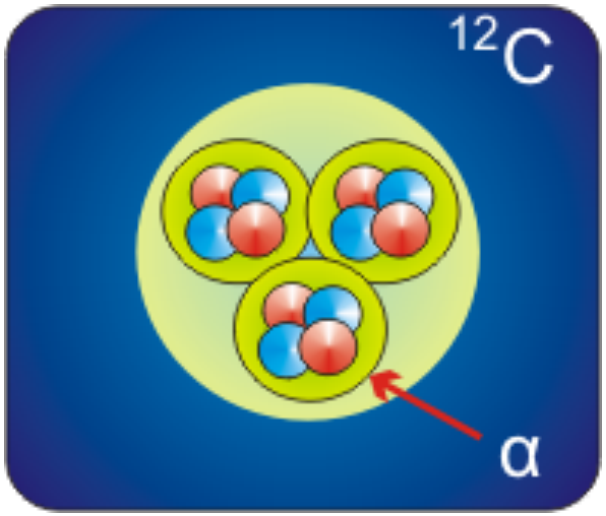
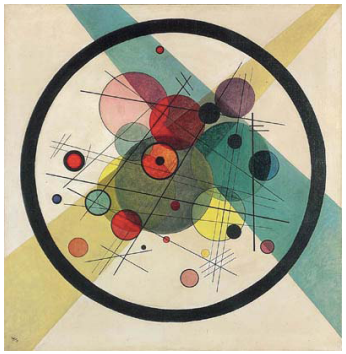
The algebraic molecular model for ^{12}C and its application to the α+^{12}C scattering: from densities and transition densities to optical potentials, nuclear form factors and cross sections

Andrea Vitturi
Dipartimento di Fisica e Astronomia, and INFN, Padova





Recent advances on proton-neutron pairing, Saclay. September 2019

Motivations (1)

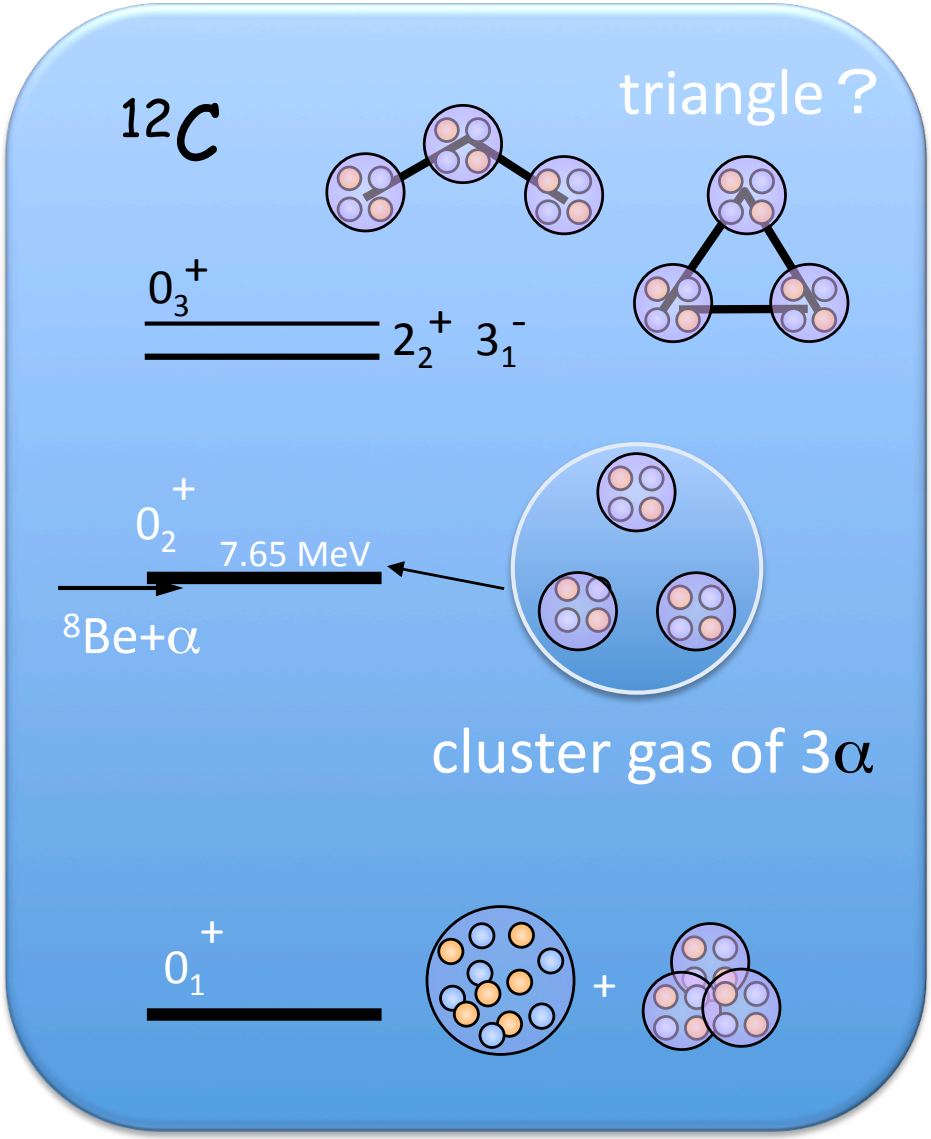
Evidence of alpha-clustering aspects in light nuclei (e.g. ^{12}C)

Coexistence of cluster and non-cluster states in the same nucleus

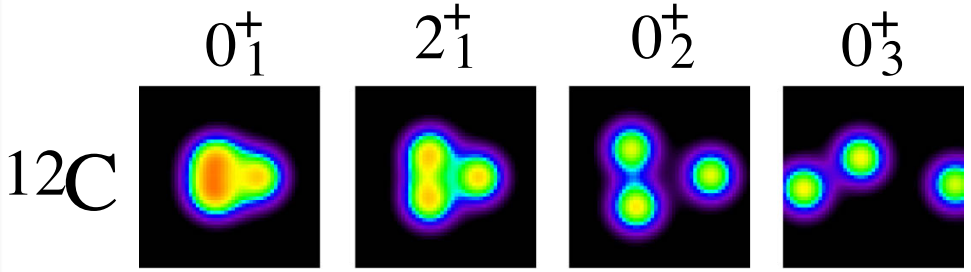
Emergence of cluster states from correlated nucleons moving in the nuclear mean field or in *ab initio* approaches

Two theoretical examples (technically rather complex): AMD and large-scale Monte Carlo shell model

Antisymmetrized Molecular Dynamics

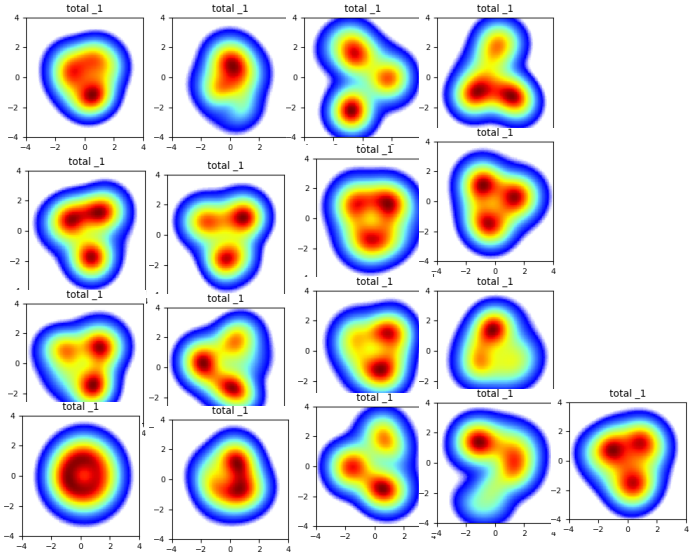


From Y. Kanada-En'yo
(and collaborators)

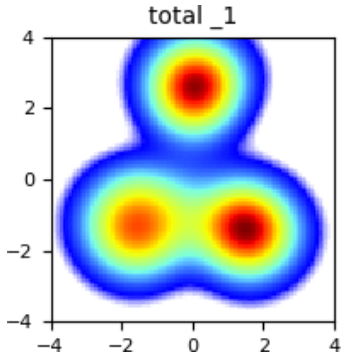


Monte Carlo Shell Model (Otsuka and the Tokyo group)

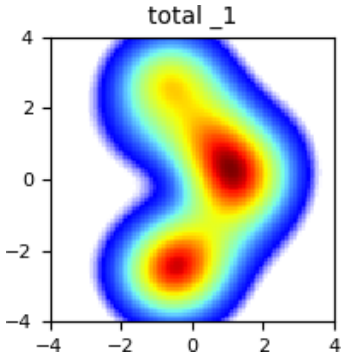
Procedure: calculation of "intrinsic" density from MCSM eigenstate



0^+_1: mixture of many "shapes"



0^+_2: main "shape"



0^+_3: main "shape"

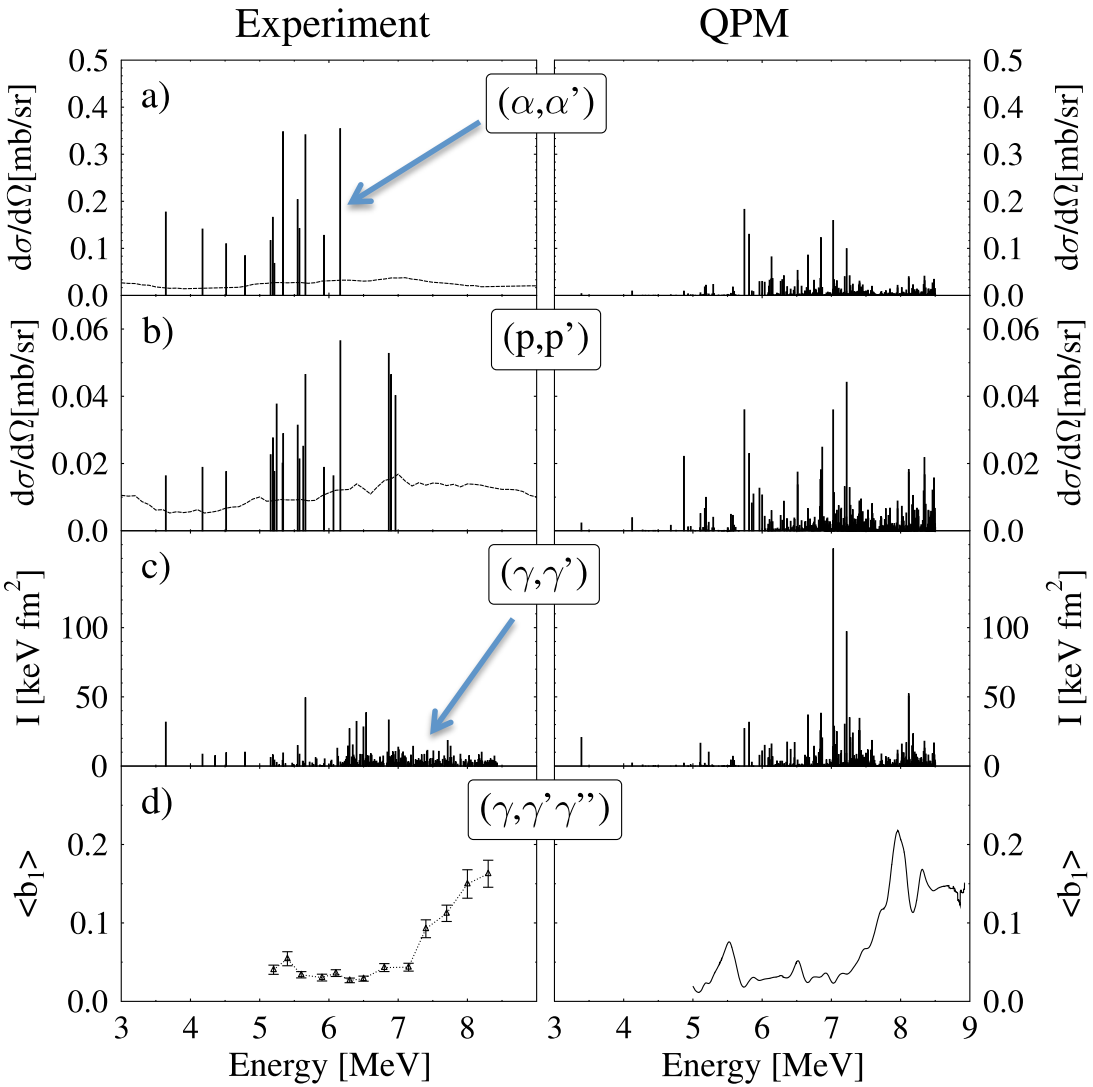
Motivations (2)

Need of combining different probes to study the different facets of a nuclear system (Multi-messenger investigation)

In the specific case of ^{12}C in addition to information based on electromagnetic probes a special role is played by NUCLEAR inelastic excitation, as for example (α, α') scattering

Cf. **Makoto Ito, Phys. Rev. C 97, 044608, 2018**
Yoshiko Kanada-En'yo and Kazuyuki Ogata, Phys. Rev. C 99, 064601,2019

An example of the success of multi-messenger analysis: the case of the Pygmy Dipole States



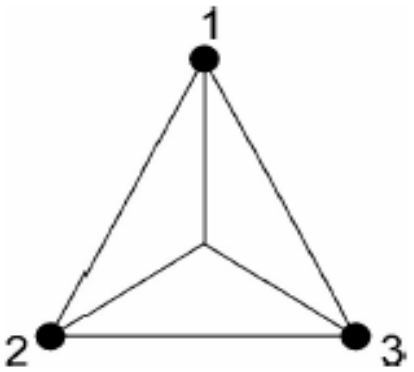
^{140}Ce

D.Savran, A. Vitturi etal, Physics Letters B 786 (2018) 16-20

Our approach: the algebraic cluster molecular model

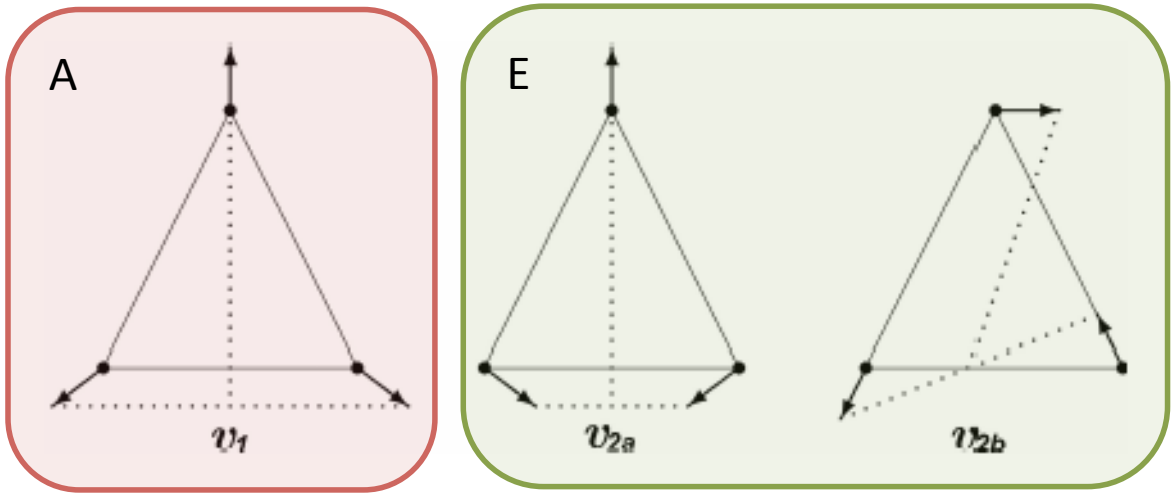
Wheeler J A, 1937
Iachello F, Bijker R. and collaborators, 2000

Three alpha-particles at the vertices of a equilateral triangle
(D_{3h} discrete symmetry)



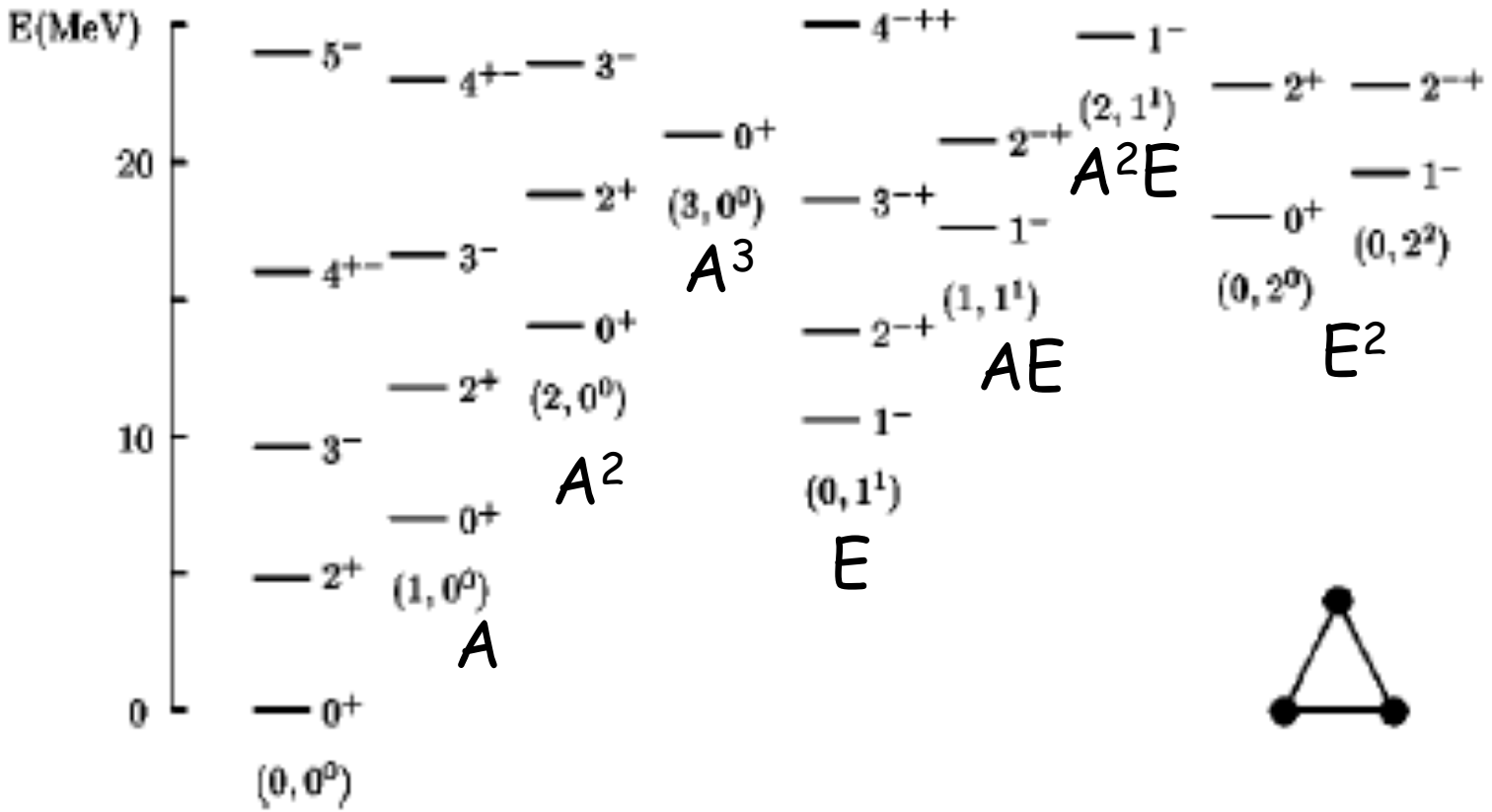
Geometry of a three-body system.

Intrinsic states are generated by the normal modes of vibration of the triangle



Vibrations of an oblate top.

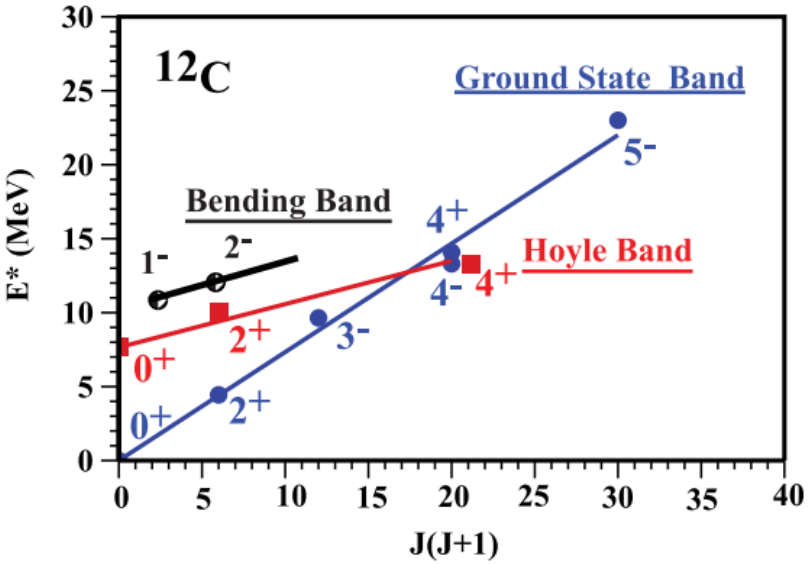
Each intrinsic state generates a rotational band, whose elements have allowed quantum numbers (angular momentum and parity) according to the rules of the point-group D3h

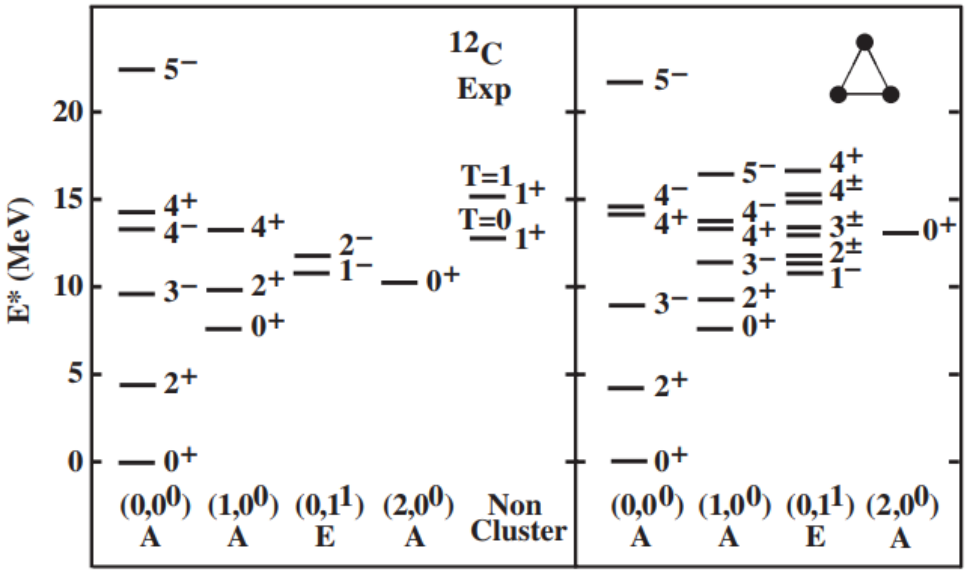


Spectrum of an equilateral triangle configuration

Notice the 'apparently strange' quantum numbers. They have a perfectly clear interpretation in the theory of point-groups ! They come from the reduction of spherical states to discrete symmetry states $SO(3) \supset D_{3h}$ and $SO(2) \supset D_{3h}$ Not all L are compatible with this operation.

Evidence for triangular D_{3h} symmetry in ^{12}C
[Marín-Lambarri PRL 113 (2014) 012502]



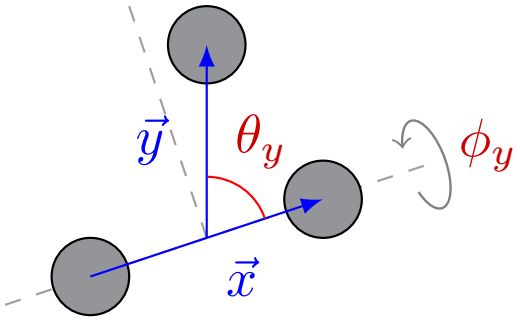


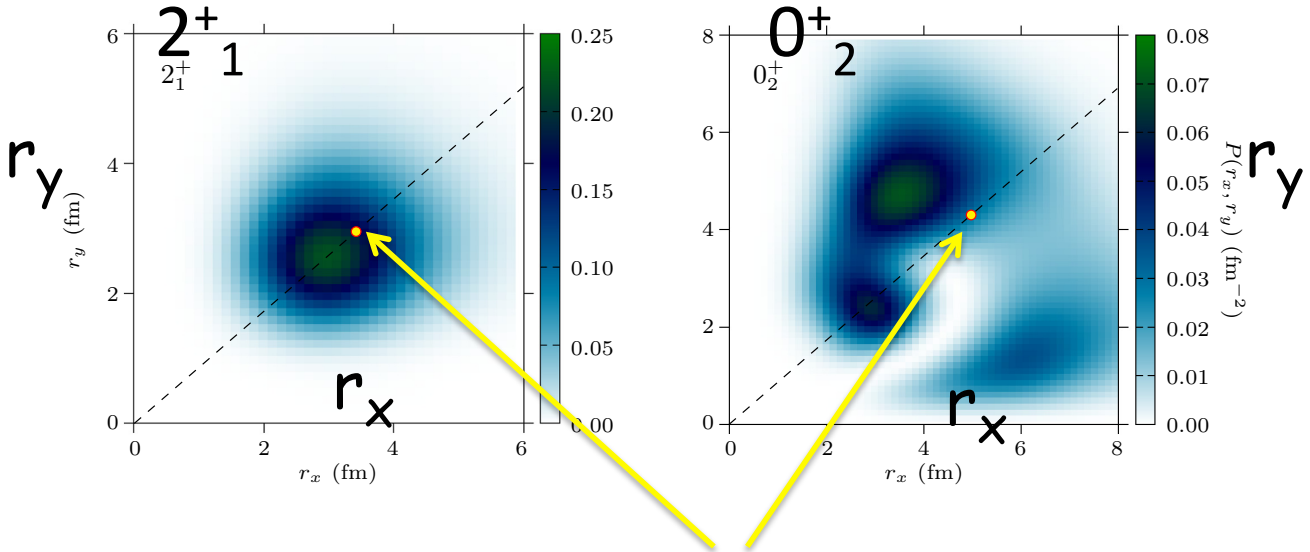
Further support to the basic triangular shape comes from our calculations based on the quantal solution of the problem of three interaction alpha's (via the two-body Ali-Bodmer plus a three-body potential) within the Hyperspherical Harmonics approach (HH), using for the continuum part pseudo-states given by Transformed Harmonic Oscillator (THO).

Results from the HH formulation support the description of the ground and 2^+_1 state of ^{12}C as three particles with the symmetry of an equilateral triangle. On the other hand the wave function for the Hoyle state seems to present a more complicated structure.

w.f. distributed around a mean triangular configuration

$|\Psi|^2$





Casal, Fortunato, Vitturi, 2019

The computed r_x, r_y r.m.s. values (yellow dots) are found to satisfy the equilateral ratio $r_y = r_x\sqrt{3}/2$ (dashed)

Densities and transition densities within the algebraic approach

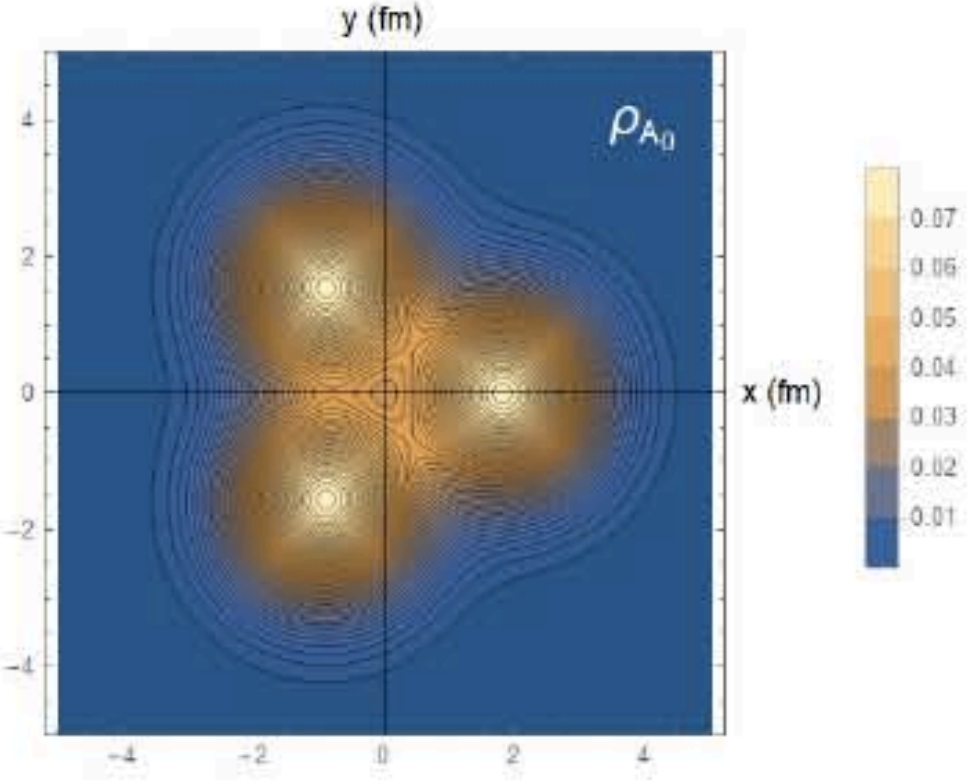
Gaussian density for an α:

$$\rho_\alpha(\vec{r}) = \left(\frac{d}{\pi}\right)^{3/2} e^{-dr^2}$$

$d = 0.56(2)$ to fit α radius

$\Rightarrow$ "static" ^{12}C g.s. density:

$$\rho_{A_0}(\vec{r}, \{\vec{r}_k\}) = \sum_{k=1}^{3} \rho_\alpha(\vec{r} - \vec{r}_k)$$



A_0: fully symmetric representation of D_{3h} with 0 quanta of excitation

$\vec{r}_1 = (\beta, \pi/2, 0)$, $\vec{r}_2 = (\beta, \pi/2, 2\pi/3)$, $\vec{r}_3 = (\beta, \pi/2, 4\pi/3)$

Radial parameter β to reproduce some ^{12}C g.s. properties

Density expanded in spherical harmonics (ground-state band A_0)

$$\rho_{A_0}(\vec{r}) = \sum_{\lambda\mu} \rho^{\lambda\mu}_{A_0}(r) Y_{\lambda\mu}(\theta,\varphi)$$

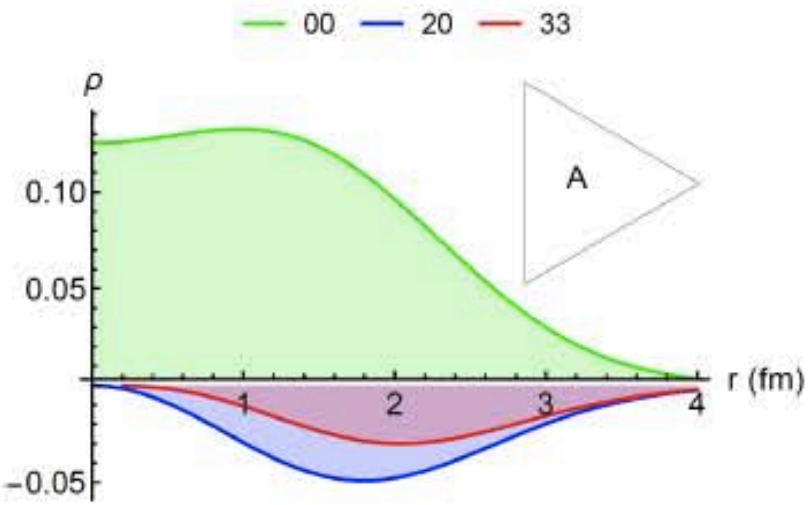
Only multipoles allowed by D_{3h}:
$\{00,20,33,\dots\}$

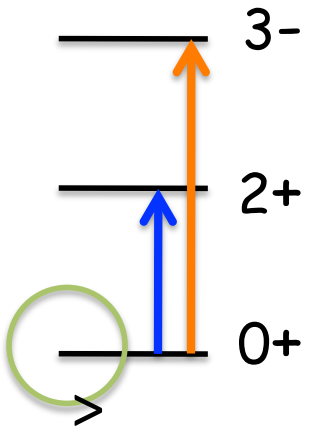
Radial transition densities

$\rho^{00}_{A_0}(r)$ intrinsic 0^+_1 g.s. density

$\rho^{20}_{A_0}(r)$ associated to 2^+_1

...





ground band

$\beta = 1.82$ fm $\Rightarrow$ reproduce the g.s. radius and the $B(E2)$ value to the 2^+_1

$$\sqrt{\langle r^2\rangle_{0^+_1}} = \frac{\sqrt{4\pi}}{3}\int r^4 \rho^{00}_{A_0}(r)dr, \qquad M(E\lambda;\lambda\to 0^+_1) = Z\int r^{2+\lambda}\rho^{\lambda}_{A_0}(r)dr$$

TABLE I: Calculated observables within the g.s. band.

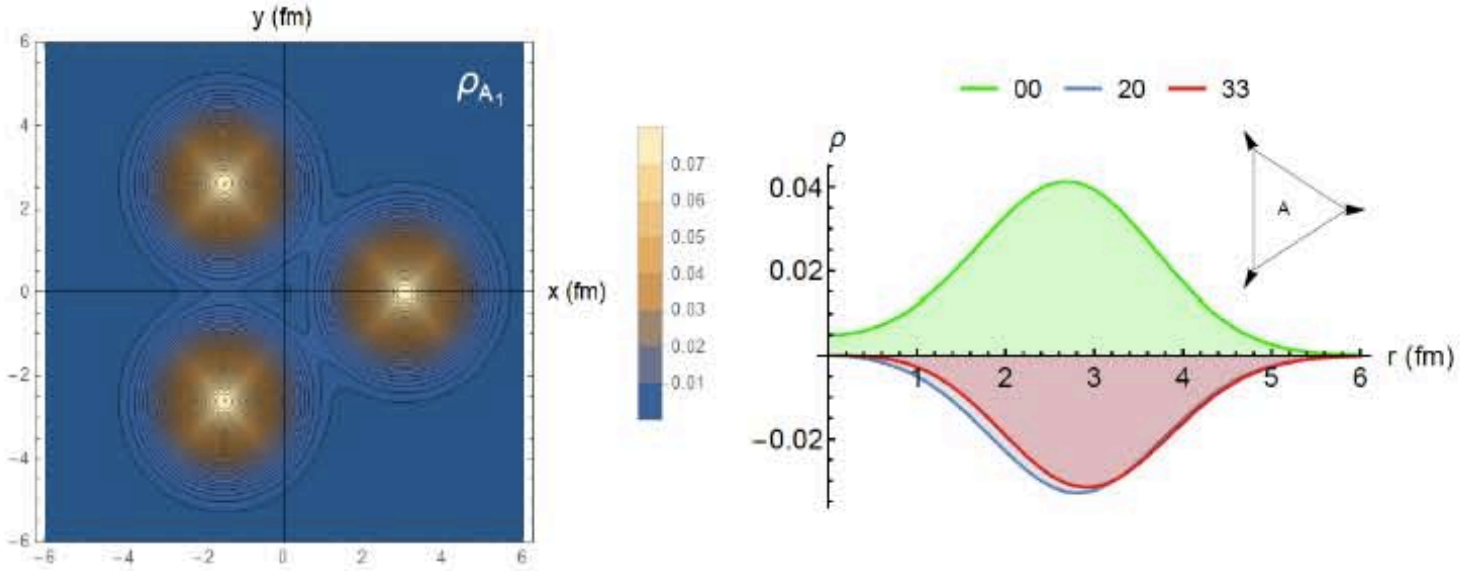
$\langle r^2 \rangle^{1/2}_{0^+_1}$	2.45 (fm)
$B(E2; 2^+_1 \to 0^+_1)$	7.86 (e^2fm^4)
$B(E3; 3^-_1 \to 0^+_1)$	65.07 (e^2fm^6)
$B(E4; 4^+_1 \to 0^+_1)$	96.99 (e^2fm^8)

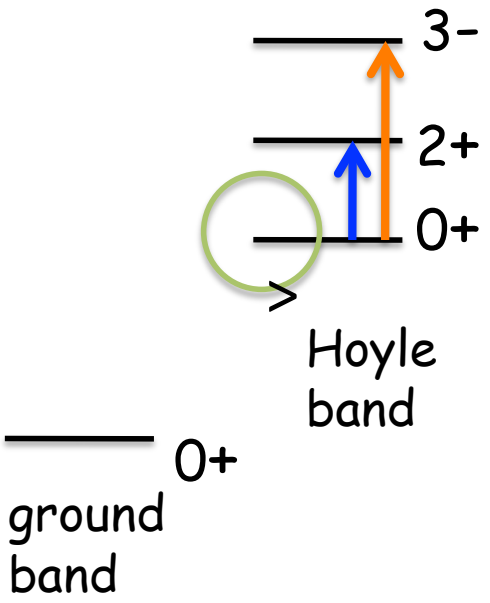
	exp		AMD		AMD+GCM		RGM	
	$B(E\lambda)$	(error)	$B(E\lambda)$	$f_{\rm tr}$	$B(E\lambda)$	$f_{\rm tr}$	$B(E\lambda)$	$f_{\rm tr}$
$E2: 2^+_1 \to 0^+_1$	7.59	(0.42)	8.53	0.94	9.09	0.91	9.31	0.90
$E0: 0^+_2 \to 0^+_1$	29.2	(0.2)	43.5	0.82	43.3	0.82	43.8	0.82
$E2: 0^+_2 \to 2^+_1$	13.5	(1.4)	25.1	0.73	24.1	0.75	5.6	1.56
$E2: 2^+_2 \to 0^+_1$	1.57[a]	(0.13)	0.39	1.99	0.49	1.93	2.48	0.80
$E2: 3^-_1 \to 1^-_1$			40.7	1	79.0	1		
$E0: 0^+_3 \to 0^+_1$			5.2	1	10.0	1		
IS1 : $1^-_1 \to 0^+_1$			2.6	1.57[b]	2.4	1.93[b]	5.7	1
IS1 : $1^-_2 \to 0^+_1$					1.5	1		
$E3: 3^-_1 \to 0^+_1$	103	(17)	71	1.20	71	1.20	125	0.91
$E4: 4^+_1 \to 0^+_1$			733	1	995	1	655	1
$E3: 3^-_1 \to 0^+_2$			428	1	1210	1	228	1
$E2: 2^+_2 \to 0^+_2$			102	1	182	1	212	1
$E2: 2^+_2 \to 0^+_3$			309	1	223	1		

A-type vibration with $n = 1$ (Hoyle-state band, A_1)

Symmetric displacements $\Delta\beta^A$; "breathing mode"

$\Delta\beta^A = 1.2$ fm $\Rightarrow \sim 1$ **fm increase in radius**
(consistent with e.g. Ito [PRC 97 (2018) 044608] and others)





Transition densities connecting the g.s. and Hoyle bands ($A_0 \to A_1$)

Expansion in small displacements $\Delta\beta$, kept at leading order:

$$\delta\rho_{A_0\to A_1}(\vec{r}) \simeq \chi_1 \frac{d}{d\beta}\rho_{A_0}(\vec{r},\beta) = \sum_{\lambda\mu} \delta\rho^{\lambda\mu}_{A_0\to A_1}(r) Y_{\lambda\mu}(\theta,\varphi)$$

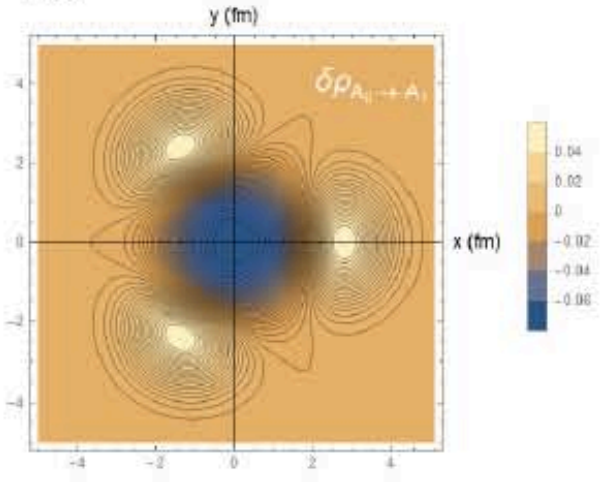
χ_1 adjusted to reproduce $E0$ strength

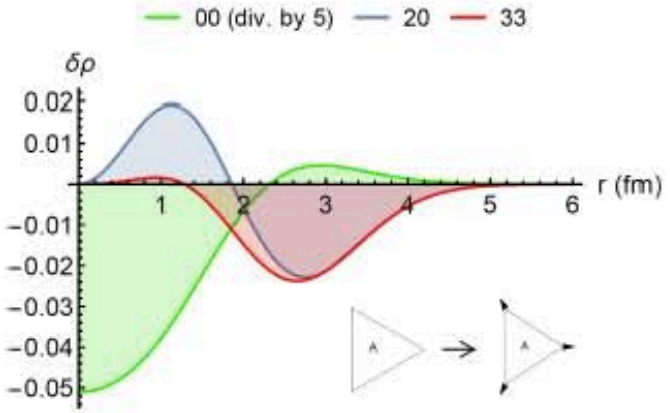
Interband radial transition densities

$\delta\rho^{00}(r)$ monopole transition 0^+_1 g.s. $\to 0^+_2$ Hoyle

$\delta\rho^{20}(r)$ quadrupole transition 0^+_1 g.s. $\to 2^+_2$

...





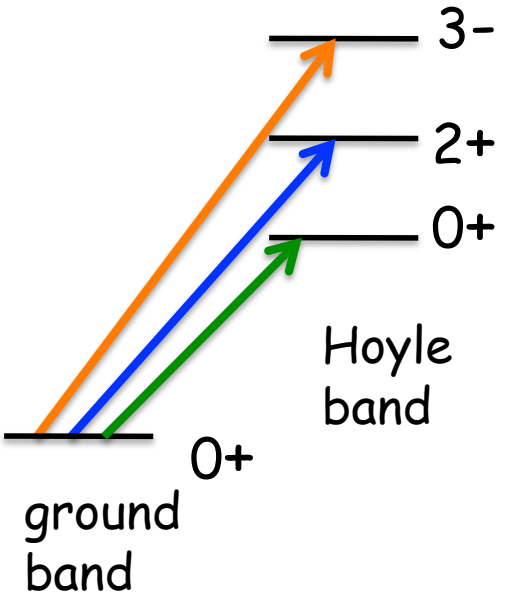
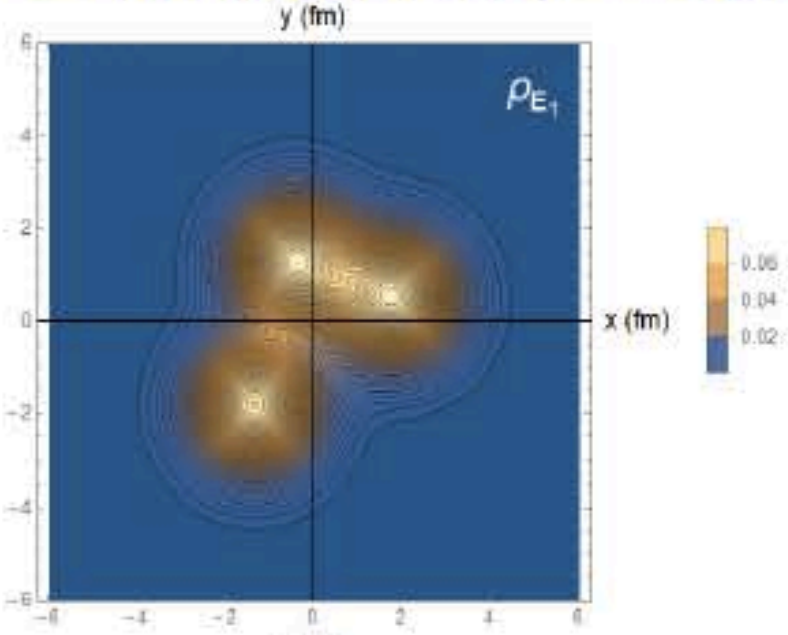


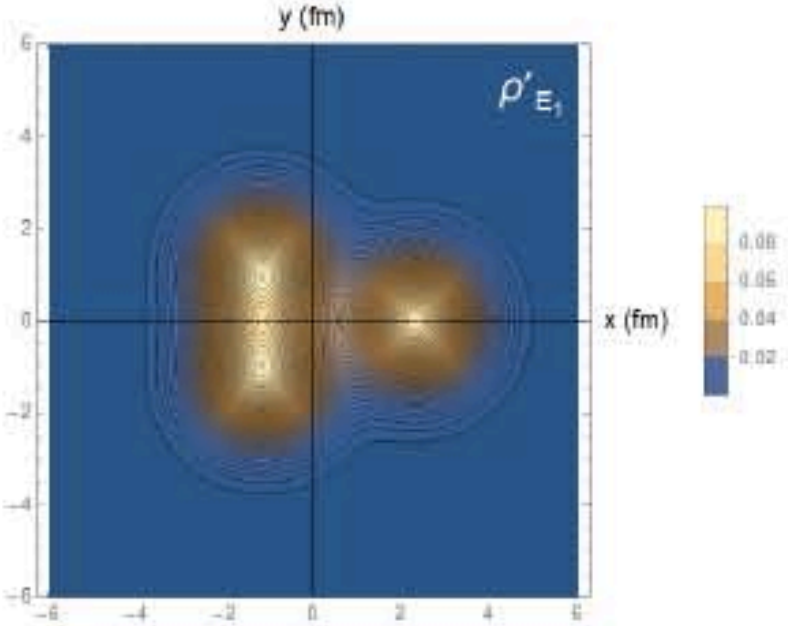
TABLE II: Quantities calculated in the present work for the Hoyle band, using the values of β, χ_1 given in the text.

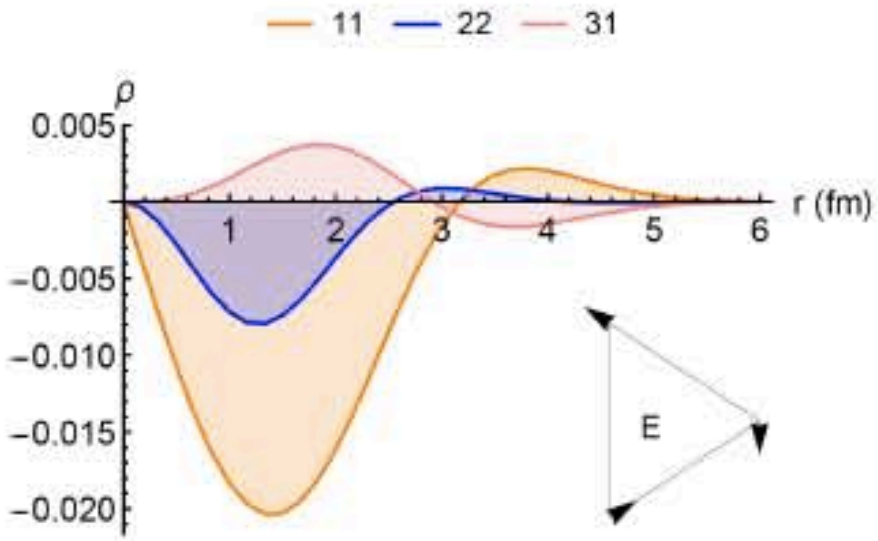
$\langle r^2 \rangle^{1/2}_{0^+_2}$	3.44 (fm)
$B(E2; 2^+_2 \to 0^+_1)$	0.58 (e^2fm^4)
$B(E2; 0^+_2 \to 2^+_1)$	2.90 (e^2fm^4)
$B(E3; 3^-_2 \to 0^+_1)$	70.42 (e^2fm^6)
$M(E0; 0^+_2 \to 0^+_1)$	5.4 (e fm^2)

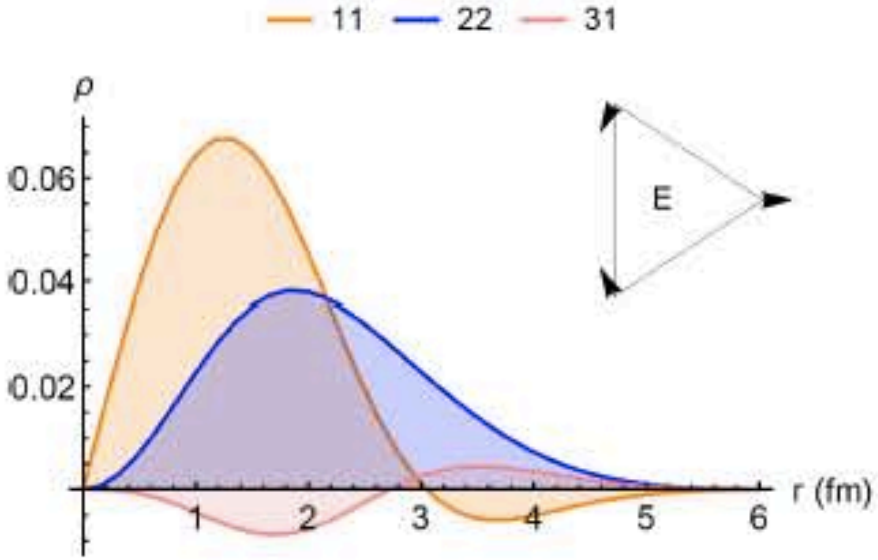
	exp		AMD		AMD+GCM		RGM	
	$B(E\lambda)$	(error)	$B(E\lambda)$	$f_{\rm tr}$	$B(E\lambda)$	$f_{\rm tr}$	$B(E\lambda)$	$f_{\rm tr}$
$E2: 2^+_1 \to 0^+_1$	7.59	(0.42)	8.53	0.94	9.09	0.91	9.31	0.90
$E0: 0^+_2 \to 0^+_1$	29.2	(0.2)	43.5	0.82	43.3	0.82	43.8	0.82
$E2: 0^+_2 \to 2^+_1$	13.5	(1.4)	25.1	0.73	24.1	0.75	5.6	1.56
$E2: 2^+_2 \to 0^+_1$	1.57[a]	(0.13)	0.39	1.99	0.49	1.93	2.48	0.80
$E2: 3^-_1 \to 1^-_1$			40.7	1	79.0	1		
$E0: 0^+_3 \to 0^+_1$			5.2	1	10.0	1		
IS1 : $1^-_1 \to 0^+_1$			2.6	1.57[b]	2.4	1.93[b]	5.7	1
IS1 : $1^-_2 \to 0^+_1$					1.5	1		
$E3: 3^-_1 \to 0^+_1$	103	(17)	71	1.20	71	1.20	125	0.91
$E4: 4^+_1 \to 0^+_1$			733	1	995	1	655	1
$E3: 3^-_1 \to 0^+_2$			428	1	1210	1	228	1
$E2: 2^+_2 \to 0^+_2$			102	1	182	1	212	1
$E2: 2^+_2 \to 0^+_3$			309	1	223	1		

E-type vibration with $n = 1$ (doubly degenerate E_1)

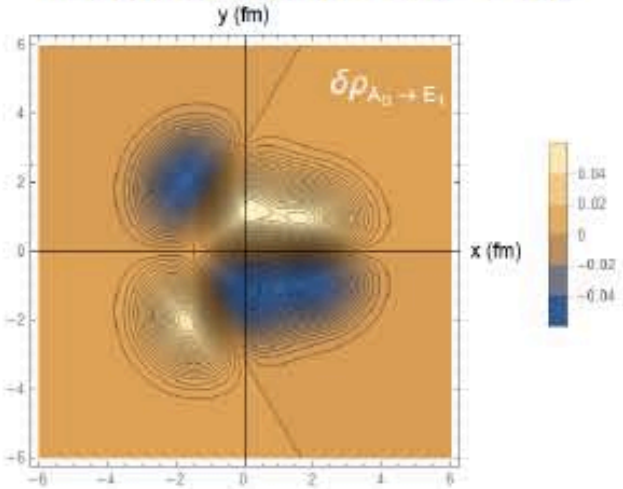


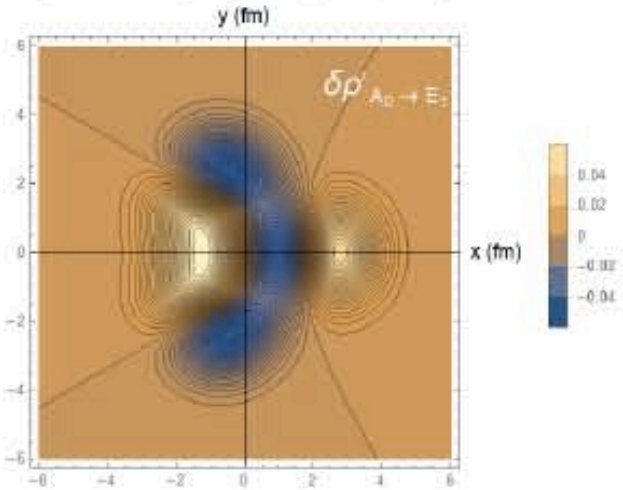


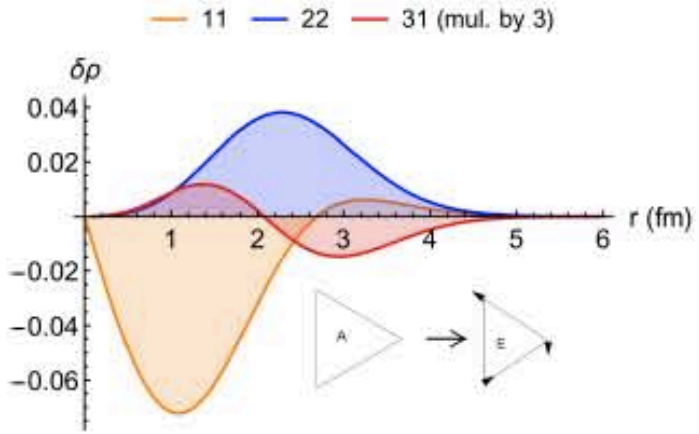


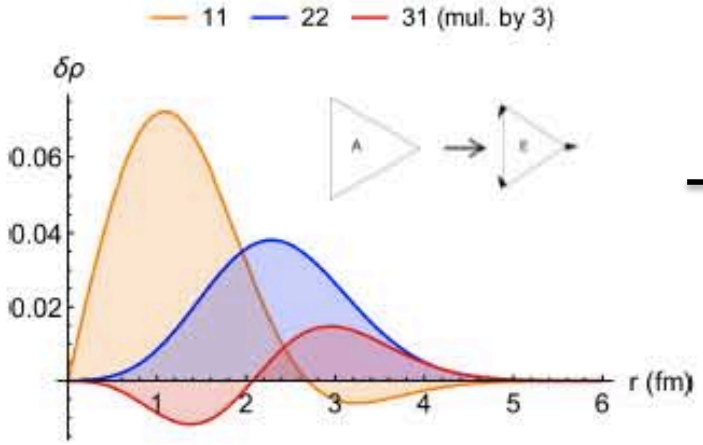


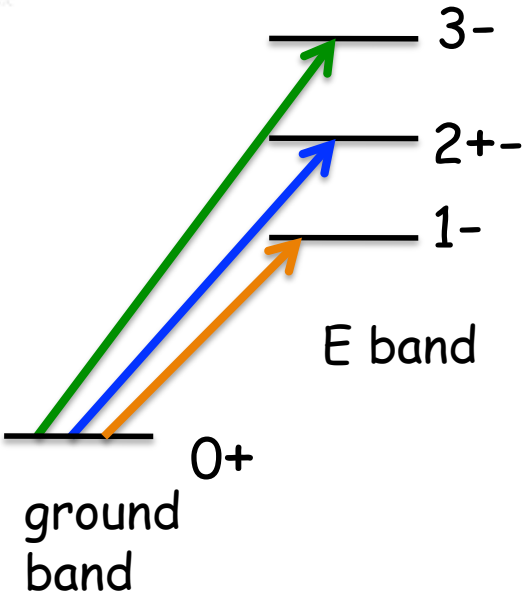
Transition densities $A_0 \to E_1$







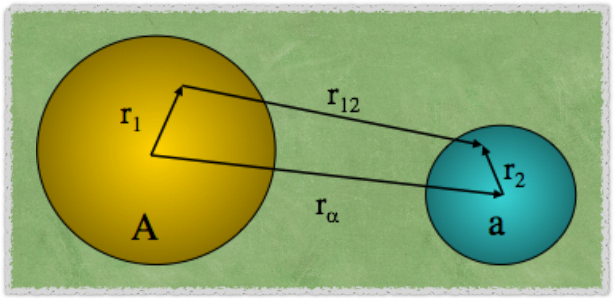




From densities and transition densities to potentials and form factors

alpha+^{12}C: folding densities and transition densities with the density of the projectile alpha

Double folding procedure



Form factors $F_{ij}(R)$, using:

- Transition densities within the algebraic cluster model

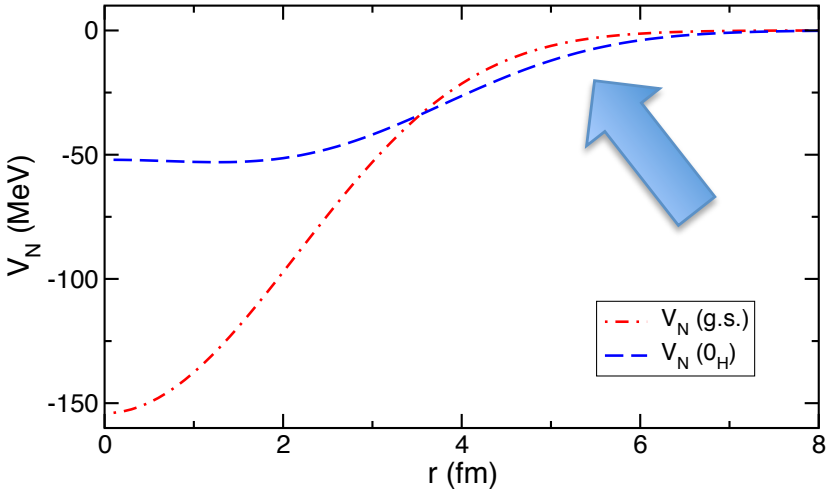
$$F_{ij}(R) = \int\int \rho_\alpha(\vec{r}_1 - \vec{R}) v_{NN}(|\vec{r}_{12}|) \delta\rho^{i\to j}(\vec{r}_2) d\vec{r}_1 d\vec{r}_2$$

(v_{NN}: Reid-type M3Y)

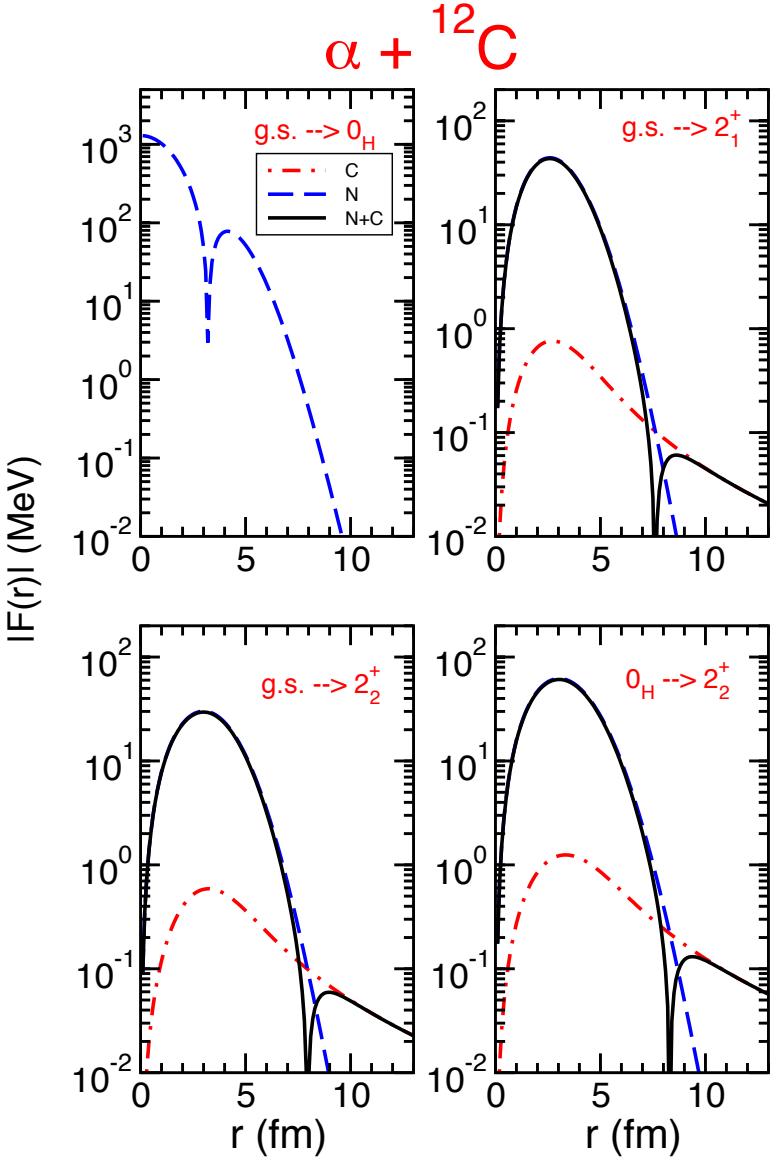
Diagonal terms: potentials

Potentials are different within the different bands

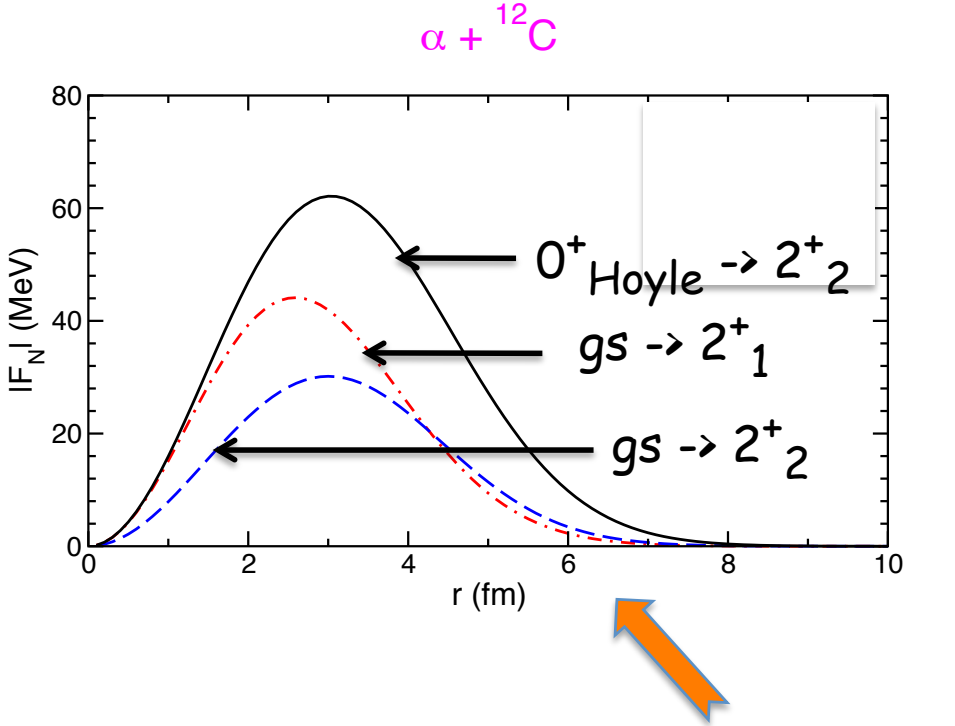
$\alpha + {}^{12}C$



Formfactors (including nuclear and coulomb contributions)



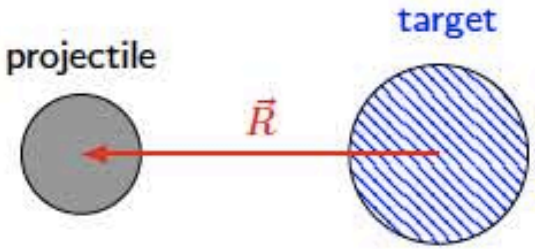
Nuclear part of the form factors with λ=2



surface region

From potentials and form factors to cross sections

Coupled-channels calculations: $\alpha + {}^{12}\mathrm{C}$ inelastic scattering



$$c \equiv \{L(nj)\},\ \boldsymbol{J} = \boldsymbol{L} + \boldsymbol{j}$$

$$\Psi_c^{JM}(\boldsymbol{\xi}, \boldsymbol{R}) = \sum_{c'} \frac{i^L}{R} \chi^J_{c,c'}(R) \Phi^{JM}_{c'}(\widehat{R}, \boldsymbol{\xi})$$

$$\Phi_c^{JM}(\widehat{R}, \boldsymbol{\xi}) = \Big[Y_L(\widehat{R}) \otimes \phi_{nj}(\boldsymbol{\xi}) \Big]_{JM}$$

$$\Big[-\frac{\hbar}{2m_r} \left(\frac{d^2}{dR^2} - \frac{L(L+1)}{R^2} \right) + E_{nj} - E \Big] \chi^J_{c,c}(R)$$
$$+ \sum_{c'} i^{L'-L} V^{JM}_{c,c'}(R) \chi^J_{c,c'}(R) = 0$$

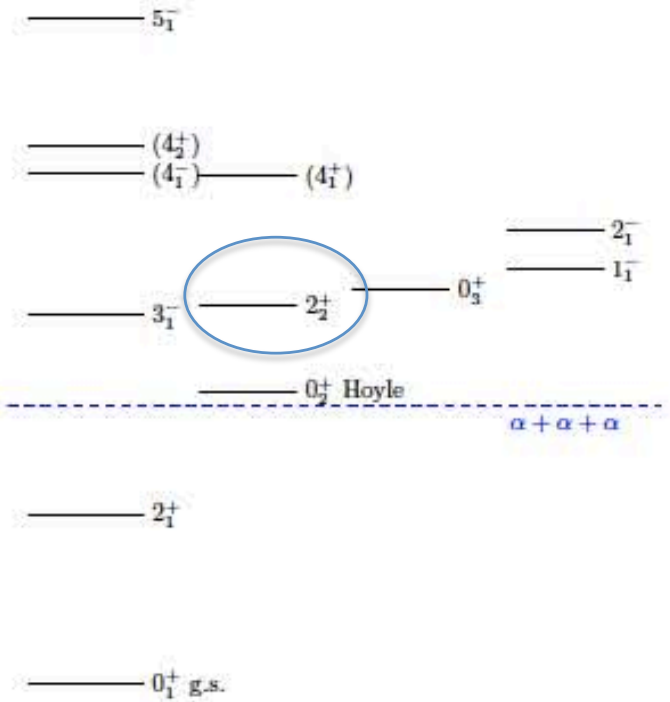
Requires $V^{JM}_{c,c'}(R) = \langle \Phi^{JM}_c | \widehat{U}_{pt} | \Phi^{JM}_{c'} \rangle$ coupling potentials;

can be written in terms of **form factors** $F_{n'j',nj}(R)$

α inelastic scattering on ^{12}C

- Population of cluster states
- Study of isoscalar monopole and dipole excitations

our model space



E.g.:

2^+_2 state measured at 9.84 MeV
[Itoh PRC84(2011)054308]

5^-_1 state measured at 22.4 MeV
[Marín-Lambarri PRL113(2014)012502]

Many theoretical coupled-channel (CC) calculations using different (microscopic) structure models for ^{12}C:
AMD [Kanada-En'yo], FMD [Neff, Feldmeier], RGM [Kamimura, Ogata], ...

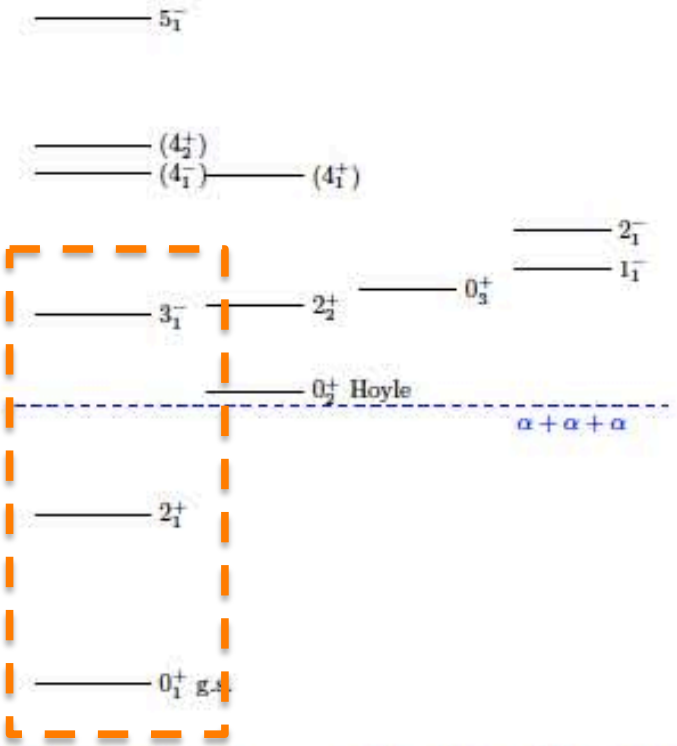
This work: Assess whether the algebraic model describes exp. data
Compare results with those from the HH framework

α inelastic scattering on ^{12}C

- Population of cluster states
- Study of isoscalar monopole and dipole excitations

our model space

first trial case



E.g.:

2^+_2 state measured at 9.84 MeV
[Itoh PRC84(2011)054308]

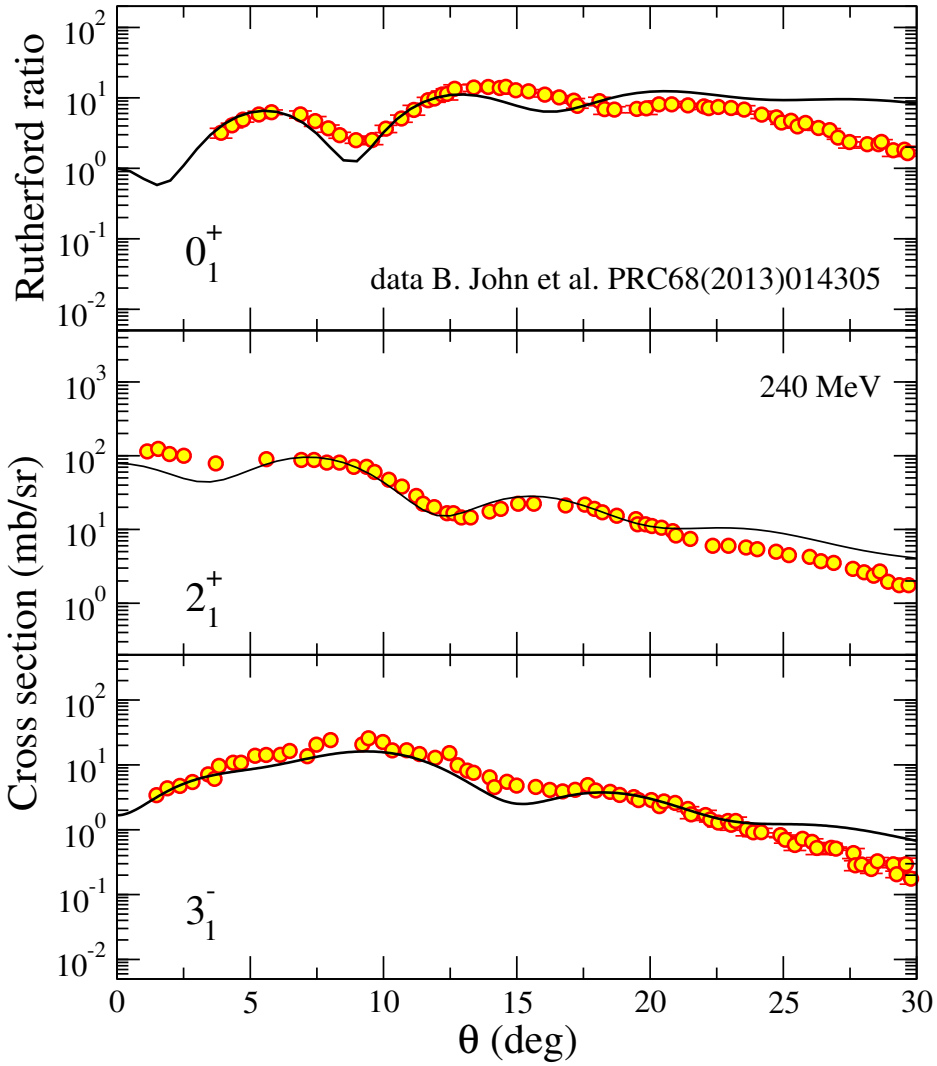
5^-_1 state measured at 22.4 MeV
[Marín-Lambarri PRL113(2014)012502]

Many theoretical coupled-channel (CC) calculations using different (microscopic) structure models for ^{12}C:
AMD [Kanada-En'yo], FMD [Neff, Feldmeier], RGM [Kamimura, Ogata], ...

This work: Assess whether the algebraic model describes exp. data
Compare results with those from the HH framework

First results within the ground band

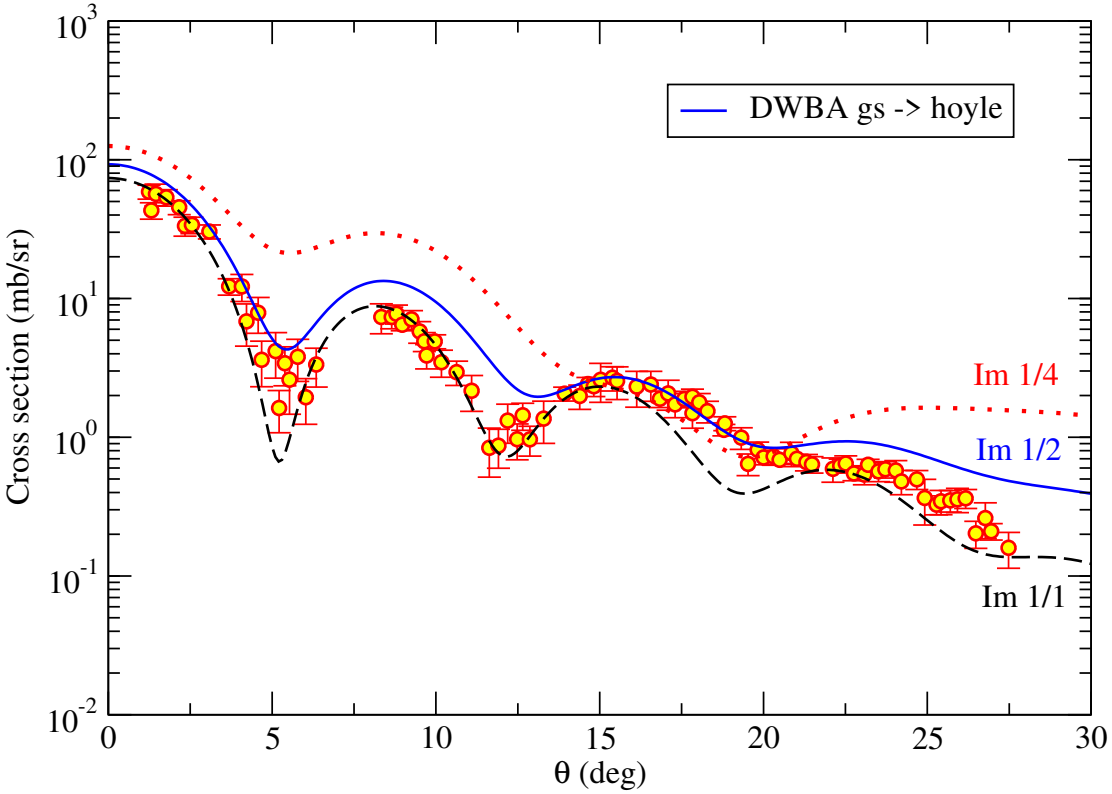
DWBA α+12C 240MeV



elastic

inelastic
(to the members of the ground-state band)

To the Hoyle state ….. others in progress ….



Obs: Sensitivity to the imaginary part of optical potentials

Thanks to
Jesus Casal and Lorenzo Fortunato (Padova)
Edoardo Lanza (Catania)
Jose' Antonio Lay (Sevilla)