

# Fingerprints of alpha-like quartet correlations in N=Z nuclei

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- Quartetting in nuclei: historical perspective
- Proton-neutron (pn) pairing and quartet correlations
  - competition between T=1 and T=0 pn pairing in N=Z nuclei
  - effect of pn pairing on Wigner energy (preliminary results)
- Quartet correlations and 4-body density matrix

**(alpha-like) quartet** = a **correlated** structure of two neutrons  
and two protons

correlations in “configuration” space (e.g., spin & isospin)

quartets as “elementary” degrees of freedom ?

- the quartets should contain a “large” correlation energy
- the interaction between the quartets should be small

# EFFECT OF QUADRUPLE CORRELATIONS IN LIGHT NUCLEI

V G SOLOVIEV

*Joint Institute of Nuclear Research, Dubna, USSR*

Received 25 December 1959

“quadruple”= two interacting pn pairs

## Fingerprints of alpha-like (quadruple) correlations

- 1) Extracting a pn pair from a even-even N=Z nucleus costs more energy than adding to it a pn pair



- 2) Extracting one neutron from a even-even N=Z nucleus costs more energy than from neighbouring nuclei

$$B(^{24}\text{Mg}) - B(^{23}\text{Mg}) = 16.6 \text{ MeV}$$

$$B(^{25}\text{Mg}) - B(^{24}\text{Mg}) = 7.3 \text{ MeV}$$

$$B(^{26}\text{Mg}) - B(^{25}\text{Mg}) = 11.3 \text{ MeV}$$

to break a quadruple (quartet) in pairs takes about 4-5 MeV

EVIDENCE FOR QUARTET STRUCTURE IN MEDIUM  
AND HEAVY NUCLEIM. DANOS<sup>‡</sup> and V. GILLET*Service de Physique Théorique, Centre d'Etudes Nucléaires de Saclay,  
B.P. no 2-91-Gif-sur-Yvette, France*

Received 1 November 1970

The second differences of the nuclear masses keeping  $T$  constant are discussed for even-even nuclei throughout the mass table. They are shown to be consistent with the quartet picture of weakly interacting tight two-proton two-neutron structures

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ENERGIES OF QUARTET STRUCTURES IN EVEN-EVEN  $N=Z$  NUCLEI

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(Received 7 August 1970)

Mass relationships are used to compute the energy of quartet excited states in  $N=Z$  even-even nuclei for  $^{12}\text{C}$  up to  $^{52}\text{Fe}$ . The states obtained are quasibound up to excitation energies of about 40 MeV and could account for the narrow structures recently observed in heavy-ion transfer experiments.

# REMARKABLE LONG-RANGE-SYSTEMATICS IN THE BINDING ENERGIES OF $\alpha$ -NUCLEI

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Phys. Lett. B130, 131 (1983)

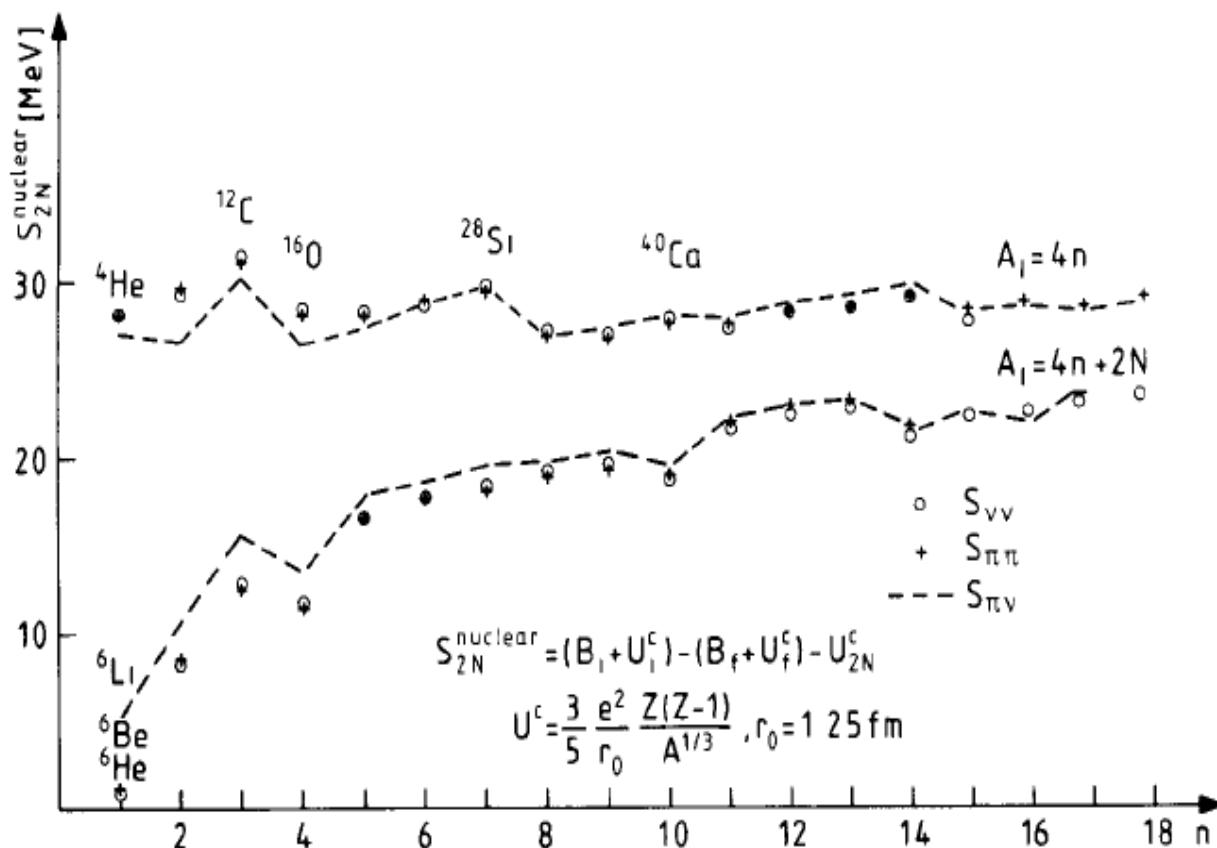


Fig. 1. The nuclear separation energies  $S_{2N}^{\text{nuclear}}$  for the separation of 2 neutrons  $\nu\nu$   $\circ$ , 2 protons  $\pi\pi$   $+$ , and a neutron-proton pair  $\nu\pi$   $--$  as a function of  $n$ , the number of  $\alpha$ -particles contained in the respective nucleus. This upper curve is for  $\alpha$ -nuclei, the lower one for  $\alpha$ -nuclei plus this nucleon pair.

## Quartet Effects in Rare-Earth Nuclei

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(Received 5 March 1986)

Quartet effects in deformed rare-earth nuclei are confronted from a phenomenological point of view. Some very simple systematic trends are evident in the experimental data when plotted as a function of a quartet number. The interacting-boson model has been modified to include quartet effects explicitly and it is able to reproduce accurately the experimental trends with fixed parameters.

PACS numbers: 21.10.Re, 21.60.Fw, 23.20.Lv

# CLUSTER OF NUCLEONS AS ELEMENTARY MODES OF EXCITATION IN NUCLEI

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CNEA, Dto. de Fisica, Avda. del Libertador 8250, 1429, Buenos Aires, Argentina

Received 9 February 1982

(Revised 10 May 1982)

Nuclear Physics A388 (1982) 606-620

**Abstract:** Conditions which must be fulfilled by clusters of nucleons to qualify as elementary modes of excitation are analysed in terms of simple criteria involving experimental binding energies. It is found that the most complex possible mode is the  $\alpha$ -like cluster.

$$T(A) = B(A) - B(A + \alpha)$$

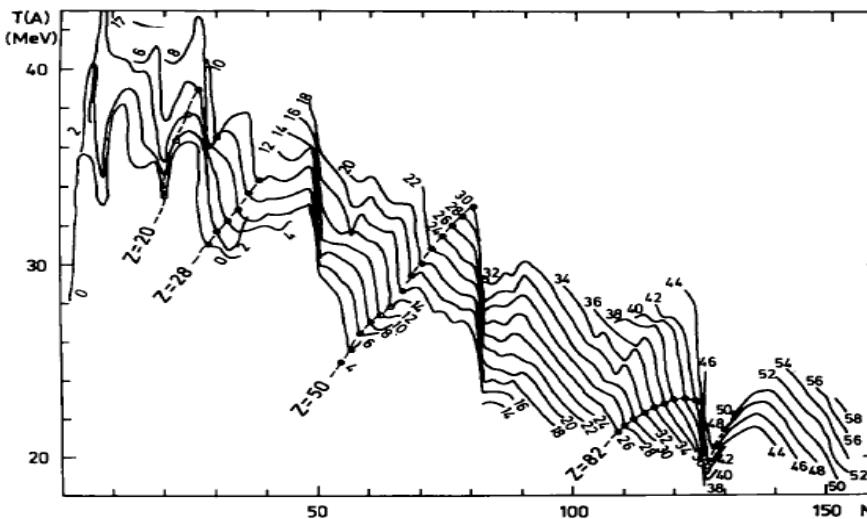


Fig. 6. Separation energy  $T(A)$  [eq. (7)]. The lines join nuclei with the same  $T_z = \frac{1}{2}(N - Z)$ . These lines are labelled by the number  $2T_z$ . The circles correspond to nuclei having a magic number of protons.

constant slope of  $T(A)$ : ground state interpretation as an alpha condensat

## proton-neutron (pn) pairing and quartetting

- isovector pn pairing:  $S=0/J=0, T=1$   
together with nn, pp  $S=0$  pairing
- isoscalar pn pairing:  $S=1/J=1, T=0$

“condensate” of pn pairs ?

SOVIET PHYSICS JETP

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*SUPERFLUIDITY OF LIGHT NUCLEI*

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Joint Institute of Nuclear Research

Submitted to JETP editor October 12, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 952-954 (March, 1960)

first BCS treatment of isovector proton-neutron pairing

"we must take into consideration the quadruple correlation of alpha-particle-like nucleons [...];  
these new correlations evidently play a very important role and somewhat mask the effect of pair correlations"

EFFECT OF QUADRUPLE CORRELATIONS IN LIGHT NUCLEI

V G SOLOVIEV

*Joint Institute of Nuclear Research, Dubna, USSR*

Received 25 December 1959

# Theoretical studies on pn pairing & alpha correlations

B. H. Flowers and M. Vijicic, NPA49(1963)

$$\Phi = \prod_{ll'mm'} \prod'_{pqrs} (U_l + V_{ll'} \epsilon_{pqrs} a_{lmp}^\dagger a_{l-mq}^\dagger a_{l'm'r}^\dagger a_{l'-m's}^\dagger) |0\rangle$$

*not actually solved !*

B. Bremond and J. G. Valatin NP41(1963)

$$\prod_\alpha (S_\alpha + V_{\alpha p} a_{\alpha p}^\dagger a_{\bar{\alpha} p}^\dagger + V_{\alpha n} a_{\alpha n}^\dagger a_{\bar{\alpha} n}^\dagger + T_\alpha a_{\alpha p}^\dagger a_{\bar{\alpha} p}^\dagger a_{\alpha n}^\dagger a_{\bar{\alpha} n}^\dagger) |0\rangle$$

*not included all relevant configurations !*

J. Eichler and M. Yamamura, NPA182(1972)

*non-collective quartets*

R. Chasman, PLB577(2003)

J. Dobes and S. Pittel PRC57(1998)

R. A. Senkov and V. Zelevinski (2011)

*quartetting for a degenerate state*

.....

almost all studies on pn pairing: in BCS/HFB approximation

Goodman , ..., Bertsch & Gezerlis

**drawbacks: particle-number and isospin are not conserved !**

beyond BCS/HFB ?

Isovector ( $T=1$ ) pairing in terms of quartets

# Isovector pairing in term of quartets

$$H = \sum_i \varepsilon_i (N_i^{(\nu)} + N_i^{(\pi)}) + \sum_{ij,\tau} V(i,j) P_{i,\tau}^+ P_{j,\tau}$$

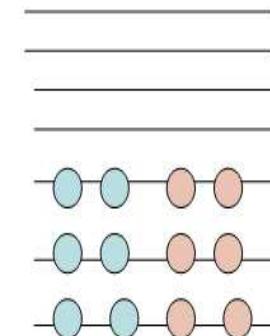
$$P_{i1}^+ \propto \nu_i^+ \nu_{\bar{i}}^+ \quad P_{i-1}^+ \propto \pi_i^+ \pi_{\bar{i}}^+ \quad P_{i0}^+ \propto \nu_i^+ \pi_{\bar{i}}^+ + \pi_i^+ \nu_{\bar{i}}^+$$

$N=Z$

***non-collective quartets***

$$Q_{ij}^+ = [P_{i\tau}^+ P_{j\tau'}^+]^{T=0} \propto P_{\nu\nu,i}^+ P_{\pi\pi,j}^+ + P_{\pi\pi,i}^+ P_{\nu\nu,j}^+ - P_{\nu\pi,i}^+ P_{\nu\pi,j}^+$$

**collective quartet**



$$Q^+ = \sum_{ij} x_{ij} [P_{i\tau}^+ P_{j\tau'}^+]^{T=0}$$

***quartet condensate***

$$| QCM \rangle = Q^{+n_q} | - \rangle \quad (\text{has } T=0, J=0)$$

# Quartet condensation versus pair condensation

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i, j) \sum_t P_{it}^+ P_{jt}$$

pairing forces extracted from SM interactions

$$|QCM\rangle \equiv (Q^+)^{n_q} |-\rangle \quad |PBCS\rangle \propto (\Gamma_{vv}^+ \Gamma_{\pi\pi}^+)^{n_q} |-\rangle \quad |PBCS0\rangle \propto (\Gamma_{v\pi}^{+2})^{n_q} |-\rangle$$

	SM	QCM	PBCS1	PBCS0
<sup>20</sup> Ne	9.173	9.170 (0.033%)	8.385 (8.590%)	7.413 (19.187%)
<sup>24</sup> Mg	14.460	14.436 (0.166%)	13.250 (8.368%)	11.801 (18.389%)
<sup>28</sup> Si	15.787	15.728 (0.374%)	14.531 (7.956%)	13.102 (17.008%)
<sup>32</sup> S	15.844	15.795 (0.309%)	14.908 (5.908%)	13.881 (12.389%)
<sup>44</sup> Ti	5.973	5.964 (0.151%)	5.487 (8.134%)	4.912 (17.763%)
<sup>48</sup> Cr	9.593	9.569 (0.250%)	8.799 (8.277%)	7.885 (17.805%)
<sup>52</sup> Fe	10.768	10.710 (0.539%)	9.815 (8.850%)	8.585 (20.273%)
<sup>104</sup> Te	3.831	3.829 (0.052%)	3.607 (5.847%)	3.356 (12.399%)
<sup>108</sup> Xe	6.752	6.696 (0.829%)	6.311 (6.531%)	5.877 (12.959%)
<sup>112</sup> Ba	8.680	8.593 (1.002%)	8.101 (6.670%)	13.064 (13.064%)

## Conclusions

- *T=1 pairing is accurately described by quartets, not by pairs*
- *there is not a pure condensate of isovector pn pairs in N=Z nuclei*

## Quartet condensation versus isospin-projected BCS

$$H = \sum_i \varepsilon_i (N_i^{(v)} + N_i^\pi) - g \sum_{ij,\tau} P_{i,\tau}^+ P_{j,\tau}$$

$$|QCM\rangle \equiv (Q^+)^{n_q} |-\rangle \quad |PBCS(N,T)\rangle = \hat{P}_T \hat{P}_N |BCS\rangle$$

$$E_{corr} = E_0 - E$$



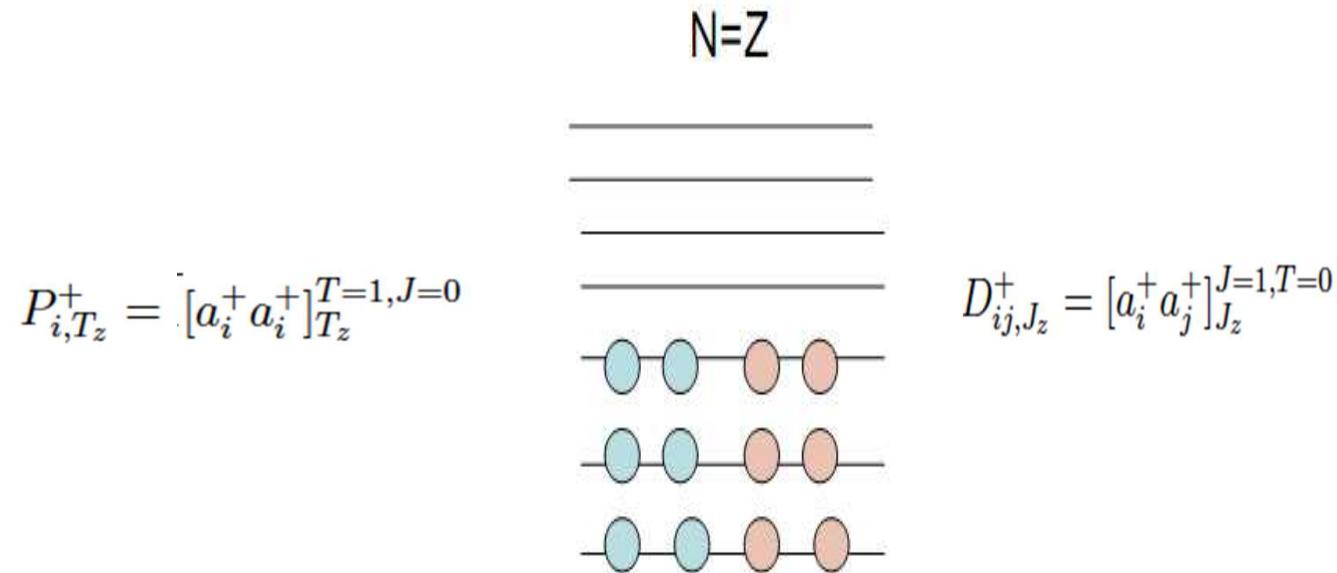
**Exact value:** 8.29 MeV

PBCS(N,T): 7.63 MeV (8%) (Chen et al , Nucl. Phys.A 1978)

QCM: 8.25 MeV (0.5%)

**QCM is not equivalent with PBCS (N,T) !**

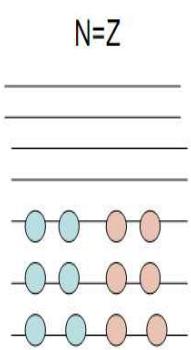
# Isoscalar and isovector pairing in N=Z nuclei



# Quartetting for isovector ( $J=0$ ) and isoscalar ( $J=1$ ) pairing

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{\tau} P_{i\tau}^+ P_{j\tau}^- + \sum_{ij} V_{J=1}^{T=0}(i,j) \sum_{\sigma} D_{i\sigma}^+ D_{j\sigma}^-$$

isovector	isoscalar	
$P_{i,T_z}^+ = [a_i^+ a_i^+]_{T_z}^{T=1, J=0}$	$D_{ij,J_z}^+ = [a_i^+ a_j^+]_{J_z}^{J=1, T=0}$	$N=\mathbb{Z}$ _____
collective quartets		_____
$Q_\nu^{+(iv)} = \sum_{i,j} x_{ij}^{(\nu)} [P_i^+ P_j^+]^{T=0}$	$Q_\nu^{+(is)} = \sum_{ij,kl} y_{ij,kl}^{(\nu)} [D_{ij}^+ D_{kl}^+]^{J=0}$	_____
generalised quartet		_____
$Q_\nu^+ = Q_\nu^{+(iv)} + Q_\nu^{+(is)}$		_____



**ground state**

$$| \text{QCM} \rangle = Q^{+n_q} | - \rangle$$

# Quartet condensation versus pair condensation for isovector & isoscalar pairing

$$H = \sum_i \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{\tau} P_{i\tau}^+ P_{j\tau}^- + \sum_{ij} V_{J=1}^{T=0}(i,j) \sum_{\sigma} D_{i\sigma}^+ D_{j\sigma}^-$$

$$(Q^+)^{n_q} |-> (\Gamma_{\nu\nu}^+ \Gamma_{\pi\pi}^+)^{n_q} |-> (\Gamma_{\nu\pi}^+)^{2n_q} |-> = (\Delta_0^+)^{2n_q} |0\rangle$$

	QCM	PBC1	PBCS0 <sub>i<sub>v</sub></sub>	PBCS0 <sub>i<sub>s</sub></sub>
<sup>20</sup> Ne	15.985 (-)	14.011 (12.35%)	13.664 (14.52%)	13.909 (12.99%)
<sup>24</sup> Mg	28.595 (0.24%)	21.993 (23.35%)	20.516 (28.50%)	23.179 (19.22%)
<sup>28</sup> Si	35.288 (0.57%)	27.206 (23.58%)	25.293 (28.95%)	27.740 (22.19%)
<sup>44</sup> Ti	7.019 (-)	5.712 (18.62%)	5.036 (28.25%)	4.196 (40.22%)
<sup>48</sup> Cr	11.614 (0.21%)	9.686 (16.85%)	8.624 (25.97%)	6.196 (46.81%)
<sup>52</sup> Fe	13.799 (0.42%)	11.774 (15.21%)	10.591 (23.73%)	6.673 (51.95%)
<sup>104</sup> Te	3.147 (-)	2.814 (10.58%)	2.544 (19.16%)	1.473 (53.19%)
<sup>108</sup> Xe	5.489 (0.20%)	4.866 (11.61%)	4.432 (19.49%)	2.432 (55.82%)
<sup>112</sup> Ba	7.017 (0.34%)	6.154 (12.82%)	5.635 (20.17%)	3.026 (57.13%)

- quartet condensation wins over Cooper pair condensates
- T=1 and T=0 pairing correlations **always** coexist in quartets

# Isoscalar and isovector proton-neutron pairing in time-reversed states

$$|\Psi\rangle = (Q_{T=1}^+ + \Delta_0^{+2})^{n_q} |-\rangle$$

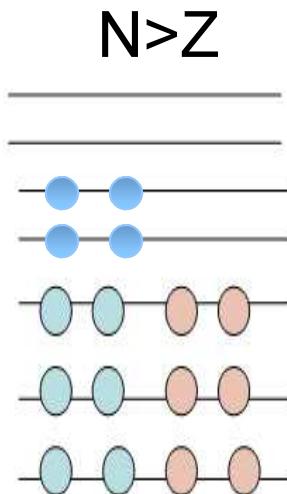
superposition of T=1 quartet condensates and T=0 pair condensates

# Isovector-isoscalar pairing and quartetting for N>Z nuclei

nuclei with  $N-Z=2n_N$

- ansatz
- all protons are correlated in alpha-like quartets
  - neutrons in excess form a pair condensate

$$|QCM\rangle = (\tilde{\Gamma}_{\nu\nu}^+)^{n_N} (Q_{T=1}^+ + \Delta_0^{+2})^{n_q} |-\rangle$$



$$Q_{T=1}^+ = 2\Gamma_{\nu\nu}^+ \Gamma_{\pi\pi}^+ - \Gamma_{\nu\pi}^{+2} \quad \Delta_0^+ = \sum y_i D_{i,0}^+$$

pn pairing and quartet correlations survive in  $N > Z$  nuclei !

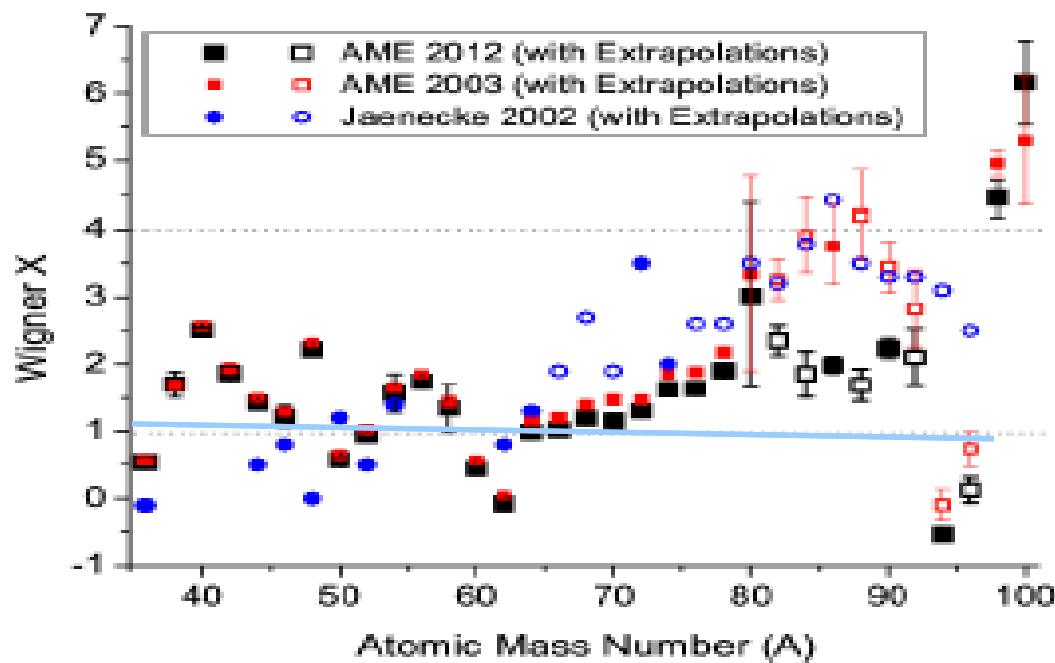
Proton-neutron pairing & Wigner energy

# Reminder on Wigner energy

$$E(N, Z) = E(N = Z) + a_s \frac{(N - Z)^2}{A} + a_W \frac{|N - Z|}{A} + \delta E_{shell} + \delta E_P$$

(no Coulomb)

$$\downarrow$$
$$E(N, Z) = E(N = Z) + \frac{T_z(T_z + X)}{2\Theta} \quad T_z = 0, 2, 4$$



I. Bentley & S. Frauendorf, PRC(2013)

# The effect of pn pairing on Wigner energy: example

$$H = \sum \varepsilon_i N_i + g_{T=1} \sum_{ij,\tau} P_{i\tau}^+ P_{j\tau}^- + g_{T=0} \sum_{ij} D_{i0}^+ D_{j0}^-$$



Skyrme  $g_1 = -24/A$   $g_0 = w g_1$

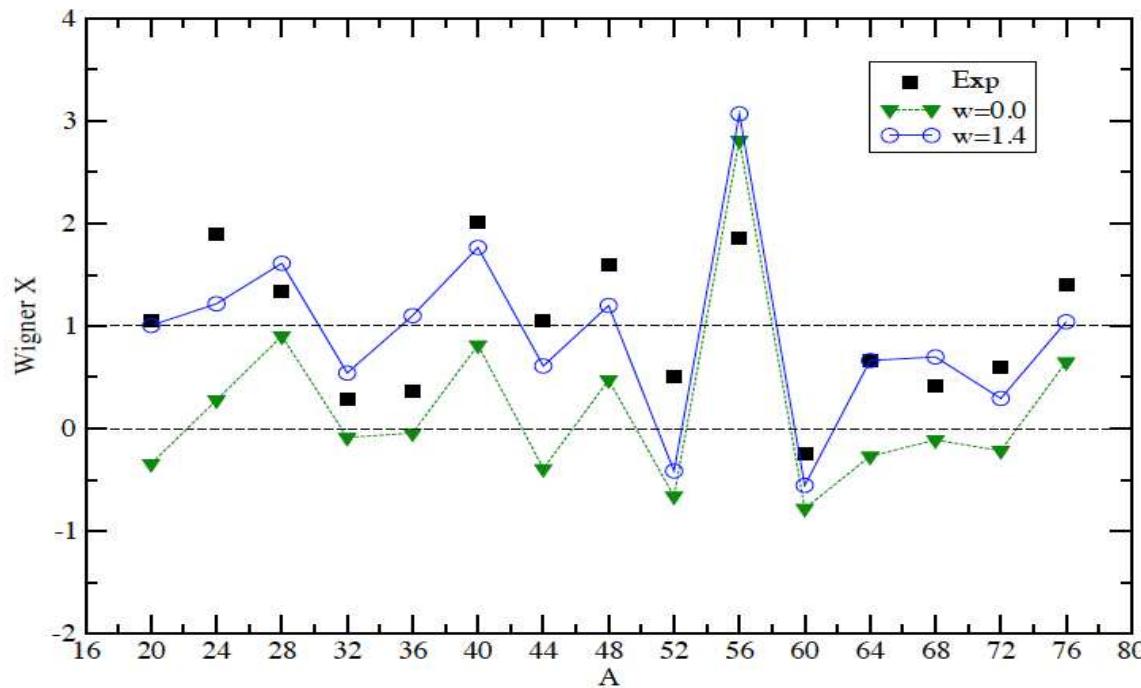
	Exp	QCM(w)	QCM(w=0)	HF+BCS	HF
<b>24Mg (3q)</b>	<b>-198.257</b>	<b>-195.420 (0.4)</b> <b>-195.905 (0.8)</b> <b>-196.500 (1.0)</b> <b>-197.577 (1.2)</b> <b>-199.609 (1.4)</b> <b>-203.514 (1.6)</b>	<b>-195.362</b>	<b>-190.333</b>	<b>-189.992</b>

## Effect of proton-neutron pairing on Wigner energy

$$H = \sum \varepsilon_i N_i + g_{T=1} \sum_{ij,\tau} P_{i\tau}^+ P_{j\tau} + g_{T=0} \sum_{ij} D_{i0}^+ D_{j0}$$

↑  
Skyrme     $g_1 = -24/A$      $g_0 = w g_1$

preliminary Skyrme-HF+QCM results\*



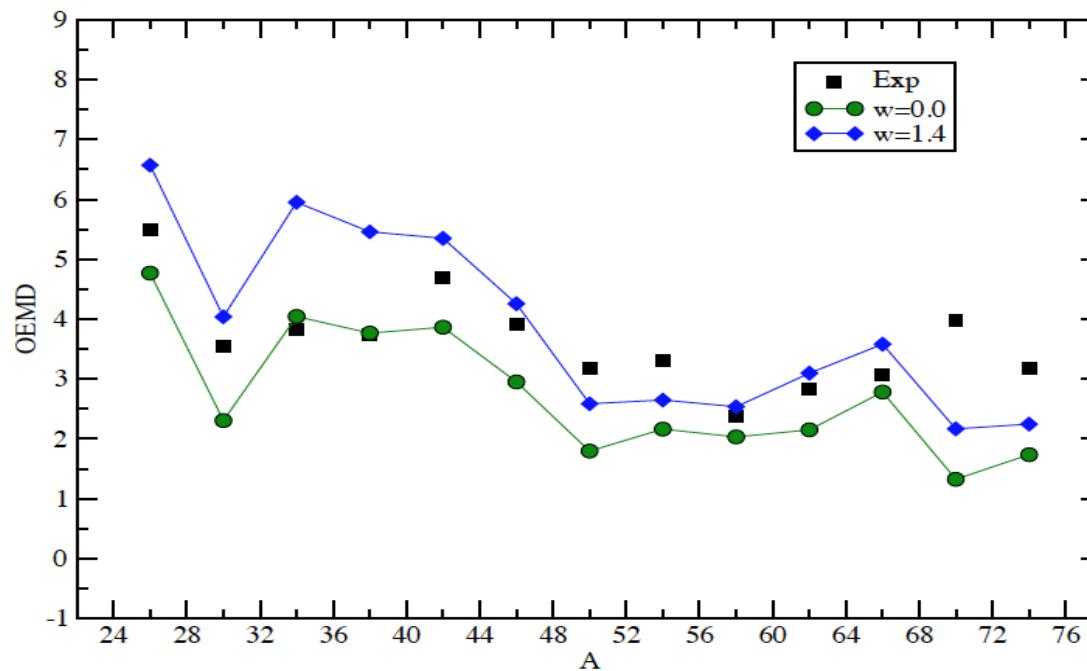
- it is suggested the need of T=0 pairing;
- the effects of T=0 and T=1 pairing are difficult to disentangle

\* N. S et all, in preparation

## Odd-even mass difference along N=Z line

$$2\Delta(N, Z) = \frac{B(N-1, Z-1) - 2B(N, Z) + B(N+1, Z+1)}{2}$$

preliminary Skyrme-HF+QCM results



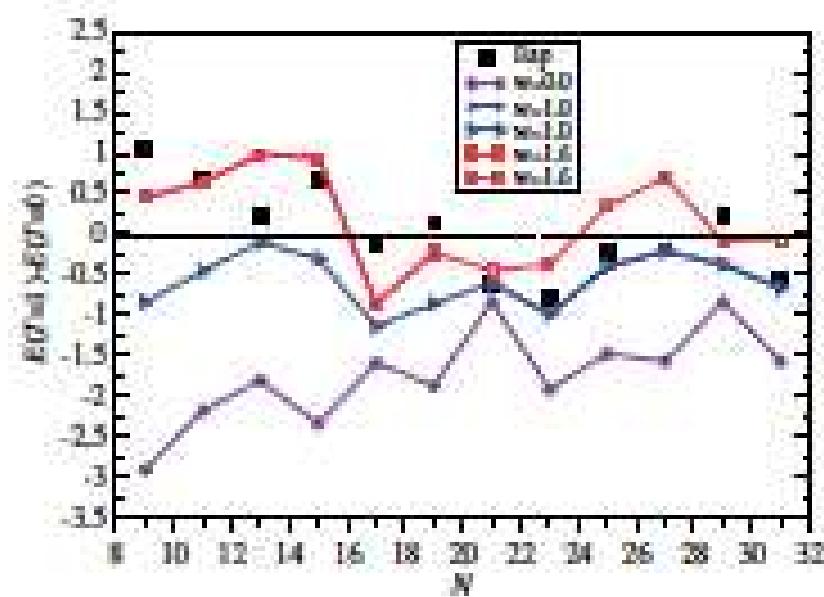
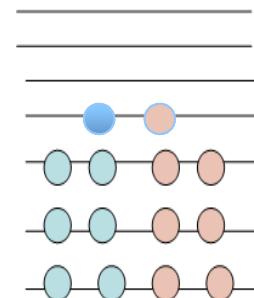
odd-odd nuclei: calculations with blocking !

# Isovector and isoscalar pairing in odd-odd N=Z

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P_{i,t}^+ P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D_{i,0}^+ D_{j,0}$$

T=1 state       $|iv; QCM\rangle = \tilde{\Gamma}_{\nu\pi}^+ (Q_{T=1}^+ + \Delta_{\nu\pi}^{+2})^{n_q} |-\rangle$

T=0 state       $|is; QCM\rangle = \tilde{\Delta}_{\nu\pi}^+ (Q_{T=1}^+ + \Delta_{\nu\pi}^{+2})^{n_q} |-\rangle$



$$W = \frac{V_0^{T=0}}{V_0^{T=1}}$$

$$V_{pairing}^{T=\{0,1\}} = V_0^{T=\{0,1\}} \delta(r_1 - r_2) \hat{P}_{S=\{0,1\}}$$

**Quartet correlations for general two-body forces ?**

# Quartet correlations for general two-body forces

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii',jj',J',T'} V_{JT}(ii';jj')[A_{ii'J'T'}^+ A_{jj'J'T'}]^{J=0,T=0}$$

$$|QCM\rangle = Q^{+n_q} |-\rangle \quad Q^+ = \sum_{ii',jj',JT} x_{ii',jj'} [A_{ii'JT}^+ A_{jj'JT}]^{0,0}$$

	$E_{corr}(SM)$	$E_{corr}(QCM)$	$E_{corr}(QM)$	$\langle SM   QCM \rangle$	
$^{20}\text{Ne}$	24.77	24.77	24.77	1	
$^{24}\text{Mg}$	55.70	53.04 (4.77%)	53.24 (4.41%)	0.85	
$^{28}\text{Si}$	88.75	86.52 (2.52%)	87.12 (1.84%)	0.86	
$^{32}\text{S}$	122.51	122.02 (0.40%)	122.29 (0.18%)	0.98	

$$E(n_q) = n_q \times E(1) + \frac{n_q(n_q - 1)}{2} \times V(n_q),$$

$E(n)$  is specific to a boson/quartet condensate

How to identify the transition to a quartet condens

# Long-range correlations of superfluidity-type and density matrix

Penrose (1951) , Penrose and Onsager (1956), C. N. Yang (1962)

n-body long-range correlations       $\longleftrightarrow$     a large eigenvalue of n-body density

Example: pair condensation

$$\rho^{(2)}(r_1, r_2; r'_1, r'_2) = \langle \Phi_0^{(N)} | \hat{\Psi}^+(r_1) \hat{\Psi}^+(r_2) \hat{\Psi}(r'_2) \hat{\Psi}(r'_1) | \Phi_0^{(N)} \rangle$$

$$\rho^{(2)}(r_1, r_2; r'_1, r'_2) = \lambda_0 \varphi_0^\square(r_1, r_2) \varphi_0(r'_1, r'_2) + \sum_{n>0} \lambda_n \varphi_n^\square(r_1, r_2) \varphi_n(r'_1, r'_2)$$

("off-diagonal long-range order")

long-range correlations:  $\gg \lambda_{n \neq 0}$

$\lambda_0$  - associated to the number of "condensed" pairs

# Eigenvalues of two-body density matrix for like-particle pairing

$$H = \sum_i \varepsilon_i N_i - k \sum_{ij} V(i,j) P_i^+ P_j \quad (k \text{ is a scaling factor})$$

two-body density  $\rho_{i,j}^{(2)} = \langle \Psi | P_i^+ P_j | \Psi \rangle$   $P_i^+ = a_i^+ a_i^+$

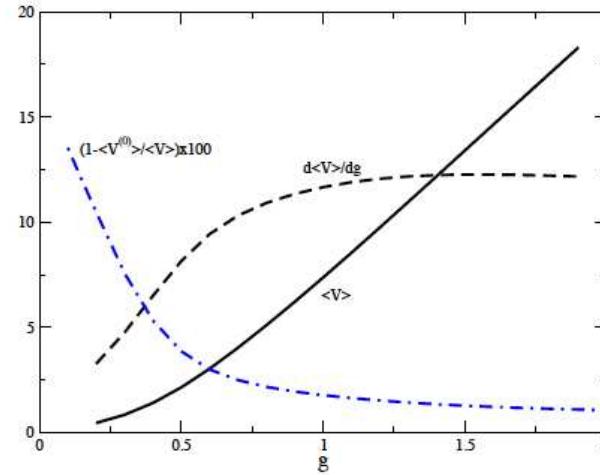
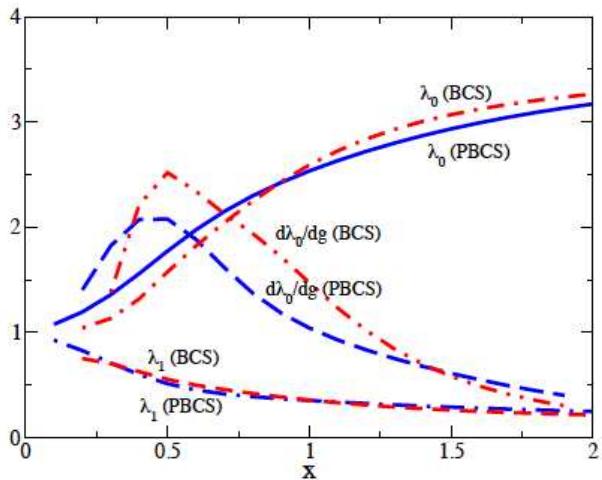
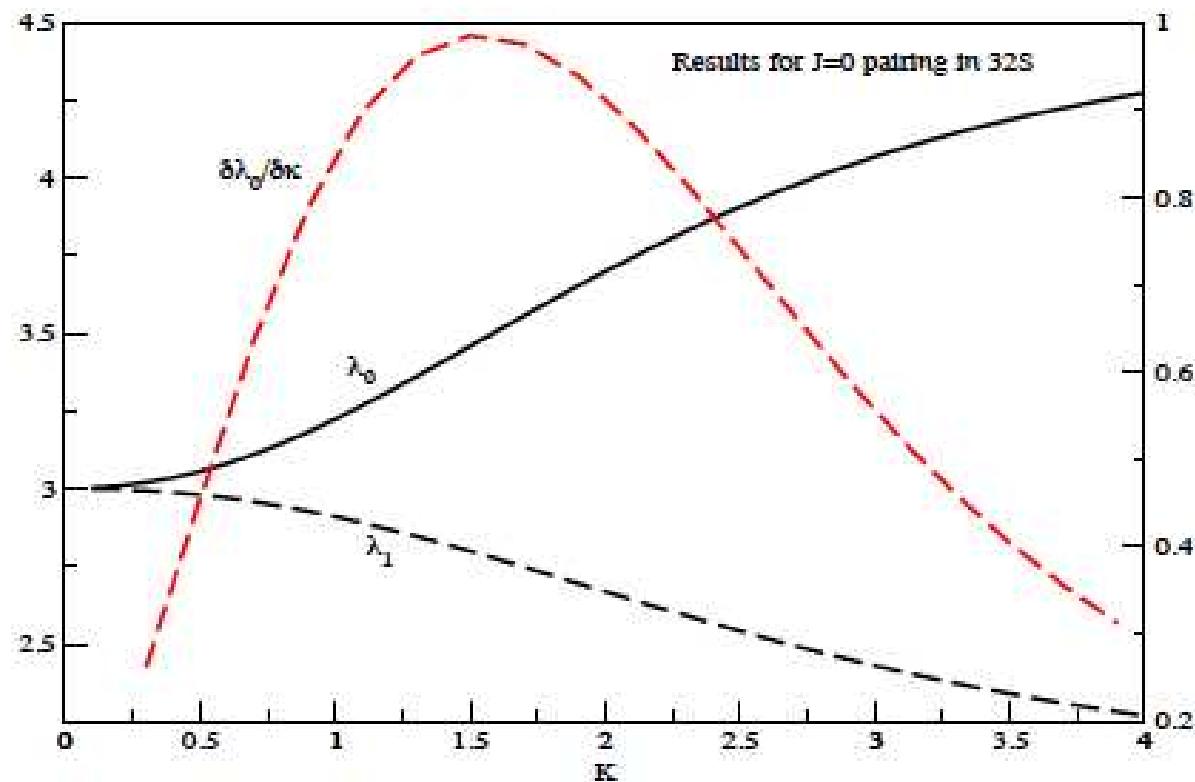
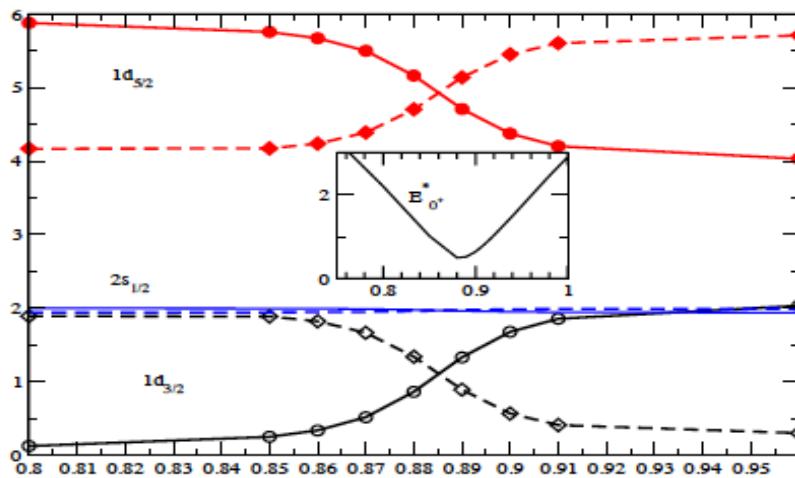
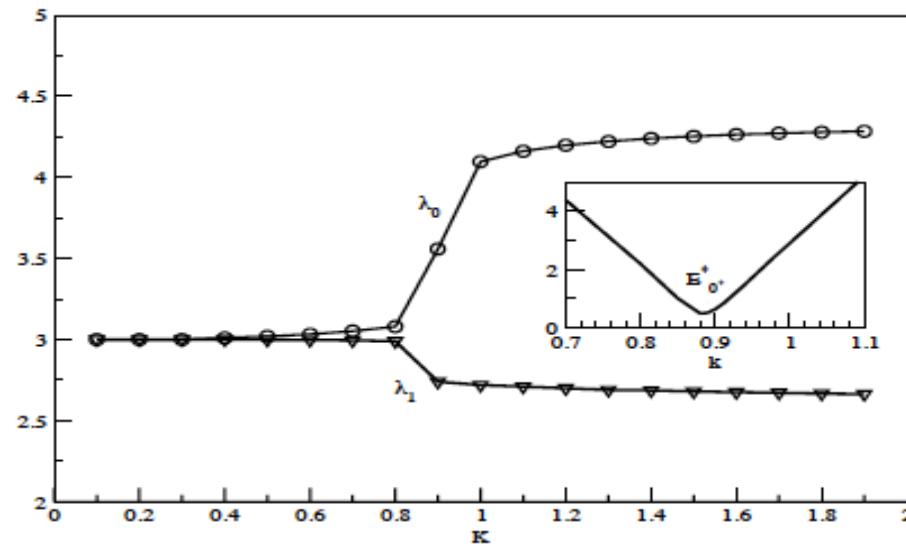


FIG. 2: Results for  $^{110}\text{Sn}$ . (a) The largest ( $\lambda_0$ ) and the second largest ( $\lambda_1$ ) eigenvalues of two-body density matrix function of the scaling factor  $k$ . (b) The average of the pairing force, its derivative to the scaling factor, and the quantity  $D$  (see text)

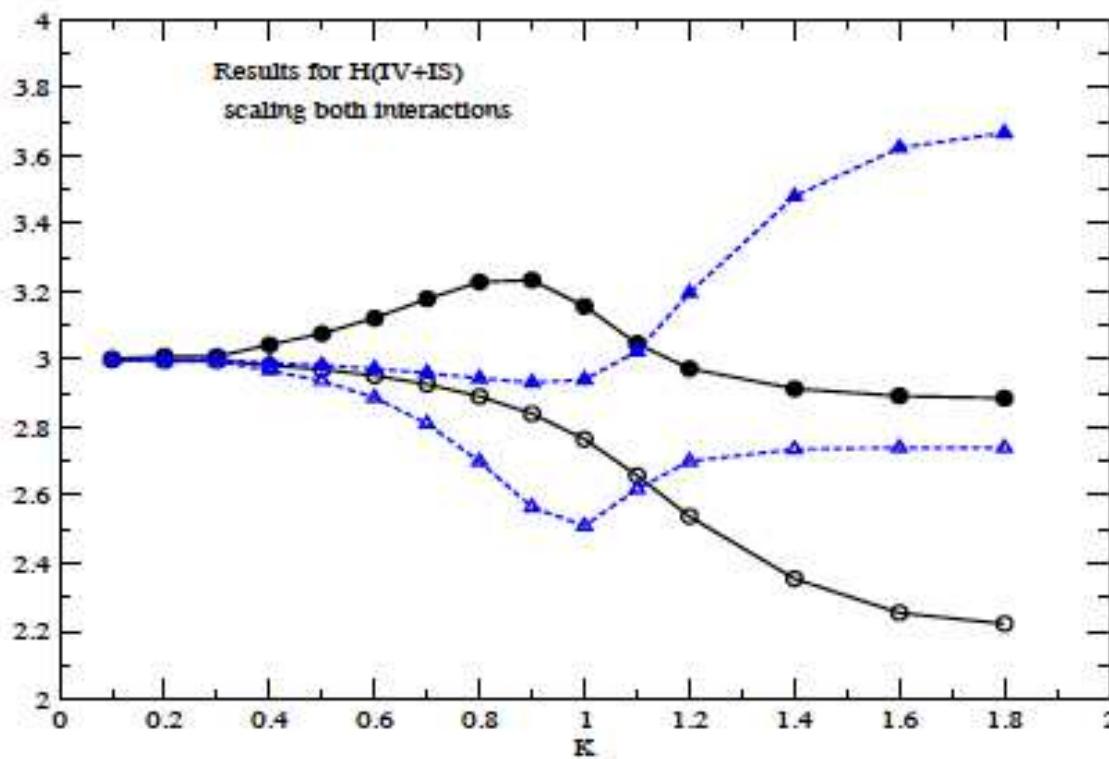
# Eigenvalues of 2-body density matrix for isoscalar pairing



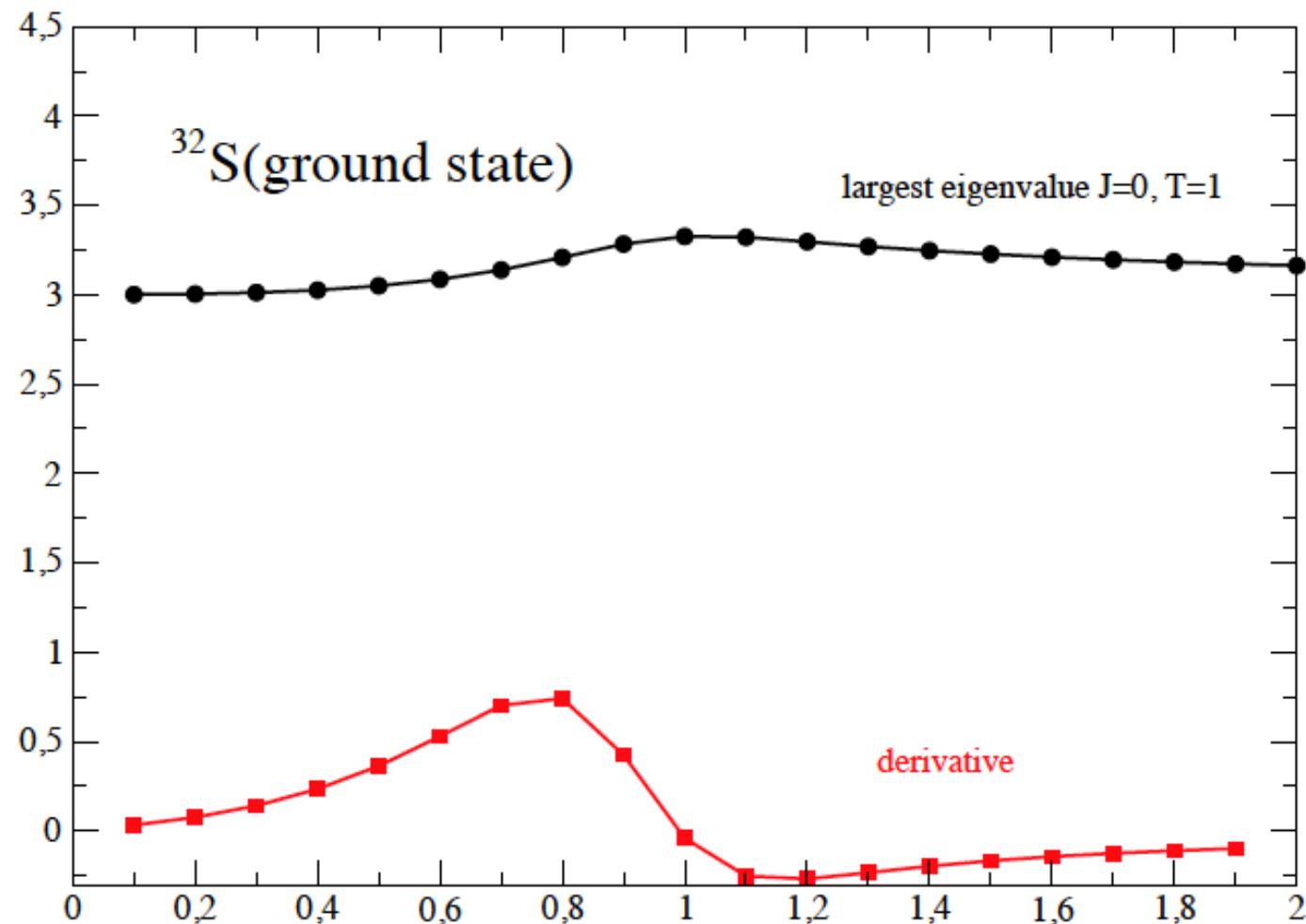
# Eigenvalues of 2-body density matrix for J=1 pairing



# Eigenvalues of 2-body density matrix for J=0 & J=1 pairing

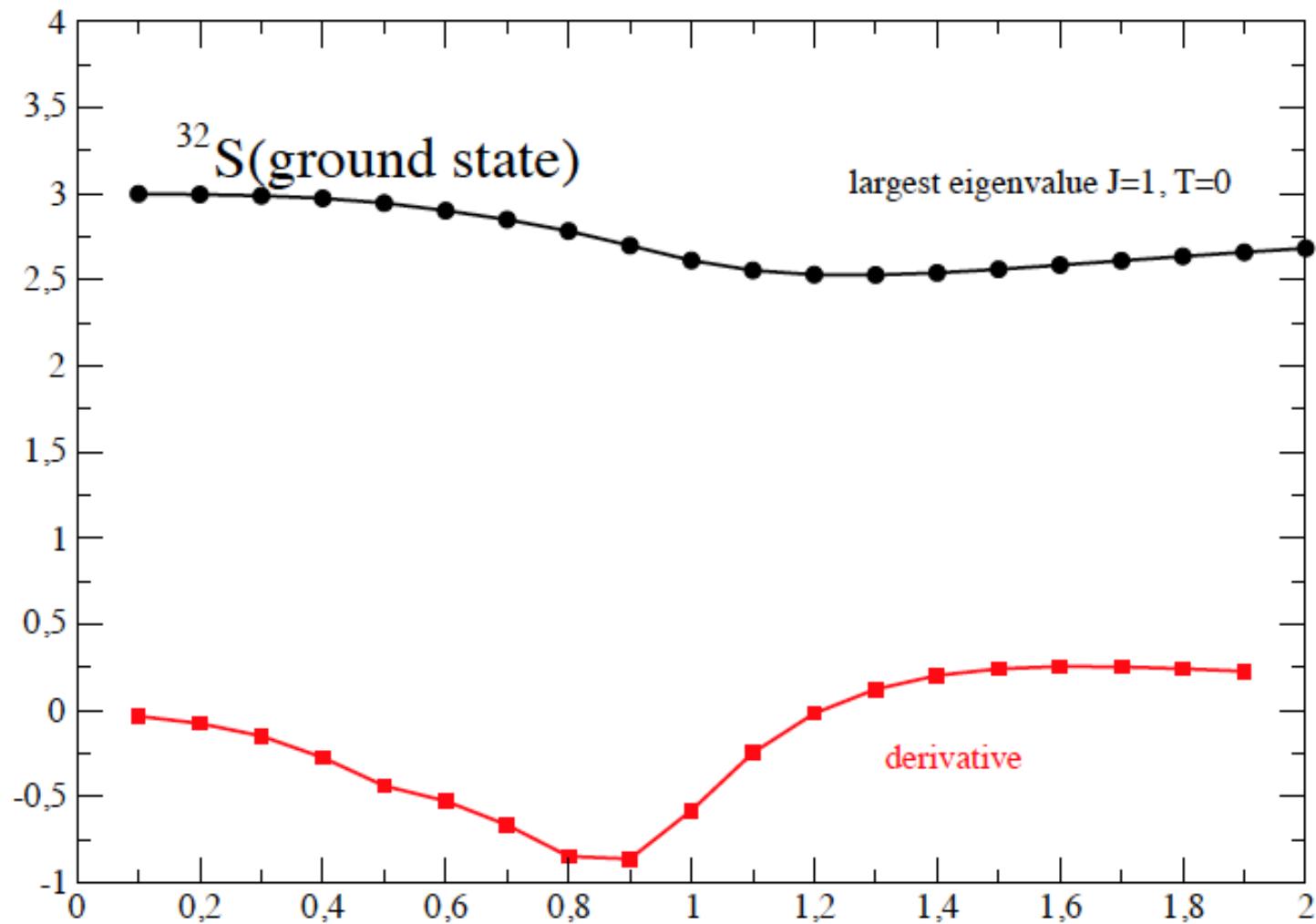


# Evolution of the largest eigenvalue for 2-body density matrix: $J=0, T=1$



signature of weak long-range correlations !

# Evolution of the largest eigenvalue for 2-body density matrix: J=1,T=0

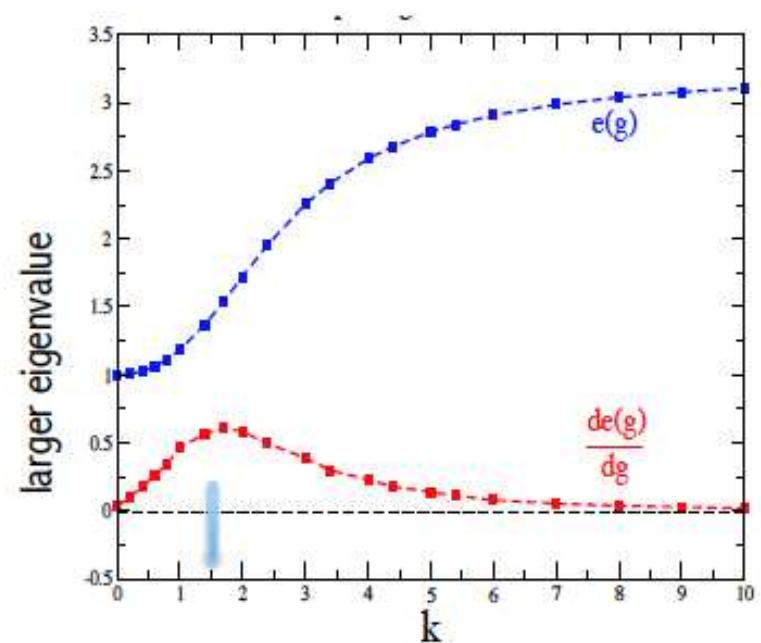
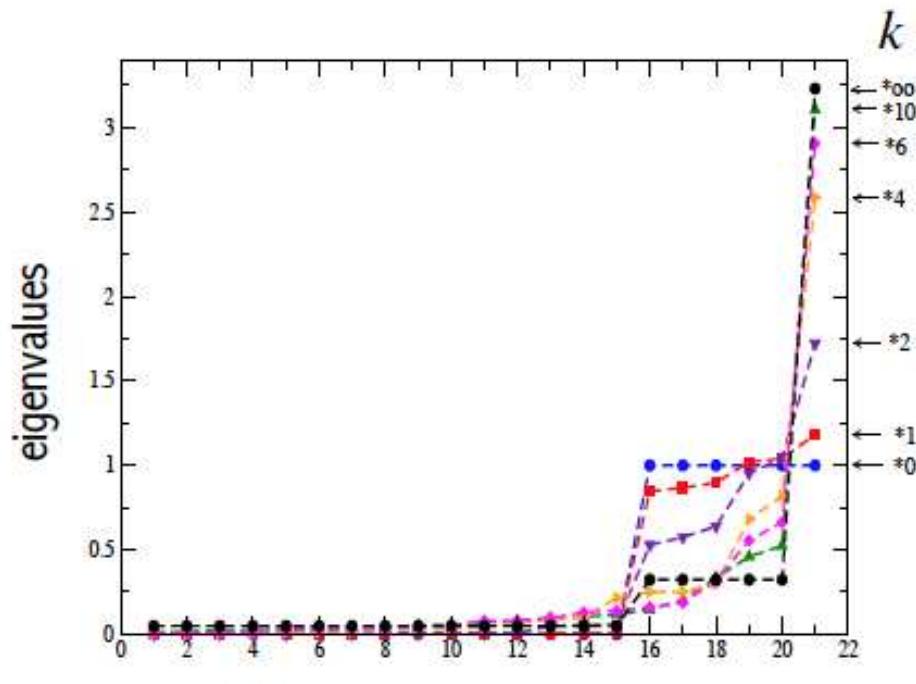
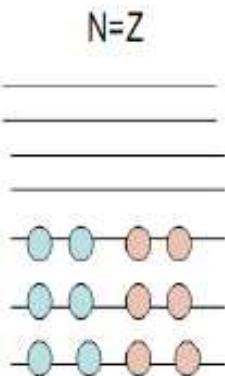


no signature of long-range correlations !

# Eigenvalues of 4-body density matrix for T=1 pairing: $^{28}\text{Si}$

$$H = \sum_i \varepsilon_i (N_i^{(v)} + N_i^{(\pi)}) - kg \sum_{ij} (P_{i,v\pi}^+ P_{j,v\pi}^- + P_{i,vv}^+ P_{j,vv}^- + P_{i,\pi\pi}^+ P_{j,\pi\pi}^-) \quad g = \frac{24}{A}$$

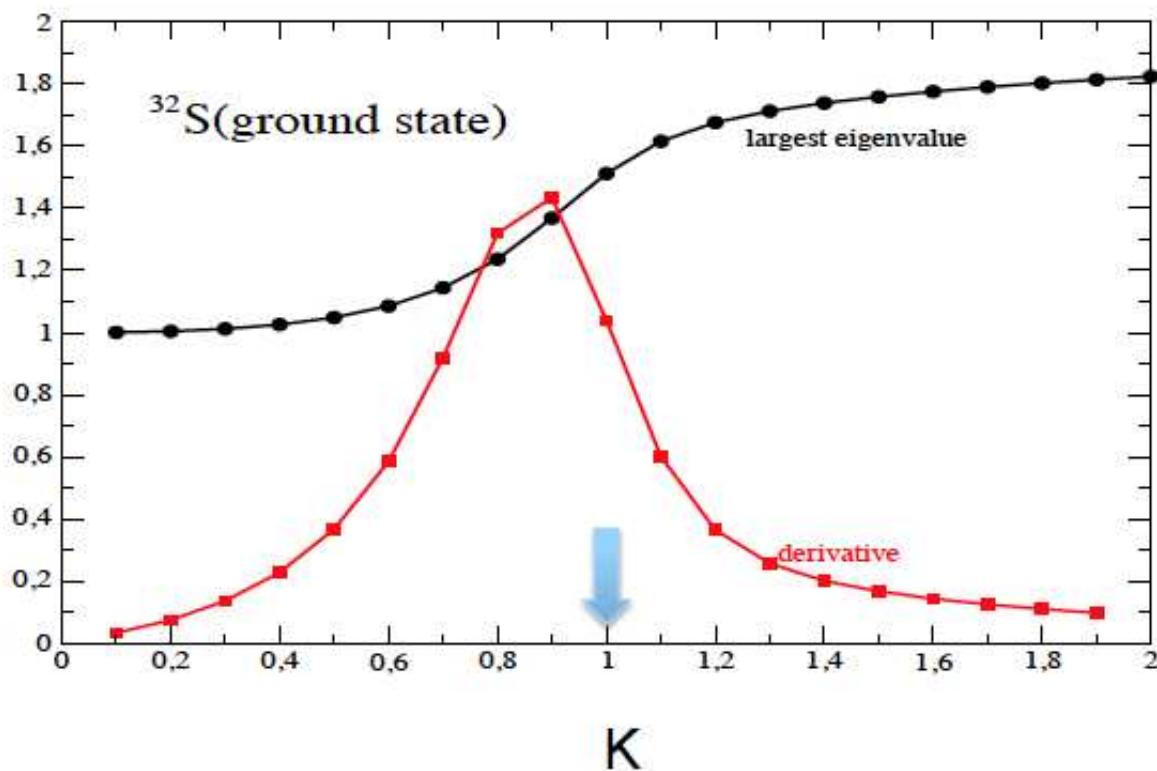
4-body density  $\rho_{i,j}^{(4)} = \langle \Psi | q_i^\dagger q_j | \Psi \rangle$   $q_i^\dagger = (a_{i_1}^\dagger a_{i_2}^\dagger a_{i_3}^\dagger a_{i_4}^\dagger)^{T=0}$



in the physical region  $\lambda_0^{(4)} > 1$

# Evolution of the largest eigenvalue of 4-body density matrix: $^{32}\text{S}$

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + k \sum_{ii',jj',J,T'} V_{JT}(ii';jj') [A_{ii'J'T'}^+ A_{jj'J'T'}]^{J=0,T=0}$$



Indication of a fast transition towards a quartet condensate !

## 4-body density for general two-body forces: sd-shell nuclei

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii', jj', J', T'} V_{JT}(ii'; jj') [A_{ii' J' T'}^+ A_{jj' J' T'}]^{J=0, T=0}$$

$$\rho_{i,j}^{(4)} = \langle SM | q_i^+ q_j | SM \rangle \quad q_i^+ = (a_{i_1}^+ a_{i_2}^+ a_{i_3}^+ a_{i_4}^+)^{T=0}$$

Largest 5 eigenvalues for sd-shell nuclei

$^{24}\text{Mg}$	<b>1.18</b>	0.15	0.03	0.29	0.01
$^{28}\text{Si}$	<b>1.19</b>	0.47	0.27	0.20	0.12
$^{32}\text{S}$	<b>1.51</b>	0.83	0.74	0.59	0.53

there is one eigenvalue larger than 1



fingerprints of long-range quartet correlations

# Summary and Conclusions

**Main message:  $T=1$  and  $T=0$  pairing forces are accurately described by alpha-like quartets, not by Cooper pairs**

- $T=1$  and  $T=0$  pn pairing always coexist in quartet-type correlations
- $T=0$  pn pairing and quartetting persist in  $N>Z$  nuclei
- proton-neutron pairing has a significant effect on Wigner energy
- 4-body density matrix indicates quartet correlations of “condensate” type

# Quartet condensation in the excited states ?

$$H = \sum_i \epsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii',jj',J',T'} V_{JT}(ii';jj') [A_{ii'J'T'}^+ A_{jj'J'T'}]^{J=0,T=0}$$

$$|0_n^+; QCM\rangle = (Q_n^+)^{n_q} |-\rangle \quad Q_n^+ = \sum_{ii',jj',JT} x_{ii',jj'}^{(n)} [A_{ii'JT}^+ A_{jj'JT}]^{0,0}$$

First excited  $0^+$

		$E_{0_1^+}(SM)$	$E_{0_1^+}(QCM)$	$\langle SM   QCM \rangle$
	$^{20}\text{Ne}$	-33.77 (6.7)	-33.77 (6.7)	1
	$^{24}\text{Mg}$	-79.76 (7.34)	-76.97 (7.47)	0.70
	$^{28}\text{Si}$	-131.00 (4.84)	-126.91 (6.71)	0.65
	$^{32}\text{S}$	-178.98 (3.46)	-178.04 (3.92)	0.95

← SM is a QCM state !?

Second excited  $0^+$

		$E_{0_2^+}(SM)$	$E_{0_2^+}(QCM)$
	$^{20}\text{Ne}$	-28.56 (11.91)	-28.56 (11.91)
	$^{24}\text{Mg}$	-77.43 (9.67)	-70.85 (13.59)
	$^{28}\text{Si}$	-128.51 (7.33)	-120.64 (12.99)
	$^{32}\text{S}$	-175.04 (7.4)	-170.84 (11.12)

superposition of many shell-model states: **cluster-type excitations** ?

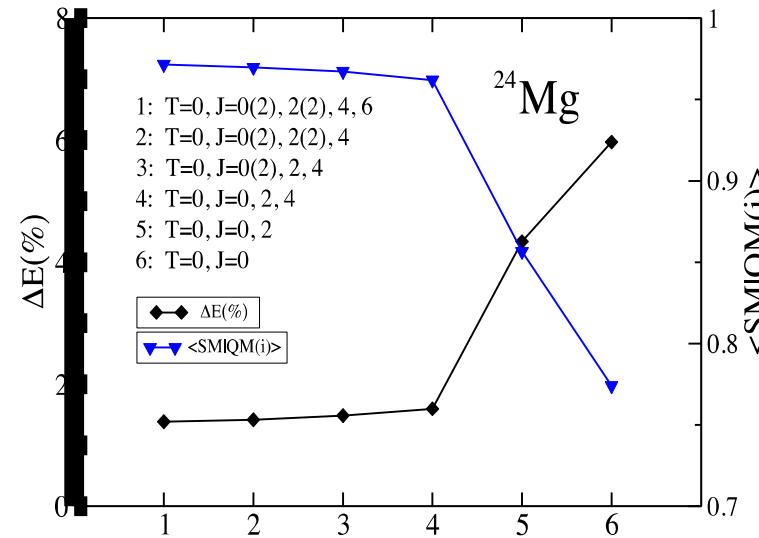
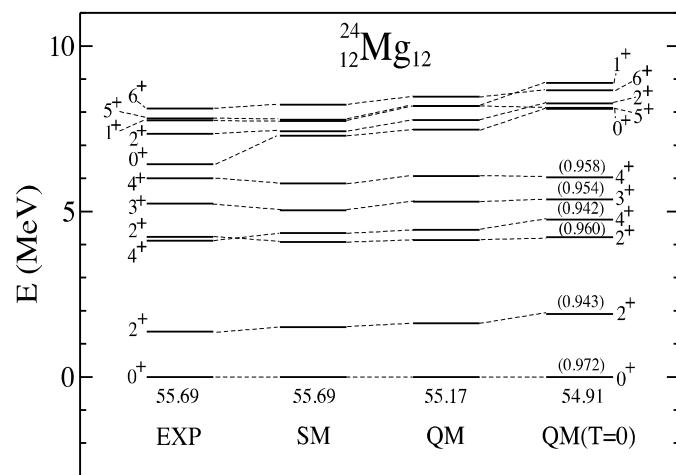
# Quartet correlations for general forces: even-even N=Z nuclei

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii', jj', J', T'} V_{JT}(ii'; jj') [A_{ii'J'T'}^+ A_{jj'J'T'}]^{J=0, T=0}$$

diagonalisation in a quartet basis

$$|QM; J\rangle = [Q_{J_1}^{(\nu_1)+} Q_{J_2}^{(\nu_2)}]^J |core\rangle$$

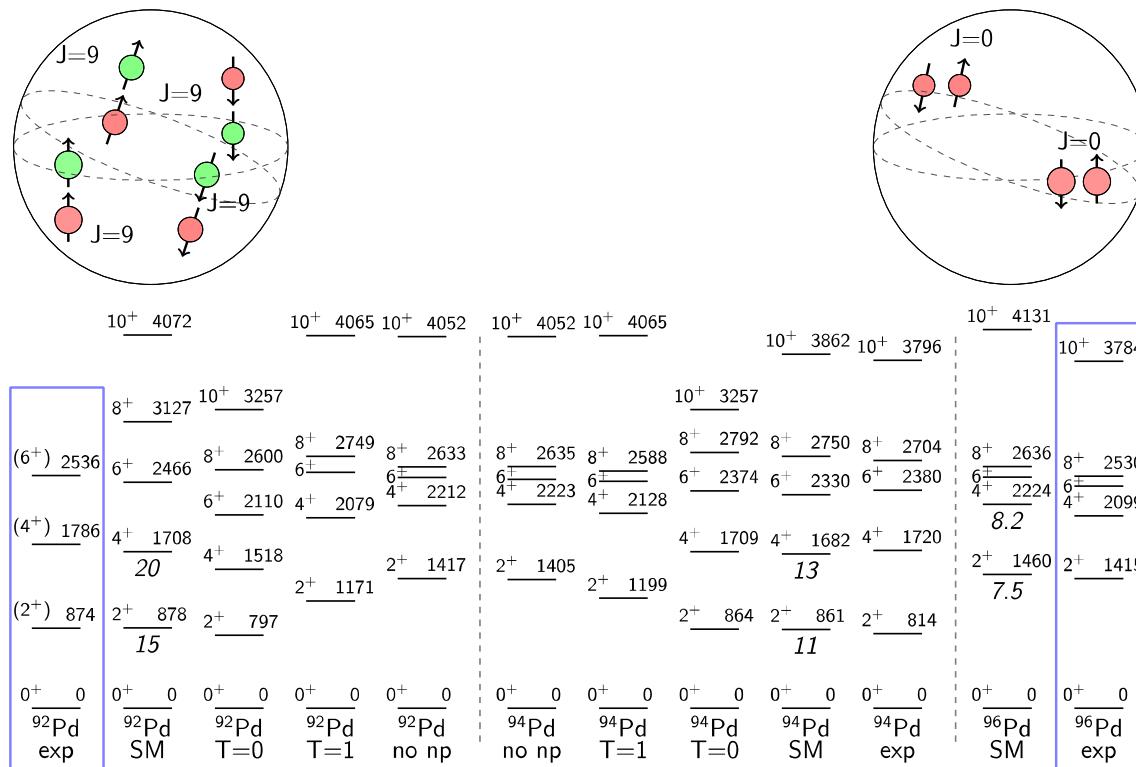
$$Q_{JT}^{(d)+} = \sum_{ii' ii' J T} x_{ii', jj'}^{(d)} [A_{ii'J_1T_1}^+ A_{jj'J_2T_2}^+]^{J,T}$$



all low-lying states can be well described by  $T=0, J=0, 2, 4$  quartets !

$T=0, J=0$  quartets give the dominant contribution in the ground state !

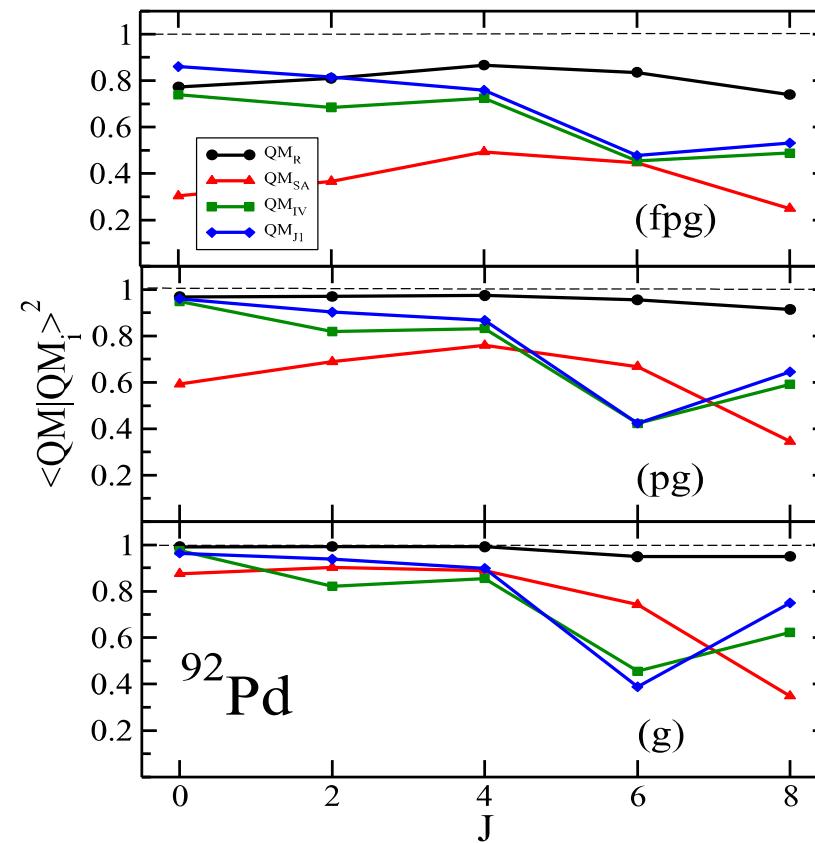
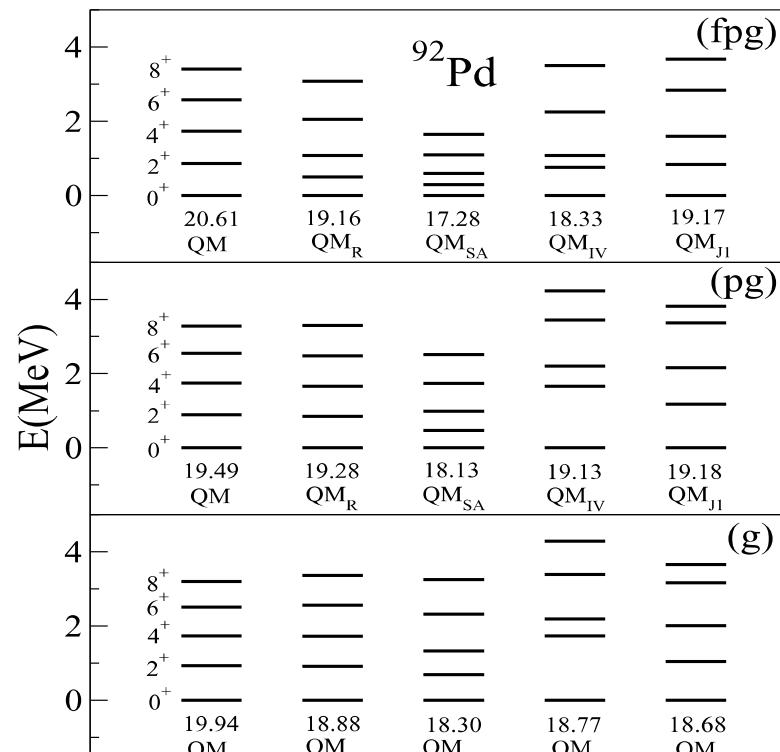
# Spin-aligned J=9 pairs in $^{92}\text{Pd}$ ?



B. Cederwall et al, Nature 469 (2011)68

# Role of spin-aligned pairs in $^{92}\text{Pd}$

$$Q_{\alpha, JM, TT_z}^+ = \sum_{i_1 j_1 J_1 T_1} \sum_{i_2 j_2 J_2 T_2} C_{i_1 j_1 J_1 T_1, i_2 j_2 J_2 T_2}^{(\alpha)} \times [ [a_{i_1}^+ a_{j_1}^+]^{J_1 T_1} [a_{i_2}^+ a_{j_2}^+]^{J_2 T_2} ]_{MT_z}^{JT}, \quad [Q_{\alpha_1, J', T'}^+ \otimes Q_{\alpha_2, J'', T''}^+]^{J, T},$$



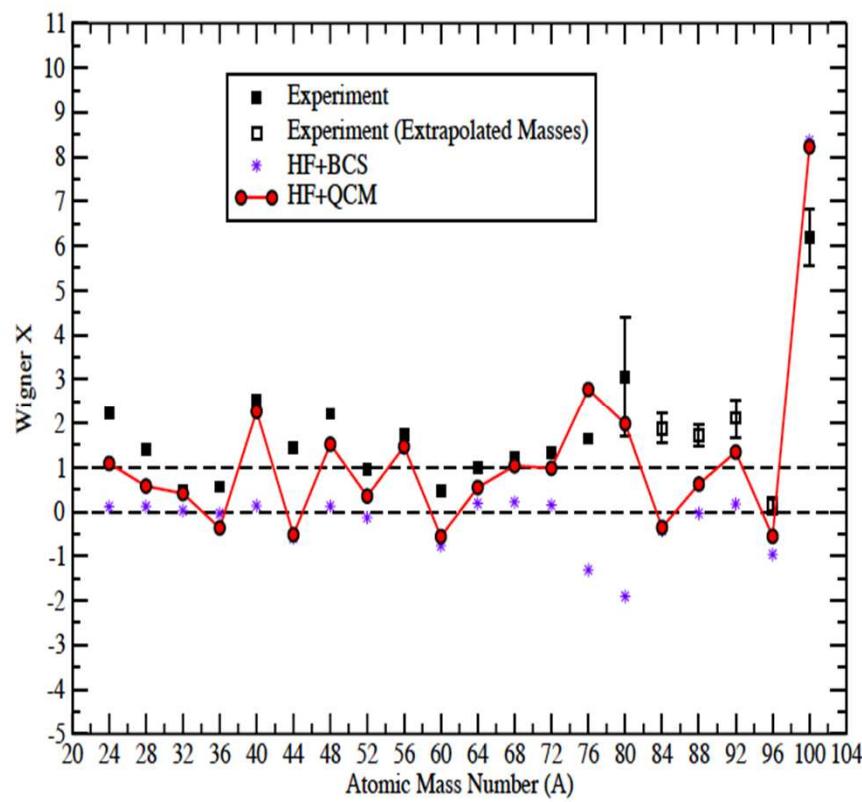
the structure of  $^{92}\text{Pd}$  is not dominated by  $J=9$  pairs

ground state is mainly built by  $J=0$  and  $J=1$  pairs

# Wigner energy with T=1 pairing interaction

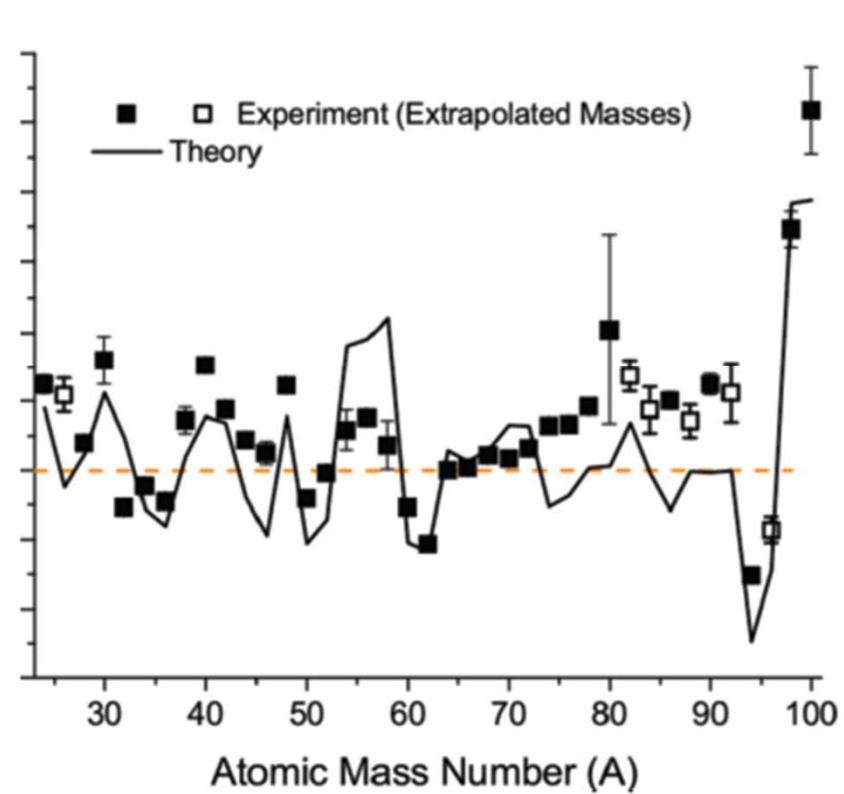
Negrea & Sandulescu PRC(2014)

$$H_V = \sum_k \epsilon_k \hat{N}_k - G_V \sum_{kk',\tau} \hat{P}_{k,\tau}^+ \hat{P}_{k',\tau}$$



Bentley & Frauendorf PRC(2013)

$$H_V = \sum_k \epsilon_k \hat{N}_k - G_V \sum_{kk',\tau} \hat{P}_{k,\tau}^+ \hat{P}_{k',\tau} + C \vec{T} \cdot \vec{T}$$



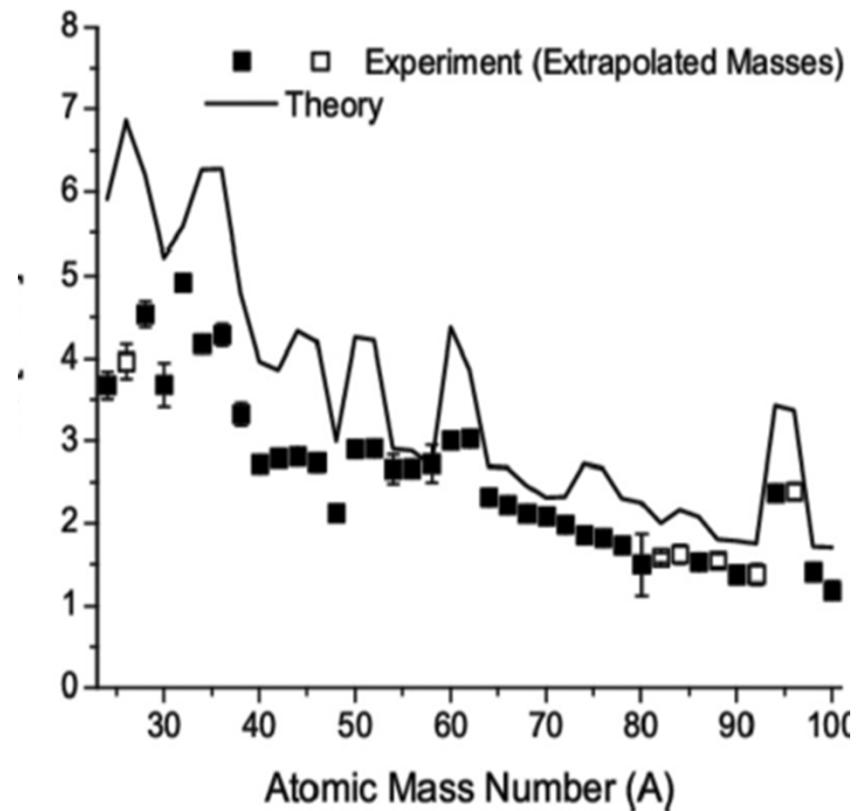
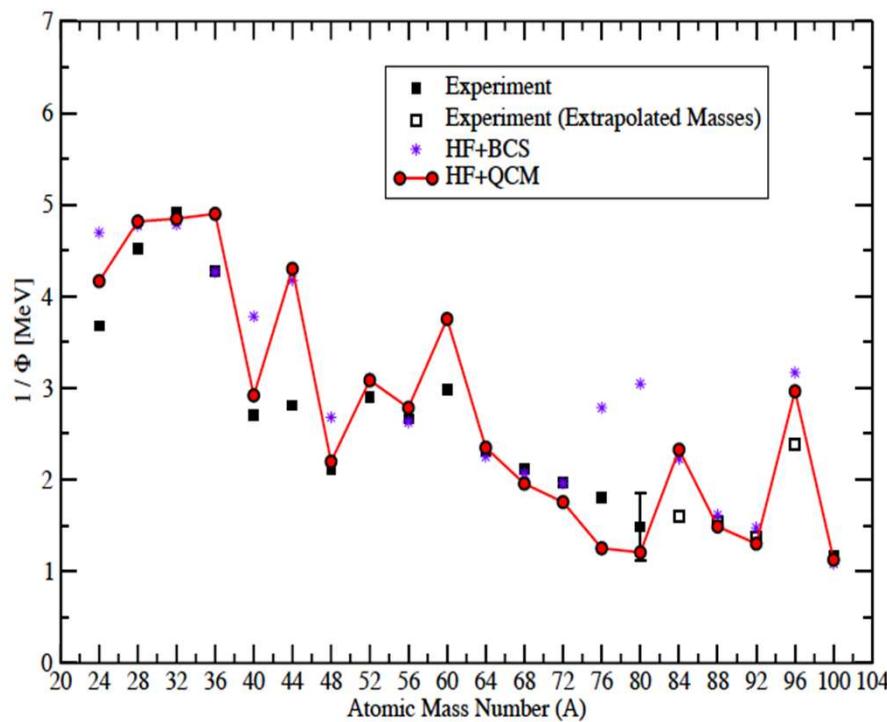
# Effect of T=1 pairing on symmetry energy

Negrea & Sandulescu PRC(2014)

$$H_V = \sum_k \epsilon_k \hat{N}_k - G_V \sum_{kk',\tau} \hat{P}_{k,\tau}^+ \hat{P}_{k',\tau}$$

Bentley & Frauendorf PRC(2013)

$$H_V = \sum_k \epsilon_k \hat{N}_k - G_V \sum_{kk',\tau} \hat{P}_{k,\tau}^+ \hat{P}_{k',\tau} + C \vec{T} \cdot \vec{T}$$



## Isovector pairing with distinct quartets

$$H = \sum_i \varepsilon_i (N_i^{(v)} + N_i^{\pi}) - g \sum_{ij,\tau} P_{i,\tau}^+ P_{j,\tau}$$

$$|QM\rangle = Q_1^+ Q_2^+ \dots Q_{n_q}^+ |-\rangle \quad |QCM\rangle = Q^{+n_q} |-\rangle$$

	exact	QM	QCM
<sup>20</sup> Ne	-6.5505	-6.5505	-6.539 (0.18%)
<sup>24</sup> Mg	-8.4227	-8.4227	-8.388 (0.41%)
<sup>28</sup> Si	-9.6610	-9.6610	-9.634 (0.28%)
<sup>32</sup> S	-10.2629	-10.2629	-10.251 (0.12%)
<sup>44</sup> Ti	-3.1466	-3.1466	-3.142 (0.15%)
<sup>48</sup> Cr	-4.2484	-4.2484	-4.227 (0.50%)
<sup>52</sup> Fe	-5.4532	-5.4531	-5.426 (0.50%)
<sup>104</sup> Te	-1.0837	-1.0837	-1.082 (0.16%)
<sup>108</sup> Xe	-1.8696	-1.8696	-1.863 (0.35%)
<sup>112</sup> Ba	-2.7035	-2.7034	-2.688 (0.57%)

QM reproduces the exact results up to the 4<sup>th</sup> digit !!

# Quartets in terms of Cooper pairs

$$Q^+ = \sum_{ij} x_{ij} [P_{i\tau}^+ P_{j\tau'}^+]^{T=0} \quad |QCM\rangle = Q^{+n_q} |->$$

$$Q^+ = 2\Gamma_{\nu\nu}^+ \Gamma_{\pi\pi}^+ - \Gamma_{\nu\pi}^+ \Gamma_{\nu\pi}^+ \quad \Gamma_\tau^+ = \sum_i x_i P_{i,\tau}^+ \quad \text{collective pairs}$$

$$|QCM\rangle = (2\Gamma_{\nu\nu}^+ \Gamma_{\pi\pi}^+ - \Gamma_{\nu\pi}^+ \Gamma_{\nu\pi}^+)^{n_q} |->$$

**‘coherent’ mixing of condensates formed by nn, pp and pn pairs**

PBCS solutions  $(\Gamma_{\nu\nu}^+ \Gamma_{\pi\pi}^+)^{n_q} \quad (\Gamma_{\nu\pi}^{+2})^{n_q}$

calculations

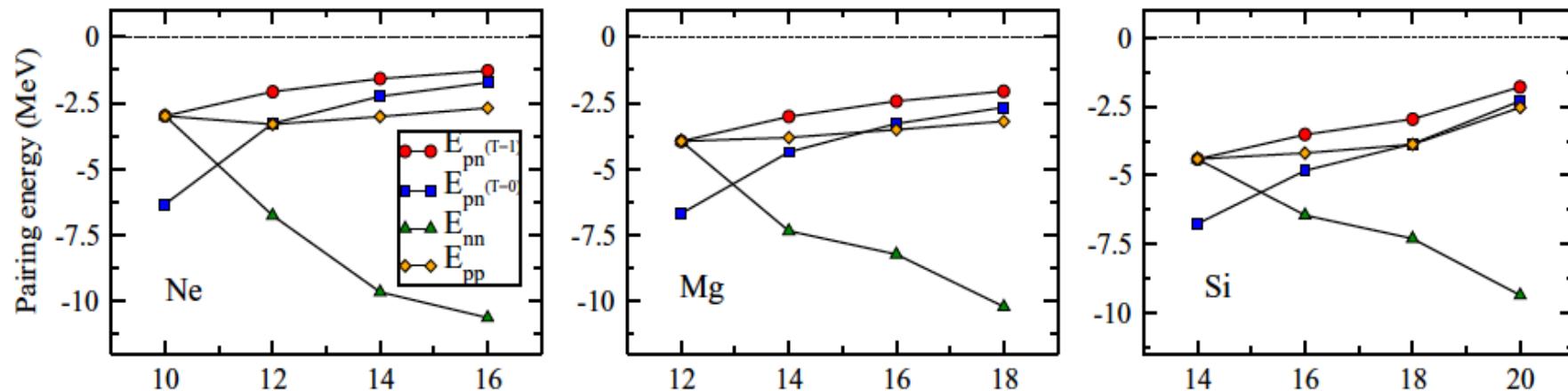
$$\delta_x \langle QCM | H | QCM \rangle = 0$$

(14 non-linear coupled equations solved iteratively)

# Isovector-isoscalar pairing and quartetting for N>Z nuclei

$$H = \sum \varepsilon_i N_i + g_{T=1} \sum_{ij, \tau} P_{i\tau}^+ P_{j\tau}^- + g_{T=0} \sum_{ij} D_{i0}^+ D_{j0}^- \quad g_{T=0} = 1.5 g_{T=1}$$

$$|QCM\rangle = (\tilde{\Gamma}_{vv}^+)^{n_N} (Q_{T=1}^+ + \Delta_0^{+2})^{n_q} |-\rangle \quad \Delta_0^+ = \sum y_i D_{i,0}^+$$



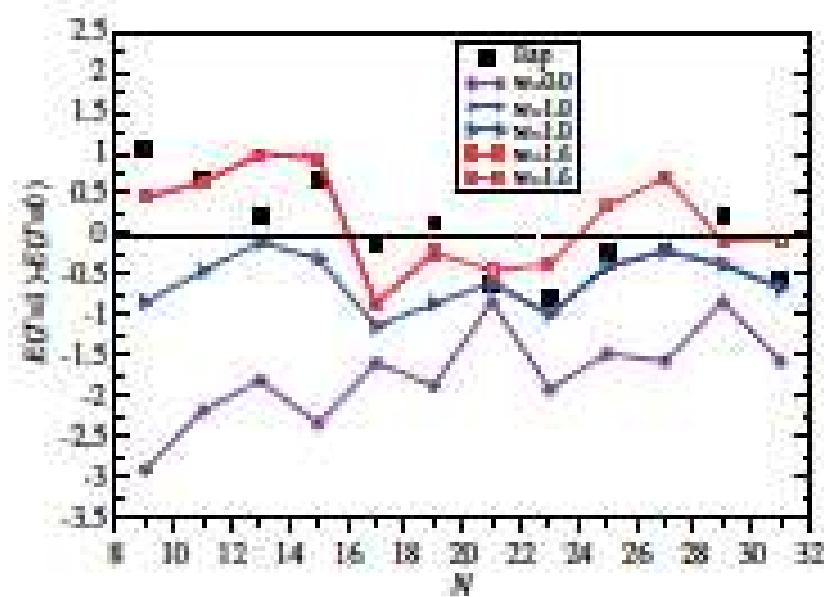
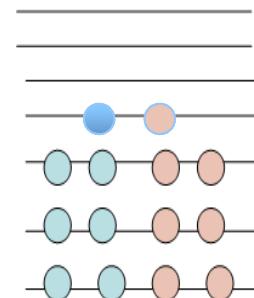
pn pairing and quartet correlations survive in N > Z nuclei !

# Isovector and isoscalar pairing in odd-odd N=Z

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P_{i,t}^+ P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D_{i,0}^+ D_{j,0}$$

T=1 state  $|iv; QCM> = \tilde{\Gamma}_{\nu\pi}^+ (Q_{T=1}^+ + \Delta_{\nu\pi}^{+2})^{n_q} |-\rangle$

T=0 state  $|is; QCM> = \tilde{\Delta}_{\nu\pi}^+ (Q_{T=1}^+ + \Delta_{\nu\pi}^{+2})^{n_q} |-\rangle$



$$W = \frac{V_0^{T=0}}{V_0^{T=1}}$$

$$V_{pairing}^{T=\{0,1\}} = V_0^{T=\{0,1\}} \delta(r_1 - r_2) \hat{P}_{S=\{0,1\}}$$

# The structure of lowest T=0 and T=1 states

T=0 ground state

		Exact	$\tilde{\Delta}_{\nu\pi}^+ (Q_{T=1}^+ + \Delta_{\nu\pi}^{+2})^{n_q}$	$\tilde{\Delta}_{\nu\pi}^+ (Q_{T=1}^+)^{n_q}$	$(\Delta_{\nu\pi}^+)^{2n_q+1}$	$\tilde{\Delta}_{\nu\pi}^+ (\Gamma_{\nu\pi}^{+2})^{n_q}$
<sup>30</sup> P	T=0	12.66	12.60 (0.44%)	12.55 (0.86%)	11.96 (5.86%)	11.94 (5.95%)

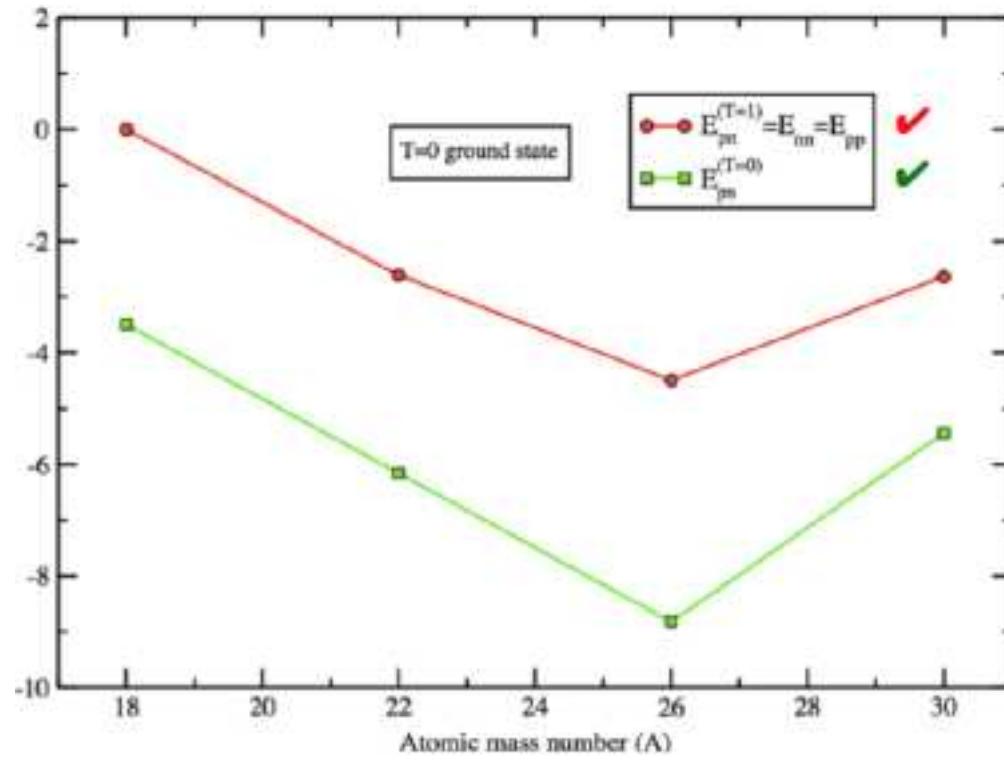
T=1 ground state

		Exact	$\tilde{\Gamma}_{\nu\pi}^+ (Q_{T=1}^+ + \Delta_{\nu\pi}^{+2})^{n_q}$	$\tilde{\Gamma}_{\nu\pi}^+ (Q_{T=1}^+)^{n_q}$	$\tilde{\Gamma}_{\nu\pi}^+ (\Delta_{\nu\pi}^{+2})^{n_q}$	$(\Gamma_{\nu\pi}^+)^{2n_q+1}$
<sup>54</sup> Co	T=1	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)

conclusion

isovector correlations are stronger in both T=0 and T=1 low-lying states

## Average value of pairing interactions



the extra T=0 pairing energy comes from the odd pn pair !