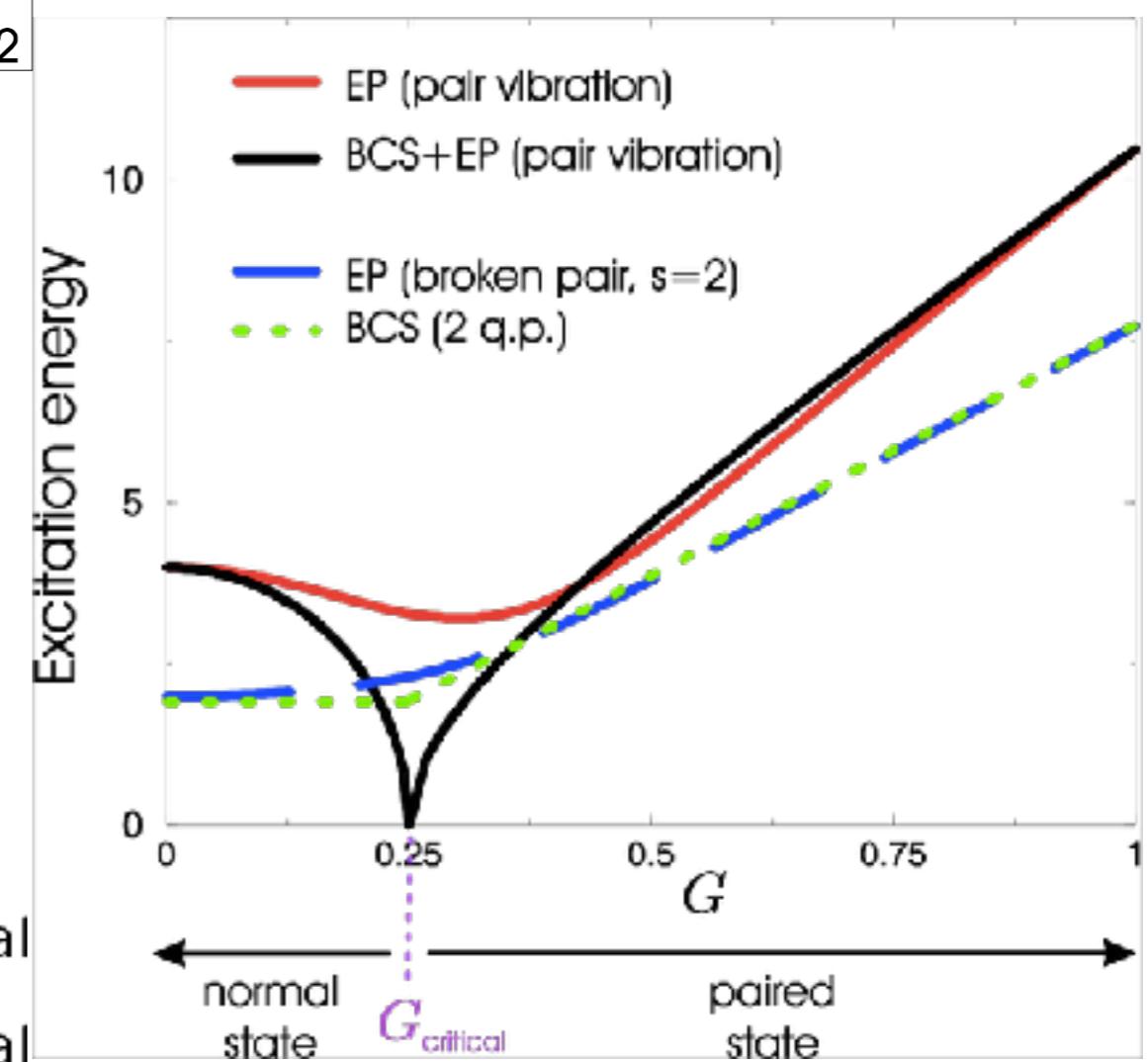
$$e_1 = e_2 = \frac{G\Omega}{4}, \quad \Delta^2 = \frac{G^2\Omega^2}{16} - \epsilon^2$$

Quasiparticle excitations

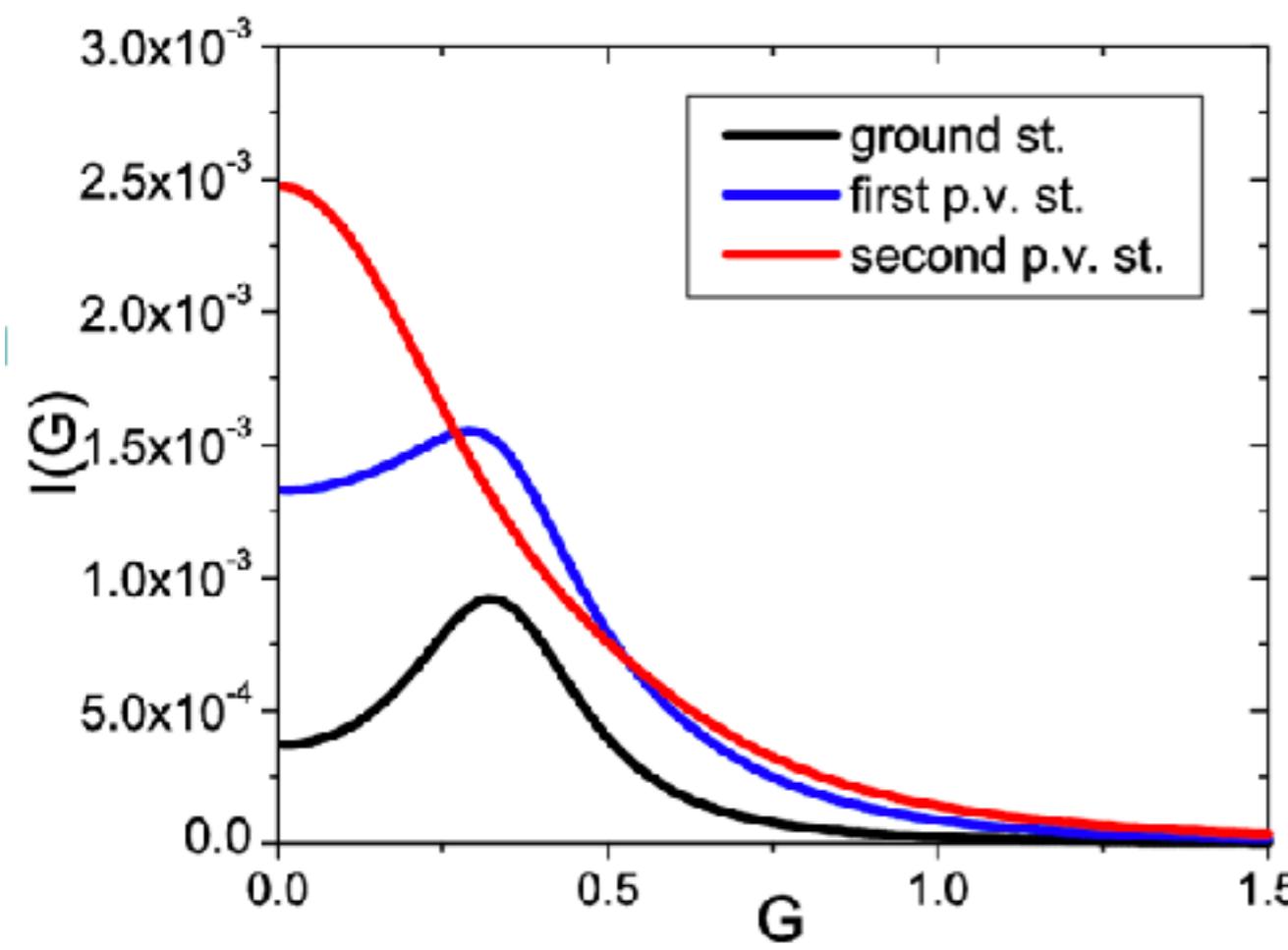
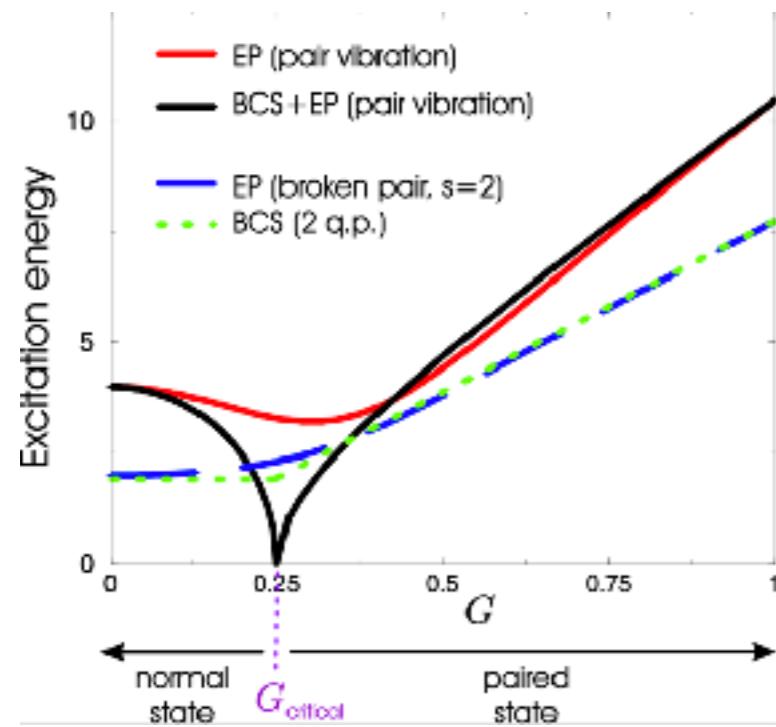
$$E_{s=2} = \begin{cases} 2e & G > G_{\text{critical}} \\ 2\epsilon & G < G_{\text{critical}} \end{cases}$$

Pair vibrations:

$$E_{\text{ex}}^2 = \begin{cases} \frac{G^2\Omega^2}{2} - 8\epsilon^2 = 8\Delta^2 & G > G_{\text{critical}} \\ 4\epsilon^2 - \frac{V^2\Omega^2}{4} & G < G_{\text{critical}} \end{cases}$$

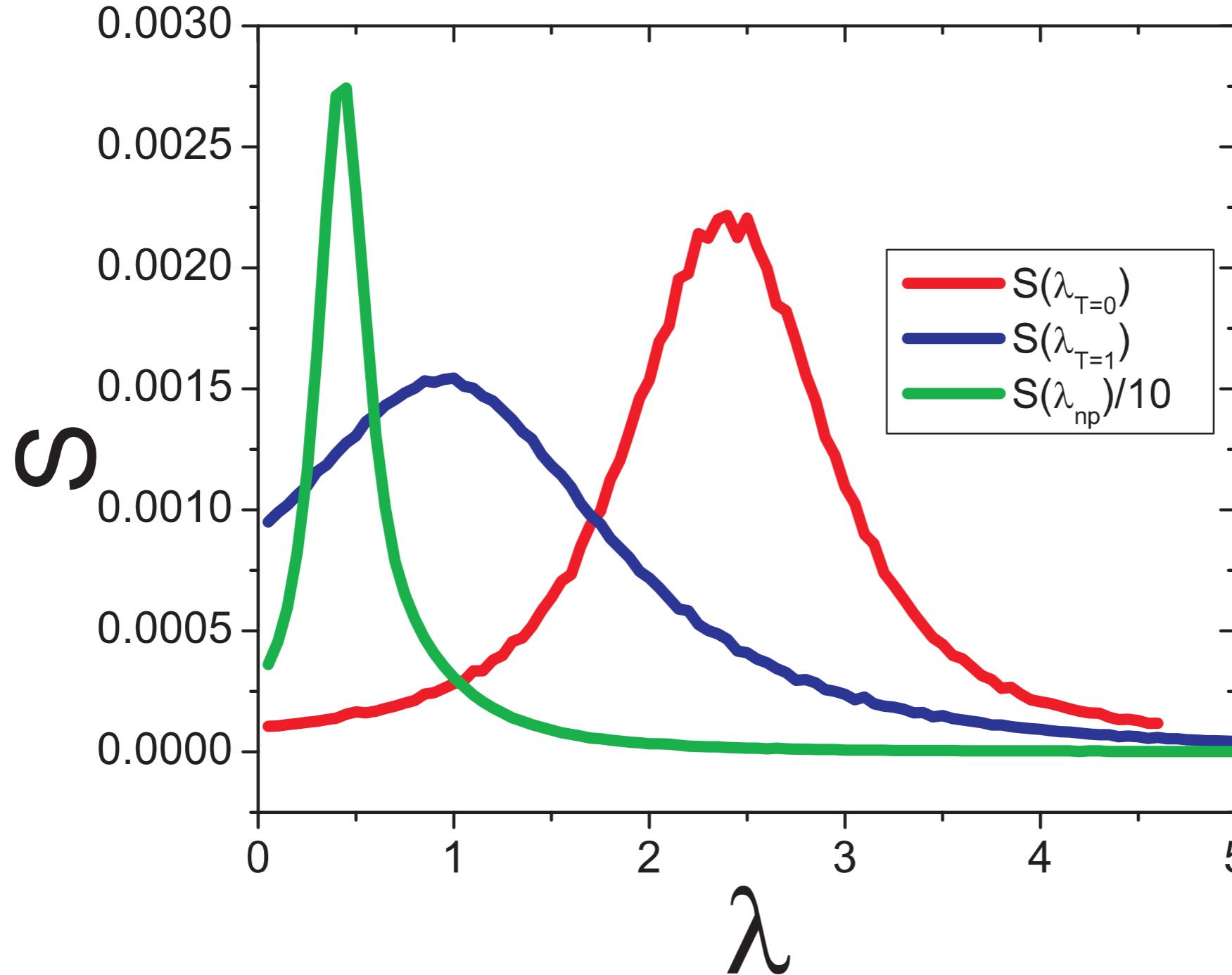


Is there a pairing phase transition in mesoscopic system?

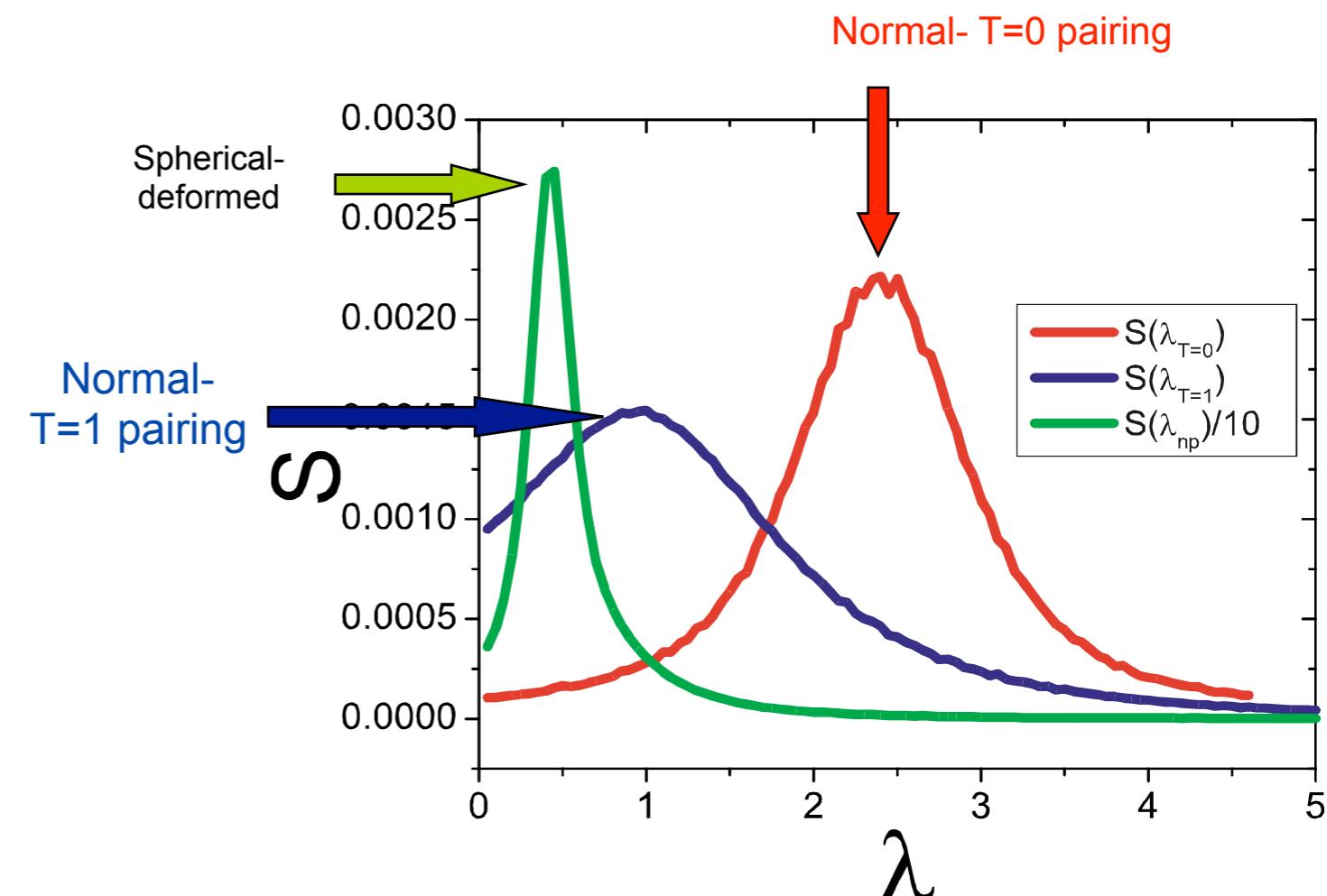
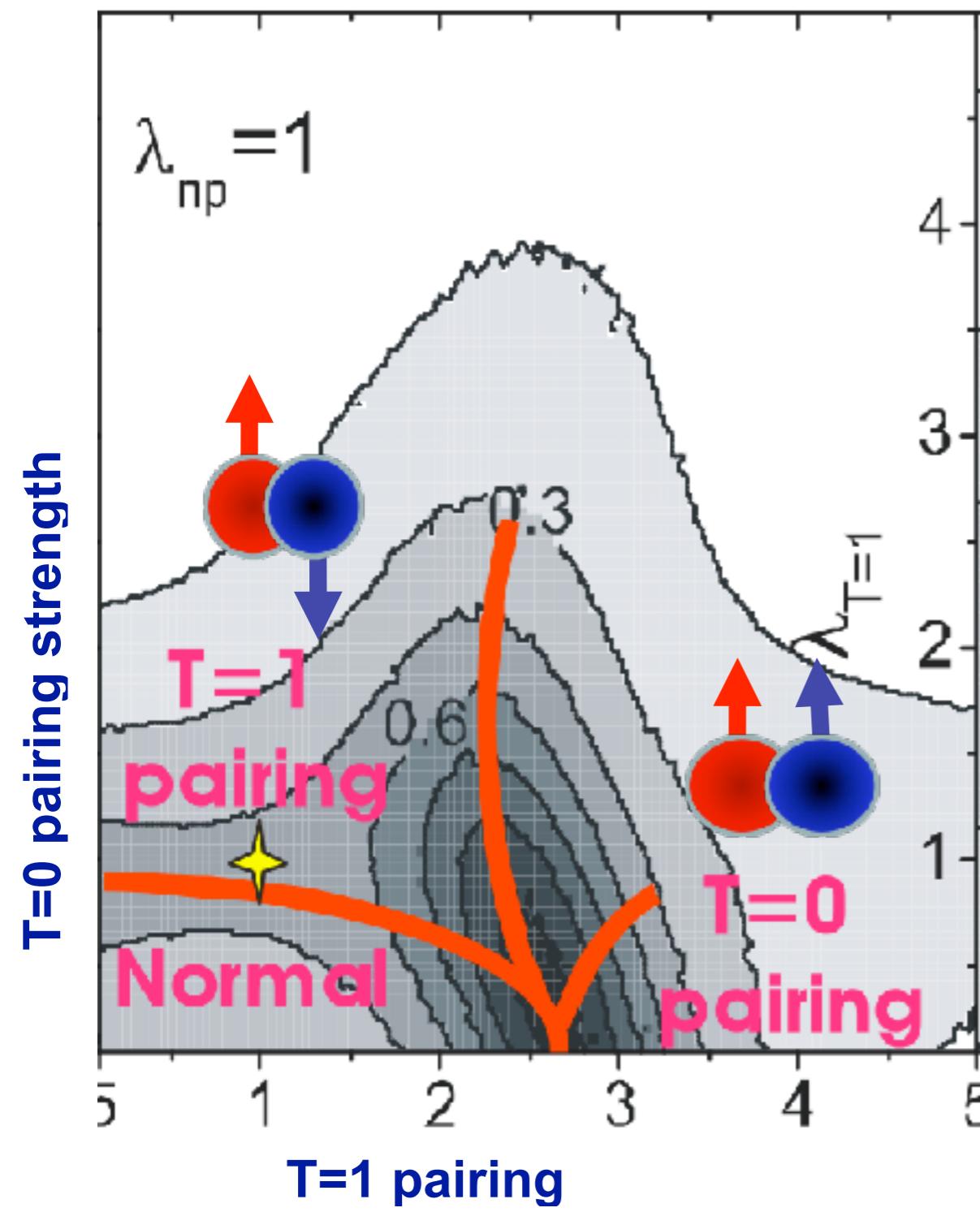


Invariant entropy

^{24}Mg phase transitions



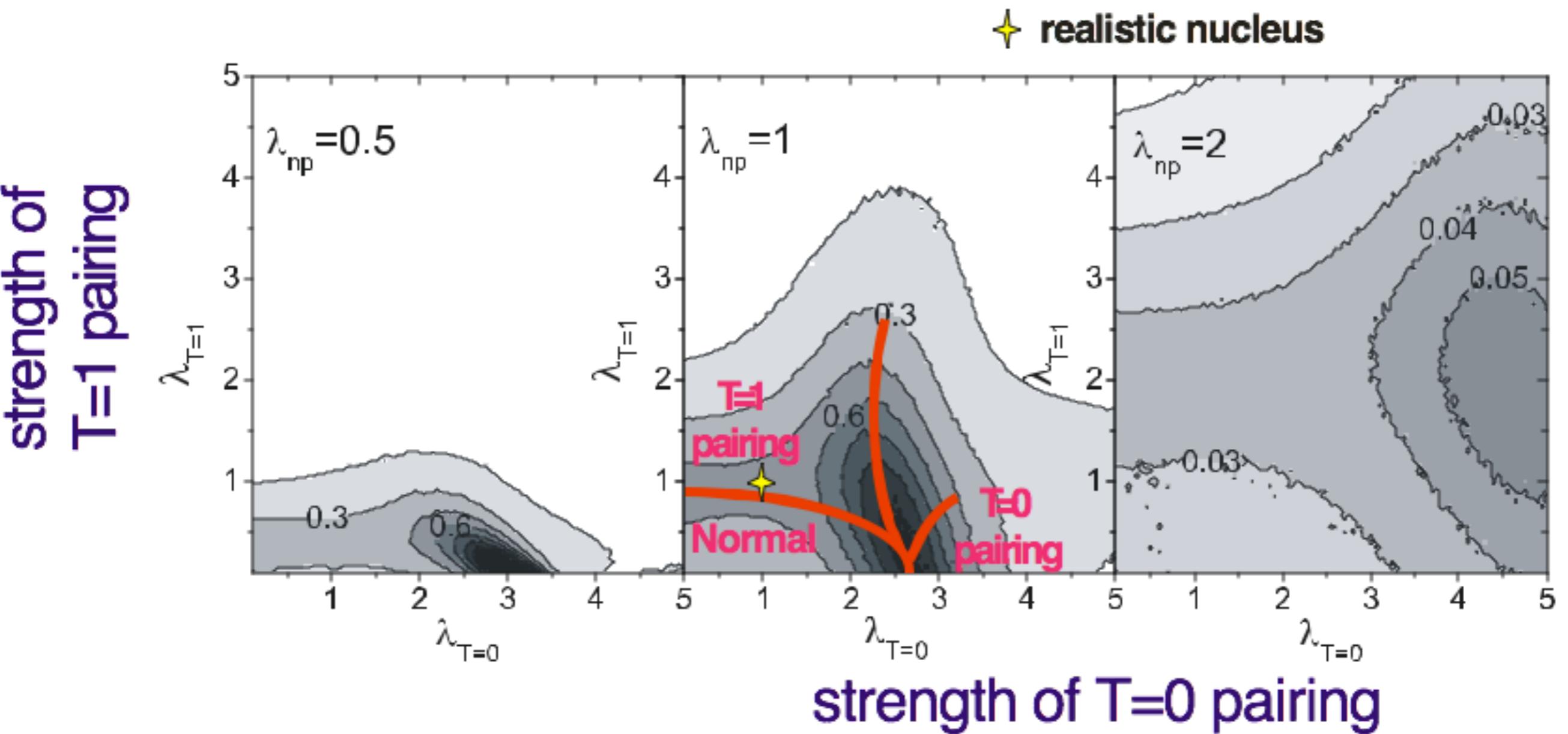
Pairing modes



Pairing modes in realistic ^{24}Mg

A. Volya, Physics Letters B 574 (2003) 27–34

^{24}Mg phase diagram

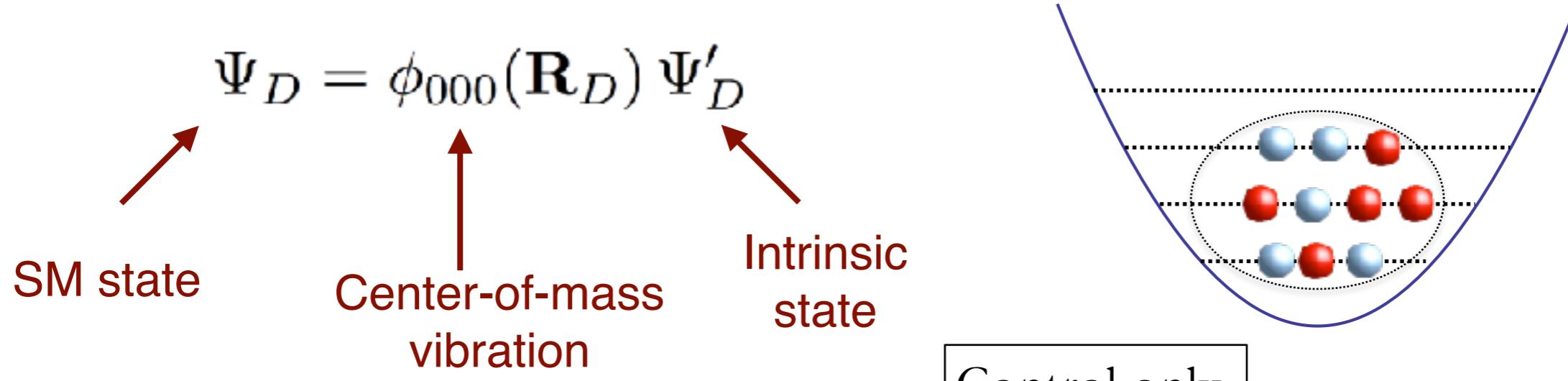


Contour plot of invariant correlational entropy showing a phase diagram as a function of $T=1$ pairing ($\lambda_{T=1}$) and $T=0$ pairing ($\lambda_{T=0}$); three plots indicate phase diagram as a function of non-pairing matrix elements (λ_{np}). Realistic case is $\lambda_{T=1}=\lambda_{T=0}=\lambda_{np}=1$

Quartets and real alphas

Translational invariance and Center of Mass (CM)

Shell model, Glockner-Lawson procedure



Controlling CM

$$D_\mu = \sqrt{\frac{4\pi}{3}} R_\mu$$

$$R_\mu = \sqrt{\frac{\hbar}{2Am\omega}} (\mathcal{B}_\mu^\dagger + \mathcal{B}_\mu)$$

Control only
CM quanta

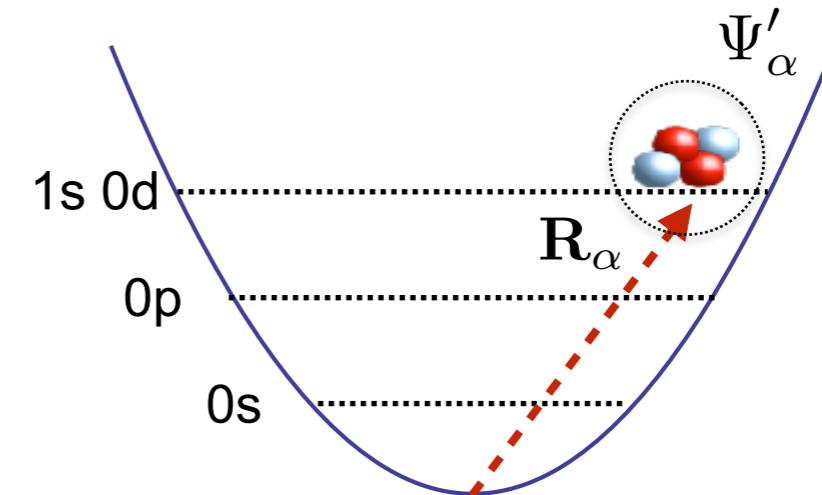
Center-of-Mass boosts

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R}) \Psi'$$

\mathcal{B}^\dagger and \mathcal{B} CM quanta creation and annihilation (vectors)

$$\Psi_{n+1\ell m} \propto \mathcal{B}^\dagger \cdot \mathcal{B}^\dagger \Psi_{n\ell m}$$

$\mathcal{B}^\dagger \times \mathcal{B}$ CM angular momentum operator

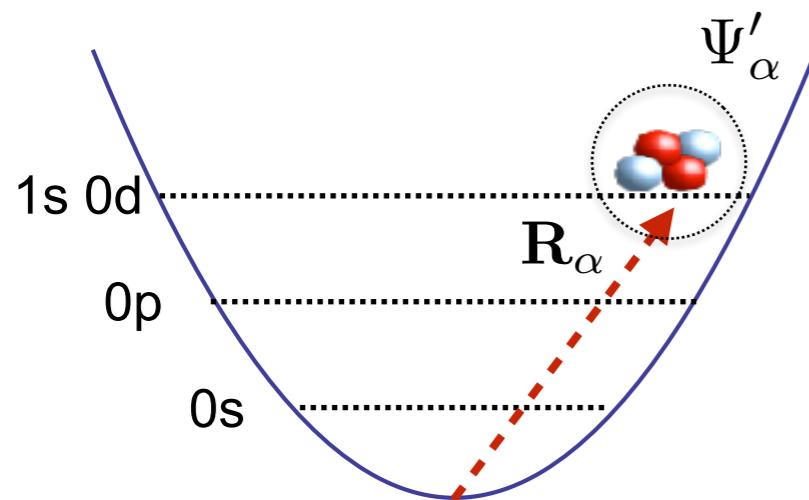


$$N = 2n + \ell$$

	$\alpha[0]$	$\alpha[4]$
Configuration	$N_{\max} = 0$	$N_{\max} = 4$
$(sd)^4$	0.038	0.035
$(p)(sd)^2(pf)$	0.308	0.282
$(p)^2(pf)^2$	0.103	0.094
$(p)^2(sd)(sdg)$	0.154	0.141
$(s)^2(sd)(sdgi)$	0.000	0.005
$(p)(sd)(pf)(sdg)$	0.000	0.009

Select configuration content of NCSM wave functions for ${}^4\text{He}$ with $\Omega = 20$ MeV boosted by 8 quanta ($L = 0$).

Approximation of $N_{\max}=0$ (s^4) Cluster coefficients for SU(3) components



Expand SU(3) 4-nucleon structure in intrinsic+ relative
all oscillator quanta of excitation are in relative motion.

$$\phi_{n\ell m}(\mathbf{R}_\alpha)\Psi'_\alpha = \sum_{\eta} X_{n\ell}^{\eta} \Phi_{(n,0):\ell m}^{\eta}$$

$$X_{n\ell}^{\eta} \equiv \langle \Phi_{(n,0):\ell m}^{\eta} | \phi_{n\ell m}(\mathbf{R}_\alpha) \Psi'_\alpha \rangle = \sqrt{\frac{1}{4^n} \frac{n!}{\prod_i (n_i!)^{\alpha_i}} \frac{4!}{\prod_i \alpha_i!}}$$

Volya and Yu. M. Tchuvil'sky, Phys. Rev. C 91, 044319 (2015).

Yu. F. Smirnov and Yu. M. Tchuvil'sky, Phys. Rev. C 15, 84 (1977).

M. Ichimura, A. Arima, E. C. Halbert, and T. Terasawa, Nucl. Phys. A 204, 225 (1973).

O. F. Nemetz, V. G. Neudatchin, A. T. Rudchik, Yu. F. Smirnov, and Yu. M. Tchuvil'sky, Nucleon Clusters in Atomic Nuclei and Multi-Nucleon Transfer Reactions (Naukova Dumka, Kiev, 1988), p. 295.

Cluster configurations

Example: alpha decay with $\ell=0$ from sd shell

21 way to make L=0 T=0 4-nucleon combination

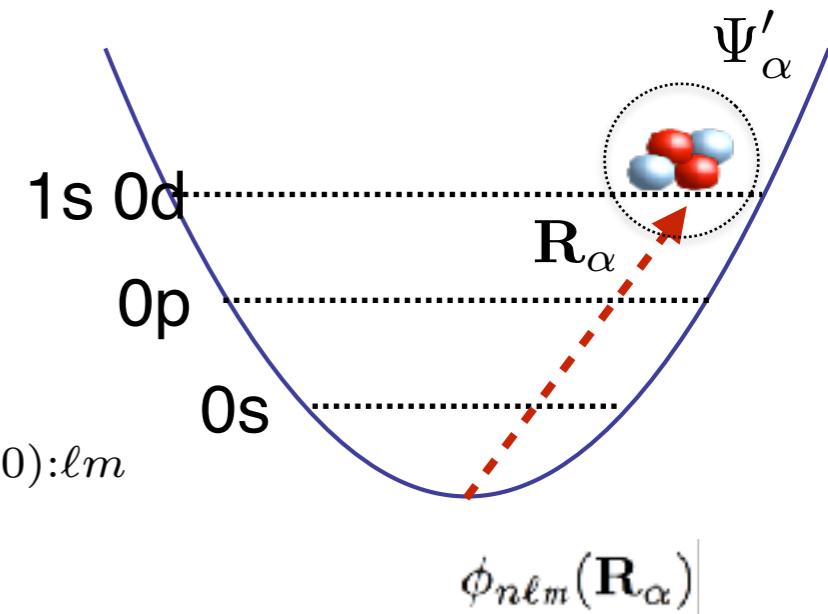
Each nucleon has 2 oscillator quanta, 8 quanta total

In oscillator basis excitation quanta are conserved

We model alpha as 4-nucleons on s-shell $(0s)^4$

Make single SU(3) operator with quantum numbers $(8,0)$ $\Phi_{(8,0):\ell m}^\eta$

Cluster coefficient is known analytically $X_{n'\ell}^\eta$



$$\underbrace{\phi_{n\ell m}(1)\phi_{n\ell m}(2)\phi_{n\ell m}(3)\phi_{n\ell m}(4)}_{\substack{4 \times 2 = 8 \text{ quanta} \\ \text{m-scheme state}}} \leftrightarrow \sum_\eta X_{n'\ell}^\eta \Phi_{(8,0):\ell m}^\eta \underset{\substack{\text{SU(3) symmetry state}}}{=} \underbrace{\phi_{n'\ell' m'}(\mathbf{R}_\alpha)}_{\substack{8 \text{ quanta} \\ \text{motion of alpha}}} \underbrace{\Psi'_\alpha}_{0 \text{ quanta}}$$

Volya and Yu. M. Tchuvil'sky, Phys. Rev. C 91, 044319 (2015).

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Center-of-Mass boosts

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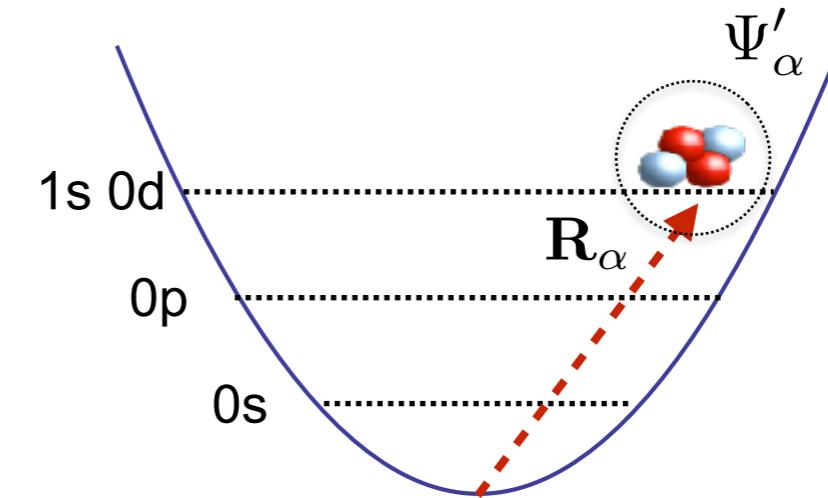
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Configuration	$N_{\max} = 0$	$N_{\max} = 4$
$(sd)^4$	0.038	0.035
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$(p)(sd)(pf)(sdg)$	0.000	0.009



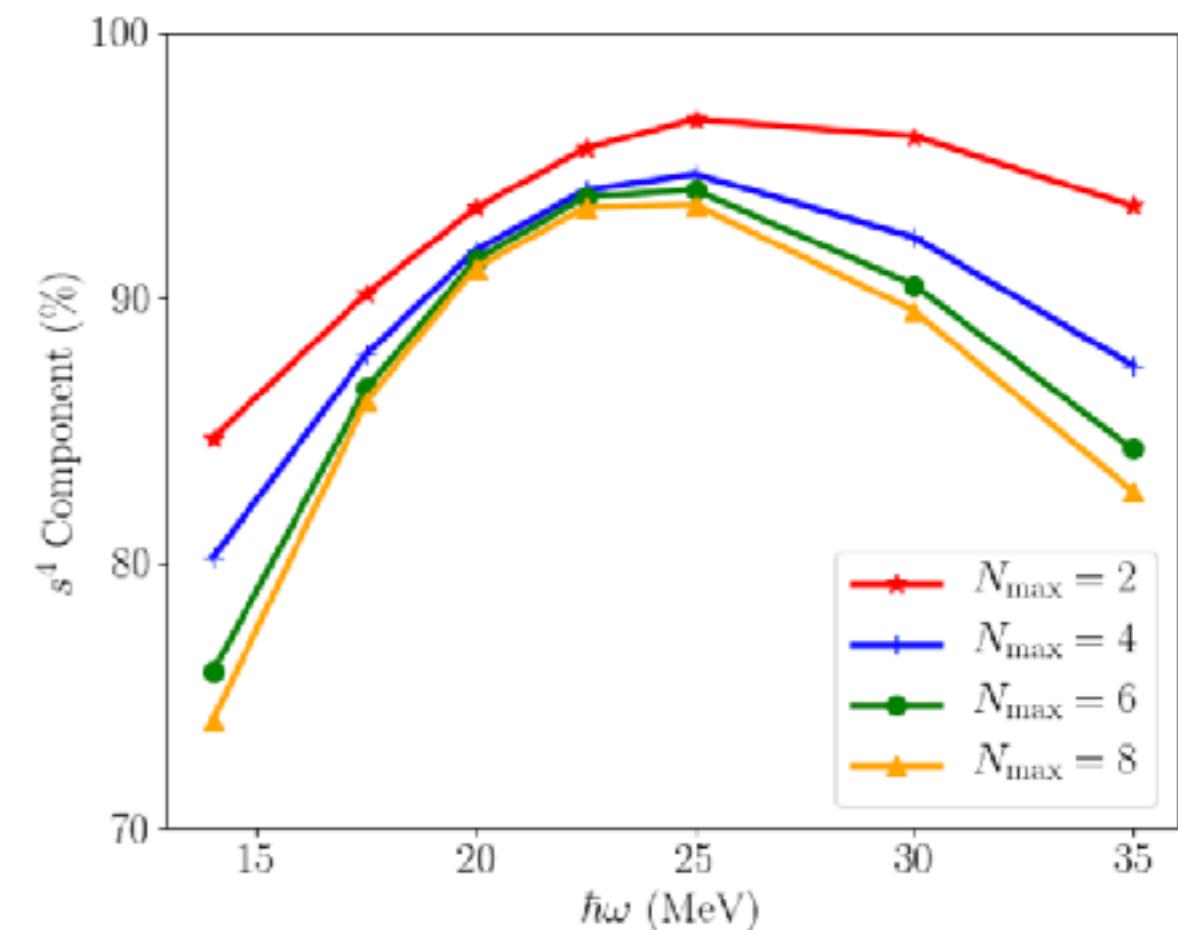
Select configuration content of NCSM wave functions for ${}^4\text{He}$ with $\Omega = 20$ MeV boosted by 8 quanta ($L = 0$).

Quartet that corresponds to alpha cluster $\ell = 0$

n	X^2	(8,0)	(4,2)	(0,4)	(2,0)
4	0.02848	1.0	0.0	0.0	0.0
3	0.00697	0.561658	0.438338	0.0	0.0
2	0.00169	0.549804	0.0451847	0.3363	0.0636439
1	0.00018	0.0693304	0.735878	0.0134005	0.147418
0	0.00011	0.0693304	0.261291	0.0990471	0.0384533

$L = S = T = 0$ $(\lambda, \mu) = (8,0), (4,2), (0,4), \text{ or } (2,0)$

$\hbar\omega = 14 \text{ MeV}$

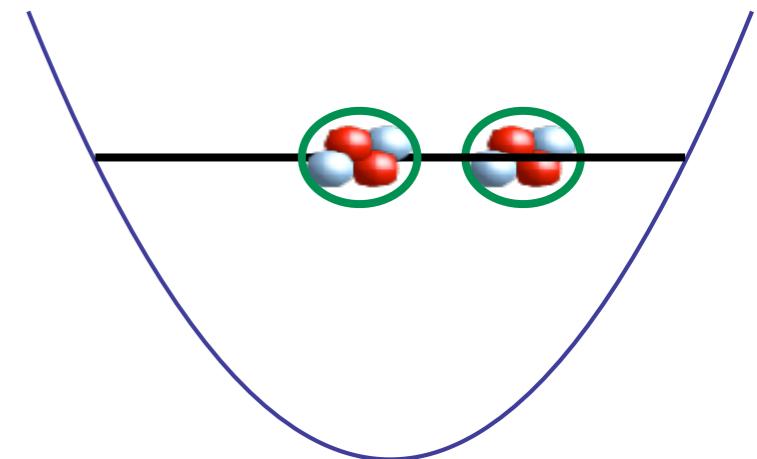


Bosonic nature of 4-nucleon operators non-orthogonality

If Φ^\dagger is thought of as being a boson then $\Phi\Phi^\dagger = 1 + N_b$

$$|\Psi_D\rangle = |\Phi\rangle \quad \langle \Phi_D | \hat{\Phi} \hat{\Phi}^\dagger | \Psi_D \rangle = \langle 0 | \hat{\Phi} \hat{\Phi} \hat{\Phi}^\dagger \hat{\Phi}^\dagger | 0 \rangle = 2$$

$$L = S = T = 0$$

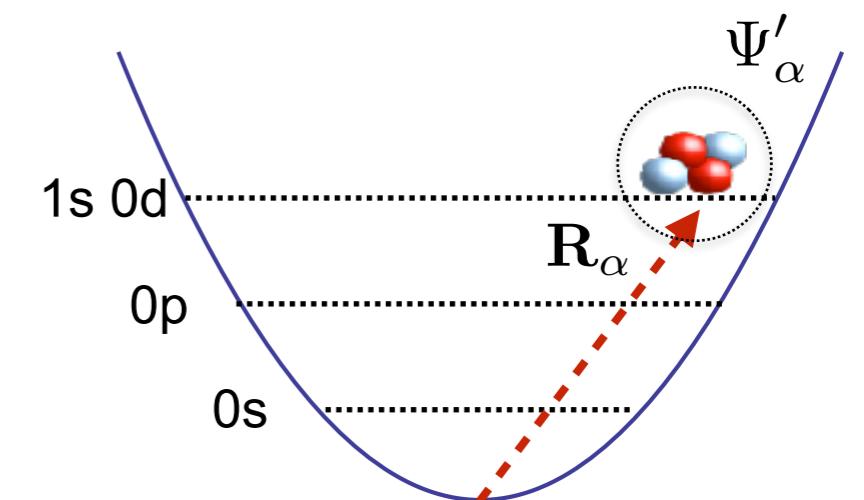
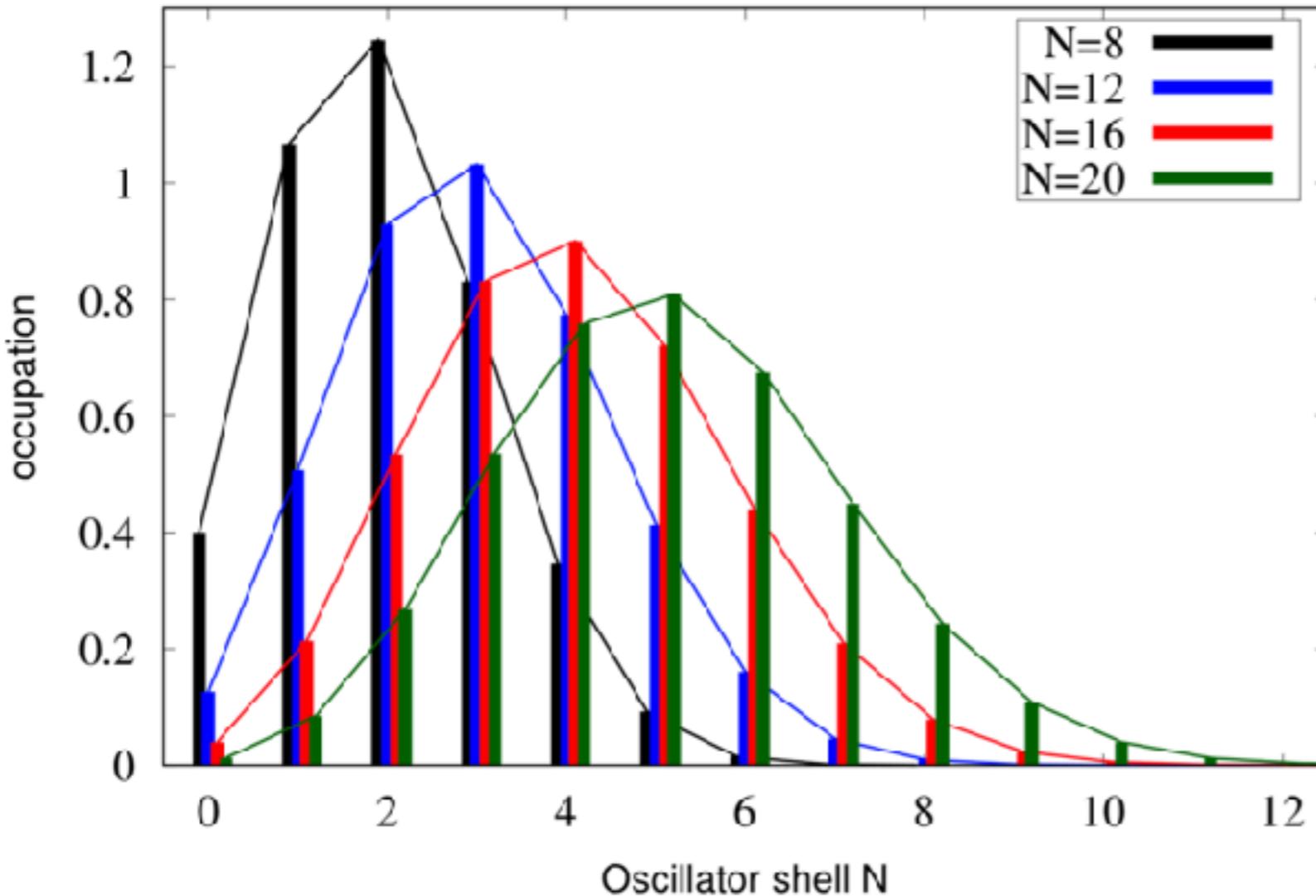


Φ	Ψ_P	$ \langle \Psi_P \hat{\Phi}^\dagger \Psi_D \rangle ^2$	$\langle 0 \hat{\Phi} \hat{\Phi} \hat{\Phi}^\dagger \hat{\Phi}^\dagger 0 \rangle$
$(p)^4 (4, 0)$	$(p)^8 (0, 4)$	1.42222*	1.42222
$(sd)^4 (8, 0)$	$(sd)^8 (8, 4)$	0.487903	1.20213
$(fp)^4 (12, 0)$	$(fp)^8 (16, 4)$	0.292411	1.41503
$(sdg)^4 (16, 0)$	$(sdg)^8 (24, 4)$	0.209525	1.5278

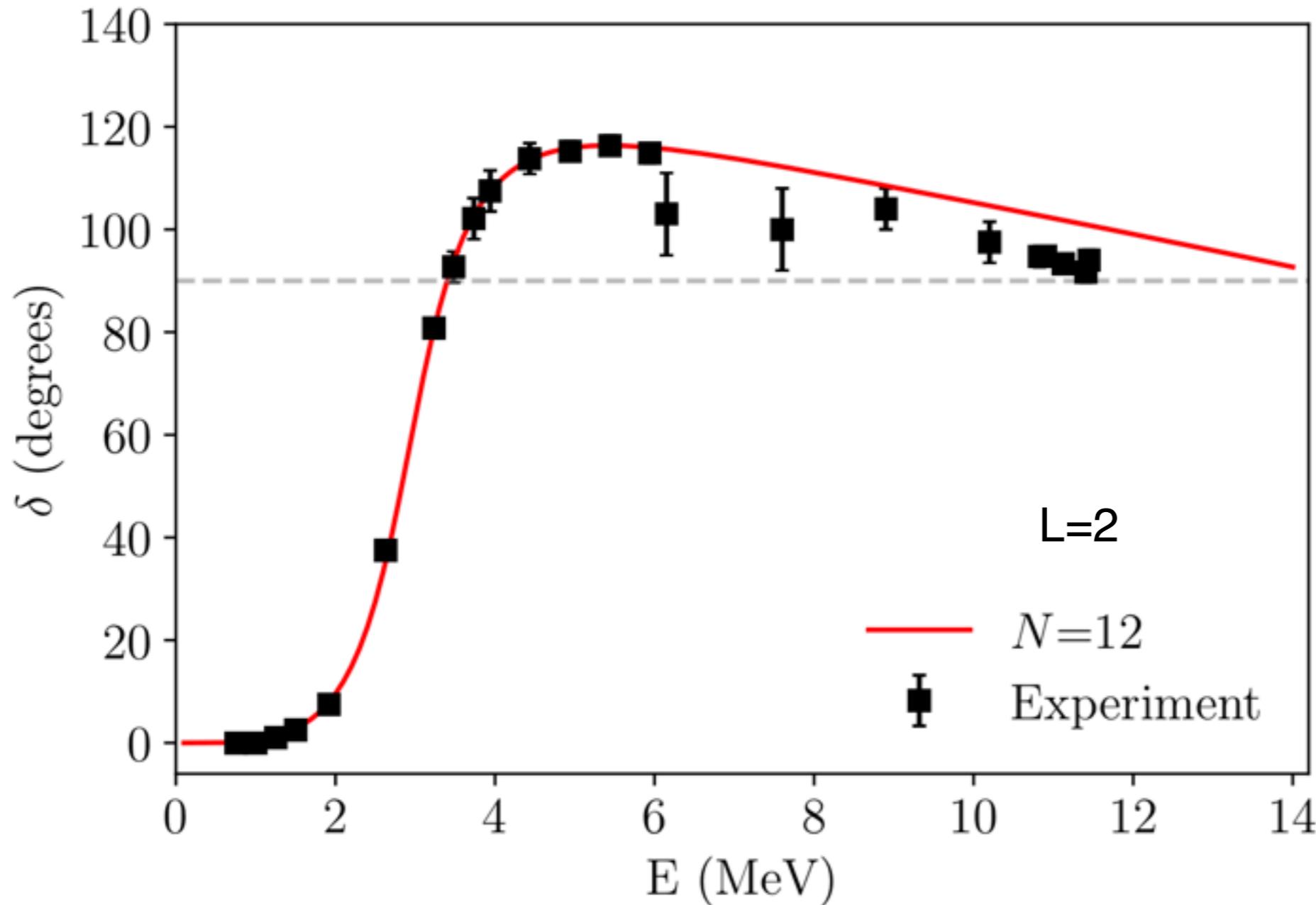
* For p-shell the result is known analytically 64/45

Effective operators (alphas) are not ideal bosons
Cluster configurations are not orthogonal and not normalized

CM-boosted configuration from shell model perspective



alpha+alpha scattering phase shifts



Experimental data from S. A. Afzal, A. A. Z. Ahmad, and S. Ali, Rev. Mod. Phys. 41, 247 (1969).

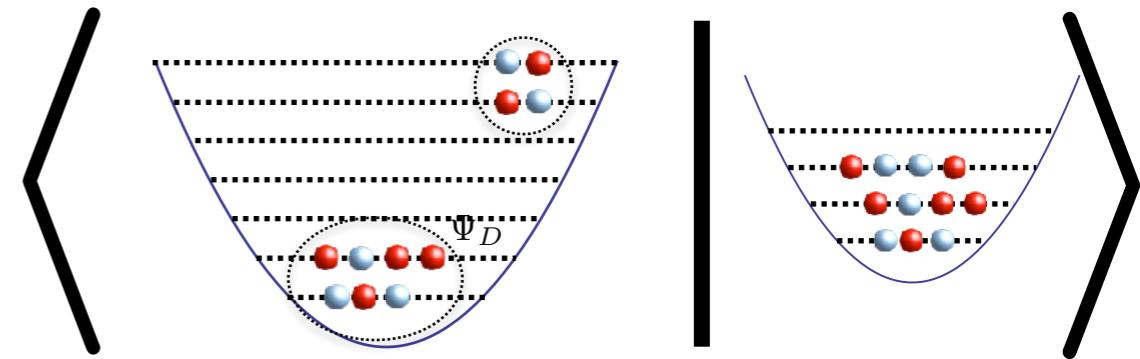
Clustering in sd-shell nuclei

- How well it compares with experiment
- Distribution of clustering strength
- Clustering and pairing

Cluster Spectroscopic Characteristics

Traditional (old) spectroscopic factor

$$\langle \phi_{n\ell} | \varphi_\ell \rangle = \langle \hat{\mathcal{A}}\{\phi_{n\ell m}(\rho) \Psi'_\alpha \Psi'_D\} | \Psi'_P \rangle =$$



$$\langle \phi_{n\ell} | \varphi_\ell \rangle = \mathcal{R}_{n\ell} \sum_{\eta} X_{n\ell}^{\eta} \mathcal{F}_{n\ell}^{\eta}$$

Recoil Factor Cluster Coefficient Fractional Parentage Coefficient

Normalized (new) spectroscopic factor

$$\psi_\ell(\rho) \equiv \hat{\mathcal{N}}_\ell^{-1/2} \varphi_\ell(\rho)$$

$$S_\ell^{(\text{new})} \equiv \langle \psi_\ell | \psi_\ell \rangle = \int \rho^2 d\rho |\psi_\ell(\rho)|^2$$

Sum of all new SF from all parent states to a given final state equals to the number of channels

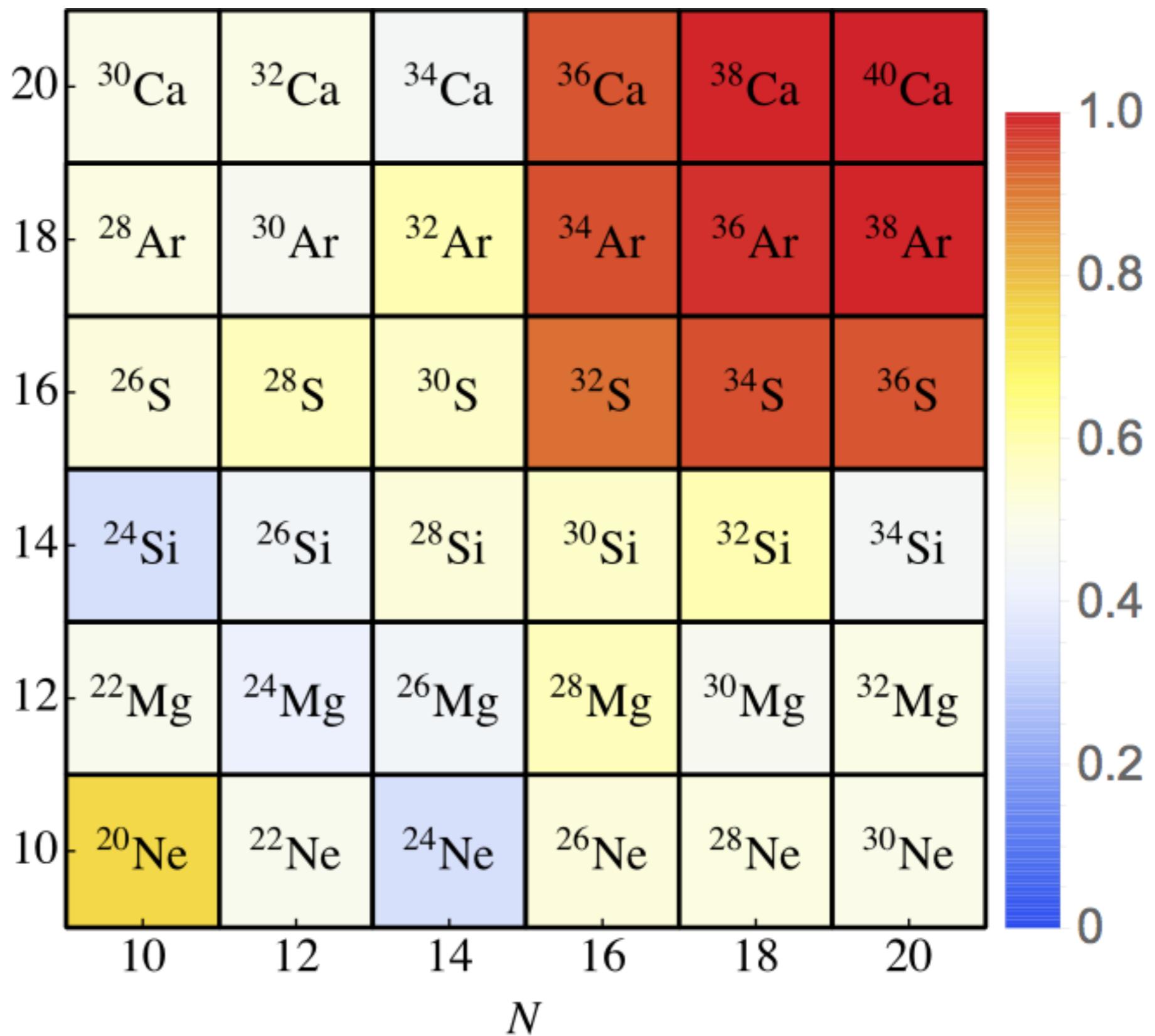
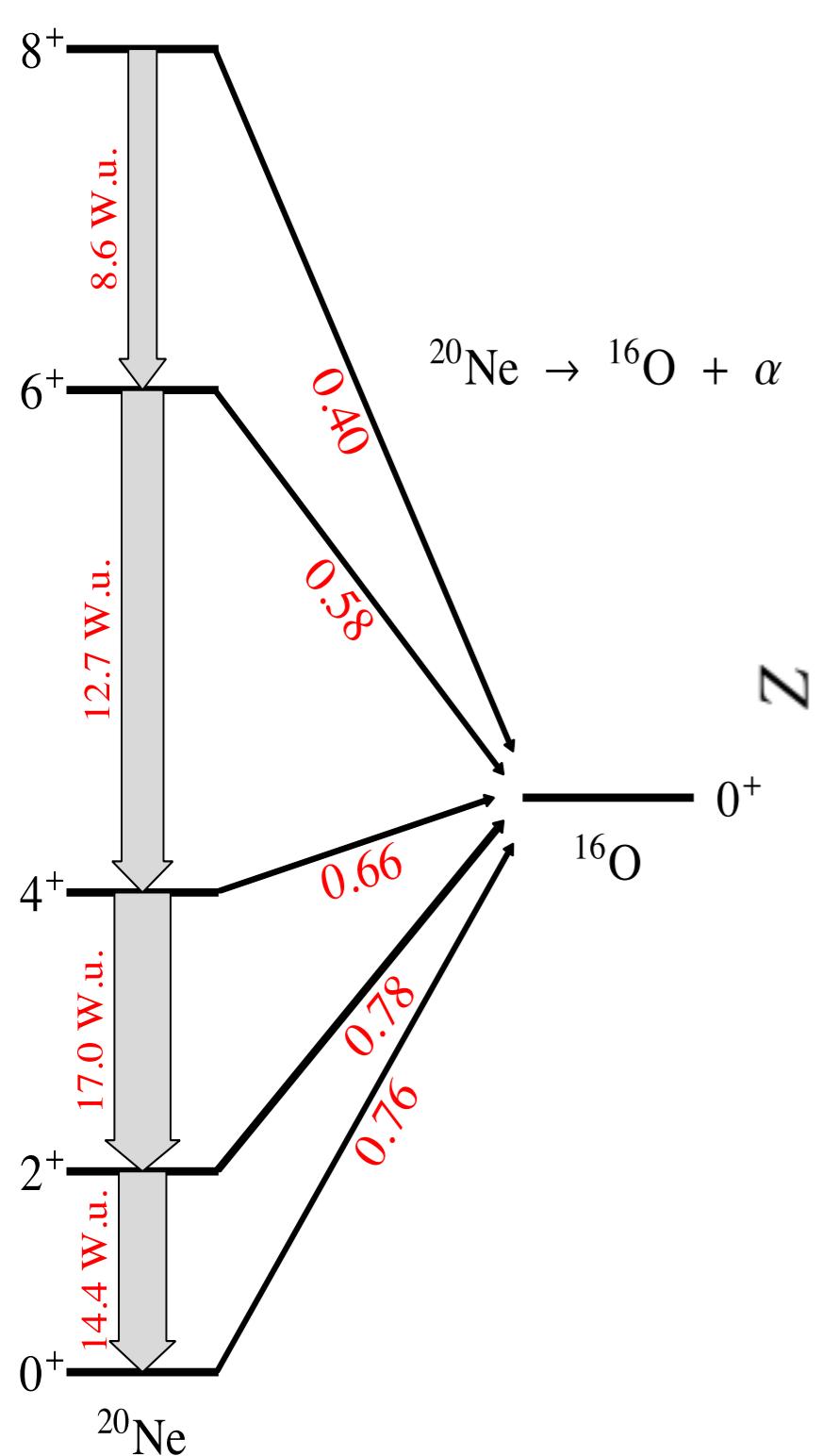
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Alpha clustering in sd-shell nuclei

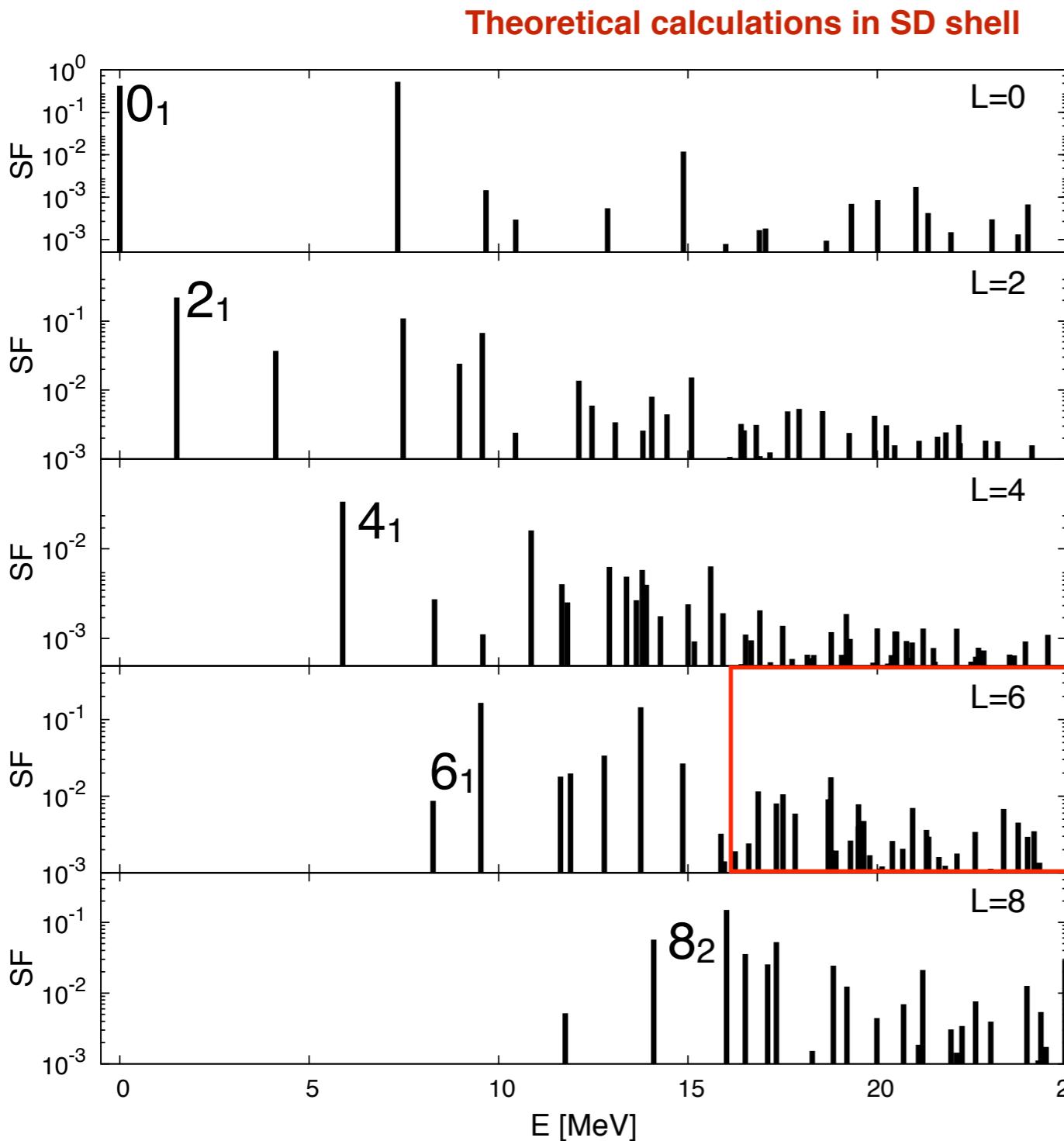
$A_P - A_D$	$S_0^{(\text{exp})}$ [1]	$S_0^{(\text{exp})}$ [2]	$S_0^{(\text{exp})}$ [3]	$S_0^{(\text{old})}$ [4]	$S_0^{(\text{old})}$ this work	$S_0^{(\text{new})}$
$^{20}\text{Ne}-^{16}\text{O}$	1.0	0.54	1	0.18	0.173	0.755
$^{22}\text{Ne}-^{18}\text{O}$			0.37	0.099	0.085	0.481
$^{24}\text{Mg}-^{20}\text{Ne}$	0.76	0.42	0.66	0.11	0.091	0.411
$^{26}\text{Mg}-^{22}\text{Ne}$			0.20	0.077	0.068	0.439
$^{28}\text{Si}-^{24}\text{Mg}$	0.37	0.20	0.33	0.076	0.080	0.526
$^{30}\text{Si}-^{26}\text{Mg}$			0.55	0.067	0.061	0.555
$^{32}\text{S}-^{28}\text{Si}$	1.05	0.55	0.45	0.090	0.082	0.911
$^{34}\text{S}-^{30}\text{Si}$				0.065	0.062	0.974
$^{36}\text{Ar}-^{32}\text{S}$				0.070	0.061	0.986
$^{38}\text{Ar}-^{34}\text{S}$			1.30	0.034	0.030	0.997
$^{40}\text{Ca}-^{36}\text{Ar}$	1.56	0.86	1.18	0.043	0.037	1

USDB interaction [5]
(8,0) configuration

- Old SF are small
- Old SF decrease with A

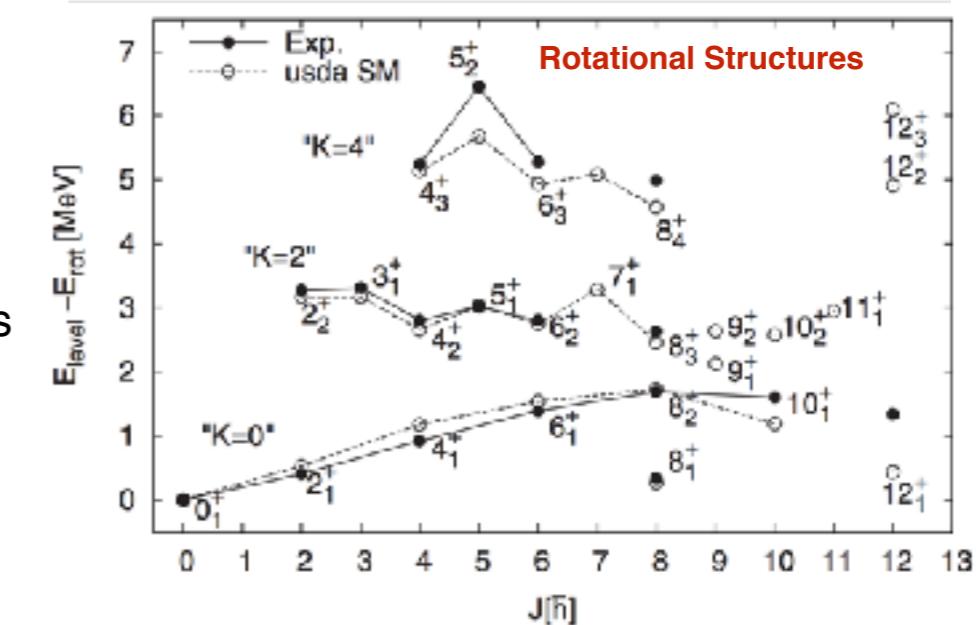
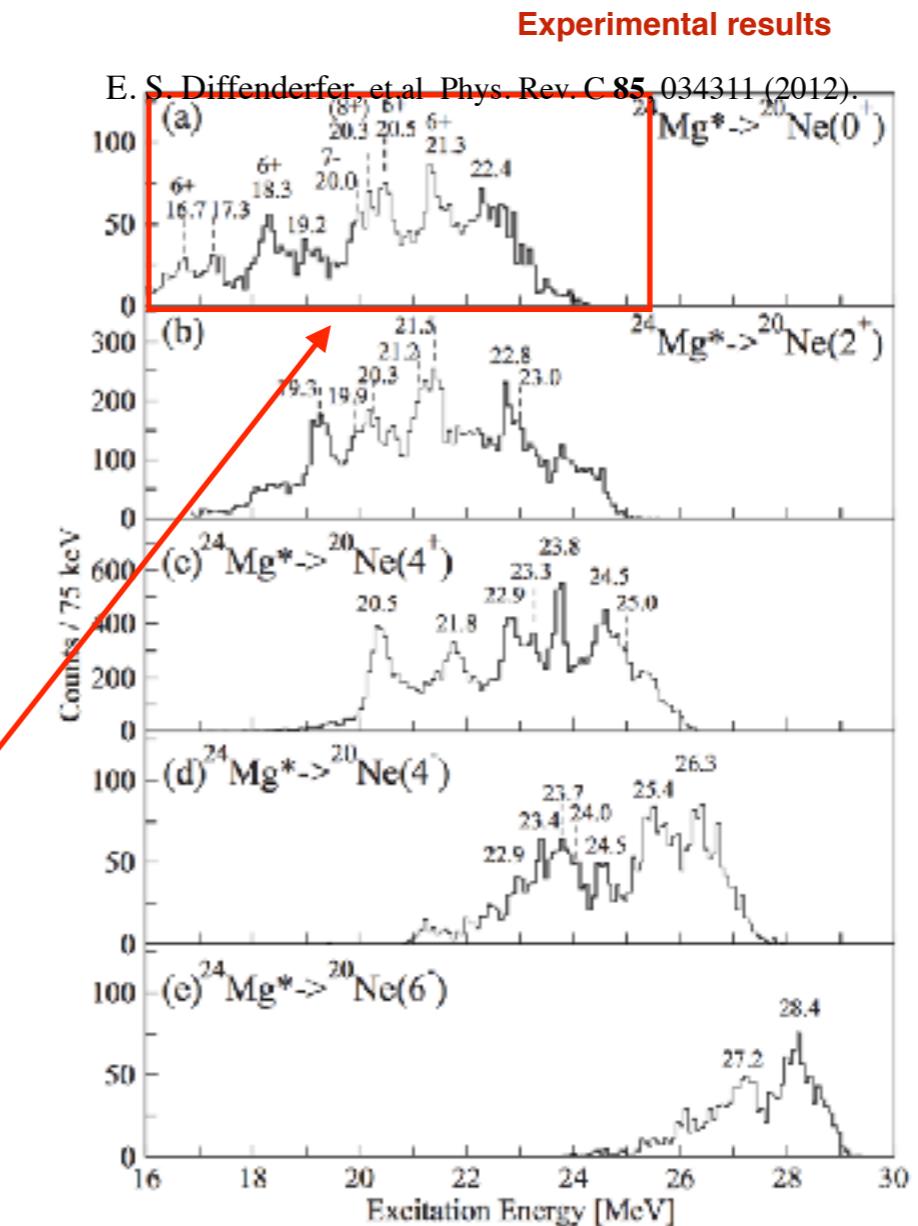
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Alpha cluster spectroscopic factors in ^{24}Mg



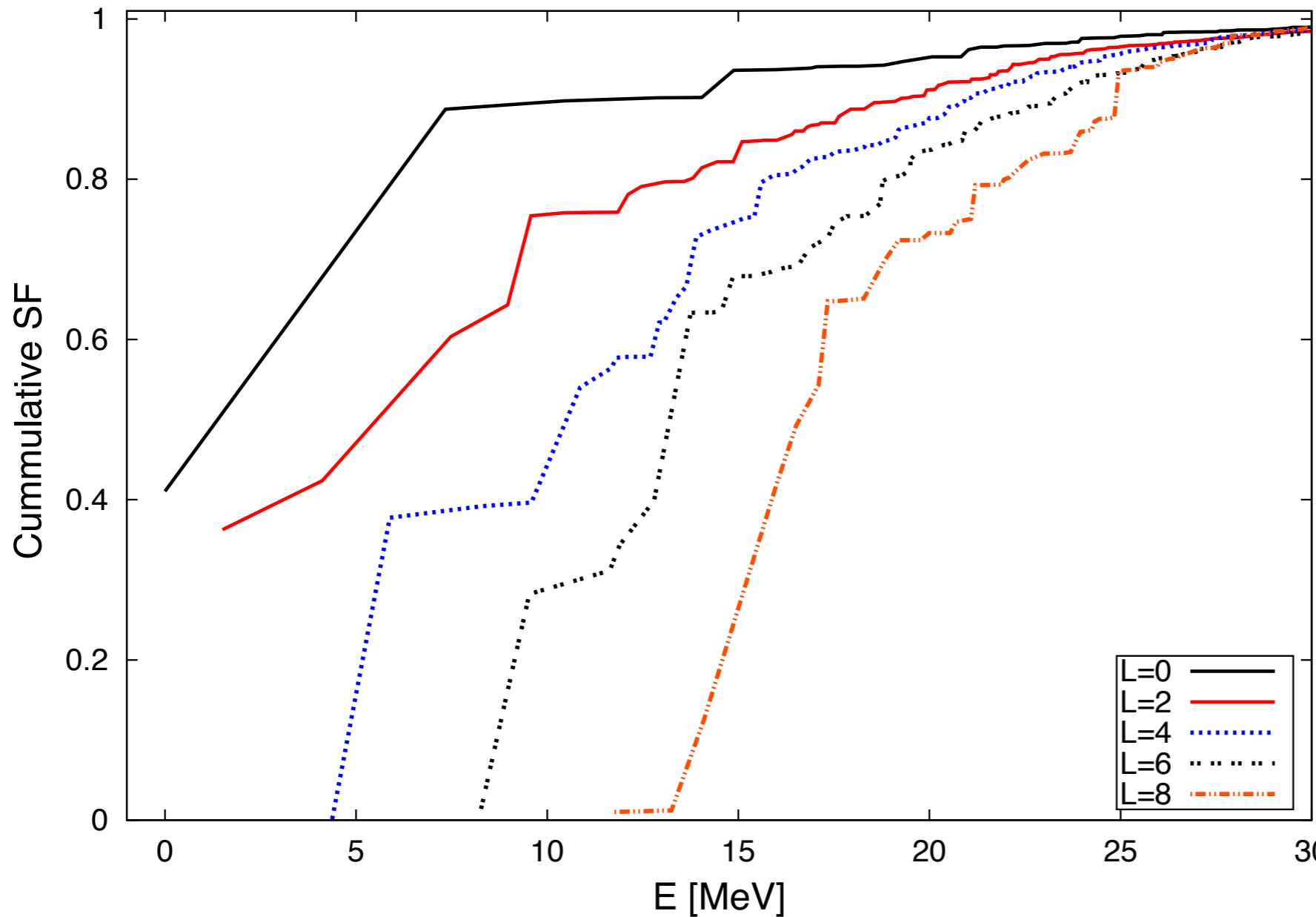
The sd valence space is considered with USDB interaction the operator is

$$|\Phi_{(8,0);L}\rangle = |(sd)^4[4] (8,0), :LS=T=0\rangle$$

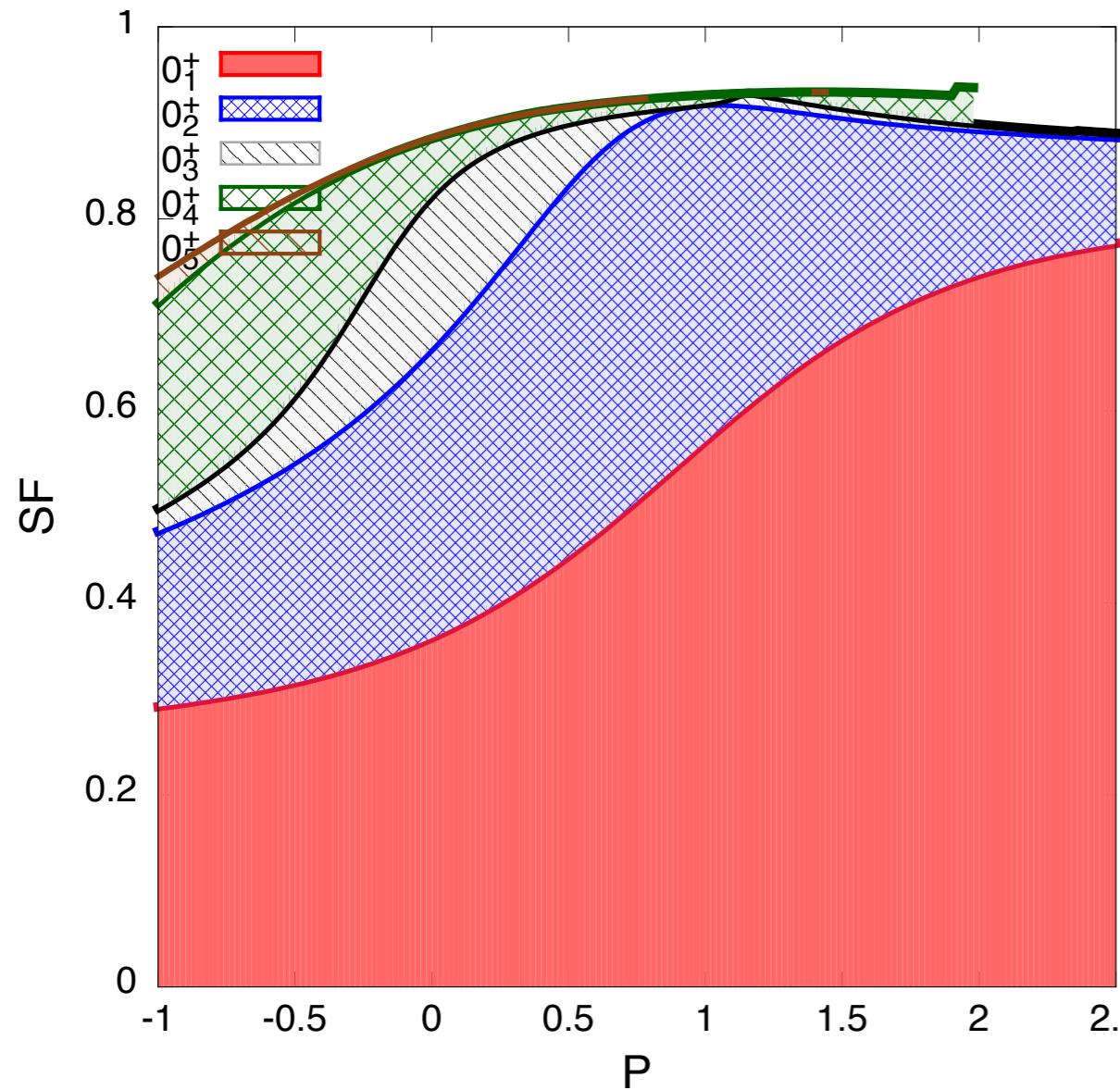


Cumulative spectroscopic strength ^{24}Mg

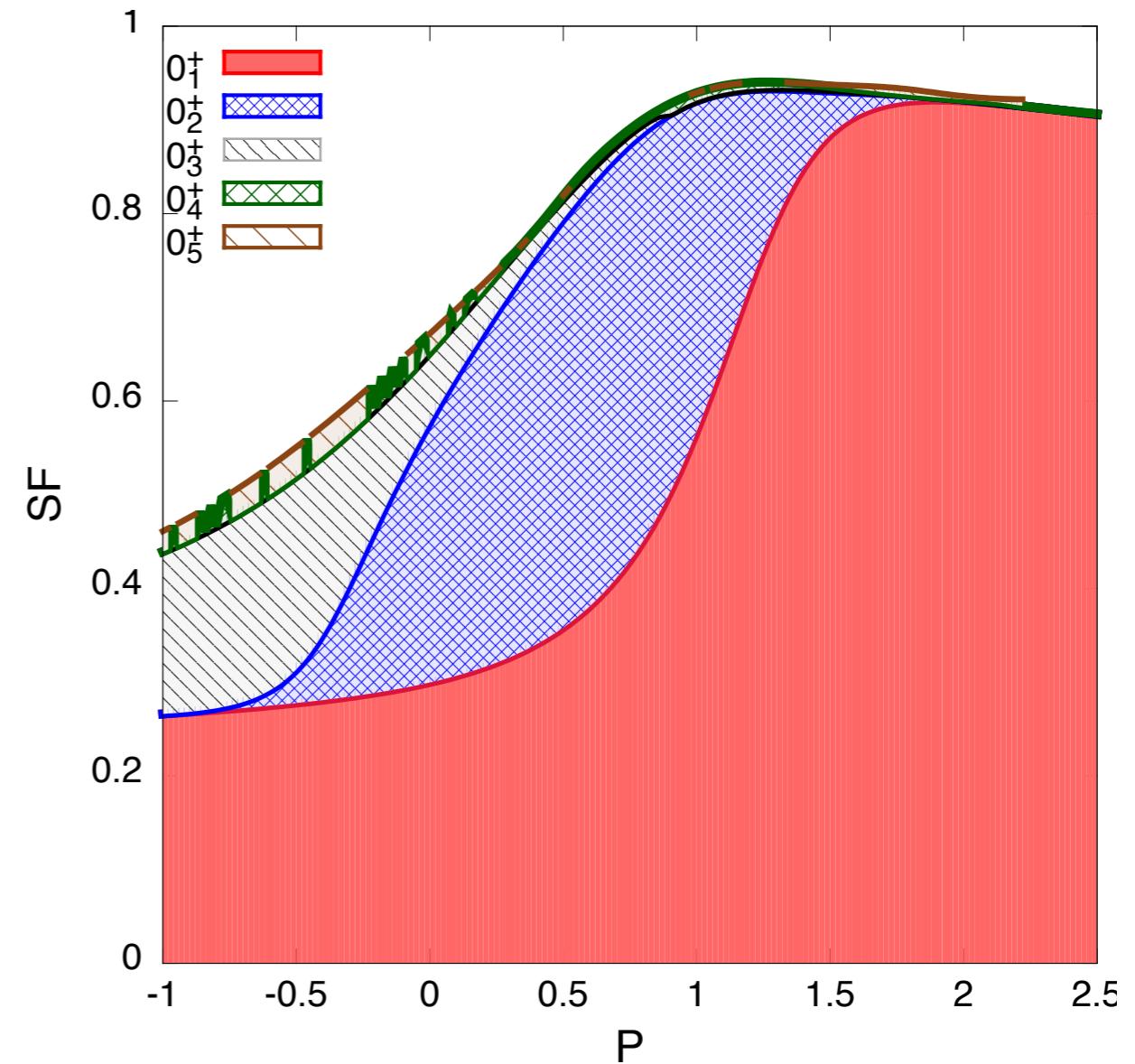
$^{20}\text{Ne}(\text{g.s}) + \text{alpha}$ channel.



Spectroscopic factors of lowest 0^+ states as a function of pairing matrix elements in ^{24}Mg



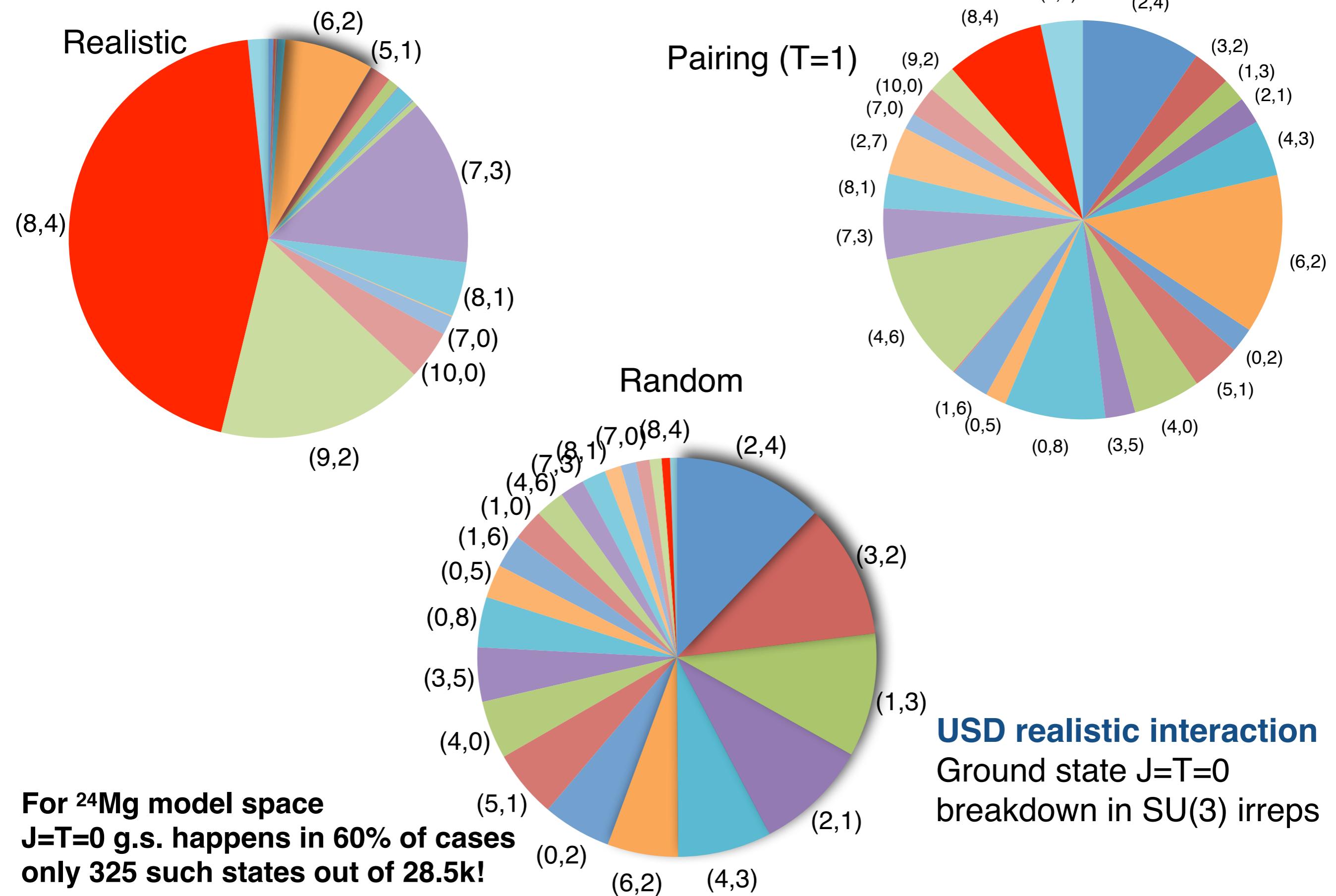
Scaling of isovector ($T=1$)
pairing matrix elements



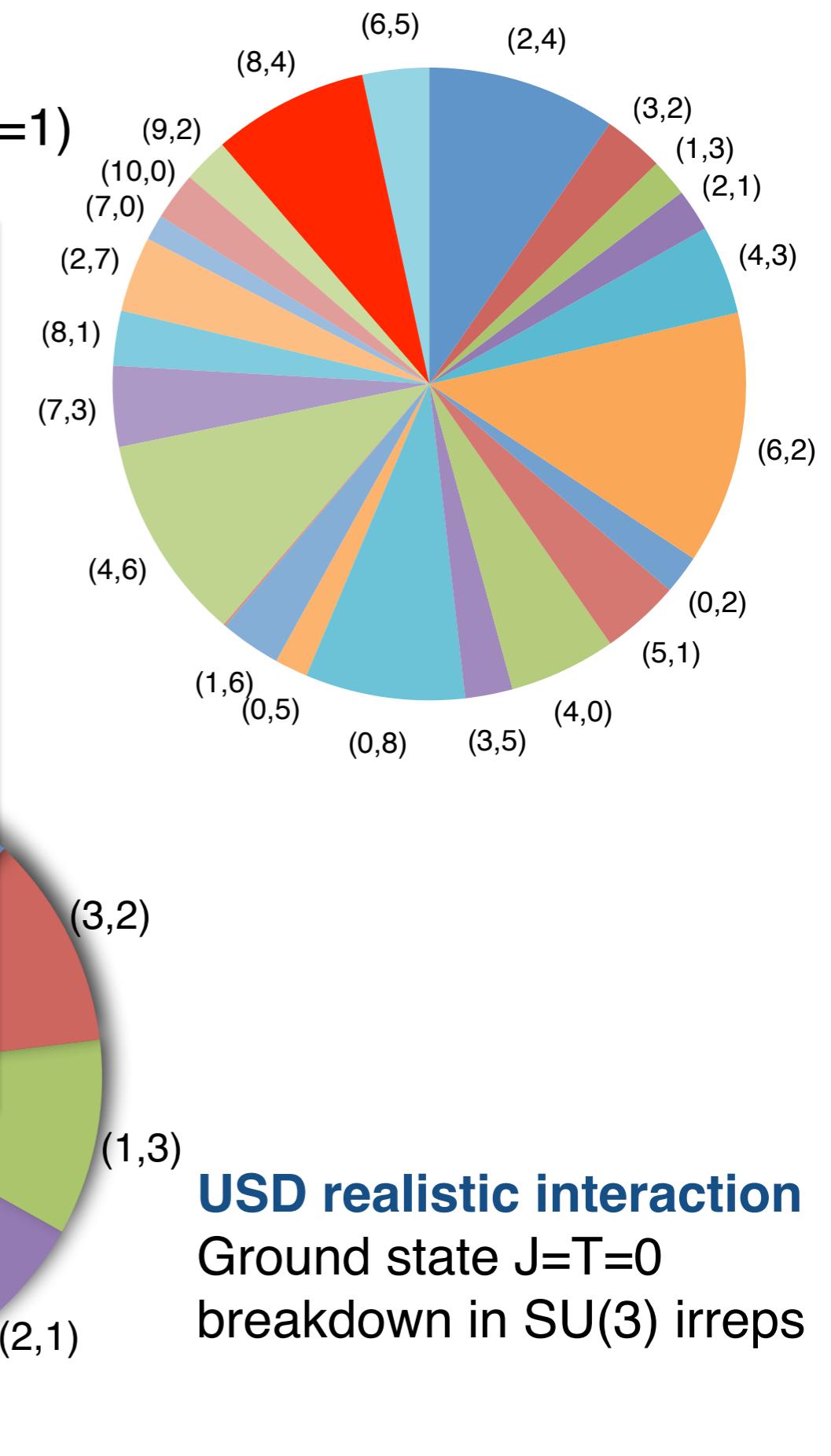
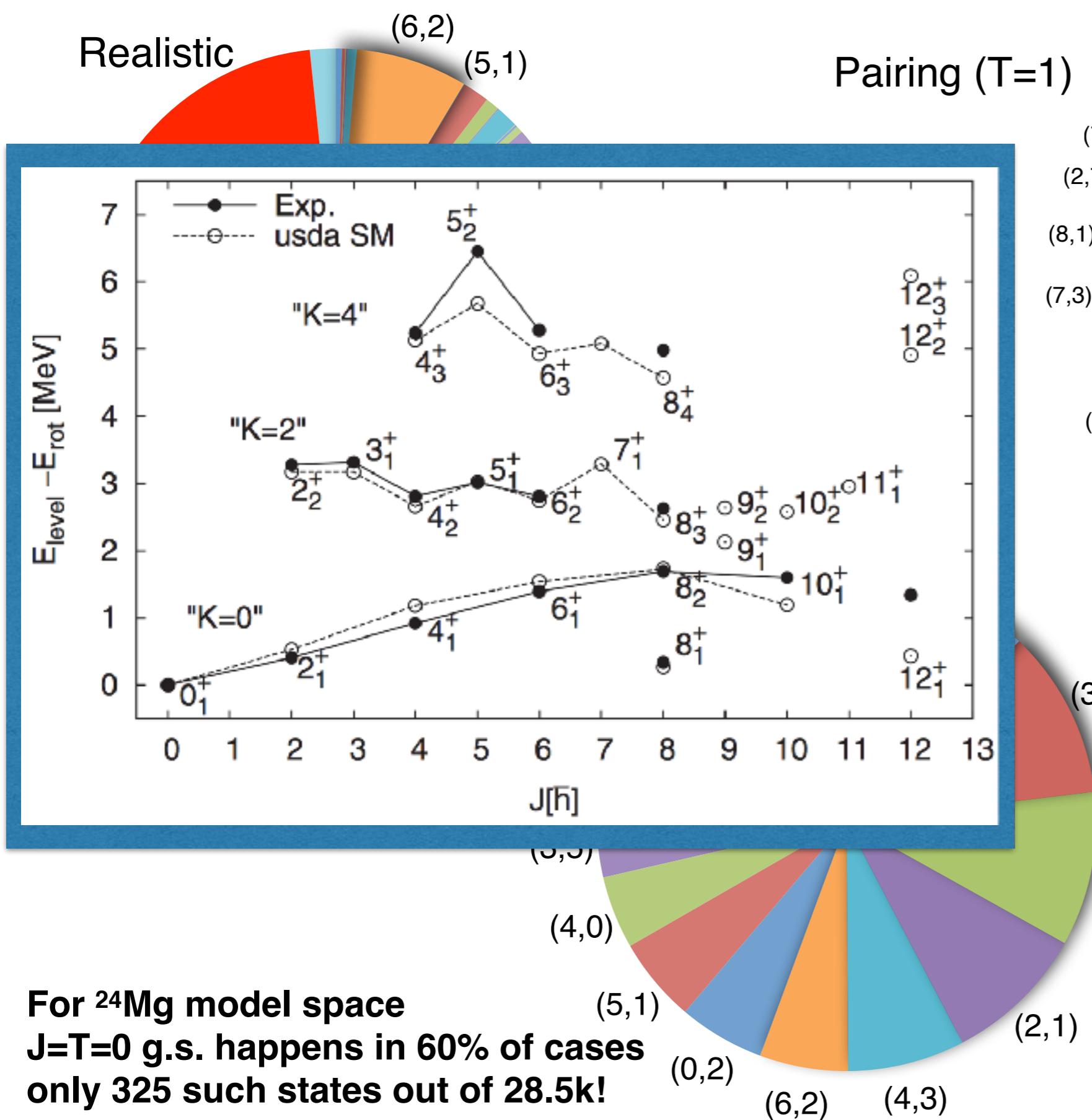
Scaling of isoscalar($T=0$)
pairing matrix elements

Interactions and symmetry of low-lying states

Classic Example: ^{24}Mg , 8 nucleons in sd-shell



Classic Example: ^{24}Mg , 8 nucleons in sd-shell



For ^{24}Mg model space
 $J=T=0$ g.s. happens in 60% of cases
 only 325 such states out of 28.5k!

Detailed study of one alpha on sd

Clustering in ^{20}Ne

4^+_3 ————— 9990

4^+_2 ————— 9945

4^+_2 ————— 9031

6^+_1 ————— 8778

6^+_1 ————— 8547

2^+_3 ————— 7833

2^+_2 ————— 7422

0^+_3 ————— 7191

2^+_2 ————— 7543

0^+_2 ————— 6725

0^+_2 ————— 6698

4^+_1 ————— 4248

4^+_1 ————— 4175

2^+_1 ————— 1634

2^+_1 ————— 1747

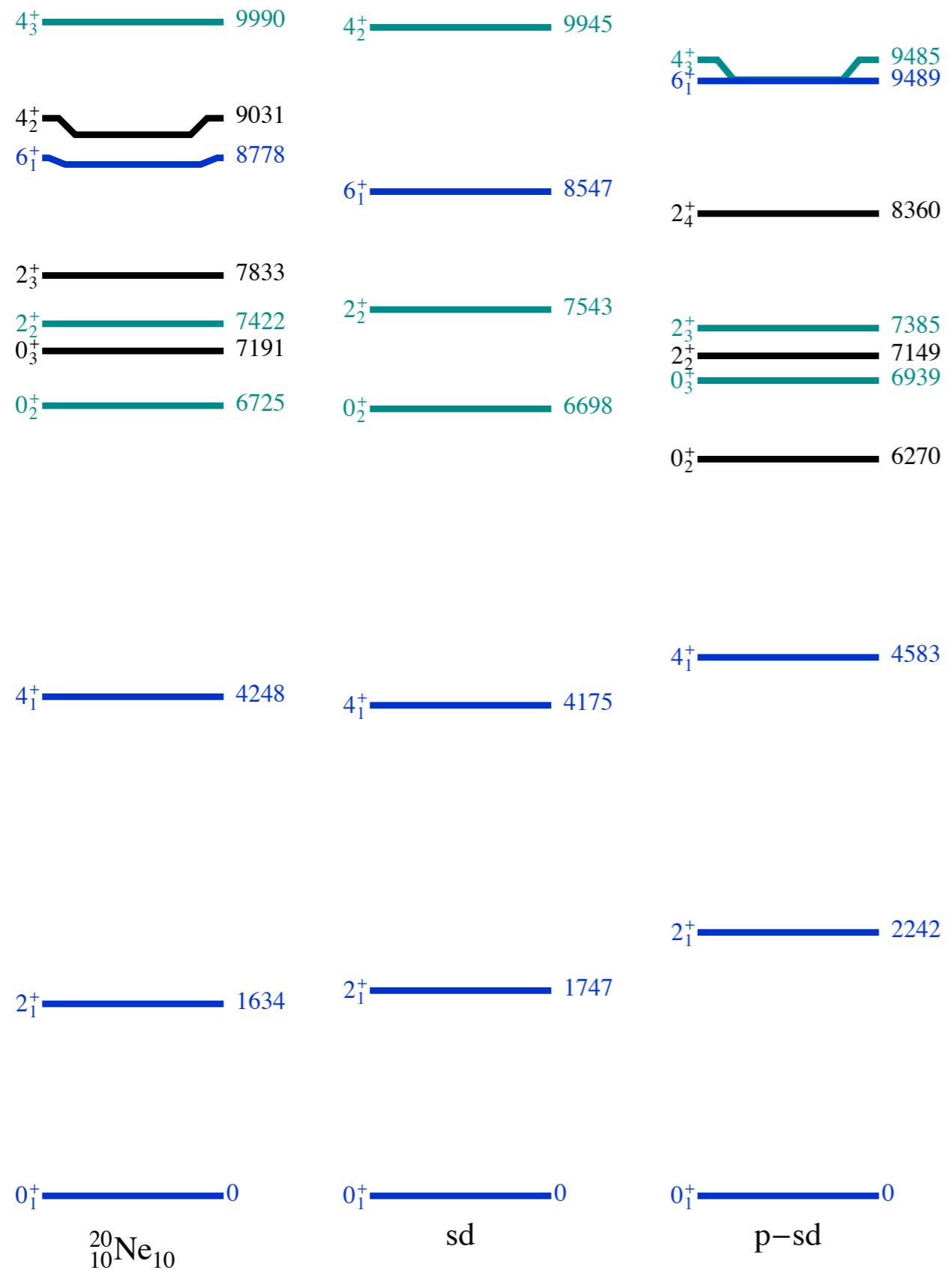
0^+_1 ————— 0

0^+_1 ————— 0

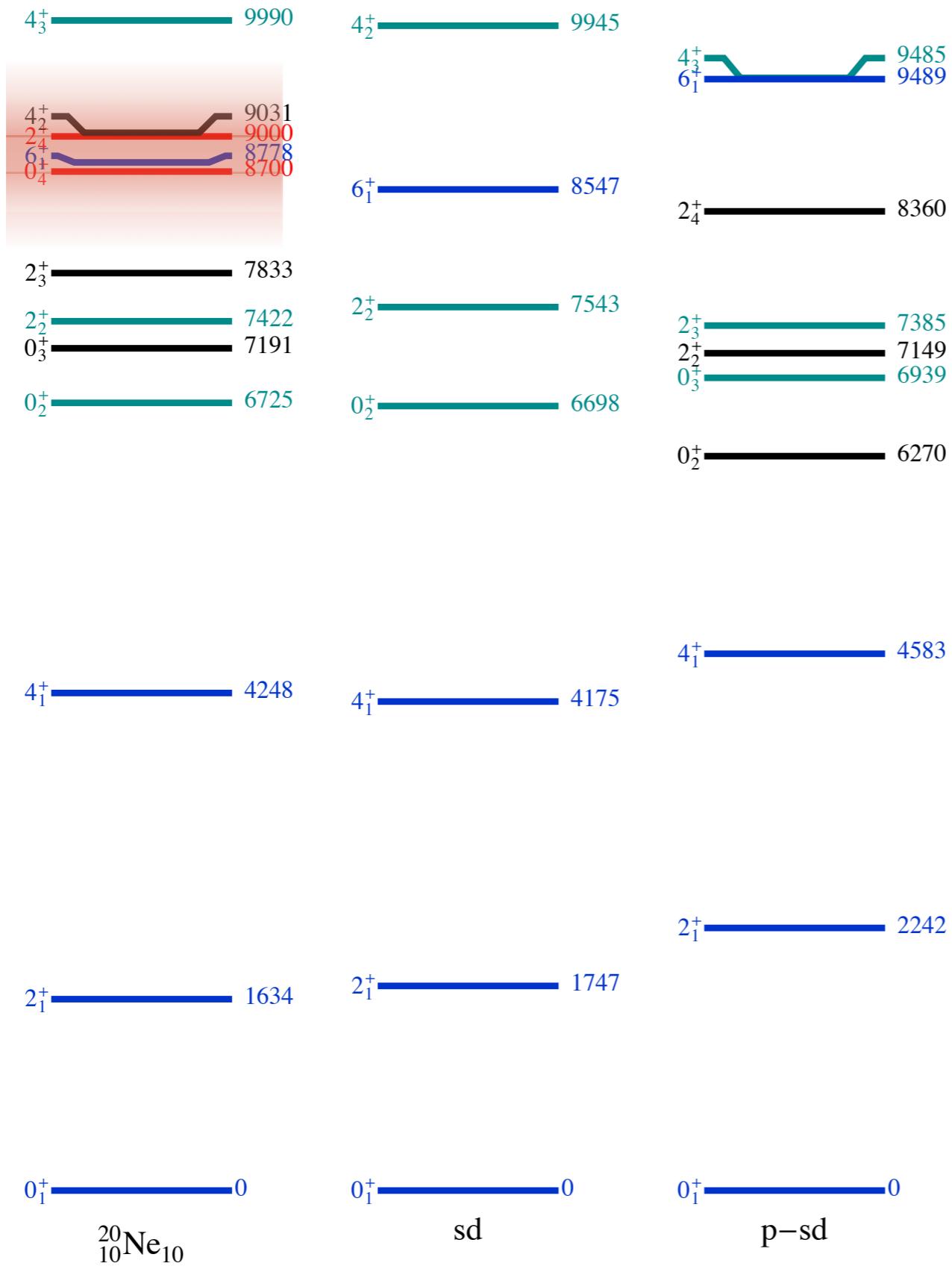
$^{20}_{10}\text{Ne}_{10}$

sd

Clustering in ^{20}Ne

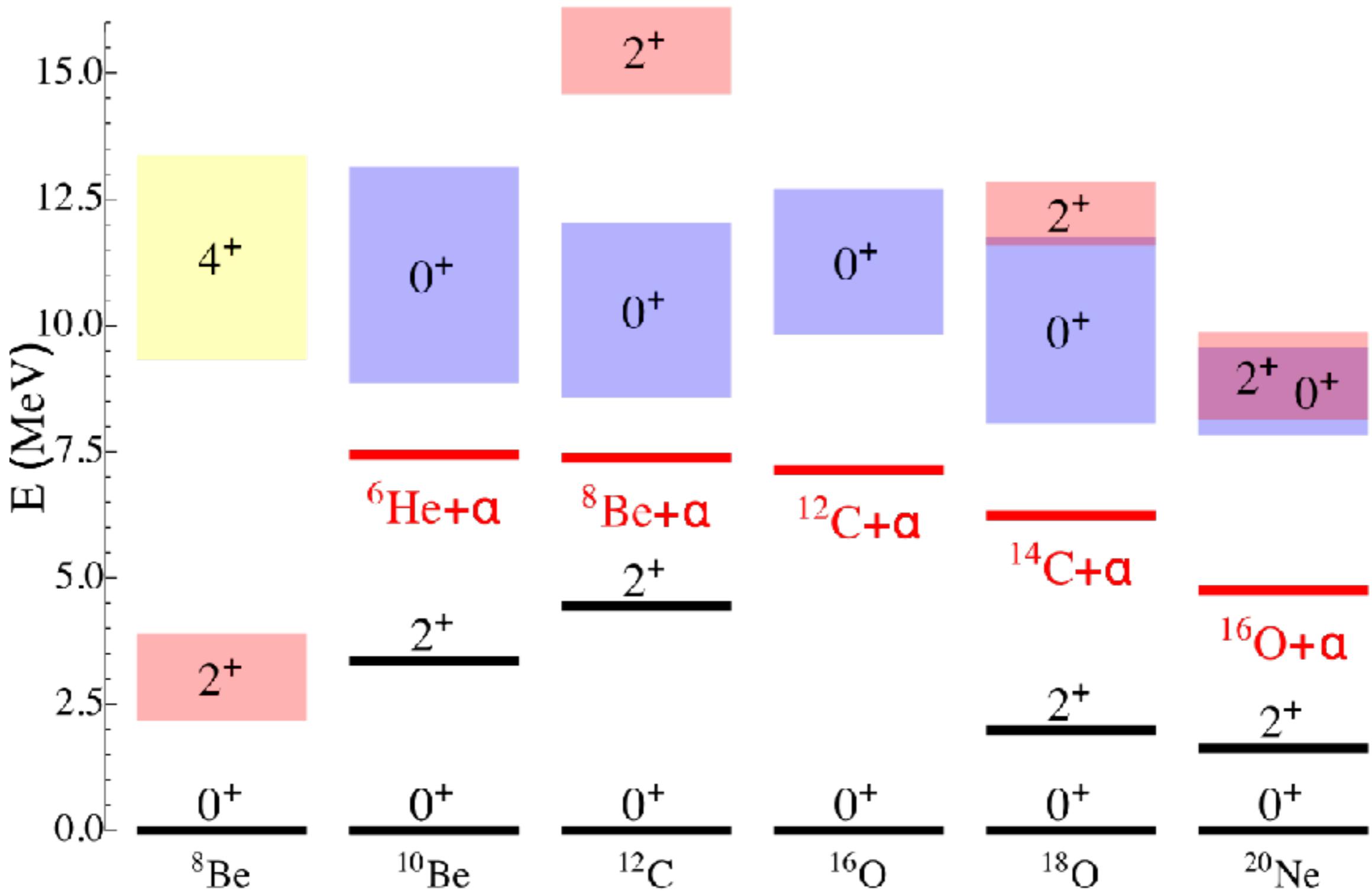


Clustering in ^{20}Ne

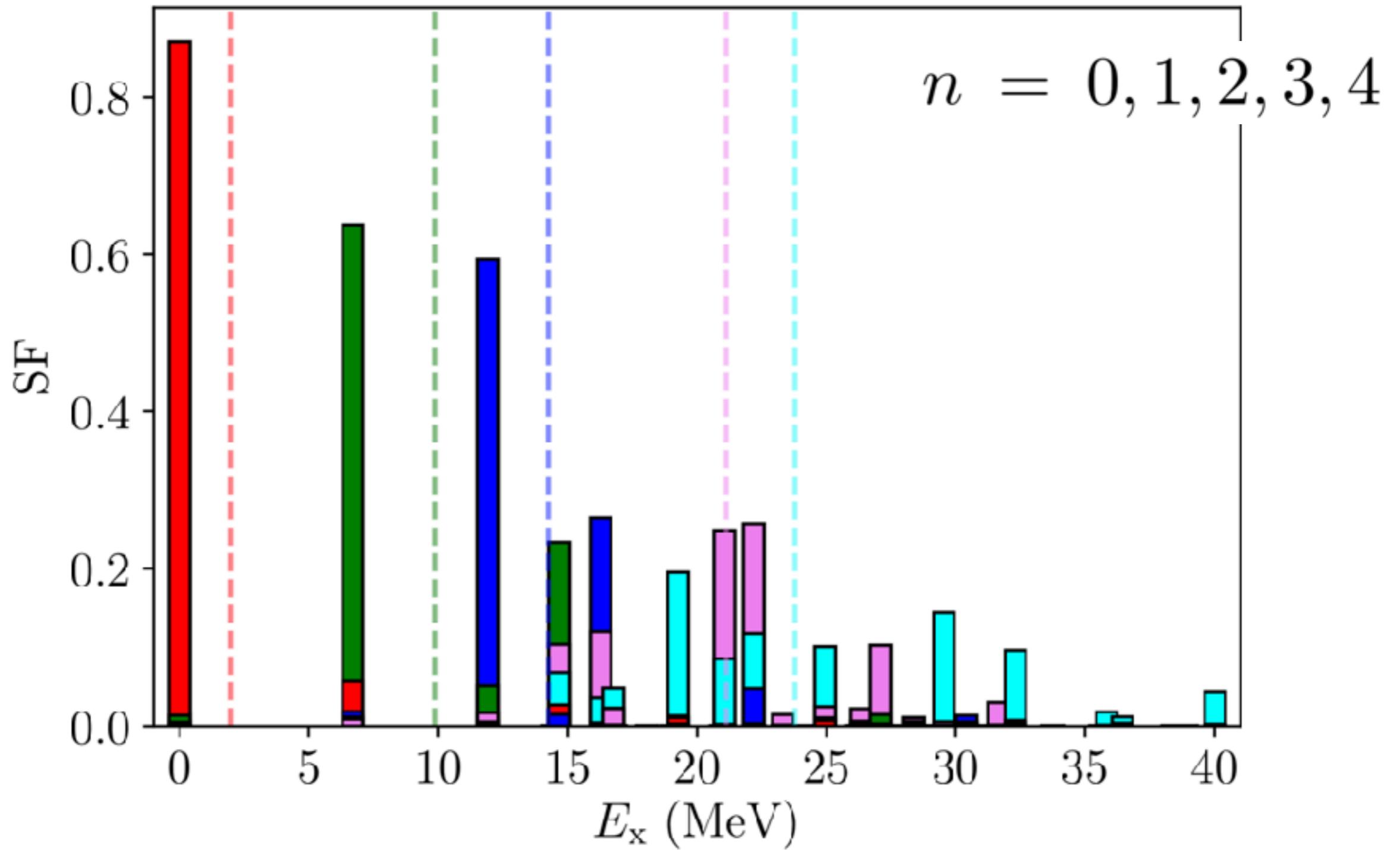


J	E MeV	Γ width	SF ex	SF th.
0^+	0	0		0.73
2^+	1.63	0		0.67
4^+	4.25	0		0.62
0^+	6.73	19	0.47	0.46
0^+	7.19	3.4	0.02	0.10
2^+	7.42	15	0.19	0.12
2^+	7.83	2	0.01	0.09
0^+	8.7	800	0.3	
6^+	8.78	0.11	0.5	0.51
2^+	9.00	800	0.86	

Clustering and continuum

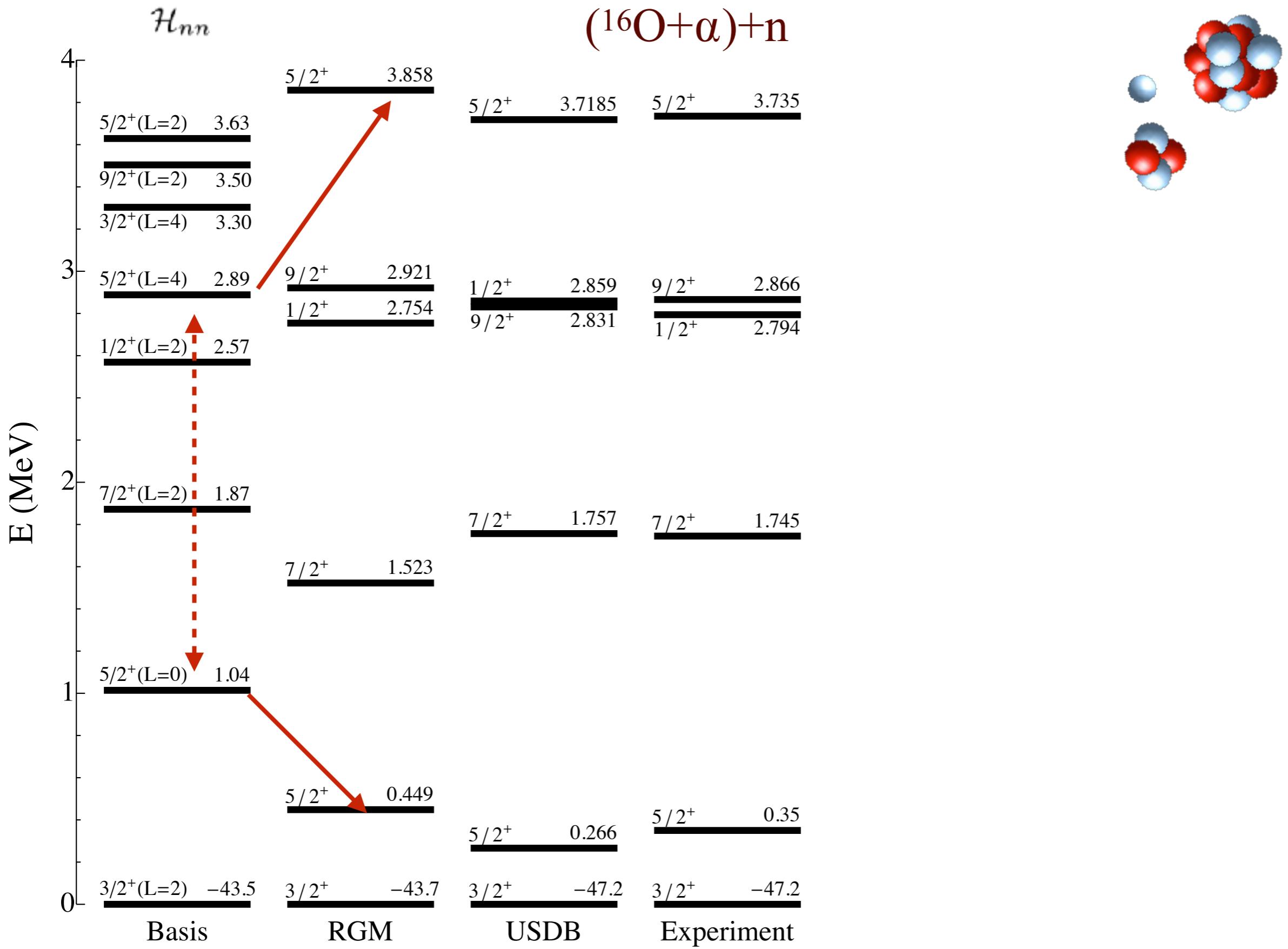


Searching for clustering strength



Distribution of dynamic spectroscopic factors for $^{20}\text{Ne} \rightarrow ^{16}\text{O}(\text{g.s.}) + \alpha$. The dashed lines correspond to the RGM energies for each decay channel.

Molecular orbits ^{21}Ne



Weak-Coupling Behavior

J^π	$S^{(new)}$			$S^{(exp)}$		
	$\ell = 0$	$\ell = 2$	$\ell = 4$	$\ell = 0$	$\ell = 2$	$\ell = 4$
$3/2+$		1.0	0.18		1.0 ± 0.05	0.42 ± 0.04
$5/2+$	0.78	0.02	0.44	1.04 ± 0.41	...	0.32 ± 0.18
$7/2+$		0.9	0.14		0.91 ± 0.08	0.23 ± 0.04
$9/2+, 1/2+$		0.81	0.33		0.9 ± 0.05	0.29 ± 0.03

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Resources: <https://www.volya.net/> (see research, clustering)

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