Isoscalar and isovector proton-neutron pairing in N > Z nuclei

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Outline of this talk

1. Introduction

2. Isovector-isoscalar quartet model for odd-odd nuclei

- proton-neutron pairing and quartetting in odd-odd nuclei;
- competition between isovector and isoscalar pairing in odd-odd nuclei.

3. Generalization of the isovector-isoscalar quartet model for N > Z even-even nuclei

- like-particle pairing and quartetting in N > Z nuclei;
- competition between isovector and isoscalar pairing in N > Z nuclei.

4. Conclusions and perspectives

Alpha-like quartet condensation offers very accurate results for isovector pairing correlations in even-even N=Z and N>Z nuclei (errors < 1%).

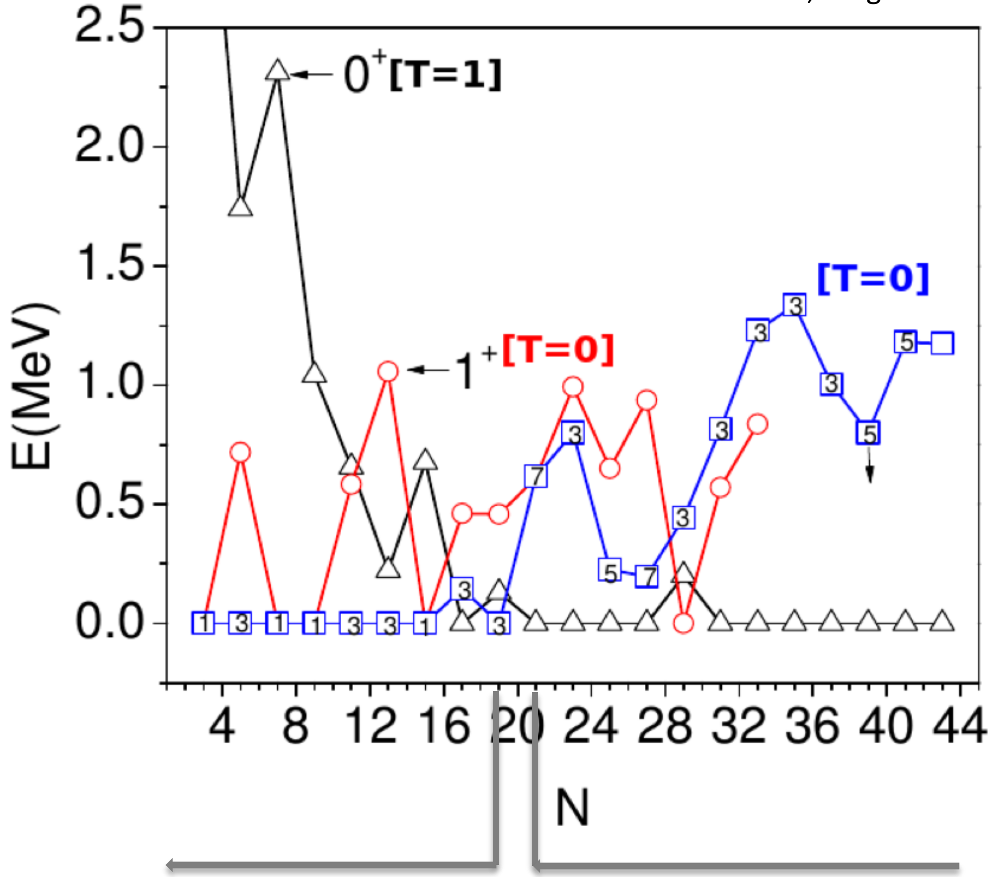
What about the isoscalar proton-neutron pairing?

Does exists such kind of pairing in N=Z nuclei?

How it competes with the isovector pairing?

Proton-neutron pairing in odd-odd nuclei

S. Frauendorf and A. O. Macchiavelli, Progr. Part. Nucl. Phys. 78, 24 (2014)



lowest T=1, 0^+ states

lowest T=0, 1^+ states

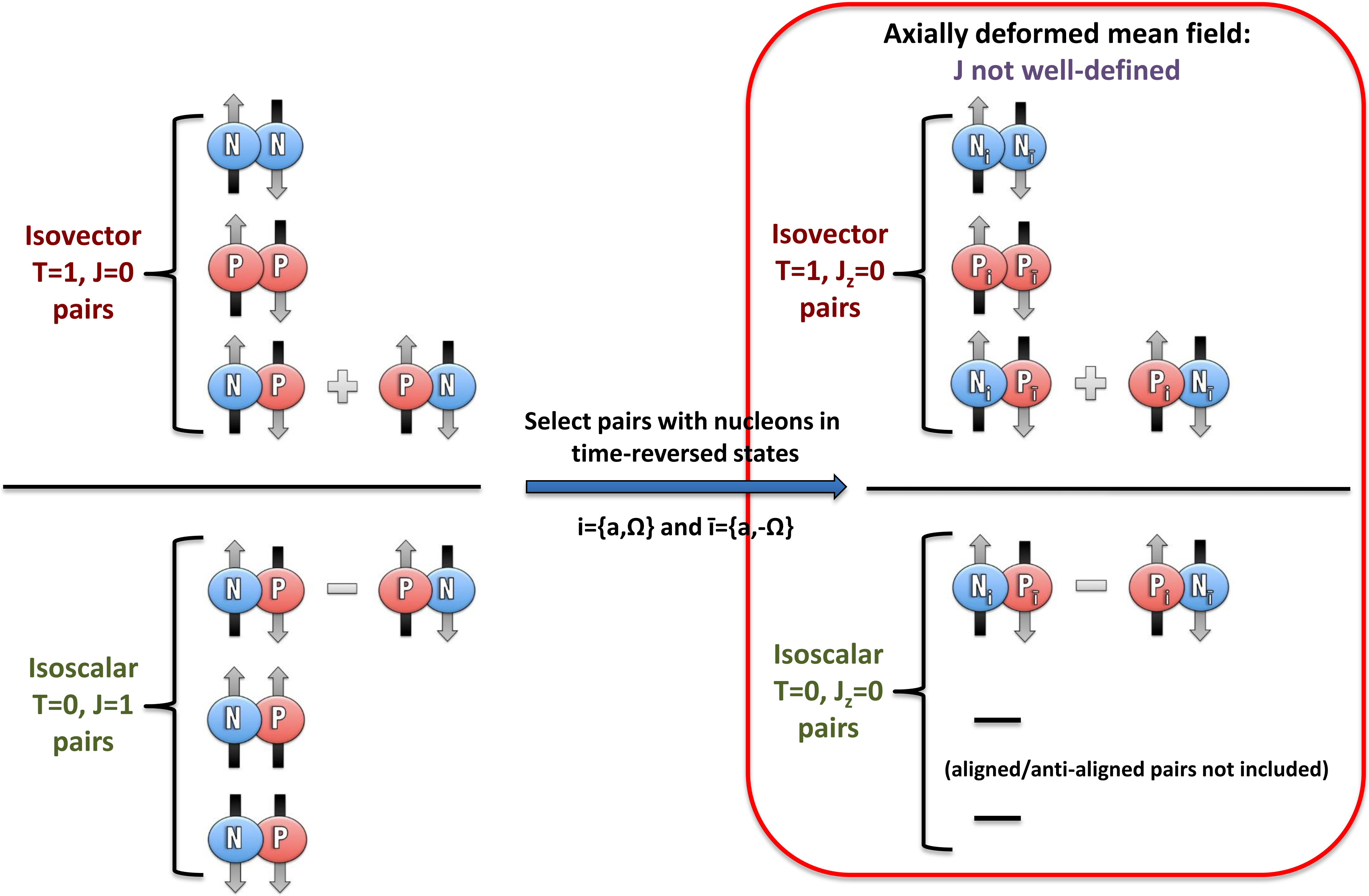
lowest T=0, I^+ states

A<40
T=0 ground state

A≥40
T=1 ground state

⟹ **Competition between T=1 and T=0 pairing !**

Isovector and isoscalar pairs in N=Z nuclei

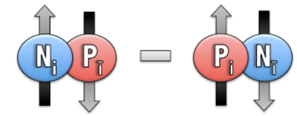


Pairing in even-even N=Z nuclei: axially deformed symmetry

Isovector T=1, J_z=0 pairs

$$P^{+}_{i,0} = \frac{1}{\sqrt{2}}(\nu^{+}_{i}\pi^{+}_{\bar{i}} + \pi^{+}_{i}\nu^{+}_{\bar{i}}) \qquad P^{+}_{i,1} = \nu^{+}_{i}\nu^{+}_{\bar{i}} \qquad P^{+}_{i,-1} = \pi^{+}_{i}\pi^{+}_{\bar{i}}$$

Isoscalar T=0, J_z=0 pairs



$$D^{+}_{i,0} = \frac{1}{\sqrt{2}}(\nu^{+}_{i}\pi^{+}_{\bar{i}} - \pi^{+}_{i}\nu^{+}_{\bar{i}})$$

Hamiltonian

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \underbrace{\sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^{+}_{i,t_z} P_{j,t_z}}_{\text{isovector}} + \underbrace{\sum_{i,j} V^{T=0}(i,j) D^{+}_{i,j_z=0} D_{j,j_z=0}}_{\text{isoscalar}}$$

Isovector quartets

$$A^{+} = \underbrace{2\Gamma^{+}_{1}\Gamma^{+}_{-1} - \Gamma^{+2}_{0}}$$

$$\Gamma^{+}_{t} = \sum_i x_i P^{+}_{i,t}$$

Isovector-isoscalar quartet

$$Q^{+} = 2\Gamma^{+}_{1}\Gamma^{+}_{-1} - \Gamma^{+2}_{0} + \Delta^{+2}_{0}$$

Collective isoscalar pairs

$$\Delta^{+}_{0} = \sum_i y_i D^{+}_{i,0}$$

⇩

Quartet condensate

$$|QCM\rangle = (Q^{+})^{n_q}|0\rangle \qquad n_q = (N+Z)/4$$

(exact solution for a set of degenerate states)

Calculation scheme

Hamiltonian: $\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^+_{i,t_z} P_{j,t_z} + \sum_{i,j} V^{T=0}(i,j) D^+_{i,j_z=0} D_{j,j_z=0}$

Quartet condensate: $|QCM\rangle = (Q^+)^{n_q}|0\rangle = (\underbrace{2\Gamma_1^+\Gamma_{-1}^+ - \Gamma_0^{+2}} + \underbrace{\Delta_0^{+2}})^{n_q}|0\rangle$

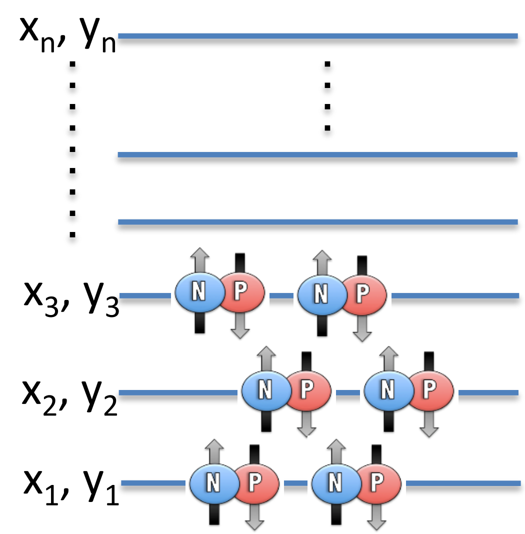
$\Gamma_t^+ = \sum_i x_i P^+_{i,t} \quad \Delta_0^+ = \sum_i y_i D^+_{i,0}$

Unknown parameters: mixing amplitudes x_i and y_i

Minimization: $\delta_{x,y} \langle \Psi | \hat{H} | \Psi \rangle = 0$

Constraint: $\langle \Psi | \Psi \rangle = 1$

The method of recurrence relations



Auxiliary states: $|n_1 n_2 n_3 n_4\rangle = (\Gamma_1^+)^{n_1} (\Gamma_{-1}^+)^{n_2} (\Gamma_0^+)^{n_3} (\Delta_0^+)^{n_4} |0\rangle$

nn pp pn(T=1) pn(T=0)

Competition between T=1 and T=0 pairing in realistic calculations

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,0} D_{j,0}$$

- s.p. states given by axially deformed Skyrme-HF calculations

- zero range delta interaction $V^{T=\{0,1\}}_{pairing}(\vec{r}_1 - \vec{r}_2) = V^{T=\{0,1\}}_0 \delta(\vec{r}_1 - \vec{r}_2) \hat{P}_{S=\{0,1\}}$ $\begin{cases} V_0^{T=1} = 465 \text{ MeV fm}^{-3} \\ V_0^{T=0}/V_0^{T=1} = 1.5 \end{cases}$ (Bertsch et al.)

$$|QCM\rangle = (A^{+} + \Delta_0^{+2})^{n_q}|0\rangle \quad |iv\rangle = (A^{+})^{n_q}|0\rangle \quad |is\rangle = (\Delta_0^{+2})^{n_q}|0\rangle$$

Correlation energies (MeV)

$$E_{corr} = E_0 - E$$

		Exact	|QCM>	$\|iv\rangle$	$\|is\rangle$	$\langle iv\|is\rangle$
^{16}O	^{20}Ne	11.38	11.38 (0.00%)	11.31 (0.62%)	10.92 (4.00%)	0.976
	^{24}Mg	19.32	19.31 (0.03%)	19.18 (0.74%)	18.93 (2.00%)	0.980
	^{28}Si	18.74	18.74 (0.01%)	18.71 (0.14%)	18.54 (1.07%)	0.992
^{40}Ca	^{44}Ti	7.095	7.094 (0.02%)	7.08 (0.18%)	6.30 (10.78%)	0.928
	^{48}Cr	12.78	12.76 (0.1%)	12.69 (0.67%)	12.22 (4.37%)	0.936
	^{52}Fe	16.39	16.34 (0.26%)	16.19 (1.17%)	15.62 (4.65%)	0.946
^{100}Sn	^{104}Te	4.53	4.52 (0.06%)	4.49 (0.82%)	4.02 (11.26%)	0.955
	^{108}Xe	8.08	8.03 (0.61%)	7.96 (1.45%)	6.75 (16.47%)	0.814
	^{112}Ba	9.36	9.27 (0.93%)	9.22 (1.43%)	7.50 (19.81%)	0.784

Conclusions:

- **QCM describes with very good precision the isoscalar-isovector pairing (errors under 1%);**
- **isovector pairing correlations are stronger than the isoscalar ones;**
- **isoscalar pairing coexist with the isovector pairing.**

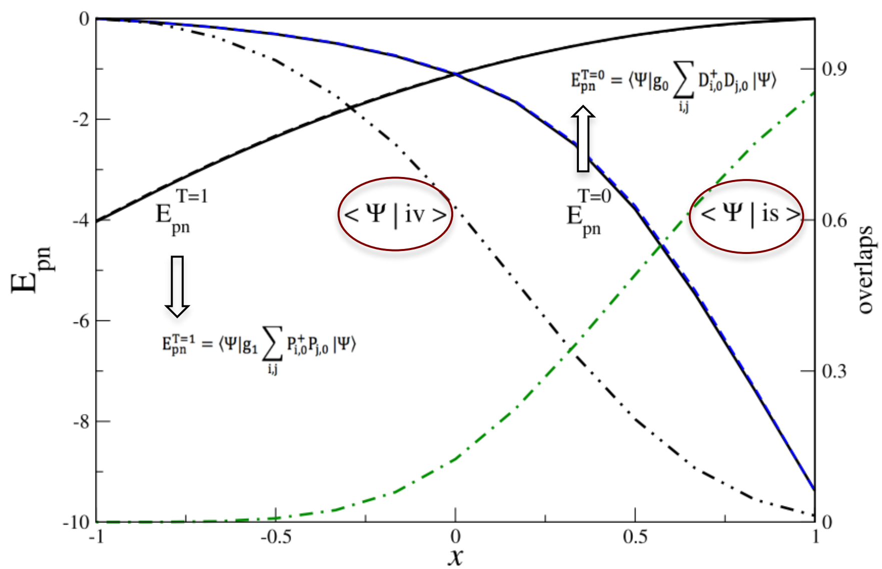
N. Sandulescu, D. N. and D. Gambacurta, Phys. Lett. B751 (2015), p. 348

Evolution of the isovector and isoscalar proton-neutron pairing correlations

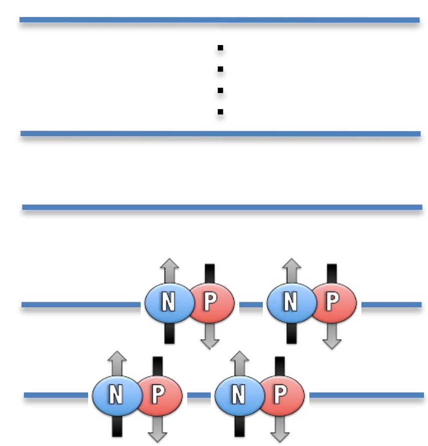
$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + g_1 \sum_{i,j} \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + g_0 \sum_{i,j} D^{+}_{i,0} D_{j,0}$$

$$\begin{cases} g_1 = g(1-x)/2 \\ g_0 = g(1+x)/2 \end{cases}$$

$$|\Psi\rangle = (A^{+} + \Delta_0^{+2})^{n_q}|0\rangle \quad |iv\rangle = (A^{+})^{n_q}|0\rangle \quad |is\rangle = (\Delta_0^{+2})^{n_q}|0\rangle$$



- 4 proton-neutron pairs;
- 10 equidistant levels.



Pairing energies:

- E(T=1) and E(T=0) follow very well the exact pairing energies (obtained by diagonalization);
- isovector and isoscalar pairing correlations coexist for any ratio between the strengths of the two pairing forces.

Overlaps:

- the overlaps show a smooth transition from a condensate of quartets to a condensate of pairs.

N. Sandulescu, D. N. and D. Gambacurta, Phys. Lett. B751 (2015), p. 348

How does the proton-neutron pairing affects the ground state of odd-odd N=Z nuclei?

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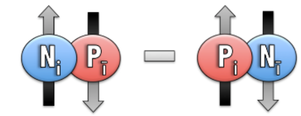
Generalization of QCM for odd-odd systems!

Pairing and quartetting in odd-odd N=Z nuclei

Isovector T=1, J_z=0 pairs

$$P^+_{i,0} = \frac{1}{\sqrt{2}}(\nu^+_i \pi^+_{\bar{i}} + \pi^+_i \nu^+_{\bar{i}}) \qquad P^+_{i,1} = \nu^+_i \nu^+_{\bar{i}} \qquad P^+_{i,-1} = \pi^+_i \pi^+_{\bar{i}}$$

Isoscalar T=0, J_z=0 pairs



$$D^+_{i,0} = \frac{1}{\sqrt{2}}(\nu^+_i \pi^+_{\bar{i}} - \pi^+_i \nu^+_{\bar{i}})$$

Hamiltonian

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \underbrace{\sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^+_{i,t_z} P_{j,t_z}}_{\text{isovector}} + \underbrace{\sum_{i,j} V^{T=0}(i,j) D^+_{i,j_z=0} D_{j,j_z=0}}_{\text{isoscalar}}$$

Isovector-isoscalar quartets

$$\Delta^+_0 = \sum_i y_i D^+_{i,0} \qquad Q^+ = 2\Gamma^+_1 \Gamma^+_{-1} - \Gamma^{+2}_0 + \Delta^{+2}_0 \qquad \Gamma^+_t = \sum_i x_i P^+_{i,t}$$

T=0 state:

$$|is, QCM\rangle = \tilde{\Delta}^+_0 (Q^+)^{n_q} |0\rangle$$

(exact solution for degenerate states)

collective isoscalar odd pair

$$\tilde{\Delta}^+_0 = \sum_i z_i D^+_{i,0}$$

T=1 state:

$$|iv, QCM\rangle = \tilde{\Gamma}^+_0 (Q^+)^{n_q} |0\rangle$$

(exact solution for degenerate states)

collective isovector odd pair

$$\tilde{\Gamma}^+_0 = \sum_i z_i P^+_{i,0}$$

Calculation scheme

Hamiltonian: $$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^{+}_{i,t_z} P_{j,t_z} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,j_z=0} D_{j,j_z=0}$$

Pair-quartet condensate:

(T=0 state)

$$|is, QCM\rangle = \tilde{\Delta}^{+}_{0} (2\Gamma^{+}_{1}\Gamma^{+}_{-1} - \Gamma^{+2}_{0} + \Delta^{+2}_{0})^{n_q} |0\rangle \qquad \tilde{\Delta}^{+}_{0} = \sum_i z_i D^{+}_{i,0}$$

$$|iv, QCM\rangle = \tilde{\Gamma}^{+}_{0} (2\Gamma^{+}_{1}\Gamma^{+}_{-1} - \Gamma^{+2}_{0} + \Delta^{+2}_{0})^{n_q} |0\rangle \qquad \tilde{\Gamma}^{+}_{0} = \sum_i z_i P^{+}_{i,0}$$

(T=1 state)

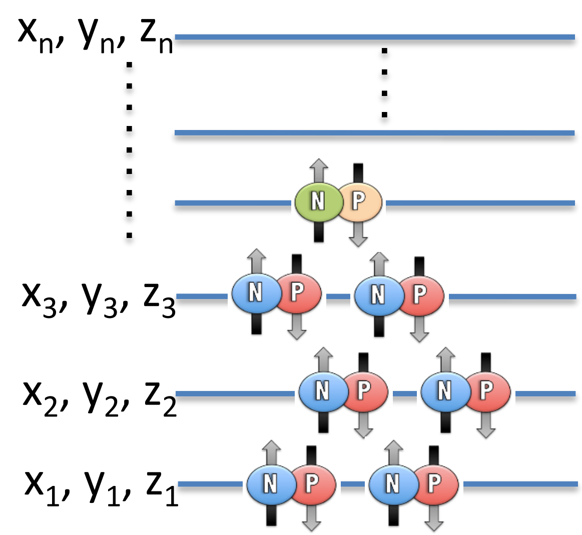
$$\Gamma^{+}_{t} = \sum_i x_i P^{+}_{i,t} \qquad \Delta^{+}_{0} = \sum_i y_i D^{+}_{i,0}$$

Unknown parameters: mixing amplitudes xi, yi and z_i

Minimization: $\delta_{x,y,z}\langle \Psi | \hat{H} | \Psi \rangle = 0$

Constraint: $\langle \Psi | \Psi \rangle = 1$

The method of recurrence relations



Auxiliary states:

$$|n_1 n_2 n_3 n_4 n_5\rangle = (\Gamma^{+}_{1})^{n_1} (\Gamma^{+}_{-1})^{n_2} (\Gamma^{+}_{0})^{n_3} (\Delta^{+}_{0})^{n_4} \tilde{\Delta}^{+}_{0} |0\rangle \quad \text{(T=0 state)}$$

$$|m_1 m_2 m_3 m_4 m_5\rangle = (\Gamma^{+}_{1})^{m_1} (\Gamma^{+}_{-1})^{m_2} (\Gamma^{+}_{0})^{m_3} (\Delta^{+}_{0})^{m_4} \tilde{\Gamma}^{+}_{0} |0\rangle \quad \text{(T=1 state)}$$

Strength of the pairing force in T=1 and T=0 channels

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^{+}_{i,t_z} P_{j,t_z} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,j_z=0} D_{j,j_z=0}$$

- s.p. states given by Skyrme-HF calculations for axially deformed m.f.

- zero range delta interaction $V^{T=\{0,1\}}_{pairing}(\vec{r}_1 - \vec{r}_2) = V_0^{T=\{0,1\}} \delta(\vec{r}_1 - \vec{r}_2) \hat{P}_{S=\{0,1\}}$

Strength of the isovector pairing force

$$V_0^{T=1} = 465 \ \text{MeV fm}^{-3}$$

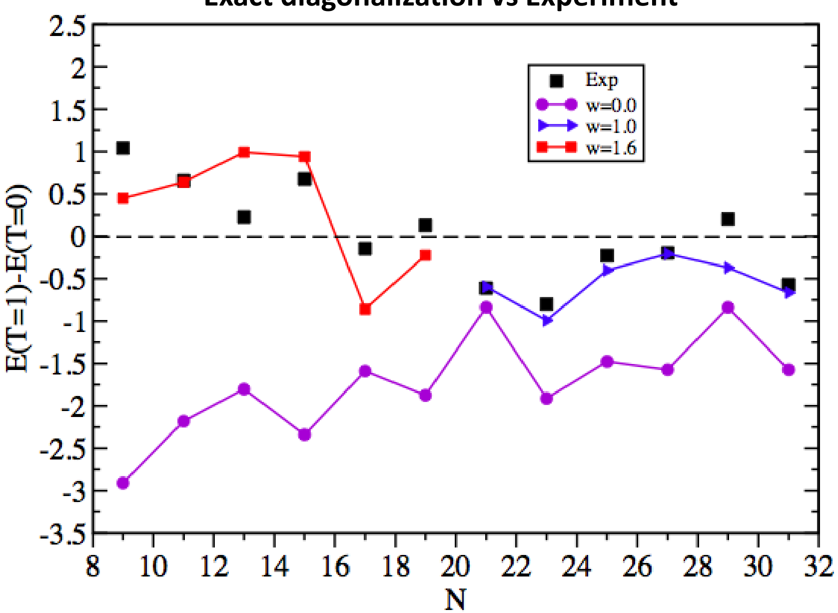
Strength of the isoscalar pairing force

$$V_0^{T=0} = w \, V_0^{T=1}$$

w = ?

A<40: w=1.6 A≥40: w=1.0

Exact diagonalization vs Experiment



The structure of the lowest T=0 and T=1 states of odd-odd nuclei

Correlation energies (MeV): $E_{corr} = E_0 - E$

T=0 ground state

		Exact	$\tilde{\Delta}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}$	$\tilde{\Delta}_0^+(Q_{iv}^+)^{n_q}$	$(\Delta_0^+)^{2n_q+1}$	$\tilde{\Delta}_0^+(\Gamma_0^{+2})^{n_q}$
^{30}P	T=0	12.66	12.60 (0.44%)	12.55 (0.86%)	11.96 (5.86%)	11.94 (5.95%)

T=1 ground state

		Exact	$\tilde{\Gamma}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}$	$\tilde{\Gamma}_0^+(Q_{iv}^+)^{n_q}$	$\tilde{\Gamma}_0^+(\Delta_0^{+2})^{n_q}$	$(\Gamma_0^+)^{2n_q+1}$
^{54}Co	T=1	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)

D.N., N. Sandulescu, D. Gambacurta, PTEP 2017, 073D05

Conclusions:

QCM describes well the low-lying states of odd-odd nuclei.

The pn pair condensates (isovector or isoscalar) are less accurate than the quartet condensates.

Isovector and isoscalar pairing correlations coexist in the even-even core.

^{50}Mn	Exact	$\tilde{\Gamma}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}$	$\tilde{\Gamma}_0^+(Q_{iv}^+)^{n_q}$	$\tilde{\Gamma}_0^+(\Delta_0^{+2})^{n_q}$	$(\Gamma_0^+)^{2n_q+1}$
$T = 1$	12.77	12.76 (0.07%)	12.75 (0.14%)	12.52 (2.02%)	12.62 (1.22%)
$T = 0$	12.37	12.36 (0.04%)	12.34 (0.24%)	12.18 (1.61%)	12.19 (1.48%)
	Exact	$\tilde{\Delta}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}$	$\tilde{\Delta}_0^+(Q_{iv}^+)^{n_q}$	$(\Delta_0^+)^{2n_q+1}$	$\tilde{\Delta}_0^+(\Gamma_0^{+2})^{n_q}$

^{54}Co	$T = 1$	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)
	$T = 0$	15.93	15.92 (0.04%)	15.89 (0.22%)	15.53 (2.56%)	15.66 (1.73%)

Core		g.s./e.s.	Exact	$\vert iv;QCM\rangle/\vert is;QCM\rangle$	$\vert iv;Q_{iv}\rangle/\vert is;Q_{iv}\rangle$	$\vert iv;C_{is}\rangle/\vert C_{is}\rangle$	$\vert C_{iv}\rangle/\vert is;C_{iv}\rangle$
^{16}O	^{18}F	T=0	3.365	3.365	3.365	3.365	3.365
		T=1	2.914	2.914	2.914	2.914	2.914
	^{22}Na	T=0	13.869	13.869 (0.00%)	13.859 (0.07%)	13.853 (0.12%)	13.848 (0.15%)
		T=1	13.230	13.226 (0.03%)	13.224 (0.05%)	12.974 (1.97%)	13.216 (0.11%)
	^{26}Al	T=0	22.058	22.052 (0.03%)	22.043 (0.07%)	21.941 (0.53%)	21.789 (1.24%)
		T=1	21.066	21.061 (0.02%)	21.051 (0.07%)	20.929 (0.66%)	20.980 (0.41%)
	^{30}P	T=0	12.655	12.599 (0.44%)	12.547 (0.86%)	11.955 (5.86%)	11.944 (5.95%)
		T=1	11.715	11.664 (0.44%)	11.620 (0.82%)	10.937 (7.11%)	10.955 (6.94%)
^{40}Ca	^{42}Sc	T=1	0.837	0.837	0.837	0.837	0.837
		T=0	0.241	0.241	0.241	0.241	0.241
	^{46}V	T=1	7.922	7.919 (0.04%)	7.914 (0.10%)	7.328 (8.11%)	7.758 (2.11%)
		T=0	6.930	6.929 (0.01%)	6.925 (0.07%)	6.729 (2.99%)	6.791 (2.05%)
	^{50}Mn	T=1	12.774	12.765 (0.07%)	12.756 (0.14%)	12.521 (2.02%)	12.620 (1.22%)
		T=0	12.372	12.367 (0.04%)	12.343 (0.24%)	12.176 (1.61%)	12.192 (1.48%)
	^{54}Co	T=1	16.138	16.116 (0.14%)	16.093 (0.28%)	15.667 (3.01%)	15.856 (1.78%)
		T=0	15.931	15.925 (0.04%)	15.896 (0.22%)	15.533 (2.56%)	15.660 (1.73%)
^{100}Sn	^{102}Sb	T=1	0.104	0.104	0.104	0.104	0.104
		T=0	0.039	0.039	0.039	0.039	0.039
	^{106}I	T=1	5.147	5.143 (0.08%)	5.135 (0.23%)	4.706 (9.37%)	4.925 (4.51%)
		T=0	4.525	4.523 (0.04%)	4.506 (0.42%)	4.196 (7.84%)	4.288 (5.53%)
	^{110}Cs	T=1	8.034	7.989 (0.56%)	7.974 (0.75%)	7.164 (12.14%)	7.589 (5.86%)
		T=0	7.096	7.064 (0.45%)	7.040 (0.80%)	6.472 (9.64%)	6.646 (6.77%)
	^{114}La	T=1	9.758	9.723 (0.36%)	9.687 (0.73%)	8.789 (11.03%)	9.273 (5.23%)
		T=0	8.954	8.929 (0.28%)	8.917 (0.42%)	8.311 (7.74%)	8.513 (5.18%)

How does the isoscalar and isovector proton-neutron pairing compete in N > Z even-even nuclei?

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Generalization of QCM for N > Z systems!

D. N., P. Buganu, D. Gambacurta, and N. Sandulescu, Phys. Rev. C98, 064319 (2018)

Pairing and quartetting in even-even N > Z nuclei

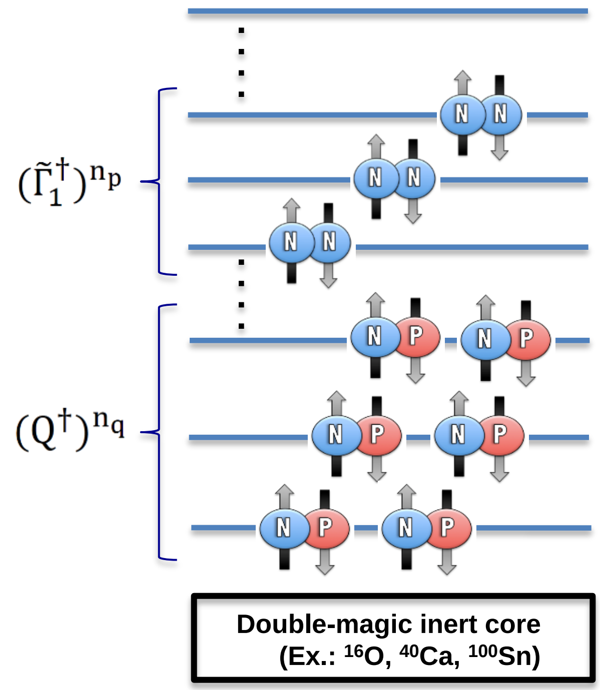
Isovector-isoscalar quartets

$\Gamma_t^+ = \sum_i x_i P_{i,t}^+$ $\qquad Q^+ = 2\Gamma_1^+\Gamma_{-1}^+ - \Gamma_0^{+2} + \Delta_0^{+2}$ $\qquad \Delta_0^+ = \sum_i y_i D_{i,0}^+$

$$|QCM\rangle = (\tilde{\Gamma}_1^\dagger)^{n_p}\,(Q^\dagger)^{n_q}|0\rangle$$

Collective nn pairs $\quad \tilde{\Gamma}_1^\dagger = \sum_i z_i P_{i,t=1}^+$

$\tilde{\Gamma}_1^\dagger$ and Γ_1^+ have different structures: $z_i \neq x_i$



Calculation scheme

Hamiltonian: $\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^{+}_{i,t_z} P_{j,t_z} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,j_z=0} D_{j,j_z=0}$

Pair-quartet condensate: $|QCM\rangle = (\tilde{\Gamma}^{\dagger}_1)^{n_p} \left(2\Gamma^{\dagger}_1 \Gamma^{\dagger}_{-1} - \Gamma^{\dagger 2}_0 + \Delta^{\dagger 2}_0\right)^{n_q} |0\rangle$

Unknown parameters: mixing amplitudes xi, yi and z_i

$$\begin{cases} \Gamma^{+}_t = \sum_i x_i P^{+}_{i,t} \\ \Delta^{+}_0 = \sum_i y_i D^{+}_{i,0} \\ \tilde{\Gamma}^{\dagger}_1 = \sum_i z_i P^{+}_{i,t=1} \end{cases}$$

Minimization: $\delta_{x,y,z} \langle \Psi | \hat{H} | \Psi \rangle = 0$

Constraint: $\langle \Psi | \Psi \rangle = 1$

The method of recurrence relations

Auxiliary states: $|n_1 n_2 n_3 n_4 n_5\rangle = (\Gamma^{\dagger}_1)^{n_1} (\Gamma^{\dagger}_{-1})^{n_2} (\Gamma^{\dagger}_0)^{n_3} (\Delta^{\dagger}_0)^{n_4} (\tilde{\Gamma}^{\dagger}_1)^{n_5} |0\rangle$

Strength of the pairing force in T=1 and T=0 channels

$$\hat{H} = \hat{H}_0 + \hat{H}_p$$

$$\hat{H}_0 = \sum_{i,\tau=\pm 1} \varepsilon_{i,\tau} N_{i,\tau}$$ - s.p. states given by Skyrme-HF calculations for axially deformed m.f.

Pairing interactions:

(I) State-independent pairing force

$$\hat{H}_p = \boxed{V^{(T=1)}} \sum_{\substack{i,j,\\ t=-1,0,1}} P^{\dagger}_{i,t} P_{j,t} + \boxed{V^{(T=0)}} \sum_{i,j} D^{\dagger}_{i,0} D_{j,0}$$

Strength of the isovector pairing force

$$V^{(T=1)} = -24/A$$

Strength of the isoscalar pairing force

$$V^{(T=0)} = \boxed{w} \cdot V^{(T=1)}$$

w = ? ⟹
- sd-shell nuclei: w=1.2
- heavier nuclei: w=0.8

(II) Zero range delta interaction

$$\hat{H}_p = \sum_{i,j} V^{(T=1)}_{i,j} \sum_{t=-1,0,1} P^{\dagger}_{i,t} P_{j,t} + \sum_{i,j} V^{(T=0)}_{i,j} D^{\dagger}_{i,0} D_{j,0}$$

$$V^{T=\{0,1\}}_{pairing}(\vec{r}_1 - \vec{r}_2) = \boxed{V_0^{T=\{0,1\}}} \delta(\vec{r}_1 - \vec{r}_2) \hat{P}_{S=\{0,1\}}$$

Strength of the isovector pairing force

$$V_0^{T=1} = 465 \ \text{MeV fm}^{-3}$$

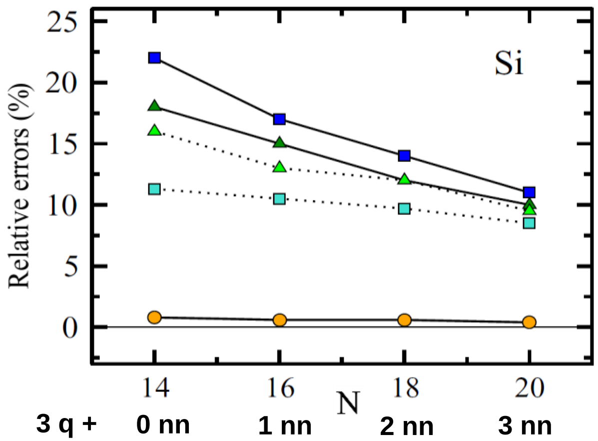
Strength of the isoscalar pairing force

$$V_0^{T=0} = \boxed{w} \cdot V_0^{T=1}$$

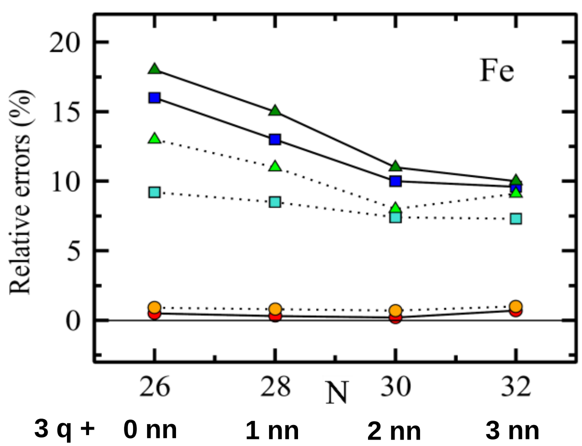
w = ? ⟹
- sd-shell nuclei: w=1.6
- heavier nuclei: w=1.0

Isovector and isoscalar pn pairing in N > Z nuclei: results (I)

sd-shell nucleus

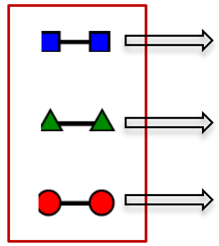


pf-shell nucleus



Correlation energies (MeV): $E_{corr} = E_0 - E$

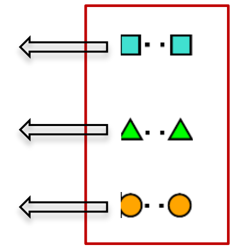
(I) State-independent pairing force **(II) Zero range delta interaction**



$|C_{iv}\rangle = (\tilde{\Gamma}_1^{\dagger})^{n_p} \left(\Gamma_0^{\dagger 2}\right)^{n_q} |0\rangle$

$|C_{is}\rangle = (\tilde{\Gamma}_1^{\dagger})^{n_p} \left(\Delta_0^{\dagger 2}\right)^{n_q} |0\rangle$

$|QCM\rangle = (\tilde{\Gamma}_1^{\dagger})^{n_p} (Q^{\dagger})^{n_q} |0\rangle$



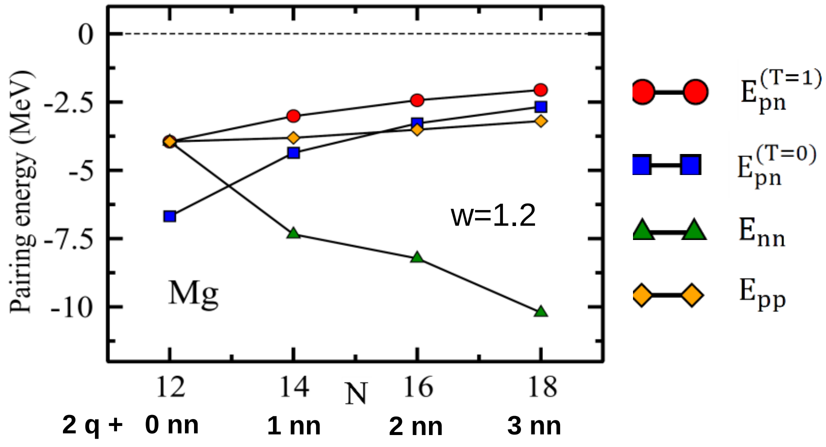
|QCM> describes well the ground state pairing correlations (errors < 1%).

|Civ> and |Cis> (larger errors): not a fast transition to a pure condensate of iv/is pn pairs.

D. N., P. Buganu, D. Gambacurta, and N. Sandulescu, Phys. Rev. C98, 064319 (2018)

Isovector and isoscalar pn pairing in N > Z nuclei: results (II)

sd-shell nucleus



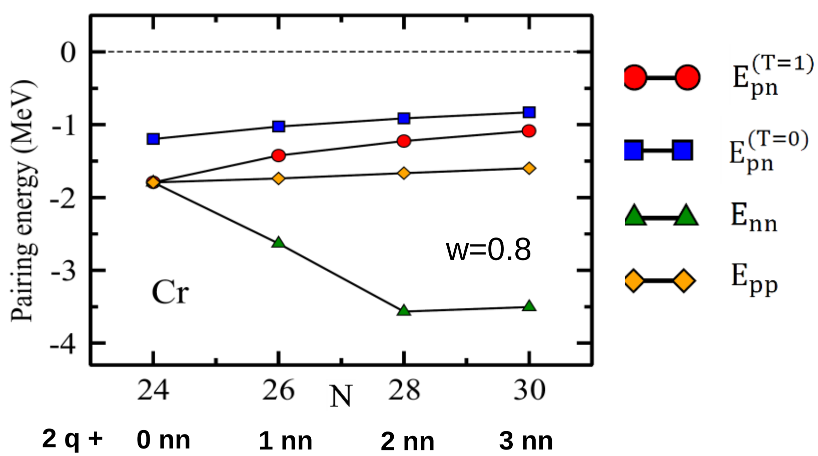
State-independent pairing force

Pairing energies (MeV):

$$E_t^{(T=1)} = V^{(T=1)} \sum_{i,j,t} \langle QCM | P_{i,t}^{\dagger} P_{j,t} | QCM \rangle$$

$$E_{pn}^{(T=0)} = V^{(T=0)} \sum_{i,j} \langle QCM | D_{i,0}^{\dagger} D_{j,0} | QCM \rangle$$

pf-shell nucleus



Pn pairing energies are decreasing, but remain significantly large even when 3 extra nn pairs are added.

Isovector and isoscalar pn pairing correlations coexist in both N = Z and N > Z nuclei.

D. N., P. Buganu, D. Gambacurta, and N. Sandulescu,
Phys. Rev. C98, 064319 (2018)

Main conclusions of this talk

QCM describes with good precision the isovector-isoscalar pairing.

Isovector pairing correlations are stronger than the isoscalar ones.

QCM describes very well the low-lying states of odd-odd nuclei.

The isovector and isoscalar pairing correlations coexist in even-even and odd-odd N = Z nuclei, but also in N > Z even-even nuclei.

Isovector and isoscalar pn pairing remain significant in systems with a few extra nn pairs.

Perspectives

Total angular momentum restoration by standard projection techniques.

Generalization of the QCM formalism to N>Z odd-odd nuclei:

$$(\tilde{\Gamma}_1^\dagger)^{n_p}\left[\tilde{\Gamma}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}\right]|0\rangle \quad \textbf{T=1 state}$$

$$(\tilde{\Gamma}_1^\dagger)^{n_p}\left[\tilde{\Delta}_0^+(Q_{iv}^+ + \Delta_0^{+2})^{n_q}\right]|0\rangle \quad \textbf{T=0 state}$$

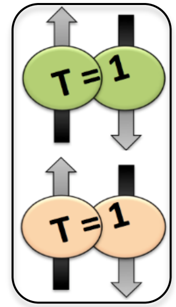
Quartetting and isospin conservation

even-even N=Z nuclei have T=0 in the ground state

$P^{+}_{i,t} \propto [a^{+}_{i} a^{+}_{\bar{i}}]^{J=0,\ T=1}_{J_z=0,\ t}$

isovector pairs

couple to **T=0**



T=0 Quartet

(correlations in spin and isospin)

$$[P^{+}_{i} P^{+}_{j}]^{T=0} = A^{+}_{ij}$$

$$A^{+} = \sum_{ij} \tilde{x}_{ij} A^{+}_{ij} = \sum_{ij} x_{ij} \left(P^{+}_{i,1} P^{+}_{j,-1} + P^{+}_{i,-1} P^{+}_{j,1} - P^{+}_{i,0} P^{+}_{j,0}\right)$$

(not a boson operator)

Collective quartets

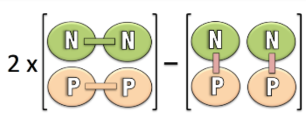
(ansatz)

$$x_{ij} \approx x_i x_j$$

Collective pairs $\Gamma^{+}_{t} = \sum_{i} x_i P^{+}_{i,t}$

Alpha-like quartets

$$A^{+} = 2\Gamma^{+}_{1}\Gamma^{+}_{-1} - \Gamma^{+2}_{0}$$



Quartet condensate

$$|QCM\rangle = (A^{+})^{n_q}|0\rangle$$

$n_q = (N+Z)/4$

(exact solution for a set of degenerate states)

Isovector pairing and quartetting in axially deformed N>Z nuclei

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} - g \sum_{t_z=-1,0,1} P^{+}_{i,t_z} P_{j,t_z}$$

(S.p. spectrum is given by deformed Skyrme-HF calculations.)

$$|QCM\rangle = (\tilde{\Gamma}_1^+)^{n_N} (A^+)^{n_q} |0\rangle \qquad |PBCS1\rangle = (\tilde{\Gamma}_1^+)^{N/2} (\Gamma_{-1}^+)^{Z/2} |0\rangle$$

Isotopes with double magic core ^{16}O

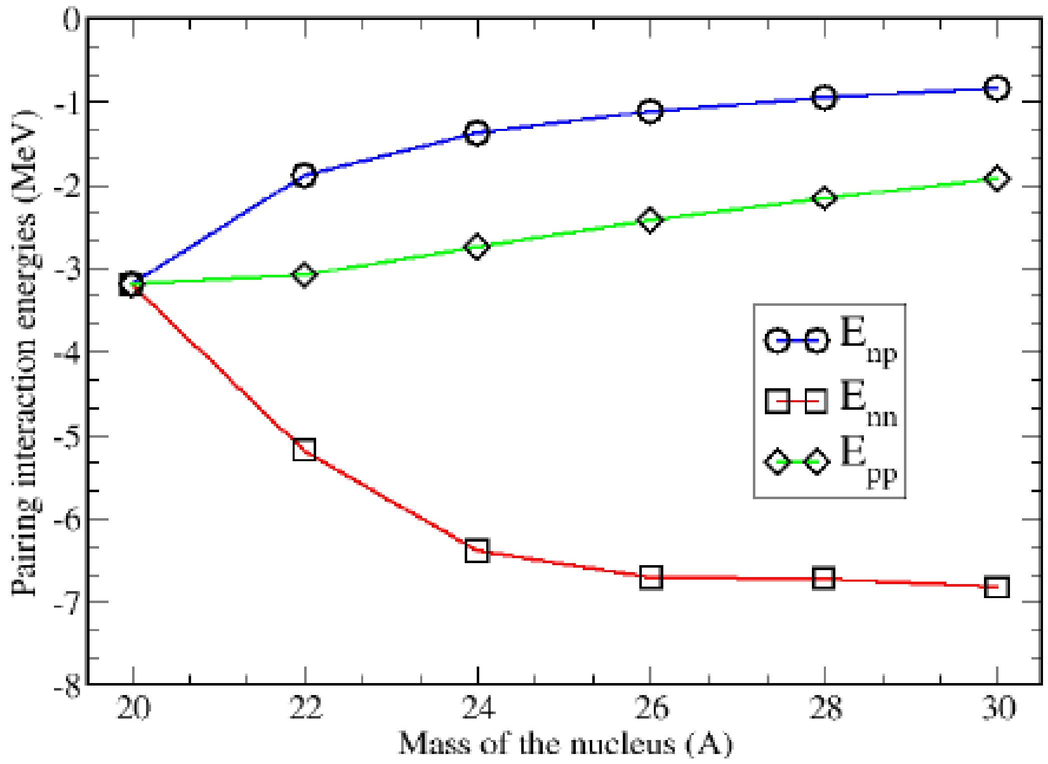
Correlation energies (MeV) $E_{corr} = E_0 - E$

	Exact	QCM	PBCS1
^{20}Ne	6.550	6.539 (0.17%)	5.752 (12.18%)
^{22}Ne	6.997	6.969 (0.40%)	6.600 (5.67%)
^{24}Ne	7.467	7.426 (0.55%)	7.226 (3.23%)
^{26}Ne	7.626	7.592 (0.45%)	7.486 (1.84%)
^{28}Ne	7.692	7.675 (0.22%)	7.622 (0.91%)
^{30}Ne	7.997	7.994 (0.04%)	7.973 (0.30%)
^{30}Si	9.310	9.296 (0.15%)	9.064 (2.64%)

	Exact	QCM	PBCS1
^{24}Mg	8.423	8.388 (0.41%)	7.668 (8.96%)
^{26}Mg	8.680	8.654 (0.30%)	8.258 (4.86%)
^{28}Mg	8.772	8.746 (0.30%)	8.531 (2.75%)
^{30}Mg	8.672	8.656 (0.18%)	8.551 (1.39%)
^{32}Mg	8.614	8.609 (0.06%)	8.567 (0.55%)
^{28}Si	9.661	9.634 (0.28%)	9.051 (6.31%)
^{32}Si	9.292	9.283 (0.10%)	9.196 (1.03%)

N. Sandulescu, D. N., C.W. Johnson, PRC 86, 041302 (R) (2012)

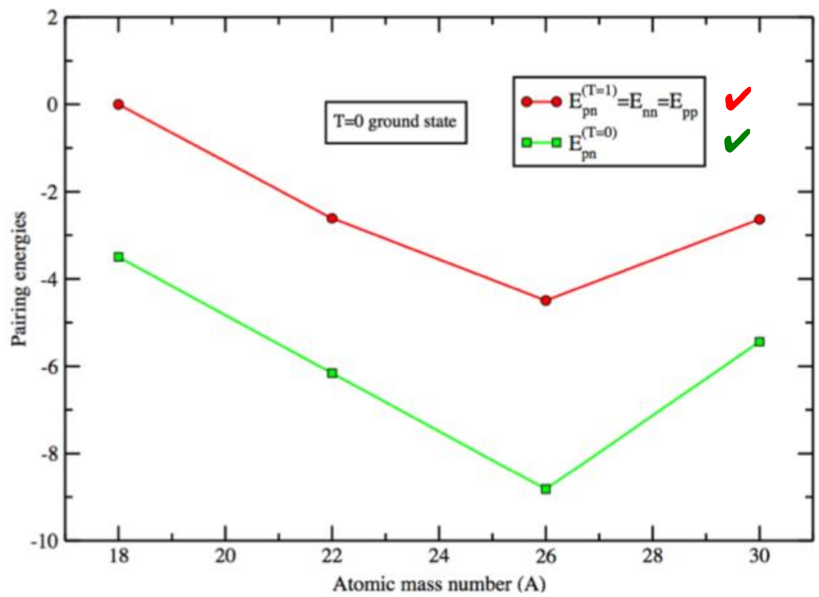
Isovector pairing energies for Ne isotopes



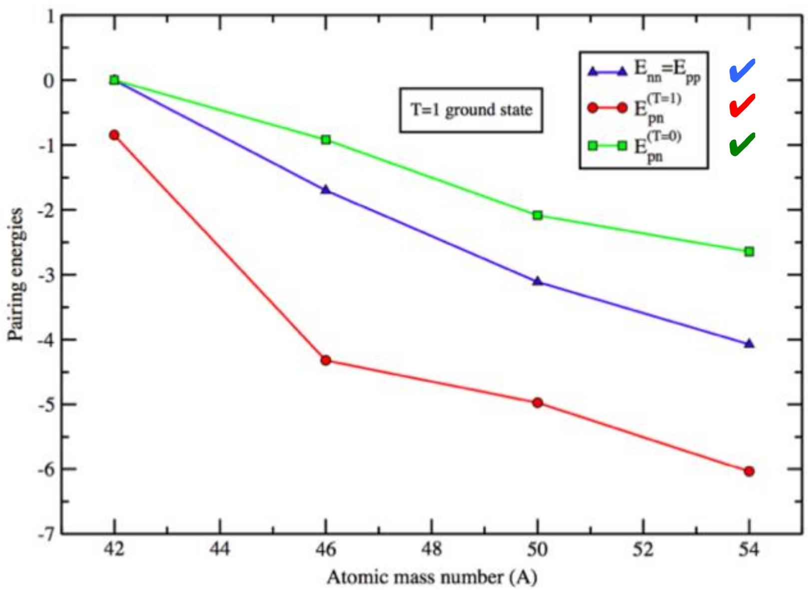
Proton-neutron correlations survive away of N=Z line!

T=1 and T=0 pairing energies

A<40



A≥40



$$E_{t_z}^{(T=1)} = \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} \langle is; QCM | P_{i,t_z}^{+} P_{j,t_z} | is; QCM \rangle$$

$$E_{pn}^{(T=0)} = \sum_{i,j} V^{T=0}(i,j) \langle is; QCM | D_{i,0}^{+} D_{j,0} | is; QCM \rangle$$

$$E_{t_z}^{(T=1)} = \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} \langle iv; QCM | P_{i,t_z}^{+} P_{j,t_z} | iv; QCM \rangle$$

$$E_{pn}^{(T=0)} = \sum_{i,j} V^{T=0}(i,j) \langle iv; QCM | D_{i,0}^{+} D_{j,0} | iv; QCM \rangle$$

Extra pairing energy in the T=0 channel: contribution from the odd T=0 pair

Extra pairing energy in the T=1 channel: caused not only by the odd T=1 pair

Table 3. Schmidt numbers for the proton–neutron pairs in the lowest $T = 1$ and $T = 0$ states of various odd–odd $N = Z$ nuclei. K_x and K_y denote the Schmidt numbers for the pairs Γ_0^+ and Δ_0^+ while K_z is the Schmidt number for the odd pair, i.e., $\tilde{\Gamma}_0^+$ for $T = 1$ states and $\tilde{\Delta}_0^+$ for $T = 0$ states.

	^{26}Al		^{30}P		^{50}Mn		^{54}Co		^{110}Cs		^{114}La	
	$T=1$	$T=0$	$T=1$	$T=0$	$T=1$	$T=0$	$T=1$	$T=0$	$T=1$	$T=0$	$T=1$	$T=0$
K_x	1.25	1.92	3.05	3.05	1.47	1.41	2.37	2.36	1.64	1.66	3.18	3.09
K_y	1.97	1.31	1.89	1.56	2.39	1.33	1.72	1.25	2.24	1.88	1.16	1.24
K_z	2.77	1.63	2.82	1.65	1.99	1.09	2.30	1.63	2.34	1.29	4.09	1.33

$$K = (\sum_i \omega_i{}^2)^2 / \sum_i \omega_i{}^4$$

$T = 0$ pairs are less collective than the isovector $T = 1$

In particular, the odd $T = 0$ pair is less collective than the odd $T = 1$ pair.

in all nuclei, except ^{50}Mn, the collectivity of the odd $T = 0$ pair is significant and comparable to the collectivity of the $T = 0$ pairs in the even–even core of the QCM states.

Isovector pairing in QCM+Skyrme-HF

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} - g \sum_{t_z=-1,0,1} P^{+}_{i,t_z} P_{j,t_z}$$

Pairing is treated as a residual interaction relative to a HF mean field.

single-particle energies: from Skyrme-HF

$$|QCM> = (\tilde{\Gamma}^{+}_{\nu\nu})^{n_N} (A^{+})^{n_q} |core>$$

$v_i^{(\nu)2}, \quad v_i^{(\pi)2}$ ⟹ **densities** ⟹ **HF**

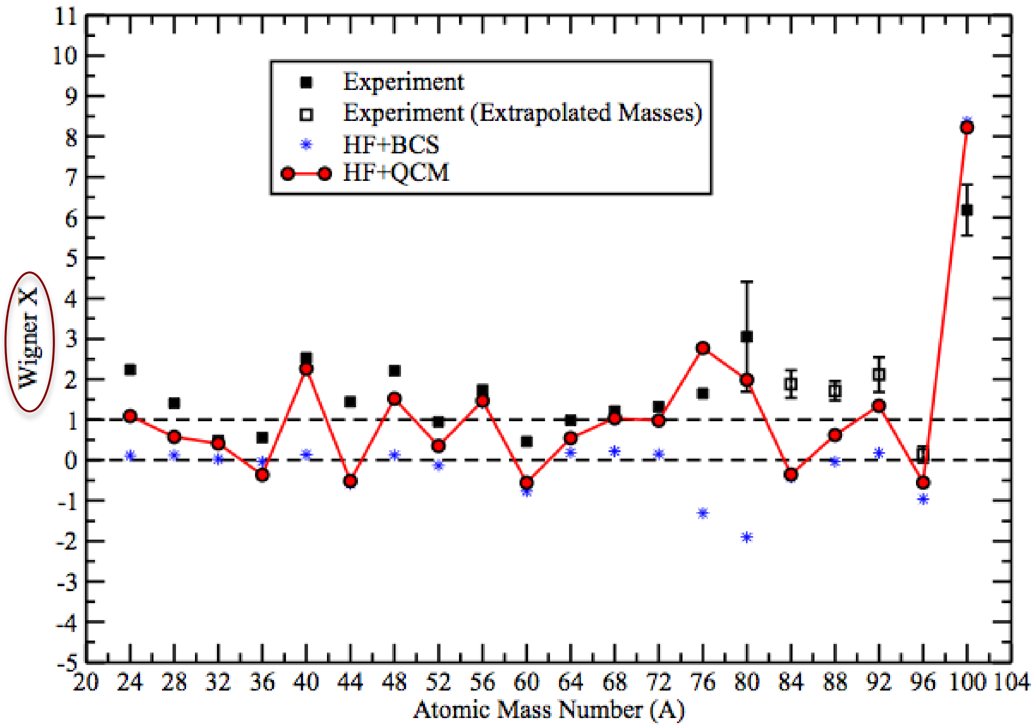
The HF and QCM calculations are iterated together until the convergence.
The pairing energy is added to the mean-field energy.

The influence of isovector pn pairing on the Wigner energy

$$E(N,Z) = E(N=Z) + \frac{T(T+X)}{2\Phi}$$

T=|N-Z|/2   T= 0, 2, 4

Sk-HF+QCM with selfconsistent calculation of deformation



D. N. , N. Sandulescu, Phys. Rev. C 90, 024322 (2014)

- HF+BCS fails to describe the Wigner energy (X close to zero for many chains).
- HF+QCM results are following well the large fluctuations of X with the mass number.
- **pn pairing is essential in describing the Wigner energy by respecting correctly the particle number and the isospin symmetries.**

Comparison between QCM and PBCS(N,T)

$|QCM\rangle = (A^+)^{n_q}|0\rangle$ ⇒ **isospin conservation**; particle number conservation

(Comparison with PBCS(N) solutions.)

$|PBCS(N,T)\rangle = \hat{P}_T \hat{P}_N |BCS\rangle$ $\quad \hat{P}_T, \hat{P}_N$ **standard projection operators**

Calculations - **application** for ^{52}Fe;
- spherical single-particle states;
- isovector pairing force of constant strength g=24/A.

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} - g \sum_{t_z=-1,0,1} P^+_{i,t_z} P_{j,t_z}$$

Correlation energy $E_{corr} = E_0 - E$	Exact value	QCM	PBCS(N,T)
	8.29 MeV	8.25 MeV	7.63 MeV

Chen, Mutter, Faessler, Nucl. Phys. A 297, 1978

- **QCM is more accurate than PBCS(N,T)**
- **QCM describes additional quartet-type correlations !**

Competition between isovector and isoscalar pairing: spherical symmetry

$$\hat{H} = \sum_i \epsilon_i (N_i^\nu + N_i^\pi) + \sum_{i,j} V_{ij}^{(T=1)} \sum_\tau P_{i,\tau}^+ P_{j,\tau} + \sum_{i,j} V_{ij}^{(T=0)} \sum_M D_{i,M}^+ D_{j,M}$$

$$|\Psi\rangle = (\alpha A^+ - \beta B^+)^{n_q} |0\rangle$$

- pairing interactions: extracted from the KB3G interaction

Valence nucleons in pf-shell, above the closed core $^{40}\mathrm{Ca}$

Nuclei	Shell Model	Quartets (errors)	β^2
^{44}Ti	4.261	4.221 (0.38 %)	0.0094
^{48}Cr	6.303	6.271 (0.50 %)	0.075

Correlation energies (MeV)

$E_{corr} = E_0 - E$

Isoscalar quartet neglected in the quartet condensate

Nuclei	Shell Model	Quartets (errors)
^{44}Ti	4.261	4.169 (2.2 %)
^{48}Cr	6.303	6.119 (2.9 %)
^{52}Fe	5.978	5.737 (4.0 %)

1. Recurrence relations method (auxiliary states)

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i,\tau} N_{i,\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t_z=-1,0,1} P^+_{i,t_z} P_{j,t_z} + \sum_{i,j} V^{T=0}(i,j) D^+_{i,j_z=0} D_{j,j_z=0}$$

$$\delta_x <\Psi|H|\Psi>=0, \quad <\Psi|\Psi>=1$$

Nr. of configurations: $Nc = \frac{(n+k-1)!}{n!(k-1)!}$

Quartet condensate (isovector pairs):

$$|QCM\rangle = (A^+)^{n_q}|0\rangle = (2\Gamma_1^+\Gamma_{-1}^+ - \Gamma_0^{+2})^{n_q}|0\rangle \qquad \Gamma_t^\dagger = \sum_i x_i P_{i,t}^\dagger$$

$$|n_1 n_2 n_3\rangle = \Gamma_1^{+n_1}\Gamma_{-1}^{+n_2}\Gamma_0^{+n_3}|0\rangle$$

$$<m_1m_2m_3|H|n_1n_2n_3>$$

Quartet condensate (isovector + isoscalar pairs):

$$|QCM\rangle = (A^+ + (\Delta_0^+)^2)^{n_q}|0\rangle \qquad \Delta_0^\dagger = \sum_i y_i D_{i,0}^\dagger$$

$$|n_1n_2n_3n_4\rangle = (\Gamma_1^+)^{n_1}(\Gamma_{-1}^+)^{n_2}(\Gamma_0^+)^{n_3}(\Delta_0^+)^{n_4}|0\rangle$$

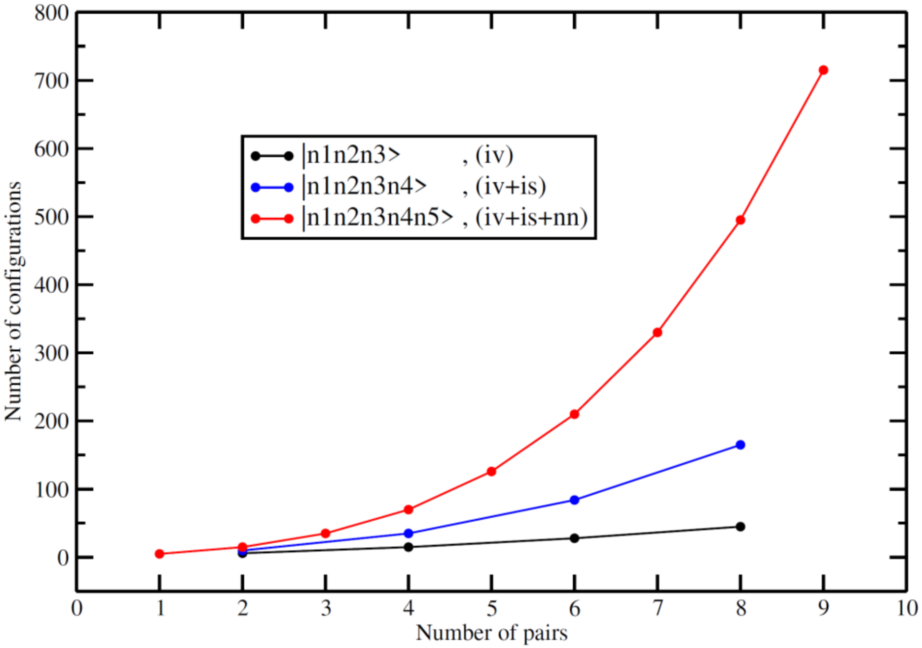
$$<m_1m_2m_3 m_4|H|n_1n_2n_3n_4>$$

Quartet condensate * neutron pair condensate

$$|QCM\rangle = (\tilde{\Gamma}_1^\dagger)^{n_N}(A^\dagger + \Delta_0^{\dagger 2})^{n_q}|0\rangle \qquad \tilde{\Gamma}_1^\dagger = \sum_i z_i P_{i,1}^\dagger$$

$$|n_1n_2n_3n_4n_5> = \Gamma_1^{\dagger n_1}\Gamma_{-1}^{\dagger n_2}\Gamma_0^{\dagger n_3}\Delta_0^{\dagger n_4}\tilde{\Gamma}_1^{\dagger n_5}|0>$$

$$<m_1m_2m_3m_4m_5|H|n_1n_2n_3n_4n_5>$$



$$\langle m_1m_2m_3m_4m_5|\, P_{i,t}^+ P_{j,t}\, |n_1n_2n_3n_4n_5\rangle$$

Total nr. of terms to calculate for 3q+2nn:

$3 * N_p * N_c(bra) * N_c(ket) * i * j = 3 * 8 * 495 * 495 * 10 * 10 = 588\ 060\ 000$

Parallel calculus? - still huge running times

'Analytic expressions' method (without auxiliary states) !

$$
\begin{aligned}
\langle n_1'n_2'n_3'|P_{i,1}^+P_{j,1}|n_1n_2n_3\rangle &= n_1x_j\langle n_1-1n_2n_3|P_{i,1}|n_1'n_2'n_3'\rangle \\
&-x_ix_j^2(n_1'n_1n_3\langle n_1-1n_2n_3-1|P_{j,0}|n_1'-1n_2'n_3'\rangle \\
&+n_1'n_1(n_1-1)\langle n_1-2n_2n_3|P_{j,1}|n_1'-1n_2'n_3'\rangle \\
&+\tfrac{1}{2}n_1'n_3(n_3-1)\langle n_1n_2n_3-2|P_{j,-1}|n_1'-1n_2'n_3'\rangle) \\
&+x_i^2x_j^2[n_1'n_3'n_1n_3[\langle n_1'-1n_2'n_3'-1|P_{j,0}^+P_{i,0}|n_1-1n_2n_3-1\rangle \\
&+\delta_{ij}(\langle n_1'-1n_2'n_3'-1|n_1-1n_2n_3-1\rangle \\
&-\tfrac{1}{2}\langle n_1'-1n_2'n_3'-1|N_{i,0}|n_1-1n_2n_3-1\rangle)] \\
&+n_1'(n_1'-1)n_1n_3[\langle n_1'-2n_2'n_3'|P_{j,0}^+P_{i,1}|n_1-1n_2n_3-1\rangle \\
&-\delta_{ij}\langle n_1'-2n_2'n_3'|T_{i,-1}|n_1-1n_2n_3-1\rangle] \\
&+\tfrac{1}{2}n_3'(n_3'-1)n_1n_3[\langle n_1'n_2'n_3'-2|P_{j,0}^+P_{i,-1}|n_1-1n_2n_3-1\rangle \\
&+\delta_{ij}\langle n_1'n_2'n_3'-2|T_{i,1}|n_1-1n_2n_3-1\rangle] \\
&+n_1'n_3'n_1(n_1-1)[\langle n_1-2n_2n_3|P_{i,0}^+P_{j,1}|n_1'-1n_2'n_3'-1\rangle \\
&-\delta_{ij}\langle n_1-2n_2n_3|T_{i,-1}|n_1'-1n_2'n_3'-1\rangle] \\
&+n_1'(n_1'-1)n_1(n_1-1)[\langle n_1'-2n_2'n_3'|P_{j,1}^+P_{i,1}|n_1-2n_2n_3\rangle \\
&+\delta_{ij}(\langle n_1'-2n_2'n_3'|n_1-2n_2n_3\rangle-\langle n_1'-2n_2'n_3'|N_{i,1}|n_1-2n_2n_3\rangle)] \\
&+\tfrac{1}{2}n_3'(n_3'-1)n_1(n_1-1)\langle n_1'n_2'n_3'-2|P_{j,1}^+P_{i,-1}|n_1-2n_2n_3\rangle \\
&+\tfrac{1}{2}n_1'n_3'n_3(n_3-1)[\langle n_1n_2n_3-2|P_{i,0}^+P_{j,-1}|n_1'-1n_2'n_3'-1\rangle \\
&+\delta_{ij}\langle n_1n_2n_3-2|T_{i,1}|n_1'-1n_2'n_3'-1\rangle] \\
&+\tfrac{1}{2}n_1'(n_1'-1)n_3(n_3-1)\langle n_1n_2n_3-2|P_{i,1}^+P_{j,-1}|n_1'-2n_2'n_3'\rangle \\
&+\tfrac{1}{4}n_3'(n_3'-1)n_3(n_3-1)[\langle n_1'n_2'n_3'-2|P_{j,-1}^+P_{i,-1}|n_1n_2n_3-2\rangle \\
&+\delta_{ij}(\langle n_1'n_2'n_3'-2|n_1n_2n_3-2\rangle-\langle n_1'n_2'n_3'-2|N_{i,-1}|n_1n_2n_3-2\rangle)]].
\end{aligned}
$$

2. '<u>Analytic expressions' method</u>: CADABRA calculations

$$\langle 2q|P^{+}_{i,0}P_{j,0}|2q\rangle = \langle 0|\,(2\Gamma_1\Gamma_2 - \Gamma_3\Gamma_3)\,(2\Gamma_1\Gamma_2 - \Gamma_3\Gamma_3)\,P^{\dagger}{}_{i3}P_{j3}\,(2\Gamma^{\dagger}{}_1\Gamma^{\dagger}{}_2 - \Gamma^{\dagger}{}_3\Gamma^{\dagger}{}_3)\,(2\Gamma^{\dagger}{}_1\Gamma^{\dagger}{}_2 - \Gamma^{\dagger}{}_3\Gamma^{\dagger}{}_3)\,|0\rangle$$

1. **Define all the collective operators:** $\Gamma^{+}_{t} \to \sum_i x_i P^{+}_{i,t}$, $\Delta^{+}_{0} \to \sum_i y_i D^{+}_{i,0}$, …

2. **Specify explicitly all the possible commutators between the non-collective operators:** $\hat{\mathcal{O}}\,\hat{\mathcal{O}}^{\dagger} \to \hat{\mathcal{O}}^{\dagger}\hat{\mathcal{O}} + [\hat{\mathcal{O}},\hat{\mathcal{O}}^{\dagger}]$

3. **Impose other necessary transformations:** $\hat{\mathcal{O}}\,|0\rangle \to 0$, $\langle 0|\hat{\mathcal{O}}^{\dagger} \to 0$, $\langle 0|0\rangle \to 1$, …

4. **Transformations to simplify the final expressions:** $\sum_i x_i^n \to a_n$, …

Final result for $\langle 2q|P^{+}_{i,0}P_{j,0}|2q\rangle$ is:

$$40x_i^7x_j + 40x_i^5x_j^3 + 40x_i^3x_j^5 + 40x_ix_j^7 - 240a_2x_i^5x_j - 240a_2x_i^3x_j^3 + 280a_4x_i^3x_j - 240a_2x_ix_j^5 + 280a_4x_ix_j^3 - 80a_6x_ix_j - 27\delta_{ij}x_i^5x_j^3 + 204\delta_{ij}x_i^3x_j^5 - 18\delta_{ij}x_i^2x_j^6 - 106\delta_{ij}x_ix_j^7 - 176\delta_{ji}x_i^5x_j^3 + 46\delta_{ji}x_i^4x_j^4 + 57\delta_{ji}x_i^3x_j^5 - 80a_2^2x_i^3x_j - 80a_2^2x_ix_j^3 - 80a_2a_4x_ix_j + 72\delta_{ij}a_2x_i^3x_j^3 - 36\delta_{ij}a_2x_i^2x_j^4 + 76\delta_{ij}a_4x_i^2x_j^2 + 76\delta_{ij}a_2x_ix_j^5 - 256\delta_{ij}a_4x_ix_j^3 + 60\delta_{ji}a_2x_i^3x_j^3 - 92\delta_{ji}a_2x_i^2x_j^4 + 160a_2^3x_ix_j - 76\delta_{ij}a_2^2x_i^2x_j^2 + 356\delta_{ij}a_2^2x_ix_j^3$$

5. **Minimize this expression with respect to the mixing amplitudes x(i).**

The running time of the new code decreases drastically!

Proton-neutron pairing and alpha-type correlations in excited states of N=Z nuclei

Ground state: $|\Psi\rangle = (Q^+)^{n_q}|0\rangle \quad Q^+ = (2\Gamma_1^+\Gamma_{-1}^+ - \Gamma_0^{+2}) - \beta(2\Delta_1^+\Delta_{-1}^+ - \Delta_0^{+2})$

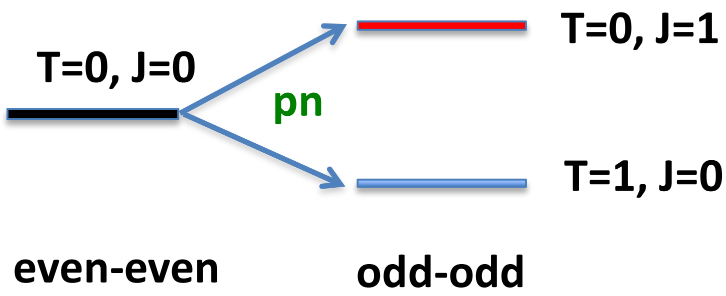
Excited states can be constructed in many ways.

Ex.: - break a quartet;
- recouple the nucleons to angular momentum J>1.

$$(Q^+)^{n_q-1}\left(\nu_i^+\nu_j^+\pi_k^+\pi_l^+\right)^{J=2}|0\rangle$$

Analysis of excitation spectra of N=Z nuclei.

Testing T=1 and T=0 pn pairing by deuteron-like transfer reactions



Relative strength of T=1 and T=0 channels:
ratio between cross sections.

Experimentally: - evaluate the transfer amplitudes from ^{52}Fe to ^{50}Mn
(higher collectivity of pn pairs for A>40)

$$\langle {}^{52}\mathrm{Fe} | \nu_i^+ \pi_i^+ | {}^{50}\mathrm{Mn} \rangle$$

Theoretically: - evaluate pn pair transfer amplitudes
- understand the influence of isovector and isoscalar pairing on the pair transfer

How? Use the generalization of QCM for odd-odd nuclei!