Exact solutions of the isovector pairing in terms of alpha-like quartets

M. Sambataro

Istituto Nazionale di Fisica Nucleare - Sezione di Catania, Italy

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Outline

- A brief history of the exact treatments of the isovector pairing Hamiltonian and the motivations behind this work
- The new formalism for its exact eigenstates
- Numerical applications
- Conclusions

in collaboration with N. Sandulescu (NIPNE, Bucharest)

Exact treatments of the isovector pairing Hamiltonian

- R.W. Richardson, Phys. Rev. **144**, 874 (1966)
  H.-T. Chen and R.W. Richardson, Phys. Lett. B **34**, 271 (1971)
  H.-T. Chen and R.W. Richardson, Nucl. Phys. A **212**, 317 (1973)
- Feng Pan and J.P. Draayer, Phys. Rev. C **66**, 044314 (2002)
- J. Links, H.-Q. Zhou, M.D. Gould, and R.H. McKenzie, J. Phys. A **35**, 6459 (2002)
- J. Dukelsky, V.G. Gueorguiev, P. Van Isacker, S. Dimitrova, B. Errea, and S. Lerma H., Phys. Rev. Lett. **96**, 072503 (2006)

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Exact eigenstates of the like-particle pairing Hamiltonian

$$H = \sum_{i=1}^{\Omega} \epsilon_i \mathcal{N}_i - g \sum_{i,i'=1}^{\Omega} P_i^\dagger P_{i'}$$

$$\mathcal{N}_i = \sum_{\sigma} a_{i\sigma}^\dagger a_{i\sigma}, \qquad P_i^\dagger = a_{i+}^\dagger a_{i-}^\dagger, \qquad (P_i^\dagger)^\dagger = P_i$$

The building blocks of the eigenstates:

$$B_\nu^\dagger = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} P_k^\dagger \qquad (E_\nu \equiv \text{“}\textit{pair energy}\text{”})$$

The (seniority-zero) eigenstates and eigenvalues:

$$|\Psi\rangle = \prod_{\nu=1}^{N} B_\nu^\dagger |0\rangle, \qquad E^{(\Psi)} = \sum_{\nu=1}^{N} E_\nu$$

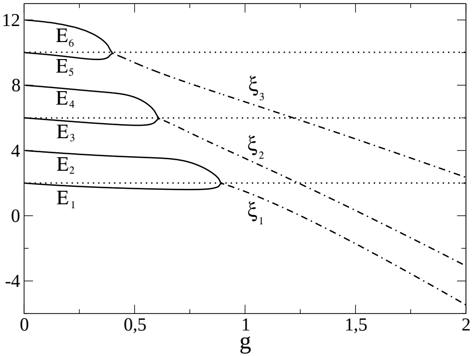
The equations to derive the E_ν's:

$$\frac{1}{g} - \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} + \sum_{\nu' \neq \nu}^{N} \frac{2}{E_{\nu'} - E_\nu} = 0$$

R.W. Richardson, Phys. Lett. **3**, 277 (1963)
R.W. Richardson and N. Sherman, Nucl. Phys. **52**, 221 (1964)

Numerical results

Pair energies for the ground state of a system of
12 particles over 12 equispaced levels



M.S., PRC 75, 054314 (2007)

The isovector pairing Hamiltonian

$$H^{(iv)} = \sum_{i=1}^{\Omega} \epsilon_i \mathcal{N}_i - g \sum_{i,i'=1}^{\Omega} \sum_{\tau=-1}^{1} P^\dagger_{i\tau} P_{i'\tau}$$

$$\mathcal{N}_i = \sum_{\sigma=\mp} \sum_{\tau=-1}^{1} a^\dagger_{i\sigma\tau} a_{i\sigma\tau}, \qquad P^\dagger_{i\tau} = [a^\dagger_{i+} a^\dagger_{i-}]^{T=1}_{\tau}, \qquad (P^\dagger_{i\tau})^\dagger = P_{i\tau}$$

$$\left(P^\dagger_{i\tau} : \quad \tau = -1 \ \ (pp), \quad \tau = 0 \ \ (pn), \quad \tau = +1 \ \ (nn) \right)$$

We confine our analysis to even-even $T = 0$ seniority-zero eigenstates.

The Hilbert space of the model is spanned by the states

$$P^\dagger_{i_1 M_{T_1}} P^\dagger_{i_2 M_{T_2}} \cdots P^\dagger_{i_N M_{T_N}} |0\rangle$$

subject to the condition

$$M_{T_1} + M_{T_2} + \cdots + M_{T_N} = 0$$

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First case: 2 protons and 2 neutrons

Building blocks:

$$B^\dagger_{\nu\tau} = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} P^\dagger_{k\tau}$$

Ansatz for the eigenstates:

$$|\Psi\rangle = [B_1^\dagger B_2^\dagger]^{T=0}|0\rangle$$

One finds:

$$\begin{aligned} H^{(iv)}|\Psi\rangle &= (E_1 + E_2)|\Psi\rangle \\ &+ \Big(1 - g\sum_k \frac{1}{2\epsilon_k - E_1} - \frac{g}{E_2 - E_1}\Big)[P^\dagger B_2^\dagger]^{T=0}|0\rangle \\ &+ \Big(1 - g\sum_k \frac{1}{2\epsilon_k - E_2} - \frac{g}{E_1 - E_2}\Big)[P^\dagger B_1^\dagger]^{T=0}|0\rangle \end{aligned}$$

being $P^\dagger_\tau = \sum_k P^\dagger_{k,\tau}$.

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being $P^{\dagger}_{\tau} = \sum_k P^{\dagger}_{k,\tau}$.

Second case: 4 protons and 4 neutrons

Ansatz:

$$|\Psi\rangle = d_1[B_1^\dagger B_2^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle + d_2[B_1^\dagger B_3^\dagger]^0[B_2^\dagger B_4^\dagger]^0|0\rangle + d_3[B_1^\dagger B_4^\dagger]^0[B_2^\dagger B_3^\dagger]^0|0\rangle$$

One finds:

$$\begin{aligned}
H^{(iv)}|\Psi\rangle &= (E_1+E_2+E_3+E_4)|\Psi\rangle \\
&+\left(d_1 - \sum_k \frac{g\cdot d_1}{2\epsilon_k - E_1} - \frac{g\cdot d_{123}}{E_2-E_1} - \frac{g\cdot d_{12}}{E_1-E_4} - \frac{g\cdot d_{13}}{E_1-E_3}\right)[P^\dagger B_2^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle \\
&+\left(d_1 - \sum_k \frac{g\cdot d_1}{2\epsilon_k - E_2} - \frac{g\cdot d_{123}}{E_1-E_2} - \frac{g\cdot d_{12}}{E_2-E_3} - \frac{g\cdot d_{13}}{E_2-E_4}\right)[P^\dagger B_1^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle \\
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&+\cdots(8\ \ \text{similar}\ \ \text{terms})
\end{aligned}$$

being: $d_{ij} \equiv d_i + d_j,\quad d_{123} \equiv d_1 + d_2 + d_3$

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$$|\Psi\rangle = d_1[B_1^\dagger B_2^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle + d_2[B_1^\dagger B_3^\dagger]^0[B_2^\dagger B_4^\dagger]^0|0\rangle + d_3[B_1^\dagger B_4^\dagger]^0[B_2^\dagger B_3^\dagger]^0|0\rangle$$

$$H^{(iv)}|\Psi\rangle = (E_1 + E_2 + E_3 + E_4)|\Psi\rangle + \sum_{i=1}^{12} F(i)[P^\dagger B_{i_1}^\dagger]^0[B_{i_2}^\dagger B_{i_3}^\dagger]^0|0\rangle$$

Variables:

$$E_1, E_2, E_3, E_4, d_2, d_3$$

$|\Psi\rangle$ is an eigenstate of $H^{(iv)}$ if a set of these variables exists such that

$$F(i) = 0 \qquad i = 1, 2, \cdots, 12$$

This set of variables must be such that

$$\min\left(\sum_{i=1}^{12}[F(i)]^2\right) = 0$$

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The general recipe for an even-even $N = Z$ system

- Adopt the collective pairs

$$B^\dagger_{\nu\tau} = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} P^\dagger_{k\tau}$$

as building blocks.

- Construct the states $|s\rangle$, product of $B^\dagger_\nu$'s arranged into $T = 0$ quartets,

$$|s\rangle = \prod_{q=1}^{N_q} [B^\dagger_{\nu(1,q,s)} B^\dagger_{\nu(2,q,s)}]^{T=0} |0\rangle,$$

such that the space $\{|s\rangle\}$ be invariant under the interchange of any two pairs.

- Expand $|\Psi\rangle$ into this basis: $|\Psi\rangle = \sum_{s=1}^{N_s} d_s |s\rangle$
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$$\frac{d_s}{g} - \sum_{k=1}^{\Omega} \frac{d_s}{2\epsilon_k - E_\nu} - \sum_{\nu' \neq \nu}^{(1,2N_q)} \frac{S_{\nu'\nu}(s)}{E_{\nu'} - E_\nu} = 0, \qquad S_{\nu'\nu}(s) = \sum_t I(t, \nu', \nu, s) d_t$$

This guarantees that

$$H^{(iv)} |\Psi\rangle = (\sum_\nu E_\nu) |\Psi\rangle$$

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Comparison with the like-particle case

The Richardson approach for the like-particle pairing:

$$|\Psi\rangle = \prod_{\nu=1}^{N} B_\nu^\dagger |0\rangle, \qquad B_\nu^\dagger = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} P_k^\dagger$$

$$\frac{1}{g} - \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} + \sum_{\nu' \neq \nu}^{N} \frac{2}{E_{\nu'} - E_\nu} = 0$$

The present approach for the isovector pairing:

$$|\Psi\rangle = \sum_{s=1}^{N_s} d_s \prod_{q=1}^{N_q} [B_{\nu(1,q,s)}^\dagger B_{\nu(2,q,s)}^\dagger]^0 |0\rangle, \qquad B_{\nu\tau}^\dagger = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_\nu} P_{k\tau}^\dagger$$

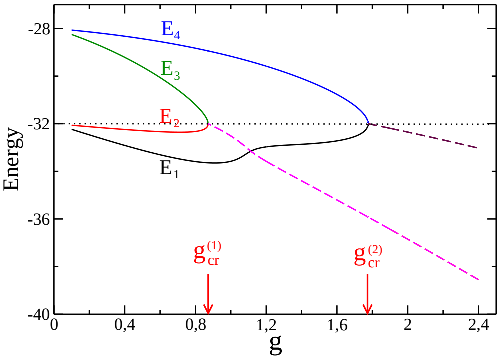
$$\frac{d_s}{g} - \sum_{k=1}^{\Omega} \frac{d_s}{2\epsilon_k - E_\nu} - \sum_{\nu' \neq \nu}^{(1,2N_q)} \frac{S_{\nu'\nu}(s)}{E_{\nu'} - E_\nu} = 0$$

In both cases:

$$E^{(\Psi)} = \sum_\nu E_\nu$$

Numerical results: 2 quartets

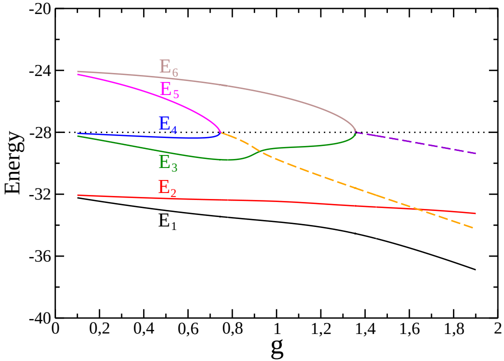
Pair energies for the ground state of a system of
4 protons and 4 neutrons over 4 equispaced levels



M.S. and N. Sandulescu, to be published

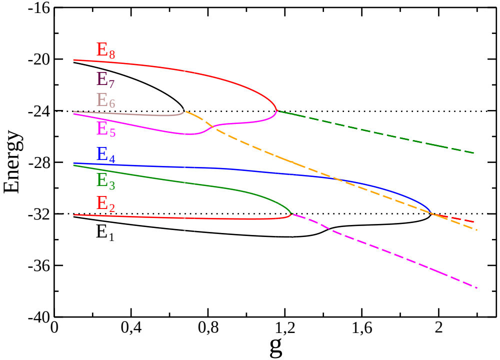
Numerical results: 3 quartets

Pair energies for the ground state of a system of 6 protons and 6 neutrons over 6 equispaced levels



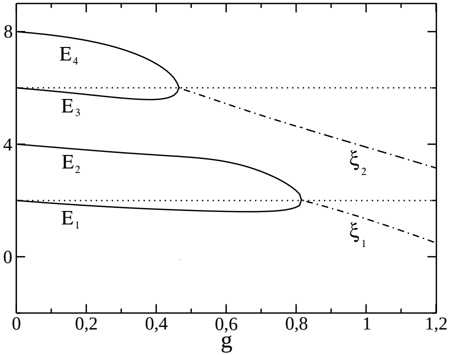
Numerical results: 4 quartets

Pair energies for the ground state of a system of 8 protons and 8 neutrons over 8 equispaced levels



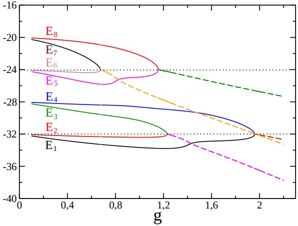
Richardson results for the like-particle pairing: 4 pairs

Pair energies for the ground state of a system of
8 like particles over 8 equispaced levels

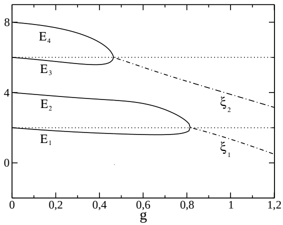


Comparison: isovector vs like-particle pairing

Pair energies for the ground state of a system of 8 protons and 8 neutrons over 8 equispaced levels



Pair energies for the ground state of a system of 8 like particles over 8 equispaced levels



Approximate treatments of pairing

Like-particle pairing

$$|PBCS\rangle = (B^\dagger)^N|0\rangle, \qquad B^\dagger_\nu = \sum_{k=1}^{\Omega} x(k) P^\dagger_k$$

Proton-neutron $T = 1$ pairing

$$|QCM\rangle = ([B^\dagger B^\dagger]^0)^{N/2}|0\rangle, \qquad B^\dagger_{\nu\tau} = \sum_{k=1}^{\Omega} y(k) P^\dagger_{k\tau}$$

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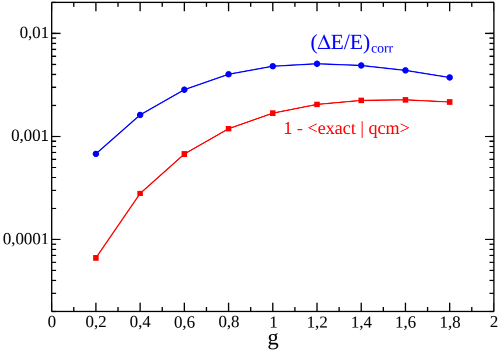
qcm vs exact results

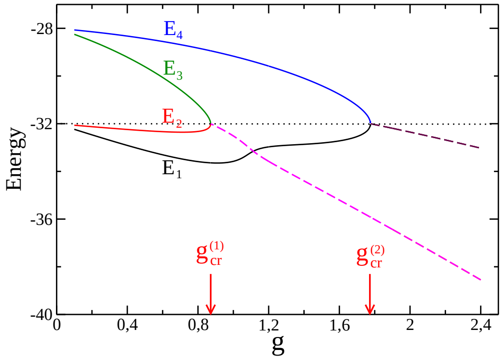
6 protons and 6 neutrons over 6 equispaced levels



Ground state for 4 protons and 4 neutrons

$$|\Psi\rangle = d_1[B_1^\dagger B_2^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle + d_2[B_1^\dagger B_3^\dagger]^0[B_2^\dagger B_4^\dagger]^0|0\rangle + d_3[B_1^\dagger B_4^\dagger]^0[B_2^\dagger B_3^\dagger]^0|0\rangle$$

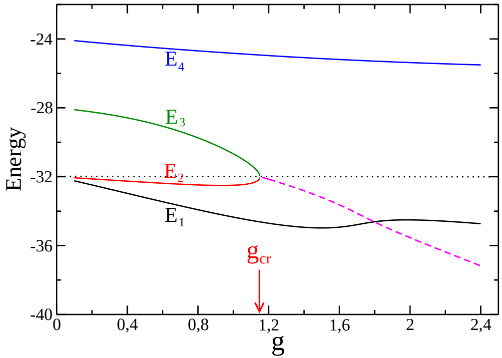
Pair energies for the ground state of a system of 4 protons and 4 neutrons over 4 equispaced levels



First excited state for 4 protons and 4 neutrons

$$|\Psi\rangle = d_1[B_1^\dagger B_2^\dagger]^0[B_3^\dagger B_4^\dagger]^0|0\rangle + d_2[B_1^\dagger B_3^\dagger]^0[B_2^\dagger B_4^\dagger]^0|0\rangle + d_3[B_1^\dagger B_4^\dagger]^0[B_2^\dagger B_3^\dagger]^0|0\rangle$$

Pair energies for the first excited state of a system of
4 protons and 4 neutrons over 4 equispaced levels



Like-particle pairing and quartets

$$|\Psi\rangle = \prod_{\lambda=1}^{N/2} B^\dagger_{2\lambda-1} B^\dagger_{2\lambda} |0\rangle$$

$$B^\dagger_i = \sum_{k=1}^{\Omega} \frac{1}{2\epsilon_k - E_i} P^\dagger_k, \qquad E_{2\lambda-1} = \xi_\lambda - i\eta_\lambda, \qquad E_{2\lambda} = \xi_\lambda + i\eta_\lambda$$

One can verify that

$$B^\dagger_{2\lambda-1} B^\dagger_{2\lambda} = \sum_{k,k'=1}^{\Omega} \gamma^{(\lambda)}_{kk'} P^\dagger_k P^\dagger_{k'}, \qquad \gamma^{(\lambda)}_{kk'} \in R$$

$$|\Psi\rangle = \prod_{\lambda=1}^{N/2} Q^\dagger_\lambda |0\rangle, \qquad Q^\dagger_\lambda \equiv B^\dagger_{2\lambda-1} B^\dagger_{2\lambda}$$

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$$|\Psi\rangle = \prod_{\lambda=1}^{N/2} Q_\lambda^{\dagger} |0\rangle, \qquad Q_\lambda^{\dagger} \equiv B_{2\lambda-1}^{\dagger} B_{2\lambda}^{\dagger}$$

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Conclusions

- We have derived the even-even $T = 0$ seniority-zero eigenstates of an isovector pairing Hamiltonian.
- We have found that these eigenstates are linear superpositions of products of $T = 1$ collective pairs arranged into $T = 0$ quartets.
- This grouping of protons and neutrons first into $T = 1$ collective pairs and then into $T = 0$ quartets is the distinctive feature of these eigenstates.
- The isovector pairing Hamiltonian favours the formation of α-like structures in $N = Z$ nuclei.